HYDRAULIC MODEL STUDIES FOR THE DESIGN OF THE 102-INCH
OUTLET IN BRENNER DAM, PROVINCE OF PUNJAB, INDIA

by

Jack Carl Schuster

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This Thesis for the M.S. degree, by
Jack Carl Schuster
has been approved for the
Department of
Mechanical Engineering

by

[Signature]

Benjamin H. Spurling, Jr.

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Schuster, Jack Carl (M.S., Mechanical Engineering)

Hydraulic Model Studies for the Design of the 102-inch Outlets in Bhakra Dam, Province of Punjab, India

Thesis directed by:

Associate Professor Benjamin H. Spurlock, Jr.

In the design of these 102-inch outlets, prevention of damage to the outlet by cavitation was the major problem. An hydraulic model, which was related to the prototype by laws of hydraulic similitude, was used to study extensively the pressure conditions and appearance of flow.

The investigation showed that in all parts of this outlet, pressure conditions were satisfactory and that subatmospheric pressures should not exceed a negative ten feet of water. Determination of the quantity of air required to maintain this pressure in the outlet was unsuccessful. Flow conditions through the outlet should be satisfactory, with the possible exception of the discharge from the elbow on the face of the spillway. A high velocity jet of water mixed with air will probably produce spray when discharged into the atmosphere. The model was not applicable in ascertaining flow conditions of this type.

Computations and pressure measurements were made in determining the forces to be used as an aid in the
structural design of the elbow and conduit opening deflector.

This abstract of about 175 words is approved as to form and content. I recommend its publication.

Signed Benjamin E. Spoolock, Jr.
Instructor in charge of dissertation
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Statements and opinions in this thesis are those of the author, and are not necessarily those of the Bureau of Reclamation.
INTRODUCTION

Purpose of Study

The purpose of this investigation was to improve the exit elbow design of the 102-inch outlets for Bhakra Dam, and to assure satisfactory operation of this outlet.

The 102-inch Outlets in Bhakra Dam

The Bhakra Dam is located on the Sutlej River in the Punjab Province of India. It is being built by the Irrigation Branch of the Public Works Department of the Punjab Province from designs prepared by the International Engineering Company, Denver, Colorado. The Bureau of Reclamation has assisted the International Engineering Company through model studies in the hydraulic laboratory in Denver, Colorado.

Bhakra Dam is similar to the Shasta Dam in California, and is comparable in size, having a height of 500 feet and a length of 1500 feet. It is a multipurpose dam, since it regulates the flow of the river for irrigation storage and for power generation. Normal quantities of irrigation water will be released through the powerhouse turbines or one of two outlet works. The first outlet works to be used will be a morning glory spillway, which will contain 4-96-inch hollow jet valves. When the demand exceeds the capacity of the turbines and spillway, additional water
may be released through the 102-inch outlets which pass through the dam to discharge upon the face of the spillway (Figure 1).

There are twenty of these outlets, ten at elevation feet 1351 and ten at elevation 1232 with a maximum head of 288 feet of water on the centerline of lower outlets. As shown in Figure 2, the original design consisted of a circular bellmouth entrance, a 36-inch control gate vented by 24-inch air pipes, a horseshoe-shaped conduit downstream from the gate, and an exit elbow which turned down into a trough in the face of the spillway. This design was patterned after the 102-inch outlets in Grand Coulee and Shasta Dam (Figure 3). The bellmouth entrance maintained a uniform drop in pressure as the flow entered the outlet. The control gate was originally developed for the outlets in Shasta Dam and was designed to operate at partial openings permitting a regulation of flow. Air pipes to vent the gate were equivalent in capacity to those in the Shasta outlets, but were arranged to suit the conditions at Ingraham Dam. Previous model tests had indicated that a horseshoe-shaped conduit would better control the discharged jet downstream of the control gate, rather than the circular conduit used in former designs. The exit elbow and trough were similar to the design of Grand Coulee Dam.

and Shasta Dams, but were horseshoe-shaped in section instead of being circular.

The proposed design of the Bhakra outlets appeared satisfactory except for the exit elbow. It was doubted that the high velocity regulated flow through the conduit could be turned down the face of the spillway without causing subatmospheric pressure and cavitation on the invert of this elbow.

