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UNITED STATES
DEPARTMENT OF THE INTERIOR
BUREAU OF RECLAMATION

AN INVESTIGATION OF THE HYDRAULICS OF THE CHUTE AT STATION 107+50

SOUTH CANAL UNCOMPAHGRE PROJECT

Hydraulic Laboratory Report Hyd-203

ENGINEERING AND GEOLOGICAL
CONTROL AND RESEARCH DIVISION

HYD 203

BRANCH OF DESIGN AND CONSTRUCTION
DENVER, COLORADO

MAY 10, 1946

UNITED STATES
DEPARTMENT OF THE INTERIOR
BUREAU OF RECLAMATION

Branch of Design and Construction
Engineering and Geological Control
and Research Division
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Laboratory Report No. 203
Hydraulic Laboratory
Compiled by: E. W. Lane and
C. W. Thomas
Reviewed by: J. E. Warnock

Subject: An investigation of the hydraulics of the chute at Station 107+50--South Canal Uncompahgre Project.

INTRODUCTION AND SUMMARY

Location, scope, and purpose of investigations. During the summer of 1931, observations were made on a chute in the South Canal of the Uncompahgre Project, of the Bureau of Reclamation, near Montrose, Colorado. These observations were taken at the same time that model studies of the Boulder Dam Spillway were being made on the location. Measurements were made, (1) to determine the roughness coefficient of the canal lining, (2) the location of the hydraulic jump at the bottom of the chute for different discharges, (3) air content in the high-velocity flow, (4) velocity distribution in the chute, and downstream from the chute, (5) entrance conditions in the transition between the earth section of canal and the concrete-lined section at the top of the chute, and (6) the curve described by free-falling water entering the chute from the spillway model.

The studies were made to determine the hydraulic properties of high-velocity flow and compare these observed results with those obtained by commonly used methods of computation.

Summary. The Uncompahgre Project has been in operation over twenty years, and therefore, much maintenance has been necessary. Between the stations observed, many patches have been made in the canal lining. For this reason a uniform grade or cross-section of canal was not obtainable and therefore, the results given have factors in them which could not be evaluated. While the accuracy was the best attainable, the results were not of laboratory caliber. The hydraulic principles of the canal were of more interest than the mathematics of these principles.

STUDIES IN THE CHUTE

Description of the chute. A profile of the canal is shown in Figure 1. The canal converges into a uniform cross-section, and from earth to concrete at Station 107+50, and flows on a fairly level grade to the top of the chute at Station 109+00. From Station 109+00 to Station 110+00 the canal drops uniformly, approximately 28½ feet. Sixteen feet in elevation is lost in the next 250 feet of length, and from there on the grade is flat (Figure 2A). The intake for the Boulder Dam Spillway model, at Station 108+22, was a turnout controlled by two 48-inch hand-operated gates. The exit of the model was at Station 111+97½. Here the water dropped 12 feet into the canal (Figure 12C).

Quantity measurements. A gaging station was established in the earth section at Station 106+50. Quantities were determined from a recording gage located approximately 1-3/4 miles upstream. A deduction of one percent was made for losses in this distance. The rating curve shown in Figure 3 was plotted from the data obtained.

An accurate profile of the cross-sections, at critical points, was taken with no flow in the canal. At these critical points the mean elevation of the water surface was determined by extending a level rod to the water surface at one-foot intervals across the canal, and reading the rod with a level. Curves were plotted, using the quantity of flow as the abscissas and elevations as ordinates. Examples of these curves are shown on Figure 4.

Roughness coefficients. Values of the roughness coefficient n, in both the Kutter-Chezy and the Manning Formulas, were computed for the following reaches of the canal; Stations 109+00 to 110+00, 110+00 to 112+50, 112+50 to 113+60, 113+60 to 115+00, 115+00 to 116+00. It was found that small errors in the area led to large discrepancies in the velocity heads in such short reaches, and introduced an error into the computations that made the results very inconsistent. Accordingly, values of n were calculated from Station 109+00, to points downstream where the water was below critical depth, by using weighted overall average values of the hydraulic radius and velocity. The error introduced was negligible. The results obtained are shown on Table 1.

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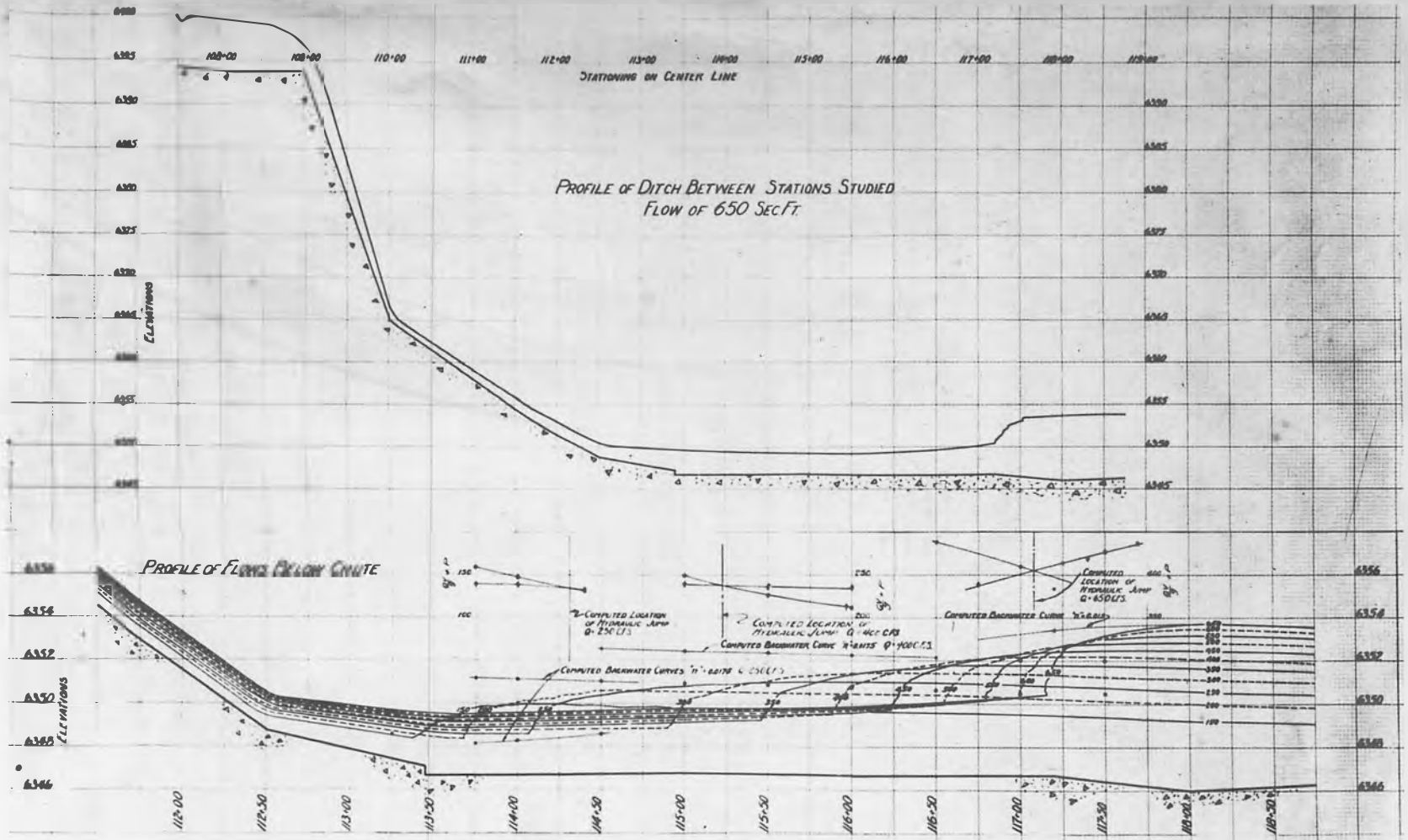

Figure 1

A - Chute and hydraulic laboratory.

