UNITED STATES
DEPARTMENT OF THE INTERIOR
BUREAU OF RECLAMATION

HYDRAULIC LABORATORY REPORT NO. HYD-2.9

Technical Memorandum No. 388

THE FLOW NET AND THE ELECTRIC ANALOGY

by

E. W. LANE,
F. B. CAMPBELL AND W. H. PRICE

Denver, Colorado
July 1, 1934

HYD 2.9

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DEPARTMENT OF THE INTERIOR

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MEMORANDUM TO CHIEF DESIGNING ENGINEER

SUBJECT: THE FLOW NET AND

THE ELECTRIC ANALOGY

By

E. W. LANE, RESEARCH ENGINEER,

F. B. CAMPBELL, ASSISTANT ENGINEER AND

W. H. PRICE, JUNIOR ENGINEER

TECHNICAL MEMORANDUM NO. 388

Denver, Colorado

July 1, 1934

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April 1, 1934

MEMORANDUM TO CHIEF DESIGNING ENGINEER

by E. W. Lane,
Research Engineer

Subject: The Flow Net and the Electric Analogy.

The following discussion of the flow net and electric analogy has been submitted for publication by the American Society of Civil Engineers, but since it will probably be found useful in this office, and since publication may be delayed, it has been put up, for office use, in the form of a technical memorandum.

THE FLOW NET AND THE ELECTRIC ANALOGY

E. W. LANE, F. B. CAMPBELL, W. H. PRICE
RESEARCH ENGINEER, ASSISTANT ENGINEER
AND JUNIOR ENGINEER, RESPECTIVELY

U. S. BUREAU OF RECLAMATION

In the solution of some of the hydraulic problems of the Boulder Dam and of the Public Works projects which were being carried out by the U. S. Bureau of Reclamation, two tools or methods, the flow net and the electric analogy have been used which were found to be very helpful and will no doubt be extensively used in the future. These methods have been employed to some extent in hydraulic work in Europe, and in this country for certain problems in other than hydraulic fields, but since hydraulic engineers in America do not seem to be generally acquainted with them, it is believed that the following account of their principles and application will be of value.

The flow net, as applied to hydraulics, is a way of representing graphically the nature of the flow of water, which permits the accompanying velocities and pressures to be computed for many conditions which heretofore have usually been determined by measurement on hydraulic models. In other words, it makes the solution of certain problems possible in the office which formerly could be solved only in the hydraulic laboratory. The electric analogy is an analogy between the flow of water and the flow of electricity in a conductor of similar boundaries, By the use of this analogy, flow nets can be readily constructed for a great variety of simple and even complicated cases.

THE HISTORY AND THEORY OF THE FLOW NET

The original and mathematical theory of the flow net rests upon the foundation of differential calculus laid down by Newton 250 years ago. The equations of motion and of continuity are based upon the work of the great contemporaries, Euler, LaGrange and Laplace, during the 18th century. Later Gauss, Stokes, William Thompson, Tait, Helmholtz, Maxwell and a host of other mathematicians and physicists enriched our knowledge of the theory and its application.

The principles of the flow net have been used to solve a variety of problems. The magnetic fields surrounding the poles of an armature have been studied by means of a flow net**. It was used by Hinderks*** for the investigation of pressure distribution in siphon spillways. The flow net was

** A. D. Moore-Mapping Magnetic and Electrostatic Fields-Electrical Journal Vol. XXIII No. 7 pp 336-362.
*** Stromungsuntersuchung on Selbstatigen Sauguberfallen-Hanover 1928.

also used at Hanover for the problem of flow under roller dams. H. Kulka records the results in an excellent treatise of the subject****. It seems first to have been introduced in this

****Der Eisenwasserbau Vol. I

country by Dr. Freeman in his summaries of these reports.*****

*****Hydraulic Laboratory Practice pp 605-618

Some work has been done with the flow net at the University of

Iowa. It has been used extensively in studying the flow of water under dams by Forcheimer, Pavlovski, Leliavsky, Harza, Terzaghi, and others. So far as could be discovered, the first application of the electric analogy to the solution of hydraulic problems to be published was by Pavlovski, although Mr. H. B. Muckleston and the late Mr. E. E. Sands, used it many years ago to investigate flow through earth dams.

