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FLOW IN STEEP CHUTES WITH SPECIAL REFERENCE TO SELF-AERATION

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Branch of Design and Construction
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Hydraulic Laboratory
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Subject: Flow in steep chutes with special reference to self-aeration - A translation of Wasserbewegung in steilen Rinnen (Schusstennen) mit besonderer Berücksichtigung der Selbstbelüftung by R. Ehrenberger in Österreichischen Ingenieur - und Architecktenvereines

Generalities and Synopsis

Waste water from the upper pool at high-head power plants is usually directed into tumble bays in which energy dissipation (transformation) is artificially achieved. The severe shocks to which such devices are subjected, as well as the large construction costs involved, have led to the suggestion that this excess flow might be diverted by means of simple, steep chutes. In such channels, a self-aeration takes place as a result of the frictional and air resistance. Such a chute was constructed to full scale at the Ruts Works in Austria. The lack of favorable experience with this type of overflow works, as well as the fact that the science of flow in steep chutes is still an almost unexplored field, suggested that an experimental study of this flow and the accompanying phenomenon of self-aeration was most desirable.

The experiments described herein were made in the Hydraulic Structures Research Institute at the instigation of the Bureau of Austrian Railways. Since existing velocity formulas are strictly limited in application to slopes of about 1 to 1.5 percent, the ultimate aim of this study was to establish a relation between the average sectional velocity, the slope, and hydraulic radius for slopes up to say 70 percent. In addition to this, it was attempted to find a relation between the average aeration at a cross section (expressed as the ratio by volume of the water portion to the whole water-air mixture), the slope and hydraulic radius. It should be mentioned at this point that the measurement of the average cross-sectional velocity meets with fundamental difficulties, for the available instruments for measuring velocity, such as current meters of various types and Pitot tubes, are based on the tacit assumption that the specific gravity of the liquid is unity. These instruments would not be applicable, should they be first calibrated in water aerated to various degrees, for, as the experiments showed, the water presented different degrees of aeration at different points of the cross section which would have to be previously determined. Before proceeding with the determination of velocities, the complete lay-out of the apparatus and the general plan of the investigation will be described in detail.

The experiments were performed on chutes with five different slopes and with four different discharges for each slope. These bottom slopes[1] in test series I to V were S = 15.5, 20.6, 32.0, 49.5, and 76.2 percent. The last value corresponds to the slope of the wasteway at the Rutz Works. An absolute fall of 3.5 meters (11.5 feet) was available in the laboratory. The length along the incline for the smallest slope (test series I) was 16 meters (52.5 feet) and for the maximum slope (test series V) was 5.5 meters (18 feet). In general, those lengths were entirely sufficient to insure uniform flow at the lower end of the chute. Only in test series V, at the higher velocities, did it appear that uniformity was not completely achieved. Hence, in this case, extrapolation had to be employed. The discharges used for each of the test series were 10, 20, 31, and 44.5 second-liters (0.353, 0.706, 1.09, and 1.57 second-feet). Occasionally, a discharge as low as three second-liters (0.106 second-feet) was used as a check. The 44.5 second-liter (1.57 second-feet) discharge was the maximum attainable under the given laboratory conditions. A baffle served to quiet

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[1]By the slope, S, is meant the tangent of the angle of inclination of the bottom of the chute. The sine of the angle of inclination had to be introduced, however, in the final formula for theoretical considerations.

the flow issuing from the high reservoir. The cross section of the channel was rectangular and in conformity to the dimensions of the Rutz waste chute (bottom width 2.5 meters or 8.2 feet) had a width of about 0.25 meters or 0.82 feet.

A certain problem arose in connection with the exact determination of the water surface. As a result of the aeration phenomenon (produced by air resistance), with the smaller slopes, the surface was much roughened, and in addition, detached drops of water moved along parallel to the water surface, while with greater velocities, a water surface, in the usual sense of the word, simply did not exist because of the gradual transition from air to water. With the larger discharges in test series V, in which surface velocities of 8.5 meters per second (27.9 feet per second) occurred, the water surface exhibited, as a result of the large amount of intermingled air, the same milky-white appearance noticeable to a greater degree at the Rutz wasteway. Figure 2 shows a schematic section through a chute. At the top, droplets of water interspersed through air are first noticed. Below this layer, there is a layer consisting of a mixture of air and water, which in turn covers a layer of water containing individual air bubbles, and finally there is a layer of unaerated water adjacent to the bottom. With steep slopes (test series IV and V), the water-air mixture extended all the way to the floor of the chute. From this description, it may be gathered that the definition of the term "water surface" is an important one. In what follows, the depth is defined as that height, above the bottom, at which the uppermost layer of water drops rebounds with considerable force against the broad side of a flat bar, held in the channel perpendicular to the direction of flow. The simple apparatus constructed for measuring the water surface permitted moving the bar for determining elevation as well as length so that both longitudinal and transverse water surface profiles could be determined.

Self-Aeration of Water

As already mentioned, the phenomenon of self-aeration is also covered briefly in the tests. By means of several glass windows placed on the sides of and extending to the bottom of the chute, this phenomenon could be directly and effectively observed. A white area indicated the aerated water, while below this strip, extending to the floor, a clear portion (air-free) was observed. Between these two layers, a more or less well defined "boundary layer" was noticed marking the limit of the aeration. If the velocity head, determined by means of a simple Pitot tube, held at various heights along a normal to the water surface, is plotted on coordinate paper, characteristic normal velocity head curves are obtained (figure 7). The elevation of that point denoted by t_1, above which the velocity head, as a result of aeration, decreases rapidly agrees well, for the small slopes (test series I-III), with the elevation of the previously mentioned "boundary layer." Pitot tube measurements made along the center line of the chute also gave information on the variation of the aeration in the longitudinal direction. Figure 4 shows a longitudinal section in which the sloping bottom is shown horizontally. The heavy curve shows the water surface, and the dashed curve indicates the position of the boundary layer. The cross-hatched part between these two curves represents the water-air mixture. It is seen from this graph that the aeration takes place gradually and not suddenly, as in certain cases of the hydraulic jump. Further, a well defined dip in the water surface is to be observed. This can be explained by the fact that the depth of water in the upper part of the channel, where no appreciable aeration is present, corresponds to an increase in velocity, while in the lower part, in spite of the further velocity increase, the depth, as a result of the aeration, continues to increase until a final constant condition is reached.