B. Discharge approximately 500 second-feet

C - Discharge approximately 500 Second-feet

Uncompahgre Project - South Canal Chute at Station 107+50

Figure 3

South Canal Discharge Curve

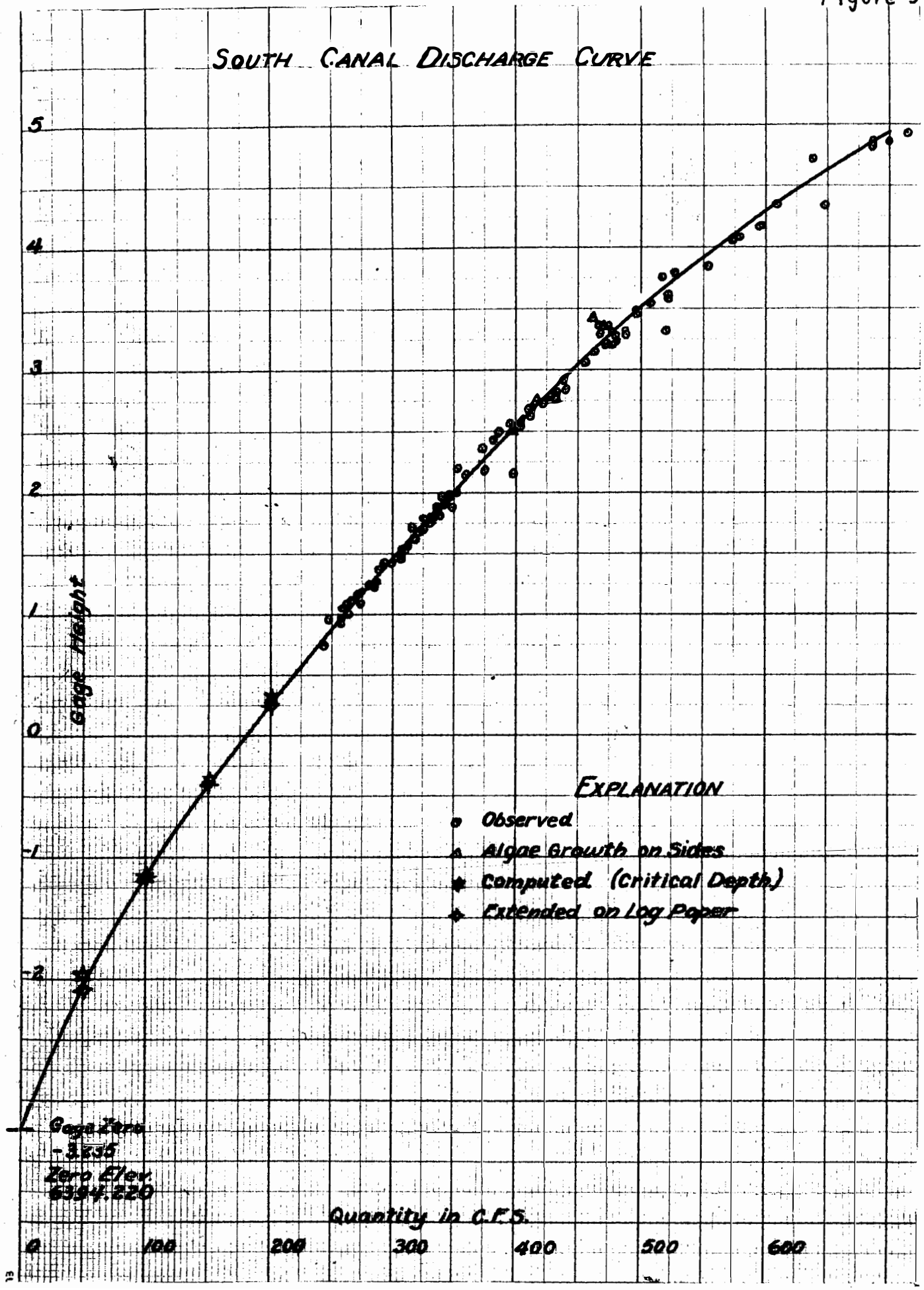

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Figure 4

WATER SURFACE ELEVATION CURVES

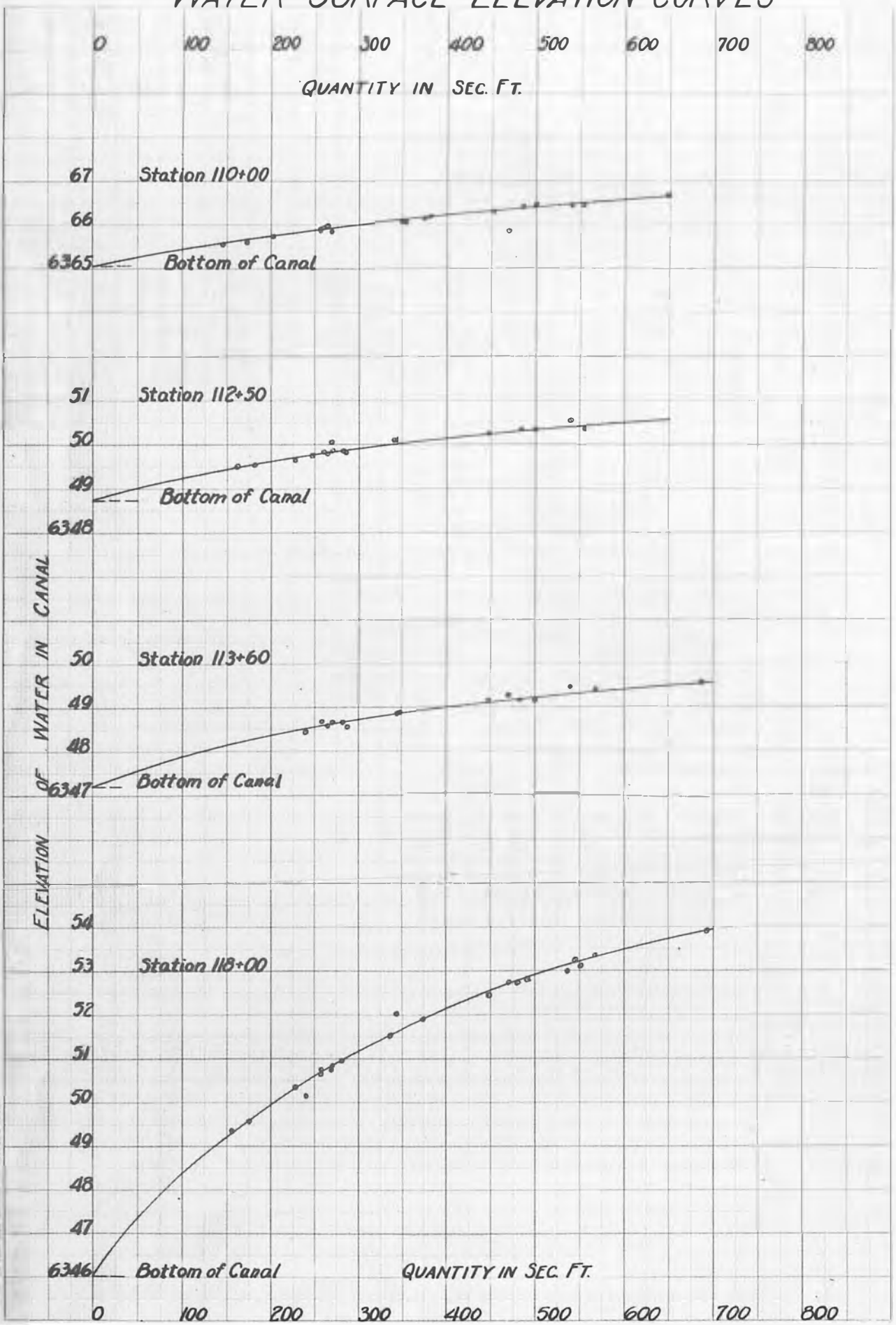

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TABLE 1

COMPUTED VALUES OF n AT STATION 109+00

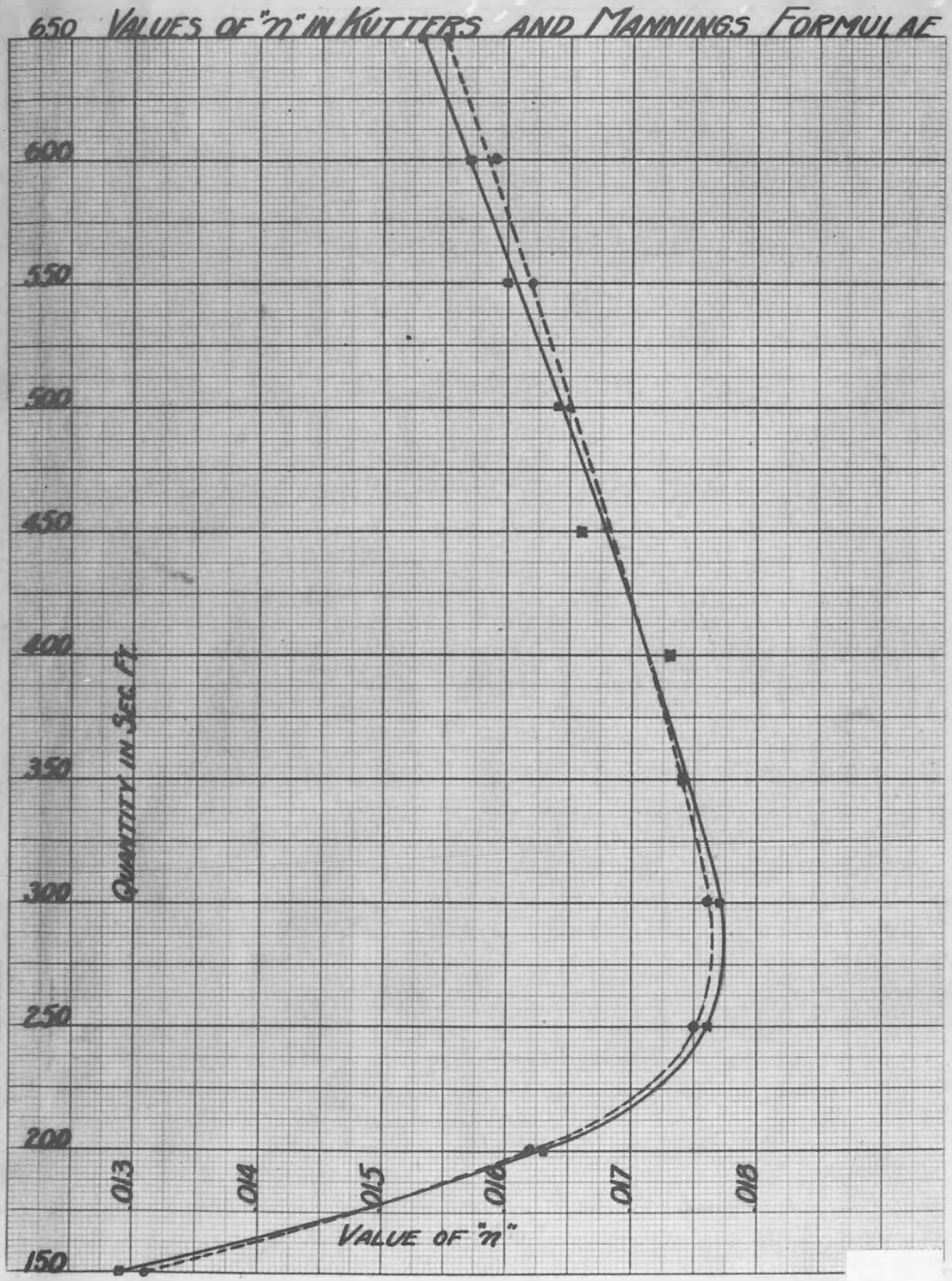
Quantity	Length of Section	Slope	R	Velocity in Ft./Sec.	Chezy "C"	Manning n	Kutter n
150	350	0.1297	0.596	29.31	105.5	0.0129	0.0131
200	460	0.1016	0.914	27.42	90.0	0.0163	0.0162
250	460	0.1055	1.047	28.27	85.0	0.0176	0.0175
300	600	0.0780	1.204	26.57	86.8	0.0177	0.0176
350	600	0.0781	1.282	28.15	89.0	0.0174	0.0174
400	600	0.0783	1.348	29.43	90.6	0.0173	0.0173
450	700	0.0669	1.372	28.70	94.7	0.0166	0.0166
500	700	0.0670	1.426	29.73	96.2	0.0164	0.0165
550	700	0.0671	1.472	31.05	98.8	0.0160	0.0162
600	700	0.0672	1.510	32.28	101.3	0.0157	0.0159
650	700	0.0673	1.537	33.45	104.1	0.0153	0.0155

The values of n obtained in both cases, plotted on Figure 5, vary uniformly. Flows up to 250 second-feet were not as accurately determined as the higher flows, and the discrepancies for the 150- and 200-second-foot flows are therefore greater than for the other discharges. The decreasing values of n from the 250-second-foot flow to the higher discharges is explainable by the fact that the bottom and the lower portion of the side linings of the canal were eroded more than the lining higher on the sides (Figures 6A and B and 7C). The average flow in the canal was around 350 to 400 second-feet, hence the erosion was low in the canal with little or no erosion higher on the sides. This variation of erosion is further discussed in Appendix I.

The hydraulic jump. The water at the bottom of the chute is above critical velocity. As it moves downstream the velocity decreases until it finally arrives at the critical point, where the phenomena, known as the "hydraulic jump," occurs (Figures 7A and B). The principles of the hydraulic jump are described by Julian Hinds in Engineering News-Record, Volume 85, No. 22, November 25, 1920, Page 1034. Mr. Hinds arrives at his conclusions by the laws of conservation of energy and of linear momentum (Appendix II). Observations were made to determine the location of the jump for different discharges. The location of the jump was plotted against discharge and a very complete curve established (Figure 8). In this figure a series of points, running nearly parallel to the curve established, shows the jump as being upstream from the plotted curve. These points represent observations taken after the water had been at a very low discharge for a considerable time and then raised to a higher discharge. During the period of low discharge an algae growth appeared on the lining above the water line. This growth probably influenced the velocity of the water to a considerable extent when it was again raised to the higher discharges.

By using the equation established by Mr. Hinds, $\frac{Q}{g} V_1 + P_1 = \frac{Q}{g} V_2 + P_2$, the theoretical position of the jump may be computed. Several unknown factors are encountered at this point. The first is the selection of the correct value of V at the entrance of the jump. The average velocity at the cross-section may not be absolutely correct. On the other hand it is not practical

Figure 5



EUGENE DIETZGEN CO., CHICAGO-NEW YORK NO. 346 B

A - Erosion at station 107 + 50

B - Erosion at Station 108+22

C - Algae Growth near entrance of chute
No flow for three days

D - Algae growth near lower end of chute
No flow for three days

Erosion and Algae Growth on Concrete Lining

Uncompahgre Project - South Canal - Chute at Station 107+50

Figure 6.

A - The hydraulic jump - Flow approximately 500 second-feet.

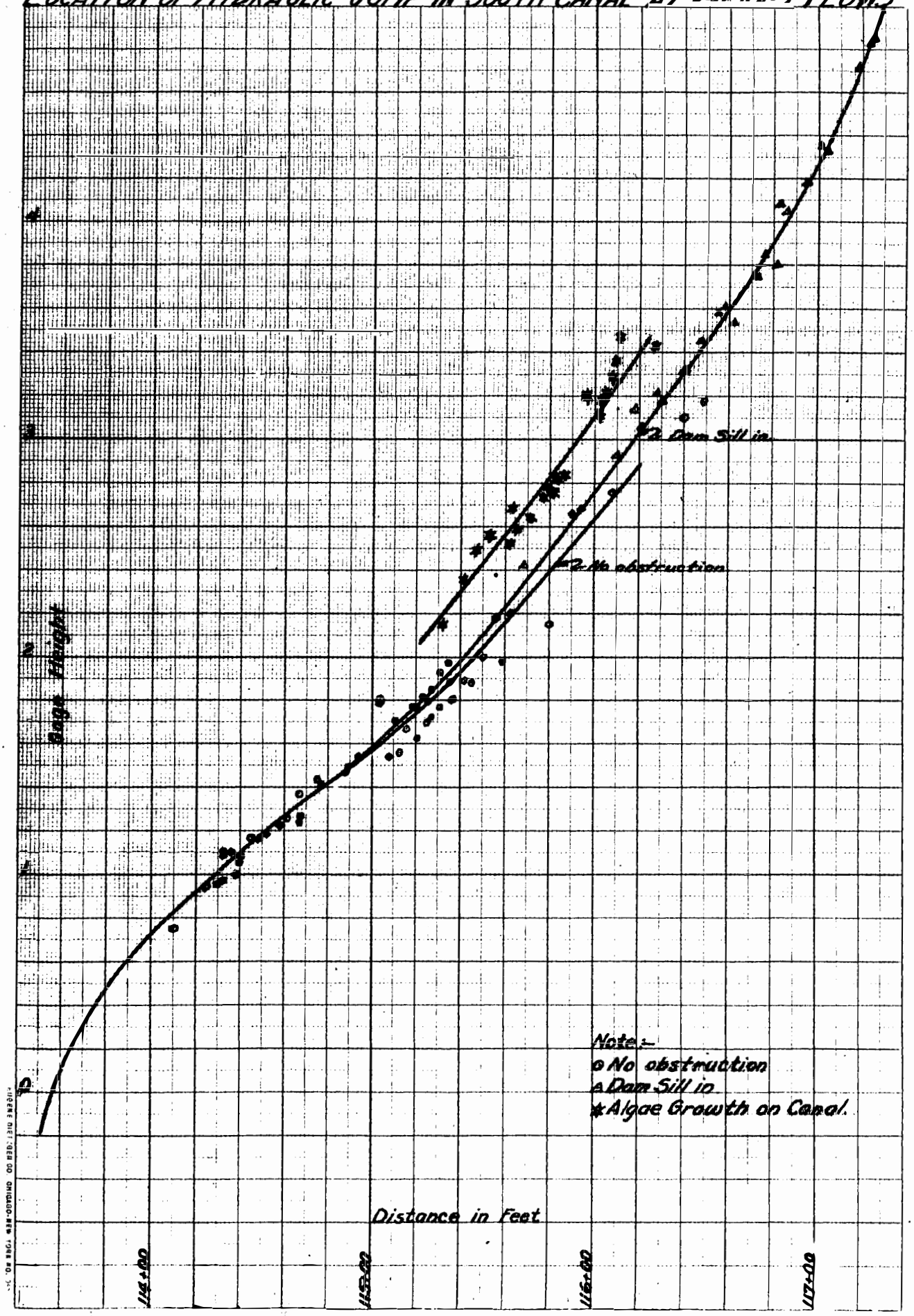
B - The Hydraulic Jump
Flow approximately 500 second-feet.

