HYD-295

Denver, Colorado

May 11, 1950

Memorandum

To: E. W. Lane

From: A. C. Carter

Subject: Critical tractive forces on channel side slopes

Reference

1. Reference is made to "Principles of Design of Stable Channels in Erodible Material," by E. W. Lane, February 10, 1950.

Summary

2. A study of channels in equilibrium has been proposed to determine the tractive force necessary for initial movement of sediment resting on the level bottom of a channel. Most canals are designed with a trapezoidal cross section, and material composing the sloping sides is acted upon by both tractive and gravity forces. This memorandum discusses the nature of the two forces and expresses their relationship for different side slopes and angles of repose of the material.

Introduction

3. In the design of a stable channel through erodible material, scour of the banks and bed must not occur. For simplicity, conditions of clear water, straight channel, trapezoidal cross section, and non-cohesive material are assumed. A particle resting on a canal side slope is acted upon by two forces: one due to the velocity of the moving water which tends to move the particle in the direction of flow and the other due to the component of particle weight which tends to move the particle down the slope. The resultant of these two forces must be less than that required to move the particle.

Forces Causing Scour

4. The effect of gravity on a particle is well understood, but that of velocity is not easily determined. Velocities are difficult to measure because of rapid fluctuations due to turbulence and steep velocity gradient near the perimeter. For this reason, the longitudinal force on a particle has been studied using the tractive force principle discovered by du Boys. Tractive force has been defined as

the component of weight of a volume of water acting in the direction of flow, or the force causing the volume of water to flow. With steady flow conditions, the tractive force is just balanced by the frictional resistance of the bed and banks. The tractive force equals the weight of the volume of water above a unit area multiplied by the slope of the canal:

$$T_o = wDS$$

T_o = tractive force lb/ft^2

w = unit wt of water lb/ft^3

D = depth ft

S = energy gradient

Critical Tractive Force Study

5. A study is to be made of the critical tractive forces which will start movement of sediment of various sizes on a level canal bottom. It is anticipated that a relation between critical tractive force and particle size will be found from the proposed investigation of existing canals which have reached a state of equilibrium.

Gravity Forces

6. Consider a trapezoidal cross section such as shown in Figure 1. The tangential component of weight of a particle resting on a side slope is:

$$W_T = W \sin \phi$$

W = weight of particle lb

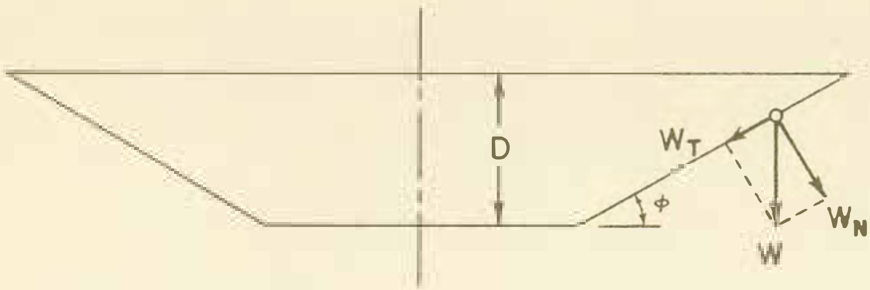
W_T = tangential component of wt lb

ϕ = side slope in degrees

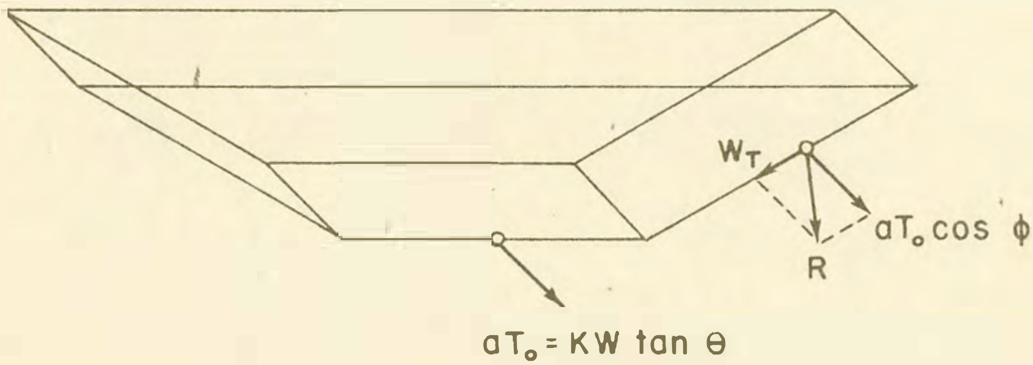
This gravity force tends to roll the particle down the slope.

Tractive Forces

7. A particle resting on the level bottom of a channel will just start to move when acted upon by the critical tractive force.



GRAVITY FORCE



TRACTIVE AND RESULTANT FORCES

FIG. 1

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However, tractive force has the dimensions of force per unit area and must be multiplied by an area to have the same dimensions as gravity forces. Assuming that critical tractive force conditions correspond to impending motion of material in the bed at the angle of repose, the critical tractive force on a particle in terms of friction force is:

$$a\,T_c = W \tan \theta$$

a = effective area of particle ft^2

T_c = critical tractive force lb/ft^2

W = weight of particle lb

θ = angle of repose in degrees

$\tan \theta$ = coefficient of friction

8. Allowable tractive forces (less than critical) on the bed of channels with given side slopes may also be expressed in terms of friction forces by direct proportion:

$$a\,T_o = K\,W \tan \theta$$

T_o = allowable tractive force on bed for a given side slope lb/ft^2

$K = T_o/T_c$ = ratio of allowable tractive force to critical tractive force

Tractive force per unit width of channel is less on side slopes than on level bottoms because the sloping area on which it acts is greater than the horizontal projected area:

$$\frac{a}{\cos \phi}\,T_o = K\,W \tan \theta \text{ or}$$

$$a\,T_o = K\,W \tan \theta \cos \phi$$

ϕ = side slope angle in degrees

Resultant of Gravity and Tractive Forces

9. A particle resting on a side slope a short distance from the intersection with the bottom is acted upon by maximum tractive and

gravity forces. Therefore, instability of the material on the perimeter of a trapezoidal cross section will first occur at this point. Since tractive and gravity forces are now in consistent units, their resultant may be found vectorially:

$$R = \sqrt{K^2 W^2 \tan^2 \theta \cos^2 \phi + W^2 \sin^2 \phi}$$

R = resultant force acting on a particle

10. For critical conditions, it is assumed that the ratio of the resultant force to the normal force on a particle is equal to the coefficient of friction of the material:

$$\tan \theta = \frac{\sqrt{K^2 W^2 \tan^2 \theta \cos^2 \phi + W^2 \sin^2 \phi}}{W \cos \phi}$$

$W \cos \phi$ = normal component of particle weight lb

Squaring both sides of the equation and simplifying:

$$\tan^2 \theta = K^2 \tan^2 \theta + \tan^2 \phi$$

Solving for K:

Cos ϕ

$$K = \sqrt{1 - \frac{\tan^2 \phi}{\tan^2 \theta}}$$

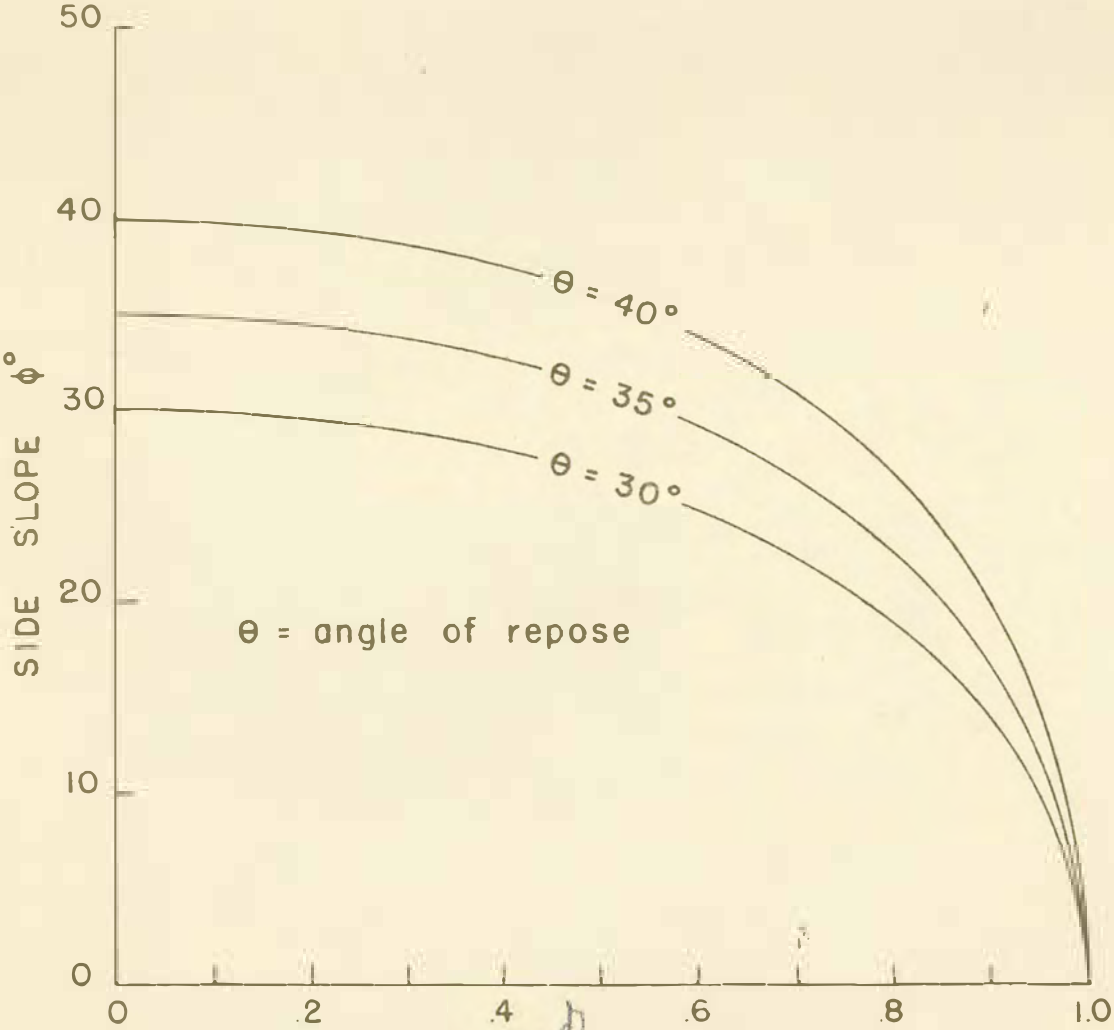
11. The K values for $\theta = 30^\circ$, 35°, and 40° were computed and plotted against side slope, ϕ, in Figure 2. In addition different K value curves are shown plotted against side slope and angle of repose in Figure 3. Knowing the K values for a certain angle of repose, the maximum allowable tractive force on the bottom of a trapezoidal channel with various side slopes in terms of the critical tractive force is:

$$T_o = K\, T_c$$

T_o = maximum allowable tractive force

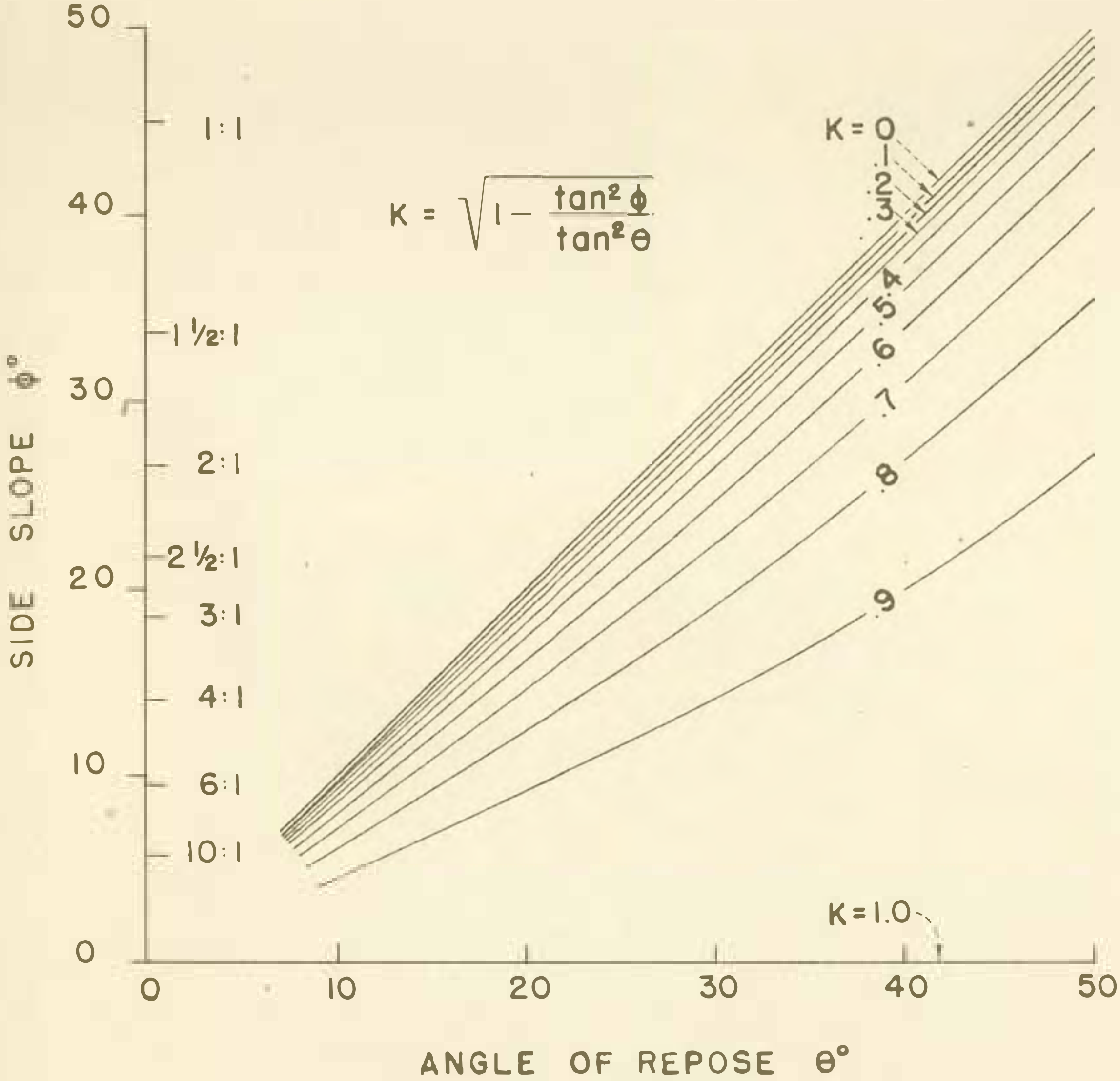
T_c = critical tractive force

12. For purposes of illustration, a critical tractive force--particle size relation was assumed:



$$K = \sqrt{1 - \frac{\tan^2 \phi}{\tan^2 \theta}}$$

VALUES OF K FOR DIFFERENT
SIDE SLOPES AND ANGLES OF REPOSE



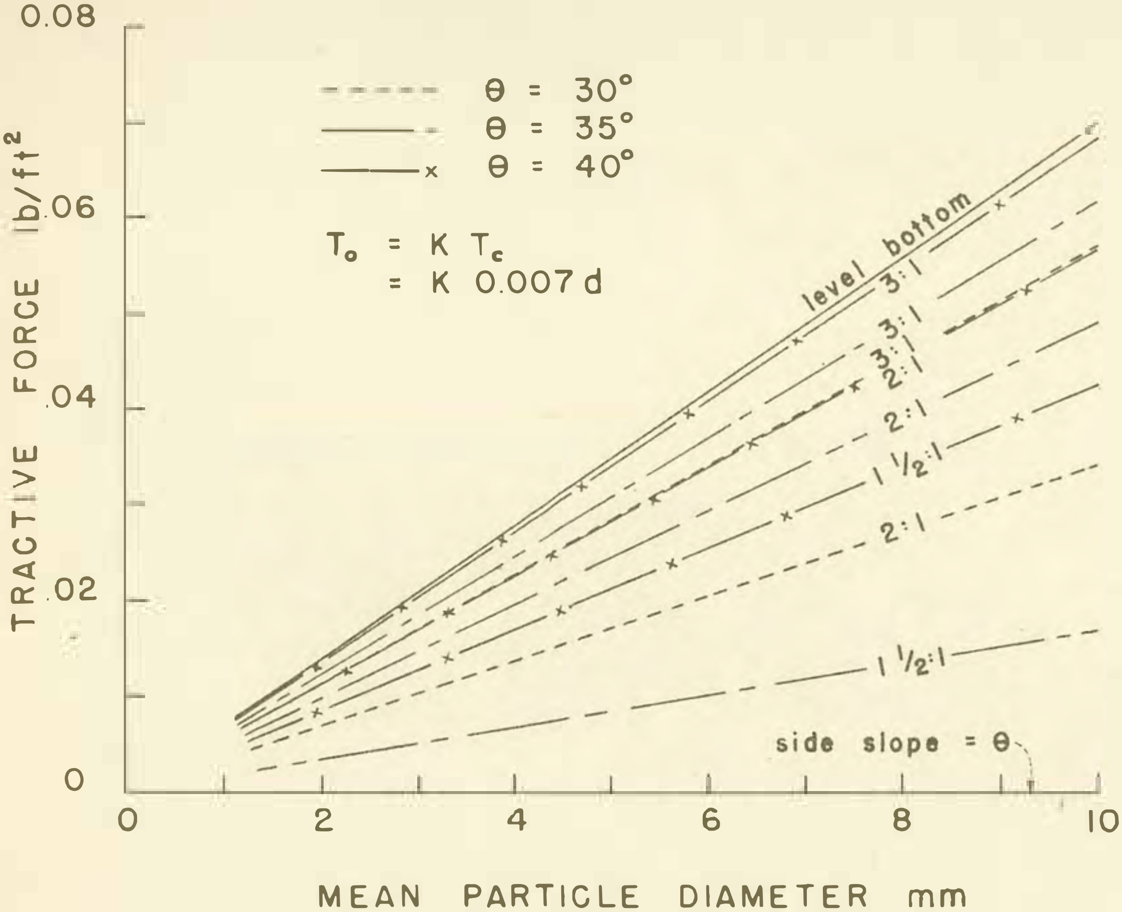
VALUES OF K FOR DIFFERENT
SIDE SLOPES AND ANGLES OF REPOSE

$$T_c = 0.007\ d$$

T_c = critical tractive force in lb/ft^2

d = mean diameter of particle in mm

Curves showing allowable bottom tractive force plotted against particle size for side slopes of 1-1/2:1, 2:1, and 3:1 and angles of repose of 30°, 35°, and 40° are shown in Figure 4.



RELATION OF ALLOWABLE TRACTIVE FORCE TO PARTICLE SIZE FOR VARIOUS SIDE SLOPES AND ANGLES OF REPOSE