THE FLOW AROUND PIERS OF DIFFERENT SHAPES
AND ITS EFFECT ON THE RIVER BED

A translation of

Strömungswirkungen an Stromspalten
von verschiedenen Grundriß Formen
und ihrer Einwirkung auf die Flusssohle

by


Translation by

Edward F. Flessy, Asst. Engineer,
U. S. Bureau of Reclamation
TRANSLATOR’S PREFACE

This translation is complete except for eight photographs which did not seem to warrant reproduction as far as an understanding of the paper is concerned. The figures appearing at the end of the translation are numbered according to the original paper. The gaps in the sequence of these figure numbers are due to the omitted photographs.

H. E. Bilney
April 22, 1937
THE FLOW AROUND PIERS OF DIFFERENT SHAPES
AND ITS EFFECT ON THE RIVER BED

(Determination of the Proper Pier Shape and Effective Prevention of Scur)

By C. Keeton

I. CERTAIN MEASURES TO BE TAKEN TO ENSURE THE STABILITY OF PIERS

For the design and construction of such structures on
streams as bridge piers, piers for dams, etc., calculations for their
particular shapes often do not satisfy the flow conditions. Tapering
the nose of a pier happens to decrease the obstruction to flow and to
avoid ice jams; for the rest, however, shapes of piers are based on
strength and architectural requirements. A stream flowing with more
or less uniform motion is contracted by the piers which cause radical
changes in the river bed in the immediate vicinity of the ob-
struction. More or less wide and deep scouring and deposition are
produced, depending on the nature of the river bed, and may endanger
the stability of the structure. The rapidity with which an unpro-
tected structure can be undermined, particularly if the flow toward
it is oblique, is shown by the failure of the abutment of the Allier
bridge at Siegburg in 1906. Until 1906 it was not clear whether

1Schmidt, Handb. der Flussbautechnik, p. 151, SIEGBURG
(Publication of the Abutment of the Allier Bridge at Siegburg, Siegbur-

dangerous scouring occurred at the nose or toe of piers. H. Engle
clarified this question in his fundamental model experiments and
proposed that the foundations of piers be protected by rock fill before
Pouring has occurred. Unfortunately, this proposal received little attention in most handbooks of hydraulic construction with the result that the foundations of piers often received no adequate protection against scour. The investigations of Angélov showed that an efficient and simple protection was afforded by a rock fill placed in the form of a horseshoe around the pier (see figure 1). Until now no empirical rules have been devised for the extent of this riprap in terms of $d_1$, $d_2$, and $h$. These dimensions depend on the one hand on the sedimentary character of the river bed, or, if the structure is built in an excavated canal, on the geological composition of the bottom, and, on the other hand on the maximum average velocity of flow.

The scour distance, $x$, which defines the upstream limit of the riprap, is considerably longer for an easily movable river bed than for a solid bed. Furthermore, different average velocities, for the same characteristics of the river bed, produce different amounts of scour. The depth of the rock fill is just as important as its area.

Angélov tested three different models of piers with various shapes of noses and tails, namely, triangular, rectangular, and circular. These three models of piers have about the most unfavorable shape as far as resistance to flow is concerned and, consequently, cause a large amount of scour. In recent practice most piers are given the form shown in figure 1. The nose is composed of circular arcs whose radius is equal to or greater than the thickness of the pier; the tail of the pier is semi-circular. In the following,
the possibility will be investigated of giving to the pier a shape which, on the one hand, possesses the smallest possible obstruction to flow, and therefore the minimum scour, and which, on the other hand, is both strong and economical. Although a rock fill is usually a safe protection, it can be ineffective if its area is such that $a > p$, thus allowing scouring action of the water to extend outside of the riprap. The individual stones at the boundary of the riprap roll over into the trough thus formed causing a deepening of the scour, which spreads either upstream or laterally from the riprap with the result that the foundation of the pier is no longer protected from the danger of underflows. Danger also exists if the stones used for the riprap are too small, thus permitting the flow to scour the riprap itself, carrying the stones downstream. Therefore, the safest and most suitable protection against underflows of all given shapes of piers should be looked for. The experiments described in the following represent only a qualitative comparison of the fundamental forms of piers. No quantitative conclusion relative to the size of the scour and siltation can be drawn. A comparison of the width and depth of scour at pier inclined to the direction of flow is likewise only qualitatively possible.

II. EXPERIMENTAL PROCEDURE AND SCALE OF THE MODEL

The following experiments were carried out in a flume at the Hydraulic Research Station of the Technical University at Darmstadt in 1958. A description of the flume and the method of measuring the discharge to within 0.1 liter per second (0.0035 second-foot) has
already been given by the author3, a point gauge reading to 0.1 milli-

meter was used to determine the water profile at the piers and along the center lines of the openings between the pier and the walls of the flume, as well as the depth of scour and the height of the sill composite. It was supported by the walls of the flume in such a way that it could be moved along the axis of the channel or at right angles to it.

A layer of sand, 0.2 meter thick (about 8 inches) was spread over the reinforced concrete floor of the flume to represent the river bottom in the model. Medium coarse sand was used for which pebbles larger than 7.0 millimeters in diameter were removed. If a large stone was placed in aarser hole, the amount action was so charged by it that the final state of the sand was reached within a shorter time. A further sifting of the sand was avoided in order that the sand develop as naturally as possible since in river bed the composition of the material is not equal in all layers. For comparable test model tests, the greatest attention must be paid to the moisture content of the material. For each test, pains were taken to avoid placing a dry layer of sand on the damp sand of a previous test; otherwise, fallacious conclusions relative to the depth and width of scour might be reached, since the specific weight of the sand and therefore its relation to the tractive force can be very different. Observance of this precaution is particularly important.
in tests on piers with very sharp noses.