Determination of the Surface Velocity by a Photographic Procedure

In order to ascertain the average velocity in a cross section, a knowledge of the surface velocity was first necessary. This was afforded by photographing a luminous float, for a short exposure (0.1 to 0.2 seconds), placed in the upper end of the chute. These floats consisted of small match boxes to which pieces of magnesium ribbon were fixed. The luminous magnesium was recorded on the film as a white strip whose real length could be easily found by comparing it with gage lines of known length placed every meter along the sides of the chute at the same elevation as the existing water surface. Match boxes were chosen for floats, in order to prevent the extinguishing of the light source by the surrounding water spray. Such a float projecting relatively high above the water surface encounters a small air resistance, which will be discussed later. Evidently, this procedure is primarily adapted to an instant when the float is in a region of uniform velocity. Only with the largest discharges of testseries V, in which absolutely uniform stretches did not exist, were several luminous floats released simultaneously close behind one another, in order to arrive at the trend of the surface velocity in the longitudinal direction.

To find the actual time of exposure, was not so simple as to measure the path described by the float. The time of exposure had to be as short as possible (about 0.1 second) for the steep chutes, considering the high velocities and the relatively short uniform paths. At first the time was measured by photographing on the same film a clock, placed near to the chute and having a luminous hand moving over a black face, and the luminous float. The elapsed time was found by simple division from the angular velocity of and the angle traversed by the hand (a stop watch was used in determining the angular velocity), the movement of the hand over the black face recording well on the film.

However, it was shown that the motion of the hand, in spite of a balance wheel, was not accurately uniform and, further, the elapsed time could not be determined with the required accuracy by simply using a stop watch. For example, if the velocity is to be determined within an accuracy of ± 5 percent, the time must be accurately measured to ± 0.005 second, assuming an elapsed time of the exposure of about 0.1 second. Since such an accuracy could not be attained with this method, the time had to be measured in some other way. This was afforded by the use of an arc lamp supplied with alternating current (25,000 CP). The light emitted by an arc lamp, as is well known, is not of constant intensity but changes regularly from maximum to minimum corresponding to the reversals of the poles (equal to twice the frequency). Accordingly, the path of the hand is recorded not as a uniform white image on a dark background, but as a series of isolated white images on a dark background. If now the frequency of the alternating current amounts to 50 cycles per second, the number of pole reversals is 100; hence, the time between two consecutive positions of the hand is accurately 0.01 second. The total time of exposure is obtained from the number of images of the hand. Using this method, it is evidently unimportant whether the hand moves with uniform angular velocity or not. Since the frequency, in consequence of the different loads imposed on the transmission network, at most varies about two percent, an accuracy in the measurement of the time between two successive positions of the hands of 0.004 second was achieved, or about four percent. Accurate experiments proved that this percentage in the most unfavorable tests should be increased perhaps by a small amount as a result of the difficulty in reading the first and last positions of the hand. The total probable error can therefore be taken as about five percent for the most unfavorable conditions. The desired velocity is given by dividing the length of the path of the float by the time thus determined. The surface velocity at a point is obtained by this method or, to be more accurate, the average over the width of the match box (about one-seventh of the total width of the chute). A knowledge of the "average surface velocity" was necessary for the further analysis. Since the float evidently did not move accurately in the middle, the corresponding reduction could not follow according to a definite, fixed ratio, but must be evaluated by considering the position of the float at a given instant. The "average surface velocities" obtained in this way are given in the following table (table I) and are denoted v_o.

It should be mentioned that the term "average surface velocity", thus obtained, is not absolutely correct because the float has various depths of immersion as a result of the different degrees of aeration in a single test. Therefore, the calculated values do not give the exact surface velocities. In the following, the assumption is made that an increase in the velocity of the float. This assumption appears justifiable because, as the tests showed, the water portion of the water-air mixture increases rapidly from top to bottom; therefore water particles at the elevation of the bottom of the float play the chief part in impelling the float. The elevation of the bottom of the float was accurately determined by stretching wires across the chute at such an elevation that the match box could float under the wire close to but not touching it. The correct elevation of the bottom of the float was given with sufficient accuracy by the elevation of the wires above the bottom and the height of match box. Check calculations for the elevation of the bottom of the float on the basis of the specific gravity of the water-air mixture and Archimedes principle gave a good agreement with the direct observations.

Nomenclature

α = angle of inclination of the chute.

A = area of the cross section.

α_W = specific weight of water

α_{WL} = apparent specific weight of the water-air mixture.

h_W = h = velocity head at a point in water.

h_{WL} = velocity head at a point in the mixture of water and air.

H = average of h_W over the entire cross section.

p_W = ratio of the volume of water to the volume of the water-air mixture at a point.

p_L = ratio of the volume of air to the volume of the water-air mixture at a point.

P_W = average pf p_W over the entire cross section.

Q = Q_W = discharge of water only.

Q_{WL} = discharge of water and air combined.

R = hydraulic radius

t = depth of flow normal to the bottom of the canal.

t' = distance of the bottom of the float above the bottom of the chute.

t" = depth measured at the edge of the Rutz chute.

t_m = average depth of flow at a cross section.

t_r = average depth of unaerated flow = $P_W t_m$.

v = velocity at a point.

V = average velocity over the entire cross section.

V_o = average "surface velocity" as measured by the float.

V_m = average velocity computed from the surface velocity.

