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SIMILITUDE RELATIONSHIPS FOR VARIABLE FLOW THROUGH EARTH DAMS

by

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Denver, Colorado,
June 24, 1935.

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MEMORANDUM TO CHIEF DESIGNING ENGINEER

SUBJECT: SIMILITUDE RELATIONSHIPS

FOR VARIABLE FLOW THROUGH EARTH DAMS

By

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Under direction of

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The two most important general considerations in selecting the material and profile for an earth dam are now usually conceded to be the resistance to seepage as such and the stability of the slopes. In addition to the usual considerations of the loss of water and erosion as a result of seepage, the position of the saturation line has an important effect on the equilibrium of both the upstream and downstream slopes.

In the absence of any conclusive and practical mathematical method for predetermining the behavior of the saturation line from properties of the material obtainable through field and laboratory tests, the use of scale models may provide a valuable guide. Current opinion is divided as to the extent to which such studies can be relied upon, but since they furnish almost the only means of increasing the knowledge of the subject except for expensive and slow full scale experiments, it is probable that their employment will continue to increase.

In applying the results of model studies to the analyses of prototypes, it is desirable to reduce the dimensional relationships to their simplest form. For the case of constant or permanent flow, these similitude equations have been presented by several writers (1)(2)(3)(4) but not, it is believed, in a form readily applicable to problems associated with varying reservoir levels. The following is a brief development from the point of view of non-permanent or variable flow.

It is convenient to predicate the entire analysis upon the stipulation that in two geometrically similar earth dams of unit length, the instantaneous flow nets shall likewise be geometrically similar; that is to say that with homologous reservoir and tailwater levels, the saturation curves shall also be homologous. It is desired to find what relationships must maintain between the physical properties of the materials, the sizes of the structures, their flow properties, and time in order that this hypothesis shall be fulfilled.

Consider the case of two such similar dams (model and prototype) in which the reservoir levels are assumed to descend after having been previously held at homologous elevations until equilibrium had been established (fig. 1). Let the "prime" or accent symbol indicate dimensions pertaining to the model, and define the notation as follows:

b = an arbitrary dimension parallel to a potential line.

d = diameter of a capillary tube.

d_h = headwater depth at beginning of period.

d_i = vertical distance between point of intersection of saturation line with downstream face and tailwater elevation at the beginning of a period.

d_t = tailwater depth at beginning of a period.

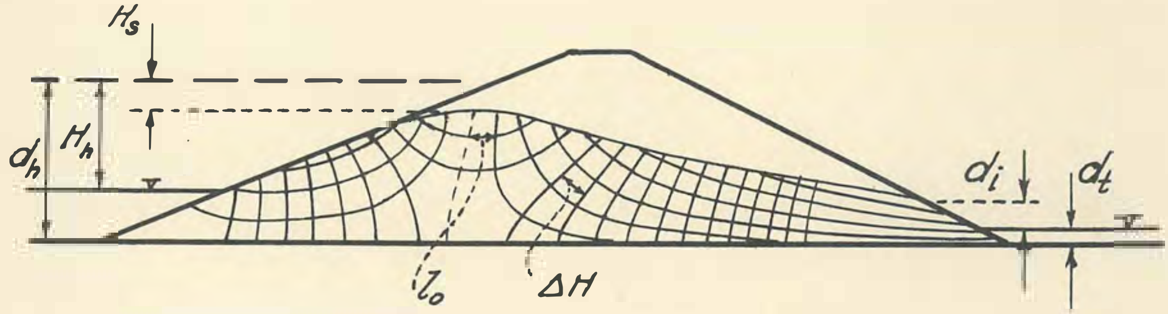

Fig. 1. Descending Reservoir

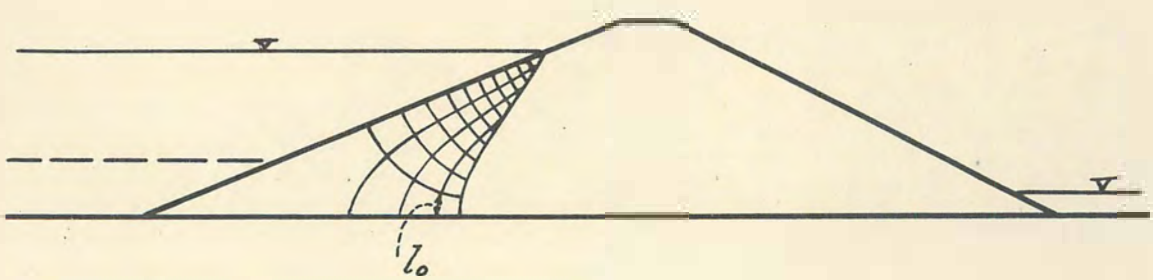

Fig. 2. Reservoir Filling

Instantaneous Flow Nets

g = acceleration of gravity

H_h = vertical drop of reservoir surface during a time lapse, t, after commencement of draw-down

H_s = vertical distance between initial reservoir level and crest of saturation line at the end of the same period, t.

