HYD-299

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HYD 299 1

Denver, Colorado
October 25, 1950

Memorandum Hyd. 299

To: E. W. Lane

From: A. C. Carter

Subject: Angle of repose of noncohesive material

Summary

Previous studies of the stable channel problem have established a need for information pertaining to angles of repose of granular materials.

A review was made of all published data available on angles of repose and internal friction for dry and submerged silt, sand, and gravel; and the results were compared graphically in an attempt to determine characteristic values. The published data did not fall into any consistent pattern, but instead were scattered over a wide range. An angle of repose of 34° appeared to be typical for dry sand, but no other specific values could be determined.

Angle of repose tests were performed in the Bureau laboratories on granular materials. Results of these measurements showed a reduction in angle of repose for sand when submerged, but practically no change for gravel. Angular gravels had angles of repose 5 to 10° greater than rounded samples.

Introduction

Studies of one phase of the stable channel problem have been made of allowable tractive forces for trapezoidal channels having various side slopes. This investigation determined the relationship between tractive and gravity forces acting on particles resting on channel side slopes. To evaluate tractive forces which would not cause scour, it was found necessary to know the angle of repose of the granular material composing the banks.

A review was made of all available data on angles of repose of noncohesive material which included published values from various sources and additional information obtained by tests performed in the Hydraulic Laboratory.

Published values of angle of repose for different types of granular material showed considerable variation, and some authors gave extremely wide ranges of values for a given type of material. Soils were usually described as earth, sand, gravel, etc., instead of being classified by particle size. Since the term "earth" is general and may refer to boulders, gravel, sand, silt, or clay, it

was assumed that the more specific term "silt" applied, and the substitution was made.

Noncohesive materials tested in the laboratory ranged in size from 0.15 to 38 mm. Angles of repose were measured for four conditions: (1) stacked in air, (2) stacked in air and then submerged, (3) stacked under water, and (4) stacked in air while damp. Rounded and angular gravels were tested separately.

Angle of Repose

Angle of repose has been defined as "the maximum angle of slope which the side of a heap or embankment of sand, earth, etc. can have without sliding." For cohesionless soils the angle of repose is usually measured in the uncompacted state and has a constant value.

Angle of Internal Friction

Angle of internal friction has been defined as "the angle whose tangent is the coefficient of friction" of a material. The coefficient of friction is the ratio of normal force to shearing force at the point of failure in shear. According to Terzaghi 20/ "***laboratory experiments have shown that the angle of internal friction of sand depends to a large extent on the initial density. The difference between the angle of internal friction of a given sand in the densest and in the loosest state may be as high as 15°." He also states that "in engineering problems the angle of internal friction of sand can usually be considered constant for practical purposes. The angle of repose *** is always approximately equal to the angle of internal friction of sand in the loosest state." Since channel side slopes have some degree of consolidation, the angle of internal friction would probably apply rather than the angle of repose. Angles of repose and internal friction were considered interchangeable for practical purposes in this memorandum.

Procedure

Angles of repose and internal friction for silt, sand, and gravel were selected for study from 24 references. In general, angles of repose were found in engineering handbooks and angles of internal friction in soil mechanics texts. The data are presented in Tables 1 through 15. Sources of information are indicated by numerical references to the bibliography. Whenever given, explanations of soil conditions are included with the original data. Angles of repose are plotted for dry materials in Figure 1 and for submerged materials in Figure 2. All types of granular soils are grouped under three main classifications: silt, sand, and gravel. Individual angle

of repose values are represented by single lines with one or more numerical bibliographic references. Ranges of values are indicated by arrows drawn between two lines which represent the limits. Explanatory notes for certain data are included in Figures 1 and 2 corresponding to the lower case letters near the reference numbers.

To illustrate the effect of compaction of noncohesive materials on their angles of internal friction, data from the U.S.W.E.S. Soil Mechanics Fact Finding Survey 22/ are shown in Figure 3 by plotting angle of internal friction against initial void ratio. Void ratio is the volume of voids divided by the volume of solids. A key to the symbols used for different sands is included in the figure. The angles of internal friction were determined by triaxial shear tests.

Similar tests were performed on sand from Zone 2 of Trenton Dam by the Bureau Earth Materials Laboratory, 24/ and the results are presented in Table 14-A. The change in angle of internal friction with density is shown by plotting internal friction angle against relative density in percent, Figure 3-A. Relative density is defined by the following equation:

$$D_r = \frac{e_{max} - e}{e_{max} - e_{min}}$$

where e_{max} is the void ratio of the soil in the loosest state, e_{min} is the void ratio of the soil in the densest state, and e is the void ratio in the state for which the relative density is desired.

A number of sand and gravel samples were obtained locally, and their angles of repose were measured by personnel of the Hydraulic Laboratory. The results of this investigation were presented by O. S. Hanson in a memorandum to E. W. Lane dated March 27, 1950. The first of these measurements were reported by E. J. Carlson in a memorandum to E. W. Lane dated July 18, 1949. Photographs of the samples tested were included in the original memorandums.

Seven different sized samples weighing approximately 80 pounds were obtained by screening. These samples were stacked in air, stacked in air and then submerged, stacked under water, and stacked in air while damp: the angles of repose were then determined by coordinate measurements of seven points on the slope. The original data are presented in Table 15 and Figures 4 and 5. Bar graphs are used in these figures to represent measured angles of repose for samples of different size ranges. Dotted, broken, and solid lines indicated whether the sample was stacked in air, stacked in air and then submerged, or stacked under water. Values for rounded material are shown in Figure 4 and for angular material in Figure 5.

Results

As shown by Figure 1, angle of repose values suggested by different authorities for dry cohesionless material do not agree very closely. Frequently, wide ranges of values were given; such data have little practical significance. An angle of repose of 34° appeared to be fairly representative for dry sand, but no single values for silt or gravel were disclosed by these data.

Relatively little information was found on angles of repose of submerged material. Selection of a typical value for silt, sand, or gravel could not be made from the few data shown in Figure 2.

Reference to Figure 3 will show clearly how degree of compaction affects angle of internal friction of five materials studied by the U.S.W.E.S. 22/ As the void ratio increases, the friction angle decreases. The average value for three sands tested was 34.9° and for gravel 32.7°.

Results of laboratory measurements reported by O. S. Hanson 23/ showed a reduction in the angle of repose for sand when submerged, but no change for gravel. Angular gravels had angles of repose 5 to 10° greater than rounded samples of the same size.

Acknowledgments

The assistance received from Mr. W. G. Holtz and Mr. H. J. Gibbs of the Earth Materials Laboratory was very helpful in the preparation of this report.

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Table 1

ANGLE OF INTERNAL FRICTION
Data from Andersen 1/

Material--dry	Percent voids	Angle of internal friction
Coarse sand	40	32° - 38°
Fine sand	46	30° - 35°
Gravel [illegible]	34	35°
Medium and coarse dry sands		30° - 35°

Table 2

ANGLE OF REPOSE DATA
from Cain 3/

Material	Angle of repose
from Leygue 10/	
Dry sand	35°
Wet sand	40° 22'
Very wet sand	59° 30'
Damp fresh earth	58° 28'
from Amer. Civil Engr's Pocket Book, Trautwine's Engr's Pocket Book, and Rankine's Treatises	
Dry sand	20° - 35°
Moist sand	30° - 45°
Wet sand	20° - 40°
Ordinary dry earth	20° - 45°
Ordinary moist earth	25° - 45°
Ordinary wet earth	25° - 30°
Gravel (round to angular)	30° - 48°
Average clean sand--dry	33° 41'
Average clean sand--moist	44°
Average clean sand--wet	33° 41'
Average clean sand--submerged	26° 34'
Ordinary earth--dry	37°
Ordinary earth--moist	39°
Ordinary earth--wet	30°
Ordinary earth--submerged	17°
Sand	34°

Table 3

ANGLE OF INTERNAL FRICTION[4]
Data from Corps of Engineers

Number of blows	Normal pressure tons/ft^2	Initial void ratio	Friction angle
5	1	.610	31.4°
5	6	.584	29.3°
25	1	.584	33.3°
25	6	.554	31.8°

Table 4

ANGLE OF INTERNAL FRICTION
Data from Hogentogler [6]

Soil types	Angle of internal friction
Silts, wet	10°
Sands, dry	34°
Sands, immersed	34°

Table 5

ANGLE OF REPOSE
Data from Howe [7]

Material	Angle of repose
from Rankine's Applied Mechanics	
Earth	14° - 45°
Dry sand	21° - 37°
Gravel	39° - 48°