TABLE I — RESULTS OF MODEL TESTS

Test Series No. tan α (sin α)	Q sec.-ft.	Adjusted Values: V_o ft. per sec.	Adjusted Values: t_m ft.	R (b=0.82 ft.) ft.	V ft. per sec.	$P_w = \frac{Q}{AV}$	$t_r = P_w t_m$	H ft.	$R_w = \frac{Q^2}{2gHA^2}$
I	0.353*	10.66	0.057	0.050	9.61(1)	0.790	0.044	-	-
155 %	0.706	13.29	0.090	0.074	11.94	0.790	0.071	1.80	0.763
(0.153)	1.09	14.93	0.123	0.094	13.29	0.807	0.099	2.20	0.813
	1.57	16.08	0.161	0.116	14.53	0.801	0.129	2.69	0.783
II	0.353*	11.65	0.054	0.048	10.24(2)	0.770	0.042	-	-
206 %	0.706	14.93	0.087	0.072	13.06	0.750	0.065	2.13	0.701
(0.202)	1.09	16.73	0.120	0.093	14.70	0.750	0.090	2.59	0.728
	1.57	18.05	0.156	0.113	16.04	0.758	0.118	3.02	0.762
III	0.353	13.45	0.051	0.045	11.48	0.729	0.037	1.71	0.638
320 %	0.706	17.06	0.082	0.068	14.47	0.716	0.059	2.46	0.678
(0.305)	1.09	19.36	0.115	0.090	16.31	0.704	0.081	2.95	0.694
	1.57	21.00	0.151	0.111	17.55	0.707	0.107	3.35	0.714
IV	0.353*	15.42	0.048	0.043	13.22(3)	0.675	0.032	-	-
495 %	0.706	19.36	0.079	0.066	16.80	0.644	0.051	2.95	0.613
(0.444)	1.09	21.98	0.112	0.088	18.73	0.631	0.071	3.54	0.612
	1.57*	24.05	0.148	0.109	20.18	0.635	0.094	-	-
V	0.353	17.39	0.052	0.047	15.78	0.513	0.027	2.17	0.481
762 %	0.706	21.33	0.085	0.071	19.46	0.512	0.044	3.18	0.486
(0.606)	1.09	24.61	0.118	0.092	21.85	0.51	0.060	3.81	0.508
	1.57*	26.87	0.156	0.113	23.62	0.513	0.080	-	-

*No Pitot tube measurements made. (1) $V = 0.9\ \bar{v}_o$ (2) $V = 0.88\ \bar{v}_o$ (3) $V = 0.86\ \bar{v}_o$

Determination of the Average Velocity at a Cross Section and the Average Aeration

As already mentioned in the foreword, the determination of the velocities at any point in the cross section meets with fundamental difficulties as a result of the aeration. Two independent variables enter at the same time (velocity and aeration). The calibration curve of the instrument is known. The average normal velocity curve (average velocity over the whole width of the chute at various depths normal to the bottom) was determined in the following manner: Since, with the exception of test series V, the aeration did not reach all the way to bottom, it was possible to establish the average normal velocity curve by direct Pitot tube measurements at least in the unaerated portion of the flow. The lower part of the curve was obtained by this means. The "average surface velocity", v_o, found photographically, gave another point on the curve at the elevation of the bottom of the float. The desired curve for v and t can be tolerably drawn through the branch rising from the bottom and through this single point. Only the top portion of the curve is doubtful. This is of no great importance in the further computations, since the amount of water in the top layers amounts to only a small percent. Finally, a comparison of the discharge computed from this normal velocity curve and the actual discharge, as will be shown later, shows a good agreement.

The average velocity at a cross section cannot be set equal to the arithmetic mean of the individual values of the velocity, because there are different degrees of aeration at different depths. Before going into the calculation of this average velocity, a mathematical expression for the term "aeration" must be derived. This seems to be necessary because the product of the cross-sectional area and the average velocity does not give the actual discharge when aeration is present. The expression consists of the inequality:

$$A\ V > Q_w$$

To produce an equality, a reduction factor, p_W, the percent of water, by volume, in the total water-air mixture, must be introduced. This gives the equation:

$$Q_W = A \cdot V \cdot p_W. \qquad (1)$$

or

$$p_W = \frac{Q_W}{A \cdot V.} \qquad (1)'$$

Were it possible to measure directly the discharge actually flowing through an isolated horizontal strip, the values of the aeration in the different layers could be determined directly. Let ΔQ_W represent the discharge passing through a horizontal strip whose area is ΔA, and let $\bar{u}'$ denote the ratio:

$$\frac{\Delta Q_W}{\Delta A} = \bar{u}' \qquad (2)$$

then it follows from equation (1) that

$$\bar{p}_W' = \frac{\Delta Q_W}{\Delta A \, \bar{v}'} = \frac{\bar{u}'}{\bar{v}'} \qquad (3)$$

in which $\bar{v}'$ represents the actual velocity taken from the average normal velocity curve at the given strip area, ΔA.

For the purpose of calibrating the layers, a sheet-metal conduit of rectangular cross section was fabricated to accurately fit the inside of the experimental chute (figure 6). The upstream end of this conduit was placed at the lower end of and at various heights above the bottom of the chute so that the flow, as it were, was cut into two parts, one of which flowed through the conduit, the other underneath it. The latter discharge was caught by a wooden calibration tank of a capacity of about six cubic meters. Thus, it was possible to determine the values of ΔA and ΔQ_W of equation 2. This method should give essentially correct values in tests with small channel slopes, for experimental difficulties, due to high velocities, did not enter. High velocities introduce errors that are not permissible. It was first assumed that using sheet metal as thin as possible, would not disturb the flow very much. However, the stiffness of the sheet metal was not sufficient to prevent excessive vibrations of the bottom of the conduit. A further difficulty lay in accurately determining the quantity of water flowing into the calibration tank in a given time. Were it possible to create a quiet horizontal water surface in the calibration tank with high velocities of flow, a further difficulty would still be encountered as a result of the large quantities of water partly aerated discharging into the tank. Much air would then be in the tank, resulting in water depths that are too large. Considering these difficulties the depth in the tank had to be measured cautiously. This method was only used for checking the tests on the flatter slopes.

In comparison with this method, the use of a Pitot tube (without an ejector) possessed greater possibilities for determining the actual value of the average aeration. The upstream leg of the Pitot tube had an opening of about two millimeters and could be moved up and down and crosswise of the chute. To reduce the incipient, large pulsations, the glass riser tubes had a relatively large diameter (50 millimeters). Since, as is well known, when a static leg (directed downstream) is also taken into account, the velocity head, $h = \frac{V^2}{2g}$, is given by the difference between the kinetic leg reading and the static leg reading. In spite of the large damping effect, pulsations persisted. Therefore, average readings were taken after observing the fluctuating water columns for several minutes. Consequently, a complete Pitot tube traverse required four or five hours.

