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DESIGN OF CURVED DRAFT TUBES BY MEANS OF AXIALLY SYMMETRICAL POTENTIAL FLOW

by

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Translated From The German
by

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Denver, Colorado

June 10, 1937

DESIGN OF CURVED DRAFT TUBES

FROM

AN AXIALLY SYMMETRICAL FLOW POTENTIAL.

A translation of

"Entwurf von Saugrohrkrümmern
mit Hilfe der achsensymmetrischen
Potentialströmung"

by F. Weinig

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Translated by

Edward F. Wilsey, Assistant Engineer
United States Bureau of Reclamation
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DESIGN OF CURVED DRAFT TUBES FROM AN AXIALLY SYMMETRICAL FLOW POTENTIAL

Inthe design of curved draft tubes, the aim is to obtain a uniform energy transformation so as to avoid recovery losses near the constricted section. The following method consists of a purely graphical treatment of curved shapes rather than experimental development of draft-tube shapes.

1. The principle of producing curved flow. With the flow of an ideal fluid the surfaces formed by enveloping stream lines can always be replaced by solid walls. Thus if stream lines are chosen which are continuous and do not loop back on themselves, in general, a correspondingly formed curved tube is obtained. Laminar flow is unsuitable for the type of flow for beginning the inquiry into curved draft tubes for with such tubes it is expedient to enlarge them gradually in a direction perpendicular to the plane of the curved tube. Thus they must be considered from the standpoint of axially symmetrical flow.

2. Properties of axially symmetrical flow. If z, r and ω are cylindrical coordinates and if z is the axis of symmetry of the flow, the stream function ψ and the potential ϕ are independent of the angle ω. In order that the stream function and potential have equal dimensions, it is to be understood that by stream function is meant the quantity of flow through a circle concentric to the axis divided by the unit length, L. The following equations for ϕ and ψ are valid for the irrotational flow of an incompressible fluid:

$$\Delta\phi = \frac{\partial^2\phi}{\partial z^2} + \frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} = 0$$

$$\Delta\psi = \frac{\partial^2\psi}{\partial z^2} + \frac{\partial^2\psi}{\partial r^2} - \frac{1}{r}\frac{\partial\psi}{\partial r} = 0$$

If ds and dn are elements of a stream line and a potential line, respectively, the true velocity is given by:

$$w = \frac{d\phi}{ds} = \frac{L}{2\pi r}\frac{d\psi}{dn}$$

The potential and stream lines being so represented (figure 1) that between all adjacent lines of the net, equal differences of stream lines, $\delta\psi$ and of potential lines $\delta\phi$ exist and if δn and δs are the lengths of the sides of the curved rectangles formed by the net of potential and stream lines such lines may be termed equidistant, so for small δs and small δn

$$\frac{\delta\psi}{\delta\phi} = 1 \longrightarrow \frac{2\pi r}{L}\,\frac{\delta n}{\delta s}$$

that is, the ratio of the length of the sides of the curved rectangle with both $\delta\psi$ and $\delta\phi$ small is,

$$\frac{\delta n}{\delta s} \longrightarrow \frac{L}{2\pi r}$$

In the limit as $\delta n \rightarrow dn$ and $\delta s \longrightarrow ds$ the ratio of the sides of the curved rectangle is inversely proportional to the distance from the axis, r.

3. Determining axially symmetrical flow with the aid of the flow net. The properties of the net of potential and stream lines just described, afford a means of computing the picture of the flow. If, for example, two stream lines are given, new stream lines and potential lines can be drawn in the space between the given stream lines by means of the flow net procedure, the net being so drawn that it has the properties described above.

4. Determining axially symmetrical flow by means of an analytic procedure. Not always, however, are flows to be determined which are bounded by two stream lines, as just described. In special cases there may be only one limiting stream line, such as the axis, for instance, of the flow in a pump intake, or a parallel to the axis at infinity, as for example, in the flow around the body of an airship. One can demand that, along a piece of giv.. stream line, say between the points a and b (figure 5c), a prescribed potential distribution exists. In this case, it is possible to draw the adjacent stream lines and then other stream lines still farther removed from the given stream line. This is applicable only insofar as no singularities enter, that is the stream lines proceed without sources (hydrodynamic sources) and turbulence, or in other words, their path is analytic. This analytic procedure can be found in the following manner:

By division into equal $\delta\phi$ intervals (figure 3), the intersections of the equidistant potential lines with the given stream lines are obtained. The pieces thus cut out from the stream lines have a length, δs . The rectangular net of adjacent stream and potential lines can be drawn by uniting the ends of the pieces.

A limit to this procedure is set by the axis in case it becomes the source of stream lines or depending on whether it pours out or takes in stream lines, a source or sink, respectively. So when the stream lines converge rapidly, or bend rapidly, this procedure leads to sources or sinks, or vortices.