Table 6

ANGLE OF REPOSE
Data from Hudson [8]

Material	Angle of repose
Earth on earth	14° - 45°

Table 7

ANGLE OF REPOSE
Data from Massey [11]

Material	Angle of repose
Dry:	
sand	1-1/2:1
gravel or soil	1-1/3:1
cobbles	1-1/4:1

Table 7--Continued

Material	Angle of repose
Deposited under water:	
sand and gravel	2:1
soil	3 and 4:1
rock	1-1/4:1

Table 8

ANGLE OF REPOSE
Data from Merriman and Wiggin 12/

Material	Angle of repose
Compiled by Rankine from Experiments by Morin and others.	
Dry earth	14° - 45°
Sand	21° - 37°
Gravel	39° - 48°
Sand	34°
Side-slope excavation:	
firm earth	1:1
very soft earth, quicksand	4:1
sand, clean	33° 41'
gravel	36° 53'
soil	33° 41'
Submerged slopes (dumped into water):	
sand, clean	26° 34'
gravel	26° 34'
soil	15° 37'
Canal banks--submerged with current (India):	
alluvial soil, very firm gravel	63° 20'
stiff earth	45°
loose earth	26° 30' to 18° 20'

Table 9

ANGLE OF REPOSE
Data from Hanswell 14/

Material	Angle of repose
Sand	22°
Gravel	31°

Table 10

ANGLE OF REPOSE
Data from Patton 15/

Material	Angle of repose
from Trautwine:	
shingle and gravel	35° - 48°

Table 11

ANGLE OF REPOSE
Data from Plummer and Dore 16/

Material	Angle of repose
Sand	30° - 40°
Muddy sand	30°
Dense sand and gravel	34°

Table 12

ANGLE OF REPOSE
Data from Terzaghi 20/

Material	Angle of internal friction	Angle of repose
Sand	30° - 50°	up to 35°

Table 13

ANGLE OF REPOSE
Data from Urguhart 21/

Material	Angle of repose
from Cain 3/	
sand, dry	20° - 35°
sand, moist	30° - 45°
sand, wet	20° - 40°
ordinary earth, dry	20° - 45°
ordinary earth, moist	25° - 45°
ordinary earth, wet	25° - 30°
gravel, round to angular	30° - 48°
gravel, sand and clay	20° - 37°
from Goodrich	
bank sand	54°
quicksand, 100 up (Tyler sieve)	42°
quicksand, 50-100 (Tyler sieve)	37°
sand, 50-100 (Tyler sieve)	29°
gravel, 1/2"	25°
gravel, 1/4"	19°
bank sand	19°
sand, 30-50 (Tyler sieve)	14°
sand, 20-30 (Tyler sieve)	10°

Table 13--Continued

Material	Angle of repose
from B. Baker earth	48°
from Steel earth	37°
from Wilson bankeand	37°

Table 14

ANGLE OF INTERNAL FRICTION
Data from U.S.W.E.S. 22/

Material	Initial void ratio	Angle of internal friction
Franklin Falls sand	0.65	36.9
	.70	36.0
	.80	34.5
	.85	33.5
Keystone Dam sand	.525	32.9
	.550	32.4
	.575	32.1
	.600	31.9
Chen sand	.6	42.0
	.7	38.0
	.8	34.2
Sardis Dam shell	.55	36.1
	.60	34.6
	.70	32.1
	.80	29.7
Pit Run gravel	.45	33.6
	.50	32.7
	.55	31.7

Table 14-A

ANGLE OF INTERNAL FRICTION
Data from Hilf 24/

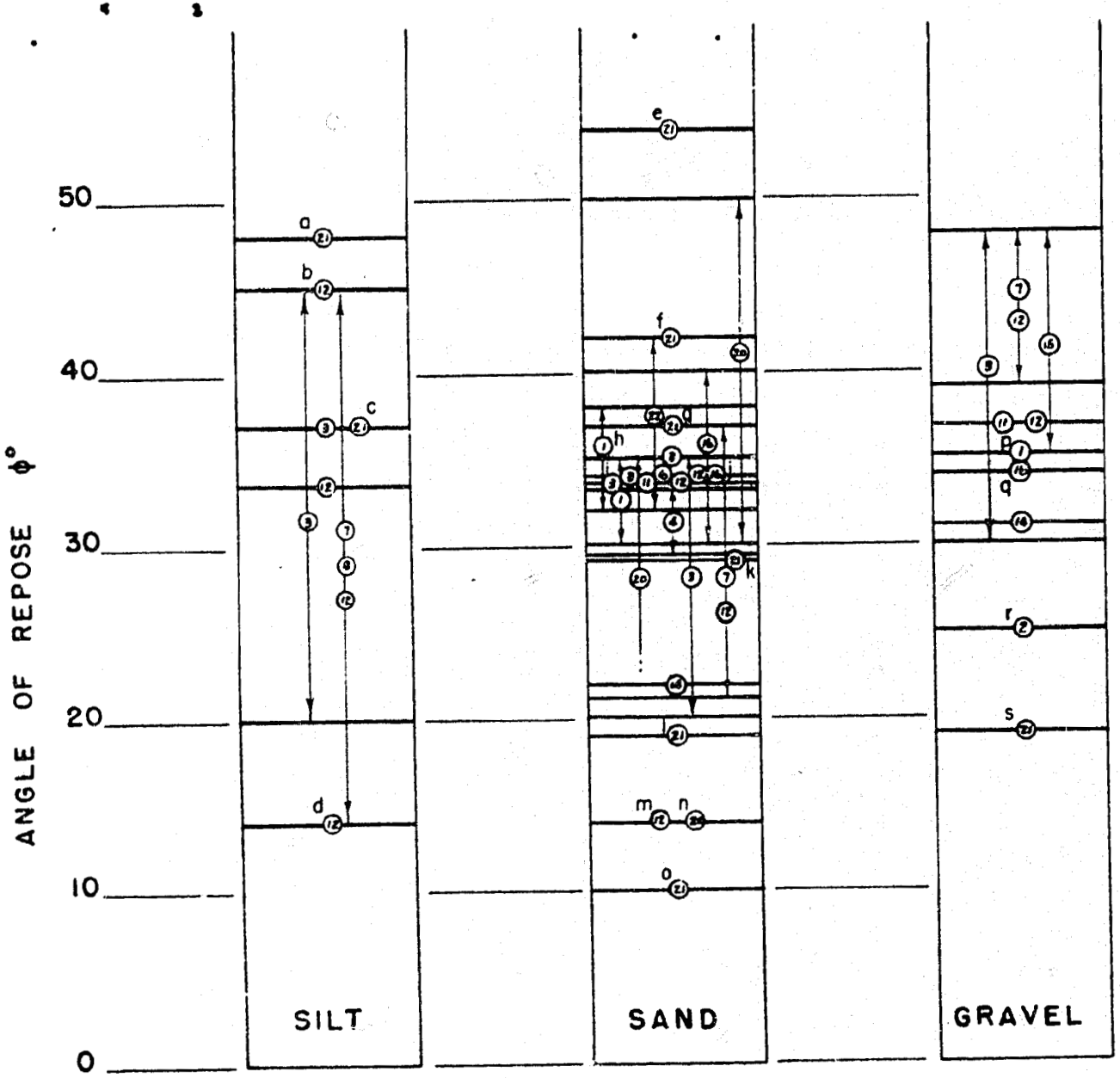
Material	Relative density--%	Internal friction angle, ϕ
Sand placed in Zone 2, Trenton Dam	23	26°
	38	30°
	50	32°
	73	36°
	84	40°

Table 15

ANGLE OF REPOSE
Data from Hanson 23/

Size mm	Shape	Stacked in air	Stacked in air and submerged	Stacked under H_2O	Stacked in air damp
19 - 38	rounded	32 35*	32	31	29
	angular	39 37*	38	40	40
9.5 - 19	rounded	24 35*/	25	29	27
	angular	37 33*	35	34	38
4.7 - 9.5	rounded	25	25	26	33
	angular	31 32*	30	32	32
1:1:1 mixt. of above		32		28	29
2.4 - 4.7	rounded	34	32	33	34
1.2 - 2.4	rounded	35	33	29	47
.15 - .42	rounded	32	27	29	90