Before computing the aeration (expressed by p_w) from the velocity head, several remarks are in order. The weight per second of the discharge, ΔQ_w, flowing through a small strip of area, ΔA can be expressed in two ways:

1. By the discharge and the specific weight, thus:

$$G = \Delta Q_W \ a_W = \Delta A \ v \ p_W a_W.$$

2. By the volume of the water-air mixture flowing per second and the apparent specific weight, a_{WL}, thus:

$$G = \Delta Q_{WL} \ a_{WL} = \Delta A \ v \ a_{WL}.$$

hence $$p_W = \frac{a_{WL}}{a_W}. \qquad (4)$$

After these short, preliminary remarks the correct procedure for computing the aeration from the velocity head should be investigated. The velocity head of the water-air mixture cannot be used for determining the apparent specific gravity, a_{WL}, directly. This velocity head is

$$h_{WL} = \frac{v^2}{2g}. \qquad (5)$$

If the rising legs of the Pitot tube do not contain a fluid of specific gravity, a_{WL}, but, on the contrary, unaerated water of specific gravity a_W, as is actually the case, since the air bubbles escape from the rising legs in a short time, the following relation obtains:

$$\frac{h_{wl}}{h_w} = \frac{a_w}{a_{WL}}$$

or $$h_{WL} = h_w \frac{a_w}{a_{WL}}$$

or from equation (4)

$$h_{WL} = \frac{h_w}{p_W} \qquad (6)$$

Equating (5) and (6), we have

$$h_{WL} = \frac{h_w}{p_w} = \frac{v^2}{2g}$$

and, finally,

$$p_w = \frac{2gh_w}{v^2} \qquad (7)$$

Five complete Pitot tube traverses were made for each experiment for finding the average value of the aeration, p_w, over the whole cross section. p_W denotes this average value. Figures 7 and 8 show the computed values for test series II, with Q = 20 liters per second (0.706 second-foot). The measurements were made at five verticals, and a total of 40 points was covered. The two graphs to the left in figure 7 show sample normal velocity-head curves, while figure 8 shows sample horizontal velocity-head curves. The averages of all the readings, including those not shown in the figures for the sake of clearness, are given to the right in figure 7 and thus represent an average normal velocity-head curve. With the aid of these curves and the normal velocity curves, earlier described, the aeration index for a given horizontal layer can be computed. From equation (7) we have

$$p_w = \frac{2gh_w}{v^2}$$

Figure 9 shows curves for both p_w and v. However, it should be noted that the lower part of the v-curve, in the region where no aeration exists as was seen through the glass windows, is found from the following equation, which is valid for ordinary conditions:

$$v = \sqrt{2g\ h_w}$$

By averaging the p_w-curve, the final average aeration factor, p_W, for the entire cross section is obtained. The p_w-curves for all tests have practically the same shape. Aeration begins at the surface where the percentage of air, $p_L = (1 - p_w) = 100\%$, and percentage of water, $p_w = 0\%$.

As the depth increases, the percentage of air decreases rapidly at first and then gradually decreases to 0 percent at the elevation of the boundary layer (see figure2). This depth corresponds approximately to the inflexion point of the corresponding vertical velocity-head curve. Evidently, this limit is not sharply defined. Although the limit, $p_w = 1$, occurred above the bottom in test series I to III, it was just at the b bottom in series IV, and in series V the aeration at the bottom was $p_w = 0.83$. Averaging the v-curve does not give the proper average velocity, V, for the whole cross section because of the various degrees of aeration at different levels in the flow. Each individual velocity must be weighted according to the aeration, p_w, at the corresponding depth. The mean velocity for the entire cross section was computed by considering narrow horizontal strips three millimeters high. Average values of v' and p_w' for each strip were taken from the proper $\bar{v}$ and p_w curves. The desired V is then computed from

$$V = \frac{\Sigma p_w' \, v'}{\Sigma \, p_w} \qquad (9)$$

This computation procedure can be checked by the discharge. If ΔQ is the discharge through a narrow horizontal strip whose area is ΔA, then

$$\Delta Q = \Delta A \; v' \; p_w'$$

and the total discharge is

$$Q = \Sigma \; \Delta Q \qquad (10)$$

The discharge computed from this equation agrees well with the discharge measured by a weir. In order to effect a complete agreement in the discharges, p_w was computed directly from

$$p_w = \frac{Q}{AV} \qquad (11)$$

and not from the average of the p_w-values as described heretofore. V is the velocity computed according to equation (9). These values of V are shown in table 1 and they do not deviate appreciably from the previously computed average values of v.

The check method just described offers the principal means of setting up the correct average vertical velocity curve for test series V, in which considerable aeration is present at the bottom of the chute. The bottom velocities were therefore determined by extrapolating auxiliary curves for v_s and S (for equal discharges).

There is another method for computing p_w without including the surface velocity measurements. It is based on the discharge measurements and the average velocity head, H, over the entire cross section, obtained by finding the mean of the h_w-curves such as the one shown to the right of figure 7. Correspondingly, equation 7, which is valid for any single point, can be made to apply, with close approximation, to the whole cross section by introducing average values of the variables over the whole cross section. Thus

$$p_w = \frac{2\,g\,H}{V^2} \qquad (7)$$

From equation (1).

$$V = \frac{Q}{A P_w}$$

Introducing this equation into (7), we have,

$$p_w = \frac{Q^2}{2gHA^2} \qquad (12)$$

The values in column 10 of table I, computed from this equation, in general, are somewhat smaller than the previous p_w obtained by using v_o. This may be explained, as will be shown at the conclusion of this investigation, by the fact that due to neglecting the air resistance of the float in measuring the surface velocity, average velocities values were obtained that are too low.

Summary and Comparison of the Experimental Results

In order to eliminate the unavoidable errors of observation and to obtain a general idea of the accuracy involved, the following graphs were prepared. On account of the limitations of space, only the most important are reproduced herewith. Figure 10 shows the relation between v_o and Q for the various slopes; figure 11 shows the relations between t_m and tan α for various discharges, Q. In both cases a group of curves are drawn arbitrarily through the plotted points. In figure 12 curves are plotted for comparing t_m and the computed values of v_o. For finding the average depth, t_m was plotted against tan α rather than against Q, for, primarily on account of the phenomenon of aeration, no coherent relation was found in the latter case. Figure 13 is applicable for finding the elevation, t', of the bottom of the float above the floor of the chute. Finally, in figure 14 the average velocity, V, for the entire cross section and the average aeration, p_w, for the whole cross section are plotted against the average depth, t_m, over the whole cross section. Table I is a summary of the results.

