

REFINEMENTS IN THE DESIGN
OF OVERFALL SPILLWAY SECTIONS

By

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THESIS

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He also wishes to thank Mr. J. E. Warnock, Director of the Hydraulic Laboratory, under whose direct supervision these studies were made, for his unfailing interest and constructive criticism in this type of research program.

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All statements contained herein are views of the author and do not necessarily constitute opinions of the Bureau of Reclamation.

I. INTRODUCTION

During the past 15 years the writer, who is in charge of the Civil Engineering Section of the United States Bureau of Reclamation Hydraulic Laboratory, has been interested in the design of overfall spillway sections for dams. Partially due to this interest, much general information has been accumulated on the subject both in the laboratory and in the field during that period of time. In addition to data obtained from the more routine hydraulic model tests, the writer has planned and supervised several test programs on basic information concerning certain phases of design for both high and low overfall dam sections. The preparation of this thesis offers an opportunity to compile a portion of these data under one cover. Some of the information has been issued in the form of interoffice reports for the design department, and the remainder is presented here for the first time. It becomes necessary to repeat portions of the above reports in order to develop the line of reasoning in the material which follows. Because of the extent of the data involved, it is intended to stress the results and their applications to design rather than the methods by which the data were actually obtained.

The following is a list of symbols as used throughout the thesis with reference to Figures 1 and 2:

- Q Total discharge, second-feet.
- q Discharge per foot of crest length, second-feet.
- W Width of test channel at gaging section, feet.
- L Length of test weir or overflow section, feet.

A Area of flow cross-section at gaging section, square feet.

V_a Average velocity of approach, feet per second.

h_a Average velocity head of approach, feet.

h_s Observed head above sharp crest of weir (measured at gaging station), feet.

$H_s = h_s + h_a$ - Total head above sharp crest of weir, feet.

E Maximum distance lower nappe rises above sharp crest of weir, feet.

h_o Observed head above high point of lower nappe, feet.

$H_o = h_o + h_a$ - Total designed head above high point of lower nappe, feet.

H Any total head above high point of lower nappe, feet.

P Depth of approach floor below sharp crest of weir, feet.

F Horizontal or vertical displacement on overhang and offset weirs, feet.

M Height of riser on offset weirs, feet.

$C = \frac{Q}{LH_o^{3/2}}$ - Coefficient of discharge for overfall section at designed head (free flow).

C_s Coefficient of discharge corrected for submergence effect.

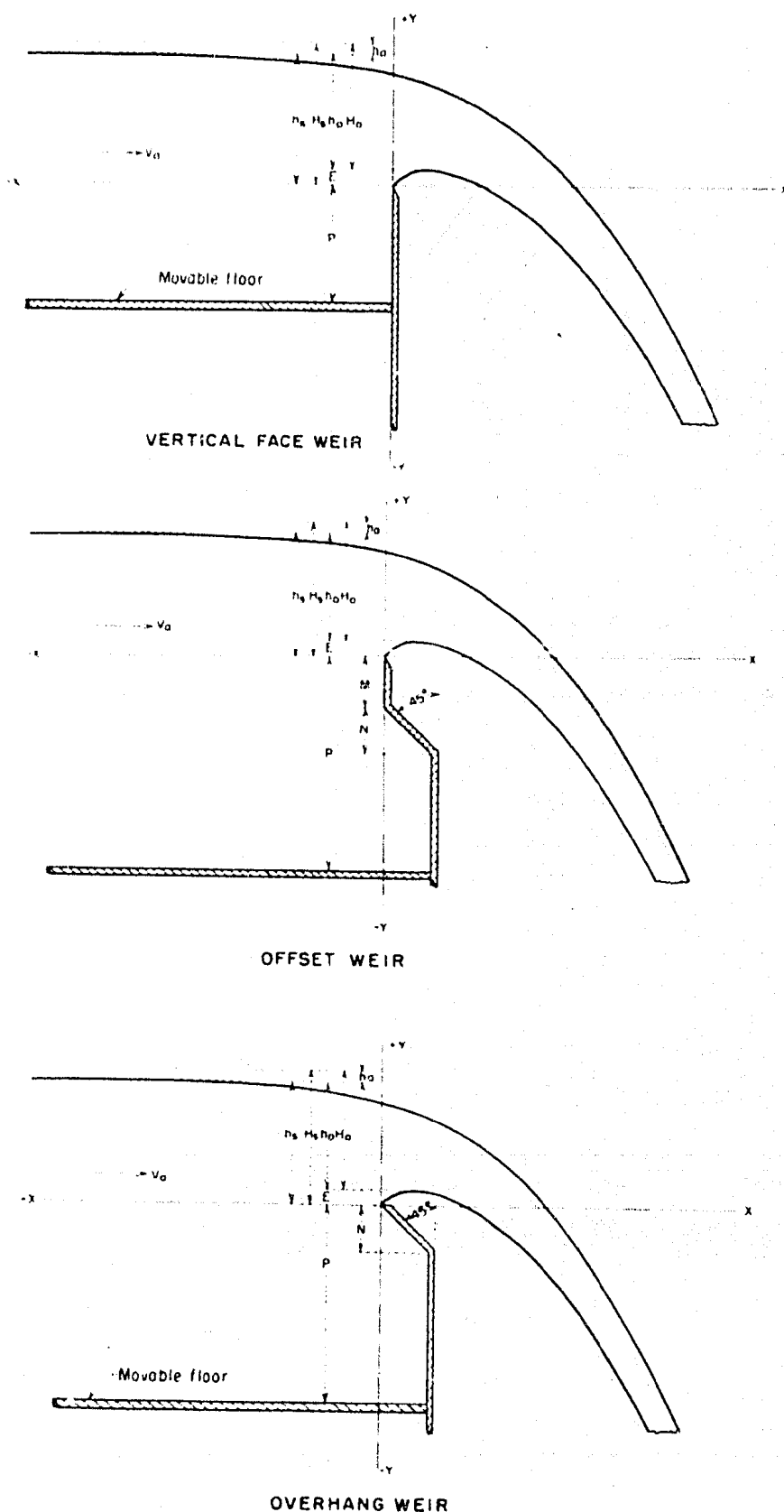
C_1 Coefficient of discharge for ideal shape of section.

C_A Coefficient of discharge for actual shape of section.

h_d Difference between total headwater elevation upstream and tailwater elevation downstream (low dams), feet.

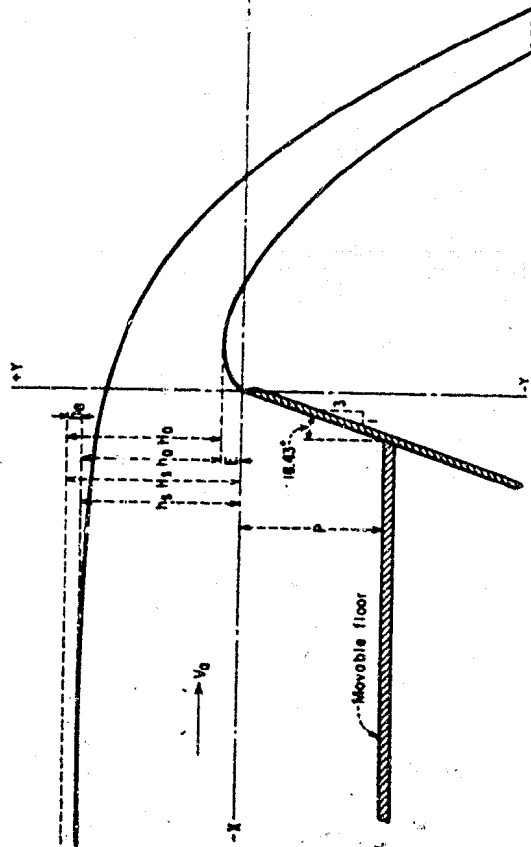
d Depth of flow on downstream apron (low dams), feet.

P_g Depth of horizontal downstream apron below crest of low dam, feet.

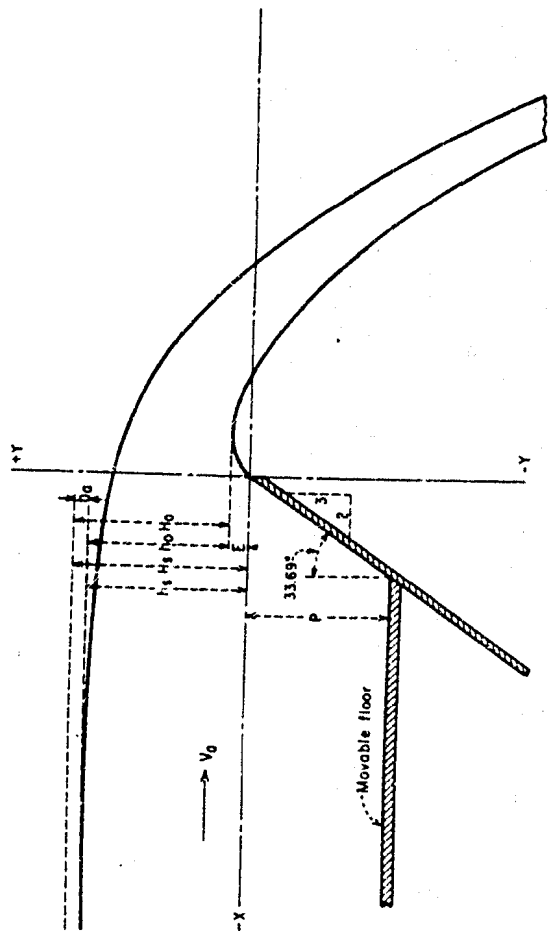


- w - Width of test channel at gaging section, feet.
 L - Length of test weir, feet.
 A - Area of flow cross section at gaging station, square feet.
 Q - Discharge quantity, second-feet.
 v_0 - Velocity of approach, feet per second

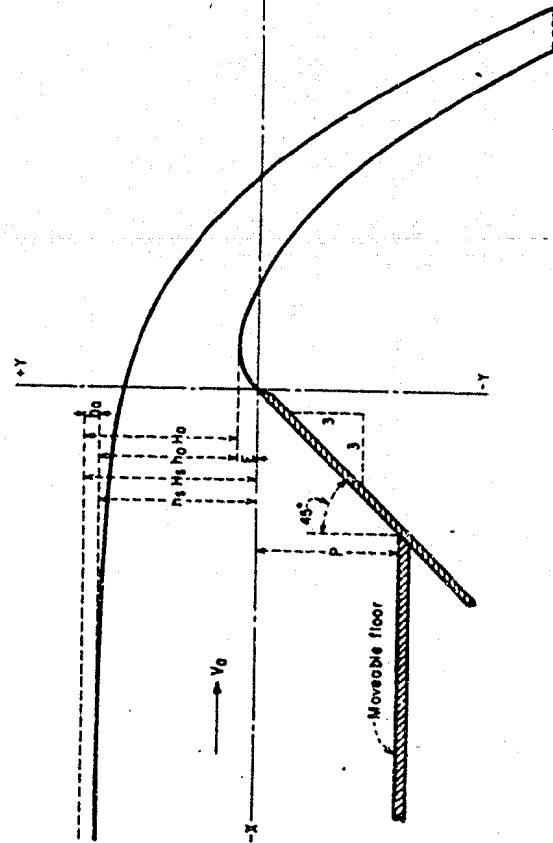
**PRINCIPAL ELEMENTS OF THE OVERFALL CREST FOR DAMS
WITH VERTICAL, OVERHANG AND OFFSET UPSTREAM FACES**



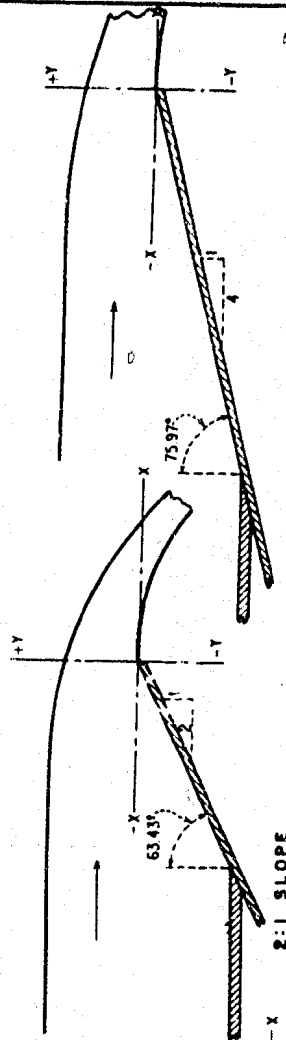
WEIR INCLINED DOWNSTREAM ON 1:3 SLOPE



WEIR INCLINED DOWNSTREAM ON 2:3 SLOPE



WEIR INCLINED DOWNSTREAM ON 3:3 SLOPE



4:1 SLOPE

WEIRS INCLINED DOWNSTREAM-BAZIN

- W - Width of test channel at gaging section, feet
- L - Length of test weir, feet
- A - Area of flow cross section at gaging station, square feet
- Q - Discharge quantity, second feet
- V_0 - Velocity of approach, feet per second

-PRINCIPAL ELEMENTS OF THE OVERFALL CREST
FOR DAMS WITH INCLINED UPSTREAM FACE

II. SHAPES FOR OVERFALL DAMS WHERE ATMOSPHERIC PRESSURE IS DESIRED ON THE DOWNSTREAM FACE

History of previous Work

With the trend in recent years toward higher dams and greater depths of flow over flood spillway sections, the importance of providing the correct overfall profile has been materially increased. It has been the accepted practice for many years to attempt to design the shape of overfall section to fit the lower nappe of the overfalling sheet of water for the maximum discharge condition. This would be the most economical as well as the most efficient shape of section on which subatmospheric pressures would be small or nonexistent. In other words, the pressures on the overfall face would approximate zero (defining zero as atmospheric pressure) for the maximum discharge condition and increase to positive values for all flows less than the maximum. It has been the practice to avoid subatmospheric pressures on the spillway face, as the resulting effects were little understood. Existing structures have proved to be temperamental and in some cases detrimental.

A deficiency of dam section at any point under the nappe will result in the formation of a subatmospheric pressure between the downstream face of the dam and the nappe of water when aeration is suppressed. With sufficient subatmospheric pressure the following undesirable conditions can obtain:

1. The resultant pressure of the spillway section may be increased, due to the reduction of back pressure, which may bring forces to act that would detract from the stability

of the dam against overturning and sliding.

2. The instability of the subatmospheric pressure, with its intermittent pressure change, can in extreme cases cause cavitation and localized disintegration of the boundary known as pitting.

3. The intermittency of the subatmospheric pressure caused by the unstable condition prevailing beneath the flow sheet can cause a state of vibration in the dam. While the amplitude of this vibration is exceedingly small, the accumulation of forces within the dam can produce secondary forces, particularly if the natural frequency of the structure bears a particular relation to that of the vibration of the nappe. This event is usually accompanied by undesirable rumblings in the proximity of the structure. This effect is most likely produced by pressure waves in the atmosphere which rattle windows, doors, and dishes for miles around.

While the design of the proper spillway overfall shape has been the subject of considerable study in the past, the results in most cases have been based either on theory entirely or on rather meager experimental data. The first, most extensive, and perhaps the most reliable studies of nappe shapes were those of Bazin,¹ made in 1886-88, in which he experimented on sharp-crested, suppressed, rectangular weirs of various shapes, and reduced,

¹ Bazin, H., "Recent experiments on the flow of water over weirs," *Annales des Ponts et Chaussées*, October 1888 (translated by Arthur Mariéchal and John C. Trautwine, Jr., and published in the *Proceedings of the Engineers' Club of Philadelphia*, Vol. VII, No. 5, 1890, p. 259, and Vol. IX, No. 3, 1892, p. 231).

ed his observations to unit head and constructed a "base curve" representing the results of his experiments. Although the results are extensive, so far as types of weirs are concerned, the range involved, for any one weir, is not sufficient for the solution of most design problems.

The curve of the lower nappe fascinated E. Boussinesq² who made it the basis of theory of flow over weirs.

So far as known, the first attempt made in American literature to develop the shape of an overfall dam to fit the overflowing sheet was that of Muller³ in 1908. He attempted to extend a curve from the upper section of the lower nappe through Bazin's data. His expression for the curve in the terminology of this thesis was

$$x^2 = 2.5 h_g y \pm 0 \text{ (Figure 1)}$$

for the thread of mean velocity, with the origin of coordinates at approximately $0.35 h_g$ above and $0.09 h_g$ downstream from the theoretical weir crest. He measured downward one-third the thickness of the nappe normal to the thread of mean velocity to locate the curve of the lower surface. Parker⁴ reproduced Muller's curve and demonstrated that it does not fit well with

²Boussinesq, E., "Comptes rendus de l'Académie des Sciences," July 4, 1897.

³Muller, H., "Development of Practical Type of Concrete Spillway Dam," Engineering Rec., Vol. 58, Oct. 24, 1908, P.461.

⁴Parker, F.A.M., "Form of the Downstream Face of Overflow Dams," The Control of Water, p.399, D. Van Nostrand Company, New York, 1916.

Bazin's curve at the upper section. Parker's comment was "The errors in details are plain. Bazin's curves refer to sharp-edged notches, under heads not exceeding 1.7 feet; and Muller applies them to thick notches, under heads of 5 or 10 feet. The principle is a good one, and the process leads to a nice curve."

Morrison and Brodie⁵ offer a parabolic equation for the lower surface of the nappe of

$$x^2 = 1.80 h_0 y$$

where h_0 is the head measured from the highest point of the lower nappe surface, Figure 1. The origin of the coordinates in this case is at the highest point of the lower nappe surface. As a factor of safety for dam design, they recommend that the equation be increased to

$$x^2 = 2.55 h_0 y$$

The equation $x^2 = 1.80 h_0 y$ was also used by the Miami Conservancy District in the design of their spillways. It was used without any increase for factor of safety. It can be shown that actually the lower nappe surface can be only approximately represented by a parabola.

Woodward⁶, in the Miami Conservancy District report, says:

"The profiles of the ogee weirs were designed to conform approximately to the profile of the lower nappe of the overflow from a sharp-crested weir as determined by Bazin's experiments. The profiles as designed agree approximately in their upper portions with the formula $x^2 = 1.8 H_0 y$, where x and y are horizontal and vertical coordinates measured from the crest of the weir and H_0 is the maximum effective head on the ogee weir, including that due to velocity of approach, Figure 1. The dis-

⁵Morrison, E., and Brodie, O.L., Masonry Dam Design, pp.120-133, 2nd Ed., 1916.

⁶Woodward, S.M., "Hydraulics of the Miami Flood Control Project," Technical Reports, Part VII, p. 223.

charge over the spillway will be computed by the formula $q = 3.0 H^{3/2}$.

Cragger proposed an equation

$$x^2 = 2.732y$$

for the line of average velocity in the nappe with the origin

of coordinates 0.063 unit upstream from the face of the weir

and 0.261 unit above the highest point of the lower nappe sur-

face. Bolin¹⁰ shows that the equation given by Cragger is

cases a line which will fall below even the lower surface if it

is continued far enough. This discrepancy, however, is less

than the increase of cross section recommended by Cragger as

a factor of safety.

Bolin¹⁰ established an equation for a portion of the low-

er surface beyond $x = 0.30$ which is

$$y = \frac{(x - 0.10)^2}{1.98} + 0.052x - 0.196$$

in which the origin of the coordinates is at the sharp crest of

the weir. Unfortunately, this equation does not cover the most

important portion of the crest, namely, the portion between the

springing point and the high point of the trajectory.

An empirical equation for the lower surface of the nappe

was derived by H. R. Bamford¹¹, Engineer, War Department, in

which

$$y = 0.525 H - 0.003 x^2 - 1.822$$

with the origin of the coordinates at the highest point of the

Cragger, E. P., Engineering for Highway Dams, pp. 105-110, 1st Ed., 1917.

Bolin¹⁰, Ing. Prof. Kthors, "Bulla forma delle vene trasversali" (on the form of the crest stream), L. Energia Elettrica, Ap. 1930.

lower surface. A similar equation with the same origin of coordinates derived by H. L. Davis, Engineer, Bureau of Reclamation, is

$$y = 0.493 H^{-0.875} x^{1.875}$$

Lamb⁹ derived a set of parametric equations for the shape of the surface of a jet issuing from a sharp-edged orifice. With some modifications and the inclusion of the effect of gravity, equations might be derived which would fit the observed data.

With the additional experimental data contained herein, the development of such equations now offers an opportunity for research to some engineer or mathematician. The foregoing disagreement, except for Basin, represents only one type of flow (that where the velocity of the water approaching the crest of the spillway is negligible). For this reason the Bureau of Reclamation performed an extensive series of laboratory tests on the subject. The resulting information is sufficient for accurate design of the most common overfall sections.

Method of Obtaining Data

The following information, which will be referred to as Bureau of Reclamation data, was obtained from suppressed, stainless-steel, sharp-crested weirs, 2 feet in length, ground to a knife edge. The weirs represented overfall spillway sections with vertical upstream face, sloping upstream faces, and also sections with overhangs and offsets on the upstream face, Fig-

⁹Lamb, H., Hydrodynamics, p. 95, 8th Ed., 1934.

ures 1 and 2. The weirs were installed one at a time in a long flume, 2 feet in width, provided with a moveable floor upstream from the weir and suitable equipment to accurately measure the profiles of the lower and upper nappe surfaces of the sheet of water flowing over the weir. Measurement of the discharge and nappe-shape profiles were made for numerous heads and approach depths upstream from the various weirs. As finally compiled, the nappe-shape coordinates are expressed in terms of the total head on the weir, and other values are expressed in dimensionless terms wherever possible. By this method the experimental data can be conveniently stepped up to prototype proportions according to Froude's Law. The coefficients of discharge herein were computed from the total head above the high point of the lower nappe surface; thus they apply to the resulting ogee section and not to the weir proper.

Nappe-shape profiles (Bureau of Reclamation Data)

The experimental coordinates for profiles of the lower and the upper nappe shapes, obtained from some 200 tests, compiled and expressed in terms of unit head, are included in the following tables for various values of $\frac{h_a}{H_0}$. (See Figure 1 for symbols).

Use upper nappe-shape coordinates for vertical-face weir.

Lower nappe : Upper nappe : coordinates, : coordinates, :		Type of weir
Table	Table	
1	9	: Vertical upstream face
2	10	: Slope 1:3 downstrm (18.43° with vertical)
3	11	: Slope 2:3 downstrm (33.69° with vertical)
4	12	: Slope 3:3 downstrm (45° with vertical)
Bazin 5	13	: Slope 2:1 downstrm (63.43° with vertical)
Bazin 6	14	: Slope 4:1 downstrm (75.97° with vertical)
7	15	: 45-degree overhang weirs
8	16	: 45-degree offset weirs with risers

One would expect to obtain the same nappe-shape profiles for a 45-degree offset weir with reasonably high riser as for the weir with vertical upstream face. This is not entirely true, as the type of flow is different on the two weirs. It was observed that the lower nappe surface, during the experiments on the vertical weir and on the three weirs tested sloping downstream, was smooth, having a glassy appearance during runs in which the head was not excessive or the approach channel too shallow. On the contrary, the nappe surfaces produced by the offset and the overhang weirs were in some cases smooth but never glassy. In every one of the latter arrangements tested, spiralling ropes played back and forth across the weir, making it difficult to traverse the lower nappe surface. These were small for the lower heads but became proportionately larger and greater in number for the higher discharges. The ropes formed at the weir and broke into spray a short distance downstream. The roughness of the nappe surfaces indicated that some energy dissipation occurred before the water left the weir and also that the resistance of the air to the rough surfaces was greater than it would have been to the smoother surfaces produced by the vertical weir. For these reasons the nappe shapes from the offset weirs do not coincide with those from the vertical weir. It so happened that the lower nappe surfaces from all reliable offset weirs could be combined and this was done to obtain Table 8. Overflow sections with plain upstream faces are conducive to stable flow conditions, whereas some of those with offsets and breaks in the upstream face are questionable, although many of the latter sections are in existence and appear to be giving satisfactory service. One

TABLE 5 - WEIR INCLINED DOWNSTREAM ON 2 TO 1 SLOPE (63.43° WITH VERTICAL)
COORDINATES OF LOWER NAPPE FOR DIFFERENT VALUES OF h_0/H_1
BAZIN'S EXPERIMENTS

Y/H_1												
h_0/H_1 X/H_1	h_0/H_1											
	0.002	0.010	0.020	0.030	0.040	0.050	0.060	0.070	X/H_1	0.002	0.010	0.020
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.080	-0.081	-0.083
0.010	0.004	0.004	0.003	0.003	0.003	0.002	0.002	0.002	0.002	-0.090	-0.091	-0.093
0.020	0.006	0.006	0.005	0.005	0.004	0.004	0.004	0.004	0.004	-0.100	-0.101	-0.103
0.030	0.009	0.009	0.008	0.008	0.007	0.007	0.006	0.006	0.006	-0.110	-0.111	-0.113
0.040	0.011	0.011	0.010	0.010	0.009	0.009	0.008	0.008	0.008	-0.120	-0.121	-0.123
0.050	0.012	0.012	0.011	0.011	0.010	0.010	0.009	0.009	0.009	-0.130	-0.131	-0.133
0.060	0.014	0.014	0.013	0.013	0.012	0.012	0.011	0.011	0.011	-0.141	-0.142	-0.144
0.070	0.016	0.016	0.015	0.015	0.014	0.014	0.013	0.013	0.013	-0.153	-0.154	-0.156
0.080	0.016	0.016	0.015	0.015	0.014	0.014	0.013	0.013	0.013	-0.164	-0.165	-0.167
0.090	0.016	0.016	0.015	0.015	0.014	0.014	0.013	0.013	0.013	-0.176	-0.177	-0.179
0.100	0.016	0.016	0.015	0.015	0.014	0.014	0.013	0.013	0.013	-0.188	-0.189	-0.191
0.110	0.016	0.016	0.015	0.015	0.014	0.014	0.013	0.013	0.013	-0.201	-0.202	-0.203
0.120	0.016	0.016	0.015	0.015	0.014	0.014	0.013	0.013	0.013	-0.214	-0.215	-0.216
0.130	0.016	0.016	0.015	0.015	0.014	0.014	0.013	0.013	0.013	-0.227	-0.228	-0.229
0.140	0.015	0.015	0.014	0.014	0.013	0.013	0.012	0.012	0.012	-0.240	-0.241	-0.242
0.150	0.014	0.014	0.013	0.013	0.012	0.012	0.011	0.011	0.011	-0.254	-0.254	-0.255
0.160	0.014	0.014	0.013	0.013	0.012	0.012	0.011	0.011	0.011	-0.268	-0.268	-0.269
0.170	0.013	0.013	0.012	0.012	0.011	0.011	0.010	0.010	0.010	-0.282	-0.282	-0.283
0.180	0.012	0.012	0.011	0.011	0.010	0.010	0.009	0.009	0.009	-0.296	-0.296	-0.297
0.190	0.011	0.011	0.010	0.010	0.009	0.009	0.008	0.008	0.008	-0.310	-0.311	-0.312
0.200	0.010	0.010	0.009	0.009	0.008	0.008	0.007	0.007	0.007	-0.324	-0.324	-0.325
0.210	0.008	0.008	0.007	0.007	0.006	0.006	0.005	0.005	0.005	-0.338	-0.338	-0.339
0.220	0.006	0.006	0.005	0.005	0.004	0.004	0.003	0.003	0.003	-0.352	-0.352	-0.353
0.230	0.004	0.004	0.003	0.003	0.002	0.002	0.001	0.001	0.001	-0.366	-0.366	-0.367
0.240	0.003	0.003	0.002	0.002	0.001	0.001	0.000	0.000	0.000	-0.380	-0.380	-0.381
0.250	0.001	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.394	-0.394	-0.395
0.260	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.408	-0.408	-0.409
0.270	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.422	-0.422	-0.423
0.280	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.436	-0.436	-0.437
0.290	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.450	-0.450	-0.451
0.300	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.464	-0.464	-0.465
0.310	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.478	-0.478	-0.479
0.320	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.492	-0.492	-0.493
0.330	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.506	-0.506	-0.507
0.340	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.520	-0.520	-0.521
0.350	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.534	-0.534	-0.535
0.360	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.548	-0.548	-0.549
0.370	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.562	-0.562	-0.563
0.380	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.576	-0.576	-0.577
0.390	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.590	-0.590	-0.591
0.400	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.604	-0.604	-0.605
0.410	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.618	-0.618	-0.619
0.420	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.632	-0.632	-0.633
0.430	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.646	-0.646	-0.647
0.440	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.660	-0.660	-0.661
0.450	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.674	-0.674	-0.675
0.460	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.688	-0.688	-0.689
0.470	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.702	-0.702	-0.703
0.480	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.716	-0.716	-0.717
0.490	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.730	-0.730	-0.731

TABLE 9 - WEIR WITH VERTICAL UPSTREAM FACE
COORDINATES OF UPPER NAPPE FOR DIFFERENT VALUES OF h_s/H_s

		Y/H_s										
h_s/H_s	X/H_s	0.002	0.020	0.040	0.060	0.080	0.100	0.120	0.140	0.160	0.180	0.200
-4.000	0.990											
-3.900	0.990											
-3.800	0.990											
-3.700	0.990		0.974	0.958								
-3.600	0.990		0.974	0.958								
-3.500	0.990		0.974	0.958	0.934	0.911						
-3.400	0.990		0.974	0.957	0.934	0.910						
-3.300	0.990		0.972	0.955	0.932	0.910						
-3.200	0.990		0.972	0.954	0.932	0.910	0.878					
-3.100	0.989		0.971	0.953	0.932	0.910	0.877					
-3.000	0.988	0.970		0.952	0.931	0.910	0.876	0.844	0.812			
-2.900	0.988	0.969		0.950	0.930	0.910	0.875	0.844	0.812			
-2.800	0.987	0.968		0.950	0.930	0.909	0.874	0.843	0.811			
-2.700	0.986	0.968		0.949	0.929	0.909	0.873	0.842	0.811			
-2.600	0.985	0.966		0.948	0.928	0.908	0.872	0.841	0.810			
-2.500	0.984	0.966	0.947	0.928	0.908	0.872	0.841	0.810	0.782			
-2.400	0.983	0.965	0.947	0.927	0.907	0.871	0.840	0.809	0.781			
-2.300	0.982	0.964	0.946	0.926	0.906	0.871	0.840	0.809	0.780			
-2.200	0.981	0.963	0.945	0.925	0.905	0.870	0.839	0.808	0.779	0.760		0.743
-2.100	0.980	0.962	0.944	0.924	0.904	0.870	0.839	0.808	0.778	0.760		0.742
-2.000	0.979	0.961	0.943	0.923	0.903	0.869	0.838	0.807	0.777	0.758		0.741
-1.900	0.978	0.960	0.942	0.922	0.903	0.869	0.838	0.807	0.776	0.758		0.740
-1.800	0.977	0.959	0.941	0.922	0.902	0.868	0.837	0.806	0.776	0.757		0.739
-1.700	0.975	0.958	0.940	0.920	0.901	0.868	0.837	0.806	0.775	0.756		0.738
-1.600	0.974	0.956	0.939	0.920	0.901	0.867	0.836	0.805	0.774	0.755		0.737
-1.500	0.972	0.955	0.938	0.919	0.900	0.866	0.835	0.804	0.773	0.754		0.736
-1.400	0.970	0.954	0.937	0.918	0.899	0.865	0.834	0.803	0.771	0.753		0.735
-1.300	0.967	0.952	0.936	0.917	0.898	0.864	0.833	0.802	0.770	0.752		0.734
-1.200	0.964	0.949	0.934	0.916	0.897	0.863	0.832	0.801	0.769	0.751		0.733
-1.100	0.961	0.946	0.931	0.914	0.894	0.861	0.831	0.800	0.768	0.750		0.732
-1.000	0.958	0.944	0.929	0.911	0.892	0.859	0.829	0.799	0.767	0.749		0.731
-0.900	0.952	0.938	0.924	0.907	0.890	0.856	0.827	0.798	0.765	0.747		0.730
-0.800	0.947	0.934	0.920	0.902	0.885	0.850	0.822	0.795	0.763	0.745		0.727
-0.700	0.940	0.926	0.913	0.896	0.880	0.846	0.820	0.793	0.761	0.742		0.724
-0.600	0.930	0.918	0.907	0.890	0.873	0.840	0.815	0.790	0.759	0.740		0.721
-0.500	0.921	0.910	0.898	0.882	0.865	0.832	0.808	0.783	0.755	0.736		0.718
-0.400	0.911	0.899	0.887	0.872	0.856	0.823	0.800	0.777	0.750	0.731		0.713
-0.300	0.899	0.886	0.873	0.858	0.844	0.812	0.791	0.770	0.744	0.726		0.707
-0.200	0.883	0.871	0.859	0.844	0.830	0.800	0.780	0.760	0.735	0.718		0.700
-0.100	0.866	0.853	0.842	0.826	0.812	0.784	0.766	0.748	0.724	0.708		0.692
0.000	0.845	0.832	0.819	0.804	0.790	0.764	0.748	0.732	0.710	0.696		0.682
0.100	0.820	0.806	0.793	0.780	0.767	0.740	0.726	0.713	0.693	0.680		0.667
0.200	0.790	0.778	0.765	0.752	0.738	0.717	0.703	0.690	0.671	0.660		0.650
0.300	0.755	0.742	0.730	0.718	0.706	0.686	0.673	0.660	0.644	0.635		0.627
0.400	0.714	0.702	0.690	0.680	0.669	0.650	0.637	0.625	0.610	0.603		0.595
0.500	0.670	0.658	0.645	0.634	0.624	0.608	0.596	0.583	0.570	0.564		0.558
0.600	0.619	0.606	0.594	0.584	0.575	0.558	0.546	0.533	0.523	0.518		0.512
0.700	0.560	0.548	0.536	0.528	0.519	0.500	0.490	0.480	0.470	0.465		0.460
0.800	0.494	0.482	0.470	0.462	0.453	0.440	0.429	0.418	0.410	0.404		0.400
0.900	0.423	0.412	0.400	0.390	0.380	0.368	0.359	0.349	0.340	0.335		0.330
1.000	0.344	0.332	0.320	0.310	0.300	0.290	0.280	0.270	0.263	0.258		0.253
1.100	0.252	0.241	0.230	0.222	0.213	0.200	0.194	0.188	0.180	0.175		0.170
1.200	0.153	0.144	0.134	0.127	0.120	0.108	0.102	0.095	0.088	0.083		0.078
1.300	0.048	0.039	0.030	0.024	0.018	0.005	0.000	-0.005	-0.012	-0.016		-0.021
1.400	-0.067	-0.074	-0.080	-0.086	-0.091	-0.104	-0.108	-0.112	-0.120	-0.125		-0.130
1.500	-0.190	-0.195	-0.200	-0.205	-0.210	-0.220	-0.225	-0.230	-0.240	-0.244		-0.245
1.600	-0.317	-0.322	-0.328	-0.332	-0.336	-0.349	-0.352	-0.355	-0.362	-0.366		-0.370
1.700	-0.450	-0.456	-0.461	-0.466	-0.470	-0.481	-0.484	-0.488	-0.496	-0.498		-0.501
1.800	-0.597	-0.601	-0.605	-0.608	-0.612	-0.622	-0.626	-0.630	-0.636	-0.638		-0.641
1.900	-0.750	-0.754	-0.758	-0.761	-0.764	-0.770	-0.774	-0.779	-0.783	-0.786		-0.790
2.000	-0.905	-0.908	-0.911	-0.916	-0.920	-0.925	-0.928	-0.930	-0.934	-0.936		-0.937
2.100	-1.080	-1.082	-1.085	-1.086	-1.088	-1.090	-1.092	-1.095	-1.092	-1.092		-1.095
2.200	-1.255	-1.264	-1.273	-1.272	-1.270	-1.271	-1.268	-1.265	-1.264	-1.262		-1.260
2.300	-1.450	-1.458	-1.465	-1.464	-1.462	-1.460	-1.452	-1.445	-1.442	-1.438		-1.433
2.400	-1.652	-1.661	-1.670	-1.666	-1.662	-1.658	-1.649	-1.640	-1.630	-1.625		-1.620
2.500	-1.865	-1.872	-1.880	-1.876	-1.872	-1.860	-1.850	-1.840	-1.826	-1.818		-1.810
2.600	-2.090	-2.090	-2.090	-2.086	-2.082	-2.068	-2.057	-2.046	-2.023	-2.016		-2.008
2.700	-2.322	-2.317	-2.312	-2.304	-2.296	-2.278	-2.264	-2.250	-2.223	-2.214		-2.207
2.800	-2.554	-2.546	-2.538	-2.526	-2.515	-2.492	-2.476	-2.460	-2.434	-2.422		-2.410
2.900	-2.790	-2.778	-2.767	-2.754	-2.740	-2.719	-2.701	-2.680	-2.655	-2.644		-2.632
3.000	-3.032	-3.021	-3.010	-2.995	-2.980	-2.958	-2.939	-2.910	-2.892	-2.881		-2.870
3.100	-3.285	-3.272	-3.260	-3.245	-3.230	-3.208	-3.192	-3.160	-3.145	-3.132		-3.120
3.200	-3.540	-3.528	-3.515	-3.502	-3.490	-3.462	-3.447	-3.418	-3.405	-3.394		-3.385
3.300	-3.803	-3.792	-3.780	-3.766	-3.752	-3.725	-3.714	-3.690	-3.676	-3.664		-3.654
3.400	-4.074	-4.062	-4.050	-4.040	-4.030	-4.008	-3.994	-3.972	-3.955	-3.946		-3.940

TABLE 13- WEIR INCLINED DOWNSTREAM ON 2 TO 1 SLOPE (63.43° WITH VERTICAL)
COORDINATES OF UPPER NAPPE FOR DIFFERENT VALUES OF h_0/H_s
BAZIN'S EXPERIMENTS

Y/H_s											
h_0/H_s X/H_s	0.002	0.02	0.04	0.06	0.07	h_0/H_s X/H_s	0.002	0.02	0.04	0.06	0.07
-3.500	0.969	0.961	0.950	0.941	0.937	-0.500	0.831	0.828	0.824	0.821	0.819
-3.400	0.968	0.959	0.950	0.940	0.935	-0.400	0.816	0.813	0.809	0.805	0.803
-3.300	0.967	0.957	0.948	0.938	0.933	-0.300	0.798	0.795	0.791	0.787	0.785
-3.200	0.966	0.956	0.947	0.937	0.932	-0.200	0.778	0.775	0.771	0.768	0.766
-3.100	0.965	0.955	0.946	0.936	0.931	-0.100	0.755	0.751	0.748	0.744	0.742
-3.000	0.963	0.954	0.944	0.935	0.930	0.000	0.728	0.724	0.721	0.717	0.715
-2.900	0.961	0.952	0.943	0.934	0.929	0.100	0.696	0.693	0.690	0.687	0.685
-2.800	0.959	0.950	0.941	0.932	0.928	0.200	0.662	0.659	0.655	0.652	0.650
-2.700	0.957	0.948	0.939	0.930	0.926	0.300	0.624	0.620	0.616	0.612	0.610
-2.600	0.955	0.946	0.937	0.928	0.924	0.400	0.580	0.577	0.574	0.570	0.569
-2.500	0.952	0.943	0.935	0.926	0.922	0.500	0.531	0.528	0.525	0.522	0.520
-2.400	0.950	0.942	0.933	0.925	0.921	0.600	0.474	0.470	0.467	0.463	0.461
-2.300	0.948	0.940	0.931	0.923	0.919	0.700	0.411	0.408	0.405	0.402	0.400
-2.200	0.944	0.936	0.929	0.921	0.917	0.800	0.342	0.339	0.335	0.332	0.330
-2.100	0.941	0.934	0.926	0.919	0.915	0.900	0.267	0.263	0.260	0.256	0.254
-2.000	0.938	0.931	0.923	0.916	0.912	1.000	0.185	0.181	0.176	0.172	0.170
-1.900	0.935	0.928	0.920	0.913	0.909	1.100	0.095	0.091	0.086	0.082	0.080
-1.800	0.930	0.923	0.916	0.909	0.905	1.200	0.000	-0.004	-0.009	-0.013	-0.015
-1.700	0.927	0.920	0.912	0.905	0.901	1.300	-0.105	-0.108	-0.112	-0.115	-0.117
-1.600	0.923	0.916	0.909	0.902	0.898	1.400	-0.214	-0.217	-0.220	-0.223	-0.225
-1.500	0.919	0.912	0.905	0.898	0.894	1.500	-0.334	-0.337	-0.340	-0.343	-0.344
-1.400	0.913	0.906	0.900	0.893	0.890	1.600	-0.460	-0.463	-0.466	-0.469	-0.470
-1.300	0.908	0.902	0.895	0.889	0.886	1.700	-0.591	-0.594	-0.596	-0.599	-0.600
-1.200	0.902	0.896	0.889	0.883	0.880	1.800	-0.730	-0.733	-0.736	-0.739	-0.740
-1.100	0.895	0.889	0.884	0.878	0.875	1.900	-0.875	-0.876	-0.878	-0.879	-0.880
-1.000	0.889	0.884	0.878	0.873	0.870	2.000	-1.026	-1.028	-1.029	-1.031	-1.032
- .900	0.880	0.875	0.870	0.865	0.862	2.100	-1.190	-1.190	-1.190	-1.190	-1.190
- .800	0.870	0.865	0.860	0.855	0.852	2.200	-1.363	-1.362	-1.361	-1.360	-1.360
- .700	0.860	0.855	0.851	0.846	0.844	2.300	-1.542	-1.540	-1.538	-1.536	-1.535
- .600	0.845	0.843	0.838	0.834	0.832	2.400	-1.730	-1.726	-1.723	-1.719	-1.717

TABLE 14- WEIR INCLINED DOWNSTREAM ON 4 TO 1 SLOPE (75.97° WITH VERTICAL)
COORDINATES OF UPPER NAPPE FOR DIFFERENT VALUES OF h_a/H_s

y/H_s											
$\frac{h_a/H_s}{x/H_s}$	0.002	0.02	0.04	0.06	0.07	$\frac{h_a/H_s}{x/H_s}$	0.002	0.02	0.04	0.06	0.07
-3.500	0.934	0.931	0.928	0.925	0.923	-0.500	0.766	0.762	0.758	0.754	0.752
-3.400	0.933	0.930	0.927	0.924	0.922	-0.400	0.749	0.746	0.742	0.739	0.737
-3.300	0.931	0.928	0.925	0.922	0.921	-0.300	0.730	0.726	0.723	0.719	0.717
-3.200	0.929	0.926	0.923	0.920	0.919	-0.200	0.708	0.705	0.702	0.699	0.697
-3.100	0.927	0.924	0.921	0.918	0.917	-0.100	0.685	0.681	0.677	0.673	0.671
-3.000	0.926	0.923	0.920	0.917	0.915	0.000	0.658	0.654	0.651	0.647	0.645
-2.900	0.923	0.920	0.917	0.914	0.912	0.100	0.627	0.623	0.619	0.615	0.613
-2.800	0.921	0.918	0.915	0.912	0.910	0.200	0.592	0.588	0.584	0.580	0.578
-2.700	0.918	0.915	0.912	0.909	0.907	0.300	0.551	0.547	0.544	0.540	0.537
-2.600	0.914	0.911	0.909	0.906	0.905	0.400	0.503	0.499	0.495	0.491	0.485
-2.500	0.912	0.909	0.906	0.903	0.902	0.500	0.451	0.447	0.442	0.438	0.436
-2.400	0.908	0.905	0.903	0.900	0.899	0.600	0.393	0.388	0.384	0.379	0.377
-2.300	0.904	0.902	0.899	0.897	0.896	0.700	0.330	0.326	0.321	0.317	0.315
-2.200	0.901	0.898	0.896	0.893	0.892	0.800	0.259	0.254	0.249	0.244	0.242
-2.100	0.898	0.895	0.892	0.889	0.888	0.900	0.178	0.173	0.168	0.163	0.161
-2.000	0.894	0.891	0.888	0.885	0.884	1.000	0.092	0.087	0.082	0.077	0.074
-1.900	0.890	0.887	0.884	0.881	0.880	1.100	-0.002	-0.007	-0.012	-0.017	-0.020
-1.800	0.886	0.883	0.880	0.877	0.875	1.200	-0.103	-0.008	-0.013	-0.018	-0.021
-1.700	0.881	0.878	0.875	0.872	0.870	1.300	-0.212	-0.217	-0.223	-0.228	-0.231
-1.600	0.876	0.873	0.870	0.867	0.865	1.400	-0.327	-0.332	-0.338	-0.343	-0.346
-1.500	0.871	0.868	0.865	0.862	0.860	1.500	-0.448	-0.452	-0.457	-0.461	-0.463
-1.400	0.864	0.861	0.858	0.855	0.853	1.600	-0.572	-0.576	-0.581	-0.585	-0.587
-1.300	0.857	0.854	0.851	0.848	0.846	1.700	-0.705	-0.709	-0.714	-0.718	-0.720
-1.200	0.848	0.845	0.842	0.839	0.837	1.800	-0.846	-0.850	-0.853	-0.857	-0.859
-1.100	0.840	0.837	0.833	0.830	0.828	1.900	-1.000	-1.003	-1.006	-1.009	-1.011
-1.000	0.830	0.827	0.823	0.820	0.818	2.000	-1.161	-1.164	-1.166	-1.169	-1.170
-0.900	0.819	0.815	0.812	0.808	0.806	2.100	-1.328	-1.331	-1.334	-1.337	-1.338
-0.800	0.807	0.804	0.800	0.797	0.795	2.200	-1.507	-1.509	-1.511	-1.513	-1.514
-0.700	0.793	0.789	0.786	0.782	0.780	2.300	-1.692	-1.694	-1.695	-1.697	-1.698
-0.600	0.780	0.776	0.772	0.768	0.766	2.400	-1.895	-1.886	-1.887	-1.888	-1.886

TABLE 15-WEIR WITH 45-DEGREE UPSTREAM OVERHANG ($M/N=0$)
COORDINATES OF UPPER NAPPE FOR DIFFERENT VALUES OF h_a/H_s

		Y/H_s							
X/H_s	h_a/H_s	0.002	0.02	0.04	0.06	0.08	0.10	0.12	0.14
-3.500	0.995	0.972							
-3.400	0.994	0.972							
-3.300	0.994	0.972							
-3.200	0.994	0.972	0.949	0.925					
-3.100	0.994	0.972	0.948	0.924	0.908	0.886	0.858	0.831	
-3.000	0.994	0.972	0.948	0.923	0.907	0.885	0.858	0.831	
-2.900	0.994	0.971	0.947	0.922	0.906	0.884	0.857	0.831	
-2.800	0.994	0.971	0.946	0.921	0.905	0.884	0.857	0.830	
-2.700	0.993	0.970	0.945	0.921	0.904	0.882	0.856	0.830	
-2.600	0.993	0.970	0.945	0.920	0.902	0.881	0.856	0.830	
-2.500	0.993	0.969	0.945	0.920	0.901	0.880	0.855	0.830	
-2.400	0.993	0.969	0.944	0.919	0.900	0.879	0.855	0.829	
-2.300	0.993	0.969	0.944	0.919	0.900	0.878	0.854	0.829	
-2.200	0.992	0.968	0.943	0.918	0.899	0.877	0.853	0.829	
-2.100	0.992	0.968	0.943	0.918	0.898	0.876	0.853	0.829	
-2.000	0.992	0.968	0.943	0.917	0.897	0.875	0.852	0.828	
-1.900	0.992	0.967	0.942	0.916	0.896	0.875	0.852	0.828	
-1.800	0.991	0.967	0.942	0.915	0.894	0.873	0.851	0.828	
-1.700	0.991	0.966	0.942	0.914	0.893	0.872	0.851	0.827	
-1.600	0.990	0.966	0.942	0.913	0.892	0.871	0.850	0.827	
-1.500	0.988	0.965	0.939	0.912	0.891	0.870	0.850	0.826	
-1.400	0.987	0.964	0.939	0.912	0.890	0.869	0.849	0.826	
-1.300	0.982	0.961	0.938	0.911	0.889	0.868	0.848	0.825	
-1.200	0.979	0.960	0.937	0.910	0.889	0.867	0.845	0.823	
-1.100	0.975	0.956	0.934	0.909	0.887	0.865	0.842	0.819	
-1.000	0.970	0.951	0.931	0.908	0.885	0.861	0.838	0.815	
-0.900	0.966	0.947	0.927	0.905	0.880	0.856	0.833	0.810	
-0.800	0.960	0.941	0.922	0.901	0.877	0.853	0.830	0.807	
-0.700	0.953	0.934	0.915	0.896	0.872	0.848	0.825	0.802	
-0.600	0.947	0.928	0.908	0.888	0.866	0.843	0.818	0.793	
-0.500	0.938	0.918	0.899	0.880	0.859	0.836	0.811	0.786	
-0.400	0.928	0.909	0.889	0.871	0.850	0.828	0.803	0.778	
-0.300	0.914	0.895	0.878	0.861	0.840	0.818	0.793	0.768	
-0.200	0.898	0.879	0.862	0.846	0.827	0.806	0.782	0.758	
-0.100	0.882	0.863	0.845	0.829	0.811	0.791	0.768	0.745	
0.000	0.860	0.842	0.825	0.809	0.792	0.772	0.751	0.730	
0.100	0.832	0.815	0.799	0.784	0.769	0.752	0.733	0.714	
0.200	0.803	0.786	0.770	0.754	0.739	0.724	0.708	0.692	
0.300	0.767	0.751	0.736	0.721	0.707	0.693	0.678	0.663	
0.400	0.724	0.709	0.695	0.680	0.667	0.654	0.643	0.632	
0.500	0.676	0.661	0.647	0.635	0.622	0.611	0.601	0.591	
0.600	0.622	0.607	0.594	0.582	0.570	0.559	0.550	0.541	
0.700	0.562	0.547	0.534	0.522	0.511	0.501	0.492	0.483	
0.800	0.496	0.481	0.469	0.458	0.446	0.436	0.429	0.422	
0.900	0.422	0.407	0.394	0.383	0.373	0.365	0.358	0.351	
1.000	0.338	0.327	0.315	0.302	0.294	0.287	0.280	0.273	
1.100	0.248	0.237	0.226	0.215	0.208	0.202	0.197	0.192	
1.200	0.149	0.138	0.129	0.120	0.114	0.108	0.103	0.098	
1.300	0.038	0.030	0.022	0.015	0.012	0.008	0.003	-0.002	
1.400	-0.078	-0.087	-0.094	-0.097	-0.102	-0.105	-0.106	-0.007	
1.500	-0.208	-0.215	-0.220	-0.222	-0.222	-0.222	-0.222	-0.222	
1.600	-0.347	-0.353	-0.355	-0.352	-0.352	-0.351	-0.350	-0.349	
1.700	-0.500	-0.501	-0.499	-0.492	-0.492	-0.490	-0.485	-0.480	
1.800	-0.650	-0.652	-0.652	-0.650	-0.645	-0.638	-0.630	-0.622	
1.900	-0.810	-0.812	-0.812	-0.811	-0.804	-0.793	-0.780	-0.767	
2.000	-0.983	-0.982	-0.980	-0.975	-0.960	-0.946	-0.935	-0.924	
2.100	-1.160	-1.157	-1.152	-1.147	-1.130	-1.115	-1.100	-0.985	
2.200	-1.350	-1.341	-1.333	-1.325	-1.308	-1.292	-1.277	-1.262	
2.300	-1.544	-1.535	-1.523	-1.508	-1.495	-1.479	-1.460	-1.441	
2.400	-1.750	-1.737	-1.723	-1.707	-1.688	-1.669	-1.650	-1.631	
2.500	-1.960	-1.947	-1.932	-1.915	-1.892	-1.870	-1.850	-1.830	
2.600	-2.170	-2.157	-2.141	-2.122	-2.101	-2.079	-2.055	-2.031	

point is definitely certain, however: offset spillway sections should not be designed with values of $\frac{H}{H_0}$ equal to less than 0.5 unless that value is zero. For values of $\frac{H}{H_0}$ between zero and 0.5, flow conditions were extremely unstable from offset weirs as the spring point was not necessarily at the sharp crest of the weir for this case.

The upper nappe-shape coordinates, shown in Tables 9, 10, 11, 12, 13, 14, and 15, are not as dependable as those for the lower surface, as other factors may alter their profiles. The upper surface of a sheet of water flowing over a dam is exposed to the atmosphere and may be altered by wind, air currents, or by insufflation of air by the water. When water, exposed to air, reaches a velocity of approximately 20 feet per second, air is entrained. As the velocity increases, the insufflation becomes more pronounced, and air actually mixes with the water. C. W. Thomas¹⁰ found, while taking measurements on a long, steep, irrigation wasteway, that water flowing at a depth of approximately 8 feet absorbed 50 percent of air, by volume, from the atmosphere for a velocity of 80 feet per second. The upper nappe-shape coordinates, therefore, represent only the ideal case where air is not considered. These experimental upper coordinates, however, may be used as a starting point from which the designer may make allowances for other influences.

Relation of E to Velocity of Approach

In the design of overflow sections it is necessary to interchange heads from those observed on the weir, h_s and H_s , to

¹⁰Thomas, C.W., "Progress Report on Studies of the Flow of Water in Open Channels with High Gradients," July 27, 1933, U.S.B.R., Report Hyd. 35.

the most economical overfall dam for practically any flow condition. Also the foregoing information it is possible to design

Design of an Overfall Dam Section with Vertical Upstream Face

reliable measurements no longer possible. critical velocities were approached in the test channel making $\frac{H_0}{H_g}$ and $\frac{H_0}{H_g + \frac{1}{2}H}$ than shown by the curves on Figures 3 and 4, the practical limits of applicability. For greater values of discharge decreases rapidly. The curves have been carried to (or as the velocity of approach increases) the coefficient of by the shape of the lower nappe. As the value of $H + \frac{1}{2}H$ decreases sharp-crested weir but for the equivalent open section outlined on Figures 3B and 4B. These coefficients are not for the $\frac{H_0}{H_g + \frac{1}{2}H}$ ed are plotted with respect to the dimensionless ratio $\frac{H_0}{H_g + \frac{1}{2}H}$. The coefficients of discharge for the various weirs tested

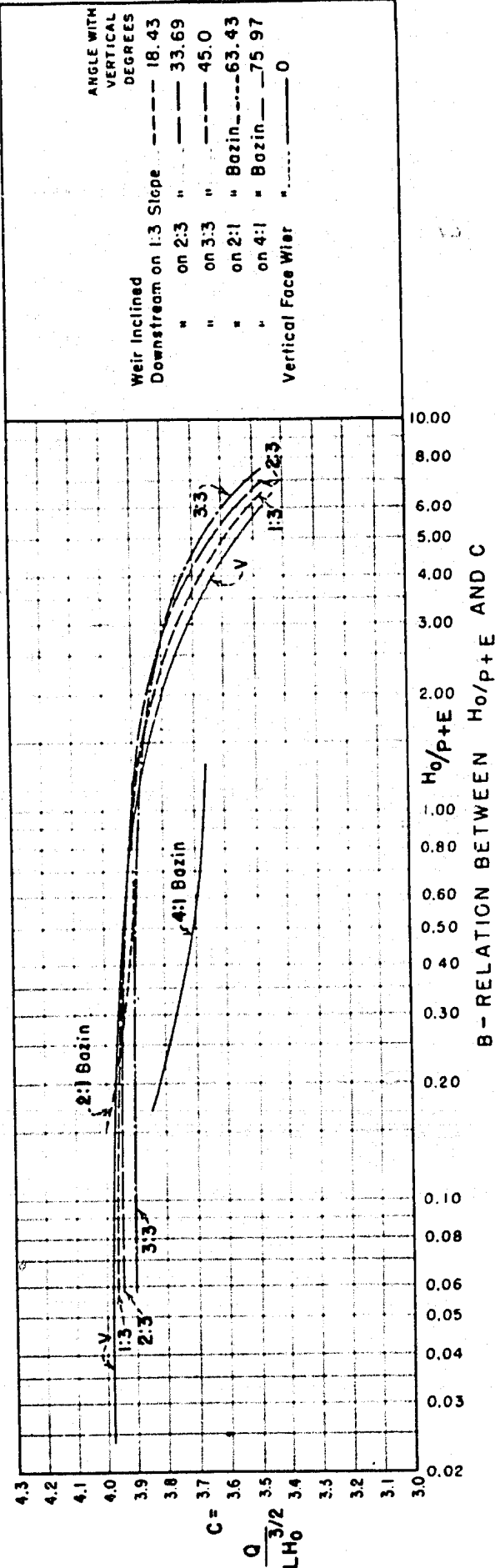
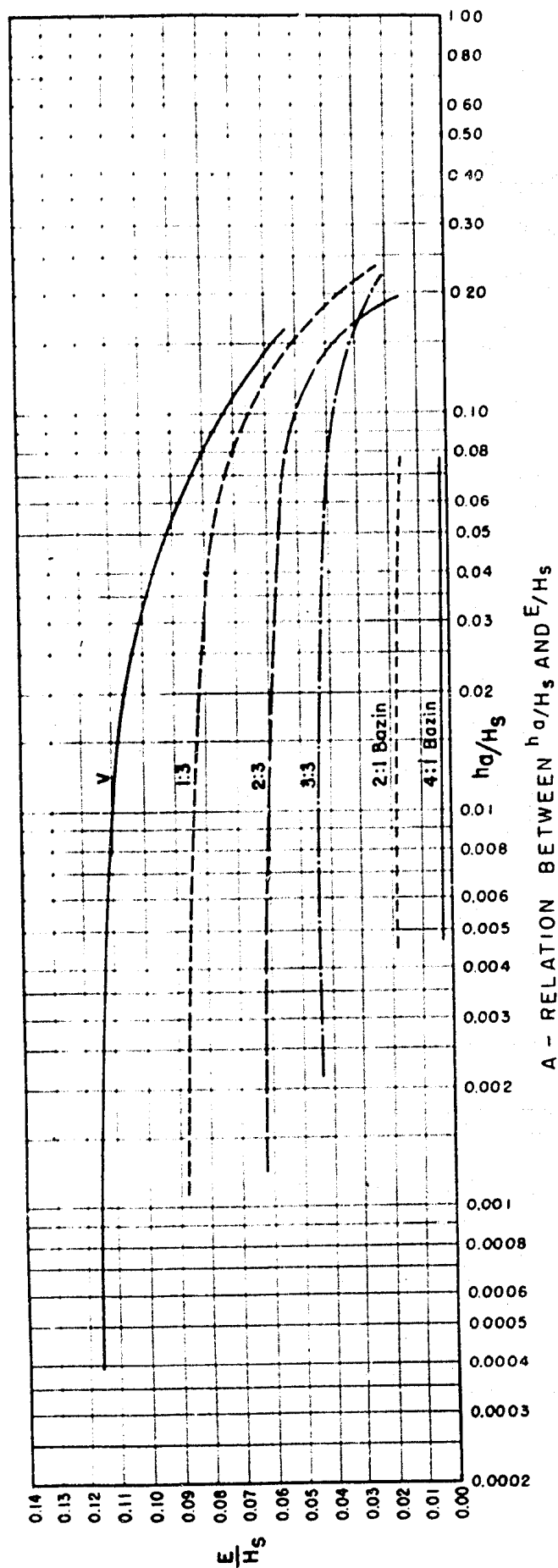
Discharge Coefficients

of the crest type. crest coordinates and applies to all weirs with the exclusion included as Figure 5. This is the same data presented with different used in eliminating out and try methods in design, is a third set of curves for transposing heads, which will only the overhang and offset weirs.

upstream faces while Figure 4 includes information concerning tested. Figure 3 applies to the weirs with vertical and sloping $\frac{H_0}{H_g}$ is plotted with respect to $\frac{H_0}{H_g}$ for all weirs are 1. This is made possible by the curves on Figures 3A and those applying to the crest of an open section, H_0 and H_g , Fig-

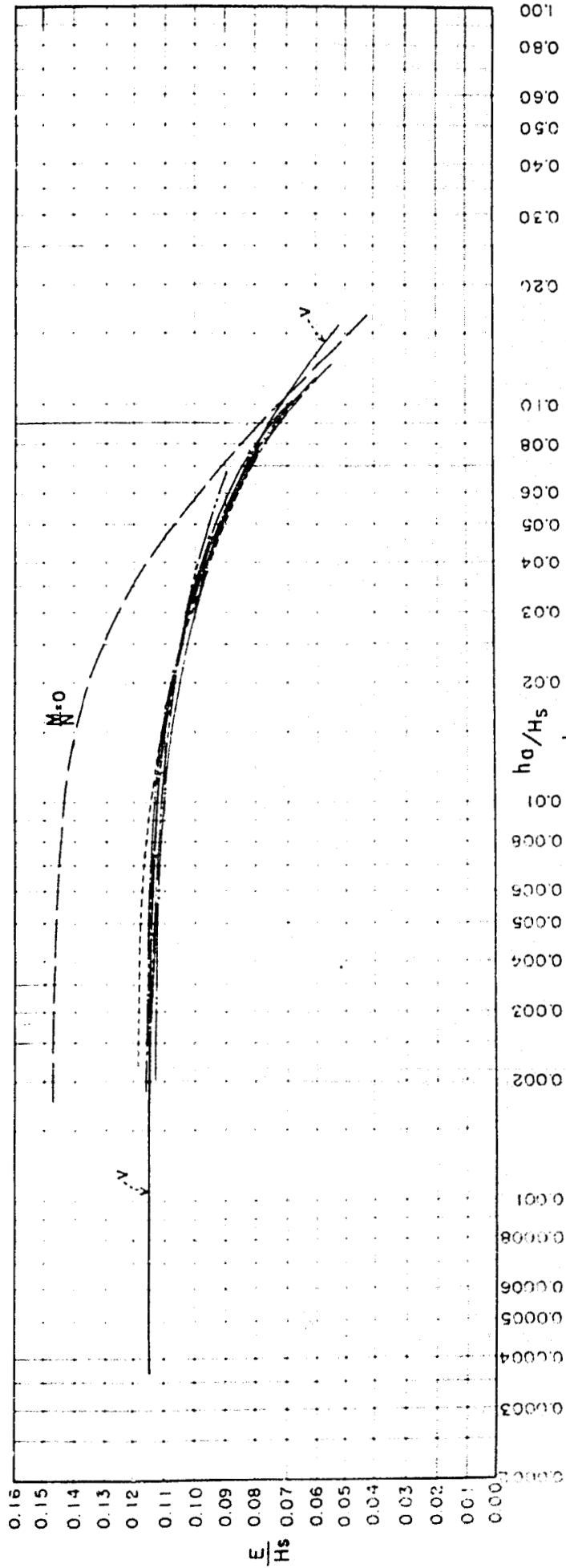
FIGURE 3

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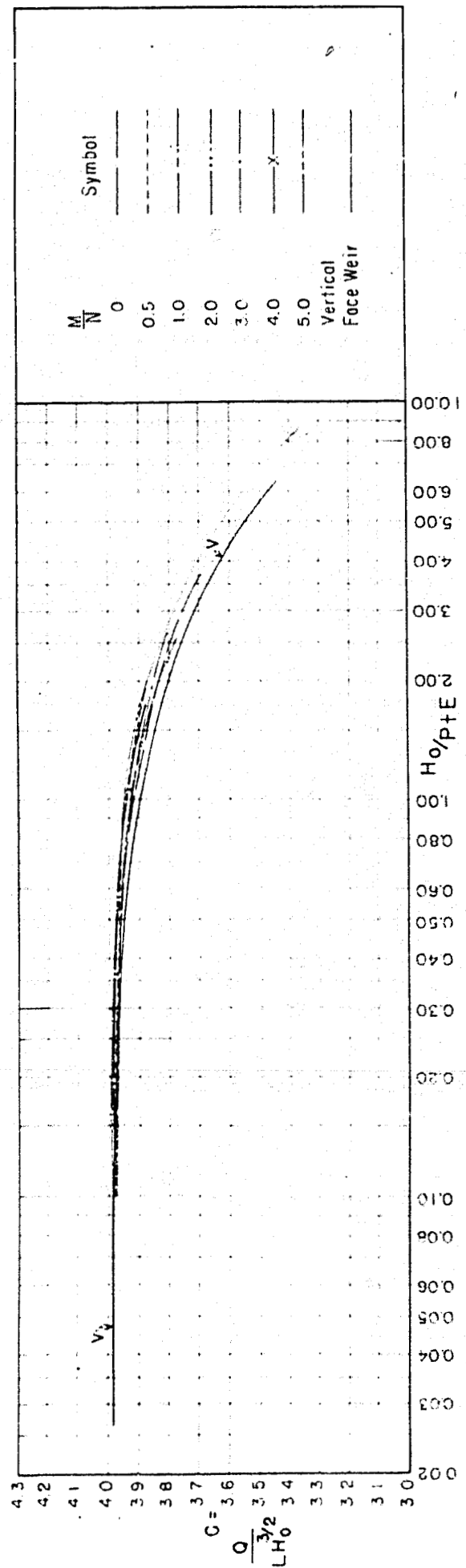


COMPARISON OF "E" AND "C" VERTICAL AND SLOPING WEIRS

31



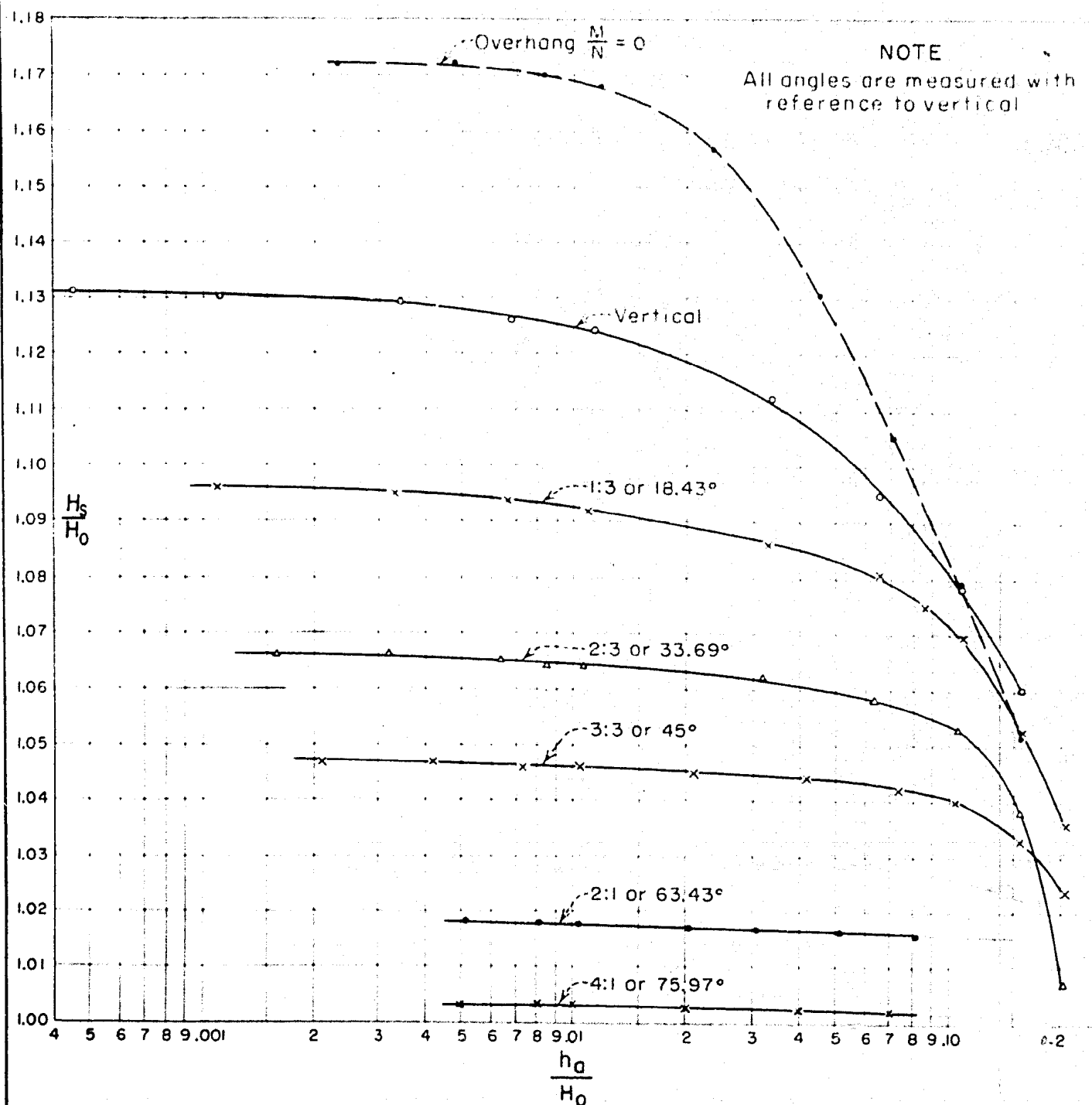
A - RELATION BETWEEN h_0/H_s AND E/H_s



B - RELATION BETWEEN $H_0/P+E$ AND C

COMPARISON OF "E" AND "C" - VERTICAL OVERHANG AND OFFSET WEIRS

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RELATION OF $\frac{H_s}{H_0}$ TO $\frac{h_a}{H_0}$ FOR VERTICAL,
SLOPING AND OVERHANG WIERS.

dition. It will be so designed that for maximum designed head on the spillway, the face of the overflow section will fit the lower profile of the overflowing sheet of water with little or no subatmospheric pressure between the sheet of water and the face of the spillway.

