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SIMILITUDE IN HYDRAULIC MODEL TESTING

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FOREWARD

The material in this report was prepared for presentation at the Fourth Hydraulic Conference to be held in Iowa, City, Iowa in June 1949. It will also be included as Chapter II - appearing under the name of J. E. Warnock, Director of the Bureau of Reclamation Hydraulic Laboratory, Denver, Colorado - in a symposium volume entitled "Engineering Hydraulics," which will be edited by Hunter Rouse, Director of the Iowa Institute of Hydraulic Research, and will be published by John Wiley and Son. All rights of a proprietary or other nature have been relinquished to the Iowa Institute of Hydraulic Research.

This report, which represents a semifinal draft of Chapter II of the book, was prepared by D. T. Hebert and J. N. Bradley. Aid in the preparation of the sections on movable-bed models and tidal and wave models was contributed by Messrs. G. B. Fenwick and R. I. Hudson respectively through the cooperation of Mr. J. B. Tiffany, Jr. of the U. S. Waterways Experiment Station, Vicksburg, Mississippi.

Seven figures are incorporated in the text of Chapter II while only two, these dealing with "Conditions for Complete Similitude" and "Types of Separation" are included herein.

ENGINEERING HYDRAULICS

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CHAPTER II

HYDRAULIC SIMILITUDE

INTRODUCTION

1. Definition of similitude

Similitude, as applied to hydraulic models, goes considerably beyond the superficial aspects of geometric similarity with which it is sometimes erroneously identified. Similitude can be defined as a known and usually limited correspondence between the behavior of a model and its prototype, with or without geometric similarity. The correspondence is usually limited because it is impossible to arrange all the conditions required for complete similitude, although these conditions are known, as will be shown subsequently. The term similitude should be qualified to indicate the general limits of correspondence or one might speak of several similitudes, each of which has a definite set of limitations.

2. Role of similitude in hydraulic model testing

The practical application of similitude in hydraulic model testing is based on the recognition that there is no pure complete similitude, but rather several imperfect similitudes which can be exploited as required. Based on assumptions now well supported by experience that any given problem can be simplified into the interplay of two major forces, a pertinent similitude may be developed by theoretical means. Each of these similitudes consists of a set of transference ratios which may be applied to model findings to predict prototype behavior.

In addition to those having a rational basis, there are other similitudes which are developed by experiment in the model to be applied in special cases such as the prediction of changes in bed configuration in erodible channels. Although attempts are being made to develop a more rational basis for transference of experimental results in such cases, the current procedure is based primarily on empirical relations. In still other cases the only basis for predicting prototype behavior from model experiments is one compounded of experience and intuition. The techniques which have been mastered by men experienced in the field permit them to adjust and apply model results in a nonformalized manner for a reasonably accurate prediction of prototype action.

This view of similitude, or similitudes, stresses the concept that the value of a model is in direct proportion to the accuracy with which it demonstrates that phase of the behavior of its prototype being investigated. The design, construction, and operation of a model reflect the same view.

3. Model-prototype comparisons

Although this discussion is mainly concerned with the practical techniques of model testing to the exclusion of any attempt at defending the value of models, a brief resumé of the results of model-prototype comparisons is included for the purpose of orienting the reader in this specialized field.

Model-prototype comparisons have clearly demonstrated that, almost without exception, there is a correspondence of behavior within and usually well beyond the expected limitations. The successful operation of many structures whose designs were developed or verified by models attest the real value of this modern tool of the hydraulic engineer.

Correspondence between model and prototype has been very satisfactory and unusually complete for overflow spillway crests, valves, gates, and outlet features. It is now customary practice in such cases to provide calibration curves based on model results in lieu of actual field calibration.

Energy dissipators, including stilling basins of various types, designed on the basis of model findings have been successfully operated in substantial agreement with model indications.

River improvement plans of tremendous magnitude have worked out successfully according to predictions based on model tests.

The high efficiencies and smooth-operating characteristics of the large modern turbines and pumps can be traced to model experiments. The improvements indicated by the models were found to be real when the prototypes were constructed. To be more specific a few representative model-prototype comparisons are presented in reference (19).

SIMILITUDE REQUIREMENTS

A thorough understanding of the principles of similitude is essential for the proper design and operation of any model. Complete similitude requires that two systems be geometrically, kinematically, and dynamically similar.

1. Geometric similarity

Two objects or systems are geometrically similar if the ratios of all corresponding linear dimensions are equal. This is independent of motion of any kind and involves only similarity in form.

2. Kinematic similarity

Kinematic similarity is similarity of motion. When the ratios of the components of velocity at homologous points in the two related or geometrically similar systems are equal, the two states of motion are kinematically similar, the paths of homologous particles will then also be geometrically similar.

3. Dynamic similarity

Dynamic similarity between two geometrically and kinematically similar systems requires that the ratios of all homologous forces (including the inertial force) in the two systems be the same.

The conditions required for complete similitude may be developed from Newton's second law of motion:

$$Ma = \text{vector sum of forces} = (F_g + F_v + F_t + F_e) \quad \text{.................(1)}$$

The term on the left, Ma, represents the mass reaction to the acting forces, and hence is considered herein as the inertial force. F_g is the force imposed on a liquid by gravity, as represented by its weight; F_v represents the resultant of the viscous forces,1/ produced by shear between neighboring zones of flow. F_t is the resultant of those forces connected with surface tension 2/ which act only on the surface of a liquid; and F_e represents those forces produced by elastic compression of the fluid.

For complete similarity the ratio of the inertial or reactive forces, model to prototype, must equal the ratio of the vector sum of the active forces:

$$\frac{M_m\ a_m}{M_p\ a_p} = \frac{(F_g + F_v + F_t + F_e)_m}{(F_g + F_v + F_t + F_e)_p} \quad \text{.................................(1a)}$$

Note: Numbers appearing, such as 1/ and 2/ above, refer to the numbered references listed at the end of this chapter.

where the subscripts m and p represent model and prototype, respectively.

Satisfaction of equation (1a) requires that:

$$\frac{M_m\, a_m}{M_p\, a_p} = \frac{(F_g)_m}{(F_g)_p} = \frac{(F_v)_m}{(F_v)_p} = \frac{(F_t)_m}{(F_t)_p} = \frac{(F_e)_m}{(F_e)_p} \quad \ldots\ldots\ldots\ldots\ldots(1b)$$

Although all fluid weights and masses are proportional under the same gravitational conditions, no model fluid is known which has the requisite viscosity, surface tension, and elastic modulus to satisfy the conditions of equation (1b). Moreover, hidden in the term for viscous forces is the effect of boundary roughness which is equally difficult to adjust for complete similitude.

Experience has shown that it is not difficult to satisfy equation (1a) as it would at first appear, as one or more of the forces may not act in the flow occurrence in question, some may act only to a negligible amount or may be related to the most prominent force. In fact, for all practical purposes a particular state of fluid motion can usually be simulated in a model by considering only one of the forces on the right of equation (1a). In at least 90 percent of all hydraulic model studies, the forces connected with surface tension and elastic compression are relatively small and can be neglected safely. Of the two remaining active forces, one or the other can usually be eliminated as being of secondary importance.

4. The Froude number

When gravitational effects predominate, a pertinent basis for similitude can be established by equating the ratio of gravitational forces to that of inertia forces and neglecting the other forces in equation (1a); the following relationship is thus obtained:

$$\frac{V_m}{\sqrt{g_m L_m}} = \frac{V_p}{\sqrt{g_p L_p}} \quad \text{or} \quad \frac{V_r}{\sqrt{g_r L_r}} = 1 \quad \ldots\ldots\ldots\ldots (2)$$

where the subscript r indicates the ratio model-to-prototype. The dimensionless expression on the left of equation (2) is the Froude number, and the equality of the number in the model and the prototype is known as the Froude law. It states that the ratio of inertia to gravity forces in a model is equal to the corresponding ratio in the prototype.

The model-prototype relationships for velocity, time, discharge, etc. are obtained directly from equation (2). For example, the discharge ratio Q_r can be obtained by combining the velocity ratio with the area ratio: Since $Q = AV$, then

$$Q_r = A_r V_r = L_r^2 \sqrt{g_r L_r} = (L_r)^{5/2} g_r$$

This and other relationships are listed for reference in Table 1.

The conditions assumed in formulating the Froude law are approximated in the case of turbulent flow with a free water surface. The effects of the gravity force predominate those of viscosity, surface tension, and elasticity. Every effort is made in the design and operation of the model to minimize the effects of the latter three forces, especially friction or viscous effects. Large models and smooth boundaries are helpful in this regard. Where the friction effects can not be ignored it is possible to make compensating adjustments which will be discussed later. The Froude similitude is used more widely than all other types combined.