C - Erosion and Algae near station 113/00

Uncompahgre Project - South Canal Chute at Station 107/50

Figure 8

LOCATION OF HYDRAULIC JUMP IN SOUTH CANAL AT VARIOUS FLOWS



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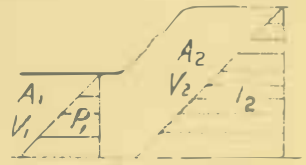
to make an infinite number of velocity measurements at a particular section. Since the purpose in computing the location of the jump was as a rough check upon the adaptability of the formula, the mean velocity entering and leaving the jump was used. This velocity was obtained by computing a backwater curve for the flow, both above and below critical velocity. The curves were computed from the closest gaging station to a point beyond the location of the jump. The values of V were computed from the formula, $Q = AV$. Table 2 shows the computations for the location of the jump. Since the actual jump covers considerable length of canal, it is questionable as to what point on the jump the established point represents. For ease in observation and uniformity this point was considered as the high point of the jump, or a point from two to three feet downstream from the central point, formed due to the higher velocity of the entering jet in the center of the canal. The observed location and the computed location of the jump check very closely (Figure 1). Therefore it may be safely concluded that for an established value of n, the location of the hydraulic jump in an open channel may be computed within practical limits of accuracy.

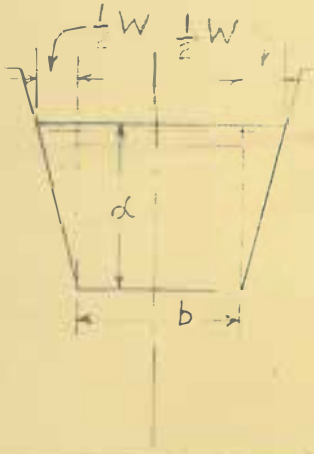
Air content in flow. Observations were made to determine the amount of air entrained in the water as it flowed down the chute. Two methods were employed. The first was to establish the velocity by the method of $Q = AV$ and the second was by the use of an air-content measuring device designed by Mr. D. C. McConaughy of the Denver Office of the Bureau of Reclamation. The McConaughy device works on the principle of measuring the amount of air left in an air-tight tank by a given amount of water passing through the tank. The entering air-water mixture came from the canal through a pitot entrance and entered the top of the tank. The water exit was at the bottom. The water settled in the tank and left all entrained air within. The first method consisted of finding V by the cross-section method and checking this velocity by pitot tube measurement (Figure 9). Pitot tube observations made at Station 10+00 checked the velocities obtained by the cross-section method. In both cases no appreciable air content was noted. However, in coming down the chute there was a froth on top of the jet which appeared to be entrained air. (Appendix I and Figure 2, Band C).

TABLE 2

COMPUTED LOCATION OF HYDRAULIC JUMP

Station	Elevation Bottom	Elevation W. S.	Water Depth	Width Bottom	Width Top	Area Sq. Ft.	Velocity Ft./Sec.	$\frac{QV}{G}$	W	Pressure $\frac{1}{2}d^2(b+1/3)$	$\frac{QV}{G}$ + Pressure	Explanation
Q = 250 Sec. Ft.												
113+75	6346.67	6348.15	1.48	8.15	9.51	13.06	19.15	149	1.36	10	159	
114+00	6346.68	6348.29	1.61	8.13	9.62	14.28	17.51	136	1.49	11	147	
114+40	6346.71	6348.56	1.85	8.15	9.85	16.65	15.02	117	1.70	15	132	
113+75	6346.67	6351.18	4.51	8.15	12.59	46.75	5.35	42	4.14	97	139	
114+00	6346.68	6351.14	4.46	8.13	12.25	45.45	5.50	43	4.12	95	138	
114+40	6346.71	6351.08	4.37	8.15	12.17	44.40	5.63	44	4.02	91	135	
Q = 400 Sec. Ft.												
115+00	6346.76	6349.24	2.48	8.10	10.23	22.70	17.60	221	2.13	28	249	
115+50	6346.70	6349.50	2.80	8.10	10.51	26.05	15.35	191	2.41	35	226	
116+00	6346.66	6349.77	3.11	8.10	10.78	29.36	13.62	169	2.68	43	212	
115+00	6346.76	6352.42	5.66	8.10	12.97	59.70	6.70	83	4.87	156	239	
115+50	6346.70	6352.35	5.65	8.10	12.96	59.50	6.72	83	4.86	154	237	
116+00	6346.66	6352.26	5.60	8.10	12.92	58.90	6.79	84	4.82	152	236	
Q = 650 Sec. Ft.												
116+50	6346.66	6350.07	3.41	8.19	11.90	34.3	18.95	383	3.71	55	438	
117+00	6346.68	6350.32	3.64	8.35	12.47	37.9	17.15	346	4.12	64	410	
117+20	6346.70	6350.45	3.75	8.35	12.60	41.6	15.62	315	4.25	69	384	
117+25	6346.65	6350.48	3.83	8.17	11.70	38.1	17.06	345	3.53	69	414	
116+75	6346.68	6353.00	6.32	8.27	15.35	74.6	8.72	176	7.08	213	389	
117+40	6346.52	6353.55	7.03	8.17	14.65	80.3	8.09	163	6.48	255	418	
117+50	6346.43	6353.60	7.17	8.25	14.89	83.0	7.83	158	6.64	269	427	
117+69	6346.27	6353.65	7.38	8.16	15.32	86.6	7.50	151	7.16	286	437	





A - The Pitot Tube Setup

B - Pitot Tube at Station 110 + 00

Uncompahgre Project South Canal Chute at Station 107 + 50

Velocity distribution. Observations were made to determine the cross-sectional and surface distribution of the velocities in the shooting and streaming jets. The cross-section distribution in the shooting jet was determined by pitot tube observations and in the streaming jet by current meter. The surface distribution was accomplished by timing wood floats for a 50-foot distance. These results are plotted in Figures 10 and 11. The velocities are very even after leaving the jump. This shows that the jump not only dissipates energy but is a means of equalizing the energy in various parts of the jet.

OTHER STUDIES

Flow conditions in the transition section. There is a very distinct drawdown of the water surface at the entrance to the concrete lining (Figure 12A). Studies were made at the entrance, for the determination of entrance coefficients. Table 3 lists coefficients of velocity found by assuming the control point to be at the bottom of the drawdown curve, at Station 107+57½, and also assuming the control to be at Station 108+22, far enough downstream to be beyond the influence of the drawdown. These are plotted on Figure 13. The coefficients at Station 107+57½ were 10 percent greater than those at Station 108+22. This indicates a loss of head between the two stations far in excess of normal conditions. As the water comes out of the drawdown there is a conversion of velocity head to elevation head which resembles a hydraulic jump. This is not a true jump because the entrance jet is above critical depth, but the phenomena evidently causes a loss of head in the regaining of the elevation head.

Table 4 shows the results obtained when computations were made to determine the back pressure at the control point caused by the centrifugal force of the water. The effect is negligible in this case, and no further study was made in this connection.

Path of jet leaving model. The return of the water from the Boulder Dam Spillway model to the canal is shown in Figure 12C. It was desired to measure the path of this jet and to establish its equation. This was done by assuming the bottom of the return tunnel as the X axis and the vertical line through the end of the tunnel as the Y axis. Coordinates were then measured as shown in Figure 12B. These values are plotted and the equation of the curve established in Figure 14. The quantity used for the test was 112 second-feet which represented the maximum discharge over the spillway.

Figure 10

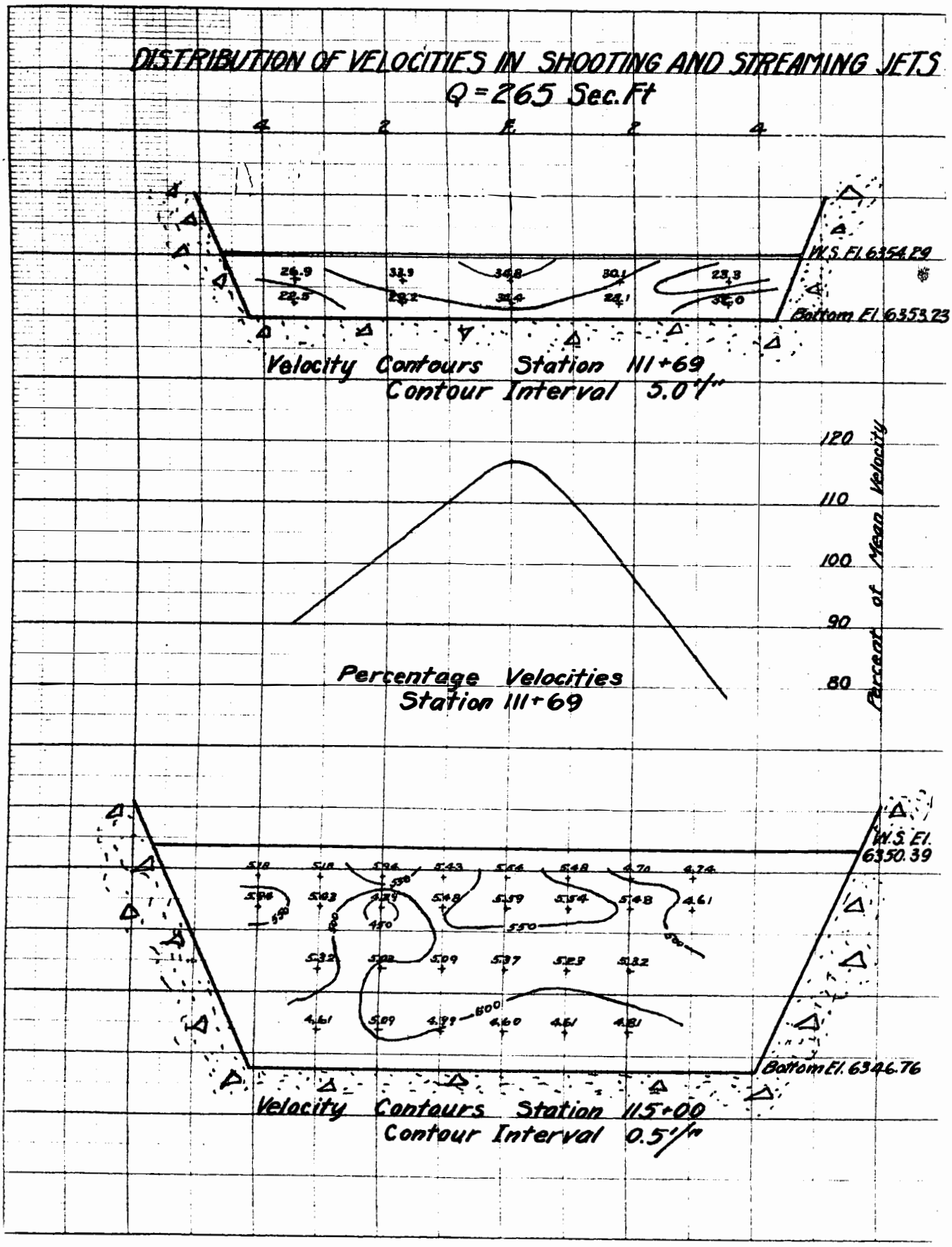


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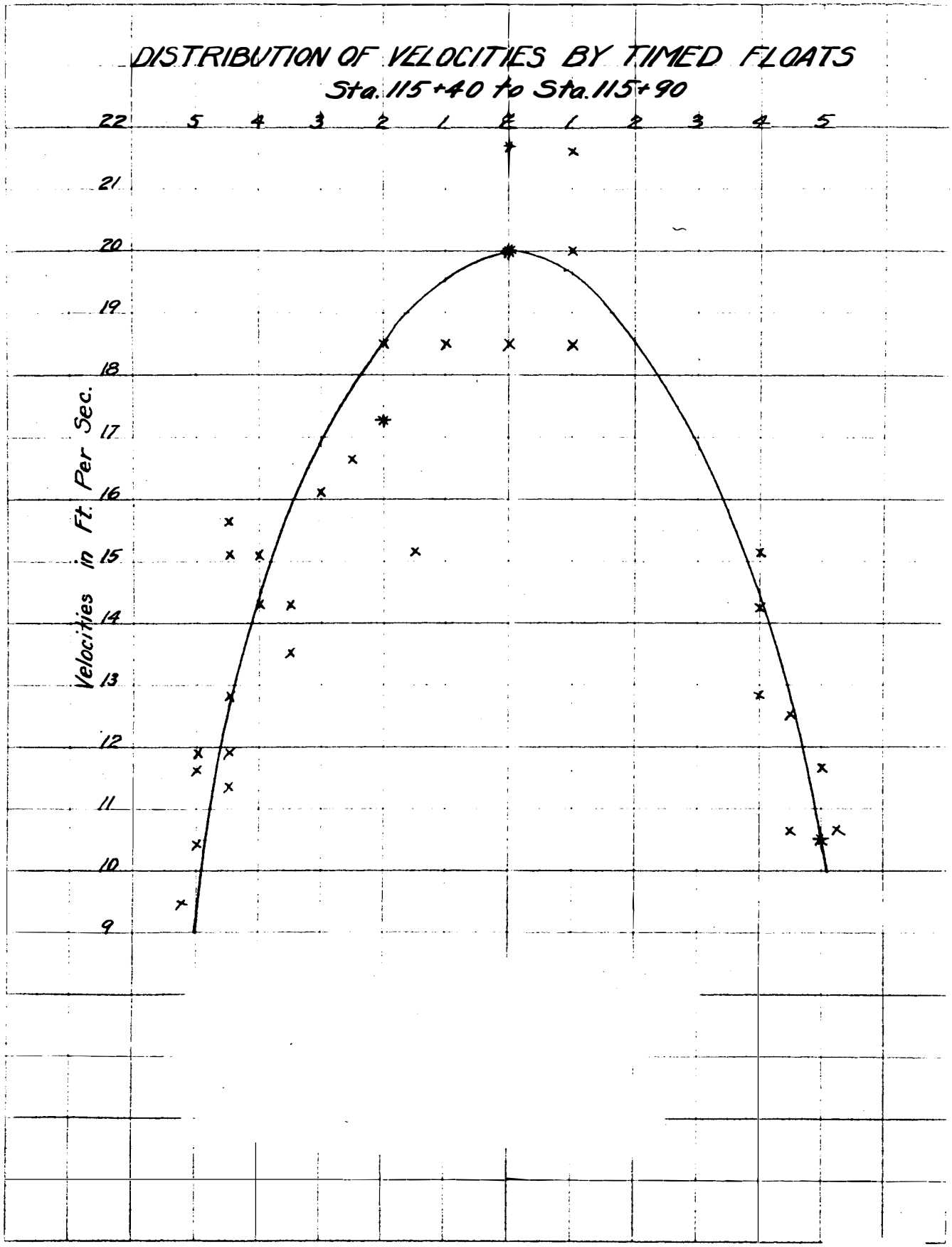
Figure 11