Example 1

Given a maximum reservoir water surface at elevation 1000 for a total discharge of 75,000 second-feet with the average approach floor at elevation 880, determine crest elevation and coordinates for shape of overflow section for a crest length of 250 feet.

Assume a coefficient of discharge, $C = 3.75$. From the expression $Q = CLH_o^{3/2}$

$$H_o^{3/2} = \frac{75,000}{3.75 \times 250} = 80.00,$$

and

$$H_o = 18.37 \text{ feet.}$$

Then

$$P \text{ } \nearrow \text{ } E = 120.00 - 18.37 = 101.43 \text{ feet}$$

$$\frac{H_o}{P \text{ } \nearrow \text{ } E} = \frac{18.37}{101.43} = 0.183, \text{ and from Fig. 3B, } C = 3.96.$$

With this new value of C ,

$$H_o^{3/2} = \frac{75,000}{3.96 \times 250} = 75.76 \text{ and } H_o = 17.90 \text{ feet.}$$

$$P \text{ } \nearrow \text{ } E = 120.00 - 17.90 = 102.10 \text{ feet.}$$

$$\frac{H_o}{P \text{ } \nearrow \text{ } E} = \frac{17.90}{102.10} = 0.175 \text{ and from Fig. 3B, } C = 3.96.$$

It has now been established that the value of $H_o = 17.90$ and the coefficient of discharge $C = 3.96$. The crest of the spillway is then $1000 - 17.90$ or elevation 982.10.

It is next necessary to determine the values of h_a and H_a . The approximate value of h_a is computed as follows:

$$H_a + P \approx 120.9 \text{ feet (approximate depth of approach).}$$

$$q = \frac{75,000}{250} \approx 300 \text{ second-feet per foot of width,}$$

then

$$V_a = \frac{300}{120} = 2.50 \text{ ft. per second (velocity of approach).}$$

hence,

$$h_a = \frac{V_a^2}{2g} = 0.098 \text{ feet.}$$

Now to obtain the value of H_a ,

$$\frac{h_a}{H_a} = \frac{0.098}{17.90} = 0.0055.$$

Entering Figure 5 with this value, $\frac{H_a}{H_o} = 1.1275$

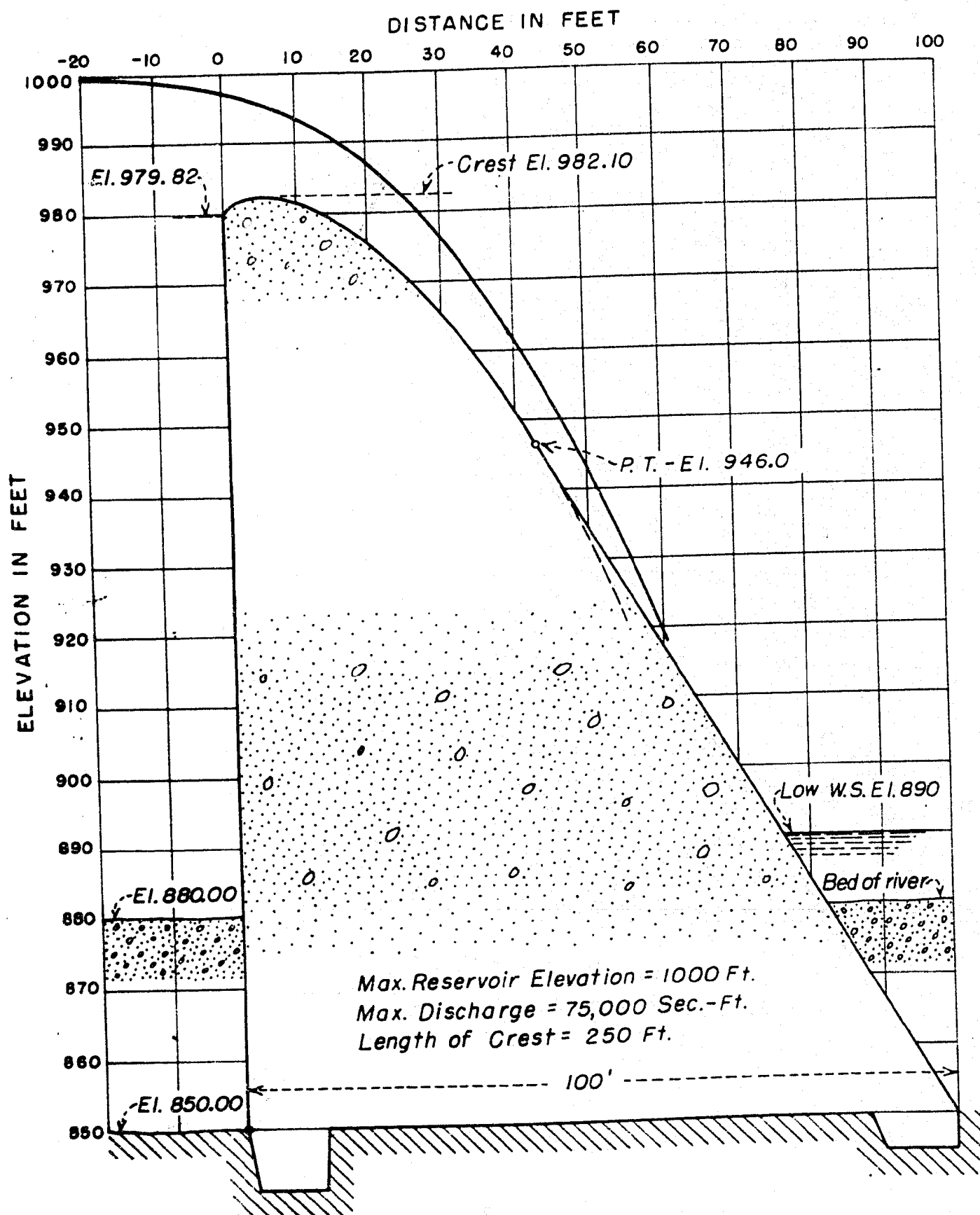
and $H_a = 1.1275 \times 17.90 \approx 20.18 \text{ feet.}$

$$\text{Now } \frac{h_a}{H_a} = \frac{0.098}{20.18} = 0.0049.$$

Referring to Table 1, values of $\frac{X}{H_a}$ are chosen (depending on the point spacing desired) and corresponding values of $\frac{Y}{H_a}$ are obtained from the Table for $\frac{h_a}{H_a} = 0.0049$. Interpolation is necessary, and the results are shown in columns 1 and 2, Table 16. Solving for X and Y from columns 1 and 2, respectively, the coordinates for the overflow section, columns 3 and 4, are obtained. Column 5 expresses the values of Y in feet of elevation. The points are shown plotted on Figure 6.

The slope of the downstream face of the overflow section

FIGURE 6



RESULTS OF EXAMPLES 1 AND 2
 VERTICAL-FACE WEIR

**TABLE 16-COORDINATES FOR OVERFLOW SECTION
EXAMPLE 1**

$\frac{X}{H_s}$	$\frac{Y}{H_s}$	X, Feet	Y, Feet	Y Elevation, Feet
1	2	3	4	5
0.000	0.0000	0.00	0.00	979.86
0.050	0.0569	1.01	1.15	981.01
0.100	0.0852	2.02	1.72	981.58
0.150	0.1014	3.03	2.05	981.91
0.200	0.1093	4.04	2.21	982.07
0.250	0.1109	5.05	2.24	982.10
0.300	0.1091	6.05	2.20	982.06
0.350	0.1045	7.06	2.11	981.97
0.400	0.0954	8.07	1.93	981.79
0.450	0.0829	9.08	1.67	981.53
0.500	0.069	10.09	1.39	981.25
0.600	0.031	12.11	0.63	980.49
0.700	-0.018	14.13	-0.36	979.50
0.800	-0.075	16.14	-1.51	978.55
0.900	-0.140	18.18	-2.83	977.03
1.000	-0.215	20.18	-4.34	975.52
1.200	-0.394	24.22	-7.95	971.91
1.400	-0.607	28.25	-12.25	967.61
1.600	-0.851	32.29	-17.17	962.69
1.800	-1.133	36.32	-22.86	957.00
2.000	-1.452	40.36	-29.30	950.56
2.200	-1.799	44.40	-36.30	943.56
2.400	-2.180	48.43	-43.99	935.87
2.600	-2.603	52.47	-52.53	927.33

beyond the curve depends on stability determinations of the structure and on the characteristics of the stilling pool at the toe of the spillway section.

Determination of wall heights on overflow dam section

A common mistake in practice is to underdesign the confining walls of a spillway. It is difficult in some cases to foresee the various factors which determine the necessary height of spillway walls, such as diagonal waves propagated by piers, waves created at different gate openings, unsymmetrical flow produced by unequal approach conditions on the two sides of a spillway, unpredictable air currents, and waves due to unsymmetrical gate operation. The latter condition can be eliminated by operating the gates in a symmetrical manner. If all gates cannot be opened an equal amount, then those that are opened should be chosen so as to produce a symmetrical pattern of flow. The extent of the effect of the remaining factors to produce waves in a spillway can be determined satisfactorily only by a model study.

Example 2

Determine the wall heights for the overfall dam section as designed in Example 1.

As H_s and $\frac{h_a}{H_s}$ have already been determined in Example 1,

it is merely necessary to choose values of $\frac{x}{H_s}$ from Table 9

and read corresponding values of $\frac{y}{H_s}$ for $\frac{h_a}{H_s} = 0.0049$. The tabulation is shown in Table 17.

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**TABLE 17-COORDINATES FOR UPPER SURFACE
EXAMPLE 2**

$\frac{X}{H_s}$	$\frac{Y}{H_s}$	X, Feet	Y, Feet	Y Elevation, Feet
1	2	3	4	5
-1.000	0.956	-20.18	19.29	999.11
-0.600	0.928	-12.11	18.73	998.55
-0.300	0.897	-6.05	18.10	997.92
0.000	0.843	0.00	17.01	996.83
0.200	0.788	4.04	15.90	995.72
0.400	0.712	8.07	14.37	994.19
0.600	0.617	12.11	12.45	992.27
0.800	0.492	16.14	9.93	989.75
1.000	0.342	20.18	6.90	986.72
1.200	0.152	24.22	3.07	982.89
1.400	-0.068	28.25	-1.37	978.45
1.600	-0.318	32.29	-6.42	973.40
1.800	-0.598	36.32	-12.07	967.75
2.000	-0.905	40.36	-18.26	961.56
2.200	-1.256	44.40	-25.35	954.47
2.400	-1.653	48.43	-33.36	946.46
2.600	-2.090	52.47	-42.18	937.64
2.800	-2.553	56.50	-51.52	928.30
3.000	-3.030	60.54	-61.15	918.67

Columns 3 and 4 were obtained by multiplying the values in 1 and 2 by H_s or 20.18. The points for the upper surface are also shown plotted on Figure 6. This curve represents the actual water surface as obtained in the experiments with near-perfect approach conditions and with little or no air absorption in the sheet of water. The amount of freeboard needed above this curve, as stated previously, depends on the particular problem. Wave heights are difficult to predict. The expansion of the sheet of water due to air entrainment increases as the water accelerates, but it can be estimated with a reasonable degree of accuracy. In this connection, reference is made to a study by H. Ehrenberger,¹¹ to actual field observations by C. W. Thomas,¹⁰ and an analysis by V. L. Streeter.¹²

Design of an overfall dam section with sloping upstream face

With material for the vertical, 1:3, 2:3, 3:3, 2:1, and 4:1 sloping weirs, it is possible to design the most economical and efficient overfall section for a dam with atmospheric pressure on the face for one of these slopes or any intermediate downstream slope.

Example 3

Design the overfall section of a low, slab concrete dam

¹¹Ehrenberger, R., "Flow of Water in Steep Chutes with Special Reference to Self-aeration." Translated by E.F. Wilsey, Oct., 1937, U.S.B.R. Report, Hyd. 29. From the German in Österreichischen Ingenieur und Architektenvereines No. 15/16 and 17/18, 1926.

¹²Streeter, V. L., "Second Progress Report on Studies of the Flow of Water in Open Channels with High Gradients," Oct. 13, 1938, U.S.B.R. Report, Hyd. 40.

with crest at elevation 180.0, for a maximum discharge of 200,000 second-feet. The approach floor is at elevation 155.0; the crest is 1,200 feet long; and the upstream face of the dam is on a 0.75:1 slope.

The solution is handled similarly to Example 1.

$$H_o = \left(\frac{Q}{CL} \right)^{2/3}$$

Assuming $C = 3.80$,

$$H_o = \left(\frac{200,000}{3.80 \times 1200} \right)^{2/3} = (43.86)^{2/3} = 12.44 \text{ feet (approx.)}$$

and

$$\frac{H_o}{P \neq E} = \frac{12.44}{25} = 0.498.$$

From Figure 3B a more accurate value of $C = 3.91$ is obtained for a slope of 0.75:1 or 36.87 degrees from the vertical.

Repeating this operation with the new value of C ,

$$H_o = \left(\frac{200,000}{3.91 \times 1200} \right)^{2/3} = (42.63)^{2/3} = 12.20 \text{ feet,}$$

and

$$\frac{H_o}{P \neq E} = \frac{12.20}{25} = 0.488.$$

From Figure 3B the coefficient of discharge remains at 3.91 for the above value of $\frac{H_o}{P \neq E}$.

The next step is to find the values of h_a and H_g .

$$q = \frac{Q}{L} = \frac{200,000}{1200} = 166.67 \text{ second-feet per ft. of crest}$$

then

$$V_a = \frac{q}{P \neq E \neq H_o} = \frac{166.67}{25 \neq 12.20} = 4.48 \text{ feet per second}$$

and

$$h_a = 0.312 \text{ feet (velocity head of approach).}$$

$$\frac{h_a}{H_o} = \frac{0.312}{12.20} = 0.0256.$$

From Figure 5, $\frac{H_s}{H_o} = 1.057$ for the above value of $\frac{h_a}{H_o}$ and a

weir sloping downstream 36.87 degrees with the vertical.

$$H_s = 1.057 \times 12.20 = 12.89 \text{ feet}$$

and

$$\frac{h_a}{H_s} = \frac{0.312}{12.89} = 0.0242$$

With these values and Tables 3 and 4, the coordinates for the overfall section can be determined. It is necessary in this case to interpolate between values in the two tables, as the upstream slope of the overfall section is 36.87 degrees. The coordinates have been tabulated in Table 18 and are shown plotted on Figure 7A.

If the upper surface coordinates are desired, these can be obtained in the same manner from Tables 11 and 12.

Design of Overfall Dam Section with Offset in Upstream Face

Example 4

Design an offset overfall dam with atmospheric pressure on the downstream face for a total head of 30 feet on the crest. The section has an 8-foot offset and a 5-foot riser on the upstream face and the average height of crest above riverbed is 25 feet. Consider crest of dam at elevation 100.0.

$$\frac{H_o}{P + R} = \frac{30}{25} = 1.20$$

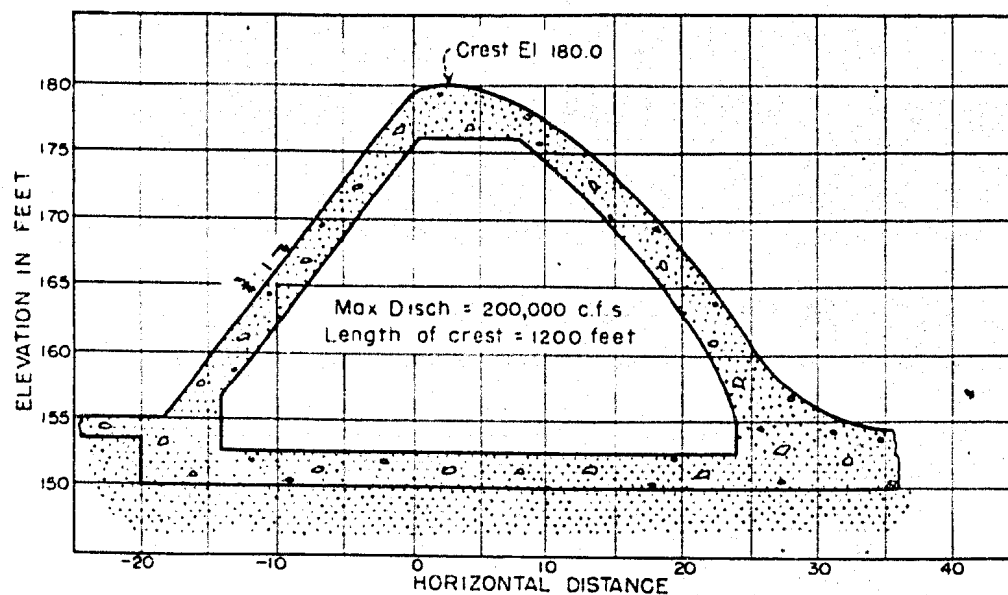
$$\text{and } \frac{R}{P} = \frac{5}{8} = 0.625$$

Entering Figure 4E with these values,

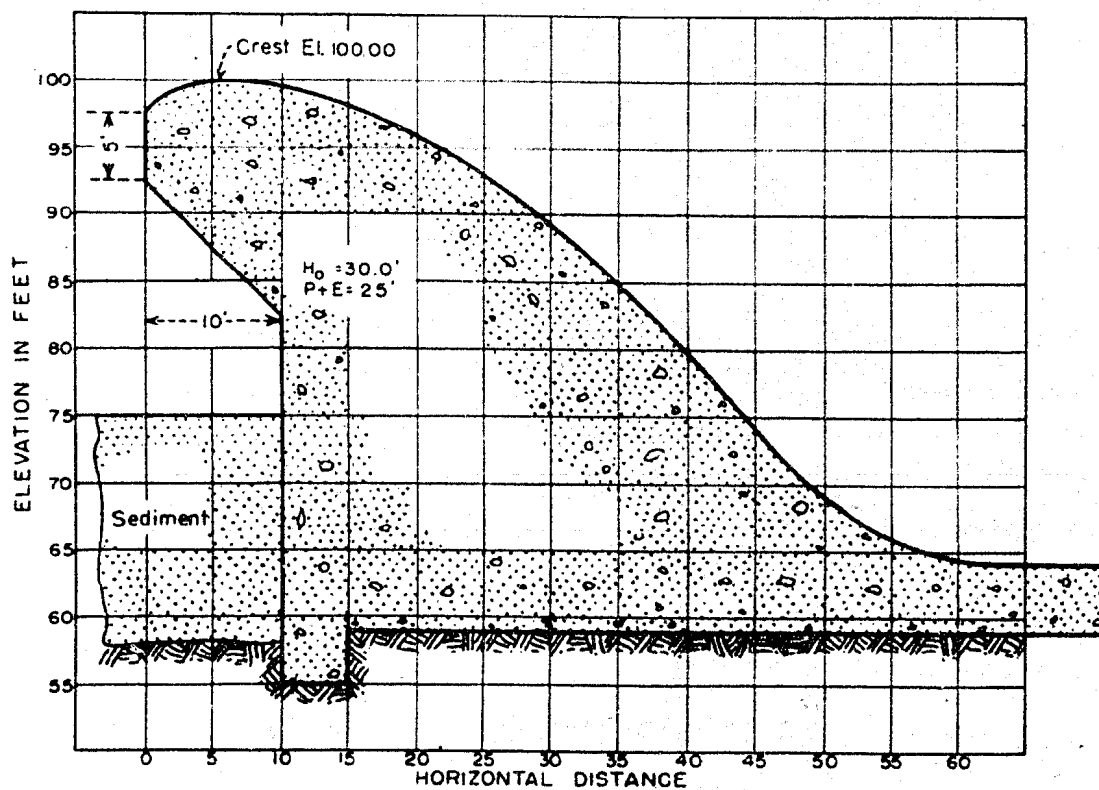
$H_o = 12.87$
 $\frac{H_o}{H_s} = 1.0242$

TABLE 18-COORDINATES FOR OVERFLOW SECTION
EXAMPLE 3

$\frac{X}{H_s}$ 1	$\frac{Y}{H_s}$			X, Feet 5	Y, 36.87-Degree Wier 6	Y Elevation, Feet 7
	33.69-Degree Wier 2	45-Degree Wier 3	36.87-Degree Wier 4			
0.000	0.000	0.000	0.000	0.00	0.00	179.28
0.050	0.032	0.018	0.028	0.64	0.36	179.64
0.100	0.050	0.034	0.046	1.29	0.59	179.87
0.150	0.059	0.041	0.054	1.93	0.70	179.98
0.180	0.061	0.042	0.056	2.32	0.72	180.00
0.200	0.062	0.042	0.056	2.58	0.72	180.00
0.220	0.061	0.042	0.056	2.83	0.72	180.00
0.250	0.059	0.039	0.053	3.22	0.68	179.96
0.300	0.051	0.032	0.046	3.86	0.59	179.87
0.350	0.041	0.021	0.035	4.51	0.45	179.73
0.400	0.029	0.008	0.023	5.15	0.30	179.58
0.450	0.015	-0.008	0.009	5.80	0.12	179.40
0.500	-0.002	-0.027	-0.009	6.44	-0.12	179.16
0.600	-0.044	-0.073	-0.052	7.73	-0.67	178.61
0.700	-0.097	-0.127	-0.105	9.02	-1.35	177.93
0.800	-0.157	-0.189	-0.166	10.30	-2.14	177.14
0.900	-0.226	-0.258	-0.235	11.59	-3.03	176.25
1.000	-0.301	-0.336	-0.311	12.88	-4.00	175.28
1.200	-0.481	-0.514	-0.490	15.46	-6.31	172.97
1.400	-0.692	-0.727	-0.702	18.03	-9.04	170.24
1.600	-0.940	-0.966	-0.947	20.61	-12.20	167.08
1.800	-1.212	-1.240	-1.220	23.18	-15.71	163.57
2.000	-1.517	-1.543	-1.524	25.76	-19.63	159.65
2.200	-1.850	-1.877	-1.858	28.34	-23.93	155.35
2.400	-2.222	-2.246	-2.229	30.91	-28.71	150.57



A - RESULTS OF EXAMPLE 3



B - RESULTS OF EXAMPLE 4

RESULTS OF EXAMPLES 3 AND 4

$$C = 3.92$$

$$Q = CH_0^{3/2} = 3.92 \times 30^{3/2} = 644 \text{ sec.-ft per foot of crest.}$$

$$V_s = \frac{Q}{A} = \frac{644}{55} = 11.71 \text{ feet per second}$$

and $h_a = 2.13 \text{ feet.}$

Again, to obtain the coordinates it is first necessary to compute the correct values of H_s and $\frac{h_a}{H_s}$. A cut and try process is necessary in the case of offset sections.

Assume

$$H_s = H_0.$$

then

$$\frac{h_a}{H_s} = \frac{2.13}{30} = 0.071$$

From Figure 4A,

$$\frac{E}{H_s} = 0.081$$

$$H_s = H_0 + E = 30 + 0.081H_s$$

$$H_s = \frac{30}{0.919} = 32.65 \text{ feet (first approximation)}$$

Repeating the above process using the last value of H_s ,

$$\frac{h_a}{H_s} = \frac{2.13}{32.65} = 0.065$$

Referring again to Figure 4A,

$$\frac{E}{H_s} = 0.084,$$

then $H_s = 30 + 0.084 H_s$

and $H_s = \frac{30}{0.916} = 32.75 \text{ feet.}$

This process can be repeated as many times as desired, but the last value of H_s is sufficiently accurate for the problem at hand.

The final value of $\frac{h_a}{H_g} = \frac{2.13}{32.75} = 0.065$

Table 19 lists the coordinates for the downstream face of the offset dam in Example 4. These were obtained by using the final values of H_g and $\frac{h_a}{H_g}$ in conjunction with Table 8.

All overfall sections designed according to the foregoing method should be checked for stability. Should the base width prove insufficient for stability, it may be broadened to the desired dimension by flattening the tangent slope.

In all of the foregoing cases the designer must decide whether to use the present approach depth, upstream from the spillway, or some future approach depth. The shallower the approach, the broader the overfall section must be to maintain atmospheric pressures on the downstream face. Therefore, if the reservoir in question is expected to fill with silt in the near future, the approach depth, which is to be used in the design, should be given serious consideration.

Table 12

COORDINATES FOR OFFSET OVERFALL SECTION (EXAMPLE 4)

$$H_s = 30.0 \quad \frac{h_a}{H_s} = 0.065$$

$\frac{X}{H_s}$	:	$\frac{Y}{H_s}$	:	X	:	Y	:	Y Elevation (feet)
(1)	:	(2)	:	(3)	:	(4)	:	(5)
0.000	:	0.000	:	0.00	:	0.00	:	97.42
0.050	:	+0.048	:	1.50	:	+1.44	:	93.86
0.100	:	0.070	:	3.00	:	2.10	:	99.52
0.150	:	0.082	:	4.50	:	2.46	:	99.88
0.200	:	0.086	:	6.00	:	2.58	:	100.00
0.250	:	0.084	:	7.50	:	2.52	:	99.94
0.300	:	0.080	:	9.00	:	2.40	:	99.82
0.350	:	0.070	:	10.50	:	2.10	:	99.52
0.400	:	0.059	:	12.00	:	1.77	:	99.19
0.450	:	0.046	:	13.50	:	1.38	:	98.80
0.500	:	+0.029	:	15.00	:	+0.87	:	98.29
0.600	:	-0.012	:	18.00	:	-0.36	:	97.06
0.700	:	0.064	:	21.00	:	1.92	:	95.50
0.800	:	0.125	:	24.00	:	3.75	:	93.67
0.900	:	0.194	:	27.00	:	5.82	:	91.60
1.000	:	0.270	:	30.00	:	8.10	:	89.30
1.200	:	0.446	:	36.00	:	13.38	:	84.04
1.400	:	0.654	:	42.00	:	19.62	:	77.80
1.600	:	0.892	:	48.00	:	26.76	:	70.66
1.800	:	1.160	:	54.00	:	34.80	:	62.62
2.000	:	-1.462	:	60.00	:	-43.36	:	53.56

III. FLOW CHARACTERISTICS, DISCHARGE, AND PRESSURES RELATIVE TO SUBMERGED DAMS

History of Previous Work

Many attempts by different individuals have been made to piece together experimental data from various sources on flow over submerged dams. As submerged flow, at its best, is unstable, it is not difficult to understand why these attempts have been only partially successful. Even if this material could be pieced together, its scope would not be sufficient to represent the picture as a whole. An attempt was made, therefore, to obtain as complete an account as the time available would permit from one laboratory study. The object of the study was to obtain general information to aid in the design of submerged dams and for this reason the results have been expressed in the most general form, that of dimensionless numbers. A bibliography of previous work on the subject is included in the back of the thesis.

Scope of Investigation

This chapter is based on experimental results from two small dams which were tested in the Bureau of Reclamation Hydraulic Laboratory. The report deals entirely with submerged flow over these dams. The study included:

1. Investigation of the various types of flow encountered
2. Determination of discharge coefficients
3. The measurement of water surfaces and pressures on the dam and in the stilling basin to aid in stability de-

terminations

Four distinct types of flow were prevalent on the downstream apron:

1. Flow at supercritical velocities
2. Flow involving the hydraulic jump
3. Flow accompanied by a drowned jump
4. Flow approaching complete submergence

Discharge coefficients were first determined for the free flow condition, then redetermined for the various conditions involving submergence. The difference between the two is termed the decrease in the coefficient of discharge due to submergence. This factor expressed in percent of the free flow coefficient has been plotted for practically all combinations of flow which can occur on small dams with horizontal downstream aprons.

Water surface and pressure measurements are included in dimensionless coordinates for a representative number of flow combinations. These plots are intended to aid the designer in picturing the type of flow to be encountered and, at the same time, offer actual values for stability determinations.

Three examples have been included to demonstrate the possible uses of the enclosed experimental information.

Test Equipment

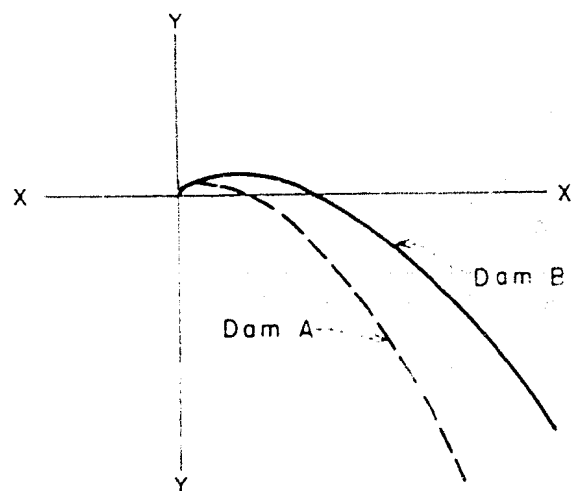
The experiments were performed on two different pieces of equipment but both were tested in the Bureau of Reclamation Hydraulic Laboratory in Denver, Colorado. The first set of experi-

In both cases the movable floors were similarly constructed, the positions of the gates were in similar locations, and the controls on the two flumes were alike in most respects. The upstream head gage connection in each case was located in the movable floor approximately fifteen ft, or fifteen times the designed head, upstream from the crest of the dam. This was connected to a transparent pot on the outside of the flume from which head readings were measured by a hook gage. The tailwater level was obtained from a point gage located 4 ft downstream from the crest of the dam. Water-surface profiles along the flume were obtained from a point gage which could be moved along a 4-inch channel iron located over the center of the flume. Pressure measurements on the bed and on the downstream floor were obtained on Dam B from piezometers located normal to these surfaces as shown in Figure 8. The discharges to the

Dam B on Figure 8. The dam was installed, constructed according to the coordinates for flume 1.95 feet in width and 30 feet long in which a sheet-metal second set of experiments was performed in another but similar the dam and sills were to prevent flow around these floors. The positions, the adjustable floors were sealed tightly against in addition to the main floor of the flume, Figure 8. For all floors were provided both upstream and downstream from the dam feet wide and approximately twenty-four feet long. Adjustable dam was installed in a rectangular sheet-metal lined flume 1.55 feet according to the coordinates for Dam A on Figure 8. The experiments were performed using a sheet metal overall dam construct-

DAM SHAPE COORDINATES
(IN FEET)

DAM A		DAM B	
X	Y	X	Y
0	0	0	0
0.034	0.038	0.052	0.001
0.067	0.057	0.104	0.096
0.101	0.068	0.156	0.117
0.135	0.074	0.208	0.131
0.168	0.075	0.260	0.138
0.202	0.073	0.312	0.140
0.235	0.073	0.364	0.139
0.270	0.064	0.416	0.135
0.303	0.056	0.468	0.128
0.336	0.046	0.520	0.118
0.404	0.021	0.572	0.106
0.471	0.012	0.624	0.091
0.538	0.050	0.702	0.065
0.605	0.094	0.806	0.022
0.673	0.145	0.910	-0.030
0.807	0.265	1.014	0.088
0.942	0.408	1.118	0.153
1.076	0.572	1.222	0.230
1.211	0.762	1.300	0.286
1.345	0.977	1.560	0.518
1.480	1.210	1.820	0.795
1.614	1.466	2.080	1.113
1.749	1.751	2.340	1.478

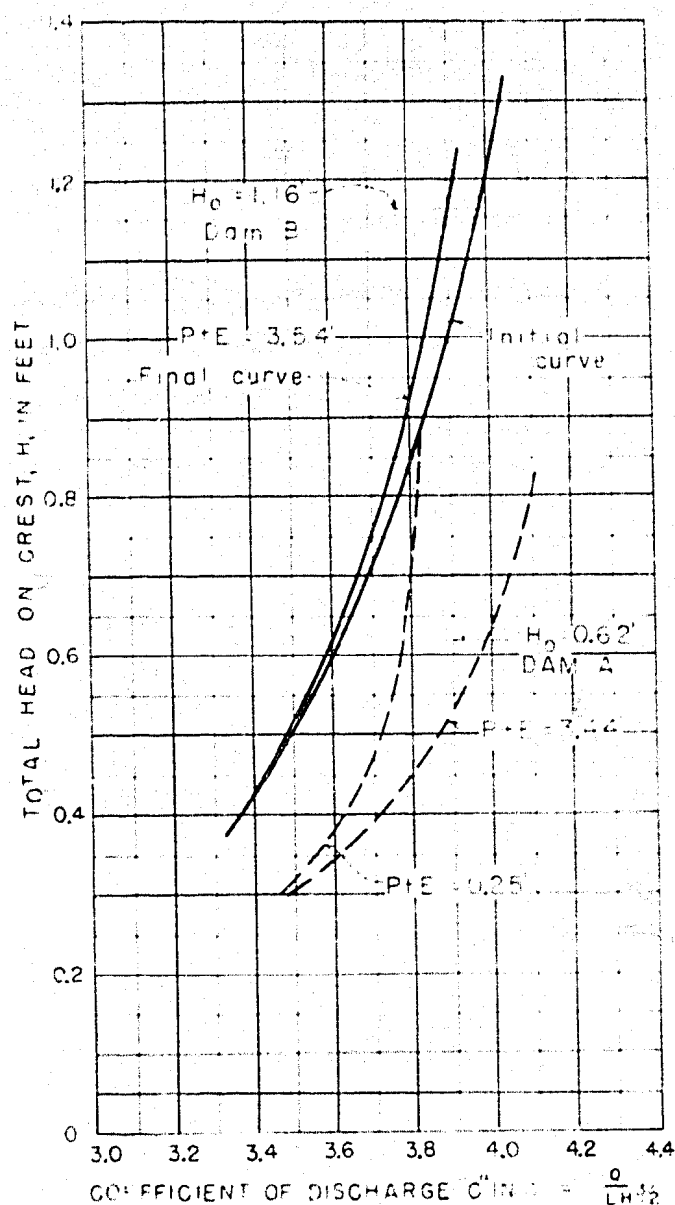


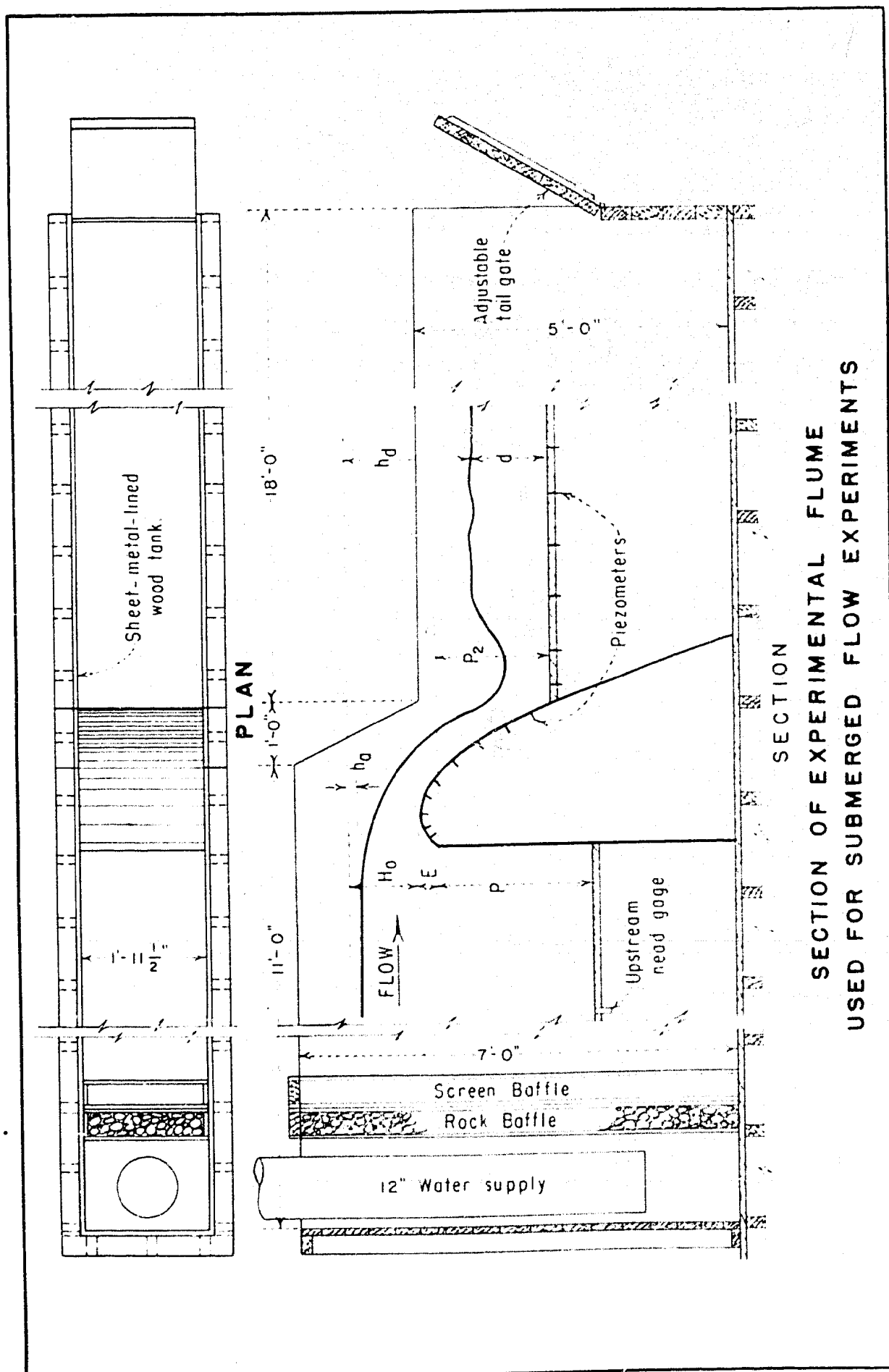
DAM A ---

Designed head, $H_0 = 0.62'$
 $P+E = 3.44'$ and $0.25'$
 Length of Crest = $152'$

DAM B —

Designed Head, $H_0 = 1.16'$
 $P+E = 3.54'$
 Length of Crest = $195'$

SUBMERGED DAM STUDIES
DAM COORDINATES AND FREE CREST
COEFFICIENT OF DISCHARGE CURVES



SECTION OF EXPERIMENTAL FLUME
USED FOR SUBMERGED FLOW EXPERIMENTS

model was measured through the accurately calibrated laboratory venturi meter system. Regulation of the tailwater on Dam B was accomplished by means of an adjustable-hinged gate located at the downstream end of the flume.

Test Procedure

The initial tests were made to obtain data required to construct curves for the free discharge condition with the upstream floor set in different positions. By this is meant the downstream floor was removed completely and free flow allowed to prevail for these tests. The curves are shown for two upstream floor positions for Dam A and one floor position for Dam B on Figure 8. The second curve for Dam B was obtained after changes were made in the approach channel to the dam. After the tests on Dam A, it was learned that the effect on the flow produced by the upstream floor position could be segregated from the effect produced by the downstream conditions. In other words, the entire effect of the upstream floor is accounted for in the free flow coefficient curves on Figure 8. These shapes were obtained by the method described in Chapter II. For this reason it was not necessary to employ more than one upstream floor position for Dam B.

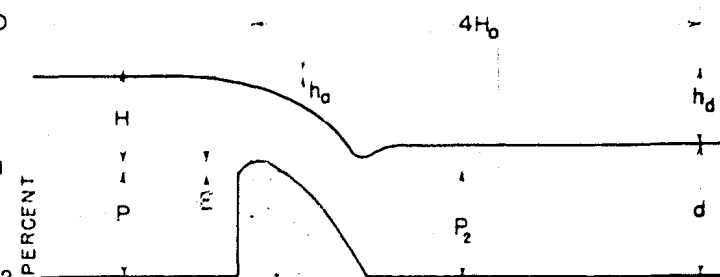
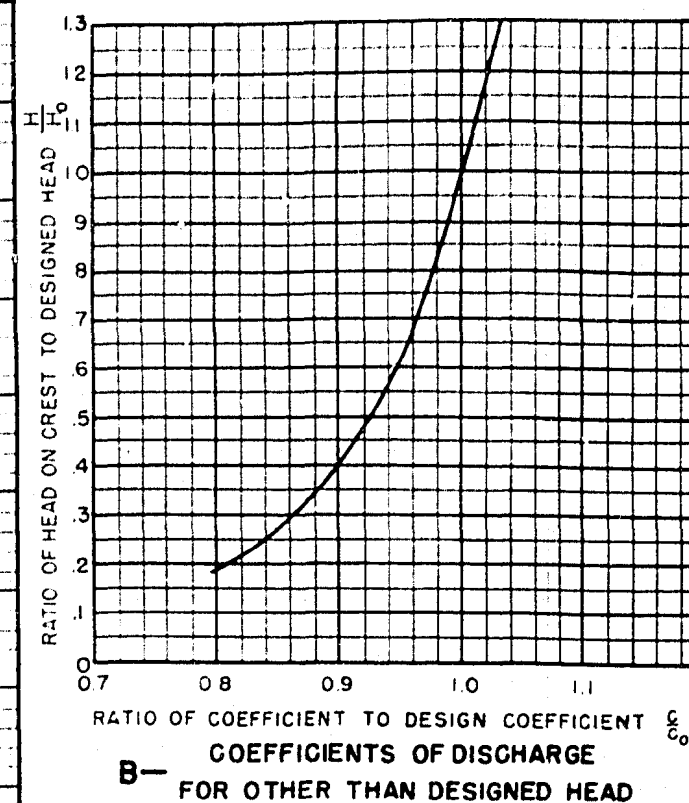
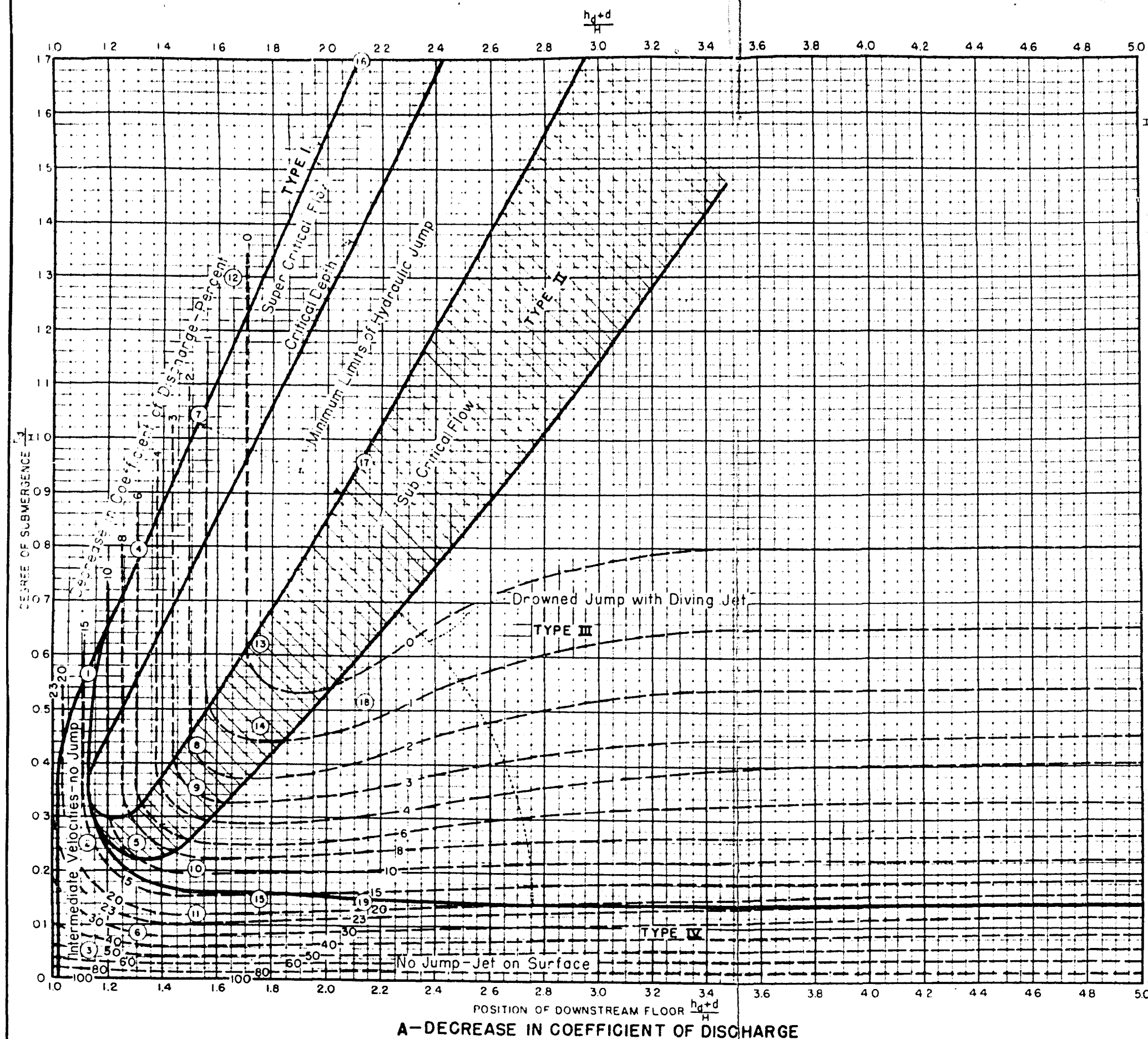
Upon completion of the free flow coefficient curves, the upstream floor was set in one of the calibrated positions and the downstream floor was fixed in a position approaching the crest of the dam. A constant head was maintained on the dam while readings of the discharge and depth of flow over the downstream floor were made for various tailwater depths. Flow conditions

encountered varied from supercritical velocities to flow at practically one hundred percent submergence, the discharge in the latter case approaching zero. This procedure involved from six to eight runs, each made with the tailgate in a different position. The same test was repeated while a second head was maintained on the crest of the dam. After this the downstream floor was lowered to a second position and the twelve to sixteen runs, outlined above, repeated. This entire routine was repeated for floor positions varying from the crest of the dam to the permanent floor of the flume. In the case of Dam A, the upstream floor was shifted to the second calibrated position and certain runs repeated. The testing on Dam A was limited to supercritical velocities on the downstream floor while that on Dam B included four types of flow. The decrease in the coefficient of discharge due to submergence and also due to the presence of the downstream floor was obtained for each run by subtracting the coefficient of discharge, thus obtained, from the free flow coefficient of Figure 8 for a corresponding flow condition. The experimental points, thus obtained, have been tabulated and included as Table 20.

The nomenclature used in the column headings of Table 20 is illustrated by the sketch on Figure 10. Column 1, Table 20, indicates the discharge per foot of dam for each run; Column 2 shows the total head on the crest of the dam, including velocity head of approach; and Column 3 is the coefficient of discharge obtained by substituting these values in the expression $C = \frac{Q}{LH^{3/2}}$. The tabulation in Column 4 represents the difference

TABLE 20

[illegible]



H_0 = TOTAL HEAD FOR WHICH CREST WAS DESIGNED
 H = TOTAL OPERATING HEAD

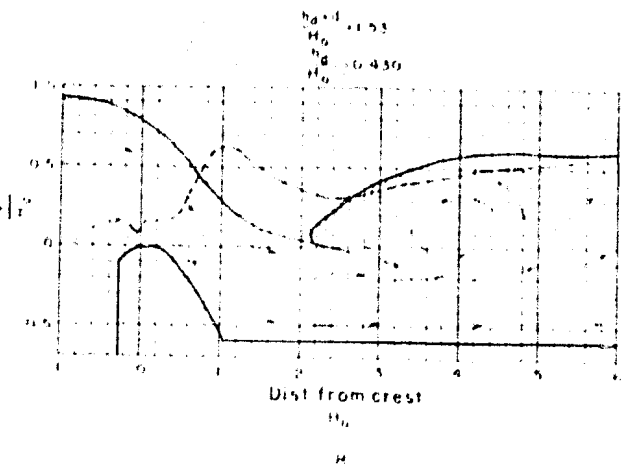
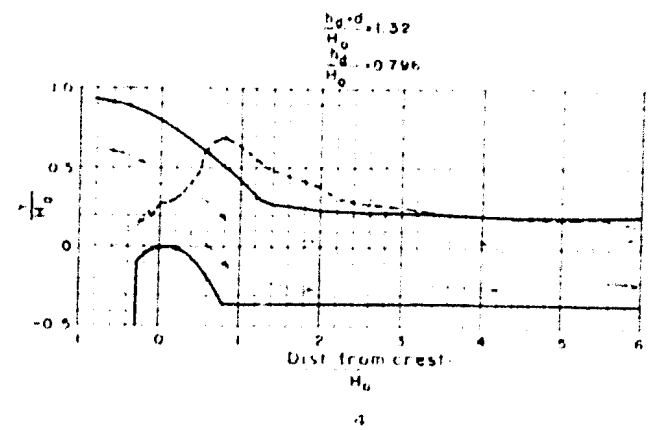
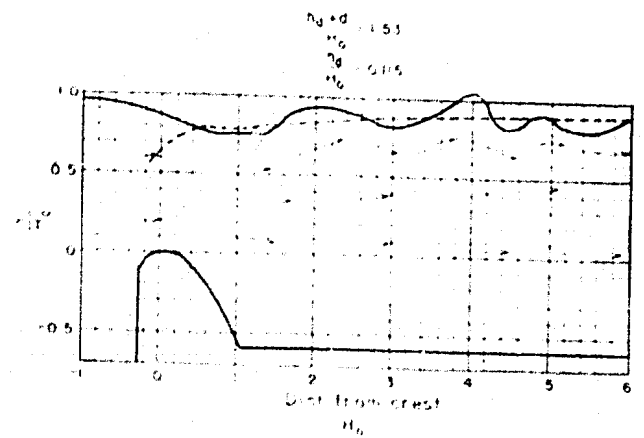
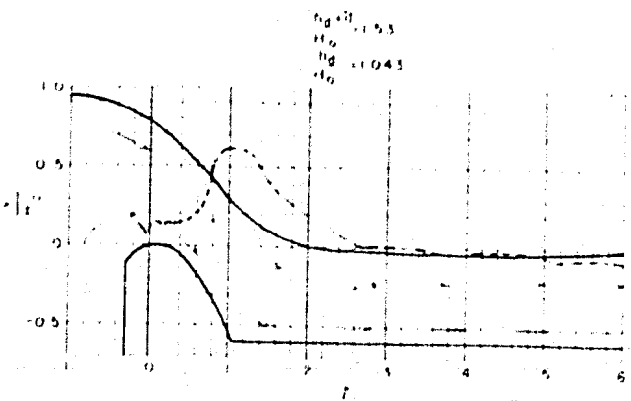
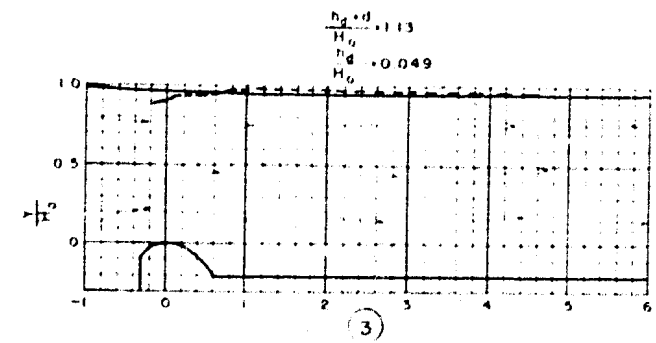
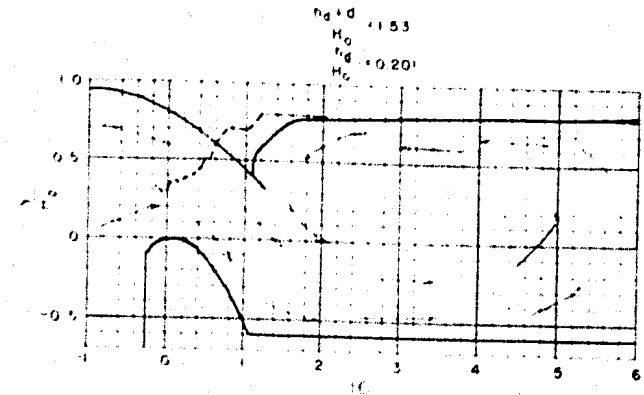
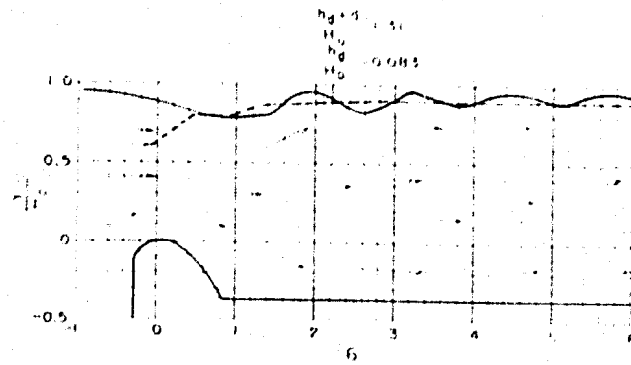
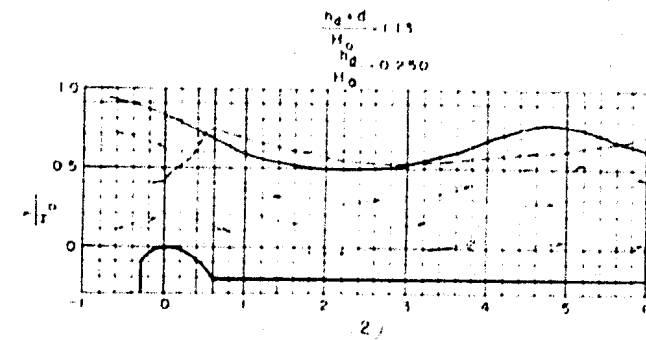
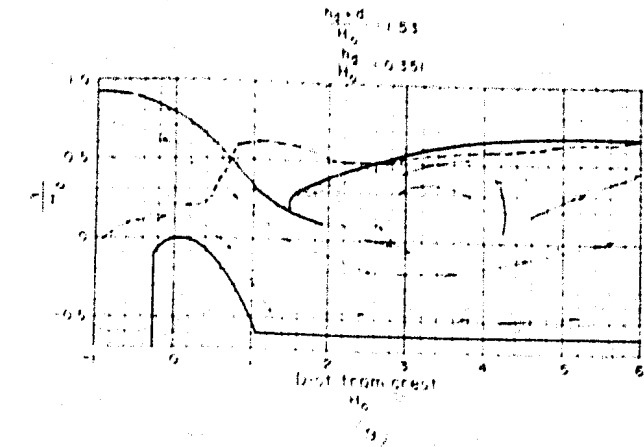
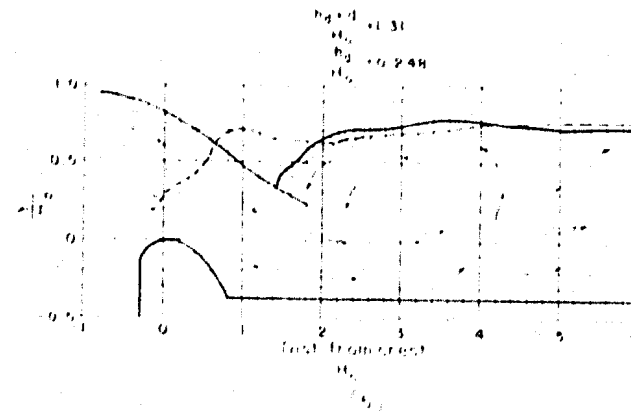
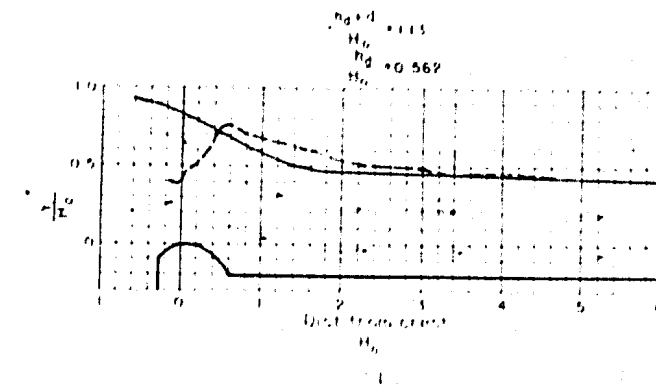
(1) to (19) Points at which water surface and pressure measurements were recorded. (See Figures 11 and 12)

CHARACTERISTICS OF FLOW OVER SUBMERGED DAMS
 GEOMETRIC RELATIONSHIPS, AND DECREASE IN THE COEFFICIENT OF DISCHARGE CAUSED BY INTERFERENCE FROM THE DOWNSTREAM FLOOR AND SUBMERGENCE

between the free discharge coefficient, Figure 8, and the coefficient obtained with submerged flow for the same dam with operation at identical heads. The positions of the upstream and downstream floors, throughout the test, are recorded in Columns 5 and 6, respectively. Columns 7 and 8 involve purely geometrical relationships, namely, the degree of submergence and the position of the downstream floor with respect to the head on the crest. Column 9 indicates the type of flow encountered downstream from the dam for each run. The symbol H_0 , when encountered, indicates only the total designed head for a given dam while H represents any total head applied to the same dam.