The ring-follower control gates in Grand Coulee Dam were to be operated fully open and the conduit and elbow always flowed full under pressure. Proving satisfactory at Grand Coulee Dam, the outlet design was then used at Shasta Dam where the tube valve and gate controls were to be operated at partial openings. This meant that the conduits would not always flow full. Model studies under those circumstances had indicated subatmospheric pressures as low as 25 feet of water on the invert of the elbow.² This was considered satisfactory, for the criterion of the above design was based on the assumption that if subatmospheric pressures did not exceed 25 feet of water, there would be no danger of cavitation. It was later found that there was a possibility of a further reduction of pressure due to local imperfections in the boundary forming the flow passage. Such a reduction must have

occurred at Shasta Dam, for vapor pressures were observed on the invert of one outlet during field tests, and pitting was reported on other outlets which have been in operation. Since the above conditions could not be considered a safe design, the criterion of design for the Bhakra outlets was to shape the flow passage and to admit air, so that in no part would the subatmospheric pressures be greater than 15 feet of water, (6.5 psi), thus reducing the possibility of cavitation and a resulting erosion.

SUMMARY

The investigation covered the conjunctive operation of the improved elbow, a horseshoe-shaped conduit, and a previously tested bellmouth entrance and jet-flow gate. Data obtained from the model were related to the prototype by the Froude and Euler numbers. The outlet model was first studied with the exit elbow removed and satisfactory flow conditions were demonstrated in this incomplete model. An elbow design was then proposed which was square in cross section. With the inner bend omitted in the first tests, the flow was observed for elbows turned through various arcs, and one having an arc of 41 degrees 19 minutes was selected for further study. The trough below the elbow and a portion of the spillway face was then attached, and positioned so that the spillway face was flush with the top of the discharge end of the elbow. The inner bend

of the inlet end of the elbow was set below the floor of
the horseshoe-shaped conduit and this formed an offset
or pocket. An air vent was placed in this offset to re-
lieve subatmospheric pressures. To improve the appearance
of the flow down the spillway, a 24-inch deflector was
placed over the trough at the end of the elbow. Detailed
pressure measurements indicated that pressures existing
in the outlet were satisfactory, and the outlet was accepted
as the final design. To show that the elbow design with
a vented offset at the inner bend was fundamentally correct
for this type outlet, a test was made by raising the inner
bend of the elbow to a position flush with the floor of
the conduit, as originally designed. A final test was
made to ascertain the pressures existing on the outer bend
of the elbow.

**Conclusions and Recommendations**

The prototype outlets will operate at any discharge
without danger of damage by cavitation, unless there is
an obstruction or irregularity in the conduit. The flow
from the outlet should have a good appearance, for it
will be turned down to lie flat in the trough and against
the face of the spillway.

Air demand measurements gave inconsistent results,
possibly because of the arrangement of the valves in the
supply line upstream from the model; but even the maximum
subatmospheric pressures obtained indicated that the vents
were adequate. The model tests cannot ascertain the possibility of noise or vibration in the vents or headers due to flow of air. No conclusions can be drawn as to the mixing of air and water in the outlet, beyond the prediction that the mixture may result in an increased amount of spray in the prototype.

Similar outlet conduits in dams built previously, were lined with steel plate; but no lining is used in a greater part of the outlets in Shakra Dam because the conduit is formed in concrete, which results in a more economical design. The model tests did not consider the material forming the conduit, for no trouble should occur if this concrete is finished smooth and hard to withstand the high velocity flow and the abrasive action of any silt in the water.

The flow from the conduit is a horizontal jet which is turned down by striking the outer bend of the elbow. Pressure measurements indicate that the concrete lining of the elbow will be satisfactory, but it seems that the outer bend of the elbow, and possibly the sides, should be lined with steel plate.

THE INVESTIGATION
The Outlet Model

This study of the Shakra outlet was made from an available 1:17 scale model of a 102-inch outlet of Shasta Dam. The Shasta model consisted of a 36-inch diameter
head tank, a flat plate at one end of this tank to represent a portion of the upstream face of the dam, a bellmouth entrance of the outlet fastened to this plate, a control gate a short distance downstream, a circular conduit, and an exit elbow (Figure 4). Few changes were necessary to revise this model to represent an outlet in Bhakra Dam (Figure 5). A wax filler was used to change the bell-shaped entrance section of the control gate to one of a conical shape. The air vent to the control gate was a 2-inch pipe equivalent in area to the two 24-inch prototype pipes. The fact that the model vent was not geometrically similar to the prototype was not considered significant. The conduit downstream from the gate was made horseshoe-shaped with a flat bottom, and of clear plastic material which permitted observation of the flow. The exit elbow, which was to be developed, was not included in the first model. Water was supplied to the model by a 12-inch centrifugal pump directly connected to a 100 horsepower variable speed motor. A gate valve was placed in the line between the pump and the head tank of the model. Because of the arrangement of the laboratory, no venturi meter could be placed in the line to measure discharge, but the model control gate had been calibrated previously and the discharge quantity could be obtained

from measurement of the head and the opening of the control gate.