The central portion of the model piers consisted of a box
12 centimeters (4-3/4 inches) wide, 25 centimeters (8-7/8 inches)
long, and 30 centimeters (10-15/16 inches) high, covered with sheet
zinc, soldered and filled with sand. The pier noses and tails con-
isting of eight distinct shapes were soldered to this box. The
assembled model pier was placed on the concrete floor of the flume
and the sand was packed with great care around the model to form a
level surface. The short fore apron usually provided for from a
static analysis of the piers, was neglected in order to simplify
the flow conditions. The first model is shown in figure 1. The
nose is formed of circular area and the tail is semicircular. The
scale of the model was 1:50 which corresponds in the prototype to a
width of 8 meters (26.2 feet) and a total length of 20 meters (65.6 feet).

Such a pier would create a vertical load on the foundation of
about 873 British short tons. This form of pier was chosen from a
large number of actual bridge piers. Its shape and size were pro-
portioned from actual piers. In contrast to this, other investiga-
tions have been carried out on the pier shapes which bear little
relation to the shapes used in practice.

The flume had a width of 0.86 meters (2.83), the piers
0.18 meters (0.59 feet). The depth of water above the model river bed
for unsubtracted flow was 0.12 meters (0.39 feet) in all experiments.
Particular attention was given to timing the tests. With a constant
depth of flow, t, the discharge, Q, and the average velocity, v, were also
constant. The cross-sectional area of the unobstructed flow is 0.65 m square meters and of the pier, 0.15 x square meters. Thus the coefficient of contraction, \( \alpha \), according to Einstein, is 0.12 / 0.65 or 0.18. The coefficient of contraction at large bridges is estimated to vary between 0.12 and 0.18. Hence, the values of \( \alpha \) in these experiments lie almost above the upper limit and in, therefore, only approximately analogous to the values used in practice. The average velocity, \( v_o \), equal to \( z \), is 17 or 0.51 meter per second (0.60 feet per second) and the flow ratio, \( v \), equal to \( b / v_o \), is 0.3375, in which \( b \) is the velocity head. The limits found in practice usually vary between 0.05 and 0.12.

In order to determine the slope of the water surface for unobstructed flow, the following analysis is necessary. L. Winkel

\[ v = \frac{k}{\sqrt{\frac{2g}{1 - \varepsilon}} \left( \frac{1 - \varepsilon}{\varepsilon} \right)} \]

Given values of \( \varepsilon \) in Chezy's formula for different bed conditions as follows:

1. Ideal times (glass wall) \( \varepsilon = 0.04 \) (m) 0.129
2. Concrete times \( \varepsilon = 0.06 \) (m) 0.092
3. Earth canals, rivets \( \varepsilon = 0.02 \) (m) 0.126

Winkel's notes: In these formulas \( v \) is expressed in m per second and \( z \) in meters. Then \( v \) is given in feet per second and \( z \) in feet, the following equations result:

1. Ideal times (glass wall) \( \varepsilon = 0.04 \) (m) 0.129
2. Concrete times \( \varepsilon = 0.06 \) (m) 0.092
3. Earth canals, rivets \( \varepsilon = 0.02 \) (m) 0.126
4. Experimental times without \( \varepsilon \)

and without \( \varepsilon \)
6. Experimental flume without a still bottom, \( C = 60 \ (\text{eq})^{0.18} \)

This last value of \( C \) is based on a large number of tests conducted in 1905.

6 E. B. Bedwell,

The floor and side walls of the flume consist in part of concrete, sheet iron in the middle of the flume, wood and glass. This combination causes a small decrease in the value of \( C \) as given for concrete flumes. If sand or coarse sand is placed on the floor of the flume the value of \( C \) ought to lie between the values given by equations 6 and 4 or: \( C = 93 \ (\text{eq})^{0.125} \) or \( C = 89.49 \). With the bed material used and the small average velocity in the tests, no large amount of transportation of bed material was noticed which should result in a decrease in \( C \). Therefore, the coefficient of friction of this layer of sand should not be greatly different from that for concrete. Therefore, the slope of the water surface, for unstratified flow in these tests, lies approximately between 0.00017 and 0.00001. An average value can be taken for determining the average backwater height.

IV. THE INFLUENCE OF THE SHAPE OF THE RISE AND TOE ON THE AMOUNT OF SEDIMENT AND SILT DEPOSITS

1. Experiments with Eclipsing Pier Heads

The pier arched was placed accurately in the center of the flume so that the sides were parallel to the sides of the flume. The first tests took 400 minutes and the later tests, 150 minutes. Flow conditions were maintained approximately constant, decreasing the duration of the test had no influence on the qualitative conclusions.
so long as the same test period was used in all experiments. Experiments on a test period only one-twentieth of the above and an average velocity twice as large. The greatest depth of scour appeared in the vicinity of the toe, the nose of sand removed being deposited at the tail. In contrast to the researches of various investigators, the deepest place of scour was observed close to the surface of the pier. (See Figure 2). Since the pier was oriented in the direction of flow, the scour and deposition were symmetrical to the longitudinal plane. The water surface profile was taken starting at the point Z on the axis of the pier, then proceeding downstream to the tip of the nose, from this point along the surface of the pier to the tip of the tail, and then downstream along the axis of the pier again (Figure 3). The depth of the scour, e, and its area as determined by x, y, and z, were considered as a function of the double tangent angle, 2 α, at the point of the nose which is a measure of the shape of the nose and indicative of its ability to deflect the flow (Figure 4). The semicircular fore possesses, then, the limiting value of 2 α equal to 150 degrees.