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A - Entrance to concrete lining

B - Method used to determine
path of water leaving model

C - Water leaving model
Discharge 112 second-feet

Figure 12

TABLE 3

COEFFICIENTS OF VELOCITY AT ENTRANCE TO CANAL LINING

CONTROL POINT AT STATION 107+57½

#1	#2	#3	#4	#5	#6	#7	#8	#9	#10	#11	#12
Quantity in Sec. Ft.	Elevation at 107+57½	Area at 107+57½	Vel. at 107+57½: V = Q/A	Elevation at Gage	Area at Gage	Vel. at Gage: V = Q/A	Velocity Head	Drop from gage to 107+57½ #5 - #2	Total Head #9 + #8	Computed Velocity 107+57½	K #4 / #11
651	6399.27	48.8	13.34	6402.08	316.02	2.06	.066	2.81	2.876	13.59	.982
560	6399.03	46.1	12.15	6401.44	275.92	2.03	.064	2.41	2.474	12.61	.964
505	6398.73	42.4	11.91	6401.00	251.00	2.01	.063	2.27	2.333	12.24	.973
450	6398.48	39.7	11.33	6400.50	223.20	2.02	.057	2.02	2.077	11.55	.981
375	6397.65	30.8	12.18	6399.75	198.67	1.89	.056	2.10	2.156	11.76	1.035
350	6397.56	29.8	11.74	6399.45	183.31	1.91	.056	1.89	1.946	11.18	1.051
Control Point at Station 107+57½											
651	6400.37	71.5	9.07	6402.08	316.02	2.06	.066	1.71	1.776	10.68	.849
560	6400.03	67.0	8.28	6401.44	275.92	2.03	.064	1.41	1.474	9.64	.850
505	6399.74	63.0	8.02	6401.00	251.00	2.01	.063	1.26	1.323	9.22	.870
450	6399.31	57.0	7.90	6400.50	223.20	2.02	.063	1.19	1.253	8.98	.880
375	6398.67	49.0	7.65	6399.85	198.67	1.89	.056	1.08	1.136	8.55	.895
350	6398.44	46.5	7.86	6399.45	183.31	1.91	.057	1.01	1.067	8.29	.948

Figure 13

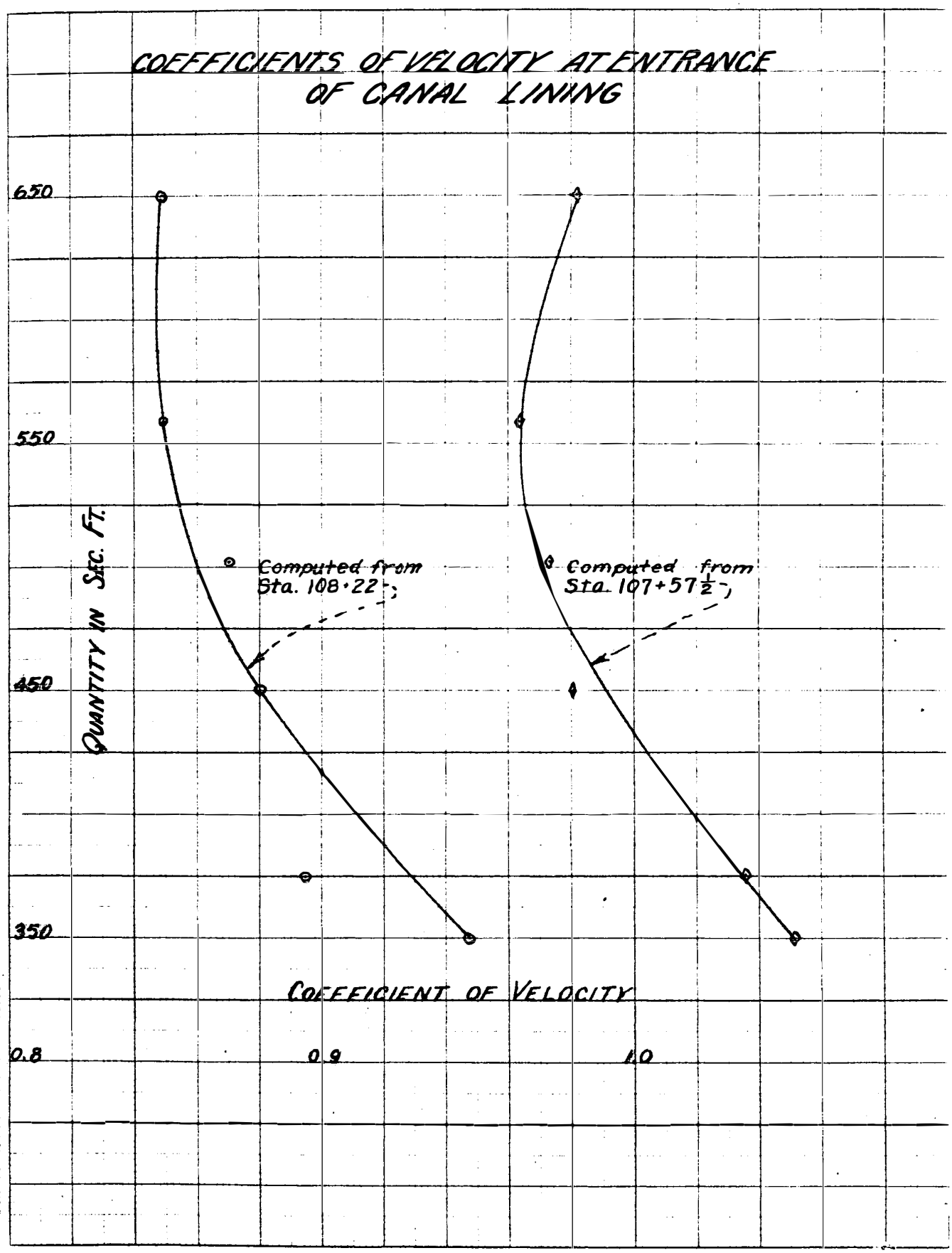

COEFFICIENTS OF VELOCITY AT ENTRANCE OF CANAL LINING
650
550
450
350
QUANTITY IN SEC. FT.
Computed from Sta. 108+22;
Computed from Sta. 107+57½;
COEFFICIENT OF VELOCITY
0.8
0.9
1.0

TABLE 4

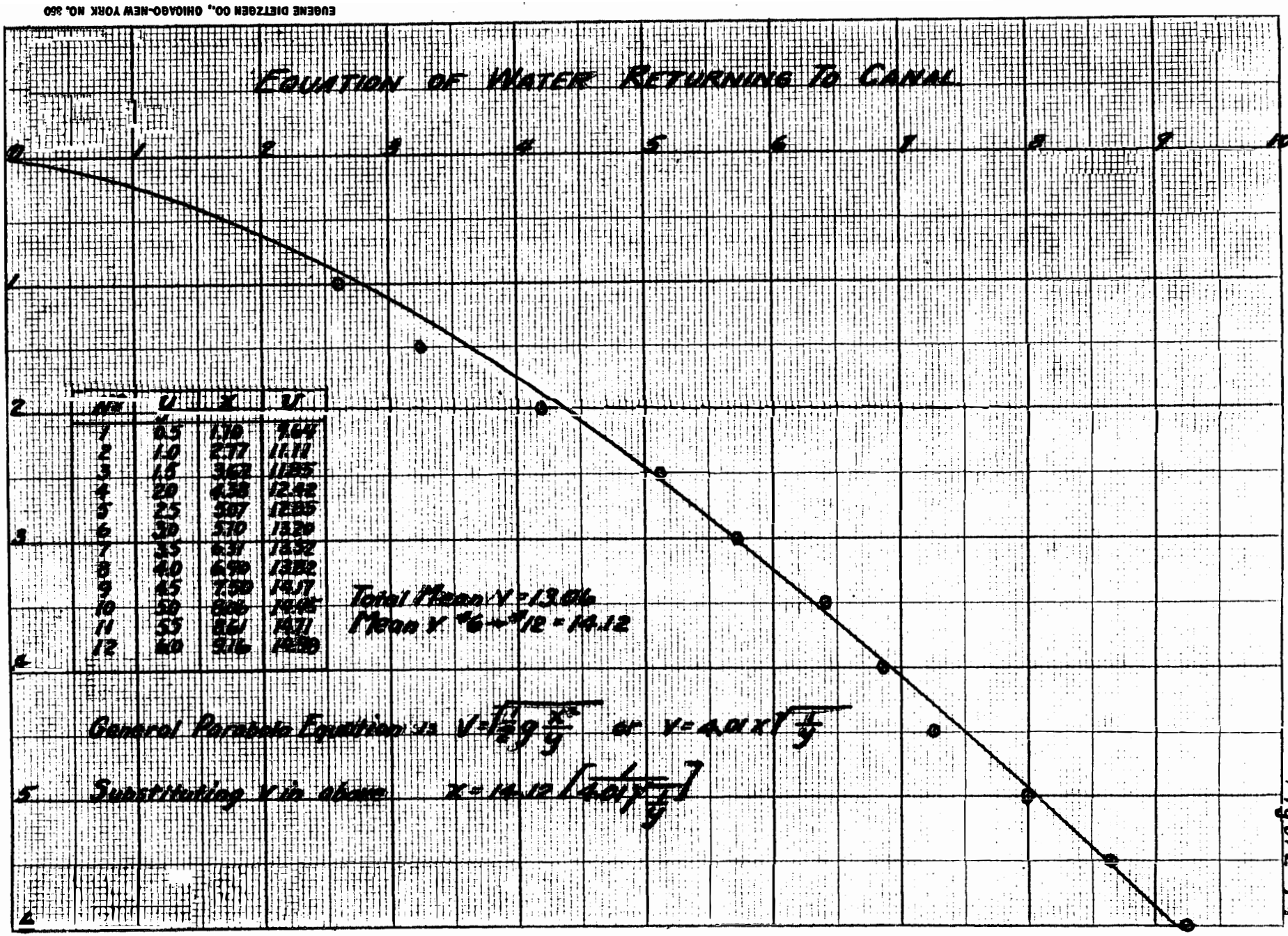
COMPUTATION OF QUANTITY AT STATION 107+57½

Considering Pressure Due to Centrifugal Force at Entrance

OBSERVED QUANTITY = 651 C.F.S.

Increment Number From W. S.	Top F_c	Bottom F_c	Mean Retarding Head	Mean Effective Head 2.876-M.R.H.	Mean Velocity $V = \sqrt{2gh}$	Area of increment	E Area (From #6 U_r)	Quantity $Q = AV$	
1	0	0.131	0.065	2.811	13.44	6.4	48.8	86.0	Total Quantity 633.6
2	0.131	0.335	0.233	2.643	13.04	11.6	42.4	151.4	
3	0.335	0.448	0.392	2.484	12.64	9.5	30.8	120.1	$K = \frac{Q\ \text{Observed}}{Q\ \text{Completed}}$
4	0.448	0.528	0.488	2.388	12.39	6.9	21.3	85.5	
5	0.528	0.575	0.552	2.324	12.23	7.2	14.4	103.7	$\frac{633.6}{651} = .974$
6	0.575	0.590	0.582	2.294	12.14	7.2	7.2	87.4	

EUGENE DIETZGEN CO., CHICAGO-NEW YORK NO. 350



No.	U	X	V
1	0.5	1.70	9.64
2	1.0	2.77	11.11
3	1.5	3.62	11.85
4	2.0	4.38	12.42
5	2.5	5.07	12.85
6	3.0	5.70	13.20
7	3.5	6.31	13.52
8	4.0	6.90	13.82
9	4.5	7.50	14.17
10	5.0	8.06	14.45
11	5.5	8.61	14.71
12	6.0	9.16	14.98

Figure 14

APPENDIX I

Recent Studies on Flow Conditions in Steep Chutes

Studies made in connection with the design of the spillway of Cle Elum Dam and the tunnels of Boulder Dam have helped to clear up some of the uncertainties in connection with flow conditions in steep channels and spillways

By E. W. Lane

Professor of Hydraulic Engineering, University of Iowa, Iowa City, Iowa, Formerly Research Engineer, U. S. Bureau of Reclamation

MORE EFFICIENT DESIGNS for steep chutes are now possible as a result of the studies made by the U. S. Bureau of Reclamation in connection with the design of irrigation water chutes and the spillways of some of its dams. In the following article the most important of the facts learned in those studies are set forth.