$\delta\phi$ This method renders usable results only if the interval, is not chosen too large.

5. Design of a curved draft tube from a given axially symmetrical flow. How to arrive at a curved draft tube starting from a given axially symmetrical flow will now be shown by a simple example. As a rule, the inlet end of the draft tube will be circular and its plane will be normal to the axis of symmetry of the given flow or nearly so. However, the circular area need not intersect this axis. As an example, we shall choose hyperbolic flow which is known to be axially symmetrical. For this:

$$\psi = \frac{4\pi}{L} r^2 z \; , \quad \phi = 2z^2 - r^2$$

After it is drawn (figure1) one can easily construct the plan and elevation of the curved draft tube perpendicular to the plane of the curve and the section (figure 2) perpendicular to the z-axis and the side elevation parallel to the z-axis. The elevation of the draft tube perpendicular to the plane of the curve is given by the two stream lines through the extremities of the diameter of the entrance. The plan is given by the entrance circle and by the tangents to it, which pass through the z-axis.

The bend found from this given type of flow appears to be suitable for medium heads, for small heads, however, it has little use since after the flow is retarded it is again accelarated. Only the part with the retardation bears a relation to the problem of a curved draft tube.

6. Design of a curved draft tube from a previously prescribed distribution of energy transformation. It is yet to be shown how to design a curved draft tube from a previously prescribed uniform transformation of energy.

After the average entrance and exit velocities and the relative positions of the entrance and exit have been decided, a middle stream line of the bend is drawn which appears to be suitable for the purpose and then the velocity distribution $w(s)$ which corresponds to the chosen course of energy transformation is fixed upon. From $w(s)$ is obtained by means of

$$\phi = \int_a^s w(s)\, ds$$

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the variation of the potential along this stream line. Proceeding now by the analytic method already describe, a net of adjacent stream lines and lines of potential is obtained and after this the desired bend can be drawn, the cross sections of which are obtained in a similar fashion to the horizontal and vertical sections shown above (figure 4).

If these are replaced by equal area sections of approximately equal ratios of width to height, the bend found thereby can be adjusted to the approximate technical requirements. Without this the energy transformation in the bend might be essentially different from that of a bend derived from axially symmetrical flow.

SUMMARY

The graphical method described makes it possible to determine axially symmetrical potential flow, which possesses a properly prescribed pressure increase along a center line (which corresponds to a stream line) of a specially curved draft tube and as a result of such a potential flow, to design a specially curved draft tube.

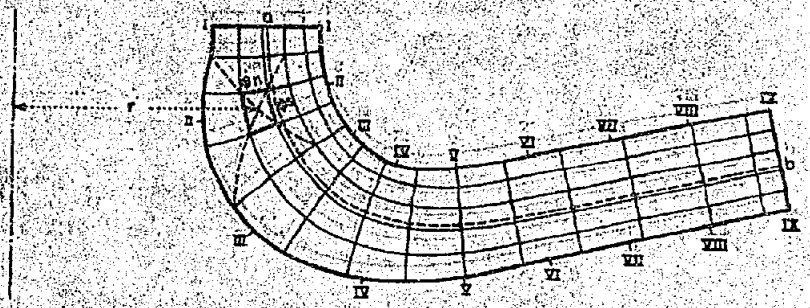


FIGURE 3 – DESIGN OF A CURVED TUBE FROM THE PRESCRIBED VELOCITY AND PRESSURE DISTRIBUTION ALONG A CHOSEN MIDDLE LINE BY THE ANALYTICAL PROCEDURE WITH THE AID OF AXIALLY SYMMETRICAL FLOW.

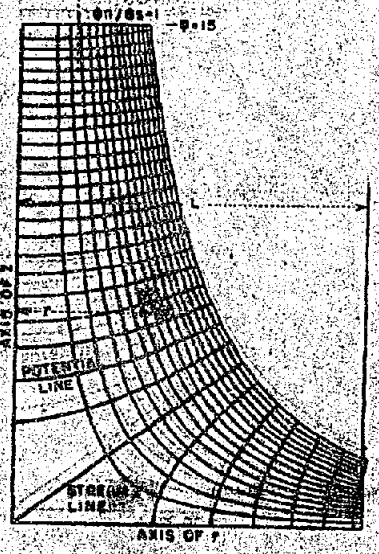
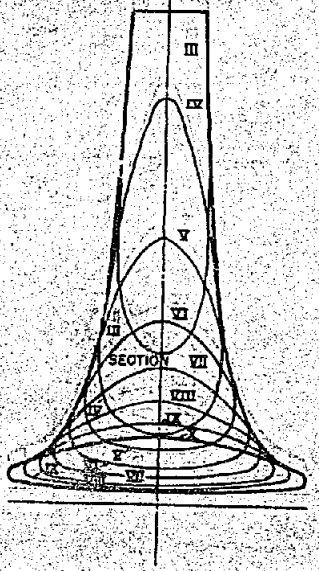