5. The Reynolds number

When viscous forces predominate a significant basis for similitude may be obtained by equating the ratio of viscous forces to inertia forces and neglecting other terms in equation (1a). Representing the viscous force by its equivalent.[1] $F_r = \mu L V$ the ratio becomes

$$\frac{L_r V_r}{\left(\frac{\mu}{\rho}\right)_r} = 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

Where $\left(\frac{\mu}{\rho}\right)_r$ is the ratio of kinematic viscosity of the model and prototype fluid, also designated by ν_r. The dimensionless expression on the left of equation (3) is the Reynolds number and the equality of the number in the model and prototype is known as the Reynolds law. The model-prototype relationships for velocity, time, discharge, etc. may be derived from equation (3) and are listed in Table 1.

Steady flow through a pressure conduit, or flow around a deeply submerged body, at ordinary velocities occurs under conditions to which the Reynolds law is applicable. It can be demonstrated that the gravity forces cancel and do not affect the flow pattern. The absence of a free water surface precludes the effects of surface tension. Steady flow conditions eliminate the need for consideration of elastic effects unless the velocities are unusually large compared to the celerity of a compression wave. It is impractical to completely satisfy the Reynolds law, but this does not seriously affect the usefulness of the similitude, as will be shown subsequently, in the discussion of closed-conduit models. The conditions under which the Froude and Reynolds laws must be compromised will be discussed in appropriate sections.

6. The Weber number

When surface tension forces predominate the pertinent similitude is obtained by equating the ratio of the surface tension forces to the inertia forces and neglecting the remaining terms in equation (1a). When the expression for surface tension force,[2] $F_t = \sigma L$, where σ is the tension per unit length, is substituted in equation (1a), the following ratio between inertia and surface tension is obtained:

$$\frac{\rho_r \, L_r \, V_r^2}{\sigma_r} = 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

This dimensionless parameter is known as the Weber number. The equality of the number in model and prototype is known as the Weber law.

Since the Weber law is rarely used in model testing, the relationships based on equation (4) are not included in Table 1. Those cases wherein surface tension forces dominate the motion, such as capillary waves in small channels, capillary movement in soils, etc. are seldom encountered as engineering problems and are hence considered to be special cases beyond the scope of this discussion. Surface tension effects do intrude themselves as troublesome factors in model testing, but these effects can usually be minimized.

7. The Mach number

Neglect of all forces other than those resulting from elastic compression permits the derivation of another dimensionless parameter which is variously known as the Mach, or Cauchy number. When F_e in equation (1a) is replaced by its equivalent,[2] $E L^2$, where E is the bulk modulous of the fluid, the following expression is obtained:

$$\frac{V_r}{\sqrt{\frac{E_r}{\rho_r}}} = 1 \quad \text{...} \quad (5)$$

The dimensionless parameter $\frac{V}{\sqrt{\frac{E}{\rho}}}$ is known as the Mach number and its required equality in model and prototype is called the Mach law.

Except for cases of unsteady flow, especially water-hammer problems, the similitude based on the Mach number has little application in hydraulic model testing. Since most water-hammer problems yield readily to analytical methods, moreover model studies are rarely needed. On the other hand, aerodynamic testing has led to an extensive development in the use of the Mach law, to deal with problems involving the flow of gases at velocities exceeding the speed $\sqrt{\frac{E}{\rho}}$ of sound, and water-entry problems of ballistics have recently brought the analysis of liquid flow to the same stage.

8. Other dimensionless numbers

There are two additional parameters which are useful in hydraulic model testing. These are the cavitation number and the Karman number. The cavitation number serves as an index by which the experimenter can predict the effects of cavitation in the prototype. The Karman number offers a means of establishing the same type of turbulent flow in the model as exists in the prototype and is useful in the design of river models. These numbers will be discussed in the sections to which they specifically apply.

For a more comprehensive treatment of hydraulic similitude, reference (3) is recommended.

TABLE 1

Characteristic	Dimension	Scale ratios for the laws of Froude	Reynolds
Geometric properties			
Length	L	L_r	L_r
Area	L^2	L_r^2	L_r^2
Volume	L^3	L_r^3	L_r^3
Kinematic properties			
Time	t	$\left(\frac{L\rho}{\gamma}\right)_r^{1/2}$	$\left(\frac{L^2\rho}{\mu}\right)_r$
Velocity	Lt^{-1}	$\left(\frac{L\gamma}{\rho}\right)_r^{1/2}$	$\left(\frac{\mu}{L\rho}\right)_r$
Acceleration	Lt^{-2}	$\left(\frac{\gamma}{\rho}\right)_r$	$\left(\frac{\mu^2}{\rho^2 L^3}\right)_r$
Discharge	$L^3 t^{-1}$	$\left[L^{5/2}\left(\frac{\gamma}{\rho}\right)^{1/2}\right]_r$	$\left(\frac{L\mu}{\rho}\right)_r$
Dynamic properties			
Mass	M	$(L^3\rho)_r$	$(L^3\rho)_r$
Force	MLt^{-2}	$(L^3\gamma)_r$	$\left(\frac{\mu^2}{\rho}\right)_r$
Density	ML^{-3}	ρ_r	ρ_r
Impulse and momentum	MLt^{-1}	$\left[L_r^{7/2}(\rho\gamma)^{1/2}\right]_r$	$(L^2\mu)_r$
Energy and work	$ML^2 t^{-2}$	$(L^4\gamma)_r$	$\left(\frac{L\mu^2}{\rho}\right)_r$
Power	$ML^2 t^{-3}$	$\left(\frac{L^{7/2}\gamma^{3/2}}{\rho^{1/2}}\right)_r$	$\left(\frac{\mu^3}{L\rho^2}\right)_r$

MODEL TECHNIQUES

Model techniques can be defined as the practical procedure, based on theory and experience, by which the similitude relationships are applied in the design and operation of hydraulic models and the interpretation of experimental results. A model study will yield valuable information if the accepted techniques are applied with care, patience, and accuracy. The following discussion deals with general procedures. Special techniques will be discussed under specific types of models.

1. Planning and construction of models

The first step in designing a model is the selection of a scale such that the similarity of action being studied is preserved in the model. Customary practice is to start with geometric similarity unless there is some definite purpose to be served by using a distortion. The scale of a model usually refers to the scale ratio of linear dimensions. Economy considerations dictate that the model be as small as possible and still yield valid results, but the burden of proof that the results of a small model are valid can be troublesome and time consuming. Current practice is to follow precedent, when available, and to err on the side of overly large models, so far as this is permitted by the limitations of available space and water supply.

The foll'wing ranges of scales have been used successfully in many model studies and are recommended for use as a point of departure in the determination of scales in any given case. Spillways for large dams may be studied on models with scale ratios (geometric) in the range from 1:30 to 1:100. Models of canal structures, valves, and gates with scales of 1:5 to 1:25 have proved to be adequate.

There is a minimum size for each type of model. Models of valves, gates, or conduits in general should be larger than 4 inches in diameter; models of canal structures should exceed 4 inches in average width; and spillway models should be scaled so that normal heads over the crest exceed 3 inches. When models smaller than these sizes are used it is difficult to build them to say nothing of the possibilities of introducing similitude defects. Valves and gates to be used in development experiments, or for rating of prototype control devices should be at least 6 inches in diameter (pipe size).

Distinct economies may be realized by choosing model scales so that standard pipe can be used for tunnel and conduit models. Parts of discarded models, such as gates and valves, may often be used to advantage. The odd scale rations which may result from such practices are not objectionable.

The data for planning a model should include an over-all plan for the proposed or existing structure, sufficient detail to determine the shape and character of all surfaces over which the flow will pass, and a complete description of proposed operating conditions, with a water-stage discharge curve for the river. For certain types of models the topography of the site and surrounding area, the results of foundation test borings, and the location and details of any other structures, particularly hydraulic structures, may also be necessary. The problems to be investigated should be thoroughly discussed with the designing engineers familiar with the project. Tangible information and background may often thus be obtained which will prove helpful in solving the hydraulic problem. For example, in the development of a particular outlet works and stilling basin it may be permissible to change the length and depth, but because the training walls also act as footings for the powerhouse, the width must remain unchanged.

Accurate and precise model drawings prevent time-consuming errors in construction. These drawings may vary from brief sketches, for more or less routine construction, to minutely detailed drawings of the more critical portions of the models. Since it is necessary to obtain greater accuracy when building small models then ordinary craftsmen are accustomed to produce careful inspection should be maintained to insure that the required tolerances are met, expecially in critical parts of the model.

In general, the greatest accuracy is needed where rapid changes in direction of flow occur, and in locations where velocities are relatively high. The structure of the model should be made sufficiently flexible to allow for considerable modification with a minimum of rebuilding. For instance, a model with a spillway basin set on or close to the laboratory floor will require extensive rebuilding if the basin is to be lowered. Provisions in the original plan for lowering the basin will eliminate major rebuilding and make the lowering a routine adjustment.