The more general conception of the flow net is sometimes called conformal representation. LaGrange studied the mathematics of this geometrical device and applied it to geographical maps. The mathematical theory is treated under the subject of conjugate functions. In conformal representation there are two families of curves. The curves of one family always intersect at right angles to those of the other family. The geographical map offers a good example; where the meridians represent one family, and the parallels of latitude the other family. In two dimensional hydraulic flow, one family of curves is called the stream lines and the other, the equi-potential lines or simply the "potential" lines. In the propogation of light and sound waves from a point source, the concentric spheres representing surfaces of equal intensity are one family while the radial lines from the point source are the other family. Thus the general conception covers many fields of applied science. The theory of conjugate functions has long been used in electri-

city and is beginning to come into general use in the field of structures. Lines of equal rate of change of stress are studied by polarized light, the slab analogy and the soap film analogy.

This article will be confined to the hydraulic flow net and the electric analogy. The hydraulic nomenclature will be used. The stream lines are briefly the paths which the particles of flowing water trace. To consider only two dimensions, a strip bounded by two stream lines give the same discharge at the inflow end as at the outflow end. The stream lines are conventionally represented by

$$\psi = \text{constant.}$$

The potential lines are usually represented as

$$\phi = \text{constant.}$$

The lines when plotted on the same sheet form a grid of curvilinear rectangles or, as a special case, curvilinear squares. The equations of continuity for the two families of curves are

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \equiv \nabla^2 \psi = 0$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \equiv \nabla^2 \phi = 0$$

where x and y are the rectilinear coordinates. The symbol ∇ is called "nabla". These are known as the Laplacian equations and as indicated in an above paragraph, have many applications.