Test Results on Coefficients of Discharge

The data contained in Table 80 were used to plot the dimensionless curves on Figure 10A. For the sake of clarity, the points are not shown on the graph. The main coordinates involve the degree of submergence and the position of the downstream floor. The heavy solid lines divide the graph into zones comprising the various types of flow encountered, such as supercritical flow, the hydraulic jump, drowned jump, and flow approaching complete submergence. The dashed lines indicate the decrease in the coefficient of discharge in percent, based on the coefficient of discharge for free flow at the same head. Beginning at the top of the sheet and reading downward, the flow designated as Type I, was at supercritical velocities illustrated by Plots 4, 7, 12, and 16 of Figures 11 and 12. The decrease in the coefficient of discharge in this region is not caused by submergence in the usual sense, but is entirely an effect produced by

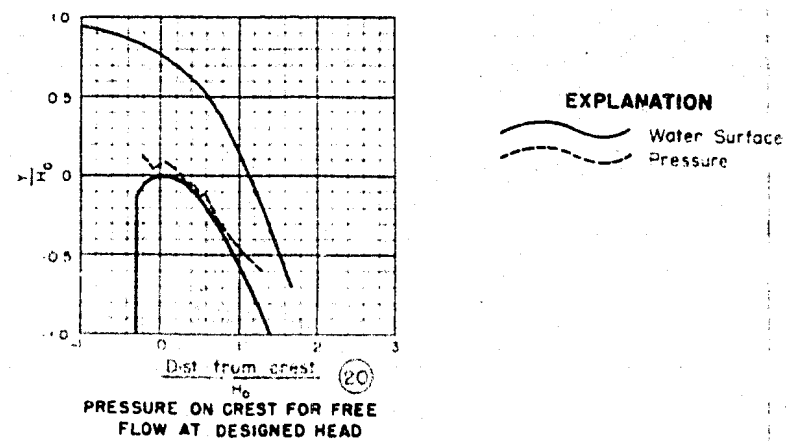
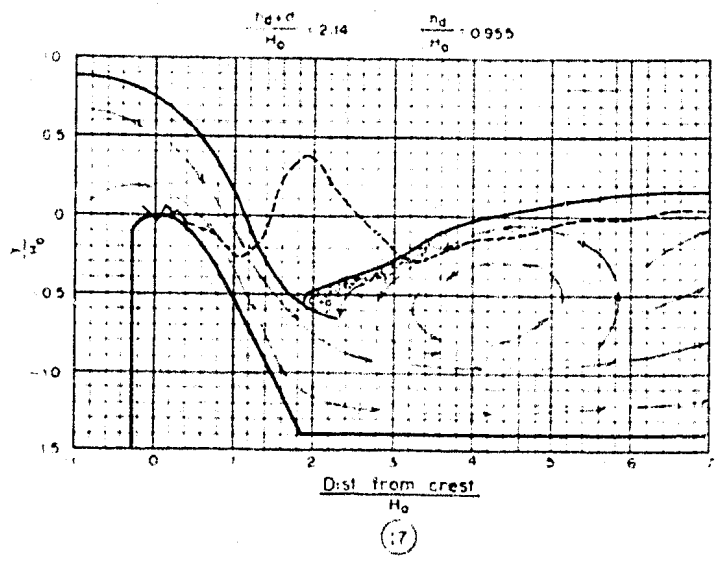
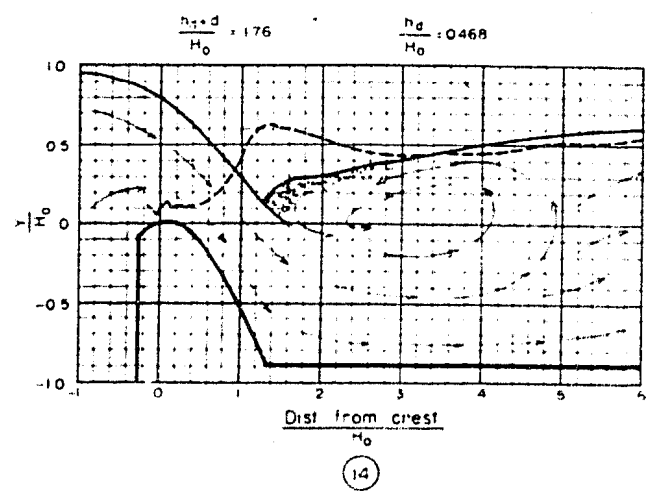
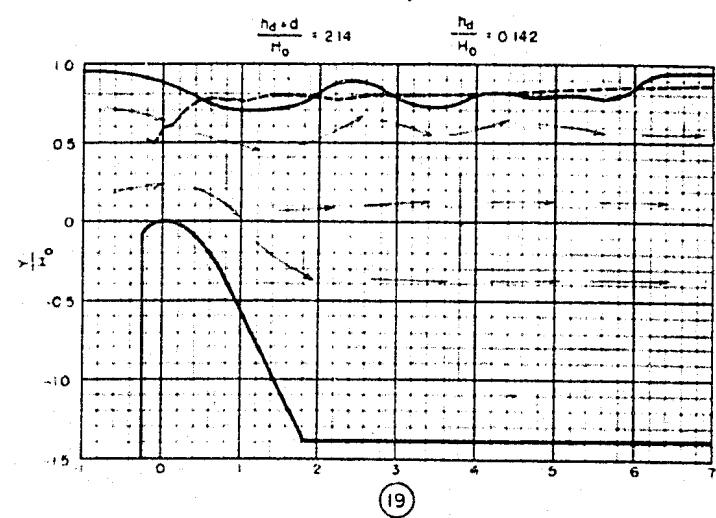
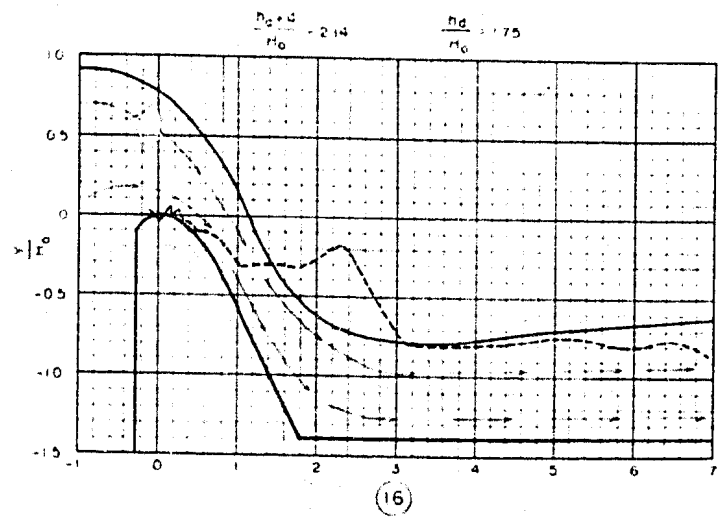
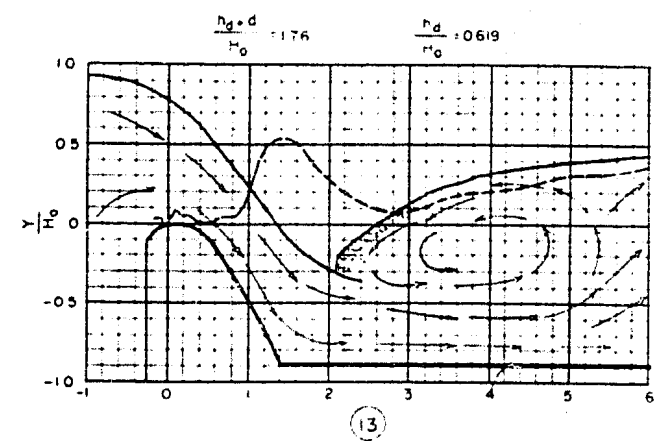
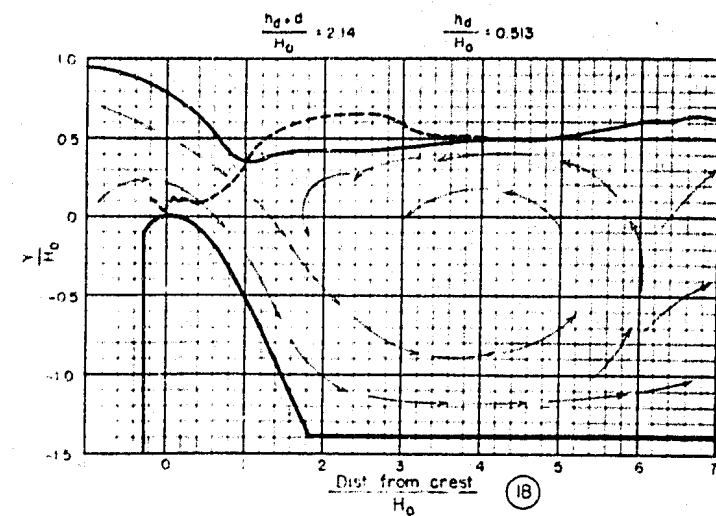
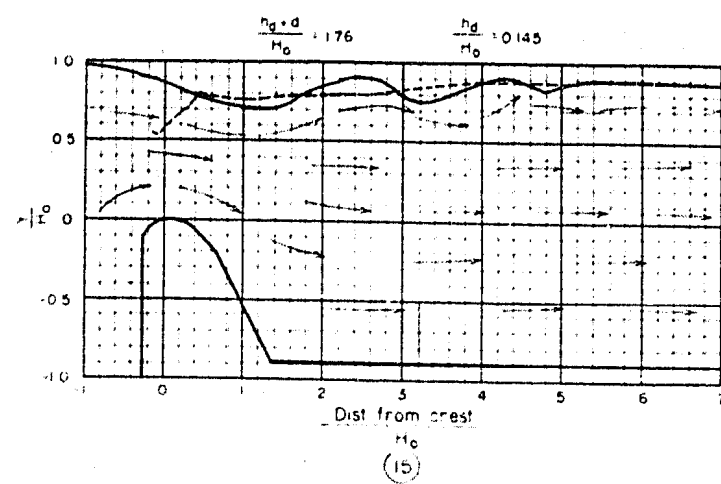
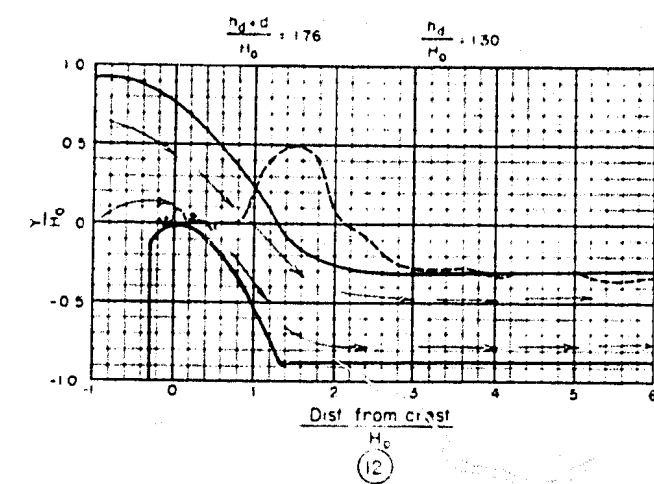


EXPLANATION

— Water surface
--- Pressure

CHARACTERISTICS OF FLOW OVER SUBMERGED DAMS
TYPICAL PRESSURE AND WATER SURFACE PROFILES
AS OCCUR AT POINTS INDICATED BY CIRCLES
ON FIGURE 10

58



CHARACTERISTICS OF FLOW OVER SUBMERGED DAMS
TYPICAL PRESSURE AND WATER SURFACE PROFILES
AS OCCUR AT POINTS INDICATED BY CIRCLES
ON FIGURE 10

the downstream apron.

As the tailwater was raised, or the value of $\frac{h_d}{H}$ decreased, an hydraulic jump occurred in which both supercritical and tranquil flow were present. The former is the Type I flow and the latter is represented by the zone designated as Type II flow, Figure 10A. The curves comprising Types I and II should not be confused with the depth versus specific energy curve, commonly associated with hydraulic jump computations, as the latter represents values of depths and energy for one discharge. Many discharges are represented in the curves on Figure 10A. The hydraulic jump is shown in Plots 5, 8, 9, 13, 14, and 17 on Figures 11 and 12.

As the value of $\frac{h_d}{H}$ continued to decrease, Figure 10A, a third type of flow became prevalent, designated as the drowned jump or Type III flow. The jet of water flowing over the dam continued to follow the dam face, but the tailwater depth was too great to allow a good hydraulic jump to form. This type of flow is illustrated in Plots 10 and 18 on Figures 11 and 12.

With still further decrease in the value of $\frac{h_d}{H}$, a fourth type of flow occurred which was truly submerged. In this case, the jet of water flowing over the dam no longer followed down the face, but separated, assuming a course ahead as indicated in Plots 11, 15, and 19 on Figures 11 and 12. This type of flow is confined to the zone designated as Type IV, Figure 10A. Except for small values of $\frac{h_d}{H}$, flow throughout Zone IV was very unstable.

An inspection of the dashed lines on Figure 10A, of con-

stant decrease in the discharge coefficient, indicates that where these lines are vertical, the decrease in the coefficient of discharge was principally due to the effect of the downstream floor and independent of submergence. As the downstream floor neared the crest of the dam, or $\frac{h_d}{H}$ approached 1.0, the coefficient of discharge decreased to 23 percent. With the downstream floor level with the crest, the dam was virtually a broad crested weir for which the theoretical decrease should approximate 23 percent.

Where the lines designated as decrease in coefficient, are horizontal for values of $\frac{h_d}{H}$ greater than 1.70, the downstream floor no longer affected the coefficient of discharge and the decrease in discharge coefficient was caused entirely by submergence. For values of $\frac{h_d}{H}$ less than 1.70 the decrease in the coefficient was produced by floor effect or a combination of submergence and floor effect.

It appears odd that the dash lines for Type III flow should rise as the value of $\frac{h_d}{H}$ increases. This was caused by a change in flow and can be clarified by reference to Plots 14 and 18 on Figure 12. Plot 14 is Type II flow and Plot 18 is Type III flow while the value of $\frac{h_d}{H}$ is practically the same for the two. In the first case, a true hydraulic jump existed and little submergence effect was present. In the second case, the tailwater depth was approximately the same, but the backwater effect was more pronounced. In other words, the point of contact between the jet falling over the dam and the tailwater occurs at $\frac{Y}{H_0} \approx 0.15$ in Plot 14 and 0.35 in Plot 18. In all cases, the depth of flow on the downstream floor was measured at a point $4H_0$, or four times

the designed head, downstream from the crest of the dam.

The ratio of the free flow coefficients $\frac{C}{C_0}$ has been plotted with respect to the ratio of heads $\frac{H}{H_0}$ on Figure 10B, where

C = Coefficient of discharge at other than the designed head

C_0 = Coefficient of discharge at designed head

H = Any total head on crest

and H_0 = Total designed head on crest.

By knowing the coefficient of discharge at the designed head it is possible, by means of Figure 10B, to determine the coefficient at other than the designed head. The data for plotting the curve was obtained principally from models tested, over a period of time, in the Bureau Hydraulic Laboratory. A check on this data from other sources^{13,14}, showed excellent agreement. By this dimensionless method of plotting all data, regardless of head or approach depth, can be represented by a single curve.

Test Results on Pressures

In addition to typing the flow and plotting the decrease in the coefficient of discharge for each case, pressures and water surface profiles were measured along the flume for a few representative types of flow on Dam B. The runs were all made with the dam operating at the designed head, H_0 , and the results, Figures 11 and 12, have been plotted in dimensionless terms with

¹³ Davis Handbook of Applied Hydraulics, Page 341 (data of E. W. Lane) and Page 23 (data of W. L. Voorduin).

¹⁴ Cox, Glen Nelson, "The Submerged Weir as a Measuring Device," University of Wisconsin Engineering Experiment Station Bulletin No. 67.

The following examples serve to illustrate the use of the preceding information on submerged dams. Each dam spillways are usually flat in longitudinal cross-section as they closely follow the profile of the dam. The gate section and upstream portion of one of these is shown by the full line on Figure 13. The coefficient of discharge for this spillway is 2.65. In contrast, the coefficient of discharge for the shape indicated by the heavy dashed line on the same figure is 3.72. The latter shape was designed so that the overflow section would fit the shape of the undernappe of the sheet of water flowing over it for the maximum discharge condition. This phase of design was covered in Chapter II. The economy involved in providing a small gate at the gate section is evident from a comparison of the above coefficients and since the completion of these tests it is now common practice to design earth dam gate sections in this manner. The following example illustrates the method of determining the position of the floor downstream from the gate.

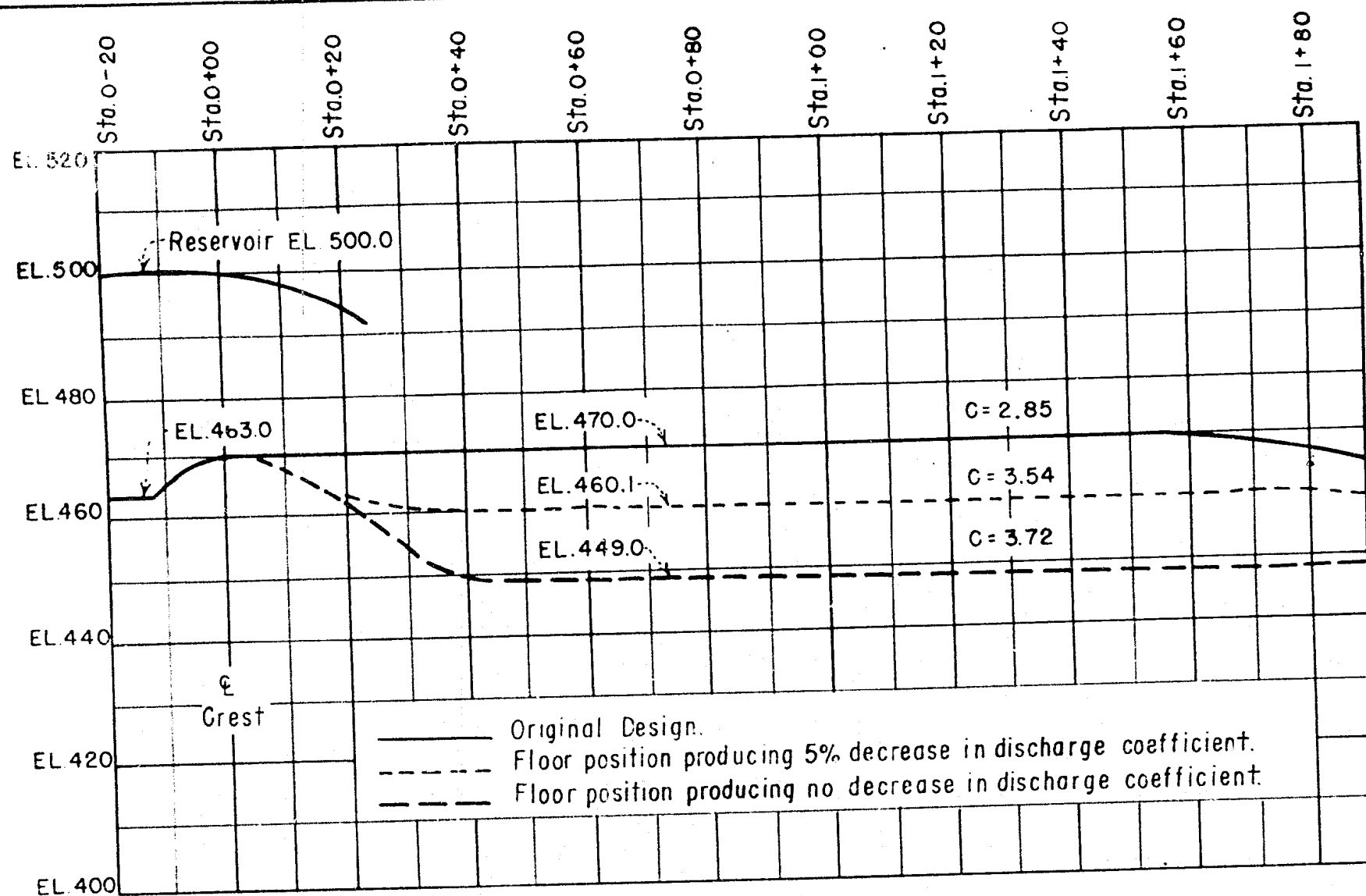
Design of Overflow Crest on Earth Dam Spillway

Example 3

Design the downstream portion of the overflow section of

Application of Results

respect to this head. The axes of coordinates originate at the crest of the dam. The conditions under which these runs were made, as to type of flow and degree of submergence, are indicated by the numbers within the circles on Figure 10A. These runs were made in an effort to provide data on stability determining conditions for small dams.



SUBMERGED DAM STUDIES
RESULTS OF EXAMPLE 5

the earth dam spillway, indicated by the solid line in Figure 13, for a minimum amount of excavation, allowing a decrease in the free flow coefficient of discharge not to exceed 5 percent for the maximum discharge condition. The crest of the overflow section is at elevation 470.0 and the total designed head on the crest $H_0 = 30.0$ feet.

The flow immediately downstream from the overflow section will occur at less than critical depth and will therefore resemble that of Type I on Figure 10A. Tailwater is not a consideration in this case.

By following down the line for Type I flow (Figure 10A), to a decrease in the coefficient of discharge of 5 percent, the values of the main coordinates at this point are

$$\frac{h_d}{H_0} = 0.81 \text{ and } \frac{h_d + d}{H_0} = 1.33$$

$$h_d = 0.81 \times 30.0 = 24.30 \text{ feet}$$

$$\text{and } h_d + d = 1.33 \times 30.0 = 39.90 \text{ feet.}$$

The position of the floor should therefore be 39.90 feet below the reservoir level for the maximum flow condition or $500.00 - 39.90 = 460.1$ feet in elevation. This floor is indicated by the lightly dotted line on Figure 13.

The depth of flow on the horizontal floor at a point $4H_0$, or 120 feet downstream from the crest, is $d = 39.90 - 24.30 = 15.60$ feet, and Graph 4 on Figure 11 indicates the type of flow to be expected. The coefficient of discharge for the downstream floor at elevation 460.1 (Figure 13) will be 3.54. Should the above procedure be repeated for the case where no decrease in the

coefficient of discharge is desired, it would be necessary to locate the downstream floor at elevation 449.0 (Figure 13); but the coefficient would be increased to 3.72.

Now should it be desired to maintain the same capacity for the new ogee section as for the former flat section, the designer has two alternatives: first, to decrease the width of the spillway or secondly, to maintain the same width and elevate the crest of the ogee section. Elevating the crest would decrease the head and also reduce the necessary excavation downstream. Either alternative would effect a considerable saving in the size of gates necessary for control of the spillway. Whether a saving would be possible in the cost of excavation and concrete would depend on the nature of the topography.

Suppose it was decided to reduce the width of the spillway as mentioned in the first case. With the downstream floor set to give a 5 percent decrease in discharge coefficient, the width of the spillway could be reduced $\frac{3.54 - 2.85}{2.85} = 19.5$ percent.

With the downstream floor positioned for no decrease in the free flow coefficient, the width of spillway could be reduced $\frac{3.72 - 2.85}{2.85} = 23.4$ percent.

On the other hand, suppose it is desired to elevate the crest of the ogee as suggested in the second alternate instead of reducing the width of spillway. Allowing a 5 percent decrease in the discharge coefficient, the crest of the ogee could be raised to elevation 474.05, thus reducing the head on the crest from 30.0 to 25.95 feet.

By positioning the downstream floor for no decrease in the free flow coefficient the crest could be raised to elevation 474.85, reducing the head over the crest to 25.15 feet.

Computation of Discharge over a Partially Submerged Dam

Example 6

Given the dam shown in Figure 14A for which the shape and free flow coefficient of discharge were determined by the method previously described in Chapter II. Compute the discharge per foot of crest length and determine the type of flow which will be encountered for the conditions shown. The free flow coefficient of discharge is 3.90.

$$\frac{h_d}{H_0} = \frac{2.5}{12} = 0.208$$

$$\frac{h_d^{3/2}}{H_0} = \frac{25}{12} = 2.168$$

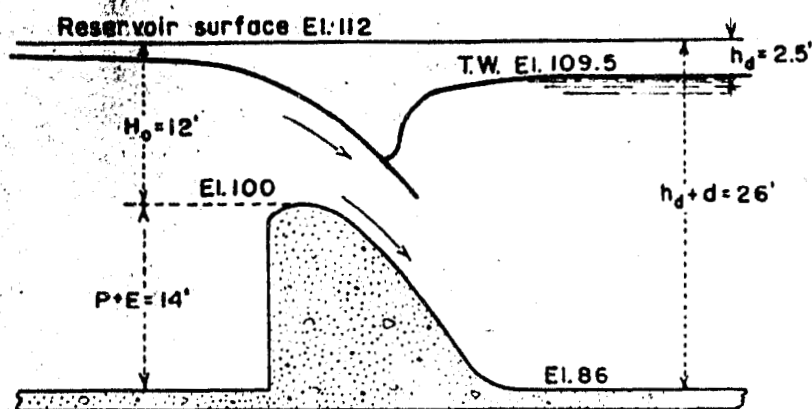
Entering Figure 10A with these values, it is found that the point falls within Region III. A drowned jump will occur and the coefficient of discharge will be 9 percent less than the free flow coefficient.

The actual coefficient of discharge for the flow conditions shown therefore will be $C_d = 3.90 \times 0.91 = 3.55$.

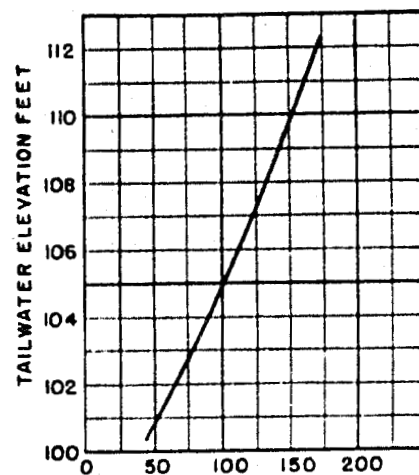
The discharge per foot of length of dam will be

$$q = C_d H_0^{3/2} = 3.55 \times 12^{3/2} = 147.5 \text{ second-feet.}$$

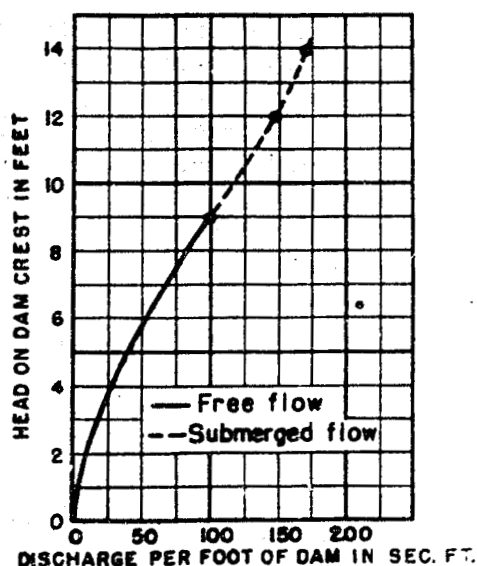
These examples serve as an introduction to the use of curves on Figure 10A. The solution of the following example requires successive approximations.



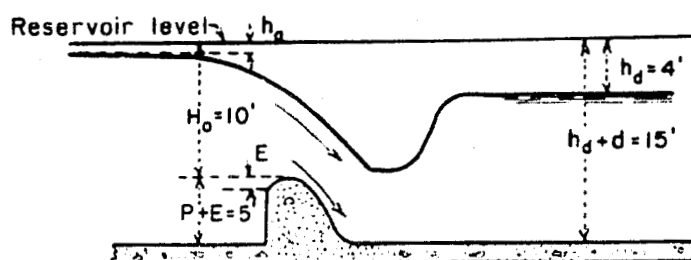
A - SUBMERGED DAM FOR EXAMPLE 6



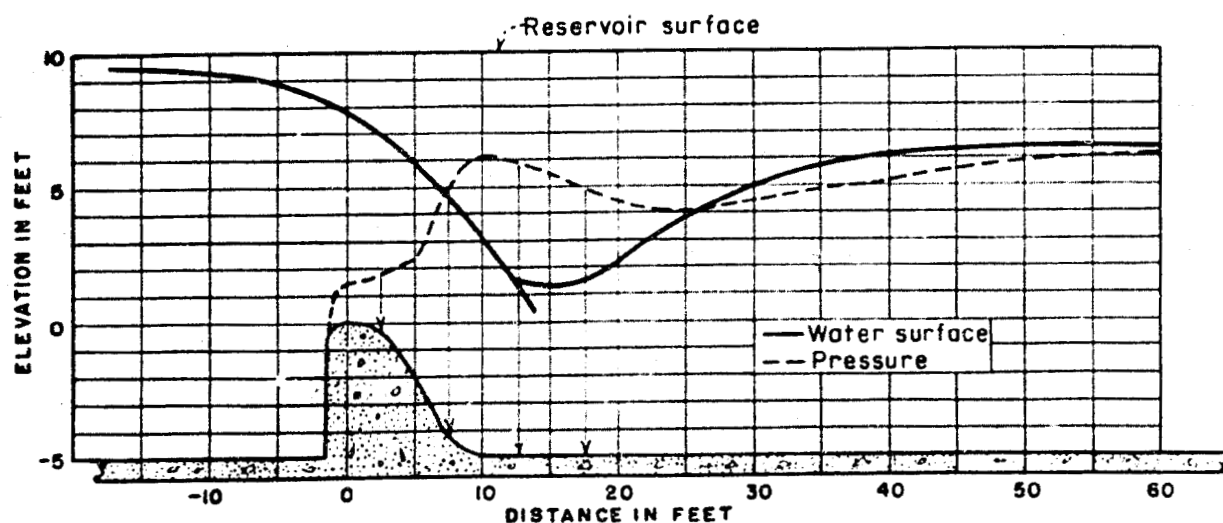
B - TAILWATER - DISCHARGE FOR EXAMPLE 7



C - RESULTS OF EXAMPLE 7



D - DAM FOR EXAMPLE 8



E - RESULTS OF EXAMPLE 8

SUBMERGED DAM STUDIES

DETAILS AND RESULTS OF
EXAMPLES 6, 7 AND 8

Computation of Headwater Curve for Low Submerged Dam

Example 7

Given the tailwater curve shown on Figure 14b for the dam

in Figure 14a. Compute a headwater curve for the same range of

discharges. The free flow coefficient at designed head is 2.90.

One point on the head-discharge curve was determined in

Example 6. A second point will be chosen for a discharge of

170 second-foot per foot of crest, which will involve greater

than the designed head on the crest. This corresponds to a tail-

water elevation of 112.0, Figure 14b.

Assuming

$$H = 13.5 \text{ feet}$$

$$\frac{H}{h_0} = \frac{13.5}{1.5} = 9.0$$

$$\frac{h_0}{H} = \frac{1.5}{13.5} = 0.111$$

and

from Figure 10a the decrease in the coefficient of dis-

charge, compared to that for a free crest, is 25 percent, and

the flow is indicative of Region IV.

The ratio of the estimated head to the designed head is

$$\frac{13.5}{12} = 1.125. \text{ Entering Figure 10b with this value, } \frac{C}{C_0} = 1.02,$$

$$\text{and } C = 1.02 \times 2.90 = 2.96.$$

The actual coefficient of discharge is therefore

$$2.96 \times 0.75 = 2.22.$$

$$H^{3/2} = \frac{Q}{C} = \frac{84.3}{2.22} = 38.0$$

and $H = 14.8$ feet, which is larger than the assumed value.

Choosing a new value of $H = 13.9$, the process is repeated. Other new values are

$$\frac{h_d}{H} = \frac{1.9}{13.9} = 0.137$$

$$\frac{h_d^{3/2}}{H} = \frac{27.9}{13.9} = 2.007$$

and $\frac{H}{H_0} = \frac{13.9}{12} = 1.16$

From Figure 10A, $C = 18$ percent decrease from free flow coefficient,

From Figure 10B, $\frac{C}{C_0} = 1.023$,

and

$$C = 1.023 \times 3.90 = 3.99.$$

The actual coefficient of discharge for the case at hand

$$C_d = 3.99 \times 0.82 = 3.27$$

$$H^{3/2} = \frac{170}{3.27} = 51.9$$

and $H = 13.93$ feet, which agrees reasonably well with the assumed value. This locates a second point on the head-discharge curve Figure 14C.

An attempt now will be made to determine the head at which submergence begins. Choosing a discharge of 150 second-feet, the tailwater, Figure 14B, is at elevation 105.

Assuming $H = 9$ feet,

$$\frac{h_d}{H} = \frac{4}{9} = 0.445$$

$$\frac{h_d^{3/2}}{H} = \frac{25}{9} = 2.583$$

and

$$\frac{H}{H_0} = \frac{9}{12} = 0.75.$$

From Figure 10A, $C = 1.7$ percent decrease from free flow coefficient.

From Figure 10B, $\frac{C}{C_0} = 0.968$

and

$$C = 0.968 \times 3.90 = 3.77$$

then

$$C_s = 3.77 \times 0.993 = 3.708$$

$$H^{3/2} = \frac{100}{3.708} = 27.0$$

and

$$H = 9.00 \text{ feet.}$$

This agrees with the assumed head and thus locates a third value very close to the point at which submerged flow begins. The remainder of the curve is completed by substituting free flow coefficients in the equation $Q = C H^{3/2}$. The free flow coefficients for less than the designed head can be obtained from Figure 10B. The completed head-discharge curve is shown on Figure 14C.

Computation of Pressure opposing Uplift

Example 3

From the dimensions shown on Figure 14D, determine the approximate water surface and the hydraulic pressures opposing uplift on the dam and apron downstream, for the head and tailwater elevations given. The free flow coefficient

$$C = 3.80$$

$$\frac{h_d}{H_0} = \frac{4}{10} = 0.40$$

and

$$\frac{h_d/d}{H_0} = \frac{15}{10} = 1.50.$$

Figure 10A shows that a hydraulic jump can be expected downstream from the dam for these conditions accompanied by a 2.5 percent decrease in the coefficient of discharge. The decrease is produced principally by floor effect and not submergence. Figure 10A also indicates that flow over the dam will be similar to that shown in Graphs 8 and 9 on Figure 11. It should again be mentioned that the profiles on Figures 11 and 12 are for flow at the designed head.

The water surface and pressure curves for Example 8 can be obtained by averaging the coordinates from Graphs 8 and 9, Figure 11, and multiplying the values by H_0 as outlined in Table 21.

The water surface and hydraulic gradient from Table 21 are plotted on Figure 14E. The pressures have been plotted vertically above the points at which they were measured. Figure 14E shows only the hydraulic pressure over the dam and downstream apron. The additional forces opposing uplift such as the weight of the dam and aprons, and the hydrostatic pressure acting horizontally against the upstream face of the dam have been purposely avoided in this example.

Table 21

COORDINATES FOR EXAMPLE 8

$H_0 = 10$

		Water surface		Pressure profile	
$\frac{X}{H_0}$	X Feet	$\frac{Y}{H_0}$	Y Feet	$\frac{Y}{H_0}$	Y Feet
-1	-10	0.92	+9.2		
-0.5	-5	0.89	8.9		
0	0	0.79	7.9	0.15	+1.5
+0.5	+5	0.59	5.9	0.24	2.4
1.0	10	0.31	3.1	0.62	6.2
1.5	15	0.13	1.3	0.55	5.5
2.0	20	0.22	2.2	0.44	4.4
2.5	25	0.38	3.8	0.41	4.1
3.0	30	0.49	4.9	0.45	4.5
3.5	35	0.57	5.7	0.49	4.9
4.0	40	0.62	6.2	0.52	5.2
5.0	50	0.64	6.4	0.58	5.8
+6.0	+60	0.64	+6.4	0.62	+6.2

IV. DETERMINATION OF DISCHARGE COEFFICIENTS FOR OTHER THAN THE IDEAL SHAPE OF OVERFALL SECTION

General

There has long been a need for a reliable method of estimating the coefficients of discharge for overfall spillway sections which differ from the ideal shape. By the ideal shape is meant, the shape of section which corresponds to the natural profile of the lower nappe surface for the designed head and given approach conditions. Pressures on the downstream face of the section will approach that of the atmosphere. This will be the most efficient overfall section, thus produce the highest discharge coefficient for a section on which subatmospheric pressures will be absent. The ideal shapes are those treated in Chapters II and III. They will be so designated for lack of a better term.

The present chapter will deal with the determination of discharge coefficients for irregular overfall spillway shapes or sections differing from the ideal shapes. The method will consist of comparing any irregular shape with corresponding ideal and irregular shapes for which the discharge coefficients are known. From the comparison, the coefficient of discharge for the unknown shape can be closely estimated for the designed head.

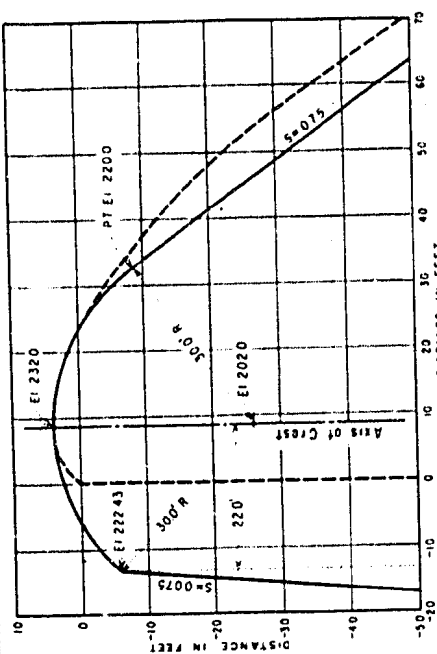
The coefficients of discharge for irregular shapes on which appreciable positive or subatmospheric pressures exist, will vary from those for an ideal shape based on the same head and approach conditions. Positive pressures on the downstream face will produce a decrease while subatmospheric pressures will

result in an increase in the coefficient of discharge. Spillway shapes are designed for both positive and subatmospheric pressure conditions and in only about 50 percent of the cases is it practical to utilize the ideal shapes.

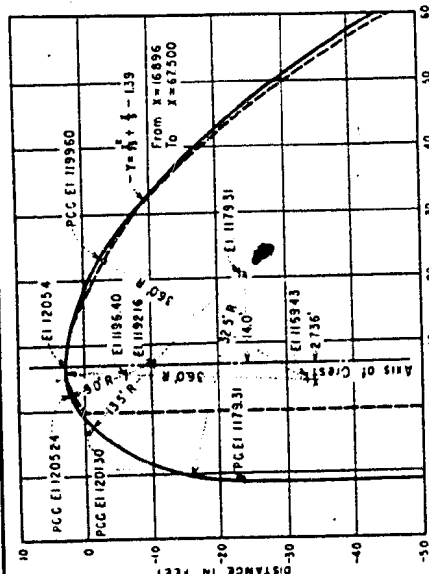
Irregular Sections with Vertical, Sloping, and Offset Upstream Faces

A comparison of various irregular shapes with corresponding ideal shapes, together with coefficients of discharge for the two, at the designed heads, are shown on Figures 15 and 16. The discharge coefficients for the irregular shapes were obtained from models tested in the laboratory over a period of some 15 years. The coefficients of discharge for the ideal shapes were computed according to the method in Chapter II.

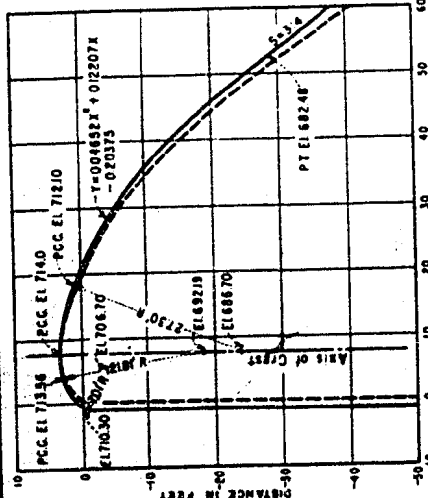
The solid line on Figure 15A represents the profile of the Wheeler Dam Spillway. Superimposed on the same Figure is the ideal shape, identified by the broken line, computed for a designed head of 17.0 feet on the crest and an average approach depth of 40 feet. The agreement in shapes is quite close and this is also true of the discharge coefficients. The coefficient for Wheeler Dam at the designed head, obtained from a 1:36 model is 3.975 and the coefficient for the ideal shape is 3.960, Figure 15A. Other cases where the actual shape closely matches the ideal shape are illustrated in Figure 15D and F and Figure 16A. The coefficients of discharge for each comparison agree reasonably well. The coefficients of discharge, prototype head, approach depth, and model scales are recorded under each graph.



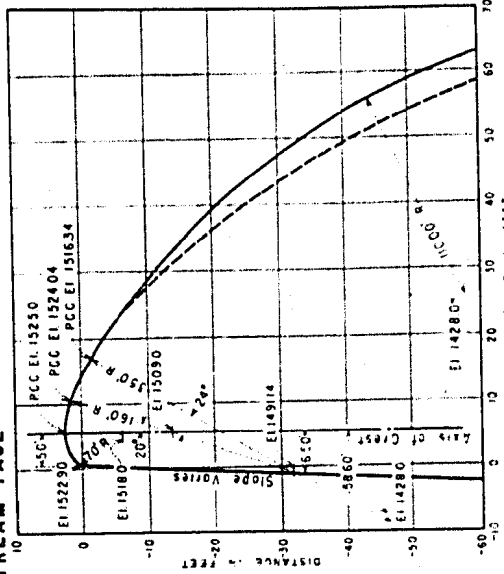
C. MADDEN DAM SPILLWAY
MODEL SCALE 1:72
RES. ELEV. 283.4
P-E = 132.0
M₀ = 31.4
C_d = 3.92
C_d = 3.97
MODEL
IDEAL



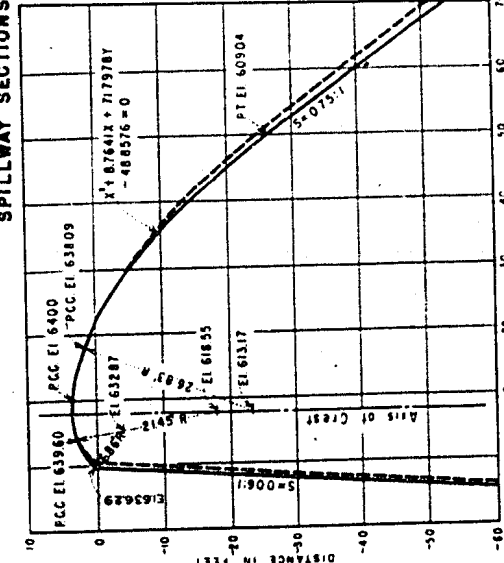
B. BOULDER DAM SPILLWAY
SHAPE 8 - FINAL MODEL C-8
MODEL SCALE 1:100
RES. ELEV. 332.0
P-E = 100
M₀ = 28.6
C_d = 3.92
C_d = 3.97
MODEL
IDEAL



A. MARSHALL FORD DAM SPILLWAY
FINAL DESIGN
MODEL SCALE 1:100
RES. ELEV. 742.0
P-E = 214.0
M₀ = 28.0
C_d = 3.92
C_d = 3.97
MODEL
IDEAL



E. ROSS DAM SPILLWAY
(2nd STEP)
MODEL SCALE 1:50
RES. ELEV. 1545.0
P-E = 125.0
M₀ = 20.0
C_d = 3.92
C_d = 3.97
MODEL
IDEAL



D. MARSHALL FORD DAM SPILLWAY
INITIAL DESIGN
MODEL SCALE 1:100
RES. ELEV. 670.0
P-E = 140.0
M₀ = 30.0
C_d = 3.92
C_d = 3.97
MODEL
IDEAL

SPILLWAY SECTIONS WITH SLOPING UPSTREAM FACE

COMPARISON OF DISCHARGE COEFFICIENTS
MODEL COEFFICIENT VERSUS IDEAL COEFFICIENT
FOR SPILLWAY SECTIONS WITH VERTICAL AND SLOPING UPSTREAM FACES

In making a comparison of this kind, it is necessary to match either the upstream faces of the actual and ideal sections or their axes. Neither method is altogether satisfactory. In the comparisons in this chapter, the axes of the actual and ideal shapes are made to coincide so that the crests or highest points on the two shapes constitute a common point.

In the case of Canyon Ferry Dam Spillway, Figure 15B, the actual profile falls outside of the ideal shape, thus positive pressures will exist on the dam face for all operating heads. A comparison of the discharge coefficients verifies this statement as the coefficient is 3.66 for the actual shape against 3.36 for the ideal shape. The spillway for Keowee Dam, Figure 15E, is a similar case.

Figure 15C shows the effect produced on the coefficient by broadening of the actual section in the upstream direction. On the other hand, Figure 16B indicates that broadening of the upstream portion of the section is not detrimental to the coefficient providing sufficient curvature is incorporated in the upstream face.

Similar comparisons of actual spillway shapes with corresponding ideal shapes are shown on Figure 16C, D, and E, and Figure 17 for overfall sections with sloping upstream faces. Similar information on overfall sections with offsets in the upstream face are included on Figure 18.

The shapes of section herein presented do not necessarily represent the final design of the dam spillways with which they are identified. Some are final designs while others represent

preliminary studies on which reliable model data were obtained. The coordinates on Figures 15 through 18 are in feet, prototype. The dimensions of the actual sections are complete and are expressed in radii, parabolas, or rectangular or polar coordinates. The coordinates of the ideal shapes have not been tabulated because of their bulky nature.

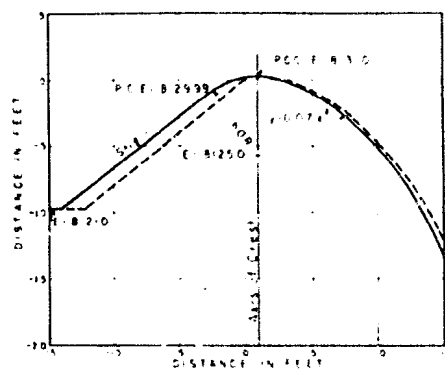
For the purpose of design, the information on Figures 15 through 18 has been replotted in dimensionless coordinates on Figures 19 through 27. All dimensions and coordinate distances have been divided by the total head, H_0 , on the respective overfall sections. By this method of plotting, similar shapes will coincide, thus actual dimensions and elevations have been omitted. Some designers may prefer the dimensionless over the straight plotting.

It is desired to call attention to a paper by W.M.Borland¹³ in which he proposes an entirely different method for the determination of discharge coefficients for irregular overfall shapes. The coefficient of discharge is expressed in terms of the ratio of the head on the overfall crest to the head that produces the best-fitting nappe shape. The method can be applied to the preceding shapes but is not applicable to the sections which follow.

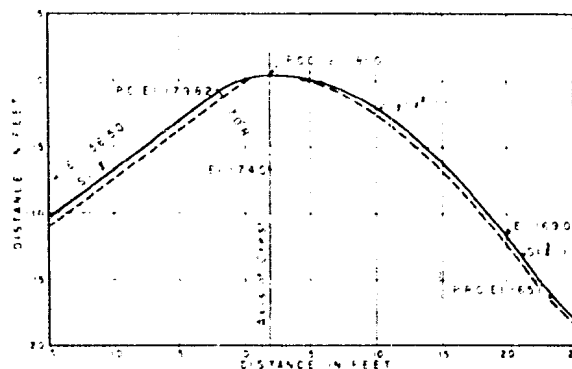
Effect of Piers on the Discharge Coefficient

Up to this point, nothing has been said concerning the effect of piers on the coefficient of discharge. In the tests de-

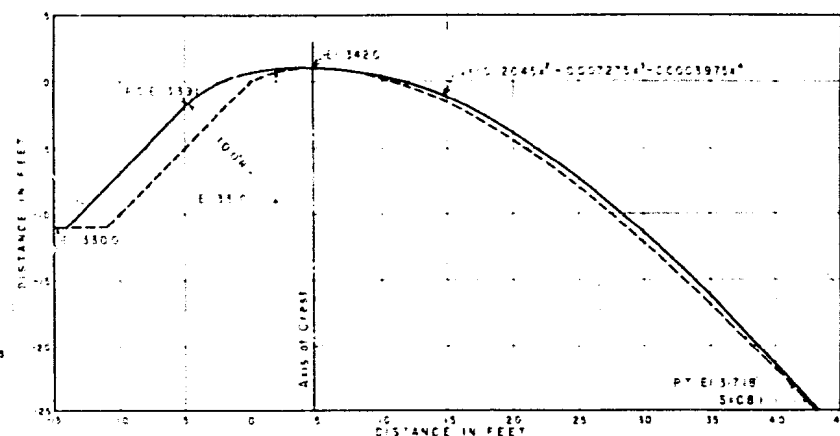
¹³ Borland, W. M. "Flow over Rounded Crest Weirs," University of Colorado, Thesis, 1938.



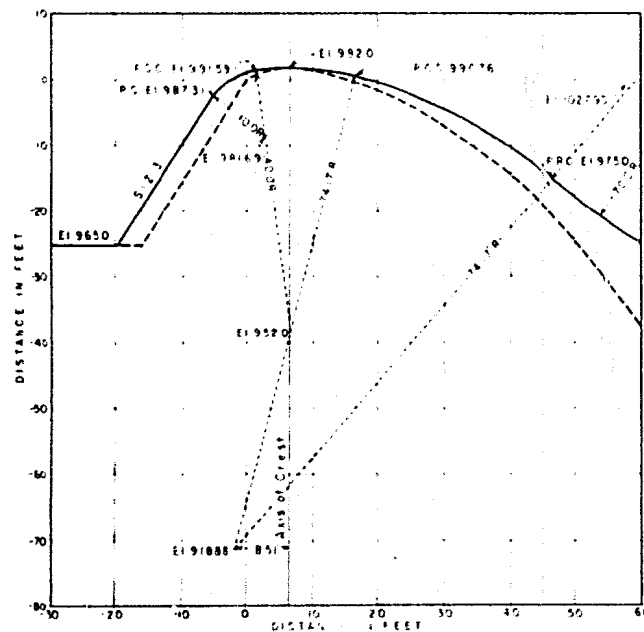
A. MOON LAKE SPILLWAY
MODEL SCALE 140
RES ELEV 8130
P+ E: 100 C_d: 3.80 — MODEL
H₀: 80 C_d: 3.89 — IDEAL



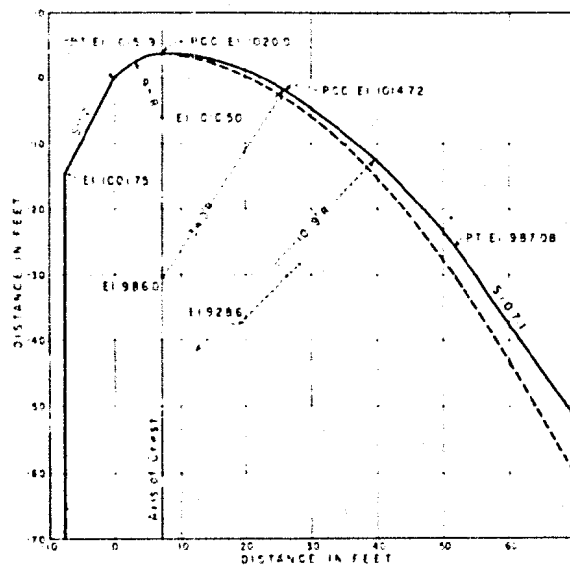
B. IMPERIAL DAM SPILLWAY
MODEL SCALE 130
RES ELEV 7910
P+ E: 270 C_d: 3.75 — MODEL
H₀: 100 C_d: 3.91 — IDEAL



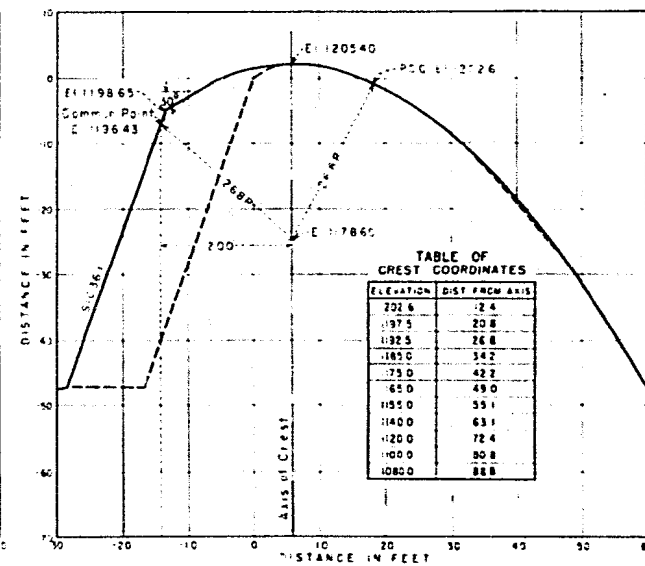
C. HEADGATE ROCK DAM SPILLWAY
MODEL SCALE 160
RES ELEV 3677
P+ E: 120 C_d: 3.57 — MODEL
H₀: 2570 C_d: 3.84 — IDEAL



D. HAMILTON DAM SPILLWAY
MODEL SCALE 160
RES ELEV 10240
P+ E: 270 C_d: 3.67 — MODEL
H₀: 320 C_d: 3.90 — IDEAL



E. NORRIS DAM SPILLWAY
MODEL SCALE 172
RES ELEV 10470
P+ E: 2010 C_d: 3.80 — MODEL
H₀: 270 C_d: 3.96 — IDEAL



F. BOULDER DAM SPILLWAY
SHAPES 384 MODEL M-3
MODEL SCALE 120
RES ELEV 12320
P+ E: 400 C_d: 3.68 — MODEL
H₀: 266 C_d: 3.92 — IDEAL

TABLE OF
CREST COORDINATES

ELEVATION	DIST FROM AXIS
202.6	12.4
197.5	20.8
192.5	26.8
187.0	34.2
175.0	42.2
165.0	49.0
155.0	55.1
140.0	63.1
120.0	72.4
100.0	80.8
108.0	88.8

COMPARISON OF DISCHARGE COEFFICIENTS
MODEL COEFFICIENT VERSUS IDEAL COEFFICIENT
FOR SPILLWAYS WITH SLOPING UPSTREAM FACES