The average values of the several experiments with a given nose size were plotted in the curves of Figure 6. The following results were obtained: Curve 5 shows that the upstream extent of the scour, x, decreases rapidly from 180° to 105° and apparently increases from there on to an angle of 25°. If the fore of the nose at the nose 15 is considered for tangent angles from 180° to 90°, it is seen that for all angles between these limits the extent of the scour upstream from the tip of the nose is approximately equal to that measured laterally from the surface of the pier (Figure 5). Then
the nose has an angle of 25°, the cover has a circular form, the center of which is at the tip of the pier. The length of cover 5 for this tangent angle is somewhat greater than that for 76.6°, which therefore can be taken as the best angle insofar as the length of cover 5 is concerned. The difference between the flow conditions at two covers with angles of 26.6° and 25°, respectively, will be investigated in section IV. Curve b shows the length of the covers for a test period of 400 minutes in contrast to curve a, which is for 100 minutes. Curve b is steeper than curve a, therefore, the final development of cover occurs later with a blunt nose than with a sharp one. For a tangent angle of 76.6° the difference between the test periods is hardly noticeable. With pointed noses, the cover develops even after the pier is put in service, later the river bed undergoes little change. The maximum depth of cover, δ, varies in a similar fashion to the length of cover for the different pier forms. In the curves given, they decrease for tangent angles from 125° to 12.5° and there on to 25° they are approximately of the same size (curve d). At an angle of 76.6° the depth of cover is approximately equal to the length of cover. Curve d shows the depth of cover at the end of a test period of 400 minutes. The same conclusions can be drawn from it as in the case of the length of cover. While the maximum depth of cover occurred at the tip of the nose for an angle of 125°, with decreasing tangent angles, further downstream it appeared along the surface of the nose. At an angle of 25° it again occurred directly at the tip of the nose. The greatest depth of cover determined by Zagol was at the corners of the rectangular nose and at the transition from the triangular nose.
to rectangular central portion. By testing the current it did not occur at the junction of the new and central part. The depth of water, \( d_0 \), at the tip of the weir. This phenomenon can be explained in terms of the flow conditions. Furthermore, the deposition downstream from the weir in the same way as the occurs at the weir, is dependent on the shape of the weir (see Figure 6).

2. Deposits with different slopes

The general nature of experience was associated with whether the shape of the tail of the weir caused any influence on the extent and depth of occur at the weir. Although in the foregoing it has been established that in general deposition occur at the end of the tail, it can also be shown that the tail line lower just at the tip of the tail than in its immediate vicinity (Figure 5). This will occur does not constitute a concern in the safety of the weir but it is known that former must have been worked on the river bed at the tail of the weir. The deposition at the end of the weir is divided into two parts by the action which occurs along the axis of the weir and spreads out more and more as it proceeds downstream and therefore covers a large area (Figure 6). This occurs, which is somewhat more shallow than that at the nose, changes with each tail shape. From a consideration of this occur, it appears that the tangential angle at the tip of the tail does not exert much influence on the direction of the flow, but it does influence considerably the extent of occurring (Figure 6). Taking the final figure four as the limiting case, the angle, \( \phi \), of the tangential at the tail with the axis of the weir is \( 90^\circ \). For values of \( \theta \) less
than 60°, the depth of scour decreases, as shown in Figure 7. The height of the deposit down stream from the end of the tail likewise decreases until an angle of 60° is reached; the deposits for a sharp angle do not lie downstream from the pier, but close to the its sides (Figure 8). From 60° to 180° the height of the deposition increases and at the latter angle starts to fall off rapidly (curve 2). The shape of the tail has no influence on the depth and extent of the scour at the nose, as Engel also proved.

5. Experiments on Plane Oblique to the Direction of Flow

In the third series of tests, a model of a pier was used whose nose consisted of circular arcs and whose tail was a semicircular. The object was to determine the relation between the depth and extent of the scour and the angle which the axis of the pier makes with the direction of the flow. In the previous experiments the axis of the pier lay in the direction of the stream, while in this series of tests the pier was placed obliquely in the flow; therefore the flow was directed toward one side of the pier. The idea behind this investigation was based on the fact that piers are seldom built in the direction of the stream flow. The construction of bridges with piers inclined to the direction of the stream is in order to avoid a more costly bridge in using a hydraulic viewpoint; for the same reason bridges over a bend in a river should not be attempted without due consideration. On the contrary, piers in reaches of a river where the radius is approximately equal to infinity should be closely investigated.
On one hand a river has a tendency to swing in a wide curve from one bank to the other, and on the other hand river curves exert an influence on the direction of flow above and below themselves. This situation was also studied in Engel's experiments and likewise influenced the arcus phenomenon. Compared to the previous experiment, the arcus was no longer symmetrical to the axis of the pier. Floods on even regulated rivers can exert a strong influence on the direction of flow. For example, it can happen that the flow before a flood may strike the pier on the left side and after the flood the right side, repeating the process from flood to flood.

In order to simplify the discussion, terms used at one are borrowed to describe the two sides of the pier, thus: leeward side, the side of the pier toward which the flow is directed, and leeward side, apparently the protected side of the pier (figure 8). The pier model was placed in the flume in these experiments so that the constricted flow sections were to the left of the nose and at the right at the tail. In all, six angles were used, namely: 5°, 10°, 15°, 21.5°, and 27° measured from the longitudinal axis of the pier to the direction of stream flow. The surface extent of the arcus at the tip of the nose was measured by "a" and "b" and "c" the distances along the chord axis of the flow toward the leeward side, and toward the leeward side, respectively, (figure 10). When the pier was set at 5° with the direction of flow, it resulted in 5° and amounted to 3.5 centimeters (0.15 inches) for the shape of pier shown in figure 10. The lee side distance increases at a rapid rate with increasing regularity (curve a) and at 57° in five times that of zero. The minimum
side distance (curve b) decreases slowly from 0° to 12.6° and then increases. At an angularity of 12.6° scour also occurs at the tail which extends outward and deepens until at 37°, at which the foundation of the tail is threatened. Figures 12, 14, and 18 show the conditions after the run at an angle of 37°; the scour measurements were taken along a line parallel to the walls of the flume to the tip of the nose, then along the sides of the pier to the tip of the tail, and finally downstream parallel to the walls of the flume again (see figure 18). Starting from 0° the length of scour upstream from the nose, t, remains practically constant up to a angularity of 3.6° and from there on increases fairly rapidly (curve a).

