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THE PERCOLATION OF WATER THROUGH EARTH DAMS

by N. N. Pavlovsky

Translation from Russian By
ANDREAS LUKSCH, DIPL. ING. M.S. Univ. of Iowa
Wm. F. BINGHAM, U. S. Bureau of Reclamation

Denver, Colorado
June 1, 1934

UNITED STATES

DEPARTMENT OF THE INTERIOR

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MEMORANDUM TO THE CHIEF DESIGNING ENGINEER

Translated from Russian

SUBJECT: THE PERCOLATION OF WATER THROUGH EARTH DAMS

BY N. N. PAVLOVSKY

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THE PERCOLATION OF WATER
THROUGH EARTH DAMS

PROF. N. N. PAVLOVSKI

CHAPTER I.

1 The hydraulic scheme of percolation through a homogeneous earth dam on an impervious foundation

We shall start the discussion of the hydraulics of percolation through a homogeneous earth dam by saying that the flow in the body of the dam can be subdivided into three charactoristic parts I, II and III, which are designated on the sketch 1. These three

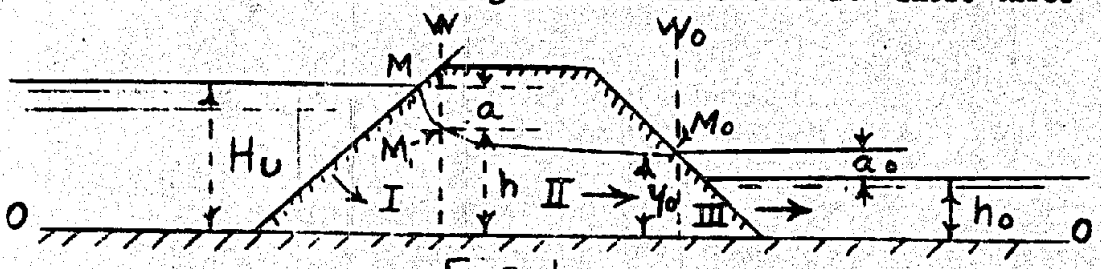

Fig. 1

parts may be called "upper wedge", "middle part" and "lower wedge." We shall point out the main hydraulic properties of the percolation flow through these parts in the following manner:

1. The "upper wedge". By percolating through the upstream face of the dam, the percolation flow enters that part of the dam which we called "upper wedge". We understand by this name that part of the dam, which is between the upstream face of the dam, and the "section line" W. (1) (foot note: About this section line see in paragraph 2) after which we find the second (middle) part of the percolation flow. While the percolation flow moves through the upstream face of the dam of course hydraulic resistances are developed. In order to overcome these the flow will have to lose a certain amount of its energy. Therefore the flow enters the second section already with a diminished specific energy (or under a smaller head).

For the underground flow the kinetic energy is only a negligable quantity as compared to the potential energy. In our case, therefore, the specific energy (or the head) in entrance section, i.e. in the plane of the upstream face, is equal to the depth of the water upstream from the dam H_u. The specific energy in the intersecting line W is expressed by the depth (h) of the percolation flow at this point. The heads (depths) are measured from the plane of reference which in the sketch is designated by the line O---O.

$$\text{Eq.(1)} \qquad a = H_U - h$$

The loss of specific energy (or the loss of head) within the boundries of the "upper wedge" therefore, can be expressed by the head (a) which is drop of the percolation curve from the point M to the point M_1 :

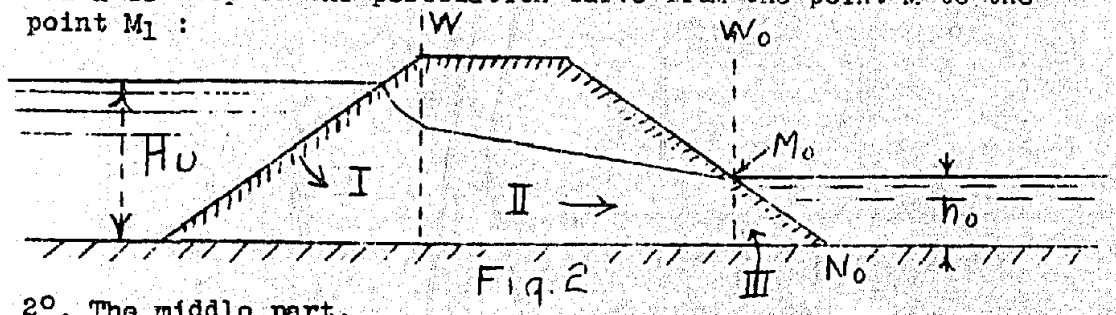

Fig. 2

2°. The middle part.

From the section line (W) on the nonuniform movement of the percolation flow on the impervious ground is beginning; throughout the whole following discussion we assume the surface of the impervious ground to be level; this corresponds to the usual condition for the actual structure in practice. The depression curve in the "middle part" of the dam is obviously the curve of the drop of the percolation surface. The depth of the percolation flow is gradually decreased from the value h (at the section W, Fig. 1) it decreases to the value y_o at the section W_o, which passes through the point M_o; i.e. through the point where the percolation flow penetrates the downstream face of the dam. For the hydraulic discussion of the problem we assume the point M_o to be higher than the elevation of the tail water level - the latter will be discussed immediately as we proceed to the "lower wedge."

3°. The "lower wedge".

If the point M_o of the depression curve could be at the elevation of the tail water surface (as is shown in Fig. 2) then in the section W_o, which designates the end of the non-uniform movement of the percolation flow in the "middle part", we would have to calculate with the head h_o. But obviously the same value of the head has to be taken for the discharge section M_o-No (Fig.2), which is in the plane of the downstream face of the dam. But if this is so we would arrive at the conclusion, that it is impossible for the water to flow from the section W_o to the section M_o- No. Actually however, such a movement takes place. The reason for this discripancy is, that in order to overcome the hydraulic resistances within the boundries of the downstream reach (that is within the boundries of that part of the dam which is below the section W_o) the percolation flow has to have some additional energy as compared to its amount in the discharging section.

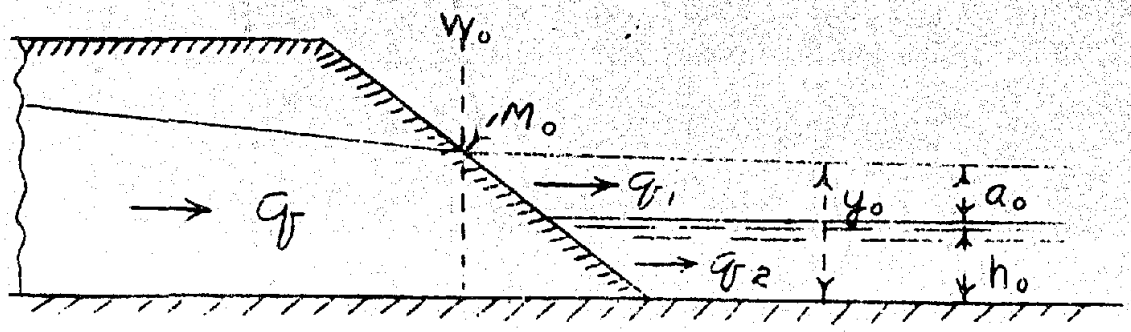

Fig 3

We have surplus energy if we assume the point M_o (like in Fig. 1) to be at an elevation which is above the elevation of the tail water level. If we designate this height of the point M_o above the tail water level by (a_o) Fig. 1. we have

(2) $y_o = a_o + h_o$

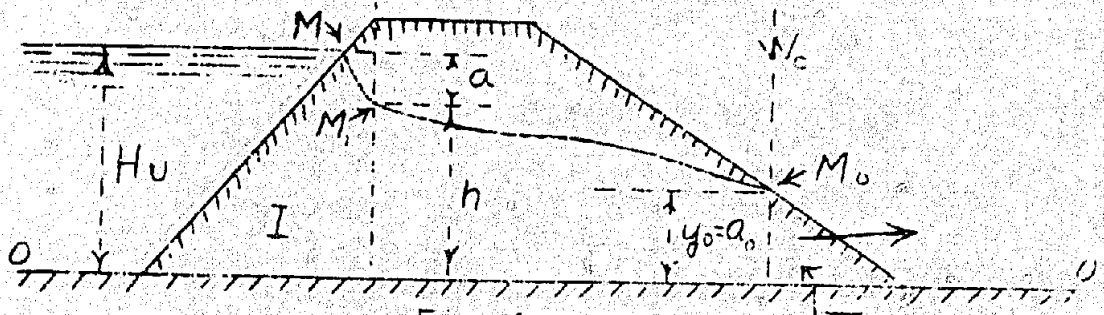

Fig. 4

For this we obtain two zones in the percolation flow within the boundries of the downstream reach (Fig. 3). In the upper zone the percolation discharge q_1, discharges into the atmosphere and in the lower zone the percolation discharge q_2 is discharged under the tail water level. Of course we have the relation

(3) $q = q_1 + q_2$

where q is the total unit-percolation discharge through the dam, i.e. the discharge per unit length of the dam (q_1 and q_2 of course also are unit discharges).

Throughout the whole previous discussion we dealt with a case where the depth in the depth of the water below the dam was $h_o > 0$, but is practice the case $h_o = 0$ is much more frequent. That is the case in which we do not have any water in the tail race channel directly below the earth dam (or no tail race channel). For this case of course the above discussed conditions in the "upper wedge" and the "middle Part" remain unchanged. For the "lower wedge" we find however, that the elevation of the discharge point (of the depression curve) Mo on the downstream face of the dam above the line O---O is equal only to (a_o), which is the head which enables the percolation flow to overcome the hydraulic resistance. Hence we have instead of equ. (2):

(4) $y_o = a_o$

This is illustrated in Fig. 4, which shows a schematical diagram of the filtration through an homogeneous earth dam for $h_o = 0$.

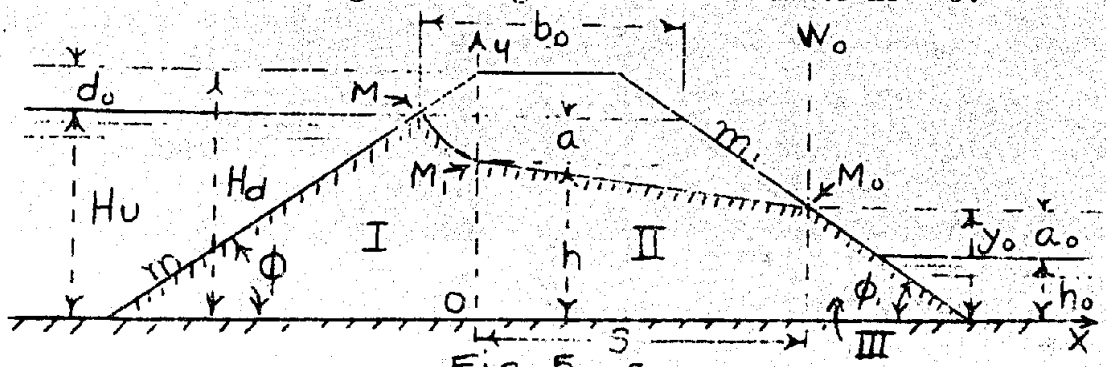

Fig. 5

2. Continuation

In continuation of the above discussion let us take up the question of the direction at which the percolation flow threads enter the boundries of the upstream reach. The line of the upstream face is the line of equal heads or the equi-potential line, since (see for example Fig. 5) the heads on this line are all equal and all have the value Hu, if they are measured from the horizontal plane of reference which are assumed to go through the abscissae axis. But since it is known that the equi-potential lines are normal to the stream lines it is clear that the stream lines of the percolation flow have to be normal to the line of the upstream slope at the point where they enter the upstream reach of the dam through its wetted surface. Further on in the upstream reach the stream lines of the percolation flow do of course change their direction. When they enter the middle part of the dam profile they have already more or less the main translative direction of the flow in the region of the ununiform motion of the flow. What has been said about the stream lines of the percolation flow is also true for the "depression curve", which is only the uppermost stream line of the discussed percolation flow.

The above pointed out entrance direction of the percolation flow, perpendicular to the upstream face of the dam is well illustrated in the Photograph No. 1. On this picture we see the beginning parts of the flow lines which were obtained by coloring during the experiments of the Leningrad Hydraulic laboratory on the dams of the lower Sviv water power plant. The same behavior at the percolation stream lines was much earlier (in the year 1908) observed by Prof. Rehbock in his experiments in the "Old" Hydraulic Laboratory of the Technical University in Karlsruhe on the models of earth dams (1)(see: "Die Wasson bau Laboratories Europae " Berlin, 1926, page 100-102(Freeman - The Hydraulic Laboratory Practice)) It should be mentioned that this behavior of the stream lines really does not have to be proved by any experiments since it is fully explained by the laws of Hydro-mechanics.

Let us now discuss the question of the separation sections W and Wo which are shown and used in Fig. 1-4. The location of the latter section (Wo) is one of the unknown quantities of the problem and we shall determine it further later. The section W is in Fig. 1-4 drawn through the upstream corner of the level surface of the dam, and this location of the line (W) has to be discussed.

For the experiments on earth dams which were carried out under our supervision during the last years, it was found that in the dam body the transition of the percolation flow into the slowly non-uniform movement, i.e. from the upstream reach into the middle part of the dam profile takes place sufficiently close to the section W, which is used in Fig. 1-4 (1) (This fact also is confirmed by the above mentioned experiments of Rehbock). The above mentioned experiments however, were not planned for the investigation of just this question. We therefore, find it necessary to estimate to what extent this or that location of (W) could influence the

obtained results for the computation of the depression curve. This has been done further below in paragraph 9. Here we think it is sufficient to point out that the very insignificant influence of the location of the line (W), for the computation of the depression curve has been established by the computations of paragraph 9.

Due to these conditions we place the section (W) through the upstream corner of the level part of the dam. Doing this we also have in mind the simplicity of a geometrical interpretation of that line by the elements of dam profile. But let us agree in this place that we select such a location of (W) only for that case when in the upper canal (Fig. 5) the full depth of water is existing. By full depth we shall understand the depth Hu for which the dam construction is planned, and for which the "reserve" height (do) of the dam above the head water level is determined according to practical experience. (1)(on the section (W) for a not filled head water level see paragraph 39.)

In concluding the preliminary remarks on the outline of the percolation flow in a homogeneous earth dam on an impervious foundation, we once more point out, that in the outline given above we regard the percolation flow in an earth dam as the hydraulic system:

Upstream reach + middle part + downstream reach,

for which of course these three parts represent one hydraulic whole (either of these three parts of the system carries one and the same discharge quantity q)

Of course the purely hydraulic approach can only furnish an approximate solution. It is, so to speak, a summarizing way of explaining the existing phenomena, without a detailed characterization of their elements. The further development and completion of the results is, according to their nature, only possible by the help of hydro-mechanic methods.

At the end of this paragraph we give a list of those symbols which we shall use in the derivation of the equations for the percolation flow in a homogeneous earth dam on an impervious foundation, and which are shown on Fig. 5 (which we shall have to use frequently throughout the following discussion).

The height of the earth dam	H_d (H_π)
The depth of the water above the dam	H_u (H_q)
The "security" height of dam over head water level	d_o
The depth of the water below the dam	h_o

The depth of the water below the dam h_o

The width of the plane surface at the dam b

The width of the dam on the elevation of the head water level b_o

The coefficient of the slope of upstream face $\cot\phi = m$

The coefficient of the slope of the downstream face $\cot\phi = m_1$

The loss of head within the upstream reach a

The depth of the percolation flow at the beginning of the middle part of the dam profile h

The depth of the percolation flow at the end of this middle part y_o

The height of the discharge point Mo of the depression curve above the tail water surface elevation ($h_o > 0$) a_o

The height of the point Mo of the depression curve above the bottom of the tail channel (for $h_o = 0$) $a_o = y_o$

The co-ordinates x, y

The section between the upstream reach and the middle part W

The section between the middle and the downstream reach W_o

The abscissa of W $x = S$

The unit percolation discharge q

The mean percolation velocity at a certain point V

The actual percolation velocity at a certain point U

The percolation coefficient of the dam body k

The main points on the depression curve M; M_1; M_o

3. The equation for the middle part.

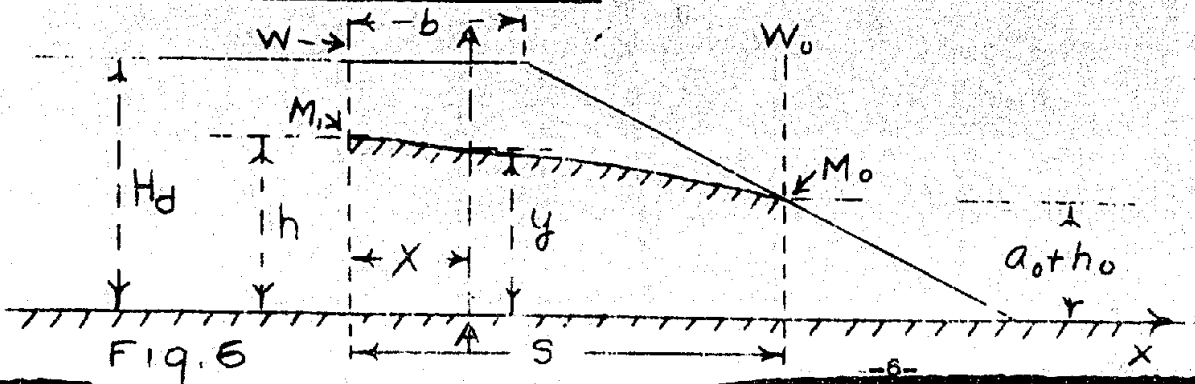

Fig. 6

In the middle part of the dam the motion of the percolation is non-uniform in the direction parallel to the impervious foundation. For the derivation of the differential equation of this flow we use the Fig. 6, which shows only the middle part of the dam.

According to the well known formula of Dupunt, the velocity of the percolation in an arbitrary section A-A is:

(5) $$V = -k \frac{dy}{dx}$$

and hence the unit discharge quantity

(6) $$q = V.y = -k \cdot y \cdot \frac{dy}{dx}$$

or

(7) $$q \frac{dx}{k} = -y \cdot dy$$

and from this we find by integration

(8) $$\frac{q \cdot x}{k} = \frac{1}{2} y^2 + C$$

The integration constant C can be determined from the condition $x = 0$, which corresponds to the ordinate of the depression curve $y = h$.

From equ. 8 we obtain with this

(9) $$C = \frac{1}{2} \cdot y^2 = \frac{1}{2} h^2$$

and

(10) $$\frac{2 \cdot q}{k} x = h^2 - y^2$$

This is already the final form of the equation for the depression curve in the middle part of the dam, and we can easily plot the curve for values of y within the limits

(11) $$h \geqq y \geqq \qquad (a_o + h_o)$$

by solving for the corresponding values of x in equation (10). In order to be able to plot this curve we have of course to know the percolation discharge q, which is for the time being an unknown quantity, but will be found below, during the further discussion

of the problem. For the construction of the depression curve in the middle part of the dam we can also select different values of (x) and determine the corresponding values of (y) from equation (10).

Since for $x = S$ (Fig. 6) we have.

(12) $$y = a_o + h_o$$

we find from (10)

(13) $$\frac{2 \cdot q\ s}{k} = h^2 - (a_o + h_o)^2$$

and

(14) $$q = \frac{K}{2 \cdot s} \left[h^2 - (a_o + h_o)^2 \right]$$

or

(14)[1] $$q = \frac{k}{2 \cdot s} \left[h^2 - y_o^2 \right]$$

Since according to equation (2)

$$y_o = a_o + h_o$$

4. <u>The equation for the "upper wedge"</u>.

Let us keep in mind what had been said about the direction at which the percolation flow enters the "upper wedge" of the dam (in paragraph 2.) on Fig. 7 (a-b-c) designates one of the stream lines of the percolation flow.

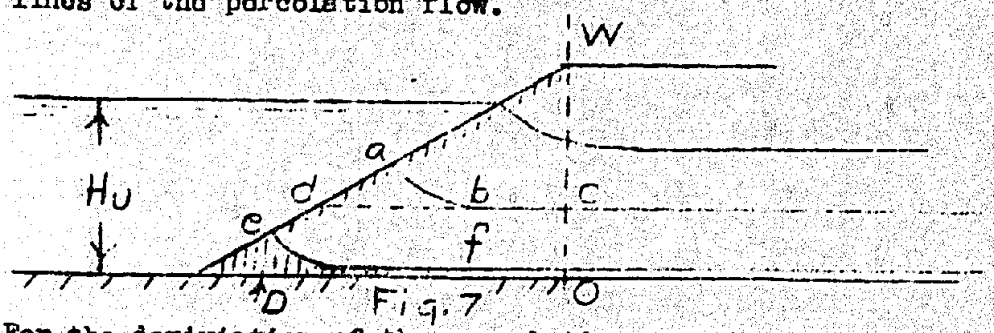

Fig. 7

For the deriviation of the percolation equation of the "upper wedge" we shall replace the curved stream lines of the type (a-b-c) by "assumed" stream lines of the type (d-b-c) which are straight and horizontal. Doing this we do to some extent increase the length of these "assumed" stream lines over that of the actual stream lines, and at the first glance it seems that we diminish the "discharge capacity" (the ability to let a discharge quantity through) of the upper wedge. Therefore, we shall have to discuss this question more thoroughly.

As has been shown by experiments which were carried out in the Hydraulic Laboratory in Leningrad, the lower percolation stream lines (as for example the line e - f on Fig. 7) after entering the upper wedge approach the impervious surface very closely. Thus the part of the upstream face which is below a certain point e has only a very little part in carrying the percolation flow. Therefore the shaded part of the dam D in reality corresponds to a rather impervious part of the dam, and thus diminishes the percolation capacity of the "upper wedge". But this decrease of the percolation capacity is accounted for in our calculation of the upper wedge only in a different way, namely; by a certain increase of the length of the "assumed" percolation stream lines.

To the above said we have to add that along the curves part of the stream line (a - b), the percolation flow has to overcome a slightly greater resistance on its unit length than on the horizontal part. This is due to the fact that the soil of the dam body is brought in in almost horizontal layers which are compacted in a vertical direction. Therefore it is quite reasonable to assume that the resistance along the somewhat shorter curved stream lines (n-) is about equal to the resistance along the longer horizontal stream lines d- b (see Fig. 7).

Of course only a comparison of the theoretical and experimental data can show to what extent the described properties of the actual stream line and the "assumed" stream line are compensated. (1) (foot note: On the very satisfactory results at such a comparison between the depression line and the horizontal stream line see below in chapter VIII.) By the above said we only intended to point out the general motive which lead us to replace the partly curved stream line by the straight line. As we shall see below such a substitution enables us to arrive at an analytical relation for the percolation flow in the upper wedge.

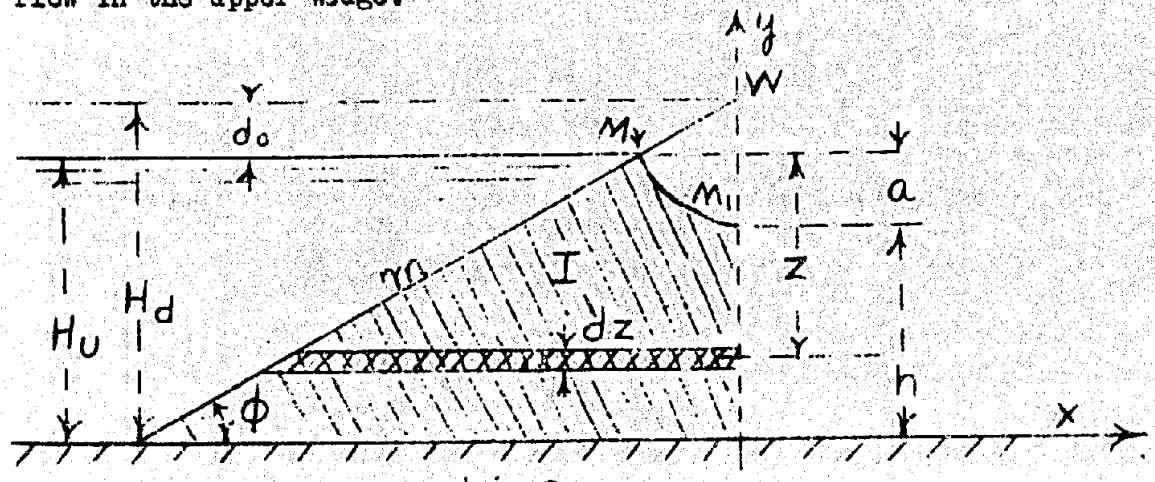

Fig. 8

By assuming the bound ry of the "upper wedge" as pointed out in paragraph 1-2, we regard the "assumed" stream line which is designated by shading in Fig. 8. The length of this stream line is:

(15) $$(d_o + z) \cot\text{an}\ \phi = m\ (d_o + z)$$

where z is the depth of the stream line below the head water level elevation. Hence the hydraulic gradient for the given case is

(16) $$J = \frac{a}{m(d_o + z)}$$

and the velocity of percolation

(17) $$U = k.J = \frac{k.a}{m(d_o + z)}$$

For the differential of the discharge dq we then may write

(18) $$dq = U.\ dz = \frac{k.a.}{m(d_o + z)}\ dz$$

and for the total percolation discharge:

(19) $$q = \int_a^{a+h} \frac{k.a}{m(d_o + z)}\ dz = \frac{k.a}{m} \int_a^{a+h} \frac{1}{(d_o + z)}\ dz$$

or

(20) $$q = \frac{ka}{m} \Big[\log\text{nat}\ (d_o + z)\Big]_a^{a+h} = \frac{K.a}{m} \log\text{nat} \left[\frac{d_o + a + h}{d_o + a}\right]$$

By substituting:

(21) $$d_o + a + h = H_d$$

(22) $$d_o + a = H_d - h\ ;$$

we obtain instead of (20):

(23) $$q = \frac{ka}{m} \ln \frac{H_d}{H_d - h}$$

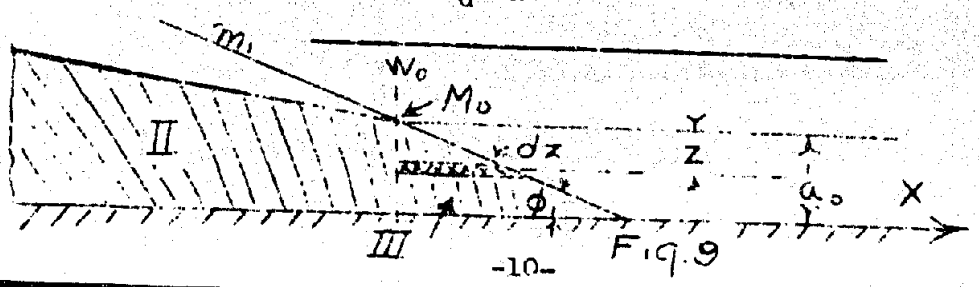

Fig. 9

By assuming the bound ry of the "upper wedge" as pointed out in paragraph 1-2, we regard the "assumed" stream line which is designated by shading in Fig. 8. The length of this stream line is:

(15) $$(d_o + z) \cot\text{an}\ \phi = m\,(d_o + Z)$$

where Z is the depth of the stream line below the head water level elevation. Hence the hydraulic gradient for the given case is

(16) $$J = \frac{a}{m(d_o + z)}$$

and the velocity of percolation

(17) $$U = k.J = \frac{k.a}{m(d_o + z)}$$

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or

(20) $$q = \frac{ka}{m}\Big[\log\text{nat}\ (d_o+z)\Big]_a^{a+h} = \frac{K.a}{m}\ \log\text{nat}\left[\frac{d_o+a+h}{d_o+a}\right]$$

By substituting:

(21) $$d_o + a + h = H_d$$

(22) $$d_o + a = H_d - h\ ;$$

we obtain instead of (20):

(23) $$q = \frac{ka}{m}\ \ln\ \frac{H_d}{H_d - h}$$

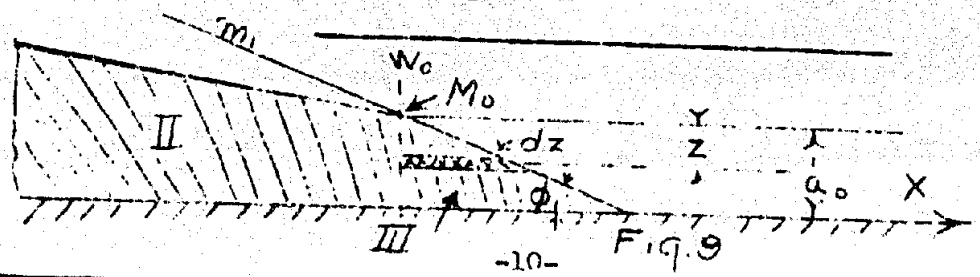


Fig. 9

5. <u>The equation for the "downstream wedge."</u>

For the "downstream wedge" we shall regard two cases: (1) the depth of the water below the dam $h_o = 0$; (2) the depth $h_o > 0$. From an analytical point of view the case (1) is but a special case of (2). But actually the case (1) is more important expecially for the use of earth dams in connection with storage basins. Therefore we shall take up its discussion first.

<u>Case 1. ($h_o = 0$)</u> In the same way as was done for the "upstream wedge" we regard again a horizontal "assumed" stream line for the "downstream"wedge". The length of the stream line designated on Fig. 9 by dark shading is

$$(24) \qquad z \cot \phi_1 = m_1 Z$$

where z is the depth of the stream line below the point M_o at which the depression curve penetrates through the downstream face of the dam. The hydraulic gradient for this stream line then is

$$(25) \qquad J = \frac{Z}{m_1 Z} = \frac{1}{m_1}$$

and the corresponding velocity is:

$$(26) \qquad U = k.J = \frac{k}{m_1}$$

The discharge element is:

$$(27) \qquad dq = U.\, d_z = \frac{k}{m_1}\, d_z$$

and the total discharge:

$$(28) \qquad q = \int_0^{a_o} \frac{k}{m_1}\, dz = \frac{k}{m_1} \int_0^{a_o} dz = \frac{k.a_o}{m_1}$$

<u>Case II. ($h_o > 0$)</u> For this case the percolation through the downstream wedge of the dam is subdivided into two zones as has been shown in paragraph 1. The first:

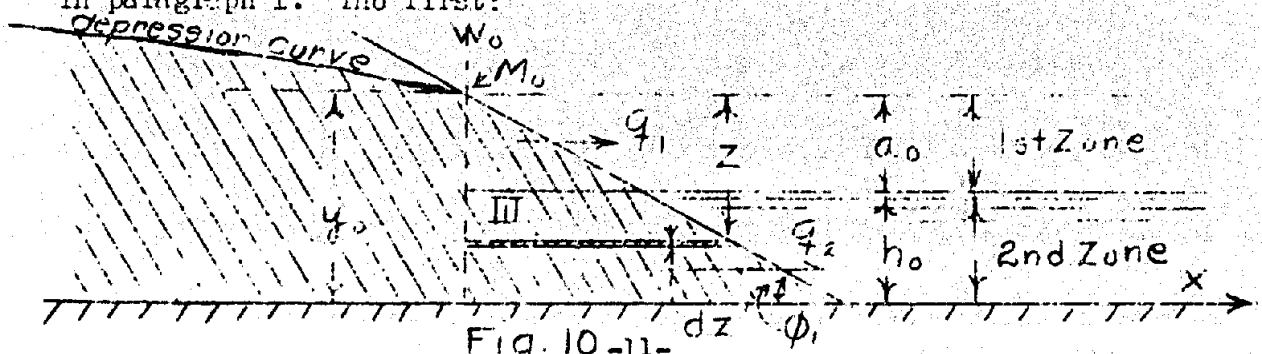


Fig. 10

zone in Fig. 10 is the triangle from the point M_o on to the horizontal line of the tail water elevation, whicle the 2nd zone is the trapezoid which is formed by the line W_o the horizontal of the impervious foundation, the downstream face, and the tail water elevation line. We let q_1 stand for the percolation discharge (specific) through the first zone. We see at once that it can be derived in the same way as was done above for case I (equation 28).

$$(29) \qquad q_1 = \frac{k.a_o}{m_1}$$

For the second zone we see from Fig. 10 that the hydraulic gradient can be derived like in (24 and 25) and hence:

$$(30) \qquad J = \frac{a_o}{m_1 \, Z}$$

and the velocity of percolation

$$(31) \qquad U = k.J = \frac{k.a_o}{m_1 . z}$$

and hence the discharge element

$$(32) \qquad dq_2 = U.\, d_z = \frac{k.a_o}{m_1 \, z} \quad d_z$$

and the total discharge for the second zone

$$(33) \qquad q_2 = \int_{a_o}^{a_o+h_o} \frac{k.a_o}{} \, d_z = \frac{k.a_o}{m_1} \int_{a_o}^{a_o+h_o} \frac{1}{z} \, d_z$$

or

$$(34) \qquad q_2 = \frac{k.a_o}{m_1} \Big[\ln z \Big]_{a_o}^{a_o+h_o} = \frac{k.a_o}{m_1} \ln \left[\frac{a_o+h_o}{a_o} \right]$$

by summarizing (29) and (34) we obtain the total percolation discharge:

$$(35) \qquad \begin{aligned} q = q_1 + q_2 &= \frac{k.a_o}{m_1} + \frac{k.a_o}{m_1} \ln \left[\frac{a_o + h_o}{a_o} \right] \\ q &= \frac{k.a_o}{m_1} \left[1 + \ln \left(\frac{a_o + h_o}{a_o} \right) \right] \end{aligned}$$

6. The system of percolation equations for the homogenous earth dam.

In the preceding paragraphs we arrived at three expressions for the percolation discharge q through the body of the dam for the first, second and third part of the dam profile and the condition $h_o \geq 0$. The corresponding equations are (23), (14), and (35), but these three equations contain 4 unknown quantities:

(36) $h;\ a_o;\ s;\ q$

We have to point out that the quantity a, which enters the equation (23) is not a fifth unknown, since it can easily be expressed from equation (22):

(37) $$a = H_d - d_o - h$$

A fourth equation however, can be readily found from the geometrical relations (see Fig. 5):

(38) $$b + m_1 H_d = s + m_1 (a_o + h_o)$$

from which we obtain:

(39) $$s = b + m_1 \left[H_d - (a_o + h_o) \right]$$

For the case $h > 0$, i. e. when the depth of the water below the dam is not zero we have the following system of 4 equations.

(40) $$\begin{cases} q = \dfrac{K\,(H_d - d_o - h)}{m} \ln \left[\dfrac{H_d}{H_d - h} \right] \\ q = \dfrac{k}{2.s} \left[h^2 - (a_o + h_o)^2 \right] \\ q = \dfrac{k.\,a_o}{m_1} \left[1 + \ln \left(\dfrac{a_o + h_o}{a_o} \right) \right] \\ s = b + m_1 \left[H_d - (a_o + h_o) \right] \end{cases}$$

and by rewriting the system 40

(41) $$\begin{cases} \text{I} \quad \dfrac{q}{k} = \dfrac{H_d - d_o - h}{m} \ln \left[\dfrac{H_d}{H_d - h} \right] \\ \text{II} \quad \dfrac{q}{k} = \dfrac{h^2 - (a_o + h_o)^2}{2.\,s} \\ \text{III} \quad \dfrac{q}{k} = \dfrac{a_o}{m_1} \left[1 + \ln \left(\dfrac{a_o + h_o}{a_o} \right) \right] \\ \text{IV} \quad s = b + m_1 \left[H_d - (a_o + h_o) \right] \end{cases}$$

We shall call this system the "system of equations" for the homogenous earth dam for $h_o > 0$. For the case for which $h_o = 0$ we can use the equations (23); (14); (28), and (39), and we obtain the following system:

$$(41)^1 \begin{cases} \text{I} & \dfrac{q}{k} = \dfrac{H_d - d_o - h}{m} \ln \left[\dfrac{H_d}{H_d - h} \right] \\ \text{II} & \dfrac{q}{k} = \dfrac{h^2 - a_o^2}{2 \cdot s} \\ \text{III} & \dfrac{q}{k} = \dfrac{a_o}{m_1} \\ \text{IV} & s = b + m_1 (H_d - a_o) \end{cases}$$

This system of course contains the same unknown quantities as the system (41).

Let us now point out some additional questions in regard to the obtained system of the percolation equations.

7. Additional remarks.

First of all let us call your attention to the fact that the system (41) or $(41)^1$ gives us an opportunity to make a rather important conclusion. If instead of taking q as an unknown quantity we regard q/k as the unknown quantity, we can find four unknown quantities from the system:

(42) $h;\ a_o;\ s;\ q/k.$

which will be expressed only by the known quantities:

(43) $H_d;\ d_o;\ h_o;\ b;\ m;\ m_1;$

that is by quantities which are given in the design of dam profile and the depths of the water above and below the dam. The selection of the quantities in (43) we shall call the determination of the "hydraulic boundary conditions" having in mind to emphasize by this name, that the hydro-mechanic boundary conditions for the percolation flow in the body of the dam (which have not been discussed here) are expressed differently.

By the conditions given in (43) the main points of the depression curve are given (Fig. 5), namely M_1, M_h and M_o. Furthermore the points of the depression curve in the part M_1-M_o can be calculated from equation (10) if we know h and q/k. Thus it becomes apparent that the shape of the depression curve in the earth dam is not dependent upon the percolation coefficient k, i.e. is not dependent upon the percolation properties of the homogenous soil, but is determined by the

hydraulic boundary conditions which were explained above.

By determining q/k from (41) we find the percolation discharge q if we know the percolation coefficient k. Therefore, it is necessary to know the coefficient k for the soil out of which the dam is built if we want to know the percolation discharge, but for the construction of the depression curve through the dam the knowledge of the percolation coefficient is not necessary.

Furthermore we have to point out that for the first part of the depression curve $M-M_1$ (see Fig. 5) we can only obtain two extreme points of the curve by the method given above, the points M and M_1: the first point is obvious (it is located at the elevation of the water surface above the dam); the second, however, is determined with the help of h or, which is the same, with the help of a, which can be found by equation (37). For the points of the curve between M and M_1 we can only give their general character: For the point M_1 the curve M_1-M_o (which can be plotted completely in all its points) and the curve $M-M_1$ should have a smooth connection (i.e. they should have one and the same tangent). In the point M on the other hand we know (according to what was said in paragraph 2) that the curve should be normal to the upstream face of the dam profile.

For this we can add that the tangent to the depression curve in the point M_1 forms an angle with the abscissa axis which can be easily determined. From equation (7) we know

$$(44) \qquad \frac{dy}{dx} = -\frac{q}{k.y}$$

For the abscissa x = 0 (of the co-ordinate origin is located as shown in Fig. 5) we obtain:

$$(45) \qquad \left(\frac{dy}{dx}\right) = -\frac{q}{k.h}$$

Since for x = C, the ordinate of the curve is

$$(46) \qquad y = h;$$

for this the tangent of the sharp angle (ξ) of the inclination of the above mentioned tangent to the abscissa axis is:

$$(47) \qquad \tan.(\xi) = \frac{q}{k.h}$$

The above said is sufficient for the practical plotting of the depression curve in homogenous earth dam, since a more accurate plotting of the curve between M-M_1, is not necessary for the design of the profile. It is worth while to remember, that if we draw the normal to the upstream face of the dam in the point M and the tangent (which we find by the above calculation) in point M_1 we obtain two straight lines which intersect at a blunt angle; of course the curved part of the depression curve M-M_1 has to be drawn within those two lines.

8. <u>The Solution of the system of the percolation equations.</u>

For the solution of the system of equations (41) we shall for the sake of writing convenience introduce a few observations:

(48) $$1 + \ln\left[\frac{a_o + h_o}{a_o}\right] = A$$

and thus changes III into:

(49) $$\frac{q}{k} = \frac{a_o}{m_1} A$$

by substituting (49) into the left side of II we obtain:

(50) $$\frac{a_o A}{m_1} = \frac{h^2 - (a_o + h_o)^2}{2 \cdot s}$$

and from this we solve for (h) in the following manner:

(51) $$h = \sqrt{\frac{2 a_o A s}{m_1} + (a_o + h_o)^2}$$

and in this expression we substitute s from equation IV of the system and obtain instead of (51):

$$h = \sqrt{\frac{2 \cdot a_o A}{m_1}\left[b + m_1\left\{H_d - (a_o + h_o)\right\}\right] + (a_o + h_o)^2}$$

(52) $$h = \sqrt{2 a_o \cdot A\left[\frac{b}{m_1} + H_d - (a_o + h_o)\right] + (a_o + h_o)^2}$$

and we call

(53) $$\sqrt{2 a_o \cdot A\left[\frac{b}{m_1} + H_d - (a_o + h_o)\right] + (a_o + h_o)^2} = D$$

and hence

(54) $$h = D$$

Using (54) the equation I of the system becomes:

$$(55) \quad \frac{q}{k} = \frac{H_d - d_o - D}{m} \ln \left[\frac{H_d}{H_d - D} \right]$$

and we again introduce an observation,

$$(56) \quad \ln \frac{H_d}{H_d - D} = E$$

which makes:

$$(57) \quad \frac{q}{k} = \frac{(H_d - d_o - D)}{M} E$$

If in (57) we substitute the left side by its value from equation (49) we obtain:

$$(58) \quad \frac{a_o A}{m_1} = \frac{(H_d - d_o - D)E}{m}$$

or

$$(59) \quad \frac{m}{m_1} \cdot a_o \cdot A = (H_d - d_o - D)E$$

we see from (48), (53), and (56) that the terms A, D and E are (for given values of H_d, b, h_o, d_o, m, m_1) only functions of a_o;

$$(60) \quad \begin{cases} A = f_1(a_o) \\ D = f_2(a_o) \\ E = f_3(a_o) \end{cases}$$

the symbols f_1, f_2, f_3 of course are those functions which are known from (48), (53) and (56).

Hence the equation (59) contains only one unknown, that is the quantity a_o, the other unknown quantities have been eliminated in the above shown manner (h, s, and q/k). Equation (59) can be solved only approximately by the method of approximation by steps. In order to shorten the work connected with this we can also apply a graphical solution of equation (59). By expressing the left side of (59) as a function of (a.) $F_1(a_o)$ and the right side as an other function $F_2(a_o)$, i.e.

$$(61) \quad F_1(a_o) = \frac{m}{m_1} a_o \cdot A$$

$$(62) \quad F_2(a_o) = H_d - d_o - D)E$$

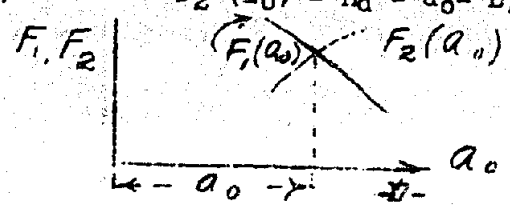


Fig. 11

We can plot a diagram of both functions within independent

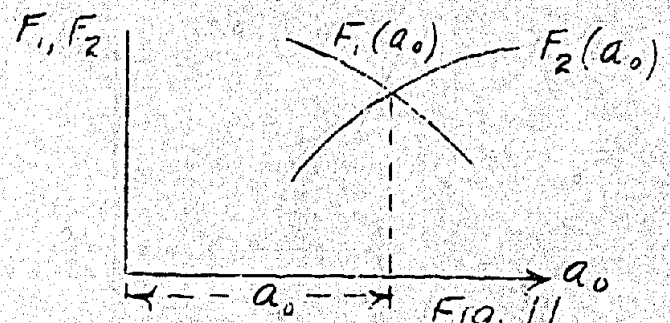


Fig. 11

limits as is shown in the schematical diagram Fig. 11. The intersection point of these two functions furnishes the desired value of (a_0).

$$a_0 = (a_0) \text{ wanted}$$

After having found (a_0) in this way it is easy to determine the remaining unknown quantities of the system (41). Thus we find from equation IV of this system S:

$$(63) \qquad S = b + m_1\left[H_d - (a_0 + h_0)\right]$$

and further, knowing A from equation (48) we can find h from (51):

$$(51) \qquad h = \sqrt{\frac{2\, a_0\, A\, .s}{m_1} + (a_0 + h_0)^2}$$

and finally from (49):

$$\frac{q}{k} = \frac{a_0\, A}{m_1}$$

from which we obtain:

$$(64) \qquad q = \frac{a_0\, A.\, k}{m_1}$$

All that has been said about the solution of the system of equations (41) referred to the case where $h_0 > 0$. For the case $h_0 = 0$ we had the system (41)[1], which is solved in the same manner only with some simplification. We find from (48) that for $h_0 = 0$

$$(65) \qquad A = 1$$

But we do not need the term A at all, since from equation III of the system (41) we obtain directly:

$$\frac{q}{k} = \frac{a_o}{m_1} \quad ; \text{ if this is}$$

introduced into the left side of equation II of the system $(41)^1$ we obtain:

$$(66) \qquad \frac{a_o}{m_1} = \frac{h^2 - a_o{}^2}{2.s}$$

From (66) we can express h in terms of a_o:

$$(67) \qquad h = \sqrt{\frac{2\, a_o \cdot s}{m_1} + a_o{}^2}$$

in which we substitute s from equation IV in the same way as this was done for $h_o > 0$; we then obtain

(68) $h = D$ in which

$$(69) \qquad D = \sqrt{2\, a_o \left(\frac{b}{m_1} + H_d - a_o\right) + a_o{}^2}$$

Equation I in the system $(41)^1$ due to (68) has the same form (55) as for the case $h_o > 0$. For this the symbol (56) remains the same:

$$(70) \qquad E = \ln \frac{H_d}{H_d - D}$$

and also (57) is unchanged

$$(71) \qquad \frac{q}{k} = \frac{(H_d - d_o - D)E}{m}$$

and it is solved in the same way as was pointed out for the case $h_o > 0$.

After finding a_o, we find s from equation IV of the system $(41)^1$ and (h) from (67). From equation III of the system $(41)^1$ we have:

$$\frac{q}{k} = \frac{a_o}{m_1}$$

and from this we find:

$$(73) \qquad q = \frac{a_o \cdot k}{m_1}$$

For the sake of better illustrating the actual functions (61) and (62) we add a diagram (Fig. 12) on which these functions are plotted for a concrete case.

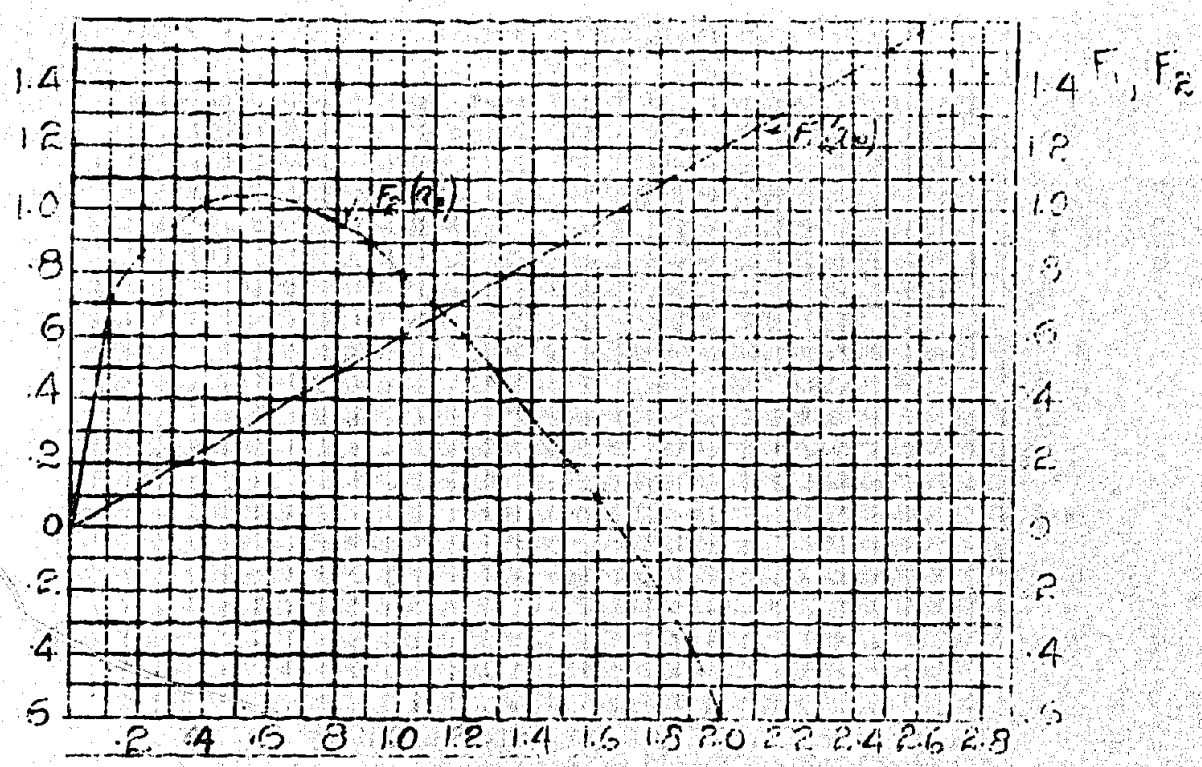

F1, F2
F2(a0)
F1(a0)
1.4
1.2
1.0
.8
.6
.4
.2
0
.2
.4
.6
.2 .4 .6 .8 1.0 1.2 1.4 1.6 1.8 2.0 2.2 2.4 2.6 2.8

Fig. 12

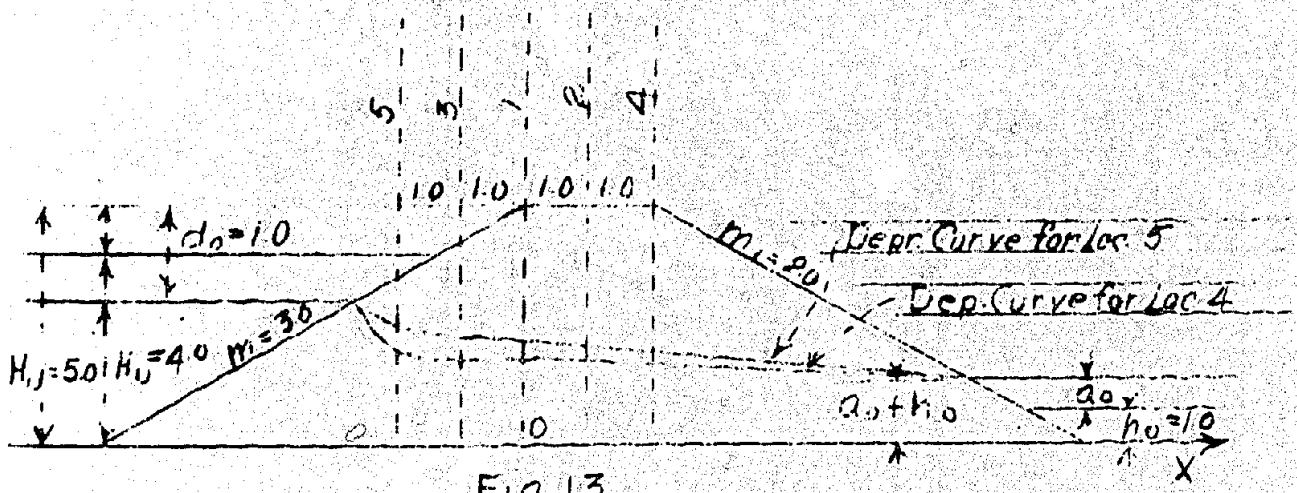

5 3 1 2 4
1.0 1.0 1.0 1.0
d0=1.0
H1=5.0 H0=4.0 m1=3.0
m1=2.0
Depr. Curve for Loc. 5
Depr. Curve for Loc. 4
a0+h0
a0
h0=1.0
0
10
X

Fig. 13

9. On the question of how to select the location of the separating section between the upper wedge and the middle part.

As was pointed out in paragraph 2 we selected the boundary between the upper wedge and the middle part as being given by the line W (in Fig. 5.) which goes through the upstream corner of the horizontal part of the dam profile. It is worth while to take up the discussion of this question in this place not only because of its general value, but also because of the special meaning which this question has for the derivation of a graphical method of the solution (1) of the four percolation equations for the earth dam. (1)(Foot note: see below in chapter 2.)

By carrying out the calculation for the depression curve for different locations of the "separating section" between the "upper wedge" and the middle part of the curve (thus for example on Fig. 13 5 different locations of the section are shown) we can convince ourselves that the selection of this section has a very small influence on the percolation discharge as well as on the shape of the depression curve (especially in its lower part which is most significant for the design of the profile of the dam). This condition was confirmed by us by a great number of numerical examples, and it is here illustrated by the numerical results which were obtained from a computation on the dam profile shown in Fig. 13. The dimensions are given in meters.

Table 1.

Table 2. (Page 22 or original)

Table I

Abscissa In Meters	Ordinates in meters for the different positions of the dividing section				
	5	3	1	2	4
- 2.0	3.172	-	-	-	-
- 1.0	3.055	2.978	-	-	-
0.0	2.925	2.855	2.795	-	-
1.0	2.800	2.732	2.673	2.616	-
4.0	2.380	2.320	2.275	2.228	2.178
6.0	2.048	2.003	1.965	1.928	1.892
8.0	1.650	1.618	1.594	1.570	1.552
9.0	1.410	1.389	1.375	1.357	1.350

Table II

Position of Dividing Section	$y_o = a_o + h_o$ (meters)	S m	q lit/sec
5	1.295	9.410	0.1282
3	1.273	9.454	0.1212
1	1.253	9.494	0.1151
2	1.235	9.530	0.1093
4	1.215	9.570	0.1028

From the figures on table I the two extreme depression curves on Fig. 13 are plotted (for location 5 and 4 of the section W). We see how from Fig. 13 how small the difference between those two curves is regardless of the fact that location 4 is removed quite far in the downstream direction; i.e. to the downstream corner of the level upper part of the dam profile. Of course the difference in the depression curves is still smaller for locations 3 and 2 of the section (W) (see table 1 and 2). From table 2 we see that for a shifting of the section (W) of 1m to one on the other side of the position 1, the value of y_o , which determines the location of the discharge point of . . the M_o of the depression curve changes only to about 2%. This of course cannot have any influence on the results of a calculation in practice. The percolation discharge changes only to about 5% due to such a change of y_o .

- - - - - -

CHAPTER II

10. <u>The law of similitude for the depression curve in an earth dam.</u> We shall show in this paragraph that for geometrically similar dam profiles the depression curves are also geometrically similar. (We do of course have to assume, that the head-water and tailwater heads (depths) are changed corresponding to the change of the dimensions of the dam profile.)

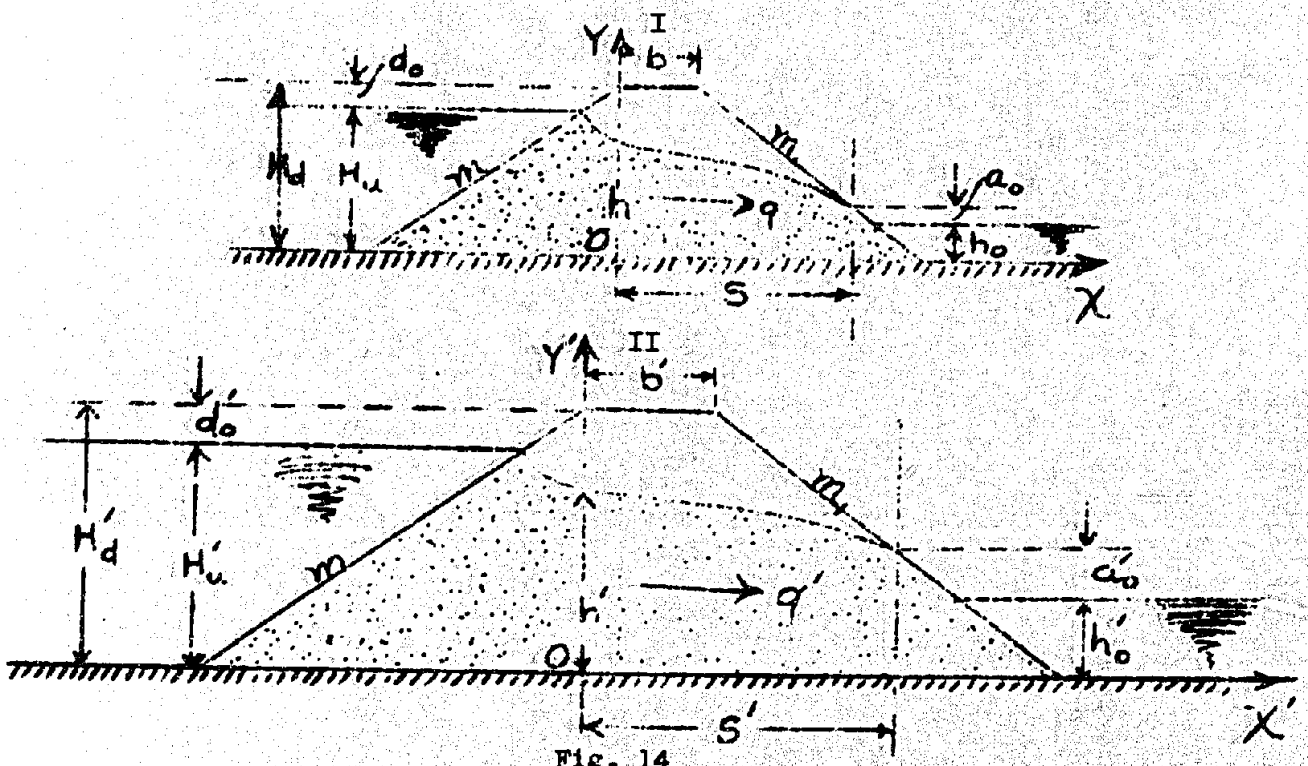


Fig. 14

For the derivation of the relations and their proof we shall use an arbitrary dam profile (See schematical sketch, Fig. 14) and we shall use the same symbols as in the previous chapter. In the sketch II we drew a geometrically similar profile. We assume that all dimensions (also those of the head and tail water depths) of it are enlarged n times as compared to sketch I. The symbols in sketch II are the same as in sketch I, the only difference being that those of sketch II are designated by a "prime" index. Of course the slope coefficients m and m_1 remain the same for both profiles.