* data from memorandum of 7-18-49
/ sample unwashed and contained some sand



TYPE OF MATERIAL - DRY

NOTES

Numbers refer to bibliography

a Baker
b firm earth
c Steel
d soft earth
e banksand
f quicksand 100
g banksand-Wilson, quicksand 50-100
h coarse, loose sand
i fine, " "
j dense sand
k sand 50-100
l banksand
m quicksand
n sand 30-50
o " 20-30
p loose gravel
q dense "
r 1/2" "
s 1/4" "

FIGURE 1

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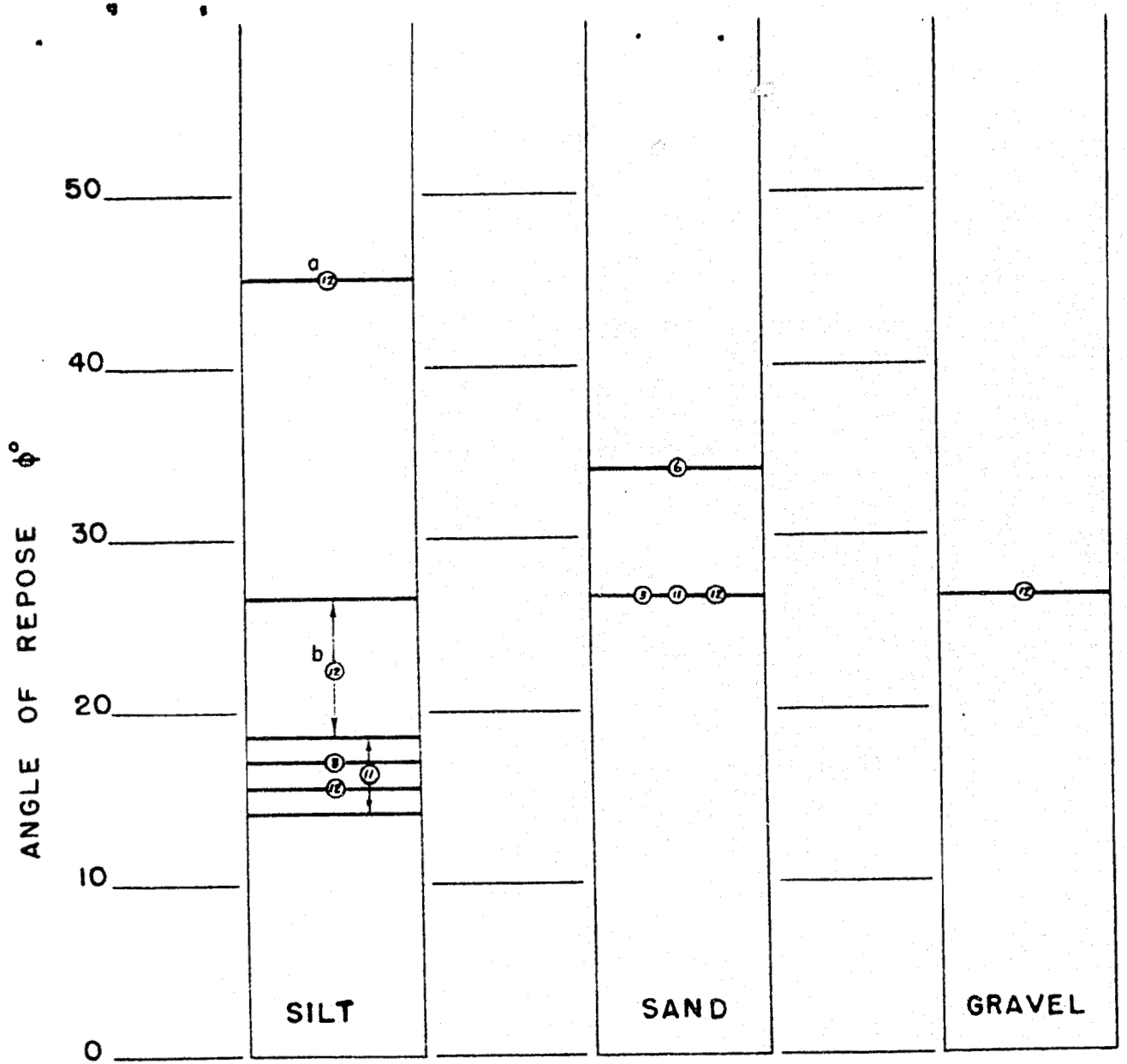
ANGLE OF REPOSE DATA FROM TABLES 1-13



TYPE OF MATERIAL - SUBMERGED

ANGLE OF REPOSE DATA FROM TABLES 1-13

NOTES

Numbers refer to bibliography.