**Instrumentation**

The general procedure of the tests was to observe the appearance of flow through the model, and to measure head, pressure, and air demand. The head, representing the water surface in the reservoir, was determined by a direct measurement of pressure in the head tank. Pressures existing in various parts of the model were transmitted by piezometers connected to a manometer or differential gage. The piezometer openings, 1/8 inch or less in diameter, were drilled normal to the surface in which they were located. These openings led outside of the model to copper or plastic tubes which were connected by rubber tubing to the manometer or differential gage. The air demand was obtained by measuring the pressure drop across a sharp-edged orifice at the entrance of the air vent.

**Flow without Exit Elbow**

The outlet model without an elbow was observed to assure a satisfactory flow in the control gate and horseshoe-shaped conduit. Only the appearance of the flow was studied in this initial test, because pressure and air demand measurements were not considered necessary. The pressures in the control gate had been checked in prior tests and severe subatmospheric pressures were
not expected in the horseshoe-shaped conduit, unless the air vent at the gate was closed.

The flow from the gate at the wide-open position was ragged, and wisps of water would leave the body of the jet. This condition was believed to be caused by turbulence created in the expanding section of the conduit upstream from the gate (Figure 5). The wisps of water were not considered important, because they were small and they disappeared as the gate closed. A small amount of water was deflected into the gate slot at partial openings. At openings between 50 and 75 percent, the jet would strike the sidewalls of the conduit, while at openings of less than 50 percent the jet would deflect down to create waves which would rise against the sidewalls. These waves extended downstream about four diameters from the gate, and met at the top of the conduit and appeared to close it. They were thin sheets of water and did not appear to affect the flow of air through the outlet.

With a model head of 16.5 feet of water and the control valve fully open, the conduit was not full and the flow was similar to that in an open channel at supercritical velocity. When the model was operated at a head of approximately 70 feet, the water was mixed with air and the conduit appeared to be filled with a water-air mixture. It was planned to investigate this condition further in later tests.
Formulation of the Exit Elbow Design

The flow from the conduit was a rectangular jet with the depth dependent upon the opening of the control gate. An exit elbow which would turn this jet down onto the face of the spillway would have to be designed upon a different basis than was the elbow for the outlets in Grand Coulee Dam. The Coulee elbow was designed for an outlet which flowed full of water under pressure. A cone-like constriction had been placed at its exit to create back pressure and to compensate for a drop in elevation, since the elbow turned down. The Bhakra elbow would have to be designed for an outlet which did not flow full, even with the control gate wide open. A constriction at the elbow exit would be useless for a varying depth of flow.

In the Bhakra outlet, with the conduit flowing partially full, the rectangular-shaped jet will shoot across the elbow to strike the surface of the outer bend at a position beyond the point of curvature and will be deflected down. This impact type of change in direction of flow cannot be avoided, because, to the author's knowledge, there is no practical design of elbow which can accommodate a jet that varies in depth as the discharge is regulated. The design of the elbow must be based upon the idea that the flow will be turned by impact against the surface of the outer bend in a manner similar to the
impact of a jet against a tilted plate. This means there will be high pressures on the outer bend of the elbow where the jet strikes, and there will be a small reversal of flow forming a backwash above the jet. Neither of these conditions should be objectionable, although the structural design of the elbow must be sufficient to withstand the pressures.

Cavitation pressures on the inner bend of this elbow must be avoided, but it was fundamental that a drop in pressure would occur where the boundary surface turns away from the flow, as does the inner bend of the elbow. Since there appeared to be no way in which to correct this condition, it was proposed that the inner bend of the elbow be omitted from the model until the nature of flow striking the outer bend of the elbow and being turned down could be further studied.

An elbow which had a square cross section and which had the same radius and turned through the same arc as the Grand Coulee elbows, was proposed for study. It was believed that the flat outer bend and vertical sides of the elbow could be adapted to the rectangular-shaped jet from the conduit. If steel plates were used to build this elbow, the problems of fabrication for a square shape would be easier to solve than those for an elbow having a circular cross section.

Since the conduit of the Bhakra outlet has an arched
top and the elbow has a flat outer bend, there will be a step-like discontinuity where the square-shaped elbow joins the arch of the conduit. No adverse pressure conditions were expected at this point, but an air vent could be provided if necessary.

**Initial Tests on Model Elbow**

The proposed elbow was built without an inner bend and was attached to the end of the conduit of the outlet model. The flow was then observed for various gate openings and heads. In general, the flow appeared to be turned by the elbow in the manner desired, and the omission of the inner bend did not appear to be important. There was little difference in appearance of the flow with a change of head, except when the velocity was not great enough to cause the jet to strike the outer bend of the elbow. In this case, at low heads the jet would fail to be turned down into the trough below the elbow, but would be propelled outward to fall on the face of the spillway.