The model need not be made strictly like the prototype. If the surfaces over which the water flows are reproduced in shape, the model will usually serve its purpose. Model spillways may be sheet-metal faces over framing, or may be solid concrete. River beds or canals may be made of cement mortar on metal lath, or of properly arranged sand and gravel molded to carefully adjusted templates. Ease of construction and results desired will affect the choice of methods for constructing the model which are treated in detail in reference (3).

The stage of the testing program will often determine the necessary accuracy of construction. In early tests, where many schemes are being studied to determine the over-all feasibility, construction need not be as finished as in the final stages where the data obtained are to be used for design and operation of the prototype structure.

Instruments are necessary adjuncts to model testing. Their proper installation and use cannot be overemphasized. Comparison of measurements constitutes the deciding factor in many hydraulic designs. Provision should be made for suitable instrumentation while the model is in the design stage. Piezometers are relatively easy to install during initial construction whereas extensive remodeling may be necessary to accomodate them after the model is completed. The number and location of measuring sections and piezometers should be carefully planned so that the measurements will completely define the action being studied. Piezometers are cheap, and should be provided in generous numbers to offset any oversight in defining the critical points. It is important that piezometer openings be flush with the surface and be smoothed inside and out to eliminate burrs which can cause erroneous readings.4/

2. Utilization of distorted scales

Distorted scales are used when a distortion serves some definite objective and the results are limited to this objective. Geometric distortion is usually required in models of river channels, floodways, harbors and estuaries, where the horizontal dimensions are large in proportion to the vertical ones. In such cases, the horizontal scales are limited by space and cost restrictions. When these scales result in model depths and slopes which are too small to yield significant results, a vertical exaggeration or a distorted vertical scale is required. Many valuable studies have been made with distorted models.

(a) Some of the advantages of geometrically distorted models are:

(1) Sufficient tractive force can be developed in the model to produce bed-load movement with a reasonably small model.

(2) Water-surface slopes and wave heights are exaggerated and therefore easier to measure.

(3) Exaggerated slopes act to offset the fact that the frictional resistance of a model is usually too large.

(4) Operation is simplified by the smaller model.

(b) Certain disadvantages which are inherent in distorted models are:

(1) Velocity and energy distribution may be seriously distorted.

(2) Slope of cuts and fills are often too steep to be molded in sand or erodible material.

(3) A model wave may differ in type and possibly in action from that of the prototype.

(4) There is an unfavorable psychological effect on the observer who views distorted models.

3. Model operation

In most model problems the operating program is obvious or has been determined before the model was built. In fact, the basis for the test is usually an operating problem of one kind or another. The program should be carefully planned to be sure that provision has been made for all contingencies. On an undistorted model the operation program can be divided into two phases; adjustment and testing. In the case of a distorted model, especially if it is the movable-bed type, an item of verification must also be considered in the adjustment phase.

Adjustment of the model consists of preliminary tests whose purpose is to reveal any defects in the model and to establish its adequacy. This phase of operation should not be hurried because it is important and economical to spend whatever time is required to establish that the model behaves as intended and that the instrumentation is satisfactory. Usually several minor changes in model design are indicated, such as: the shifting of or addition to the measuring equipment, the change in erodible material, or a partial redesign of a model component. During the preliminary testing the experimenter becomes acquainted with the idiosyncrasies of the model. In addition, it is often possible to eliminate obviously inadequat designs in the preliminary tests. When the model is of the movable-bed type, the adjustment phase involves verification tests in which the recorded action of a prototype is duplicated in the model. Because such tests are often the only basis for similitude, verification is an extremely important part of model operation. It will be discussed more fully under the topic of movable-bed models.

Testing consists of the systematic operation of each proposed design during which the experimenter must be continually on the alert to improve the design, reduce the cost of construction of the prototype, or both. It is important that the experimenter use care, patience, ingenuity, and imagination, and that he be able to interpret the model results correctly. It is wise for him to work in close cooperation with the designing engineers, since keeping all interested parties informed will result in more ideas for the determination of a definite and satisfactory test program. The test data should be analyzed concurrently with the testing to eliminate unsound results. Such an analysis also makes it possible to reduce the amount of data necessary to a solution. When possible, functional relationships among the different variables should be applied to the data to aid in spotting erroneous measurements.

Since the end product of any model study is the report transmitting the findings and recommendations, it is essential that the experimenter maintain a complete and accurate set of notes, including a diary. Negative as well as positive results must be recorded. A complete photographic record of all important tests is indispensable to a convincing report. In addition a complete set of photographs may many times eliminate the necessity of repeating a series of tests.

4. Limitation and interpretation of results

No matter how carefully a model is designed and constructed, it does not contribute an automatic solution, but provides data and information which requires intelligent interpretation based upon the experimenter's knowledge of basic mechanics and hydraulics as well as upon his experience.

Interpretation of model results in terms of the prototype, within the limitations of the similitude prevailing, is the most critical phase of the model study. While any model serves as a vehicle for demonstration of fundamental principles that apply equally well to a prototype, any direct translation of results must be accomplished with restraint. A model is designed and operated according to a similitude which is necessarily limited, and the limitations must be respected in the prediction of prototype behavior. For example, in using the Froude similitude which ignores the effect of viscosity any action involving frictional resistance should not be scaled to prototype terms without adjusting for the distorted model friction. The various expedients by which similitude defects may be adjusted in interpreting model results will be discussed under appropriate topics.

CLOSED-CONDUIT MODELS

Closed-conduit flow is distinguished by the absence of a free water surface or at least by the absence of the effects of a free water surface. The latter condition prevails in the case of flow around a deeply submerged body. The absence of a free water surface introduces an appreciable simplification into similitude considerations. The forces of gravity are everywhere balanced and surface tension is not involved. There remains only the viscous and inertia forces, and in the case of unsteady flow, elastic forces. In steady closed-conduit flow the Reynolds law specifically applies. If two systems have similar boundaries of the same relative roughness, the flows will be similar in every detail when the Reynolds numbers are equal. In most cases, it is impractical, if not impossible, to arrange equal values of Reynolds number but fortunately a rather complete similitude can be obtained by approximating the equality.

1. Representation of prototype resistance

The most difficult problem in closed-conduit models is that of accomplishing a true representation of prototype resistance. The effects of fluid resistance in models of closed conduits are best presented by referring to a Reynolds number--friction coefficient--graph. Flow along a boundary may take one of three forms: (1) laminar, (2) smooth-wall turbulent, and (3) rough-wall turbulent. The pronounced difference in the resistance law for each type of flow necessarily influences model procedure.

When the Reynolds number of a prototype is less than 2000, as it may be in the case of a small conduit carrying a relatively viscous fluid like oil, the motion will be wholly laminar. If such motion is both steady and uniform, inertial forces will not be involved, and complete similitude will be attained under all conditions. If the flow is either unsteady or non-uniform, however, inertial forces become important, and the model should be designed to have the same Reynolds number as its prototype. The similitude relationships for such a study, which derive from the Reynolds law, have been listed in Table 1.

In the majority of model studies the Reynolds number of the prototype will range between values of 1,000,000 to 20,000,000, which is well beyond the transition zone between laminar and turbulent flow. In this Reynolds number range the roughness of the prototype boundary is a controlling factor, in that it determines which of the two types of surface resistance will prevail. Complete Reynolds similitude requires not only that the resistance coefficient of model and prototype be equal but as well that the type of resistance be the same. When the prototype boundaries are hydraulically smooth the model boundaries are likewise made smooth, but equal resistance coefficients will be obtained only if the Reynolds numbers are the same. If the Reynolds number of the model is lower - as is often necessarily the case - the resistance will be disproportionately high in the model and must be adjusted according to theory when prototype behavior is predicted. When the prototype boundaries are relatively rough, on the other hand, the mode of flow will generally be rough-turbulent, which is characterized by a constant resistance coefficient equal to that of the prototype - for the same relative roughness - and the results may be transferred without reservations.

When the required value of $\underline{R}$ cannot be obtained in the model, which is the usual case, the following expedient may be used. The resistance coefficient of the prototype can be duplicated in the model by making the model relatively smoother than the prototype. This equality will only exist for a particular discharge or value of $\underline{R}$, because the model will have a mode of flow differing from that of the prototype. For all other discharges the resistance coefficient will be higher or lower than that of the prototype, and the data will hence require adjustment before transference to the prototype.