Analysis of the Results

Thus far, for the sake of simplicity and the fact that a rectangular chute was employed, the average depth, t_m, for the entire cross section has been used rather than the usual hydraulic radius, R. However, in order to generalize the test results, t_m was replaced by R, in the logarithmic graphs in figure 15. Five parallel, straight, dashed lines corresponding to different slopes pass through the plotted points. For comparison, the computed values from Rehbock's[1] revised and complete

[1] Rehbock: Betrachtung über Abfluss, Stau - und Walzenbildung bei fleissenden Gewässern. (Observations on Discharge, Backwater and Roller Formation in Flowing Streams)- Julius Springer, Berlin, 1917, p.46.

form of the Ganguillet-Kutter formula (n = 0.010) are given in the same figure (dashed lines without points). From the flatter slope of these latter lines as compared with the first set of lines, the amount of energy dissipation due to self-aeration is to be seen.

It is worth mentioning that the intersections of the two sets of dashed lines fall between approximately 3.00 and 3.50 meters per second (9.8 and 11.5 feet per second). In consequence of the smallness of the angles of intersection, the position of the intersection points, as read, are subject to error. The choice of Kutter's n also has a moderate influence on the position of these intersections. As has already been mentioned, if the difficulty of the defining "water surface" is considered, it seems possible that the intersections should lie somewhat lower, namely, at approximately 2.0 meters per second (5.1 feet per second). In spite of this, this graph does show that self-aeration begins at a definite velocity. This velocity may be considered to be from 2 to 3 meters per second (6.6 to 9.8 feet per second) for smooth, wooden flumes; that is, the valid range of the existing velocity formulas vary probably extends only up to this limit. At some greater velocity the braking effect of aeration comes into play.

Some Supplementary Remarks on the Phenomenon of Aeration

The underlying purpose of the preceding investigation was not to study the causes of aeration, yet there are some interesting facts which may contribute to clarifying its behavior. If glass plates are attached to the end of the wooden chute so as to form an extension of the two side walls, aeration of the under side of the jet will be produced, since the jet on leaving the bottom of the chute comes in contact with the air. In order to study the conditions on the bottom and at the end of the chute better, pieces were cut out of the side walls, extending all the way to the bottom, and replaced by glass plates. Aeration was produced with a bottom velocity of 3.00 meters per second (9.8 feet per second). This test corresponded to test series I with Q = 44.5 liters per second (1.57 second-feet). A similar condition should be observed in a water jet discharging from a pipe line under pressure. For this purpose a short, wooden pipe of square section (inner dimensions 1 x 1 cm.) was fabricated and connected to a pipe line under pressure. The two side walls of the wooden pipe were not carried all the way to the end but, as in the previous case of the chute, were supplemented by glass plates which projected out for some distance from the end of the pipe. With this arrangement it was possible to observe the sides of the flow before and after it emerged from the end of the pipe proper. Again characteristic aeration was observed at the top and bottom of the jet with an unaerated portion between. If the jet is discharged under a lower pressure, it retains its shape, there being no disintegration of the upper and lower surfaces into individual water drops (beginning of aeration). As the velocity was gradually increased, the two surfaces first exhibited considerable roughness and then decomposed into individual drops of water, and finally, as the velocity was increased still more, a typical state of aeration was produced. By careful observation, that velocity was sought at which the first separation of drops of water (beginning of aeration) occurred. Although the beginning of this condition was not sharply indicated, this velocity may be said to be approximately three meters per second (9.8 feet per second) which is somewhat smaller than the limiting value determined previously from figure 15. Whether the cause of this difference is to be attributed to the fact that in a steep chute the air directly over the water surface is already in motion, while in the case of the pipe, the jet, after discharging from the end of the pipe comes into contact with quiet air; or is to be attributed to the uncertainty in determining the "water surface" (in steep chutes); or whether other conditions play a part, cannot be answered here. Finally, let us turn back for a moment. Aeration, as determined from the earlier investigation, began in every case at 100 percent ($P_w = 0$) at the water surface and at a definite depth fell to 0 percent ($P_w = 1$). With the aid of p- and v-curves plotted from the results of single measurements, it was attempted to find that velocity up to which aeration increased or at which air bubbles were still being absorbed. From figure 9 it is seen that the aeration increases, for example, down to a depth of 7 mm. (0.023 feet). This depth corresponds to a velocity of 3.80 meters per second (12.5 feet per second). If all other such cases are investigated, values are obtained which vary between the relatively narrow limits of 3.5 meters per second (11.5 feet per second); and 4.5 meters per second (14.8 feet per second); the mean lies around 4.00 meters per second (13.1 feet per second).

Verification and Generalization of the Results Found for the Model Chute on the Basis of Measurement of an Actual Chute at the Rutz Works

As already mentioned at the beginning, the chute in series V of the experiments had the same slope as the wasteway at the Rutz Works. Since the completion of this structure, several measurements have been made of the surface velocity by:

1. Rümelin and Angerer, three measurements in 1913.
2. der Versuchsanstalt für Wasserbau (Vienna Hydraulic Laboratory) by order of the Bureau for the Electrification of the Austrian State Railways, four measurements in 1923.
3. der Wasserkraftwerks - A.G (Water Power Plant Company, WAG), five measurements in 1923.

The discharge for the first case was determined from a calibrated weir; in the last two cases from the inflow and outflow of a reservoir.

The storage action of the reservoir and intake tunnels was not taken into consideration. For the determination of the discharge from the reservoir in the second case, two complete current meter measurements made in the tailrace of the turbine draft tubes served, and in the third case, the discharge diagram of the turbines was used. None too great an accuracy could be expected in the discharge thus measured. A somewhat greater accuracy was attained in the surface velocity measurements. The depth measurements were made at only one side of the chute in the second and third cases, while in the first they were made at both sides. Rümelin found that the water surface in a transverse profile was not horizontal, but was dish-shaped, and furthermore, the depths measured at the two banks of the trapezoidal section showed a difference up to 10 centimeters (0.328 feet) at a discharge of 3.4 cubic meters per second (120 second-feet). Likewise, in spite of the symmetrical arrangement, similar conditions were observed in the model tests. With this circumstance in mind, as well as a later one concerning the average aeration, it appears that with the exception of the data furnished by Rümelin, the depth of flow, in general, appears to be too large. It seems permissible, therefore, to reduce the measured depths, t", to correspond. Several considerations lead to the following relation for finding the adjusted depth of flow:

$$t = 0.8\ t''$$

The difference t"-t amounts to 5.5 centimeters (0.18 feet) for the maximum case. This value can be accepted in view of the difficulty in determining the water surface. The complete results of these measurements are given in table III.