ΔH = potential drop or loss of piezometric elevation between two adjacent equipotential lines of a flow net

j = percent gain of water on filling expressed as a fraction of the volume occupied by the unsaturated soil; a "filling" constant

k = coefficient of permeability or the volume of flow through a unit cross-sectional area under a unit potential gradient in unit time

k_s = coefficient of permeability as determined experimentally at an accepted standard temperature

K, K_1, K_2 = experimental constants

ℓ = length of a side of a square of the instantaneous flow net

ℓ_o = length of a side of a square at the crest of a descending or at the toe of an advancing flow net

m = percent loss of water on draining expressed as a fraction of the volume occupied by the saturated soil; a "drainage" constant

n = scale ratio

μ = absolute viscosity (dimensions, lb. sec./ft^2)

q = volume of water discharged per unit time, or flow

q_a = flow through an area having the dimensions b and unity

ρ = density of water (dimensions, mass/ft^3)

t = time lapse from the beginning of the period during which the reservoir level descends or rises

T = Temperature, degrees Fahrenheit

v = velocity with which any point on the saturation curve translates

v_h = velocity with which the reservoir surface descends or rises

v_o = velocity of percolation

v_r = velocity with which the toe of the saturation curve advances

v_s = velocity with which the crest of the saturation curve descends

ν = kinematic viscosity (dimensions, ft^2/sec.)

If we assume that a flow net constructed of squares could exist, then, as Terzaghi has pointed out (3) the quantity, q, which can flow through a rectangular area perpendicular to the direction of flow and having the dimensions l_o and unity must be

$$q = k \times 1 \times l_o \frac{\Delta H}{l_o} \tag{1}$$

But if $m\, l_o^2 \times 1$ is the volume of water which must flow out of the volume $l_o^2 \times 1$ at the crest as the soil passes from the saturated to the drained state then $\frac{m\, l_o^2 \times 1}{q}$ is the time required for the saturation boundary to descend through the distance l_o, and the velocity with which the crest descends is

$$v_s = \frac{l_o}{\frac{m\, l_o^2 \times 1}{q}} = \frac{q}{m\, l_o \times 1}, \text{ or}$$

$$v_s = \frac{k\, l_o \times 1}{m\, l_o \times 1} \frac{\Delta H}{l_o} = \frac{k}{m} \frac{\Delta H}{l_o} \tag{2}$$

From our hypothesis that in the two structures the flow nets shall be similar

$$H_s = nH_s' , \tag{3}$$

from which

$$dH_s = n\, dH_s' \tag{4}$$

But since

$$H_s = \int_0^t v_s\, dt, \tag{5}$$

$$dH_s = v_s\, dt \tag{6}$$

From (4) and (6)

$$\frac{dH_s}{dH_s'} = n = \frac{v_s\ dt}{v_s'\ dt'} \tag{7}$$

Also, considering the original hypothesis,

$$H = n \Delta H' \tag{8}$$

and

$$\tau_o = n \, \tau_o' \tag{9}$$

so that

$$\frac{\Delta H}{\tau_o} = \frac{\Delta H'}{\tau_o'} \tag{10}$$

Referring to equations (2) and (10) we see therefore that

$$\frac{v_s}{v_s'} = \frac{\frac{k}{m}}{\frac{k'}{m'}} \tag{11}$$

Substituting (11) in (7) we obtain

$$n = \frac{\frac{k}{m}}{\frac{k'}{m'}} \frac{dt}{dt'} \tag{12}$$

which upon integration yields

$$nt' = \frac{\frac{k}{m}}{\frac{k'}{m'}} t + C \tag{13}$$

But if t = o when t' = o, the constant disappears, leaving

$$\frac{t}{t'} = n \frac{\frac{k'}{m'}}{\frac{k}{m}} \tag{14}$$

This is a fundamental relationship which must be satisfied if in two geometrically similar dams with descending reservoir levels, the instantaneous flow nets are also to be similar.

If $\frac{k}{m}$, which is an indication of the rate at which a material can pass from the saturated to the drained state under unit potential gradient (see equation 2), be described as the "drainage index" of that material, we see that, given the drainage indices of the materials to be used in prototype and model and given the scale ratio, the ratio between the time lapses at which similar flow conditions will obtain in model and prototype is fixed. Conversely, if it is desired to reproduce in a model within a specified time the conditions which existed in a prototype at the end of another known time, the model materials must be so chosen that their drainage indices satisfy equation (14).