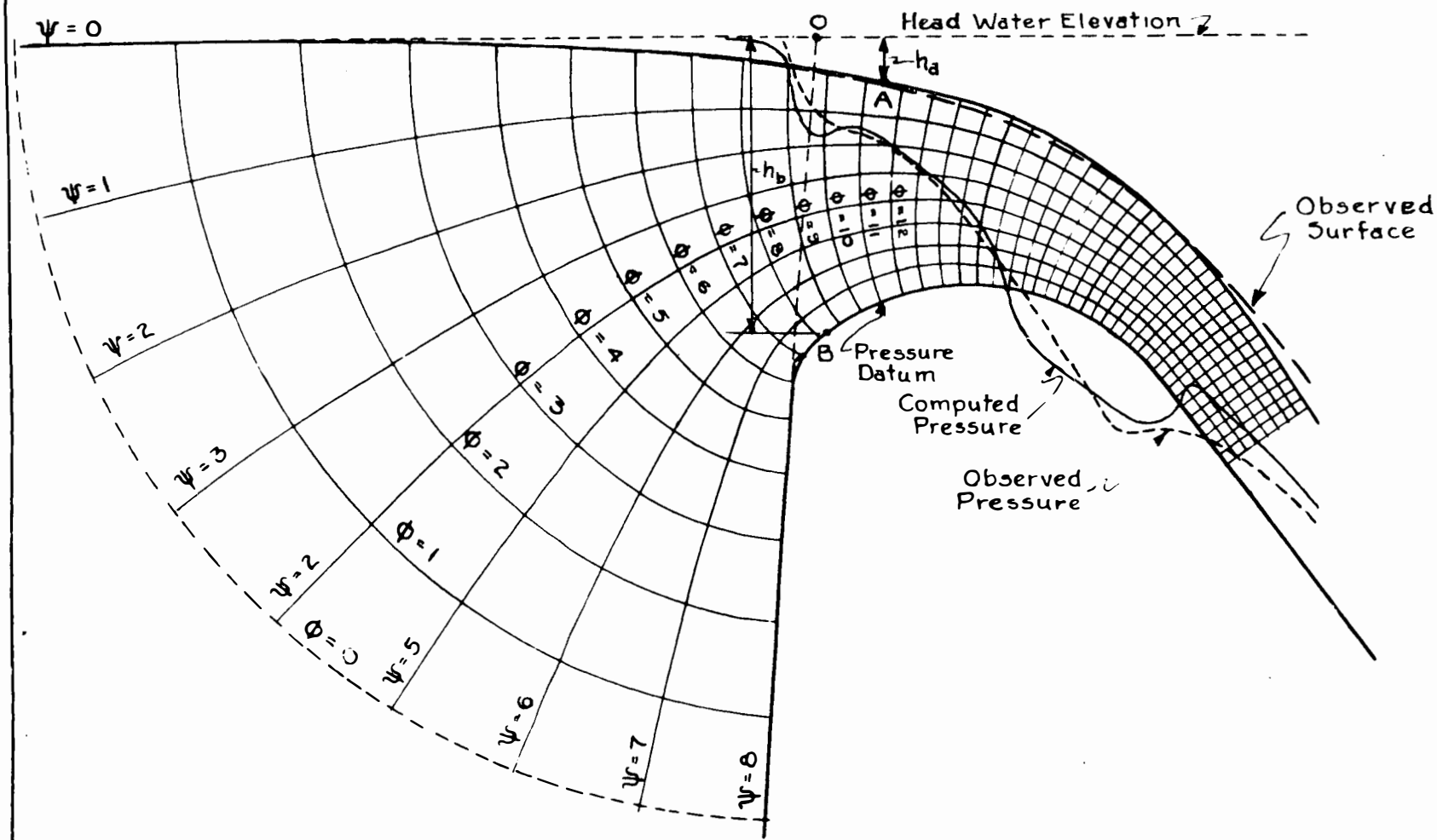


Fig. 1 - Flow Net for Discharge over a Dam

XD-991

By mathematical reasoning, it can be shown that the families of curves represented by ψ and ϕ intersect at right angles to each other. It can be further shown by a vector interpretation of Stoke's integral equation that the ratio of length to breadth of the curvilinear rectangles is a constant throughout the flow net.*

* A paper on the mathematical theory by F. B. Campbell is filed in the Society library.

SOLVING HYDRAULIC PROBLEMS WITH THE FLOW NET

The flow net can be applied to the solution of a variety of hydraulic problems. Its use in determining the flow conditions in the cases of siphon spillways and roller dams has already been mentioned. It has been used by the Bureau of Reclamation to determine the pressure on the crest of an overflow dam. Figure 1 shows a flow net developed by Mr. Clarence Rawhouser, Junior Engineer, for one shape of overflow dam and the pressures on the crest which he predicted by means of it. The piezometric pressure found at any point on the crest is indicated by the height of the pressure lines above or below the crest at that point, heights above the crest being positive pressures and below being negative pressures respectively. A model of the crest was also constructed and the shape of the nappe and the pressures on the crest were observed. The agreement of the predicted and observed results is very striking. Similar flow nets have been constructed to determine the pressures exerted on drum gates on a dam crest, to aid in their structural design.

The flow net is also useful in studying the flow of water under masonry dams on pervious foundations. Figure 2 shows a net constructed to show the flow under a dam on a homogeneous foundation material. By means of such a net the action of the water may be more clearly visualized. A series of tests are under way to study in a similar manner the flow through various types of earth dams.

GRAPHICAL CONSTRUCTION OF THE FLOW NET

A flow net may be constructed graphically by a rather laborious procedure, without resorting to the use of the electric analogy. Theoretically such a net might be employed in the solution of both two and three dimensional flows, but the complications introduced by the third dimension are so great that the graphically constructed net is practically limited to a two dimensional analysis. The flow of water over the crest of an overflow dam, as shown on Fig. 1 is typical of problems which can be solved by a two dimensional analysis.

To construct such a flow net the upstream face of the weir is projected upward to the intersection with the headwater level at point O. The potential line $\phi = 0$ is then drawn from the upstream face of the weir to the water surface, as a segment of a circle with C as the center. Theoretically the radius should be infinite, but practically a radius two or three times the head is sufficient.

The flow net may be started from the potential line $\phi = 0$. The net can be made up of curvilinear rectangles but the curvilinear

squares are found most convenient. The line $\phi = 0$ should be laid off in equal divisions. Later, it may be necessary to adjust these divisions. They are shown on Fig. 1 from $\psi = 0$ to $\psi = 8$. The net should be sketched roughly in its entirety from beginning to end. Much time can be wasted in attempting to perfect a single square or small group of squares before proceeding to the remainder. By a cut and try process it is then adjusted until it meets all the required conditions. Since this involves a great many changes in the position of the lines, heavy paper which will stand repeated erasures should be used. The speed with which a net can be sketched increases rapidly as one gains a little experience in the procedure.

If the boundaries of the flowing water are exactly known, the problem is greatly simplified. If not they should be sketched in as accurately as possible and can be located more closely as the refinement proceeds. In working with the flow of water over weirs or dam crests, the classical experiments of Bazin* will be found very useful in helping to estab-

* Recent Experiments on the Flow of Water over Weirs--Translated by A. Marichal and J. C. Trautwine--Proc. Engrs. Club of Phila. Vol. IX pp 231-238-1892 and Vol. X pp 136-164-1893.

lish the boundaries. Where the boundaries are not definitely located certain conditions are often known which aid in fixing them. For example, in the flow over a weir, or dam crest the velocity at any point in the free surfaces is equal to the velo-

city head corresponding to the distance of that point below the reservoir level. (If the velocity of approach to the weir is large, the head corresponding to this velocity will of course have to be added to the reservoir level). By adjusting the net until the free surfaces at all points fulfill this condition, their position can be accurately located. In other words, these relations become a condition which the adjusted flow net must fulfill, in addition to the requirements necessary when the boundaries are definitely fixed.