In order to discuss the average cross-sectional velocity in terms of the float velocity, it is necessary to find a relation between V and v_0. Evidently, here we are dealing with guesses. Rümelin s view is that there is a considerable difference between these velocities; others think them to be about equal. The model tests of the Rutz wasteway to a scale of 1:10 show a probable ratio of $\frac{V}{V_0} = 0.9$. Therefore, this was used in all further calculations.

Since the results of the measurements in table II are evidently subject to inevitable errors of observation, the corresponding values of t and V, and V and Q are plotted in figure 18 and smooth curves drawn through the points. For practical considerations, a curve for R and V was substituted for a curve for t and V. The adjusted values of V and Q for various values of R in table III are taken from figure 18. The computed values of the average aeration, expressed by

$$P_W = \frac{Q}{AV}$$

are given in the last line. If the adjusted values of R and V in table III are plotted on logarithmic paper, as in figure 15, a straight line may be drawn through the points obtained. This line fits the points well and from it the effect of aeration is readily discernible. Its slope deviates somewhat from the slope of the lines found from the model tests and this may be attributed to neglecting the air resistance of the luminous float. The maximum deviation occurs with Q = 44.5 liters per second (1.57 second-feet). If the deviation of the velocity, ΔV, is taken as a measure of the air resistance, then, since the air resistance is proportional to the square of relative velocity between the float and the surrounding air, we have

$$\Delta V = K V^2$$

If the line found from the model tests (figure 15) is prolonged upward and the velocity differences computed from this line and the line for the Rutz measurements, a relation between ΔV and V is obtained. When plotted logarithmically, a straight line variation is clearly given. The tangent of the angle between this line and the horizontal axis is 1.6, so that the equation of the line becomes:

$$\Delta V = K V^{1.6}$$

Thus it is seen, that the velocity difference caused by the air

TABLE II

RESULTS OF MEASUREMENTS ON THE RUTZ WASTEWAY

b = 8.2 ft. see p. 3

Made by	(1) Rumelin, Angerer[x]			(2) V. F. W. und E. A.				(3) W A G				
t' in ft.								0.33	0.47	0.49	0.56	0.92
				Computed from A								
t = 0.8 t' in ft.	0.10	0.16	0.49	.18	.22	.29	.41	0.26	0.38	0.39	0.45	0.74
R in ft.*	0.097	0.157	0.446	0.210	0.272	0.361	0.512	0.249	0.348	0.364	0.407	0.640
A in ft.[2]*	0.82	1.40	4.20	1.83	2.42	3.29	4.90	2.22	3.18	3.35	3.80	6.37
v_o in ft./sec. +	26.2	35.8	62.3	39.4	45.3	52.5	63.3	44.0	53.5	53.2	55.8	69.2
$v_m = 0.9\ v_o$ in ft./sec.	23.6	32.2	56.1	35.5	40.8	47.2	57.0	39.6	48.2	47.9	51.2	62.3
Q in sec.-ft. x	17.7	28.3	120.0	27.9	36.0	52.3	129.2	32.8	56.6	62.5	70.3	151.9

*Computed using t. + Measured. x Water only.

TABLE III

RUTZ WASTEWAY: ADJUSTED MEASUREMENTS AND

COMPUTED AERATION

Made by	Rümelin, Angerer (1)			V. F. W. und E. A.				W A G				
R in ft.	0.097	0.157	0.446	0.210	0.272	0.361	0.512	0.249	0.348	0.364	0.407	0.640
V in ft. per sec.*	23.6	30.2	53.2	35.4	40.7	47.2	57.1	38.7	46.6	47.6	50.5	64.0
Q in sec.-ft.*	12.4	18.7	86.5	26.5	37.1	57.6	113.0	31.8	54.7	58.3	72.4	176.6
$Q' = A.$ [illegible]	19.4	42.0	223.5	65.0	98.5	155.7	279.3	85.8	148.7	159.3	192.5	406.8
$P_w = \frac{Q}{Q'}$	0.64	0.44	0.39	0.41	0.38	0.37	0.40	0.37	0.37	0.37	0.38	0.43

*Adjusted values.

resistance is approximately proportional to the square of the average cross-sectional velocity. The unimportant deviation in the exponents may be explained by the circumstance that the air directly over the water surface is not at rest but is actually moving in the direction of the water. Thus there is a complete justification for making a small correction in the curve for R and V obtained by model experiments (dashed lines in figure 15). By doing this, the computed values of P_w in column 7 of table I show a small decrease, so that an excellent agreement is obtained with the values of P_w in column 10, which are computed from the average velocity head. The agreement with these latter values of P_w which are independent of the float measurements and their attendant errors, is a further reason for justifying this correction. The corrected curves for R and P_w are shown on the right-hand side of figure 15. From these lines, it is seen that the percentage of water in the total water-air flow decreases slowly with an increasing hydraulic radius. The more or less complete scattering of the points relating to the Rutz wasteway may be attributed to the probable inaccuracies in the discharge measurements. The corrected curves for V and Q (dashed lines) for the Rutz wasteway, using the new relations for R and V, and R and P_w, are plotted by way of comparison in figure 18.

The Two New Formulas

The two final relations,

$$V = f_1 (R, \sin \alpha) \text{ and } P_w = f_2 (R, \sin \alpha),$$

which we have been searching for, are obtained directly from the logarithmic graph in figure 15. Although the first function may be represented by a single equation, the second requires two parts, according to whether sin α is smaller or greater than 0.476 (28.5°). This limit seems to correspond to that slope at which the aeration extends all the way to the bottom of the chute. This view is corroborated by directly observing the flow through the glass window in test series IV ($\sin \alpha = 0.444$), in which this aeration condition was approximately reproduced, while for test series I to III it was not the case.