As we know from § 8, the basic equation by which the quantity a_o is determined for sketch I (Fig. 14) has the form of equation (59). Hence we have the corresponding equation for the determination of a_o' for sketch II:

$$(74) \quad \frac{m}{m_1} \; a_o' \, A' = (H_d' - d_o' - D') \, E'$$

where the quantities A', D' and E' are found from equations (48), (53) and (56) as follows:

$$(75)\quad A' = 1 + \ln \frac{a_o' + h'o}{a_o'}$$

$$(76)\quad D' = \sqrt{2a_o' A' \left[\frac{b'}{m1} + H_d' - \left(a_o' + h'o\right)\right] + (a_o' + h_o')^2}$$

$$(77)\quad E' = \ln \frac{H'd}{Hd' - D'}$$

We shall show that equation (74) is fulfilled for

$$(78)\quad a_o' = n\, a_o$$

where n is the "similarity modulus" for the transition from profile I to profile II in Fig. 14. For the transfer from one profile to the other the relations between the quantities H_d, b, and h_o and the quantities Hd', b^1 and h_o' are determined by the following equations:

$$H_d' = n \cdot H_d$$

$$(79)\quad b' = n \cdot b$$

$$h_o' = n \cdot h_o$$

From equations (75) to (77) we see that:

$$A' = A$$

$$(80)\quad D' = n \cdot D$$

$$E' = E$$

By multiplying both sides of equation (59) by n we obtain:

$$(81)\quad \frac{m}{m_1} \cdot n \cdot a_o \cdot A = (n \cdot H_d - n \cdot d_o = n\, D)\, E_1$$

and if we remember the relations (78) to (80) we obtain instead of (81)

$$\frac{m}{m_1} a_o' A = (H_d' - d_o' - D')\, E',$$ i.e. we find that equation (74) is fulfilled for

$$a_o' = n \cdot a_o$$

The quantities s' and h' for the profile II are obtained in the following manner. In accordance with the general relation of s in equation IV of the system (41) we obtain for s' the following formula:

(82) $s' = b' + m_1 \left[H_d' - (a_o' + h_o')\right]$ from which we see that according to (78) and (79) that

(83) $s' = m \cdot s$

From (51) we find for h':

(84) $h' = \sqrt{\frac{2a_o' A's'}{m_1} + (a_o' + h'_o)^2}$

with (78), (79), (80) and (83) we then obtain:

(85) $h' = n \sqrt{\frac{2a_o \cdot A \cdot s}{m_1} + (a_o + h_o)^2}$

and compared with (51) we again see that:

(86) $h' = n \cdot h$

Thus we see that the basic elements of the depression curve of the profile II (Fig. 14), i.e., the quantities a_o', s' and h', are in fact proportional to the corresponding quantities of the profile I.

Let us discuss the intermediate points of the second part of the depression curve. At first we shall discuss the change of the unit discharge in the profile II, q'. According to equation (64) we can write (assuming that the percolation coefficient k does not change):

(87) $q' = \frac{a_o' A'k}{m_1}$

Since $A' = A$ and $a_o' = n \cdot a_o$ we obtain

(88) $q' = \frac{na_o A k}{m_1}$

and according to equation (64):

(89) $q' = n \cdot q$

From these equations we see that the unit discharge q' is n times larger than the unit discharge q.

For the intermediate points of the second part of the depression curve of the profile II we can write an equation which is corresponding to equation (10):

(90) $\frac{2q'}{k} s' = h'^2 - y'^2$

in which x' and y' are the coordinates for the profile II. From this we obtain:

$$(91)\quad y'^2 = h'^2 - \frac{2q'\,x'}{k}$$

and by substituting into the right side:

$$h' = nh_1\,;\ q' = nq$$

we find

$$(92)\quad y'^2 = n^2 \,.\, h^2 - \frac{2qnx'}{k}$$

If we also substitute the value for the abscissae

$$(93)\quad x' = nx$$

we see that

$$(94)\quad y'^2 = n^2\left(h^2 - \frac{2qx}{k}\right) = n^2y^2$$

and hence

$$(94')\quad y' = n \,.\, y$$

Thus for the whole second part of the depression curve of the profile II the similarity is retained.

With this we conclude the proof of the law of similitude for the depression curve in earth dams. We may add that since according to ¶ 7 the shape of the depression curve is not dependent upon the coefficient of percolation, this coefficient also has no influence on the above discussed law of similitude. If we would have taken a percolation coefficient $k' \lessgtr k$ the percolation discharge would be taken instead of from equation (89) from the equation:

$$(95)\quad q' = \frac{k'}{k} \,.\, nq$$

11. <u>A graphic method of solving the system of percolation equations.</u> The fundamental assumptions on which this graphical method can be based are the following:

(1) For the geometrically similar profiles the corresponding depression curves are also similar to each other.

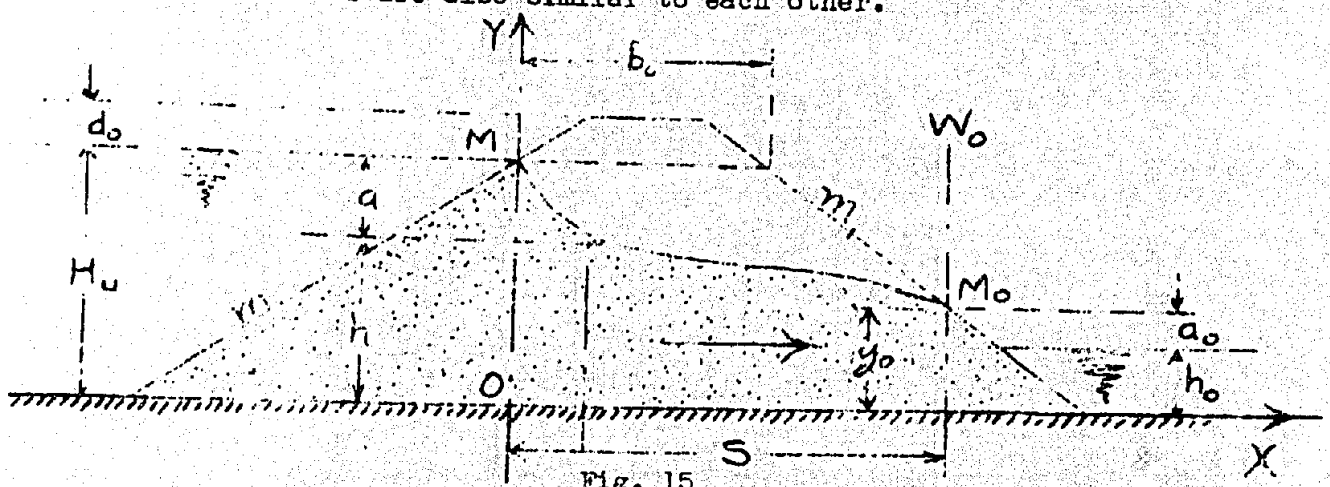


Fig. 15

(2) It is accurate enough to assume that the relation of the quantity (a_o) to the width of the flat top of the dam (b) is linear.

(3) For the graphical solution of the system of percolation equations, the boundary vertical between the first and the second part of the percolation flow in the body of the dam can be drawn through point M (Fig. 15). The point M is the intersection between the head water level and the upstream face of the dam.

Let us now explain these three fundamental assumptions. The first of them is a short formulation of the law of similitude which was discussed in § 10. The second is based on a sufficiently great number of computations, which were carried out according to the analytical method of the solution of the percolation equations as shown in § 8. The third assumption finally follows from what was said in § 9.

To the third assumption we may say that it introduces a simplification of the graphical method in the sense that the quantity d_o (the elevation of the top of the dam above the headwater surface level) does not enter the graphical diagrams, for which thus one of the quantities is eliminated. The y-axis is placed as is shown in Fig. 15., and from this axis on the construction of the second part of the depression curve starts. After this part has been plotted the beginning part of the curve has to be graphically corrected in the following manner (in accordance with what has been said about the first part of the curve in

P 7). We connect the point M with the plotted part of the curve by drawing the normal to the upstream face of the dam in point M, and then draw a smooth curve which has this normal and the plotted curve as tangents.

For the location of the separating vertical according to the **third** assumption the quantity (a) (See Fig. 15) expresses the loss of head (or the loss of specific energy) for the flow of filtration thru the "upper wedge" of the dam. This is the part from the upstream face to the division vertical according to its new position. This new position of the vertical differs from the position at which we placed it in all previous paragraphs. We have to mention that by moving the division vertical in the direction to the upstream face a certain amount of surplus enters our calculations, since according to tables 1 and 2 (P 9) the quantities q and a_o are slightly increased by doing so.

We have yet to prove, that for the new position of the division vertical instead of the width of the top of the dam b, the width of dam at the elevation of the headwater surface has to enter our calculations (see Fig. 15). We shall designate this width by the symbol b_o, all the remaining symbols however are not changed. For b_o we can write the following relation:

$$(96) \quad \ldots\ldots b_o = b + (m + m_1)\, d_o$$

After we thus have discussed the three fundamental presuppositions we can start to exphin the graphical method for the system of percolation equations. For a trianguler profile with a flat top at the elevation of the head water surface we would have a relation for the quantity (a_o)

$$(97) \quad a_o = \alpha \cdot H_u$$

according to the first presupposition (the law of similitude). In this equation α is a coefficient which depends upon the shape of the triangular profile, (i.e. upon the slope ratios (m) and (m_1)) and upon the ratio of the head water and tailwater, i.e., upon the quantity $\frac{h_o}{H_u}$. If we go over to the trapezoidical profile of the dam we have to account for the influence of the quantity b_o on the quantity a_o. Based on the second presupposition we can express this influence by the formula:

$$(98) \quad a_o = \alpha \cdot H_u - \beta \cdot b_o$$

since it is clear that for increasing values of b_o the value a_o has to decrease. The coefficient (β) is dependent, like (α) upon the values of m and m_1, and upon the ratio of $\frac{h_o}{H_u}$. Further below, we shall

discuss the determination of the coefficients (α) and (β) from the graphs. In this place we shall discuss the general method of determination for the other unknown quantities.

Knowing (a_o), and keeping in mind that the ordinate of the discharge point Mo is

(99) $y_o = a_o + h_o$

we can find the value of y_o. In accordance with equation IV of the system (41) we find the length (s) for a location of the division vertical which has to be chosen for the graphical computation (according to Fig. 15) from the expression:

(100) $s = b_o + m_1 (H_u - y_o)$.

Then, knowing a_o we find the value of A from (48), and we can also write:*

*Further below a nomographic chart for the determination of the values of A is given. (Graph No. 4)

(101) $A = \quad + \ln \frac{y_o}{a_o}$

After this we find (h) from equation (51) or by introducing (y_o) from the formula

(102) $h^2 = \frac{2a_o A s}{m_1} + y_o^2$

and

(103) $h = \sqrt{\frac{2a_o A s}{m_1} + y_o^2}$

The percolation discharge q finally can be found from equation (64).

12. Auxiliary diagrams and tables. After having explained the basic relations for the graphical solution of the system of percolation equations, we shall now proceed to discuss the graphs (and one table) themselves. We have to point out that these diagrams were plotted based on the numberical results of a great number of analytical computations of the solution* for system (41) and (41'). Thus they represent a short systematic derivation of these solutions.

* This work as has been pointed out in the preface was done by the engineer R. R. Tschugajef.

The graphs Nos. 1 to 3 give the values of the coefficients α and β (which enter the equation (98)) for different values of $\frac{h_o}{H_u}$ and for the most commonly used values of the slopes (m) and (m_1).

The graph No. 1 is plotted for m = 2.0 and the values of m_1

(104) $m_1 = 1.5; 2.0; 2.5; 3.0; 3.5$

The graph No. 2 is plotted for m = 2.5 and the same values of m_1 as in No. 1 and finally graph No. 3 is plotted for m = 3.0 and same values of m_1 as in equation (104), only that also the value $m_1 = 4.0$ is added.**

** For values which are between those values of m and m_1 which are used in the graphs, we can interpolate using the same graphs.

The notes which accompany the graphs give those limits of the value $\frac{h_o}{H_u}$ within which the values of (α) and (β) are calculated. The limits given have to be understood in such a manner that within these limits the second presupposition (see ¶ 11) is sufficiently accurate, and the basic formula (98) which is derived from it also is sufficiently accurate. Outside of those limits the graphical computation can be used only as an orientation after which we would have to use the analytical solution (¶ 8).

On graph No. 4 we have a simple nomographic chart for the determination of the value of (A) in relation to the ratio $\frac{y_o}{a_o}$. The nomographic chart is constructed according to the formula (101). Using Brigg's logarithms, we can, if we wish calculate the value of A from the formula

(105) $A = 1 + 2.303 \, \mathrm{Log}_{10} \frac{y_o}{a_o}$

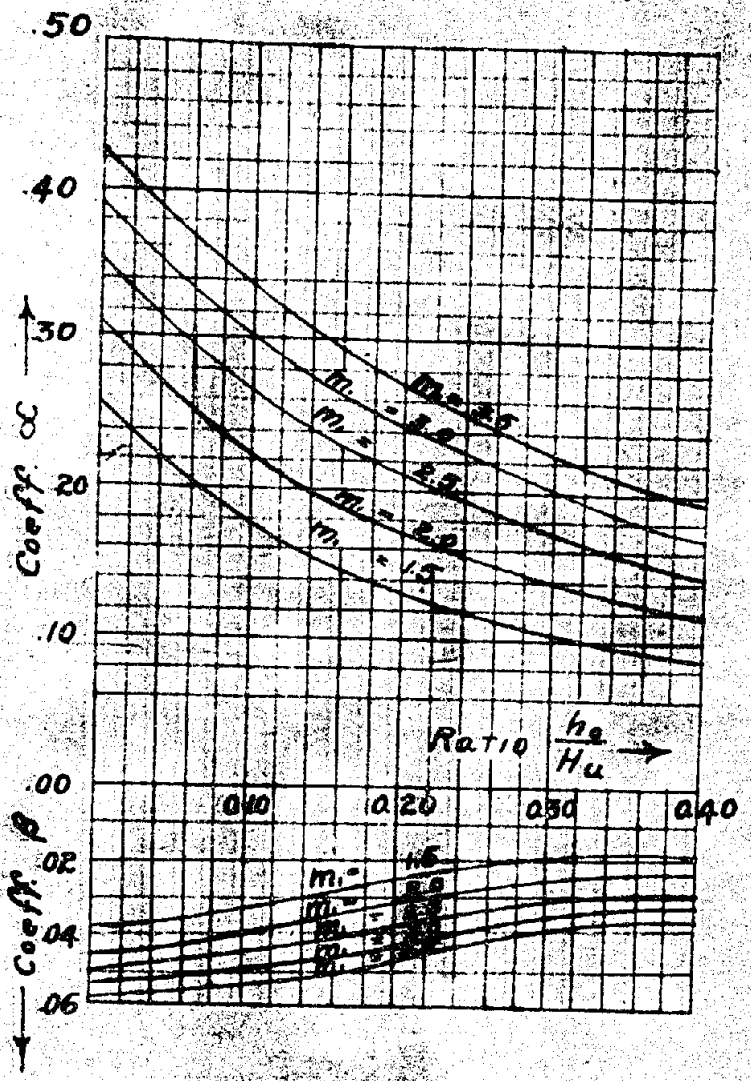
Finally the table (3) serves for the determination of the coefficients (α) and (β) for the case where the depth of the water downstream from the dam is zero (i.e. $h_o = o$). We can for this case determine the values (α) and (β) also from the graphs Nos. 1 to 3, by reading them directly from the ordinate axis (i.e. for $\frac{h_o}{H_u} = o$). But for this rather frequently existant case, the table (3) furnishes more accurate results. The note which is accompanying table (3) has the same meaning as the notes under the graphs Nos. 1 to 3.

In Tables 4 and 5 the formulas which are connected with the graphical method, are tabulated. The order in which they are listed at the same time denotes the order in which they are used for the computation.

Graph No. 1

for the determination of the Coefficients α and β in relation to $\frac{h_e}{H_u}$

$m = 2.0$



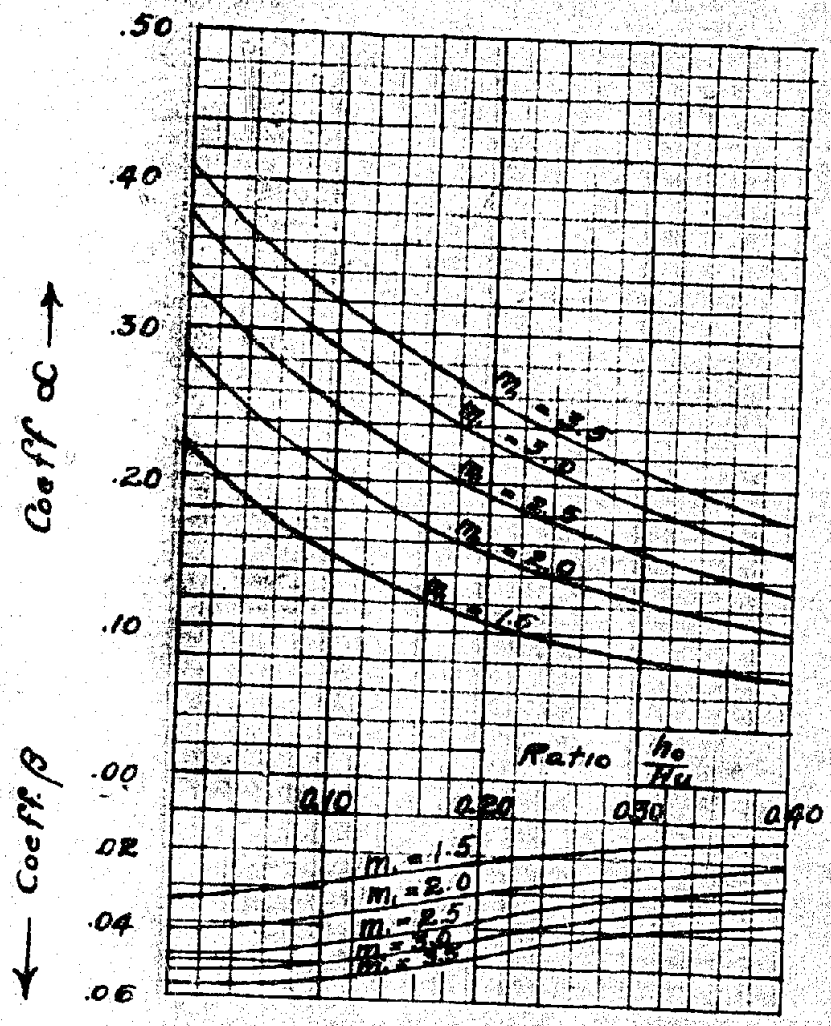
Notice: The graph is plotted for the cases where the ratio of $\frac{b_e}{H_u}$ is within the limits of 1.25 to 3.00

If this ratio is outside of the denoted limits the graphs will in many cases furnish results which are not accurate enough.

Graph No. 2

For determination of the coefficients α and β in relation to the Ratio $\frac{h_o}{H_u}$

$m = 2.5$



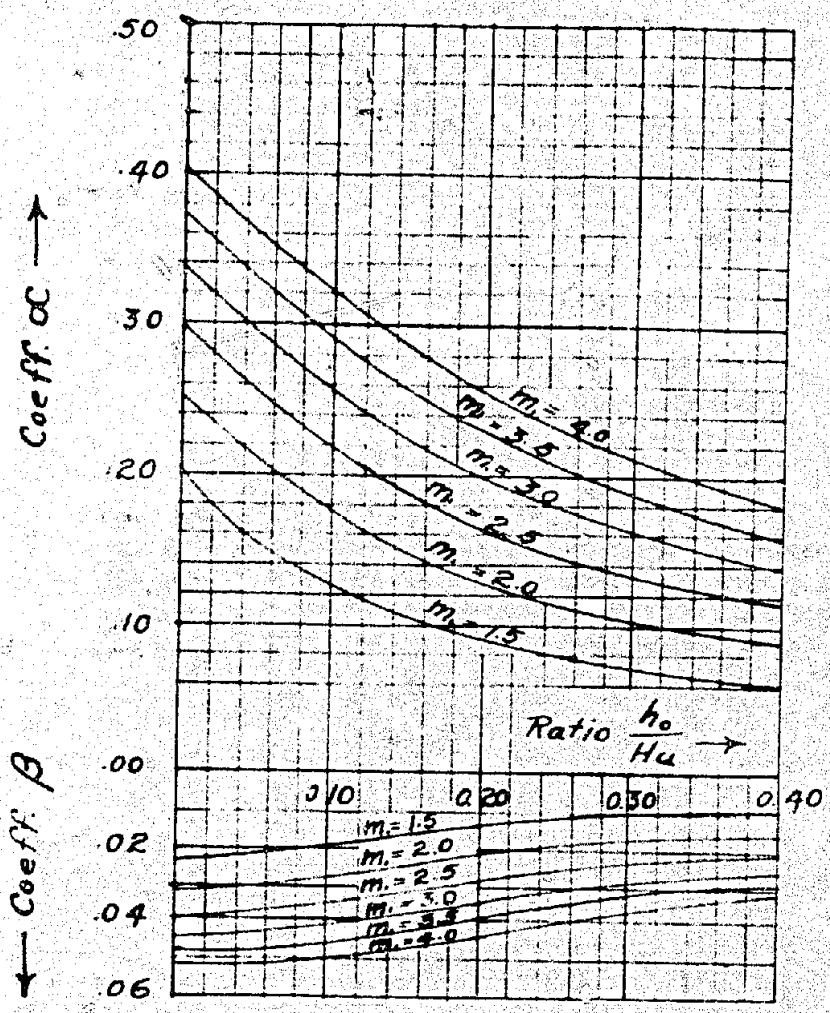
Notice: The curves "$m_1 = 1.5$" are plotted for a ratio $\frac{b_o}{H_u}$, which is within the limits 1.25 to 3.00

The remaining curves are plotted for values $\frac{b_o}{H_u}$ which change from 0.75 to 2.20. If the ratio $\frac{b_o}{H_u}$ is not within the designated limits, the graphs will in many cases furnish values which are not accurate enough

Graph No. 3

For the determination of the coefficients α and β in relation to $\frac{h_o}{Hu}$

$m = 3.0$



Notice: The curves "$m_1 = 1.5$" are computed for a ratio of $\frac{b_o}{Hu}$ within the limits 1.25 to 3.00 the remaining curves for $\frac{b_o}{Hu}$ varying from 0.70 to 2.20. If $\frac{b_o}{Hu}$ is not within the given limits the values furnished by the graphs are in most cases not accurate enough.

Graph No. 4
Homographic chart for the determination of the Coefficient A in relation to the ratio $\frac{y_0}{a_0}$

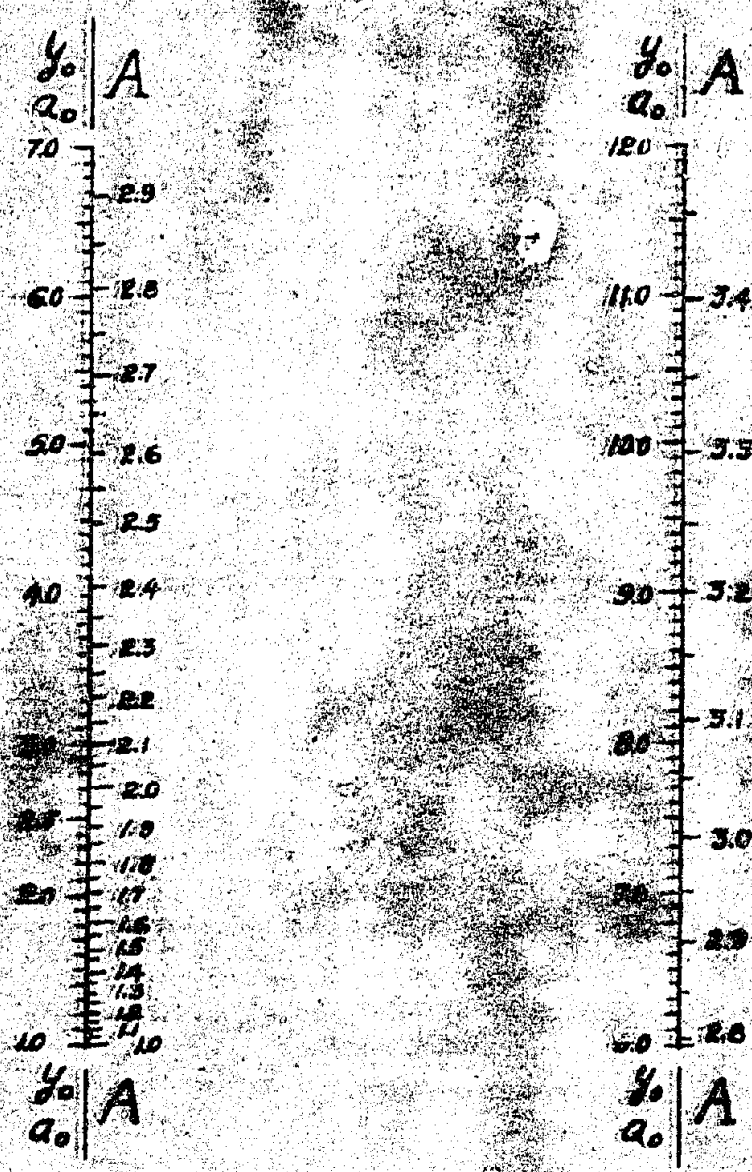


Table 3

The values of the coefficients α and β as used in the formula (98) for the case $h_0 = 0$.

No. of Profile	m	m_1	α	β
1*	2.0	1.5	.258	.039
2*	2.0	2.0	.312	.046
3*	2.0	2.5	.355	.050
4*	2.0	3.0	.393	.054
5*	2.0	3.5	.426	.057
6*	2.5	1.5	.233	.033
7	2.5	2.0	.286	.041
8	2.5	2.5	.334	.048
9	2.5	3.0	.375	.054
10	2.5	3.5	.408	.057
11*	3.0	1.5	.198	.023
12	3.0	2.0	.252	.031
13	3.0	2.5	.299	.038
14	3.0	3.0	.339	.043
15	3.0	3.5	.374	.048
16	3.0	4.0	.404	.051

NOTE: A sufficiently accurate determination of the value of a_0 with Table 3 values of α and β is possible as long as the ratio of h_0 is within the limits: (1) of 1.25 to 3.0 for the profiles which are marked*, and (2) of 0.75 to 2.2 for the profiles which are not marked.

$\frac{h_0}{H_0}$

TABLE 4

The derivation and list of formulas for the case $h_o > 0$.

Order of procedure	The quantity which is determined	The formula
1°	b_o	$b_o = b + (m + m_1)\, d_o$
2°	α	Determined from the graphs No 1 + 3.
3°	β	
4°	a_o	$a_o = \alpha . H_u - \beta . b_o$
5°	y_o	$y_o = a_o + h_o$
6°	S	$S = b_o + m_1 (H_u - y_o)$
7°	A	is determined from chart graph No. 4 or from the formula: $A = 1 + 2.303 \log_{10} \frac{y_o}{a_o}$
8°	h^2	$h^2 = \frac{2a_o . A . s}{m_1} + y_o^2$
9°	h	$h = \sqrt{\frac{2a_o . A . s}{m_1} + y_o^2}$
10°	q_k	$\frac{q}{k} = \frac{a_o A}{m_1}$
11°	q	$q = \frac{a_o . A . k}{m_1}$
12°	q_d	$q_{daily} = 8640 \frac{A}{m_1} . a_o . k$
13°	The equation of the depression curve	$y^2 = h^2 - \frac{2q}{k} x$

Note: On the quantity q_d see further below § 13. formula (108).

TABLE 5

The derivation and list of the formulas for the case $h_o = 0$

Order of procedure	The quantity which is determined	The Formula
1°	b_o	$b_o = b + (m + m_1) \cdot d_o$
2°	α	taken from table 3.
3°	β	
4°	a_o	$a_o = \alpha H_u - \beta \cdot b_o$
5°	S	$S = b_o + m_1(H_u - a_o)$
6°	h^2	$h^2 = \frac{2 \cdot a_o S}{m_1} + a_o^2$
7°	h	$h = \sqrt{\frac{2 \cdot a_o S}{m_1} + a_o^2}$
8°	q_k	$\frac{q}{k} = \frac{a_o K}{m_1}$
9°	q	$q = \frac{a_o k}{m_1}$
10°	q_d	$q_d = \frac{8640}{m_1} a_o k$
11°	The equation of the depression curve.	$y^2 = h^2 - \frac{2q}{k} x$

Note: On the quantity q_d see further below § 13. equati.. (111).

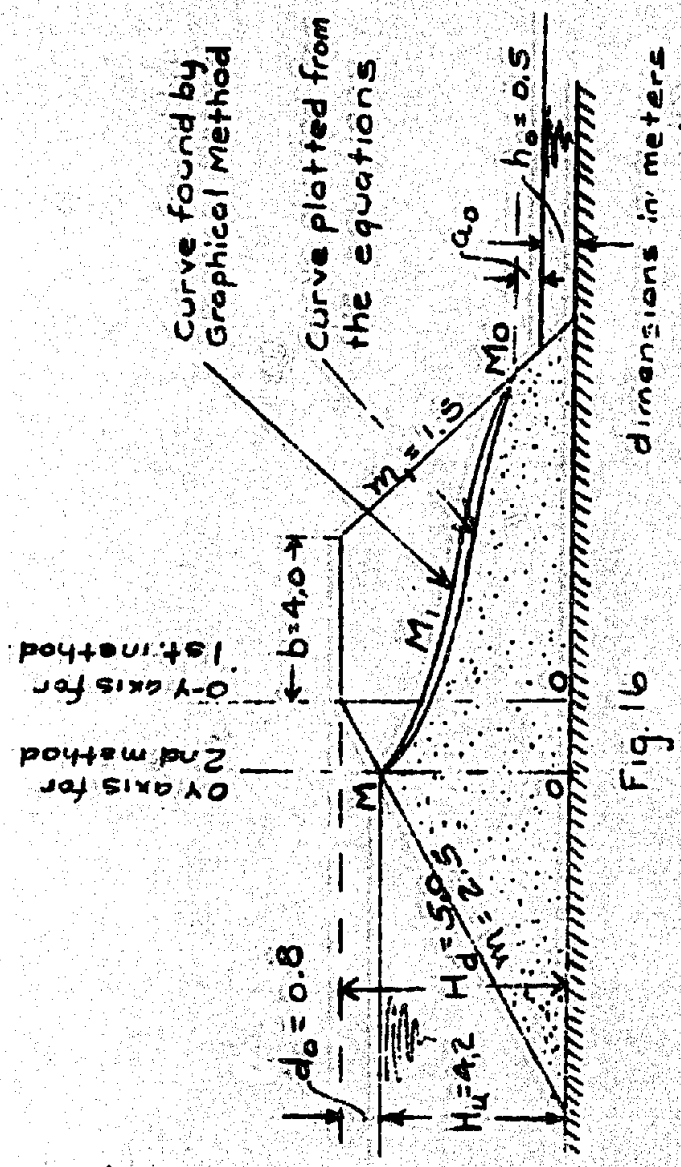


Fig. 16

§13. Example for the construction of the depression curve for $h_0 > 0$.

Let us use the following example of an earth dam (Fig. 16):

$H_d = 5.0$ m
$b = 4.0$ m
$d_0 = 0.8$ m
$H_u = 4.2$ m
$h_0 = 0.5$ m
$m = 2.5$
$m_1 = 1.5$ (1)

the filtration coefficient $k = 0.00005$ cm/sec.

(foot note):1

The numerical values of the slopes of the dam faces and the percolation coefficient are taken only for this example. These values (and the values which will be used in all following examples) should not be regarded as such that it would be recommended for any practical project of a dam construction.)

For those numerical values we shall now solve the system of percolation equations as given in (41), which serves for the construction of the percolation curve and the determination of the percolation discharge q.

1. Analytical solution.

According to what has been said in paragraph 8 we have at first to determine the value a_0 from equation (59) by way of approximation ie. using different values a_0 and comparing from them the different values of F_1 (a_0) and F_2 (a_0) of the left and right side of equation (59). Previous to this we determine the constant values for the present example:

(106) $$\frac{m}{m_1} = \frac{2.5}{1.5} = 1.67$$

and

(107) $$\frac{b}{m_1} + H_d = \frac{4.0}{1.5} + 5.0 = 7.67$$

All other steps for the approximation of the value a_o which satisfies the equation (59) can be seen from table (6). In the third horizontal row of the table the expressions for α_o are given in an abreviated way of writing. The numbers in heavy print stand for the mathematical formulations which are given in second row, the numbers refer to the number of the column in which the complete formula is written.

As we can see from table (6) the equation (59) can be regarded as being satisfied by a value of

$$a_o = 0.33 \text{ m.}$$

since for this value

$$F_1(a_o) = \approx F_2(a_o)$$

Furthermore we find the other unknown quantities as is shown in paragraph 8. Thus from equation (68) we find:

$$S = 4.0 + 1.5\,[5.0-(0.33 + 0.50)] = 10.25 \text{ m.}$$

and from (51):

$$h = \sqrt{\frac{2 \times 0.33 \times 1.93 \times 10.25}{1.5} + (0.50 + 0.33)^2}$$

$$= 3.07 \text{ m}$$

and finally from (49):

$$\frac{q}{k} = \frac{0.33 \times 1.93}{1.5} = 0.425 \text{ m} = 42.5 \text{ m}$$

so that according to (64):

$$q = 42.5 \times 0.00005$$

$$= 0.002125 \text{ cm}^2/\text{sec.}$$

for which the unit discharge is referred to 1 cm of the width of the flow. It is, however, better to express the quantity q in liters per day and per one meter of the width of the flow. For this we obtain instead of the equat. (64), the equation:

(108) $$q_d = 8640 \frac{A}{m_1} \gamma\, a_o\, k$$

where A and m_1 are dimensionless quantities, a_o is expressed in cm, and the percolation coefficient is expressed in cm/sec. For the given example we find:

$$q_d = 8640 \times \frac{1.93}{1.5} \times 33 \times 0.00005$$

$$= 18.3$$

expressed in liters per day per 1 meter of the width of the flow (i.e.) 1 meter of the length of the dam). (1)

(Foot note: (1) Usually the earth dams have a variable height along the length of the crest (the crest being horizontal the dam follows differences in ground etc.). For the determination of the total percolation discharge therefore we have to subdivide the dam in its length, into a number of reaches which have as nearly as possible the same height. After having determined the percolation discharge for each of these parts we add all of them and obtain the total percolation discharge).

TABLE 6

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
a_o	$y_o = a_o + h_o$	$\frac{y_o}{a_o}$	$A = 1 + 2.303 \log \frac{y_o}{a_o}$	$F_1(a_o) = \frac{m}{m_1} a_o . A$	$\left(\frac{b}{m_1} + H_d\right) - y_o$	$2 a_o A \left[\left(\frac{b}{m_1} + H_d\right) - y_o\right]$	$(y_o)^2$	$D^2 = 2 a_o A \left[\left(\frac{b}{m_1} + H_d\right) - y_o\right] + y_o^2$	$D = \sqrt{2 a_o A \left[\left(\frac{b}{m_1} + H_d\right) - y_o\right] + y_o^2}$	$H_d - D$	$\frac{H_d}{H_d - D}$	$E = 2.303 \log_{10} \frac{H_d}{(H_d - D)}$	$H_d - d_o - D$	$F_2(a_o) = (H_d - d_o - D) E$
a_o	**(1)** + 0.5	$\frac{\mathbf{(2)}}{\mathbf{(1)}}$	$A = 1 + 2.303 \log$ **(3)**	$F_1(a_o) = 1.67$ **(1)** **(4)**	7.67 − **(2)**	2 **(1)** . **(4)** . **(6)**	$(\mathbf{2})^2$	$D^2 =$ **(7)** + **(8)**	$D = \sqrt{\mathbf{(9)}}$	5.0 − **(10)**	$\frac{5.0}{\mathbf{(11)}}$	$E = 2.303 \log_{10}$ **(12)**	**(11)** − 0.8	$F_2(a_o) =$ **(14)** **(13)**
.60	1.10	1.83	1.61	1.61	6.57	12.70	1.21	13.91	3.73	1.27	3.94	1.37	0.47	.65
.45	.95	2.11	1.75	1.32	6.72	10.59	.90	11.49	3.39	1.61	3.10	1.13	.81	.92
.35	.85	2.43	1.89	1.10	6.82	9.03	.72	9.75	3.12	1.88	2.66	.98	1.08	1.06
.33	.83	2.52	1.93	1.06	6.84	8.71	.69	9.40	3.07	1.93	2.59	.95	1.13	1.07

Note: The numbers in heavy type refer to the column in which the complete formulas are written. values given in meters

For the obtained values of h and q_k, the equation (10) for the second part of the depression curve (from the point M_1 to M_o) then has the following form (in meters):

$$(109) \quad Y^2 = 9.40 - 0.84\,x$$

Table (7) is computed from this equation.

TABLE 7

X	0.85 x	$Y^2 = 9.40-0.85\,x$	Y
0	0.00	9.40	3.07 = h
2	1.70	7.70	2.78
4	3.40	6.00	2.45
6	5.10	4.30	2.07
8	6.80	2.60	1.61
9	7.65	1.75	1.32
S= 10.25	8.71	0.69	$0.83 = a_o + h_o$

II Solution with graphical charts.

Let us arrange the solution in the order which is given in the list of the formulas on table 4 (see paragraph 12). We place the Y-axis through the point M as this is shown in Fig. 16.

At first we determine b_o (the meanings of the symbols are given in paragraph 12):

$$b_o = 4.0 + (2.5 + 1.5)\,(0.8) = 7.2$$

From graph No. 2 we find

$$\alpha = 0.137, \quad \beta = 0.0237$$

and have $m = 2.5$

$m_1 = 1.5$

$$\frac{h_o}{H_u} = \frac{0.50}{4.20} = 0.119$$

Furthermore (see table 4) we have

$$a_o = 4.20 \times 0.137 - 7.20 \times 0.0237 = 0.40 \text{ m}$$

and from this we find:

$$y_o = 0.40 + 0.50 = 0.90 \text{ m}$$

$$S = 7.20 + 1.50\,(4.20 - 0.90) = 12.15 \text{ m}$$

$A = 1.81$ (from graph No. 4 for $\frac{y_o}{a_o} = 2.25$)

$h^2 = 12.53 \text{ m}^2$ (from formula 8° of table 4)

$$h = \sqrt{12.53} = 3.54 \text{ m}$$

and finally from formula 10° and 11° of the same table 4 we obtain

$$\frac{q}{k} = \frac{a_o A}{m_1} = \frac{0.40 \times 1.81}{1.5} = 0.483 \text{ m} = 48.3 \text{ cm.}$$

$$q = 48.3 \times 0.00005 = 0.0024 \text{ cm}^2/\text{sec.}$$

and from formula 12° of table 4:

$$q_d = 8640 \frac{A}{m_1} a_o k = 8640 \frac{1.81}{1.5} \times (.40) \times .00005$$
$$= 20.8$$

given in liter per day and per 1 meter of the length of the dam.

Hence the equation of the despression curve is (in meters):

$$Y^2 = \underline{12.53 - 0.966 x} \tag{110}$$

for this equation the depression curve is computed making also the correction which was pointed out in paragraph 11. On Fig. 16 both depression curves are drawn to scale, the curve which was found by the analytical method and the curve which was found by the graphical method.

<u>Paragraph 14.</u> <u>Example for the computation of the depression curve for the case.</u>

$$h_o = 0$$

For this case we take the following example:

$H_d = 8.0$ m

$d_o = 1.50$ m; $H_u = 6.50$ m.

$b = 5.00$ m

$m = 3.0$; $m_1 = 2.0$ ($h_o = 0$)

$k = 0.00005$ cm/sec. (see beginning of Para. 13)

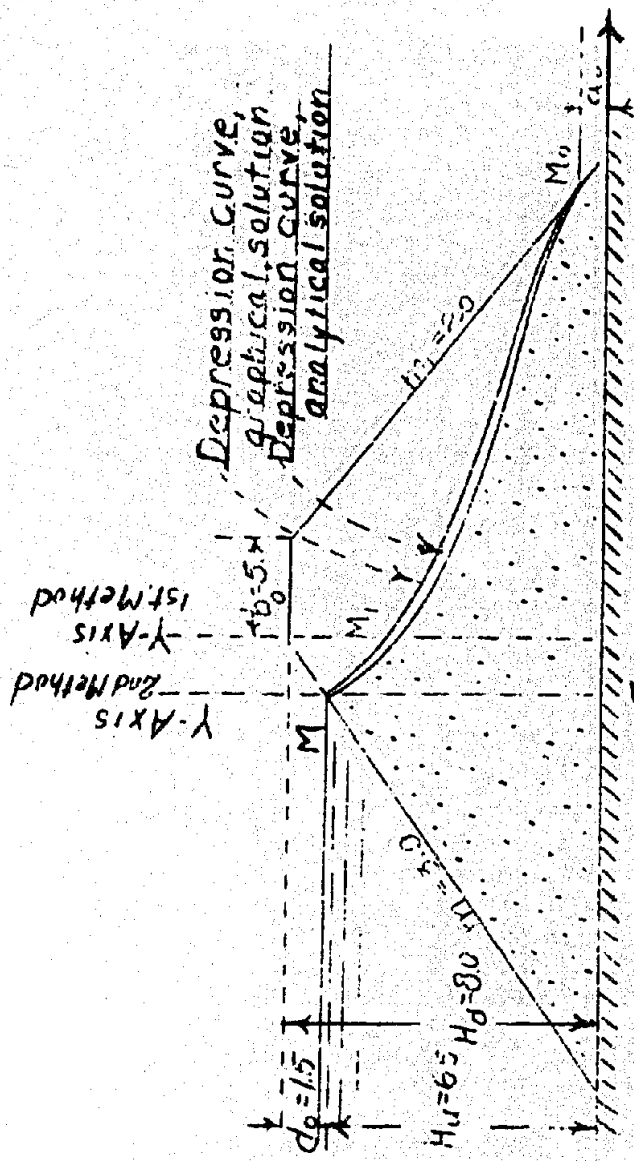


Fig. 17

1. Analytical Solution.

Let us keep in mind all that has been said in paragraph 8 for the case $h_o = 0$.

As first we find

$$\frac{m}{m_1} = \frac{3.0}{2.0} = 1.50$$

$$\frac{b}{m_1} + H_d = \frac{5.0}{2.0} + 8.0 = 10.50 \text{ m}$$

and after that we make up table 8 as was done in the preceding paragraph. Table 8 is arranged in the same manner, showing the complete mathematical formulas in one column and the abreviated formulas in another.

From the table we see that for the given example

$$a_o = 1.08 \text{ m}$$

and we find the value for S from formula IV of the system of equations (41^1):

$$S = 5.00 + 2.0\left[8.00 - 1.08\right] =$$

$$= 18.83 \text{ m.}$$

and h from equation (67):

$$h = \sqrt{21.52} = 4.65 \text{ m}$$

furthermore from equation III of the system (41^1):

$$\frac{q}{k} = \frac{a_o}{m_1} = \frac{1.08}{2.0} = 0.54 \text{ m} = 54 \text{ cm}$$

and from (73):

$$q = (54.)\,0.00005 = 0.0027 \text{ cm}^2/\text{sec.}$$

But instead of (108) we obtain for q_d:

$$(111)\quad q_d = \frac{8640}{m_1}\, a_o\, k$$

and hence

$$q_d = \frac{8640}{2.0}(108)(0.00005) = 23.4$$

For the values of h and $\frac{q}{k}$, computed above, the equation for the second part of the depression curve equation (10) has the following form:

$$(112) \quad y^2 = 21.52 - 1.08\,x$$

According to this equation the part of the depression in Fig. 17 is constructed.

II. Solution according to table 3.

In accordance with table (5) we determine first of all the width of the dam at the elevation of the head water surface: (1)

(Foot note: The location of the Y - axis is shown on Fig. (17); the axis goes through the point M)

$$b_o = 5.00 + (3.0 + 2.0)\,1.50 = 12.50 \text{ m}$$

From table 3 we then obtain:

$$\alpha = 0.252; \; \beta = 0.031$$

For our example m = 3.0 and m_1 = 2.0 and furthermore according to the order of procedure given on table (5):

$$a_o = 0.252 \times 6.50 - 0.031 \times 12.50$$

$$= 1.25 \text{ m}$$

$$S = 12.50 + 2.0\,(6.50 - 1.25) = 23.0 \text{ m}$$

$$h^2 = 30.31 \text{ m}^2 \text{ (from formula 6° of table 5)}$$

$$h = \sqrt{30.31} = 5.50 \text{ m}$$

and finally from formula 8° of table (5):

$$\frac{q}{k} = \frac{a_o}{m_1} = \frac{1.25}{2.0} = 0.625 \text{ m} = 62.5 \text{ cm.}$$

$$q = 62.5(0.00005) = 0.0031 \text{ cm}^2/\text{sec.}$$

$$q_d = \frac{8640}{m_1}\,a_o\,k = \frac{8640}{2}(1.25)(0.00005)$$

$$= 27.0$$

given in liters per day and per meter of the length of the dam.

The equation for the depression curve for the location of the y-axis is as shown in Fig (17) has the form:

TABLE 8

1	2	3	4	5	6	7	8	9	10	11	12
a_o	$F_1(a_o) = \frac{m}{m_1} a_o$	$\left(\frac{b}{m_1} + H_d\right) - a_o$	$2 a_o \left(\frac{b}{m_1} + H_d\right) - a_o$	a_o^2	$D^2 = 2a_o\left[\left(\frac{b}{m_1} + H_d\right) - a_o\right] + a_o^2$	$D = \sqrt{2a_o\left[\left(\frac{b}{m_1} + H_d\right) - a_o\right] + a_o^2}$	$H_d - D$	$\frac{H_d}{H_d - D}$	$E = 2.303 \log_{10} \frac{H_d}{H_d - D}$	$\left[(H_d - d_o) - D\right]$	$F_2(a_o) = \left[(H_d - d_o) - D\right] E$
a_o	$F_1(a_o) = 1.5$ (1)	10.5 - (1)	2 x (1) x (3)	$(1)^2$	$D^2 = (4) + (5)$	$D = \sqrt{(6)}$	8 - (7)	8 / 8	$E = 2.303 \log_{10}$ (9)	(8) - 1.50	$F_2(a_o) = (11)$ (10)
1.50	2.25	9.00	27.00	2.25	29.25	5.41	2.59	3.09	1.13	1.09	1.23
1.40	2.10	9.10	25.48	1.96	27.44	5.24	2.76	2.90	1.07	1.26	1.34
1.20	1.80	9.30	22.32	1.44	23.76	4.87	3.13	2.56	0.94	1.63	1.54
1.10	1.65	9.40	20.65	1.21	21.86	4.67	3.33	2.40	0.88	1.83	1.61
1.08	1.62	9.42	20.35	1.17	21.52	4.65	3.35	2.39	0.87	1.85	1.62

Dimensions given in meters

(113) $Y^2 = 30.31 - 1.25\ x$

From this equation the depression curve is constructed and the correction according to paragraph 11 is made afterwards in its first part.

Paragraph 15. Additional examples.

In addition to the two examples which have been discussed above we shall give a few short calculations which illustrate the law of similitude for the percolation flow through an earth dam. The law of similitude also has a number of practical applications for the projecting of earth dams especially if a great number of earth dams has to be built as for example in the lower and middle Volyor provinces, the Northern Kankersus provinces, etc.

1°. Let us assume that the depression curve has to be plotted in an earth dam of the following given dimensions. (the symbols are the same as used in paragraph 13-14):

$H_d = 16.0$ m

$d_o = 3.0$ m; $H_u = 13.0$m

$b = 10.00$ m

$m = 3$; $m_1 = 2$; $h_o = 0$

We see that this profile is geometrically similar to the dam profile which was used as an example in Para. 14. Also the heads are geometrically similar. The modulus of similarity is:

$n = 2.0$

We use this similarity coefficient (scale ratio) in changing the formulas of Para. 10, and we denote the unknown quantities of the given example by a_o^1; S^1; h^1.

(Foot note: On the plotting of the second part of the depression curve, see further below in section 3°)

Knowing a_o, S and h from the analytical calculation in Para. 14 we find:

(114) $(a_o^1 = n.a_o = 2 \times 1.08 = 2.16$ m
$(S^1 = n.\ S = 2 \times 18.83 = 37.66$ m
$(h^1 = n.h = 2 \times 4.65 = 9.30$ m

2°. If the same percolation coefficient $k = 0.00005$ cm/sec. is used as was used in the case given in Para. 14, the unit percolation discharge q^1 is obtained from equation (89):

$$(115) \quad q^1 = n.q = 2 \times 0.0027 = 0.0054 \text{ cm}^3/\text{sec.}$$

and in the same way:

$$(115^1) \quad q_d^1 = n.q_d = 2 \times 23.4 = 46.8$$

taken in liters per day (24 hrs.) per one meter of the length of the dam.

If the percolation coefficient is different from the one which was used in Para. 14, and is for example equal to $k^1 = 0.00001$ cm/sec., we obtain from equation (64):

$$(116) \quad q^1 = \frac{a_o^1 . A^1 . k^1}{m_1}$$

in which (see paragraph 10):

$$A^1 = A; \; a_o^1 = n . a_o$$

so that:

$$(117) \quad q^1 = \frac{n.a_o . A}{m_1} k^1 = \frac{n.a_o . A.k}{m_1} \frac{k^1}{k}$$

from which we obtain according to equation (64):

$$(118) \quad q^1 = n.q \frac{k^1}{k}$$

and hence also

$$(118^1) \quad q_d^1 = n.q_d \frac{k^1}{k}$$

and for our case:

$$q_d^1 = 2 \times 23.4 \frac{0.00001}{0.00005} = 9.4$$

again taken in liters per day and per 1 meter of the length of the dam.

3°. For the plotting of the second part of the depression curve we use equations (10) and (92) i.e.:

$$(119) \quad \begin{cases} y^2 = h^2 - \frac{2 q x}{k} \\ (y^1)^2 = n^2 h^2 - \frac{2 q . n}{k} x^1 \end{cases}$$

The equation with y^1 and x^1 refers to the present example which the equation with x, y refers to the example of Para. 14. We can write the equation (119) in the form:

$$(120) \quad \begin{cases} y^2 = k - N x \\ (y^1)^2 = k^1 - N^1 x^1 \end{cases}$$

where:

(121) $K^1 = n^2 k; \; N^1 = n.N$

From equation (112) we know that $k = 21.52$ and $N = 1.08$ and therefore for the given example we obtain:

$$K^1 = 4\,k = 86.08$$
$$N^1 = 2N = 2.16$$

and thus the depression curve for our case has the equation:

(122) $(y^1)^2 = 86.08 - 2.16\,x^1$

For this of course the coordinate axes ($0-x^1$) and $0-y^1$) are located in the same way as the axes (0-X) and 0-Y).

Since the shape of the depression curve is not dependent upon the percolation coefficient (see Para. 10), the equation (122) is correct for the conditions of the given profile(dimensions given in the beginning of this paragraph) for an arbitrary coefficient k (if Darcy's law is valid).

4°. If we take as another example the profile on Fig. 16, all dimensions are reduced by 20 % (with the exception of the values for m and m_1 which remain constant), and thus we have the scale ratio:

$$n = 0.8$$

We shall not repeat what was said under 1° and 3° of this paragraph. But we can point out that for the plotting of the depression curve we can in an easy way use the table (7) which was made in paragraph 13, by only increasing the values of the abscissae and ordinates by the factor $n = 0.8$. After that the curve can be plotted by points for the given profile (Fig. 16).

CHAPTER III

16. The percolation through a core of rectangular cross section.

Turning now to the question of percolation in a dam which is

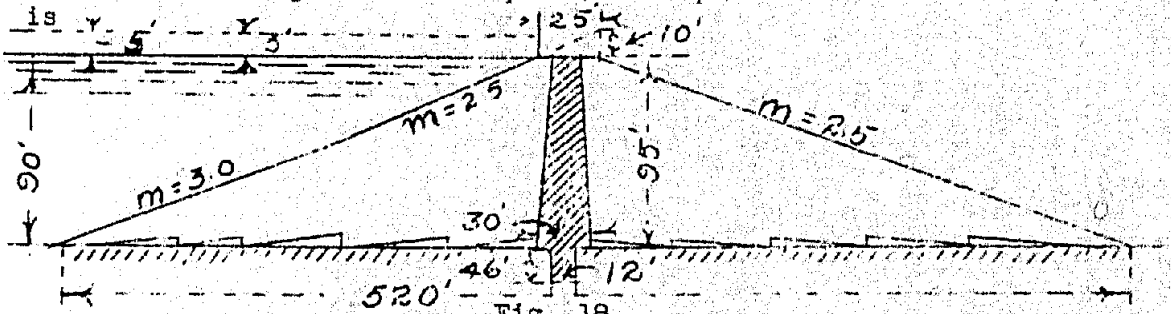


Fig. 18

equipped with a so-called "core" we have at first to explain what we understand by core. We understand by it a vertical wall of comparatively small thickness which is placed inside of the dam body, and which consists of comparatively impervious material like lime-cement, or very well rolled sand-clay soil which is well suited for the purpose etc. A typical example of a core is given by the dam at the Pilarcitos in California, which is shown in Fig. 18. The slope of the sides of the trapezoidal core is here $m_0 = \sim \frac{1}{3}$ where m_0 is the cotan of the angle of inclination of the sides of the core. Generally for such structures the value of m_0 varies within the limits:

$$(123) \qquad m_0 = \frac{1}{12} - \frac{1}{6}$$

and as a mean value one can assume

$$(124) \qquad m_0 = \sim \frac{1}{8}$$

We show a rectangular core on a separate sketch. We assume a constant thickness of t. the depth of the underground flow upstream (Foot note: The trapezoidial core will be discussed further below. 20-21) from the core is h_ξ and below the core h_c. The loss of head (or of the specific energy) which is caused by the percolation through the core is called c_c, and the percolation coefficient of the material of which the core is made is called k_0. We subdivide the percolation flow through the core into two zones, as will be shown below. We make this subdivision in the following manner:

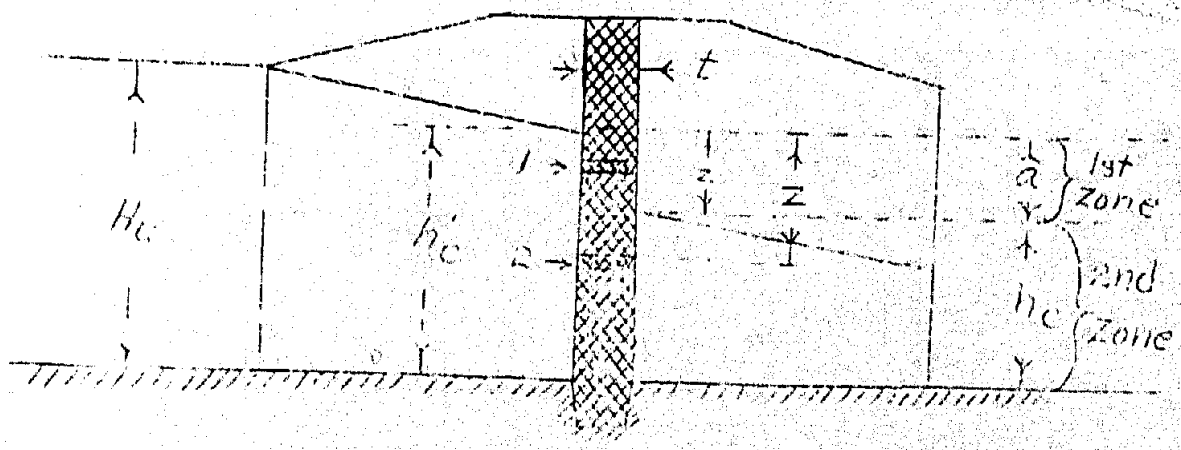


Fig. 19.

The first zone covers the upper part of the core. In this part all the percolation flow-layers are analogous to small pipes which are under a somewhat different head among each other and open into the atmosphere. It is obvious that in the ground behind the core we have atmospheric pressure so far as it is above the depression curve. After the flow-layers of this zone have moved through the core they form something like springs at the downstream face of the core, and from here the "drops" downward directly along the face of the core. We can explain this by saying that back of the core the depression line runs along the downstream face of the core itself.