The maximum depth of scour is just as important as its area as far as the safety of the foundation of the pier is concerned. The depth of scour, h, remains approximately constant up to an angle of 3.6°, then increases (see curve d in figure 16) at 57° it attains a value twice as large as that at 0°. Another measure of scour is afforded by a comparison of the area of the surface exposed by the scour on the two sides of the pier. These areas are computed roughly (figure 16) by dividing the respective areas into triangles and rectangles. When the axis of the pier is in the same direction as the stream flow, the areas exposed on the two sides are approximately equal, and about 11.4 square centimeters (0.6 square inches). With increasing angularity they grow rapidly until on the upstream side at 57° the area is about five times larger than that at 0° but (curve d) but on the leeward side the increase is less rapid. Using the curves in figures 16 and 18, the following conclusions can be drawn: the length of scour increases
rapidly on the lee side, the surface exposed by the scour less rapidly; on the windward side the length of scour even decreases initially in contrast to the surface of scour which grows rapidly. The greatest depth of scour occurs, as before, at the tip of the nose. At the larger angularities the depth of scour at the tail increases more and more and finally overtakes that at the nose. The scour on the leeward side is therefore more shallow and less dangerous than that on the windward side. A large amount of deposition occurs at the leeward side of the tail, which increases in height and extent with greater angularity, and hence, on this side, the safety of the pier is increased. The results of the experiments can be summarized as follows: an angularity of 5.5° between the axis of the pier and the direction of flow is no more dangerous scour than one of 0°. With increasing angularity the danger of underscouring at the nose grows. If underscouring actually occurs, the pier will probably tilt upstream. With an angularity of 21.5° and larger, the tail as well as the nose is endangered to an ever-increasing degree. With underscouring at the nose and tail the pier will turn about the longitudinal axis of the pier toward the windward side. Certain construction measures on the leeward and windward side of the nose as well as on the windward side of the tail should be taken to insure safety against angularities likely to be met with. The failure, already mentioned, of the abutment of the Allner bridge at Siegburg is covered by these conclusions relative to the process of underscouring.

4. Experiments on the so-called "Fish Nose" in Two Different Positions

In the fourth series of experiments a supplementary investi-
gation was undertaken on a pier shape which possesses, according to the
tests of other experimentors, a very small resistance to flow but which
from the standpoint of strength should not come into view. The pier was
formed from circular arcs while the rest was included between two
planes intersecting at an angle of 120° (Figure 16). The angle between
the tangent to the arc at the junction between head and tail and the
adjacent plane amounted to 60°. According to other investigators, the
existence of this discontinuity causes the flow to separate from sur-
face of the pier. This form of pier because of its similarity to the
shape of a fish is denoted as “fish form”. The “fish form” was first
placed accurately in the center of the channel and in the direction
of flow with the blunt nose upstream (Figure 18). A similar develop-
ment of scour was observed as in the other tests with the same form of
nose. However, the extent of the scour, that is the scour length in
front of and laterally from the tip of the nose were smaller. The
scour length, \( r \), was about 71 percent and the depth of scour, 66 percent
of the corresponding values for the other pier with the same nose
shape. No scour (Figure 18) was observed near the tail as in the former
tests (Figure 7). This form of pier is, therefore, around 50 percent
efficient than the other as regards scour.
For the purpose of comparison, the "fish form" was turned face about so that the sharp end was pointing upstream (figure 15). The scour at the upstream tip was similar to that in the earlier series of tests on the model with a tangent angle of 20°; it was circular in shape, the center being at the tip at which the greatest depth of scour appeared. (Figure 5, curves e and o). Therefore, a model of a pier with a tangent angle at the nose of 20° is comparable to the "fish form" since the flow conditions are of the same kind.

IV. THE RELATION BETWEEN THE WATER SURFACE PROFILE AND SCOURING

Compare the results in these figures with the Computations of Army Berechnung des Stauens infolge von Querschnittswidersprüngen (Calculation of the Backwater Caused by Channel Contractions); Zentralblatt der Bauverwaltung, 1919, p. 472.

1. Profiles of the Water Surface

Water surface profiles were measured in a similar fashion as the scour beginning at a point (1 meter; 5.28 feet) upstream from the tip of the nose, then proceeding downstream along the axis of the pier to the tip of the nose, then along the sides of the pier to the extremity of the tail and finally downstream again along the axis of the pier. Characteristic water-surface profiles are shown in figure 17.

Let the slope of the surface for unobstructed steady flow be denoted by $S$ and the length of the pier by $L$. Because of the contraction of the cross section the water surface rises in front of the nose of the pier, the amount of the rising being the vertical distance between the elevated profile, $S_0$, and the profile for unobstructed flow, $S$. $S_0$ can be calculated approximately assuming the cross section of the river to vary as a parabola. A short distance upstream from the


nose of the pier $S_0$ is elevated to the sharp peak of $A$. The difference in height between this point and the corresponding point on the profile, $S$, is denoted by $Z_0$. As the flow is contracted, the velocity increases and the water surface drops its lowest point being at $B$. From $B$ the profile rises to form a standing wave and after that the flow follows the slope, $S_1$. $S_1$ is larger than $S_0$; this increased friction slope is the result of the velocity increase from $v_1$ to $v_2$. Just before the flow emerges from the contracted section, the water surface rises again and at the end of the pier approaches the profile for unobstructed flow. Downstream from the pier and along its axis, it drops once more to the point $F$ and finally reaches the profile for unobstructed flow at $G$. In section II the average slope, $S_0$, for unobstructed steady flow has already been computed. If a line having this slope is drawn to scale through the point $G$, the approximate maximum rise of the water surface upstream from the nose will be obtained. Proceeding in this way the amount of the elevation of the water surface for the various pier shapes was obtained. The pier with a nose formed from circular arcs (tangent angle 155°) and a semicircular tail, caused an elevation of 5.4 millimeters (0.165 inch). For the first position of the "fish form" with the same tangent angle at the blunt end $Z_0$ was about 3 millimeters (0.115 inch) and in the second position, only 1.6 millimeters (0.065 inch). Since the "fish form" represents a special case, the maximum rise observed at the nose of a pier with a tangent angle of 125° was compared with the rise at a 90° nose (figure 3). The maximum rise, $Z_{max}$ for pier noses having