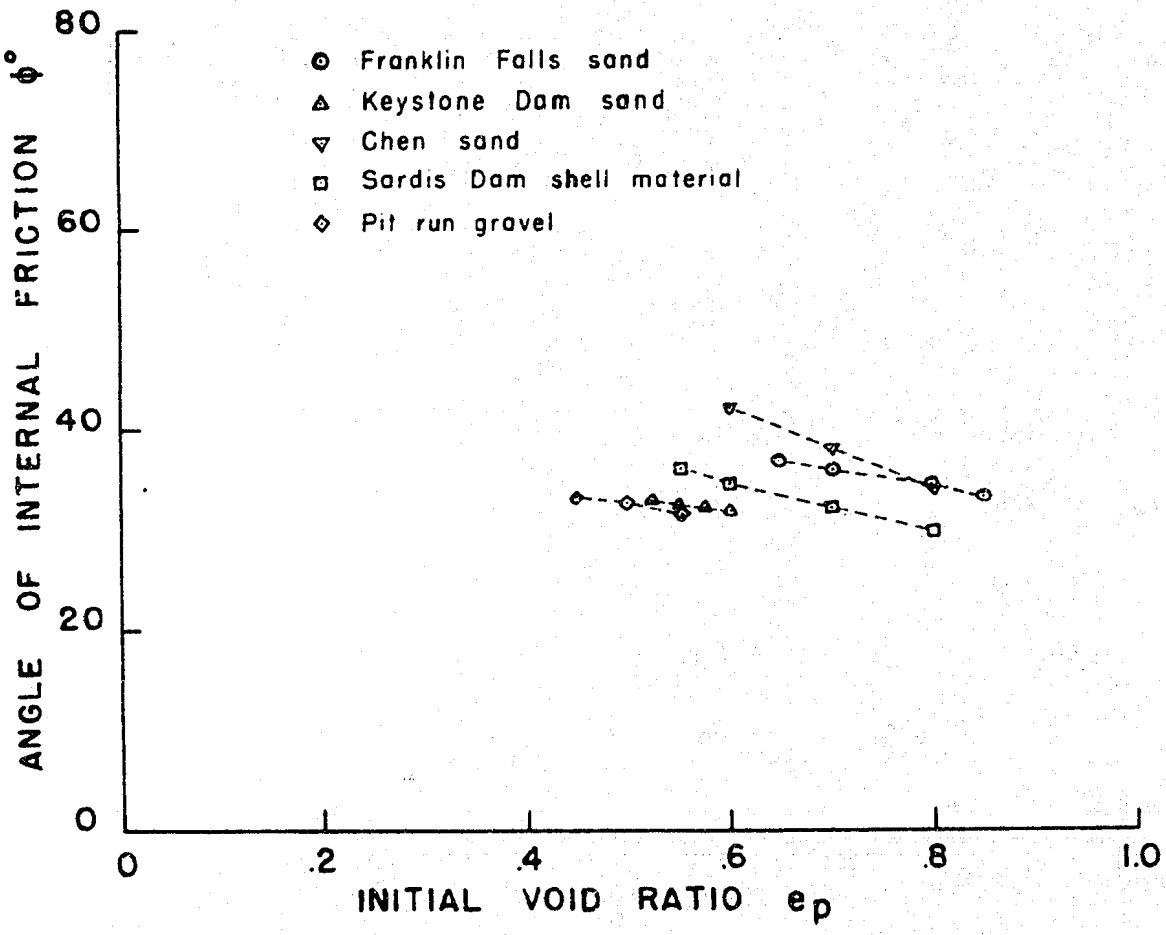
a stiff earth, current - India

b loose earth, current - India

FIGURE 2

404

FIGURE 3

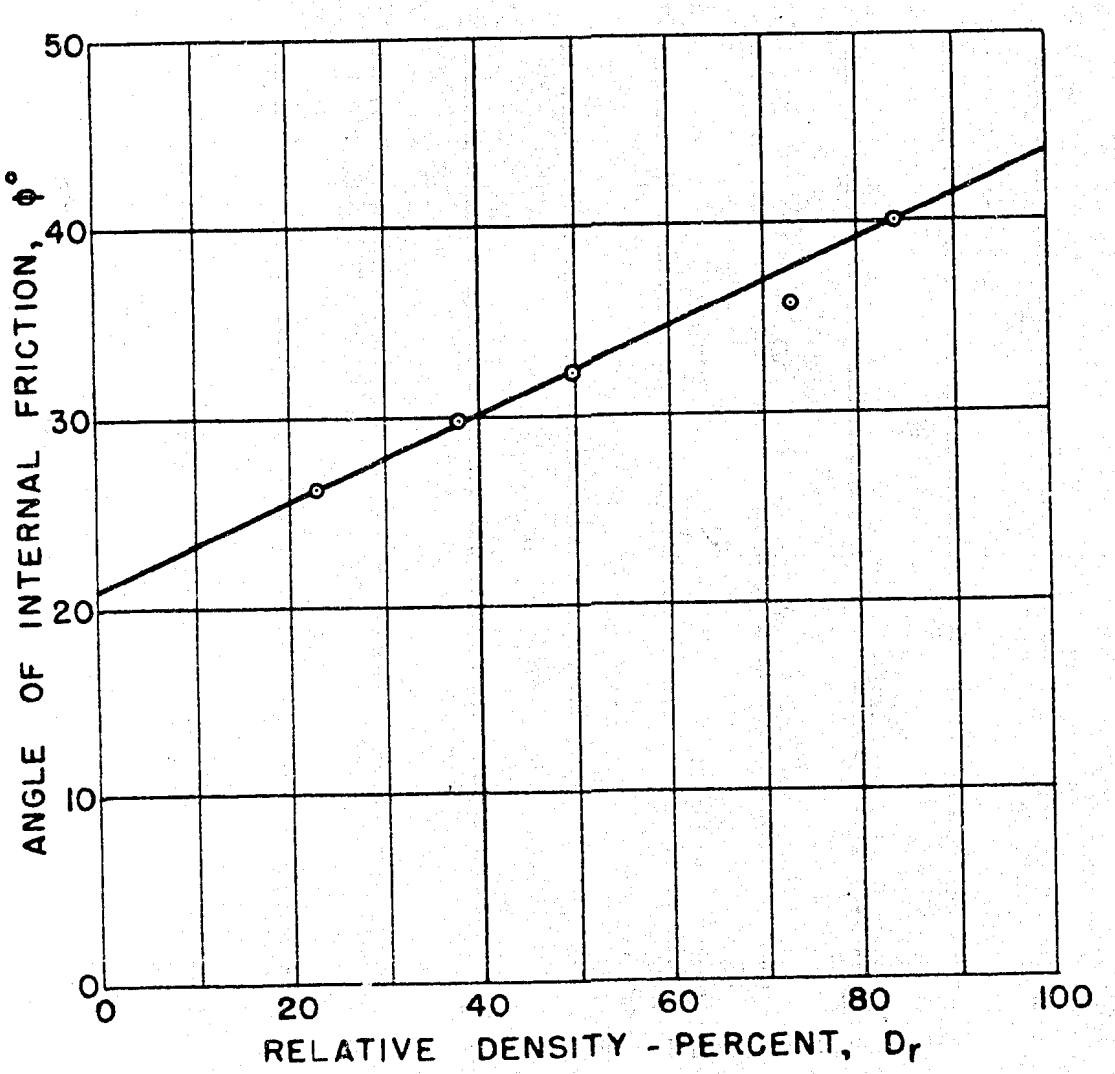


RELATIONS OF ANGLE OF INTERNAL FRICTION TO VOID RATIO

DATA FROM TABLE 14

404

FIGURE 3-A



RELATION OF INTERNAL FRICTION ANGLE
TO RELATIVE DENSITY

404 ○ SAND, ZONE 2, TRENTON DAM

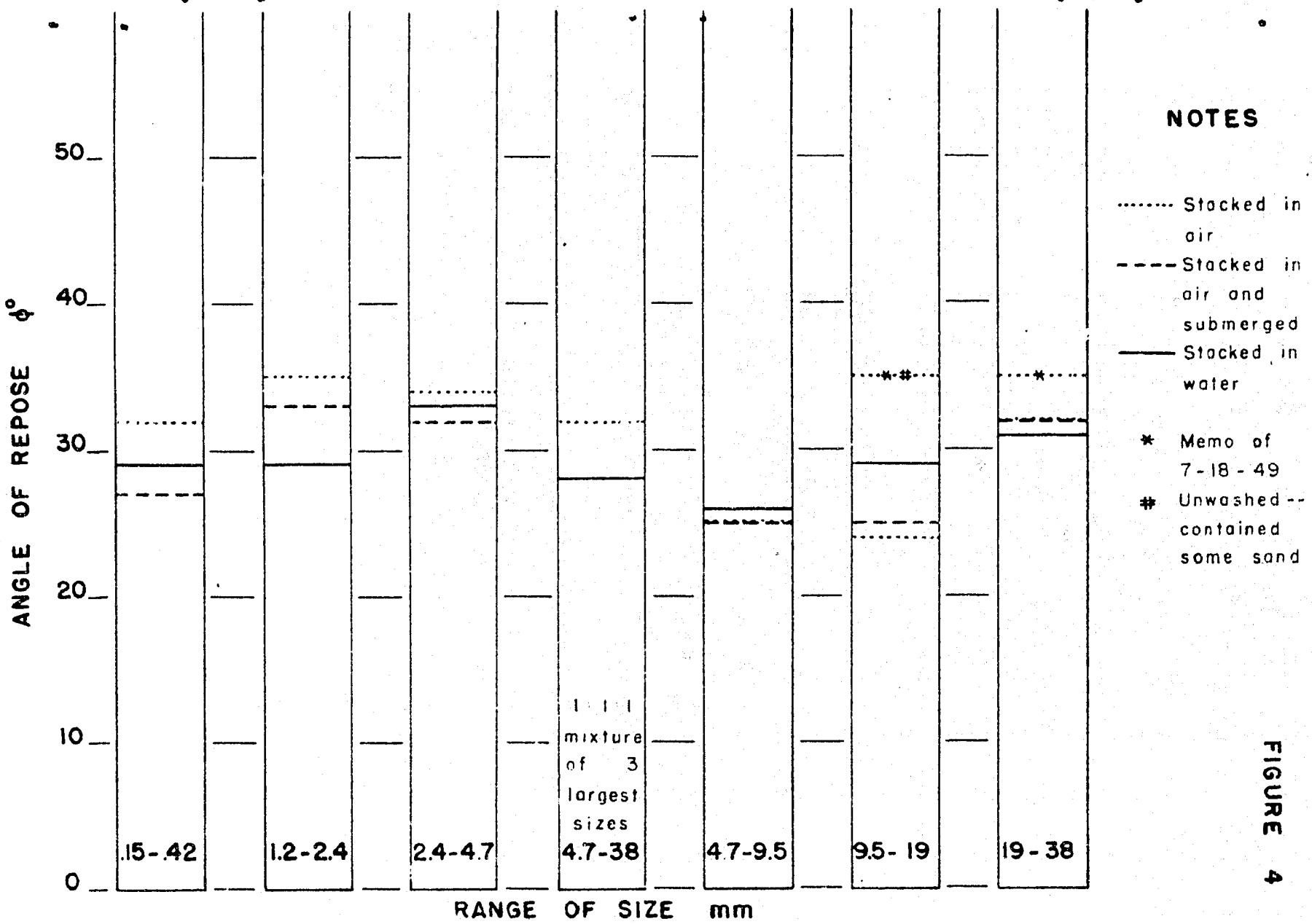


FIGURE 4

ANGLES OF REPOSE FOR ROUNDED MATERIAL
FROM TABLE 15

404

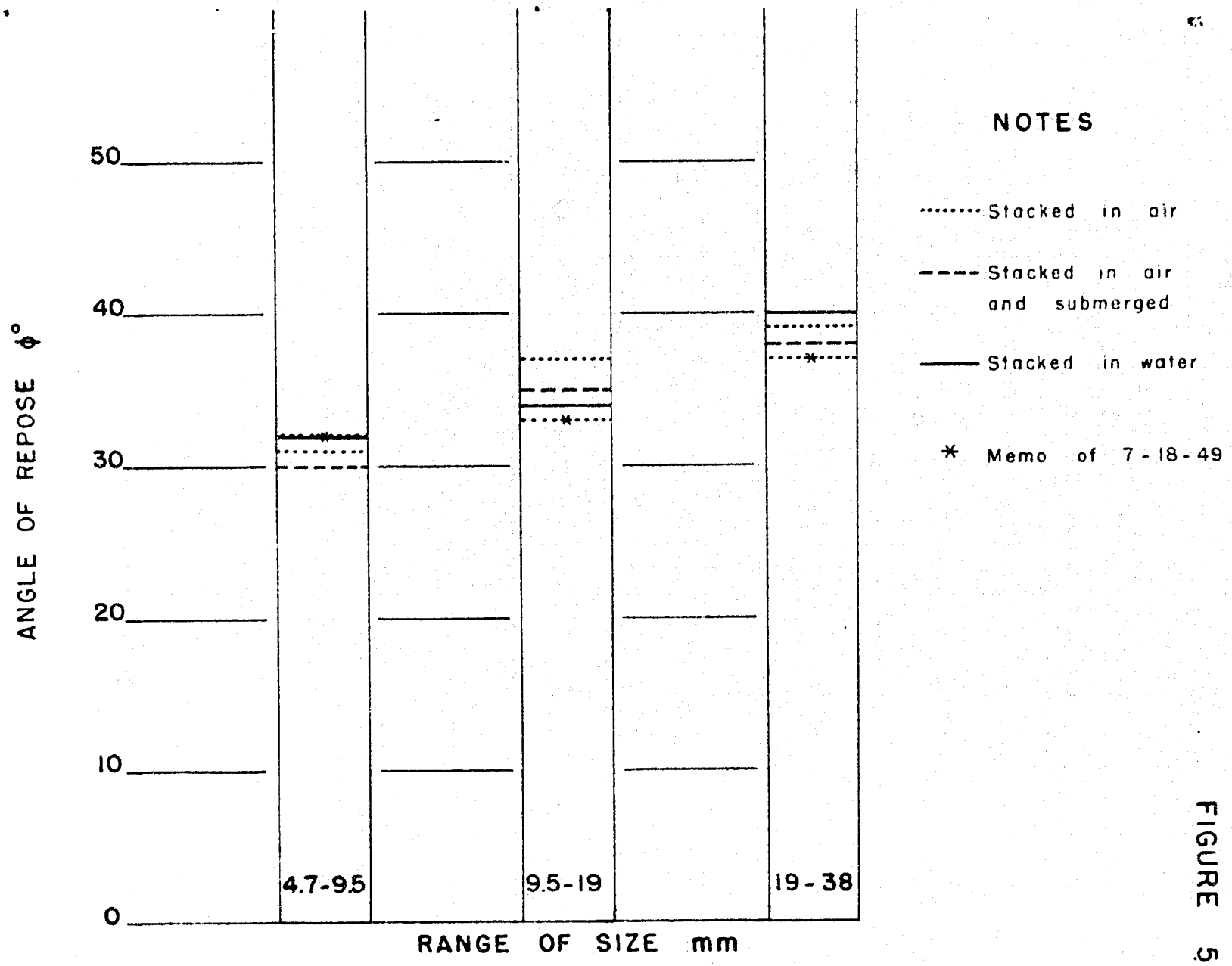


FIGURE 5

ANGLES OF REPOSE FOR ANGULAR MATERIAL FROM TABLE 15

404

Denver, Colorado

March 27, 1950

To : E. W. Lane

From : O. S. Hanson

Subject: Angle of repose of gravel and sand samples

1. In connection with the preliminary work on stable channel investigations the angle of repose of various gravel and sand samples was measured under a variety of conditions. The first of these measurements were made by E. J. Carlson and reported in a memorandum to you dated July 18, 1949. These original measurements were all made on dry gravel samples stacked in air.

2. The program was continued at a later date by measuring these and additional samples stacked in air, stacked in air and then submerged, stacked under water, and stacked in air while damp. These measurements were made using approximately 80 pounds of each sample and stacking the gravel against the wall of a tin lined box. The angle of repose was determined from natural tangents calculated by making horizontal and vertical measurements at seven points on the slope.