Although it is highly desirable to arrange a close representation of prototype resistance in the model, experience has shown that the value of R for the model exceeds 1,000,000 the influence of the errors caused by a minor discrepency in frictional resistance will not materially affect the validity of the model results. In most model studies of engineering problems involving gates, valves, transitions, etc. the inertial effects or those resulting from changes in magnitude or direction of velocity, clearly dominate the friction effect. The exceptions usually involve long lengths of conduit wherein the difference in friction coefficients results in serious discrepancies. Usually arrangements can be made to offset such a condition. For example, in a model test of an outlet including an entrance, a conduit, and a valve or gate at the end of the line, the problem usually requires detailed study of the entrance and valve. As the entrance is short, the frictional resistance is a minor factor and may be safely ignored. The distorted friction loss in the conduit may be compensated by reducing the roughness or shortening the length. Friction effects in the valves will be negligible. Another way of approaching such a problem is to regard the conduit as an appurtenance for the sole purpose of providing proper approach conditions to the valve. Then all data on the valve performance are related to the conditions at its entrance. For example, the discharge coefficient will be based on the total head one diameter upstream from the valve. When the prototype discharge is predicted the pertinent head will be computed on the basis of known or computed prototype resistance. Pressure data may also be expressed in percent of the total head one diameter upstream from the valve. These percentages which are dimensionless may be applied directly to prototype conditions.

2. Separation effects

Under conditions in which the flow separates from the boundaries, the geometry of the boundaries loses its primary significance and similitude is largely dependent on the dynamic action of the fluid. Separation 2/, 5/ may occur when: (1) there is an abrupt change or discontinuity of a boundary; (2) velocities exceed those required for incipient cavitation; and (3) the boundary layer expands in the face of an adverse pressure gradient.

In the first case the point of separation is fixed and will be similar in model and prototype and the eddy formation in the model will not be too dissimilar from that of the prototype if the Reynolds numbers are in the same order of magnitude. Separation due to cavitation is a special case, and the significant parameter for similitude is the cavitation number.

Adverse pressure gradients--those wherein the pressure increases in the direction of flow--occur whenever diverging boundaries or curvatures are involved. Unless the boundary layer is sufficiently turbulent to withstand the adverse pressure gradient, separation will occur. Conditions which determine the location of a point of separation are complex for they include the geometry and roughness of the boundary, the Reynolds number and the turbulence of the flow. Moreover a pronounced shift in separation point often occurs when the boundary layer changes in type from laminar to turbulent. If a model is operated at velocities high enough to insure boundary layer turbulence the distortion in the location of the point of separation will be small. In the case of flow around an obstacle the boundary layer will generally be turbulent at all values of Reynolds number above 500,000. Only in the rare case when the relative roughness and the values of Reynolds number are equal in model and prototype is there definite assurance that the separation point will be similarly located.

It is good practice to vary the velocities over a wide range in a model to establish that a turbulent layer actually prevails. This can be detected by an increase in pressure when the layer shifts from turbulent to laminar.

3. Cavitation studies

Cavitation in a hydraulic structure [6] resulting from the existence of pressures approaching the vapor tension of the fluid at some point in the flow may cause: (1) damage by pitting, (2) serious vibration, and (3) marked changes in pressure distribution due to separation. The particular effect of cavitation being studied determines the type of model test. In general, there are two techniques: (1) operation of model at atmospheric pressure; and (2) operation at scaled atmosphere, or reduced pressure.

When the problem involves the prediction or elimination of cavitation erosion, the first technique is suitable. Detailed measurements of pressure distribution in the model will reveal subatmospheric pressures which, scaled to prototype terms, may approach vapor tension. This may be regarded as evidence of cavitation in the prototype. For example, tests of a model valve indicate subatmospheric pressures of 5 feet of water at a model head of 50 feet. If the prototype head is 300 feet, subatmospheric pressures of 30 feet--approaching vapor pressure--may be predicted and cavitation is possible in the prototype. In practice the model shape would be revised to eliminate the subatmospheric pressures, or in special cases vacuums not to exceed one-half atmosphere in the prototype are tolerable. Negative pressures greater than a vacuum of one-half atmosphere are considered to be in the realm of potential cavitation since irregularities of the boundary may create local reductions in pressure and thus local cavitation. In this type of testing the location of piezometers is of major importance. They must be located so that all critical areas will be delineated. In general, the piezometer openings should be located in expanding sections, in curved surfaces, and downstream from discontinuities.

The vibration and energy loss effects of cavitation can be studied best in reduced pressure models. Tests are performed in water tunnels 7/ or any closed system wherein the pressure at the test section can be maintained at any desired degree of vacuum. Such a test facility makes it possible to duplicate cavitation conditions in the model by arranging equal values of the cavitation number in model and prototype. The cavitation number may take many forms the most useful of which is:

$$\underline{K} = \frac{h_o - h_v}{\frac{V_o^2}{2g}}$$

where

h_o is the piezometric head at some reference section, x;

h_v is the piezometric head corresponding to the vapor pressure of the model fluid at ambient temperature; and

$\frac{V_o^2}{2g}$ is the reference velocity head.

For the same values of K in model and prototype, the pattern of cavitation--and hence the flow efficiency--will be the same. Since the frequency of vibration and the rate of pitting will vary in proportion to the velocity, however, and since the physical properties of the boundary materials will also be involved, such factors must be given proper consideration in predicting the structural behavior of the protytype from model tests.

4. <u>Problems of unsteady flow and compressibility effects</u>

Problems in which compressibility of the liquid is important are seldom encountered in the hydraulics except in cases of unsteady flow. For example, design and operation of closed-conduit systems, such as pumping plant effluent lines, power stations, and surge tanks, involve knowledge of the elastic behavior of liquid and conduit under transient conditions, and detailed model studies of such problems must be based on the Mach law of similutide. However, most of these problems can be handled adequately by analytical or graphical methods and model studies are seldom necessary. When they are required, usually in conjunction with analytical studies, only qualitative findings are involved. Under such conditions the problem is usually reduced to one of steady state and the laws of similitude previously discussed are applicable. For example, the problem of eliminating slamming of flap gates on pump and discharge lines may be worked out on a model operated according to the Froude law without representing the true pump and conduit characteristics. Demonstration in the model of the causative factors is usually an adequate basis for developing antislam devices such as dash pots or air-injection equipment.

OPEN-CHANNEL MODELS

The technique of open-channel models is complicated by the presence and effects of an open water surface which is free to change in position and shape. Usually little is known about the free surface except that it is a surface of constant pressure comprising one of the flow boundaries. The dominating force is that of gravity or weight, and thus the Froude law of similitude is pertinent. The solution of an open-channel problem involves the application of the Froude law with adjustment or limitations and often requires scaled distortions to accommodate the effects of the other forces.

Problems in open-channel flow involving hydraulic structures generally consist of three parts: (1) overflow and underflow sections, (2) transitions, and (3) energy dissipators, any or all of which may be included in a single model. It is customary to carefully preserve complete geometric similarity and model heads are adjusted to the values required by the Froude law. The testing technique as well as the interpretation of model results varies somewhat for the different parts of a structure each of which will be discussed separately. Problems of river hydraulics, on the other hand, involve the prediction of stage-discharge relationships, scour and deposition, and the passage of floods, and can seldom be studied with undistorted models. Although the Froude law is still the basic similitude criterion, surface roughness and viscous resistance must be given careful consideration. Each of these factors will receive special attention in the last three articles of this section.

1. Overflow and underflow structures

The most common types of overflow and underflow structures are spillways and outlet works for dams, diversion dams, sluiceways, canal intakes, wasteways, chutes, checks, and drops. These structures may vary greatly in size and shape but all embody a control over or under which the flow changes from less than critical to greater than critical velocity. The model boundaries generally consist of planed wood, sheet metal, or smooth concrete which are acceptable as an approximation to the required smoothness. With scale ratios of 1:30 to 1:60, the effects of distorted friction in the model are small in comparison to the inertial effects and the results are transferable with few reservations. Water-surface profiles, pressure distribution and velocities closely resemble those of the prototype for both free and submerged flow. Rating curves obtained from model calibrations are valid for prototype use provided the model control section conforms to the previous recommendations as to size. More precise calibrations may be obtained with enlarged sectional models. Such models, which may have a scale ratio of 1:5 to 1:10, should be at least 2 feet in width and the approach conditions should be arranged with care. The best shape of overflow profile as well as the water-surface profile may also be obtained with a higher degree of accuracy in such a model.

When the problem involves overfall shapes with high coefficients and subatmospheric pressures, the existence and effects of adverse pressure gradients likely to cause separation should be investigated. The procedure for this purpose follows that previously discussed in conjunction with closed-conduit models. The model should be sufficiently large, possibly a sectional model, to insure valid boundary layer conditions.

In addition to the physical measurements, the general appearance of flow should be carefully noted as it will often indicate the need for revision not apparent in the physical data. Poor approach conditions, excessive end contractions, and improper pier design, which are usually obvious, may create disturbances which affect the operation of the entire structure. Small surface disturbances which in the model appear to be attached to the main sheet of water may in the prototype become detached and result in excessive spray formation which will influence the location of gate pins and counterweights. Allowance for this scale effect, which cannot be placed on a rational basis, is dependent on the operator's experience and engineering judgment.

2. Transition structures

Transition structures are designed to accomplish changes in flow from one set of conditions to another. They may consist of changes in elevation such as in chutes or superelevated channels or of changes in plan such as in canal width and depth. Methods and information for the design of transition structures are noticeably deficient. For this reason models of transitions are truly design tools, especially for supercritical flow. For example, models supply reliable data for the design of superelevated channels and chutes with converging or diverging training walls. The model data on water-surface profiles and pressure distribution may be transferred to the prototype by the Froude relationships.