FIGURE 1 – NET OF STREAM AND POTENTIAL LINES FOR AXIALLY SYMMETRICAL FLOW



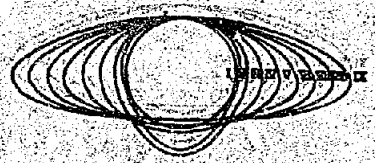
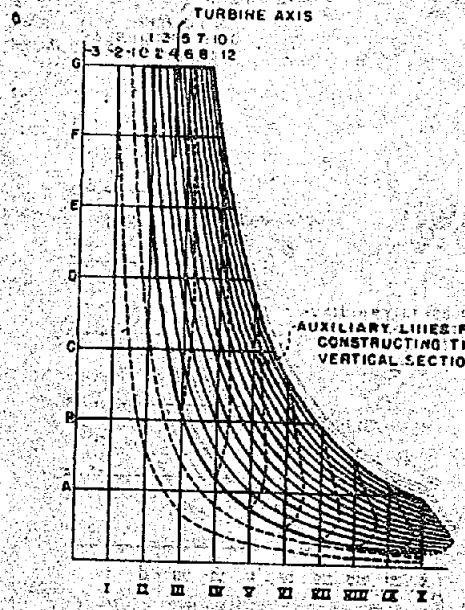


FIGURE 4 – SECTIONS OF THE CURVED DRAFT TUBE AT PLANES I, II, III, ETC. OF FIGURE 3



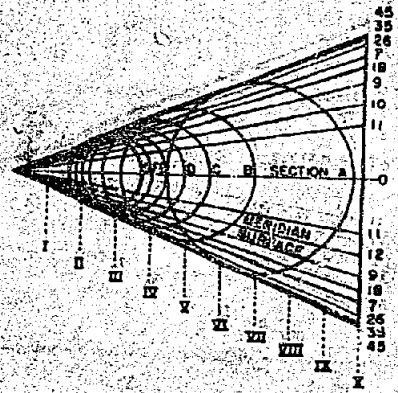


FIGURE 2 – PLAN, ELEVATION AND SIDE ELEVATION OF A CURVED DRAFT TUBE AS DETERMINED FROM A GIVEN AXIALLY SYMMETRICAL FLOW

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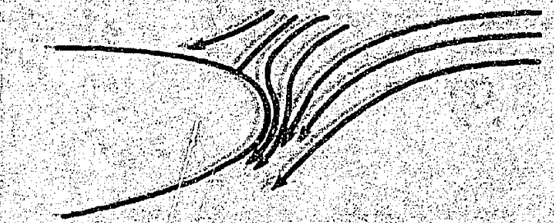
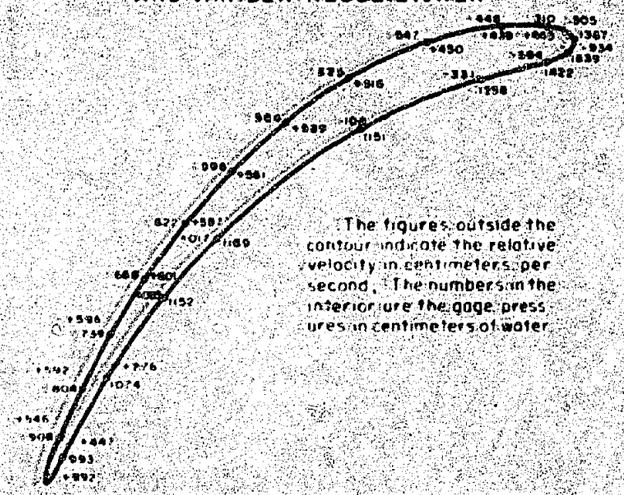


FIGURE 4 - SOME STREAM LINES RELATIVE TO THE VANE ABOUT TO ATTACK AN IMPELLER OF ONE OF THE CENTRIFUGAL PUMPS AT MEDEMBLIK ACCORDING TO BURGERS AND VAN DER HEGGE ZIJNEN



The figures outside the contour indicate the relative velocity in centimeters per second. The numbers in the interior are the gage pressures in centimeters of water

FIGURE 5 - DISTRIBUTION OF VELOCITY AND PRESSURE AROUND A VANE OF AN IMPELLER OF ONE OF THE CENTRIFUGAL PUMPS AT THE "LELY" STATION AT MEDEMBLIK