In adjusting the net after the first trial, some engineers prefer to use diagonals which extend continuously through the squares. This diagonal net of curvilinear squares possesses the same properties as the original net.

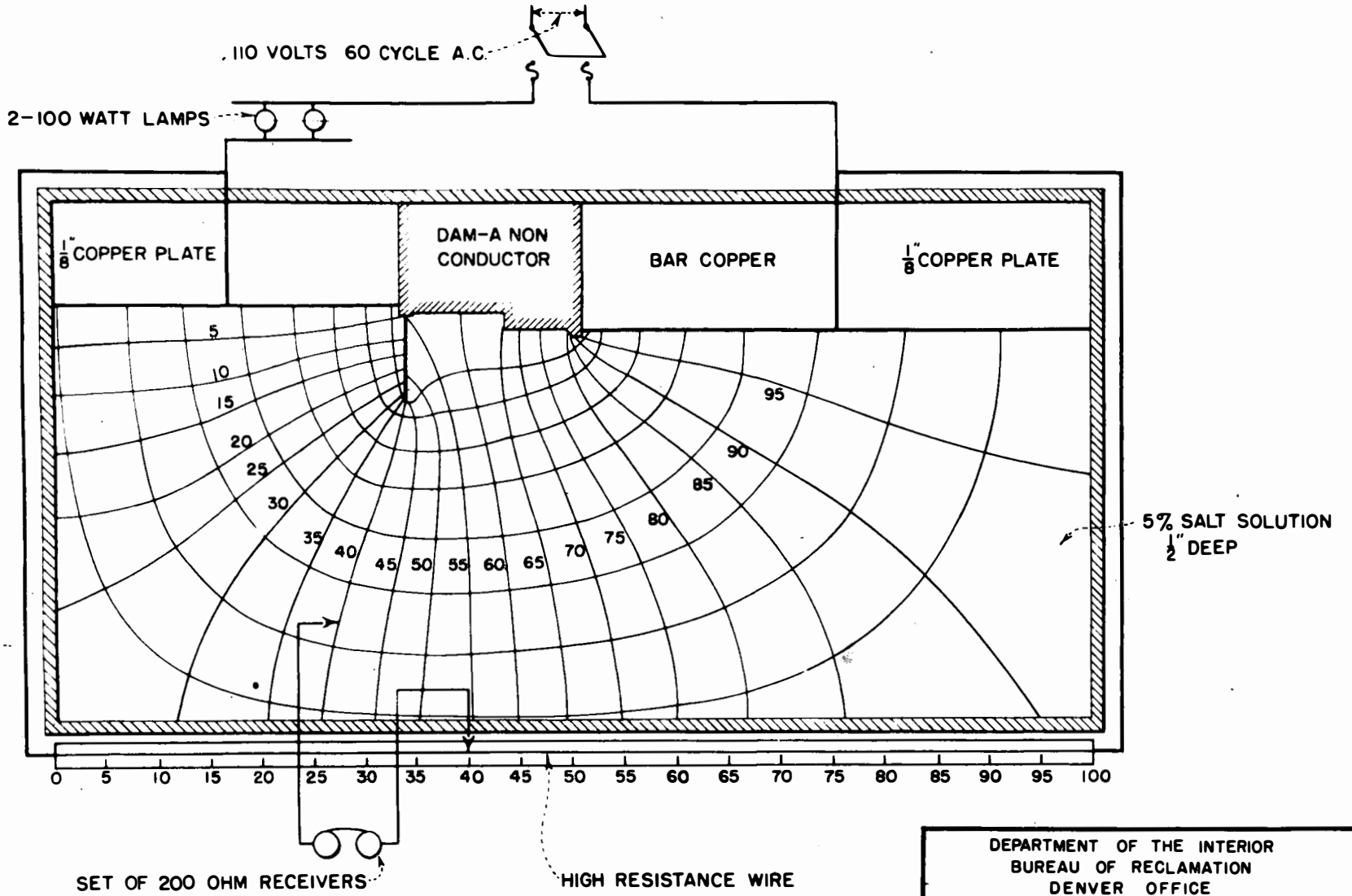
In order to compute the pressures at a certain point, two general principles are employed; First, the total energy at all points is the same and is composed of elevation, pressure and velocity heads; second, the velocity at any point is inversely proportional to the length of the side of the curvilinear square at that point. It is necessary first to establish the scalar value of velocity on the flow net. This can be done at the free water surface if its location is accurately known, since the velocity at any point A in the free water surface is that corresponding to its distance below the reservoir level. The velocities in all other points of the net can then be computed, since they are to the velocity at A in the