The new formulas (in English units) are:

$$V = 97 \; R^{0.52} \; \sin \alpha^{0.4}$$

or

$$V = \frac{97}{\sin \alpha^{0.1}} \; R^{0.02} \; \sqrt{R \sin \alpha}$$

The percentage of water in the total water-air mixture is expressed by the following two equations:

$$P_w = 42 \; R^{-0.05} \; \sin \alpha^{-0.26} \quad \text{for } \sin \alpha < 0.476$$

$$P_w = 30 \; R^{-0.05} \; \sin \alpha^{-0.74} \quad \text{for } \sin \alpha > 0.476$$

These equations are valid for artificial chutes of dressed wood and approximately rectangular cross section. They are represented logarithmically in figure 19 for practical use. They are valid up to:

$R = 0.30$ m. (0.98 feet) and $\sin \alpha = 0.707$ ($\alpha = 45°$).

It is seen from this graph that within the given valid range an upper, limiting value for the velocity, independent of the hydraulic radius and the slope, is not yet reached. The question, whether the curve of R and V with greater R-values acquires a steeper slope and finally ends vertically, cannot be answered on the basis of the experiments made heretofore. It should also be mentioned that the computed velocities for a free overfall into an air-filled space cannot be applied to our study without further analysis, since for steep chutes in the limiting case when $\sin \alpha = 1$ (vertical chute), the air enters at only one side, the other three sides being protected from the surrounding air by walls. As to the development of the aeration, it is to be emphasized that with equal wall roughness, the slope is the most important factor, and the hydraulic radius and hence the depth are only minor factors.

Finally, attention should be called to a circumstance which may be adapted to clarifying the problem of self-aeration from another point of view. If the simultaneous values of t_m and V are taken from the results of a single experiment and plotted on logarithmic paper, the graph on the left-hand side of figure 20 is obtained. The frictional effect of aeration is clearly shown in that at the larger slopes, other conditions remaining constant, the depth suddenly increases with increasing slope. If the average depth at a cross section is multiplied by the aeration ratio, P_w or

$$t_r = P_w t$$

and the simultaneous values of t_r and V are plotted in a similar fashion (right-hand side of figure 20), the disturbing action of aeration is again brought out. It should be remembered that the values of V and P_w are taken from the corrected lines of figure 15 and that the chute is rectangular in cross section.

Conclusions

The foregoing investigation was undertaken to clarify the principles of flow in steep chutes, and the phenomenon of aeration. The degree of self-aeration is denoted by the percent of water, P_w, in the total water-air mixture. The relation:

$$Q = A\ V\ P_w$$

serves to express the measurements undertaken in a chute, V is the average velocity at a cross section, and P_w is the average aeration in the two principal equations I and II or the graphical representation in figure 19. These equations are valid for artificial chutes of dressed wood, with approximately reactangular cross section. Although no statement of their absolute accuracy can be given, in view of the difficulties of establishing a single argument for such, they should serve a useful purpose to the practicing engineer in designing new structures. In the interest of the continuous development of the knowledge concerning flow in steep chutes, let the desire be expressed that the results of velocity and discharge measurements at all such existing structures be given the widest distribution possible.

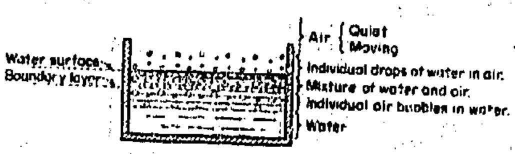

FIGURE 2 - SCHEMATIC CROSS SECTION OF THE FLOW IN A CHUTE

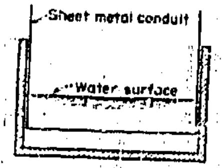

FIGURE 6 - CALIBRATION OF THE FLOW LAYERS
(CROSS SECTION OF THE CHUTE)