When unusually large divergences are involved the influence of the distorted boundary layer on separation should be investigated by the methods previously discussed.

In long flat transitions a distortion in slope or length may be required to offset the disproportionately high friction resistance of the model. The distortion can be accomplished on the basis of computed losses in model and prototype as will be explained in conjunction with fixed-bed models.

When the prototype velocities are high, as in the case of a long steep chute or spillway, the sheet of water entrains appreciable percentages of air which increases the depth of flow. This phenomenon which is known as insufflation is not duplicated in the ordinary model based on the Froude similitude. Special models wherein high velocities are initiated by pressure flow under a gate discharging in the model chute may be used to accomplish insufflation for qualitative studies.[18]

3. Energy dissipators

The term energy dissipator is used in a broad sense to include all types of stilling basins designed to protect the main structure from damage by the fast-flowing water. Although the purpose of a stilling basin is to protect the main structure the basin itself should not be considered expendable. The conventional method of evaluating the performance of a stilling basin design is by the general appearance of the action. This is especially true for preliminary designs. The presence of extreme turbulence, waves, high-exit velocities, high-velocity eddies or pulsating flow in the downstream portion of the structure all indicate the need for refinement of design or abandonment of the scheme. Each design should be tested over the complete range of discharge and tailwater conditions. Basins that do not have a fairly wide range of permissible tailwater are not practical, ordinarily, since many unknown variables such as degradation of stream beds or power-plant discharge are not reflected in the preparation of the tailwater curve. Refinement in designs of stilling basins may be evaluated by comparing measured velocities, wave heights, pressures, and erosion of the downstream river bed. The general operation, preferably recorded on photographs, will also help to evaluate the effectiveness of a particular design.

Wave and surge heights are particularly important where the discharge is passed directly from the stilling basin into an unlined canal since the waves have a destructive effect on the sloping banks. Maintenance costs can be reduced considerably if waves and surges can be held to a minimum. In such cases it is often desirable to record wave heights and periods by motion pictures, oscillograph records, or other instantaneous methods.

The most impartial criterion available with which to judge stilling basin behavior is that of comparative erosion. It may be made either in a qualitative or semiquantitative manner. For preliminary tests the qualitative method is usually sufficient, but before recommendations are made for a final design it is often advisable to investigate erosion more carefully by a semiquantitative method.

To make qualitative erosion studies, a readily erodible sand or other material is used to represent the channel downstream from the stilling basin. The measured depth and extent of erosion indicates the relative effectiveness of the basin under test when compared with erosion patterns from other basins if the same movable-bed material is used. Whether or not the erosion pattern represents the depth of erosion to be expected in the prototype depends upon the choice of model-bed material and the duration of the test. Regardless of the choice, however, the model will indicate the erosion tendency if enough time is allowed. The model is operated for a time interval long enough to develop measurable erosion. This time interval, which has no fixed relation to the time scale of the Froude similitude, is used in the evaluation of alternate designs.

A semiquantitative method for evaluating the depth and extent of prototype erosion utilizes a stiff mixture of sand, cement, and water as the bed material. The resistance of the material, as determined by samples in a small flume, can be varied by changing the proportions of sand and cement. By trial and error a mixture is obtained which begins to erode at a model velocity corresponding to the estimated eroding velocity of the prototype. The model erosion with a bed of this mixture will approximate the prototype erosion to an extent which depends on the accuracy of the estimate of eroding velocity for the prototype. A bed mixture consisting of 50 to 100 parts of sand to one part of Lumnite cement will produce a desirable semistable material. It is important, however, that the mixing operations be prolonged (approximately 10 times that for regular concrete) to insure a uniform texture. As the Lumnite cement gives approximately 28-day strength in 24 hours little change in strength will be evident in the bed material after that period.

4. Fixed-bed channels

Problems involving relatively long stretches of either a canal or a river, wherein actual changes in bed configuration are not critical, are usually studied in fixed-bed models. The influence of wall friction is generally of major importance in such problems and cannot be ignored or adjusted when the results are interpreted.

Unless an unusually large model is involved a distortion in slope is required to: (1) offset the disproportionately high friction; (2) to obtain a sufficiently high value of Reynolds number which will insure turbulent flow; or (3) to accommodate the model in the available space. In the first case the vertical exaggeration is designed to compensate model friction. In the other cases extra or artificial model roughness may be required to compensate the exaggerated model slope.

The required distortion of slope can be computed from the Manning equation:[3]

$$V = \frac{1.486}{n} R^{2/3} S^{1/2}$$

Where V, R, and S represent velocity, hydraulic radius, and slope, respectively, and n is the Manning roughness coefficient. In order that the velocity ratio be the square root of the depth scale as required for inertial effects according to the Froude similitude,

$$V_r = \sqrt{D_r} = \frac{R_r^{2/3} S_r^{1/2}}{n_r}$$

Substituting $\frac{D}{L}$ for S results in the expression:

$$\sqrt{D_r} = \frac{R_r^{2/3} D_r^{1/2}}{n_r L_r^{1/2}} \quad \text{or} \quad \frac{D_r}{L_r} = \frac{n_r^2 D_r}{R_r^{4/3}}$$

If n is known for the model and prototype then n_r is known and the exaggeration $\frac{D_r}{L_r}$ can be computed for a given depth D and hydraulic radius R. In models where the slope distortion is dictated by other considerations an adjustment of model roughness is required to duplicate prototype conditions. If the distortion and the value of n for the prototype are known, the required value of n for the model can be computed from the equation. Unless basic data are available in a particular laboratory on the values of n for different types of roughness, the required roughness must be obtained by trial and error. The model with roughness adjusted for a particular depth will yield dependable results for flow at or near that depth. If a problem involves several depths, model roughness should be adjusted to give an average friction that is approximately right for each depth or the roughness may be varied with depth for a closer approximation at all depths. To be noted in this connection is the fact that the Manning η is strictly a roughness characteristic, and that the Manning formula hence applies only at sufficiently high values of R for viscous effects to be negligible.

5. Movable-bed channels

There are many open-channel problems involving scouring, deposition, and transportation of channel-bed material through the action of flowing water. Such problems are usually studied by means of movable-bed models; however, in some cases limited studies of problems of this type can be made by investigating current directions and velocities in fixed-bed models. Despite the limitations of the similitude attainable with movable-bed models, they have proved to be invaluable aids in the solving of complex problems involving the shifting of stream-bed materials. Similitude in movable-bed models defies the mathematical analysis which can be applied to models involving hydraulic structures or other fixed-boundary studies. Instead of arranging the various hydraulic forces involved to meet definite requirements laid down in any law of similitude the successful prosecution of a movable-bed model study requires that the combined action of the hydraulic forces bring about similitude with respect to the all important phenomenon of bed movement, which is the essence of this type of model study.

The general design approach is that of selecting scales and bed material which will result in bed movement of a nature generally similar to that in the prototype, taking into account the relative effect of the various discharges from minimum to maximum. There are two prerequisites to such a design approach: first, a thorough knowledge of the characteristics of the prototype based upon the collection and study of hydraulic and hydrographic data; and second, experience in the field of movable-bed hydraulic models.

When the dimensions of a watercourse are scaled down to model dimensions the reduced hydraulic forces are no longer sufficient to move bed material of a workable size (unless very large models are used, which is ordinarily not practicable due to considerations of cost, available space, water supply, etc.), and it is usually necessary to distort the linear scale ratio. Distortion of the scale ratio means that the vertical scale ratio is made larger than the horizontal scale ratio, resulting in exaggeration of vertical dimensions, and thus all slopes in the model. Distortion (vertical scale divided by horizontal scale) may vary from two or less to as high as seven, or even more in special cases. Generally, distortion should be kept as low as possible without reducing bed movement too much.

Although vertical exaggeration is usually necessary from the standpoint of obtaining sufficient bed movement, it does introduce certain undesirable effects which have been listed previously but which warrant additional discussion at this point. The exaggeration may increase the slopes of the model banks beyond their angle of repose so that they will no longer stand. This condition may be avoided, in cases where permanence of the banks can be assumed, by forming them in a rigid material. Distortion also increases the longitudinal slope of the stream, and thus tends to upset the flow regimen to a point where artificial model roughness is required to restore it. However, since channel-bed roughness is a function of the model-bed material, which is not amenable to adjustment, a distortion in the discharge and velocity scales would automatically result. Such a distortion away from the dynamical ratio of the square root of the vertical scale makes the model invalid with respect to water-surface curvatures in general. Another effect of vertical exaggeration is a distortion of the lateral distribution of velocity and kinetic energy. Therefore, when the problem to be studied involves this distribution as an important factor (as would be the case when modeling the confluence of two streams, channels split by islands, or even very sharp bends) the distortion must be restricted to not more then four or five unless the prototype channel is unusually wide for its depth. In recent model studies, the difficulties of compromising resitance and bed movements have been minimized by using various model-bed materials of lower specific gravities so that less scale distortion is required to produce proper movement. Among such materials are coal, pumice, sawdust, and ground plastics.