inverse proportion of the sides of the squares at these two points. If the position of the free water surface is not accurately known the velocity may be approximately established and refined as the adjustment to the known conditions is made by the cut and try process. The velocity can also be computed if the discharge is known, since equal discharges are included between each set of stream lines. If the discharge coefficients can be closely estimated, the velocities can also be accurately computed.

Having determined the scalar value of the velocity, the pressure P_b at any other point B (Figure 1) may be computed by the formula $P_b = h_b - \frac{V_b^2}{2g}$. The head h_b is measured down from the reservoir elevation and V_b can be computed from the distance between the points ϕ_B, ψ_B and ϕ_B, ψ_9 using the flow net for coordinate designations.

CONSTRUCTING A FLOW NET BY THE ELECTRIC ANALOGY

The flow net as developed by the electric analogy and that derived by the more laborious graphical method are identical. The electric analogy apparatus may take many forms. One of the simplest used is shown on Fig. 2 which was set up to study the flow through the soil beneath a structure acting as a dam. It consists of a shallow trough with plate glass bottom and sides, provided with means for accurate leveling, and filled to



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ELECTRIC ANALOGY APPARATUS

DRAWN W.H.P. SUBMITTED
TRACED D.S.Y.-O.R.S. RECOMMENDED
CHECKED APPROVED

DENVER, COLO., APRIL 24, 1934

X-D-1010

a depth of say 1/2 of an inch with an electrolyte, usually a solution of common salt. At each side of this "dam" were placed copper terminals at the position of the contact of the headwater and tailwater with the foundation material. These were connected to a 110-volt alternating current source thru a lamp bank which reduced the voltage drop across the terminals to about 18 volts. At the opposite side of the tray over a meter scale, was a high resistance wire, one meter long, each end of which was connected to one of the terminals by a heavy copper wire. A spring was necessary at one end of this wire to keep it in tension under all conditions of temperature introduced by the current flowing thru it. On this resistance wire was a sliding contact connected thru a set of earphones to a probing pencil. In passing from one terminal to the other the current followed paths as shown by the light line connecting the terminals. The heavier lines crossing these represent lines of equal potential plotted for increments of 5% of the total potential drop. It was the purpose of an apparatus to determine these lines of equal potential. To determine the position of any desired potential line, the sliding contact on the resistance wire was placed at the point giving this potential and the probing pencil moved around in the salt solution until the telephone receivers indicated

that no current flowed thru the wire from the sliding contact to the probing pencil. The position of this point was observed by means of a system of coordinates ruled on a paper, placed beneath the bottom of the plate glass tray. For example, suppose it was desired to find the position of the potential line representing a drop of potential from the "headwater" terminal of 40% of the total drop between the terminals. The sliding contact was placed on the resistance wire at 40% of its length or 40 c.m. from the end connected to the "headwater" terminal. Since the resistance of the heavy wires between the terminals and the ends of the resistance wire was negligible, and the resistance wire was uniform in cross section and resistivity, the resistance from the headwater terminal to the sliding contact was 40% of the total resistance between terminals and therefore the potential drop to this point was 40% of the total potential drop. When the probing point was placed at any position in the salt solution a current would flow from the resistance wire to the solution or visa versa if there was a difference of potential between these points. When the probing point reached a point where the hum in the receiver stopped, it indicated no current flowed and that the potential at the probing point was the same as that at the contact point on the resistance wire; namely, a 40% of the potential drop. By determining the coordinates of

several points of equal potential and joining them with a line, the position of the line of equal potential could be plotted. If it was desired to determine the potential at any designated point in the salt solution, by placing the probe at this point and moving the sliding contact until it reached a point of equal potential as indicated by no flow thru the receiver, the potential of the probing point could be determined from the position of the sliding contact. Experience with this apparatus showed that potential lines could be located with a high degree of accuracy. To check it an experiment was performed with the tray clear and terminals entirely across each end. The potential lines with this setup for equal potential increments should be straight, equally spaced lines parallel to the terminals; and the results obtained agreed very closely with their theoretical position.