With the gate fully open, the elbow was almost filled and the water was turned down in the desired direction with a minimum of spray (Figure 6). The step-like discontinuity, where the flat outer bend of the elbow joined the arched top of the conduit, was filled with water under pressure. As the gate closed the jet skipped across the elbow to strike the outer bend, and most of the water
A. Control gate 100 percent open.

B. Gate 40 percent open.

C. Gate 15 percent open.

Flow from 41 degree-19 minute elbow
Bhakra Dam 1:17 model
was directed down along the curve of this elbow; however, part of the water was deflected down the sides to form fins resulting in a fan-shaped jet below the elbow (Figure 6B). These fins were not thick, and at small openings of the gate they disappeared (Figure 6C).

To observe the effects of turning the jet through a greater angle, the elbow was extended through arcs of 50, 60, and 70 degrees. The water was successfully turned through all angles, and the appearance of flow was somewhat improved as the angle of the bend increased (Figure 7). None of the tests showed that the inner bend of the elbow was an essential part in restricting the flow to its proper path. It was concluded from this test that the flow would be turned down by impact against the outer bend of the elbow, both at the full and at partial openings of the control gate.

**Tests with Trough and Face of Spillway Included in Model**

The tests related above did not determine the angle of the elbow that was the best suited to turn the water into the trough and onto the spillway. To better visualize this problem, the outlet model was extended by adding a discharge trough and a portion of the face of the spillway (Figure 8A). The trough and spillway face were simplified in construction so that they could be quickly revised. Tests were visual only, and consisted of different arrangements of the trough and elbow with
A. Gate 100 percent open.

B. Gate 40 percent open.

FLOW FROM 70-DEGREE ELBOW
BHAKRA DAM 1:17 MODEL
a. The model.

b. Discharge with control gate 100 percent open.

BHAKRA DAM 1:17 MODEL
respect to the spillway face.

The first test of this series was with an elbow which turned down through an angle of 41 degrees 19 minutes (Figures 8B and 9). The appearance of the flow did not differ greatly with the control gate at various openings. There was a tendency for the jet to fan out on the face of the spillway, as shown in Figure 9B, because the jet struck the rough corners of the trough. This condition would be eliminated in subsequent models.

It was found that the top of the elbow exit should be placed flush with the face of the spillway. If the elbow exit were placed below the face of the spillway, the length of the trough would be increased; if it were set above, it would complicate the design of a conduit opening deflector which will be constructed over the outlet exit (Figure 2). This deflector was a rise in the face of the spillway over the elbow, and caused the water coming from the spillway crest above to skip the elbow trough. A design of this deflector had been established by prior model studies.

An elbow which turned through 45 degrees was next studied, but the appearance of the flow did not differ greatly from that of the 49 degree 19 minute elbow (Figures 10A and 10B). The appearance of the flow in the photographs in Figure 10 were different from those in Figure 9 since they were made by different cameras and with
A. Control gate 100 percent open; 45-degree elbow.

B. Control gate 25 percent open; 45-degree elbow.

C. Discharge stopped by high speed photography. Gate 100 percent open; 41-degree-19 minute elbow.

BHAKRA DAM 1:17 MODEL
different lighting. To further emphasize the possible
difference of appearance of the model through the use
of photography, a special picture was made by stopping
the action of the water with high speed flash lamps
(Figure 10C).

A 50 degree elbow was proposed, but the increased
length of the trough made this impractical. Either the
41 degree 19 minute elbow or the 45 degree elbow would
be used in further tests, but the observed difference in
the two designs was so slight that it was impossible to
state that one was better than the other.

The Final Elbow Design

The 41 degree 19 minute elbow was selected for fur-
ther study because of the shorter length of the discharge
trough. It was believed that this elbow would be the
final design, and that further tests would consist of re-
fining this design and of measuring pressures and air
demand. An exit section resulting from the visual tests
was built and attached to the outlet model. It included
the square exit elbow, the discharge trough, and a
portion of the spillway face (Figure 5D and 11). The
inner bend of the elbow, which had been omitted in the
previous studies, was now included. To avoid subatmos-
pheric pressures on this bend, it was set 18 inches
(prototype) below the floor of the horseshoe-shaped
conduit. This formed an offset or step, and the water
A. The model showing exit elbow.

B. Pressure through outlet with gate 80 percent open.

BHAKRA DAM 1:17 MODEL
could spring clear in the same manner as it did when the inner bend was not in place. Beginning at this offset, the elbow bend curved through an arc of 41 degrees 19 minutes to become parallel with the outer bend of the elbow at the exit end, and to form a 102-inch square opening (Figure 5D). The rectangular shaped discharge trough was tangent to the edge of the inner bend and sides of the elbow and was faired into the face of the spillway. Air vents were used in this design to relieve regions of subatmospheric pressure. A 1-inch vent was placed in the offset at the inner bend of the elbow, three ½-inch vents were placed on the spillway face below the trough and a 1-inch vent was placed at the top of the conduit two feet upstream from the elbow. To study pressure conditions in this design, piezometers were located in the conduit, elbow, trough, and on the spillway face (Figures 5, 11, and 13).