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10 The slope line, $S_0$, was drawn through a point on the backwater curve, $S_0$, (0.60 meters, 2.00 feet) upstream from the tip of the nose. Therefore, it gives a value of $\triangle Z_0$ to which $S_0$ must be added in order to obtain the total rise of the water surface, $Z_{max}$, at the tip of the nose.
the following tangent angles: 135°, 75°, (12° "fish form") equal 2.4 millimeters, 2.8 millimeters, 1.3 millimeters, respectively. Therefore, the maximum rise \( Z_a \) is dependent on the shape of the nose. The more pointed the nose the smaller \( Z_a \) will be.

Figure 8 shows the water surface profiles for piers having the same shape of nose (tangent angle 136°) but with tails of different degrees of pointedness. It is seen that \( \Delta Z \) in all of the experiments is approximately equal and, since \( \Delta Z + Z_1 = Z_a \), the total rise in depth is approximately constant and about 3.4 millimeters (0.133 inches). The shape of the tail neither exerts an influence on the rise of the profile at the nose nor on the occurrence as was pointed out in section III. The special case of the "fish form" with a tangent angle of 135° at the same nose has already been shown to produce a rise in depth of about 3.3 millimeters (0.132 inches).

As \( Z_a \) decreases, the drop to \( L \) decreases (Figure 17). The height of this standing wave can be very large relative to the total en-amount-of its depth of flow at high velocities. Soldan reports\(^\text{11}\) that this height was measured at the bridge below the Waldeck Dam.

\(^{11}\)Soldan: "Über die Berechnung des Rückenwettes (Computation of Backwater at Bridges);" Zeitschrift der Bauverwaltung, 1913, p. 422.

It amounted to 1.15 meters (3.77 feet) at a depth of flow of about 2.5 meters (8.20 feet) and an average velocity of 3.2 meters per second (10.5 feet per second). The flow in this case was streamwise since the corresponding wave velocity was 4.9 meters per second (16.1 feet per second). A rise of the water surface occurs in place of the drop under certain conditions with shooting flow\(^\text{12}\). The size of the drop

\(^{12}\)Karch-Herders: "Zur Bewegung des Wassers und der stehenden Flachen (The Motion of Water and the Forces Arising Thereby);" Bau-

- \text{Zeitschrift der Bauverwaltung, 1913, p. 422.}
depends on the resistance to flow of the piers but this will not be considered here. The steep slope $S$ between the two points $A$ and $B$ causes erosion of the river bed. Figure 5 shows that $S$ decreases with a decreasing $e_o$.

The water surface profiles on the two sides of a pier inclined to the direction of flow are very interesting. The measuring procedure was the same as for the former measurements given in figure 13. The peak does not occur at the tip of the pier but along the windward side and from there on the profile is similar to that for a pier placed parallel to the stream flow. On the lee side, on the contrary, the water surface drops rapidly because of the sudden large increase in the velocity. As the cross section of flow increases the average velocity decreases and the profile rises only to fall again from the high point near the tail. It is seen that a pier oblique to the direction of flow produces two wholly different profiles on its two sides and therefore the flow conditions on the two sides also deviate from each other. With a pier inclined $27^\circ$ to the stream direction, a rise in the water surface of 4 millimeters (0.16 inch) was observed. This relatively small rise ($h_4$ is about 4 millimeters for a pier placed parallel to the direction of flow) is caused by the sharp decrease in the cross-sectional area of flow on the lee side of the pier.

2. The Water Surface Profiles along the Center Lines of the Canoines

The second set of profiles was accurately measured along the center lines of the openings to the left and right of the pier, or at a distance of 0.25 meters (8.21 inches) from the wall of the flume.
The profiles had two portions common to those observed in the first series of tests along the axis and sides of the pier, namely: first, the rising slope $S_0$, and, second, the friction slope $S_1$. Both $S_0$ and $S_1$ are due to the obstruction offered by the pier (Figure 17). The velocity, $v_0$, of the flow decreases to $v_1$ as the pier is approached. At the beginning of the contraction the velocity increases to $v_1$ accompanied by a drop in the water surface. This drop begins at a certain distance upstream from the beginning of the contraction.

Therefore, when the contraction begins, the water surface has already dropped a certain amount, and continues to drop for some distance upstream from the beginning of the contraction, in the contracted opening until it is terminated by a standing wave. From there on it follows very closely the increased friction slope, $S_1$, observed at the boundaries of the pier. Downstream from the pier although it follows in a general way the profile taken along the axis of the pier, the depression in the profile along the center line of an opening is not as deep nor as long; therefore, the two profiles do not agree very well in this reach. The backwater height $2$, caused by a pier obstruction, was determined in the same way as $I_0$ in the first tests. For the first shape of pier and the "fish form" in two positions, 3 (Figures 5 and 15) had approximately the same value namely 0.7 millimeters (0.027 inches). This height is only a fraction of the height of the rise, $I_0$, at the nose of the pier and is almost constant in value for different forms of pier noses. Therefore, the backwater height for streamline flow is practically independent of the shape of both the nose and tail of the pier.
Rehbock gives the following empirical equation for the backwater height \( h \) with streaming flow, \( h = \beta \lambda k \)

According to this, \( \beta \) is the pier shape coefficient; \( \lambda \) the channel contraction ratio; and \( k \) the velocity head downstream from the pier.