The formation of a boundary layer

As the water starts down the steep section of a chute the portion of it which is not close to the bottom or sides is rapidly accelerated and soon attains a high velocity. There is a narrow zone on the bottom and at the sides, however, in which the velocity adjacent to the walls and floor is zero and that at the outside of the zone reaches that of the center section. In this narrow boundary layer there is therefore a rapid increase in velocity with increasing distance from the side walls, or in other words, there is a high velocity gradient. In the side strips the flow is very turbulent and air is entrained, giving the water a white appearance, as shown in Fig. 1, which pictures the conditions in a steep chute on the Uncompahgre irrigation project in Colorado. There is a similar zone in contact with the bottom, but it is not as apparent since it is not in contact with the atmosphere, and air is therefore not entrained. This boundary layer, both on the bottom and sides, is narrow at the top of the chute but widens as the water flows down, as shown in Fig. 1. The water in the central swift-flowing portion has a relatively smooth surface, and the acceleration resulting from moving down the chute is reduced but little by the effect of friction on the sides. This central portion becomes narrower and thinner as the boundary layer increases in thickness, and if the chute is long enough a point is reached where the velocity throughout the entire cross-section is considerably retarded by side friction. When this point is reached the surface becomes rough, since the turbulent zone has extended through to the water surface.

In long chutes, changes of gradient frequently occur. These are usually formed with convex or concave vertical curves. Convex vertical curves must not be made sharper than the trajectory which the moving water would have in falling under the action of gravity. If the flume bottom is steeper, the flowing water will not follow the bottom but will leave it and rise possibly even above the tops of the side walls. In determining the shape of the trajectory for any case it is not sufficient to use the mean velocity, with a friction loss computed by one of the well-known velocity formulas, since each particle will tend to take a trajectory depending upon its individual velocity, and some of the particles will have a greater velocity and therefore will assume a flatter trajectory than the mean velocity would indicate. Since the higher velocities occur near the surface, these particles will be the ones with the flattest trajectory, and there will be few particles above them to prevent them from following their natural courses. Because the water near the surface, especially near the upper end of the chute, is retarded little by friction in the boundary layer, as previously explained, it will have a velocity nearly equal to that due to its total fall. To be sure that the flowing water will follow convex vertical curves, it is desirable that chutes be designed to be as flat or flatter than the trajectory of a projectile with an initial direction of that of the flume above the curve, and a velocity equal to that which would be produced by the total fall down to that point. In some cases, however, this may prove very expensive, and somewhat sharper curves have been used, as will be explained later.

When the vertical curve is concave, a double spiral motion is set up, similar to that which takes place in bends in an enclosed channel. This is caused by the centrifugal force, as shown in Fig. 3, which represents the pressures set up by a concave vertical curve in a rectangular flume. The depth of the water at the sides and center of the channel approaching the bend being the same, the pressures due to the weight of the water, represented by the areas *AA* and *CC* are equal. As the water flows around the concave bend the centrifugal force of the water sets up a pressure, represented by *BB* and *DD*. The radius of curvature causing these forces is the same at the center and sides of the flume, but the velocity is greater at the center, thus giving rise to greater centrifugal-force pressures at the center of the flume and a resultant centrifugal-force effect acting toward the sides. This pressure, being greater near the bottom, gives rise to a double spiral motion, as shown in the lower drawing, Fig. 3. This motion was very prominent at the concave curve in the chute of Fig. 1, as shown by the rapid movement

FIG. 1—A STEEP CHUTE on the Uncompahgre irrigation project well shows how turbulent flow develops along the side walls.

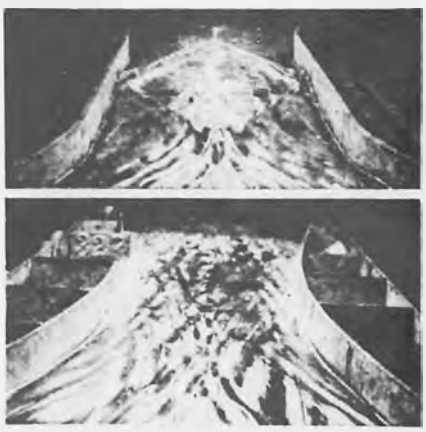


FIG. 2—DIAMOND-SHAPED FIGURES are formed by the higher waves that develop on the surface of flumes where the sides converge, as shown in the upper picture of a model of the spillway of Cle Elum Dam. By using a flume with flexible sides (lower picture) it was found possible to develop a transition section in which the flow, though still producing a complex wave pattern, produced no high waves. The shape of the contracting part of the flume exerts the greatest influence on the patterns.

of the white water from the sides across the surface in the flume below the bend.

Effects of non-uniform width

If the chute or trough is not of uniform width, a peculiar wave action is apt to result. Fig. 2 shows the conditions occurring in a model of the preliminary design of the spillway of the Cle Elum Dam in Washington. Similar diamond-shaped figures have been observed in a number of full-sized structures. It appeared as though a stream of water, of a depth considerably greater than the average in the flume, was deflected by each side of the converging section and moved across the flume to the opposite side, where it impinged on the wall and was deflected back, at the center of the flume meeting and passing through the similar stream deflected from the opposite side. The introduction of coloring matter into the water flowing down this flume, however, immediately demonstrated that the water itself flowed nearly directly down the channel, and did not follow the apparent direction at all. This proved that these ridges were not currents but merely waves that moved across the channel. The diagonal direction of the wave was due to the longitudinal movement of the water in the flume. The direction was the resultant of the motion of the wave across the flume and the movement along the flume of the water through which the wave was traveling. A comparison was made of the direction of the ripples observed in a flume of uniform width and those computed on this basis. The cross waves were assumed to move at right angles to the sides of the flume at the wave velocity $\sqrt{gd}$, where d is the depth of the water. For depths of about 0.1 ft. or less the wave velocities seemed to be somewhat less than computed by the formula, but for depths of about 0.2 ft. or more the law seemed to hold.

The study of this wave motion was not sufficiently comprehensive to enable a complete analysis of the phenomena to be made from which a satisfactory shape of spillway with decreasing width could be computed, but by means of a model with flexible sides such a shape was worked out. It had a section contracting from 200 to 100 ft. width, and expanding to 200 ft. width again at the lower end. In it the wave pattern was complex, but gave no high waves at any point, as shown in the lower photograph, Fig. 2. It was found that the shape of the contracting part of the flume exerted a great influence on the wave patterns set up, but the shape of the expanding section had little effect.

In the design of chutes, and in fact, in all structures in which the water moves at less than critical depth, it should be remembered that changes in the horizontal direction of flow are apt to be difficult to make, and therefore should never be made unless model tests have proved that the desired results will be secured. Where the depths are greater than the critical, changes of direction can be made with comparative ease. This is most easily visualized by a conception that is frequently brought out in European literature but does not seem to have been largely used in this country. The critical depth of water flowing in a rectangular channel is related to velocity of flow by the formula $V = \sqrt{gd}$. The velocity of the wave motion in the channel is also $V = \sqrt{gd}$. If, in a stream of water moving at a velocity greater than the critical and therefore at a velocity greater than $\sqrt{gd}$, an obstruction is placed in a manner that would cause a wave to move up the stream, this wave can move upstream with respect to the flowing water at a velocity of only $\sqrt{gd}$, which is less than the downstream motion of the water, and therefore the wave can

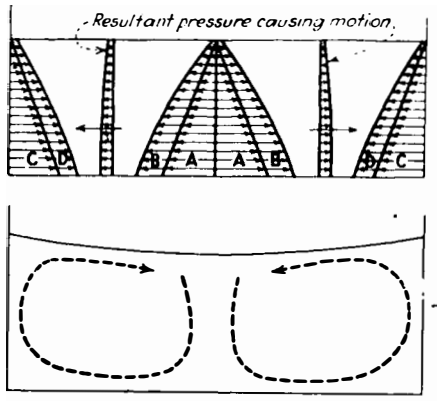


FIG. 3—CENTRIFUGAL FORCES at vertical curves set up pressures, as shown diagrammatically in the upper line, that result in a double spiral motion, as indicated in the lower line.

This conception explains why water moving at greater than critical velocity is so hard to control. It is also a valuable aid in visualizing the reason for many other phenomena of fast-moving water in channels—for example, why the level of the water below a section of a flume in which the velocity is greater than the critical has no effect upon flow conditions above the critical velocity section.

Friction losses at high velocities

Data on friction losses in open channels at high velocity are scarce, and there is considerable difference of opinion regarding such losses. The problem is complicated by the inclusion of air in the flowing water, which gives rise to the condition where Q is not equal to AV, if A is the measured water cross-section, including the air. In investigating this subject for the design of the tunnel of the Boulder Dam, observations were made on the flow through a chute on the Uncompahgre irrigation project in Colorado. The profile of this structure is shown on Fig. 4. The discharge was taken from a rating curve based upon a large number of current-meter measurements. Water-surface elevations were measured by means of a point gage, the average elevation of about ten points across the surface being taken as the mean for the section. Observations of water-surface elevations for about 20 different discharges were made at a number of stations along the flume, and an elevation-discharge curve was constructed for each station. A curve of the position of the jump vs. the discharge was also plotted. From these data the profiles for various discharges in the flume were

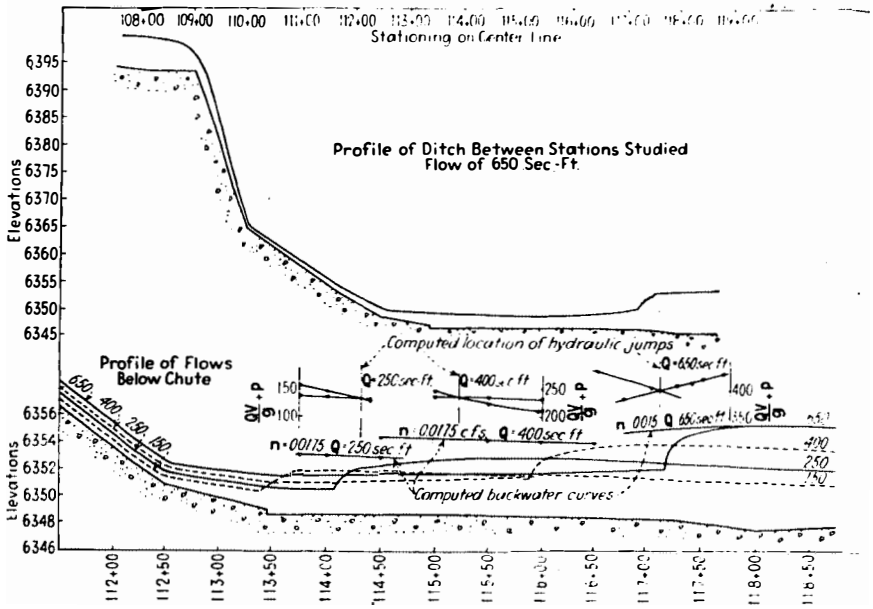


FIG. 4—SECTIONS OF A CHUTE on the Uncompahgre irrigation project used in the study of friction losses at high velocities.

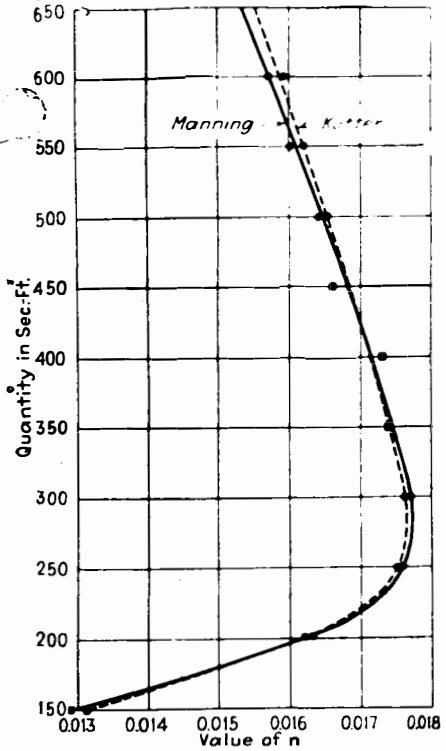


FIG. 5—ROUGHNESS VALUES for the tests section of the Uncompahgre flume as determined by the Manning and Kutter formulas.

were computed by the Kutter and Manning formulas as shown on Fig. 5. The velocity of flow in this portion of the flume is above the critical. An attempt was made to compute the roughness values for short sections, but the velocity-head errors involved in a slight inaccuracy of the observation of the water-surface levels were so great that reliable values of friction loss for short sections could not be secured. The values represented in the figure were determined from as great a length as possible, ranging from 350 to 700 ft. The roughness values are not noticeably different from those that would be expected for flows at low velocity or subcritical velocity. The bottom of the flume was of medium smooth concrete. The side walls near the bottom were roughened by frost action and growth of moss, but higher up they were smoother. This accounts for the roughness values shown on Fig. 5, starting at approximately 0.013, increasing to a maximum about 0.0177 and then decreasing to about 0.0154, as the discharge increased.

It seems to be generally agreed that where the area of flow of the combined air and water is used in the computations the friction coefficients will be substantially the same at high velocities as low velocities. However, the actual velocities of the water particles are greater than those computed on this basis by an amount equal to the per cent of entrained air. In computing the trajectories for the design of vertical curves if the use of full theoretical velocity of total drop is impractical, the velocities used should be higher than those based upon the ordinary friction values. In designing the chutes of the Bureau of Reclamation it has been customary to use Kutter's N of 0.014 for computing the depth of flow of the water including the air and an N of 0.008 for computing the velocities used in determining the trajectories limiting vertical curves and the entering velocities of stilling pools.

It has been argued that water at greater than critical velocity should show a different friction loss than at subcritical velocity because under the former condition the velocity is uniform throughout the flowing stream. There seems to be no good basis, however, for believing that the velocity is the same throughout the stream when flowing at greater than critical velocity. It seems probable that this idea originated because equal velocity distribution is usually found with supercritical velocity, since channels are rare in which supercritical velocity continues for a long enough distance to cause the formation of the unequal velocity distribution that occurs in most channels with the ordinary velocity conditions. Measurement by pitot tube and float in the Uncompahgre ditch (Fig. 6) showed that as the water proceeded down the ditch the velocity distribution became more uneven, reaching a distribution closely resembling that observed in rough pipes.