If we consider any flow element (layer) in this zone and define its position by the value of (z) (see Fig. 19, element 1) we obtain the following conditions:

The length of the layer is (t), the acting head for it is (z), and the gradient is:

$$(125) \qquad J = \frac{z}{t}$$

and hence the percolation velocity:

$$(126) \qquad U = k_o \cdot J = \frac{k_o \cdot z}{t}$$

and the corresponding element of the unit discharge is:

$$(127) \qquad d\,q_1 = \frac{k_o \cdot z}{t} \cdot dz$$

So that the filtration discharge q_1 for the whole first zone is:

$$(128) \qquad q_1 = \int_0^{a_c} \frac{k_o \cdot z}{t}\,dz = \frac{k_o}{t}\int_0^{a_c} z\,dz = \frac{k_o\,a_c^2}{2t}$$

The second zone is located in the lower part of the core; the height of this part is = h_c (Fig. 19). For the flow element in this zone we see the gradient:

$$(129) \qquad J = \frac{a_c}{t}$$

and the percolation velocity:

$$(130) \qquad U = k_o \cdot J = \frac{k_o \cdot a_c}{t}$$

Also the location of this element will be defined by the value of (z) (see Fig. 19), and we see that for this zone the value of (z) can vary between the limits

$$(131) \qquad a_c \leq z \leq h'_c$$

For this the element of the unit discharge is:

$$(132) \qquad d\,q_2 = U.\,dz = \frac{k_o.a_c}{t} \cdot dz$$

and the total discharge q_2 is:

$$(133) \qquad q_2 = \int_{a_c}^{h\delta} \frac{k_o.a_c}{t}\,dz = \frac{k_o.a_c}{t}(h\delta - a_c)$$

By adding the discharge q_1 and q_2 we obtain the total percolation discharge:

$$(134) \qquad q = q_1 + q_2 = \frac{k_o.a_c}{t}\left(h'_c - \frac{a_c}{2}\right)$$

This can also be written in a different way, i.e. from Fig. 19 we see:

(135) $a_c = h\delta - h_c$ and if we substitute this into eq. 134 we find:

$$(136) \qquad q = \frac{k_o.(h'^2_c - h_c^2)}{2t}$$

or

$$(137) \qquad \frac{q}{k_o} = \frac{h\delta^2 - h_c^2}{2t}$$

17. <u>The systems of percolation equations for a dam with rectangular core.</u>

If we regard the percolation through a dam with rectangular core as a whole (Fig. 20) and keep in mind what has been said in the first chapter and in the previous paragraph, we can subdivide the percolation flow into the following parts: (1) The entrance or (upper) wedge which makes up the first part of the depression curve m- n_1;

(2) The part behind

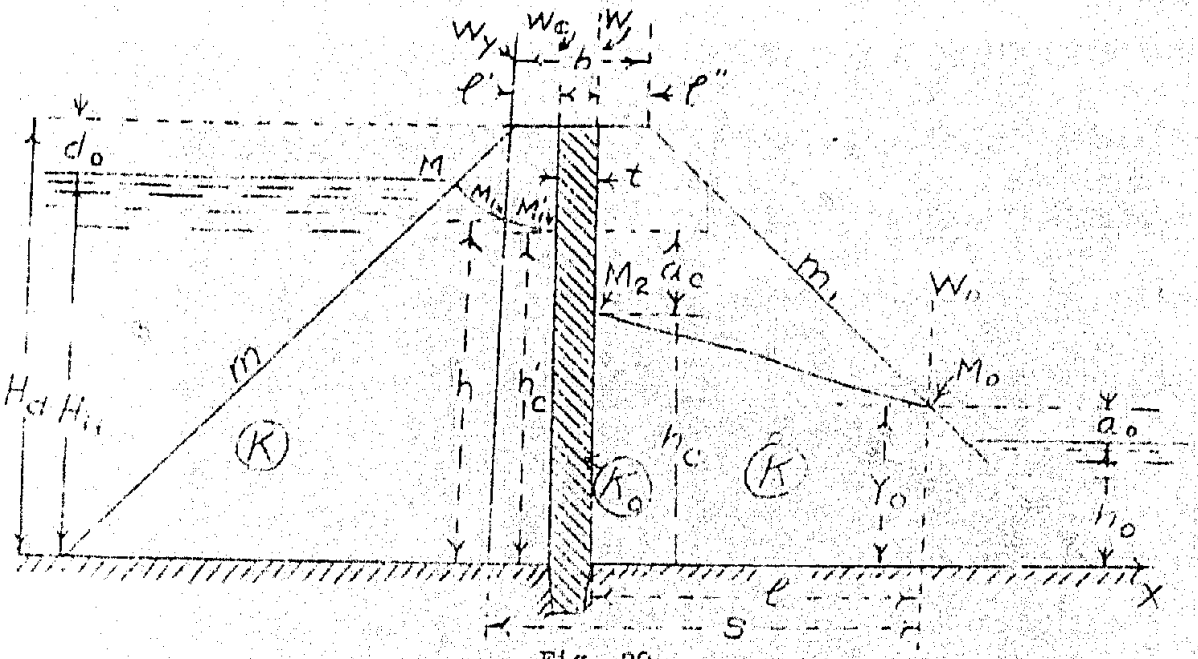


Fig. 20.

the upper wedge up to the core (the depression curve from the point $M_1 - M_1'$), (3) the percolation directly through the core with a loss of head of $= a_c$; (4) the part of the percolation flow behind the core until the "lower wedge", with the depression curve from M_2-M_o; (5) the lower (or discharge) wedge to the right side from the vertical line W_o which is laid through the point M_o. Knowing the quantities:

(139) H_d; d_o; h_c; b; ℓ'; t; k, and k_o; m and m_1

we have to find the following 7 independent variables

(139) h; h_c'; h_o; a_o; s; ℓ and q (or $\frac{q}{k}$)

For our case of the dam with core the equations I, III and IV of system (41) remain unchanged; instead of eq. II of this system we obtain two equations which correspond to the second and fourth part of the depression curve which is shown in Fig. 20. Based on eq. (10) for the non-uniform motion of the underground flow along a horizontal underlying layer, we obtain for the second part of our depression curve ($M_1 + M_1'$):

(140) $$\frac{q}{k} = \frac{h^2 - h_c'^2}{2\ell'}$$

and similarly for the fourth part (M_2-M_o):

(141) $$\frac{q}{k} = \frac{h_c^2 - (a_o + h_o)^2}{2\ell}$$

From Fig. 20 we also see that

(142) $$\ell' + t + \ell = S$$

and if we remember the relationship (137) of the preceding paragraph we obtain the system of percolation equation for the dam with core as follows:

$$
(143)\quad \begin{cases}
\dfrac{q}{k} = \dfrac{H_d - d_o - h}{m} \ln \dfrac{H_d}{H_d - h} \; : \\[2ex]
\dfrac{q}{k} = \dfrac{h^2 - h_c'^2}{2\ell'} \\[2ex]
\dfrac{q}{k_o} = \dfrac{h_c'^2 - h_c^2}{2t} \\[2ex]
\dfrac{q}{k} = \dfrac{h_c^2 - (a_o + h_o)^2}{2\ell} \\[2ex]
\dfrac{q}{k} = \dfrac{a_o}{m_1}\left[1 + \ln\left(\dfrac{a_o + h_o}{a_o}\right)\right] \\[2ex]
S = b + m_1\left[H_d - (a_o + h_o)\right] \\[2ex]
\ell' + t + \ell = S
\end{cases}
$$

This system of equations then serves for the determination of the seven unknown quantities in (139).

In order to decrease the number of unknown quantities in this system we eliminate s from the last two equations of the system. We then obtain one expression instead of two

$$
(144)\quad \ell = b + m_1\left[H_d - (a_o + h_o)\right] - (t + \ell')
$$

In the same way we can treat the first two equations of the system (143) by eliminating h. With this purpose in view we shall keep in mind that the second part of the depression line is rather short (M_1-M'_1; Fig. 20), and we have to consider what was said in paragraph 9. We shall then regard the <u>whole</u> upper part of the dam profile as the "entrance wedge", and we then apply for this part the same method of determining the loss of head which was used in paragraph 4.

We then see that the length of all elements of the percolation fig. (see paragraph 4) is increased to ℓ', where ℓ' (see Fig. 20) is the distance of the upstream wall of the core from the vertical plane W wich is laid through the point M_1, and hence for the discharge q we have instead of (1):

$$(145) \qquad q = \int_{a}^{a+h_c'} \frac{ka}{m(d_o+z)+\ell'}\,dz = ka \int_{a}^{a+h_c'} \frac{1}{m(d_o+z)+\ell'}\,dz$$

from which we obtain

$$(146) \qquad \frac{q}{k} = \frac{a}{m}\,\ell n\,\frac{m(d_o+a+h_c')+\ell'}{m(d_o+a)+\ell'}$$

or

$$(147) \qquad \frac{q}{k} = \frac{a}{m}\,\ell n\,\frac{d_o+a+h_c'+\frac{\ell'}{m}}{d_o+a+\frac{\ell'}{m}}$$

since

$$(148) \qquad \begin{cases} (d_o+a+h_c' = H_d \\ (d_o+a = H_d-h_c' \\ (a = H_d-d_o-h_c' \end{cases}$$

we have instead of (147):

$$(149) \qquad \frac{q}{k} = \frac{H_d-d_o-h_c'}{m}\,\ell n\left[\frac{\frac{\ell'}{m}+H_d}{\frac{\ell'}{m}+H_d-h_c'}\right]$$

We thus obtain instead of the system (143) the system of percolation equations:

$$(150) \qquad \begin{cases} \text{I} & \frac{q}{k} = \frac{H_d-d_o-h_c'}{m}\,\ell n\,\frac{\frac{\ell'}{m}+H_d}{\frac{\ell'}{m}+H_d-h_c'} \\ \text{II} & \frac{q}{k_o} = \frac{h'^2-h_c^2}{2t} \\ \text{III} & \frac{q}{k} = \frac{h'^2-(a_o+h_o)^2}{2\ell} \\ \text{IV} & \frac{q}{k} = \frac{a_o}{m_1}\left[1+\ell n\left(\frac{a_o+h_o}{a_o}\right)\right] \\ \text{V} & \ell = b+m_1\left[H_d-(a_o+h_o)\right]-(t+\ell') \end{cases}$$

In this system we have five unknown quantities:

(151) h_c'; h_c; a_o; ℓ; q (or $\frac{q}{k}$) and the known quantities given in (138).

The system (150) has to be solved by approximation. Thus for example we can regard (a_o) as the basic unknown quantity as in paragraph 8, and give different numerical values to it. By taking some numerical value of (a_o) we find from (IV eq. 150) the corresponding value of $\frac{q}{k}$ and from V the value of C. After that we determine the h_c from III and h_c' from eq. II of the system (150). After this eq. I will furnish a certain value of $\frac{q}{k}$ which of course would be equal to the value of $\frac{q}{k}$ which had been found by eq. IV if the value chosen for a_o in the beginning of the computation was correct. For the first trial, however, we never succeed in choosing the correct value. If we continue this method of approximation (trial) we finally find the correct value of (a_o), and in order to simplify the work we can use graphs for $\left(\frac{q}{k}\right)_{IV}$ and $\left(\frac{q}{k}\right)_{I}$, i.e. graphs for the values of $\left(\frac{q}{k}\right)$ I which we obtain from eq IV and I for the different values of (a_o). In Fig. 21 a schematical sketch of such a graph is shown (not the actual shape of the curves). The intersection point N of the curves $\left(\frac{q}{k}\right) = f(a_o)$ for $\left(\frac{q}{k}\right)$ taken from eq IV and I respectively furnishes the wanted value of a_o. If we have the value of (a_o) we can easily determine the remaining unknown quantities from the equations given in system (150).

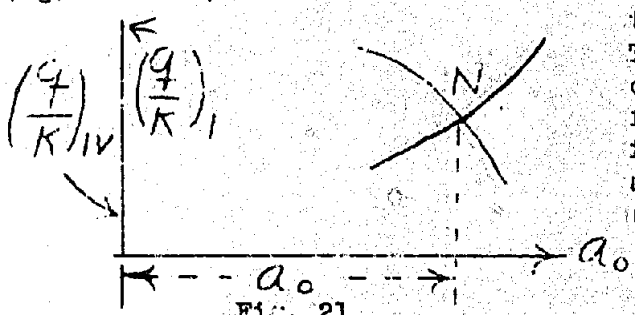


Fig. 21.

From the equations of system (143) as well as those of (150) we see that the shape of the depression curve in the earth dam with core does not depend on the absolute values of the percolation coefficients k and k_o, but only on the relation $\left(\frac{k}{k_o}\right)$. We can easily convince ourselves that this is true if we reason as in paragraph 7, and if we express the percolation equation of the core in the following form:

$$(151') \qquad \frac{h_c'^2 - h_c^2}{2t} = \frac{q}{k_o} = \frac{k}{k_o} \cdot \frac{q}{k}$$

¶ 18. On the "imaginary" lengths of the percolation stream.

As a basis for the second ("imaginary") method of the determination of the depression curve in an earth dam with core, we shall at first regard the theoretical (schematic) percolation on a horizontal impervious layer which is shown in Fig. 22 (1). For this we use a nomenclature which is in no way connected with the previously used designation.

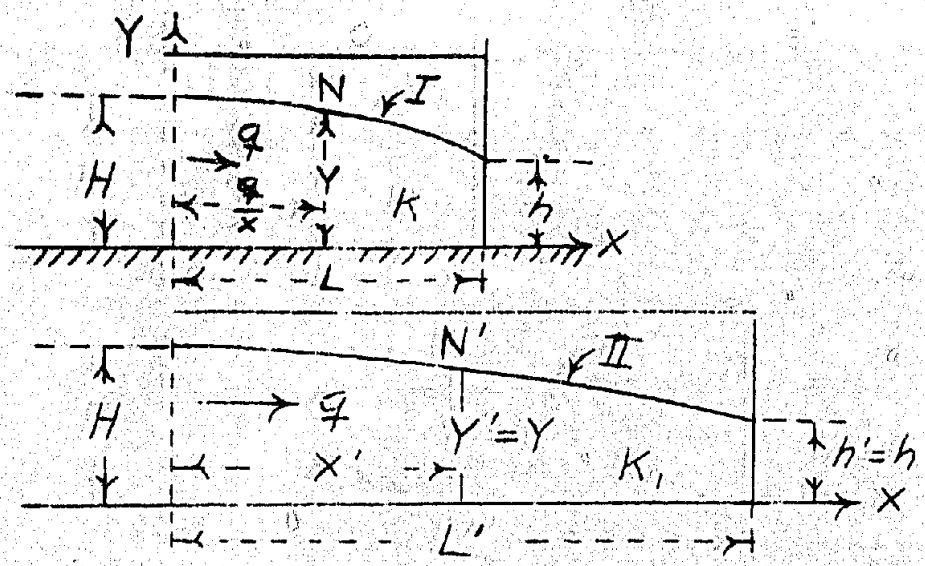


Fig. 22.

If we use the axes Ox and Oy the equation of the depression curve for the present case can be written (according to what was said in paragraph 3) in the following form:

$$(152) \quad Y^2 = H^2 - \frac{2q}{k} x$$

where H is the beginning depth of the under-ground flow, (q) is the unit discharge, (k) the coefficient of percolation. For the final depth (h) we obtain from (152) for $x = L$:

$$(153) \quad h^2 = H^2 - \frac{2qL}{k}$$

If now we take a different soil with a percolation coefficient k', while the beginning depth H and the unit discharge q remain the same and the length of the flow now is L', we obtain instead of (153) for the end depth h' the following expression:

$$(154) \quad (h')^2 = H^2 - \frac{2qL'}{k'}$$

For this the loss of head along the length L' will be:

$$(155) \quad h'_f = H - h'$$

But if we want that for the second kind of soil (Fig. 22, II) the loss of head along the new length L' remains the same as the previous (in Fig. 22, I), i.e. that it is $h_f = H - h$, we have to select such a length L' that $h' = h$. We see that this condition is fulfilled for (153) and (154) if

$$(156) \quad \frac{L}{k} = \frac{L'}{k'}$$

or

$$(157) \quad \frac{L'}{L} = \frac{k'}{k}$$

Thus if we want to obtain equal losses of head in I and II (Fig. 22) along L and L' these lengths have to be directly proportional to the percolation coefficients k' and k. We have the equation (152) for the depression curve in the soil I, and the equation of the depression curve for II under the same initial head H and for the same unit discharge will be:

$$(158) \quad (y')^2 = H^2 - \frac{2q}{k'} x'$$

and if y' = y (Fig. 22, II)

$$(159) \quad y^2 = H^2 - \frac{2q}{k'} x'$$

and comparing this with 152 we see that

$$(160) \quad \frac{x'}{x} = \frac{k'}{k}$$

We call the ratio of the percolation coefficients :

$$(161) \quad \frac{k'}{K} = \sigma$$

and hence we obtain from (157) and (160) that

$$(162) \quad - - - - - \quad L' = \sigma L$$

and

$$(163) \quad - - - - - \quad x' = \sigma x$$

From this we see that if we have the depression curve in some flow (Fig. 22, I) we obtain the depression curve in the "equivalent" flow (Fig. 22, II), by leaving the ordinates of the first curve the same and by increasing the abscissae by the factor (σ), i.e. by the ratio of the two percolation coefficients $\frac{k'}{k}$. In other words, we obtain the depression curve in one of the two "equivalent flows" by the corresponding scale of distortion of the other depression curve. i.e. so to speak by "compressing" or "stretching" it, in the horizontal direction. We call this method the "method of distortion of scale." For the transfer from the depression curve I to the curve in II we have the relations of eqts. (162)-(163); for the reverse operations we have:

$$(164) \quad L = \frac{L'}{\sigma}$$

and

$$(165) \quad x = \frac{x'}{\sigma}$$

For the further discussion we take the hydraulic percolation diagram shown in A, Fig. 23. For this we have, for a horizontal impervious lower layer, a percolation flow which can be subdivided into 3 parts: In the first part the percolation coefficient is k_1, in the second k_2 and in the third k_3. The letters designating the lengths and depths are given on the sketch. We regard part I as the basic (i.e. not changing) flow, and the other parts we substitute by "equivalent" flows with a percolation coefficient = k_1 in the above shown manner. That is we retain the initial depth H, the unit discharge, and the total loss of head for the system $h_f = H-h$. For this process the length of the first part remains unchanged (Fig. 23, B).

(166) =

the length of the second part is according to (163):

(167) $$\ell'_{2'} = \sigma_2 \ell_2$$

and that of the third part

(168) $$\ell'_{3'} = \sigma_3 \ell_3$$

where

(169) $$\sigma_2 = \frac{k_2}{k_1}$$

$$\sigma_3 = \frac{k_3}{k_1}$$

The total length of the whole "equivalent" system is equal to:

(170) $$L' = \ell_1 + \ell_2 \sigma_2 + \ell_3 \sigma_3$$

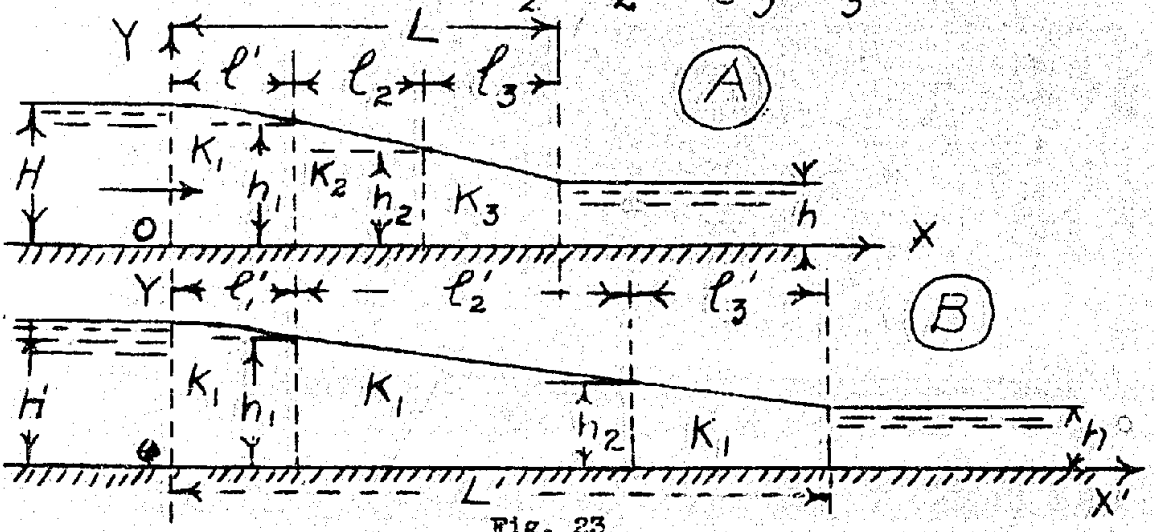


Fig. 23

and the soil of this system has a uniform percolation coefficient of k_1. We then can easily plot the depression curve by points in a coordinate system x', y from the equation (152) which now takes the following form:

$$(171) \qquad y^2 = H^2 - \frac{2q}{k_1} x'$$

From this it is also easy to obtain the depression curve for the first, initial system (Fig. 23 A) by the above discussed "method of distorted scale."

For the sake of brevity and clearness we shall introduce the term "imaginary length of the flow" (virtual = apparent length of flow), and it shall be defined in the following manner: The imaginary length of the whole given underground hydraulic system or of its parts is the geometrical length of the corresponding equivalent system or of parts of the corresponding equivalent system. We shall in the following discussion designate the "imaginary" lengths by the subscript v (standing for "virtual"). Thus according to (166)-(170) the imaginary lengths for the arrangement (fig. 23) A are, for the geometrical lengths ℓ_1; ℓ_2; ℓ_3 and L:

$$(172) \qquad \begin{aligned} &\text{for the first part } (\ell_1)_v = \ell_1 \\ &\text{for the second part } (\ell_2)_v = \sigma_2 \ell_2 \\ &\text{for the third part } (\ell_3)_v = \sigma_3 \ell_3 \\ &\text{for the whole system } L_v = \ell_1 + \sigma_2 \ell_2 + \sigma_3 \ell_3 \end{aligned}$$

where again σ_2 and σ_3 are formed from (169).

19. The "imaginary method" for the construction of the depression curve in an earth dam with a rectangular core.

Returning now to the earth dam with rectangular core which has already been shown in Fig. 20, we select the "hydraulic system" which is enclosed between the entrance and the discharge "wedge", i.e. between the verticals W and We. For this system then we find the "imaginary length" according to what has been said in the preceding paragraph. We have, however, at first to discuss a preliminary question:

In the preceding paragraph the meaning of the "equivalent flow" and the "imaginary length" has been explained on the basis of equation (152) for a depression curve of the non-uniform slowly changing motion of the underground flow along the horizontal impervious layer. The equation is, as is well known, based on the formula of Dupit for slowly changing movement of the underground flows. On the other hand we have, in paragraph 16, discussed a special scheme for the percolation within the core of dense material which is not directly connected with the formula of Dupois. Therefore, it is natural that the question arises: Can we expand what was said in the preceding paragraph to the case of a dam with core? We find a simple affirmative answer in the fact that the percolation through the core as given in paragraph 16, has lead to the expression (137):

$$\frac{q}{k_o} = \frac{(h'_c)^2 - h^2_c}{2t}$$

where: h'_c stands for the depth of the underground flow directly upstream from the core, h_c directly downstream from the core, and t stands for the thickness of the rectangular core or, which is the same, for the length of the percolation flows which pass through the core; furthermore, k_o was the percolation coefficient of the material of the core, and q the unit discharge. From the above equation we can derive:

$$(173) \qquad h^2_c = h'^2_c - \frac{2q}{k_o} t$$

But we would have obtained the same relation (173) between h'_c and h_c directly from equation (152) which was used as the basis of the discussion in the preceding paragraph, if we replace

$$(174) \qquad y = h_c \text{ and } H = h'_c$$

From this we see that the loss of head in the core is equal to $(h'_c - h_c)$. That means that the main characteristic for the "equivalent flow" and the "imaginary length" is expressed on the basis of one and the same relation. This, then, is the loss of head on the elements of the slowly changing movement. It also proves the correctness of applying the method of the "imaginary lengths" for the case of a dam with a rectangular core. We have, however, to remember that this is correct only so far as the losses of head in the core are concerned, but can not be extended to the shape of the depression curve in this core. According to the scheme given in paragraph 16, we do not think of such a depression curve in the core, but only of a free surface of the underground flow in the core. Therefore, we shall agree that with the method of "imaginary lengths" we shall generally speaking only determine the loss of head in the core, which we have to know together with the other elements from which we can construct the depression curve for the whole dam. But the details of the flow through the core are not important for this aim.

We now turn to the "hydraulic system" which we picked out in the earth dam at the beginning of this paragraph, i.e. to the percolation flow between the entrance and discharge "wedge" (Fig. 20). We see that this "system" corresponds very well to the "system" which was shown in Fig. 23. and discussed in paragraph 18. The only, however, not important difference is that in the latter all three parts had different coefficients of percolation, and that in the given case the coefficient of the core is k_o, and the two parts of the dam which are connected with the core have the percolation coefficient k.

In accordance with (172) we obtain for the present case the following relations for the "imaginary lengths":

$$(175) \qquad \ell'_v = \ell'; \ \ell''_v = \ell''$$

$$(176) \qquad t_v = \sigma' t$$

(177) $\ell_v = \ell$

(178) $S_v' = \ell' + \sigma t + \ell = S + (\sigma - 1)t$

Furthermore, we find for ℓ_v, i.e. for the imaginary width of the dam top:

(179) $b_v + \ell' + t_v + \ell'' = \sigma . t + \ell' + \ell''$

where in all these relations:

(180) $= \frac{k}{k_o}$

In this way we reduce the problem of the percolation flow through a dam with core to the percolation flow through the "equivalent" homogeneous earthdam without core.

We design the cross section through this profile of the "equivalent" dam knowning σ_v from (179) and keeping the quantities

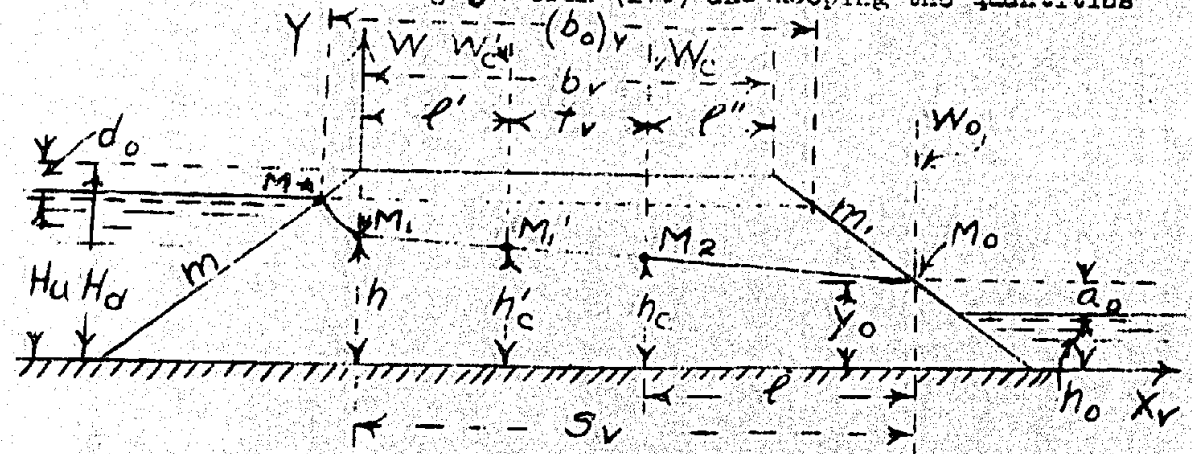


Fig. 24.

H_d; d_o; h_o, m and m_1 unchanged. For the two last quantities (m and m_1) we can mention that their constancy can be explained by the fact that the percolation coefficient of the entrance and discharge wedge is = k, i.e. the same as the one of the parts which are directly adjacent to the core. The profile of the equivalent dam is shown in figure 24. The dimensions which differ from those of the original profile (Fig. 20) are designated by the subscript (v), while the other dimensions are given in the same letters as those of Fig. 20. (see the relations in equation (175) to (179)). The depression curve and some of its points shall be discussed further below.

For the "equivalent dam" we have the following unknown quantities (see chapter 1):

(181) a_o; h; Sv; q (or $\frac{q}{k}$)

For these we have for $h_o > 0$ according to (41) the following system of percolation equations:

$$\text{I} \quad \frac{q}{k} = \frac{H_d - d_o - h}{m} \ln \frac{H_d}{H_d - h}$$

$$\text{II} \quad \frac{q}{K} = \frac{h^2 - (a_o + h_o)^2}{2\, S_v}$$

(182) $$\text{III} \quad \frac{q}{k} = \frac{a_o}{m_1}\left[1 + \ln \frac{(a_o + h_o)}{a_o}\right]$$

$$\text{IV} \quad S_v = b_v + m_1\left[H_d - (a_o + h_o)\right]$$

and for the case of $h_o = 0$ according to (41')

$$\text{I} \quad \frac{q}{k} = \frac{H_d - d_o - h}{m} \ln \left[\frac{H_d}{H_d - h}\right]$$

$$\text{II} \quad \frac{q}{k} = \frac{h^2 - a_o^{\,2}}{2\, S_v}$$

(182') $$\text{III} \quad \frac{q}{k} = \frac{a_o}{m_1}$$

$$\text{IV} \quad S_v = b_v + m_1 (H_d - a_o)$$

We solve these equations either analytically, as has been shown in paragraph 8 or with the help of the graphs and table 3 as was shown in paragraph 10-12. For the latter method we have to remember, that for the width of the dam on the elevation of the head water level, we have to introduce the corresponding imaginary $(b_o)_v$, which is, as we easily see from Fig. 24.

(183) $$(b_o)_v = b_v + (m + m_1)\, d_o$$

Furthermore, we have for the graphical method of solving the systems (182) and (182') to keep in mind what had been said to the graphics No. 1 to 3 and to the table 3.

By determining the unknown quantities of (181), we obtain the main points of the depression curve of the "imaginary" dam: M_o; M_1, and M (the last of which of course is already previously known). Afterwards we construct point by point the depression curve within the region between M_1 and M_o using the equation of the non-uniform movement on the horizontal underlaying layer (for the co-ordinate axes which are shown in Fig. 24).

(184) $y^2 = h^2 - \frac{2q}{k} x_v$

Doing this we remember that for the graphical solution of the systems (182) and (182') the ordinate axis has to go through the point M, and not through the point M_1, as in Fig. 24. It is not necessary to construct the depression curve by point, along the length t_v, as becomes clear from what was said in the beginning of this paragraph. It is sufficient to obtain the points M_1 and M_2. For this the abscissa x_v for the point M_1' in equation (184) is:

(185) $x_v = \ell'$

and for the point M_2 the abscissa is:

(186) $x_v = \ell' + t_v$

In order to transfer to the given dam profile with core (Fig. 20) we now have only to apply the "ratio of distortion of the scale", which in our case can be done quite easily(1) the part of the equivalent dam to the left side of the vertical line W'_c (see Fig. 24) or leave unchanged, also the depression curve from M to M_1' is left unchanged; (2) the part which is between the points W'_c and W_c has the horizontal length t_v, and we have to "contract" it to the length t, i.e. we reduce the length of this part by the factor σ ; (3) the part of the "equivalent" dam laying to the right side of the vertical line W_c, remains unchanged, and is only shifted to the left side by the distance $= t_v - t$ (together with the part of the depression curve between M_2 and M_0).

Thus we obtain the dam profile given in Fig. 20, in which we then obtain the construction of the depression curve, and the amount of losses of head in the core, according to equation (135) equal to $a_c = h'_c - h_c$.

As a conclusion we can point out that although the proposed second ("imaginary") method of the construction of the depression curve in an earth dam with core is somewhat simpler than the first method which had been shown in paragraph 17, and which consists in the solving of the systems of percolation equations, this first method still retains its significance for the problem in hand. It does show clearly the method of interpretation of the percolation in an earth dam with core, and furnishes the basic characteristic of the working of such a dam with respect to percolation, as a completely connected and whole hydraulic system. Thus also the "virtual" method becomes quite clear. Furthermore, we think that the above given systems of percolation equations for the earth dam with core can serve as a basis for a more comprehensive explanation of the problem at hand. Finally the first method can also be used for practical computations in many cases.

§ 20. On the percolation through a trapezoidical core.

Due to its construction advantages over the rectangular core the trapezoidical core is almost exclusively applied in the practice of dam construction. For this reason it is necessary for us to discuss the question of percolation through such a core (Fig. 25), and we shall for the time being not deal with the percolation in the remaining part of the dam. Let us at first explain the symbols which are used on the (figure 25);

H_u = the depth of the headwater upstream from the dam.

h'_c = the depth of the underground flow directly before the core.

h_c = the depth of the percolation flow behind the core.

a_c = the loss of head in the core.

t_o = the thickness of the core on the elevation of the headwater surface.

T = the thickness of the core at its base.

m_o = the slope of the core (see paragraph 16).

t_c = the thickness of the core on the elevation h'_c, i.e. on the elevation of the percolation flow level in front of the core.

t = the variable thickness of the core on any arbitrarily assumed elevation which is given by the quantity (z) (see Fig. 25).

In accordance with the scheme given in paragraph 16 for the rectangular core, we subdivide the percolation through the core also here into two zones.

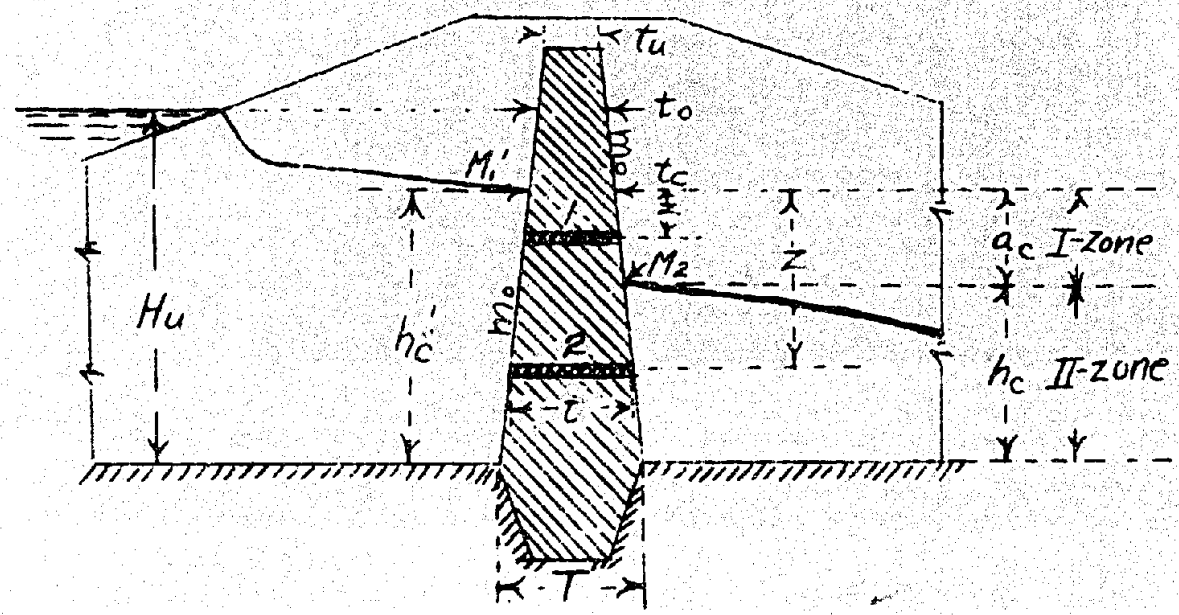


Fig. 25.

The first zone (the upper). In selecting in this zone an arbitrary flow element 1 (see Fig. 25) we obtain for this element the same expressions as were given in equation (125)-(127) in paragraph 16, for the rectangular core, only that for the present case the quantity (t) is variable and dependent upon (z):

$$(187) \qquad t = t_c + 2\, m_D\, Z$$

so that according to (127) we obtain for the element of the discharged q_1:

$$(188) \qquad dq_1 = \frac{k_o\, Z}{t_c + 2 m_o\, Z}\, dZ$$

and consequently :

$$(189) \qquad q_1 = \int_a^{a_c} \frac{k_o\, z}{t_c + 2\, m_o z}\, d_z = \frac{k_o}{2\, m_o} \int_a^{a_c} \frac{z}{z + \frac{t_c}{2\, m_o}}\, dz$$

and since:

$$(190) \qquad \frac{z}{z + \frac{t_c}{2\, m_o}} = 1 - \frac{\frac{t_c}{2\, m_o}}{z + \frac{t_c}{2\, m_o}}$$

we find from (189):

$$(191) \qquad q_1 = \frac{k_o}{2\, m_o} \left[a_c - \frac{t_c}{2\, m_o} \ln \left(\frac{a_c + \frac{t_c}{2\, m_o}}{\frac{t_c}{2\, m_o}} \right) \right]$$

The second (lower) zone. For this zone we select the flow element 2(see Fig. 25) and we obtain the same expressions as (129)-(132) of paragraph 16, only that again (t) is related to z in the way given in (187). We then have:

$$(192) \qquad dq_2 = \frac{k_o a_c}{t_c + 2 m_o z}\, dz$$

and q_2 is from this determined to:

$$(193) \qquad q_2 = \int_{a_c}^{h'_c} \frac{k_o\, a_c}{t_c + 2 m_o z}\, d_z = k_o\, a_c \int_{a_c}^{h'_c} \frac{1}{t_c + 2\, m_o z}\, dz$$

or

$$(194) \qquad q_2 = \frac{k_o\, a_c}{2\, m_o} \ln \left[\frac{h'_c + \frac{t_c}{2\, m_o}}{a_c + \frac{t_c}{2\, m_o}} \right]$$

By adding the partial discharge we obtain the total discharge q:

$$(195) \qquad q = q_1 + q_2 = \frac{k_o\, a_c}{2\, m_o} \left[1 - \frac{t_c}{2\, m_o a_c} \ln \left(\frac{a_c + \frac{t_c}{2\, m_o}}{\frac{t_c}{2\, m_o}} \right) + \ln \left(\frac{h'_c + \frac{t_c}{2\, m_o}}{a_c + \frac{t_c}{2\, m_o}} \right) \right]$$

In this expression we now have to express t_c in terms of h'_c and some geometrical elements of the dam profile.

Let us now regard the geometrical relations in the core on a special diagram (Fig. 26) where the trapezoid is completed to a triangle by the dobbed lines which meet in the vertex point N. We see that

(196) $$t_c = 2\, m_o\, H_c$$

and from this we find

(197) $$\frac{t_c}{2\, m_o} = H_c$$

where H_c can be expressed in terms of h'_c and the height of the complete triangle (KNL) H:

(198) $$H_c = H - h'_c$$

for which the quantity H is known for the given core and is equal to:

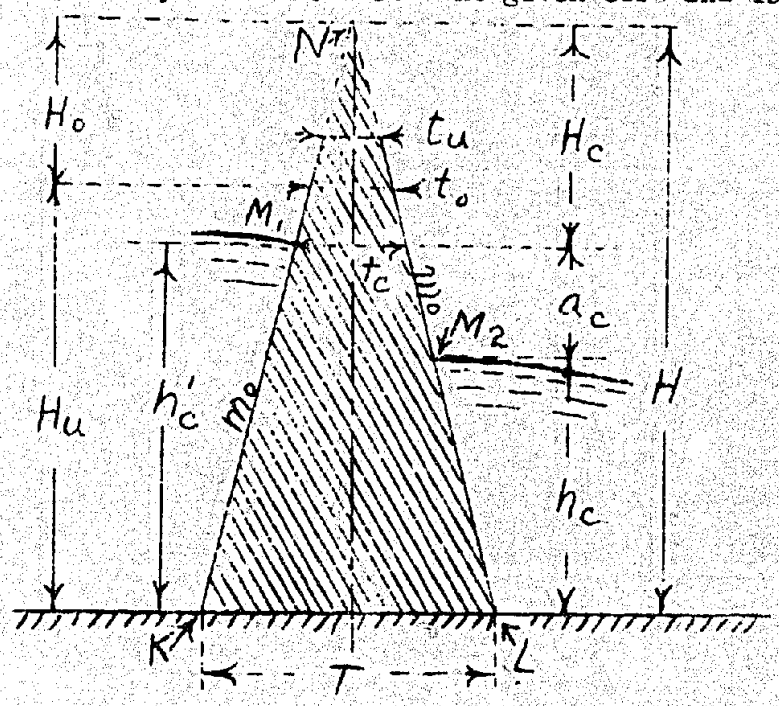


Fig. 26.

(199) $$H = H_u + H_o = H_u + \frac{t_o}{2\, m_o}$$

By substituting (197) into the expression for the discharge (195) we obtain

(200) $$q = \frac{k_o\, a_c}{2\, m_o}\left[1 - \frac{H_c}{a_c}\, \ln\left[\frac{a_c + H_c}{H_c}\right] + \ln\left(\frac{h'_c + H_c}{a_c + H_c}\right)\right] ;$$

and if we express in this equation H_c according to (198) and keep in mind that $a_c = h'_c - h_c$, we obtain the following expression:

$$(201) \qquad q = \frac{k_o(h'_c-h_c)}{2\,m_o}\left[1 - \frac{H-h'_c}{h'_c-h_c}\,\ell n\left(\frac{H-h_c}{H-h'_c}\right) - \ell n\,\frac{H}{H-h_c}\right]$$

and thus

$$(202) \qquad \frac{q}{k_o} = \frac{h'_c-h_c}{2\,m_o}\left[1 - \frac{H-h'_c}{h'_c-h_c}\,\ell n\left(\frac{H-h_c}{H-h'_c}\right) + \ell n\left(\frac{H}{H-h'_c}\right)\right]$$

to which corresponds for a constant thickness of the core the above obtained formula (137), the equation (134) on the other hand corresponds to (200).

¶ 21. The introduction of a mean thickness of the trapezoidical co

By obtaining the relations given in equation (202) we can set up a system of percolation equations for the dam with trapezoidal core. But the solution of such a system would be comparatively complicated for practical purposes. Therefore, we shall discuss here the question of the possibility of replacing the trapezoidical core by a core which has the uniform average thickness of the latter for the calculation.

With this aim in mind we take the equation (202) for $\frac{q}{k_o}$ and the equation (137) into which we now introduce the average thickness t_{av} :

$$(203) \qquad \left(\frac{q}{k_o}\right)_{av} = \frac{h'^2 - h^2_c}{2\,t_{av}}$$

and dividing (203) by (202) we obtain:

$$(204) \qquad \varepsilon = \left(\frac{q}{k_o}\right)_{av} : \frac{q}{k_o}$$

$$(205) \qquad \varepsilon = \frac{m_o\,(h'_c + h_c)}{t_{av}\left[1 - \frac{H-h'_c}{h'_c-h_c}\,\ell n\left(\frac{H-h_c}{H-h'_c}\right) + \ell n\left(\frac{H}{H-h_c}\right)\right]}$$

In order to obtain a more general form for the comparison of $\left(\frac{q}{k}\right)$ & $\left(\frac{q}{k}\right)_{av}$ we introduce some ratios which refer to a certain basic quantity of reference. As the latter we introduce the depth of the water in front of the dam H_u. We thus call

$$(206) \qquad \frac{h'_c}{H_u} = \eta'_c$$

(207) $$\frac{h_c'}{H_u} = \eta'_c$$

and obtain

(208) $$h'_c = \eta'_c \cdot H_u$$

(209) $$h_c = \eta_c \cdot H_u$$

Furthermore, it is for our purpose sufficiently accurate to assume that the thickness of the core at the elevation of the head water level (t_o) is approximately 1/3 of the width of the dam crest b; i.e.:

(210) $$t_o = \sim \frac{1}{3} b$$

In connection with this we remember the commonly used relation between the depth of the water upstream from the dam H_u and the width b, and for our comparison we can use the quantity (1)

(211) $$t_o = \sim 0.1 \; H.$$

(foot note: If the thickness (t_o) is for some reason taken to be more than this is given in (211) then the core is more closely rectangular, and for this reason the results of the above carried out comparison would come out more in favor for the recommended method of replacing the trapezoidical core by a rectangular core of average thickness. We therefore want to emphasize that the relation (211) is given only for the sake of the impartial carrying out of the above comparison)

We have from (199) see (Fig. 26):

(212) $$H = H_u + \frac{0.1\, H_1}{2\, m_o} = \left(1 + \frac{0.05}{m_o}\right) H_u$$

and for the quantity t_{av} we find (see Fig. 26):

(213) $$t_{av} = 1/2\,(t_o + T) = 1/2\left[t_o + (t_o + 2m_o H_u)\right] = (0.1 + m_o) H_u$$

We can write (212) in the simplified manner:

(214) $$H = \eta \cdot H_u$$

where

(215) $$\eta = 1 + \frac{0.05}{m_o}$$

we can now introduce (208), (209) (213) and (214) into (205) and find

(216) $$\text{----}\; \varepsilon = \frac{m_o}{0.1 + m_o} \cdot \frac{\eta'_c + \eta_c}{\phi\,(\eta'_c, \eta_c)}$$

where for the given quantity m_o:

(217) $$\phi\,(\eta'_c, \eta_c) = 1 - \frac{\eta - \eta'_c}{\eta'_c - \eta_c} \ln\left(\frac{\eta - \eta_c}{\eta - \eta'_c}\right) + \ln\left(\frac{\eta}{\eta - \eta_c}\right)$$

The tables further below are computed from the equations 216, 217 and 215 (table 9-11). The first is made for $m_o = 1/12$, the second for $m_o = 1/8$ and the third for $m_o = 1/6$. In the last columns of these tables the quantities:

$$a_c = \eta_c' - \eta_c = \frac{h_c' - h_c}{H_u} = \frac{a_c}{H_u}$$ are

calculated in addition to the above listed values; they expose the relative head losses in the core (i.e. within the limits of the depth of the water H_u).

For the comparison let us at first turn our attention to those quantities (ε) of table 9-11, which are obtained from $\eta_c' = 1$ in the following manner.

The mean thickness of the core t_{av} has for each given case to be found at the beginning of the computation from the formula

$$(218) \qquad t_{av} = \frac{t_o + T}{2}$$

although actually the core influences the filtration not within the limits of its thickness (t_c) to T but within the limits between t_c to T (Fig. 25-26). But at the beginning of the computation we do not know the quantity t_c beforehand, and therefore we cannot calculate the average thickness of the core from the formula:

$$(219) \qquad t_{av} = \frac{t_c + T}{2}$$, and thus we introduce into the computation a somewhat reduced thickness t_{av} according to formula (218). only for $\eta_c' = 1$, i.e. $h'_c = H_u$, are the formula (218) and (219) analogous. From this discussion it arises that for the characterisation of the results of the substitution of the trapezoidical core by the rectangular core only those values of tables 9-11 are important which are related to the value $\eta_c' = 1$.

TABLE 9: $Mo = \frac{1}{12}$

No.=	η_c'	η_c	$\eta_c' + \eta_c$	$\Phi(\eta_c, \eta_c')$	ε	$a_c = \eta_c' - \eta_c$
1	1.00	0.00	1.00	0.410	1.11	1.00
		0.10	1.10	0.456	1.10	.90
		0.30	1.30	0.544	1.08	.70
		.50	1.50	0.648	1.05	.60
		.70	1.70	0.765	1.01	.30
2	.90	.00	.90	.357	1.14	.90
		.10	1.00	.399	1.13	.80
		.30	1.20	.483	1.12	.60
		.50	1.40	.582	1.10	.40
		.70	1.60	.698	1.06	.20
3	.80	.00	.80	.306	1.19	.80
		.10	.90	.346	1.18	.70
		.30	1.10	.430	1.16	.50
		.50	1.30	.523	1.13	.30
		.70	1.50	.633	1.07	.10
4	.70	.00	1.29	.259	1.23	.70
		.10	1.50	.297	1.22	.60
		.30	2.25	.379	1.20	.40
		.50	4.50	.473	1.15	.20
5	.60	.00	.60	.215	1.27	.60
		.10	.70	.254	1.25	.50
		.30	.90	.333	1.22	.30
		.50	1.10	.420	1.19	.10

TABLE 10

$m_o = \frac{1}{8}$

No.	η_c'	η_c	$\eta_c' + \eta_c$	$\Phi(\eta_c, \eta_c')$	ε	$a_c = \eta_c' - \eta_c$
1	1.00	.00	1.00	.499	1.12	1.00
		.10	1.10	.551	1.11	.90
		.30	1.30	.663	1.09	.70
		.50	1.50	.794	1.05	.50
		.70	1.70	.946	1.00	.30
2	.90	.00	.90	.427	1.17	.90
		.10	1.00	.478	1.16	.80
		.30	1.20	.584	1.14	.60
		.50	1.40	.709	1.10	.40
		.70	1.60	.853	1.04	.20
3	.80	.00	.80	.364	1.22	.80
		.10	.900	.413	1.21	.70
		.30	1.10	.513	1.19	.50
		.50	1.30	.631	1.15	.30
		.70	1.50	.763	1.08	.10
4	.70	.00	.70	.307	1.27	.70
		.10	.80	.349	1.26	.60
		.30	1.00	.449	1.24	.40
		.50	1.20	.563	1.18	.20
5	.60	.00	.60	.253	1.32	.60
		.10	.70	.296	1.30	.50
		.30	.90	.394	1.27	.30
		.50	1.10	.498	1.22	.10

TABLE 11

$m_o = \frac{1}{6}$

No.	η_c'	η_c	$\eta_c' + \eta_c$	$\Phi(\eta_c, \eta_c')$	ε	$q_c = \eta_c' - \eta_c$
1	1.00	.00	1.00	.553	1.13	1.00
		.10	1.10	.616	1.12	.90
		.30	1.30	.747	1.09	.70
		.50	1.50	.895	1.05	.50
		.70	1.70	1.081	.99	.30
2	.90	.00	.90	.475	1.19	.90
		.10	1.00	.530	1.18	.80
		.30	1.20	.651	1.15	.60
		.50	1.40	.791	1.11	.40
		.70	1.60	.960	1.04	.20
3	.80	.00	.80	.403	1.24	.80
		.10	.90	.454	1.23	.70
		.30	1.10	.568	1.21	.50
		.50	1.30	.701	1.16	.30
		.70	1.50	.857	1.09	.10
4	.70	.00	.70	.336	1.30	.70
		.10	.80	.386	1.29	.60
		.30	1.00	.494	1.27	.40
		.50	1.20	.625	1.20	.20
5	.60	.00	.60	.278	1.35	.60
		.10	.70	.326	1.34	.50
		.30	.90	.431	1.30	.30
		.50	1.10	.560	1.24	.10

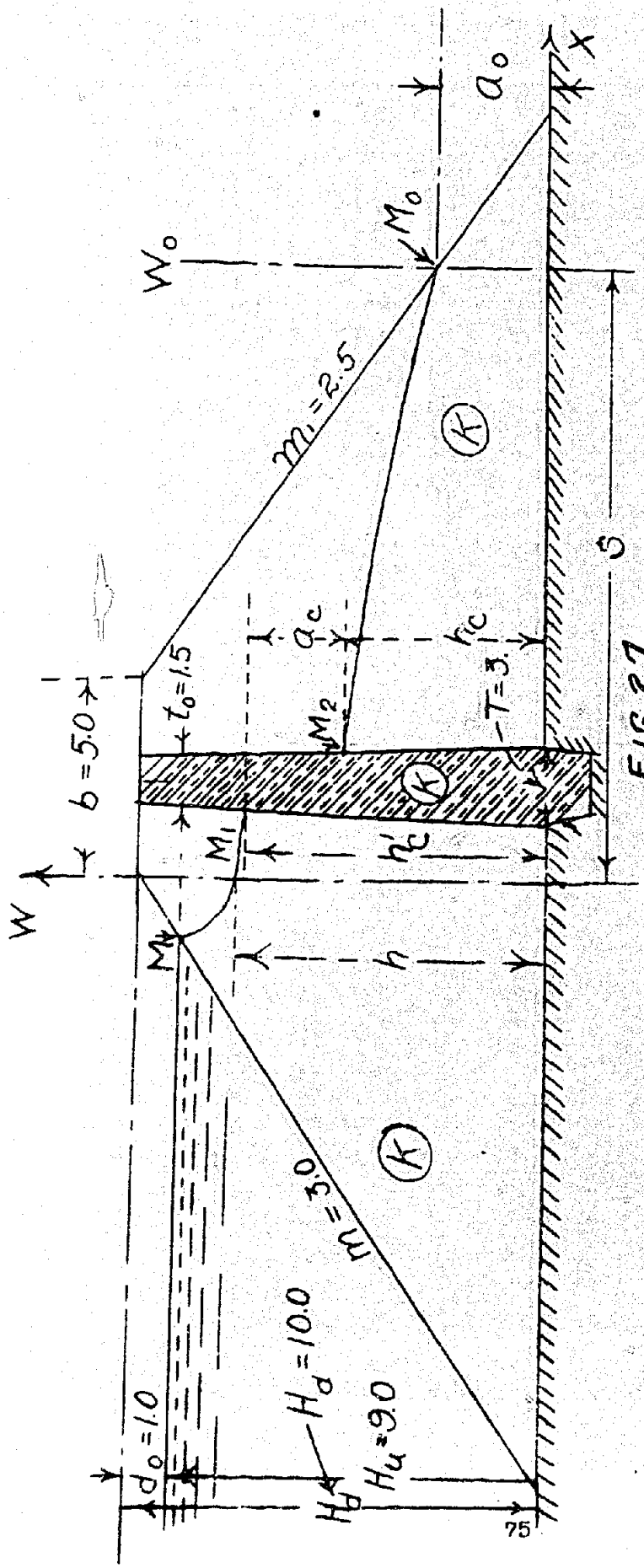


FIG. 27

From those values we also see that for the cases which are taken up in the tables the quantity ε changes approximately between 1.0 & 1.13 and has an average of about $\sim$ 1.06. From this we can conclude that for practical cases we can simply substitute the trapezoidical core by a rectangular core of the average thickness t_{av}.

However, the other values of the table 9-11 (for m_1, L1) show the necessity of the following correction. In computing the depression curve for an average thickness from eqs. (218) we obtain values for t_c, so that we can after this find now (more accurate) values of t_{av} according to (219) and can again carry out the computation for plotting the depression curve and the determination of the percolation discharge q. Of course the computation for an average thickness is carried out as this has been shown in paragraph 17-19.

We can finally point out, that it is not necessary to use the method of "averaging the thickness" of the trapezodical core at all, if we turn directly to equation (202), if this seems to be desirable for some reason in a particular practical case, then (202) is included into the system of the more complicated percolation equations for earth dams with core. But this case shall not be taken up more completely.

22. Example of plotting the "depression curve" for an earth dam with core.

Let us consider the following example of an earth dam with inner core (Fig. 27). We shall again use the adopted notations:

H_d = 10.0 m	d_o = 1.0 m	h_o = 0 (no water in the tail race)
b = 5.0 m	H_u = 9.0 m	t_o = 1.5 m
T = 3.0 m	m = 3.0	m_1 = 2.5

The percolation coefficient of the core be k_o = 0.00001 cm/sec. that of the material of the dam body k = 0.0001 cm/sec.

('Foot note: On the question of the values of the coefficient k_o, and k, m and m_1 see the foot note to the example given in paragraph 13)

At first we determine the average thickness of the core

$t_{av} = \frac{t_o + T}{2}$ = 2.25 m, and following the quantities ℓ' and ℓ'' for the average thickness of the core (see Fig. 27 and also the preceding Fig. 20 for the dam with rectangular core).

$$(220) \quad \ell' = \ell'' = \frac{1}{2}(b - t_{av}) = 1.375 \cong \underline{1.37 \text{ m}}$$

we now find the coefficient (σ) from (180): $\sigma = \frac{k}{k_o}$

$\sigma = \frac{k}{k_o}$ = 10.00 and have from equation (176) the imaginary thickness of the core t_v, which in the given example corresponds to its average geometrical thickness (= 2.25 m).

$$(221) \quad t_v = \sigma t_{av} = 10(2.25) = 22.5 \text{ m}$$

and the imaginary (virtual) width of the dam top from equation (179):

$$b_v = \ell' + t_o + \ell'' = 25.25 \text{ m.}$$

Thus the profile of the homogeneous "equivalent dam" (see Fig. 24) has the following dimensions: width of dam top (crest) = b_v = 25.25 m, the values H_d, H_u, d_o, h_o, m and m_1 remain the same as for the "basic" (given) dam with core. The coefficient of percolation for the "equivalent" dam is k = 0.0001 cm/sec. For this dam we do now compute the depression curve as has been shown in chapter II where also an example (paragraph 14) was given. For this purpose it is in the given case necessary to solve the system of equations given in (182').

For the analytic solution, the same as in paragraph 14, we have at first to find

$$\frac{m}{m_1} = \frac{3.0}{2.5} = 1.20$$

$$(222) \quad \frac{b_v}{m_1} + H_d = \frac{25.25}{2.5} + 10.0 = 20.1 \text{ m}$$

After that we compute table 12 using different values of a_o. In regard to this table we can say the same has been said already in paragraph 14 in regard to table 8. We see from table 12 that for the "equivalent dam"

$$a_o = 1.56 \text{ m}$$

we now find the value of S_v from equation IV of system (182'):

$$S_v = b_v + m_1 (H_d - a_o) = 25.25 + 2.5(10.0 - 1.56) = 46.35 \text{m}$$

and h from equation (67):

$$h = \sqrt{\frac{2\, a_o\, S_v}{m_1} + a_o^2} = \sqrt{\frac{2 \times 1.56 \times 46.35}{2.5} + 1.56^2}$$

$$(223) \qquad h = 7.76 \text{ m.}$$

Furthermore from III of system (182'):

$$\frac{q}{k} = \frac{a_o}{m_1} = \frac{1.56}{2.5} = 0.624 \text{ m} = 62.4 \text{ m}$$

and from (73):

$$q = \frac{a_o\, k}{m_1} = 62.4 \times 0.0001 = 0.00624 \text{ cm}^2/\text{sec.}$$

or from (111):

$$q_c = \frac{8640}{m_1}\, a_o\, k = 8640 \times 0.00624 = 53.9$$ measured in liters per day (24 hrs) and per 1 meter of the length of the dam.

For the determined values of h and $\frac{q}{k}$ the equation (184) for the second part of the depression curve in the "equivalent" dam has, referre to the axes shown in Fig. 24 the following form:

$$(224) \qquad y^2 = h^2 - \frac{2q}{k} x_v = 60.24 - 125\, x_v$$

From this equation we then can find the coordinates of several points. Thus we find for M_1' the abscissa x_v from (185):

$$x_v = \ell' = 1.37 \text{ m}$$

and for the point M_2 the abscissa is according to (186):

$$x_v = \ell' + t_v = 23.87 \text{ m}$$

and for M_1' we find that the ordinate of the depression curve is:

$$y = h'_c = 7.65 \text{ m and for } M_2 \quad y = h_c = 5.50 \text{ m.}$$

For the section of the curve between M_1 and M_1' we take only one intermediate point (see table 13), and for this section $x_v = x$, where x is the abscissa of the depression curve in the given dam with core (Fig. 27).

