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GENERAL GRAPHICAL METHOD FOR CALCULATING THE PROPAGATION OF PLANE WAVES

by

M. LOUIS BERGERON

- - -

TRANSLATED BY
D. J. HEBERT, ASSOCIATE ENGINEER

- - -

Denver, Colorado
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GENERAL GRAPHICAL METHOD FOR CALCULATING THE PROPAGATION OF PLANE WAVES

A TRANSLATION OF PAGES 407 TO 413, AND 462 TO 480
of

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Translated by D. J. HEBERT, ASSOCIATE ENGINEER

Under direction of

J. E. WARNOCK, ENGINEER
and
R. F. BLANKS, SENIOR ENGINEER

Denver, Colorado
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TRANSLATOR'S PREFACE

The following is but a partial translation of Bergeron's paper, and, for this reason, an outline of the entire paper was compiled by translating division and subdivision headings and is included to indicate the contents other than the parts which have been translated.

The only major parts translated were chapter I which deals with general principles and chapter 5 which deals with applications of these principles to hydraulic problems.

The last paragraph of the original paper has also been translated because of the general nature of its contents.

Underscored words in this translation are italicised words in the original article.

OUTLINE

Chapter I. (pp. 407 to 413, incl.)

1. General discussion.
2. General equations.
3. Law of propagation of plane waves.

Chapter II. (pp. 414 to 427, incl.)

4. Longitudinal waves in a metallic prism.
5. Calculation of the velocity a.
6. Applications.
   (a) Collision of two bars of the same material, of the same section S, of the same period, traveling at different speeds C_o and C_o'.
   (b) Collision of two bodies of the same weight ω, the same wave velocity a, with sections S and S' inversely proportional to their velocities C_o and C_o', and the same period.
   (c) Collision of two bodies with the same (weight) ω, wave velocity, and section S but with different period and velocity.
   (d) Shock of a prism against an infinite plate.
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7. Transverse waves along a taut string.
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   (a) Plucked string (harp).
   (b) Struck string (piano).
   (c) Fall of a weight on a string.
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10. Torsion waves along a rotating right cylinder.

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12. Applications.

(a) Shaft with initial applied torsion suddenly released.

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13. Pressure waves along a pipe full of water, otherwise called water hammer.

14. Calculation of velocity a.

15. Applications.

(a) Sudden closure of an orifice creating a cavitation on the return stroke.

(b) Sudden afflux and sudden suppression of flow in a canal.

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16. Waves in electrical conduits.

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18. Characteristic lines (in graph).

19. Applications.

(a) Closing of an accumulator or battery on a line with one end grounded to earth through an ohmic resistance $R > \sqrt{\frac{\ell}{c}}$ (fig. 35).

(b) Same as case (a) but with an alternator instead of the accumulator or battery and with $R < \sqrt{\frac{\ell}{c}}$ (fig. 36).

(c) Same as (a) with an inductance L in place of resistance R (fig. 38).

(d) Same as (a) but with a capitance C in place of the resistance R and gradually charged with current (fig. 40).

20. Conclusion.

CHAPTER I

1. General discussion

In our universe all phenomena are propagated from place to place, in the medium where they are produced, in the form of a wave whose appearance is familiar to us on a water surface or along a cord under slight tension, which has been agitated at one of its ends.

In a great number of cases where the wave is not subjected to deformation during its propagation, it is called (plane). It remains identical to itself forever if the medium is homogeneous and limitless; but the medium is always limited, and each limit imposes particular conditions which refract, reflect, or transform the waves upon their arrival.

In other cases the waves undergo a deformation arising from the homogeneous medium itself in proportion to the advance of the wave. This modification is then continuous; but, one can sometimes conceive of it as being produced by sudden jumps at points in space, equidistant and sufficiently close, and thus one constructs an approximation which falls on the preceding case where the limits would be the locations of the sudden jumps.

In principle, a wave is then a physical phenomenon in motion; started from some point, it remains the same for an observer who travels with its velocity, and this velocity is constant between the limits of the homogeneous medium where the phenomenon has occurred.

Every phenomenon is characterized by the variation of two physical dimensions or variables, which should define the state of the bodies considered at any instant, and not by any single dimension, as a superficial view of things often leads one to believe. It is for this reason that in a liquid, or, more generally, in any medium of elastic material, a force is always indissolubly connected with a velocity. These two dimensions are the pressure and the discharge in a water conduit; the longitudinal force and the velocity of longitudinal displacement of a transverse wave in a metal bar; the transverse force and the velocity of transverse displacement of a

section normal to the axis of a tight cord; the torsional moment and the angular velocity of a section normal to the axis of a bar in rotation. The same is true along an electric conduit where the tension (volts) is connected with the current (amperes).

In addition, these two dimensions are always the ones which enter into the equation of equilibrium of the system being studied. Therefore, the desire to consider alone, for example, a tension wave, as one readily does, is a pure abstraction which warps judgment because it has no more reality than to wish to talk only of ordinates of a curve and to ignore its abscissa. There is no phenomenon which is a simple propagation of a tension wave. There are only phenomena of tension-intensity, pressure-discharge, force-velocity, and so forth, and it is the return to this complete and real concept which is the philosophy of the method which we are going to discuss.