The observations on this ditch gave an unusual opportunity to check the accuracy of computations, giving the location of the hydraulic jump by the method described by Julian Hinds in his paper on "The Hydraulic Jump and Critical Depth in the Design of Hydraulic Structures," *ENR*, Nov. 25, 1920, p. 1034.

The results for three discharges are given in Fig. 4. For discharges of 250

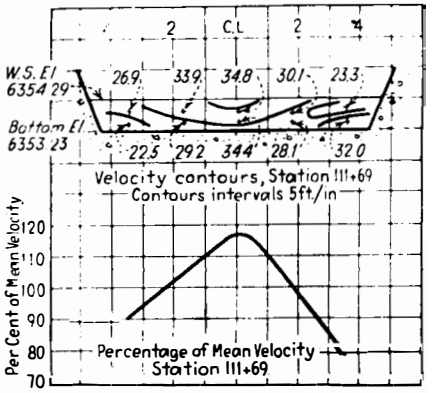


FIG. 6—DISTRIBUTION OF VELOCITIES in a shooting jet, as determined by measurements made in the Uncompahgre chute.

and 650 sec.-ft. the agreement was reasonably close but was not so good for 400 sec.-ft. The reason for this difference in accuracy is not apparent. The conditions in the chute were particularly unfavorable to accurate determination, however, since the location of the jump was controlled largely by the uncertain factor of friction. Under such conditions too much dependence should not be placed on computed positions. For conditions where there is a considerable change in channel section or elevation, the position of the jump can no doubt be computed with considerable accuracy.

The experiments on this flume were made by G. C. Wright, assistant engineer, under the direction of the writer. The designing of all canal structures of the Bureau of Reclamation is under the direction of H. R. McBirney, senior engineer. All designing and research are under J. L. Savage, chief designing engineer, and R. F. Walter, chief engineer, Denver, and all activities of the bureau are under the direction of Elwood Mead, commissioner, Washington, D. C.

Cement Industry Employment Increased Under PWA

MORE THAN 70,000,000 man-hours of employment have been created since August, 1933, and over $130,000,000 of the amount paid for the producing and handling of cement used on public-works projects went as wages, according to a report issued by the Bureau of Labor Statistics. These figures are for indirect labor and do not include work developed in placing concrete on the construction site.

This study is the second of a series that the bureau is conducting in an effort to secure accurate measures of indirect employment. Reports made by contractors to the Department of Labor show that 62,000,000 barrels of cement were required for use in the first PWA program up to October, 1935.

Breaking down the figures further, the report shows that an average of 1.26 man-hours of labor is required for each barrel of portland cement produced. Transportation of the cement to the construction site accounts for the larger share of labor requirements, involving 0.503 man-hours per barrel, while the manufacturing process requires 0.390 man-hours for each barrel produced. The remaining 0.074 man-hours are credited to administration and selling. This is held to be only a partial picture of the increase in employment that was brought about by the demands for cement on PWA projects, because each barrel of cement produced involved 0.283 man-hours of employment producing and transporting raw materials

APPENDIX II

Appendix II

The Hydraulic Jump and Critical Depth in the Design of Hydraulic Structures

How Established Principles May Be Applied to the Design of Canals and Other Works—A Study Based on the Laws of Conservation of Energy and of Linear Momentum

By Julian Hinds
Engineer, U. S. Reclamation Service, Denver, Col.

THE hydraulic jump and the critical depth have recently come to be recognized as factors of considerable importance in the design of open channels and related hydraulic structures. An excellent technical discussion of this subject will be found in the *Transactions* of the American Society of Civil Engineers, Vol. LXXX, p. 338, in a paper on "The Hydraulic Jump in Open Channel Flow," by Karl R. Kennison, with discussions by a number of prominent engineers. Also, Messrs. Ward, Riegel and Beebe in "Technical Reports, Part III," issued by the Miami Conservancy District, in 1917, present an interesting discussion of the problem, and submit valuable experimental data on the action of the jump below reservoir outlet works. E. W. Lane, in *Proceedings* of the American Society of Civil Engineers, December, 1919, discusses the occurrence of the hydraulic jump in connection with experimental work on flow through contractions.

FIG. 1. STEADY FLOW WITH CHANGING VELOCITY, UNIFORM CHANNEL

It is not the intention of this paper to add to the fundamental theories already advanced, but an attempt will be made to show how the established principles may be applied to the design of canals and canal structures.

It is assumed throughout this discussion that the kinetic head is truly represented by the velocity head as computed from the mean velocity.

Practically all formulas previously proposed for the solution of hydraulic jump problems are limited to rectangular sections. While such a limitation simplifies the computations it in no way simplifies the fundamental conceptions, and an attempt will be made to keep the discussion general.

The notation used herein is as follows:

A Area of section
b Width of rectangular channel
d Depth of water
d_1, d_2, d_3 etc. Depths at stations 1, 2, 3 etc.
d_c Critical depth
F Force or pressure or change in momentum
F' Force per unit of volume
g Acceleration of gravity
H Energy
h_f Loss of energy or friction or other loss
h_v Velocity head
h_{v_1}, h_{v_2}, h_{v_3} Velocity heads at stations 1, 2, 3 etc.
h_c Velocity head for critical depth
M Mass
P Hydrostatic pressure on end of water prism
P_1, P_2, P_3 Hydrostatic pressures at stations 1, 2, 3
Q Discharge
s Slope of energy gradient
s' Slope of channel
T Width of water surface
t Time
V Velocity
V_c Critical velocity
W Weight of a cubic unit of water
Z Ratio of T to d for a triangular channel

As a fundamental basis for discussion it will be necessary to accept the law of the conservation of energy and the law of the conservation of linear momentum. The former law will appear as Bernoulli's theorem, i.e., the elevation of the water surface at any point in a channel plus the velocity head at that point is equal to the same functions at any other point, plus (or minus) intervening losses. Using the notation already given and as shown in Fig. 1 and taking the bottom of the canal at B as datum, this relation may be expressed thus:

$$h = d_1 + h_{v_1} = d_2 + h_{v_2} + h_f \qquad (1)$$

There is no exception to this rule and it is independent of the form or slope of channel, or of channel changes occurring between the two points.

The second law requires that the momentum of a system of particles, considered collectively, cannot be altered by the particles impinging upon each other, but can only be changed by the influence of an external force. The change produced in momentum depends upon the magnitude of the external force and its duration. Stated simply, the law requires that force must equal rate of change of momentum or that force is equal to mass times acceleration. If the acceleration between two stations as A and B, Fig. 1, be uniform, the relation may be written

$$F = \frac{MV - MV_1}{t} \qquad (2)$$

where F equals force, MV and MV_1 are the momentum at A and B respectively and t is the time through which F is applied. If F' is the force acting on a unit volume of water, of weight W, then from equation (2)

$$F' = \frac{W}{gt}(V - V_1) \qquad (3)$$

The total force acting on Q units per second for time t will be

$$F = F'Qt = \frac{QW}{g}(V - V_1) \qquad (4)$$

Fig. 2 shows graphically the relation existing between the energy of flow (depth plus velocity head), the momentum and the depth for a given discharge in a given channel. This diagram is not affected by the slope or roughness of the channel, and is independent of the method of producing or maintaining flow. The diagram is constructed to scale for 150 sec.-ft. flowing in the irregular channel shown in Fig. 3. The lower of the two curves is obtained by platting values of $d + h_c$ over corresponding values of d, and may be called the energy curve, since it represents the energy of flow corresponding to various depths, the bottom of the canal being taken as datum. Values of V and h_v

in ge area. It will be observed that there is one point, C, on the curve for which the value of $d + hv$ is a minimum. The depth, d_c, corresponding to this point is called the critical depth. If flow is taking place at any depth, d_a, other than d_c, there will be a corresponding depth, d_b, having the same value of $d + hv$. The depths d_a and d_b will be called alternative energy depths or alternative energy stages.

Let Fig. 4 represent the profile of a portion of a channel of uniform cross-section having functions as represented in Fig. 2. Then from the energy curve, Fig. 2, it appears that the depth at S may be either d_a or d_b and that it may be made to change from one of these depths to the other at will, provided some means for making the change without depressing the energy line through the point C be supplied.

Such a change in depth with no loss of energy, however, involves a change in momentum and can only be

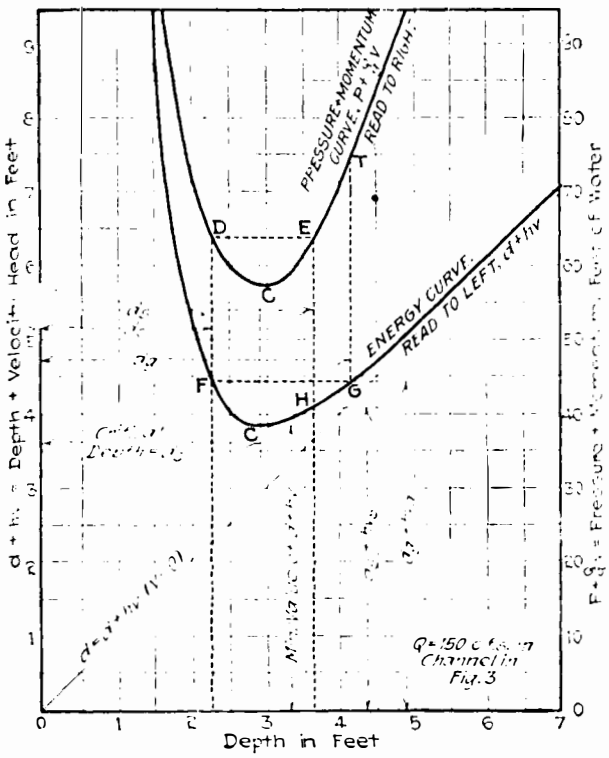


FIG. 2. RELATION BETWEEN ENERGY OF FLOW, MOMENTUM AND DISCHARGE

effected by the intervention of some external force. The required force may be supplied by gravity, friction, unbalanced pressures or by a combination of these factors.

In Fig. 1, let P_a be the hydrostatic pressure on the plane of the cross-section at A, P_b being the corresponding pressure at B, and let F''' equal friction, or any other external force applied between A and B. From (4), taking $W = 1$

$$\frac{Q}{g}(V_b - V_a) = P_a - P_b + F''' \qquad (5)$$

If the change in depth occur without the intervention of the force F''', then equation (5) placing $F''' = 0$ and transposing gives the relation

$$\frac{Q}{g}V_a + P_a = \frac{Q}{g}V_b + P_b \qquad (6)$$

which must hold if a change in depth occurs under the influence of the external forces P_a and P_b only.

The momentum curve in Fig. 2 gives values $\frac{Q}{g}V + P$ for various values of d. The abscissæ are the same as used for the energy curve and values $\frac{Q}{g}V + P$ are shown on the right. For a given depth there is always one other depth having an equal value of $\frac{Q}{g}V + P$, this point falling in all cases beyond the critical depth. Therefore, for any depth of flow there is always another depth which we will call the alternative momentum stage, to which the flow may change without the intervention of an external force. Such a change, however, requires a change in the energy of flow.

Since for a given change in depth the change in $d + hv$ is not proportional to the change in $\frac{Q}{g}V + P$ it follows that a change in depth cannot occur without the introduction of some factor to preserve a balance. A change between alternative energy stages without loss may be effected by the application of an external force only, and a change between alternative momentum stages may be accomplished by a change in energy only. All other changes in depth, involving a change in velocity, require both an external force and a change in energy.

There are numerous causes which may produce a change in stage in a canal, but if the channel is straight, of uniform cross-section and roughness, and free from obstructions, changes in stage are generally caused by changes in grade. An analysis of a simple case will be made to show where changes may be expected. Let Fig. 5 be the profile of a canal of uniform cross-section. Let the slopes to the left of K and to the right of N be sufficient to maintain flow at normal depths, d_a and d_b, respectively, both greater than the critical depth, the slope between K and N being sufficient to maintain a normal depth, d_m, less than the critical. Let I be a sufficient distance upstream from K not to be affected by the "drawdown," and let M be sufficiently far below K for uniform flow to be established. Flow at I will be at high stage while at M it will be at low stage. Somewhere between it must pass through the critical depth.

Before proceeding to locate the point of passage it will be well to investigate the properties of the "energy gradient" shown on the profile. This gradient is determined by plotting the velocity head above the water surface at all points. At a given point the total energy, i.e., the sum of the static and kinetic energies, is represented by the elevation of this line. It follows from Bernoulli's theorem that the fall in this gradient between any two points represents the sum of all losses occurring between the two points. The slope of this line at any point represents the slope required to overcome friction and other losses at that point. The energy gradient can never rise in absolute elevation in the direction of flow, since there can be no increase in energy. The distance from any point on the energy gradient vertically downward to the bottom of the canal is $d + hv$. It will be clear from Fig. 2 that there will in all cases be two water depths corresponding to

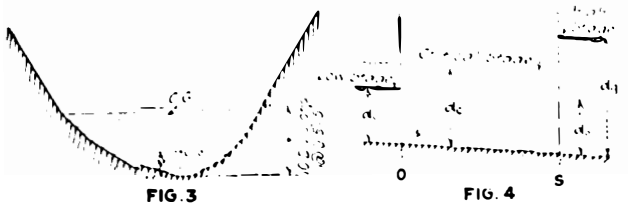


FIG. 3. SHOWING SUBDIVISIONS FOR COMPUTING AREAS AND PRESSURES. FIG. 4. ALTERNATE STAGES

any possible energy gradient. It is also evident that the energy gradient cannot be brought to within less than a certain minimum distance from the bottom of the canal, the two corresponding depths becoming equal at that point. The gradient will generally be a continuous line and can make an abrupt change in height or slope only where a sudden loss occurs. An example of sudden loss, caused by an abrupt change in stage, is shown at *O*, Fig. 5. So-called sudden changes in open channels are actually more or less extended, but for simplicity they are assumed in all computations to be instantaneous.