1	a_0	a_0	1.40	1.50	1.53	1.56	
2	$F_1(a_0) = \frac{m}{m_1} \cdot a_0$	$F_1(a_0) = 1.2(1)$	1.68	1.80	1.84	1.87	
3	$\left[\left(\frac{bv}{m_1} + Hd\right) - a_0\right]$	$20.1 - (1)$	18.70	18.60	18.57	18.54	
4	$2a_0\left[\left(\frac{bv}{m_1} + Hd\right) - a_0\right]$	$2.(1)(3)$	52.40	55.80	56.80	57.80	
5	a_0^2	$(1)^2$	1.96	2.25	2.34	2.44	
6	$D^2 = 2a_0\left[\left(\frac{bv}{m_1} + Hd\right) - a_0\right] + a_0^2$	$D^2 = (4) + (5)$	54.36	58.05	59.14	60.24	
7	D	$D = \sqrt{(6)}$	7.37	7.63	7.70	7.76	
8	$Hd - D$	$10 - (7)$	2.63	2.37	2.30	2.24	
9	$\frac{Hd}{Hd - D}$	$\frac{10}{(8)}$	3.80	4.22	4.35	4.46	
10	$E = 2.303 \log \frac{Hd}{Hd - D}$	$E = 2.303 \log_{10} (9)$	1.33	1.44	1.47	1.50	
11	$Hd - d - D$	$(8) - 1$	1.63	1.37	1.30	1.24	
12	$F_2(a_0) = [Hd - d - D] + E$	$F_2(a_0) = (10)(11)$	2.17	1.97	1.91	1.86	

TABLE 13

(dimensions in meters)

$x_v = x$	$1.25 \times X_v$	y^2	y
.00	.00	60.24	7.76 = h
.70	.88	59.36	7.70
1.37 = ℓ'	1.71	58.53	7.65 = h'_c

For the remaining points of the depression curve we select the abscissae (225) $x_v > (\ell' + t_v)$

i.e. $x_v > 23.87$ m

for which of course these abscissae have to be taken within the limits.

(226) $23.87 \text{ m} < x_v < S_v$

or $23.87 \text{ m} < x_v < 46.35$ m.

The computations of the ordinates (y) for the part of the depression curve between M_2 and Mo according to (224) are shown in table 14. In the last column also the abscissae x are listed which belong to the corresponding points of the given dam profile with core. Based on what has been said about the "distortion of the scale" at the end of paragraph 19, the abscissae x (table 14) are for our example obtained from the abscissae x_v by reducing the latter by the quantity $= t_v - t_{av}$ i.e.:

(227) $x = x_v - (t_v - t_{av})$

or (in the given example):

(228) $x = x_v - 20.25$ (in meters)

The depression curve is shown on Fig. 27 where we can clearly see the amount of head loss due to the core, which is in the given example equal to:

(229) $a_c = h_c' - h_c = 2.15$ m.

TABLE 14
(dimensions in meters)

x_v	$1.25x_v$	y^2	y	$x = x_v - 20.25$
$\ell' + t_v$ = 23.87	29.85	30.39	5.50 = h_c	3.62
25.00	31.25	28.99	5.37	4.75
27.00	33.75	26.49	5.15	6.75
29.00	36.25	23.99	4.90	8.75
31.00	38.75	21.49	4.63	10.75
33.00	41.25	18.99	4.35	12.75
35.00	43.75	16.49	4.06	14.75
37.00	46.25	13.99	3.74	16.75
39.00	48.75	11.49	3.39	18.75
41.00	51.25	8.99	3.00	20.75
43.00	53.75	6.49	2.55	22.75
45.00	56.25	3.99	2.00	24.75
S_v = 46.35	57.80	2.44	1.56	26.10=S
			$y_o = a_o + h_o$	

For the graphic solution of the systems (182) or (182')for the conditions of the "equivalent dam" which have a rather great width of the dam crest, the graphs No. 1-3 and table 3 (which are shown in Para. 12) and set up expressively for dams with ordinary crest widths, can give only orientating results. These results have to be, generally speaking, corrected by an analytic solution of the systems (182) or (182') as has been shown for the above example.

For this example the use of the graphs No. 1-3 or better of table 3 has to be carried out along the following lines:

After finding (in the same way as shown above) the quantities

(230) t_{av}, ℓ', ℓ'', σ, t_v and b_v we determine the virtual width of the dam $(b_o)_v$ on the elevation of the head water level (from 183):

(231) $(b_o)_v = b_v + (m + m_1)\, b_o = 30.75$ m and then (see Para. 14-II) from table 3

$\alpha = \sim 0.299$; $\beta = \sim 0.038$ corresponding to $m = 3.0$ and $m_1 = 2.5$ for the discussed example. Furthermore we obtain (using the guiding table 5 (f))

(Footnote: Substituting of course the notations bo & s by $(b_o)_v$ & s_v)

$a_o = 0.299 \times 9.0 - 0.038 \times 30.75 = 1.52$ m

$s_v = 30.75 + 2.5\,(9.0 - 1.52) = 49.46$ m

$h^2 = 62.41$ (from formula 6°) table 5) $h = 7.90$ m

and finally from 8° - 10° (table 5): $\frac{q}{k} = \frac{a_o}{m_1} = \frac{1.52}{2.50} = 0.61$ m $= 61$ cm

$q = 61 \times .00001 = \underline{0.0061\ cm^2/sec.}$

(End of Chapter III)

CHAPTER IV

¶ 23. Filtration through the upstream face of the dam.

Aside from an inner core in the dam, as discussed in the third chapter, we find another means of diminishing the perviousness of an earth dam: the construction of an earth dam with an upstream face of a less pervious material than the rest of the dam (see Fig. 28). We shall designate this type of dam construction by the term "dam with impervious upstream face layer" and call the impervious layer on the dam "protection layer."

In discussing the percolation through such a "protection layer" we shall have in mind a pavement of a constant "average thickness" and we shall use the following designations (see Fig. 28):

H_U - depth of water upstream from dam

δ - the thickness of the pavement

h_p - depth of the percolation flow in the body of the dam, directly beyond the upstream face protection layer.

A_p - head loss in pavement

k_o - percolation coefficient of the material of this layer

m' (= cotang. ϕ') - the slope coefficient of the layer. In regard to the last quantity we can say that m' may generally speaking be different from the value of m which determines the slope of the upstream face if the impervious "layer" is at the upstream side covered with some sort of pervious material as shown in Fig. 28. But if it is sufficient to cover the protection layer with, for example, crushed stone pavement then

$$m' = m \tag{232}$$

But this condition will not enter our discussions since we shall have to deal with cases for which we can neglect the hydraulic resistances in the part above the "protection layer" as compared to the resistances of the layer itself.

We shall discuss the percolation through the protection layer along the same general lines as the percolation through the core in ¶ 16. We subdivide the percolation flow into two parts (see Fig. 29).

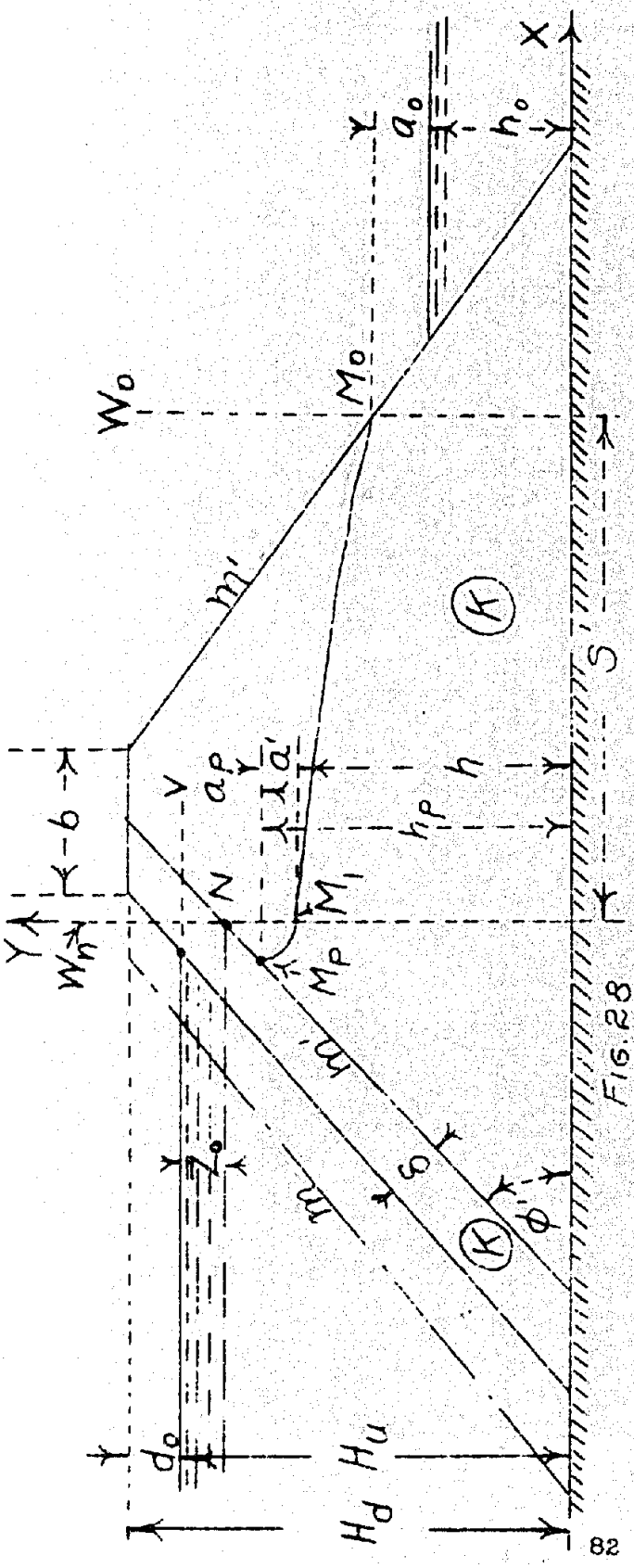


Fig. 28

The first (upper) zone.

In this zone the flow element $N_o M_o$ (see Fig. 29) has the length δ, which is measured in the direction normal to the slope of the protection layer, corresponding to what had been said in ¶ 2, about the flow elements entering the upstream face. The gradient of this flow element is

$$(233) \quad J = \frac{Z}{\delta}$$

Since the active head for the layer (the difference of head at its ends) is equal to Z, where Z is the depth of submergence of the outcoming end of the flow element below the level of the head water upstream from the dam. For this the velocity of percolation of the flow element is:

$$(234) \quad U = k \cdot J = \frac{k_o Z}{\delta}$$

and the corresponding elementary percolation discharge:

$$(235) \quad dq_1 = u\,d\xi = \frac{k_o Z}{\delta}\,d\xi$$

where $d\xi$ is the thickness of the flow element.

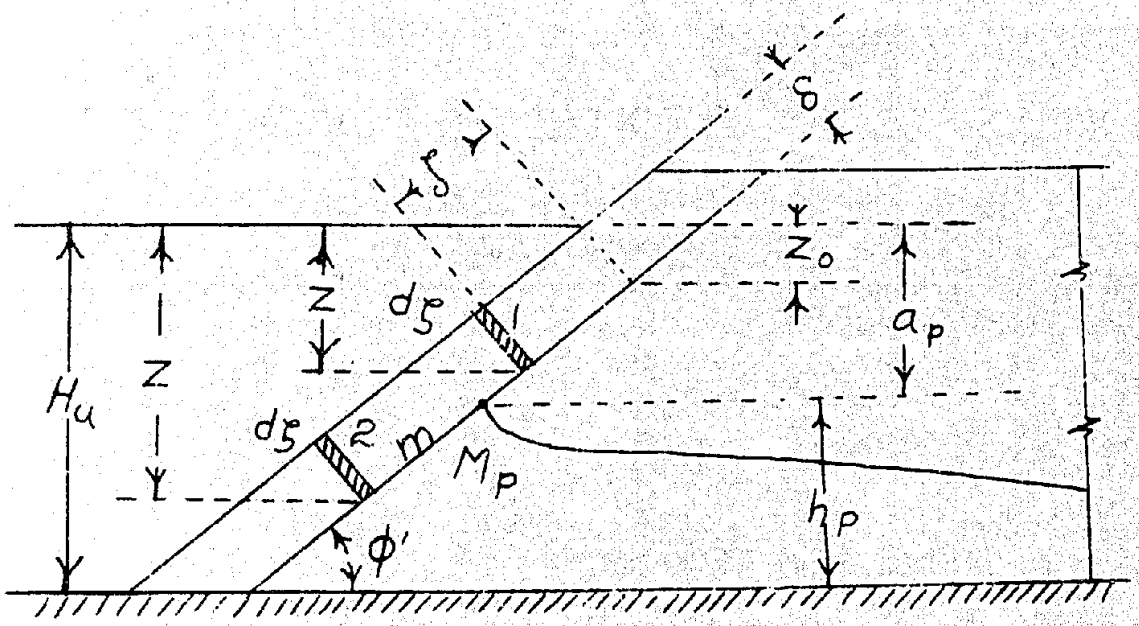


FIG. 29

From Fig. 29 we obtain:

(236) $$Z = \zeta \sin\phi' + \delta \cos\phi'$$ and

(237) $$\zeta \sin\phi' = Z - \delta \cos\phi'$$

and differentiating this expression:

(238) $$d\zeta = \frac{dZ}{\sin\phi'}$$

and introducing this result into the equation (235) for the elementary discharge dq_1 we obtain:

(239) $$dq_1 = \frac{k_o Z}{\delta \sin\phi'} dZ$$

whence we obtain the percolation discharge

(240) $$q_1 = \int_{Z_o}^{a_p} \frac{k_o Z}{\delta \sin\phi'} dZ = \frac{k_o}{\delta \sin\phi'} \int_{Z_o}^{a_p} Z dZ$$

or

(241) $$q_1 = \frac{k_o}{2\delta \sin\phi'} \left[a_p^2 - Z_o^2 \right]$$

where the quantity Z_o can be seen to be

$$z_o = \delta \cos \phi' \qquad \text{from Fig. 29.}$$

<u>The second (lower) zone.</u>

Here for the percolation of the flow through the protection layer the gradient of the flow element 2 is:

$$(243) \qquad J = \frac{a_p}{\delta}$$

the percolation velocity

$$(244) \qquad U = k_o J = \frac{k_o a_p}{\delta}$$

and the corresponding elementary discharge

$$(245) \qquad dq_2 = \frac{k_o a_p}{\delta} \, d\zeta$$

or in correspondence to (238):

$$(246) \qquad dq_2 = \frac{k_o a_p}{\delta \sin \phi'} \, dZ$$

whence we obtain the discharge q_2:

$$(247) \qquad q_2 = \int_{a_p}^{H_u} \frac{k_o a_p}{\delta \sin \phi'} \, dZ = \frac{k_o a_p}{\delta \sin \phi'} \, (H_u - a_p)$$

The total flow discharge through the protection layer is from (241) and (247):

$$(248) \qquad q = q_1 + q_2 = \frac{k_o a_p}{\delta \sin \phi'} \left(H_u - \frac{a_p}{2}\right) - \frac{k_o z_o^2}{2\delta \sin \phi'}$$

in which we substitute

$$(249) \qquad a_p = H_u - h_p$$

so that instead of (248) we obtain:

$$(250) \qquad q = \frac{k_o\left(h_u^2 - h_p^2\right)}{2\,\delta \sin\phi'} - \frac{k_o Z_o^2}{2\,\delta \sin\phi'}$$

or:

$$(251) \qquad q = \frac{k_o\left[\left(H_u^2 - Z_o^2\right) - h_p^2\right]}{2\,\delta \sin\phi'}$$

where again Z_o is a known quantity which is determined by (242)

$$Z_o = \delta \cos\phi' .$$

24. Equations for the upper wedge.

Analoguous to the way in which this term was used in Cahpter I, we shall by "upper wedge" designate that part of the dam body which is located between the upstream face and some dividing vertical line. As the latter we shall assume the line W_n (Fig. 30) which goes through the intersection of the inner boundary of the protection layer and the normal on it M-N. The depth of the percolation flow in the division line W_n is h and the loss of head within the "upper wedge" is called a_1 where

$$(252) \qquad h_p - h = a'$$

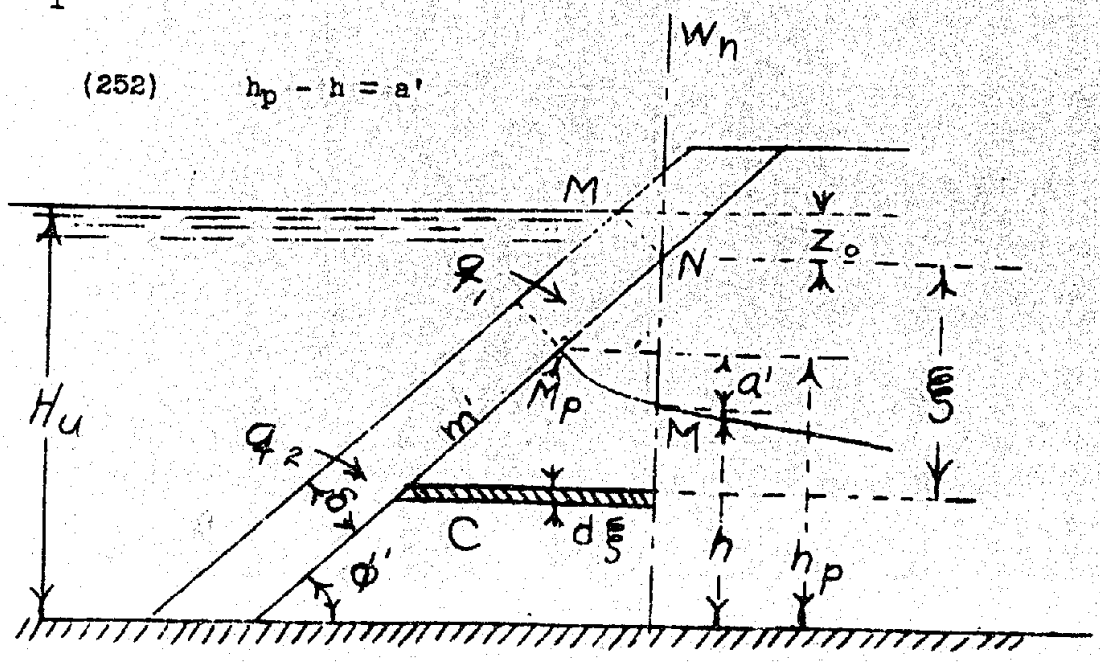


FIG. 30

In the same was as in ¶4, we shall again regard horizontal percolation flow elements. The length of such a flow element C (see Fig. 30) is $m' \cdot \xi$ and hence the hydraulic gradient of it:

$$(253) \qquad J = \frac{a'}{m' \xi}$$

and the velocity of percolation

$$(254) \qquad U = k \cdot J = \frac{ka'}{m' \xi}$$

where k is the percolation coefficient of the body material of the dam as compared to k_o which is the percolation coefficient of the material of the protection layer.

From (254) the corresponding elementary discharge is:

$$(255) \qquad dq_2 = U d\xi = \frac{k.a'}{m' \xi} \cdot d\xi$$

whence we obtain the discharge q_2 which leaves the second zone of the protection layer as:

$$(256) \qquad q_2 = \int_{\xi_1}^{\xi_2} \frac{k.a'}{m' \xi} d\xi = \frac{k.a'}{m'} \ln \frac{\xi_2}{\xi_1}$$

where the upper and lower limit of the integration are (see Fig. 30):

$$(257) \qquad \begin{aligned} \xi_2 &= H_u - Z_o \\ \xi_1 &= H_u - (h + Z_o) \end{aligned}$$

so that

$$(258) \qquad q_2 = \frac{k.a'}{m'} \operatorname{Ln} \frac{H_u - Z_o}{H_u - h - Z_o}$$

Let us notice that for this manner of analysis of the percolation flow in the boundaries of the "upper Wedge" we do not include a certain loss of head which arises from the fact that flow elements which percolate through the first (upper) zone drop on the surface of the percolation flow of the lower zone and somewhat increase its discharge on a short distance. Disregarding this loss of head we introduce a

certain reserve into the calculation, since doing so the resulting total discharge of percolation becomes somewhat greater, and the point M_o where the depression curve leaves the downstream face of the dam is somewhat raised (see Fig. 28).

Equating (258) and 247) we obtain:

$$(259)\quad \frac{ka'}{m'} \ln \frac{H_u \quad Z_o}{H_u - Z_o - h} = \frac{K_o a_p}{\delta \sin \phi'} (H_u - a_p)$$

in which we can express a' by (252) and a_p by(249) and obtain

$$(260)\quad \frac{k(h_p - h)}{m'} \ln \frac{H_u - Z_o}{H_u - Z_o - h} = \frac{k_o h_p (H_u - h_p)}{\delta \sin \phi'}$$

or:

$$(261)\quad \frac{h_p(H_u - h_p)}{\delta \sin \phi'} = \frac{k}{k_o} \cdot \frac{h_p - h}{m'} \ln \frac{H_u - Z_o}{H_u - Z_o - h}$$

and introducing the notation:

$$(262)\quad \frac{k}{k_o} = \sigma$$

we obtain instead of (261):

$$(263)\quad \frac{h_p(H_u - h_p)}{\sigma \delta \sin \phi'} = \frac{h_p - h}{m'} \ln \frac{H_u - Z_o}{H_u - Z_o - h}$$

which gives us a relation between h_p and h which are the two unknowns.

25. The system of percolation equations for a dam with "protection layer."

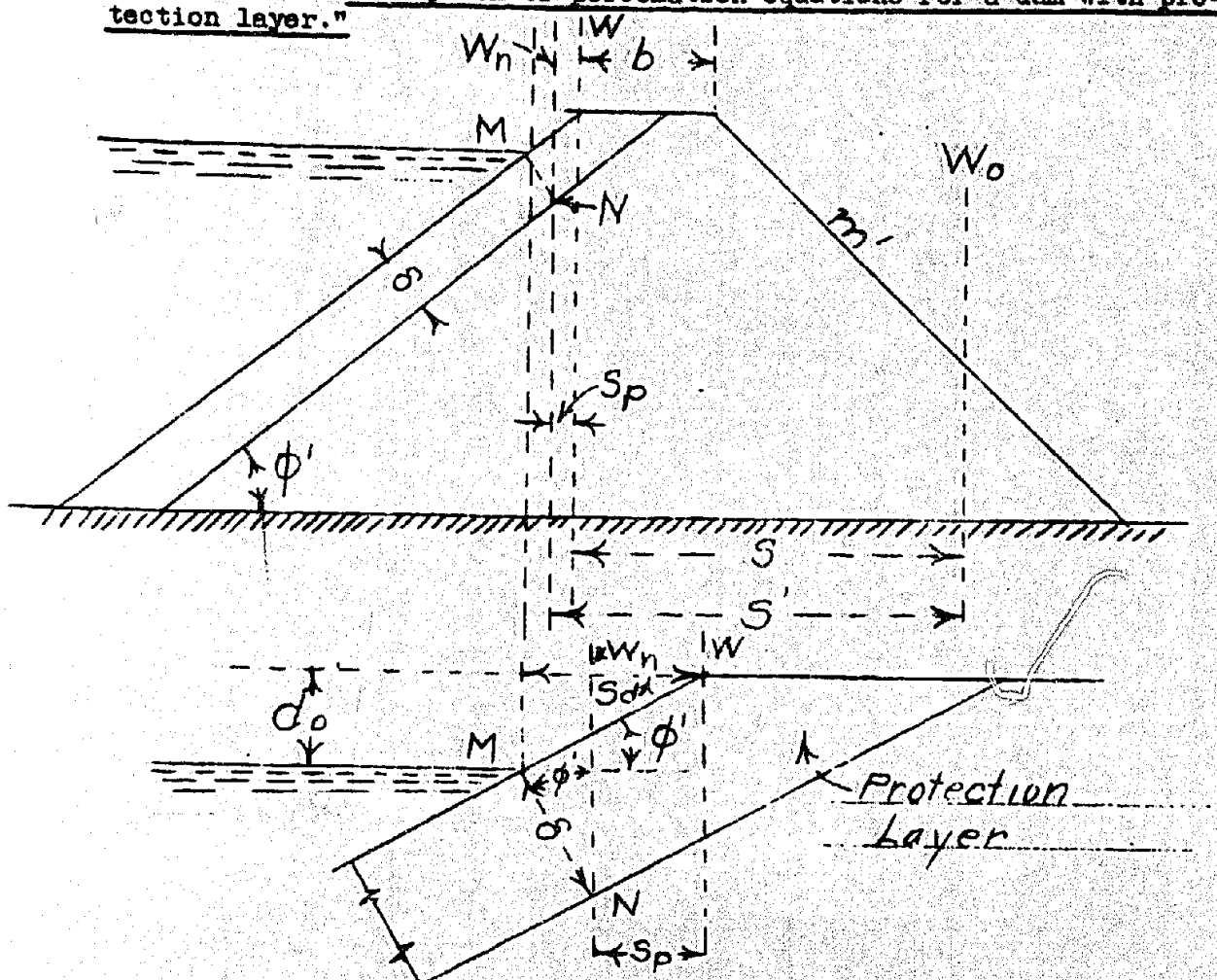


FIG. 31

After having discussed in the two preceeding paragraphs the percolation through the upstream face (protecting) layer and through the "upper wedge" we turn to Fig. 28, on which all parts of the depression curve are represented. From the "division vertical" Wn on down to the point where the depression curve leaves the downstream face of the dam we have the same case of a nonuniform movement along the horizontal sublayer like in ¶ 3 for the homogeneous earth dam without core or protecting layer. Therefore we obtain the equation of the depression curve like in (13):

$$\frac{qs'}{k} = \frac{1}{2}\left[h^2 - (a_0 + h_0)^2\right] \tag{264}$$

Instead of the distance s which enters equation (13) we introduce here the distance s' which is taken from the vertical W_n to the section W_o; in the given case we use as the end section of the upper wedge W_n and not W as in AP3.

Using the symbols of Fig. 31 which represents the upper part of the dam and a detail of it we obtain

(265) $$s' = s + s_p$$

(266) $$s = s_p + \delta \sin \phi'$$

(267) $$s\delta = d_o \operatorname{cotan} \phi' = m'd_o$$

whence

(268) $$s_p = s\delta - \delta \sin \phi' = m'd_o - \delta \sin \phi'$$

and according to (265) we have:

(269) $$s' = s + m'd_o - \delta \sin \phi'$$

Since we have for s obtained already in Chapter I (equation 39) the expression:

$s = b + m_1 \left[H_d - (a_o + h_o) \right]$, where m_1 is the slope of the downstream face we find from (269):

(270) $$s' = m_1 \left[H_d - (a_o + h_o) \right] + b + m'd_o - \delta \sin \phi'$$

and since $H_d = H_u - d_o$

(271) $$s' = m_1 \left[H_u - (a_o + h_o) \right] + b + (m' + m_1)d_o - \delta \sin \phi'$$

The pieces a_o and h_o are shown on Fig. 28. They have the same meaning as they had for the homogeneous dam. The last equation (35) which was found for the uniform "lower wedge" of the dam remains the same in the case discussed here, i.e.:

(272) $$\frac{q}{k} = \frac{a_o}{m_1} \left[1 + \ln \frac{a_o + h_o}{a_o} \right]$$

By making use of the expression (262) and transforming (251) into the form:

(273) $$\frac{q}{k} = \frac{H_u^2 - Z_o^2 - h_p^2}{2\sigma\delta \sin \phi'}$$

We now collect the equations (273), (263), (264), (272) and (271) into one system and obtain:

$$\text{I} \quad \frac{q}{k} = \frac{H_u^2 - Z_o^2 - h_p^2}{2\,\sigma\,\delta\,\sin\phi'}$$

$$\text{II} \quad \frac{h_p(H_u - H_p)}{\sigma\,\delta\,\sin\phi'} = \frac{h_p - h}{m'} \ln \frac{H_u - Z_o}{H_u - Z_o - h}$$

$$(274) \quad \text{III} \quad \frac{q}{k} = \frac{h^2 - (a_o + h_o)^2}{2\,\varkappa\,s'}$$

$$\text{IV} \quad \frac{q}{k} = \frac{a_o}{m'} \left[1 + \ln \frac{a_o + h_o}{a_o} \right]$$

$$\text{V} \quad s' = m_1 \left[H_u - (a_o + h_o) \right] + b + (m' + m_1)\, d_o - \delta \sin\phi'$$

This is the system of the percolation equations for the core of an earth dam with a protection layer (almost impervious layer) see Fig. 28. In this system of equations we have 5 unknown quantities:

$$(275) \quad h_p;\ h;\ a_o;\ s';\ q \ \left(\text{or } \frac{q}{k}\right)$$

and the quantities:

$$(276) \quad H_u;\ Z_o;\ h_o;\ b;\ d;\ \delta;\ k;\ \sigma;\ \phi;\ m\text{I and } m_1$$

are known.

The system (274) can be solved by an approximation method in the following manner: Using any given value of "a_o" we find from IV a corresponding value of "$\frac{q}{k}$" and from equation V a corresponding value of "s'." After this we determine the quantity "h" with equation III and from I we then find the "h_p" which corresponds to the value of "$\frac{q}{k}$" obtained by IV and we can for an assumed value of "a_o" determine the corresponding values of "h" and "h_p" which we can then introduce into equation II of the system. If "a_o" is chosen correctly this latter equation will be satisfied by the values "h" and "h_p" which we introduce; this of course can be attained only after several trials.

In closing this ℙ we can point out that for the case where no water is below the dam, i.e.:

$$h_o = 0$$

the first two equations of the system remain unchanged while the last three equations obtain the form:

$$\frac{q}{k} = \frac{h^2 - a_o^2}{2s'}$$

(277) $$\frac{q}{k} = \frac{a_o}{m_1}$$

$$s' = m_1 (H_u - a_o) + b + (m' + m_1) d_o - \delta \sin \phi'$$

"virtual method"

26. The "imaginary method" of discussing the percolation through the "protection layer" and the "upper wedge."

For the sake of simplification of the above derived method of analysis we shall now treat the percolation through the protection layer and the upper wedge simultaneously (Fig. 32). For this of course in the first zone of the layer the scheme given in ℙ 23 remains the same but in the second zone we have to take the flow elements as shown on Fig. 32, i.e. we select flow elements which extend from the surface of the upstream face to the division line W_n.

Let us at first point that if we take any one dimensional flow of the length ℓ_1 and a percolation coefficient of k_1 we find, according to the law of Darcy a percolation velocity of U_1 of:

(278) $$U_1 = \frac{K_1 \cdot H_1}{\ell_1}$$

where H_1 is the difference of heads at the beginning and end of the flow (or the "loss of head"). From this equation we obtain:

(279) $$H_1 = \frac{U_1}{k_1} \cdot \ell_1 \qquad \text{or}$$

(280) $$H = \frac{q\,\ell_1}{k_1 w_1}$$

where Q is the percolation discharge and w_1 is the geometrical area of the cross section through the flow. For any other one dimensional percolation flow we have:

$$(281) \qquad H_2 = \frac{Q\,\ell_2}{k_2 w_2}$$

for the same discharge Q.

From this we see that the head losses H_1 and H_2 will be equal to each other; i.e., the two flows will be equivalent in regard to the loss of head of the conditions:

$$(282) \qquad \frac{\ell_1}{k_2 w_1} = \frac{\ell_2}{k_2 w_2} \qquad \text{or}$$

$$(283) \qquad \frac{\ell'}{\ell_2} = \frac{k_1 w_1}{k_2 w_2}$$

is fulfilled.

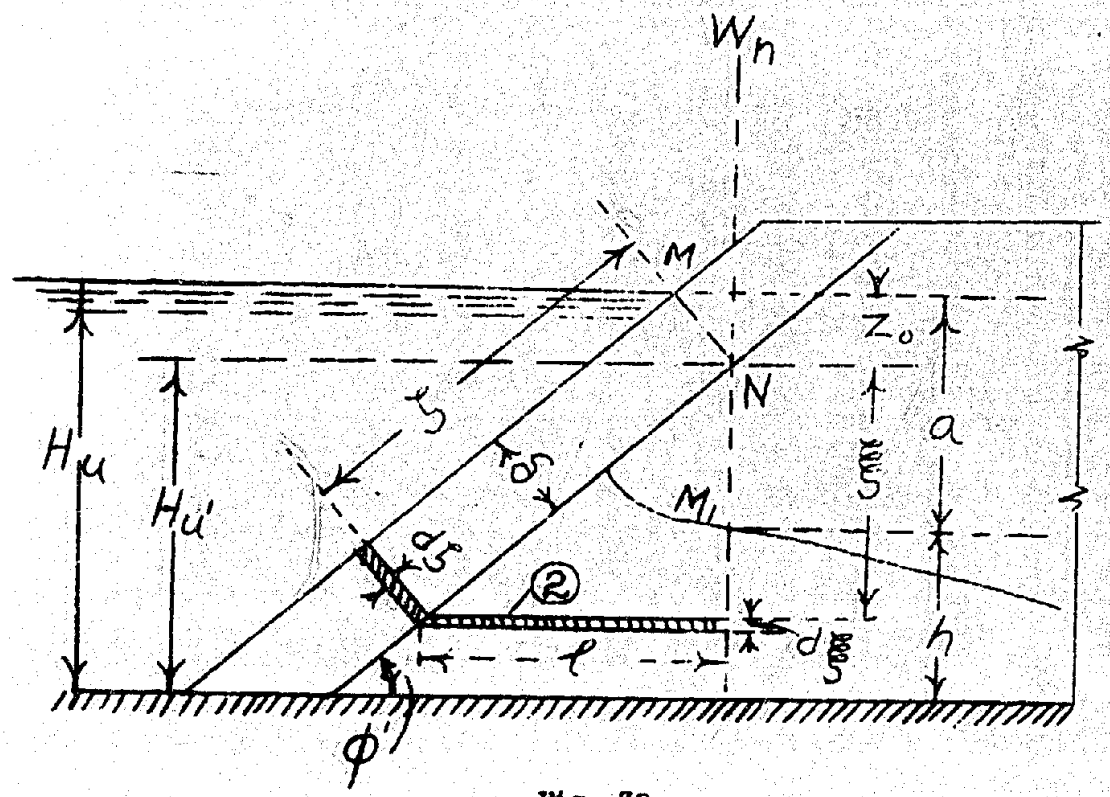


Fig. 32

Now we can easily derive a formula for the "virtual" thickness δ_v of the protection layer. In fact, if we stretch out the flow element ② as shown in Fig. 32 we obtain a flow element as shown in Fig. 33. For this the length of its first part is δ and the percolation coefficient k_o, and the cross sectional area $d\zeta$. For second part the length is $= \ell$, the percolation coefficient is $= k$ and the cross sectional area $d\xi$ (diagram A, Fig. 33). Substituting the first part by an "equivalent part," i.e. by a flow element having the cross section $d\xi$ and the percolation coefficient $= k$ (based on the transformation given in (283), we find the length of this substituted flow element δ_v (diagram B, Fig. 33) form the relation:

$$(284) \qquad \frac{\delta_v}{\delta} = \frac{k}{K_o} \frac{d\xi}{d\zeta}$$

in which we have from Fig. 32:

$$(285) \qquad \frac{d\xi}{d\zeta} = \sin \phi'$$

and from equation (262)

$$\frac{k}{k_o} = \sigma$$

whence we obtain from (284):

$$(286) \qquad \delta_v = \sigma \delta \sin \phi'$$

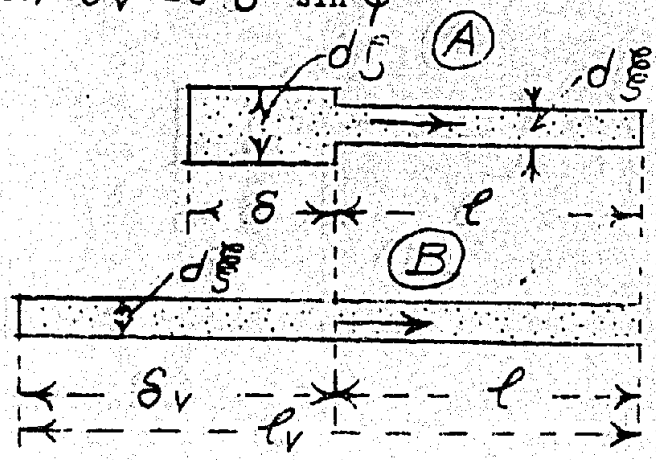


Fig. 33

In this expression we can call the value δ_V the "virtual dimension" of the "virtual thickness" of the protection layer.

Footnote: We can in this connection speak about thickness only in a somewhat limited way, since the equivalent flow element, which has the length δ_V is horizontal, due to which fact also the thickness of the "equivalent protection layer" has to be measured in a horizontal direction, and not in the direction normal to its slope.

The total length ℓ_V of the obtained "equivalent" percolation flow element is (see Fig. 33B):

$$(287) \qquad \ell_V = \delta_V + \ell$$

and we can call this the "virtual length" of the flow element (2) shown in Fig. 32.

Now we find the discharge through the second zone (Fig. 32) q_2 in the following manner: the velocity of percolation of the flow element 2 is:

$$(288) \qquad U = \frac{k \cdot a}{\ell_V}$$

where (a) is the loss of head, calculated from the point at which the percolation flow enters the protection layer to the point at which it meets the division section W_n (Fig. 32) thus that:

$$(289) \qquad dq_2 = U d\xi = \frac{ka}{\ell_V} d\xi$$

and the discharge q_2 will be:

$$(290) \qquad q_2 = \int_{\xi_1}^{\xi_2} \frac{k \cdot a}{\ell_V} d\xi = k \cdot a \int_{\xi_1}^{\xi_2} \frac{d\xi}{\ell_V}$$

where in the same way as in ¶ 24, the upper and lower limit of the integration can be found from (257). The length ℓ_v is according to (287) and to Fig. 32

$$(291) \qquad \ell_v = \delta_v + \ell = \delta_v + m'\xi$$

and from (290) we thus obtain:

$$(292) \qquad q_2 = k \cdot a \int_{\xi_1}^{\xi_2} \frac{1}{\delta_v + m'\xi} \, d\xi = \frac{ka}{m'} \ln \frac{\delta_v + m'(H_u - Z_o)}{\delta_v + m'(H_u - Z_o - h)}$$

(293) and abbreviating: $H_u - Z_o = H_u'$ we obtain

$$(294) \qquad q_2 = \frac{k \cdot a}{m'} \ln \frac{\frac{\delta_v}{m'} + H_u'}{\frac{\delta_v}{m'} + H_u' - h}$$

For the discharge q_1 in the first zone we have the same expression as was found in ¶ 23 in equation (241):

$$q_1 = \frac{k_o}{2\,\delta \sin\phi'} (a_p^2 - Z_o^2)$$

in which expression the symbols used are the same as discussed in ¶ 23 and shown on Fig. 29. But we have for the present case to change the expression for q_1 slightly because for the above discussed "virtual "method" of analyzing the percolation through the second zone we have avoided the value of a_p (Fig. 28) as being an intermediate value, and have introduced "a" which is calculated (as shown above) from the point at which the percolation flow enters the protection layer, to the point on the division section W_n. Therefore we shall now assume that $a_p \cong a$ a since the difference between the two is only very small. We thus introduce a certain surplus into our computations (i.e. we increase the value of q_1 by using instead of "a_p" the slightly greater value "a").

Equation (241) thus changes into

$$(295) \qquad q_1 = \frac{k_o}{2\,\delta \sin\phi'} (a^2 - Z_o^2)$$

which can be written:

$$(296) \qquad q_1 = \frac{k}{\frac{2\delta k}{k_o} \sin\phi'} (a^2 - z_o^2)$$

or due to (262) and (286)

$$(297) \qquad q_1 = \frac{k}{2\delta_v} (a^2 - z_o^2)$$

and adding q_2 (294) and q_1 (297) we obtain the total discharge q:

$$(298) \qquad q = \frac{ka}{m'} \ln \frac{\frac{\delta_v}{m'} + H_u'}{\frac{\delta_v}{m'} + H_u' - h} + \frac{k}{2\delta_v} (a^2 - z_o^2)$$

(299) where $a = H_u - h$ $\qquad$ d is used for δ, d_v for δ_v

whence we obtain

$$(300) \qquad \frac{q}{k} = \frac{H_u - h}{m'} \ln \frac{\frac{d_v}{m'} + H_u'}{\frac{d_v}{m'} + H_u' - h} + \frac{(H_u - h)^2 - z_o^2}{2d_v}$$

which replaces the two first equations of the system (274) and eliminates the unknown "h_p" from the computation.

P 27. <u>The system of percolation equations for the dam with protection layer if the "virtual method" is used.</u>

With the aim of simplifying its form we introduce the notation

$$(301) \qquad \frac{\delta_v}{m'} = \delta_v' \qquad \text{into equation (300).}$$

The geometrical meaning of δ_v' becomes apparent from Fig. 34. In Fig. 34 we see also the meaning of the "equivalent thickness" δ_v. From the Fig. we obtain:

$$(302) \qquad \delta_v' = \delta_v \tan \phi' = \frac{\delta_v}{m_1}$$

i.e. δ_v' is the length of the segment of a vertical line between the boundaries of the "equivalent" protection layer.

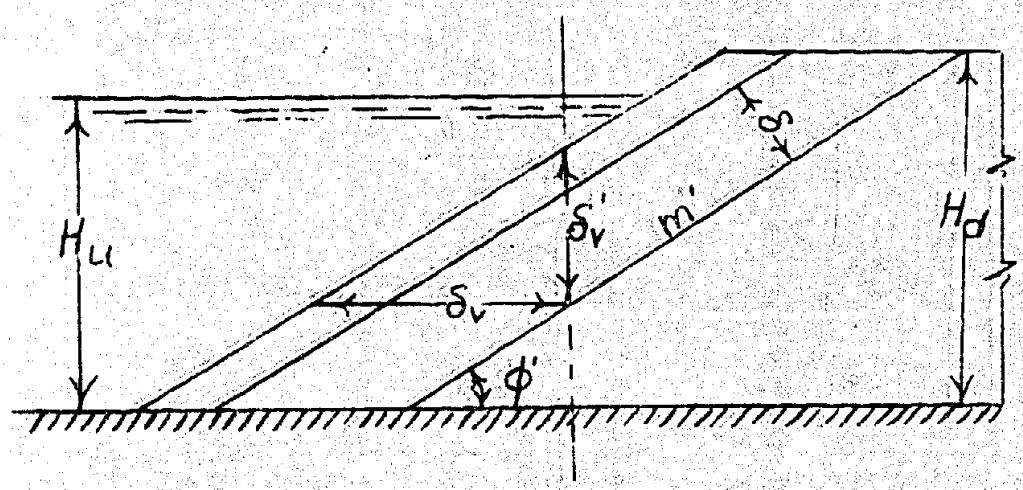


FIG. 34

We obtain introducing (301) from (300):

$$(303) \qquad \frac{q}{k} = \frac{H_u - h}{m'} \left[\ln \frac{\delta_v' + H_u'}{\delta_v' + H_u' - h} + \frac{H_u - h}{2\,\delta_v'} - \frac{z_o^2}{2\,\delta_v'\,(H_u - h)} \right]$$

and the system of percolation equations (274) is substituted by the following:

$$\text{I} \quad \frac{q}{k} = \frac{H_u - h}{m'}\left[\ln \frac{\delta_v' + H_u'}{\delta_v' + H_u' - h} + \frac{H_u - h}{2\,\delta_v'} - \frac{z_o^2}{2\,\delta_v'(H_u - h)}\right.$$

$$\text{II} \quad \frac{q}{k} = \frac{h^2 - (a_o + h_o)^2}{2\,s'}$$

(304)

$$\text{III} \quad \frac{q}{k} = \frac{a_o}{m_1}\left[1 + \ln \frac{a_o + h_o}{a_o}\right]$$

$$\text{IV} \quad s' = m_1\left[H_u - (a_o + h_o) + b + (m' + m_1)\,d_o - \delta \sin \phi'\right.$$

In this system the known quantities are given in equation (276) and the unknown are (see Fig. 28):

(305) h; a_o; s' and q (or $\frac{q}{k}$) and they can be determined in the same way as was shown in ¶ 8 for the homogeneous earth dam.

At first we obtain a rolation analogeous to (51) for the quantity h:

$$(306) \quad h = \sqrt{\frac{2 \times a_o \times A \times s'}{m_1} + (a_o + h_o)^2}$$

where

$$(307) \quad A = 1 + \ln\left[\frac{a_o + h_o}{h_o}\right]$$

and introducing the value at s' from equation IV of system 304 we obtain:

(308) $h = D'$ where D' has the value

$$(309) \quad D' = \sqrt{2a_oA\left[\frac{b'}{m_1} + H_u - (a_o + h_o)\right] + (a_o + h_o)^2} = D'$$

if we also introduce for the sake of abbreviation

$$(310) \qquad b' = b + (m' + m_1)d_o - \delta \sin \phi'$$

Using (308) we transform the first equation of the system (304) into the form:

$$(311) \qquad \frac{q}{k} = \frac{(H_u - D')\ E'}{m'}$$

denoting by E' the following term:

$$(312) \qquad E' = \ln \frac{\delta_Y' + H_u'}{\delta_Y' + H_u' - D'} + \frac{H_u - D'}{2\,\delta_Y'} - \frac{z_o^2}{2\,\delta_Y'\,(H_u - D')}$$

and substituting into (311) instead of $\frac{q}{k}$ its value according to (49) we find:

$$(313) \qquad \frac{a_o A}{m_1} = \frac{(H_u - D')\ E'}{m'}$$

or

$$(314) \qquad \frac{m'}{m_1}\ a_o\ A = (H_u - D')\ E'$$

in which A, D', and E' are functions of "a_o" alone, and from what has been said in ¶ 8 in regard to equation (39) it is easy to see how equation (314) is solved in regard to a_o.

After determining a_o, we find s' from IV of system (304) knowing A and h from (306). From equation (49) we know the expression of $\frac{q}{k}$:

$$(315) \qquad \frac{q}{k} = \frac{a_o \cdot A}{m_1}$$

if we also introduce for the sake of abbreviation

$$(310) \qquad b' = b + (m' + m_1)d_o - \delta \sin \phi'$$

Using (308) we transform the first equation of the system (304) into the form:

$$(311) \qquad \frac{q}{k} = \frac{(H_u - D')\,E'}{m'}$$

denoting by E' the following term:

$$(312) \qquad E' = \ln \frac{\delta_Y' + H_u'}{\delta_Y' + H_u' - D'} + \frac{H_u - D'}{2\,\delta_Y'} - \frac{Z_o^2}{2\,\delta_V'\,(H_u - D')}$$

and substituting into (311) instead of $\frac{q}{k}$ its value according to (49) we find:

$$(313) \qquad \frac{a_o A}{m_1} = \frac{(H_u - D')\,E'}{m'}$$

or

$$(314) \qquad \frac{m'}{m_1}\, a_o\, A = (H_u - D')\,E'$$

in which A, D', and E' are functions of "a_o" alone, and from what has been said in ¶ 8 in regard to equation (39) it is easy to see how equation (314) is solved in regard to a_o.

After determining a_o, we find s' from IV of system (304) knowing A and h from (306). From equation (49) we know the expression of $\frac{q}{k}$:

$$(315) \qquad \frac{q}{k} = \frac{a_o \cdot A}{m_1}$$

whence

(316) $$q = \frac{a_o \cdot A \cdot k}{m_1}$$ as was also found

for the case of the homogeneous earth dam.

If there is no water below the earth dam ($h_o = 0$) the last three equations of (304) are replaced by the three equations of (277) and instead of (304) we now have the system:

(317)

I $$\frac{q}{k} = \frac{H_u - h}{m'}\left[\ln \frac{\delta_v' + H_u'}{\delta_v' + H_u' - h} + \frac{H_u - h}{2\,\delta_v'} - \frac{z_o^2}{2\,\delta_v'(H_u - h)}\right]$$

II $$\frac{q}{k} = \frac{h - a_o^2}{2\,s'}$$

III $$\frac{q}{k} = \frac{a_o}{m_1}$$

IV $$s' = m_1\,(H_u - a_o) + b + (m' + m_1)\,d_o - \delta \sin\phi'$$

This system of equations is solved as shown in P 8 and it is:

(318) $$A = 1$$

(319) $$D' = \sqrt{2a_o\frac{b'}{m_1} + H_u - a_o) + a_o^2}$$

the value of E' is given by equation(312) and remains the same as for $h_o > 0$ only that for the computation D' has to be taken from equation (319). In order to determine a_o we solve, instead of (314) the following equation:

(320) $$\frac{m'}{m_1}\,a_o = (H_u - D')\,E'$$

We have also to point out, that for a "protection layer" of changing thickness the value of δ which has been used in this chapter represents an "average thickness" of the layer as has already been mentioned in the beginning of ¶ 23; this mean thickness is determined by the formula:

$$\delta = \frac{\delta_o + \Delta}{2} \tag{322}$$

in which δ_o is the thickness of the layer at the elvation of the head-water level, and Δ the thickness of the layer at its base. In regard to the coefficient m' (slope) we can say that for a layer of changing thickness the slope of the axial plane through the layer 0' - 0' is called m' (see Fig. 35).

We shall now discuss a numerical example. Let us assume that we have the given dimensions of an earth dam with protection layer of variable thickness, and we have determined the average thickness of that layer according to (322). Further we shall assume that we have found the angle of inclination ϕ' of the line 0' - 0' in reference to the horizontal (Fig. 35) so that

$$m' = \cot\text{an}\ \phi' \qquad \text{is known.} \tag{323}$$

Knowing these values we can draw the dam profile with "reduced protection layer" (i.e. protection layer of equal mean thickness and at a given ϕ'). We do not give a design of this profile here, but we shall use Fig. 28 and assume that for our example the given dimensions are according to Fig. 28:

$H_d = 16.5$ m

$b = 7.0$ m

$d_o = 1.5$ m ; $H_u = 15.0$ m

$h_o = 0$ (no water below the dam)

$\delta = 2.7$ m

The coefficient of percolation for the protection layer is k_o = 0.000006 cm/sec; the coefficient of percolation of the dam body is

footnote: see footnotes to the examples for ¶ 13 and 22).

$k = 0.00009$ cm/sec. We know than $m' = \cot\phi' = 2.8$, and we find:

$$(324) \qquad \sin\phi' = \frac{1}{\sqrt{1 + \cot^2\phi'}} = \frac{1}{\sqrt{1 + m'^2}} = 0.336$$

$$(325) \qquad \cos\phi' = \frac{1}{\sqrt{1 + m'^2}} = 0.941$$

Whence we obtain from (242):

$$z_o = \delta \cos\phi' = 2.7 \times 0.941 = 2.54 \text{ m}$$

and from (286):

$$\delta_V = \sigma \cdot \delta \cdot \sin\phi' \qquad \text{in which}$$

according to (262)

$$\sigma = \frac{0.00009}{0.000006} = 15.0$$

whence the "vertical thickness" of the protection layer is:

$$\delta_V = 15.0 \times 2.7 \times 0.336 = 13.61 \text{ m}$$

from (302) we further more find

$$\delta_V' = \frac{\delta_V}{m'} = \frac{13.61}{2.8} = 4.86 \text{ m}$$

and we can now start the solution of the system of percolation equations of (317) in the manner shown in ¶ 27.

We find for this purpose from (293)

$$H_u' = H_u - z_o = 15.0 - 2.54 = 12.46 \text{ m}$$

and from (310)

$$b = b + (m' + m_1)\, do - \delta \sin \phi' = 14.02 \text{ m}$$

furthermore we determine:

$$\frac{m'}{m_1} = \frac{2.8}{2.5} = 1.12$$

$$\frac{b_1}{m_1} + H_u = \frac{14.02}{2.5} + 15.0 = 20.60 \text{ m}$$

$$\delta_Y' + H_u' = 4.86 + 12.46 = 17.32 \text{ m}$$

$$2\,\delta_Y' = 2 \times 4.86 = 9.72 \text{ m}$$

and

$$\frac{z_o^2}{2\delta_V'} = \frac{2.54^2}{9.72} = 0.665 \text{ m}$$

Selecting different values of a_o we now compute the table 15. From this table we see that a value of $a_o = 3.67$ m satisfies the system of equations given under (317).

Now we compute the value of s' in making use of equation IV of the system (317) and remembering also equation (310):

$$s' = m_1 (H_u - a_o) + b = 2.5\,(15.0 - 3.67) + 14.02 = 42.35 \text{ m.}$$

Furthermore we find the value of h from equation (321):

$$h = \sqrt{\frac{2a_o s'}{m_1} + a_o^2} = \sqrt{\frac{2 \times 3.67 \times 42.35}{2.5} + 3.67^2}$$

$$h = 11.74 \text{ m}$$

From formula III of system (317) we now find $\frac{q}{k}$:

$$\frac{q}{k} = \frac{a_o}{m_1} = \frac{3.67}{2.50} = 1.47 \text{ m} = 147 \text{ cm.}$$

The discharge quantity is computed from (73):

$$q = \frac{a_o}{m_1} k = 147 \times 0.00009 = 0.0132 \text{ cm}^2/\text{sec.}$$

or from (111)

$$q_{daily} = \frac{8640}{m_1} a_o k = 8640 \times 0.0132 = 114.0$$

in liters per 24 hrs. and per linear foot of dam length.

The equation of the second part of the depression curve has the same form as equation (10) of the homogeneous earth dam (the coordinate axes as shown in Fig. 28)

$$y^2 = h^2 - \frac{2q}{k} x$$

For the values which we have found above

$$(326) \qquad y^2 = 137.57 - 2.94\, x$$

Table 16 is computed from this equation and allows to plot the depression curve.

TABLE 15

1	a_o	a_o	3.20	3.50	3.65	3.67
2	$F_1(a_o) = \frac{m'}{m_1} a_o$	$F_1(a_o) = 1.12(1)$	3.58	3.92	4.08	4.11
3	$\left(\frac{b'}{m_1} + H_u\right) - a_o$	$20.60 - (1)$	17.40	17.10	16.95	16.93
4	$2\, a_o\left[\left(\frac{b'}{m_1} + H_u\right) - a_o\right]$	$2(1)(3)$	111.55	119.70	123.70	124.20
5	a_o^2	$(1)^2$	10.24	12.25	13.31	13.47
6	$D^{12} = 2a_o\left[\left(\frac{b'}{m_1} + H_u\right) - a_o\right] + a_o^2$	$D^{12} = (4) + (5)$	121.39	131.95	157.01	137.57
7	D^1	$D^1 = \sqrt{(6)}$	11.02	11.48	11.71	11.74
8	$H_u - D'$	$15 - (7)$	3.98	3.52	3.29	3.26
9	$(\delta_v + H_u') - D'$	$17.32 - (7)$	6.30	5.84	5.61	5.58
10	$\frac{\delta_v' + H_u'}{(\delta_v' + H_u') - D'}$	$\frac{17.32}{(9)}$	2.75	2.96	3.09	3.10
11	$2.303\ \mathrm{Log}_{10}\frac{(\delta_v' + H_u')}{(\delta_v' + H_u') - D'}$	$2.303\ \mathrm{Log}_{10}(10)$	1.01	1.09	1.12	1.13
12	$\frac{H_u - D'}{2\,\delta_v'}$	$\frac{(8)}{9.72}$	.41	.36	.34	.34
13	$\frac{Z_o^2}{2\,\delta_v'(H_u - D')}$	$\frac{.665}{(8)}$	.17	.19	.20	.20
14	E'	$E' = (11) + (12) - (13)$	1.25	1.26	1.26	1.27
15	$F_2(a_o) = (H_u - D')\,E'$	$F_2(a_o) = (8)(14)$	4.97	4.43	4.15	4.13

CHAPTER V

29. Percolation in a homogeneous earth dam with drainage system when $h_o = 0$.

In modern hydraulics the drainage of the downstream part of an earth dam has been adopted as a successful and economic means to lower the depression curve within the dam. This is especially true

Footnote: See for example Creager and Justin "Hydro-electric Handbook" New York, 1927, p. 229).

for such dams which are built on an impervious foundation, so that there can be no percolation of water from the dam body into the ground.

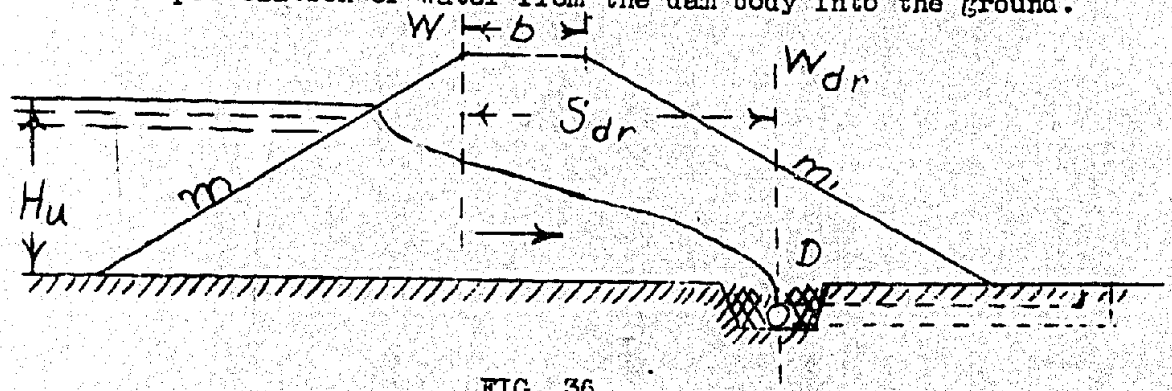


FIG. 36.

We shall not discuss here the construction details, but we can point out that the drainage of earth dams can effectively be realized by one of the following methods:

(1) A longitudinal system of drain pipes or drainage ditches can be used (see the schematical design Fig. 36, where (D) is shown as the main longitudinal drainage line, drain pipe, laid on the bottom of a ditch and covered with an "inverted filter." One of the side channels to this drainage line is shown by a dotted line).

(2) A rock fill or a fill of some other very pervious material (with a filter F between the dam body and the backfill. If the dimensions of this back fill are large enough it also can serve as a constructive means of stabilizing the dam structure (Fig. 37).

Footnote: Let us note that the backfill may play the part of filter only if it has comparatively small dimensions which would be the case for dams of small heights.