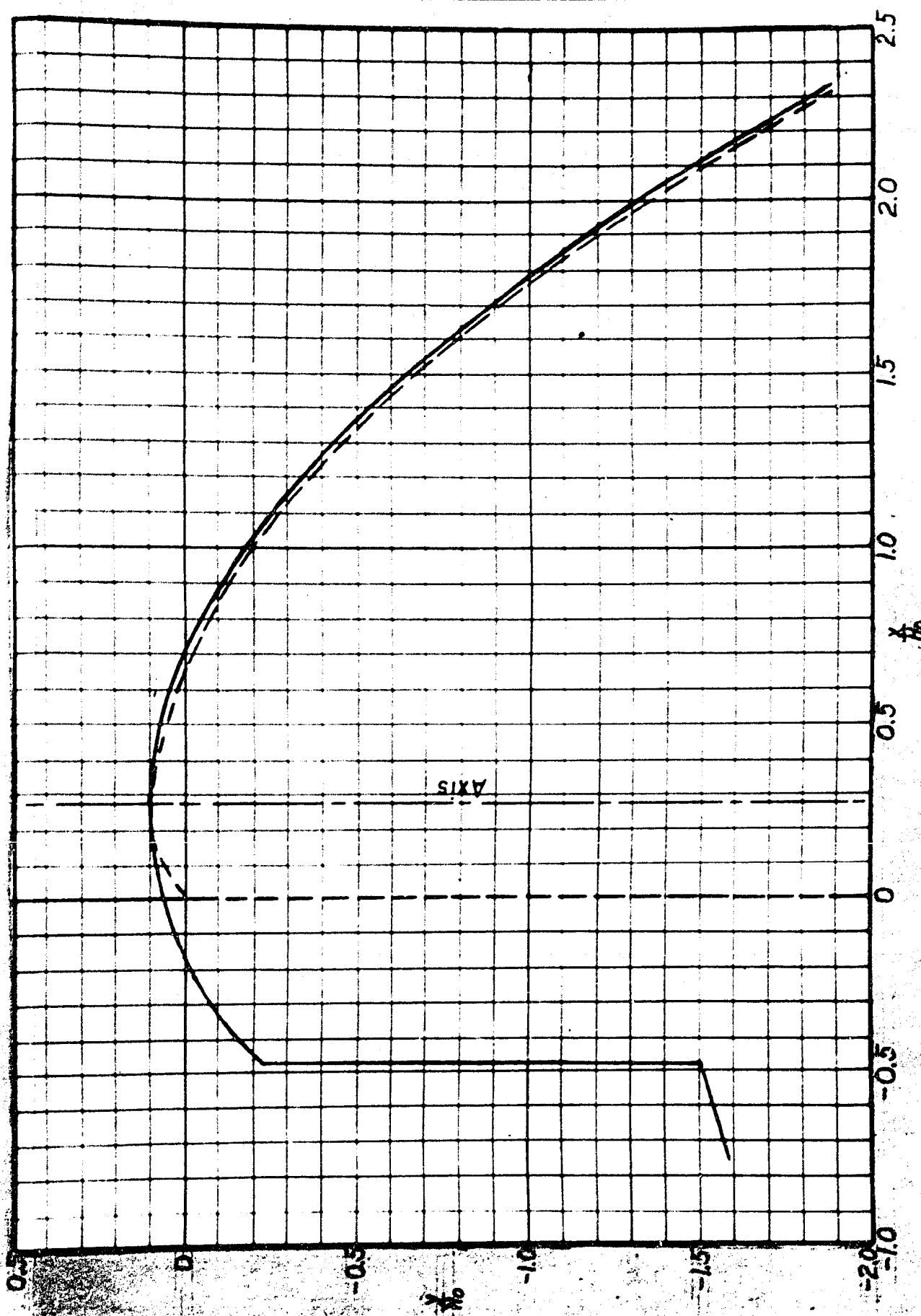
15B. CAYNON FERRY DAM SPILLWAY

$$\frac{H_0}{P+E} = 0.2615$$

$$C_A = 3.66$$

$$C_I = 3.96$$

$$\text{MODEL}$$

$$\text{IDEAL}$$


15C. BOULDER DAM SPILLWAY

$$\frac{H_0}{P+E} = 0.6650$$

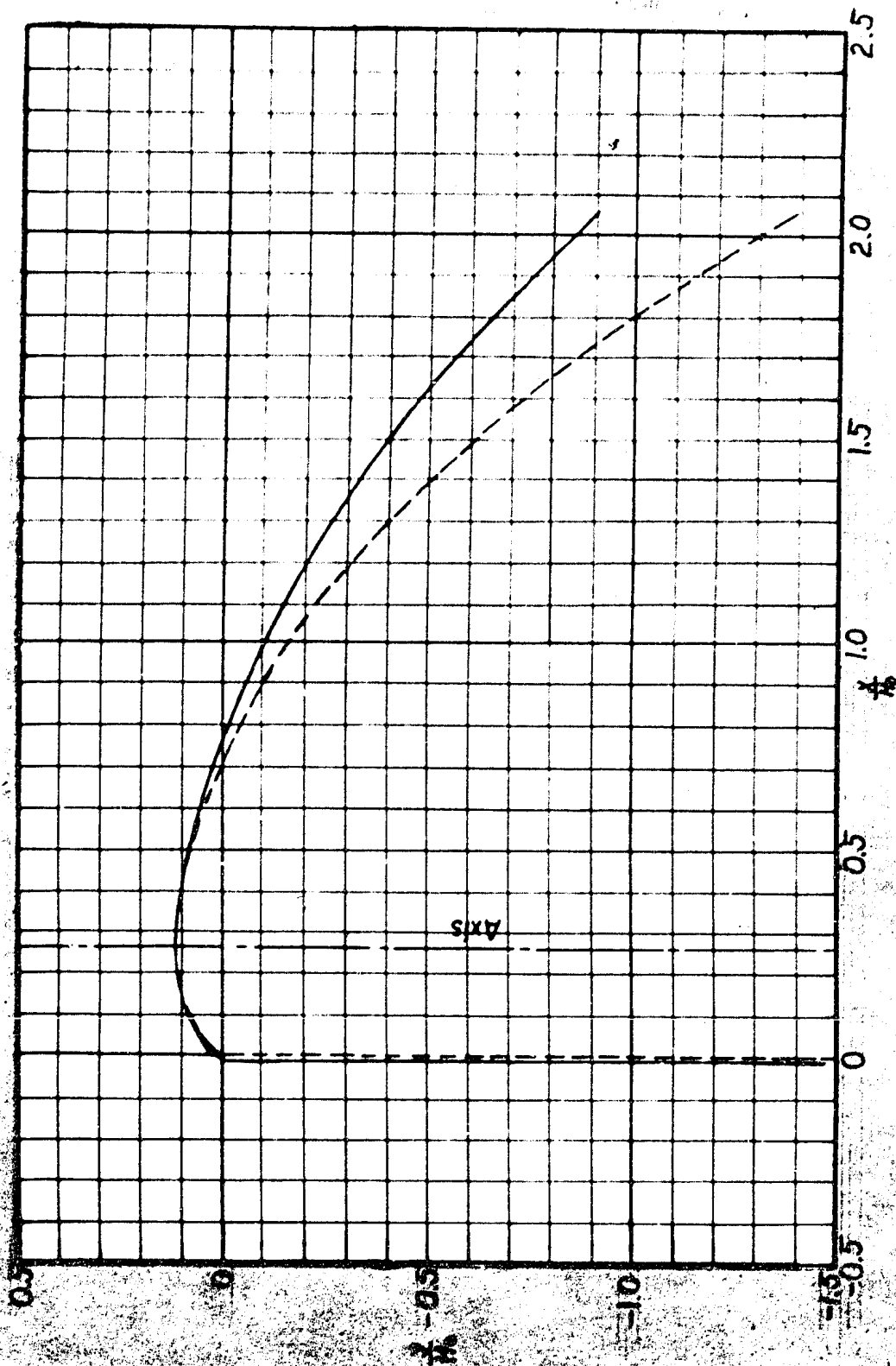
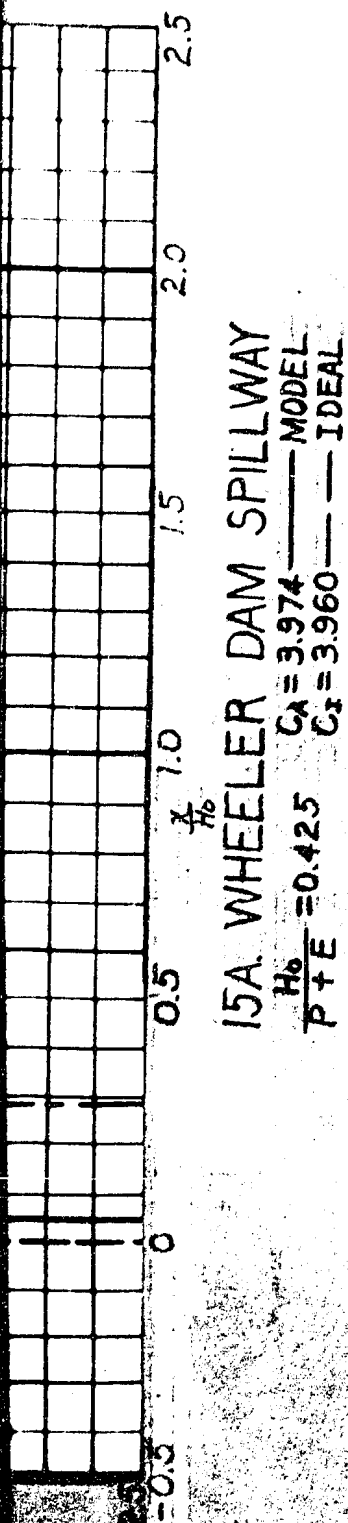
$$C_A = 3.575$$

$$C_I = 3.930$$

$$\text{MODEL M-1}$$

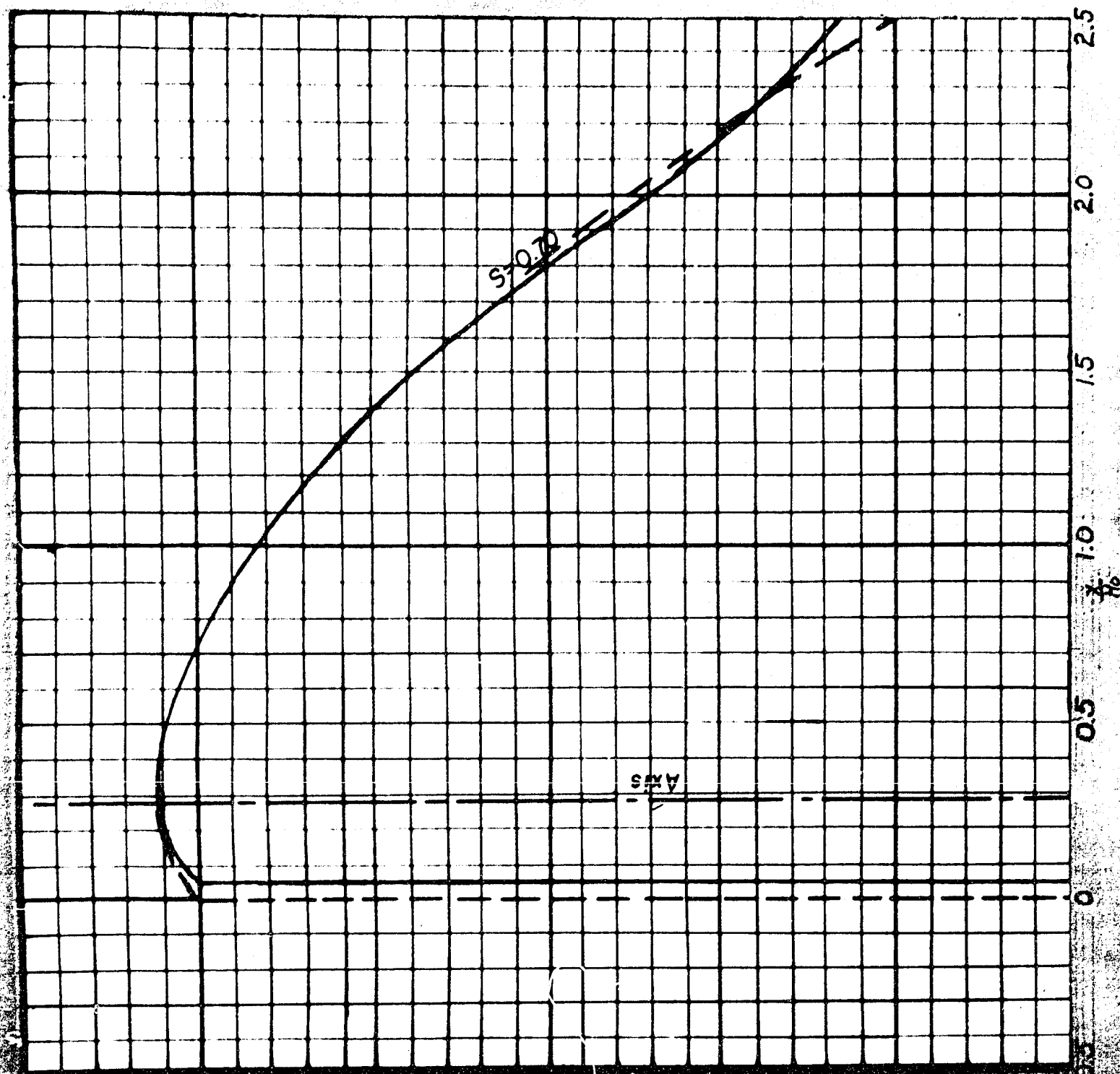
$$\text{MODEL}$$

$$\text{IDEAL}$$



COMPARISON OF DISCHARGE COEFFICIENTS

DIMENSIONLESS PLOTTING — SCALE: 2.5" = 1.0

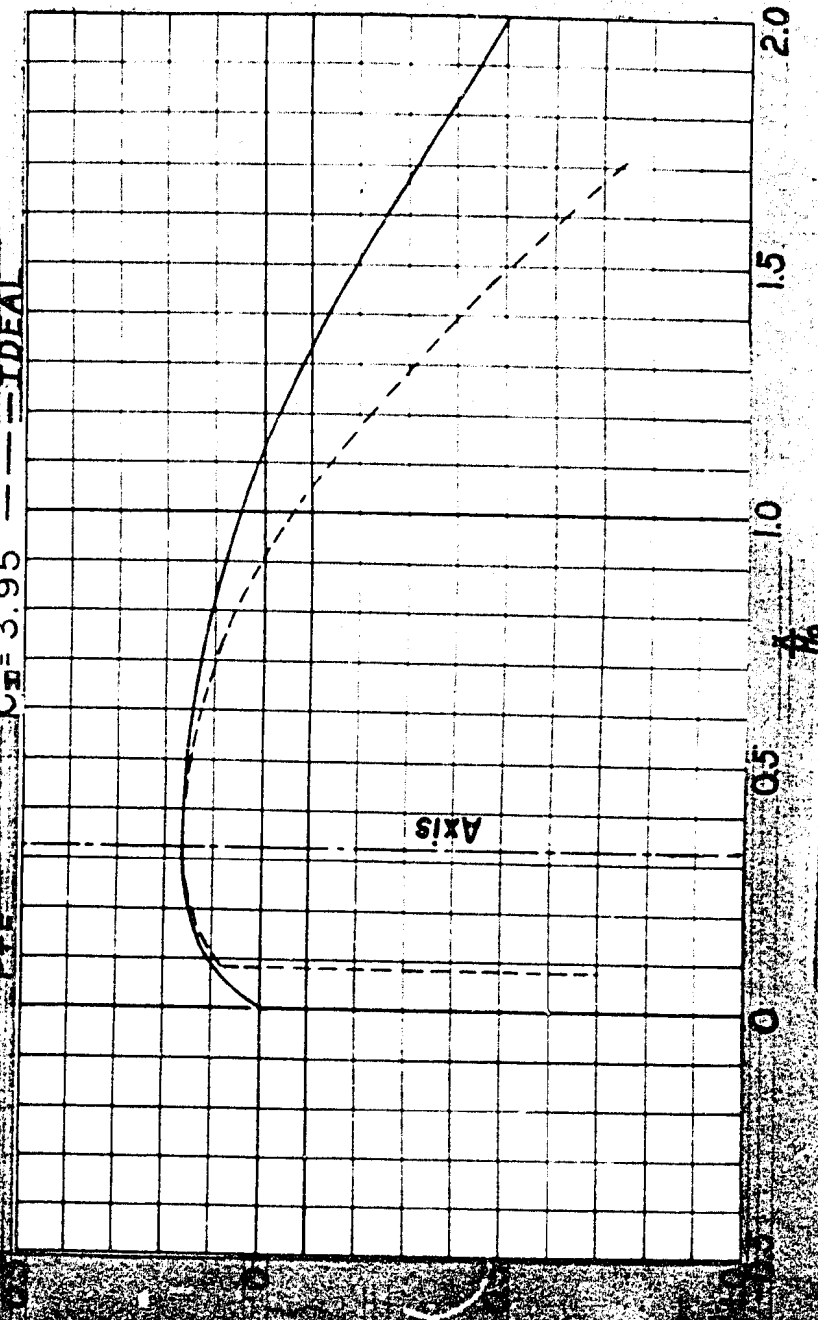


15A WHEELER DAM SPILLWAY

$\frac{H_0}{P+E} = 0.425$ $C_1 = 3.974$ MODEL
 $C_2 = 3.960$ IDEAL

15D. BOULDER DAM SPILLWAY

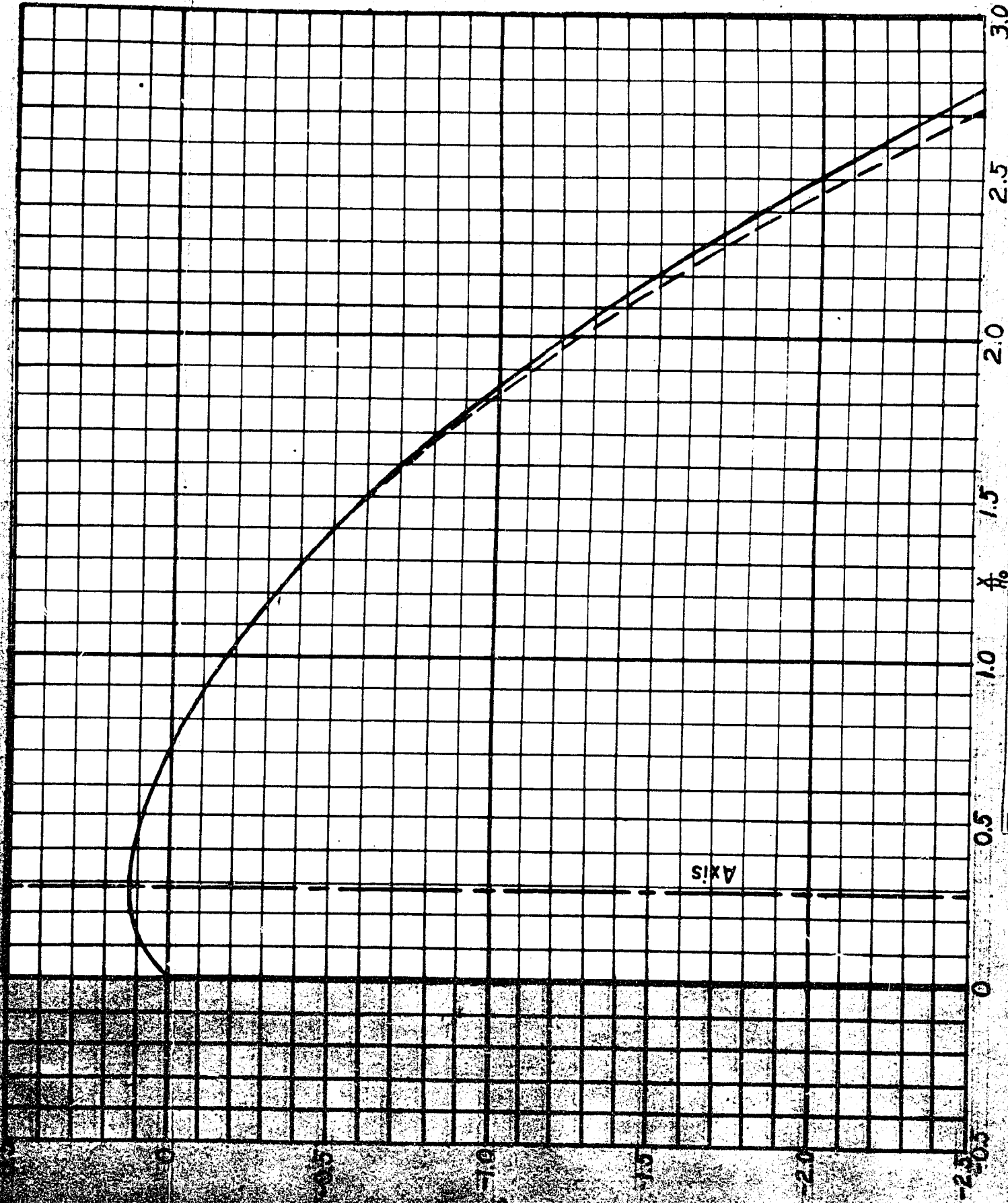
SHAPE 1 MODEL C-2
 $\frac{H_0}{P+E} = 0.5619$ $C_A = 3.875$ MODEL
 $C_I = 3.95$ IDEAL



COMPARISON OF DISCHARGE COEFFICIENTS

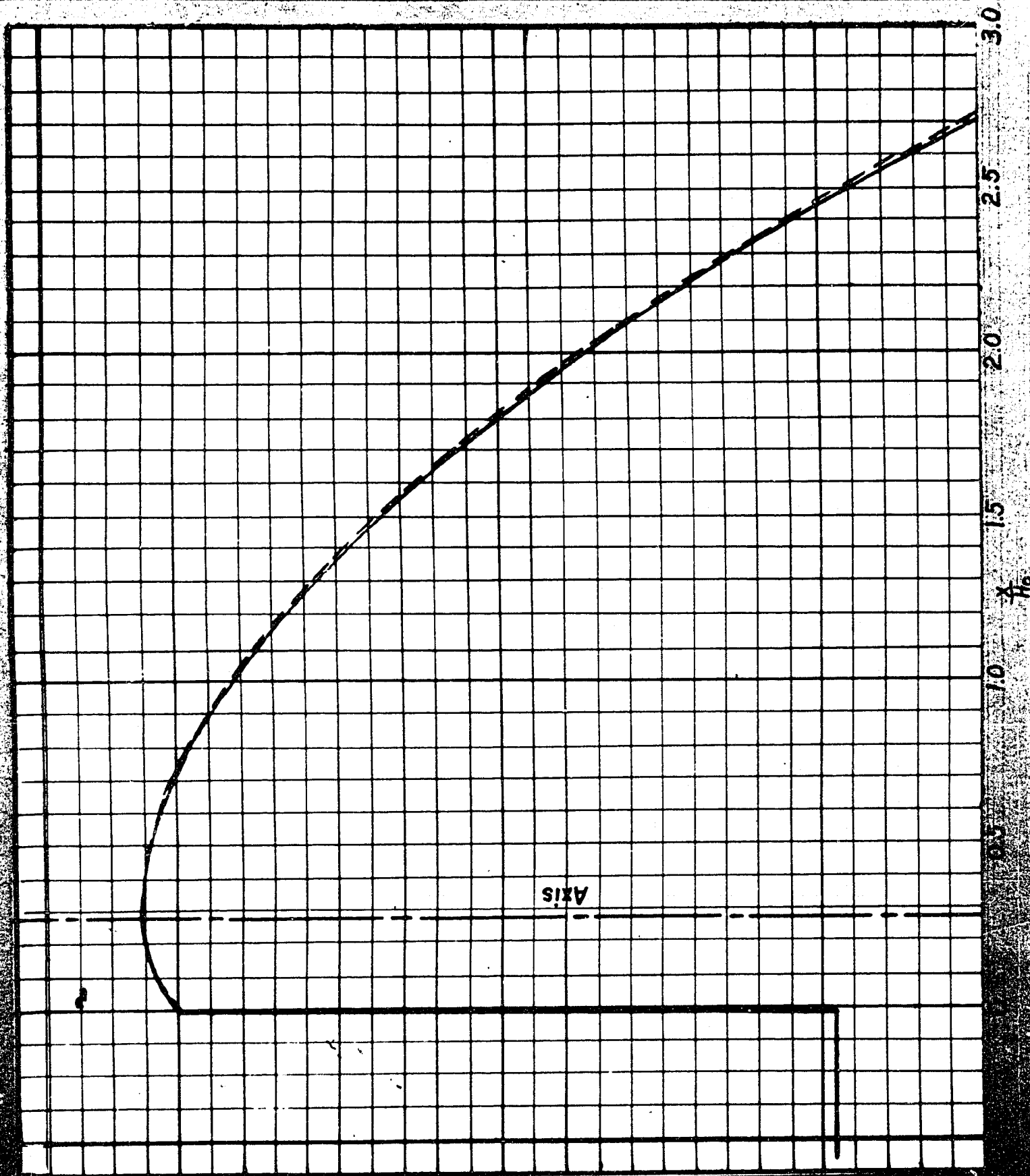
15E. KESWICK DAM SPILLWAY

$\frac{H_0}{P+E} = 1.471$ $C_A = 3.45$ MODEL
 $C_I = 3.85$ IDEAL



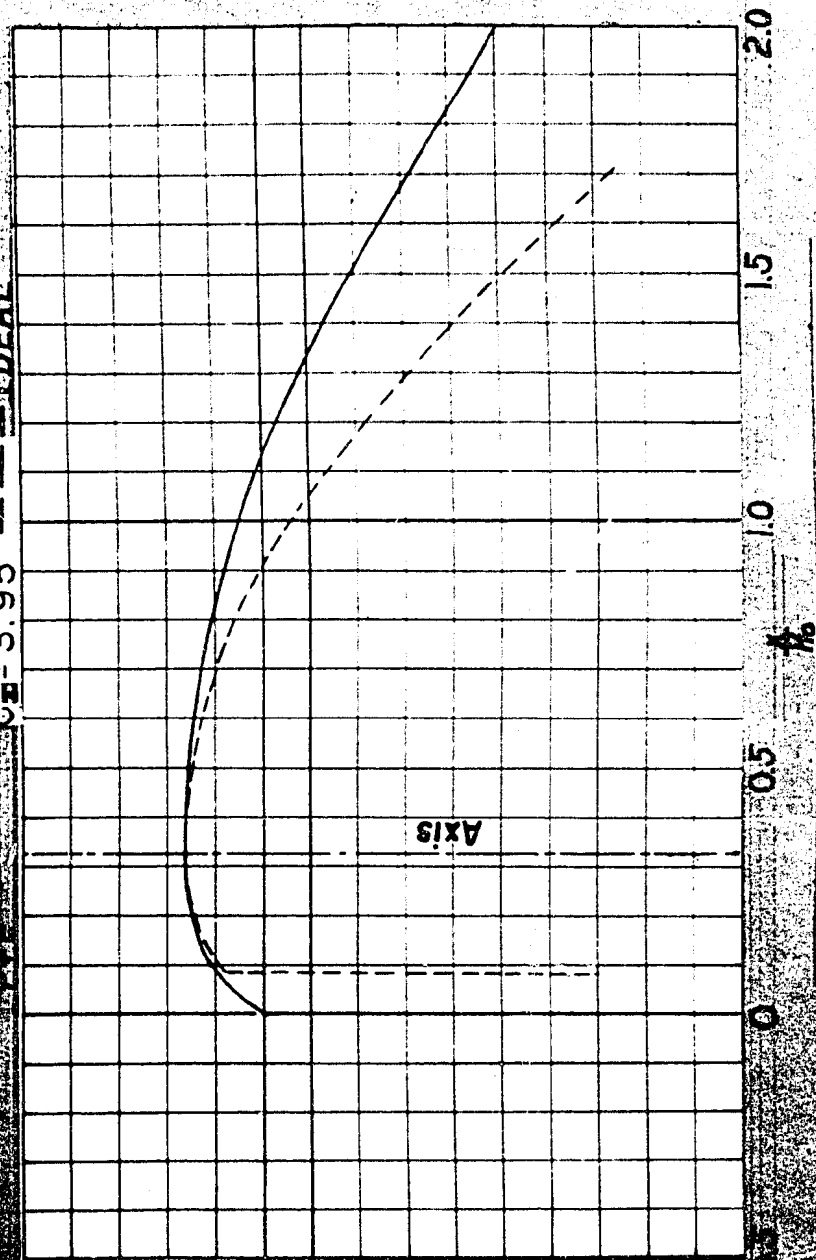
15F. BOULDER DAM SPILLWAY

SHAPE 2 MODEL C-3
 $\frac{H_0}{P+E} = 0.08917$ $C_A = 3.91$ MODEL
 $C_I = 3.91$ IDEAL



15D. BOULDER DAM SPILLWAY

SHAPE I MODEL C-2
 $C_A = 3.875$ ——— MODEL
 $C_E = 3.95$ - - - IDEAL



15E. KESWICK DAM SPILLWAY

$C_A = 3.15$ ——— MODEL
 $C_E = 3.85$ - - - IDEAL

COMPARISON OF DISCHARGE COEFFICIENTS

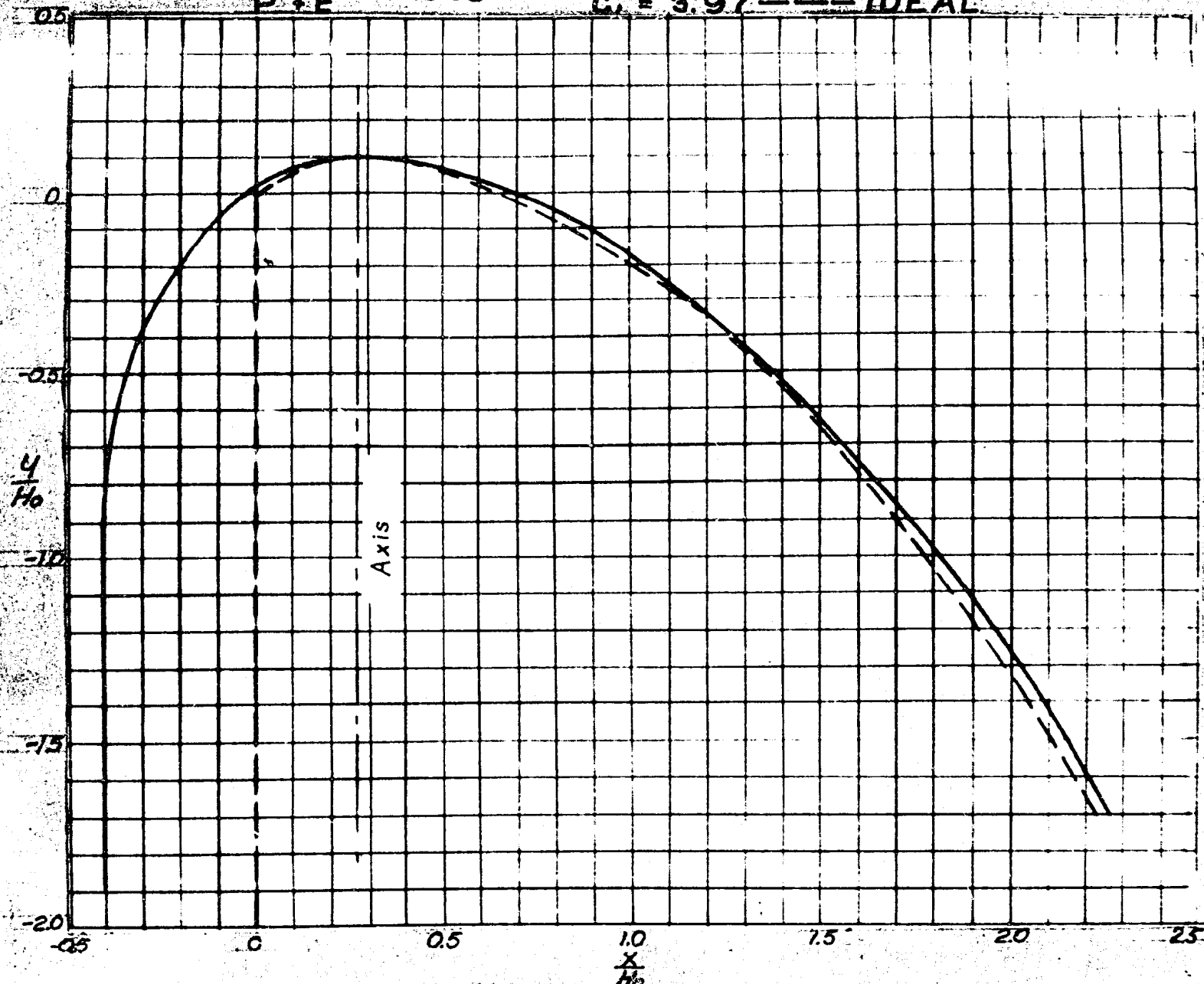
DIMENSIONLESS PLOTTING — SCALE: 2.5" = 1.0

16 A. MARSHALL FORD DAM SPILLWAY
(FINAL DESIGN)

$$\frac{H_o}{P+E} = 0.1908$$

$$C_A = 3.955 \text{ — MODEL}$$

$$C_I = 3.97 \text{ — IDEAL}$$



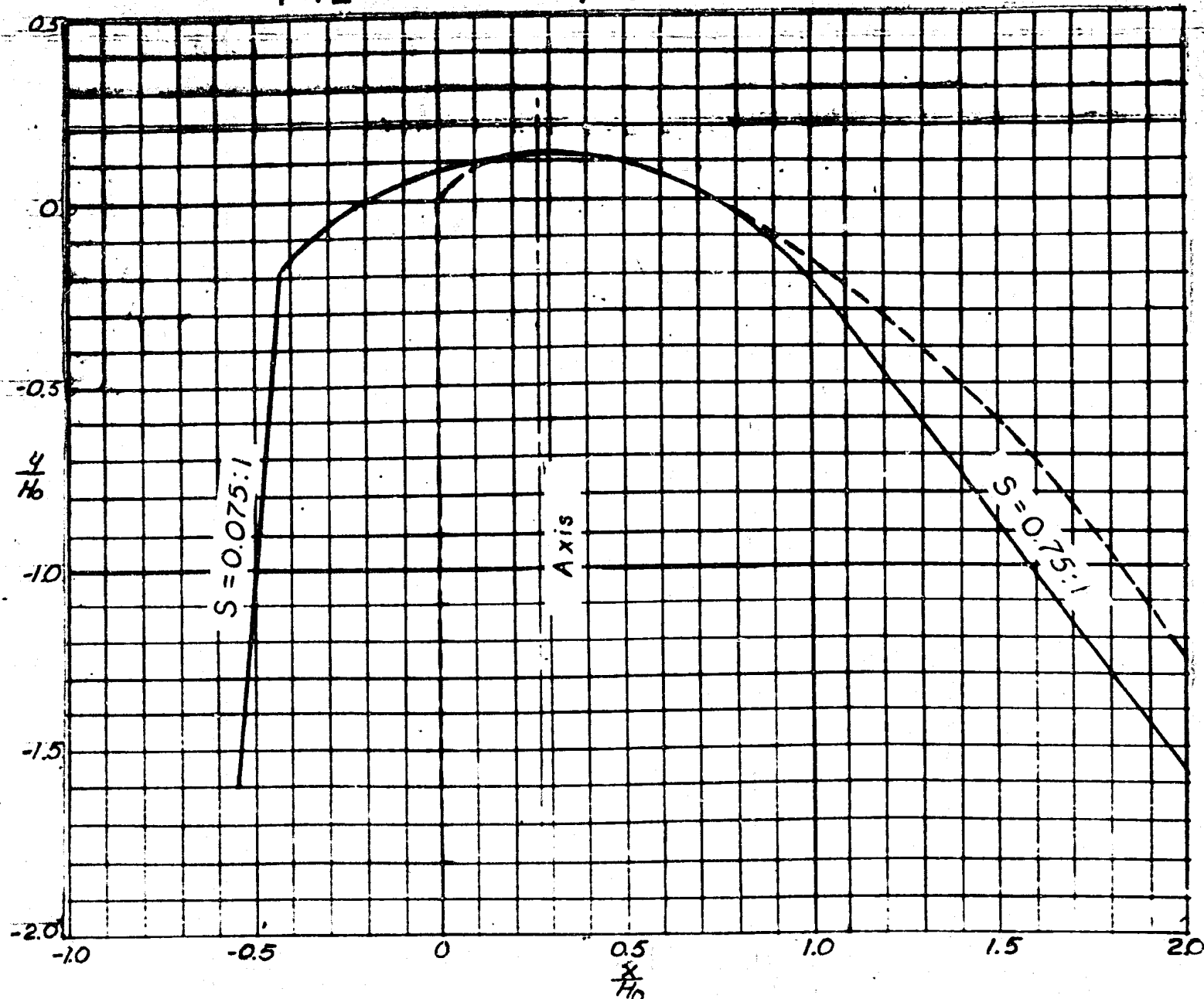
16 B.

BOULDER DAM SPILLWAY
SHAPE 8 — FINAL MODEL C-8

$$\frac{H_o}{P+E} = 0.6650$$

$$C_A = 3.925 \text{ — MODEL}$$

$$C_I = 3.92 \text{ — IDEAL}$$



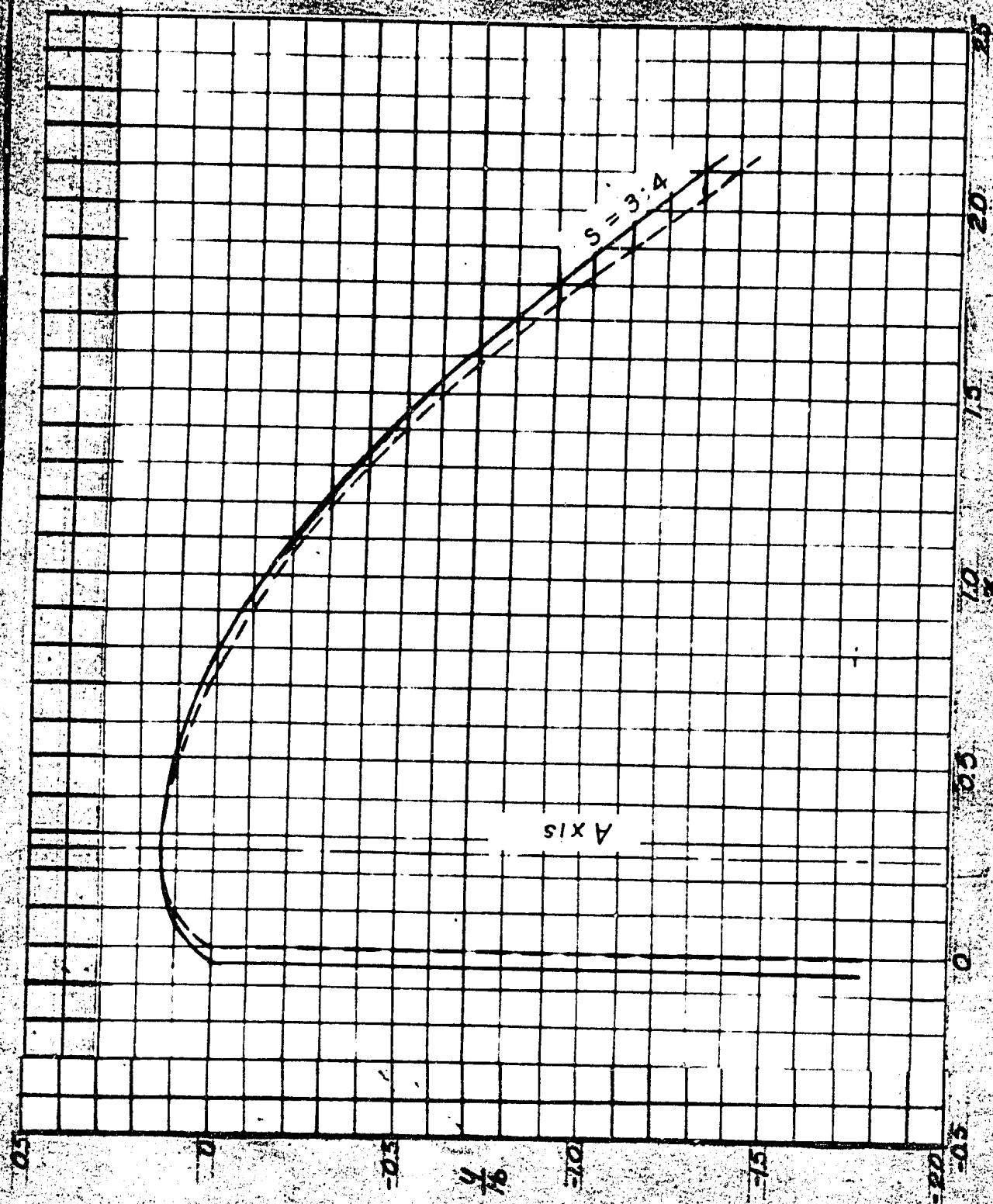
16 C.

MADDEN DAM SPILLWAY

$$\frac{H_o}{P+E} = 0.2379$$

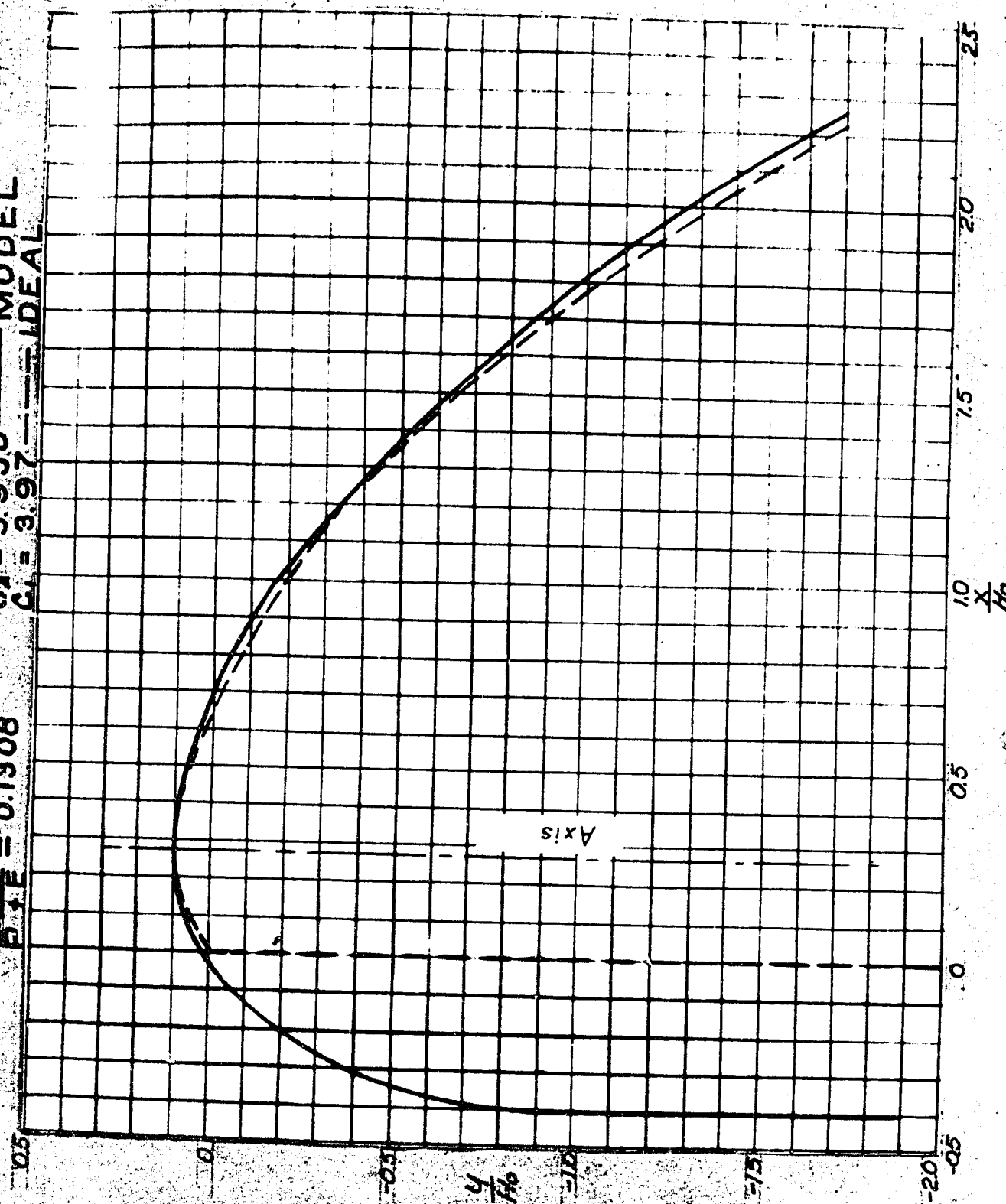
$$C_A = 3.82 \text{ — MODEL}$$

$$C_I = 3.97 \text{ — IDEAL}$$



16 A. MARSHALL FORD DAM SPILLWAY
(FINAL DESIGN)

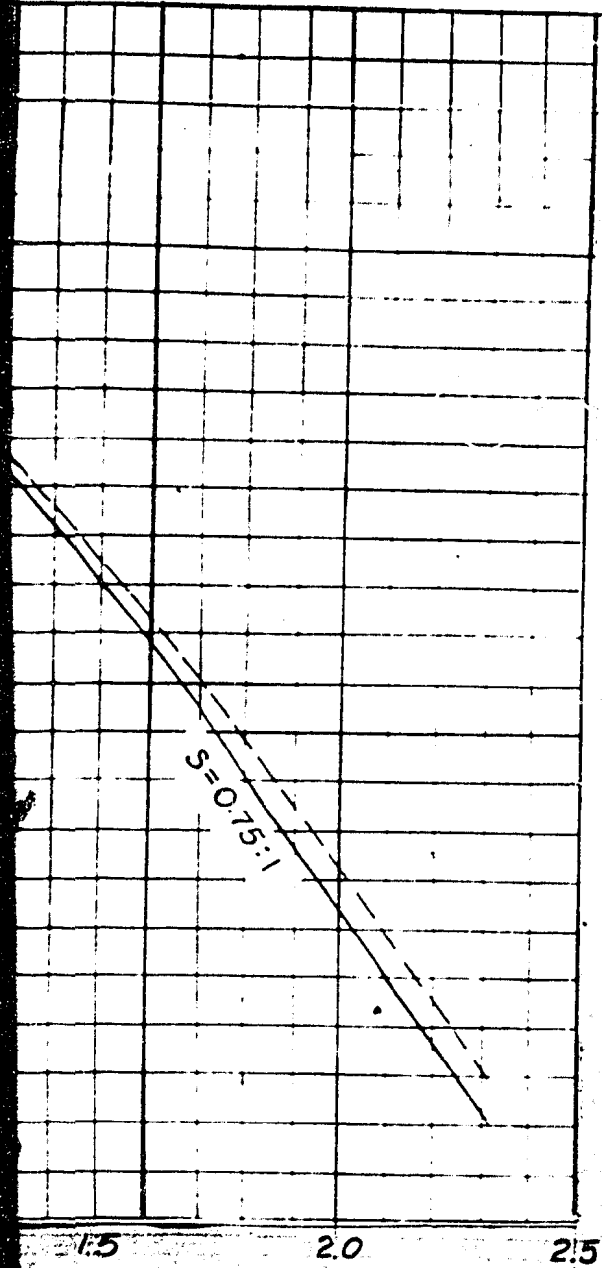
$H_0/P+E = 0.1308$ $C_d = 3.955$ — MODEL
 $C_d = 3.97$ — IDEAL



16 B. BOULDER DAM SPILLWAY
SHAPE 8 — FINAL MODEL C-8

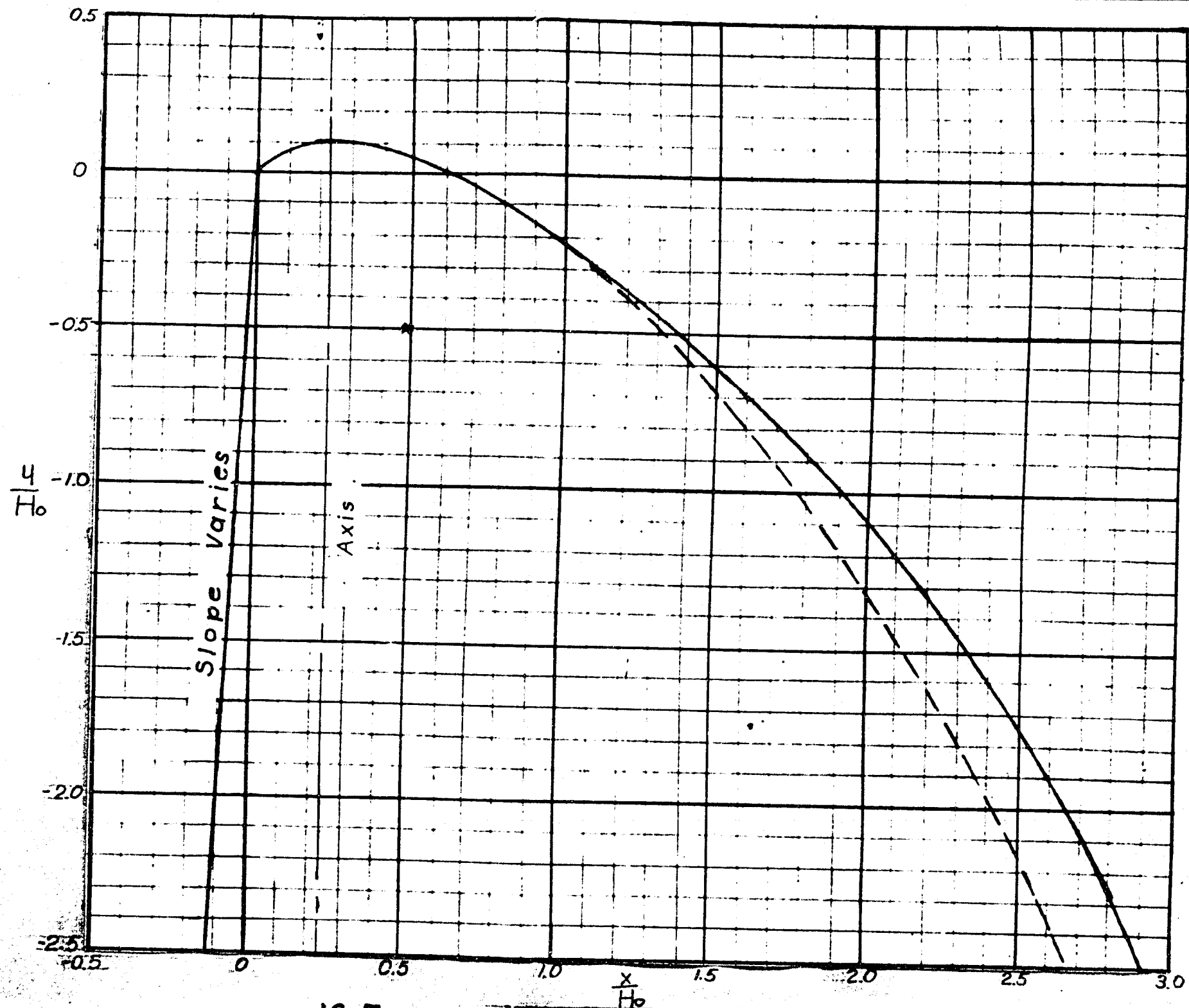
$H_0/P+E = 0.6650$ $C_d = 3.925$ — MODEL
 $C_d = 3.92$ — IDEAL

COMPARISON OF DISCHARGE COEFFICIENTS
DIMENSIONLESS PLOTTING — SCALE: 2.5"=1.0



DAM SPILLWAY
(IGN)
6 ——— MODEL
7 ——— IDEAL

COMPARISON OF DISCHARGE COEFFICIENT
DIMENSIONLESS PLOTTING — SCALE: 2.5" = 1.0



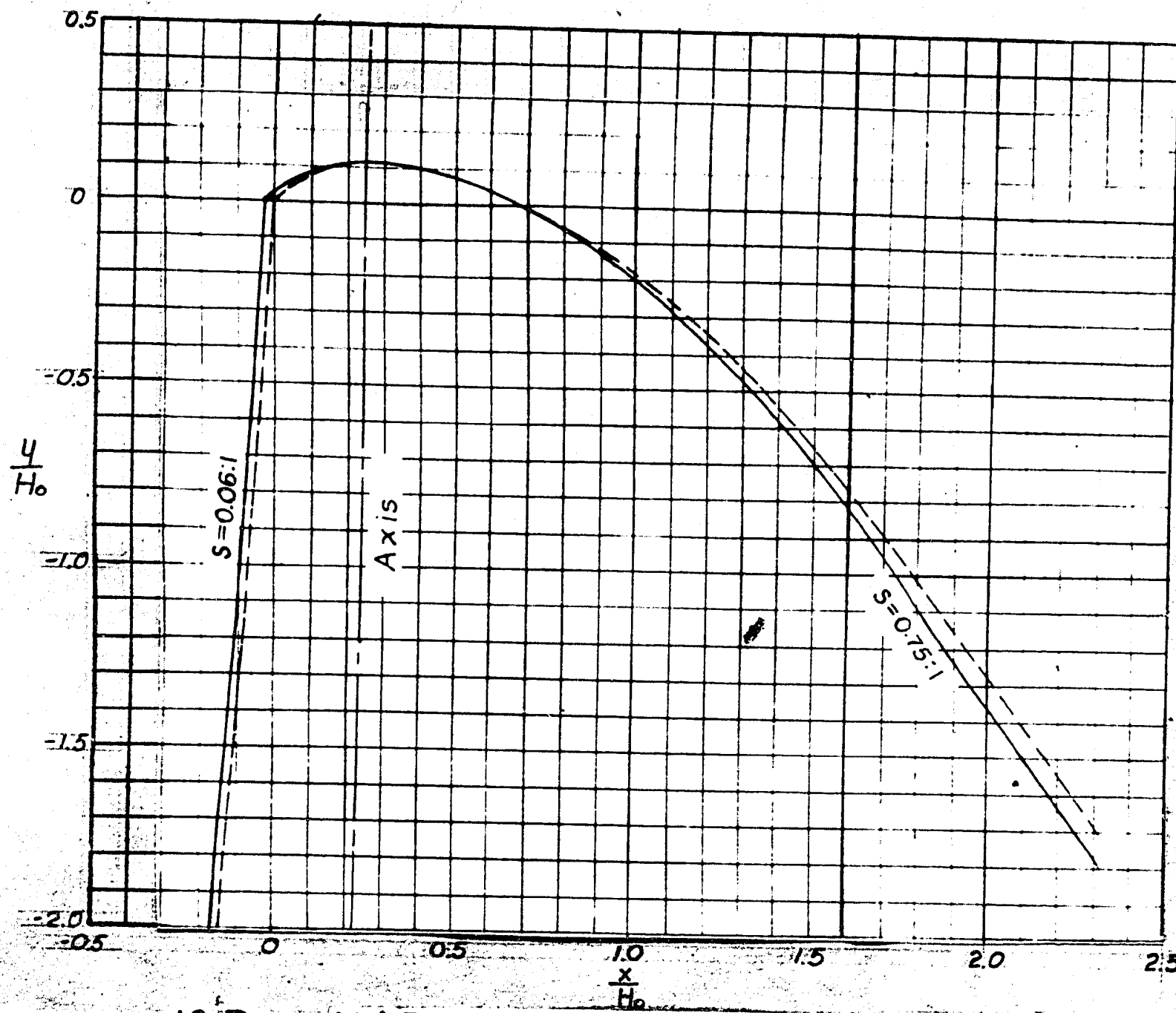
16 E.

ROSS DAM SPILLWAY
(2nd STEP)

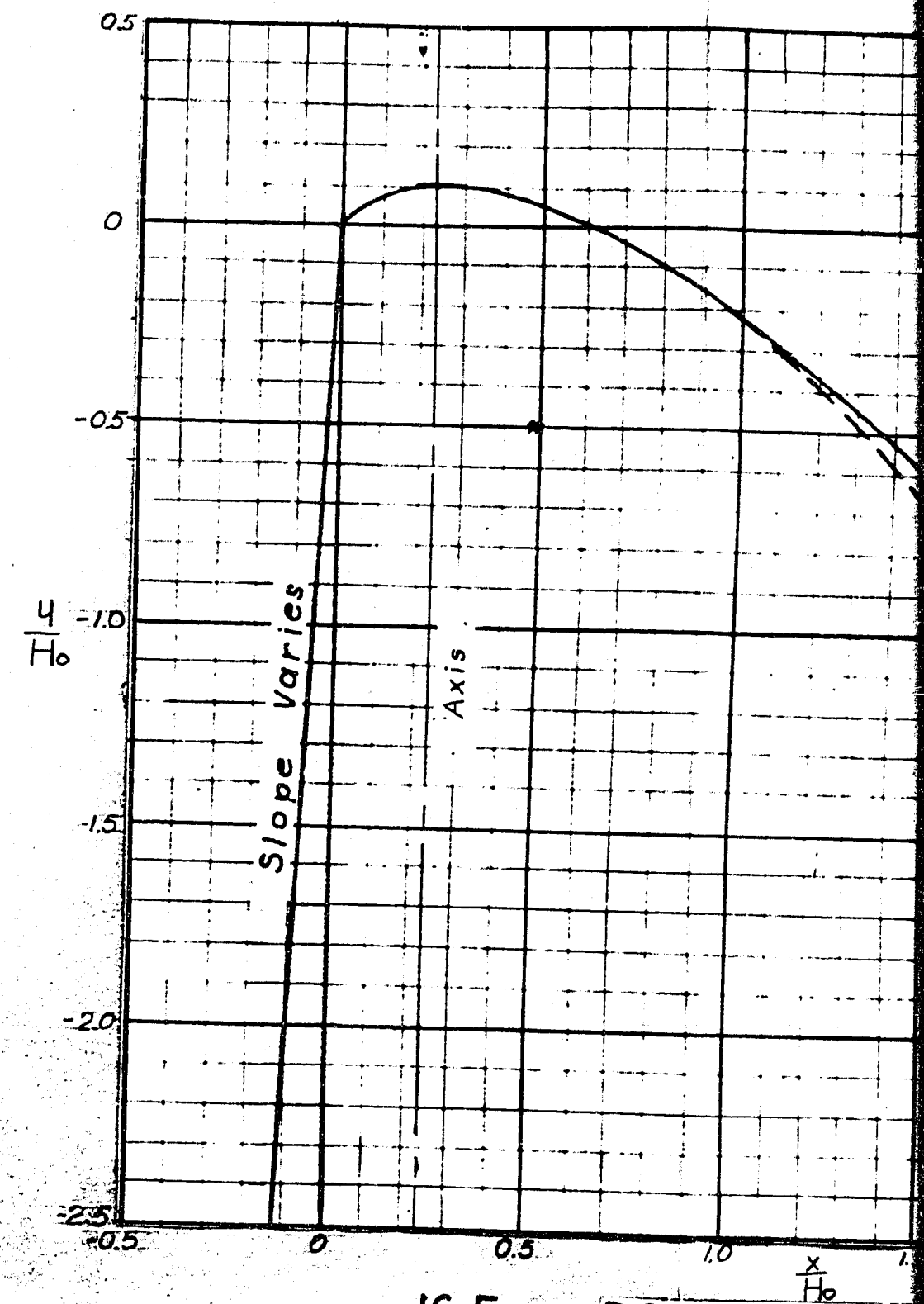
$$\frac{H_0}{P+E} = 0.1600$$

$$C_d = 3.835 \text{ ——— MODEL}$$

$$C_d = 3.97 \text{ ——— IDEAL}$$

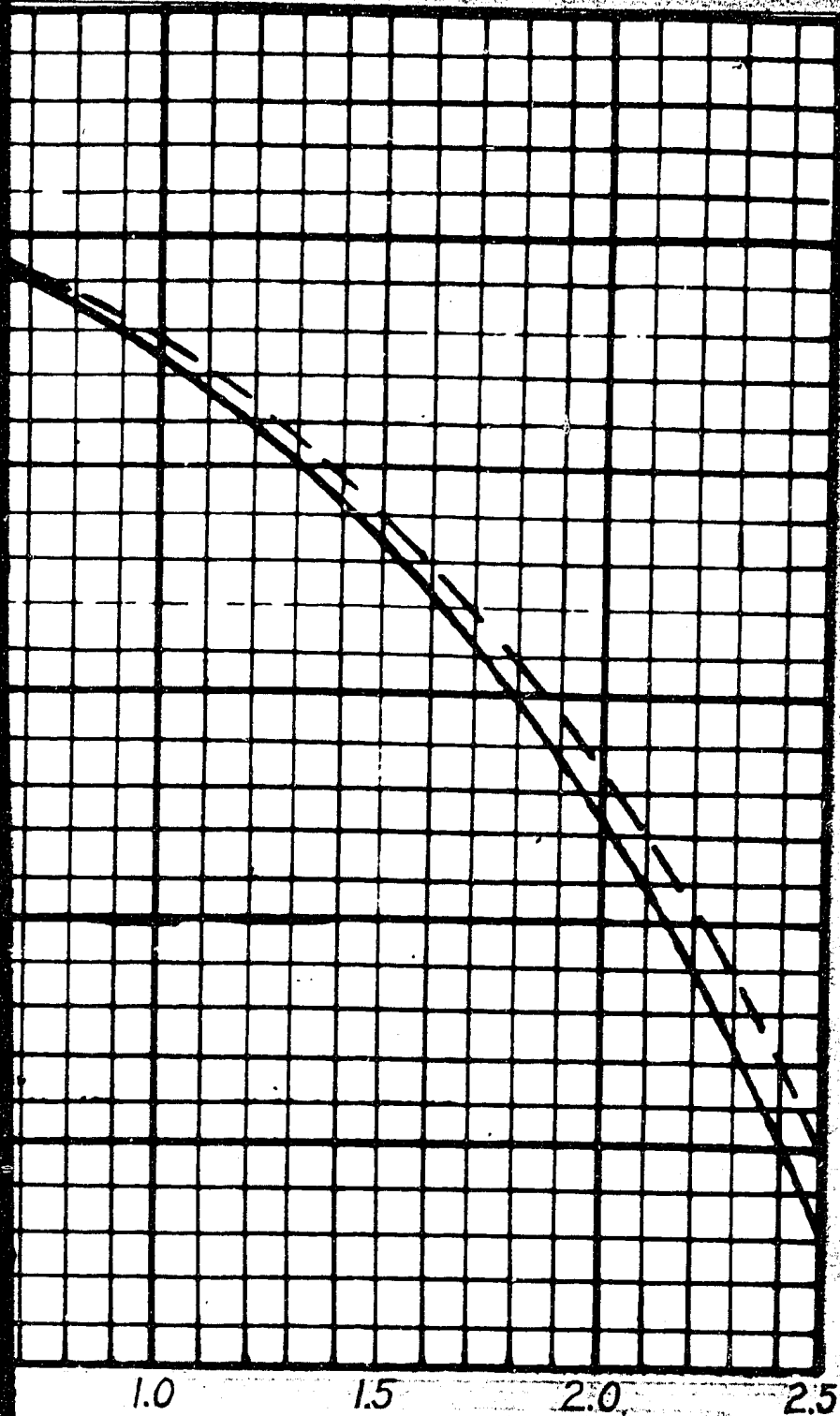


16 D. MARSHALL FORD DAM SPILLWAY
(INITIAL DESIGN)
 $\frac{H_0}{P+E} = 0.2143$ $C_d = 3.95$ — MODEL
 $C_d = 3.97$ — IDEAL



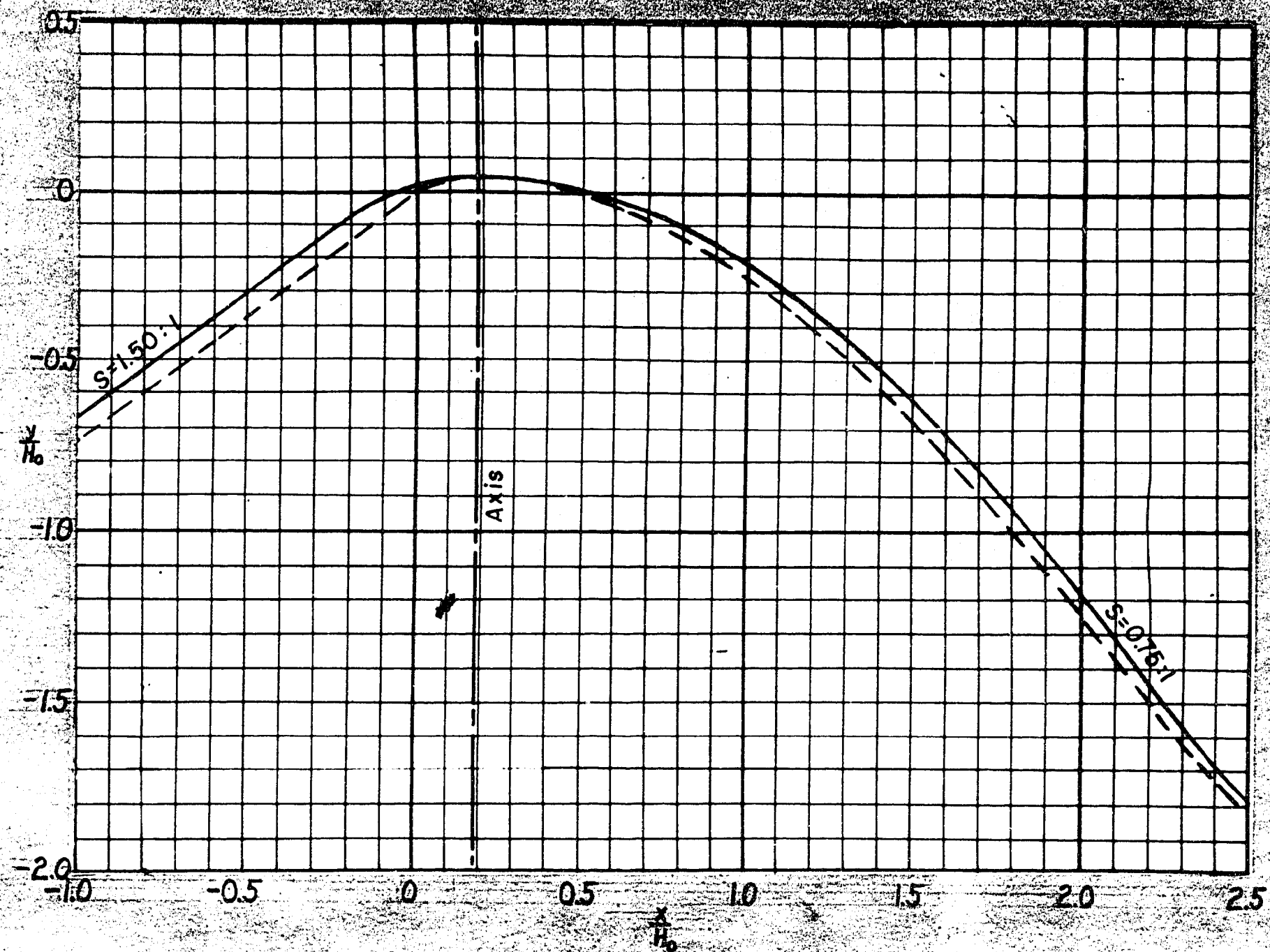
16 E. ROSS DAM
(2nd STEP)
 $\frac{H_0}{P+E} = 0.1600$ $C_d = 3.95$ — MODEL
 $C_d = 3.97$ — IDEAL