The writing of the equations of the physical phenomena provoked in a continuous medium is never done without considering the two dimensions or variables in question, and it is only in the discussions of the solution that they are often dissociated.

These equations are well known, and, although it is easy and perhaps preferable, as we shall see, to neglect them, we will recall them first in order to show the connection which links the graphical method, which is the object of this study, to the former algebraic methods.

2. General equations

If one considers an infinitely thin slice dx of a water conduit and writes the equation of the forces acting and the equation of continuity, one sets up the following two differential equations:[1]

[1] See: L. Allievi, Theorie du coup de belier (Donod); L. Bergeron, Revue General d'Electricite, 14 mai 1932, Bulletin des Ingenieurs Civils, mai 1926, Congrès de Mechanique de Liege, 1930.

$$\frac{dh}{dx} = \frac{1}{g}\frac{dv}{dt}; \qquad \frac{dv}{dx} = \frac{g}{a^2}\frac{dh}{dt} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

where h is the pressure in meters of water and v the velocity of flow.

Along an electric line, by considering the linear capacity C and the linear inductance of the line, one sets up the equations

$$\frac{du}{dx} = l\,\frac{di}{dt}; \qquad \frac{di}{dx} = C\,\frac{du}{dt} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

where u is the voltage and i the current.

It is apparent that these two systems are identical and the same is true for all the cases cited previously (where we will see that the medium does not undergo deformation during the propagation of the phenomenon).

The solution of these equations is:

$$u = u_o + F(x - at) + f(x + at) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$$i = i_o + K\left[f(x + at) - F(x - at)\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

where u_o and i_o are the values of the two variables or characteristic dimensions (voltage-amperage, for example) during the initial permanent state preceding the variation which constitutes the phenomenon to be studied. In these equations F and f are functions which can be absolutely anything; no limit is imposed on them except that the independent variables, space and time x, t, shall always be grouped by the relation $x \pm at$ where a has the dimensions of a velocity. It is easy to see that this relation simply implies the following physical fact:

> The function $F(x - at)$ has a constant value for an observer who travels at the velocity a, since for him $x = at$, with the result that $x - at = 0$ and thus suppresses any variation of the function F.
>
> Likewise, the function $f(x + at)$ has a constant value for an observer who travels at the velocity $(-a)$ because this makes $x + at = 0$.

But it is these same equations which define a wave being propagated from place to place without changing, with the result that equations (3) and

(4) simply express the following double physical fact:

(a) The increase $(u - u_o)$ of the tension or voltage at any plane M is, at a time t, equal to the sum of any two waves (of tension) which cross each other at this place at this time.

(b) At the same time, the increase $(i - i_o)$ of the intensity is K times the difference between the same two waves, where K depends on the properties of the physical medium being considered.

Thus the two dimensions u and i are indissolubly connected during the propagation of the same phenomenon. In the investigation of the waves F and f of tension, this is only a step toward a complete knowledge of u and i, but for computation this step is necessary and one can only continue by always knowing the waves F and f in terms of time. However, the reflections and refractions which take place at the limits, or changes in the medium, rapidly make the calculations inextricable and the more serious rough errors can occur without the operator's knowledge. One can say that this difficulty is invalidating and that the calculation is impossible because one has to deal with complex groupings and with conditions at any limits.

However, a very large simplification which leads directly to the graphical method is obtained by noting that the double fact expressed by equations (3) and (4) resolves itself into a <u>very simple and unique law from which the waves F and f are eliminated for an observer who travels with the velocity of the waves</u>.

In effect, let M be any point in the medium XY, figure 1 (which could be, for example, an electric conduit), where the characteristic dimensions u and i have the values u_{t_M} and i_{t_M} at any time t.

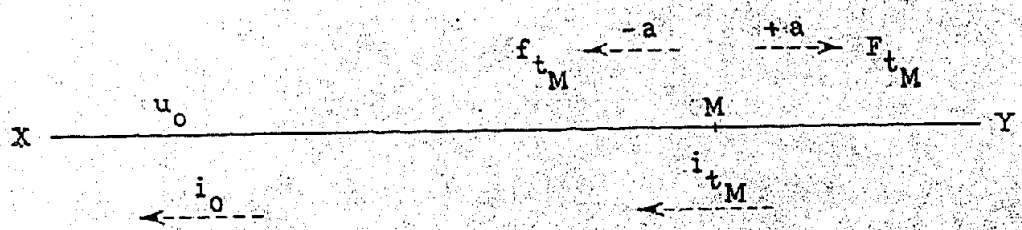
Equations (3) and (4) give the relations