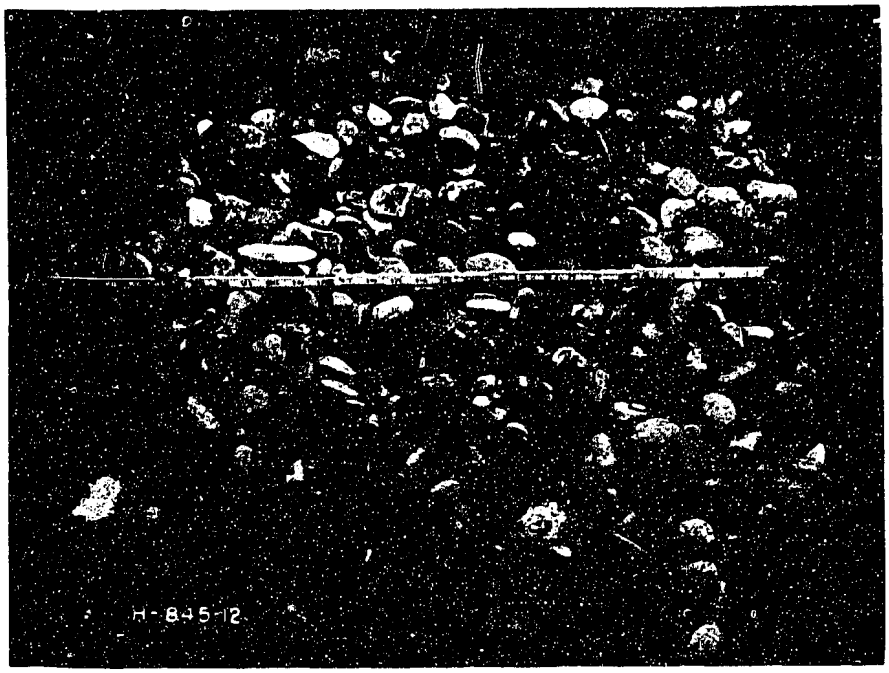
3. The following table shows the results of these measurements including those covered by the memorandum of July 18, 1949. Photographs of samples measured are attached.

Size	Shape	Angle in degrees from the horizontal: Stacked in air	Stacked in air and submerged	Stacked under water	Stacked in air damp	Photograph No.
19 mm to 38 mm rounded (3/4 to 1 1/2 in.)		32 35*	32	31	29	H-845-12
	angular	39 37*	[illegible]	40	40	H-845-7
9.5 mm to 19 mm rounded (3/8 to 3/4 in.)		34 35*#	25	29	27	H-845-11
	angular	37 33*	35	34	38	H-845-8
4.7 mm to 9.5 mm rounded (0.185 to 3/8 in.)	angular	25 31 32*	25 30	26 32	33 32	H-845-10 H-845-9
1:1:1 mixture of above		32		28	29	
2.4 mm to 4.7 mm rounded		34	32	33	34	M-5676
1.2 mm to 2.4 mm rounded		35	33	29	47	M-5675
.15 mm to .42 mm rounded		32	27	29	90	M-5589

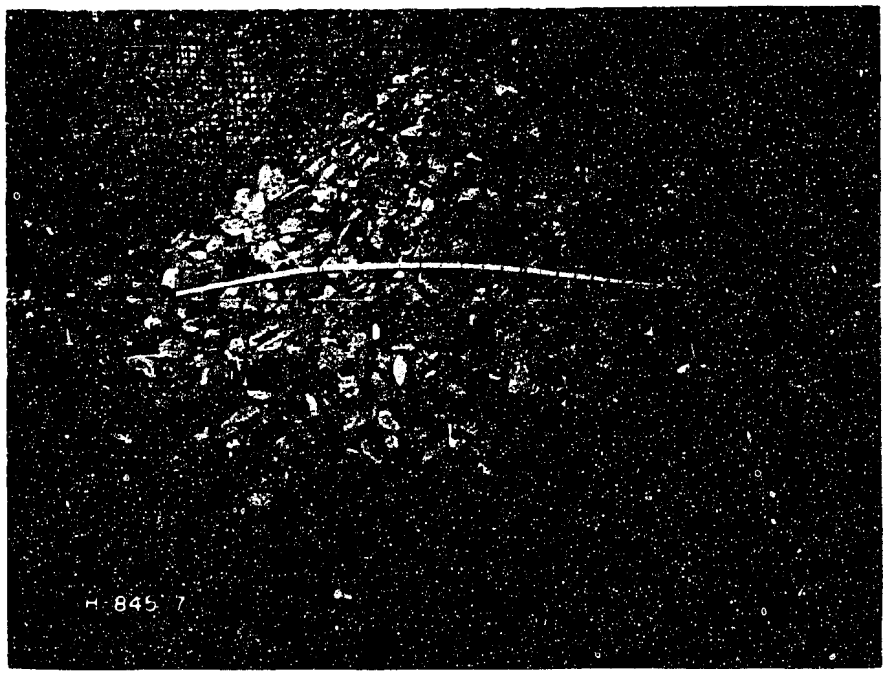
* data from memorandum of July 18, 1949
# this sample was unwashed and contained some sand

4. Results of these measurements show that for the gravel samples there is practically no change in the angle of repose in air or under water. The sand samples, however, do show a reduction in angle of repose when submerged. The magnitude of this reduction increases as the diameter decreases. The smaller sands also showed an appreciable increase in angle when piled while damp with the 0.15 mm to 0.42 mm sand standing vertical. The effect of shape is shown by the angular gravels having a greater angle of repose than the rounded samples by 5 to 10 degrees.