Using the conditions for the experiments mentioned in the preceding paragraph and assuming \( \beta \) to be unity, \( h \) as computed by Rehbock's equation is 0.6 millimeters as compared to 0.7 millimeter by experiment.

The height of backwater caused by the obstruction of a channel by bridge piers for streaming flow, which one is most concerned in practice, is dependent on the contraction ratio and only in a small degree on the shape of the pier, and is proportionally smaller than the rise in the water surface at the nose of a pier. As to the location of the maximum backwater height, Rehbock's results should be consistent with those of this investigation. According to Rehbock the water surface elevation should be measured at one pier length upstream from the nose (figure 6).

The point of measurement should agree with the point at which the drop begins, therefore, it should be measured at a lesser distance than the one pier length. The location of this point is not at a fixed distance for all shapes of noses. With a very pointed nose, the initial drop begins further downstream than for a blunt nose (compare figures 3 and 16, second position of the "fish form"). This can be explained as follows: With a blunt nose the contraction is more or less sudden. Since the entrance velocity, \( v_e \), is considerably larger than \( v_0 \), the water surface begins to drop at
some distance above the pier, depending on the difference of velocity. With a very pointed nose, as in the second position of the "fish form", this entrance velocity is not much greater than \( v_0 \), therefore, in such cases the water surface begins to drop near the tip of the pier; however, more thorough experiments are needed to define this point.

**B. The Transverse Face as a Source of Stress**

By comparing the two water surface profiles when drawn in the same figure, a difference can be easily detected. Figure 19 shows the profiles, a and b, along the axis and boundary of the pier, and along the center line of the opening, respectively. From a point upstream from the tip of the nose as far as a certain point downstream from the nose, profile a lies higher than profile b. Downstream from this point first one, then the other lies uppermost. Because of the difference in the water surface elevations at the same cross section, transverse slopes exist from the pier and toward the pier. When the pier is oblique to the direction of the stream these differences in elevations are larger on the lee side than those in figure 19. The "fish form" with the pointed and upstream (figure 15) shows the smallest differences in between the two profiles. According to the investigations of different experimenters, transverse slopes produce rollers about horizontal axes. Engels observed rollers at the nose of a bridge pier and at the nose of a dike\(^\text{14}\), E. Müller and Beyeren in \(^\text{15}\).

\(^\text{14}\)Engels: Handbuch des Wasserbaus (Handbook of Hydraulic Construction) Leipzig, 1890, p. 622, fig. 182; p. 632, fig. 248.
\(^\text{15}\)Compare also with: Biopoj Geotechnik 1930, p. 289.


\(^\text{16}\)Müller, M.: Beitrag zur Wirbelbewegung in Strömungen (Contribution to Vortex Motion in Water Streams) Mathematische Dissertation, Kopenhagen, 1892.
These rollers due to their bottom currents scour the river bed, and the greater the transverse slope and with that the difference in pressure, or, in other words, the greater the difference in head, the more potent the scouring action on the river bed. The slope of these transverse drops (figure 17) depends, according to the tests herein, on the shape of the nose of the pier. The tip of the nose is the point at which several noteworthy rollers begin. Roller 1 caused by the reverse slope (figure 18) causes scouring upstream from the nose and opposite to the direction of stream flow. Roller 2 resulting from the transverse slope $S^1$ scour along front surfaces of the pier. In addition to these two rollers with horizontal axes, many others occur which cause a large amount of scour at the nose of the pier and which are also distributed around the surface of the pier. The material secured from the river bed at the nose is transported by the rapid flow through the contracted opening and deposited at some distance downstream depending on the shape of the tail. The pier noses with angles of $25^\circ$ and $12^\circ$, respectively, show different scour phenomena than blunt ones. The rise in the water surface at the nose with pointed nose is small and hence the local longitudinal and transverse slopes are flat. Roller 1 causes the greatest scour while the rollers in the other direction are not apt to be very active at the river bed.

In order to conclude these deliberations the position along the pier of the greatest scour ought to be investigated. A large number of experiments showed that the distance of the greatest depth of scour from the tip increased with decreasing tangent angles of the nose and reached the maximum distance for a $63.8^\circ$ nose. The opposite
tendency was shown by the drop, $S_d$, and the slope, $S'$, therefore with flatter slopes, the location of the maximum attack on the bed recedes downstream and with it the maximum depth of scour.

4. The Surface Flow at Various Shapes of Piers

According to Krey, a fixed body immersed in a flowing stream:

1Krey: Der Widerstand von Einbauten in Flüssen und anderen offenen Strömen auf das strömende Wasser (The Resistance to the Flow of Water of Structures Built in Rivers and Other Open Channels);
Benachricht, 1925, p. 418.

...compels the stream lines to deviate from their usual courses which cause an excess pressure, $P_1$, in front of the body and a reduced pressure, $P_2$, behind it. The excess pressure has already been thoroughly considered in this paper but for the investigation of the reduced pressure it is necessary to determine the surface flow under streaming conditions at the various shapes of piers.

The first pier tested had a nose composed of circular arcs and a semicircular tail. The stream lines curved around the arch-shaped nose (see the stream lines at the nose in Figure 21). The greatest deflection occurred in these stream lines which struck the tip of the nose. The nearer they lay to the center of the contracted opening the smaller was their deviation from a straight line. The innermost stream lines or those found nearest to the surface of the pier broke away from the nose at a certain distance downstream from the tip and then flowed approximately parallel to the sides of the pier between A and B. In this region they are separated from the pier by reliefs
with vertical axes and further downstream they turn and finally meet each other at D (figure 18). The material removed from the river bottom by the rollers at the nose and transported downstream by the current is partially deposited by these side rollers, as Hobrock has named them¹⁹ at the sides of the pier (figure 5). At the end of the tail at B where the contracted cross section expands to its full value again the stress lines deviate from their parallel direction by about 6° and approach each other. These observations have been confirmed by E. Winkel¹⁹. Rollers with vertical axes termed under rollers by Hobrock, fill the triangle formed by the converging stress lines between B and D. The water surface slopes seawards from D to C opposite to the normal slope of the stream; from B to C it slopes in the normal direction. These under rollers deposit material eroded from the nose. Besides this deposition, two kinds of scouring can be observed at the tail. First, a shallow scour occurred at the tail, and, second, an elongated scour appeared along the axis of the pier between the deposits. This scour, which flattened out and expanded laterally in the downstream direction (figure 18). The shallow scour at the tail apparently was caused by rollers with horizontal axes produced by the slope from B to C.