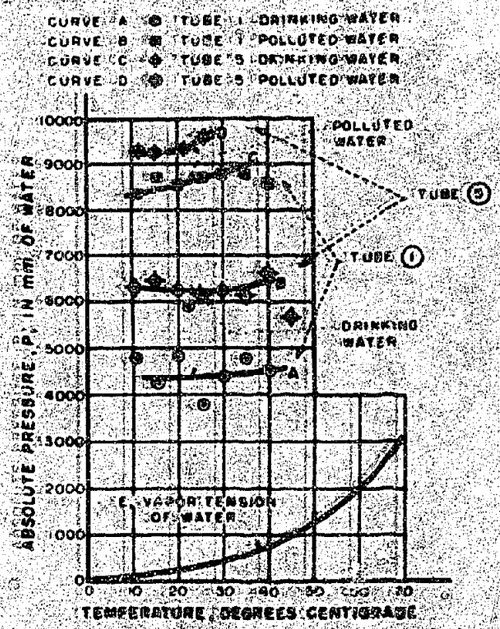


FIGURE 7 - ABSOLUTE PRESSURE AT THE INCEPTION OF CAVITATION FOR BOTH PURE AND POLLUTED WATER AS A FUNCTION OF THE TEMPERATURE

FIGURE 8 - THE AIR PRESSURE WITHIN THE BUBBLE FLOATING IN WATER SATURATED WITH AIR AT ATMOSPHERIC PRESSURE ACQUIRES EQUILIBRIUM WITH THE WATER PRESSURE AND SURFACE TENSION

FIGURE 9 - EQUILIBRIUM OF OF A SURFACE EXTENDED BY INTERIOR PRESSURE

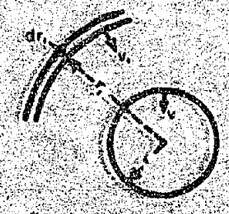


FIGURE 11 - BUBBLE CONTRACTING CONCENTRICALLY

FIGURE 10 - AIR BUBBLES ADVANCING ALONG THE THROAT OF A VENTURI TUBE. THE POINT TO THE RIGHT INDICATES WHERE THEY SUDDENLY DISAPPEAR

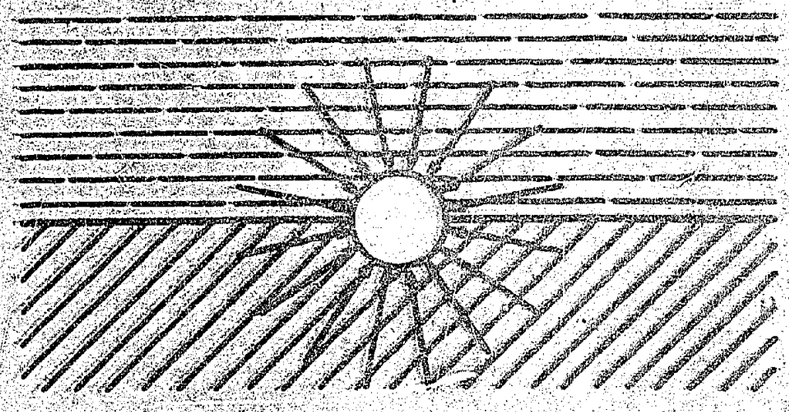


FIGURE 15 - IMPACT OF A HEMI-SPHERICAL BUBBLE CONTRACTING ON AN INCOMPRESSIBLE, WEIGHTLESS SPHERE RESTING IN A SURFACE CAVITY

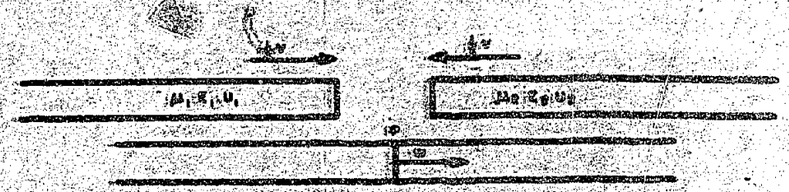


FIGURE 18 - AXIAL IMPACT BETWEEN TWO BARS OF DIFFERENT MATERIALS

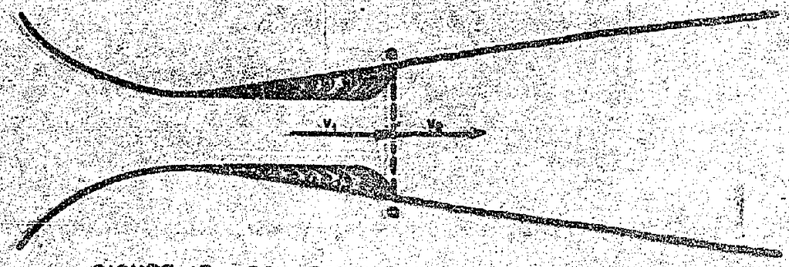


FIGURE 17 - CAVITATION OF THE FIRST CATEGORY CHARACTERIZED BY THE SPONTANEOUS DISAPPEARANCE OF AIR BUBBLES AT THE FRONTIER, a-a, WHICH OSCILLATES BACK AND FORTH