/s/ Odin S. Hanson

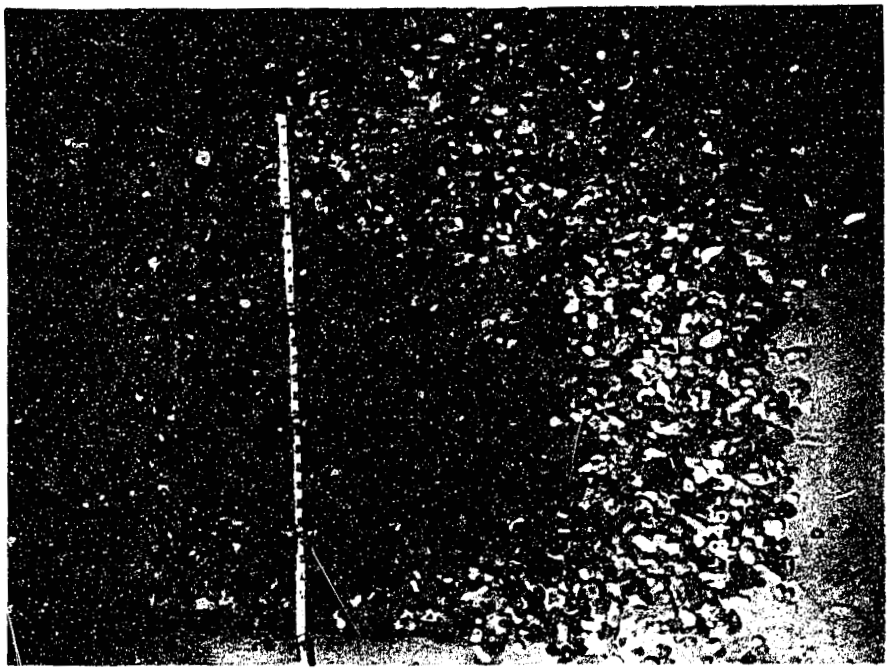


ROUNDED

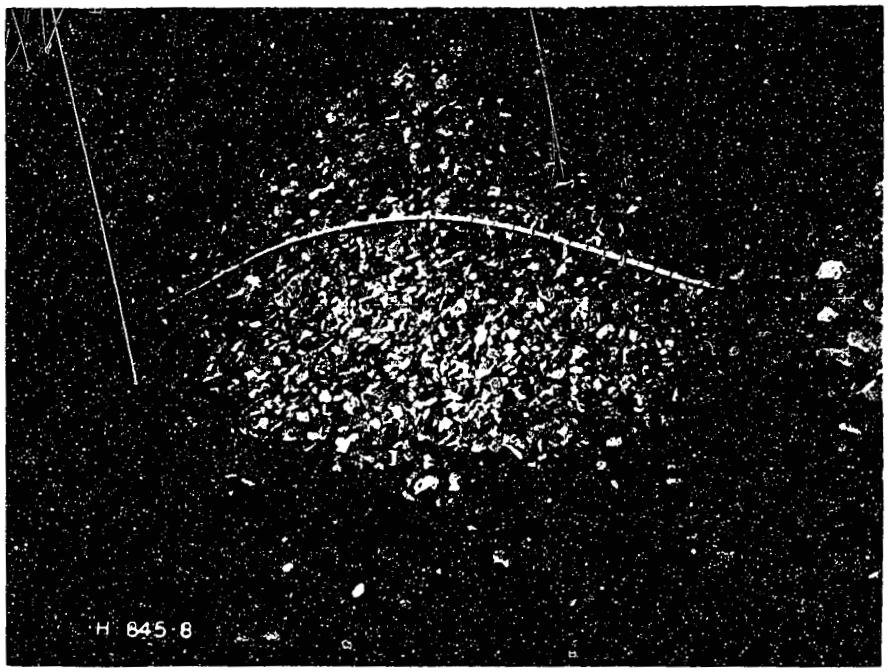


ANGULAR

19 mm TO 38 mm
3/4 in. TO 1/2 in.

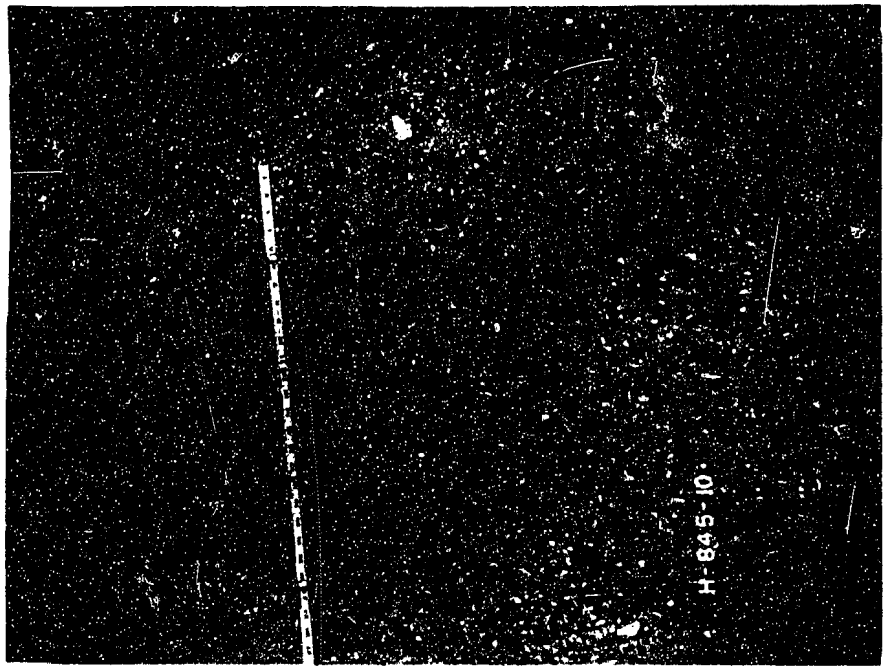


ROUNDED

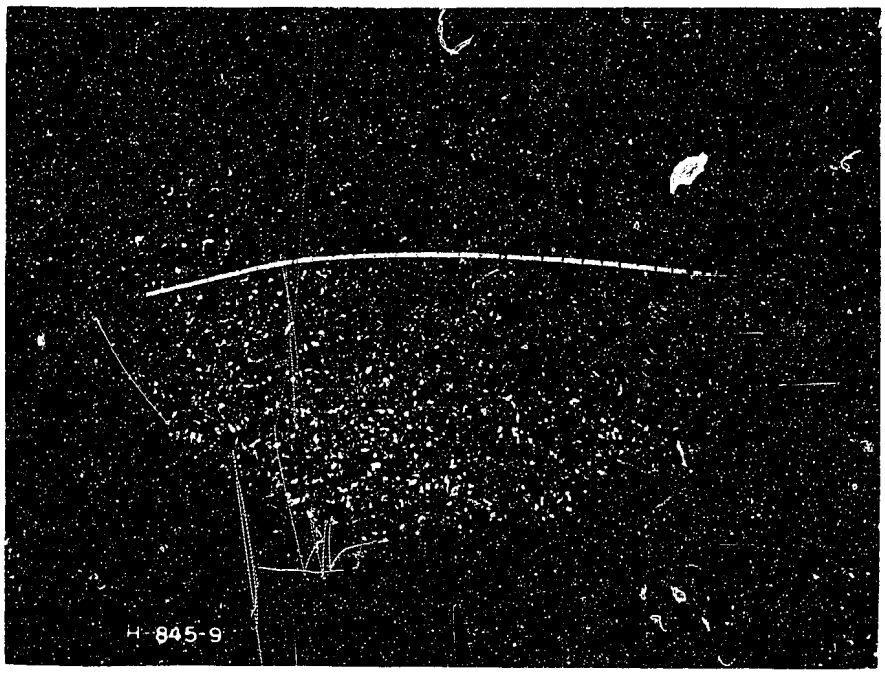


ANGULAR

9.5 mm TO 19 mm
3/8 in. TO 3/4 in.