Instead of recommending definite scale ratios for this type of model a few representative examples are listed:

<u>Table 2</u>

Movable-bed model studies made at United States Waterways Experiment Station

(a) Boston Bar vicinity, Mississippi River.

<u>Horizontal</u> scale 1 to 600. <u>Vertical</u> scale 1 to 100.

<u>Bed material:</u> Processed coal, specific gravity 1.3, average grain, diameter 0.8 mm, and gravel areas represented by haydite.

(b) Pryors Island Reach of Ohio River.

<u>Horizontal</u> scale 1 to 600. <u>Vertical</u> scale 1 to 150.

Supplemental slope of 0.00015 added to obtain correct tractive force.

<u>Bed material:</u> Crushed coal, mean diameter 1.23 mm, and specific gravity 1.30.

<u>Other studies</u>

(c) Rhone River study, shoaling study in vicinity of River Yseron.*

<u>Horizontal</u> scale 1 to 250. <u>Vertical</u> scale 1 to 50.

<u>Bed material:</u> Very clean quartzy sand, mean diameter 0.295, sand chosen from considerations of time, delivery, and price.

Supplemental slope added.

Many other examples of movable-bed model studies may be found in published reports of the Waterways Experiment Station, Corps of Engineers, Vicksburg, Mississippi, and reference (8).

The correct model discharge scales required to maintain correct depth for the various stages of flow must be determined experimentally in the model itself, as must the time scale. The technique of experimental determination of scales, other than physical scales, will be discussed in more detail subsequently in connection with the model verification.

Construction methods for movable-bed models are generally similar to those used in the case of open-channel fixed-bed models. The principal difference is in the technique of molding the bed. A common procedure is that of providing curb rails which are roughly parallel to the model channel and are set at any convenient datum. Templets mounted on cross-rails and supported on the curbs provide the control for molding the bed. The curbs also provide a means for supporting a sounding board which is used to survey the model beds. A portion of the model channel at each end should be molded in rigid material in order to reproduce correct entrance and exit guides for the model flow.

In addition to the usual model equipment, a bed-load-feeding apparatus should also be provided at the channel entrance to supply bed load to the model. The rate of feeding should be adjustable so that it can be varied as required to accomplish proper duplication of prototype action. Special methods and devices to aid in mapping model beds are discussed in reference (3).

The verification of a movable-bed model is an intricate cut-and-try process of progressively adjusting the various hydraulic forces and varying the model operating technique until the model will reproduce with acceptable accuracy changes in bed configuration which are known to have occurred in the prototype between certain dates. In this manner the accuracy of the functioning of the model is established and certain of the scale relationships with the prototype are worked out. This procedure usually consists, in general, of the following steps: (a) two prototype bed surveys of past dates are selected--the time between these two dates being known as the verification period--and the model bed is molded to conform to the earlier survey; (b) the hydraulic phenomenon which occurred in the prototype during the verifications period are simulated in the model to the proper time scale (estimated, to begin with) all of regulated measures undertaken in nature during that period being reproduced in the model at their proper time; and (c) the model bed is surveyed at the end of the period, and the model is considered to be satisfactorily verified only when this survey checks the later prototype survey with acceptable accuracy.

During cut-and-try verification tests it may be found necessary to manipulate the time scale, discharge scale, rate and manner of bed-load feeding, perhaps the slope scale (as applied to the water surface) and perhaps the gradation of the bed material. Often it is necessary to use time and discharge scales which vary with stage in order to make each model stage effect its proportional share of bed movement. It is evident, therefore, that the verification phase of a movable-bed model calls for an intimate knowledge of the prototype, as well as experience with this type of model study.

After a satisfactory verification of a movable-bed model, the degree of similitude attained (and this may vary considerably for different sections of the same model, and for different tests of proposed improvement plans for the prototype) remains largely a matter of engineering judgment. The similitude obtained in the model is based upon this general reasoning: If the model accurately reproduces changes which are known to have occurred in the bed of the prototype, it can be relied upon to predict changes of a similar nature which can be expected to occur in the future. As a word of caution, however, it is pointed out that the dynamic distortion inherent in the verification of a movable-bed model places an important limitation upon the type of test which can be made subsequently, and thus, upon the type of results which can be obtained from the study. Since the verification is achieved on the basis of an adjusted simulation of recorded prototype phenomena, the model cannot be expected to respond accurately to conditions which involve to drastic departures from those involved in its verification.

In model tests to determine the effects which various tentative plans of channel regulation will have in the prototype, it is necessary that the scale relationship and the model operating technique developed during the verification phase be maintained. The interpretation of the results of these tests is also a highly important phase of the model study. In those cases where sufficient field data are available to verify the model through a series of significant bed changes, the results of model tests may be used to predict the quantitative behavior of the prototype with respect to any action reasonably similar to that reproduced in the verification period. However, the need for and the value of experience in making such predictions is in no sense replaced by the model verification. Valuable experience may be gained by studying comparative results in models and prototypes.

In the final analysis, the validity of the results of a movable-bed model study, and the interpretation of the results thereof, are largely dependent upon general judgment and reasoning, the basis of such reasoning being the verification of the model, a knowledge of the prototype, and familiarity with the general characteristics of such models.

6. Problems of unsteady flow

When the state of flow in an open channel is unsteady, the additional factor of accolration head is introduced. Since all parameters change with time when the flow is unsteady, the equation of motion takes the form of partial differential equations, which can be solved only by approximate methods. Under such conditions a model can be regarded as an integrating machine.

As in the case of steady open-channel flow, the Froude law is used to accomplish similitude, and the frictional resistance is adjusted as required. In general, a model that is valid for steady flow at different stages is equally valid for unsteady flow. Since velocities and depths change with time, the adjustment of model friction becomes more significant and more difficult. To accomplish correct transient conditions, the frictional resistance should be similar, model to prototype, at all stages of flow. The roughness of the model may be adjusted so that steady flow at all stages is correctly reproduced, although the actual roughness over different areas of the model channel may vary. A procedure for accomplishing this condition has been developed by Professor Thomas.[9]

Distortion of the linear scale does not alter the suitability of the model if the frictional resistance is adjusted accordingly.

A somewhat different approach to accomplishing a suitable model is to establish the scale in such a way that the same type of flow is obtained in the model as prevails in the prototype. A criterion for assuring this condition, which amounts to a special Reynolds number, has been given by Danel.[10] In a natural water course the flow is, almost without exception, rough-turbulent flow. If the flow in the model is also rough turbulent, the the variation of frictional resistance with velocity will be proportional to the velocity squared as in the prototype, and the model will be suitable for study of unsteady flow. When the value of $\frac{V_* k}{\nu}$ is greater than 50 or 60 for roughness of the sand-grain type and greater than 100 for any type of roughness, rough-turbulent flow will prevail. Danel has called the dimensionless expression the "Karman" number, where

V_* = shear velocity = $\sqrt{Rig}$;

k = height of equivalent sand roughness; and

ν = kinematic viscosity.

Unless the prototype has a steep slope, it is necessary to distort the model slope by introducing vertical exaggeration. There is a limit to the permissible distortion based on the curvature of the water surface. The distortion must be kept below the value which will introduce a curvature in the model of such a degree that the pressure distribution departs appreciably from hydrostatic.

TIDAL AND WAVE MODELS

The design of tidal and wave models to study problems involving harbor improvements or protection works, channel improvement estuaries, beach erosion, etc., proceeds from the basic considerations that have been discussed with either fixed- or movable-bed models. In addition, the influence of tides or waves, or both, must be included. When the principal factor is tidal flow, it is usually necessary to use a distorted or vertically exaggerated model. A tide will have an amplitude of several feet and in order to obtain a measurable range in the model, the vertical dimensions should be made disproportionately large compared to a reasonable horizontal scale based on considerations of available space. The larger depth may also be required to give velocities high enough to move bed material. In the case of problems involving waves the particular type of wave controls the allowable distortion.

Preliminary operation of a model of this type should be directed toward making such adjustments in model tides or waves, roughness, bed materials, or even in vertical scale, as are required to bring the model performance into conformity with either the known or predicted behavior of the prototype. In complicated problems, which are the rule rather than the exception, the procedure of developing relative worth is recommended, instead of attempting to transfer model results directly to prototype except where purely qualitative considerations are involved. The relative worth of some proposed scheme is developed in the model by reference to a standard or base test. The performance of each scheme is determined by operating the model from some standard initial condition with identical tides, waves, river flow, littoral current, rate of adding sediment, etc. Such a procedure is usually adequate for selecting the most appropriate scheme and presents no formidable difficulties.