Essentially different features enter the second case. The discharge


attains its maximum average velocity \( v_2 \) at the end of the contracted opening. At cross section 3 the discharge again flows in the full cross section and the velocity \( v_2 \) diminishes gradually to \( v_3 \), the average velocity in the unobstructed cross section. In the triangle CDF the flow is opposite to the direction of the main stream. The exit velocity \( v_2 \) acts on the mass of water in CDF, producing a bottom flow beginning at 6 on the axis of the pier and which seems to spread out in the directions GH and GH'. This causes a scouring of the river bed which has already been investigated in section III (see figure 18). This bottom flow could be clearly seen during a test on a square pier at the Technical University at Hannover. Therefore, downstream from the pier, two entirely different types of flow occur, a surface flow in the direction FD and a bottom flow in the direction GH.

If the tail is made more pointed but the same form of nose retained, the following observations can be made. The separation of the stream lines from the nose, the divided parallel flow along the sides of the pier, and the existence of the side rollers are all similar as in the investigation on the first shape of pier, but the triangle BEE in figure 19 becomes smaller. A portion of the under rollers become side rollers at the tail. The maximum deposition no longer appears downstream from the pier at a definite tangent angle but occurs along the sides of the tail. The ground.
flow was also observed with a pointed tail; it no longer began on the apex of the pier but along the sides of the tail at a distance from the end of the tail which increased with decreasing angles while the depth of scour downstream from the tip of the tail decreased (figure 7 curve a). By making the tail more pointed, the reduced pressure behind the pier was relieved and the scouring reduced.

When the nose is semicircular it can be observed that the stream lines cling to the sides of the pier for a certain distance. By making the nose more pointed, they are bent less and the point where they leave the surface of the pier recedes more and more downstream. (See point A figure 19). The length AB becomes shorter and the region filled by the rollers smaller. This has also been observed by Babcock. Since the flow slings to the sides of the nose and then separates from it at a certain distance downstream, the deposition produced by the side rollers in this region of the nose is completely suppressed. By making the nose more pointed, the scouring becomes flatter but more elongated.

Similar flow phenomena are observed for piers inclined to the direction of the stream (figure 68). The stream lines are deflected sharply on the windward side, but they follow the outline of the pier. The angle of deviation downstream from the tail amounts to about 60°. The characteristic signs of a bottom flow at the tail cannot be observed here since the main stream acts very strongly on the bed. The stream lines on the lee
side separate from the nose close to the tip. This separation is caused by the high velocity already mentioned in connection with the elevating of the water surface at the tip of the pier. The stream lines spread out in the shape of a fan with an angle of deviation of about 6°. In the region between the surface of the pier and the main stream, rollers with vertical axes were observed which deposited the scouring material there (figure 15).

The flow conditions observed in the case of the "fish form" were somewhat different. The bluntness was first placed upstream. The stream lines hugged the surface of the nose as far as the junction with the converging planes (figure 7). They then flowed approximately in the direction of the main stream for a certain distance downstream and then were deflected by an angle of about 1.5° (compared to 6°/the previous pier shape). Side rollers appear in the region between the surfaces of the pier and the main stream. The material secured out at the nose was deposited by these rollers immediately below the scour (Figure 16). Farther downstream, the river bed along the boundary of the tail showed no noticeable change. A ground flow was disclosed by a very small region of scour.

If the pointed end is placed upstream the stream lines cling to the sides of the nose throughout its entire length departing from the pier at the junction with the tail. The entrance velocity \( v_1 \) is not much larger than \( v_0 \). The material secured out at the tip of the nose is deposited immediately downstream from the scour and thus is not carried away by the stream. An increasing velocity through
the opening will cause erosion of the river bottom over its entire width. This scouring increases to a maximum at the transition from the nose to the tail (figure 15). In the region downstream from this between the converging stream lines, deposition can again be observed. The flow conditions for a nose with a tangent angle of 25° are similar. Downstream from the blunt tail of the "fish farm" an angle of deflection of about 60° was measured; the effects of a bottom flow could also be seen.