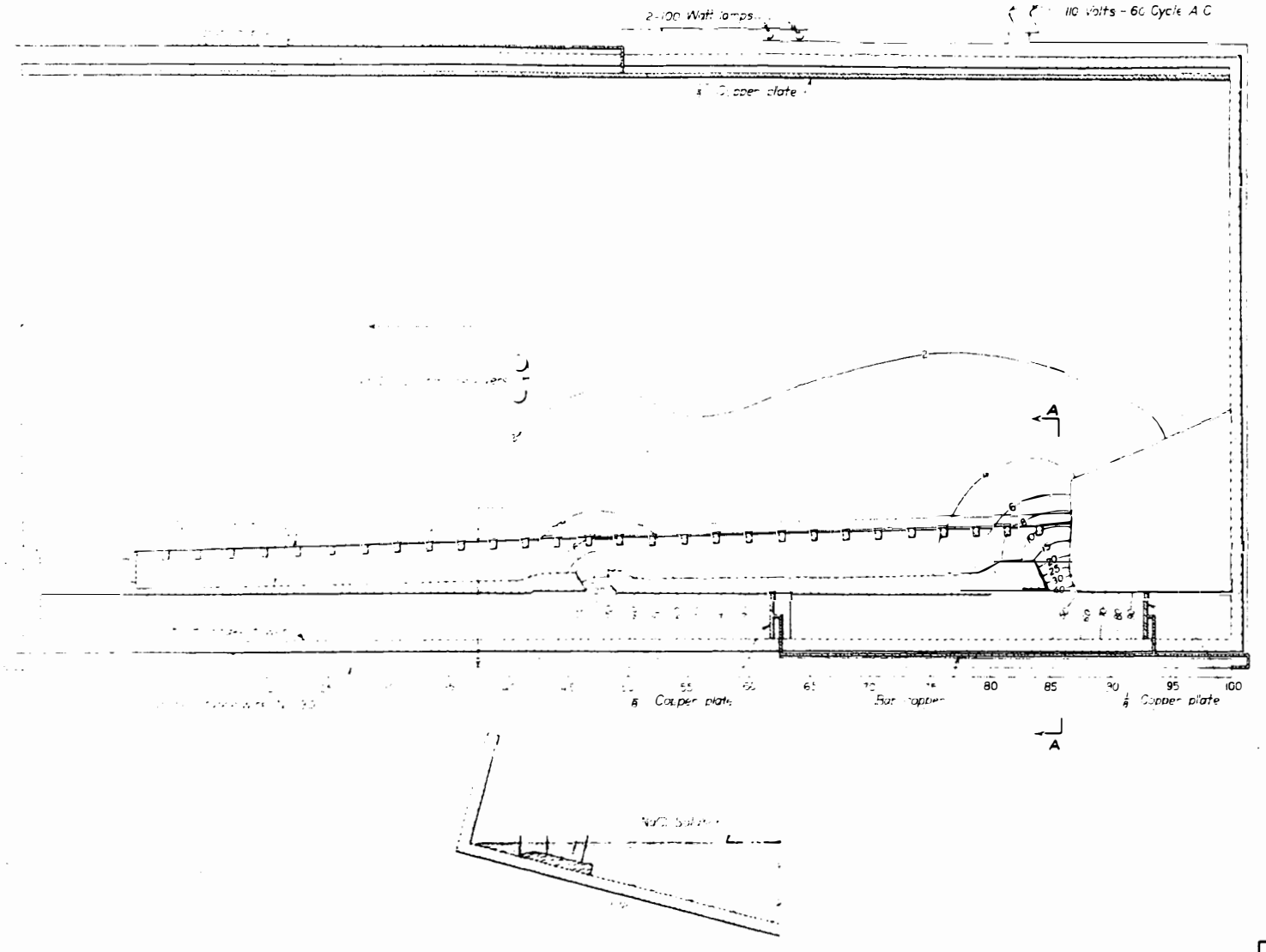
Having obtained a complete set of potential lines, the flow lines could be drawn in graphically with little difficulty. They could however be constructed electrically by changing the terminals, making the dam and sheet piling of metal to form one terminal, and placing along the bottom and side edges of the tray a metal strip to form the other terminal, the former two terminals being replaced by a non-conductor. With these new terminals the set of potential lines would be identical with the flow lines for the previous terminal setup.