**Flow Appearance in the Final Design**

In a trial run, it was found that a lip placed across the top of the trough at the elbow exit directed the flow down into the trough, and materially improved the appearance of the jet at partial gate openings. Several types of lips were tried, and the most practical appeared to be a 1.41-inch flat extension of the conduit opening deflector in the plane of the face of the spillway. This extension of the deflector to form a
lip was included in the final design (Figure 5D).

The appearance of the flow from the elbow was satisfactory in all respects, being turned and directed down the spillway without the formation of fins or large amounts of spray (Figure 12A). At openings less than 15 percent, a small fin and splash did occur in the center of the trough, but this condition did not appear to be objectionable and there was no way of eliminating it.

To observe this flow in a different manner, high speed photography was employed to stop the movement of the water (Figure 12B). The exposures were made in approximately 1/20,000 of a second with a General Electric No. FT 20 flash tube, synchronized with a 4x5 Speed Graphic camera. The appearance of flow in these photographs was similar to the flow that one might expect in a prototype structure. These photographs suggested that high speed photography could be used to study the flow in the conduit upstream from the elbow. As previously described, when the model was operated under a head of 70 feet, the flow appeared to be insufflated with air and to fill the conduit. The resulting mixture seemed comparatively uniform and regular as shown in Figure 11B, but this was not the case as shown in Figure 13. The flow in the photographs revealed a turbulent mixture that was neither uniform nor homogeneous.
FIGURE 12

Gate 100 percent open.

Gate 50 percent open.

Gate 25 percent open.

A. Flow as observed.

B. Flow stopped by high-speed photography.

(Behind=-1/20,000 second; velocity 20 feet per second.)

BHAKRA DAM 1:17 MODEL.
A. Gate 25 percent open.

B. Gate 50 percent open.

C. Gate 100 percent open.

FLOW IN CONDUIT ABOVE ELBOW, DIRECTION OF FLOW TO LEFT
BHAKRA DAM 1:17 MODEL
Close inspection of the photographs suggests that water tends to move through the conduit in slugs, because of an irregular mixing of the air.

**Pressure Measurements in the Conduit**

The pressures in the conduit upstream from the elbow were not studied in detail, but measurements were made to ascertain the general conditions. Subatmospheric pressures, which would not be greater than eight feet of water in the prototype were measured in the conduit immediately downstream from the gate. This pressure was expected because of the air requirements of the gate. An unexpected pressure condition occurred in the conduit at the vent in the top near the elbow—the pressure was positive instead of negative. At scale head, the space in the conduit above the flowing water was essentially continuous from the valve to the elbow, and it would appear that the air pressure at one end should be almost the same as at the other. Because the exit was blocked by the jet striking the outer bend of the elbow and forming a backwash, the shear action of the water tended to build up the pressure by pulling the air to the downstream end of the conduit. The result was that the vent on top of the conduit next to the elbow ejected air. This resulted in the suggestion that air be taken from this vent and diverted into the vent at the offset in the inner bend of the elbow.
This was not recommended, because the insufflation of air into the water caused an air and water mixture to be ejected from the vent on top of the conduit when the model was operated at a 70-foot head. This made the vent's efficiency questionable since the action in the prototype with respect to the insufflation of air was uncertain. It may be concluded that there will be no adverse pressure conditions in this conduit unless there is a misalignment of the floor or walls. Any irregularity or projection will result in severe subatmospheric pressures.

Pressure Measurements in Elbow, Trough, and on the Face of the Spillway

The piezometers located in the exit elbow, trough, and on the spillway face, as shown in Figures 5 and 14A, permitted a detailed pressure study of the exit section. The results of these studies, based on a design head of 28 feet, are shown in Figure 14. The minimum pressure on the inner bend of the elbow was found to be not less than five or six feet of water below atmospheric, and at no point was there subatmospheric pressures sufficiently intense to cause cavitation in the prototype. There were two regions of subatmospheric pressure, one on the inner bend of the exit elbow, and the other on the face of the spillway immediately below the trough, but both of these regions were vented.
A. PIEZOMETER LOCATIONS

B. GATE 100% OPEN

C. GATE 80% OPEN

D. GATE 60% OPEN

E. GATE 40% OPEN

F. GATE 20% OPEN

NOTE - ALL PRESSURE IN FEET OF WATER BASED ON 200 FOOT HEAD PRESSURE CONTOURS OBTAINED FROM INTERPOLATION OF PIEZOMETER READINGS