COMPARISON OF DISCHARGE COEFFICIENT
DIMENSIONLESS PLOTTING — SCALE: 2.5" = 1.0



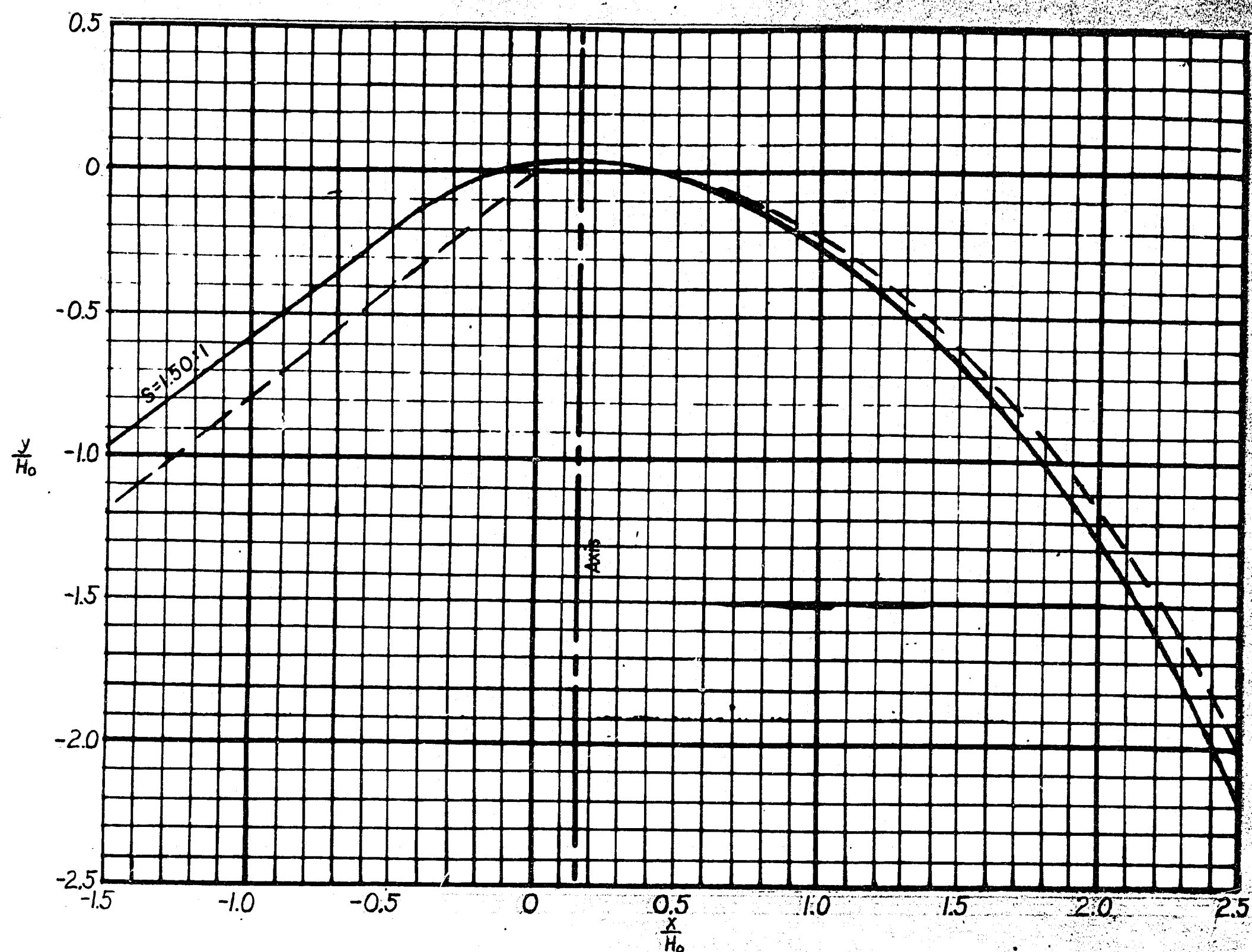
SPILLWAY
 90 ——— MODEL
 89 ——— IDEAL

COMPARISON OF DISCHARGE COEFFICIENTS
 DIMENSIONLESS PLOTTING — SCALE 2.5"=1.0



17B. IMPERIAL DAM SPILLWAY

$\frac{h_0}{P+H} = 0.3704$ $C_d = 3.75$ ——— MODEL
 $C_d = 3.81$ ——— IDEAL



17A. MOON LAKE SPILLWAY

$$\frac{H_0}{P+E} = 0.600$$

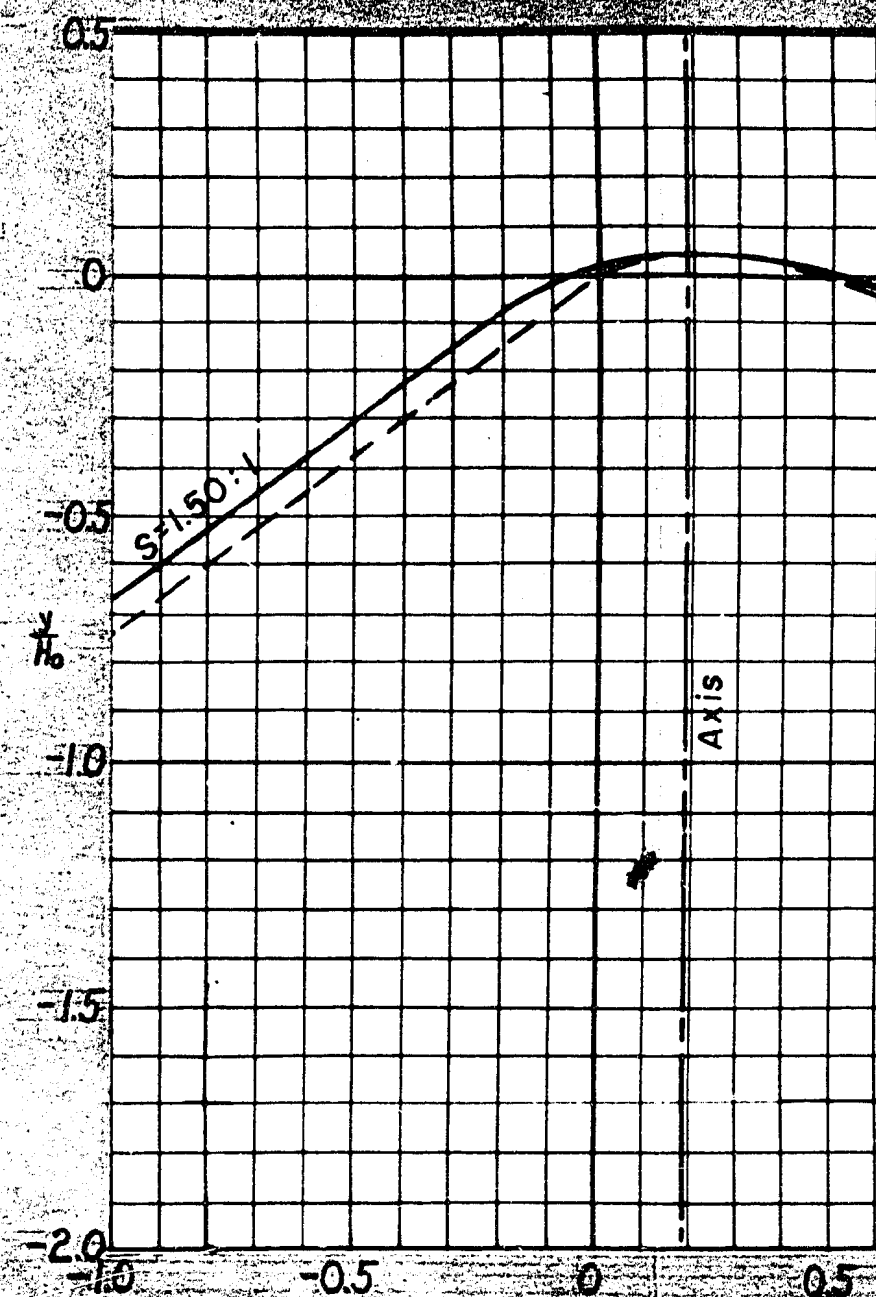
$$C_A = 3.90$$

MODEL

$$C_I = 3.89$$

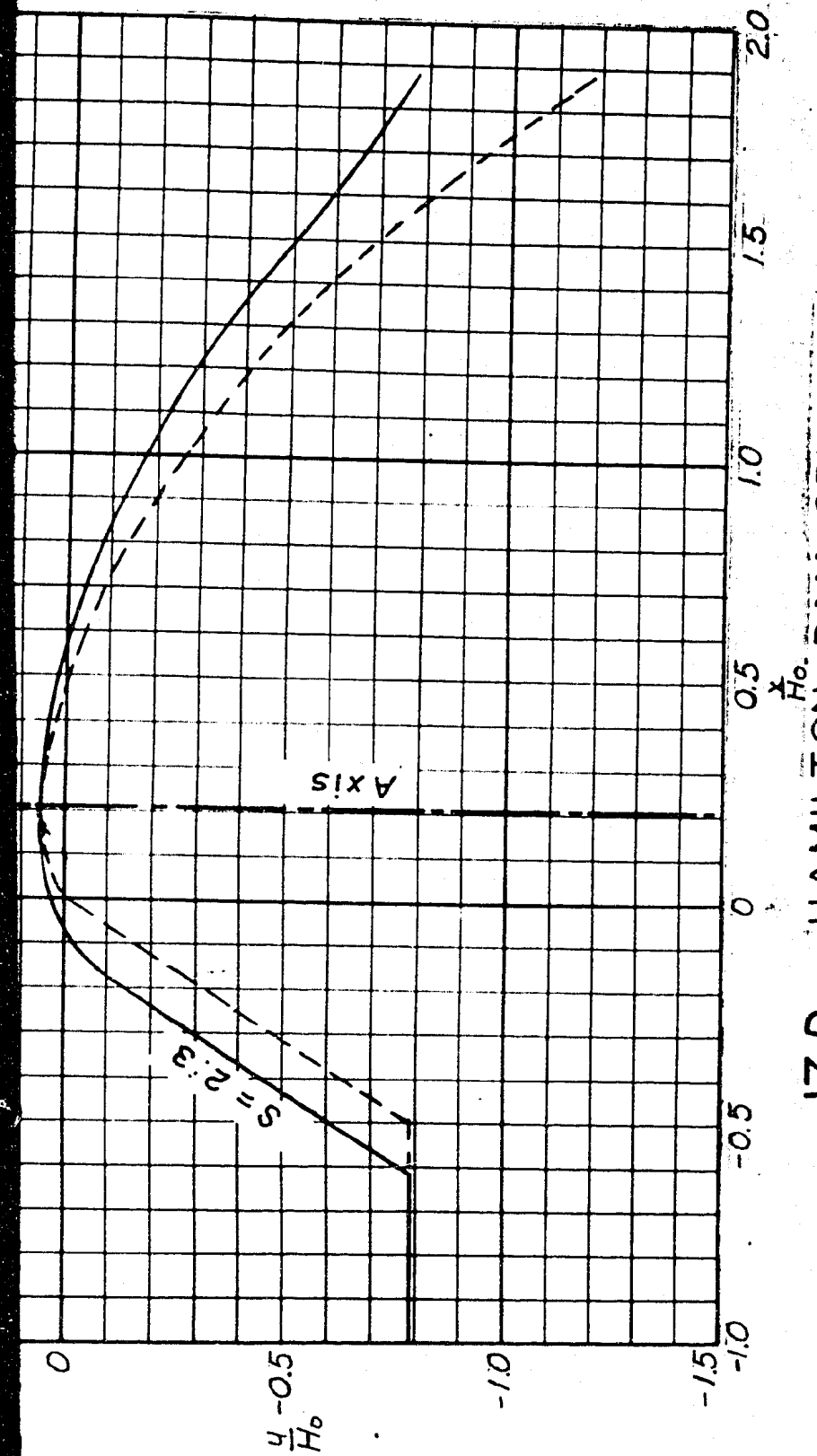
IDEAL

COMPARISON OF DISCHARGE COEFFICIENTS
DIMENSIONLESS PLOTTING — SCALE 2.5"=10'

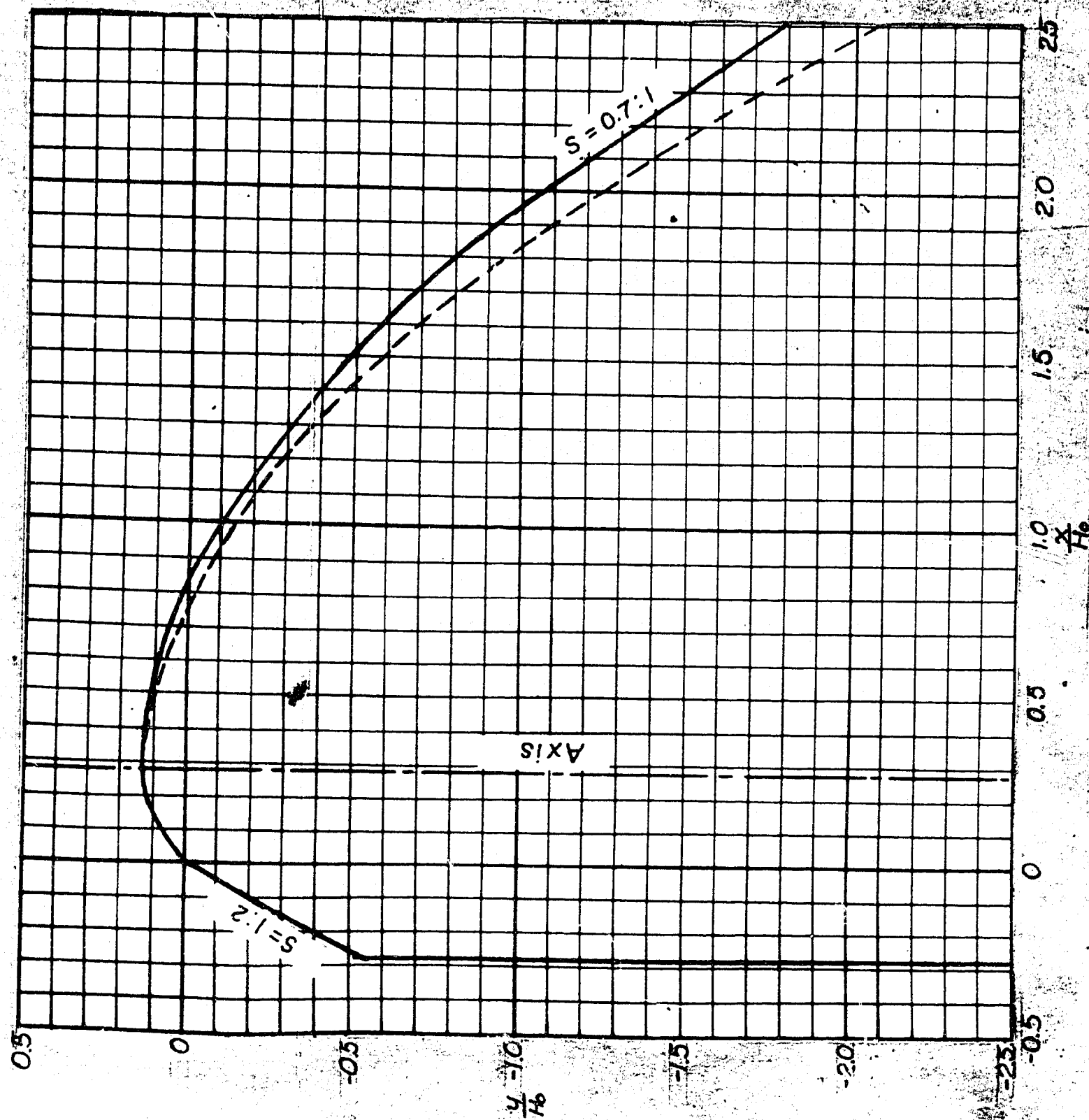


17B. IMPERIA

$$\frac{H_0}{P+E} = 0.3704$$



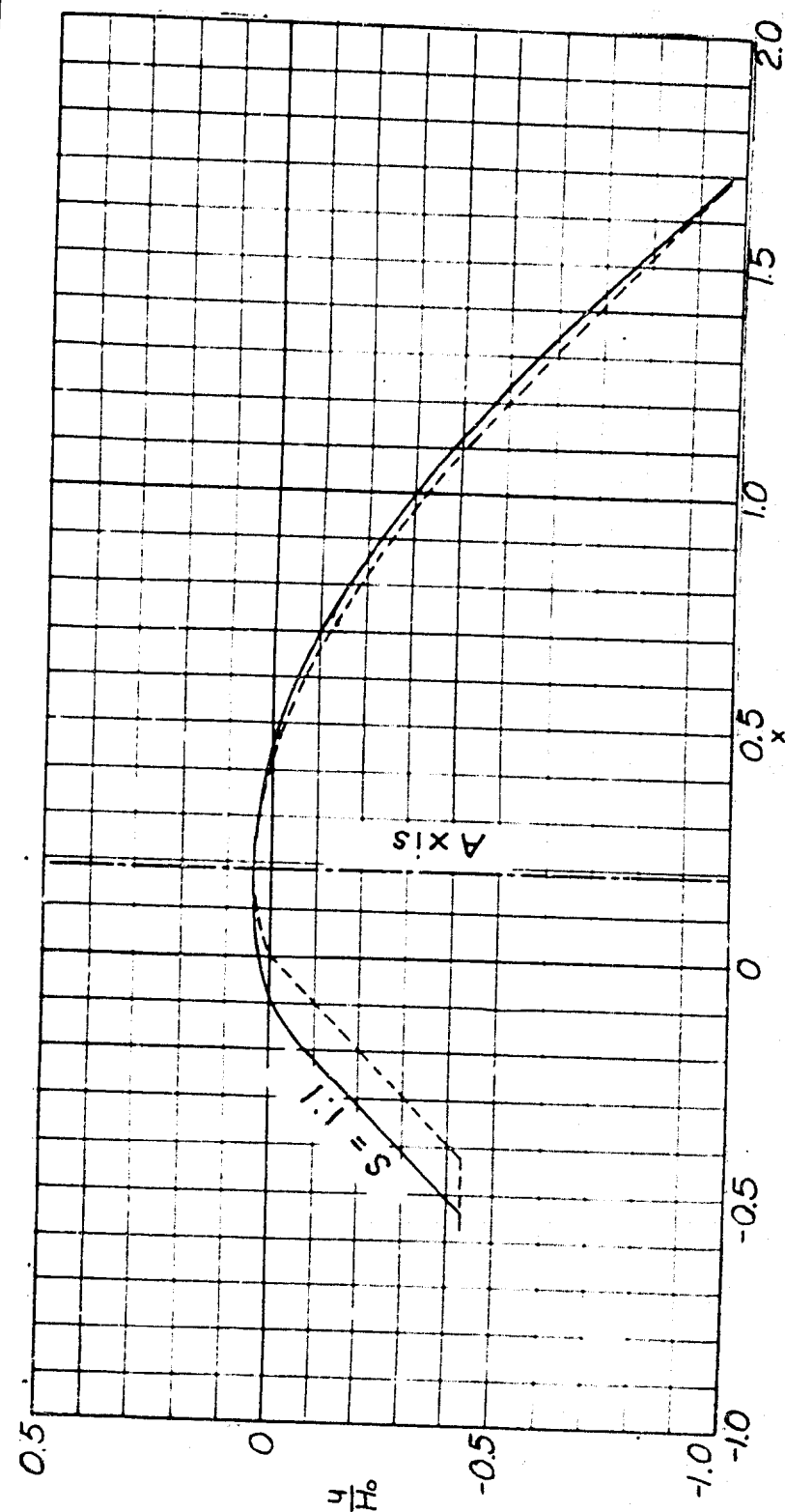
17 D. HAMILTON DAM SPILLWAY
 $\frac{H_0}{P+E} = 1.185$ $C_1 = 3.67$ ——— MODEL
 $C_1 = 3.90$ ---- IDEAL



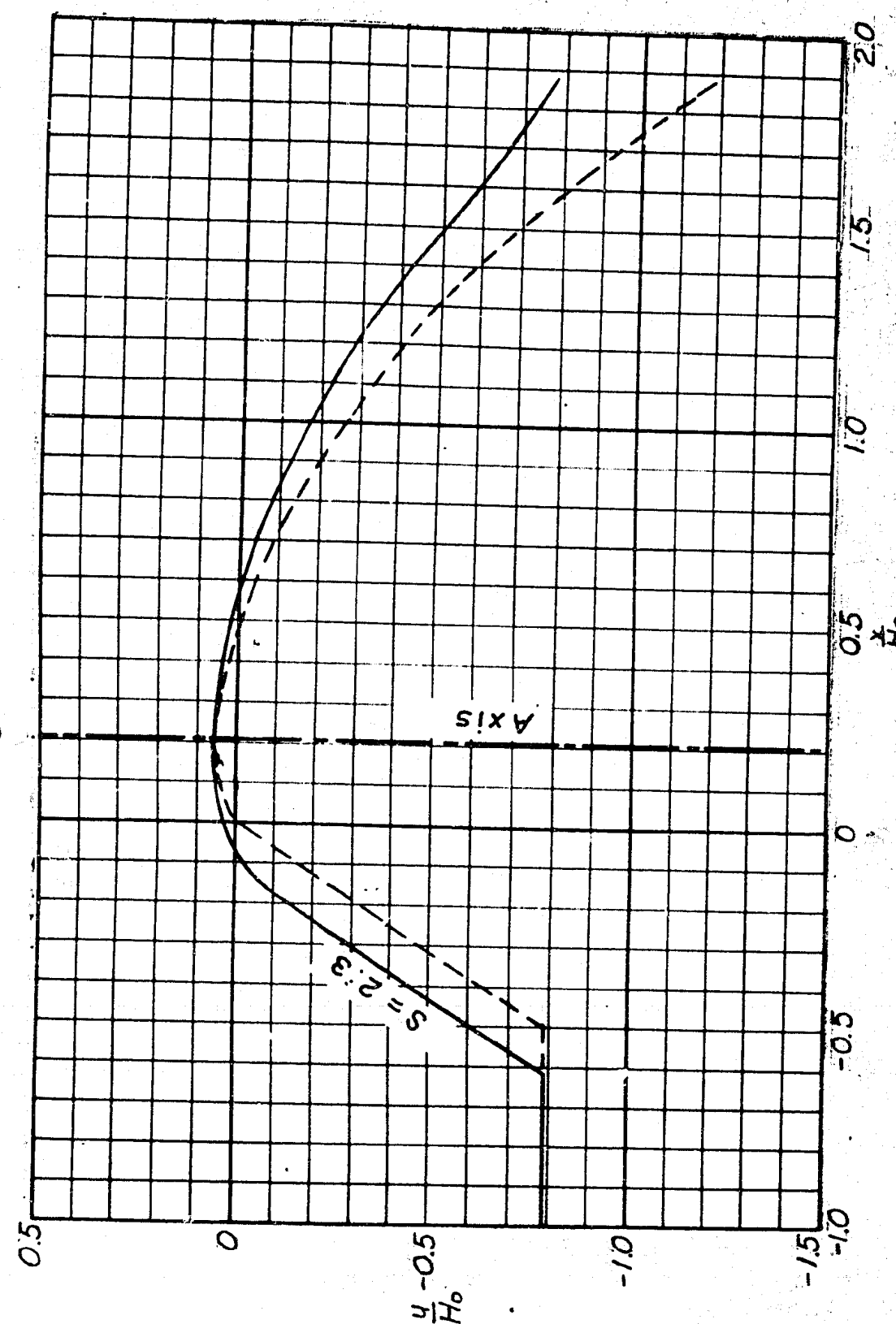
17 E. NORRIS DAM SPILLWAY
 $\frac{H_0}{P+E} = 0.1343$ $C_1 = 3.80$ ——— MODEL
 $C_1 = 3.96$ ---- IDEAL

COMPARISON OF DISCHARGE COEFFICIENTS

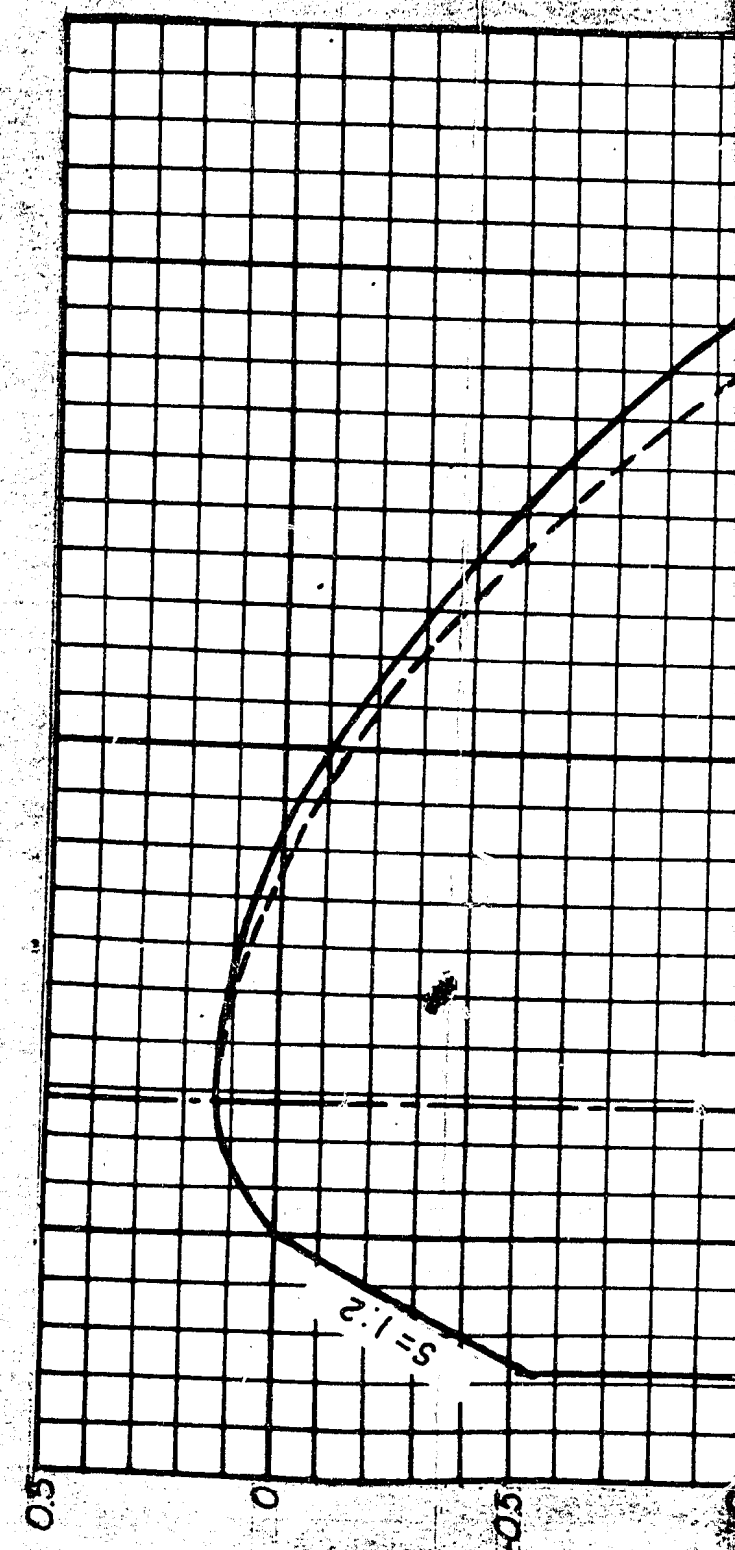
DIMENSIONLESS PLOTTING — SCALE: 2.5" = 1.0



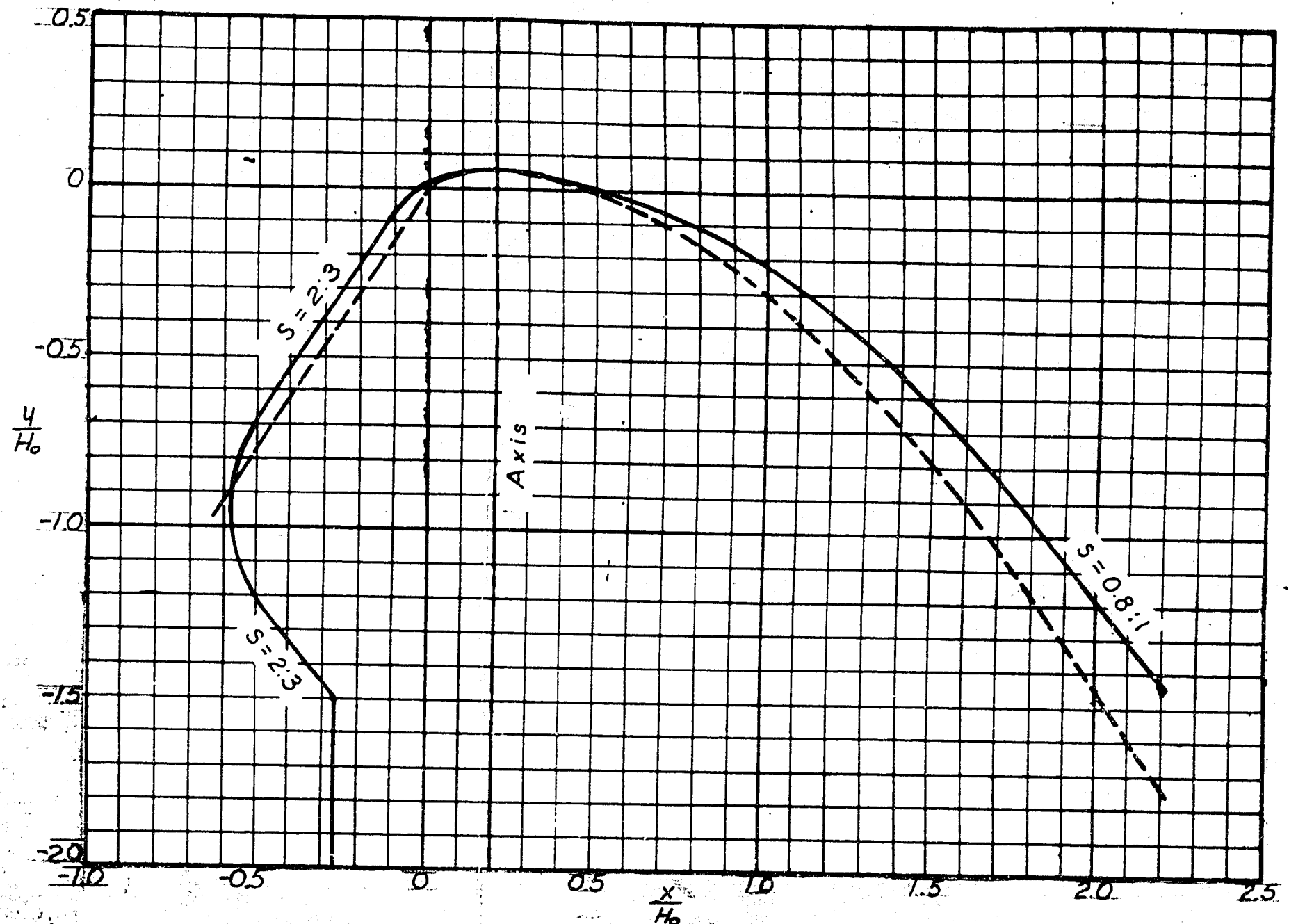
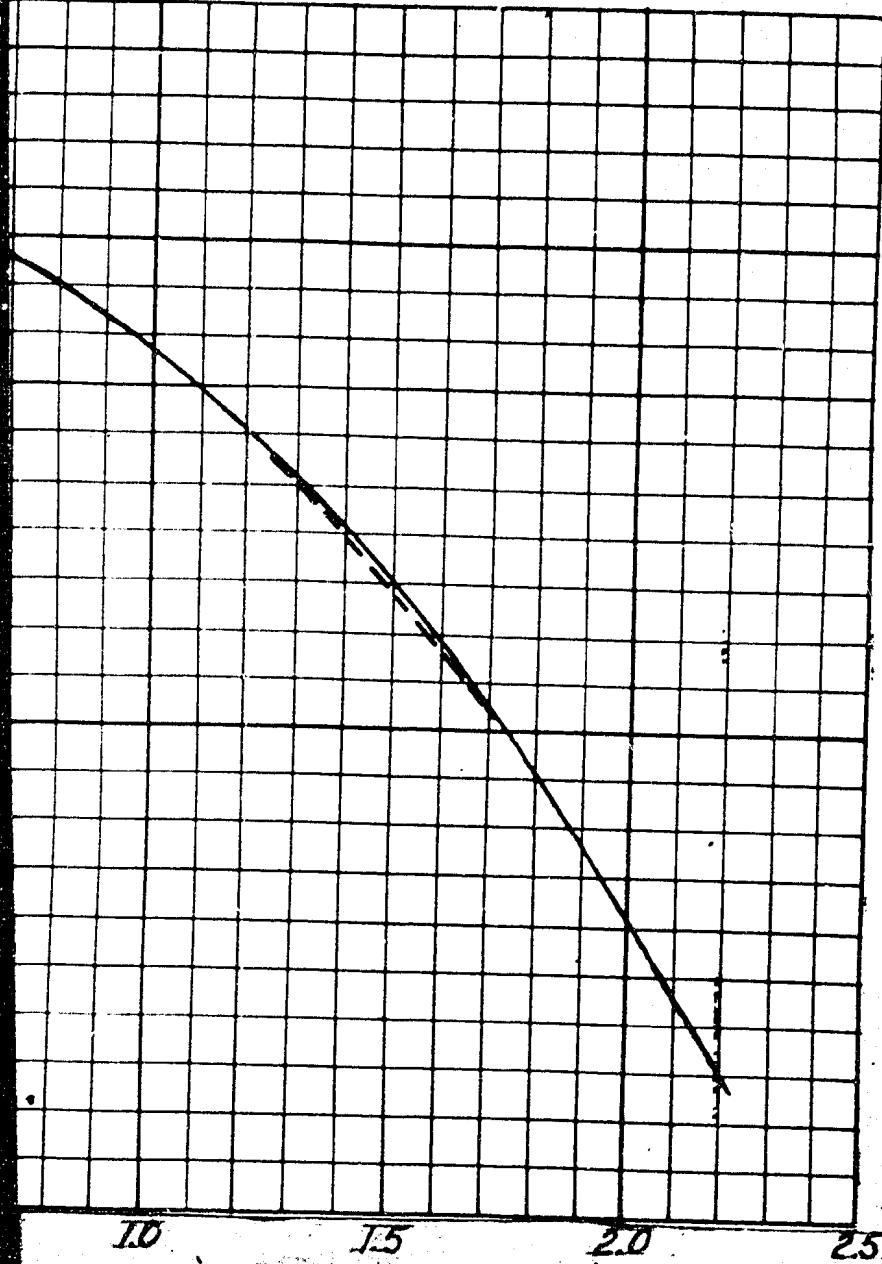
17 C. HEADGATE ROCK DAM SPILLWAY
 $\frac{H_0}{P+E} = 2.1417$ $C_d = 3.57$ — MODEL
 $C_d = 3.84$ --- IDEAL



17 D. HAMILTON DAM SPILLWAY
 $\frac{H_0}{P+E} = 1.185$ $C_d = 3.67$ — MODEL
 $C_d = 3.90$ --- IDEAL



COMPARISON OF DISCHARGE COEFFICIENTS
 DIMENSIONLESS PLOTTING — SCALE: 2.5" = 1.0

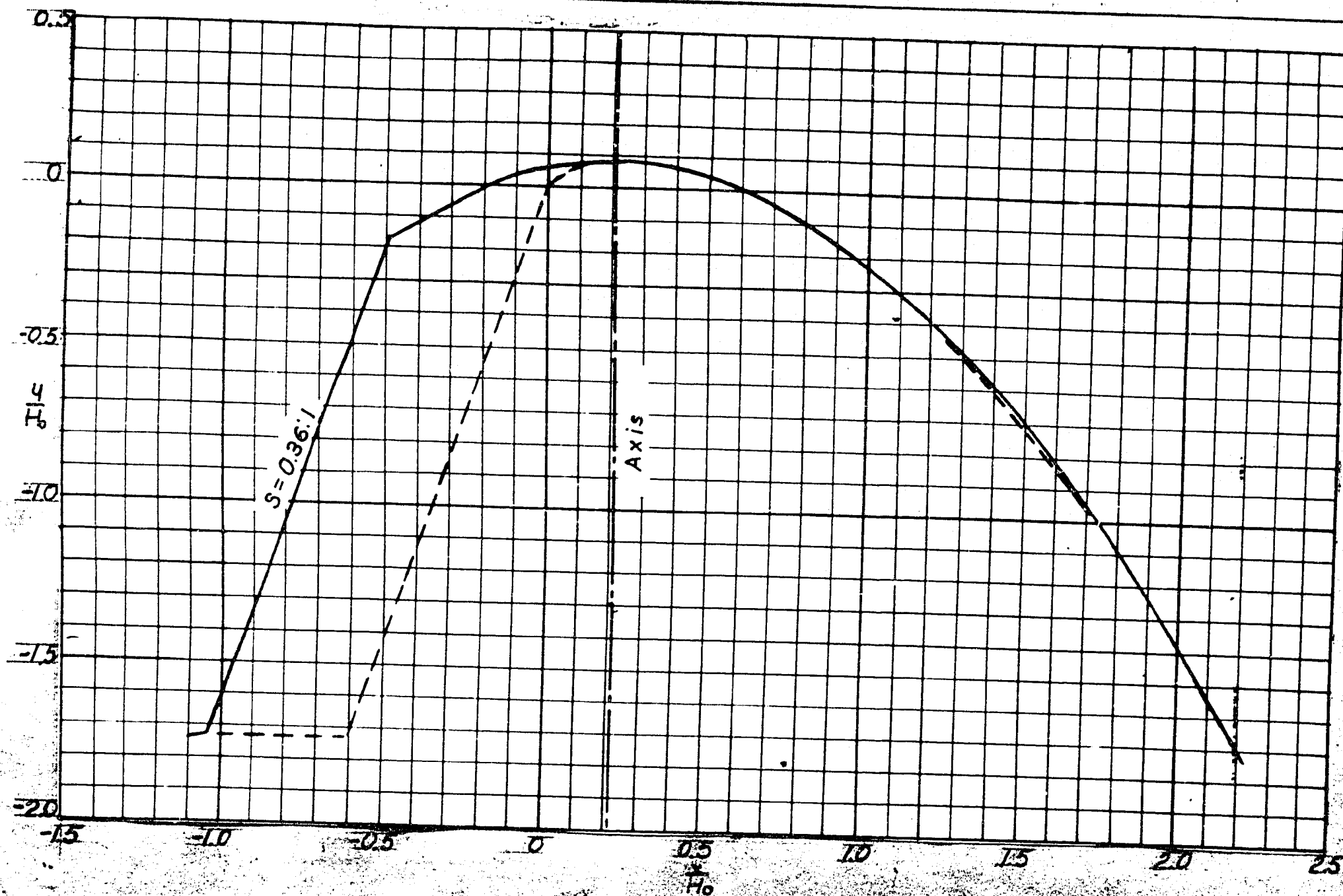


M SPILLWAY
 ODEL M-3
 68 ——— MODEL
 92 ——— IDEAL

18 A. GRAND COULEE DAM SPILLWAY
 $\frac{H_0}{P+E} = 0.08792$ $C_A = 3.78$ ——— MODEL
 $C_I = 3.95$ ——— IDEAL

COMPARISON OF DISCHARGE COEFFICIENTS

DIMENSIONLESS PLOTTING — SCALE: 2.5" = 1.0

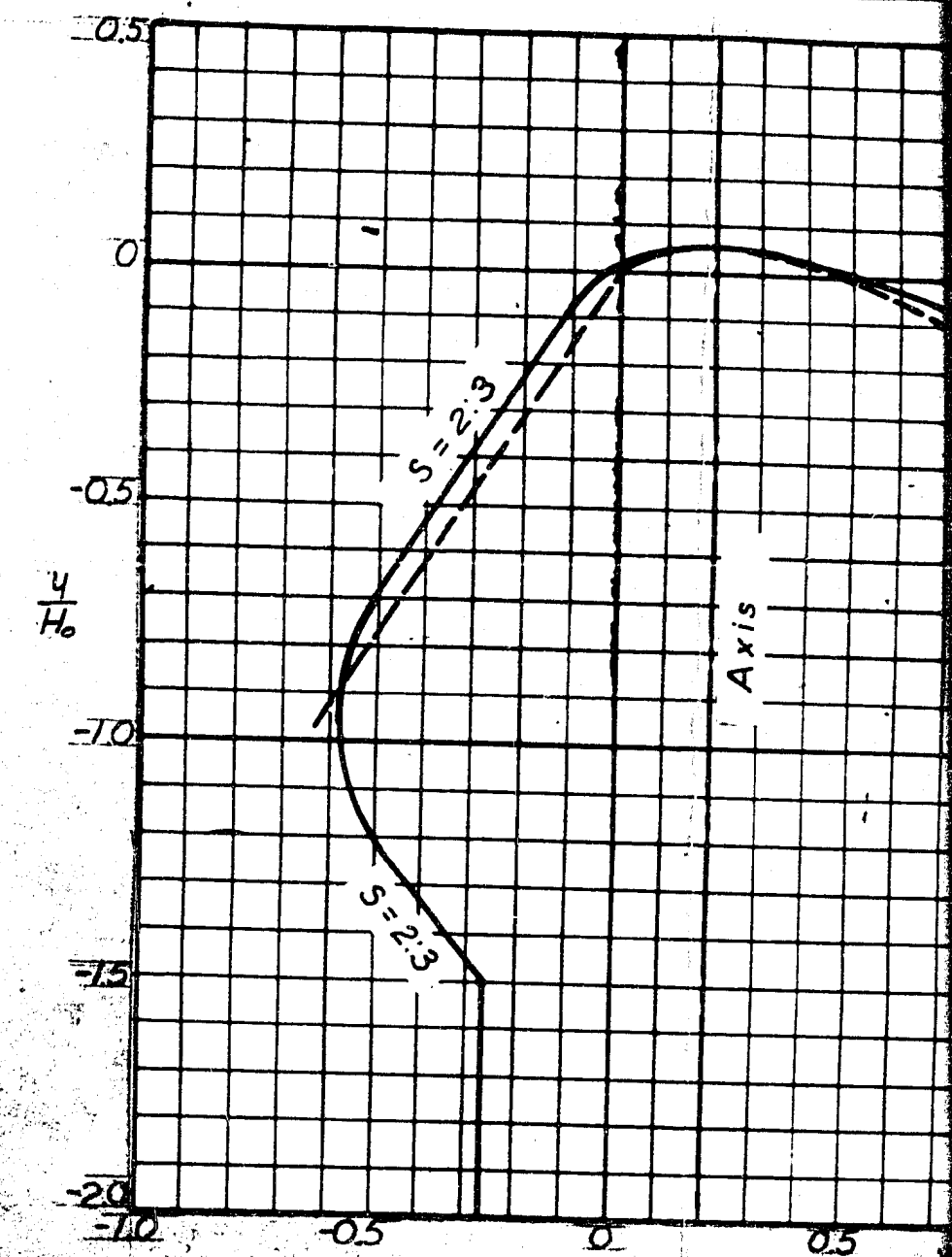


17 F.

BOULDER DAM SPILLWAY
SHAPES 384 MODEL M-3

$$\frac{H_0}{P+E} = 0.6650$$

$C_A = 3.68$ ——— MODEL
 $C_I = 3.92$ ——— IDEAL



18 A.

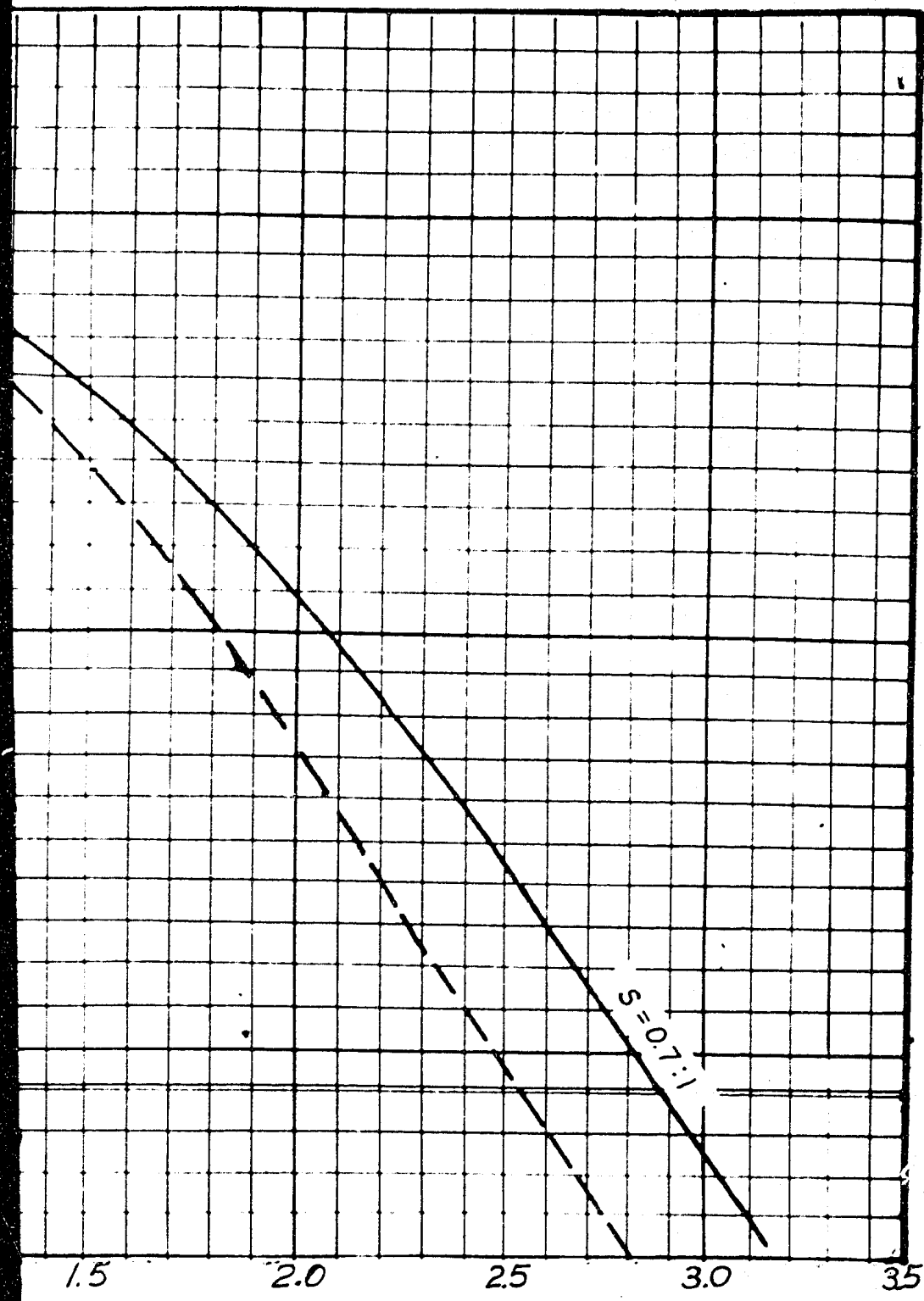
GRAND COULEE

$$\frac{H_0}{P+E} = 0.08792$$

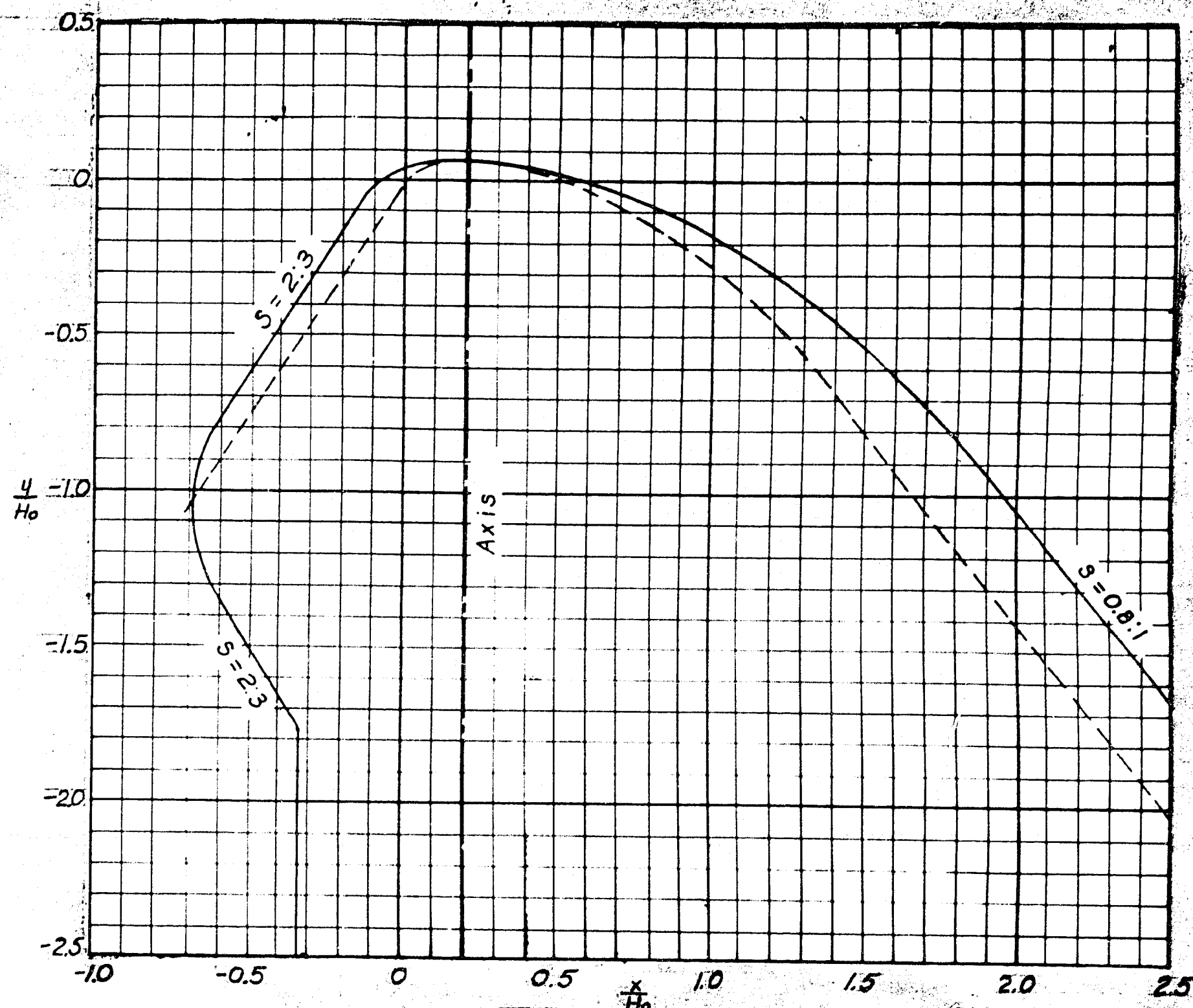
C_A
 C_I

COMPARISON OF DISCHARGE COEFFICIENTS

DIMENSIONLESS PLOTTING — SCALE 25" = 1.0

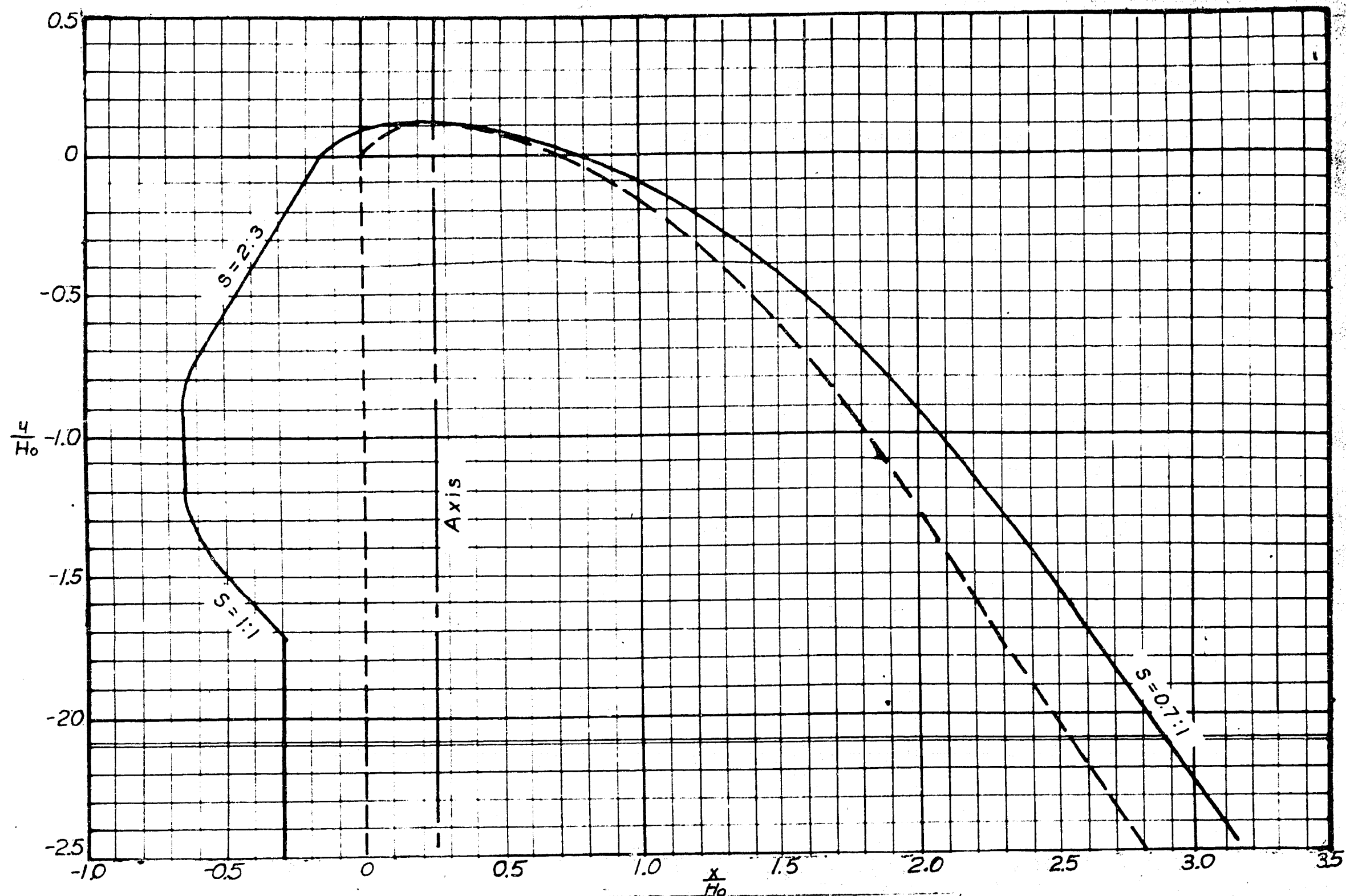


DAM SPILLWAY
 $C_d = 3.64$ ——— MODEL
 $C_d = 3.97$ ---- IDEAL

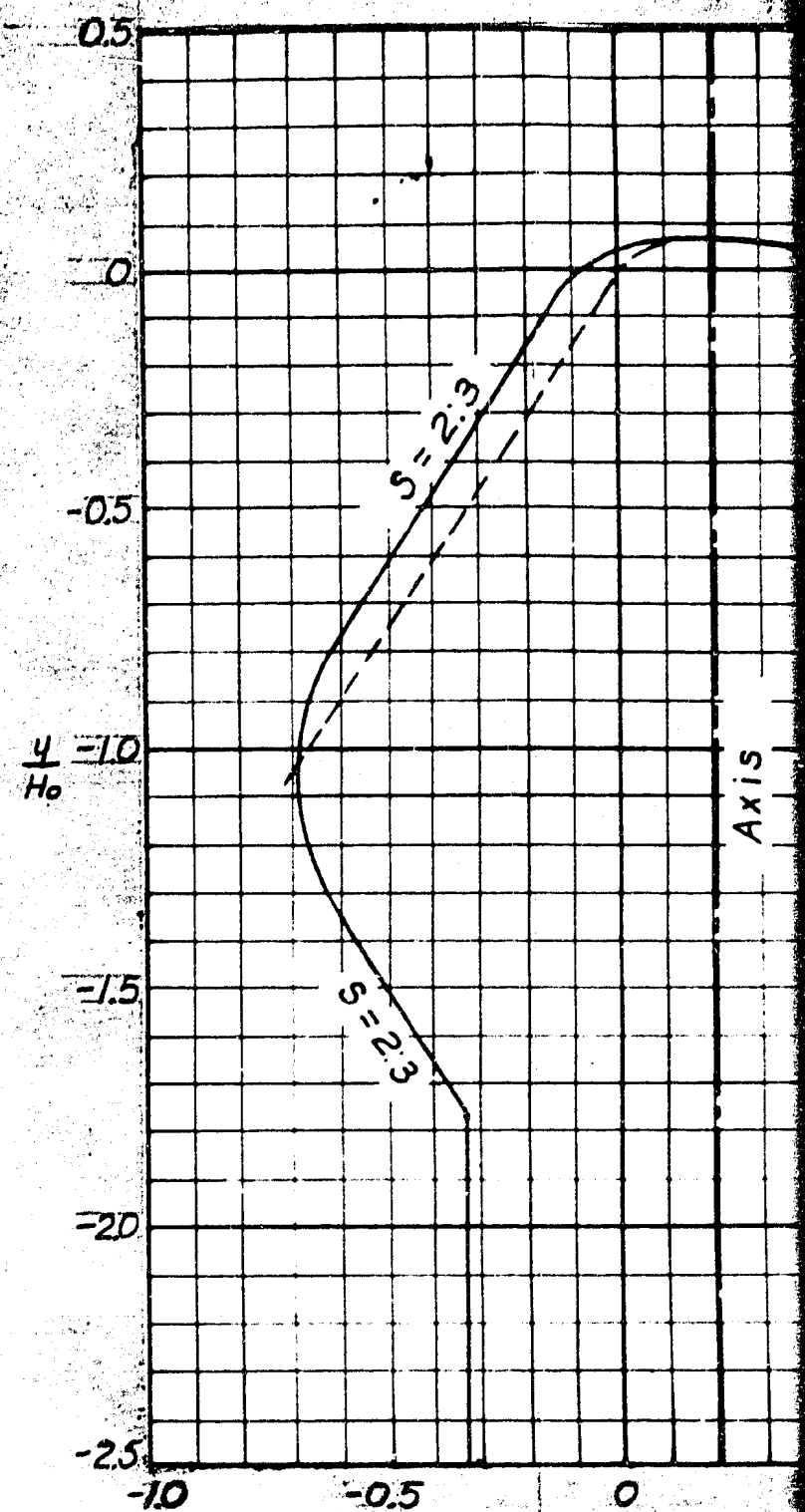


18C. SHASTA DAM SPILLWAY
 $\frac{H_o}{P+E} = 0.07235$ $C_d = 3.76$ ——— MODEL
 $C_d = 3.95$ ---- IDEAL

COMPARISON OF DISCHARGE COEFFICIENTS
 DIMENSIONLESS PLOTTING — SCALE: 2.5" = 1.0



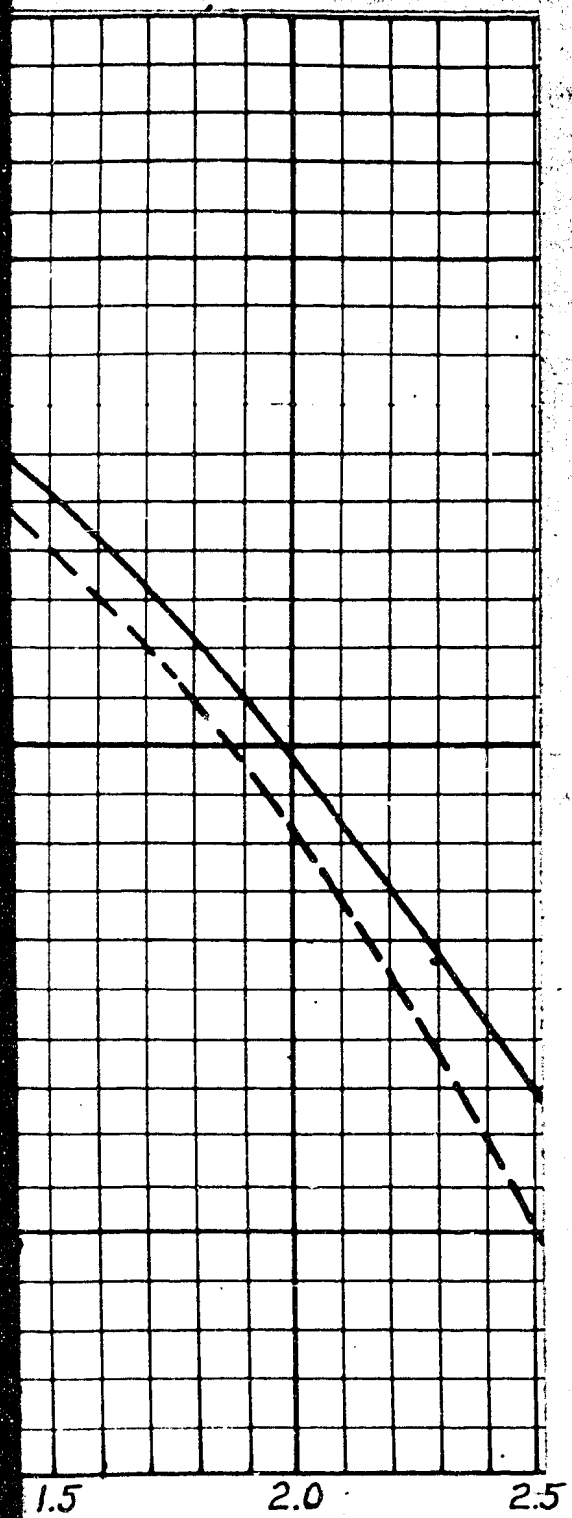
18 B. FRIANT DAM SPILLWAY
 $\frac{H_0}{P+E} = 0.07600$ $C_A = 3.64$ ——— MODEL
 $C_I = 3.97$ ---- IDEAL