Many variations in the electrical setup are possible. By using a longer resistance wire a higher voltage can be used which permits a more accurate location of the potential line. The voltage limit is that which will not cause sufficient current to flow thru the resistance wire to heat it unduly. The concentration of the electrolyte can be changed to keep the current flowing thru it to a reasonable amount. With vacuum tube amplification a loud speaker may be used instead of the telephone receiver. Sheets of very thin tin foil have been used in place of the electrolyte. Their shape can be more easily altered than the shape of the electrolyte, but their resistance is so low that in order to prevent heating a very low voltage across the terminals must be used and a very sensitive detector is required. A pantographic arranged to plot the position of the probing point on a sheet of paper will materially reduce the labor of graphically presenting the results.

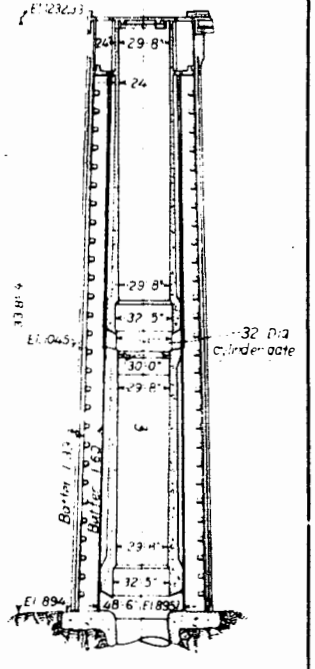
The electric analogy apparatus has several advantages over the graphical method in addition to the fact that it often permits a more rapid construction of the flow net. In the case of flow beneath dams, as illustrated by Figure 2, it gives the pressures directly. Since the mathematical expression for the law of flow thru an electrical conductor has the same form as that for the flow of water thru soils at ordinary velocities the drop in electrical potential in the analogy

apparatus will represent directly the drop of piezometric pressure in the water flowing under the dam. Thus in Figure 2, the potential lines can represent directly the percentage of the total piezometric drop from the headwater to the tail-water, as well as the percentage of the potential drop between the two electrical terminals.

One of the greatest advantages of the electrical analogy is that it permits a solution of the problems with three dimensions. For example, in the design of the intake towers for the Boulder Dam it was desired to determine the direction of the currents approaching the gate openings in order to reduce the obstruction offered by the structure supporting the trashracks to a minimum. No practical way was found to do this on a hydraulic model of reasonable size. A model of a sector of the tower and canyon wall was set up with a salt solution representing the water space and a non-conductor representing the tower Figure 3. The division of flow between the two gates, as determined by hydraulic experiment was simulated by a corresponding division of the electrical current, secured by adjusting the position of the electrode D. The potential lines shown on the figure, were determined by means of the analogy. The direction of flow at any point is at a right angle to these lines. Other three dimensional problems which can be attacked, with the electrical analogy are the pressures beneath dams in



SECTION A-A



SECTION ON ℄ OF TOWER

DEPARTMENT OF THE INTERIOR
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BOULDER CANYON PROJECT

BOULDER DAM

INTAKE TOWER STUDIES
ELECTRIC ANALOGY MODEL

DRAWN	A. H. P.	SUBMITTED
TRACED	D. S. Y.	RECOMMENDED
CHECKED		APPROVED

DENVER, COLO. APRIL 16, 1934

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a narrow canyon, or the effect of drainage holes beneath a dam in such a situation, and the flow of water around the abutments of dams on pervious foundations.

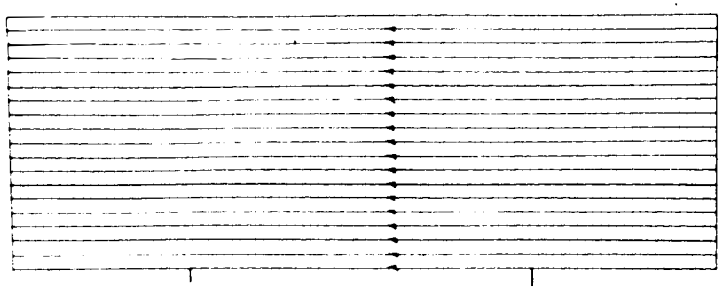
ANOTHER METHOD OF FLOW NET CONSTRUCTION

A method of developing a flow net is used by aerodynamical engineers known as the method of sources and sinks.*