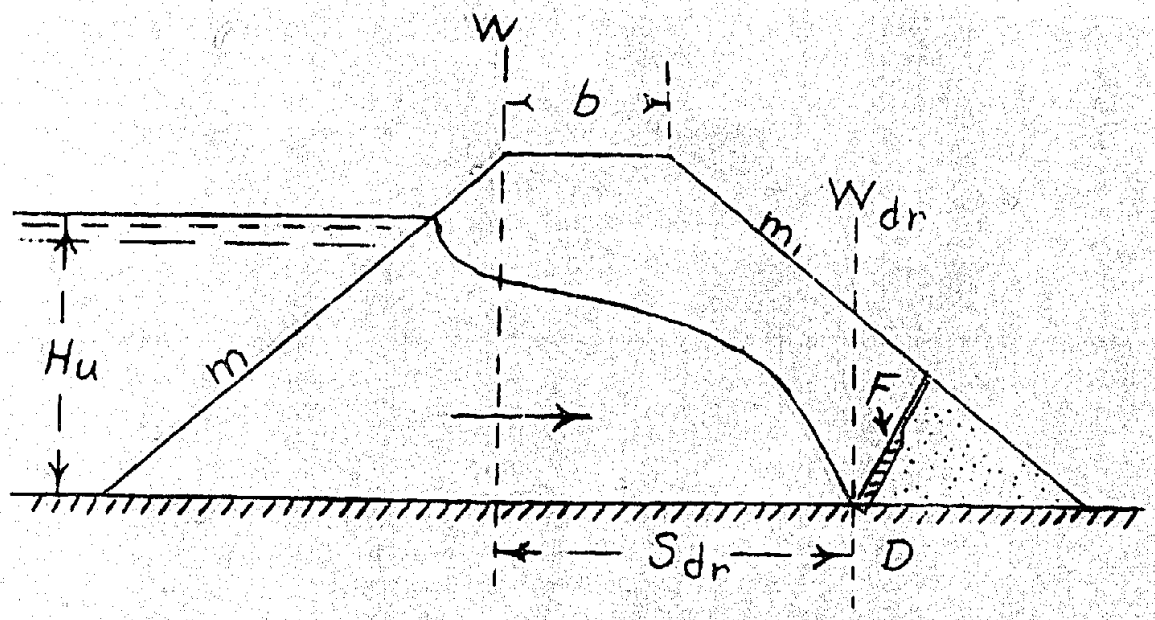


FIG. 37

(3) A combination of the two methods stated under (1) and (2).

In the cases pointed out the depression curve bends down into the region of D_1 and so to speak leaves the dam at that point. We therefore place the "outlet section" W_d through this point, as is shown on Fig. 36 and 37. We now can start to discuss the depression curve in a uniform earth dam with drainage system. The drainage is schematically represented as shown in Fig. 38 by the point D.

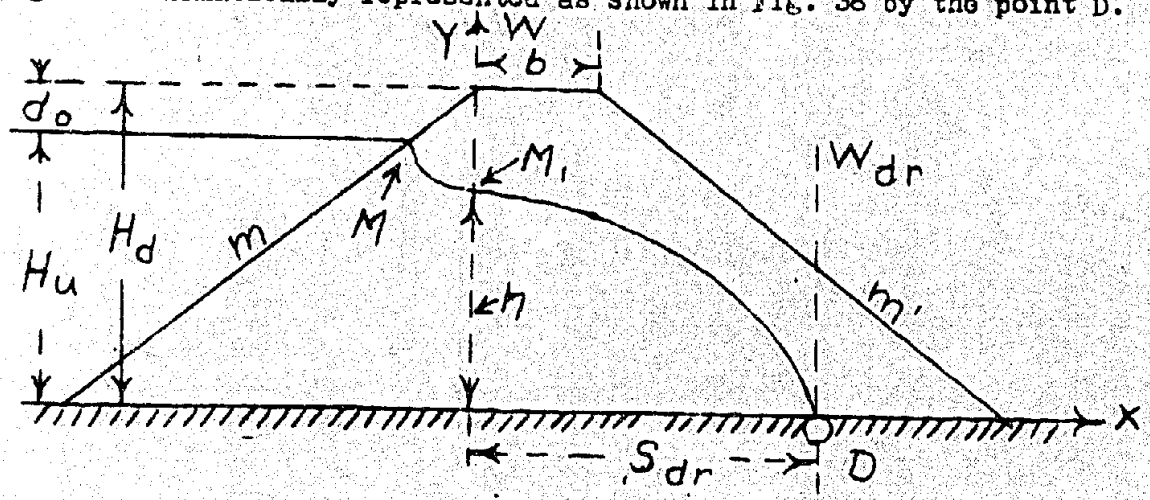


FIG. 38

For this condition the "lower wedge" is in regard to percolation non-existent, and we have only two parts of the depression curve: $M - M_1$ (for the upper wedge) and $M_1 - D$ (from the dividing section W to the section W_d). For the first part the equation (23) remains unchanged as given in ¶ 4; but for the second part however we have to substitute into equation (14') (see also Fig. 38):

$$y_o \cong 0$$

$$s = s_{dr}$$ and thus obtain:

(327) $$q = \frac{kh^2}{2s_{dr}}$$

in which s_d is known.

Thus for the case which is discussed here the system of percolation equations is simplified to only two equations:

(328)

I $$\frac{q}{k} = \frac{H_d - d_o - h}{m} \ln \frac{H_d}{H_d - h}$$

II $$\frac{q}{k} = \frac{h^2}{2s_{dr}}$$

The symbols have the same meaning as given in Chapter I (and seen in Fig. 38). The unknown are:

(329) h and q (or $\frac{q}{k}$)

while the quantities

(330) H_d; d_o; s_{dr}; m and k are known.

If we introduce the expression for $\frac{q}{k}$ from equation II and I we have

(331) $$\frac{h^2}{2s_{dr}} = \frac{H_d - d_o - h}{m} \ln \frac{H_d}{H_d - h}$$ i.e. an equation with only one unknown. From it we find:

$$(332) \qquad h = \sqrt{\frac{2s_{dr}}{m}(H_d - d_o - h)\ \ln \frac{H_d}{H_d - h}}$$

and we let

$$(333) \qquad F = \frac{2s_{dr}}{m}(H_d - d_o - h)\ \ln \frac{H_d}{H_d - h}$$

whence

$$(334) \qquad h = \sqrt{F}$$

Now we can let h take different values and can thus compute a graph of the function $\sqrt{F}$ as is shown on Fig. 39. If now we draw a line OA under an angle of 45° against the horizontal the intersection of this line and the curve of the function $\sqrt{F}$ furnishes the desired value of h:

$$(335) \qquad h = h_{wanted};$$

now we can find $\frac{q}{k}$ and q from equation II of the system (328).

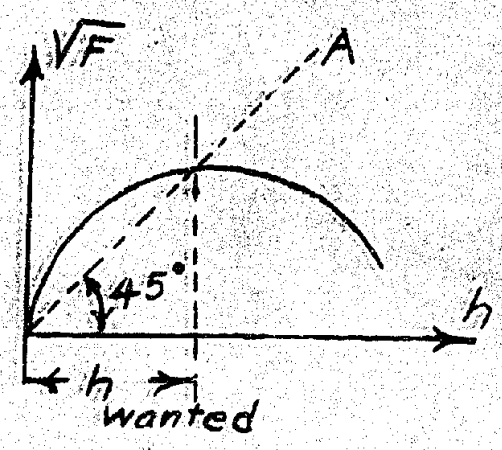


FIG. 39.

If the value h_d (i.e. the depth of the percolation flow at the section W_d) is not identical zero, as this is shown on Fig. 40, then the depression curve can be constructed in the following manner. At first we assume that $h_d = 0$ (since h_d will in all cases only be small) and compute the percolation discharge according to the system of equations (328), and after we have the value q_o we determine by a hydraulic calculation the value h_d which is necessary if the computed q shall be carried out of the dam. Knowing h_d we obtain instead of the system (328) the system:

$$\frac{q}{k} = \frac{H_d - d_o - h}{m} \ln \frac{H_d}{H_d - h}$$

(336)

$$\frac{q}{k} = \frac{h^2 - h_{dr}^2}{2s_{dr}}$$

which gives instead of (334):

$$h^2 - h_{dr}^2 = F \tag{337}$$

and

$$h = \sqrt{F + h_{dr}^2} \tag{338}$$

where F again is expressed by (333) and instead of the graph for $\sqrt{F}$ we now have to plot a graph of the function $\sqrt{F + h_{dr}^2}$ In order to find h we again draw the (45°) line OA as in Fig. 39 and the intersection of the curve and this line will be the wanted value of h.

In order to obtain a greater accuracy of the calculation we can, after having found the depth h in the above manner once more find q from II of system (336) and once more make the computation for h_d and h. We can point out here that often the value of h_d is so small as compared to h that it is not necessary to use system 336, and that even for $h_d = 0$ we can use system (328).

The intermediate points of the depression curve between M_1 and D (Fig. 38 and 40) can be computed from equation (10) and the values are found by

$$y^2 = h^2 - \frac{2q}{k} x \tag{339}$$

in which we let x take different values and find the corresponding values of y.

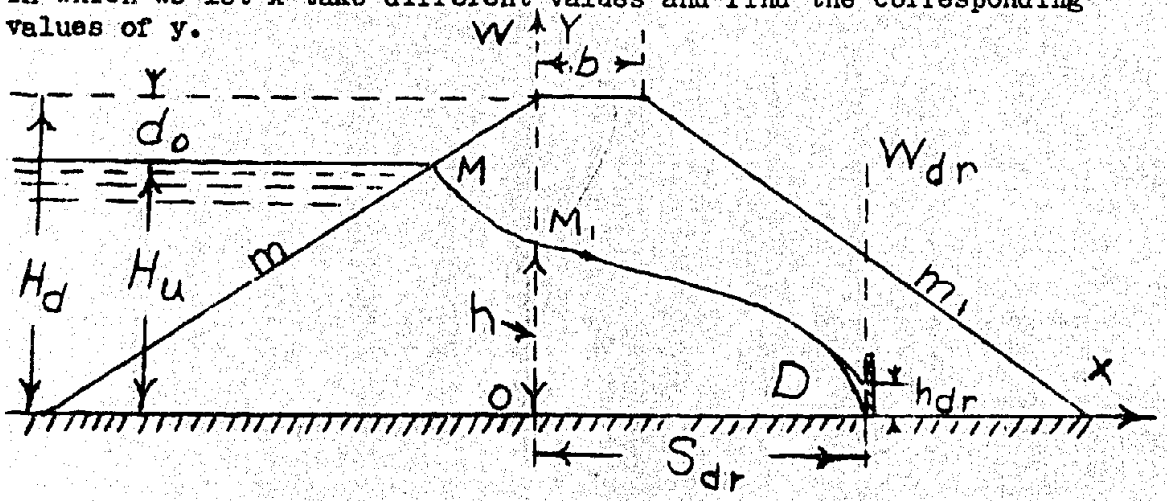


FIG. 40

From the system of percolation equations (328) or (336) we conclude that the shape of the depression curve does not depend on the percolation coefficient k also when drainage is used in the dam. That is again it is independent from the percolation properties of the material out of which the dam is built (see also what has been said in this connection in Chapter I.).

¶ 30. Continuation.

In broadening out the preceding paragraph we shall now discuss the more complicated calculation of the depression curve for the case where the drainage of the earth dam is obtained by a backfill of pervious material. If this backfill is made of crushed stone or some other very pervious material (see Fig. 37) then the depression curve can be computed as shown in the preceding paragraph in the same way as for the case $h_d = 0$ of Fig. 38. If however, the backfill is made of less pervious material (see Fig. 41) and we want to compute the hydraulic resistance of the drainage itself then we can do this in the following manner, assuming from the start $h_{dr} > 0$.

For the first approximation of the solution we assume again that $h_d = 0$ (as in ¶ 29: see what has been said about the case shown in Fig. 40). We then determine h and $\frac{q}{k}$ from the system (328), taking s_d as on Fig. 37. We then notice that the backfill as shown in Fig. 41 may be regarded as a small earth dam of uniform material with a percolation coefficient k_d. For it we can thus write the system of 4 percolation equations as given under (41') in Chapter I. We shall not write out those equations here in detail, but we can point out that for the "lower wedge" of the backfill we obtain:

(340) $\frac{q}{k_{dr}} = \frac{a_o}{m_{dr}}$

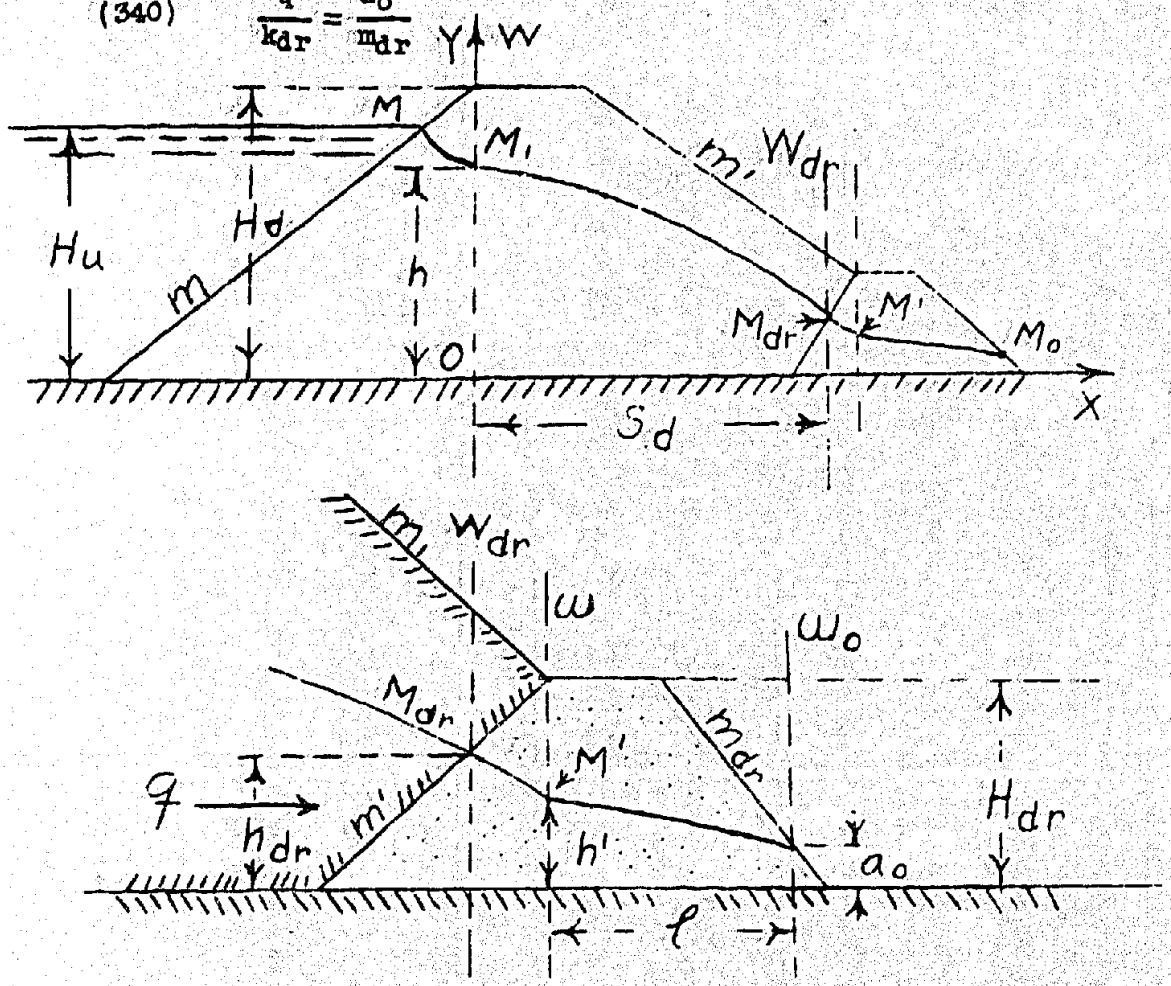


FIG. 41

in which m_d is the slope coefficient of the downstream wedge and generally

(341) $m_d \neq m_1$

and letting

(342) $\sigma = \frac{k}{k_d}$

we obtain from (340)

$$(343) \qquad \frac{q}{k} = \frac{a_o}{\sigma \cdot m_d}$$

and

$$(344) \qquad a_o = \sigma \cdot m_d \cdot \frac{q}{k};$$

now we write according to equation (10) for the middle part of the backfill (see detail to Fig. 41) i.e. between the sections w and w_o:

$$(345) \qquad \frac{2k}{k_{dr}} l = h'^2 - a_o^2$$

$$(346) \qquad \frac{2q}{k} l \sigma = h'^2 - a_o^2$$

from which we can find the depth h'.

Using the symbols of Fig. 41 we now write equation I of the system (41') and obtain:

$$(347) \qquad \frac{q}{k} = \frac{h_{dr} - h'}{\sigma \cdot m_{dr}} \ln \frac{H_{dr}}{H_{dr} - h'}$$

from which we can directly determine h_{dr}, since the remaining quantities in this equation are known (m_{dr} is the inside slope coefficient of the fill). We now also obtain the location of the point M_{dr} on the depression curve, since it corresponds to the value h_{dr}.

We can now correct the value of $\frac{q}{k}$ using the system (336) and using in it the distance s_{dr} as shown in Fig. 41, and the depth h_{dr} from the preceding computation. Determining h from (338) we find the value of $\frac{q}{k}$ from equation II of system (336) after which we again make the calculation according to formulas (340) - (347); obtaining the points M_o; M' and M_{dr} of the depression curve. Point M_1 is determined by means of the obtained value of h. If for this second computation the value h_{dr} is substantially different from the value h_{dr} found by the first calculation then a third computation is necessary; for this of course we

start with the h_{dr} value found by the second approximation. We have to solve this problem by this method of approximation by steps. It is not difficult to do this since the formulas (340) - (347) are rather simple. The intermediate point of the depression curve can be easily found from equation (10) after the main points M_o; M'; M_{dr} and M_1 have been determined.

§ 31. Percolation in a homogeneous earth dam with drainage system when $h_o > 0$.

If the depth of the tailwater $h_o > 0$ (see Fig. 42) the drainage of the dam retains a certain significance insofar that it keeps the depression curve at some distance from the downstream face of the dam, and thus can be regarded as advantageous even in this case. One of the

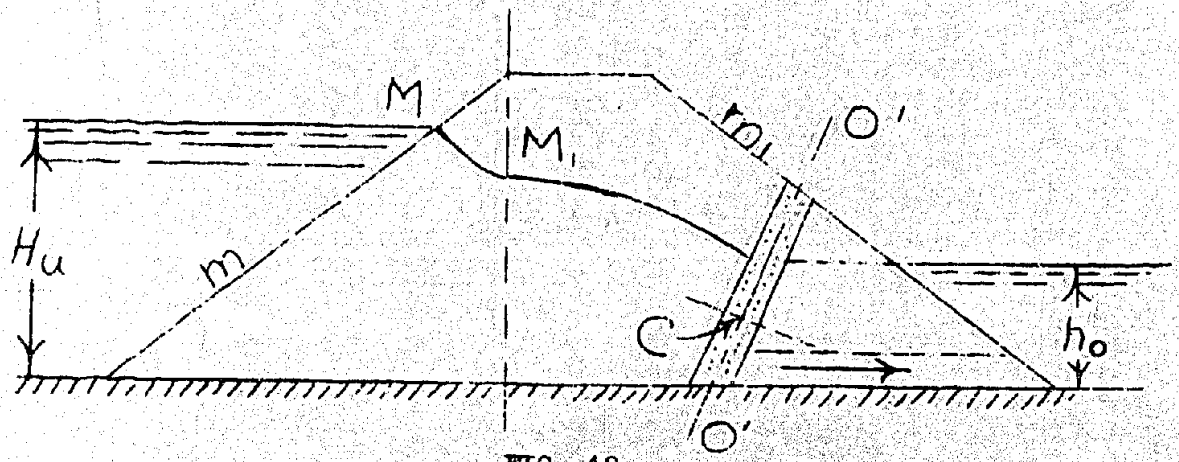


FIG. 42

possible designs of such a drainage is shown in Fig. 42. The discharge part of the drainage can be made as a layer of pervious material which is connected with the slanting drainage layer, or it can be made of separate parts connecting the heel of the dam with the drainage layer. For a calculation of the depression curve the above design has to be slightly changed, namely: the axis O'-O' (Fig. 42) has to be rotated around the center point C so that it acquires a vertical direction. This is shown in Fig. 43.

If the drainage is sufficiently pervious we can neglect its hydraulic resistance and we can (see Fig. 43) let:

(348) $$h_{dr} = h_o$$

for the system of equations (336) we now have the system of percolation equations as follows:

$$\frac{q}{k} = \frac{H_d - d_o - h}{m} \ln \frac{H_d}{H_d - h}$$

(349)

$$\frac{q}{k} = \frac{h^2 - h_o^2}{2s_{dr}}$$

This system is solved like the system (336) and therefore no further discussion for the case $h_{dr} = h_o$ is necessary. But if for some

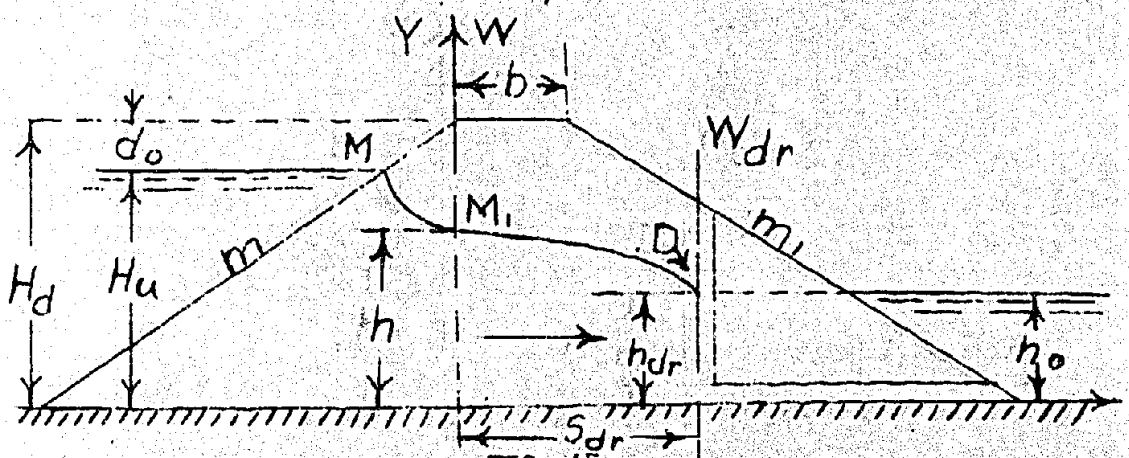


FIG. 43

reason it should be necessary to account for the hydraulic resistance

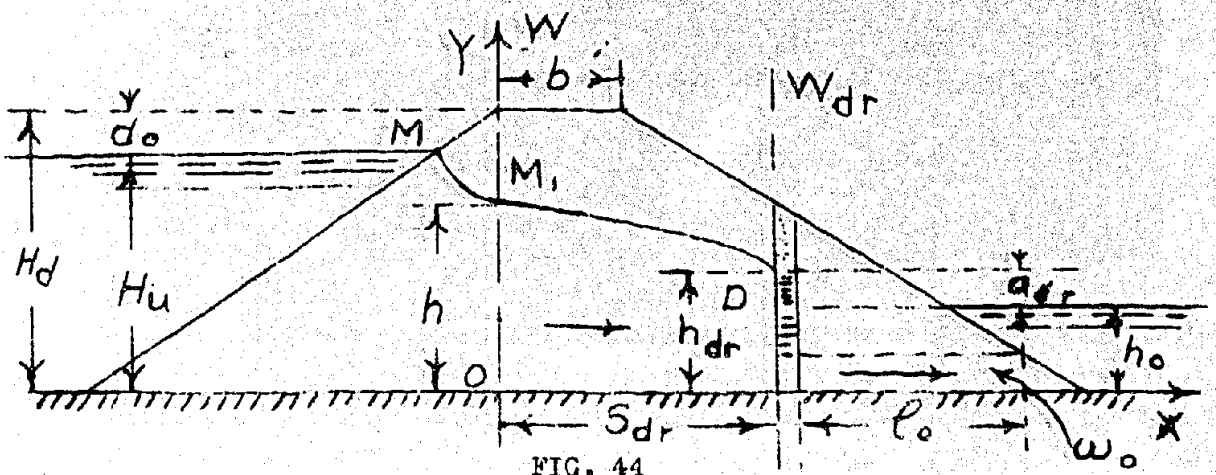


FIG. 44

of the drainage system like shown in Fig. 44 we have to add a few things to the above discussion. We shall for this purpose call

l_o = the length of the outlet part of the drainage (see Fig. 44)

a_{dr} = loss of head of outlet part of drainage

w_o = cross sectional area of outlet (taken per linear foot of dam length)

k_{dr} = the percolation coefficient of the material which forms the drainage system

The following discussion pertains to both the cases: (1) where the drainage outlet is formed by a continuous layer of pervious material connecting the dam heel with the drainage layer; (2) where the drainage outlet is formed by separate drainage ditches filled with pervious material, and connecting the dam heel with the drainage layer.

In the outlet part of the drainage the percolation velocity is

$$(350) \qquad = \frac{k_{dr} \cdot a_{dr}}{l_o}$$

if we neglect the percolation capacities of the lower wedge (i.e. the part of the dam at the right side from $\bar{w}_{dr}$) we obtain a discharge q:

$$(351) \qquad q = w_o \quad = k_{dr} \cdot w_o \frac{a_{dr}}{l_o}$$

or

$$(352) \qquad \frac{q}{k_{dr}} = \frac{w_o a_{dr}}{l_o}$$

$$(353) \qquad \text{introducing } \frac{k}{k_{dr}} = \sigma < 1$$

where k as in all previous discussions is the percolation coefficient of the dam material. Thus (352) changes into:

(354) $$\frac{q}{k} = \frac{w_o a_{dr}}{\sigma\, l_o}$$

for which the quantity $\sigma\, l_o$ could be called the "virtual length" of the outlet part of the drainage and we shall call it l_v:

(355) $$l_v = \sigma\, l_o$$

thus (354) becomes:

(356) $$\frac{q}{k} = \frac{w_o a_{dr}}{l_v}$$

or since

(357) $$h_{dr} - h_o = a_{dr}$$

it follows:

(358) $$\frac{q}{k} = \frac{w_o (h_{dr} - h_o)}{l_v}$$

By combining this one equation with the two equations of (336), the system of percolation equations for the discussed case is:

(359)

I $$\frac{q}{k} = \frac{H_d - d_o - h}{m} \ln \frac{H_d}{H_d - h}$$

II $$\frac{q}{k} = \frac{h^2 - h_{dr}^2}{2s_{dr}}$$

III $$\frac{q}{k} = \frac{w_o (h_{dr} - h_o)}{l_v}$$

The unknown quantities in this system are:

(360) h; h_{dr}; q (or $\frac{q}{k}$); and the quantities

(361) H_d; d_o; s_{dr}; h_o; w_o; l_v; k; m are known.

From system (359) we have:

$$\frac{H_d - d_o - h}{m} \ln \frac{H_d}{H_d - h} = \frac{h^2 - h_{dr}^2}{2s_{dr}} = \frac{w_o(h_{dr} - h_o)}{l_v} \tag{362}$$

We find from the first two numbers:

$$h_{dr} = \phi_1(h) \tag{363}$$

and from the first and third:

$$h_{dr} = \phi_2(h) \tag{364}$$

where

$$\phi_1(h) = \sqrt{h^2 - 2s_{dr} \cdot \frac{H_d - d_o - h}{m} \ln\left(\frac{H_d}{H_d - h}\right)} \tag{365}$$

and

$$\phi_2(h) = h_o + \frac{l_v}{w_o} \cdot \frac{H_d - d_o - h}{m} \ln\left(\frac{H_d}{H_d - h}\right) \tag{366}$$

By introducing different values of h we can compute the curves for the functions $\phi_1(h)$ and $\phi_2(h)$ from (365) and (366). The intersection point of the two curves determines the value of h (on the abscissa axis) and the value of h_{dr} (on the ordinate axis) as is shown in Fig. 45. The value of $\frac{q}{k}$ can after this be found from equations II and III of the system (359); the intermediate points of

the depression curve again are computed from equation (10).

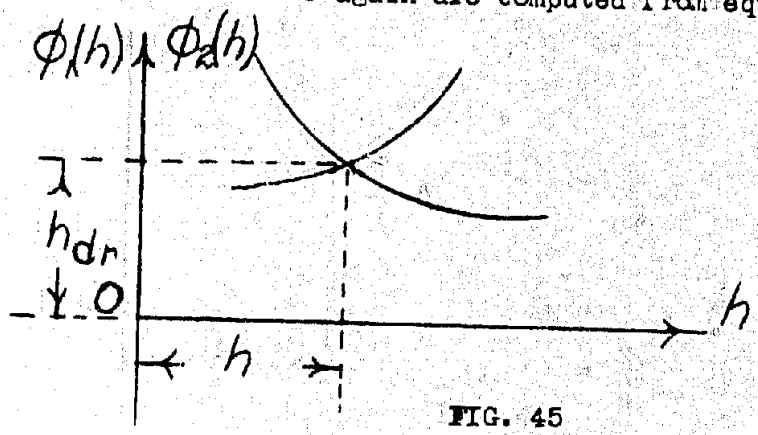


FIG. 45

32. Percolation in an earth dam with the core or protection layer and drainage.

If a drainage system is built in an earth dam with core wall, we make use of the "virtual method" which was pointed out in chapter III and change the problem to one of a uniform ("equivalent") earth dam. For it we can make the computation of the depression curve in the same manner as in § 29. Thus it will not be difficult to apply this method also for the given dam with core, and therefore we shall because of the clearness of the problem not go into any further discussion.

For the case of a dam with a protection layer (see Fig. 46) the system (317) is changed into a system of only two equations of drainage is also used in the dam (for $h_d = 0$ on the main drainage line D):

$$\frac{q}{k} = \frac{H_u - h}{m'} \left[\ln \frac{\delta_v' + H_u'}{\delta_v' + H_u' - h} + \frac{H_u - h}{2\delta_v'} - \frac{z_o^2}{2\delta_v'(H_u - H)} \right.$$

(367)

$$\frac{q}{k} = \frac{h^2}{2s_{dr}}$$

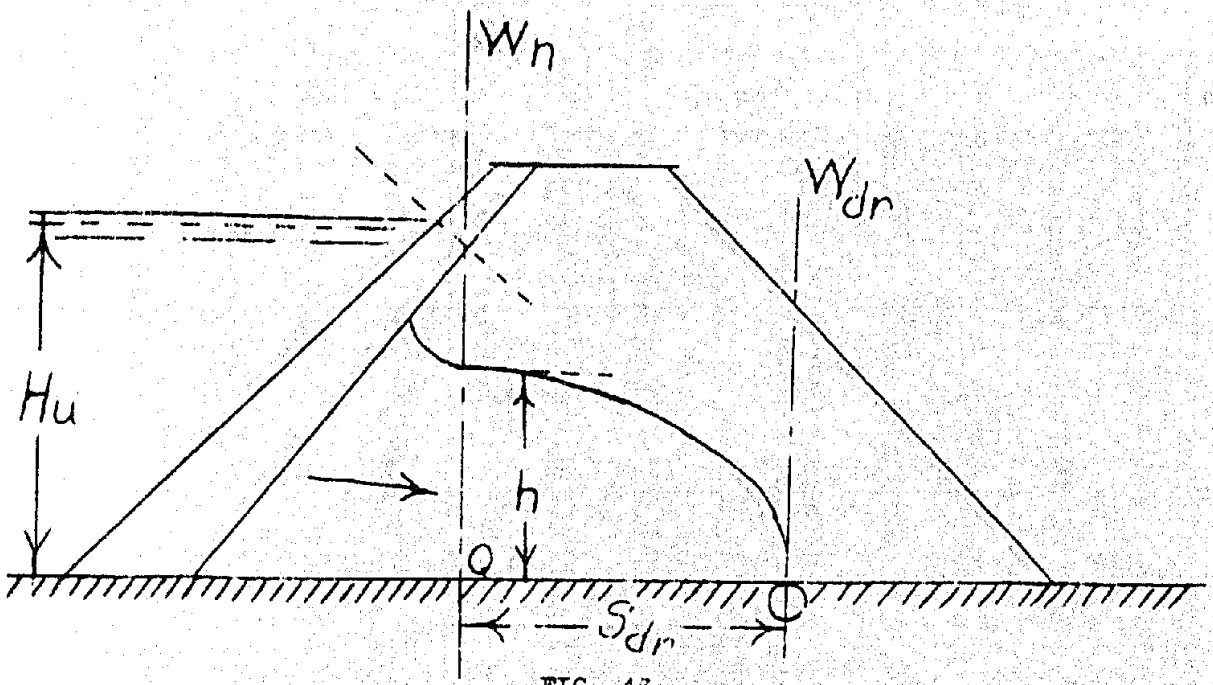


FIG. 46

The symbols used are the same as in system (317) only that instead of s' which is used in (317) we now use s_{dr} which stands for the distance between w_n and w_{dr} and is a known quantity (Fig. 46). The unknowns h and q (or $\frac{q}{k}$) enter the equation (340). They can be found in the same manner which was used for the same quantities in ¶ 29.

If we substitute the value of $\frac{q}{k}$ from the second equation into the first equation of (367) we obtain:

$$(368) \qquad \frac{h^2}{2s_{dr}} = \frac{H_u - h}{m'}\left[\ln\frac{\delta_v' + H_u'}{\delta_v' + H_u' - h} + \frac{H_u - h}{2\,\delta_v'} - \frac{z_o^2}{2\,\delta_v'(H_u - h)}\right]$$

and introducing

$$(369) \qquad F_o = \frac{2s_{dr}\,(H_u - h)}{m'}\left[\ln\frac{\delta_v' + H_u'}{\delta_v' + H_u' - h} + \frac{H_u - h}{2\,\delta_v'} - \frac{z_o^2}{2\,\delta_v'(H_u - h)}\right]$$

we have

$$(370) \qquad h = \sqrt{F_o}$$

We now compute a graph of the function $\sqrt{F_o}$ by using different values of h, and draw a straight (45°) line OA like we did in ¶ 29. Fig. 39.

After having in this manner found the depth h we determine the value of $\frac{q}{k}$ and q from the second equation of the system (367). The computation of the intermediate points of the depressive curve was explained already in Chapter IV.

If the depth h_{dr} of the percolation flow in the section w_n cannot be assumed to be equal zero (for example if water is in the tail channel directly below the dam) then, in the same way as this was shown in ¶ 29, we use instead of the second equation of the system (367) also for this case the equation:

$$(371) \qquad \frac{q}{k} = \frac{h^2 - h_{dr}^2}{2s_{dr}}$$

But the first equation of the system remains unchanged. For the depth h we have instead of (370) the expression:

$$(372) \qquad h = \sqrt{F_o + h_{dr}^2}$$

where F_o again is the same as in equation (369). Again we compute and plot a diagram of the function $\sqrt{F_o + h_{dr}^2}$, plotting h as the abscissae and $\sqrt{F_o + h_{dr}^2}$ as the ordinates. Drawing now the 45° straight line OA through the origin (like in Fig. 39) we again find the value of h and can now from equation (371) compute the values of $\frac{q}{k}$ and q.

In conclusion of the discussion of earth dams with drainage system we have to call attention to the following fact: for $h_{dr} \cong 0$ or for small values of h_{dr} (see Fig. 38 and 46) the condition of the "slow changeability" of the flow when approaching the section w_{dr} is correctly speaking not existing; much in the same way as this condition is not fulfilled in some similar cases of the hydraulics of surface flows when these approach the sections of critical depth (such a case is for example the overflow which is located at the end of a canal section of which has a bottom slope which is less than the critical slope). This situation does not influence the points of the depression curve which are further away from the section w_n, but due to it the computation of points of the depression curve in the neighborhood of the drain line has only the value of an approximating orientation. But we can not avoid this if we try to approach the given problem hydraulically. The greater the depth h_{dr} of the percolation flow at the section w_{dr} the more will the flow satisfy the conditions of "slowly changing flow" in that region, and better will be the results obtained by a hydraulic computation of the depression curve.

¶ 33. Examples.

Using Fig. 38, we shall as the first example take up the computation of the depression curve for a homogeneous earth dam with drainage. There is no water downstream from the dam. We shall use the same dimensions as were used in ¶ 14 (example of homogeneous earth dam without drainage). Using the lettering of Fig. 38 we have the following dimensions given:

$H_d = 8.0$ m

$d_o = 1.5$ m

$H_u = 6.5$ m

$b = 5.0$ m

$m = 3.0$; $m_1 = 2.0$

percolation coefficient $k = 0.00005$ cm/sec.

furthermore we assume:

$s_{dr} = 15.0$ m.

For the locations of the drain line D shown in Fig. 38 and for a sufficient perviousness of the drainage we assume that the depth of the percolation flow in the section w_{dr} is:

$h_{dr} \cong 0$

and therefore we shall make use of the system of percolation equations given under (328). We have, as has been shown in ¶ 29 for the function F (from 333)) the value:

$$F = \frac{2.15.0}{3.0} (8.0 - 1.5 - h) \ln\left(\frac{8.0}{8.0 - h}\right)$$

or

$$(373) \qquad F = 10.0(6.5 - h) \ln\left(\frac{8.0}{8.0 - h}\right)$$

or changing to Briggs logarithms:

$$(374) \qquad F = 23.0(6.5 - h)\ \mathrm{Log}_{10}\left(\frac{8.0}{8.0 - h}\right)$$

We now make a table of the values of this function for different values of h (see table 17) and plot the corresponding graph as $\sqrt{F} = f(h)$ (see Fig. 47)

Table 17

h	F	$\sqrt{F}$	Remarks
6.00	6.93	2.63	Depths h given in meters
5.00	14.68	3.84	
4.00	17.30	4.16	
3.00	16.43	4.06	
2.00	12.83	3.56	

Drawing a straight line at an inclination of 45° through the origin we find from its intersection with the curve that for our case

$$h = 4.15 \text{ m}$$

and now we can use the second equation of the system (328):

$$\frac{q}{k} = \frac{h^2}{2s_{dr}} = \frac{4.15^2}{2.15.0} = 0.575 \text{ m} = 57.5 \text{ cm}$$

whence:

$$q = 0.00005 \cdot 57.5 = 0.002875 \text{ cm}^2/\text{sec.}$$

or

$$q_{daily} = 8640q = 8640 \times 0.002875 = 24.8$$

where q_{daily} is measured in liters per 24 hours and per 1 linear meter of the length of dam.

For the computation of the depression curve in the interval between the points M_1 - D we use equation (339):

$$y^2 = h^2 - \frac{2q}{k} x$$

in which our present case has the constants:

$$(375) \qquad y^2 = 17.22 - 1.15\ x$$

The values of table 18 are computed from this equation.

Table 18

No. of point	The coordinates in inches	
	x	y
1	.0	4.15 = h
2	2.0	3.86
3	4.0	3.56
4	6.0	3.21
5	8.0	2.83
6	10.0	2.39
7	12.0	1.85
8	14.0	1.06
9	s_{dr} = 15.0	0.0 = h_{dr}

For the second example we make use of the Fig. 43, and we take the main dimensions as given in the example of № 13:

H_d = 5.0 m

d_o = 0.8 m

H_u = 4.2 m

h_o = .5 m

b = 4.0 m

m = 2.5; m_1 = 1.5

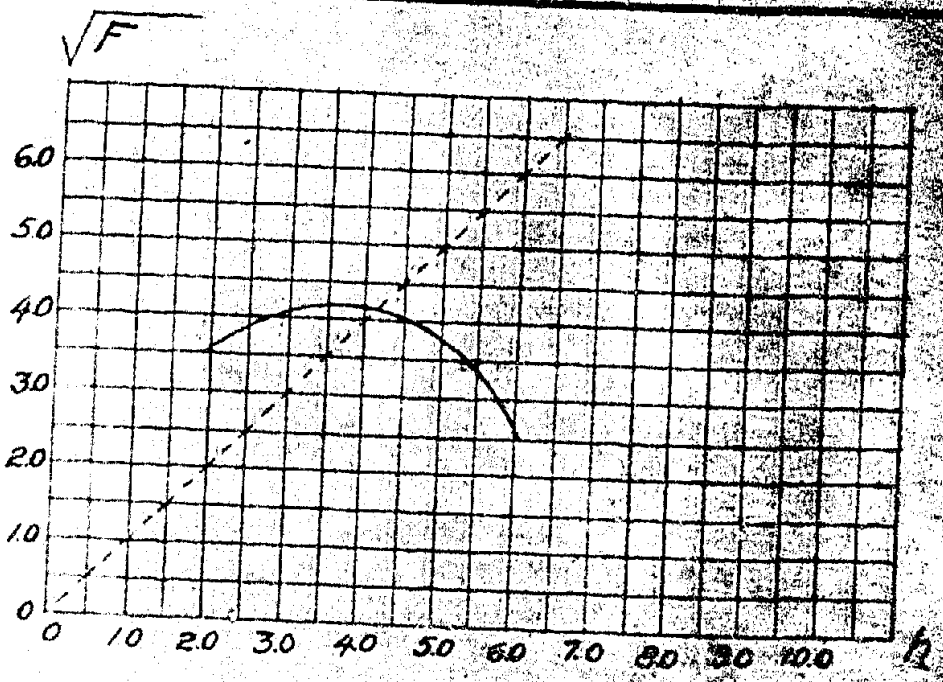

$\sqrt{F}$
6.0
5.0
4.0
3.0
2.0
1.0
0
0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.0
h

Fig. 47

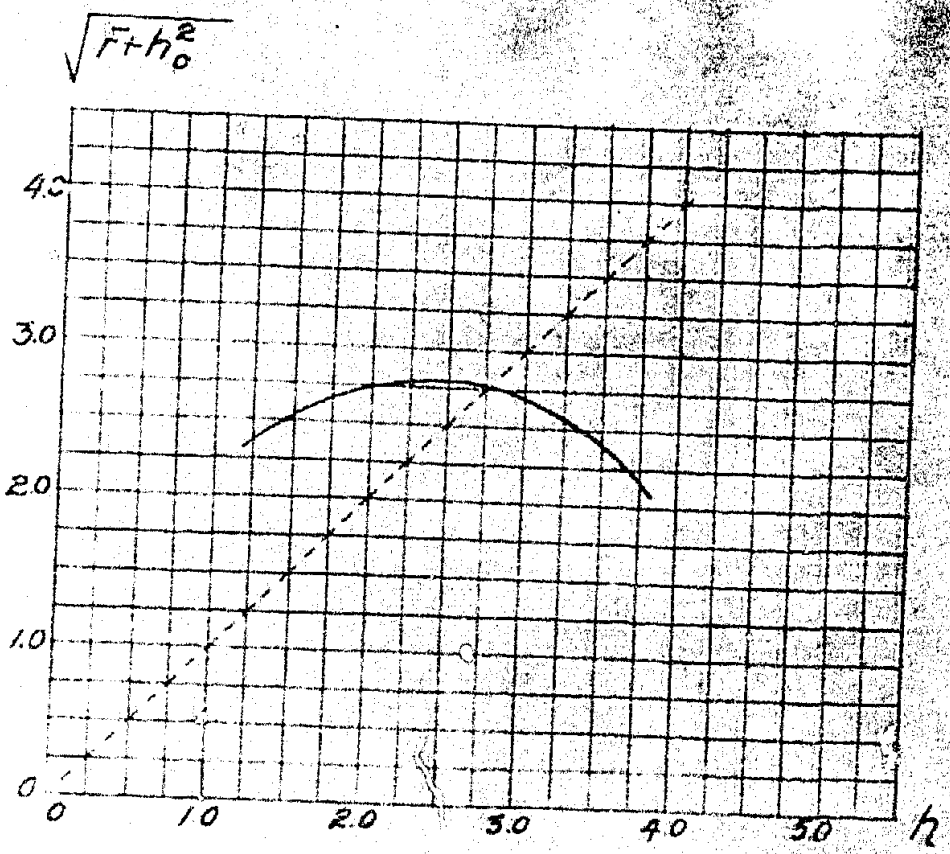

$\sqrt{F+h_o^2}$
4.0
3.0
2.0
1.0
0
0
1.0
2.0
3.0
4.0
5.0
h

Fig. 48

$k = 0.00005$ cm/sec.
$s_{dr} = 8.0$ m.

We shall take up the case where $h_{dr} \cong h_o = 0.5$ m, and we can therefore use the system of percolation equations given as (349) in ¶ 31. It is solved in the same manner as the system (336).

From (333) we know the expression for the function F:

$$F = \frac{2 \times 8.0}{2.5} (5.0 - 0.8 - h) \ln \left(\frac{5.0}{5.0 - h} \right)$$

(376) or $$F = 6.4 (4.2 - h) \ln \left(\frac{5.0}{5.0 - h} \right)$$

and

(377) $$F = 14.74 (4.2 - h) \operatorname{Log}_{10} \left(\frac{5.0}{5.0 - h} \right)$$

Furthermore we have to plot a graph for the function $\sqrt{F + h_o^2}$ using different values of the depth h. In table 19 the values for plotting Fig. 48 are tabulated.

Table 19

h	F	$\sqrt{F + h_o^2}$	
4.00	2.06	1.52	Depths h given in meters
3.50	5.39	2.38	
3.00	7.04	2.70	
2.50	7.54	2.79	
2.00	7.16	2.72	
1.50	6.18	2.54	

Drawing again a straight line under 45° through the origin we obtain for this case:

$$h = 2.75 \text{ m}$$

and now we find from the second equation of system (349):

$$\frac{q}{k} = \frac{h^2 - h_o^2}{2s_{dr}} = 0.457 \text{ m} = 45.7 \text{ cm.}$$

whence

$$q = 0.00005 \times 45.7 = 0.002284 \text{ cm}^2/\text{sec.}$$

or

$q_{daily} = 8610 \times 0.002284 = 19.8$ measured in liters per day and per linear meter of length of dam.

The equation of the depression curve between the points M_1 and D (see Fig. 43) is according to (339):

$$(378) \qquad y^2 = 7.56 - 0.914\,x$$

and the values of table 20 are computed from this last equation.

Table 20

No. of point	Coordinates in meters	
	x	y
1	.0	2.75 = h
2	2.0	2.39
3	4.0	1.97
4	6.0	1.44
5	7.0	1.08
6	s_{dr} = 8.0	.50 = h_{dr}

CHAPTER VI

34. Barrages with "lower wedge" for $h_o > 0$.

The theories and methods which were discussed in the preceding chapters can be applied not only for the design of an earth dam, but also for the design of certain types of barrages (coffer dams) which are built in order to keep the site of some hydraulic structure dry during the construction period. The part of this barrage which is only little pervious can have a common trapezoidal profile like other earth dams, and in that case of course the depression curve can be calculated in the same way as has been shown for the earth dams. But for

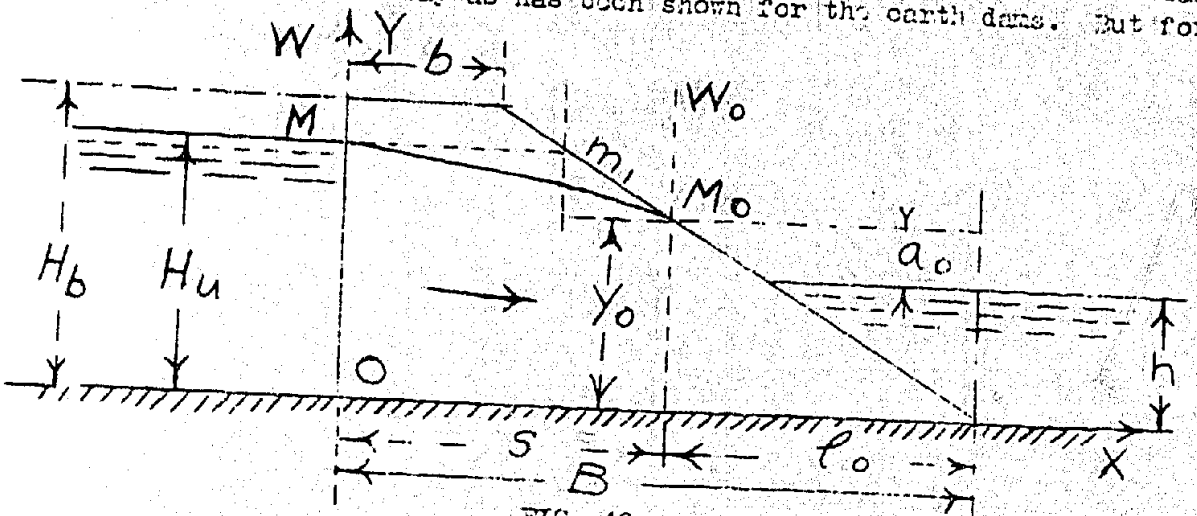


FIG. 49

barrages/which either the upstream or the downstream face are vertical the problem is somewhat changed and the discussion of its solution demands some supplementary remarks.

In this paragraph we shall take up the barrage with a vertical upstream face "Barrage with lower wedge") for the case of $h_o > 0$, i.e. there is water downstream from the barrage.

In Fig. 49 not the whole profile of such a barrage is shown but merely its little pervious part, which of course alone is significant for the calculation of the depression curve. The height of the barrage is denoted by H_b and its width at the bottom is called b. All the other dimensions are denoted by the same symbols which were used for the earth dams. A few additional quantities which enter the calculation will be explained further below.

Since for the barrage as shown in Fig. 49 (partly) the "upper wedge" is missing, we obtain the necessary system of percolation

equations as a special case of the system (41) in which we have to assume (according to Fig. 49):

Footnote: The vertical upstream face of the barrage is for this purpose regarded as being pervious so that we can say that $h = H_u$. But if this upstream face is so constructed that it is very dense (impervious) the question of the percolation of water through the barrage does not arise at all.

(379)
$$h = H_u$$
$$H_d = H_B$$

Now we have a system of three equations since the first equation of (41) disappears because no "upper wedge" is involved in the problem.

(380)
$$\text{I} \quad \frac{q}{k} = \frac{H_u^2 - (a_o + h_o)^2}{2s}$$
$$\text{II} \quad \frac{q}{k} = \frac{a_o}{m_1}\left[1 + \ln \frac{a_o + h_o}{a_o}\right]$$
$$\text{III} \quad s = b + m_1\left[H_B - (a_o + h_o)\right]$$

The three unknown quantities in this system are:

(381) a_o; s and q (or $\frac{q}{k}$)

and their determination will follow the general lines pointed out in §§. Of course the solution will be simplified corresponding to the character of the problem in hand. At first we rewrite equation (380) using:

(382) $y_o = a_o + h_o$

(382') $a_o = y_o - h_o$

In the last equation of the system we can furthermore replace from (Fig. 49):

(383) $$l_o = m_1 \cdot y_o$$

where l_o is given on the Fig., and we see that

(384) $$s = B - l_o = B - m_1 y_o$$

Hence the system (380) changes into the system:

$$\text{I} \qquad \frac{q}{k} = \frac{H_u^2 - y_o^2}{2(B - l_o)}$$

(385) $$\text{II} \qquad \frac{q}{k} = \frac{y_o - h_o}{m_1}\left[1 + \ln\frac{y_o}{y_o - h_o}\right]$$

$$\text{III} \qquad l_o = m_1 y_o$$

in which the unknown quantities are:

(386) $$y_o;\ l_o;\ \frac{q}{k} \ \text{(or } q\text{)}$$

Letting

(387) $$A_o = 1 + \ln\frac{y_o}{y_o - h_o}$$

we obtain from equation II of the system (385):

(388) $$\frac{q}{k} \quad \frac{A_o(y_o - h_o)}{m_1}$$

and introducing this into I we obtain:

$$(389) \qquad \frac{A_o(y_o - h_o)}{m_1} = \frac{H_u^2 - y_o^2}{2(B - l_o)}$$

whence we have using equation III (in substituting l_o):

$$(390) \qquad \frac{A_o(y_o - h_o)}{m_1} = \frac{H_u^2 - y_o^2}{2(B - m_1 y_o)}$$

whence:

$$(391) \qquad y_o = h_o + \frac{m_1(H_u^2 - y_o^2)}{2A_o(B - m_1 y_o)}$$

If we now let:

$$(392) \qquad Y = h_o + \frac{m_1(H_u^2 - y_o^2)}{2A_o(B - m_1 y_o)}$$

we finally have from (391):

$$(393) \qquad y_o = Y$$

We can now by introducing different values of y_o plot a graph of the function Y. By drawing a straight line under an angle of 45° with the horizontal through the origin we find the demanded value of y_o as the intersection of this straight line and the curve for Y. (See Fig. 50).

$$(394) \qquad y_o = (y_o)_{wanted}$$

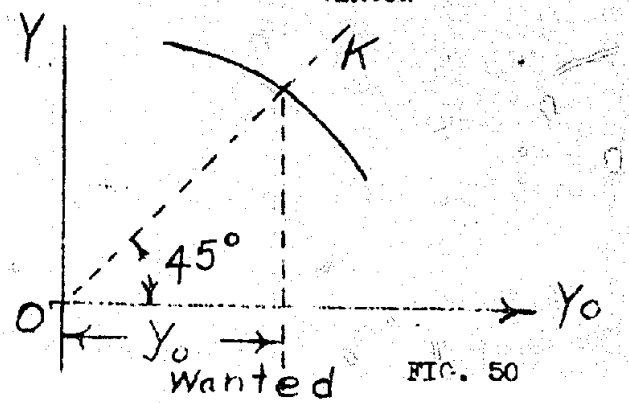


FIG. 50

From equation III of the system (385) we now can determine the value of l_o and furthermore from equation I, $\frac{q}{k}$ (or q). If we now want to compute the intermediate points of the depression curve for the coordinate axes as given in Fig. 49 we can do this by using the equation of the non uniform motion along the horizontal ground layer:

$$\frac{2q}{k} x = H_u^2 - y^2 \tag{395}$$

whence

$$y^2 = H_u^2 - \frac{2q}{k} s \tag{396}$$

in which the abscissae have to be within the limits

$$s > x > 0 \qquad \text{or} \tag{397}$$

$$(B - l_o) > x > 0 \tag{398}$$

35. Barrage with "lower wedge" for $h_o = 0$.

The case $h_o > 0$ which has been discussed in the preceding paragraph may occur in practice when for example the site of the structure is partly flooded (the water may not be completely pumped out of the (dug-out) or some leak may have developed in the coffer dam enclosing the site, etc.). For the normal case of the use of a barrage we have to assume the condition $h_o = 0$, which is illustrated in Fig. 51.