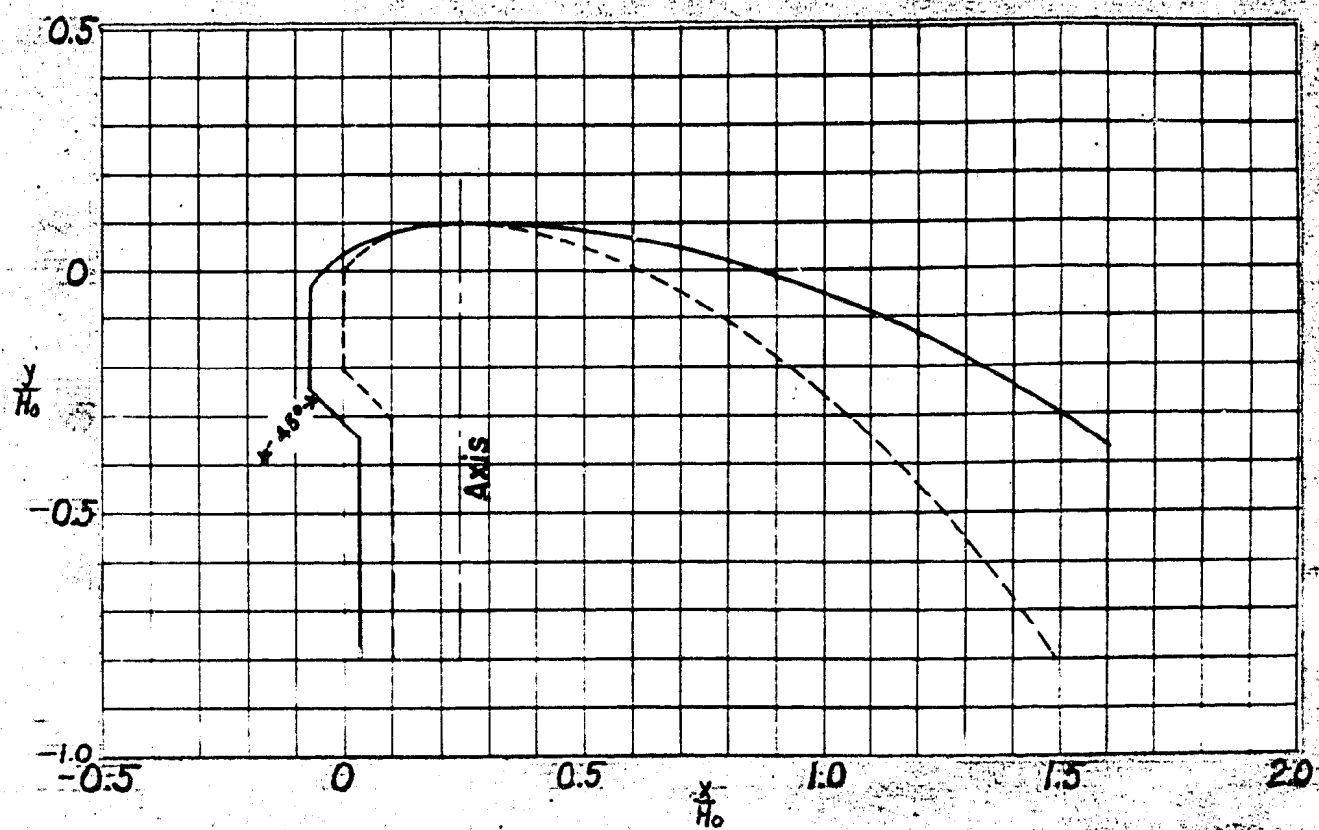
18 C.
 $\frac{H_0}{P+E} = 0.07$

COMPARISON OF DISCHARGE COEFFICIENTS

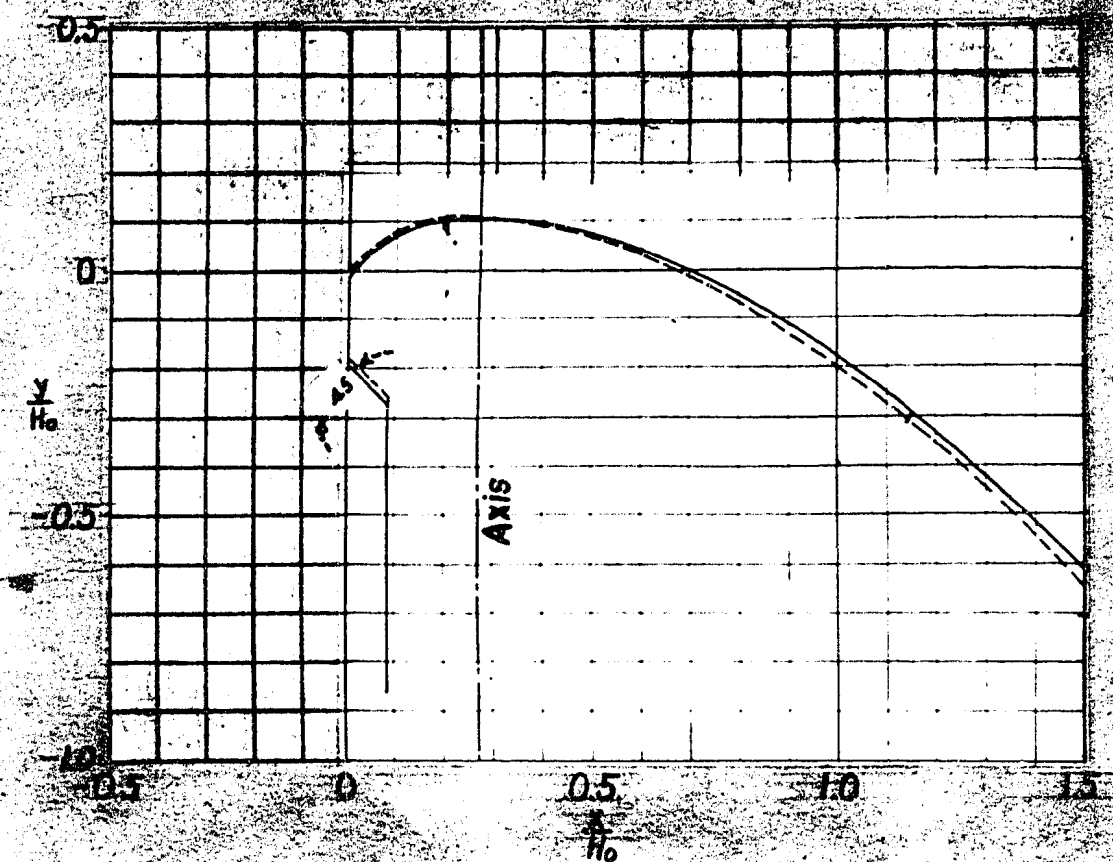
DIMENSIONLESS PLOTTING — SCALE: 2.5" = 1.0



WAY
ODEL
EAL



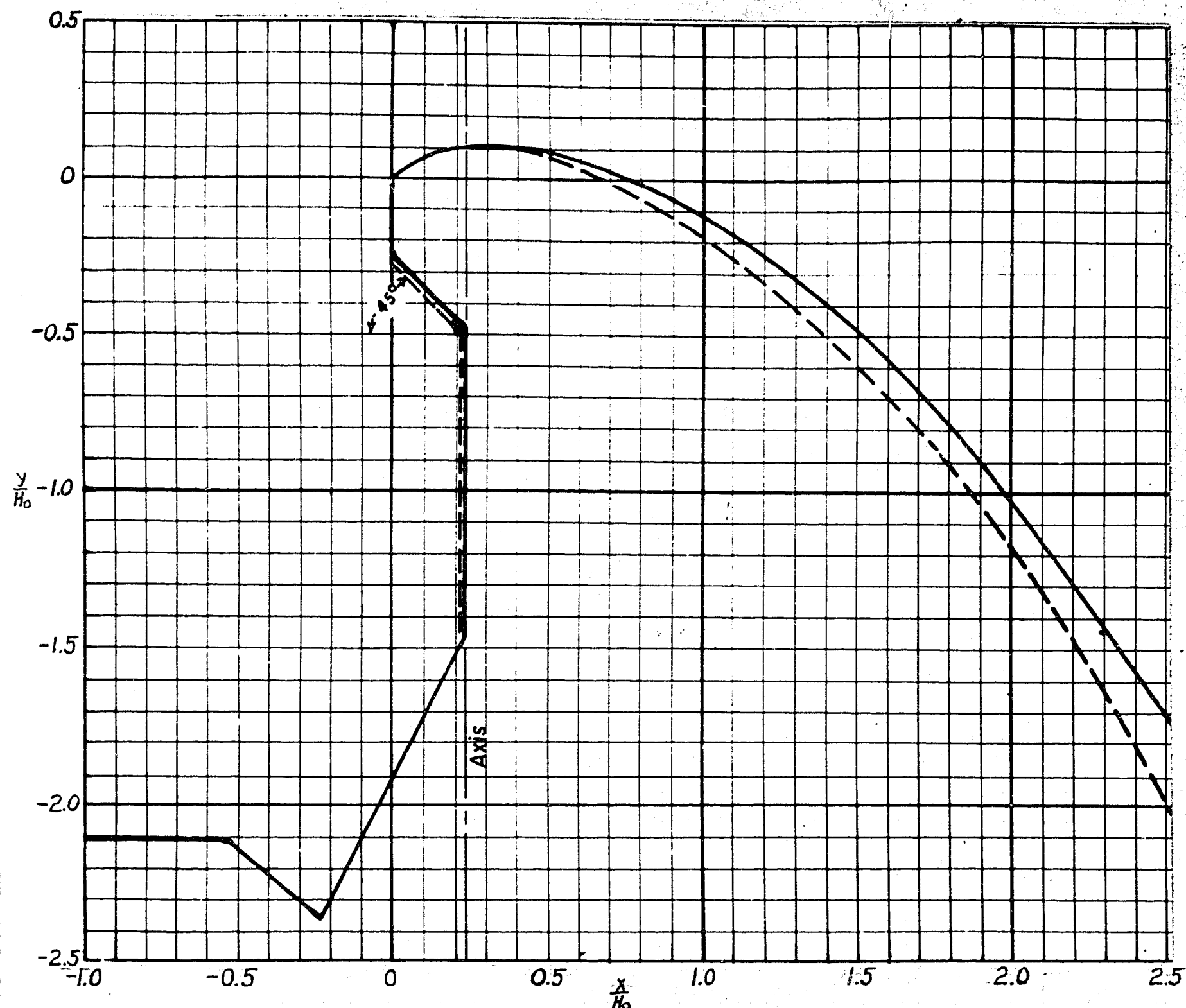
18 E. DAVIS DAM SPILLWAY
 $\frac{H_0}{P+E} = 0.5555$ $C_A = 3.51$ ——— MODEL
 $C_I = 3.96$ ——— IDEAL



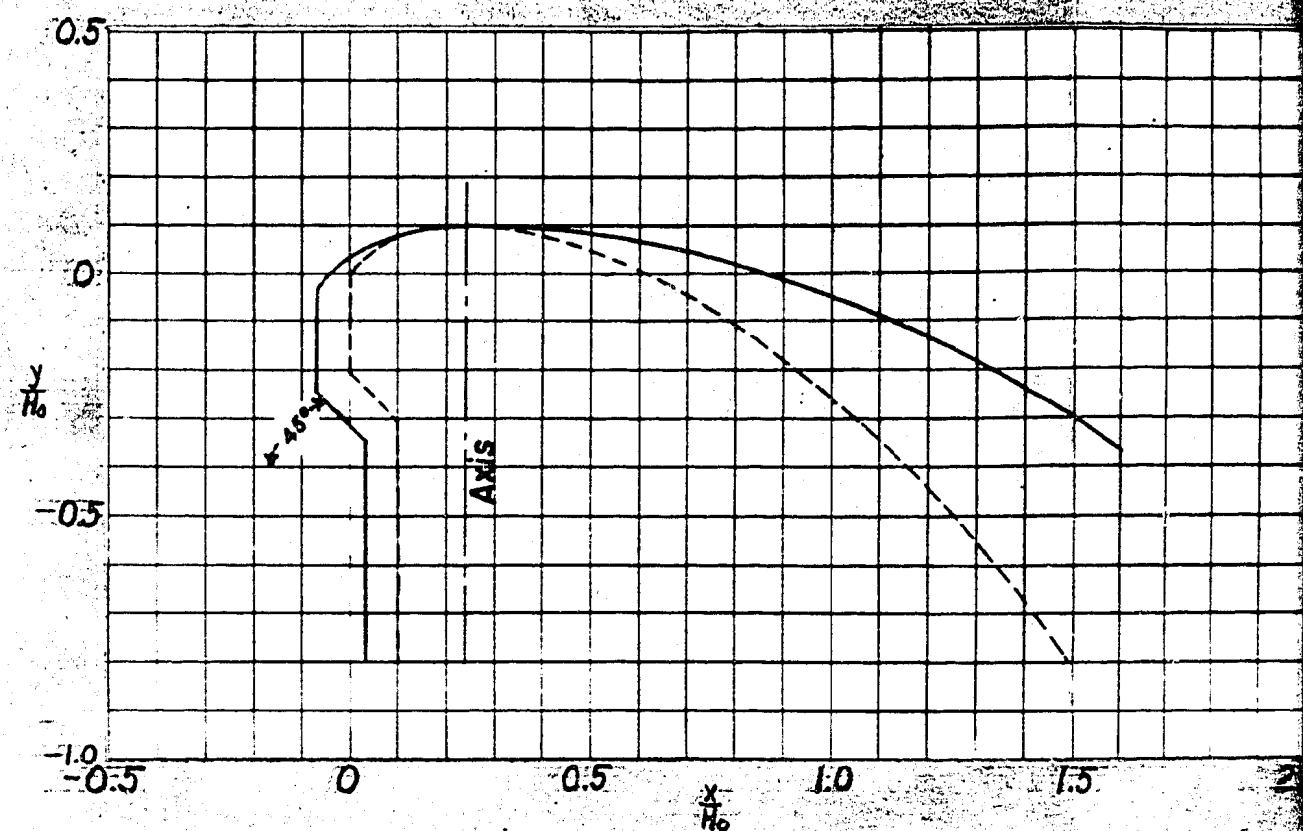
18 F. DAVIS DAM SPILLWAY
 (FINAL DESIGN)
 $\frac{H_0}{P+E} = 0.6444$ $C_A = 3.94$ ——— MODEL
 $C_I = 3.95$ ——— IDEAL

COMPARISON OF DISCHARGE COEFFICIENTS

DIMENSIONLESS PLOTTING — SCALE: 2.5" = 1.0



18 D. ANGOSTURA DAM SPILLWAY
 $\frac{H_0}{P+E} = 0.4474$ $C_A = 3.85$ ——— MODEL
 $C_I = 3.97$ ---- IDEAL



18 E. DAVIS DAM SPILLWAY
 $\frac{H_0}{P+E} = 0.5555$ $C_A = 3.51$ ——— MODEL
 $C_I = 3.96$ ---- IDEAL

COMPARISON OF DISCHARGE COEFFICIENTS
 DIMENSIONLESS PLOTTING — SCALE: 2.5" = 1.0

scribed in Chapters II and III, piers were absent. All coefficients of discharge in the present chapter have been computed using the net length of crest (width of piers excluded). In the design of spillways with piers, it may be necessary to make an adjustment in the discharge coefficient when obtained from the method in Chapters II and III. For blunt-nose piers, the adjustment may be appreciable, while for well-streamlined piers, a correction may not be necessary.

Many experiments have been performed by various agencies and individuals on the effect of piers on flowing water. Much of this experimentation has concerned broad-bridge piers in which submergence was a factor. No attempt is made here to correlate this data as it alone would constitute a lengthy study. Instead, the results of a short study on piers, made in connection with the spillway for Grand Coulee Dam, will serve the present purpose. The results of the study, which was limited to the case of no backwater or submergence effect downstream from the piers, are shown on Figures 28 and 29. A profile of the overfall section showing the position of the piers with respect to the axis of the spillway has been drawn on Figure 28. The tests included piers with square, triangular, rounded, and streamlined upstream noses, each nose being identified with a letter ranging from A through L. Each nose was tested on piers of different lengths, the lengths being identified by the numbers 1, 2, 3, and 4, the smallest number representing the longest pier.

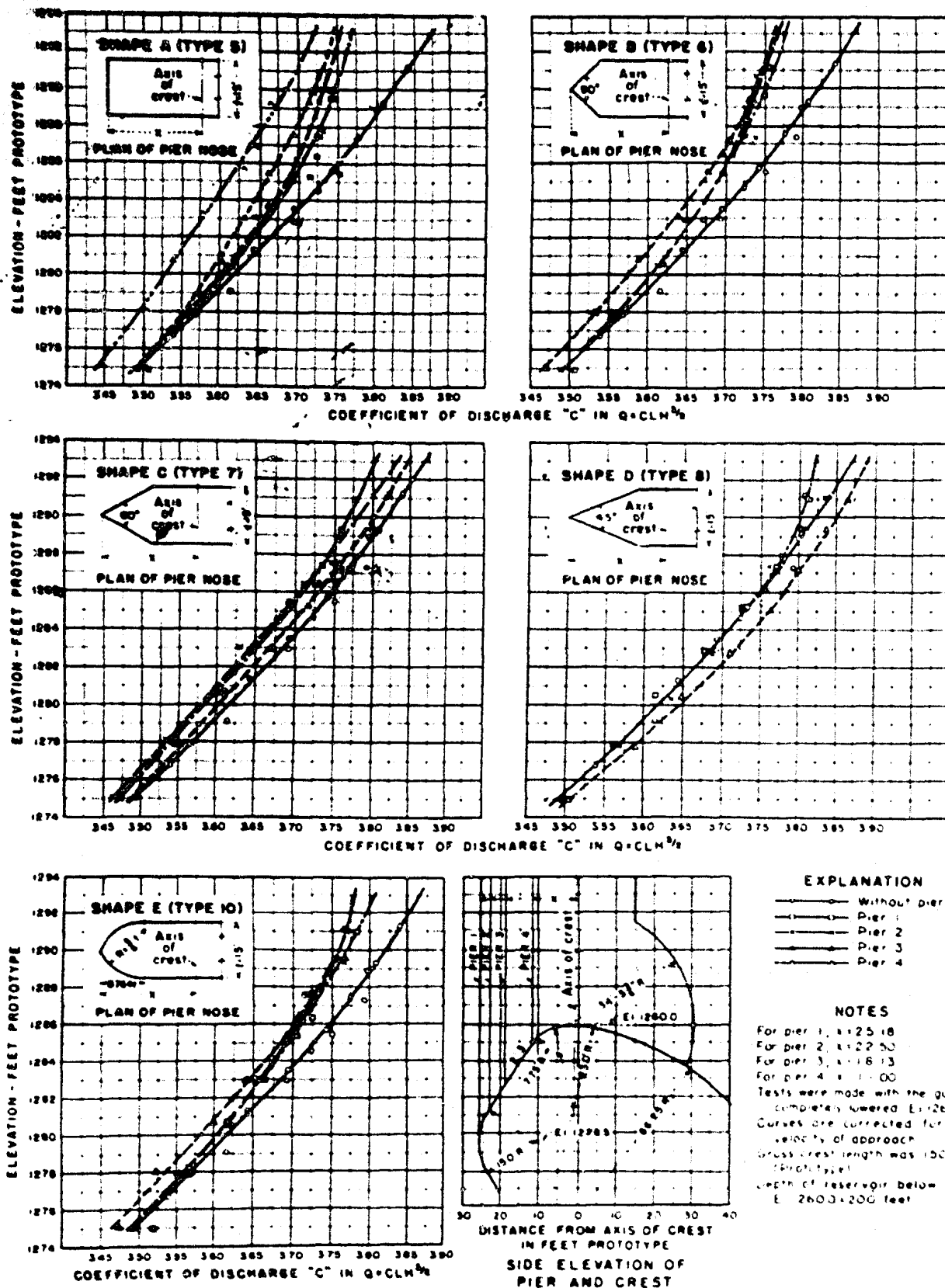
In the case of Pier A, with a square nose (Figure 28), the coefficient of discharge increased as the pier was lengthened.

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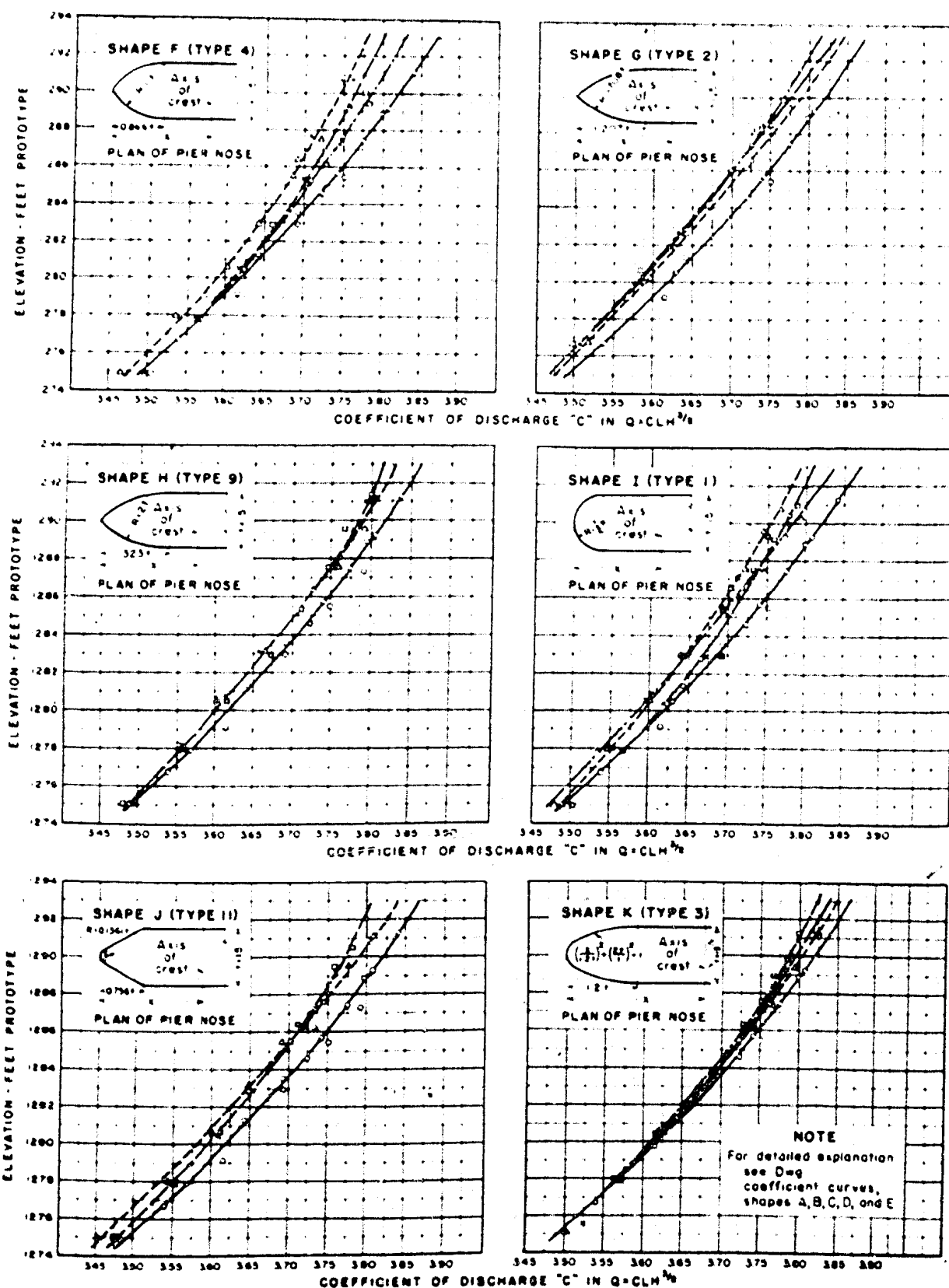
Many experiments have been performed by various agencies and individuals on the effect of piers on flowing water. Much of this experimentation has concerned broad-bridge piers in which submergence was a factor. No attempt is made here to correlate this data as it alone would constitute a lengthy study. Instead, the results of a short study on piers, made in connection with the spillway for Grand Coulee Dam, will serve the present purpose. The results of the study, which was limited to the case of no backwater or submergence effect downstream from the piers, are shown on Figures 28 and 29. A profile of the overfall section showing the position of the piers with respect to the axis of the spillway has been drawn on Figure 28. The tests included piers with square, triangular, rounded, and streamlined upstream noses, each nose being identified with a letter ranging from A through K. Each nose was tested on piers of different lengths, the lengths being identified by the numbers 1, 2, 3, and 4, the smallest number representing the longest pier.

In the case of Pier A, with a square nose (Figure 28), the coefficient of discharge increased as the pier was lengthened.

FIGURE 28



GRAND COULEE DAM
HYDRAULIC MODEL STUDIES-PIER SHAPES
SCALE 1/40
COEFFICIENT CURVES-SHAPES A,B,C,D, AND E



GRAND COULEE DAM
 HYDRAULIC MODEL STUDIES-PIER SHAPES
 SCALE 1:40
 COEFFICIENT CURVES-SHAPES F, G, H, I, J, AND K

The solid line with small circles, present in all graphs, represents the coefficient of discharge curve for flow over the free crest (piers removed). The results in general follow expectations; that is, the longer and better-streamlined piers are the more efficient, except for Shape D, Figure 23. In this case, the coefficient curve for Pier 3 is consistently better than that for the free crest. This can be explained by the fact that critical depth occurred adjacent to the tapered portion of the pier, the effective length of crest was increased, for this one case, resulting in a slightly larger discharge than for the free crest for a given head. Length 4, Shape D, which should have indicated a still better coefficient curve, was not tested as the short pier would not have allowed the installation of a drum gate on the Grand Coulee Spillway crest.

With the exception of the Shape D nose, the streamlined piers were the better. Shape H, a truly streamlined pier (Figure 29), shows a decrease in the coefficient of discharge of 0.6 percent when compared with the free flow coefficient for a head of 30 feet on the crest. The radius of the nose was equal to twice the width of the pier and the length of pier, in this case, was not important.

Making a similar comparison for the K pier (Figure 29) with parabolic nose, the discharge coefficient was reduced 1.0 percent for the long No. 1 length and 0.5 percent for the shorter No. 3 length when compared to that for 30 feet of head on the free crest.

The round blunt nose, Shape I pier (Figure 29), shows a

decrease in the coefficient of 1.8 percent for Length 3 and 1.2 percent for Length 1 at 30 feet of head. The extreme case is the square-nose pier, Shape A, (Figure 28) which shows a decrease in coefficient of 3.8 percent for the short No. 4 length and 2.0 percent for the long No. 1 length.

The conclusion to be drawn from the foregoing study is that for well-streamlined piers of medium length or longer, the pier effect can be neglected as its magnitude is within the experimental accuracy of the data in Chapters II and III. Should, however, the piers consist of a blunt-nose type, it is advisable to apply a correction to the coefficient of discharge. This can be done by making use of Figures 28 and 29.

Computation of Approach Velocity

The question invariably arises as to the computation of the velocity head of approach to spillways. Throughout this study, the velocity head is based on the average velocity of approach or discharge of spillway divided by corresponding average approach area. The Coriolis coefficient, or velocity head correction, has not been applied as computation of the present and future cross-sectional areas of most rivers, which is a factor, does not warrant this accuracy.

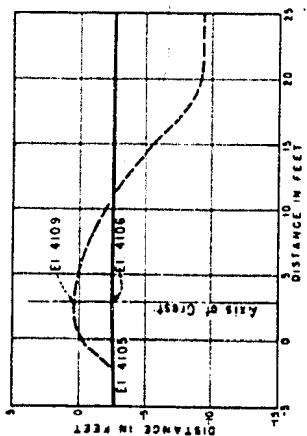
Irregular, Flat, (Earth-dam Type) Overflow Spillway Sections

Earth-dam spillways usually follow closely the downstream profile of the dam, consequently they are rather flat in longitudinal profile and the approach channel to the spillway is shallow. It was shown in Example 5, Chapter III, that the efficiency

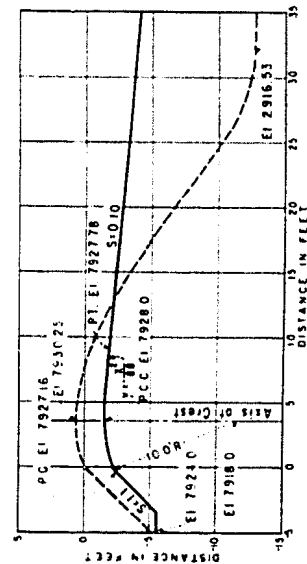
of a spillway with a flat grade can be increased considerably by making use of a small ogee or overflow crest at the gate section. This is extensively illustrated on Figures 30, 31, and 32. As it is often difficult to estimate the coefficient of discharge to be expected on flat, irregular spillway sections, the present information was prepared in an endeavor to aid the designer in this task.

Referring to Figure 30A, the solid line represents the actual gate section while the broken line is the ideal shape computed for the maximum head and prevailing approach conditions. Both sections are plotted with a common vertical axis, but the better efficiency in the case of the ideal shape allows this crest to be elevated as shown. In other words, either shape will accommodate the same discharge per foot of length at the maximum reservoir head; however, to completely accomplish this economy, it is necessary to drop a portion of the spillway chute as indicated.

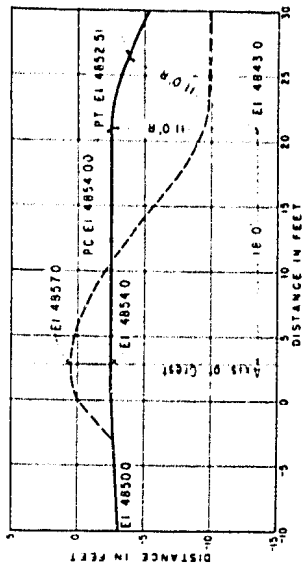
The ideal shapes on Figures 30, 31, and 32 were computed to obtain the maximum discharge coefficient in each case. The ideal overflow sections were computed according to the method outlined in Chapter II and the positions of the chute floors downstream were determined as explained in Chapter III. In the case of Figure 30A, the coefficient of discharge for maximum head was increased from 2.81 to 3.77 by the above procedure. This makes possible a reduction in the height of the gates on the crest or, as an alternative, the crest could be held to its original elevation, thus taking advantage of a corresponding re-



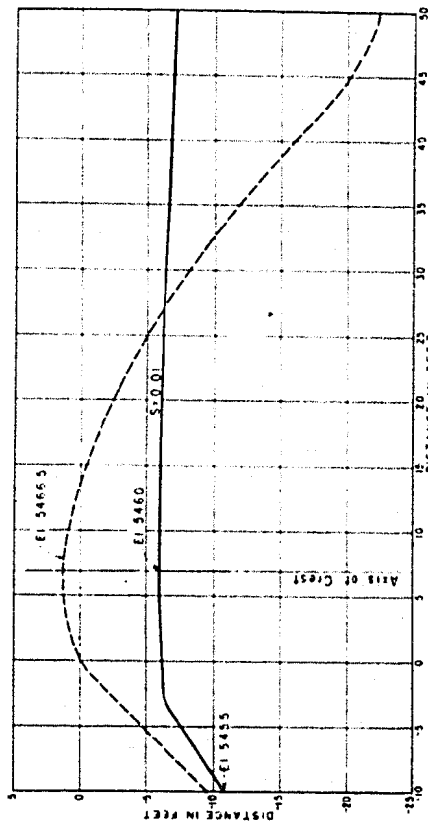
A. RYE PATCH DAM SPILLWAY
MODEL SCALE 1:50
RES ELEV 4125.0
P.E. = 4.0' C₁ = 2.72
M₀ = 14.11' C₂ = 3.77
MODEL
IDEAL



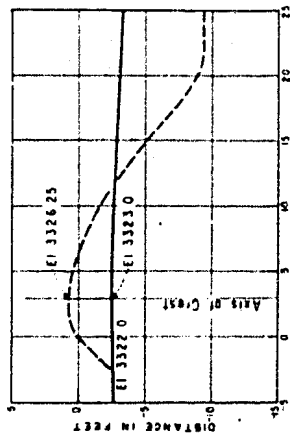
B. GREEN MOUNTAIN DAM SPILLWAY
(FINAL DESIGN)
MODEL SCALE 1:40
RES ELEV 7950.0
P.E. = 8.23' C₁ = 3.21
M₀ = 19.75' C₂ = 3.785
MODEL
IDEAL



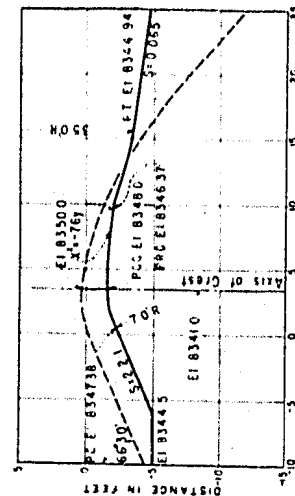
C. PINE VIEW DAM SPILLWAY
MODEL SCALE 1:30
RES ELEV 4870.0
P.E. = 7.0' C₁ = 2.74
M₀ = 12.81' C₂ = 3.86
MODEL
IDEAL



E. ALCOVA DAM SPILLWAY
MODEL SCALE 1:72
RES ELEV 5500.0
P.E. = 11.3' C₁ = 2.85
M₀ = 33.5' C₂ = 3.80
MODEL
IDEAL

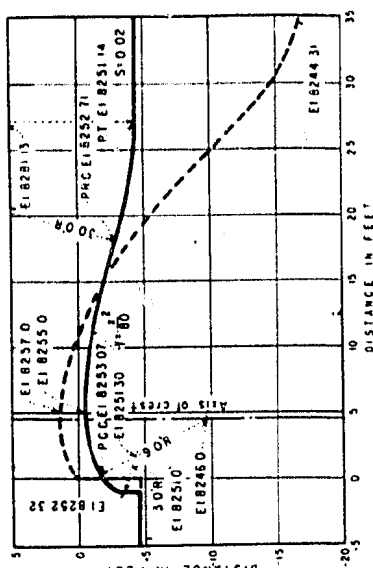


D. AGENCY VALLEY SPILLWAY
MODEL SCALE 1:30
RES ELEV 3340.0
P.E. = 4.85' C₁ = 2.73
M₀ = 13.75' C₂ = 3.78
MODEL
IDEAL



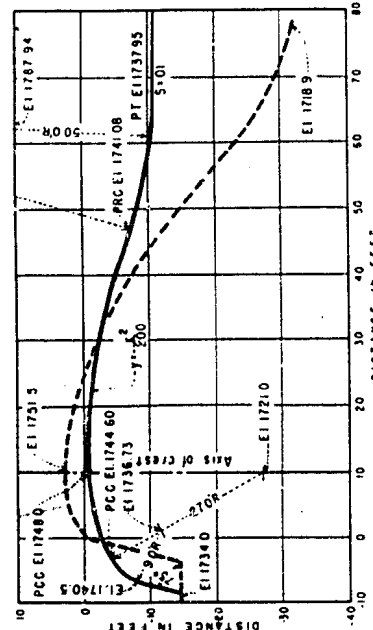
F. SHADOW MOUNTAIN DAM SPILLWAY
MODEL SCALE 1:30
RES ELEV 8367.0
P.E. = 4.3' C₁ = 2.74
M₀ = 18.0' C₂ = 3.78
MODEL
IDEAL

COMPARISON OF DISCHARGE COEFFICIENTS
MODEL COEFFICIENT VERSUS IDEAL COEFFICIENT
FOR EARTH DAM SPILLWAY SECTIONS WITH SHALLOW APPROACH DEPTH



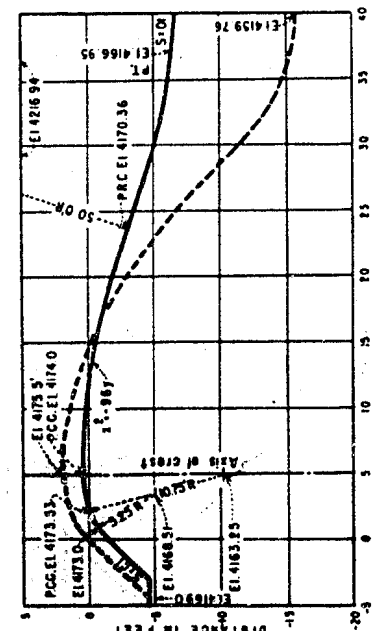
G. GRANBY DAM SPILLWAY

(FINAL DESIGN)
MODEL SCALE 1:100
RES. ELEV. 8275.0
P.E. = 6.0' C_d = 3.20 MODEL
M₀ = 18.10' C_d = 3.71 IDEAL



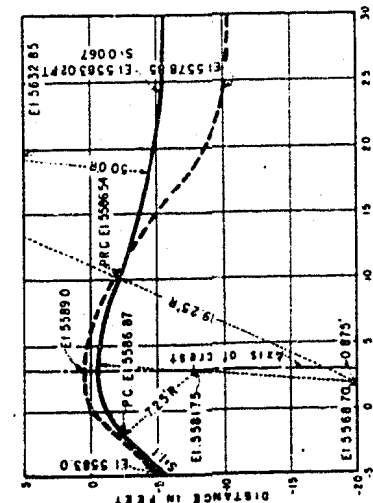
B. BARTLETT DAM SPILLWAY

(FINAL DESIGN)
MODEL SCALE 1:100
RES. ELEV. 1798.0
P.E. = 12.5' C_d = 3.78 MODEL
M₀ = 46.4' C_d = 3.78 IDEAL



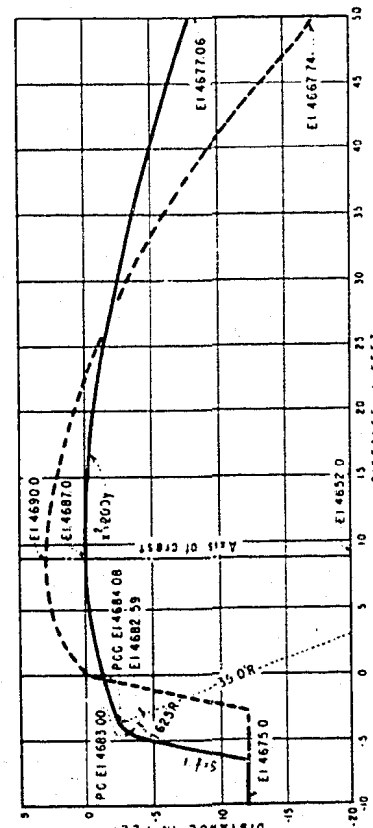
A. ANDERSON RANCH DAM SPILLWAY

(FINAL DESIGN)
MODEL SCALE 1:48
RES. ELEV. 4166.0
P.E. = 6.8' C_d = 3.40 MODEL
M₀ = 22.46' C_d = 3.70 IDEAL



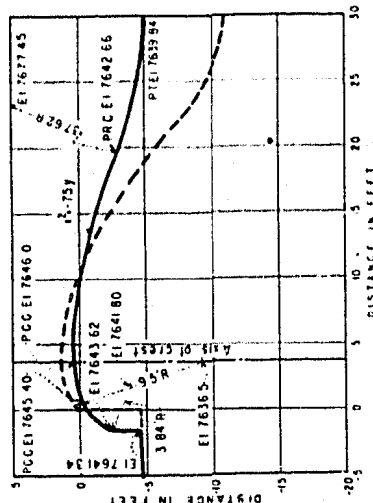
D. BOCA DAM SPILLWAY

(FINAL DESIGN)
MODEL SCALE 1:48
RES. ELEV. 5803.40
P.E. = 6.0' C_d = 3.37 MODEL
M₀ = 14.55' C_d = 3.67 IDEAL



E. BOYSEN DAM SPILLWAY

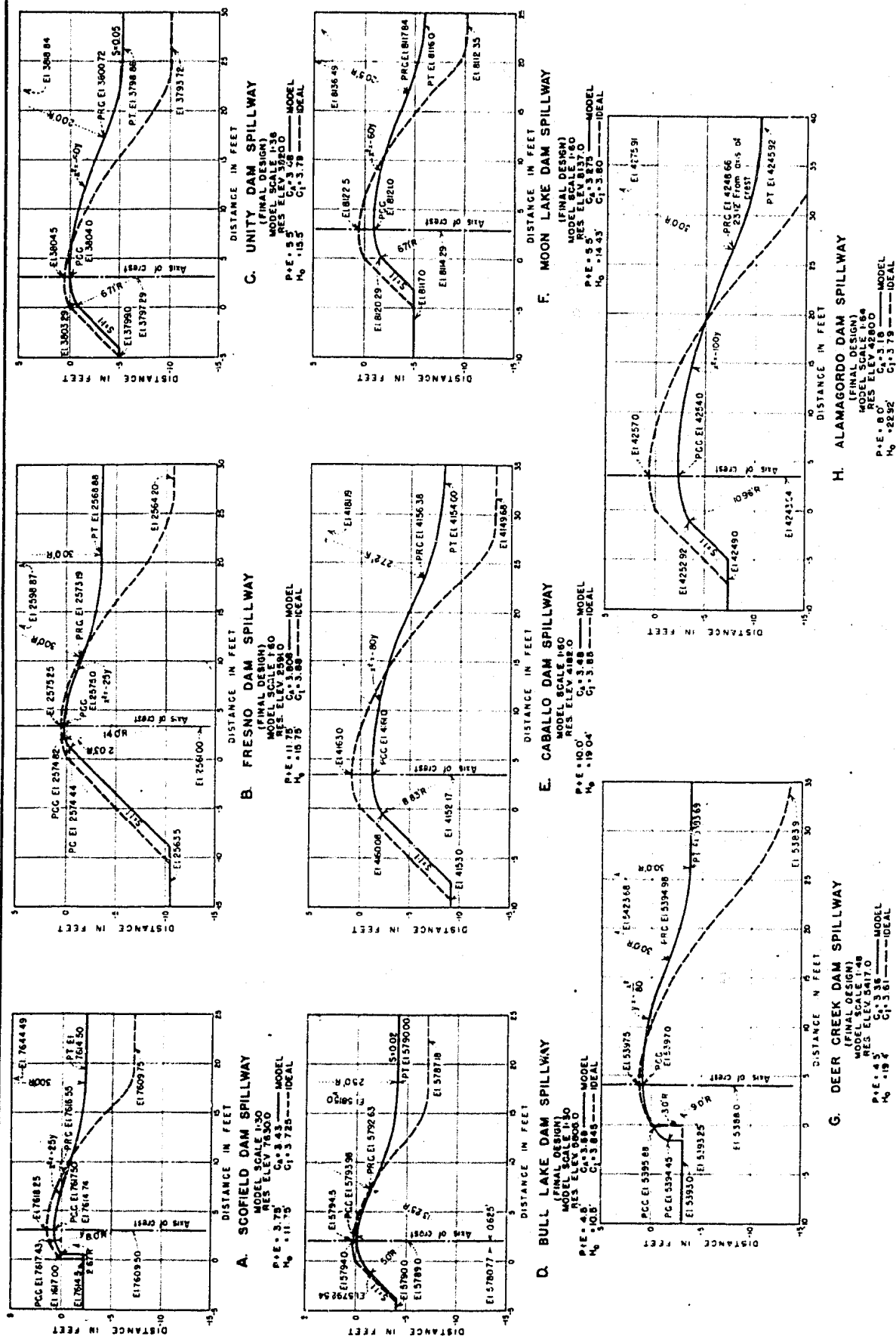
(FINAL DESIGN)
MODEL SCALE 1:60
RES. ELEV. 4731.5
P.E. = 13.0' C_d = 3.76 MODEL
M₀ = 41.9' C_d = 3.76 IDEAL



F. VALLECITO DAM SPILLWAY

(FINAL DESIGN)
MODEL SCALE 1:20
RES. ELEV. 7685.0
P.E. = 6.0' C_d = 3.423 MODEL
M₀ = 17.96' C_d = 3.72 IDEAL

COMPARISON OF DISCHARGE COEFFICIENTS MODEL COEFFICIENT VERSUS IDEAL COEFFICIENT FOR EARTH DAM SPILLWAY SECTIONS WITH SHALLOW APPROACH DEPTH



COMPARISON OF DISCHARGE COEFFICIENTS

MODEL COEFFICIENT VERSUS IDEAL COEFFICIENT
FOR EARTH DAM SPILLWAY SECTIONS WITH SHALLOW APPROACH DEPTH

duction in the length of the gates.

Practically all of the actual spillway sections on Figure 30 are flat, well illustrating the value of the small ogee refinement. The sections on Figures 31 and 32 show an attempt by the designers to increase the efficiency of this flat-type spillway. The comparisons are interesting but the increase in efficiencies is not of course, as striking as in Figure 30.

Figures 30, 31, and 32 show true dimensions for the actual spillway shapes and plotted coordinates for the ideal shapes. To aid the designer in estimating discharge coefficients on irregular, flat, overfall shapes, the above information has also been plotted in dimensionless coordinates on Figures 33 through 37. Actual dimensions have been omitted in the second plotting.

Application of Results

Example 2

Determine the head-coefficient curve for the spillway shown on Figure 38 for free flow. The designed head on the crest is 17.0 feet.

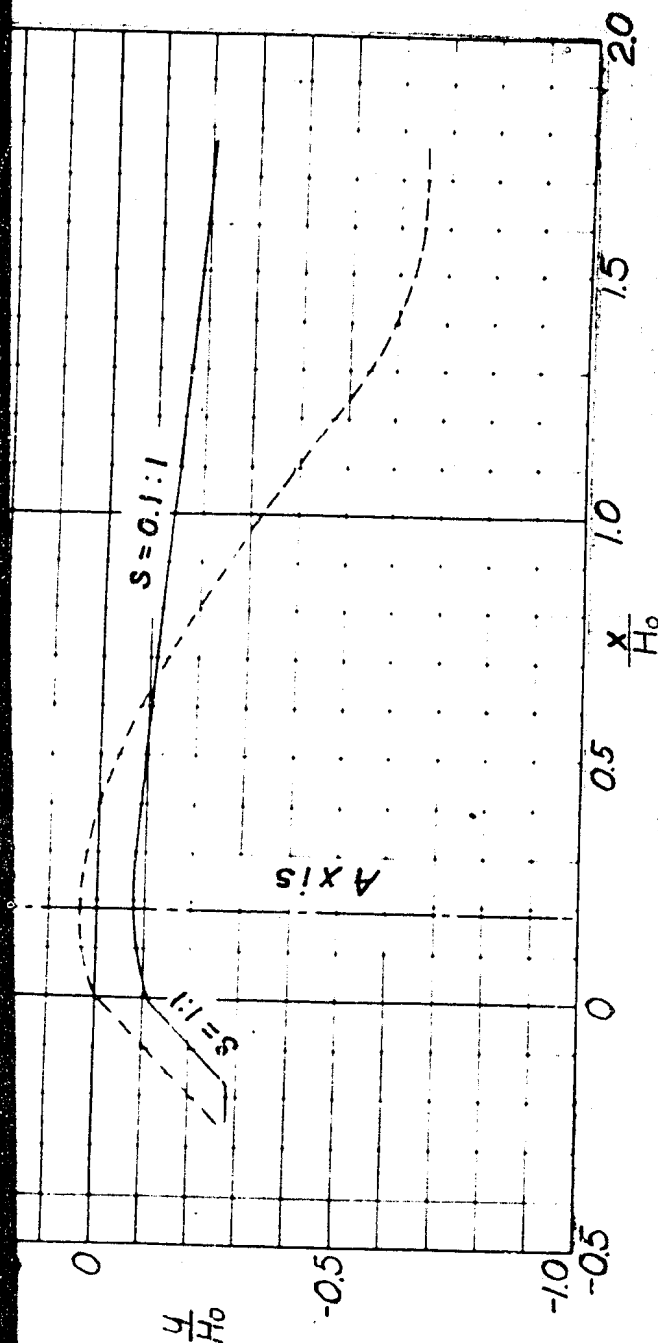
The coordinates and coefficient of discharge are first determined for the ideal shape. Assuming an average approach depth of 60 feet,

$$\frac{H_0}{P+H_0} = \frac{17}{60} = 0.284$$

Entering Figure 38 with this value,

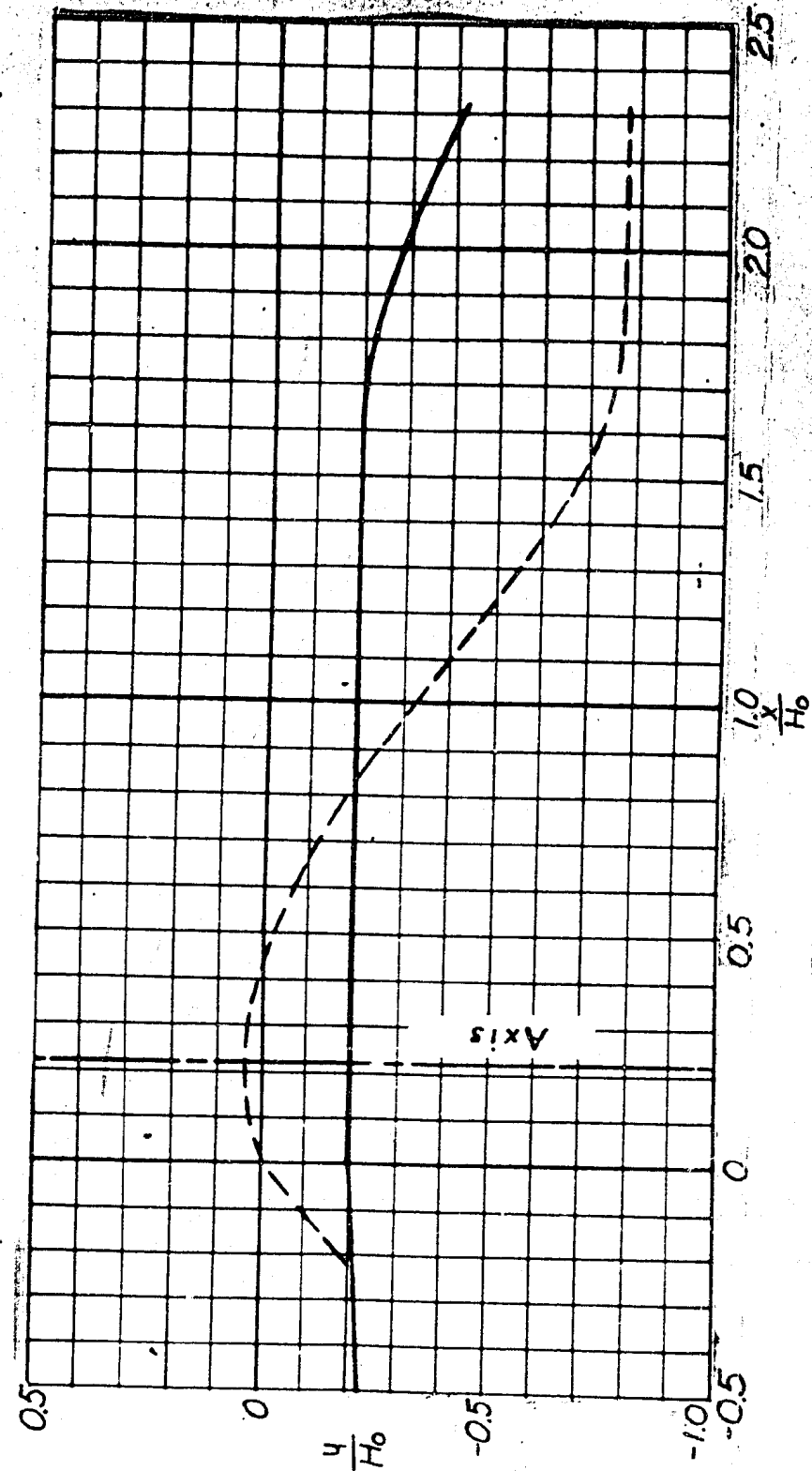
$$C_1 = 3.95$$

$$q = C_1 H_0^{3/2} = 3.95 \times 17^{3/2} = 277 \text{ second-feet per foot}$$



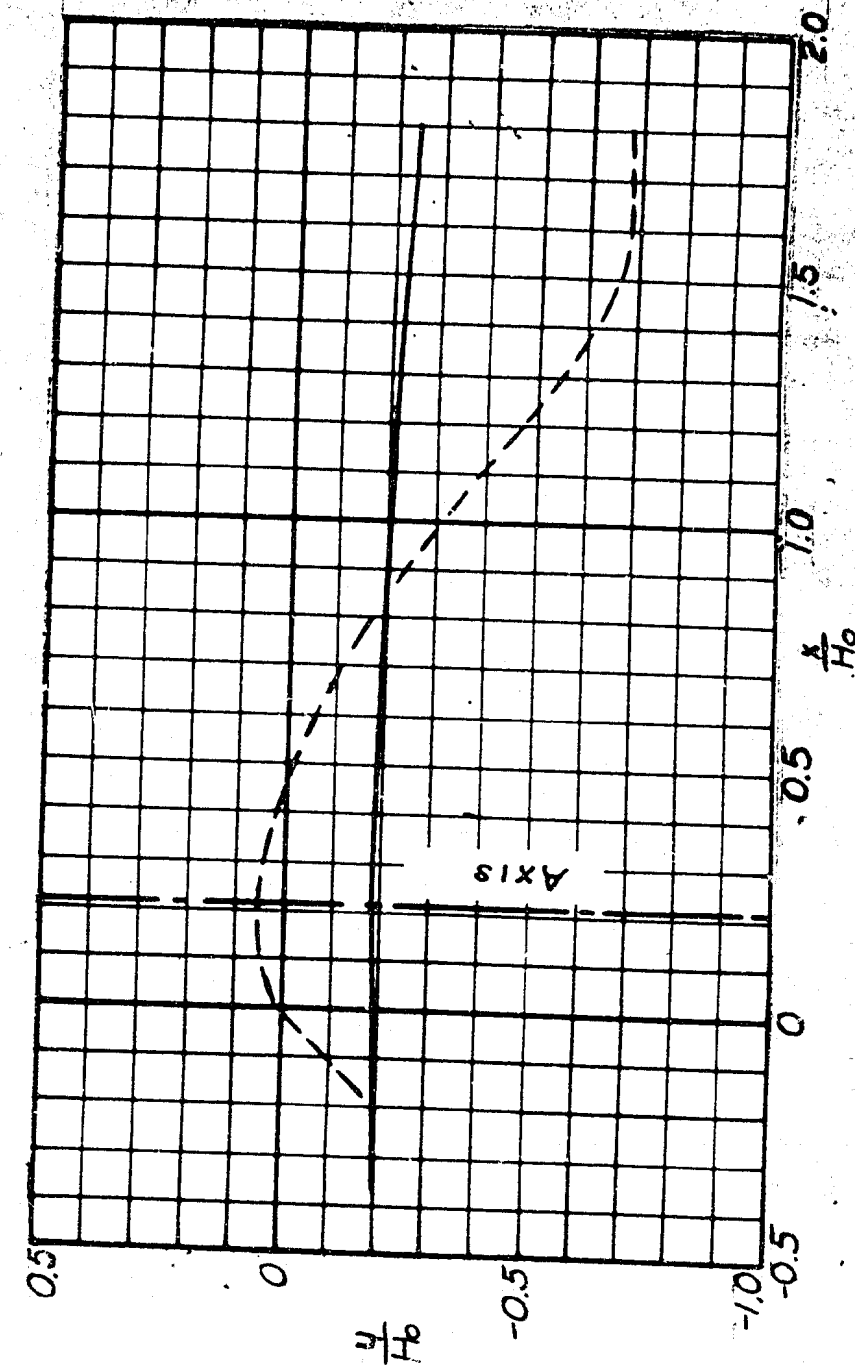
30 B. GREEN MOUNTAIN DAM SPILLWAY
(FINAL DESIGN)

$\frac{H_0}{P+E} = 3.160$ $C_A = 3.21$ — MODEL
 $C_I = 3.785$ --- IDEAL



30 C. PINE VIEW DAM SPILLWAY

$\frac{H_0}{P+E} = 1.844$ $C_A = 2.74$ — MODEL
 $C_I = 3.86$ --- IDEAL

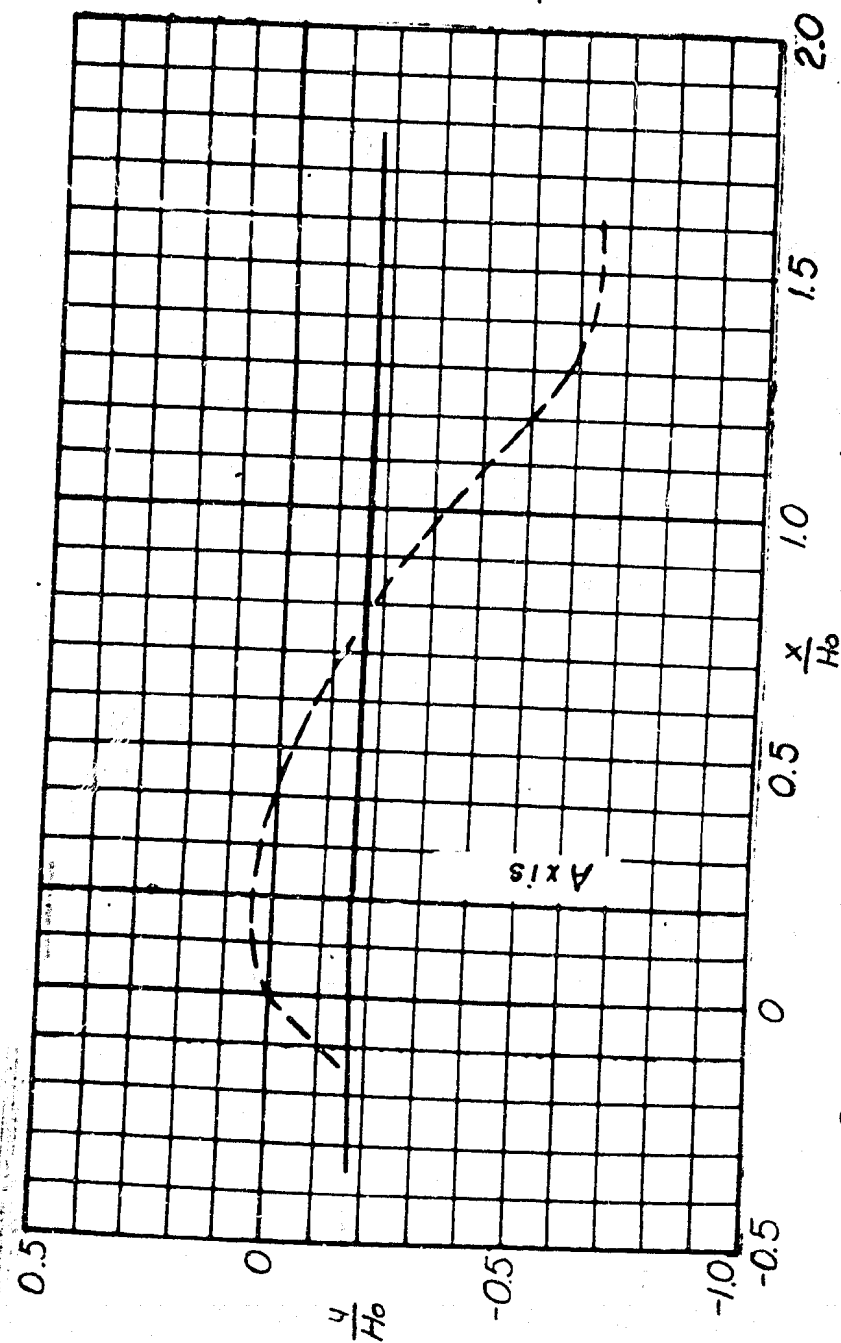


30 D. AGENCY VALLEY SPILLWAY

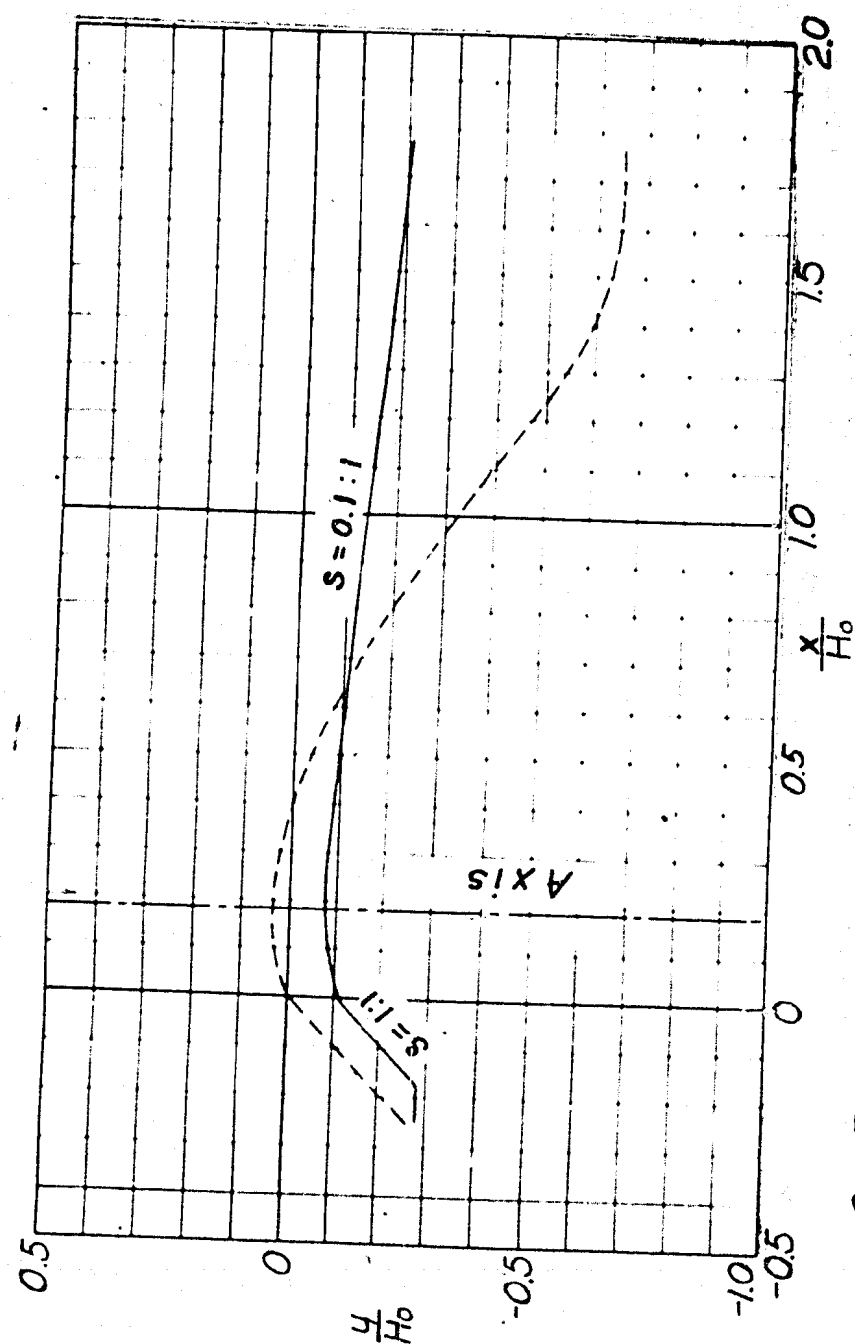
$\frac{H_0}{P+E} = 3.235$ $C_A = 2.73$ — MODEL
 $C_I = 3.78$ --- IDEAL