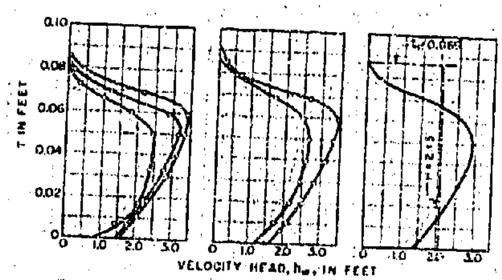

FIGURE 7 - NORMAL VELOCITY HEAD CURVES

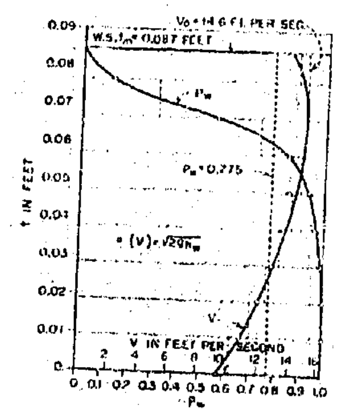

FIGURE 9 - CONSTRUCTION OF THE V AND P_w CURVES

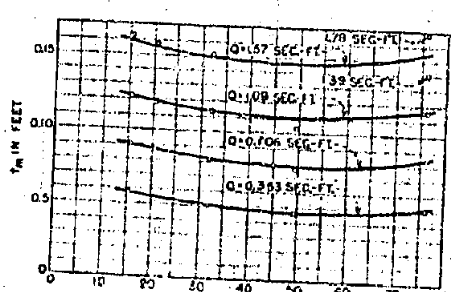

FIGURE 11 - CURVES FOR FINDING THE AVERAGE CROSS-SECTIONAL DEPTH

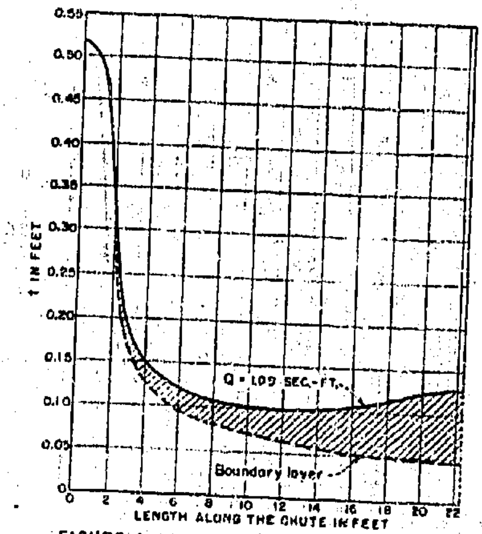

FIGURE 4 - LONGITUDINAL SECTION THROUGH THE CHUTE

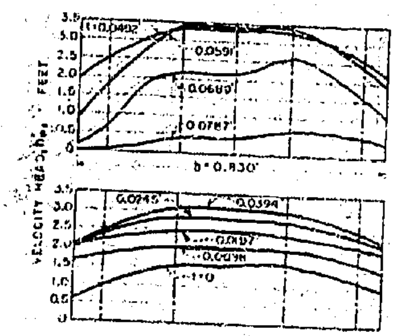

FIGURE 8 - HORIZONTAL VELOCITY HEAD CURVES

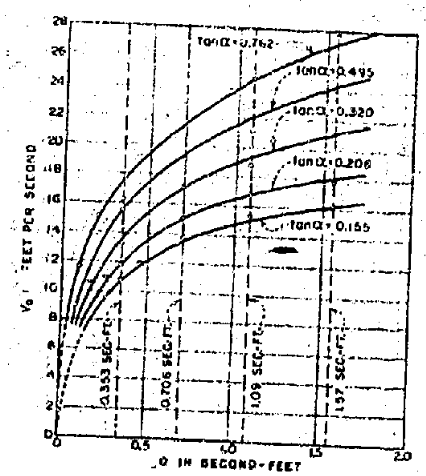

FIGURE 10 - CURVES FOR FINDING THE AVERAGE SURFACE VELOCITY

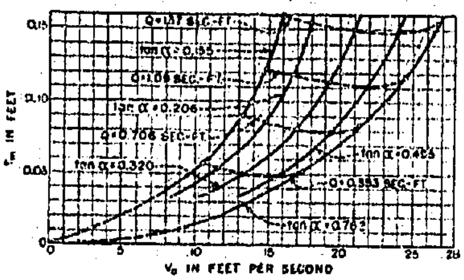

FIGURE 12 - RELATION BETWEEN t_m AND V_0

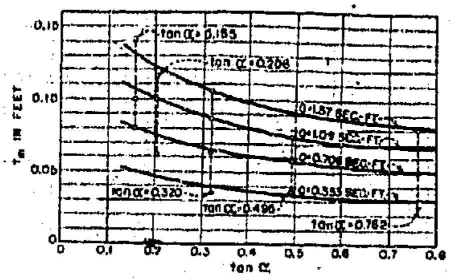

FIGURE 13 - DETERMINATION OF THE ELEVATION, t, OF THE FLOAT BOTTOM

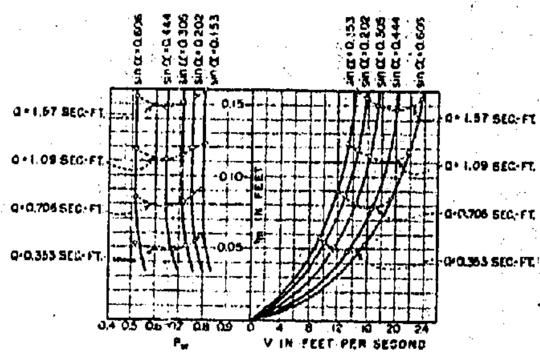

FIGURE 14 - RELATIONS BETWEEN THE AVERAGE CROSS-SECTIONAL VELOCITY AND DEPTH AND THE AVERAGE CROSS-SECTIONAL AERATION AND DEPTH

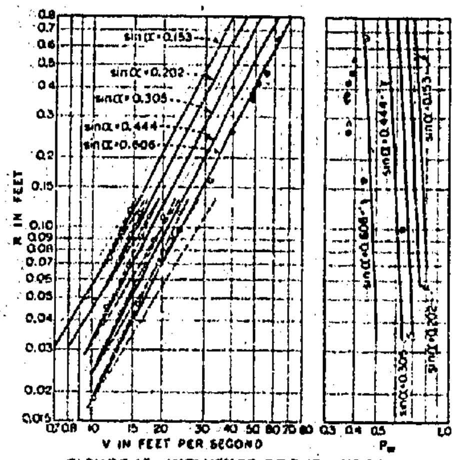

FIGURE 15 - INFLUENCE OF THE HYDRAULIC RADIUS R, ON V AND ON P_w

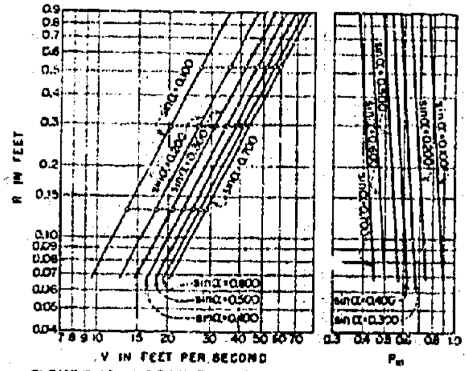

FIGURE 19 - LOGARITHMIC REPRESENTATION OF THE TWO FORMULAS, $V = f_1(R,S)$ AND $P_w = f_2(R,S)$

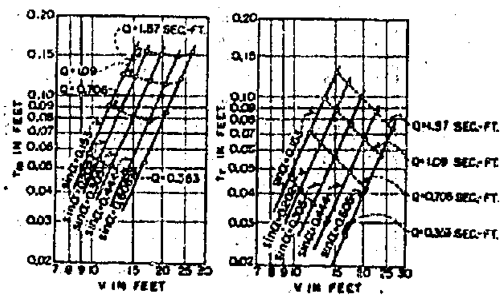

FIGURE 20 - COMPARISON OF THE RELATIONS BETWEEN t_m AND V, AND t_r AND V IN WHICH $t_r = P_w t_m$

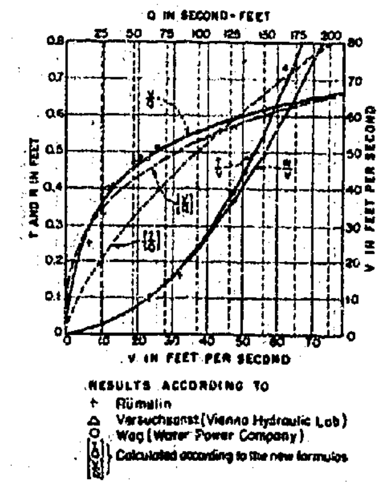

FIGURE 18 - RESULTS OF MEASUREMENTS ON THE RUTZ WASTEWAY