In this case remembering equation (379) we obtain from system (41') the following system of percolation equations:

$$\frac{q}{k} = \frac{H_u^2 - a_o^2}{2s}$$

$$\frac{q}{k} = \frac{a_o}{m_1} \tag{399}$$

$$s = b + m_1 (H_B - a_o)$$

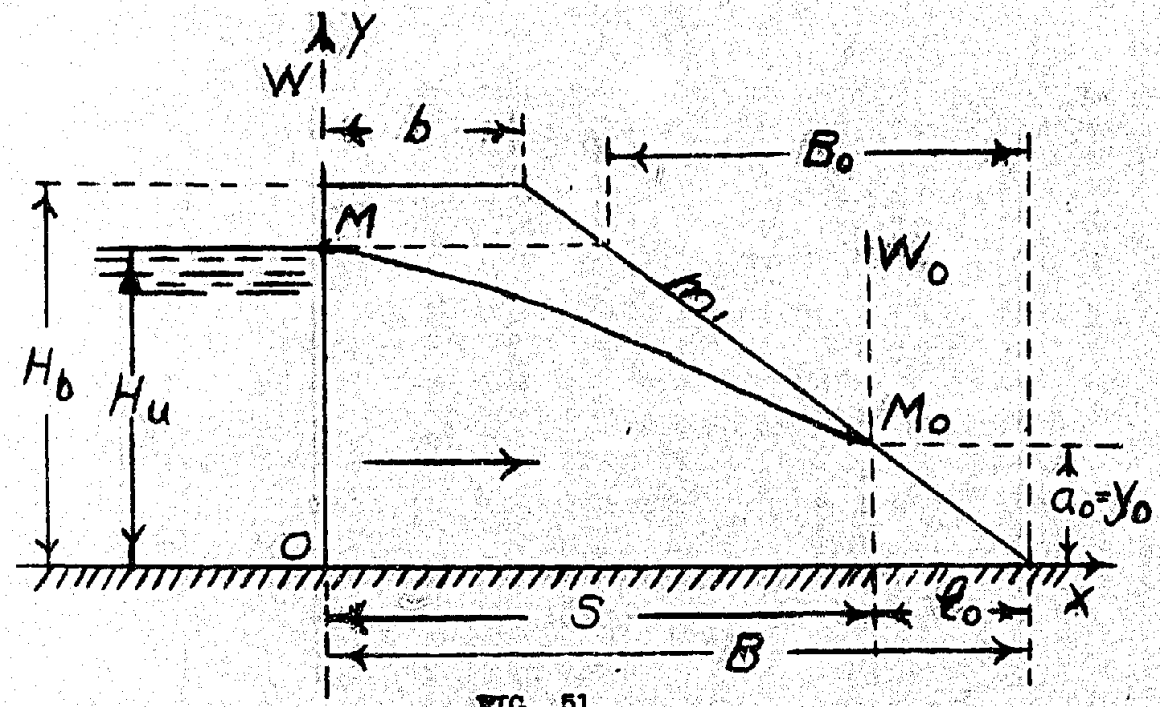


FIG. 51

Or if we remember equations (383) - (384) and notice that for this case $y_o = a_o$ we obtain:

$$\text{I} \qquad \frac{q}{k} = \frac{H_u^2 - y_o^2}{2(B - l_o)}$$

$$(400) \qquad \text{II} \qquad \frac{q}{k} = \frac{y_o}{m_1}$$

$$\text{III} \qquad l_o = m_1 y_o$$

This system can be solved in a rather simple way without using methods of approximation. From equation III we introduce the value for l_o into equation I:

$$(401) \qquad \frac{q}{k} = \frac{H_u^2 - y_o^2}{2(B - m_1 y_o)}$$

and equating (401) with II we obtain:

$$(402) \qquad \frac{y_o}{m_1} = \frac{H_u^2 - y_o^2}{2(B - m_1 y_o)}$$

whence we obtain:

$$(403) \qquad m_1 y_o^2 - 2By_o + m_1 H_u^2 = 0$$

and solving for y_o we obtain:

$$(404) \qquad y_o = \frac{B}{m_1} \pm \sqrt{\left(\frac{B}{m_1}\right)^2 - H_u^2}$$

Since in this expression

$$(405) \qquad \frac{B}{m_1} = T$$

where T is the height of the triangle shown in Fig. 52 it is obvious that always

$$(406) \qquad \frac{B}{m_1} = T > H_u > y_o$$

and therefore the correct solution for y_o is the root:

$$(407) \qquad y_o = \frac{B}{m_1} - \sqrt{\left(\frac{B}{m_1}\right)^2 - H_u^2}$$

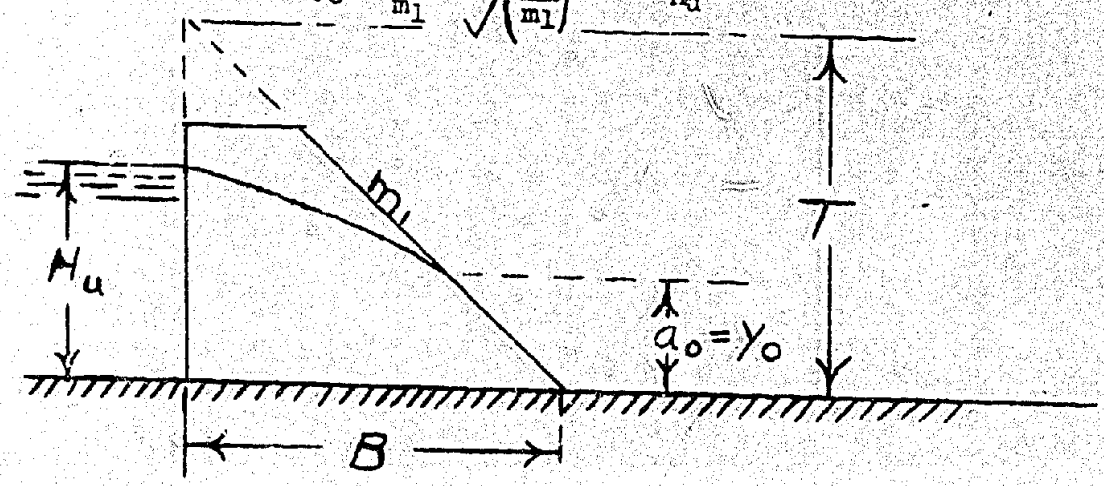


FIG. 52

or

$$(408) \qquad y_o = \left[\frac{B}{m_1 h_u} - \sqrt{\left(\frac{B}{m_1 H_u}\right)^2 - 1}\right] H_u$$

We see that in this expression

$$(409) \qquad m_1 H_u = B_o$$

where B_o is the horizontal projection of that part of the downstream face which is below the elevation of the head-water level (see Fig. 51). Thus we obtain from (408):

$$(410) \qquad y_o = (\beta_o - \sqrt{\beta_o^2 - 1}\,)\, H_u$$

in which

$$(411) \qquad \beta_o = \frac{B}{B_o}$$

β_o is a known quantity for given dimension of the profile of the barrage and for a given depth at the head water H_u.

Let us add that for a further simplification of the computation of the depression curve for a barrage with "lower wedge" (for the case $h_o = 0$) we can introduce the notation

$$(412) \qquad \beta_o - \sqrt{\beta_o^2 - 1} = \eta_o$$

and that we can plot a diagram for the quantity η_o in dependence of β_o. The curve $\eta_o = f(\beta_o)$ is plotted on graph No. 5.

Knowing β_o we find η_o from the graph and determine y_o from equation (410) writing this equation (by using (412)) in the form

$$(413) \qquad y_o = \eta_o H_u.$$

Furthermore we have from equation III of (400):

$$(414) \qquad l_o = m_1 y_o = m_1 \eta_o H_u$$

whence

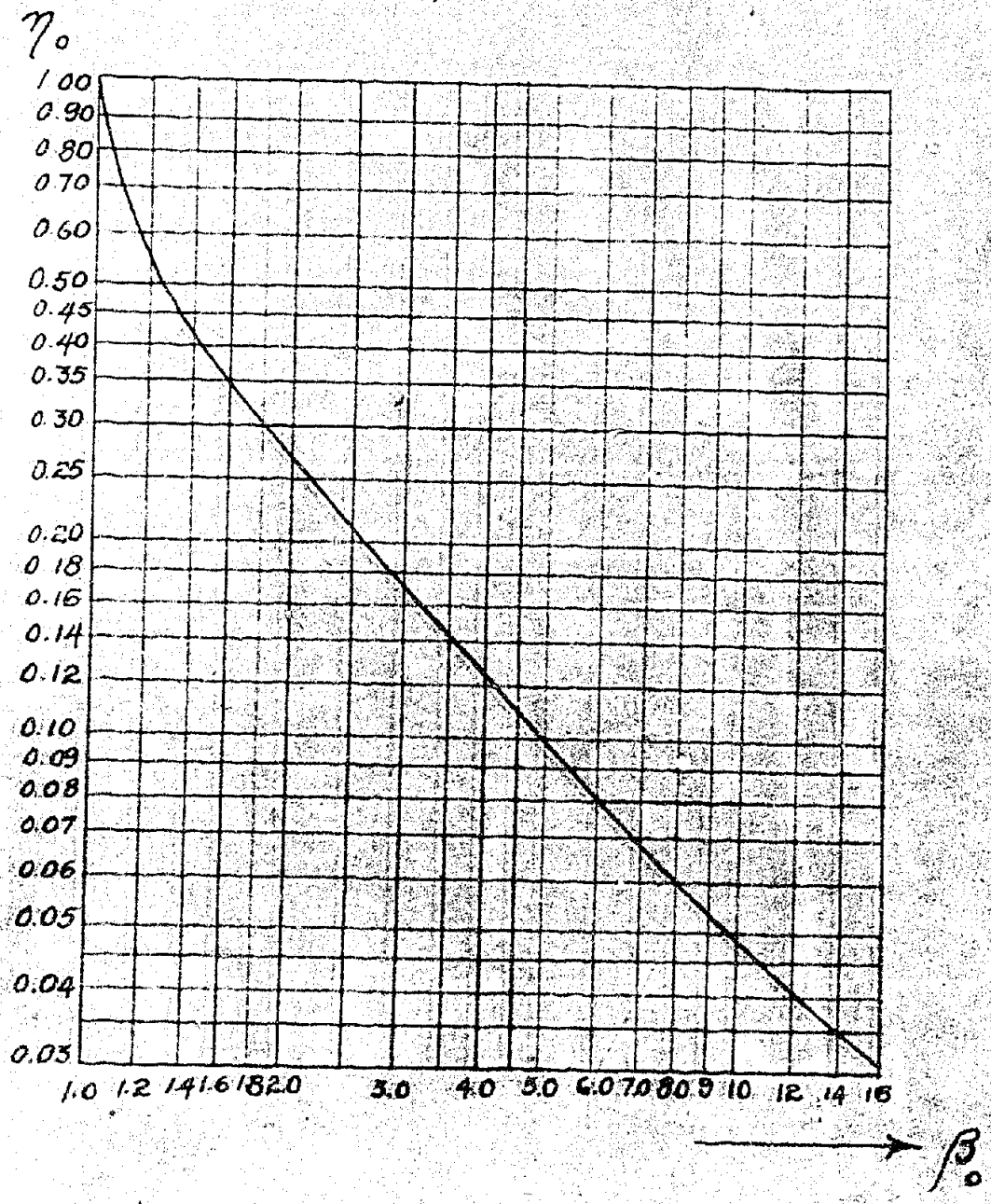
$$(415) \qquad \frac{q}{k} = \frac{y_o}{m_1} = \frac{\eta_o}{m_1} \cdot H_u$$

$$(416) \qquad q = \frac{k y_o}{m_1} = \frac{k \cdot \eta_o}{m_1} \cdot H_u$$

i.e. using graph No. 5 we can determine the quantities:

(417) y_o; l_o; $\frac{q}{k}$ (or q) in any desirable order.

Graph No 5


η_o
1.00
0.90
0.80
0.70
0.60
0.50
0.45
0.40
0.35
0.30
0.25
0.20
0.18
0.16
0.14
0.12
0.10
0.09
0.08
0.07
0.06
0.05
0.04
0.03
1.0 1.2 1.4 1.6 1.8 2.0 3.0 4.0 5.0 6.0 7.0 8.0 9 10 12 14 16
β_o

36. Barrage with "upper wedge."

For such barrages (Fig. 53) we have a system of percolation equations which is a special case of (41) when $h_o > 0$. Since for such a barrage the "lower wedge" does not exist equation III of system (41) drops out, and since in the given case(s) is known equation IV also drops out; $s = b$ where b is the width at the top of the barrage (Fig. 53). The other symbols used in Fig. 53 will be understood without any further explanation.

We also have to notice that for the present case the quantity a_o in equation II of the system (41) vanishes

$$a_o = 0 \tag{418}$$

since while there is no "lower wedge" the loss of head in this wedge also does not exist. Naturally (418) is true only if the downstream (vertical) face of the barrage is built to be pervious. If the downstream face should be impervious one could not talk about a percolation through the body of the barrage at all. If on the other hand the downstream face is of an "intermediate" nature, i.e. if it is pervious only to a certain degree, this condition may be accounted for in calculating the depression curve. But we shall not go into those details and limit our discussion to the case characterized by equation (418).

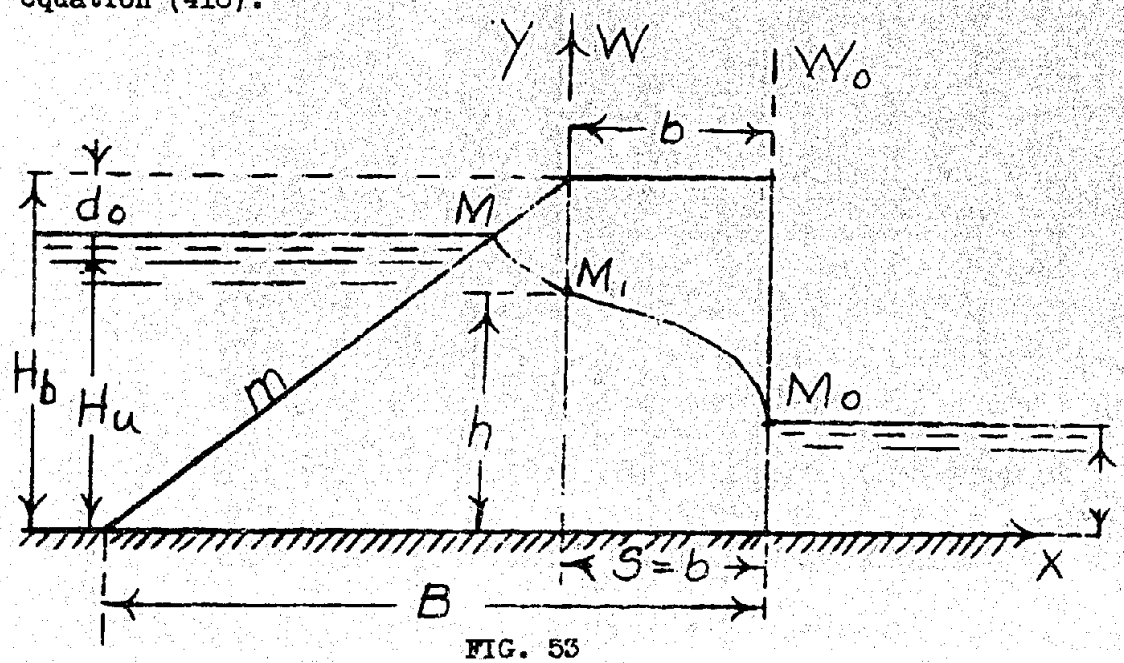


FIG. 53

If we now substitute H_d by H_B in equation I of the system (41) we obtain for the given case a system of two equations:

$$(419)\qquad \begin{aligned} &\text{I} \qquad \frac{q}{k} = \frac{H_B - d_o - h}{m} \ln \frac{H_B}{H_B - h} \\ &\text{II} \qquad \frac{q}{k} = \frac{h^2 - h_o^2}{2b} \end{aligned}$$

in which the unknown quantities are:

$$(420)\qquad h \text{ and } \frac{q}{k} \text{ (or } q)$$

This system of equations is solved like the analogous systems (336) and (349) which we obtained in Chapter V. If we introduce the notation

$$(421)\qquad F = \frac{2b}{m} (H_B - d_o - h) \ln \frac{H_B}{H_B - h}$$

we have analogous to (338) an expression for the depth h:

$$(422)\qquad h = \sqrt{F + h_o^2}$$

From what has been said in ¶ 29 we know how to plot a graph of the function $\sqrt{F + h_o^2}$ and how to find the desired value of h from this graph (see also Fig. 48). We also know from the previous discussion how to find $\frac{q}{k}$ and q and the intermediate points of the depression curve.

We now can take up the case where $h_o = 0$, and see that for it instead of system (419) we have:

$$(423)\qquad \begin{aligned} &\text{I} \qquad \frac{q}{k} = \frac{H_B - d_o - h}{m} \ln \frac{H_B}{H_B - h} \\ &\text{II} \qquad \frac{q}{k} = \frac{h^2}{2b} \end{aligned}$$

for which instead of (422) we obtain

$$h = \sqrt{F} \tag{424}$$

in which F again is expressed by (421) so that the further procedure does not have to be discussed again.

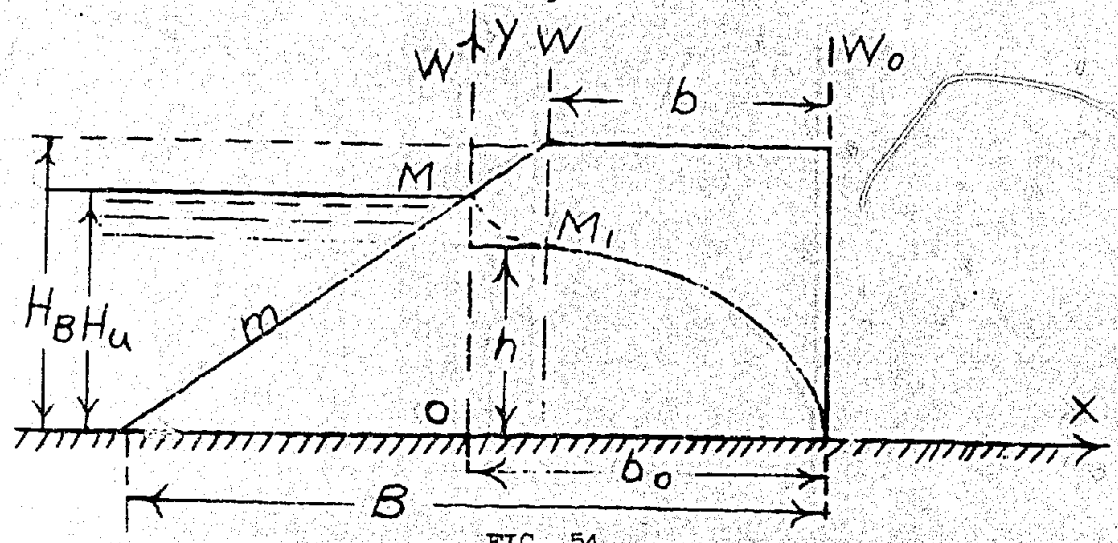


FIG. 54

The case $h_o = 0$ is the normal condition under which a barrage is used in practice, and therefore it seems justified to discuss it some more. We can move the division line from W to W' (see Fig. 54), i.e. we place the division W' through the intersection of the upstream face and the head water elevation (M). Following the same line of reasoning which was adopted in ¶ 4 (Chapter I) we have to substitute into equations (423):

Foot note: see also further below ¶ 39 - 40.

$$\begin{aligned} d_o &= 0 \\ H_B &= H_u \end{aligned} \tag{425}$$

and with this we obtain the system:

$$\begin{aligned} \frac{q}{k} &= \frac{H_u - h}{m} \ln \frac{H_u}{H_u - h} \\ \frac{q}{k} &= \frac{h^2}{2b_o} \end{aligned} \tag{426}$$

We see that into the second of the two equations not b, but b_o is introduced, i.e. the width of the barrage at the elevation of the head water level. From Fig. 54 we see that this has to be so for a division section placed in the way W' is placed.

From (426) we see that:

$$(427) \qquad \frac{H_u - h}{m} \ln \frac{H_u}{H_u - h} = \frac{h^2}{2b_o}$$

If we let:

$$(428) \qquad \frac{h}{H_u} = \eta$$

we change (427) into:

$$(429) \qquad \frac{1 - \eta}{m} \ln \frac{1}{1 - \eta} = \frac{\eta^2}{2} \cdot \frac{H_u}{b_o}$$

or

$$(430) \qquad \frac{2(\eta - 1) \ln (1 - \eta)}{\eta^2} = \frac{mH_u}{b_o}$$

From Fig. 54 we see:

$$(431) \qquad \frac{mH_u}{b_o} = \frac{B - b_o}{b_o} = \frac{B}{b_o} - 1$$

whence we obtain from (430):

$$(432) \qquad 1 + \frac{2(\eta - 1) \ln (1 - \eta)}{\eta^2} = \frac{B}{b_o}$$

or

$$(433) \qquad f(\eta) = \frac{B}{b_o}$$

in which

$$(434) \qquad f(\eta) = 1 + \frac{2(\eta - 1)\ln(1 - \eta)}{\eta^2}$$

The values of η can according to (428) only vary between the limits

$$(435) \qquad 1 \geqq \eta \geqq 0$$

since $\eta = \frac{h}{H_u}$ cannot be greater than unity. In regard to the corresponding limits between which the function $f(\eta)$ can vary we can show that:

$$(436) \qquad \lim_{\eta = 1} \left| f(\eta) \right| = 1$$

for which we obtain

$$(437) \qquad B = b_o \qquad \text{from equation (435).}$$

This would correspond to a rectangular barrage which has a vertical upstream and downstream face. For $\eta = 0$ we can show

$$(438) \qquad \lim_{\eta = 0} \left| f(\eta) \right| = \infty$$

The curve of the variation of the function $f(\eta)$ is shown on graph No. 6 for variations of the argument between $\eta = 0.2 \div 1.0$. This graph can be used in the following manner: We determine the value of $\frac{B}{b_o}$ for the given barrage with upper wedge; we now use this value as an ordinate and then as an abscissae and read the corresponding values of η. After this we obtain from (428):

$$(439) \qquad h = \eta \cdot H_u$$

Furthermore we determine $\frac{q}{k}$ from equation II of the system (426) and the percolation discharge q. If necessary we can now plot the depression curve also from the equation:

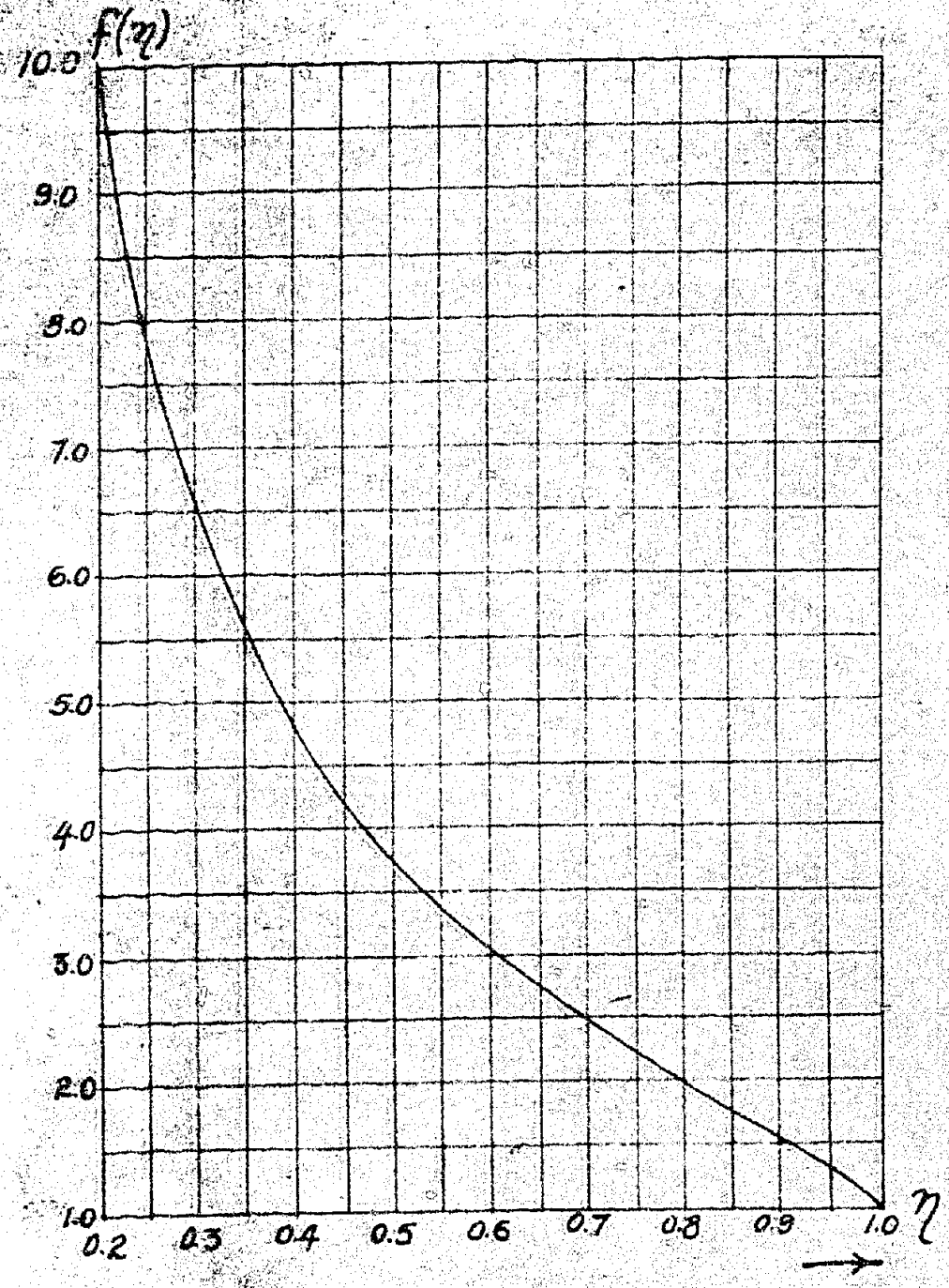
$$(440) \qquad y^2 = h^2 - \frac{2q}{k} x$$

as has been previously discussed several times. For this the abscissae x vary between the limits:

$$(441) \qquad b_o > x > 0$$

For the first (upper) part of the depression curve the plotting and drawing has to be done according to what has been said in ¶ 7 of Chapter I.

Graph No. 6



¶ 37. Barrages composed of several different materials.

The barrages which we have discussed above may also be composed of a number of different materials. Such cases, as we shall soon see, can be transformed into cases for which the material of the structure is uniform, if we use the same "virtual method" which was discussed in Chapter III. Let us consider several different cases.

(1) In Fig. 55 a barrage with lower wedge consisting of two parts is shown. The percolation coefficients of these two parts are equal to k_1 and k_2 respectively. Let us for example assume that:

$$k_1 < k_2 \tag{442}$$

and let us transform the problem into one for which we have a uniform material which has the percolation coefficient of k_2. With this aim in mind we find at first (corresponding to what has been said in Chapter III (¶ 18 and 19)) the coefficient:

$$\sigma = \frac{k_2}{k_1} \tag{443}$$

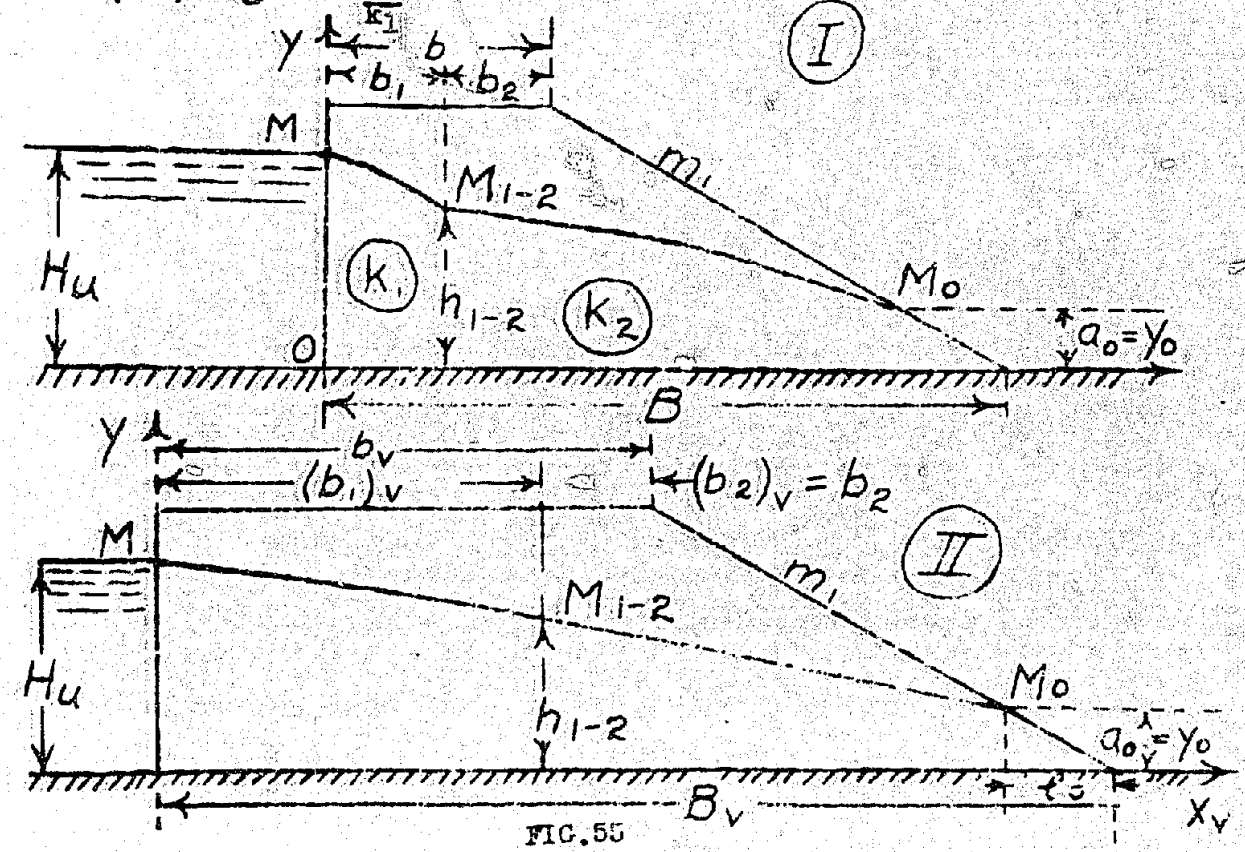


FIG.55

since this coefficient is necessary in order to determine several "virtual dimensions" of the barrage.

The given profile (Fig. 55, I) has the main dimensions:

(444) b; b_1; b_2; B; m_1, and the "virtual" dimensions which are denoted by the subscript v will be:

$$(445) \qquad (b_1)_v = \sigma \cdot b_1$$

$$(446) \qquad (b_2)_v = b_2$$

$$(447) \qquad b_v = \sigma \cdot b_1 + b_2$$

We can from these last values design the profile of the "equivalent" barrage, for which of course the slope m_1 remains unchanged (see Fig. 55, II).

For the resulting "equivalent" barrage we now determine the hydraulic and percolation quantities y_o, l_o and $\frac{q}{k}$ and q as shown above. All those quantities can, without any change, also be applied for the original profile for which we did not have a uniform material as has been shown in Chapter III. But we have to consider that for the given barrage the shape of the depression curve in the original profile will be different from that in the "equivalent" profile. We shall therefore take up this question for the design as shown in Fig. 55.

For the equivalent barrage the equation for the depression curve is:

$$(448) \qquad y^2 = H_u^2 - \frac{2q}{k_2} x_v$$

and hence the depth h_{1-2} in the division point M_{1-2} (see Fig. 55) is equal to:

$$(449) \qquad h_{1-2} = \sqrt{H_u^2 - \frac{2q}{k_2}(b_1)_v} = \sqrt{H_u^2 - \frac{2q}{k_2}\sigma b_1} = \sqrt{H_u^2 - \frac{q}{k_1} b_1}$$

so that we can directly plot the point M_{1-2} of the depression curve on the profile of the given barrage.

The abscissae x_v of the equivalent barrage and the abscissae x of the actual barrage are obviously connected by the following relations:

1) for the branch $M \div M_{1-2}$, i. e. for $x \leqq b_1$:

(450) $$x_v = \sigma \cdot x$$

2) for the branch $M_{1-2} \div M_o$, i.e. for $x \geqq b_1$:

(451) $$x_v = \sigma \; b_1 + (x - b_1)$$

If we now introduce (450) and (451) into (448) the equation of the depression curve of the given barrage has the following form:

1) for the branch $M \div M_{1-2}$

(452) $$y^2 = H_u^2 - \frac{2q}{k2}\sigma x = H_u^2 - \frac{2q}{k_1} x$$

2) for the branch $M_{1-2} \div M_o$:

(453) $$y^2 = h_{1-2}^2 - \frac{2q}{k2}(x - b_1)$$

The formulas (452) and (453) can for the given example also be obtained directly, i.e. without using the "virtual method." But the latter is necessary in order to find the basic unknown quantities (y_o, l_o, q) as has been pointed out above. Furthermore we can add that if the depression curve is plotted point by point for the "equivalent barrage" from the equation (448), this depression curve can in a simple manner, also be plotted for the given barrage without using equation (452) and (453) by merely using the "method of the distorted scale," as it was discussed in Chapter III. For the given example in the branch M-M_{1-2} the ordinates of the "equivalent" depression curve remain unchanged while its abscissae have to be correspondingly "compressed," i.e. they have to be diminished at the ratio of σ. The part $M_{1-2} - M_o$ of the "equivalent" depression curve on the other hand, is without change transferred to the given profile.

(2) Let us now take the case of a rectangular barrage which is composed of three different materials (see Fig. 56) which have coefficients of percolation of k_1; k_2; k_3 respectively. Since such a case

has already been discussed in ℙ 18 in the general discussion of the "virtual method" we shall in this paragraph only give the dimensions of the "equivalent" barrage using the notations of Fig. 56, and assuming that

$$k_1 < k_2 < k_3 \tag{454}$$

We reduce the problem to a barrage of uniform material with a percolation coefficient of k_1 and introduce the notations:

$$\sigma_2 = \frac{k_2}{k_1}$$
$$\sigma_3 = \frac{k_3}{k_1} \tag{455}$$

The "virtual" dimensions which obtain are:

$$(b_1)_v = b_1 \tag{456}$$

$$(b_2)_v = \sigma_2 \cdot b_2 \tag{457}$$

$$(b_3)_v = \sigma_3 \cdot b_3 \tag{458}$$

$$B_v = b_1 + \sigma_2 b_2 + \sigma_3 b_3 \tag{459}$$

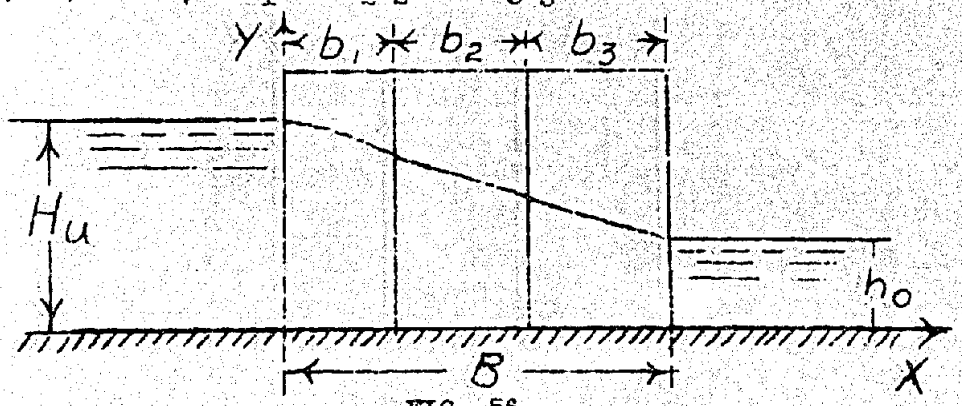


FIG. 56

We thus come to the "equivalent" rectangular barrage of uniform material with a width at the base of B_v. For such a rectangular barrage we determine the quantity $\frac{q}{k}$ from the known equation (10) and we obtain:

$$(460) \qquad \frac{q}{k_1} = \frac{H_u^2 - h_o^2}{2B_v}$$

where H_u and h_o are the depths of the head water and the tail water respectively. If $h_o = 0$ we have:

$$(461) \qquad \frac{q}{k_1} = \frac{H_u^2}{2B_v}$$

We find the percolation discharge from (460) or (461) knowing the percolation coefficient k_1. We then plot the depression curve in the "equivalent" barrage for the coordinate axis Ox_v and O_y from the equation:

$$(462) \qquad y^2 = H_u^2 - \frac{2q}{k_1} x_v$$

From this we can easily transfer to the depression curve for the given profile by using the "method of the distorted scale" (Ф 18). We show the shape of this depression curve on Fig. 56, without going into the details of the computation.

(3) Let us at last regard a barrage with upper wedge, which again is composed of three different materials (as shown in Fig. 57.) The percolation coefficients of the three parts are k_1; k_2 and k_3 respectively, and we furthermore assume that:

$$(463) \qquad \begin{cases} k_1 > k_2 \\ k_2 < k_3 \end{cases}$$

and reduce the problem to one for a homogenous barrage the material of which has a percolation coefficient of k_1 (i.e. that of the material of the upper wedge). Using the notations of (455) we obtain the

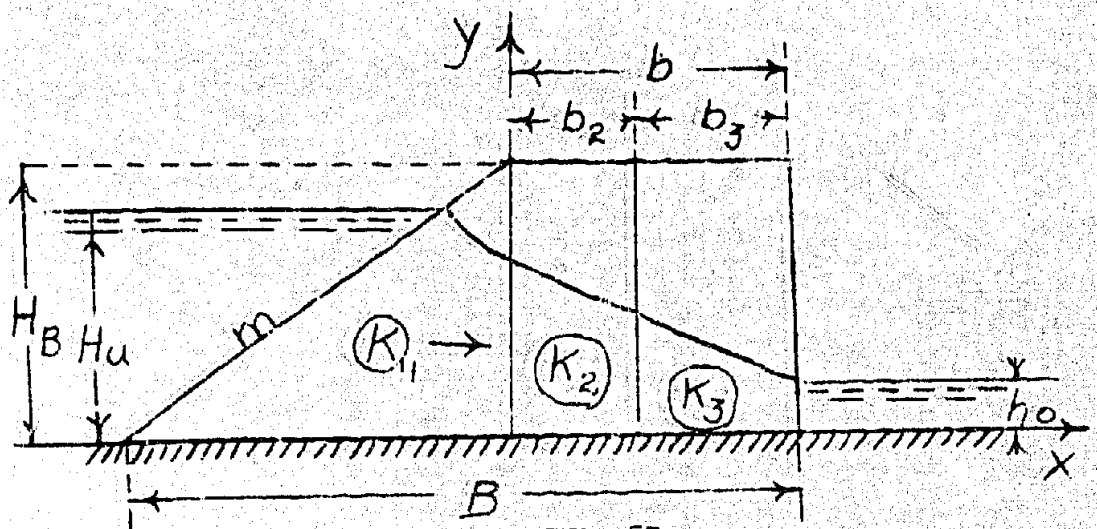


FIG. 57

following virtual dimensions of the barrage (see Fig. 57):

(464) $b_v = (b_2)_v + (b_3)_v = \sigma_2 b_2 + \sigma_3 b_3$

(465) $B_v = b_v + mH_B$

From these dimensions we can design the profile of the "equivalent" barrage (retaining the upstream slope (m) unchanged), and for it we can obtain the percolation quantities etc. in the same manner as shown in ¶ 36. The transfer from the "equivalent" barrage to the given is made in the same manner as described under case (1) of this paragraph.

¶ 38. Barrage with drainage system.

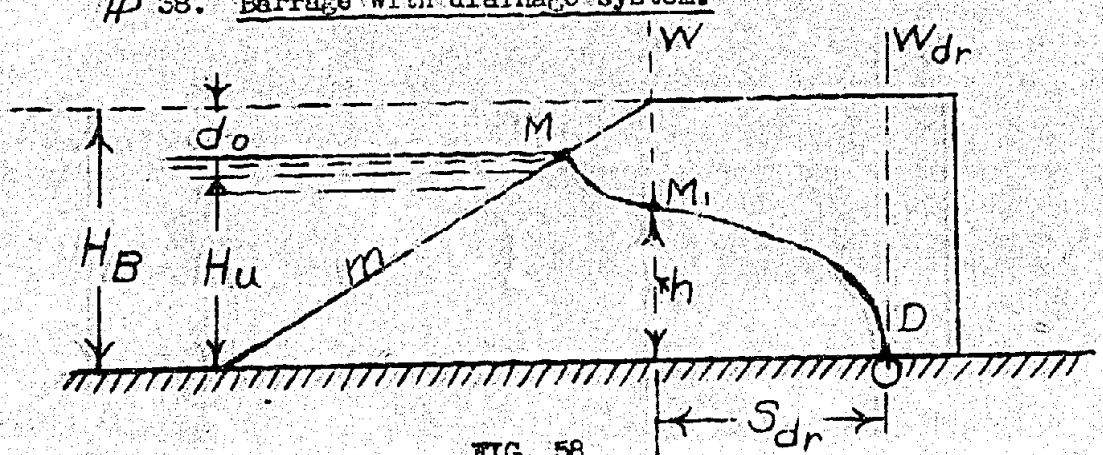


FIG. 58

We now turn to barrages with drainage system and shall take up two different cases:

(1) Barrage with upper wedge and drainage system D as shown in Fig. 58. If we compare this case with the case of drainage in an earth dam as given in Fig. 38 (Chapter V) we see that in regard to percolation there is no difference between the two. Therefore we can use what has been said about the construction of the depression curve in ¶ 29. Without any change also for the present problem. The notations used on Fig. 58 have the same meaning as those used on Fig. 38. The height of the barrage is, as has been done already in the preceding paragraphs, denoted by H_B.

(2) We now shall take up the case of a barrage with lower

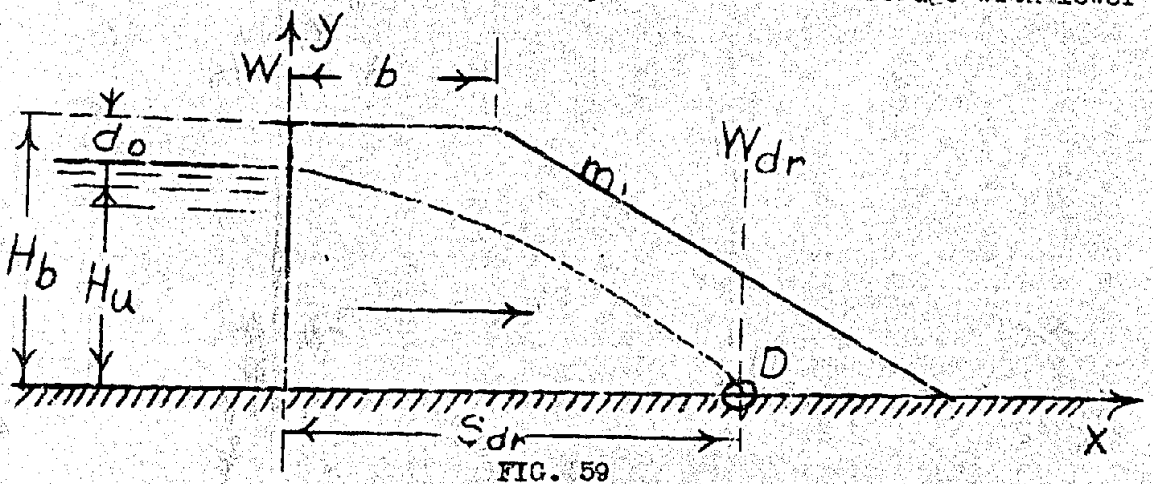


FIG. 59

wedge and drainage system as shown in Fig. 59. We see that the lower wedge has no meaning in regard to percolation, since the water is collected in the drainage pipe before it can go through the downstream wedge. Due to this fact the analysis of the percolation through this barrage is very much simplified.

From equation (10) we have for the non uniform flow of underground water on a horizontal lower layer a value of $\frac{q}{K}$ given by the equation:

$$\frac{q}{k} = \frac{H_u^2}{2s_{dr}} \tag{466}$$

We assume, as is shown in Fig. 59 that the depth of the percolation flow above the drain pipe (in the section W_{dr}) is zero:

(467) $h_{dr} \cong 0$

and we obtain $\frac{q}{k}$ directly from (466) and knowing k also q.

If $h_{dr} \neq 0$ we find from the same equation (10) that:

(468) $$\frac{q}{k} = \frac{H_u^2 - h_{dr}^2}{2s_{dr}}$$

in which h_{dr} may or may not be known beforehand.

The first case (i.e. h_{dr} known beforehand) can exist for example if water is below the barrage (tail water) and since we can neglect the hydraulic resistance of the drainage we have for this case:

(469) $h_{dr} = h_o$

(no special Fig. given.)

In this h_o is the depth of the tail water, directly downstream from the barrage. From equation (468) we can now directly obtain the value of $\frac{q}{k}$ and q.

The second case (i.e. h_{dr} not known beforehand) may exist if we want to calculate the hydraulic resistance of the drainage of one or the other type of barrage design. In this case we have to solve the problem in the same manner as has been shown in Chapter V for analogous cases of earth dams (see ¶ 29, 30 and 31). Of course the solution will be simplified due to the fact that for the present case no upper wedge has to be taken in account. We have nothing to add to the discussion in the above mentioned paragraphs. After having found h_{dr} we then can determine $\frac{q}{k}$ and q from equation (468).

The depression curve for barrages with lower wedge and drainage system (see Fig. 59) is also computed from equation (10) which for the present case has the form:

(470) $$y^2 = H_u^2 - \frac{2q}{k} x$$

in which x can be given different values within the limits:

$$(471) \qquad s_{dr} \geqq x \geqq 0$$

¶ 39. Remarks to the percolation of water through earth dams under the condition of a lowered head water level.

We shall close the present chapter (and with it really the whole discussion of our analysis of percolation of water through earth dams) with some remarks which, while they have no immediate bearing on the subject of this chapter, are of some significance for several parts of this book.

Footnote: The following chapters VII and VIII can by their general nature be regarded as a conclusion since no further new cases of percolation of water through earth dams are discussed in them.)

The question which will be discussed below is the percolation of water through earth dams when the head water does not have its full depth.

In Chapter I it has been shown that we lay the division section (W) between the upper wedge and the middle part of the earth dam through the upstream corner of the dam top (Fig. 5) if we want to compute the depression curve for given head water depth H_u. This depth of course has to be calculated from the conditions of river site etc., and from it the additional height of the dam d_o is determined (see Fig. 5). But it may happen that for some reason we have to compute the depression curve for the case that the head water does not have the depth H_u but only the depth H_1, i.e.

$$(472) \qquad H < H_u$$

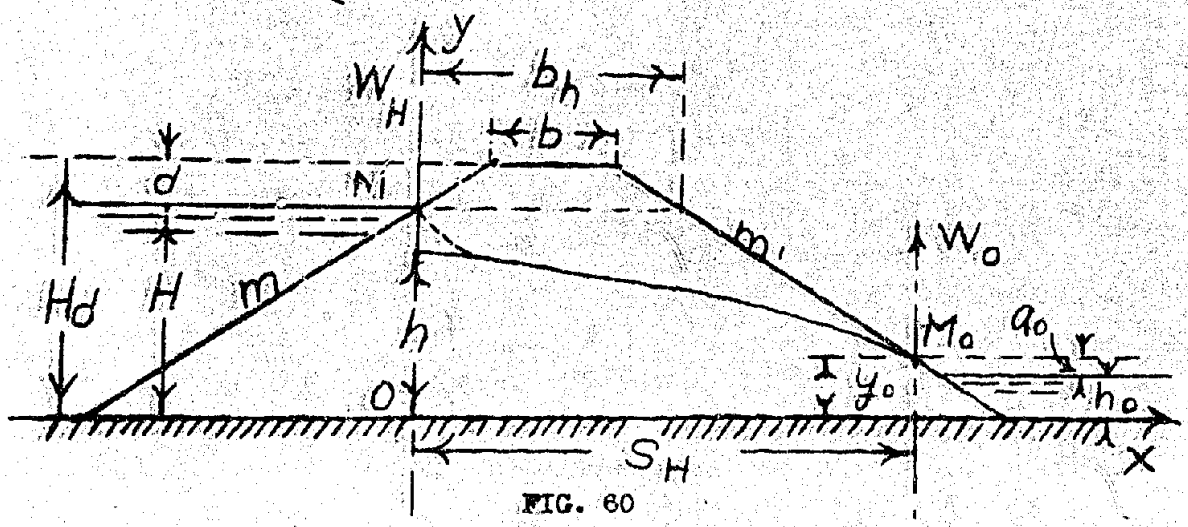


FIG. 60

For this case we have to notice that the point where the head water level line intersects the upstream face is more or less far removed from the line W, or, what is the same, the line W is removed downstream in regard to the point M. Let us remember what has been said (in ¶ 9) about the influence of the location of the division section W on the elements of the depression curve. We notice that for the given case (i.e. $H > H_1$) it will be safer to lay the division section with "a certain storage margin" not through the upstream corner of the dam top, but through the intersecting point of the head water level and the upstream slope of the dam. Such a location of the division section is shown in Fig. 60 as W_H.

We can add to this that also for a full depth of the head water, in the case than an especially high degree of safety of the dam is demanded, the height d_o has to be selected much greater than for the ordinary design. In such a case it is also advisable to use a location of the division line like in Fig. 60 (W_H) and not like in Fig. 5 (W) if the depression curve shall be computed.

We shall now set up the system of percolation equations for a homogenous earth dam under the conditions of lowered head water level H (Fig. 60). It is obvious that equation III of the system (41) remains unchanged. Equation II is changed (using the notations of Fig. 60) into:

$$\frac{q}{k} = \frac{H^2 - (a_o + h_o)^2}{2s_H} = \frac{H^2 - y_o^2}{2s_H} \tag{473}$$

Equation IV, which is obtained from purely geometrical relations is for the given case:

$$s_H = b_H + m_1(H - y_o) \tag{474}$$

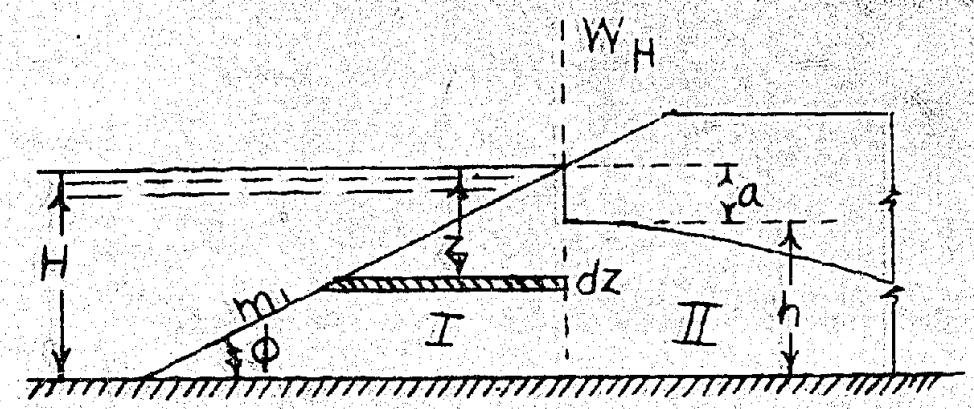


FIG. 61

in which b_H is the width of the dam at the elevation of the head water level at depth H; b_H is known and equal to:

$$(475) \qquad b_H = b + (m + m_1)d$$

using the notations of Fig. 60.

It remains to set up equation I for the present case, and we shall do this in a somewhat more detailed manner than was used in ℙ 36. For this purpose we consider an arbitrary flow element (see Fig. 61) within the region of the upper wedge, and using the same line of reasoning as in ℙ 4 we only have to observe that the length of the flow element for the present problem is expressed in a form which is slightly different from that of ℙ 4.

Footnote: It is necessary to go back to ℙ 4 and remember the derivation given there.

The length of the separate flow element (Fig. 61) is equal to:

Footnote: The notations z, m, ϕ, a, h which are shown in Fig. 61 can also be understood from what is said in Chapter I.

$$(476) \qquad z \cot\text{an}\, \phi = mz$$

The corresponding hydraulic gradient is:

$$(477) \qquad J = \frac{a}{mz}$$

and hence the velocity of percolation:

$$(478) \qquad U = k \cdot J = \frac{k \cdot a}{mz}$$

The elementary percolation discharge for this is:

$$(479) \qquad dq = Ud_w = \frac{ka}{mz}\, dz$$

and the total discharge:

$$(480) \qquad q = \int_a^H \frac{k.a}{m.z}\, dz = \frac{k.a}{m} \ln \frac{H}{a}$$

whence

$$(481) \qquad \frac{q}{k} = \frac{a}{m} \ln \frac{H}{a}$$

or since

$$(482) \qquad a = H - h$$

$$(483) \qquad \frac{q}{k} = \frac{H - h}{m} \ln \frac{H}{H-h}$$

¶ 40. Continuation.

As we see from the preceding paragraph the system of the four percolation equations for the case of lowered head water level is:

$$(484) \qquad \begin{aligned} &\text{I} \quad \frac{q}{k} = \frac{H - h}{m} \ln \frac{H}{H - h} \\ &\text{II} \quad \frac{q}{k} = \frac{H^2 - (a_o + h_o)^2}{2s_H} \\ &\text{III} \quad \frac{q}{k} = \frac{a_o}{m_1}\left[1 + \ln\left(\frac{a_o + h_o}{a_o}\right)\right] \\ &\text{IV} \quad s_H = b_H + m_1\left[H - (a_o + h_o)\right] \end{aligned}$$

If there is no water downstream from the dam ($h_o = 0$) the corresponding system of percolation equations is:

$$\text{I} \qquad \frac{q}{k} = \frac{H - h}{m} \ln \frac{H}{H - h}$$

$$\text{II} \qquad \frac{q}{k} = \frac{H^2 - a_o^2}{2s_H}$$

(484')

$$\text{III} \qquad \frac{q}{k} = \frac{a_o}{m_1}$$

$$\text{IV} \qquad s_H = b_H + m_1 (H - a_o)$$

in which we have to remember that a_o is the height of the point at which the depression curve leaves the downstream face above the horizontal plane of the foundation.

The solution of the obtained systems of percolation equations (484) and (484') is clear from the discussions given in Chapters I and II. The same is true for the "correction" of the upper part of the depression curve which is shown by a dotted line on Fig. 60 (a discussion of this part of the depression curve is given in ¶ 7).

Since the case of an earth dam with core wall can be reduced to the case of an "equivalent" homogeneous earth dam (Chapter III), it will be clear (from the above said), how we can solve the percolation problem for an earth dam with core for the condition of a lowered head water level.

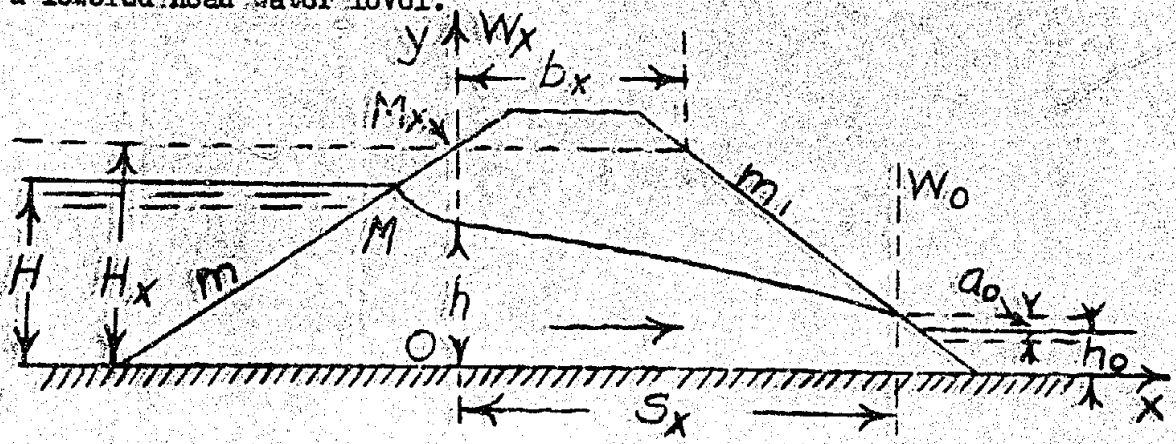


FIG. 62

We shall conclude this paragraph by discussing the "intermediate" location of the division section, i.e. the division section is located between the position of W and W_H. This "intermediate" location is shown on Fig. 62 and denoted by W_x. The vertical line W_x is laid through the point M_x (not M) which is somewhat above the head water level elevation of the lowered head water. This arrangement is more like that which was chosen for the general discussion of percolation in the upper wedge presented in ¶ 4. The vertical W_x is as compared to W_H moved somewhat downstream which was also true for the vertical W which was, in the same sense, somewhat removed from the vertical through the intersection point of the head water level line and the upstream face. The point M_x can for example be so chosen that the vertical distance between M and M_x (see Fig. 62) is about 10 ÷ 15% of the depth H.

It is obvious that instead of equations (41) we obtain (for $h_o > 0$) the following system of percolation equations: (see Fig. 62).

$$\text{I} \qquad \frac{q}{k} = \frac{H - h}{m} \ln \frac{H_x}{H_x - h}$$

$$\text{II} \qquad \frac{q}{k} = \frac{h^2 - (a_o + h_o)^2}{2s_x}$$

(485)

$$\text{III} \qquad \frac{q}{k} = \frac{a_o}{m_1}\left[1 + \ln\left(\frac{a_o + h_o}{a_o}\right)\right]$$

$$\text{IV} \qquad s_x = b_x + m_1\left[H_x - (a_o + h_o)\right]$$

and for $h_o = 0$

$$\text{I} \qquad \frac{q}{k} = \frac{H - h}{m} \ln \frac{H_x}{H_x - h}$$

$$\text{II} \qquad \frac{q}{k} = \frac{h^2 - a_o^2}{2s_x}$$

(485')

$$\text{III} \qquad \frac{q}{k} = \frac{a_o}{m}$$

$$\text{IV} \qquad s_x = b_x + m_1 (H_x - a_o)$$

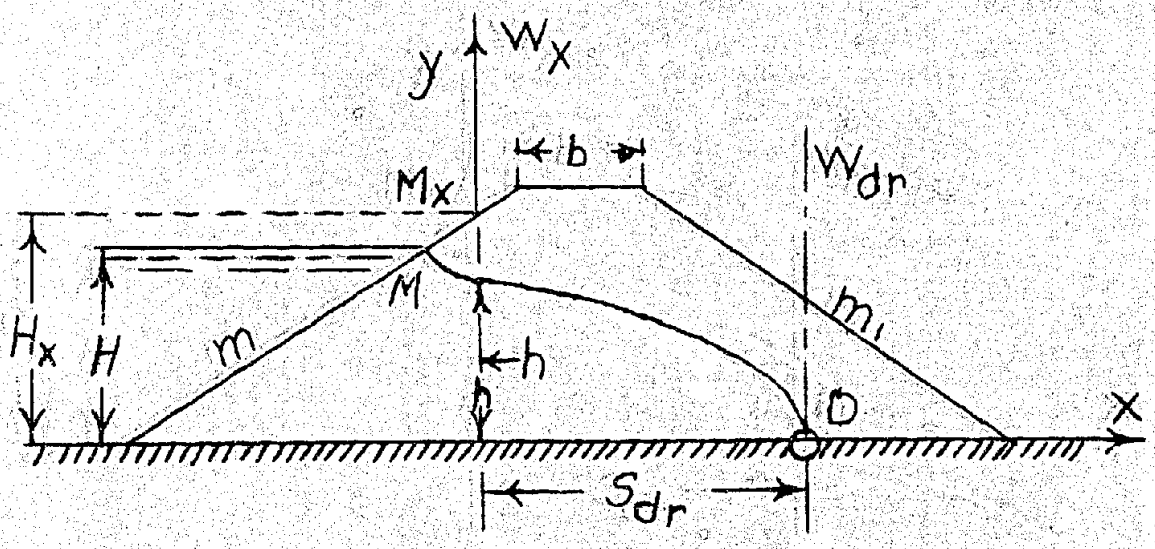


FIG. 63

The "intermediate" location (η_{π}) of the division section can also be applied for earth dams with cores and earth dams with drainage system under the condition of a lowered head water level. For the dam with drainage (Fig. 63) we obtain instead of equations (328) a system of two percolation equations:

$$\text{I} \qquad \frac{q}{k} = \frac{H - h}{m} \ln \frac{H_x}{H_x - h}$$

(486)

$$\text{II} \qquad \frac{q}{k} = \frac{h^2}{2s_{dr}}$$

CHAPTER VII

41. Method of representing the depression curve as a straight line.

In this chapter we shall discuss the methods of determining the depression curve which were recommended by other authors. It seems justified to analyze these methods on the basis of the hydraulic theory of the whole question which was developed in the previous chapters. Let us at first take up a method which up to a recent time has been widely used in America as well as in S.S.S.R. and is in fact even at the present time often used in projecting earth dams. Since it is impossible to connect this method with the name of one particular author we shall call it "method of the straight line depression curve."