Since the various physical quantities are unsteady, that is, vary with time, special instrumentation is required. Such instruments may include a clock-driven drum or oscillograph, sensitive devices for measuring wave heights, motion picture cameras for recording the movement of paper confetti or reflector floats, or special equipment for adding and measuring the concentration of salt or dye solutions to define current directions.

1. Simulation of tide and current

The effectiveness of tidal and wave models depends in large measure on proper reproduction of natural tides or waves in the model. Design of mechanical devices for generating tides or waves is a major problem and the cost of such devices is an important part of the total cost of a model study. Tide machines of two distinct types have been used. In one case the variation in water level is accomplished by displacement with a mechanically actuated plunger. In a machine used by Gibson in the Severn Estuary study,[8] the variation in tidal amplitude from spring to neap was obtained by varying the stroke of the plunger through a train of epicyclic gearing. In a machine used by the Bureau of Reclamation[11] to study salinity intrusion into the Sacramento-San Joaquin Delta, a mixed tide and its variations throughout a lunar cycle was obtained by controlling the plunger through a cam which was cut to represent a lunar cycle of tides. Both designs are adapted to problems where the use of a mean tide is not advisable. The plunger type machine is limited to small models in which the tidal prisms require a reasonably small plunger. In the other type of tide machine the required variation in water level is accomplished by adjustment of a weir or a valve supplied by a constant flow of water. The height of a weir or the opening of a valve are regulated by a controlling device which is actuated whenever the model level, as indicated by a float, differs from the required level as indicated by a cam. The effects of inertia are compensated by a power interrupter which is adjusted by trial and error. This type of machine is well adapted for use in larger models where the total flow of water during flood and ebb tide is relatively large and has been used in many studies made at the Waterways Experiment Station, Vicksburg, Mississippi.[12]

2. Investigation of wave action

The fundamental type of wave involved in a model study has an important bearing on the choice of vertical scale or exaggeration. The wave velocity is represented by the expression

$$c^2 = \left(\frac{g\lambda}{2\pi}\right) \tanh 2\pi \frac{D}{\lambda}$$

where c equals velocity of wave or celerity, in feet per second, λ equals wave length in feet, D equals mean depth in feet. For short waves in which $\frac{D}{\lambda}$ is greater than about one-half the hyperbolic tangent approaches unity and the wave velocity depends principally on the wave length.

For long waves, $\frac{D}{\lambda}$ of one-tenth or less, the value of c approaches $\sqrt{gD}$. Since the type of wave depends upon the ratio of $\frac{D}{\lambda}$, a vertical to a horizontal dimension, it is possible that too much vertical exaggeration will alter the type of wave and this possibility should be checked. In the case of solitary waves, or tidal waves, which are very long, the possibility is remote and quite large exxaggerations are permissable as far as similarity of wave form is concerned.

In harbor models built to investigate wave and surge action, the wave from which protection is desired may range in period from a few seconds to as much as 5 minutes or more. The short-period waves (usually from about 6 to 18 seconds) may range from only a foot or so to more than 20 feet in height. Long waves (usually from about 1 to 5 minutes in period) are generally less than 3 feet in height. If the problem is primarily one concerning short-period waves, an undistorted-scale model should be used. However, as the period of the primary waves increases, the amount of scale distortion which can be tolerated increases. For instance, a scale distortion of five (ratio of length scale to depth scale) will not result in distorted modes of oscillation in a model when the prototype wave periods are about 2 minutes or more. If the primary problem has to do with long-period waves, and the effects of short-period waves are secondary, it is possible to use a distorted-scale model and still obtain results as accurate as warranted by the circumstances. A good understanding of the prototype problem is required before the use of a distorted-scale model can be sanctioned.

In models designed for the study of stability of rubble mounds or pressures on impervious breakwater, an undistorted model should be used and the results interpreted according to the Froude law. If care is exercised in sizing and proportioning the breakwater material and constructing the model breakwater, results of considerable accuracy can be obtained. It has been found that a scale of 1:45 is about the smallest scale that can be used for this type model (where the prototype breakwater is situated in at least 50 feet of water) when considerable accuracy is required.

Machines for generating model waves are usually based on the vertical movement of a displacement plunger.[13/] An older type using an eccentric roller has now largely been replaced by the plunger type which gives better reproduction of natural waves. The plunger is made with a triangular cross section having the apex down. The back face is usually vertical and is placed close to a wall to prevent any wave formation from it. The front face, which is active in generating the desired wave, is sloped at an angle which is usually about 45 degrees but may be variable. Additional flexibility should be provided by making the displacement as well as the speed of oscillation adjustable. Unless the problem involves waves in a constant direction the wave machine should be mobile so that its position can be altered to generate waves from any required direction. For slow-period waves, of relatively large amplitude, the surge chamber type used at the California Institute of Technology, Hydraulic Structures Laboratory, is more suitable than the plunger type.[14] In this type, water is alternately stored and released by regulation of the air pressure in an air-tight chamber. The frequency can be made very low and the shape of the wave can be controlled by manipulation of the valve which regulates subatmospheric pressure in the chamber. The keynote to be sounded in the design of wave machines is flexibility, so that proper action developed by operation of the model may be obtained by suitable adjustment.

3. Movement of sediment

In those problems where the waves constitute an agency for performing some other action, such as movement of bed, or semisuspended material, or the diffusion of salinity, the characteristics of the wave are not of primary importance. It may be necessary to distort a model to obtain proper bed movement despite the fact that the wave type is changed. In such cases the similitude relations are developed experimentally as previously described. To obtain proper movement of bed material in tidal or wave models it is usually necessary to make the model large, use a high distortion, or use a lightweight sediment. If space permits and time allows, a large model will be satisfactory but expensive. If tidal currents alone are to be considered, distortion of the order of 85 will accomplish movement with sands having a mean diameter of about three-fourths of that of the prototype sand.[8] If the wave type must be preserved, at least partially, a lightweight sediment may be used. A material used with much success at the Waterways Experiment Station, Vicksburg, Mississippi, is a lightweight asphaltic material called gilsonite, with a specific gravity as low as 1.03. Such a material will move readily under the action of model waves to represent suspended or semisuspended material in nature. The material is difficult to handle and similarity must be developed by extensive verification tests. For these reasons the material is not recommended unless many studies are contemplated, which would justify the development of the specialized technique required for its use.

HYDRAULIC-MACHINERY MODELS

Experience has demonstrated that the performance of a hydraulic machine may be obtained by model tests with a minimum of time and expense. The need for performance tests arises from the fact that flow conditions in machines, such as pumps or turbines, are so exceedingly complex as to make it impossible to predict performance in the required detail from analytical considerations alone.

1. Performance tests

Performance tests may be required and can be appropriately carried out by means of models to: (1) guide and verify theoretical developments in design; (2) evaluate performance under special conditions, such as the effect of the setting on cavitation; and (3) perform acceptance tests on competitive designs in lieu of full-scale tests.

Since hydraulic machines involve flow within closed boundaries, similitude considerations and limitations closely parallel those developed for closed-conduit flow. In general, the problem of attaining proper representation of frictional resistance is not critical in the case of hydraulic machines except for an accurate reproduction of efficiency. The principal action in a hydraulic machine is a dynamic transfer of energy between rotating elements of the machine and the moving fluid. Aside from the occurrence of separation, cavitation, and certain details of flow near the boundary the general characteristics of the energy transfer are similar in model and prototype machines under the following conditions. The ratio between fluid velocities and peripheral velocities of the rotating elements must be equal in the model and prototype at geometrically similar points. Fluid velocities are proportional to $\frac{Q}{D^2}$ where Q is the discharge and D is the diameter of the rotor, and peripheral velocities are proportional to ND, where N is the speed in revolution per second. The ratio between fluid velocities and peripheral velocities may be expressed as

$$\frac{\frac{Q}{D^2}}{ND} \quad \text{or} \quad \frac{Q}{ND^3}$$

and the condition for kinematic similarity is that

$$\left(\frac{Q}{ND^3}\right)_r = 1$$

In addition to this condition, complete similarity of flow requires that the Reynolds number and relative roughness of the surfaces be equal in model and prototype. These conditions cannot be satisfied in practice but they are approached by making the model surfaces smooth and by operating the models, especially pump models, at prototype velocities.

A significant parameter used widely in the design and typing of hydraulic machines is the specific speed.15/ For turbines the specific speed takes the form:

$$N_s = \frac{N \sqrt{P}}{H^{5/4}}$$

where

P = horsepower, and

H = operating head in feet of water,

and for pumps the form:

$$N_s = \frac{N \sqrt{Q}}{H^{3/4}}$$

The difference in form derives from the fact that power is significant for turbine design, whereas discharge is significant in pump design. Since the specific speed describes the operating conditions (N, Q, and H) required for similar flow conditions, in geometrically similar machines, it must be equal in model and prototype. When this condition is satisfied the previously stated requirement is also satisfied.