The values of certain other useful ratios may be obtained directly. Since

$$H_h = nH_h' , \tag{15}$$

$$dH_h = n\, dH_h' \tag{16}$$

But

$$H_h = \int_0^t v_h \, dt \tag{17}$$

from which

$$dH_h = v_h \, dt \tag{18}$$

and

$$\frac{dH_h}{v_h} = dt \qquad (19)$$

From (12)

$$\frac{dt}{dt'} = n \frac{\frac{k'}{m'}}{\frac{k}{m}} , \qquad (20)$$

so that combining 16, 19, and 20, we have

$$n = \frac{v_h}{v_h'} n \frac{\frac{k'}{m'}}{\frac{k}{m}} \qquad (21)$$

therefore

$$\frac{v_h}{v_h'} = \frac{\frac{k}{m}}{\frac{k'}{m'}} \qquad (22)$$

or, in words, the draw-down velocities are directly proportional to the drainage indices.

The discharge, q_a, through any area perpendicular to the direction of flow and having the dimensions, b, and unity, will of course be the product of that area by the coefficient of permeability and the potential gradient, or

$$q_a = 1 \times b \times k \frac{\Delta H}{l} \qquad (23)$$

But by hypothesis,

$$b = n\, b' \qquad (24)$$

and as shown for equation (10)

$$\frac{\Delta H}{l} = \frac{\Delta H'}{l'} \qquad (25)$$

so that obviously

$$\frac{q_a}{q_a'} = n \frac{k}{k'} \qquad (26)$$

If b is conceived as the length of any potential line in the flow net and $\frac{\Delta H}{l}$ is a weighted effective gradient applicable to the full length of that line, then q_a represents the total flow through a unit length of dam when we consider the case of constant reservoir level. Thus the rates of percolation through two similar dams are in proportion to the product of the scale ratio and the ratio of their coefficients of permeability.

Consider the case of a rising reservoir level and an advancing saturation boundary. At the impervious foundation line the saturation boundary will be perpendicular to the flow lines so that the same argument used for the descending reservoir may be applied: viz., the quantity, q, which can flow through a rectangular area perpendicular to the direction of flow is

$$q = 1 \times l_o \times k \frac{\Delta H}{l_o} \qquad (1)$$

If $j\, l_o^2 \times 1$ is the volume of water which must flow into the volume $l_o^2 \times 1$ at the toe of the advancing saturation line in order for that volume to pass from its initial to the saturated state, then, following the same reasoning with which equation (2) was developed,

$$v_r = \frac{k}{j} \frac{\Delta H}{\rho_o} \tag{27}$$

Similarly all of the equations determined for the descending reservoir level can be duplicated for the rising level using j instead of m.

Recapitulating, the significant equations are therefore, for descending reservoir,

$$\frac{t}{t'} = n \frac{\frac{k'}{m'}}{\frac{k}{m}} \tag{14}$$

$$\frac{v_h}{v_h'} = \frac{\frac{k}{m}}{\frac{k'}{m'}} \tag{22}$$

$$\frac{v_s}{v_s'} = \frac{\frac{k'}{m}}{\frac{k'}{m'}} \tag{11}$$

for rising reservoir,

$$\frac{t}{t'} = n \frac{\frac{k'}{j'}}{\frac{k}{j}} \tag{28}$$

$$\frac{v_h}{v_h'} = \frac{\frac{k}{j}}{\frac{k'}{j'}} \tag{29}$$

$$\frac{v_r}{v_r'} = \frac{\frac{k}{j}}{\frac{k'}{j'}} \tag{30}$$

and for constant reservoir level,

$$\frac{q_n}{q_n'} = n \frac{k}{k'} \tag{26}$$

Naturally

$$\left.\begin{matrix} d_h \\ d_t \\ d_i \end{matrix}\right) \begin{matrix} \text{must also} \\ = \end{matrix} \; n \left(\begin{matrix} d_h' \\ d_t' \\ d_i' \end{matrix}\right. \tag{31}$$

In the preceding discussion no allowance has been made for the effects of temperature upon k, m, and j; that is to say, the equations apply only to the case of constant and identical temperatures in model and prototype. Furthermore, various investigators (5)(6)(7) do not agree as to the range within which Darcy's law, upon which equation (1) is based, may be valid. Poiseuille's formula of which Darcy's may be said to be a special case, is

$$v_o = \frac{gd^2}{32 \nu} \frac{\Delta H}{l} \tag{32}$$

If Darcy's law,

$$v_o = k \frac{\Delta H}{l} \tag{33}$$

is to be accepted, the coefficient of permeability corresponds to

$$k = \frac{2gd^2}{K \nu} \qquad (34)$$

but according to Lindquist (5) this is not the case, his experiments indicating rateher that

$$k = \frac{2gd^2}{\nu K_1 + v_o dK_2} \qquad (35)$$

It may nevertheless be desirable to introduce a correction for the effect of temperature upon the permeability, accepting tentatively the validity of Darcy's law. Since ν is defined as $\frac{\mu}{\rho}$ and Poiseuille (8) has given