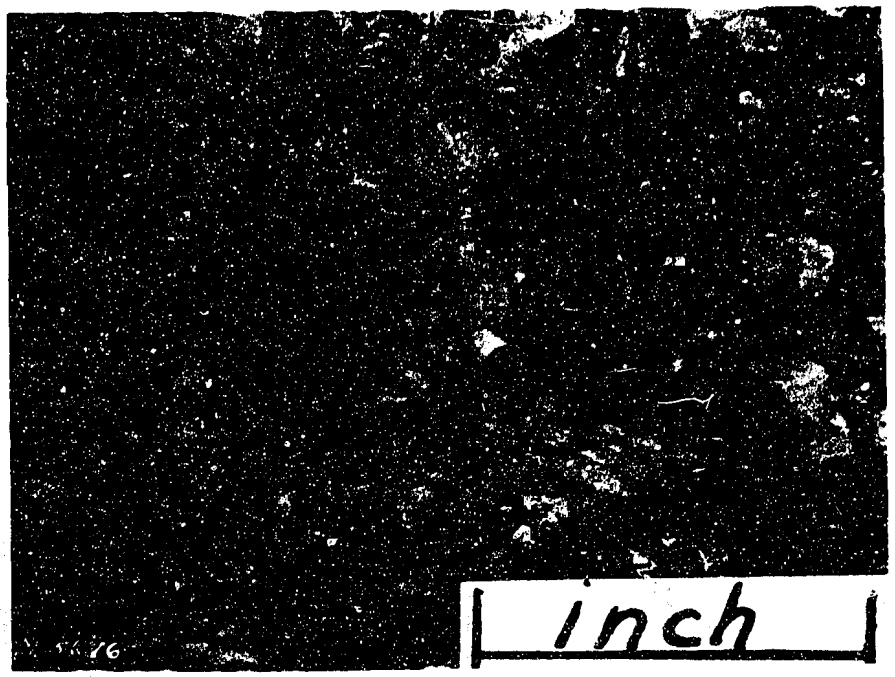


ROUNDED

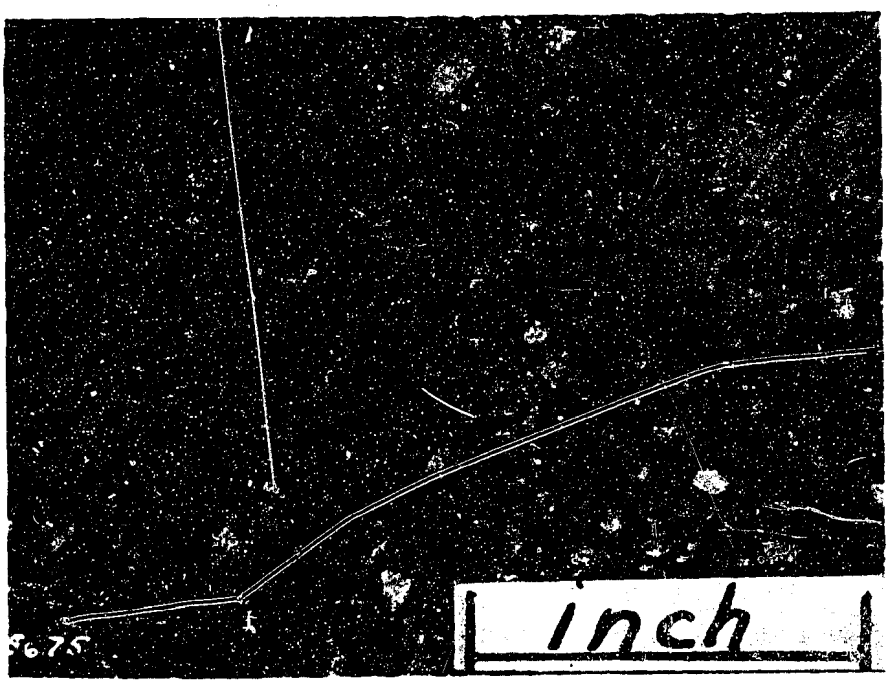


ANGULAR

4.7 mm TO 9.5 mm
0.185 in. TO 3/8 in.

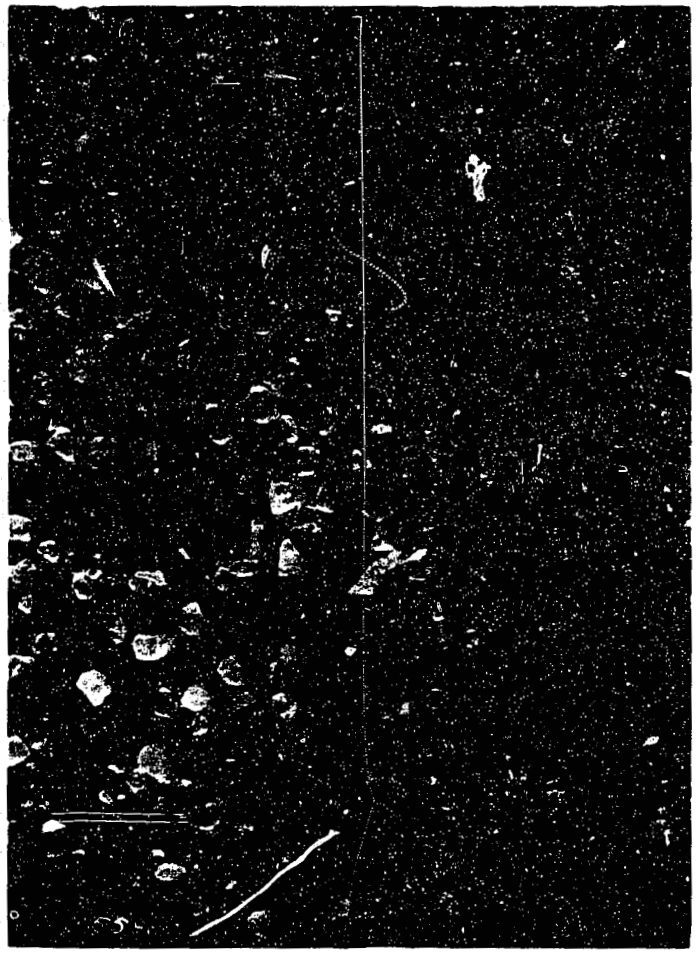


2.4 mm TO 4.7 mm



1.2 mm TO 2.4 mm

ENLARGED PHOTOGRAPHS OF SAND SAMPLES



0.15 mm TO 0.42 mm

ENLARGED PHOTOGRAPH OF SAND SAMPLE

Denver, Colorado

July 18, 1949

To : E. W. Lane

From : E. J. Carlson

Subject: Angle of repose of gravel samples

1. On July 5, the angle of repose was measured of several gravel samples that were on the Bureau Laboratories unloading docks. To determine this angle an adjustable square with a protractor head, together with a 5-foot metal straight-edge was used in the manner shown in enclosed Photograph H-845-1.

2. Tabulated below is a list of the samples giving the angle of repose (in air), size classification, and the number of the enclosed photograph which is a picture of the material.