Model results may be extended to the prototype by relationships derived from the equality of specific speed together with the actual ratios of head or speed used in the test. A convenient form which is being used to express experimental results is to use dimensionless plottings wherein the parameters are expressed in terms of normal head, discharge, power, etc. For example, a head-discharge curve would have for ordinate, head divided by normal head, and for abscissa, discharge divided by discharge at normal head.

Performance characteristics demonstrated by model tests apply directly to the prototype with the exception that the model efficiency is one or two points lower than that of the prototype. Much turbine testing is accomplished at model heads scaled to geometric scale ratio. On the other hand, modern pump testing is usually carried out at prototype velocities and head [16] by making the model speed equal to the prototype speed multiplied by the ratio of diameters prototype to model. The higher values of Reynolds number obtained by such practice yields a higher model efficiency and the model performance is more significant with respect to separation and cavitation effects.

2. Prediction of cavitation effects

A specific test for the effect of cavitation is generally included in a modern model study of a hydraulic machine, especially a pump. A special cavitation number designated as σ is used for such a test where

$$\sigma = \frac{h_s + h_a - h_{vp}}{H}$$

where h_s equals suction head, h_a equals atmospheric pressure, h_{vp} equals vapor pressure of the fluid, and H equals total head. When the value of σ for the model is equal to that for the prototype the model will demonstrate the effects of cavitation that can be expected in the prototype. The required value of σ can be obtained by manipulating the suction head or the model atmosphere. In the case of pump models operating at prototype heads, the model suction head can be readily varied to embrace the expected range of prototype suction heads and the critical value of σ may be established. The data from such tests made at different values of σ may be plotted as efficiency versus σ, or head versus σ. A break in either curve indicates the inception of cavitation and defines the permissible prototype value of suction head.

3. <u>Air-flow studies of pressure distribution</u>

Studies of hydraulic machinery utilizing air as a testing fluid at relatively low velocities have been made to evaluate characteristic curves or to study pressure distribution over the vane elements. If the air velocities are kept below 300 feet per second, the effects of compressibility can be ignored and the data can be computed by hydraulic formulas. If the results are expressed in dimensionless form, they are directly applicable to the prototype.

Testing with air offers a means for accomplishing quick and inexpensive tests. Cheap, light material can be used and changes can be readily made. Modeling clay serves admirably as a boundary for cut-and-try experiments. Measurements require greater precision in air studies since deflections of liquid columns used to indicate pressure or velocity are only a few inches or less. An entire laboratory devoted to aerodynamic testing of hydraulic machines is maintained by Escher Wyss Company and has been described by Keller.[17]

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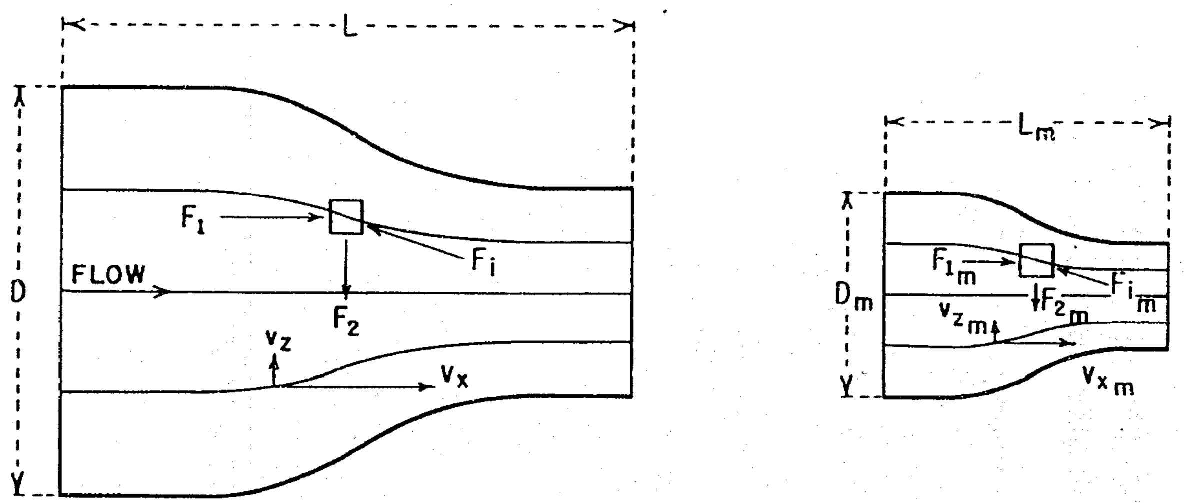
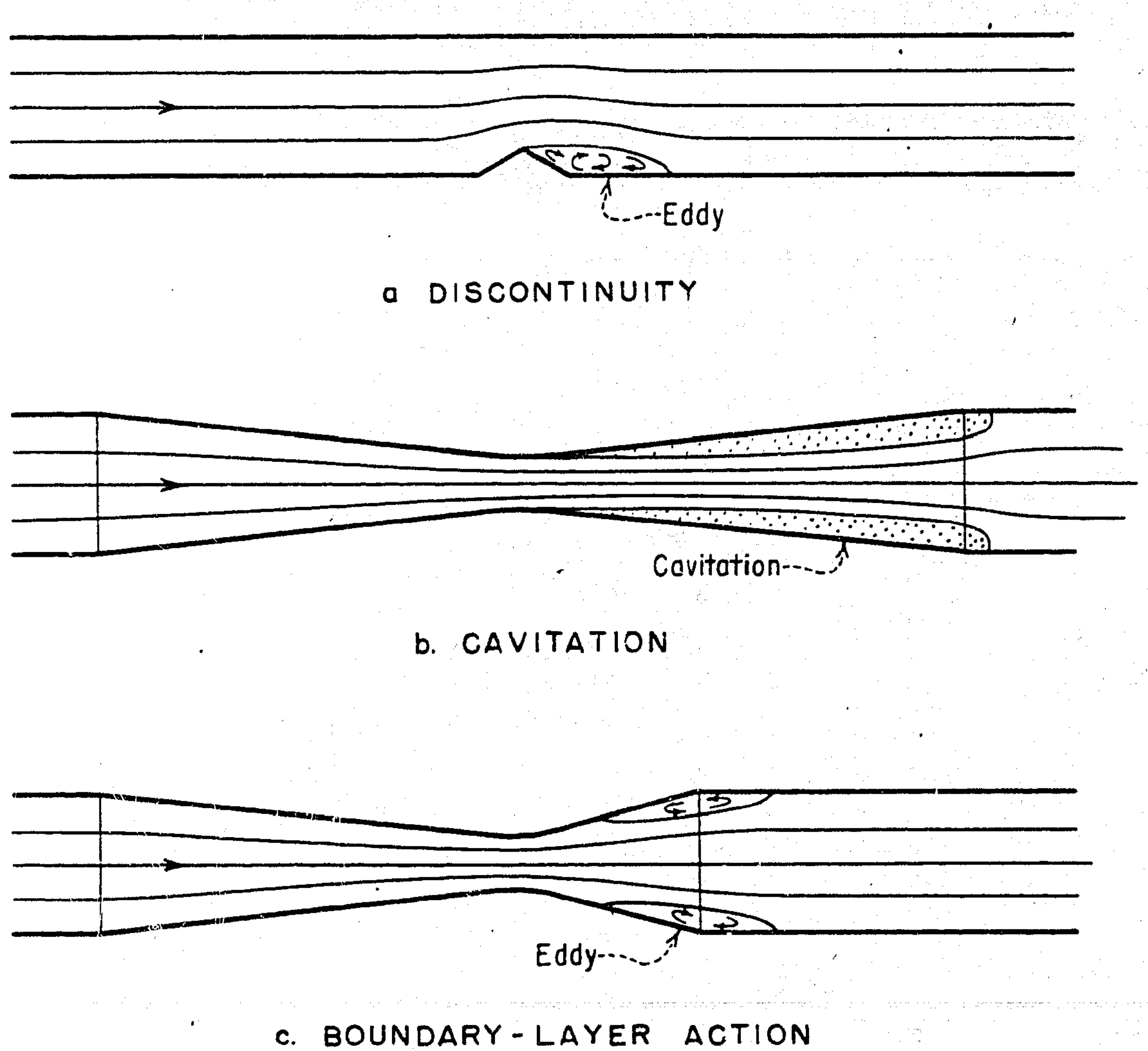
FIGURE 1



Geometric $(L)_r = \frac{L_m}{L} = \frac{D_m}{D}$ Kinematic $(v)_r = \frac{(v_z)_m}{v_z} = \frac{(v_x)_m}{v_x}$ Dynamic $(F)_r = \frac{(F_1)_m}{F_1} = \frac{(F_2)_m}{F_2} = \frac{(F_i)_m}{F_i}$

CONDITIONS FOR COMPLETE SIMILITUDE

FIGURE 2



TYPES OF SEPARATION