Returning to Fig. 5, the water surface must pass through the critical depth between *I* and *K*, at *K* or between *K* and *M*. Assume the passages to occur at some point *J* above *K*. The energy gradient must under this assumption drop down at *J* to a minimum height above the bottom of the channel. Therefore, the friction slope from *J* to *K* cannot be steeper than the slope of the base of the canal, and, since the slope of the canal is only sufficient to maintain flow at the normal depth, d_i, which is greater than the critical depth, the energy available is not sufficient to overcome friction losses from *J* to *K* and maintain flow at or below the critical depth. Hence the critical depth cannot exist at any point above *K*. If the passage occurs at some point, *L*, below *K* the water surface from *K* to *L* must be at or above the critical, the velocity will be less than normal, and therefore the friction slope will be flatter than the canal slope; that is, the value of $d + hv$ at *L* will be greater than at *K*, whereas if the critical depth occurred at *L* it should be less. Therefore, the passage cannot occur below *K*. If the point of passage is at *K*, $d + hv$ will increase from *K* to *M* and the fall in the gradient will be less than the fall in the canal grade. This is logical since the velocity is less than the normal velocity. The fall in the energy gradient from *I* to *K* will be greater than the fall in the canal grade, to balance the increase in friction due to velocities in excess of the normal velocities. The point of critical flow will, therefore, come at *K*.

No reference has been made to values of the slope from *K* to *N* except to state that it is sufficient to maintain flow at a depth less than the critical. As long as this slope is sufficient to support flow at a normal depth equal to or less than the critical depth, it may be varied at will without affecting flow conditions above *K*. For this reason *K* is called a control.

Flow to the right of *M* will be uniform at the depth d_m until the flatter slope at *N* is encountered. For simplicity the loss through shock due to the vertical angle at *N* is neglected. In actual construction this angle, if sharp, would be relieved by a vertical curve to reduce the shock. Flow to the right of *N* cannot continue at the depth d_m, since the slope of the canal is insufficient to overcome friction at that depth. The excess of the frictional resistance over the force due to the slope of the bottom of the channel produces a retarding force tending to reduce the velocity and momentum of flow and causing the depth to gradually increase, the velocity head and the depth plus velocity head being decreased to supply the energy necessary to overcome friction.

If this gradual rise in water surface be assumed to continue, along the dotted line *YZ* in the figure, until the normal depth is reached at *Z*, there will be some point, as at *S*, where the depth is equal to the critical depth. Since $d + hc$ is a minimum at *S* and is not a minimum at *Z* it follows that the available friction slope from *S* to *Z* must be less than the canal slope. But the velocity from *S* to *Z* is greater than the normal velocity, hence the required friction slope is greater

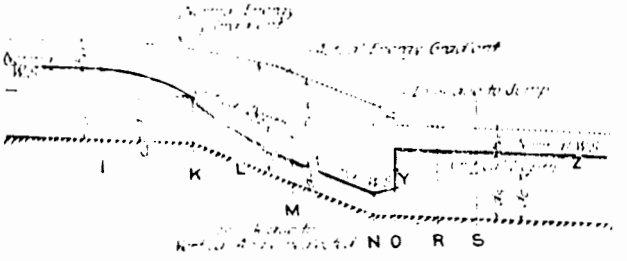


FIG. 5. CHANGES OF STAGE IN UNIFORM CHANNEL

than the canal slope, and the depth cannot change from d_c at *S* to d_n at *Z*. It is necessary that the water depth change, between *N* and *Z*, from d_m to d_n, but it must not at any point have the intermediate depth d_c nor in fact any depth for which $d + hc$ is less than at *Z*. The change occurs suddenly through what is known as the hydraulic jump, from some low-stage depth to the depth d_n. The depth d_r and the low-stage depth at *R*, where the energy gradient for the water surface *YZ* intersects the normal energy gradient, are alternate energy depths, similar to d_y and d_n in Fig. 2, and if the jump involved no loss of energy it would occur at that point. Referring to Fig. 2 it will be seen that in order for the change to occur at *R* there must be an increase in $P + \frac{Q}{g}V$, similar to the change from *D* to *T*. Such a change in momentum requires the application of an external force. The only external forces available aside from P_r and P_t are the forces of gravity acting through the canal slope and resistance due to friction. These tend to neutralize each other and are negligible in amount. Therefore, *F'''* in equation (5) may be assumed to be zero, equation (6) must hold, and the jump cannot occur at *R*, but must take place at some earlier stage where $P + \frac{Q}{g}V$ is equal to the final value of that function. This requirement apparently conflicts with Bernoulli's theorem, but there is automatically introduced a disturbance which produces an internal loss of proper magnitude to preserve the equilibrium of equation (1). The low-stage depth at the jump will be d_y, corresponding to the point *D* on the momentum curve in Fig. 2. The loss of energy in the jump is equal to $(d_y + hv_y) - (d_r + hv_r)$.

This loss is unavoidable for a change in stage in a channel of constant cross-section and falling grade.

By properly adjusting the shape or vertical alignment of the channel additional external forces may be introduced, and the jump, with the attendant loss of head, may be reduced or eliminated, as will be pointed out later.

The two curves in Fig. 2 approach each other indefinitely to the left of C, while to the right of C they diverge rapidly. A little study of these curves will show that for small heights of jump the loss approaches zero, but that the loss of head increases rapidly as the height of jump increases. For example, in Fig. 2, if the jump occurs from d = 2.5 to d = 3.38 the loss in depth will be from d = 3.47 to d = 3.38 or 0.09 ft. The corresponding energy loss is about 0.05 ft. If the jump occurs from d = 2 ft. to d = 4 ft., the loss in depth will be from 5 ft. to 4 ft. or 1 ft., the energy loss being 5.17 — 4.33 = 0.84 ft. If the jump occurs from d = 1.75 ft. to d = 4.55 ft. the loss in depth will be 6.30 — 4.55 = 1.75 ft. and the energy loss will be 6.40 — 4.80 = 1.60. Taking the discharge as 150 sec.-ft. this last loss requires the continuous destruction of energy equivalent to about 27 horsepower.

After the depth, d_n, from which the jump will take place has been determined the location of the jump may be obtained by finding the point at which flow will be retarded to that depth. This point may be conveniently found from the following equation, derived from Fig. 1:

$$Ls_y + d_a + hv_a = d_b + hv_b + Ls_f \qquad (7)$$

By assuming values for d_a and d_b all functions at A and B can be computed, including the friction slopes. If the canal slope, s_w, be known and if the friction slope from A to B, be assumed equal to the average of the slopes at these points, all factors in equation (7), except L, become known and L is readily found.

Equation (7) is applicable to any variable flow. The only approximation involved is in the assumption that the average slope is equal to the average of slopes at the computed points. By assuming depths sufficiently close together the error from this source can be reduced as far as desired. However, ordinary friction formulæ are not known to apply accurately to variable flow and extreme refinement in computation is not justified. The friction slope may be determined by Kutter's formula, or by any other friction formula. This equation should not be applied through a control section or a hydraulic jump, but may be used to find the water surface at J, L or N, Fig. 5, or above a check or dam.

The changes in canal slope at points K and N, Fig. 5, are purposely assumed to be great, so that the changes in stage will be marked, but when the normal depth is near the critical, troublesome fluctuations are often produced by very slight unintentional irregularities in the channel. The fluctuations in such cases appear to be out of all proportion to the offending irregularities. This is due to the fact that in the vicinity of C, Fig. 2, the momentum and energy curves are approximately horizontal so that if the amount of energy ($d + hv$) required at a given point is changed slightly, a comparatively great change in depth must occur to preserve a balance. The possibility of trouble from this source is discussed by J. S. Longwell in an article on "Flow Conditions, Congo Low Line Flume, North Platte Project," published in the *Reclamation Record*, August, 1917, and reprinted in *Engineering News-Record*, Jan. 3, 1918, p. 38. If in the design of a channel it is found that the depth is at or near the critical the shape or slope of the channel should, if practicable, be changed to secure greater stability. Usually the critical velocity can be changed by widening or narrowing the channel, or the normal velocity by altering the slope. If such changes are not practicable, liberal freeboard should be allowed, and extreme care should be used in construction to secure uniformity in grade and cross-section.

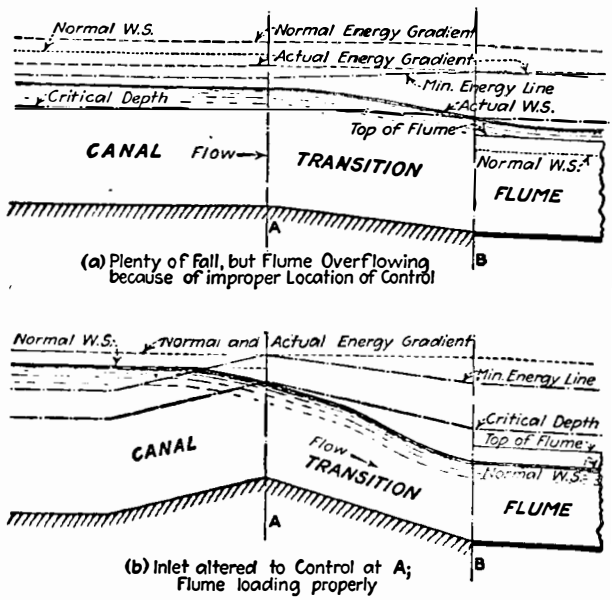


FIG. 6. VARIABLE CROSS-SECTIONS CANAL TO FLUME. FOR TRANSITION FROM HIGH TO LOW STAGE

Changes in stage which occur at transitions between canals and flumes, tunnels or other high-velocity conduits, where the cross-section of the channel is variable, involve only the principles already discussed, but additional factors are introduced which affect the mathematical treatment. It will be convenient to consider these transitions under six headings, as determined by the stages between which changes occur, as follows:

(a) Changing from high stage to low stage, increasing velocity.

(b) Changing from one low stage to another, increasing velocity.

(c) Changing from one high stage to another, increasing velocity.

(d) Changing from low stage to high stage, reducing velocity.

(e) Changing from one low stage to another, reducing velocity.

(f) Changing from one high stage to another, reducing velocity.

Case (a) is similar to the example presented in the discussion of Fig. 5, and by arguments already used it can be shown that the control section cannot be above A or below B, in Fig. 6, AB being a variable transition between a canal and a flume. To locate the point of control plot a minimum energy line, as shown in (a) and (b), Fig. 6. This line is obtained by plotting the minimum values of $d + hv$ above the canal bottom, and it represents the minimum possible elevation of the energy gradient at any point. The actual energy gradient cannot fall below this line and if the two gradients intersect it must be at the highest point on

the minimum line. Hence, in Fig. 6 (a), although there is apparently sufficient drop from the normal water surface in the canal to that in the flume, the flume will overflow because of the incorrect location of the control. The drop which should produce velocity head is used up in friction through the lowering of the water surface in the canal. By humping the bottom, as in (b), to bring the minimum energy line at the upper end of the transition into coincidence with the normal energy gradient, the trouble is avoided. Racing in the canal is prevented by the throttling effect of the control at *A*. The same effect can be secured, if desired, by narrowing the section at *A*, rather than by raising the grade. The inlet structure may be made to act as an automatic check by shaping it so that the head required to pass any quantity of water at the critical depth is equal to the normal head in the canal above for that quantity. It is theoretically possible to construct such a control check so that it will exactly control all quantities of flow in a given channel, but it is sufficient, for all practical purposes, to design the structure to fit exactly for two discharges, usually full discharge and one-fourth discharge, as in the case of a notched drop.

In changing from one low stage to another, having a greater velocity (case b) it is possible, by contracting the channel or by raising the bottom, or both, to force the water surface up to the critical depth and under extreme conditions a jump may be produced within the transition or in the canal above. Such a contingency is, however, remote and ordinarily this type of transition will not be effected by the critical depth or the hydraulic jump.

The transition from one high stage to another, having a higher velocity (case c) is often accompanied by disturbances attributable to an incorrect control. Fig.