COMPARISON OF DISCHARGE COEFFICIENTS

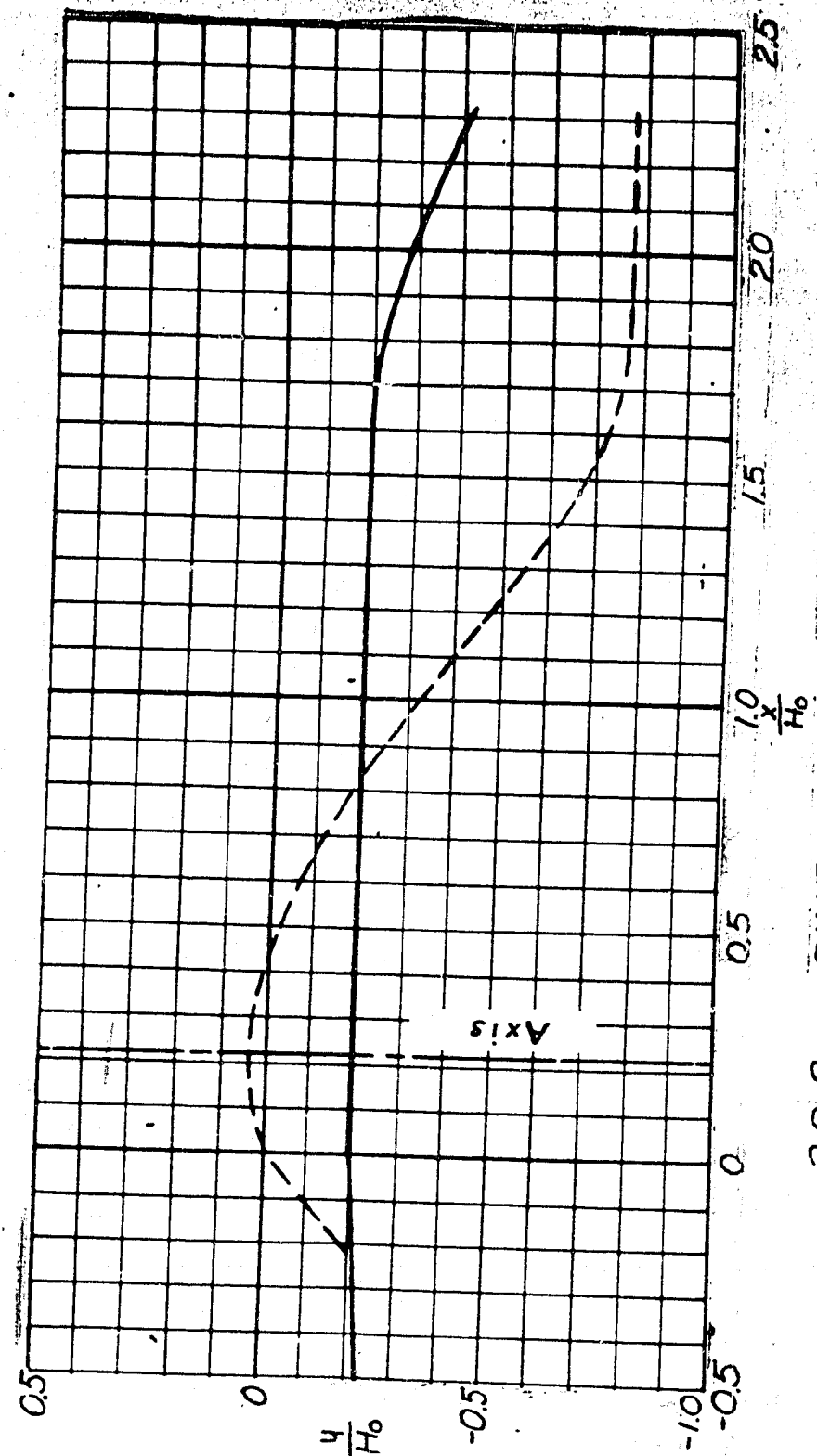
DIMENSIONLESS PLOTTING — SCALE: 2.5" = 1.0



30A. RYE PATCH DAM SPILLWAY
 $\frac{H_0}{P+E} = 3.527$ $C_d = 2.81$ ——— MODEL
 $C_d = 3.77$ ——— IDEAL



30B. GREEN MOUNTAIN DAM SPILLWAY
 (FINAL DESIGN)
 $\frac{H_0}{P+E} = 3.160$ $C_d = 3.21$ ——— MODEL
 $C_d = 3.785$ ——— IDEAL

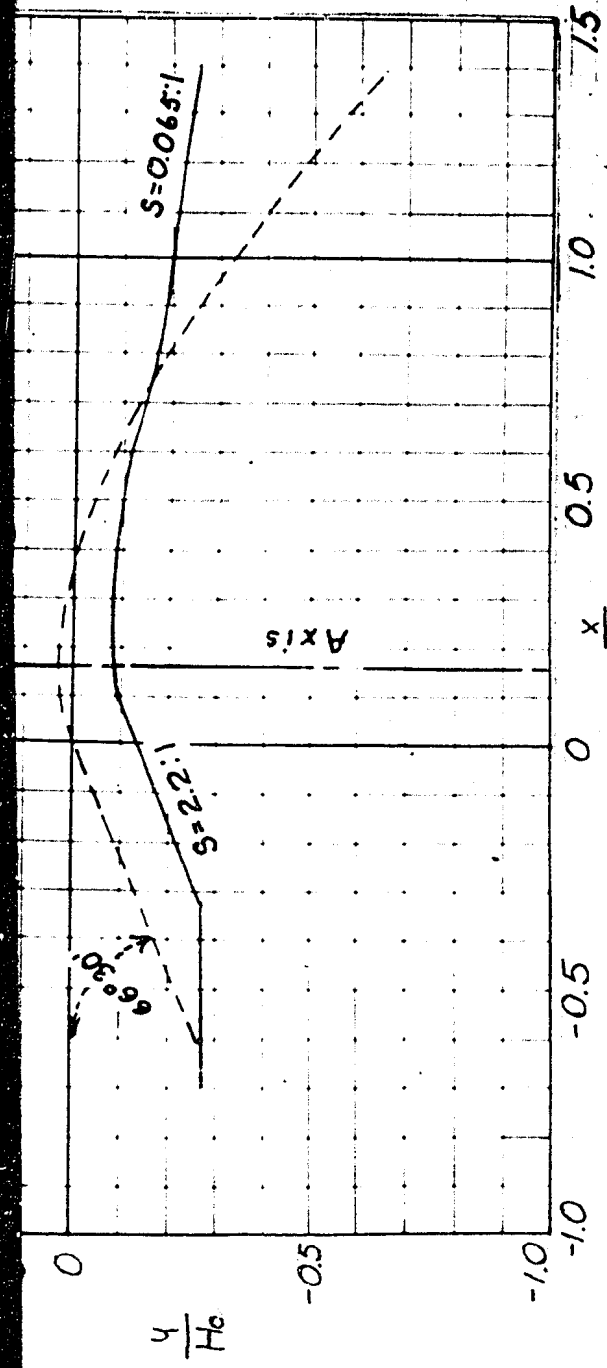


30C. PINE VIEW DAM SPILLWAY
 $\frac{H_0}{P+E} = 1.844$ $C_d = 2.74$ ——— MODEL
 $C_d = 3.86$ ——— IDEAL

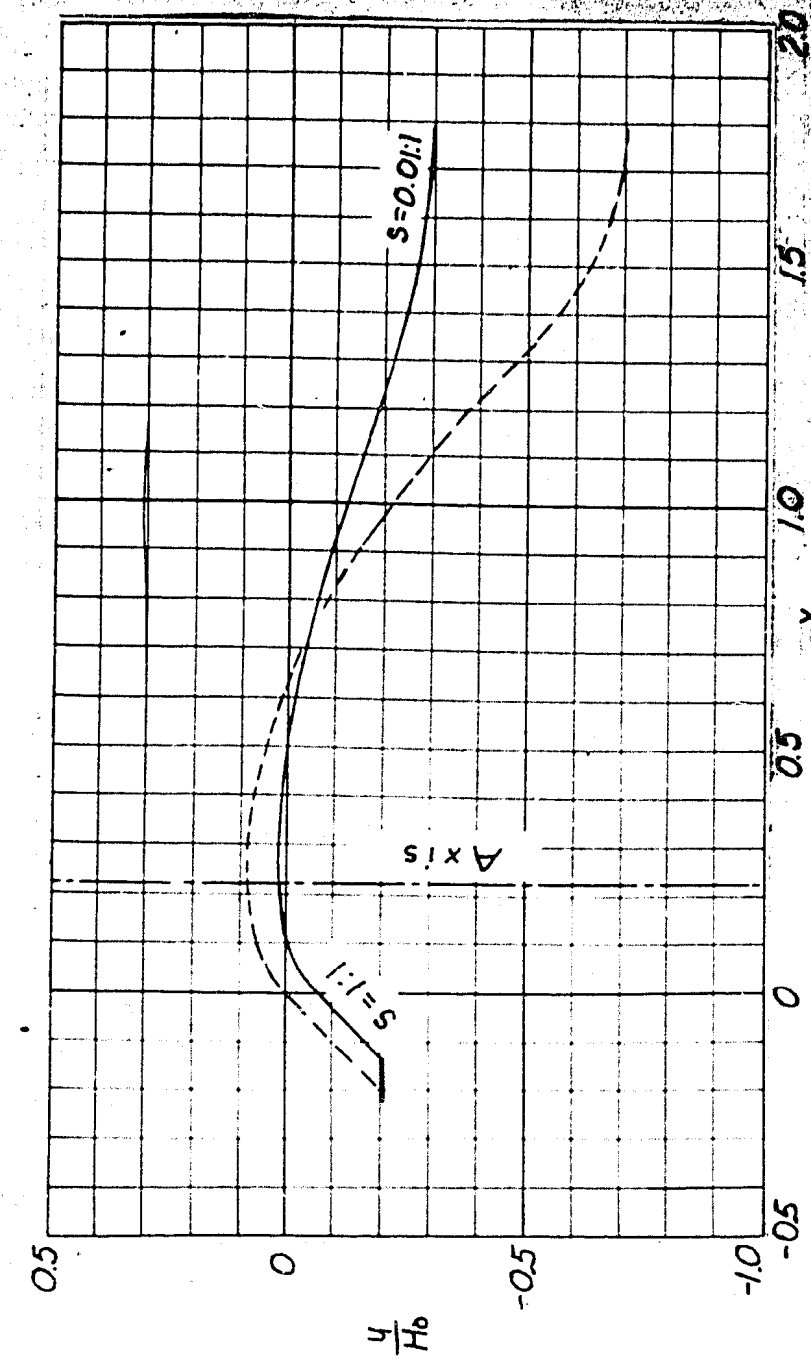
COMPARISON OF DISCHARGE COEFFICIENTS

DIMENSIONLESS PLOTTING — SCALE: 2.5" = 1.0

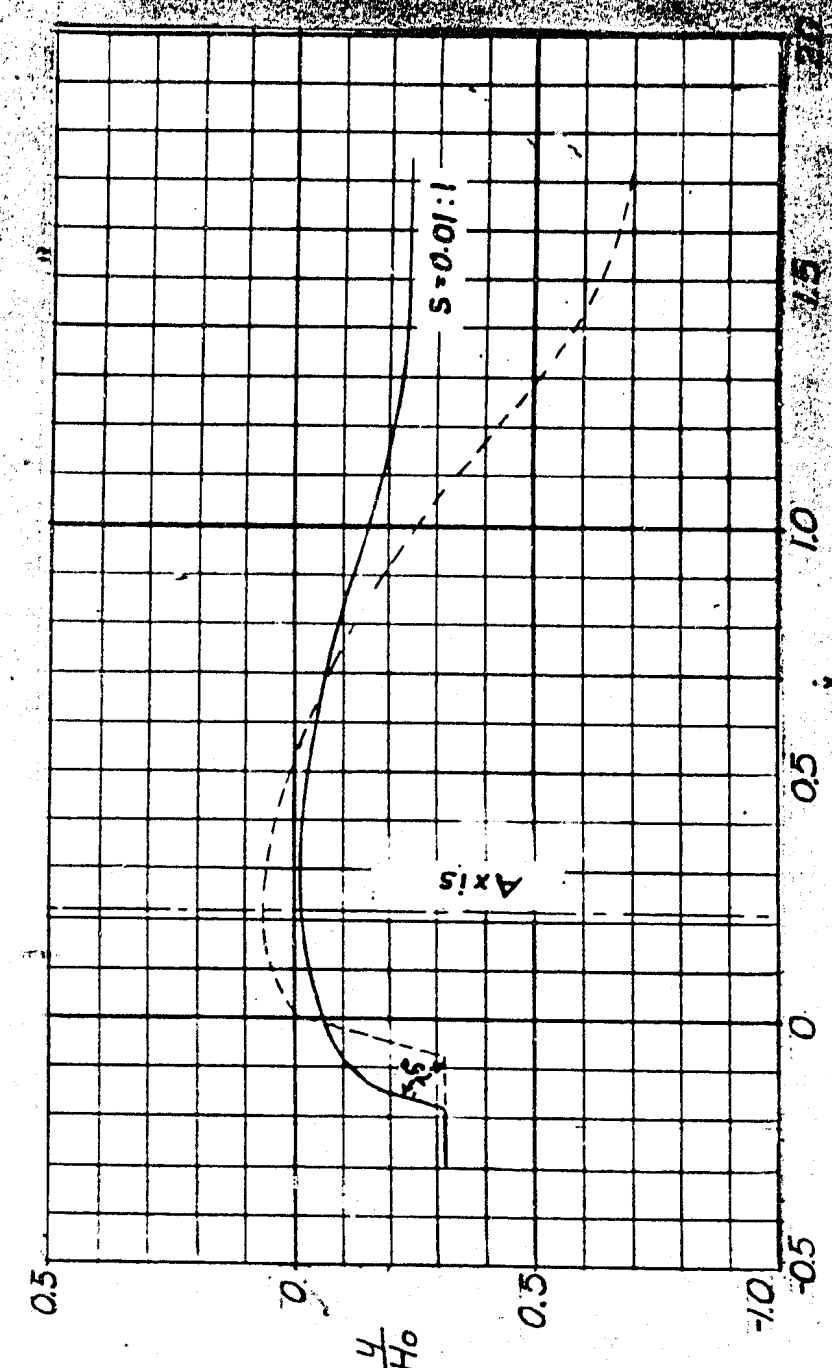
$C_1 = 3.86$ --- IDEAL



30. F. SHADOW MOUNTAIN DAM SPILLWAY
 $\frac{H_0}{P+E} = 4.000$ $C_1 = 3.25$ — MODEL
 $C_1 = 3.78$ --- IDEAL



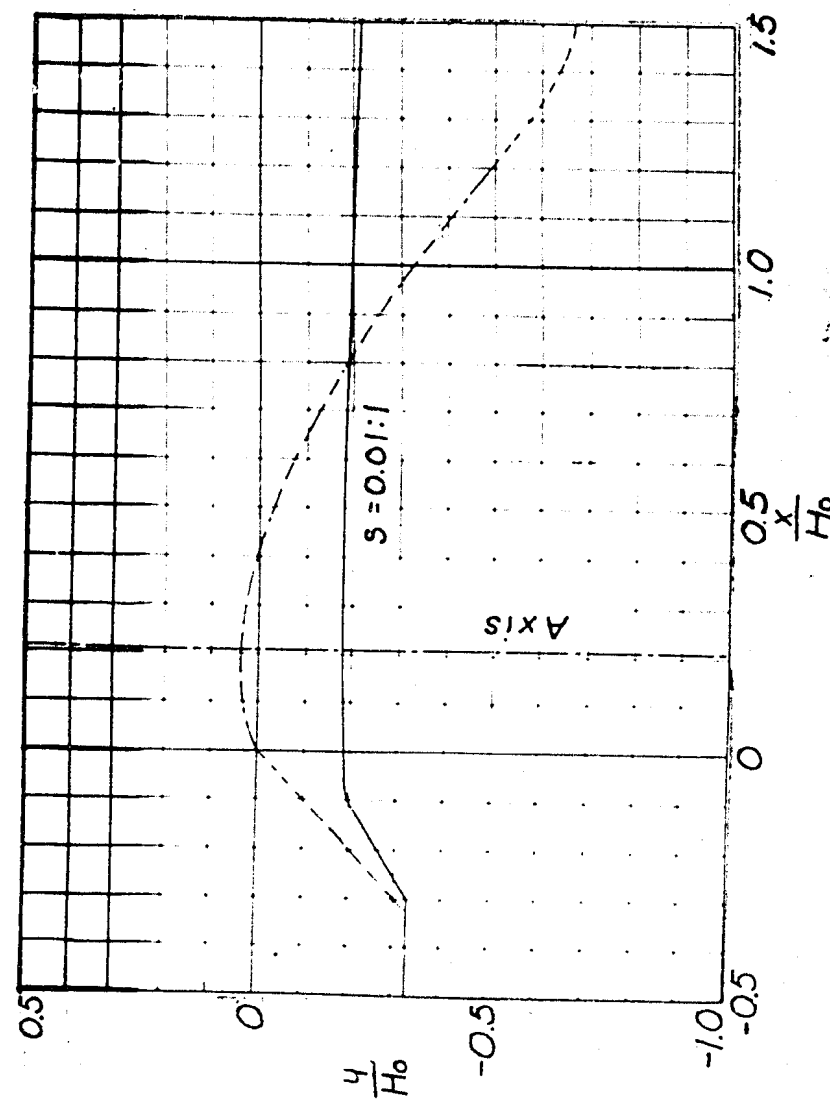
31. A. ANDERSON RANCH DAM SPILLWAY
 (FINAL DESIGN)
 $\frac{H_0}{P+E} = 3.455$ $C_1 = 3.40$ — MODEL
 $C_1 = 3.76$ --- IDEAL



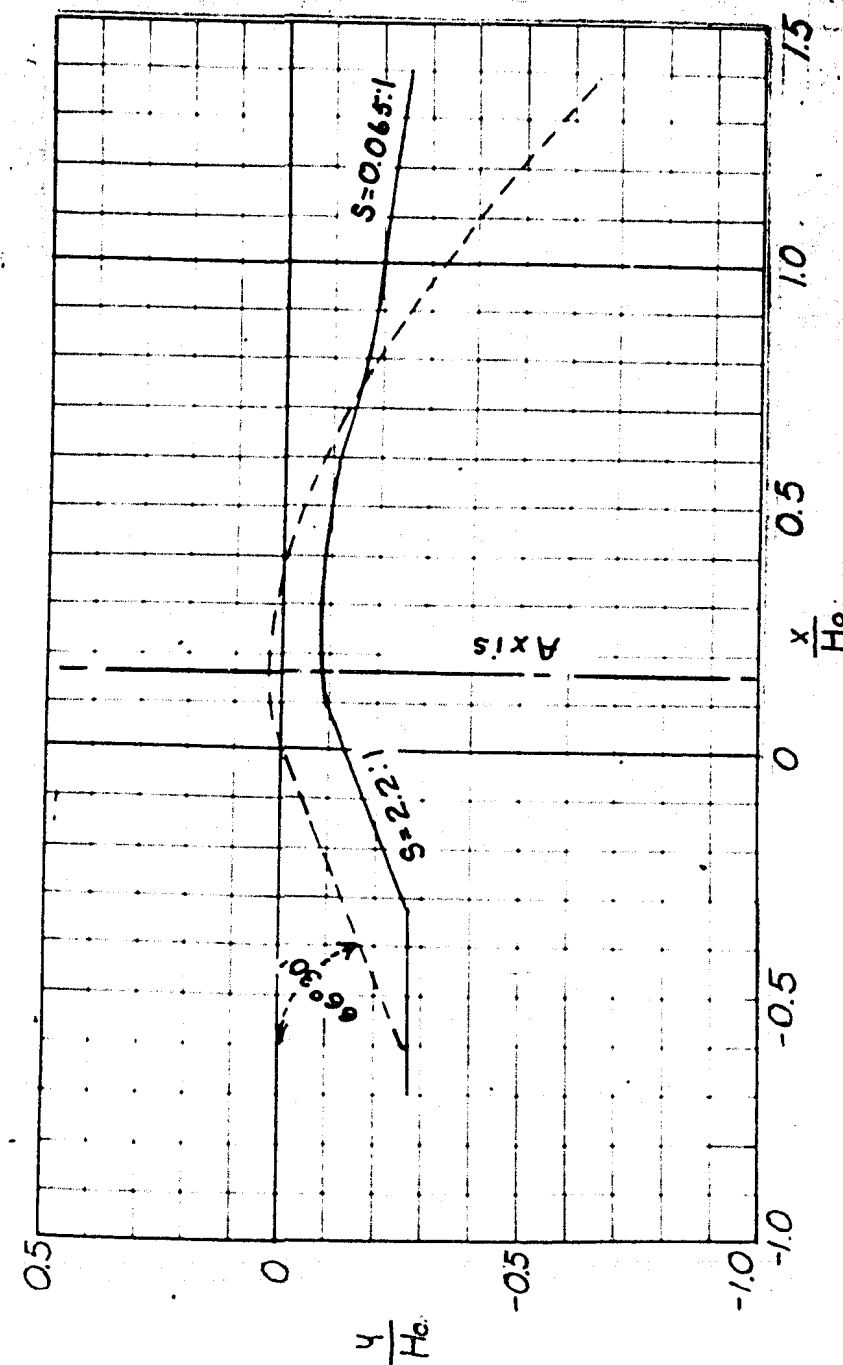
31. B. BARTLETT DAM SPILLWAY
 (FINAL DESIGN)
 $\frac{H_0}{P+E} = 2.651$ $C_1 = 3.482$ — MODEL
 $C_1 = 3.76$ --- IDEAL

COMPARISON OF DISCHARGE COEFFICIENTS

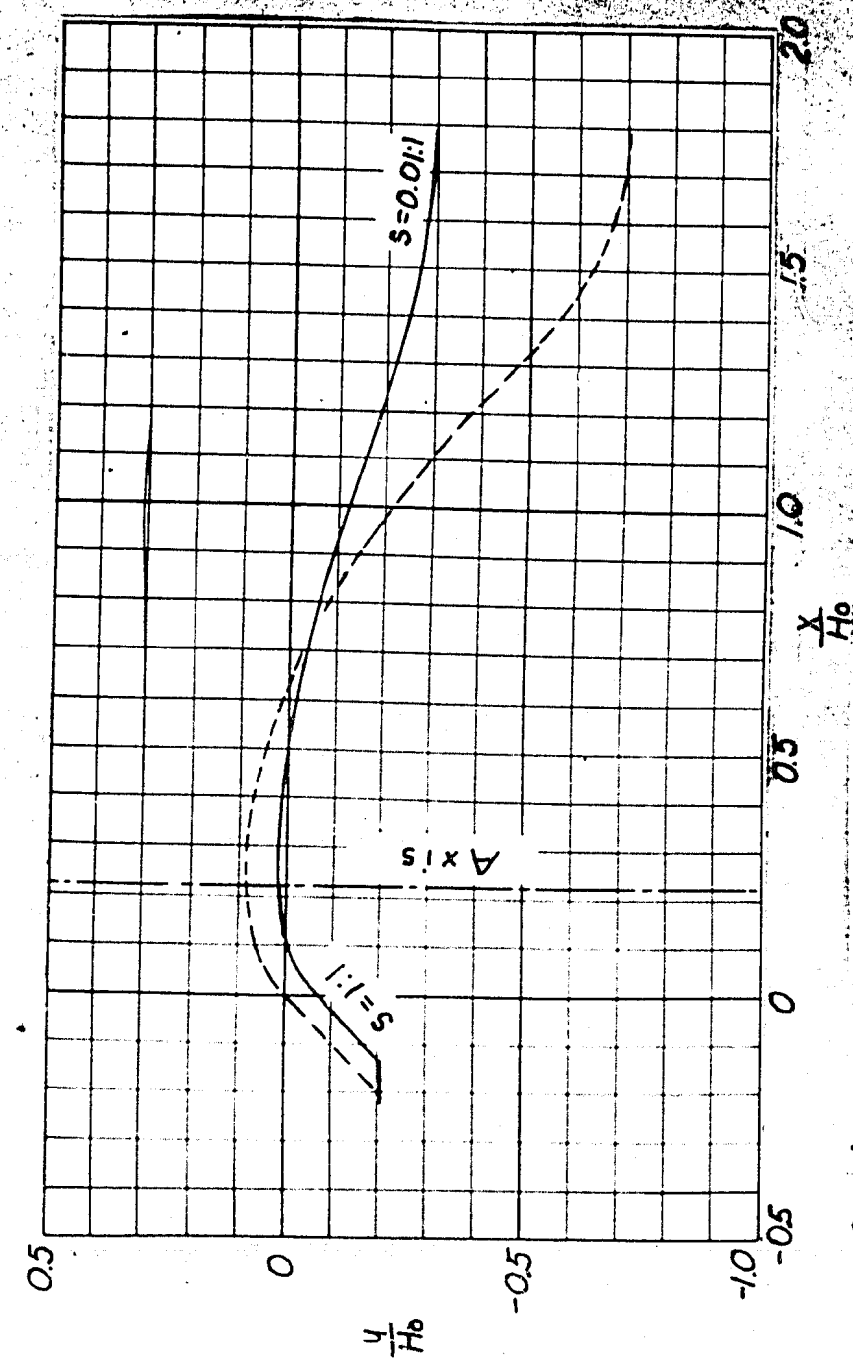
DIMENSIONLESS PLOTTING — SCALE: 2.5" = 1.0



30 E. ALCOVA DAM SPILLWAY
 $\frac{H_0}{P+E} = 2.913$ $C_A = 2.85$ — MODEL
 $C_I = 3.80$ ---- IDEAL



30 F. SHADOW MOUNTAIN DAM SPILLWAY
 $\frac{H_0}{P+E} = 4.000$ $C_A = 3.25$ — MODEL
 $C_I = 3.78$ ---- IDEAL



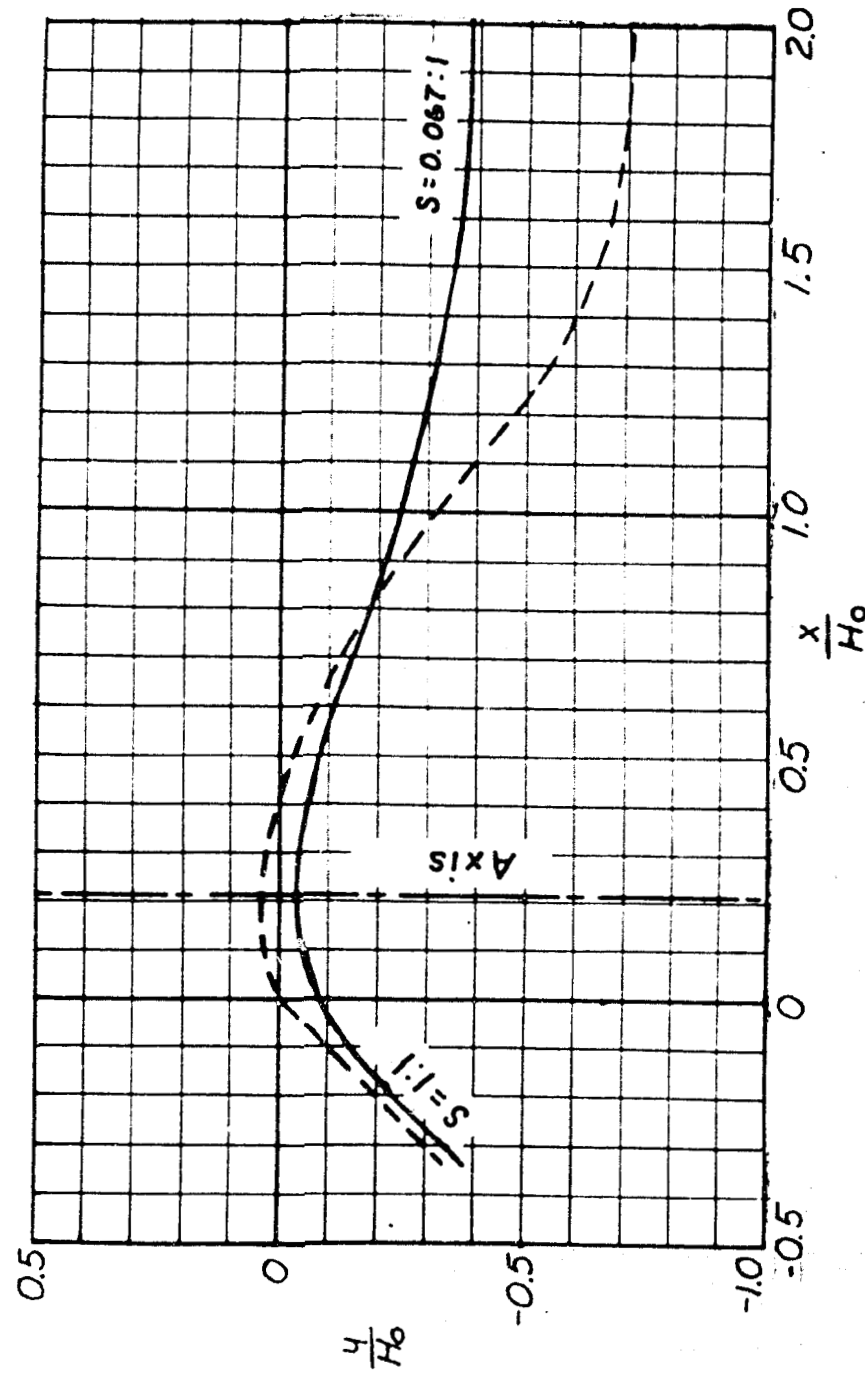
31 A. ANDERSON RANCH DAM SPILLWAY
 (FINAL DESIGN)
 $\frac{H_0}{P+E} = 3.455$ $C_A = 3.40$ — MODEL
 $C_I = 3.76$ ---- IDEAL

COMPARISON OF DISCHARGE COEFFICIENTS

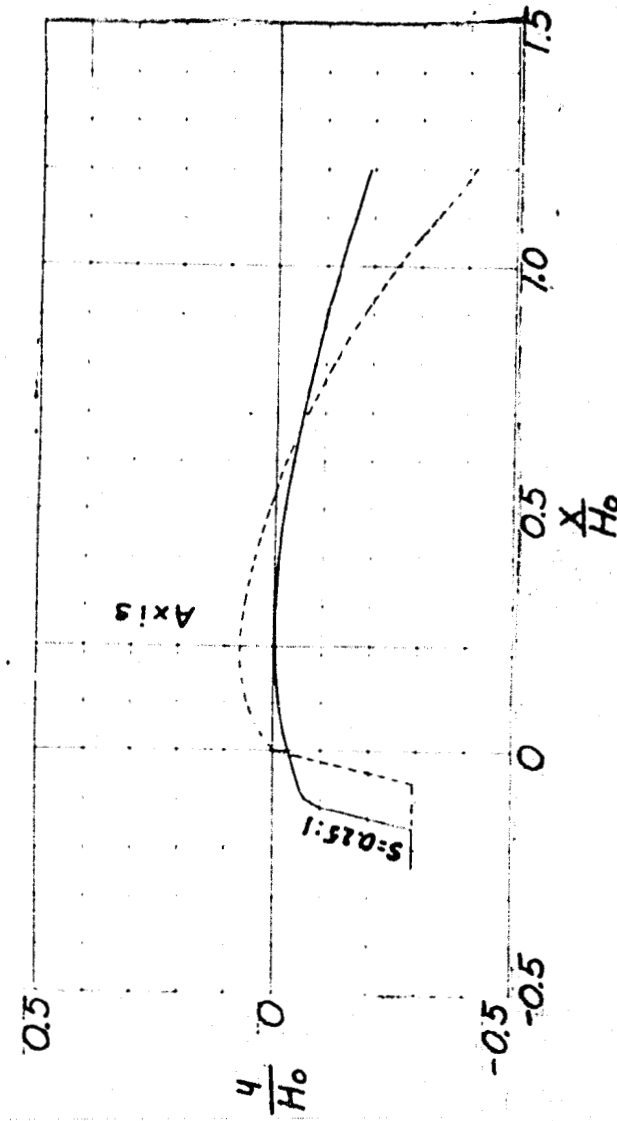
DIMENSIONLESS PLOTTING — SCALE: 2.5" = 1.0

COMPARISON OF DISCHARGE COEFFICIENTS

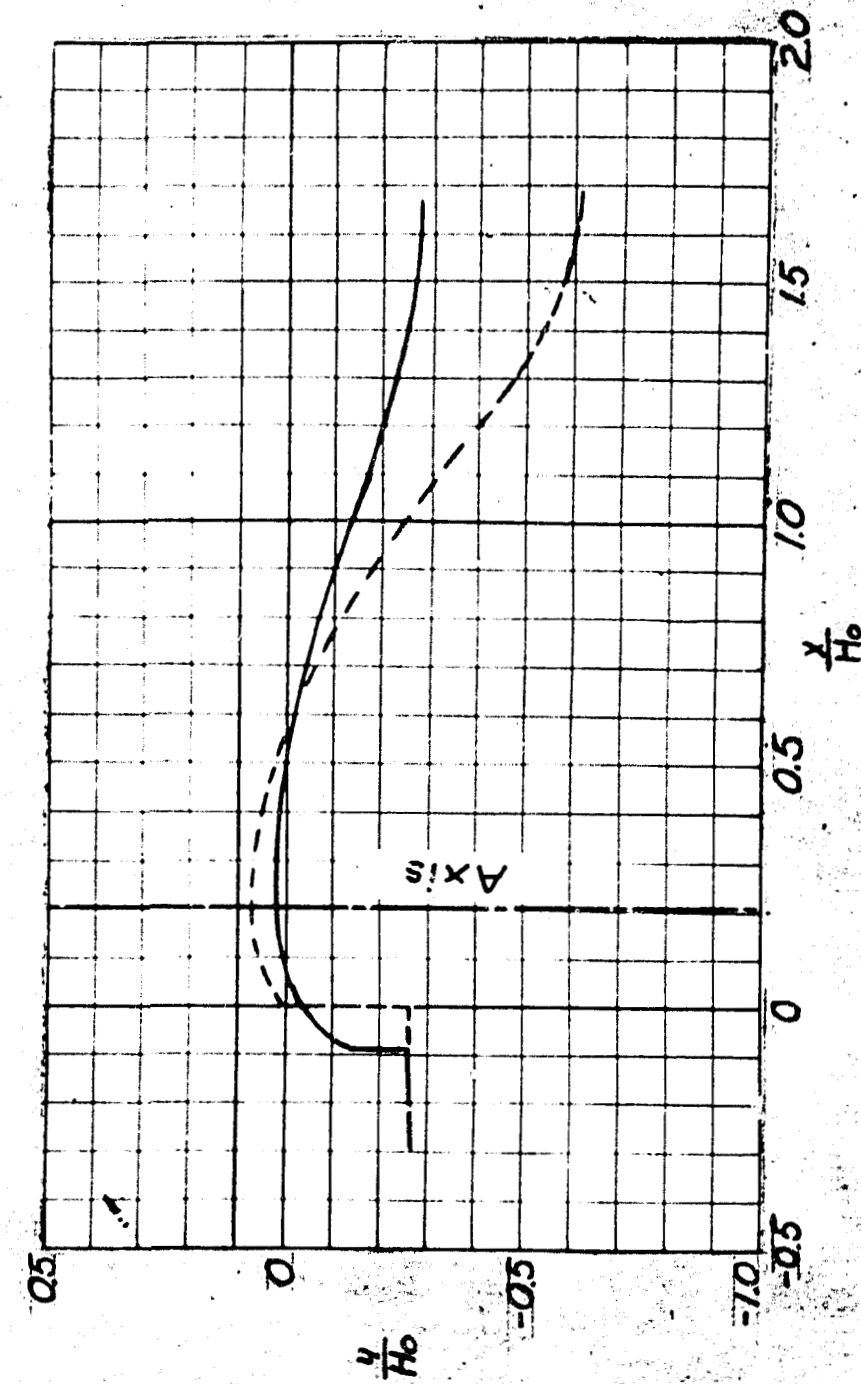
DIMENSIONLESS PLOTTING — SCALE: 2.5" = 1.0



31 D. BOCA DAM SPILLWAY
(FINAL DESIGN)
 $\frac{H_0}{P+E} = 2.425$ $C_d = 3.40$ — MODEL
 $C_i = 3.81$ --- IDEAL



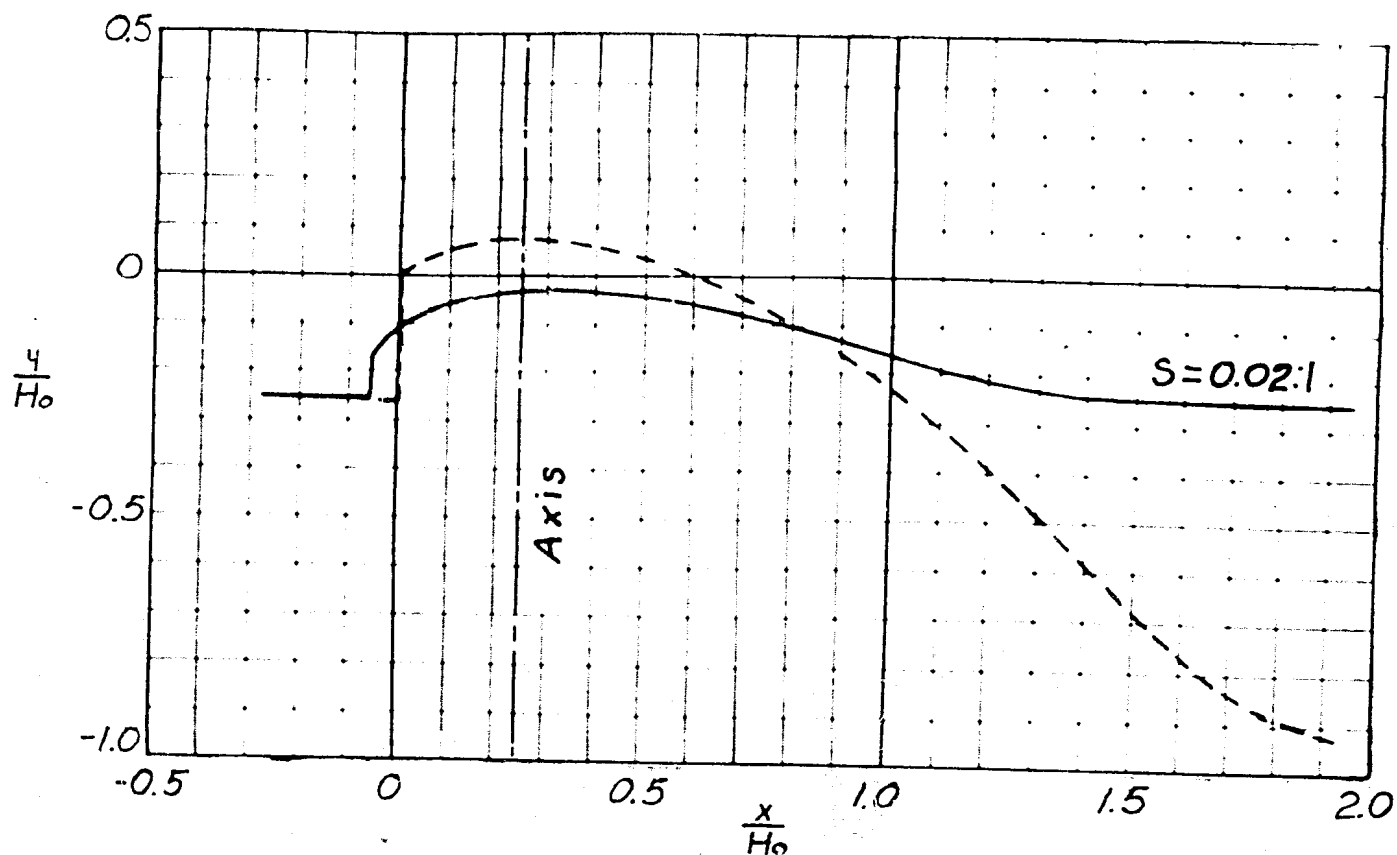
31 E. BOYSEN DAM SPILLWAY
(FINAL DESIGN)
 $\frac{H_0}{P+E} = 2.767$ $C_d = 3.37$ — MODEL
 $C_i = 3.76$ --- IDEAL



31 F. VALLECITO DAM SPILLWAY
(FINAL DESIGN)
 $\frac{H_0}{P+E} = 2.993$ $C_d = 3.423$ — MODEL
 $C_i = 3.72$ --- IDEAL

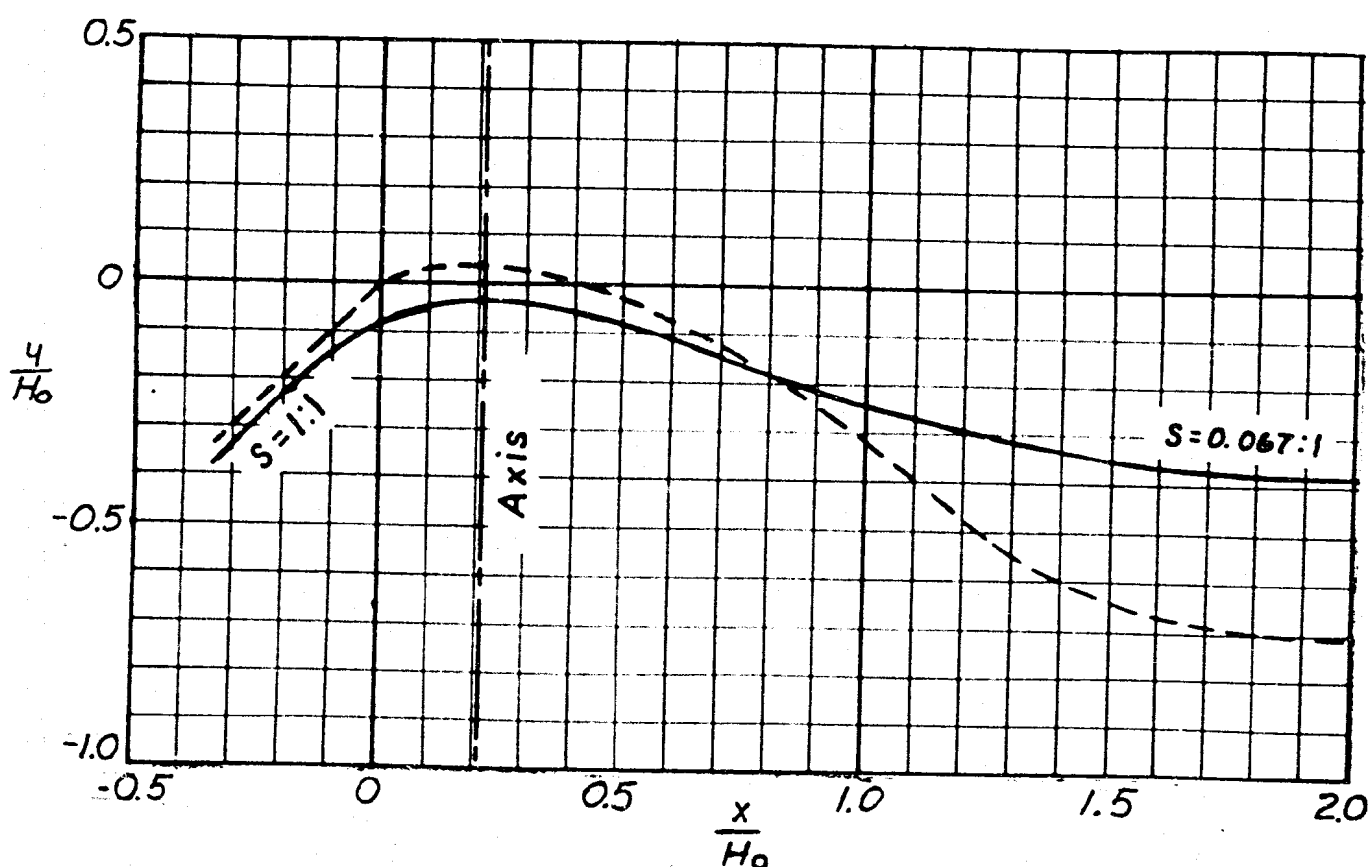
COMPARISON OF DISCHARGE COEFFICIENTS

DIMENSIONLESS PLOTTING — SCALE: 2.5" = 1.0



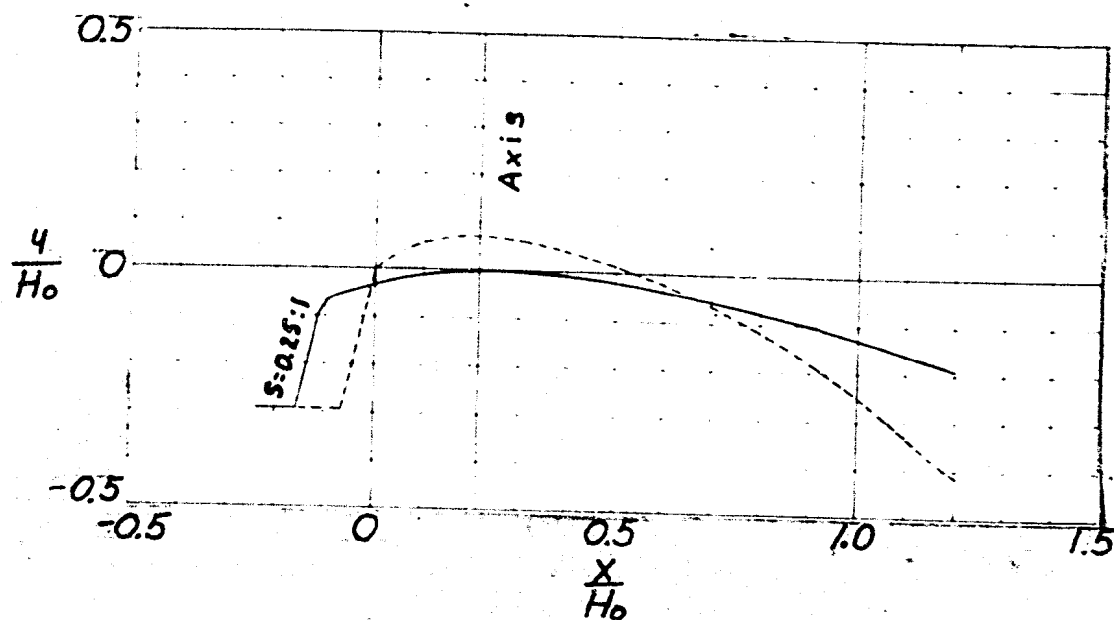
31 C. GRANBY DAM SPILLWAY
(FINAL DESIGN)

$$\frac{H_0}{P + E} = 3.017 \quad C_A = 3.20 \text{ — MODEL} \\ C_I = 3.71 \text{ — IDEAL}$$



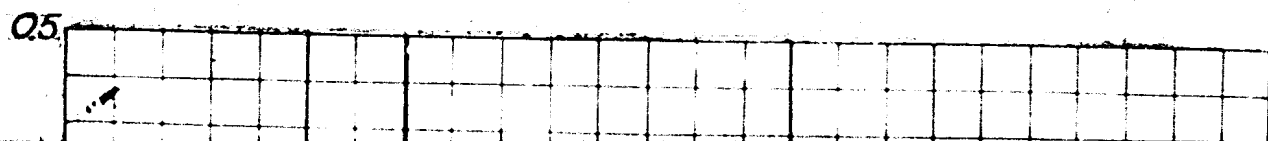
31 D. BOCA DAM SPILLWAY
(FINAL DESIGN)

$$\frac{H_0}{P + E} = 2.425 \quad C_A = 3.40 \text{ — MODEL} \\ C_I = 3.81 \text{ — IDEAL}$$



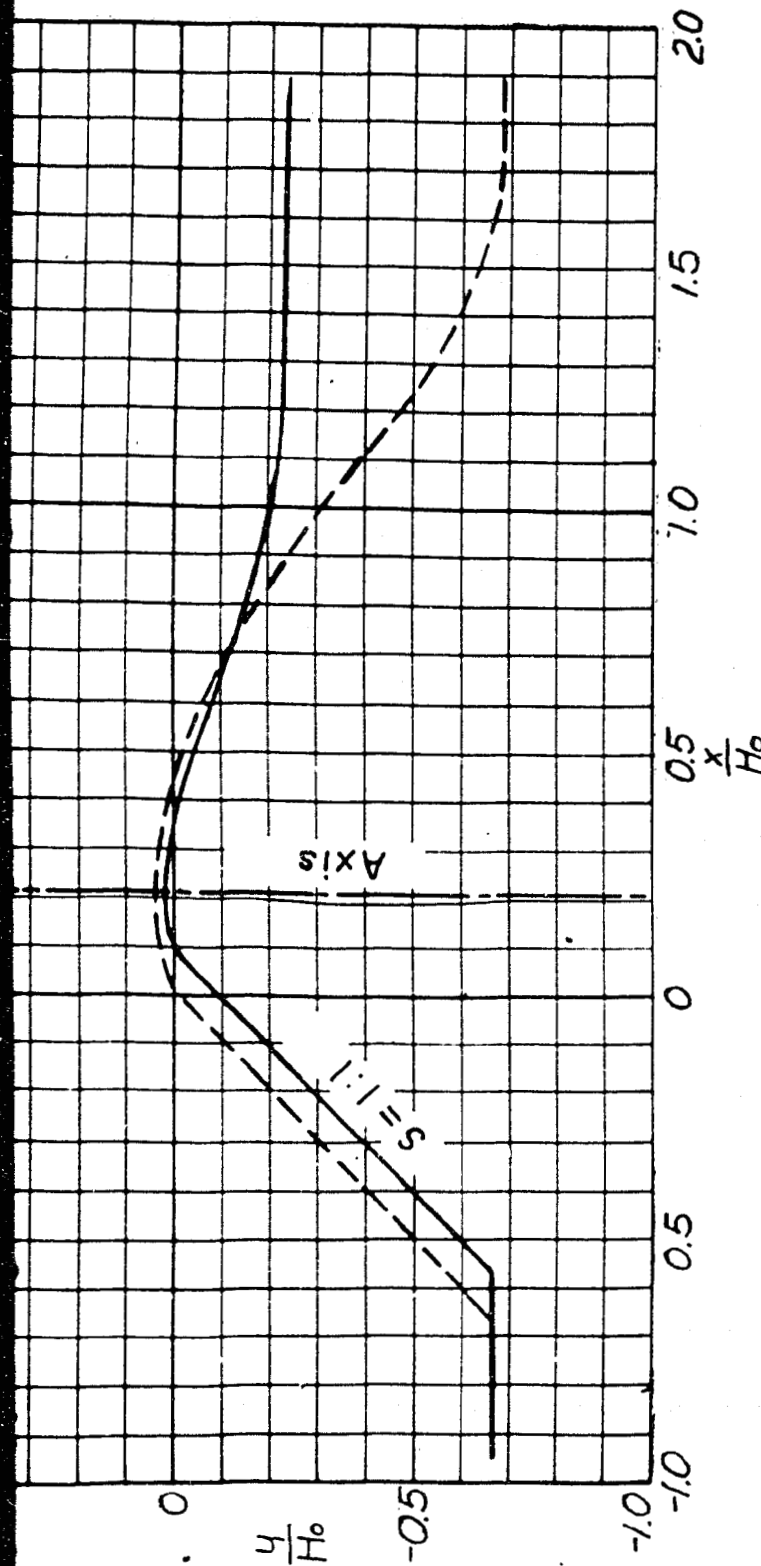
31 E. BOYSEN DAM SPILLWAY
(FINAL DESIGN)

$$\frac{H_0}{P + E} = 2.767 \quad C_A = 3.37 \text{ — MODEL} \\ C_I = 3.76 \text{ — IDEAL}$$

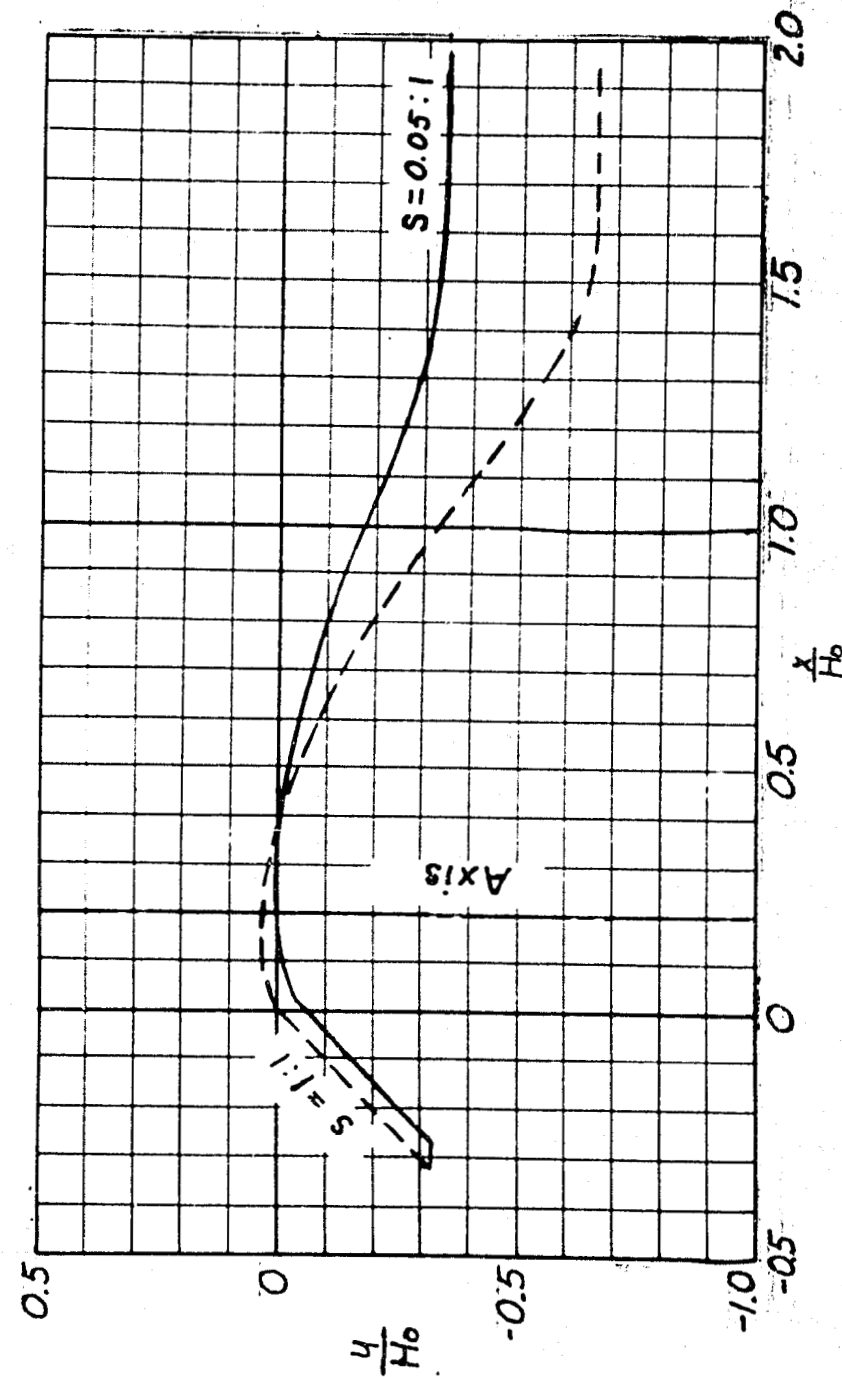


COMPARISON OF DISCHARGE COEFFICIENTS

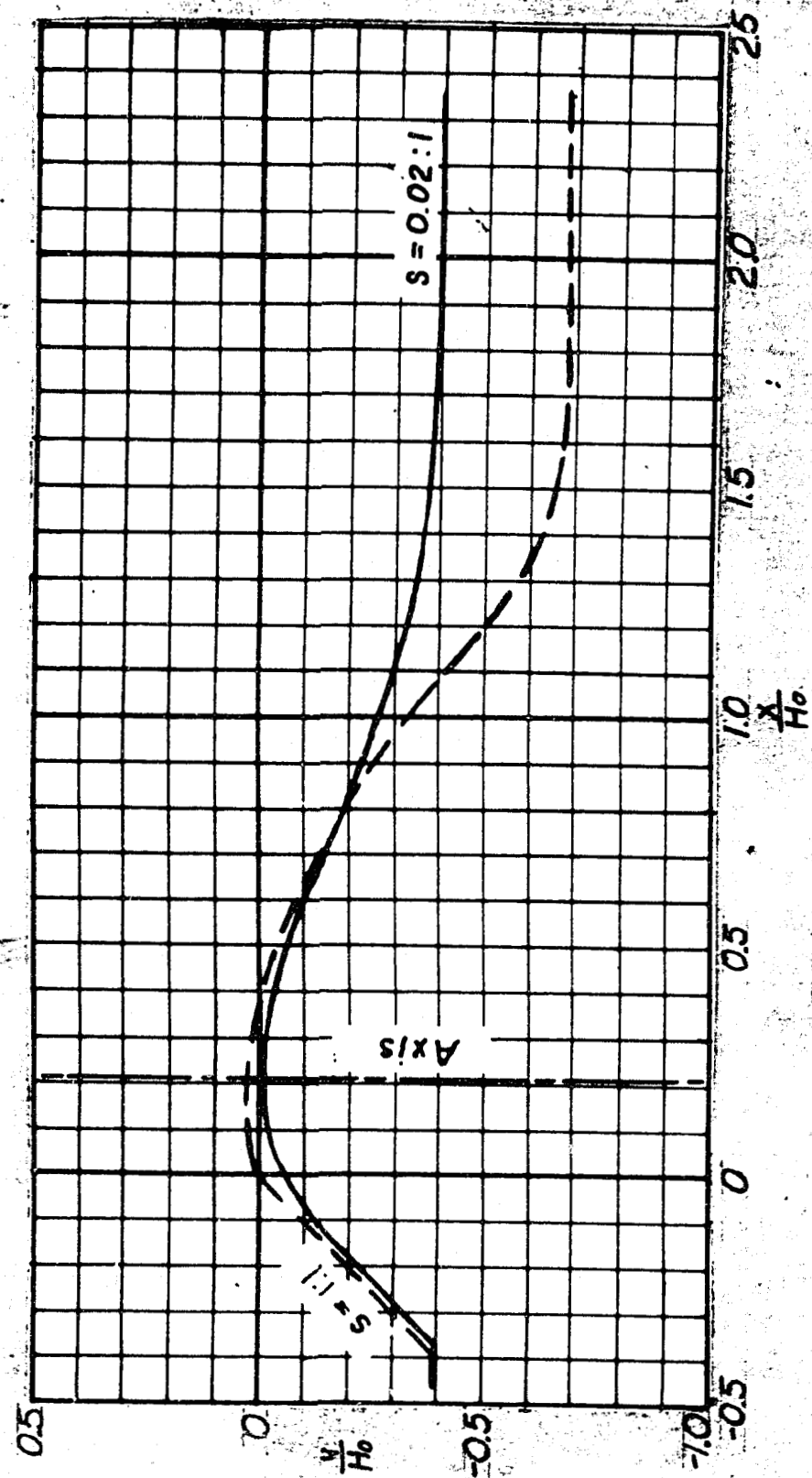
DIMENSIONLESS PLOTTING — SCALE: 2.5" = 1.0