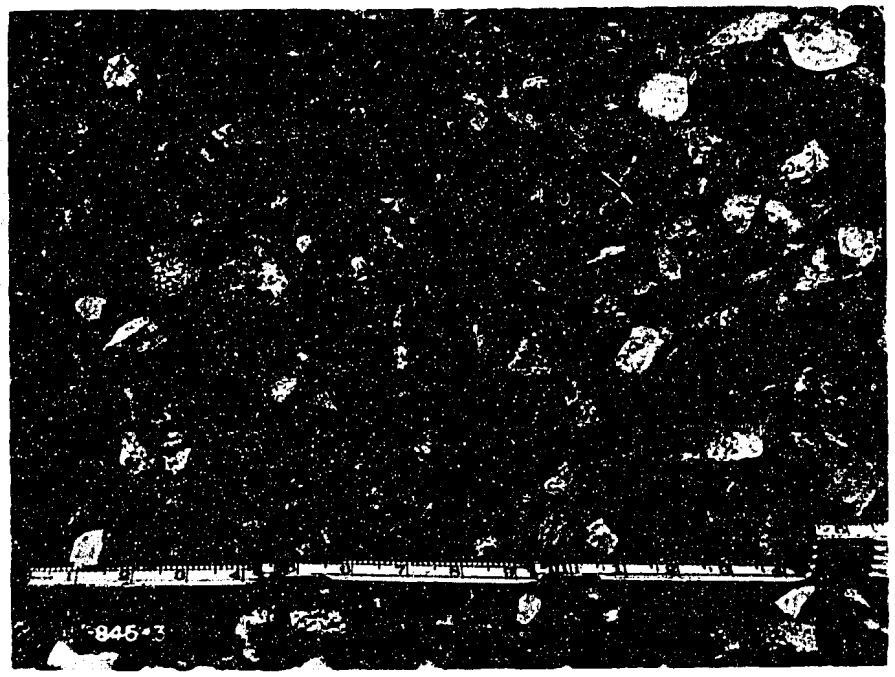
3. All the samples were obtained from the J. B. Bertrand Sand and Gravel Company, 4295 York Street, on Clear Creek. Samples No. 1 and 2 were in the condition as delivered. Samples No. 3 to 5, inclusive, were rewashed and graded in the Bureau's Materials Laboratory. Sample No. 2 had some sand remaining in the gravel.

LIST OF SAMPLES

Sample No.	Angle of repose average of 3 msmts	Size classification	Photograph No.
1	35.0°	3/4 in to 1-1/2 in	H-845-2
2	34.9°	1/4 in to 3/4 in	H-845-5
3	32.2°	No 4 mesh to 3/8 in	H-845-6
4	33.3°	3/8 in to 3/4 in	H-845-4
5	37.0°	3/4 in to 1-1/2 in	H-845-3

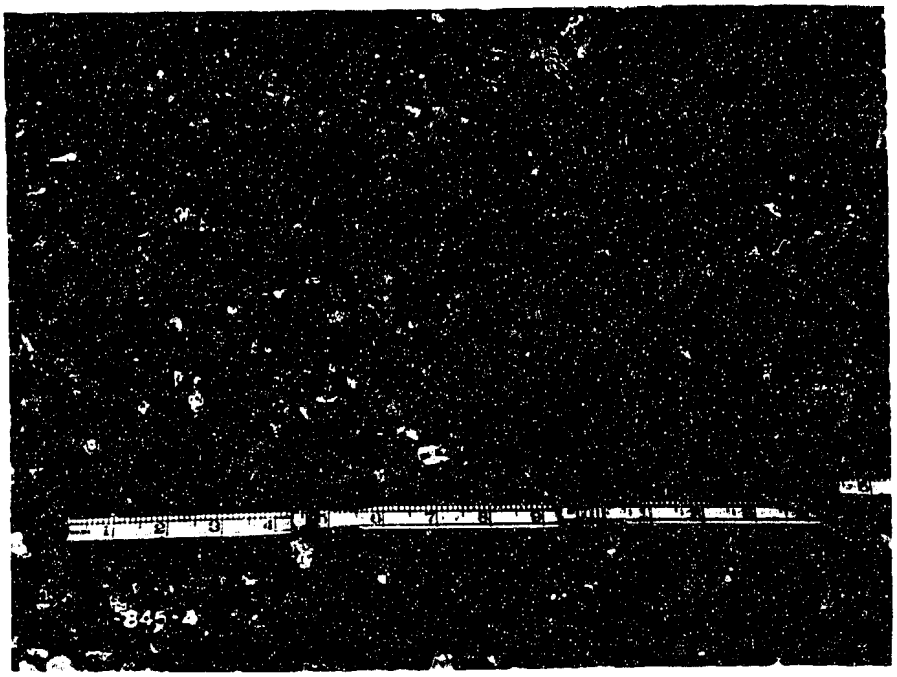
Enclosure (1)

ROUNDED
ANGLE OF REPOSE 34.0°

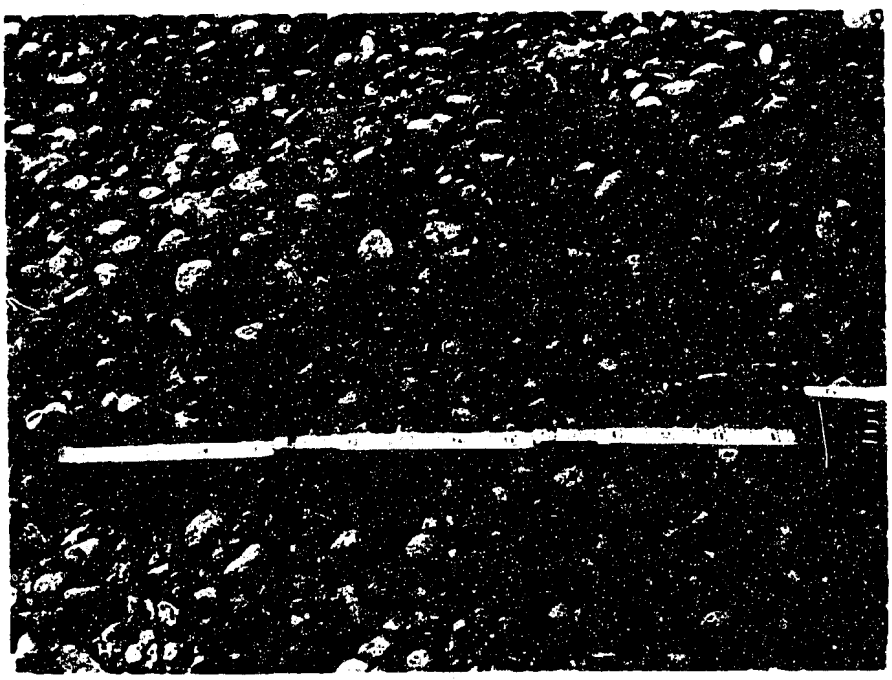


ANGULAR
ANGLE OF REPOSE 37.0°

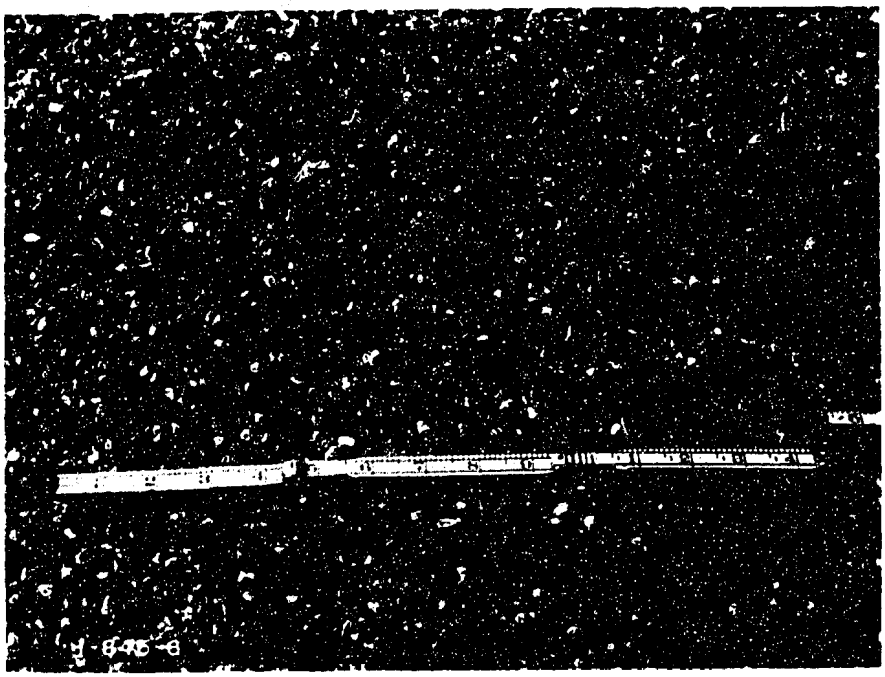
3/4 in. TO 1-1/2 in. GRAVEL



3/8 in. TO 3/4 in. GRAVEL
ANGLE OF REPOSE 33.3°



1/4 in. TO 3/4 in. GRAVEL
ANGLE OF REPOSE 34.9°



0.185 in. TO 3/8 in. GRAVEL

ANGLE OF REPOSE 32.2°