BHAKRA DAM
HYDRAULIC MODEL STUDIES 1:17 SCALE
PRESSURES ON OUTLET ELBOW

MODEL SCALE IN INCHES
The size and location of the vent in the elbow was questioned, for it was believed to be larger than necessary. Tests were made by decreasing the size of this opening and there was no appreciable differences in pressure on the inner bend of the elbow until the vent was completely closed. Even with the vent closed, the subatmospheric pressure was not severe at most gate openings. This demonstrated that the vent size was not critical, and when it was decided to use 15-inch vents in the prototype (0.882-inch model), this diameter was considered adequate. This 15-inch vent (Figure 1) was located in a different position in the offset than was the model vent, but such a change of position would not create any adverse flow conditions.

To check the effect of the three 8-inch vents in the face of the spillway below the trough, they were closed. The pressures on the spillway were reduced to the extent that cavitation would occur in the prototype, and it was concluded that venting in this location was necessary. Other types of vents were proposed, but the three 8-inch pipes were simple structurally, and no changes were made.

To prove that the step at the beginning of the inner bend of the elbow was necessary, the model was revised by filling the step with wax until it was flush with the floor of the conduit, similar to the elbow of the original design, as shown in Figure 2. Piezometers were installed
in the inner bend and pressures were measured. Severe negative pressures occurred, so severe in fact, that even on the model after the third, the wax pulled out of place. This was considered proof that the offset in the inner bend of the elbow was a necessary feature of this design.

Air Demand in the Model

The model indicated that subatmospheric pressures would not be below 5 or 6 feet of water, and it was concluded that the air vents were adequate and would operate satisfactorily. The subatmospheric pressures under the most adverse conditions, barring misalignment, should not be lower than 10 feet of water for an operating head of 288 feet.

To have the information available, it was planned to measure the air demand of the outlet to afford a basis of model-prototype comparison. A test program was drafted, but during the course of the testing, erratic and results were obtained. At a constant head/control gate opening, the air demand would vary as much as 100 percent. The tests were carefully rerun, and it was thought that the gate valve in the line and the variable speed pump were partially responsible. The procedure for operating the model was to set the control gate at the desired opening, then set the speed of the pump until the head was greater than desired, and then to set the required head by closing the gate.
valve upstream from the head tank. The air demand of the model would change whenever this procedure was changed, and as a result it was assumed that the air demand was affected by the turbulence of flow in the approach con- 
cit.

Another explanation of the erratic air-demand re-
sults might lie in the fact that there were two air 
vents in the shaker outlet. The elbow could be likened 
to an air-ejector that had an inlet such as the air vent 
in the off-set at the inner bend. With certain flow 
conditions in the outlet, it is possible that this vent 
could not supply the air demand to the elbow, thereby 
making it necessary for additional air to be drawn 
from the vent at the control gate. Although air-demand 
data was taken at scale head, the same condition of 
irregular mixing, which has been shown by the high-speed 
photography, could be present at this lower head. The 
water and air flow through the conduit and elbow may be 
nonuniform. This condition in itself would cause an 
irregular air-pumping action. At present, the writer 
is not prepared to comment further upon this condition, 
or can data obtained be submitted until the problem 
is further studied and the flow conditions are better 
understood.

**Verification of Pressures on the Outer Bend of the Outlet Elbow**

The preliminary structural design of the outer bend
of the elbow and deflector were based on pressures which were greater than those observed by the model tests, and a check was desired to ascertain the accuracy of the model measurements. It was possible to do this in a manner entirely independent of the pressure measurements, because the total pressure vector, a force obtained by summation of the measured pressures, was equal to the force required to turn the water in the elbow. This latter force could be computed by application of the laws of momentum if the discharge or velocity and the angle through which the water turned were known. A comparison was made in this manner for the condition when the control gate upstream was fully open, for then the pressures on the outer bend of the elbow were at a maximum.

The force necessary to turn the water, expressed as horizontal and vertical components, \( F_x \) and \( F_y \), was first estimated by use of the relationship:

\[
F_x = M v (1 - \cos \Theta);
\]

\[
F_y = M v \sin \Theta;
\]

where

\[
M = \text{the mass of the fluid per second, that is } M = \frac{w a v}{g}
\]

\( w \) = the specific weight of the fluid, that is 62.4 lbs. per cubic feet,

\( v \) = the velocity in feet per second,

\( a \) = the area of the flowing jet, and

\( \Theta \) = the angle of deflection of the jet.