V. THE PREVENTION OF SCOUR BY SILLS AND BY A SPECIAL SHAPE OF THE NOSE

Following his experiments on piers, Engel proposed that in addition to a rock fill around the pier, a sill be placed downstream from the pier in cases where the river bed is easily eroded. He placed special emphasis on raising the water level upstream in order to facilitate the passage through the openings between the piers. Acting on this proposal, a sill was placed across the contracted openings at three different positions. The elevation of the crest of the sills in each position was 5 centimeters above the same elevation or about one-third of the water depth of undisturbed flow. The first position was at the end of the nose. An elongated shallower scouring was observed from the tip of the nose as fuses the sill (figure 25). This resulted from raising the water level which reduced the rise of the water surface at the nose and consequently the reverse slope; but a drop of the water surface along the center line of the openings was not prevented by the placing of a sill in this position. The flow
over the sill caused the rollers to form downstream from the sill, which secured the river bed to a considerable extent even along the sides of the pier. In the second experiment the sill was placed at the beginning of the tail. The scour at the tip of the nose was smaller than in the first experiment. The increase in the depth of the water in the contracted openings decreased the transverse slope compared to the previous location of the sill. Deep scouring occurred downstream from the sill because of the rollers produced by the overflow at the sill. The depth of scour was greater along the center lines of the openings M, than along the axis of the pier P. However, the deepest scour was measured in line with the sides of the pier, P. The end of the pier was chosen for the next position of the sill. The sill now in one piece was placed against the tail of the pier. The scouring at the nose as well as along the sides of the pier was reduced to a minimum for this position of the sill. Scouring in the tumble bay below the sill along the center line of the opening, M, attained about the same depth as in the second experiment. The scour along the axis of the pier P had become a maximum, because the flow conditions below the pier and sill for this position were aggravated by the flow over the sill. The position of the sill at the end of the pier was the most favorable thus far attained. However, the deep scouring downstream from the sill endangered the tail to such an extent that neither this nor the second position are desirable from a practical standpoint. On account of this the sill was placed at a distance downstream from the tail of the pier equal to \( b \) being the total length of the pier (Figure 20). In this position a study
was made of the influence of the height of the crest of the sill on the scouring at the pier and downstream from the sill, using four different heights. The scouring at the nose of the pier was greater for the first sill height than in the first experiment. As the sill was lowered, the depth of scour at the nose of the pier increased while the scouring below the sill decreased rapidly. At the nose both the length of scour, \( r \), and the width of scour, \( w \), increased. When the crest of the sill was at the zero elevation the depth and area of scour were about of the same amount as in the experiments without sills. The raising of the water surface by the above sills caused a decrease in the reverse slope in the profile along the axis of the pier at the nose and therefore a reduction in the effect of the rollers created there. When the height of the sill was at the zero elevation, the effect of the transverse slope was unchanged from the effect of the comparable experiment without a sill.

Finally the form of the nose was completely changed and the scouring investigated. The top of the nose (50 centimeters above the river bed) was formed from circular arcs, the tangent angle being about 155° (figure 25), while at the river bed the nose had a tangent angle of about 90°. The front edge of the nose had a slope of about 1:2.5. Hence, the sides of the nose were warped surfaces, and the nose resembled, perhaps, a snow plow. The tangent angle at the section where the water surface met the nose was about 85°. The question lay in what way the tangent angle affected the depth and area of scour.

It is seen in figure 6 that for a tangent angle of about 90° the depth of scour is 2.4 centimeters and for 85° 2.3 centimeters.
measured depth of scour now amounted to only 1.55 centimeters. According to the wave curve this is about the smallest depth of scour which can be obtained with a pointed arch-shaped nose. The length and width of scour were changed. The scour in front of the edge of the nose along the axis of the pier is no longer pronounced. The river bed begins to drop steeply at a greater distance downstream than in the other experiments (compare this distance with those in Figure 2). However, the lateral extent of the scour at the nose increased in comparison with that at a tangent angle of $60^\circ$. The scouring action of the rollers with horizontal axes at the nose was transformed by this shape of nose into a surface effect. Construction to insure the safety of a pier against the scouring effects of surface rollers is easier than against rollers which extend deep down under water surface.

VI. CONCLUSIONS

1. The backwater caused by pier which is of particular significance relative to the flooding of river bottom land in general should be estimated. The backwater height and the drop, $h_r$, can be determined only approximately for a river with a movable bed, since the backwater height is strongly influenced by scouring and deposition. Scouring should never be permitted without deep channels. Its depressive action is shown by an example taken from practice.\[32\] After a flood the right-hand pier of this bridge, which has been previously mentioned, was unsecured. The average velocity of the flood water
was about from 4 to 8 meters (13 to 26 feet per second). The scour
projected below the foundation 1.5 meters (4.9 feet) although the erosion
occurred in clay. The piers of this oblique bridge were placed in the
direction of the stream, thus each pier was set back from the preceding
one. This arrangement caused scouring just as dangerous as though the
piers themselves were inclined to the direction of the stream, especi-
ally at the pier farthest upstream.

2. Flow conditions at piers are strongly dependent on the
shape of the pier. With small models of piers, the flow phenomena are
such that it is hardly possible to draw conclusions from model bed
transportation experiments which are applicable quantitatively in the
prototype.23 Quantitative investigations can be most easily undertaken
by the construction of large models of piers in model rivers. The
river bed of the model must correspond to the sedimentary composition
of the river bed for which the structure is planned. The scale of the
model must be thoroughly investigated. However, relative variations
in the scour produced by different nose and tail shapes can be obtained
from the curves in figure 5.

23 Sleney, 'Riederstandmessungen an umströmten Zylindern von Kreis-
und Rechteckquerschnitt (Resistance Measurements of the
Flow around Circular Cylinders and Other Cylinders with Cross
Sections Similar to Bridge Piers); Berlin, 1878, p. 98, footnote
123.
is threatened. On account of this, piers already constructed before
the regulation of a stream are a matter of concern.

4. For beds easily transported, a sill can be recommended
because of its favorable effect on the scouring at the nose of the
pier. However, with such bed, the sill itself due to the tumble bay
below can be scarcely protected against underscouring and overturning.

5. The safety of piers can be better secured either by the
pier-shaped nose or by making the nose so pointed that the scouring
line upstream from the pier foundation. The scouring in the first
case can be treated combatted by an extended flat rock fill. In the
second case three possibilities present themselves. For instance, the
nose of the pier may be built on a caisson foundation up to a level
slightly above the maximum flood and may be made as pointed as pos-
sible; above this level the pier may have a blunt nose. Or, a pointed
triangular-shaped casting may be driven into the river bed in front of
a blunt pier shaping the casting so that the transition from it to the
pier corresponds to the stress lines. Such castings should project
above the maximum flood level; they may be filled with any properly
tamped material covered with a layer of concrete. The third possibility
is to drive the forms of the concrete nose to a considerable depth
below the river bed. These forms are not removed on completion of the
pier and hence allow shaping the nose similarly as in the second case.

6. From a hydraulic standpoint, wide span bridges with the
most slender piers possible, are to be preferred.
Figure 7

Figure 8

Figure 9

Figure 10