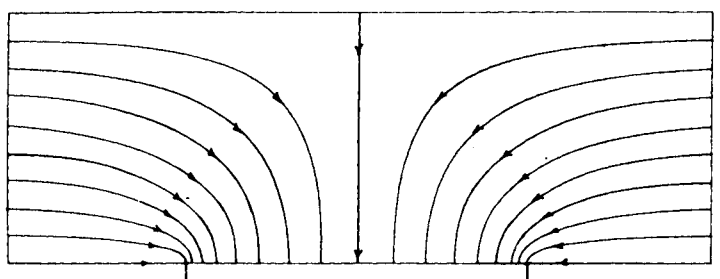
* Applied Aerodynamics - Leonard Barstow P. 351

Its application to the problems of practical hydraulics is limited. In connection with it however there has been developed a method of combining flow nets which will be found to be valuable in the solution of certain problems. For example, the flow in a rectangular flue with a branch taking off at right angles may be analyzed as a combination of a flow straight down the flume (Figure 4A) and a flow out of the branch, drawing water in both directions from the main flume (Figure 4B). The directions of the water currents in the combined flow may be found by superimposing the two stream line systems and drawing the lines joining the intersections of the stream lines of the two systems as shown on Figure 4C. It will be seen that the combined flow direction at any point is the resultant of the directions in the two simpler conditions. The agreement between the currents thus predicted and that actually observed for a case experimented upon at the Institution of Hydraulic Engineering in Vienna**, is shown on Figure 4D.

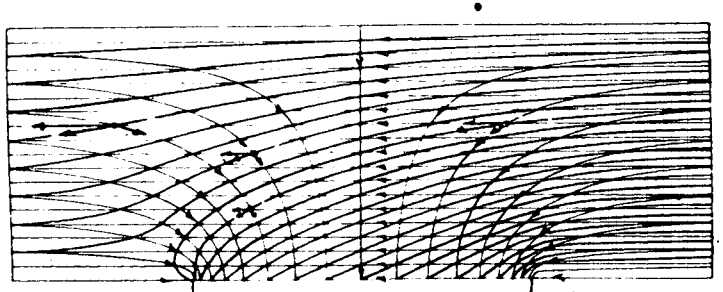
** Hydraulic Laboratory Practice - Freeman Page 428



A



B



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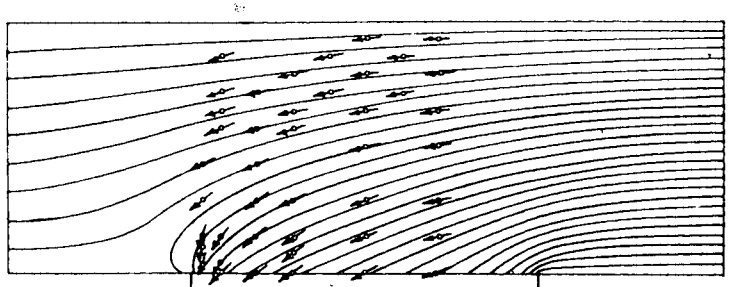

D

NOTE: ARROWS SHOW DIRECTION OF STREAM LINES DETERMINED BY EXPERIMENT.

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METHOD OF COMBINING
FLOW LINES

DRAWN: E.J.H. SUBMITTED:
TRACED: E.J.H. RECOMMENDED:
CHECKED: APPROVED:

DENVER, COLO. 2-3-38 XD-992

LIMITATIONS OF FLOW NET SOLUTIONS

The theory of the flow net, as applied to hydraulic problems, assumes the existence of a perfect fluid. Water is not a perfect fluid, but in many cases its action is so nearly that of a perfect fluid that no appreciable error is involved in the assumption. It can therefore not be applied where friction or impact losses are appreciable. Where water is accelerating, for example, as flow over a weir or dam crest, thru an orifice or under a gate, the flow net analysis can be used with considerable accuracy, but where the water is decelerating impact or eddy losses are more likely to be set up and the results are apt to be less reliable. Flow thru sudden expansions, such as would be likely to cause eddies, could not be accurately analyzed. The accuracy of the agreement of the flow net analysis with the flow of the water in the actual case will depend upon the closeness with which the flow of the water approaches the action of a perfect fluid. This should be kept in mind in deciding on the probable reliability of a flow net analysis.

ACKNOWLEDGEMENTS

In its use by the Bureau of Reclamation of the flow net and electric analogy for the solution of hydraulic problems, assistance has been given by many persons, among whom are the following: R. E. Glover, Clarence Rawhouser Jun. Am. Soc. C. E.,

F. H. Sweaney Assoc. M.A.I.E.E., Prof. R. W. Powell, Assoc. M. Am. Soc. C. E., Ford Kurtz, M. Am. Soc. C. E. and, J. Danel.

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