Since it was necessary to include the effects of gravity and the pressure change around the bend, the formulae
must be modified to include those factors, thus:

\[ F_x = M \cdot v (1 - \cos \theta) + p \cdot a; \quad F_y = M \cdot v \sin \theta - w \cdot a \cdot L; \]

where

\[ p = \text{the average pressure at the upstream end of the elbow,} \]

\[ L = \text{the average length of the elbow.} \]

In the computation, \( \theta \) was considered as 44 degrees to account for the effect of the lip at the end of the elbow; \( L \) was estimated to be 27.0 feet, and \( p \) was approximately 15 feet of water. The values of the area of the jet \((a)\) and the velocity \((v)\) were more uncertain, and had to be computed in the following manner: In prototype terms, with the control gate wide open and the head at 288 feet, the discharge through the outlet will be 5,410 cubic feet per second, as obtained by the coefficient curve of Figure 4:

\[ A = C \cdot A \cdot \sqrt{\frac{2g}{h}} = 0.70 \times 56.74 \times \sqrt{\frac{64.4}{288}} = 5,410 \text{ c.f.s.} \]

Assuming that the velocity of the jet at the control gate was 95 percent of the theoretical velocity, the jet velocity will be 129.5 feet per second:

\[ v = 0.95 \times \sqrt{64.4 \times 288} = 129.5 \text{ feet per second} \]

in flowing down the pipe, the velocity of the jet is reduced. This reduction was estimated in two ways; first, by frictional losses, as if the flow were in a closed pipe. Assuming that \( f = 0.015 \) in the expression; \( h_f = f \cdot \frac{1}{2} \cdot \frac{v^2}{g} \);

then \[ h_f = \frac{(0.015)(196)(129.5)^2}{(8.5)(64.4)} = 90 \text{ feet} \]

The head at the elbow would then be 170 feet, and the velocity will be 104.6 feet per second.
\[ H = \text{head at gate loss losses} = \frac{(129.5)^2}{64.4} = 90 = 170 \text{ feet} \]

\[ V = \text{velocity at elbow} = \sqrt{2g \times 170} = 104.6 \text{ feet per second} \]

This value for the velocity was uncertain because of the mixing of the air and water; therefore, the velocity was determined in another manner. Air demand measurements indicated that the water-air demand ratio was approximately 3.8:1 at the full gate opening, and it was observed that the air and water mixed in the conduit downstream from the control gate at the 100 percent opening.

The area of the conduit was 64.5 square feet:

\[ a_c = \frac{\pi}{8} (8.5)^2 + 4.25 \times 8.5 = 64.5 \text{ square feet} \]

It follows by proportion of air to water that the area of the water jet was approximately 51.0 square feet at the exit of the conduit where the water and air were mixed.

\[ a = (64.5) \frac{3.8}{1} = 51.0 \text{ square feet} \]

The velocity of the jet will then be 106 feet per second.

\[ v = \frac{5110}{51.0} = 106 \text{ feet per second} \]

This checks the computations determined by the pipe loss method.

With the determination of \( \theta, L, a, \) and \( v, \) the forces on the elbow will be:

\[ F_x = 32.2 \times 51.0 \times (106)^2 \times (1 - 0.710) + 15 \times 51 \times 62.5 = 360,000 \text{ pounds} \]
\[ F_y = \frac{62.5}{32.2} \times 51.0 \times (106)^2 \times 0.935 - 62.5 \times 51 \times 27 \approx 687,000 \text{ pounds} \]

To check these forces by the summation of pressures, the pressure data for the 100 percent opening was integrated over the elbow to give forces of 335,000 pounds for \( F_x \), and 713,000 pounds for \( F_y \), (Figure 15). With this comparison, it may be concluded that the pressure measurements as obtained by the model were reliable.
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APPENDIX

HYDRAULIC SIMILITUDE

A perfect hydraulic model, except from the standpoint of pure geometrical proportions, is a rarity seldom achieved. Yet the mechanics of similitude plays an increasingly important role in the present day concepts of fluid mechanics. Models used by engineers can produce results that may save many times the cost of the model, and produce a certainty of design unobtainable from computations alone.

Similarity of flow occurs between a prototype and its model and may exist between various natural phenomena, if the proper laws of similitude are satisfied. The laws of similitude offer a means of correlating the results obtained from similar flow occurrences in prototype and model. The application of these laws leads to a more comprehensive solution, and to a clearer understanding of the flow phenomena. For complete similarity in fluid flow, it is necessary that the system be geometrically, kinematically and dynamically similar.

**Geometric Similarity**

Two objects or systems are geometrically similar if the ratios of all corresponding linear dimensions are equal. This is independent of motion and involves only similarity of form. Length similarity can be expressed as follows:

\[
\frac{L_m}{L_p} = L_r
\]
where \( L_m \) and \( L_p \) are corresponding linear dimensions in model and prototype respectively, and \( L_r \) is the linear scale ratio.