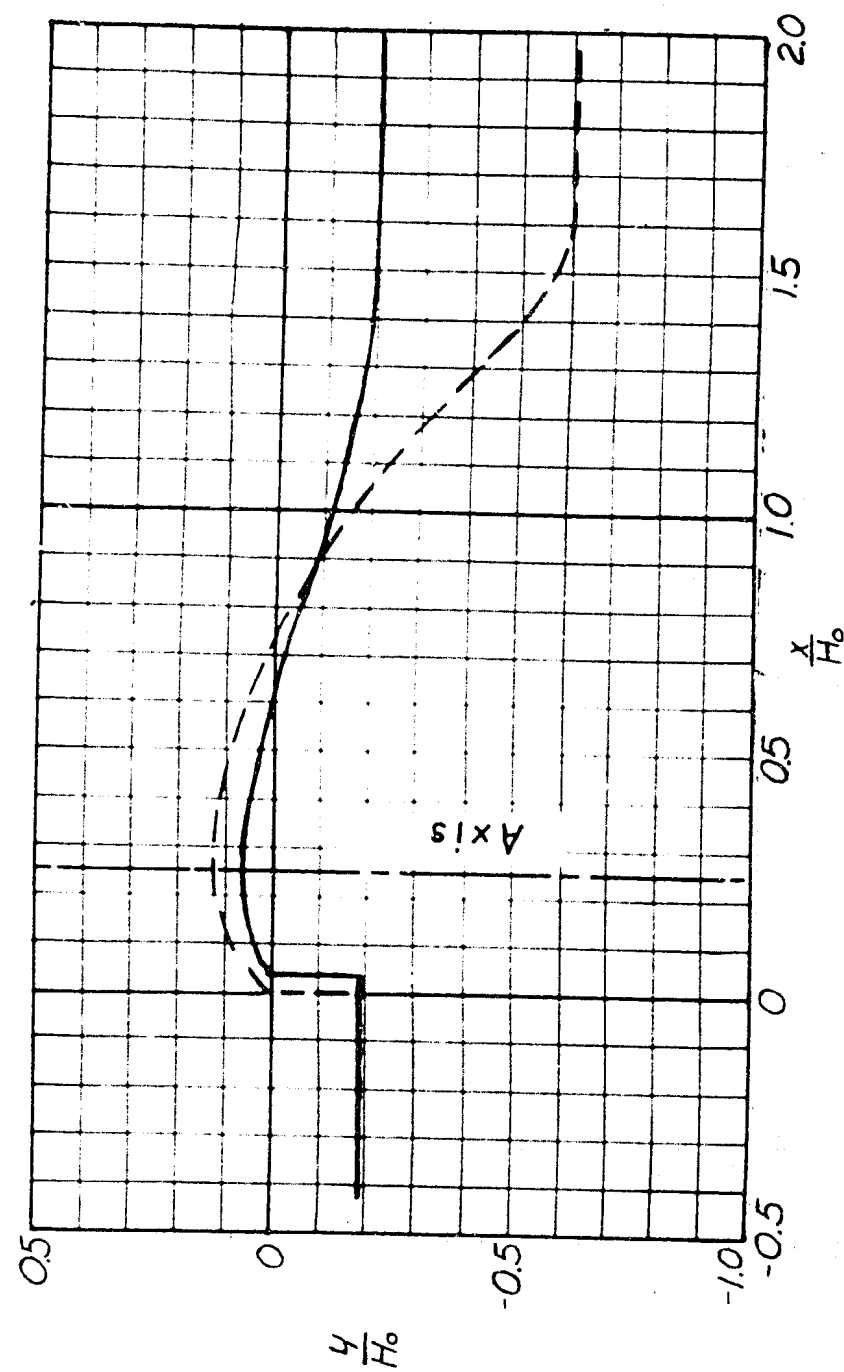
32B FRESNO DAM SPILLWAY
(FINAL DESIGN)
 $\frac{H_0}{P+E} = 1.34$ $C_d = 3.808$ — MODEL
 $C_d = 3.88$ — IDEAL



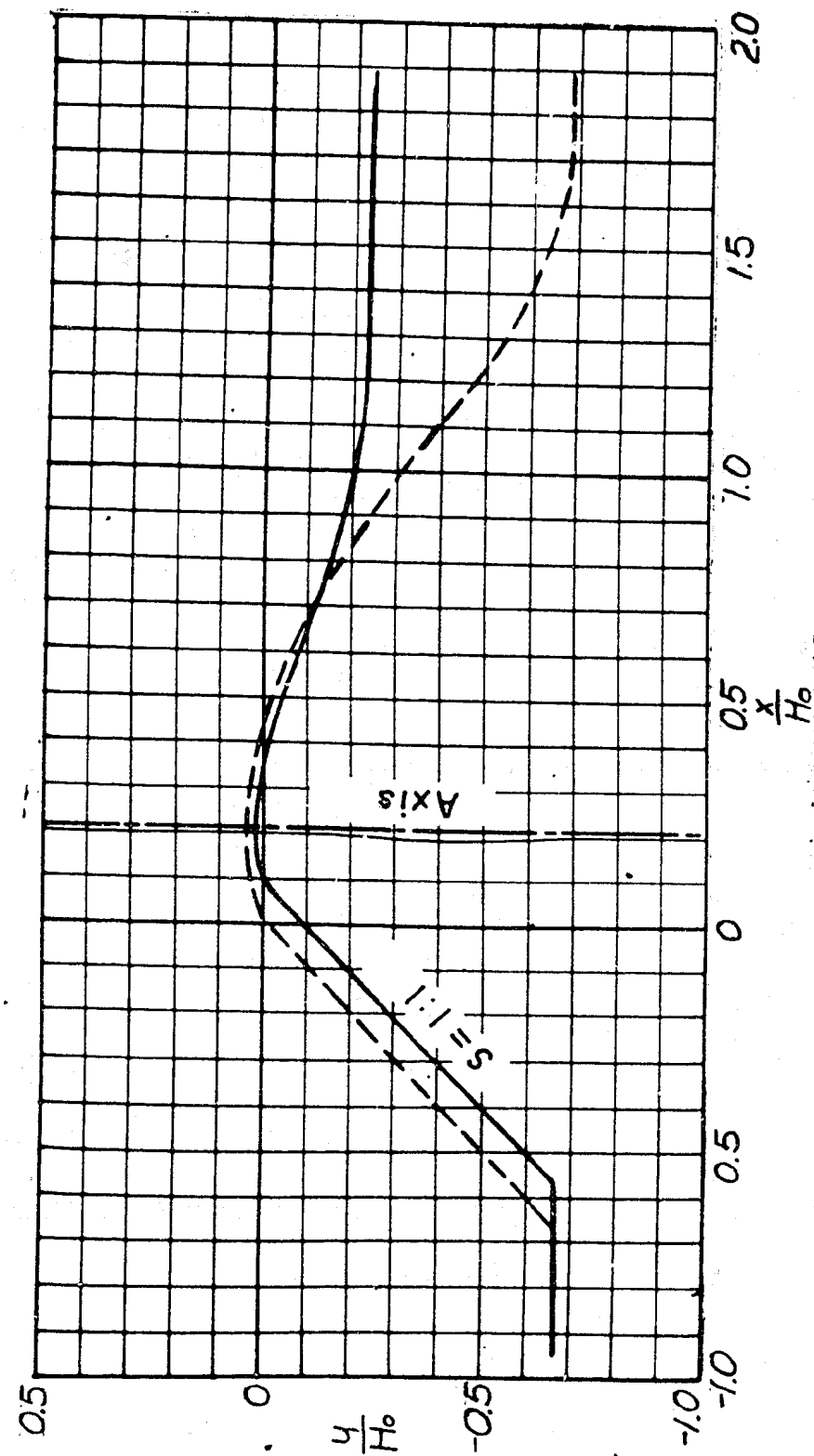
32C UNITY DAM SPILLWAY
(FINAL DESIGN)
 $\frac{H_0}{P+E} = 2.82$ $C_d = 3.48$ — MODEL
 $C_d = 3.79$ — IDEAL



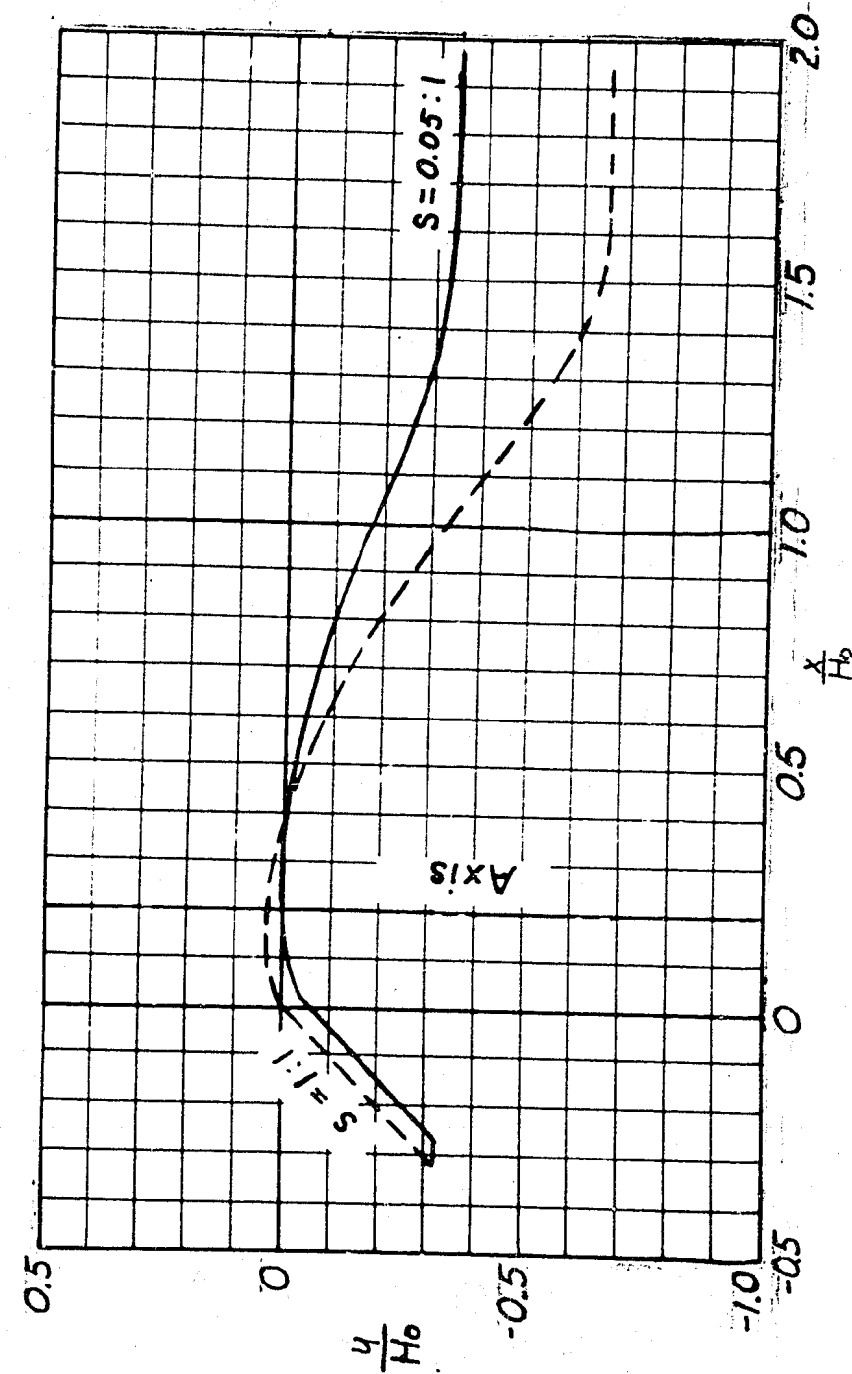
32D BULL LAKE DAM SPILLWAY
(FINAL DESIGN)
 $\frac{H_0}{P+E} = 2.33$ $C_d = 3.58$ — MODEL
 $C_d = 3.845$ — IDEAL



32A SCOFIELD DAM SPILLWAY
 $\frac{H_0}{P+E} = 3.13$ $C_A = 3.43$ — MODEL
 $C_I = 3.725$ --- IDEAL



32B FRESNO DAM SPILLWAY
 (FINAL DESIGN)
 $\frac{H_0}{P+E} = 1.34$ $C_A = 3.808$ — MODEL
 $C_I = 3.88$ --- IDEAL



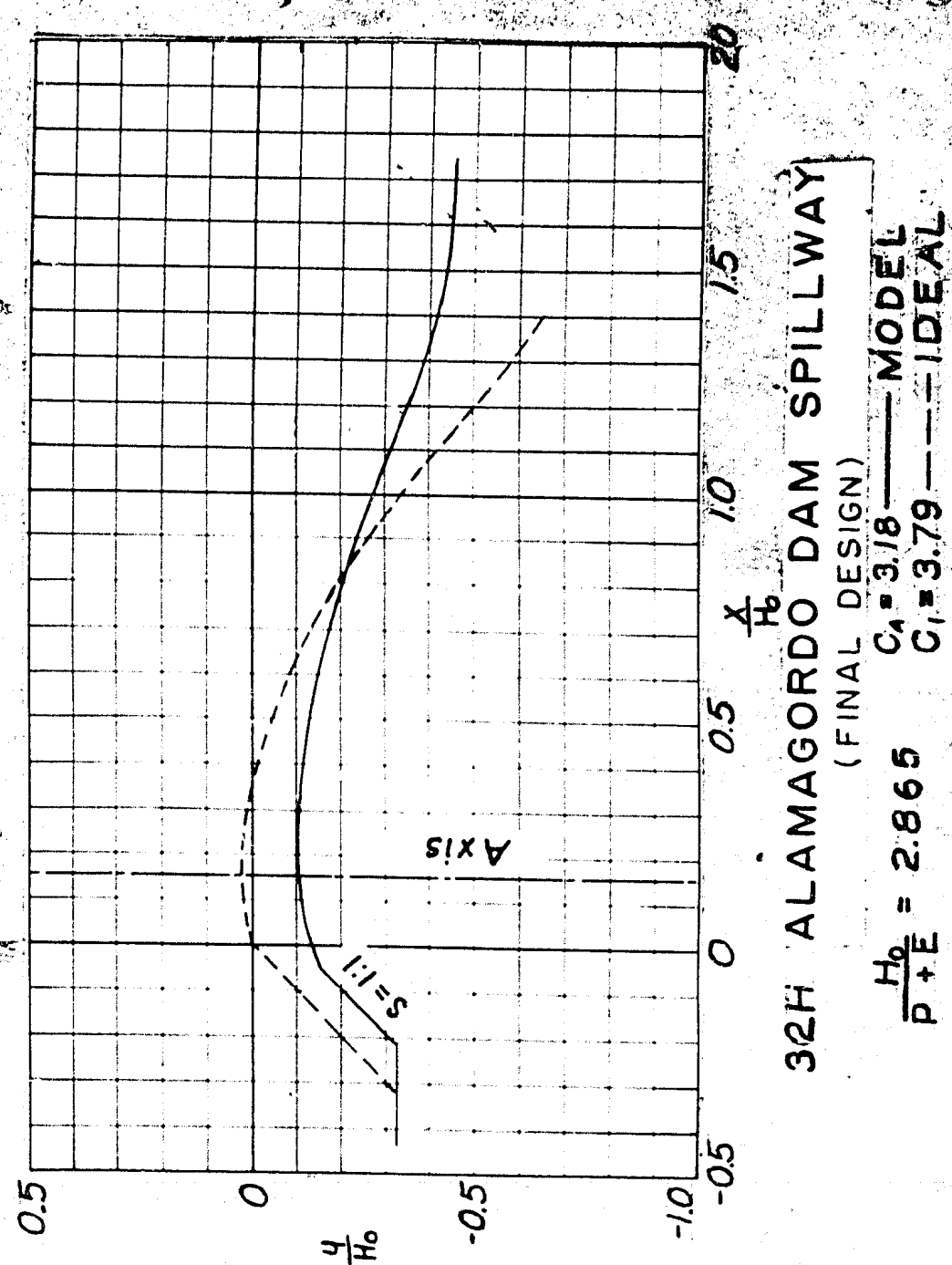
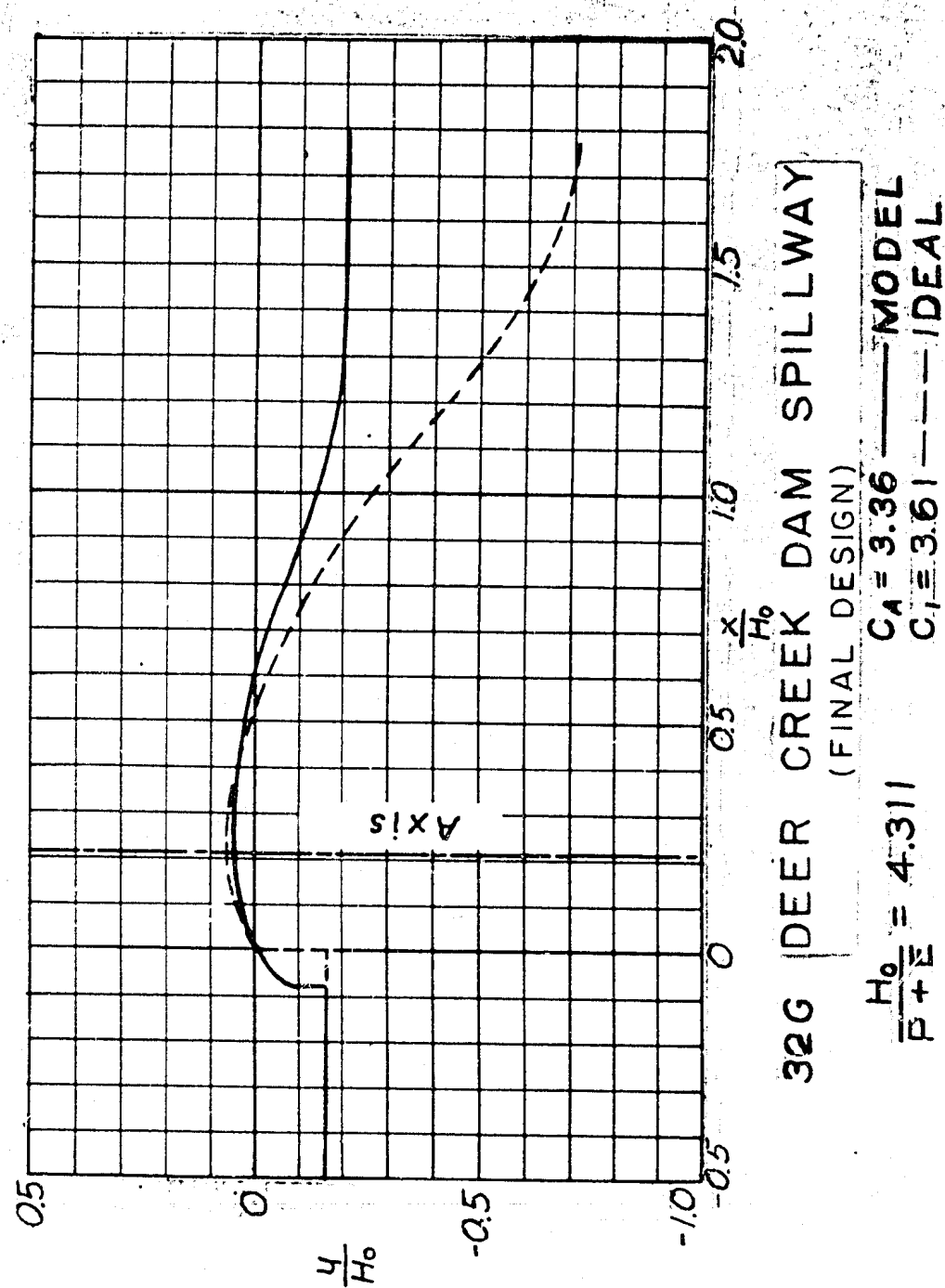
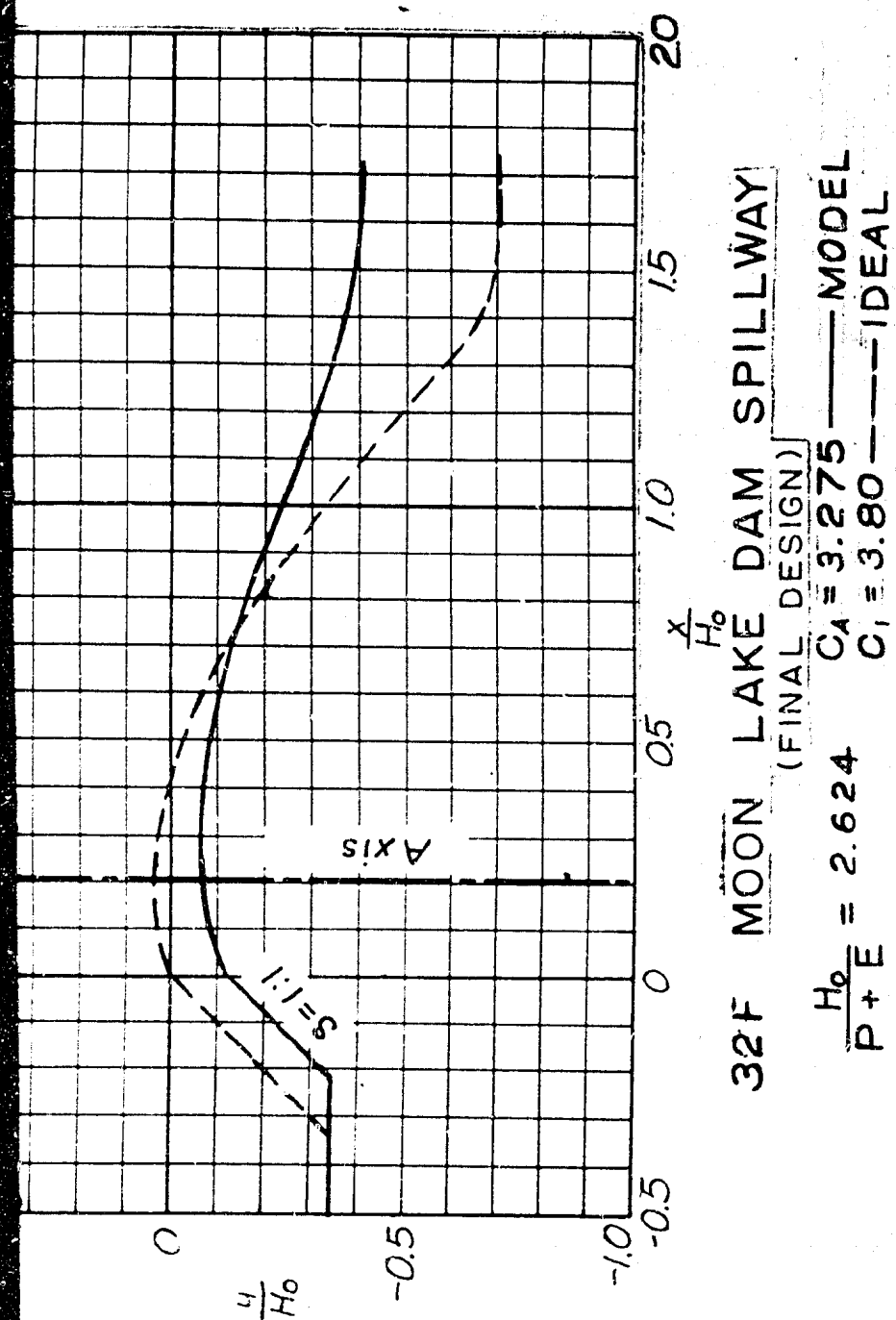
32C UNITY DAM SPILLWAY
 (FINAL DESIGN)
 $\frac{H_0}{P+E} = 2.82$ $C_A = 3.48$ — MODEL
 $C_I = 3.79$ --- IDEAL

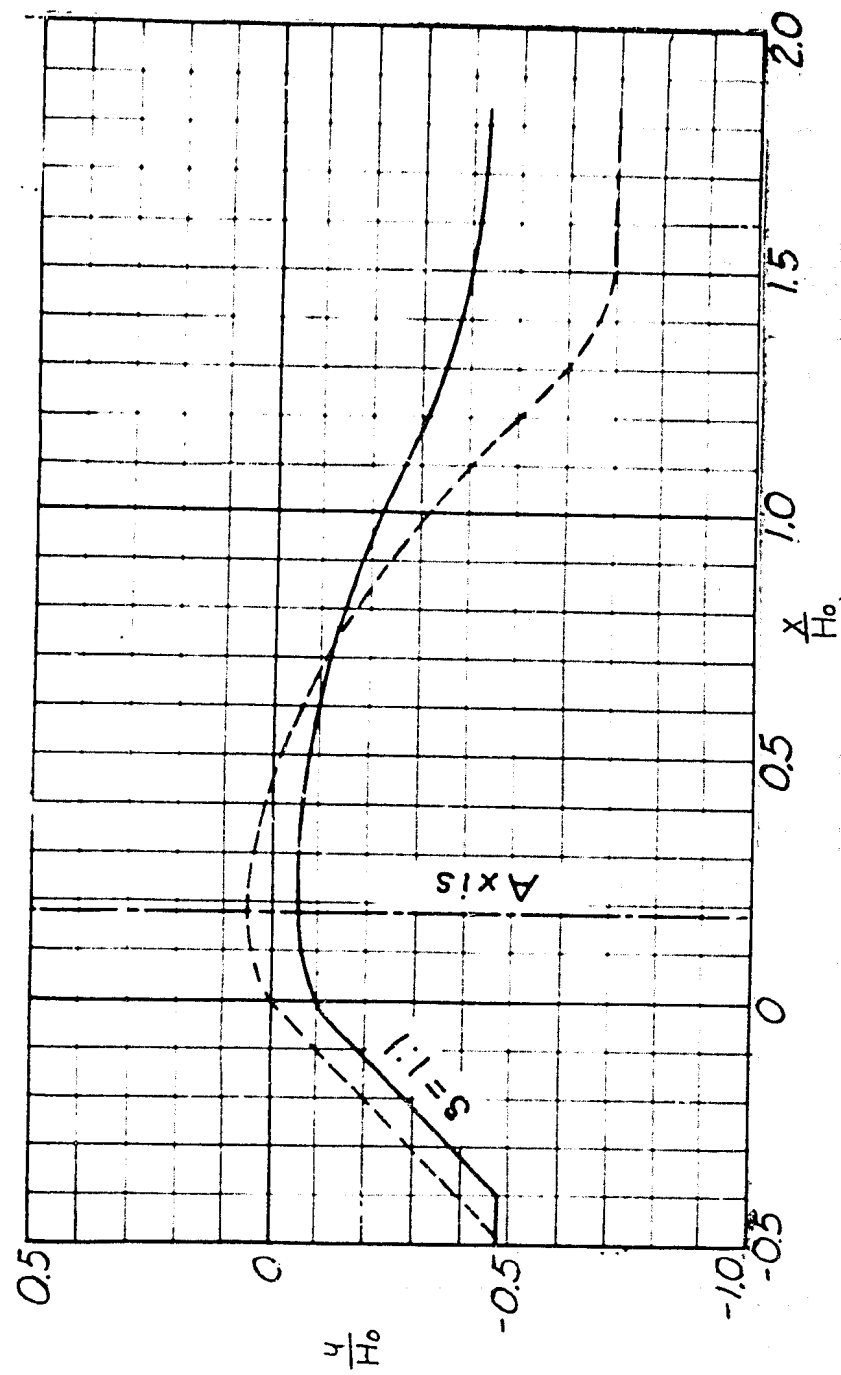
COMPARISON OF DISCHARGE COEFFICIENTS

DIMENSIONLESS PLOTTING — SCALE: 2.5" = 1.0

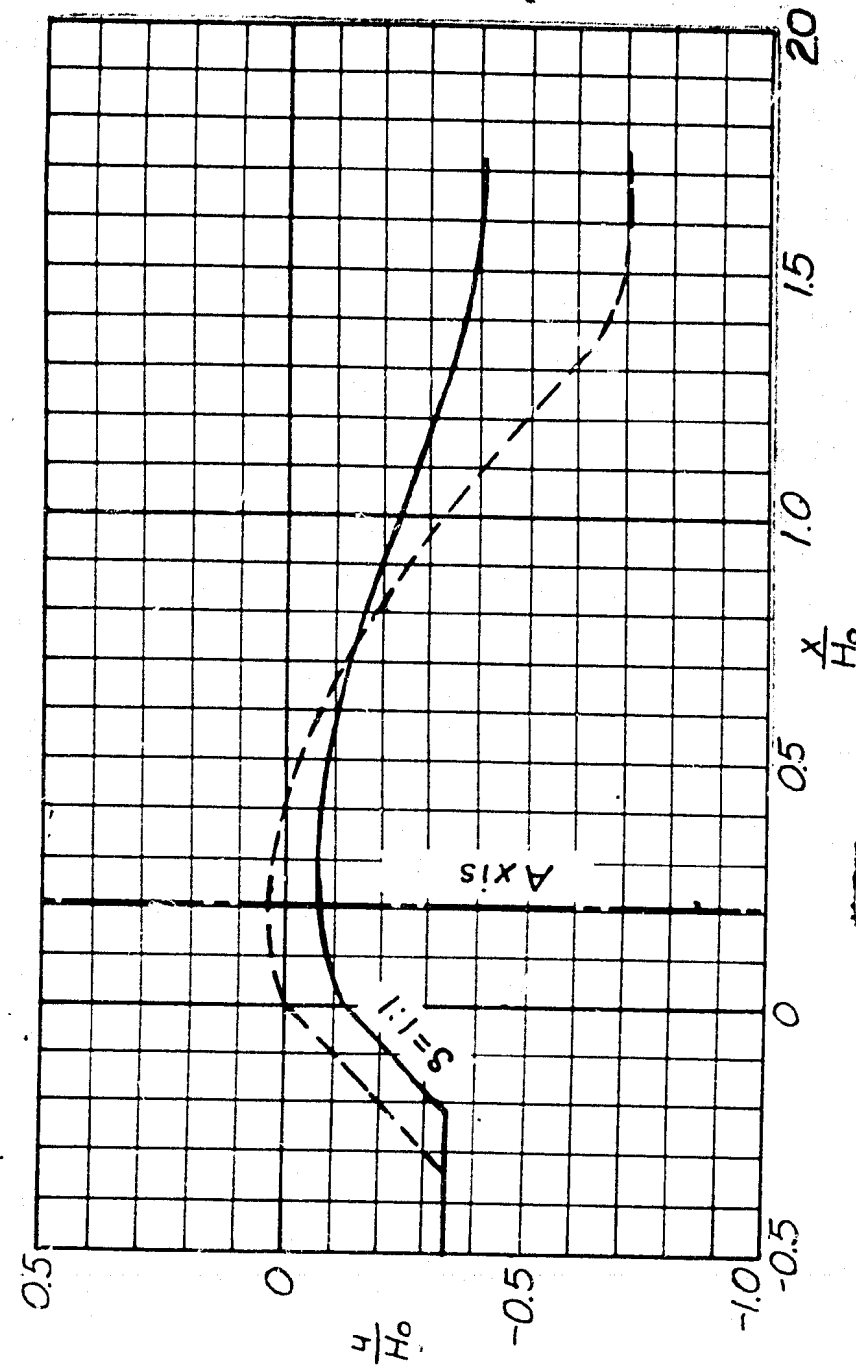
COMPARISON OF DISCHARGE COEFFICIENTS

DIMENSIONLESS PLOTTING — SCALE: 2.5" = 1.0

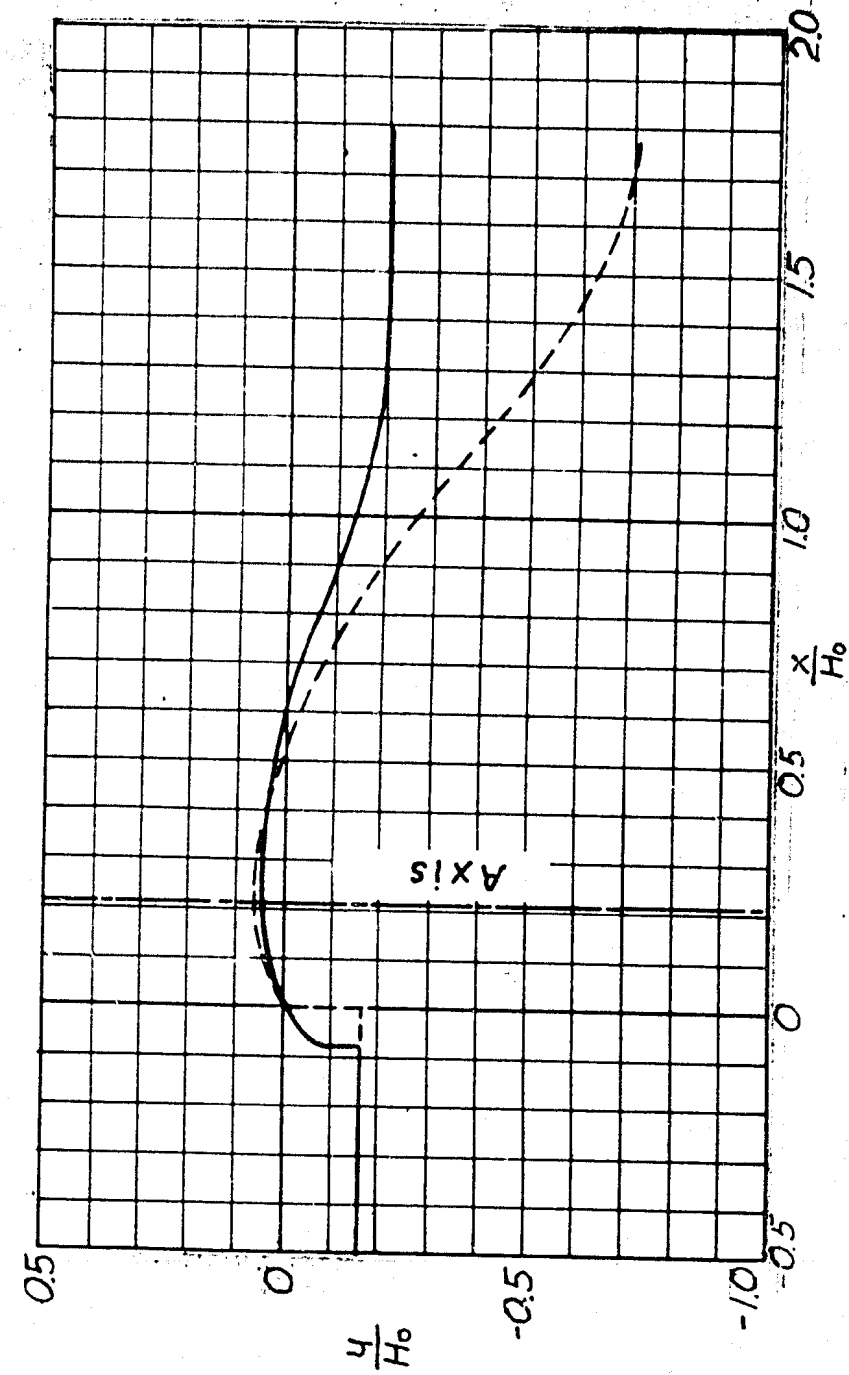




32E CABALLO DAM SPILLWAY
 $\frac{H_0}{P+E} = 1.904$ $C_A = 3.48$ — MODEL
 $C_I = 3.85$ --- IDEAL



32F MOON LAKE DAM SPILLWAY
 (FINAL DESIGN)
 $\frac{H_0}{P+E} = 2.624$ $C_A = 3.275$ — MODEL
 $C_I = 3.80$ --- IDEAL



32G DEER CREEK DAM SPILLWAY
 (FINAL DESIGN)
 $\frac{H_0}{P+E} = 4.311$ $C_A = 3.36$ — MODEL
 $C_I = 3.61$ --- IDEAL

COMPARISON OF DISCHARGE COEFFICIENTS

DIMENSIONLESS PLOTTING — SCALE: 2.5" = 1.0

Solving for H_s and $\frac{h_a}{H_s}$ as outlined in Chapter II,

$$H_s = 19.05 \text{ feet, } \frac{h_a}{H_s} = 0.0106 \text{ and } \frac{u_s}{H_o} = 1.12.$$

The coordinates, obtained from Table 1, are tabulated in columns 1 and 2 of Table 22. Columns 3 and 4 show the same coordinates in terms of H_o . The latter, identified by the broken line, are plotted on Figure 39, using a 40 engineer's scale.

Superimposed on the same Figure is the actual shape of section, for which the radii and dimensions of Figure 38 have been divided by the quantity H_o .

A comparison of the actual and ideal overfall shapes now exists on Figure 39 with one coefficient known, that for the ideal case which is 3.95. The shapes in question should be plotted on transparent paper so that they can be compared directly, and to the same scale, with the stock shapes on Figures 19 through 27. When one is found which closely corresponds to the one in question, the coefficient of discharge for the actual shape is obtained from the stock shape. In case a good comparison is not available, a certain amount of judgment is necessary. Proficiency, however, is acquired with practice.

The shapes for the Example, Figure 39, bear a fair resemblance to stock shape 16D on Figure 22, although the actual and ideal shapes are reversed. Nevertheless, the differences in the two cases are about the same and it can be estimated that the actual coefficient should be very close to 3.98 as compared with 3.95 for the ideal shape.

With the actual coefficient for the dam in question estab-

DETAILS OF SPILLWAY FOR EXAMPLES 9 AND 10

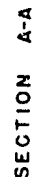


Table 22

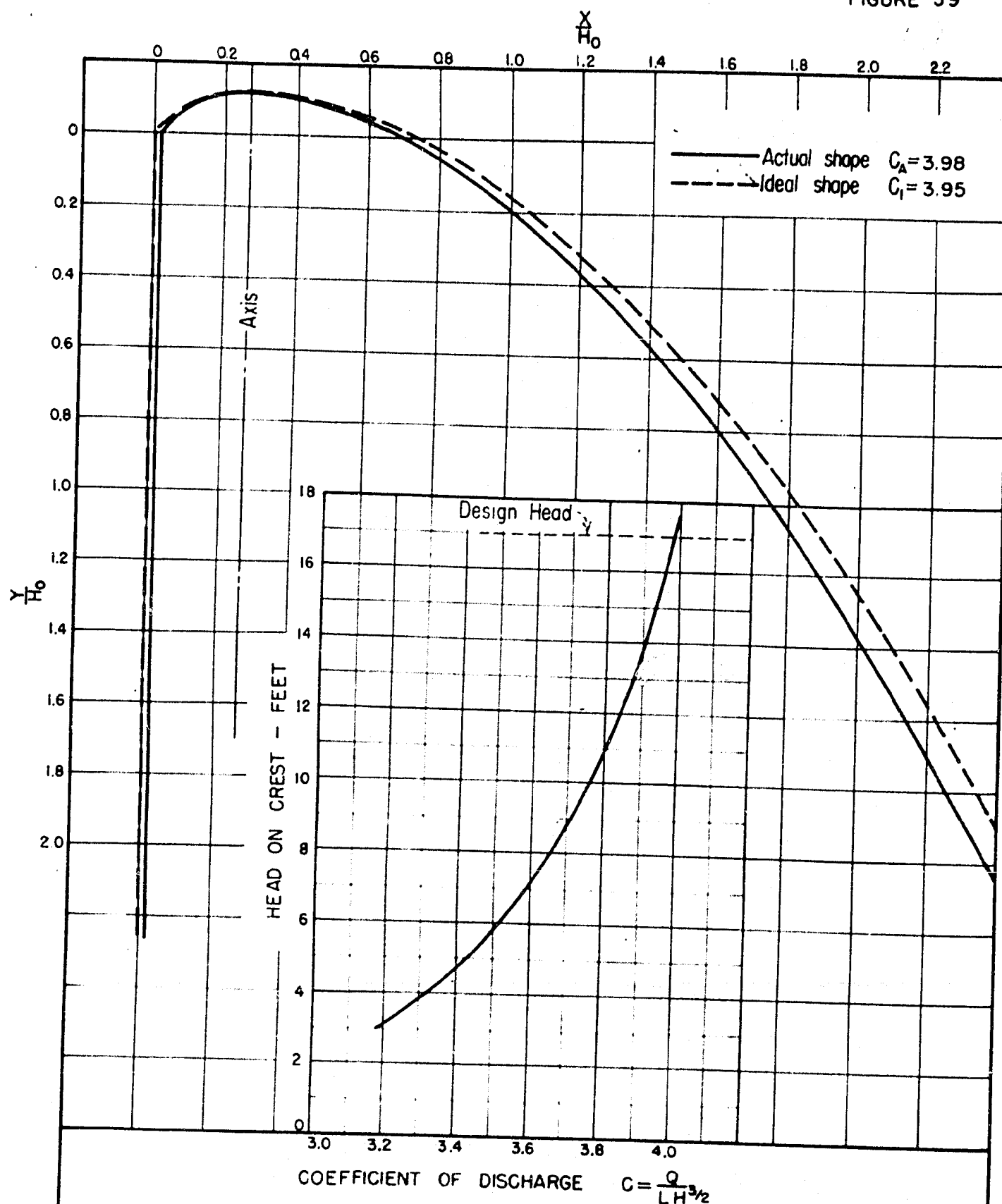
COORDINATES FOR IDEAL OVERFALL SHAPE (EXAMPLE 9)

$$\frac{h_a}{H_s} = 0.0106$$

$$\frac{H_a}{H_o} = 1.12$$

$\frac{X}{H_s}$	$\frac{Y}{H_s}$	$\frac{X}{H_o}$	$\frac{Y}{H_o}$
0	0	0	0
0.05	+0.0559	0.056	+0.063
.10	.0836	.112	.094
.20	.1069	.224	.120
.30	.1064	.336	.119
.40	.0923	.448	.103
.50	.066	.560	.074
.60	+0.028	.672	+0.031
.70	-0.021	.784	-0.024
.80	.078	.896	.087
.90	.143	1.007	.160
1.0	.218	1.120	.244
1.2	.397	1.344	.445
1.4	.610	1.567	.683
1.6	.854	1.791	.957
1.8	1.136	2.015	1.272
2.0	1.455	2.240	1.630
2.2	1.802	2.462	2.020
2.4	2.183	2.683	2.445
2.6	-2.606	2.910	-2.920

FIGURE 39



DETERMINATION OF DISCHARGE COEFFICIENTS
 FOR IRREGULAR SECTION
 RESULTS OF EXAMPLE 9

lished at 3.98, and a designed head of 17.0 feet on the crest, the coefficients for other heads are obtained from the curve on Figure 10B. The completed head-coefficient curve for the spillway section in Example 9 is shown plotted on Figure 39.

metrical operation. Experience has shown, both from models and by the design department and checked in the laboratory for symmetrical operation, drops and chutes are carefully proportioned of our hydraulic structures such as dam spillways, outlet works, absent on the majority of gates on Bureau projects. Almost all indicators installed on them. Unfortunately this equipment is however, it is necessary that these gates have accurate position in order to use radial and slide gates as metering devices.

cont. Gates can be computed with an estimated accuracy of 1 to 5 per cent section, rating curves for flow at all positions of the all of which should be available for practically any dam or non-gates. Given the designed head, approach conditions, dimensions of the overall shape, and overall measurements of the gates, measurement of water flowing under radial or vertical slide based on the results of model studies, is proposed for accurate as a contribution to this quest for information a method, in the system.

Flow is desired on the main streams. In fact, the field is design of metering the flow at practically all control devices main canals and irrigation laterals but better regulation of for methods of water measurement. Not only is this true on with each succeeding year, there is an increasing demand for better water becoming more valuable in the western states

Introduction

V. RATING CURVES FOR FLOW UNDER RADIAL AND VERTICAL SLIDE GATES

field structures, that in most cases unsymmetrical flow greatly intensifies the maintenance problem. The structures can be designed for unsymmetrical flow conditions but this requires additional training walls or devices which increase initial costs. Directions are continually sent from laboratory to the field instructing operators to regulate their gates to produce symmetrical flow patterns whenever possible. Yet without indicators on the gates it is impossible for even a good operator to obtain symmetry of flow.

It would not be a difficult task to install inexpensive indicators on existing gates, and position indicators should certainly be considered essential equipment on future installations. Now with the possibility of utilizing gate structures as metering stations, as well as regulating devices, position indicators take on additional importance.

Sources of Information

The late Mr. Robert E. Horton developed a formula, by means of model and field tests, for the measurement of flow under radial gates where a flat floor was involved and a hydraulic jump occurred downstream.¹⁴ The work constitutes one condition of flow and the paper ably covers this case. Figure 40A shows the gate arrangement diagrammatically.

Other arrangements requiring investigation are also shown on Figure 40. These differ principally in the approach to the

¹⁴Horton, Robert E., "Discharge coefficients for Tainter Gates", Engineering News-Record, January 4, 1934.

gate and the shape of section ^{of gate} downstream. This chapter will deal with the development of a flow equation for the gate arrangements shown in Figure 40B, D, E, and F. Arrangements B and D are typical of gate sections for earth dam spillways and sluiceways, while E and F are for concrete dam spillways using radial and slide gates, respectively, where considerable approach depth is present. Arrangement 40C shows a radial gate located downstream from the crest of a spillway. This type of setting remains to be investigated. It will not be treated here but was placed on Figure 40 as a reminder.

Referring to Figure 40G and H, the discharge in a stream tube under a gate is

$$dq = V da = VL dh$$

As the effective velocity is increased for angles of the gate less than 90 degrees, the arbitrary expression for velocity has been chosen,

$$V = K \frac{\sqrt{2gh}}{\sin \theta} \quad (\text{with } \theta \text{ limited between } 45 \text{ and } 90 \text{ degrees})$$

where $K = f \left(\frac{D}{H}, \theta \text{ and } C_o \right)$.

The expression for the total discharge under the gate is then

$$Q = K \frac{\sqrt{2g} L}{\sin \theta} \int_{H-D}^H h^{1/2} dh = \frac{2K \sqrt{2g} L}{3 \sin \theta} \left[H^{3/2} - (H-D)^{3/2} \right] \quad (1)$$

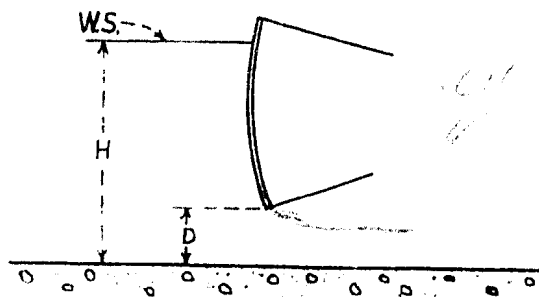
Letting $\frac{D}{H} = N$ and replacing $\frac{2}{3} K$ by η , $\eta = \frac{2}{3} K$

$$Q = \eta \left[1 - (1-N)^{3/2} \right] \frac{\sqrt{2g} L H^{3/2}}{\sin \theta} \quad (2)$$

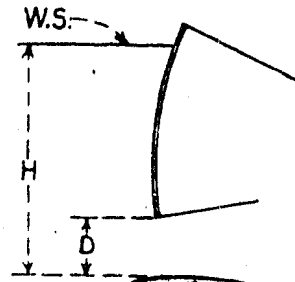
Replacing $\left[1 - (1-N)^{3/2} \right]$ by the letter F ,

$$Q = \eta \frac{F}{\sin \theta} \sqrt{2g} L H^{3/2} \quad (3)$$

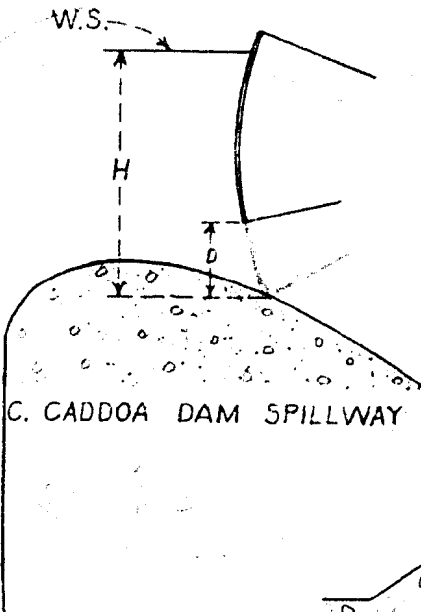
which is the equation for flow at partial gate openings.



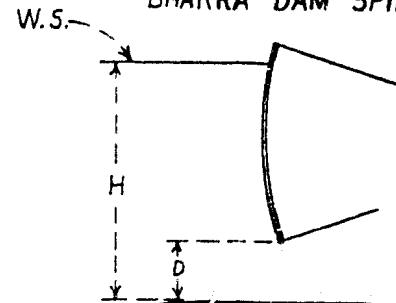
A. HORTON'S EXPERIMENTS



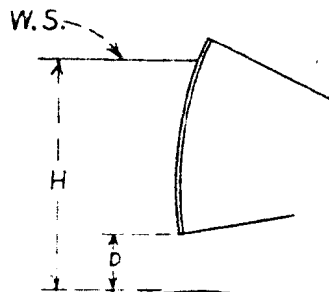
B. BULL LAKE DAM SPILLWAY
CASCADE DAM SPILLWAY
BHAKRA DAM SPILLWAY



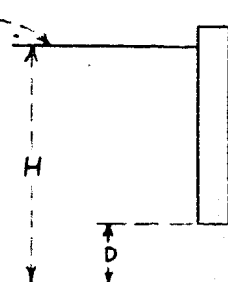
C. CADDOA DAM SPILLWAY



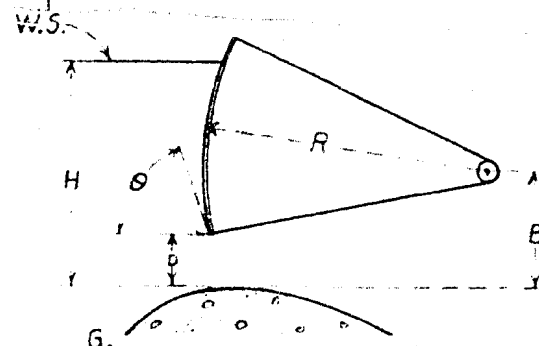
D. IMPERIAL DAM SLUICWAY
ENDERS DAM SPILLWAY



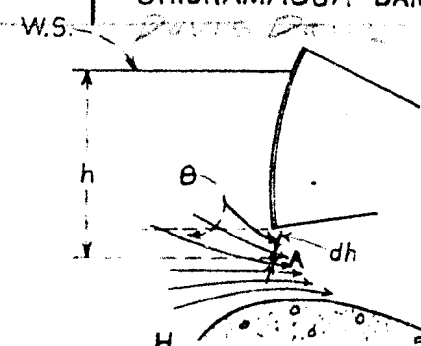
E. STEWART MOUNTAIN DAM
ROSS DAM



F. PICKWICK DAM
CHICKAMAUGA DAM



G.



H.

TYPES OF RADIAL AND VERTICAL GATE INSTALLATIONS

From the experimental data obtained from the spillway models listed on 403, D, E, and F, it was possible to express the coefficient, γ , in terms of the free flow coefficient, C_o , for the designed head, H_o , as follows:

$$\gamma = \frac{1}{\sqrt{2g}} (1.537 C_o - 1.203 \frac{C_o^2}{\sqrt{2g}}) \quad (4)$$

To compute discharges under gates by the above method, it is first necessary to know the value of the coefficient of discharge for free flow at the designed head for the spillway section under examination. The development of equation (3) is based on this coefficient which can be obtained for practically any shape of section by the method outlined in the preceding chapters.

Values of F are tabulated for corresponding values of N , up to 0.80, on Table 23. Equation (3) is limited to values of N up to approximately 0.80 as the lip of the gate leaves the water surface for values of N approaching this value.

The angle, Θ , in equation (3) is measured with respect to the horizontal and is limited to angles from 45 to 90 degrees. When the angle of the gate moderately exceeds 90 degrees, use $\sin \Theta = 1.0$.

The development of equation (3) may not bear critical examination. Many variables are involved; some have been eliminated from the immediate problem by basing the computation on the free flow coefficient, C_o . The effects of the remaining variable factors have been lumped into the coefficient γ .

Equation (3) has the advantage that it can be applied to most any radial or open slide gate problem. The result will be

Table 23

VALUES OF F AND N

N	F	:	N	F	:	N	F
0	0	:	.30	.414	:	.60	.747
.01	.015	:	.31	.427	:	.61	.756
.02	.030	:	.32	.439	:	.62	.766
.03	.045	:	.33	.452	:	.63	.775
.04	.059	:	.34	.464	:	.64	.784
.05	.074	:	.35	.476	:	.65	.793
.06	.089	:	.36	.488	:	.66	.802
.07	.103	:	.37	.500	:	.67	.810
.08	.113	:	.38	.512	:	.68	.819
.09	.132	:	.39	.524	:	.69	.827
.10	.146	:	.40	.535	:	.70	.836
.11	.160	:	.41	.547	:	.71	.844
.12	.174	:	.42	.558	:	.72	.852
.13	.188	:	.43	.570	:	.73	.860
.14	.202	:	.44	.581	:	.74	.867
.15	.216	:	.45	.592	:	.75	.875
.16	.230	:	.46	.603	:	.76	.882
.17	.244	:	.47	.614	:	.77	.890
.18	.258	:	.48	.625	:	.78	.897
.19	.271	:	.49	.636	:	.79	.904
.20	.284	:	.50	.646	:	.80	.911
.21	.298	:	.51	.657	:		
.22	.311	:	.52	.668	:		
.23	.324	:	.53	.678	:		
.24	.337	:	.54	.688	:		
.25	.350	:	.55	.698	:		
.26	.363	:	.56	.708	:		
.27	.376	:	.57	.718	:		
.28	.389	:	.58	.728	:		
.29	.402	:	.59	.738	:		

Additional values of F for values
of $N > .80$;
 $F = 1 - (1 - N)^{1.5}$

N	$(1 - N)^{1.5}$	F	
.81	.083	.917	1.000 -.083 ----- .917
.82	.076	.924	1.000 -.076 ----- .924
.83	.070	.930	
.84	.064	.936	
.85	.058	.942	
.86	.052	.948	
.87	.046	.954	
.88	.041	.959	
.89	.036	.964	
.90	.032	.968	
.91	.027	.973	
.92	.023	.977	
.93	.019	.981	
.94	.015	.985	
.95	.011	.989	
.96	.008	.992	
.97	.005	.995	
.98	.003	.997	
.99	.001	.999	

well within the accuracy with which the operator can set the gate opening and observe the reservoir head in the field. With the accumulation of additional experimental information the above procedure may be, at a later date, subject to a more rigorous analysis.

Determination of Discharge Curves
for Partial Gate Openings

Example 10

With the free-flow, head-coefficient curve determined in Example 9, for the spillway on Figure 38, construct a family of curves for flow under one gate, plotting reservoir elevation against discharge for gate openings at intervals of two feet. The free flow coefficient $C_0 = 3.98$, and the length of gate $L = 21.0$ feet.

From equation (4),

$$\eta = \frac{1}{\sqrt{2g}} \left(1.537 \times 3.98 - \frac{1.208 (3.98)^2}{\sqrt{2g}} \right) = 0.465.$$

From three to five heads are substituted in equation (3) for each two-foot gate interval and the computation has been tabulated in Table 24.

One limitation needs surveillance; that the computation be discontinued when the lip of the gate leaves the water surface, as equation (3) is not valid for free flow. The point at which free flow begins can be determined approximately by reference to Tables 1 and 9.

Entering Table 1 with the value $\frac{h_g}{H_g} = 0.0106$ (determined in Example 9), the axis, or high point of the section, occurs at $\frac{x}{H_g} = 0.25$. Entering Table 9 with the latter value and

Table 24

SPILLWAY DISCHARGE FOR ONE GATE--COMPUTATIONS FOR EXAMPLE 10

$C_o = 3.98$

$L = 21.0'$

$\mu = 0.465$

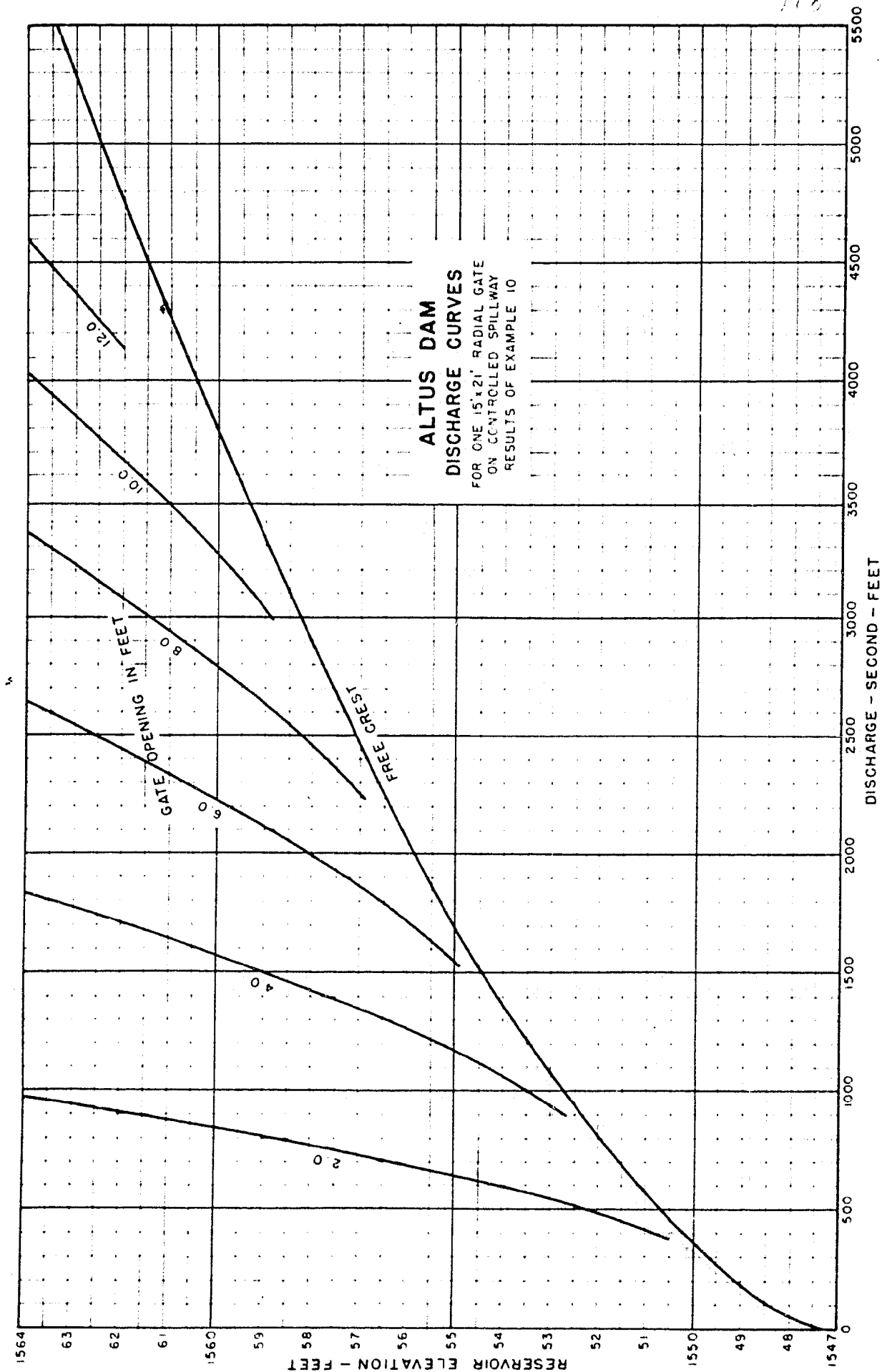
D Feet	H Feet	N	F	ϕ degrees	$\sin \phi$	Q c.f.s.	Reservoir elevation
2	4	0.500	0.646	73.5	0.960	422	1551
	7	.286	.397			600	1554
	10	.200	.284			732	1557
	14	.143	.208			886	1561
	17	.118	.171			978	1564
4	6	0.667	0.808	79.5	0.983	946	1553
	10	.400	.535			1350	1557
	14	.286	.397			1650	1561
	17	.235	.330			1840	1564
6	8	0.750	0.875	85.5	0.994	1560	1555
	11	.545	.693			2000	1558
	14	.429	.569			2340	1561
	17	.353	.480			2650	1564
8	10	0.800	0.911	< 90	1.00	2260	1557
	12	.667	.808			2630	1559
	14	.571	.719			2940	1561
	17	.470	.614			3370	1564
10	12	0.833	0.932	< 90	1.00	3030	1559
	14	.714	.847			3470	1561
	16	.625	.770			3860	1563
	17	.588	.736			4030	1564
12	13	0.922	0.978	< 90	1.00	-----	-----
	15	0.800	.911			4140	1562
	17	0.705	.840			4600	1564

$\frac{h_a}{H_s} = 0.0106$, $\frac{Y}{H_s} = 0.766$. Converting this to $\frac{Y}{H_o}$, or $\frac{D}{H_o}$ to agree with the symbols for the problem at hand,

$$\frac{D}{H_o} \text{ or } N = 0.766 \times 1.12 = 0.86.$$

The computations should be discontinued before this value of N is reached (see Table 24). The completed family of discharge curves is included as Figure 41.

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