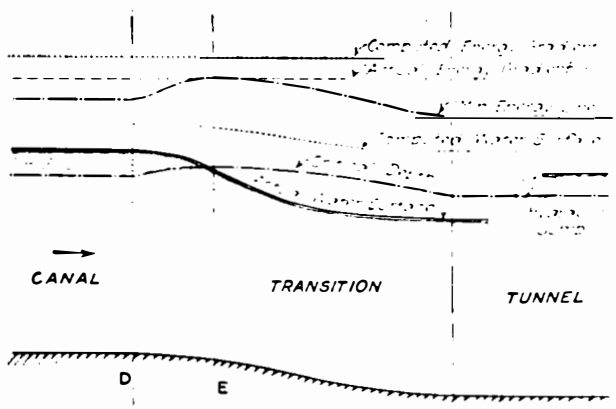


FIG. 7. FAULTY INTAKE: ONE HIGH STAGE TO ANOTHER

7 represents to an exaggerated scale a faulty design recently prepared by the writer for a transition from a segmental open channel to a circular tunnel. The hydraulics for this transition were computed at 2-ft. intervals and no discrepancies were found. However, liberal allowance was made for transitions and friction losses, and a "safe" coefficient of roughness was used to determine the depth in the tunnel. After construction it was found that transition losses were negligible and that the normal depth in the tunnel immediately below the entrance was considerably less than the assumed normal depth. As a result the energy gradient

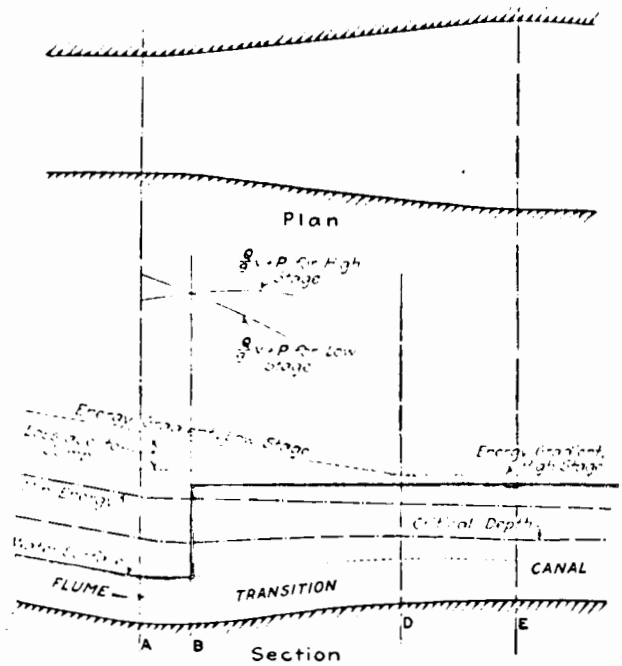


FIG. 8. JUMP WITH VARYING CHANNEL SECTION AND BOTTOM SLOPE

for the tunnel dropped below the summit of the minimum energy line, and the flow passed to low stage at *E*, causing a jump to occur just below the end of the transition. The transition should have been proportioned to keep the summit of the minimum energy line below the lowest possible position of the energy gradient at *F*. The jump was particularly objectionable at this location, and was eliminated by bolting cross timbers to the bottom of the channel, thus increasing the friction and bringing the energy gradient up to its computed position.

Transition from low stage to high stage, (case *d*) may be accomplished either with or without the hydraulic jump. Unless the section of the channel is properly varied or the bottom "humped" the jump is inevitable. Fig. 8 represents a transition in which the variation in channel section is not sufficient to avoid the jump. The energy gradient for low stage is computed from *A* toward *E*, using equation (1), the gradient for high stage being computed backwards, from *E* toward *A*, in the same way. After these gradients and their corresponding depths are found values of $\frac{Q}{g}V + P$ for the two stages are computed, and plotted to any convenient scale and datum. The jump must occur where this function is equal for the two depths, or at *B*, the intersection of the plotted lines. By varying the cross-section or the elevations of the flume, transition or canal, the location of the intersection, *B*, may be varied at will. If this point falls to the left of *A* the jump will occur in the flume and may cause it to overflow. If it falls to the right of *E* the jump will occur in the canal section where the resulting disturbances may be objectionable.

If the transition in Fig. 8 be so altered that the minimum energy line at *D* becomes tangent to the two energy gradients at their point of intersection, the two gradients automatically changing to become tangent to

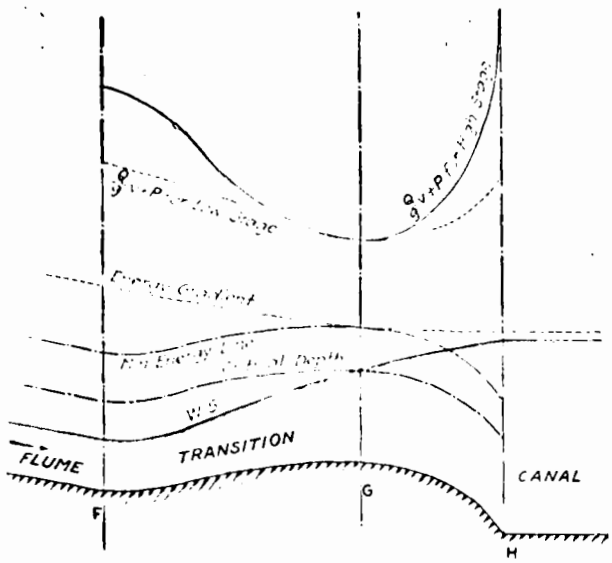


FIG. 9. TRANSITION WITHOUT JUMP

each other at that point, the transition may be accomplished without the jump. Such a transition is illustrated in Fig. 9. The $\frac{Q}{g}V + P$ lines intersect and become tangent at the point G. The excess of the pressure in an upstream direction over that in a downstream direction on the hump in the bottom or on contractions in the sides of the channel supplies the force F'' required in equation (5).

In changing from one low stage to another with a lower velocity (case *c*) the hydraulic jump and the control section are not often encountered. A contraction in the sides of the channel or a hump or obstruction in the bottom may cause the water surface to rise temporarily above the critical depth, but such contraction or obstruction is not likely to exist in an artificial channel, except by deliberate design. The low secondary dam sometimes placed below an overflow dam to break up the high velocity constitutes such an obstruction, but the normal depth below the secondary dam is usually above the critical so that the conditions of (case *d*) obtain.

The most usual form of canal transition for reducing velocity is from one high stage to another and such structures are often subject to unexpected irregularities. It is usual in designing transitions of this type to provide for only a partial recovery of head, to allow a factor of safety to take care of imperfections in the structure and of fouling in the canal below. This results in an excess of energy. If the minimum energy line is at all points well below the energy gradient the water surface in the high-velocity channel will be lowered and the excess head will be consumed in increased friction, but if the minimum energy line is high a control is likely to be formed, resulting in low stage flow for a short distance, followed by a jump back to normal. An actual instance of such an outlet is shown in Fig. 10, where a very gradual change from a 6.1-ft. diameter tunnel to a 8.3-ft. segmental lined section is effected in a length of 100 ft. If the critical depth in the tunnel and in the transition had been lower the water surface would have been further drawn down at J to make the energy gradient from above coincide with that from below, but the high position of the minimum-energy line at J limits the draw down. As a result the flow passes to low stage at J, returning to high stage through the hydraulic jump at K. The location and height of the jump may be determined as in case (*d*) Fig. 10 is plotted from actual observations. If the required water surface elevation at L were to be increased to bring the energy gradient at that point above the elevation of the minimum-energy line at J, the control at J and the jump at K would be avoided. It will be noticed that the velocities increase from J toward K, reaching a maximum somewhere near K, the effective length of transition being reduced to KL. Under proper conditions the point K may fall to the right of L, and in any event the turbulence below L will be greater than if the transition were effected without the jump. If turbulence is objectionable the outlet should be proportioned to avoid the formation of a control.

The critical depth may be found by constructing either the energy or momentum curve, and finding its lowest point, as in Fig. 2, but it can be more readily determined by means of the equation

$$\frac{A^3}{T} = \frac{Q^2}{g} \qquad (8)$$

where A and T are respectively the area and top width of the water sections for the critical depth. Using

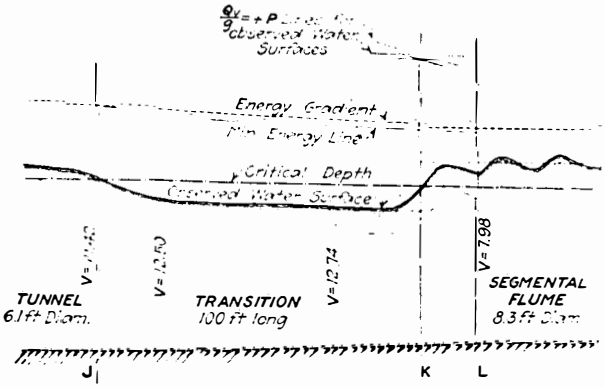


FIG. 10. TRANSITION FROM ONE HIGH STAGE TO ANOTHER; JUMP CAUSED BY EXCESS HEAD

the notation already established, and letting H = $d + hv$, and A = area of section = some function $f(d)$ of the depth, equation (8) is derived as follows:

$$H = d + hv \qquad d + \frac{V^2}{2g}$$

$$= d + \frac{1}{A^2} \times \frac{Q^2}{2g}$$

$$= d + \frac{1}{f^2 d} \times \frac{Q^2}{2g};$$

differentiating, $\frac{dH}{dd} = 1 \qquad \frac{f' d}{f^3 d} \times \frac{Q^2}{g}$,

where $f'(d)$ = A^2 and $f'(d)$ = the first derivative of A with respect to d, T, the width at the water surface. H is a minimum when

$\frac{dH}{dd}$ is zero, or when $\frac{A^3}{T} = \frac{Q^2}{g}$ as given above.

By substituting AV for Q this equation may be written

$$\frac{A}{2T} = \frac{V^2}{2g} = hv \qquad (9)$$

In a rectangular section of width, b, A is equal to bd, T is equal to b, and (8) may be reduced to

$$\frac{Q^2}{g} = b^3 d_c^3 \text{ (for rectangular sections)} \qquad (10)$$

from which d_c is readily determined. Equation (9) reduces to the well-known form

$$hv_c = \frac{1}{2} d_c \text{ (for rectangular sections)} \qquad (11)$$

In a triangular section, where the ratio of T to d is X, equation (8) reduces to

$$\frac{Q^2}{g} = \frac{d^5 X^2}{8} \text{ (for triangular section)} \qquad (12)$$

and (11) becomes

$$hv_c = \frac{1}{4} d_c \text{ (for triangular section)} \qquad (13)$$

Similar formulæ may be deduced for any channel having a known mathematical relation betwen A, t and d, but generally the resulting equations are of the $\frac{5}{2}$ deg., and are complicated, and it is preferable to use equations (8) and (9) without further reduction. If d_c is known and Q required these equations may be solved directly, but if d_c is the unknown the solution can best be made by trial.

Valuable assistance in the preparation of this paper has been rendered by D. C. McConaughy and W. H. Nalder and other engineers in the Denver Office of the Reclamation Service.

To Discuss Labor Conditions and Hours of Work in Steel Industry

In order to learn the results of the three-shift system in steel plants Horace B. Drury, formerly of the economics department of Ohio State University and recently with the Industrial Relations Division of the Shipping Board, has been spending some months in visiting steel plants in the United States, collecting technical data covering the details of the operation of the system.

Mr. Drury has put the results of his observations into a paper which will be presented at a joint meeting held under the auspices of the Taylor Society in New York, Dec. 3 at 8 p.m., in the Engineering Societies' Building. The other organizations participating in the meeting will be the metropolitan and management sections of the American Society of Mechanical Engineers, and the New York section of the American Institute of Electrical Engineers.

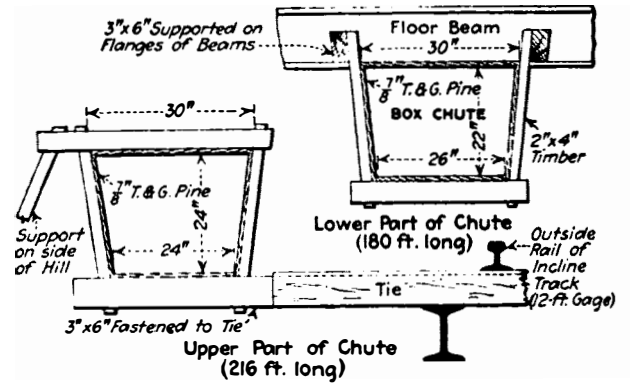
The purpose of presenting the paper is to assist the steel industry in America to prepare for the three-shift system which, judged by the general tendency toward the shortening of hours of labor and the fact that in other countries steel production has been put upon the three-shift basis will probably come here. It is therefore the part of wisdom for managements to prepare for it.

At the same meeting William B. Dickson, vice-president of the Midvale Steel & Ordnance Co., will discuss the subject from the point of view of the manufacturer, and the general discussion will be led by Robert B. Wolf, consulting engineer, New York.

Landslip Material Successfully Chuted Into Railroad Cars

REMOVAL of a hillside slide by passing the material down a steep chute into cars has been carried out successfully on the Pennsylvania System near the Birmingham station at Pittsburgh. At this point the rocky face of the hill rises abruptly from the track level for 150 ft. and then continues on a steep slope for about 200 ft. to the top. During the thaw early this year, springs in the hillside caused a slip near the top, the material blocking two of the four tracks.

Two steam shovels mounted on flat cars were sent to the work, laborers secured by ropes tied to trees being stationed at the top to shovel down the loose earth. After a few days a fire hose was led over from the top and an attempt was made to wash the dirt down.



INCLINED CHUTE USED IN CLEARING LANDSLIDE

This proved unsatisfactory, but the steam shovels had cleared the tracks and drying weather had set in, so that there was no immediate danger of further sliding. The engineers in charge of the work finally decided upon a system of removing the loose material by hand and passing it down an inclined chute, a ¾-in. jet of water being used simply as a lubricant. This method proved satisfactory and was carried on during the summer, when conditions were favorable.

The steel inclined structure of the Monongahela Inclined Plane Co. is near the slip and was utilized to support the chute, which was 396 ft. long and had a slope of 1 on 3. The construction is shown in the accompanying drawing. For the upper 216 ft. of the length, timbers 3 x 6-in. were placed in the track of the incline, projecting on one side about 5 ft. beyond the ends of the ties and carrying frames for the box chute. For the lower 180 ft. the chute was suspended beneath the deck of the incline, the frames being spiked to timbers resting on the flanges of the steel floor beams. The chute was a closed box of ¾-in. tongued and grooved flooring, but as this was rough enough to cause clay to adhere to the sides it was afterward lined with sheet iron. With this lining and the lubricating effect of water operation was successful.

Traps or movable doors in the top provided for dumping the material into the chute, twelve men with wheelbarrows being engaged in this work, and loading on an average of two cars daily. These openings were useful also in removing occasional stoppages when the material consisted mainly of clay. At the lower end of the chute a bottom trap delivered the material into the