Therefore, \( \text{Area } A_r = \frac{L_m^2}{L_p^2} \)

\[ \text{Volume } V_r = \frac{L_m^3}{L_m^3} \]

This means that areas in the model multiplied by the square of the scale ratio give prototype areas, and volumes multiplied by the scale ratio cubed gives prototype volume, etc.

**Kinematic and Dynamic Similarity**

Kinematic similarity is a similarity of motion. When the ratios of the components of velocity in two related systems are equal and the paths of motion of the particles are geometrically similar, the two motion occurrences are kinematically similar.

Two motion occurrences are dynamically similar if they are kinematically similar, if the ratios of the forces causing or restraining the motion are equal, and if the ratios of the masses are also equal. This follows Newton's fundamental law of motion which can be written:

\[ M_a = \text{vector sum of forces acting in the direction of flow, or} \]

\[ M_a = (F_g + F_v + F_t + F_s) \]

where \( M_a \) = inertia force

\( F_g \) = gravitational force

\( F_v \) = viscous force
$$F_t = \text{surface tension}$$

$$F_e = \text{elastic forces}$$

For complete similarity in two fluid motion occurrences, the following relationship must be satisfied:

$$\frac{\mu_1 a_1}{\rho_1 a_1} = \frac{(F_g + F_v + F_t + F_e)_1}{(F_g + F_v + F_t + F_e)_m}$$

By separately equating the forces on the right of the equation to the inertia force on the left, it can be shown that the resulting development will yield the Froude number, the Reynolds number, the Weber number, and the Mach Number.

**Significance of Dimensionless Parameters**

Four dimensionless parameters resulting from the above possible development should be satisfied in any fluid flow occurrence. This is physically impossible, and often it is difficult to satisfy even one of the four parameters. In the case of the 102-inch outlets in Shakra Dam, two of the above parameters were partially satisfied, the Reynolds number and Froude number. The Weber number and the Mach number could be neglected, for in the case of the Weber number, the model's flow quantities and velocities were large enough to preclude any effects of surface tension. Since all movements of the jet flow gate will be controlled to prevent a water hammer shock in the outlet, the Mach number could also be neglected.

When viscous forces predominate such as in steady flow through a pressure conduit (section of the outlet
from the bellmouth entrance to the orifice in the jet flow gate), the gravity force, although acting on all particles of the fluid, does not effect the flow picture, and the Froude number can be neglected. Reynolds Law predominates, but was not completely satisfied. Fortunately this was not necessary. If a curve for the dimensionless friction factor "f" in the Darcy-Weisbach formula

\[ h_f = f \frac{L V^2}{D^2 g} \]

plotted against Reynolds number is used, it will be noted that "f" varies considerably for values of Reynolds number between 2,000 and 100,000. Between a Reynolds number of 100,000 and 1,000,000 the slope is more gradual, and beyond 1,000,000 the friction factor levels off to a more or less constant value. Thus, if a model can be constructed sufficiently large to obtain Reynolds numbers which equal or exceed 1,000,000 for all pertinent test flows, the friction factor for the prototype, although the Reynolds number may range up to 20,000,000 will be little different from that of the model.

When gravity forces predominate, such as in the case of the conduit downstream of the jet flow gate where there is a free water surface, the Froude law can be satisfied. By scaling the head producing flow to the gate by a direct linear ratio, and neglecting the small difference in friction loss, the velocity of the entering flow to the conduit was proportional to the prototype velocity. Considering the depth from the free water surface to the
invert of the conduit as "d" and replacing L in the Froude number by "d" then:

$$F_g = \frac{V}{\sqrt{gd}}$$

which would define the free water surface for both model and prototype since boundaries were geometrically similar.

The Euler number, which is a definition of the discharge coefficient for similar boundaries and flow conditions, also holds for the model-prototype relationship.

Reynolds Number Computations

Based on upstream conduit diameter:

$$V = C \sqrt{2gh} \quad h = 26.6 = 16.93 \text{ feet}$$

$$V = 0.7 \sqrt{2 \times 16.93} = 23.1 \text{ fps Figure 4}$$

$$R = \frac{Vd}{\nu} = \frac{23.1 \times 0.5}{1.2 \times 10^{-5}} = 962,000$$

Based on gate orifice diameter:

$$Q = AV = 0.1964 \times 23.1 = 4.54 \text{ cfs}$$

$$V = \frac{Q}{A} = \frac{4.54 \times 4}{\pi \times 0.47^2} = 26 \text{ fps}$$

$$R = \frac{Vd}{\nu} = \frac{26.0 \times 0.47}{1.2 \times 10^{-5}} = 1,020,000$$