$$\mu = \frac{0.00003716}{0.4712 + 0.01435\ T + 0.0000682\ T^2} \qquad (36)$$

for water, we may write

$$\frac{\nu_1}{\nu_2} = \frac{0.4712 + 0.01435\ T_2 + 0.0000682\ T_2^2}{0.4712 + 0.01435\ T_1 + 0.0000682\ T_1^2} \qquad (37)$$

where $\frac{\nu_1}{\nu_2}$ is the ratio between the kinematic viscosities of any two bodies of water at temperatures T_1 and T_2, the variation of density with temperature being considered negligible. For any two identical soils, the, the ratio of their permeabilities at different temperatures is, from (34)

$$\frac{k_1}{k_2} = \frac{\nu_2}{\nu_1} \qquad (38)$$

The permeability of a given soil at any temperature may therefore be expressed in terms of its permeability at some standard temperature and the ratio of the kinematic viscosities, so that

$$k_T = k_s \frac{\nu_s}{\nu_T} \tag{39}$$

in which the subscripts, s, denote a standard temperature (e.g., 75° F) and the subscripts, T, signify any temperature.

The ratio between the permeabilities of any two materials at any temperature can readily be seen from equation (39);

$$\frac{k}{k'} = \frac{k_s \dfrac{\nu_s}{\nu_T}}{k_s' \dfrac{\nu_s}{\nu'_T}} = \frac{k_s}{k_s'} \frac{\nu'}{\nu} \tag{40}$$

The previously summarized equations which contain the ratio $\frac{k}{k'}$ could thus be rewritten to include the effects of temperature on the permeability as follows:

$$\frac{t}{t'} = n \frac{k_s'}{k_s} \frac{m}{m'} \frac{\nu}{\nu'} \tag{14a}$$

$$\frac{v_h}{v_h'} = \frac{k_s}{k_s'} \frac{m'}{m} \frac{\nu'}{\nu} \tag{22a}$$

$$\frac{v_s}{v'_s} = \frac{k_s}{k'_s} \frac{m'}{m} \frac{\nu'}{\nu} \qquad (11a)$$

$$\frac{t}{t'} = n \frac{k_s}{k'_s} \frac{j}{j'} \frac{\nu}{\nu'} \qquad (28a)$$

$$\frac{v_h}{v'_h} = \frac{k_s}{k'_s} \frac{j'}{j} \frac{\nu'}{\nu} \qquad (29a)$$

$$\frac{v_r}{v'_r} = \frac{k_s}{k'_s} \frac{j'}{j} \frac{\nu'}{\nu} \qquad (30a)$$

$$\frac{q_a}{q'_a} = n \frac{k_s}{k'_s} \frac{\nu'}{\nu} \qquad (26a)$$

Still lacking is any correction for the influence of temperature on the drainage and filling constants, m and j. It is reasonable to suppose that both may be considerably affected by changes in the viscosity, and that they are related in some simple way to the capillary height. Additional experimental work is needed, however, to further clarify this relationship.

Throughout the preceding discussion, the presence and effect of the capillary zone above the saturation line have been neglected. That this zone exerts a significant influence on the flow which cannot practically be made to conform to the other similitude relations is clearly demonstrated by B. Körner (2). The same writer

List of References

(1) N. N. Pavlovsky, "On the Flow of Water Through Earth Dams" Bulletin of the Institution of Hydrotechnics, Leningrad, 1931 (In Russian)

(2) B. Körner, "Erforschung der physikalischen Gesetze, nach welchen die Durchsickerung des Wassers durch eine Talsperre oder durch den Untergrund stattfindet" (Investigation of the Physical Laws According to Which the Percolation of Water through Dams or Through the Ground Takes Place), 1er Congres des Grands Barrages, Vol IV, Stockholm, 1933

(3) Charles Terzaghi, "Retaining Wall Design for the Fifteen Mile Falls Dam", Engr. News-Record, May 17, 1934

(4) H.C.P. DeVos, "Das Strömen von Wasser durch Erddämme und deren Unterlage" (The Flow of Water through Earth Dams and their Foundations), 1er Congres des Grands Barrages, Vol IV, Stockholm, 1933

(5) Erik Lindquist, "On the Flow of Water Through Porous Soil", 1er Congres des Grands Barrages, Vol V, Stockholm, 1933

(6) Josef Donat, "Ein Beitrag zur Durchlässigkeit der Sande" (Contribution on the Permeability of Sands) Wasserkraft und Wasserwirtschaft, no. 17, 1929.

(7) O.E. Meinzer and V. C. Fishel, "Tests of Permeability with Low Hydraulic Gradients", National Research Council, Trans. Amer. Geophysical Union, April, June, 1934, Part II

(8) A. H. Gibson, "Hydraulics and its Application", 1930, p. 10.