This method was developed in a number of Anglo-American handbooks for the construction of earth dams and was taken over into the Russion hydraulic literature and practice without any changes. The method is based on the following condition--or rather hypothesis: In a homogeneous earth dam on an impervious foundation the "line of saturation," i.e. the line of the percolation flow is a straight line, the inclination of which depends only upon the nature of the material of which the dam body is made. The inclination of this line increases with the degree of imperviousness of the material.

Footnote: This is exactly how this form is used (line of saturation) in the well-known handbook: Creager and Justin "Hydro-Electric Handbook, New York, 1927, p. 223; however the method of the straight line depression curve itself is not explained in this handbook (about this see further below ¶ 42.)

In the American (and hence also in our) practice the slope of the "line of saturation" used for projecting earth dam structures is usually assumed to be:

$$i = 0.35 \div 0.20 \qquad (487)$$

according to a higher or lesser degree of imperviousness of the dam material.

Hence for a given dam profile the "line of saturation" can have different positions. (See Fig. 64).

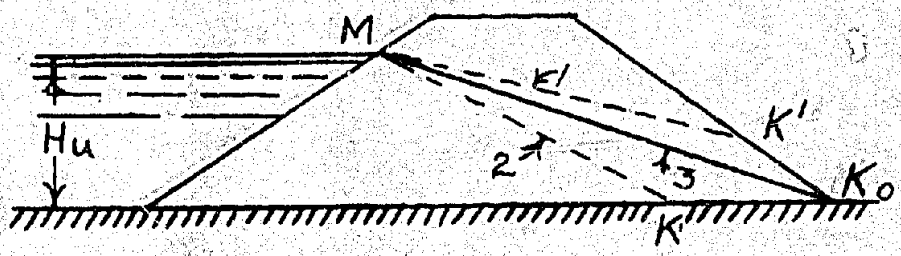


FIG. 64

(1) Position M-K', for which the downstream face is intersected at the point K'.

(2) Position M-K, for which the dam foundation is intersected at the point K.

(3) An intermediate position M-K_o (special case) for which the "line of saturation" goes through the point K_o. In other words, for one and the same dam profile all three characteristic positions of the "line of saturation" are possible, and only the nature of the material of the dam determines which of the three cases will exist in the given dam structure. The position M-K is regarded as the most desireable position of the "line of saturation" and seems to be necessary in order to guarantee the safety of the dam.

From this point of view the conditions of imperviousness of the dam can be "verified," in a very simple way: selecting arbitrarily the profile of the dam structure we draw a straight line having the slope (i) through the point M (Fig. 64). The slope is given according to the material used for the structure; we can thus easily see which of the three characteristic cases exists for the structure. If the line intersects the downstream face we say that this condition is not acceptable and we can according to the above described method get out of the difficulty in the most simple manner: all we have to do is to add a backfill to the downstream face (Fig. 65 I), or to make the slope coefficient of the downstream face greater than m_1 ($m_{11} > m_1$) see Fig. 65 II. If we do this the line of saturation is already "within" the dam profile and does not intersect the downstream face. Thus we add to the original dam profile the parts which are shown in Fig. 65, as the shaded areas.

After having pointed out the characteristic features of the "method of the straight line depression curve" we shall discuss a general comparison between this method and the theory developed in this book. We shall limit our discussion to homogeneous earth dams, since

the method of the "straight line depression curve" was suggested just for that type of earth dam.

Footnote: On the attempts of using this method for dams with a core wall we shall say a few words in ¶ 43.

The hydraulic method of analyzing the percolation flow in earth dams, which was developed by us, leads to the following conclusions (see ¶ 7): The shape of the depression curve is independent independent of the percolation coefficient of the material of the dam (i.e. it does not depend on the porosity properties of the material). The shape of the depression curve is determined by the geometrical and hydraulic elements of the dam profile (43), which enter the equations of systems (41) and 41') as the best known quantities. From these systems of equations we see that the depression line is curved and not straight.

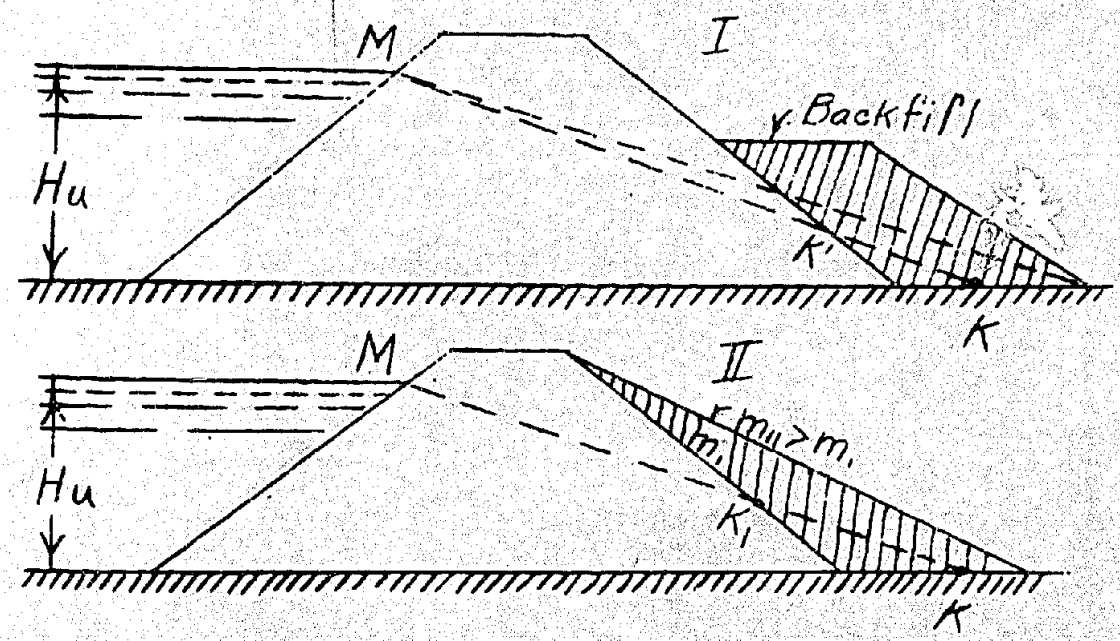


FIG. 65

Let us see now how the percolation coefficient influences the elements of the percolation flow. Providing we have found q^k from equations (41) or (41') we shall call it "the computed discharge". Using the notation

(488) $q_r = \frac{q}{k}$ we find that

(489) $q = k \cdot q_r$

that means, the percolation discharge (q) changes (for given hydraulic boundary conditions) proportionally to the value of the percolation coefficient (k.) On the other hand we can point out that by the method of the "straight line depression curve" the percolation discharge can not be determined at all, since it is not possible to say at what depth of the percolation flow one can write the general equation for the discharge $q = w \cdot V$ (the depths along the flow according to this method change from H_u in the point M to zero in the point K or to some finite value in the point K', see Fig. 64).

Thus the fundamental conclusions which we drew from our hydraulic method of solving the percolation problem in earth dams are contradicting the basic conditions of the "method of straight line depression curve."

¶ 42. Discussion of the "straight line depression method."

Since the method of the "straight line depression curve" does not have any theoretical basis which could be compared with our own basic theory, we have to examine said method mainly from the point of view of the logic of the hydraulic reasoning.

(1) We see from Fig. 64 and 65 that the cross sections available to the percolation flow decrease (in the downstream direction) and hence the velocities V must increase. But the increase of velocities demands also an increase of the hydraulic gradients along the flow while according to the given method we have a straight depression line which from the hydraulic point of view should result in a constant velocity of percolation V along the whole percolation flow. Thus we can find a hydraulic inconsistency already in the basic assumption of a straight depression line. From this it becomes clear that the depression line ("line of saturation") in a homogeneous earth dam cannot be straight.

(2) A still more greater logic impossibility is contained in the following "idea" of the method here discussed. Let us pick out that position of the depression line which for the discussed method is regarded as the most advantageous and even necessary for the stability of the dam--the position M-K (Fig. 64). For this it is quite justified to ask oneself the question: When the percolation flow has reached the point K, what happens to it afterwards? The flow cannot be "swallowed" in this point K, since there is no drainage at that point and the foundation of the dam is supposed to be impervious, but the flow can also not move onward, since according to the assumption of the discussed method the straight line M-K represents the whole flow (Fig. 64). But this line "ends" for some mysterious reason in

the point K and beyond the point K the "dry" part of the profile begins (the main characteristic of the line M-K is that it does not go through" the downstream face of the dam and does not even "approach" it). Thus the question asked above is not answered if we stay within the boundaries given by the method of the straight line depression curve.

In reality the percolation flow will not behave quite as politely as the authors of this method want it: After having reached the point K the flow has to move on in the only possible direction--the direction towards the tail water, with an outlet on the downstream face of the dam. There it actually discharges, and the "point of discharge" of the depression curve has been determined by us in Chapter I. The position of the depression line M-k is simply impossible--that is the only answer to the question asked above.

(3) In the discussion of the last point we had assumed that the depth of the water directly below the dam was zero ($h_o = 0$). Let us now take the case when $h_o > 0$ (Fig. 66). If the material of the dam is comparatively impervious the "line of saturation is steep,

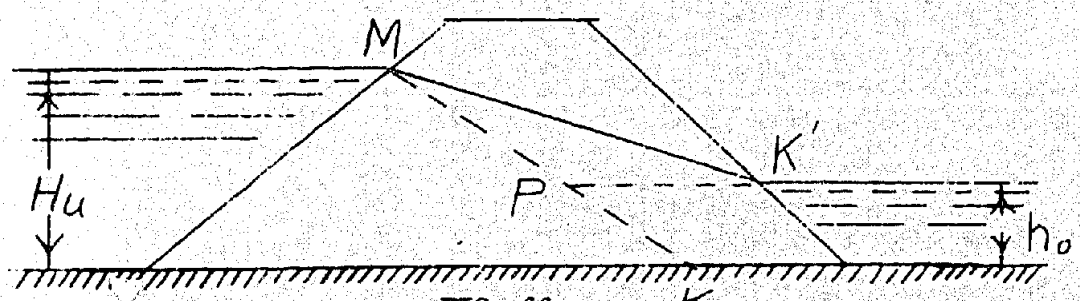


FIG. 66

according to the adopted method. It will have the position M-K which is shown for $h_o = 0$ on Fig. 66. But if we now assume that for the same dam (i.e also for the same material of the dam) the tailwater has risen for some reason to a depth $h_o > 0$, it seems that any engineer will draw this "straight line of saturation" to the point K'. That is this line will be less steep than the line M-K although the soil of which the dam is made has not been changed. Doing this we find that only due to the rise of the tailwater to a depth h_o the percolation properties of the dam material have for some unknown reason become decidedly worse.

Of course there do not exist such reasons, and the above said again only reveals the logic inconsistencies of the here discussed method.

These inconsistencies also cannot be eliminated for a different manner of drawing the straight depression line for the case of $h_o > 0$: If we draw the line M-P (see Fig. 66) for the upper and middle part of the profile in accordance with the percolation properties of the material, and from P on the horizontal straight line P-K', which is on the elevation of the tail water level the question arises: How is it possible that the percolation flow can move to the right from point P since the hydraulic gradient there is zero? It is obvious that such a movement is not possible. If we draw the line M-P-K' in such a manner that the point P is slightly higher than the elevation of the tailwater level (i.e. so that the piece P-K' may have some slope) the line of saturation has a break at the point P, for which in a homogeneous earth dam no explanation whatever can be given. Finally if we could have a straight saturation line M-K (Fig. 67) then for $h_o > 0$ we would with the same right be justified to draw also a straight line of saturation A-B from the downstream side. If these two saturation lines would have a somewhat less steep slope they might intersect at a point C. But it is obvious that a construction of lines of saturation according to Fig. 67 are contrary to common sense; thus they can also not get us around the above pointed out discrepancies.

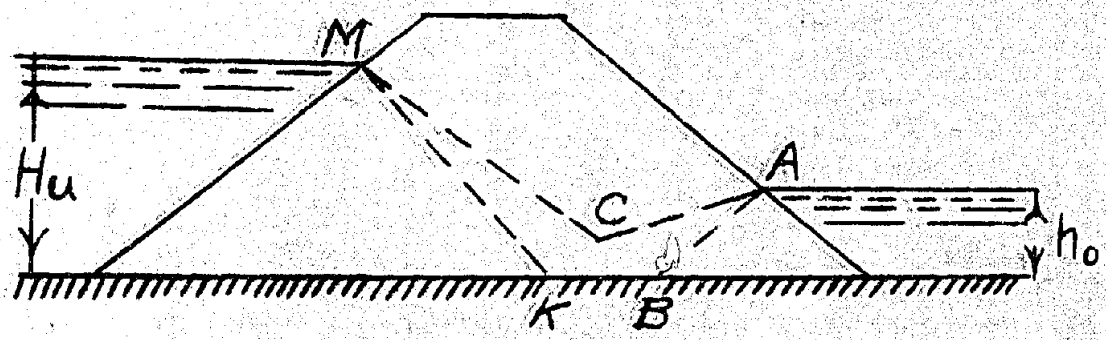


FIG. 67

Hence we see that also for the case $h_o > 0$ we cannot find a satisfactory answer to the critical remarks brought out against the method of the "straight line of saturation."

(4) In paragraphs (2) and (3) we have shown that depression lines of the type M-K (see Fig. 64 and 66) are not possible in a homogeneous earth dam without a drainage system. In paragraph (1) we have furthermore shown why in such an earth dam the depression curve cannot be a straight line at all. From this we then obtain the following rather compact formulation: <u>According to the method discussed here none of of the three characteristic positions of the depression lines shown in Fig. 64 is impossible.</u>

The above given remarks to the method of the "straight line of saturation" are rather simple and elementary, but the discussed method can not even stand such slight criticism, and we have to say that in point K (Fig. 64) not only the "individual saturation" "comes to an end," but also the last remains of logic of the authors and supporters of this method.

In concluding the present paragraph it is interesting to notice that not only in the Russian but also in the foreign (Anglo-American) hydraulic literature we do not find any sufficient amount of criticism of the method of the "straight line of saturation" from a generally hydraulic point of view; some serious handbooks of most recent date simply avoid mentioning this method. We shall shortly dwell upon two of them. Of course there is no necessity of discussing those handbooks and publications which accept the method discussed here without any reservations.

At first let us mention the well known encyclopedia of Parker (Parker: "The Control of Water," 2nd edition London, 1925). In the chapter on dams he discusses (Chapter VII (Section A), p. 328) the profile of one of the dams (depth of water 46'; m = 4 for upstream face and m = 2 for downstream face), and does not object to the slope of the depression line i = 1/6 which was used as the basis for the design, but says that it would be better either to mutually interchange the slopes of the upper and lower face and make m = 2 and m_1 = 4 or to make both slopes equal $m = m_1 = 3$. Thus, Parker points out the depression line with the slope 1/6 would be run much lower than the point K_0 (see Fig. 64) i.e. in other words it would intersect the base line of the dam like the saturation line M-K. We see that in this case apparently even Parker is far from being free from a direct influence of the "method of the straight line of saturation."

In the handbook of Creager and Justin which we shall shortly mention ("Hydro-Electric Handbook," N. York, 1927) the author of the chapter on earth dams (Justin) almost exclusively deals with the calculation and construction of earth dams on a pervious foundation and does not mention the "method of the straight line of saturation." But giving a schematical (Chapt. XIII, pp. 227-228) design of an earth dam on an impervious foundation he points out that if the material of the dam is but little pervious, the saturation line may intersect the downstream face at a point which is considerably higher than the point K_0 (see Fig. 64). The author adds that "such a dam is most probably destroyed," and as a remedy against this he recommends the construction of a concrete core in the dam body, and also mentions the effect of a carefully planned and constructed drainage system.

We shall not dwell upon the two latter preventive means since their effectiveness from a technical point of view is beyond any doubt, but we want to underline the fact that Justin speaks of a rise of the point K' (see Fig. 64) in connection with a less pervious material of the dam body. This remark is as we have seen contrary to the basic assumption of the method of the straight line of saturation. Based on the theory of percolation flow which we developed in this book we can neither agree with the remark of Justin as it stands nor with the conclusion which he draws from it in regard to the safety of the homogeneous dam of the given design. We believe that the short mentioning of the statements of this author illustrate the unclassified points which will arise in this question if we do not approach it from at least a hydraulic point of view.

43. <u>The practical application of the "method of the straight saturation line."</u>

The basic practical application of "the straight saturation line method" is clearly seen from Fig. 65, and from what has been said about that in ¶ 41. There it was shown how the safety of the dam structure in regard to percolation can be examined with the help of this method. The use of a backfill B and the enlargement of the downstream slope (lengthening of the profile in downstream direction) see Fig. 65 is meant, as is known, to provide for a dam profile in which the saturation line stays within the profile and does not intersect the downstream face. This condition is even said to be absolutely necessary in order to guarantee the durability of the dam. But in the preceding paragraph we have already pointed out the impossibility of the location of the depression line according to the straight line M-K in Fig. 64 and 65. And thus it is obvious that the recommended practical means for securing the "safe" design of the profile (backfilled and enlarged downstream slope) cannot possibly have the desired results.

Furthermore such a real significance is assigned to these means that the question about the "limiting height" of the earth dam of a given trapezoidal profile is distinctly asked and answered. We think it necessary to give a somewhat more detailed discussion of this fallacious idea.

If we take a trapezoidal profile of an earth dam (see Fig. 68) we see that if the depression line M-K "lies within the profile" (using the terminology of the discussed method)

$$\overline{NK} < \overline{NK_0}$$

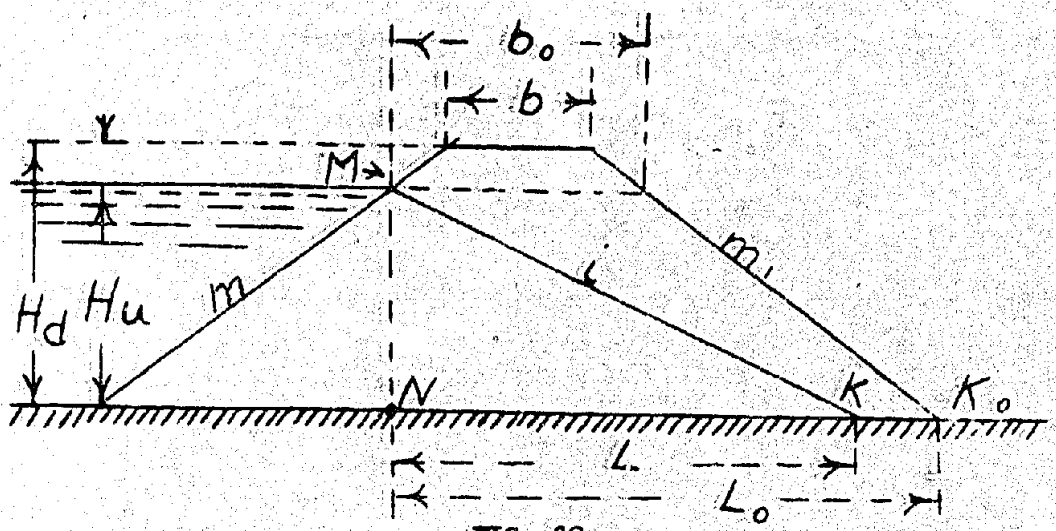


FIG. 68

or, if we use the notations of Fig. 68

(490) $L < L_o$

where

(491) $L = b_o + m_1 H_u$ and

(492) $L = \frac{H_u}{i}$

where i is the slope of the line M-K (which in the present case is determined as the ratio of the depth H_u to the length L).

With (491) and (492) 490 changes into

(493) $\frac{H_u}{i} < b_o + m_1 H_u$

from which

(494) $H_u < \frac{b_o}{\frac{1}{i} - m_1}$

that means the depth of the water upstream from the dam has to be

smaller than

$$(495) \qquad \frac{b_o}{\frac{1}{i} - m_1}$$

in order to satisfy the "durability condition" (490).

Footnote: It should be mentioned that in regard to a necessary (or at least desirable) length K-Ko (see Fig. 68) no indications whatsoever are given in the discussed method.

This condition is not satisfied if the depth H_u is equal to or greater than the value given in (495), and therefore the "limiting depth" in the discussed method is given by the expression:

$$(496) \qquad H_{u(limiting)} = \frac{b_o}{\frac{1}{i} - m_1}$$

From this depth we can also find the corresponding "limiting" height of the dam $H_{d(limiting)}$:

$$(497) \qquad H_{d(limiting)} = d_o + H_{u(limiting)} = d_o + \frac{b_o}{\frac{1}{i} - m_1}$$

For the illustration of this we have worked out a few numerical examples (table 21) for the following given conditions:

$m = m_1 = 2.5$;
$b = 5.0$ m;
$d_o = 1.0$ and 2.0 m;
$i = 1/3$; $1/4$; and $1/5$

for which (see Fig. 68):

$$b_o = b + (m + m_1)\, d_o = b + 5d_o$$

Table 21

	"Limiting height" of the dam in meters	
i	for d_o = 1.0 m	for d_o = 2.0 m
1/3	21.0	32.0
1/4	7.7	12.0
1/5	5.0	8.0

As we see from the table even for medium slopes (and more so for small) of the "depression straight line" the limiting heights of the dam become very small for the above given conditions. But we can by no means recommend these values which have been obtained by the method of the straight line depression for practical dam design. We want to emphasize that they are just as unfounded as the "method" itself and the same is true for the very idea of "the limiting height of the dam with a trapezoidal profile."

If we think of an application of the discussed method to earth dams with inner core and base this on the original idea of this method the construction of a depression line might be the following: From the intersection point of the water level line with the upstream face we draw a line with the slope ; i; within the region of the core the depression line drops rapidly by the value a_c (loss of head in the core); below the core the line is again parallel to the slope which has been chosen according to the porosity qualities of the material of the dam from (487) or by some other such means. In the lower part of the dam we thus obtain one of the three "discharge points" (K, K', K_o) which have been shown in Fig. 64 for the homogeneous dam.

This method, which is sometimes used for practical designing, but is not directly mentioned in serious handbooks does not contain any additional inconsistencies besides those which have already been mentioned in the above discussion of the application of the "straight depression line method" for homogeneous earth dams. We can only point out that the loss of head within the core can of course never be calculated by any definite and well established method as a certain percentage of the whole head within the dam. If therefore in

some hydraulic handbooks, we find that a core of "well rolled clay" or clay-cement has a head loss of 17 per cent, then this information includes a two fold mistake. In the first place the commission of experts which was formed by the administration of the Croton water development in New York determined the loss in the core of the dam at New Croton to be 17 per cent of the depth in the reservoir for a core

Footnote: See for example Bassell's book "Earth Dams" 1917, page 67

of "rock fill" and not "rolled clay" or "clay cement." Secondly it is obvious that the head loss in the core is dependent upon the thickness of the latter, the relation of the percolation coefficients of the core material and the dam material and from other conditions which have been discussed by us in Chapter III of this treatise. Only the application of the percolation equations gives us the possibility to make a hydraulic calculation of the head loss in the core of a dam for any given conditions, and as we have seen in Chapter III this calculation is quite simple.

The above said seems to be sufficient in order to convince the reader that the method of the straight saturation line is not applicable for the calculation of earth dams with cores, and, because of the same reasons, for earth dams with "protection layers" etc. The impossibility of giving even "average values" of the amount of head loss in the core of a dam and "protection layer" robs the discussed method for these cases even of the formal "correctness" which it has for the homogeneous earth dam where, no matter how wrong it may be, one still can construct the "straight saturation line."

If we now turn to the earth dam with drainage system we can only say that for this case the method of the straight saturation line is given another chance to disclose its logic defficiency. If for example for the dam without drainage the material of the dam is characterized by the straight depression line M-K' (see Fig. 69) then anybody who would design the dam with a drainage would draw the depression line according to the line M-D of Fig. 69, while the material in the dam has not been changed. We see thus, that when drainage is used, the dam material acquires quite unexpectedly different percolation properties, which are characterised by a steeper depression line (slope i). But this does not yet end the strangeness of this method; If no artificial drainage is present, and if the depth of the water below the dam is $h_c = 0$ such a dam has a "natural drainage" in the neighbor hood of the point N (the discharge point into the atmosphere); but for that case the straight depression line is not drawn to this point but either higher or lower, depending on the material of the dam.

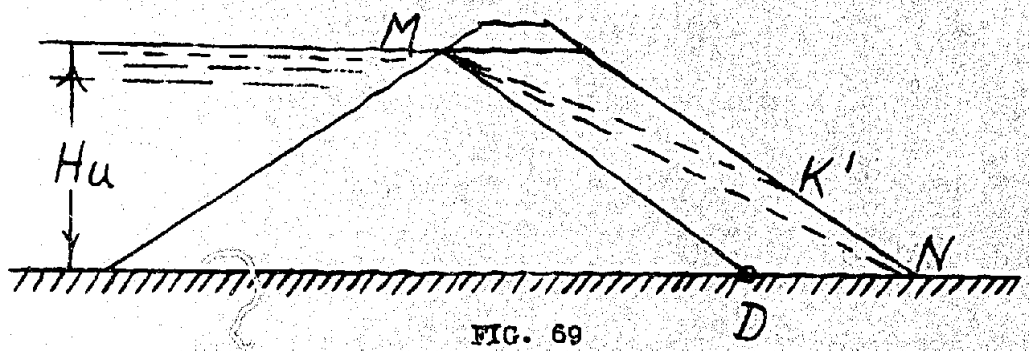


FIG. 69

In connection with the last question we have to say that we can by no means agree with Prof. N. J. Anisen who says:

Foornote: "Dams" second edition, page 21, 1928.

"In successfully working hydraulic structures one can assume that the saturation line has a slope which is steeper than that of the straight line MN." But in fact, if we have to talk about a "straight saturation line" then it can only have a slope which is "less steep" than the line M-N, since for a steeper saturation line we would obtain a line of the type M-K (Fig. 64), and we have above shown its impossibility.

44. Supplementary remarks. We shall now shortly take up a method which was sometimes (if not often) used by Russian designers of hydraulic structures, and which represents nothing but a slightly changes "method of the straight saturation line" and therefore can be discussed in this place. We think of the practical method which is based on the well known method of Bligh who developed it only for the percolation of water below hydraulic structures. By this method we find the sufficient width of the dam base B(see Fig. 70), if we regard only the lowest percolation layer A-A', from the expression:

$$B = c \cdot H_u \qquad (498)$$

where c is the known coefficient of Bligh.

The fact alone that Bligh developed this method for the movement of underground waters under pressure and below hydraulic structures, and not for the movement without pressure in the body of an earth dam is not sufficient to discard the above mentioned method: The latter could have been regarded as being correct if the method of Bligh itself would be corredt. But already in 1922 we have carefully

discussed Bligh's method and have shown its fundamental incorrectness and summarized our opinion in the following words:

Footnote: N. N. Pawlowsky "The Theory of the Movement of Underground Waters Under Hydraulic Structures" Edition of the Scientific Melcoration Institute 1922, p. 741.

"The hypothesis in regard to the 'path of percolation', which is used as the basis for the method (the whole method is called after this hypothesis 'the theory of the path of percolation' or 'theory of the contour line') is in its nature entirely arbitrary, physically--not explainable, matermatically--impossible, and hydromechanically--inconsistent; but the two central ideas which form the basis for the method (the line of the distribution of the heads (pressures) under the hydraulic structure, and the possibility of extending their known length of their underground contour at random) are theoretically as well as experimentally disproved."

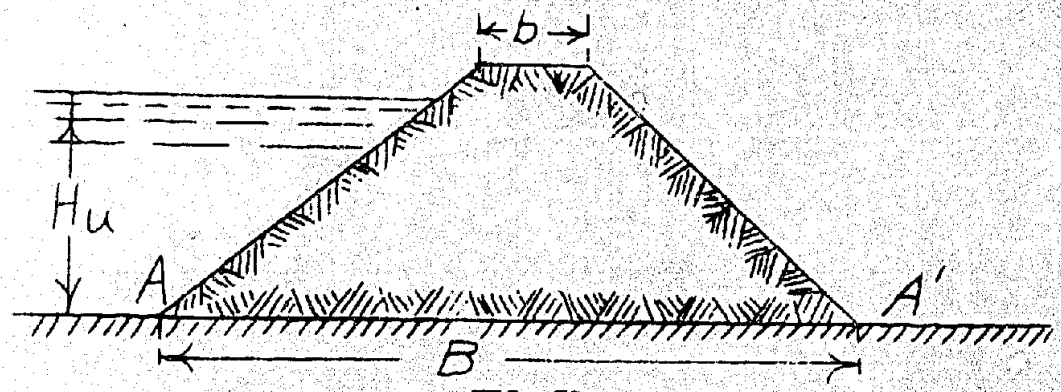


FIG. 70

After what has been said above we regard Bligh's method completely fallacious also if applied to earth dams, and do not think it is necessary to say any more in regard to this.

In connection with our above stated opinion of Bligh's method we have however to clarify a misunderstanding. Such a misunderstanding of this question may arise for the reader because in his book "Hydraulic Dams" N. J. Anisimow(2) refers the reader to one of our articles(3)

Footnote (2): "Hydraulic Dams" 2nd edition, Reprint NKPS, 1931, page 114.

Footnote (3): "On the Determination of the thickness of the Channel Bed; No. VI, of the Bulletin of the Scientific Melcoration Institute, 1923.

for the purpose of acquainting the reader with Bligh's method of determining the thickness of the channel. He does this in such unclear terms, that the reader may conclude that the author of this book is a supporter of Bligh's method. Therefore we find it necessary to point out here that in fact our discussion of the question in the above named paper is in no sense connected with Bligh's method (i.e. with his "theory of the path of percolation") nor with his formulas for the determination of the thickness of the tumbling basin slab, and therefore also can not serve as a means to get acquainted with the question of determining the thickness of the tumbling basin slab according to Bligh, neither from a historical nor from any other point of view.

That this is true will be easily seen if we remember what was said in the respective article. By the way already in the first paragraph is said: "Let us assume, that throughout the whole further discussion we shall regard the pressure distribution under the tumbling basin as already being determined by some method, since the below given relations can be applied for any existing law of the pressure distribution for example also for the straight line "law" (more correctly--hypothesis) of Bligh or a more rough assumption of a constant pressure under the tumbling basin (tail race), equal to the pressure head of the upper basin, or, at last the more complicated--but from our point of view more correct--law at which we arrived in our above[1] named article."

Footnote: We have in mind: "The theory of the movement of underground water under hydraulic structures" - 1922.

If in concluding we return to the method of the "straight saturation line" we have to point out that we think its critical discussion was necessary especially since it seems that this American method has been introduced [2] into the Russian hydraulic literature without any criticism so that it is not possible for the practical engineer to judge its real value. Of course in its time this method represented a certain improvement of hydraulic engineering since it furnished some way to at least an elementary calculation instead of the "construction experiment" upon which the whole design of an earth dam was based up to that time. This "experimental construction"

Footnote (2): See for example N.I. Anisimow "Dams, First Part." 2nd edition - 1928 - pp. 19, 20, 21,

is in this field as well as in other fields of construction so expensive and involves as a rule a great loss of material, money and time.

Therefore we have 15 years ago found it necessary to acquaint the Russian hydraulic engineers with the, for that time, excellent book of the American engineer Bassel "Earth Dams" (3) in which at that time among other things also the discussed method is presented.

Footnote (3): See my translation of the book "Bassell, "Earth Dams" 1917

But as we have tried to point out this method has now to be regarded as obsolete, and it seems to me that a better name for it would be "the method of the impossible straight saturation line." So far as it is still retained in our own literature and practice, we have to say that the "practical engineers" who use it attempt to attain the impossible, that the engineer-realists tend to the unreal-- attempt to obtain a saturation line which stays within the dam body when no drainage is provided for.

¶ 45. The influence of the width of the dam crest on the percolation flow.

The engineer who designs an earth dam often wants to know what influence a change of width of the dam crest and a change of slopes of the up- and downstream faces might have on the elements of the percolation flow in the earth dam. After we have closed our discussion of the "straight saturation line" method we therefore think that it will be worth while to take up a hydraulic discussion of some points of this method, and we shall so that we can in the course of these arrive at some practical results.

The location of the discharge point of the depression curve M_o (see for example Fig. 5, in the first chapter) is determined, as has been shown by the quantity (a_o) which we shall here express by the relation given in Chapter II (98):

$$a_o = \alpha H_u - \beta b_o$$

for which the meanings of the symbols are explained in Chapter II. From equation (96) we have for (b_o):

$$b_o = b + (m + m_1) d_o$$

so that

$$(499) \qquad a_o = \alpha H_u - \beta\left[b + (m + m_1) d_o\right]$$

From this we can express the influence of increasing the width of the crest beyond b. On the quantity a_0 by differentiating (499) with respect to b:

$$(500) \qquad \frac{da_0}{db} = -\beta$$

i.e. with increasing crest width the "discharge point" is lowered (and vice versa), but their influence of an increase of b is not very great (for practical cases). We can see this easily by looking up the numerical values of β in the graphs 1-3 of Chapter II.

Thus the increase of the crest width of the dam can not be recommended as a practical means of lowering the downstream part of the depression curve.

Footnote: As is obvious from the discussion this result pertains to earth dams without cores and without an upstream protection layer (clay puddle). If we have a core as a protection layer and its width is increased then the head loss of the percolation flow which is caused by these parts of the dam can materially lower the downstream part of the depression curve, this lowering of the depression curve in its downstream part can be calculated by using the equations of Chapters III and IV.

Exactly the same thing happens when the whole dam is not designed to be wider but, only a downstream backfill (see Fig. 71) is used, since according to the systems of equations (41) and 41') for a widening of the dam of course the quantity b' enters the equations, regardless of the fact whether the whole dam is made wider, or only a backfill is used which has the same slope of its lower part as the downstream face of the dam (m_1).

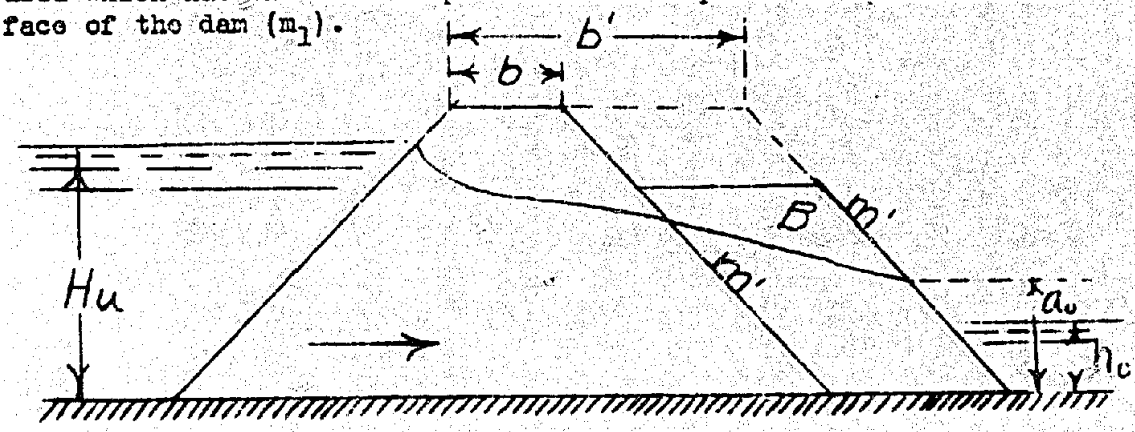


FIG. 71

For the influence of the crest width of the dam on the percolation discharge (q), we find from equation 8° (table 5, Chapter I) similar to (500):

$$(501) \qquad \frac{d}{db}\left(\frac{q}{k}\right) = \frac{d}{db}\left(\frac{a_o}{m_1}\right) = -\frac{\beta}{m_1}$$

for the case of $h_o = 0$. For the case $h_o > 0$ we find from equation 10° (table 4.)

$$(502) \qquad \frac{d}{db}\left(\frac{q}{k}\right) = \frac{d}{db}\left(\frac{a_o A}{m_1}\right) = \frac{1}{m_1}\left[A\frac{da_o}{db} + a_o\frac{dA}{db}\right]$$

in which according to equation (48)

$$(503) \qquad A = 1 + \ln\left(1 + \frac{h_o}{a_o}\right)$$

and hence for this case:

$$(504) \qquad \frac{d}{db}\left(\frac{q}{k}\right) = -\frac{\beta}{m_1}\left[1 + \left(\ln\frac{a_o + h_o}{a_o} - \frac{h_o}{a_o + h_o}\right)\right]$$

From the relations which we obtained in (501) we see that for $h_o = 0$ the increase of the crest-width of the dam serves--as could be expected--to decrease the percolation discharge. But this influence of the width (b) on the discharge (q) (for practical purposes) is still more insignificant than the influence of b on the value of a_o, since according to (501) the coefficient β is divided by m_1. We therefore can repeat what was said about the practical use of widening the dam crest for lowering the "discharge point" also for diminishing the discharge quantity.

The same is true also for the case $h_o > 0$. We can rewrite equation (504) in the following way:

$$(505) \qquad \frac{d}{db}\left(\frac{q}{k}\right) = -\frac{\beta}{m_1}\left[\frac{a_o}{a_o + h_o} + \ln\frac{a_o + h_o}{a_o}\right]$$

in which according to (2):

$$y_o = a_o + h_o$$

and hence:

$$(506) \qquad \frac{d}{db}\left(\frac{q}{k}\right) = -\frac{\beta}{m_1}\left[\frac{a_o}{y_o} + \ln\frac{y_o}{a_o}\right]$$

in which of course

$$(507) \qquad \frac{y_o}{a_o} \geqq 1$$

If by $f\left(\frac{y_o}{a_o}\right)$ we designate the function which is enclosed by the brackets of equation (506) and compute a table of the $f\left(\frac{y_o}{a_o}\right)$ values (see table 22), we see that the numerical values of

$$(508) \qquad \frac{1}{m_1} f\left(\frac{y_o}{a_o}\right)$$

are for practical values of y_o and a_o are not greater than 1.0 to 1.5. Therefore the numerical values of the derivative (506) of $\frac{q}{k}$ with respect to b will be of the same order as the numerical values of the coefficient β, i.e. negligible for practical purposes.

Table 22

$\frac{y_o}{a_o}$	$f\left(\frac{y_o}{a_o}\right)$	Values of the quantity $\frac{1}{m_1} f\left(\frac{y_o}{a_o}\right)$		
		$m_1 = 2.0$	$m_1 = 2.5$	$m_1 = 3.0$
1.00	1.000	.50	.40	.33
1.10	1.195	.60	.48	.40
1.25	1.473	.74	.59	.49
1.50	1.900	.95	.76	.64
2.00	2.693	1.35	1.08	.90
3.00	4.100	2.05	1.64	1.37
4.00	5.385	2.70	2.16	1.80
5.00	6.610	3.30	2.64	2.20
7.50	9.515	4.75	3.80	3.17
10.00	12.303	6.15	4.92	4.10

¶ 46. The influence of the slope of the dam faces on the elements of the percolation flow.

For a discussion of the influence of the upstream and downstream face slope coefficients m and m_1 on the elements of the percolation flow we again start with the relation of (499):

$$a_o = \alpha H_u - \beta \left[b + (m + m_1)\, d_o \right]$$

and by dividing through with H_u:

$$\frac{a_o}{H_u} = \alpha - \beta \left[\frac{b}{H_u} + (m + m_1) \frac{d_o}{H_u} \right] \qquad (509)$$

If we want to obtain a general solution of the question it is sufficient [1] to take:

$$(510) \qquad \frac{d_o}{H_u} = \sim \cdot 1$$

$$(511) \qquad \frac{b}{H_u} = \sim \cdot 3$$

Footnote: The numerical value of the second term of the right member of (509) is small as compared to α

and if we introduce the notation

$$(512) \qquad \zeta = \frac{a_o}{H_u}$$

and call it the "reference elevation" of the discharge point of the percolation flow it is according to (509):

$$(513) \qquad \zeta = \alpha - \beta \left[0.3 + 0.1 \, (m + m_1) \right]$$

Limiting our discussion to the case for $h_o = 0$ [1] we now compute a graph from equation (513) by plotting ζ as a function of

Footnote: For ($h_o \neq 0$) the above shown relations will on the whole remain the same.

the slope coefficient of the downstream face of the dam (m_1). Doing this we use three different values of m (m = 2.0; 2.5; 3.0) and take the α and β values from table 3 in Chapter II. (Fig. 72). From Fig. 72 we can draw the following conclusions:

(1) For a given slope of the "wet" (upstream face) face of the dam the downstream part of the depression curve is raised for increasing slope coefficients of the "dry" face (downstream face).

(2) For a given slope of the "dry" face of the dam the downstream part of the depression line is lowered for increasing slope coefficients of the "wet" face. This second conclusion is more clearly illustrated in Fig. 73 in which ζ is plotted as a function of (m) for different values of (m_1).

If we want to bring out the "intensity" of the raising of the depression curve (downstream part) for increasing the slope coefficients of the "dry" dam face we can regard the value of ζ for $m_1 = 1.5$ to be 100 per cent and designate it by $\zeta_{1.5}$. From Fig. 72 we can now compute the ratios $\frac{\zeta}{\zeta_{1.5}}$ and tabulate them in table 23. We can compute a similar table 24 for the ratios $\frac{\zeta}{\zeta_{2.0}}$ for which we regard the value of ζ for m ; 2 $\zeta_{2.0}$ as 100 per cent. Thus we can express the intensity of the lowering the downstream part of the depression curve for increasing slope coefficients of the "wet" dam face.

Table 23

m	Values of $\frac{\zeta}{\zeta_{1.5}}$ in %.				
	$m_1 = 1.5$	$m_1 = 2.0$	$m_1 = 2.5$	$m_1 = 3$	$m_1 = 3.5$
2.0	100.0	120.5	136.5	150.0	162.5
2.5	100.0	122.0	142.0	158.0	171.0
3.0	100.0	125.5	147.5	166.0	182.0

Table 24

m_1	Values of $\frac{\zeta}{\zeta_{2.0}}$ in %		
	m = 2.0	m = 2.5	m = 3.0
1.5	100.0	89.5	77.5
2.0	100.0	91.5	81.5
2.5	100.0	93.0	84.0
3.0	100.0	94.0	86.0
3.5	100.0	94.5	87.0

As we see from table 23 the increase of slope coefficient of the downstream face results in a rather substantial relative elevation

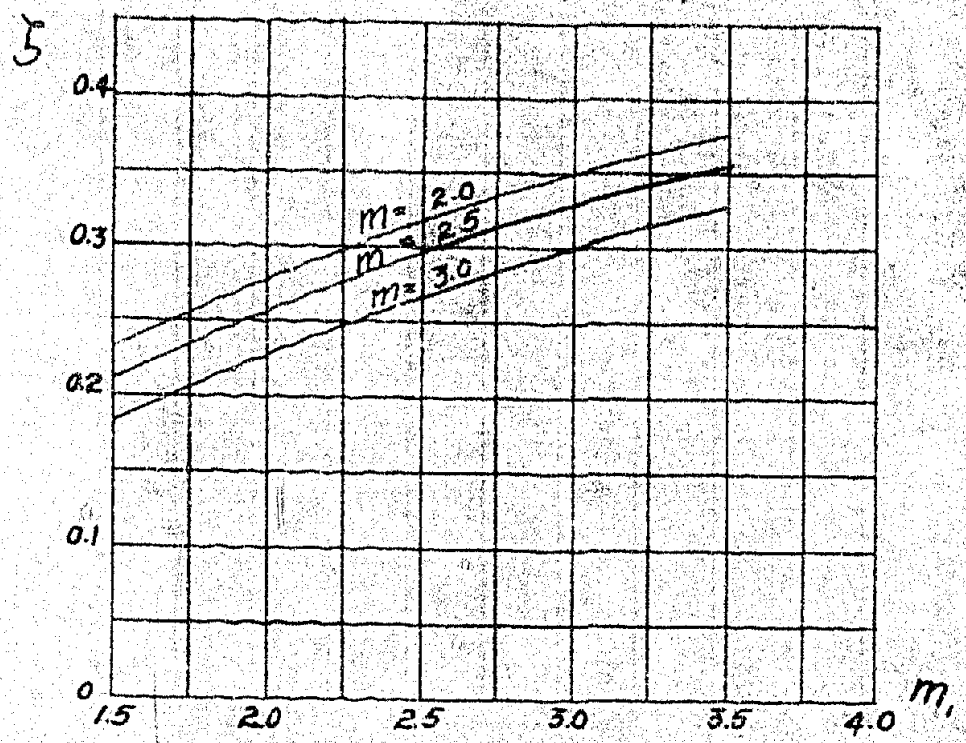

ζ
0.4
0.3
0.2
0.1
0
m = 2.0
m = 2.5
m = 3.0
1.5
2.0
2.5
3.0
3.5
4.0
m₁

Fig. 72

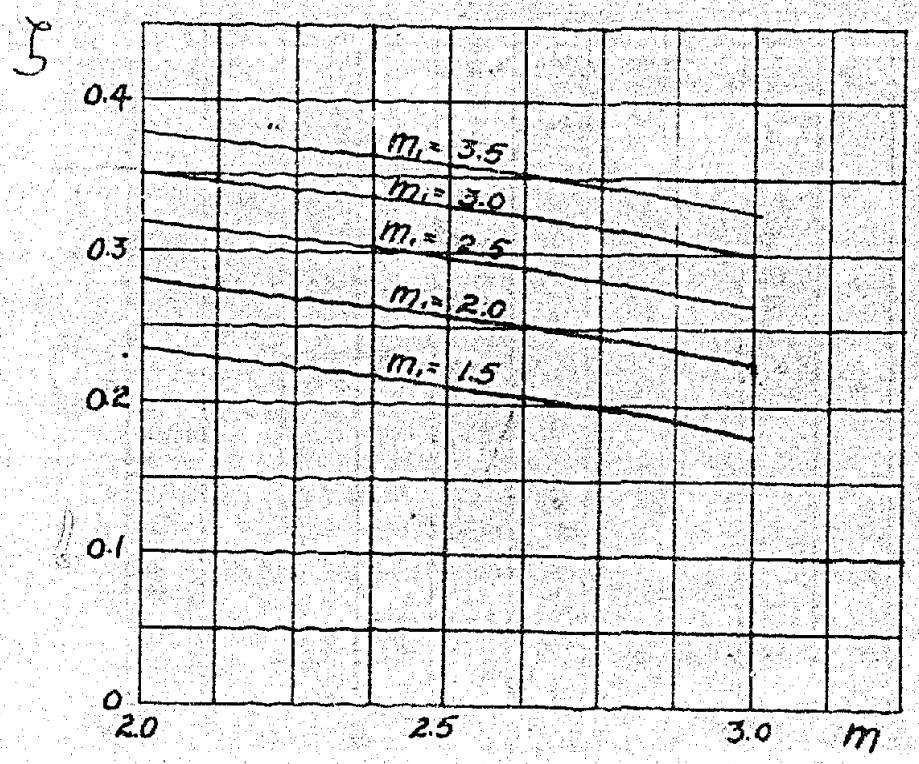

ζ
0.4
0.3
0.2
0.1
0
m₁ = 3.5
m₁ = 3.0
m₁ = 2.5
m₁ = 2.0
m₁ = 1.5
2.0
2.5
3.0
m

Fig. 73

of the downstream part of the depression curve. That means that less steep downstream faces of the dam do not only prevent the location of the depression curve "within the dam profile," but even raise the depression curve in the downstream part of the dam (contrary to the "method of the straight saturation line"). Therefore, when we select the slope of the downstream face of the dam we should be guided by the stability of the given dam material in the downstream part and not by the desire to "lower" the depression curve.

In regard to the influence of (m) and(m_1) on the percolation discharge we can make use of the relation which we obtained in equation III of the system of equations (41'):

$$\frac{q}{k} = \frac{a_o}{m_1} \quad ;$$

using the notation introduced in (512) we have:

$$(514) \qquad \frac{q}{k} \quad \frac{\zeta}{m_1} H_u \qquad \text{or}$$

$$(515) \qquad q = \frac{\zeta}{m_1} \cdot K \cdot H_u$$

We now plot a graph for the factor $\frac{\zeta}{m_1}$ on Fig. 74 similar to that which was plotted above for ζ alone in Fig. 72. From Fig. 74 we see that flat downstream slopes reduce the percolation discharge to some extent. Flat upstream slopes affect the percolation discharge also in the sense of decreasing it, as can be seen from the curves on Fig. 75 which represent the relation of $\frac{\zeta}{m_1}$ (of equation 515) to the coefficient m for different values of m_1. For flatter slopes of either of the dam faces, the hydraulic resistances of the upper and the lower "wedge" are increased, and therefore that fact alone should have told us that the influence of the slopes of the faces, would affect the percolation discharge in the way shown above.

The discussion which was given in paragraphs 45 and 46 also throws some light on the question of dams built of sandy materials. If the designer used the method of the "straight saturation line" such dams demand a very considerable width at their base, and involve a great amount of work (and material). We have seen that according to this method the slope of the "straight saturation line" should be about .12 ÷ .10 for sandy materials. But we think that what we have

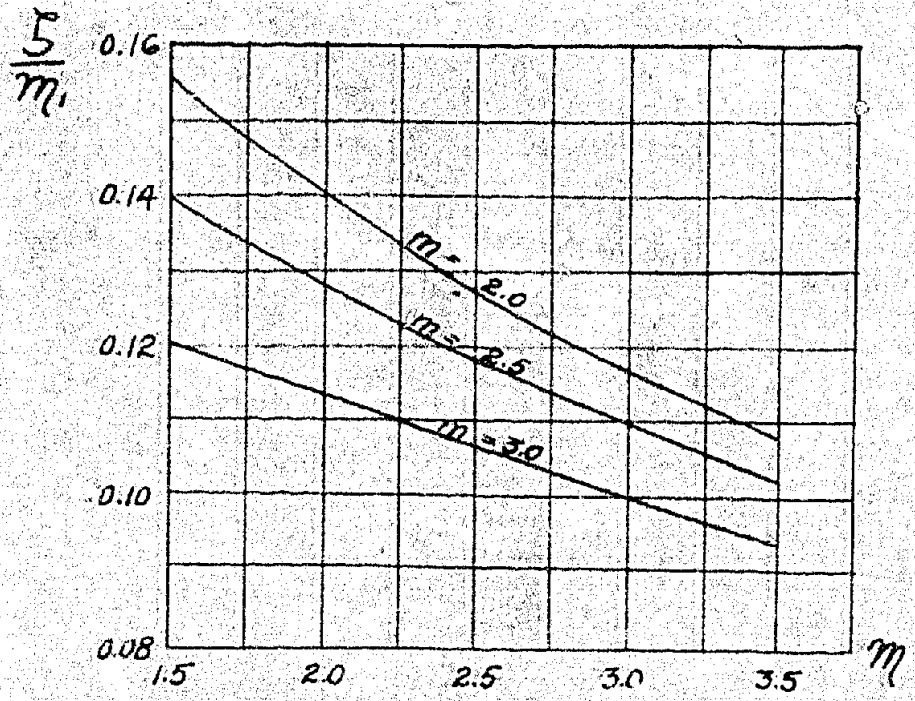

$\frac{\zeta}{m_1}$
0.16
0.14
0.12
0.10
0.08
1.5
2.0
2.5
3.0
3.5
m
$m = 2.0$
$m = 2.5$
$m = 3.0$

Fig. 74

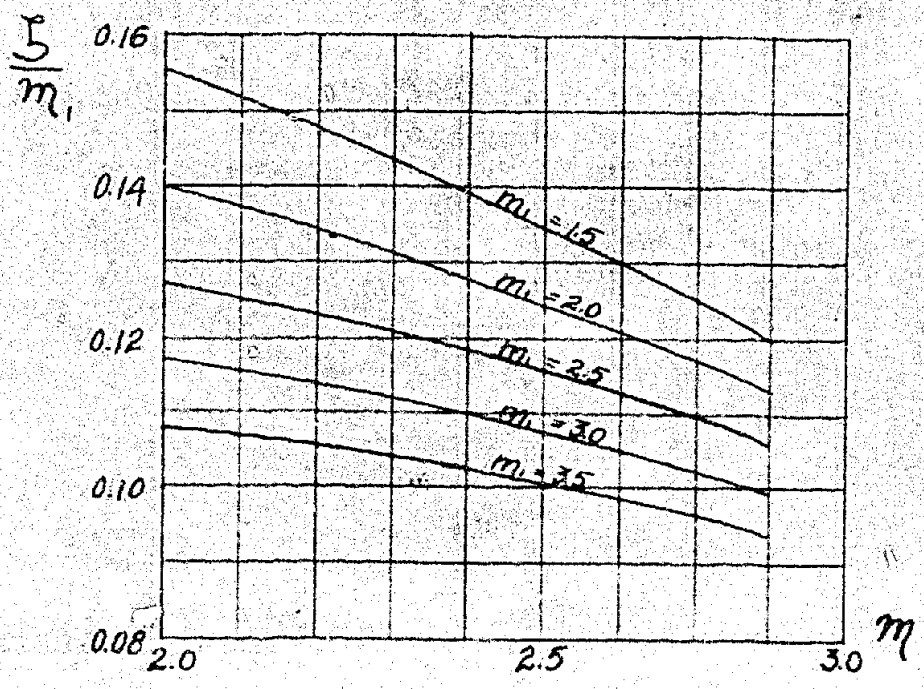

$\frac{\zeta}{m_1}$
0.16
0.14
0.12
0.10
0.08
2.0
2.5
3.0
m
$m_1 = 1.5$
$m_1 = 2.0$
$m_1 = 2.5$
$m_1 = 3.0$
$m_1 = 3.5$

Fig. 75

said about this method, and what we have shown in our discussion of the percolation problems in earth dams, enables us to consider the question of building sand dams with smaller base widths. At least, we think, that it should be enough to stimulate a serious interest of the practical engineer in the design of such dams.

In the near future it would be very desirable to use several sand dams of the "shortened type" (narrower base), and these would have to be built with the help of experimental constructions (models). This is necessary because the condition of washing out of the dam material in the downstream part are not known, and generally the mechanical action of the percolation flow on the ground through which it flows are not sufficiently investigated either theoretically or experimentally. The practical importance of this lack of knowledge however is substantially minimized if a drainage system is provided for in the design of the earth dams (and in this case sand dams). A drainage system is an excellent means to avoid or at least substantially reduce the damaging effect of the percolation flow on the downstream part of the dam.

Because of this fact the future model experiments should by all means investigate the question of drainage not only for the conditions of the commonly used dam of sand-and-clay material, but also for the dam of sand material of the "narrowed base" design. Besides this one should also have in mind dams with an inner core or a protection (puddle-clay) layer; both of these designs can greatly reduce the percolation discharge. This is especially true for dams if such a reduction of the percolation discharge is important for reasons of the water supply.

Let us conclude the present paragraph with a discussion of the relations between the slope coefficient of the downstream face of the homogeneous earth dam and the slope of the depression curve at the point of the vertical section line W_0 (see Fig. 5 for the whole profile of the dam and Fig. 76 for a detail of it.)

From equation (7) we have:

$$\frac{dy}{dx} = \frac{q}{k \cdot y} \tag{516}$$

and the value of the ordinate at the point M_0 is

$$y = y_0 = a_0 + h_0 \tag{517}$$

whence the derivative will have the value:

(518) $$\left.\frac{dy}{dx}\right._o = -\frac{q}{k(a_o + h_o)}$$

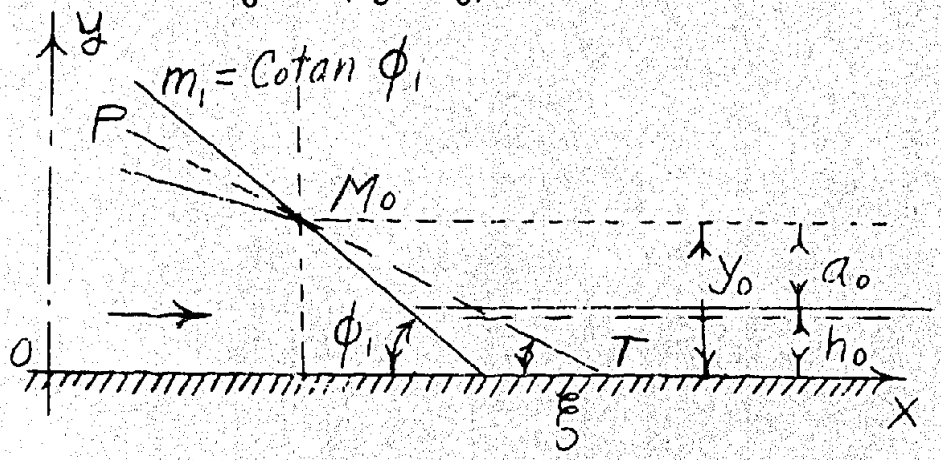


FIG. 76

The tangent P-T to the depression curve at the point M_o forms an acute angle ξ with the abscissa axis (see Fig. 76). For this angle:

(519) $$\tan \xi = -\left.\frac{dy}{dx}\right._o = \frac{q}{k(a_o + h_o)}$$

Remembering the relations which we obtained in paragraph 8 (49) and (48) we see that:

(520) $$\tan \xi = -\frac{a_o}{m_1(a_o + h_o)}\left[1 + \ln \frac{a_o + h_o}{a_o}\right]$$

whence we obtain for the case that $h_o = 0$:

(521) $$\tan \xi = \frac{1}{m_1}$$ and hence

(522) $$\text{cotan}\ \xi = m_1 = \text{cotan}\ \phi_1$$

and

(523) $$\xi = \phi_1$$

That means for $h = 0$ the contour line of the downstream face is the tangent to the depression curve at the point M_o (the "discharge point").

But for $h_o > 0$ (see Fig. 76) this is not true, and since $a_o + h_o = y_o$ we can write (520) in the form:

$$(524) \qquad \tan \xi = \frac{a_o}{m_1 y_o} \left(1 + \ln \frac{y_o}{a_o}\right)$$

whence

$$(525) \qquad \operatorname{cotan} \xi = \frac{m_1 \cdot y_o}{a_o\left(1 + \ln \frac{y_o}{a_o}\right)}$$

and hence

$$(562) \qquad \frac{\operatorname{cotan} \xi}{\operatorname{cotan} \phi_1} = \frac{y_o}{a_o} \left(\frac{1}{1 + \ln \frac{y_o}{a_o}}\right)$$

In Fig. 77 the relation $\frac{\operatorname{cotan} \xi}{\operatorname{cotan} \phi_1}$ is plotted for different values of $\frac{y_o}{a_o}$, and from the curve we see that for the discussed case

$$(527) \qquad \frac{\operatorname{cotan} \xi}{\operatorname{cotan} \phi_1} > 1.$$

That means when $h > 0$ the depression curve is, in its downstream part, less steep than the downstream face of the dam (see Fig. 76).

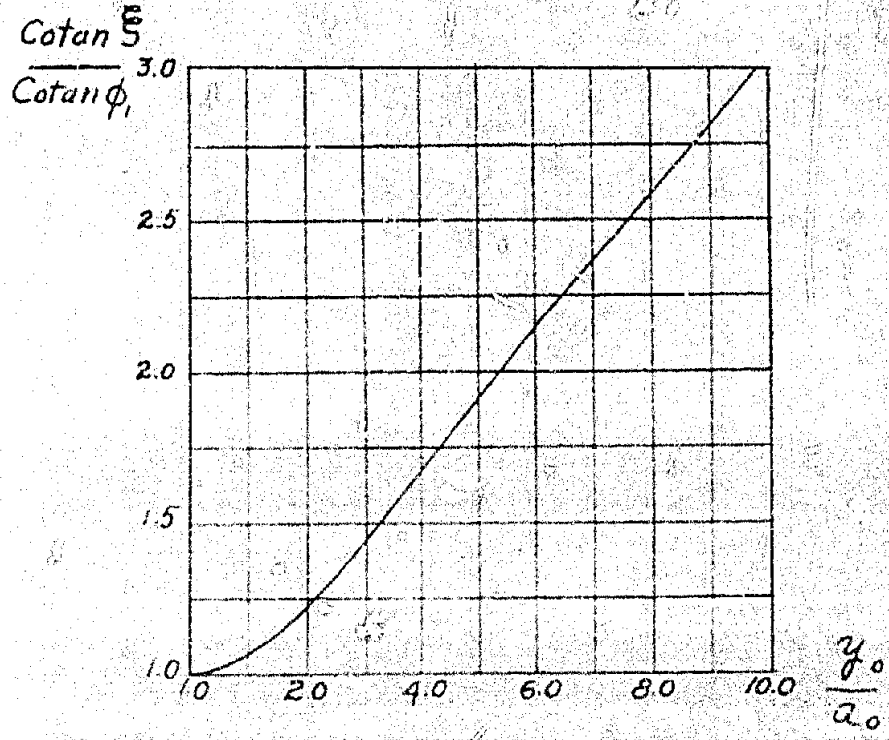


Fig. 77

47. The method of Lorenz and the method of Schaffernak.

As the first, if not very complete, attempt to attack the percolation problem in homogeneous earth dams by means of hydraulic considerations we can regard the application of the well known equation of the non-uniform movement of underground flows on a horizontal layer. For this the equation was used only for the middle part of the profile while the upper and lower "wedge" were completely left out of the discussion.

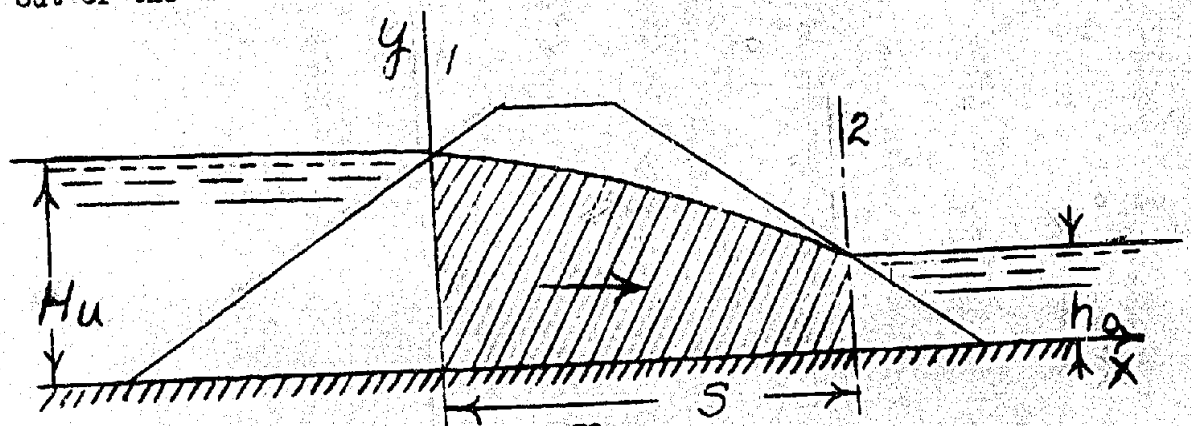


FIG. 78

1°. Such a method which we find in H. Lorenz ("Technische Hydromechanik" 1910, p. 438.) is illustrated on Fig. 78. The depth in section "1" is assumed to be equal to H_u (for our notation), and in section "2" equal to h_o. And if we do not consider the hydraulic resistances between the boundaries of the upper and lower wedge we obtain from (10):

$$\frac{2q}{k} \cdot S = H_u^2 - h_o^2 \qquad (528)$$

in which $\frac{q}{k}$ is the only unknown quantity which can be determined from (528). After that the depression curve can be plotted from the equation:

$$\frac{2q}{k} x = H_u^2 - y^2 \qquad (529)$$

for the coordinate axes as shown in Fig. 78.

How crude this method is can be seen from the numerical table 25, in which we show how substantial the percentage of the upper and lower wedge is in making up the total head loss. Table 25 is compiled from the data of the examples which have been solved according to our own method in ℘ 13 and 14. The total head loss is $H_u - h_o$, the loss in the upper wedge $= H_u - h$ (see Fig. 5) in the middle part $= h - y_o$, and in the lower wedge $y_o - h_o = a_o$.

Table 25

Data from	Losses of head in the different parts of the dam body in % of the total loss		
	$\frac{H_u - h}{H_u - h_o}$	$\frac{H - y_o}{H_u - h_o}$	$\frac{a_o}{H_u - h_o}$
Example from ℘ 13	31.0	60.0	9.0
Example from ℘ 14	28.5	55.0	16.5

Lorenz does not draw any practical or theoretical conclusions from his presentation of the percolation problem in dams and he also does not make any remarks about the case $h_o = 0$, although it is the most important for practical problems. Thus we can say that the given method of discussion, since it does not include but the middle part of the dam (shaded portion on Fig. 78) and that only unsatisfactory, can hardly be regarded as a first step towards a more or less satisfactory solution of the problem. In the above named book, Lorenz devotes only very little space and attention to this problem.

2°. In his short article which was published in 1917[1] Schaffernak investigates the part of a dam profile which is shown as a shaded area in Fig. 79.

Footnote: "Allgemeine Bauzeitung," 1917, Heft IV. When I worked out my method of solving the percolation problem this work, which was published in 1917 in the form of a magazine article (in Vienna) was unknown to me. As we shall see further on, the solution of Schaffernak is only a very special case of the problem which is in this book presented in a much more comprehensive and general manner.

We see that Schaffernak neglects the hydraulic resistances in the upper wedge. For the lower wedge (according to our terminology) the

discussion of Schaffernak is analogous to our discussion in /P 5. Schaffernak has limited his discussion to the homogeneous earth dam.

From the above given table 25 we also see, that it is erroneous to neglect the upper wedge of the dam. We see that the head losses in the uppwe wedge constitute a substantial part of the total head loss of the percolation flow in the earth dam. Furthermore we have to notice that the losses in the upper wedge are about two or three times greater than those of the lower wedge, which is not neglected by Schaffernak. This seems theoretically as well as practically very unlogical. The author does not give any explanations in regard to this question, but we can never the less notice a tendency to "avoid" this question. Thus the sketch of the dam profile in his article shows an upstream face with a slope of about 3:4; but in the experiments the models had a vertical upstream face.

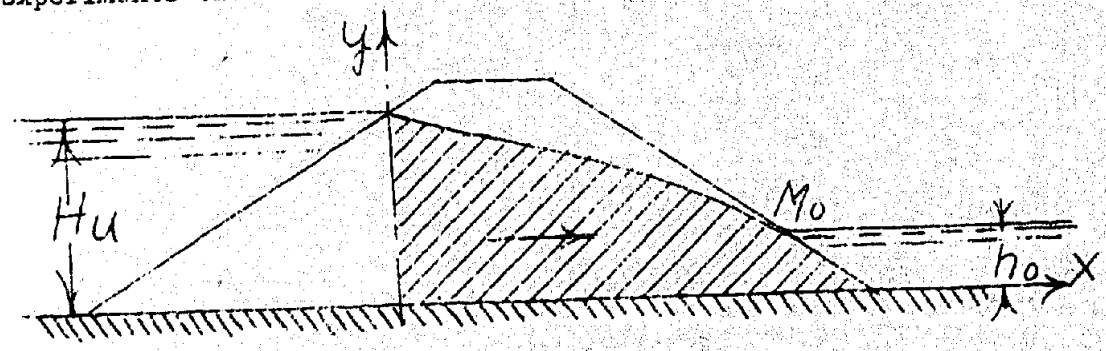


FIG. 79

Thus the method of Schaffernak is useful only for this last case; but for the design of earth dams this case is without interest. Furthermore the method cannot be used at all for dams with protection layers (puddle-clay etc.) or inside cores, and therefore it is easy to see why these cases were not taken up, and could not be taken up by the above mentioned author. Schaffernak gives a short discussion of one case of a drainage system (in the form of a downstream backfill of pervious material). For the main body of the dam he uses Lorenz's method of analysis, and for the backfill his own method. But especially for the case of a drainage system in the dam the losses in the lower wedge are very small or even zero, and for this case it would be important to consider the influence of the upper wedge, but Schaffernak does not do this.

From the previous pages it becomes evident that the work of Schaffarnak covers only a very special case of the much more general discussion of the different main characteristics of percolation in earth dams, which have been presented in this book.

¶ 48. The method of Schoklitsch and Schmidt.

Although the methods of Schoklitsch and Schmidt have been published only recently we do not find in them anything that will be essentially important for the further development of the question of percolation in earth dams. We shall see this in the short review of those methods which is given below.

1°. Analizing the percolation through barrages, Schoklisch determines the "discharge ordinate" of the depression curve on the basis of the postulate of the maximum percolation discharge, disregarding the upper wedge. Thus we have essentially the same scheme as used by Schaffernak. The difference between the two methods of approach consists merely in a different way of analyzing the percolation in the lower wedge. We shall therefore only take up the characteristic feature of Schoklitsch's method--the postulate of the maximum percolation discharge.

The formula which Schoklitsch gives for the ordinate (y_o) at the discharge point of the depression curve is (using our own notations):

$$y_o = H_u + \frac{b_o}{m_1} - \sqrt{\left(H_u + \frac{b_o}{m_1}\right)^2 - H_u^2} \qquad (530)$$

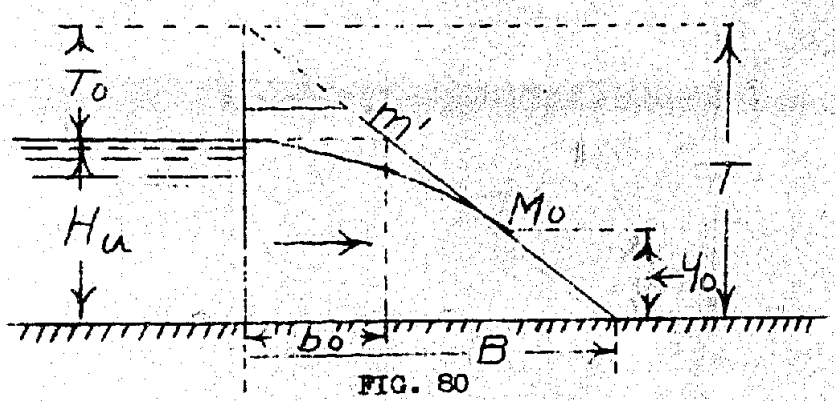


FIG. 80

In this formula H_u = depth of the headwater

m_1 = the slope coefficient of the downstream face

b_o = the width of the profile at the elevation of the upstream headwater level.

According to what was said in ¶ 47 (2°) this formula can not be correct for dams and barrages with an upper wedge, and therefore we shall

refer it only to the case of a vertical upstream face (see Fig. 80).

From Fig. 80 we see that:

(531) $$\frac{b_o}{m_1} = T_o$$ and

(532) $$T_o + H_u = \frac{b_o}{m_1} + H_u = T = \frac{B}{m_1}$$ and

hence formula (530 can be changed into:

(533) $$y_o = \frac{B}{m_1} - \sqrt{\left(\frac{B}{m_1}\right)^2 + H_u^2}$$

that means we obtain the formula which we derived in ¶ 35 (407) for a barrage with lower wedge for the case $h_o = 0$.

For the discussion of the applicability of the postulate of maximum percolation discharge we use formula I and III of the equation system (400) of ¶ 35:

(534) $$\frac{q}{k} = \frac{H_u^2 - y_o^2}{2(B - \ell_o)}$$

(535) $$\ell_o = m_1 y_o$$

whence:

(536) $$\frac{q}{k} = \frac{H_u^2 - y_o^2}{2(B - m_1 y_o)}$$

For the determination of the quantity (y_o) we apply the postulate of the maximum discharge quantity (doing this we notice the rather close analogy to the known postulate of Belanger who presents it in his discussion of the broad crested overflow).

As we see from (536) the percolation discharge q is a function of the variable (y_o), and therefore we write it in the form:

$$(537) \qquad q = f(y_o) = \frac{k(H_u^2 - y_o^2)}{2(B - m_1 y_o)}$$

and taking the derivative of $f(y_o)$ with respect to (y_o), and equating it to zero we obtain:

$$(538) \qquad \frac{df(y_o)}{dy_o} = k\frac{m_1(H_u^2 - y_o^2) - 2y_o(B - m_1 y_o)}{2(B - m_1 y_o)^2} = 0$$

whence:

$$(539) \qquad m_1 H_u^2 = 2By_o - m_1 y_o^2$$

From this equation we obtain that value of (y_o) for which the percolation discharge is a maximum, and we see that the solution is the same as (533). In this way the postulate of the maximum discharge quantity for h = 0 leads to the same result which we obtain if we derive the hydraulic resistances within the boundaries of the lower wedge (for a vertical upstream face).

Thus we see that for this case (h = 0) equation II of the equation system (400) can be replaced by the above quoted postulate. But the above developed line of reasoning remains exactly the same also for $h_o > 0$, and we see that also for this case the law of the maximum discharge is equivalent only to the same equation II of the system (400). For a discussion of the hydraulic resistances in the lower wedge for the case of $h_o > 0$ however, we arrived--in ¶ 34--to a different equation, namely equation II of the system (380).

In this way the law of maximum discharge furnishes the same solutions for the cases of $h_o > 0$ and $h_o = 0$. This can not be accepted as being correct since the pressure of the water behind the dam $(h_o > 0)$ obviously affects the elements of the percolation flow through the body of the dam: It will "press" upon the depression curve and reduce the percolation discharge.

In view of these facts we can say that the law of maximum discharge has only a very limited significance for the discussion of

percolation through earth dams. While this law was pointed out by us as one of the methods of solving the problem of percolation in homogeneous earth dams at the end of 1927 (in our work in the Scientific-Meliorization Institute), it was left out completely in the final draft of the work since it was regarded as not being of sufficient general importance for the solution of the problem at hand.

2°. The nature of Schmidt's book[1] on earth dams is almost exclusively experimental.

Footnote: Schmied: "Die Wasserbewegung im Dammkörper" Wien, 1928.

The theoretical (hydraulic) discussion of the form of the depression curve is given very little space in the book[2], since Schmied bases it on the schematical drawing shown in Fig. 81. In the same way as Lorenz does, the author bases his construction of the depression curve on the equation of nonuniform movement of underground waters on a horizontal impervious layer, but he refers the initial depth H_u to the vertical section OA (Fig. 81) which also is used as the ordinate axis of the coordinate system. Therefore the equation derived by Schmied has (using our own notations) the form:

$$\frac{c}{k} x = \frac{1}{2} (H_u^2 - y^2) \qquad (540)$$

It is obvious that this formula does not refer to the real dam profile, but to a fictitious profile which is bounded by the two vertical sections OA and M_oN_o so that the percolation takes place between the limits OA M_oN_o, as this is shown by the shaded area on Fig. 81. But the author does not make any remarks about this situation, and only when he describes his laboratory experiments he makes in one place the rather hazy remark that, "only that part of the depression curve, which lines within the body of the dam, should be regarded as being real" (the author regards the curve M-M_o as such a "real" part; see Fig. 81).

But the part M-M_o is also not real, it is fictitious, as the whole scheme of the author. We shall see why. The "wedge" OAM is in reality of course not filled with soil and by introducing such an "additional" resistance we naturally increase the drop (lowering) of the depression curve in the upstream part of the dam, decrease the percolation discharge quantity. As a consequence of this also the "discharge point" M_o is lowered. It is clear that such a procedure cannot be justified even by the wish to "introduce some safety margin" into our calculations, since we do not obtain any "safety surplus" by using it. Furthermore in the scheme of Schmied the point M (see Fig.

81) is located lower than the point A, i.e. lower than the headwater level, which is impossible; at the point M the depression curve is not

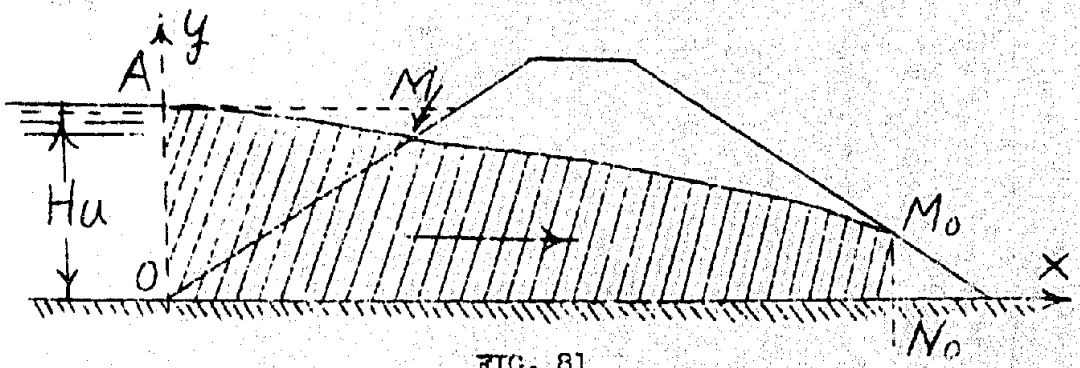


FIG. 81

normal to the upstream face; finally the influence of the lower wedge on the depression curve is theoretically not accounted for.(1)

Footnote: On page 120 of his work Schmied gives the discussion of Schaffernak of the conditions in the lower wedge for $h_o = 0$, but he does this only in connection with a remark that the influence of the downstream face on the discharge point M_o does not contradict the laboratory experiments (which were made by Schmied.)

Schmied does not discuss the case $h_o > 0$ at all.

This discussion characterizes the insufficiency and artificially of the theoretical scheme of Schmied, and we think a further discussion of his formulas for the depression curve unnecessary and shall only add a few remarks about his laboratory experiments.

The main part of the experiments was carried out already in the year 1909-1910, but for different reasons they were worked out and published only in the year 1928. The experiments were made in a wooden flume and mainly with sand models of a height of only 8 cm. A small part of the experiments was carried out with clay and mixed materials. Unfortunately the author does not give us photographs of the experiments, nor drawings of the flume and its details--with the exception of a diagram for the computation of the pressures, which were measured in 3 to 4 points along the base of the dam model. Each experiment had a duration of 3/4 of an hour on the average; the total number of experiments is 100.

As we see the technical and material means which Schmied had at his disposal were very limited, but the experiments themselves were carried out with great accuracy, carefulness and attention. The author says that the "harmonic play" at the piezometer readings which

he observed gave him the same impression of beauty as if he were listening to a musical performance.

We are the last to turn down such an impression, but we have to mention, that for the given case we discovered some dissonances in this "music," and one of them is, that the piezometers in Schmied's experiments have shown the pressures on the ground (bottom), and therefore these readings have not given the outline of the depression curve, but only the distribution of the pressure along the foundation of the dam. These circumstances, in connection with the smallness of model scale (the height of the model = 8 cm) do not allow us to compare the theoretical derivations of the author with the laboratory experiments which he carried out, and which in other respects are very instructive.

We can add that the methods of Schoklitsch and Schmied do not consider the cases of earth dams with integral core wall or protection layer (-clay puddle) or drainage system. With this we conclude our discussion of the methods of solving the percolation problem in earth dams which have been presented by other authors and go over to the concluding chapter of the present work.

CHAPTER VIII

49. A comparison of the theoretical and experimental depression curves.

Under our general supervision R. N. Davidenkow carried out experiments of the percolation in the body of homogeneous earth dams in the years 1928-29 at the Hydraulic Laboratory of the Scientific-Melioration Institute. For these experiments a special wooden frame of a length of 3.0 m, a width of 0.7 m and a height of 1.0 m was built. The experiments were carried out with dam models of a height of about 0.50 m. As a material for the models we used rather fine sand, the mean grain size being about 0.4 mm and their effective diameter being about 0.25 mm. In most of the experiments the part below the dam was dry ($h_0 = 0$), but several experiments were made also for the case of $h_0 > 0$. Aside from these we also carried out experiments with earth dams with drainage systems. The drainage was provided for by a backfill of gravel in the downstream part of the dam model.

We shall in this place not go into a detailed description of the above mentioned experiments

Footnote: See for these in R. N. Davidenkow: "Experiments on the percolation through earth dams."

but shall compare several theoretically computed and experimentally observed depression curves. In Fig. 82 we show 4 cross-sectional profiles of earth dam models, with an upstream face at m = 2 and a downstream face at m, varying from 2 to 3. The slope coefficients of the dam faces as well as the main dimensions of the model and the depths of the headwater are denoted on the drawings; the dimensions are given in cm. The depression curves are plotted on the basis of the theoretical method which we recommended above, and are shown as dotted lines; the experimental curves are shown as solid lines.

On profile I of Fig. 82 the experimental and theoretical depression curves coincide so closely, that it was not possible to show them separately, and therefore on this profile only one common depression curve is shown. In the profiles II to IV the lower part of the theoretical curves is somewhat higher than the experimental curve. The middle and upper part of the theoretical curve is either higher or lower, but also here the theoretical and experimental curves are extremely close together. We have to point out that it was not possible to increase the depth of the headwater beyond the amount which is shown on the drawings, because if we did the downstream face was

rapidly washed away. Thus the elevation of the crests of the dam models above the headwater level was greater than it would be in the corresponding dam structures of natural size, and full headwater depth. Therefore the theoretical depression curves were computed from the system of percolation equations given in (484').

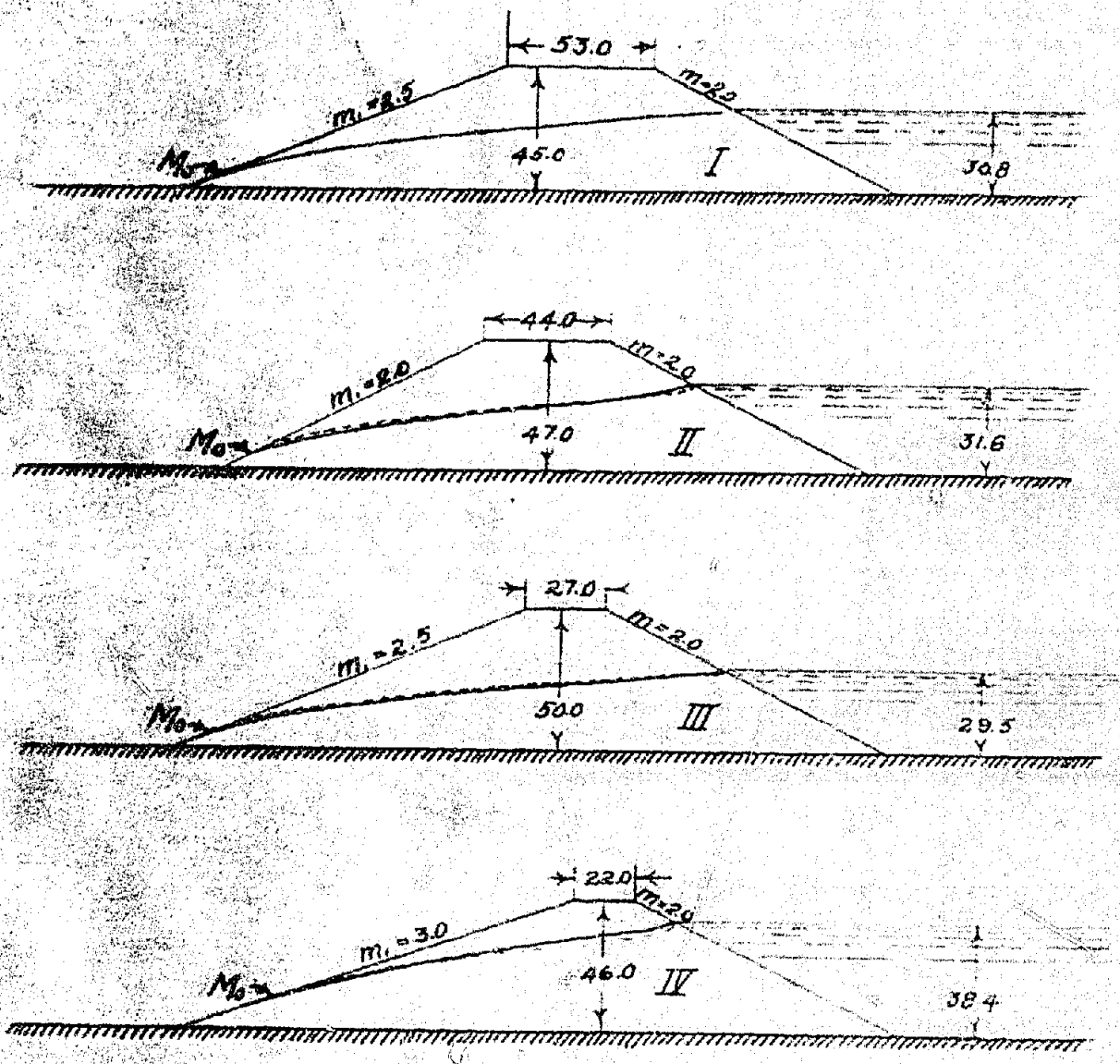
On the whole we can notice that the shape of the theoretical depression curves, which have been computed from the method developed in this book is (as we can see from Fig. 82) very close to the shape of the depression curves which have been determined by direct laboratory measurements.

The discharge point of the depression curve (M_0) which we obtain from the theoretical computation is slightly higher than the experimentally determined point (see Fig. 82), thus the theoretical computation provides for a small safety margin. But this fact has no substantial significance in regard to the shape of the theoretical depression curves as a whole, as we can easily see from Fig. 82. The question of the exact location of the discharge point M_0 for the case of $h_0 > 0$ was not fully answered by the model experiments, but this fact is not essentially important for judging the value of the recommended hydraulic method of computing the depression curve: for such cases the theoretical and experimental curves also show a sufficient similarity of shape, as is shown for one experiment on Fig. 83.

Footnote: A more exact answer to the question about the "discharge point" can only be given by a hydromechanical analysis.

The experiments were made mainly in order to compare the theoretical depression curves with the actual depression curves, and they do not allow to check up on the comparative quantitative values of the theoretical percolation discharge quantity and the actual percolation discharge quantity. This is mainly due to an incomplete knowledge of the percolation coefficient of the material of which the dam models were made, and because no special device was used in order to face the dam models from the air enclosed therein. Presence of air in the dam model causes the percolation discharge of the experiments to decrease substantially. Thus the theoretical discharge quantities for our experiments are also greater than the experimentally measured percolation discharges, which does at least indicate that the recommended method of calculating the percolation discharge through the dam body is sufficiently safe. Because of the just stated reasons we shall not reproduce the numerical values of the percolation discharge quantities as they have been measured in the model tests.

Footnote: These data are given in the above mentioned report on the experiments by R. M. Davidenkow.



Saturation Curves

——— Experimental

- - - - - By Pavlovsky

Dimensions in Cm.

Fig. 82

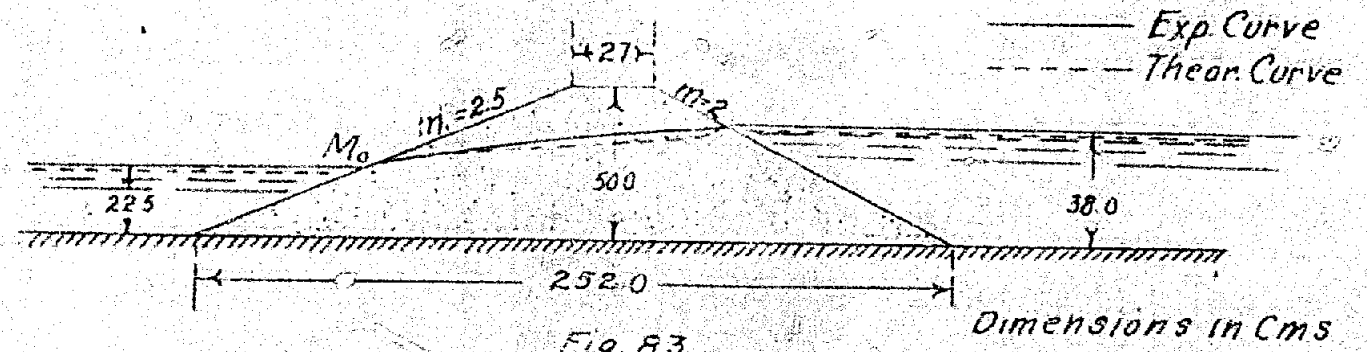

Exp. Curve
Theor. Curve
27
m=2.5
m=2
M_0
22.5
50.0
38.0
252.0
Dimensions in Cms

Fig. 83

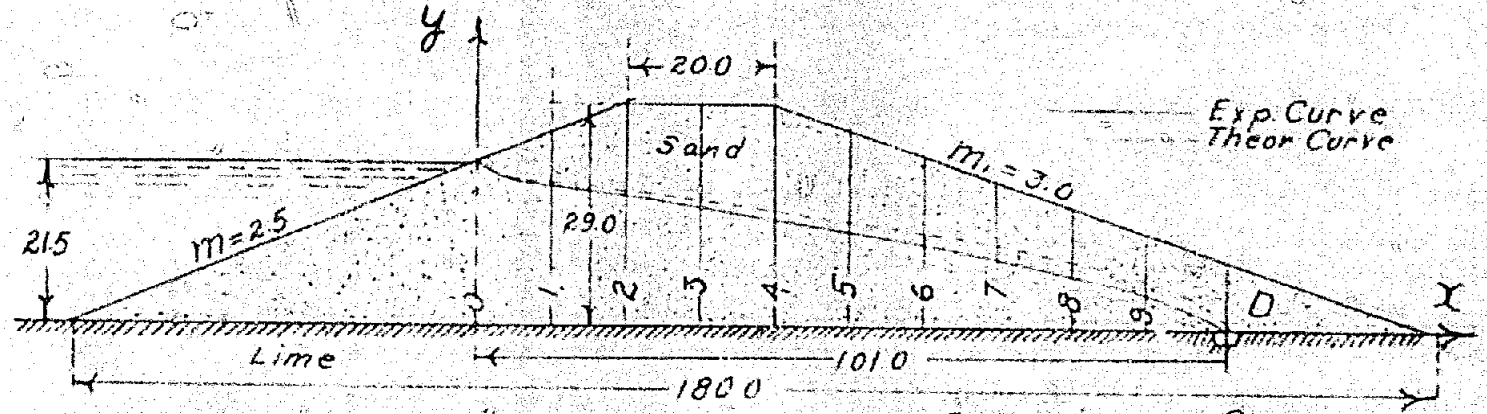

y
20.0
Exp. Curve
Theor. Curve
Sand
m=3.0
21.5
m=2.5
29.0
0
1
2
3
4
5
6
7
8
9
D
X
Lime
101.0
180.0
Dimensions in Cms

Fig. 84

50. Continuation.

Turning now to the results of the experiments with dam models which were furnished with a drainage system we can point out that aside from the work done in the Hydraulic Laboratory of the Scientific-Melioration Institute such experiments were also carried out in the Hydroengineering Laboratory of the Leningrad Hydraulic Institute under our supervision (in the year 1929). Those latter experiments in order to investigate the dam structures of the Lower-Swirsk Hydroelectric plant. We shall first take up those experiments. The experiments were made in a laboratory flume of a scale of 1:20, and their aim was the study of the pressureless flow of underground water. The length of the flume was 5.0 m, the width 0.40 m, and the height 0.80 m. The front (side) wall was made of plate glass, so that it was possible to make the flow lines of the percolation visible by coloring the water which percolated through the model.

We shall in this place show the result of the experiments on the right (bank) levee of the Svir Water Power Plant.(1)

Footnote: The experiments were made under the direction of S. N. Izbask.

The natural depth of the water was $H_u = 4.30$ m, the height of the levee $H_d = 5.80$ m, the width of the crest $b = 4.0$ m and the slope coefficients: $m = 2.5$ (upstream) and $m_1 = 3.0$ (downstream). The dimensions of the sand model are given on Fig. 84 in (cm). The drainage system in the models was made of a perforated concrete pipe of a diameter of about 1.5 cm, which was embedded in a layer of gravel.

On the photographs No. 2-4 the percolation flow lines are shown by coloring the flow and then tracing the flow lines directly on the glass wall of the flume, for "tracing" the lines on the glass a special pencil was used, which would be visible on the glass. Photograph No. 2 shows the path of percolation lines which were obtained 1 hour after the coloring material was introduced into the region of the upstream face. Photo No. 3 shows the same lines after 2 hours and 45 minutes, photo 4 after 5 hours and 20 minutes, when the color lines had reached the drainage line. On this last photograph the upper dotted line denotes the upper boundary in which a capillary action of the water was observed in the model. In photo No. 5 we show the picture of the percolation flow lines for a somewhat different dam profile (and a somewhat larger scale of the photo) and a very long time after the coloring material had been introduced into the region of the upstream face for the first time. The time was 15 days.

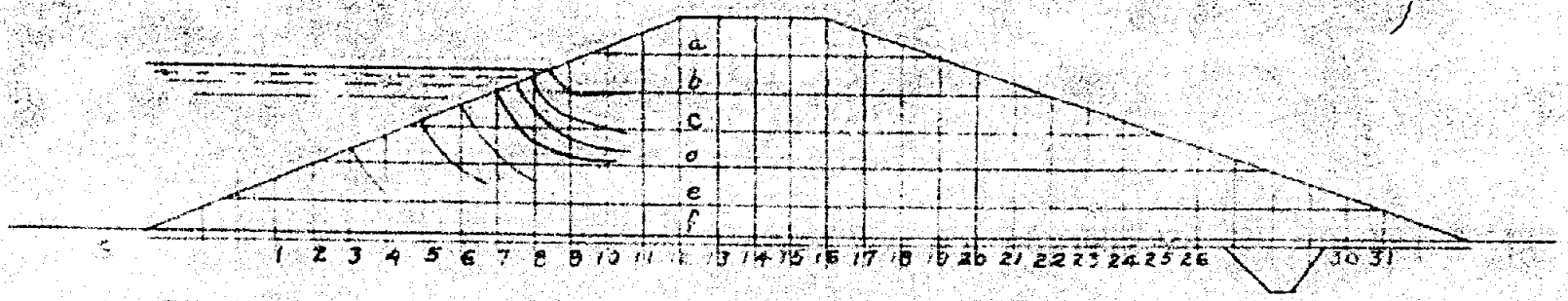


Photo No 2 - Percolation flow lines obtained 1 hr after introduction of color

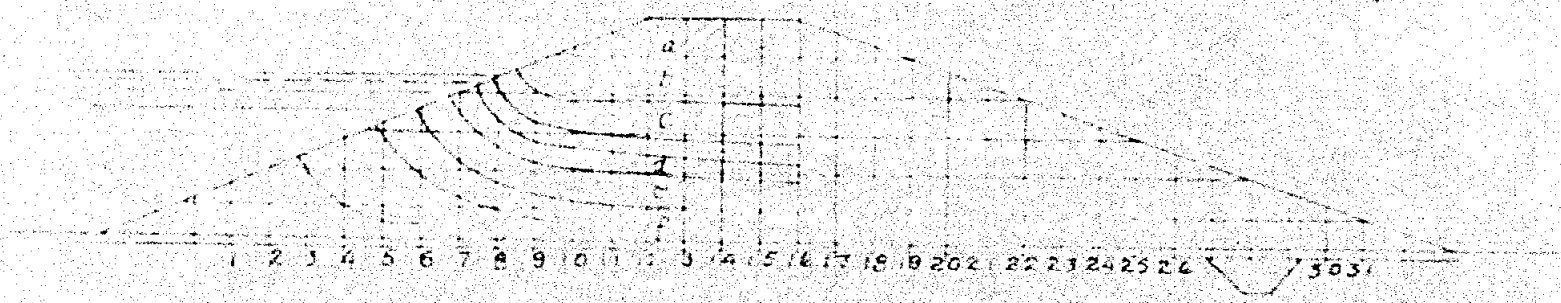


Photo No 3 - Percolation flow lines obtained 2hrs and 3.15hr after introduction of color

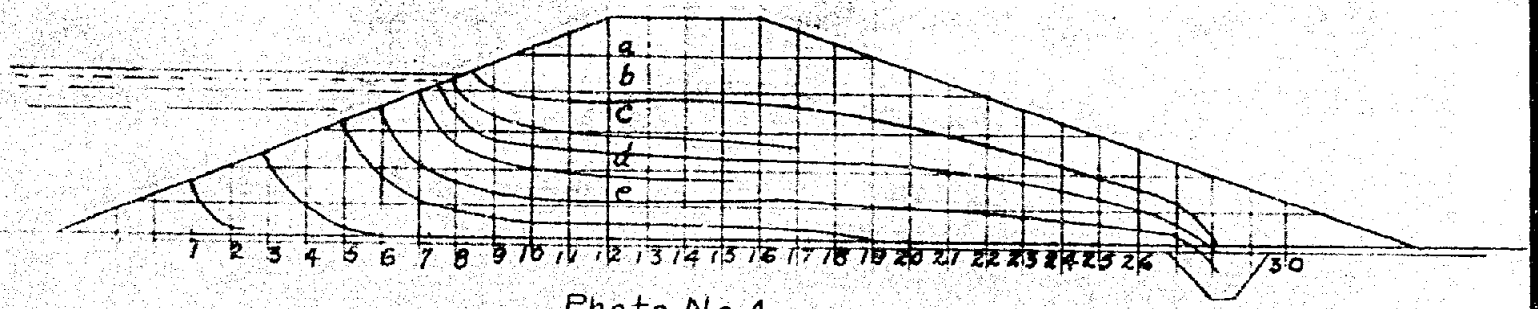


Photo. No. 4

Percolation flow lines obtained 5 hrs. and 20 min. after introduction of color
[Moment when flow reaches the drainage point]

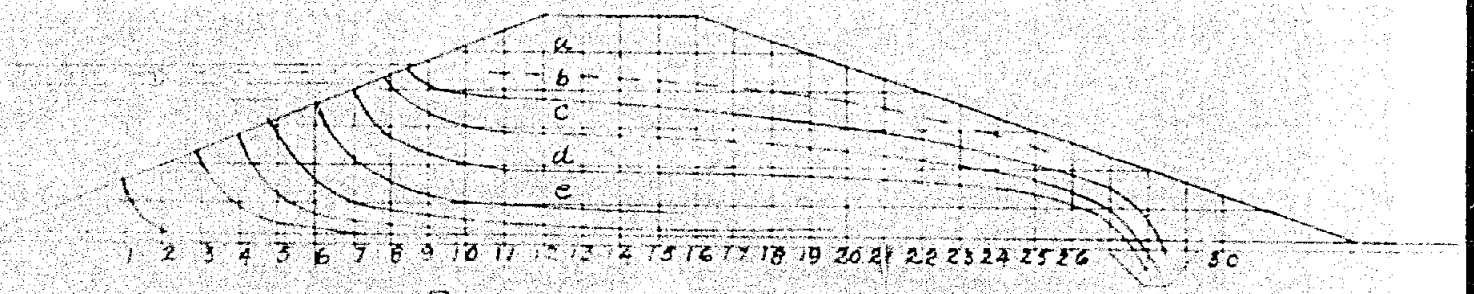


Photo. No. 5

Final picture of the percolation flow lines

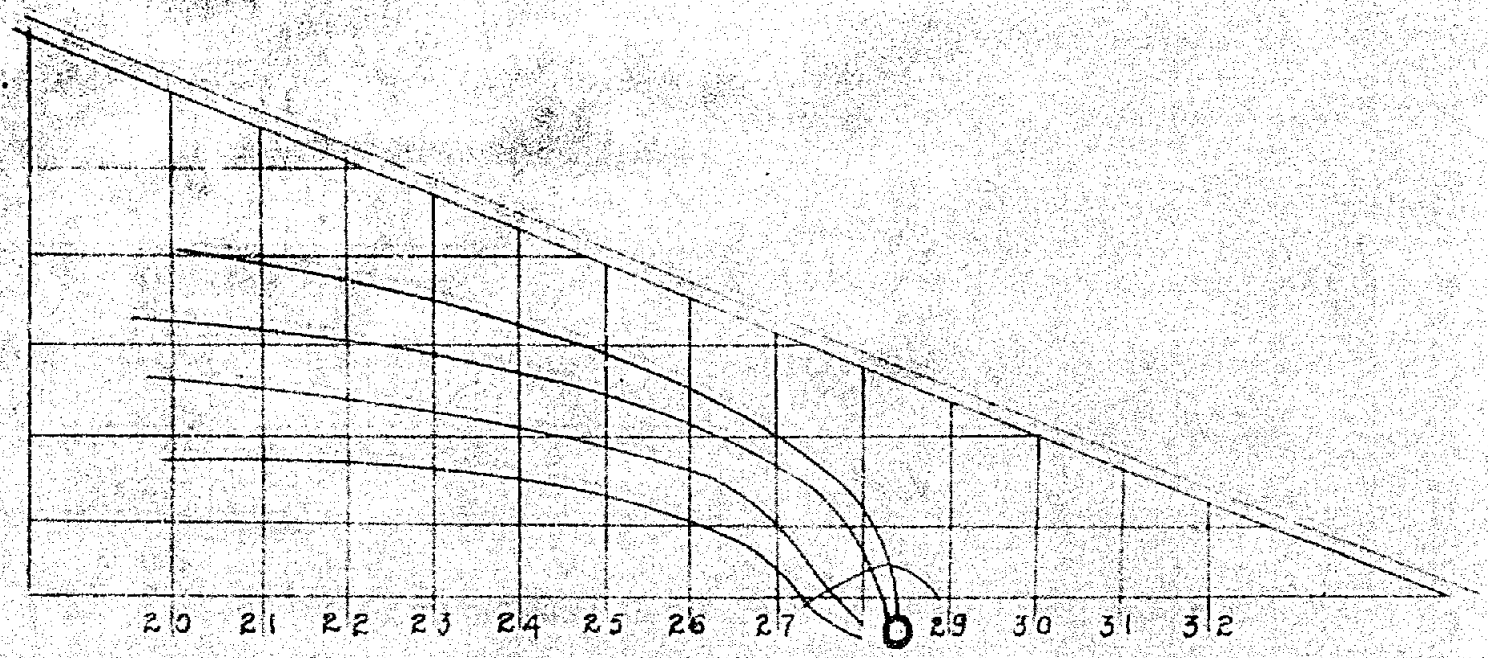


Photo. No 6 A detail of filtration near the drain

The percolation took place continuously throughout this period, while of course the coloring material was put in only on some of the days. On photo No. 6 at last we show a detail of the percolation flow lines of the pervious dam profile directly at the drainage point. The uppermost lines on photo No. 5 and 6 (dotted line on No. 5 and solid line on No. 6) have the same meaning as the uppermost line on photo No. 4, which we explained above. The experimentally determined depression curve is plotted on the cross sectional view of the levee (see Fig. 84) together with the theoretical depression curve. The theoretical curve is plotted according to our derivations for the percolation flow in earth dams with drainage system. As we can see from photo No. 5 and 6, the depth of the percolation flow directly above the drainage point can be assumed to be $h_{dr} = 0$. The theoretical computations for the depression curve of this case were made according to the equation system (486) which is derived in ¶ 40.[1]

Footnote: Also when the depression curve is for this case computed from equation system (328) the curve differs but very little from that which is shown on Fig. 84.

The theoretical depression curve in Fig. 84 is shown as a dotted line, the experimental curve as a solid line, and the location of the coordinate axes O_x and O_y is marked.

From the above shown experimental results on dam models with drainage system, we see that the general character of the depression curve, and the way in which the percolation works in the dam profile, check very satisfactorily with the previously established theoretical ~~depressionxxxxxx theoretical~~ analysis. Also quantitatively the geometrical elements of the theoretical depression curve coincide very closely with the experimental results. The theoretical curve is on the whole a little higher and thus provides for a small "reserve."

For the experiments on dam profiles with drainage, which were made by R. N. Davidenkov the theoretical and experimental curves are shown on Fig. 85. The dimensions of the models are given in centimeters. Again the theoretical curves are shown as dotted lines while the experimental curves are shown as solid lines. For the profile I the theoretical curve is computed from equation system (328) and for profiles II and II from system (486). This comparison also supports our theoretical analysis.

We can add that by the experiments which were carried out in the Hydraulic Laboratory of the Scientific-Melioration Institute also shows the question of the comparatively small influence of the crest width of the dam on the depression curve is clarified. The experiments correspond in this respect completely to our theoretical results

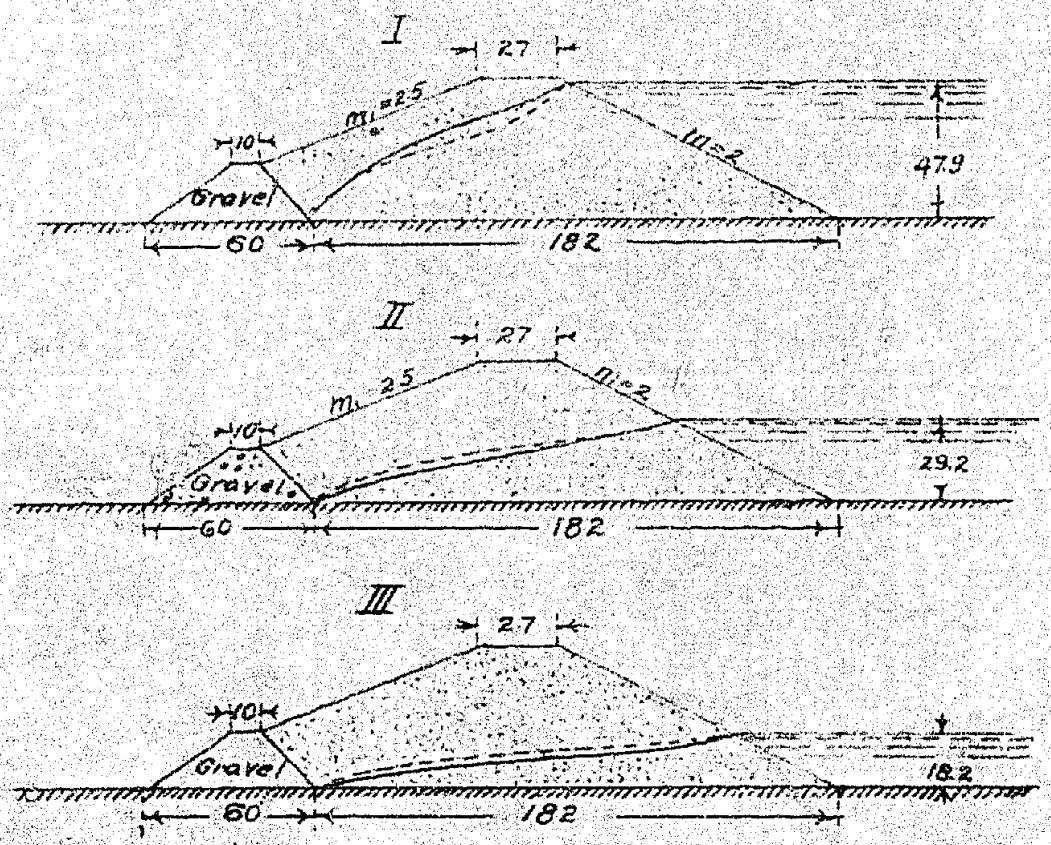

I
27
m1=2.5
10
m=2
47.9
Gravel
60
182
II
27
m1 2.5
m=2
10
29.2
Gravel
60
182
III
27
10
Gravel
18.2
60
182
Dimensions in Cms
Exp. Curve
Theor. Curve

Fig. 85

(see ¶ 45).

Footnote: For the experimental data see the above mentioned work of R. N. Davidenkow.

We now conclude the short discussion of the experimental material connected with our work on earth dams which was up to this time at our own disposal. At the same time we want to point out that the most important question for further experimental investigation is a comparison of the shapes of depression curves for different dam materials, for homogeneous dam structures. For this purpose we would have to investigate dam profiles models of exactly equal dimensions and made of fine, medium and coarse sand, and models made of sand-clay mixtures etc. It also would be important to make experiments on dam models with inner core wall and protection layer (clay-puddle layers). While we want to emphasize the necessity of making these further experiments we are fully aware of the fact that the suggested tests on models do by no means include all the questions of percolation through earth dams for which an experimental investigation would be desirable.

51. Summary of the fundamental conclusions drawn from the discussion of percolation through earth dams.

Throughout the chapters of this book we have in many places pointed out those theoretical conclusions which, we think, could be drawn directly from the discussions of the different aspects of the question of percolation through earth dams. In this final paragraph we think that it is desirable to give a short summary of the important conclusions; doing this we shall in some place name the conclusions in a somewhat different order as they appear throughout the chapters of this book. We shall always refer to the chapter, paragraph, etc. of this book in which derivation explanation, etc. of the respective conclusions is given.

1°. In the method of analysis which is recommended by us, the earth dam is treated as some hydraulic system, the parts of which together form an integral whole, through which the movement of the percolation flow takes place. For the homogeneous earth dam this system is formed by three parts: upper wedge, middle parts, and lower wedge. For earth dams with inner core wall or protection layer the system is correspondingly more complicated and for dams with drainage and some cases of barrages the system is simplified.

2°. The basic elements of the percolation flow in earth dams are determined by the systems of four percolation equations given in (41) and 41') corresponding to $h_0 > 0$ and $h_0 = 0$ respectively (see Fig. 5), if we regard the question from a hydraulic point of view.

From these equations we can determine the main points of the depression curve and the percolation discharge. The intermediate points of the depression curve in the middle part of the dam can be computed from equation (10). By determining the main points of the depression curve we do at the same time determine the head losses in the upper wedge, the middle part and the lower wedge.

3°. In regard to the method of solving the systems of equations (41) and(41') for the homogeneous earth dam we have to say this: If we use the "analytical method" we arrive at the basic equation (59) which contains only one unknown (a_0) which determines the location of the "discharge point" (M_0) of the depression curve (see Fig. 5); after knowing (a_0) it is easy to find the other unknown quantities. Aside from this we show in Chapter II a somewhat simplified "graphical method" of solving the systems (41) and (41') for the commonly used dimensions of earth dams (mainly of the type used for power development). With this method the main elements of the percolation flow in the dam body are found in the most simple way, this is especially important for earth dams of the above named type because of the practical conditions of their construction.

4°. The equation systems (41) and (41') which we obtained from our hydraulic method of analysis allow us to say that the shape of the depression curve in a homogeneous earth dam is not dependent upon the percolation coefficient of the material out of which the dam body is made (¶ 7); the shape of the depression curve is determined by the hydraulic and geometrical conditions (43). The percolation discharge quantity is proportional to the percolation coefficient k. Furthermore the systems (41) and (41') lead to the following law of similitude for the depression curve in homogeneous earth dams (¶ 10); for dam profiles which are geometrically similar the corresponding depression curves are also similar (the condition is that together with a change of the dimensions of the profile also the depths of the water in the head and tail channels are proportionally changed.)

5°. In the design of homogeneous earth dams the increase of the crest width (b) can not be recommended as a practical means to lower the "discharge part" of depression curve and to diminish the percolation discharge in the dam body (¶ 45), since within the practically possible limits the influence of the width (b) on these two elements of the percolation flow is negligible. The influence of the slope m_1 and m of the downstream and upstream faces of the dam on the two elements of the percolation flow is the following; the downstream part of the depression curve is lifted when the coefficient m_1 is increased and lowered when the coefficient m is increased (¶ 46). The numerical values of these conditions can be calculated from the solutions of the

systems (41) and (41') (see Fig. 72 and 73 and tables 23 and 24); the increase of the slope coefficient (flattening of the slopes) of both the upstream and downstream dam face decreases the percolation discharge through the dam body (see Fig. 74 and 75.).

6°. For earth dams with inner core wall our hydraulic method furnishes a percolation system of seven equations (143) or for some slight simplification of the analysis a system of five equations (150). The method of solving these equations is shown in ℘ 17. The determination of the elements of the percolation flow for earth dams with core walls is substantially simplified if we use the "virtual method" which is recommended by us. By this method the question of percolation through a dam with core wall is reduced to the question of percolation through an "equivalent" homogeneous earth dam (℘ 18 and 19). The shape of the depression curve in a given dam profile with core wall does not depend upon the absolute values of the percolation coefficient in the core wall (k_o) and the percolation coefficient of the other parts of the dam (k), but depends upon the ratio

$$\sigma = \frac{k}{k_o} .$$

7°. For an earth dam with an (upstream) protection layer (clay puddle etc.) we obtain a system of five percolation equations (274). In order to simplify the determination of the elements of the percolation flow we again recommend a "virtual method" (℘ 26 and 27) which is somewhat different from the method used for dam profiles with inner core wall. In regard to the influence which the percolation coefficient (k_o) of the protection layer and (k) of the remaining part of the dam have on the shape of the depression curve, we can repeat what has been said about the same question in 6°. Furthermore we can point out that the recommended method gives us the possibility to compute the losses of head in the inner core wall as well as in the(upstream) protection layer.

8°. The developed method can also be used for analyzing the percolation through dams with drainage system. This is shown in Chapter V. There we show the systems of percolation equations corresponding to several different cases: For the tailwater depth $h_o = 0$ and $h_o > 0$, for the homogeneous dam and for dam profiles with core wall and protection layer etc. The systems of percolation equations which we obtained, theoretically characterize the drainage as an excellent means to lower the downstream part of the depression curve and make it possible to carry out a hydraulic calculation of the elements of the percolation flow in dam profiles with drainage.

9°. The ideas which we developed for the percolation in the bodies of earth dams can be applied to some of the types of barrages (cofferdams) which are used in order to protect the construction pits of hydraulic structures. In Chapter VI we give systems of percolation equations for a number of such barrages ("with downstream wedge," with upstream wedge" and with different backfills etc.) which correspond to special cases of trapezoidical dam profiles. A substantial simplification of the solution is obtained by using a graphical method for barrages "with lower wedge" (and $h_o = 0$), and for barrages "with upper wedge" (and $h_o \geq 0$) ¶ 35 and 36.

10°. In regard to the methods of constructing the depression curves in earth dams, which have been offered by other authors we can say the following: The well known method "of the straight saturation line" which is often used in practice is from the point of view of our hydraulic conception of the percolation problem absolutely worthless (and wrong) for any practical applications (¶ 41 - 44). Some of the other methods show attempt at an hydraulic approach to the percolation problem, and we have discussed the methods of: Lorenz, Schaffernak, Schoklitsch and Schmeid (¶ 47 - 48). In view of the errors in these methods we can not regard any of them as being applicable even for the homogeneous trapezoidical earth dam without drainage--to say nothing of dam profiles with inner core or upstream-protection layer.

11°. Our method of analyzing the percolation in earth dams, it seems to me, should at least be sufficient in order to be able to bring up the question of sand dams with "reduced profiles"--especially when drainage is used since drainage substantially cuts down the damaging effect of the percolation flow on the downstream part of the dam. Due to this it is especially desirable that an experimental study of the sand dams with "reduced profile" should be included in the future experimental research program.

12°. In ¶ 49 and 50 we reproduce several comparisons of experimental results and results which were obtained on the basis of our proposed theory. We see that the shape of the theoretical depression curves agree in a sufficiently satisfactory manner with the shapes of the curves which were obtained by direct measurement on the model. The experiments made on percolation through dam models in the Hydraulic Laboratories of the Scientific-Meleoration Institute and in the Hydraulic Laboratory of the Leningrad-Hydrotechnic Institute do not yet cover all cases which we examined theoretically. It is therefore very desirable that the program of experimental investigation should be continued and expanded. In ¶ 50 we point out which of the questions seem to deserve to be given preference in the future experimental studies.

The summary which was given in this paragraph does not include all conclusions which we could make in this book. Furthermore we can add that the method of solving the question of percolation which is proposed in this book can also be applied for several other types and shapes of earth dams, and generally dams of the "gravity type." But this would necessitate further theoretical and experimental investigation in this field. In the same manner, a further development of the proposed hydraulic approximation method by hydromechanical methods has to be left to the future.

We hope that our work furnishes a sufficient hydraulic explanation of the problem of percolation through earth dams of different types--and especially of the problem of constructing the depression curve--which is of such great importance for the design of any dam. In regard to this we can only hope that the hydraulic analysis which was developed in this work may find an application in the design of dams for experimental (model) constructions which are absolutely necessary in order to solve one of the most difficult problems of hydraulic structures--the problem of the earth dam.

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