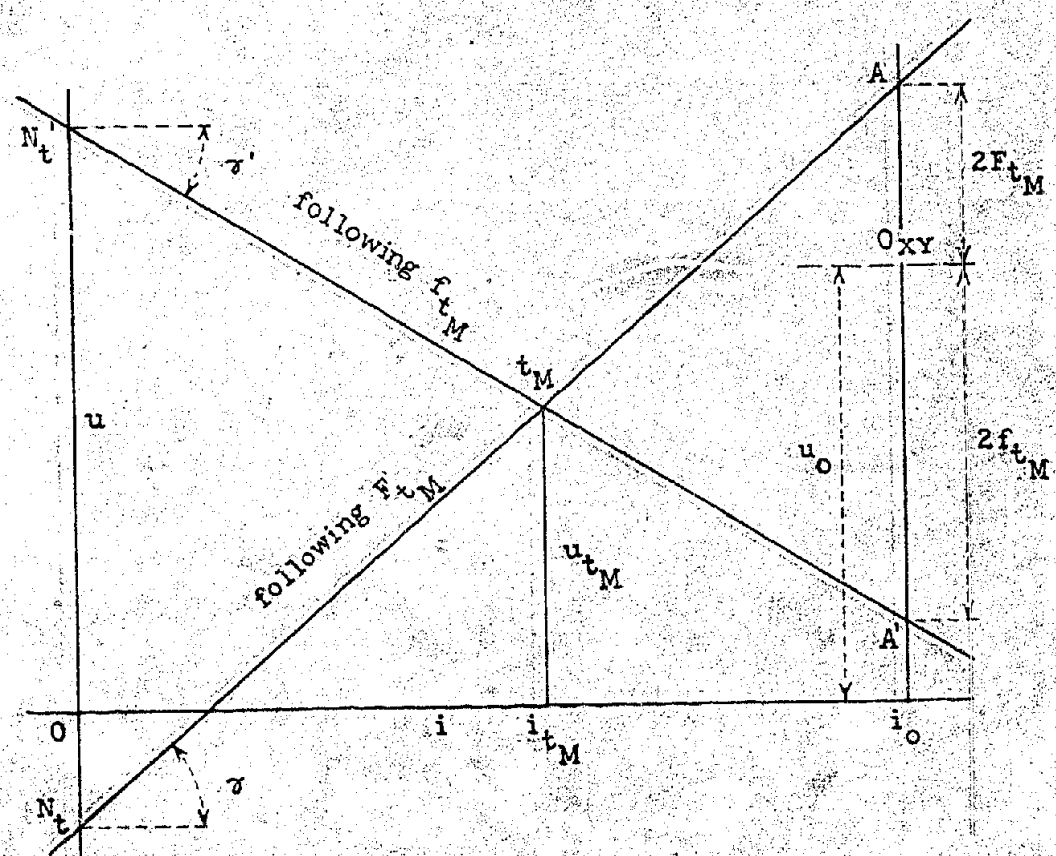
$$(u_{t_M} - u_o) = F_{t_M} + f_{t_M} \quad \text{.....(5)}$$

and

$$\frac{1}{K}\left(i_{t_M} - i_o\right) = f_{t_M} - F_{t_M} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

where F_{t_M} and f_{t_M} are the waves which cross each other at this place and at this time.





Figures 1 and 2

Let us suppose, now, that an observer leaves M at time t following the wave F_{t_M}. For him this wave will maintain its value and equations (3) and (4) will give:

$$(u - u_o) = F_{t_M} + f \quad \text{.....(7)}$$

$$\frac{1}{K}(i - i_o) = f - F_{t_M} \quad \text{.....(8)}$$

Subtracting (5) from (7), then (6) from (8), and equating the results, one finds:

$$(u_{t_M} - u) = \frac{1}{K}(i_{t_M} - i) \quad \text{.....(9)}$$

an equation from which F and f are eliminated and which is the desired law, represented by the line $t_M N_t$ with the angular coefficient $\tan \gamma = \frac{1}{K}$ (fig. 2).

Likewise, for another observer leaving M at time t and following the wave f_{t_M}, the equations (3) and (4) will give:

$$(u - u_o) = F + f_{t_M} \quad \text{.....(10)}$$

$$\frac{1}{K}(i - i_o) = f_{t_M} - F \quad \text{.....(11)}$$

Subtracting (5) from (10) and (6) from (11) and equating the results, one finds:

$$(u_{t_M} - u) = \frac{1}{K}(i - i_{t_M}) \quad \text{.....(12)}$$

represented by the line $t_M N_t'$ (fig. 2) with angular coefficient $\tan \gamma' = -\frac{1}{K}$, that is, symmetrical to the preceding one with respect to a horizontal through point t_M.

3. Law of propagation of plane waves

Equations (9) and (12) express a universal property common to all propagations of plane waves as follows:

For an observer who travels at the velocity of the waves, the increase in u and the increase in i are always in a constant ratio, or, in other words, the physical dimension u expressed as a function of the physical dimension i lies on a straight line which passes through the figurative point t_M of the regimen u_{t_M}, i_{t_M} at time t and at position M of the starting point.

The coefficient of proportionality, that is, the slope of this line, is $\pm \frac{1}{K}$, the sign depending on the sense of displacement of the observer. The term K is a constant of the medium in which the phenomenon is produced.

In attempting to consolidate this law one must realize that during its propagation a wave F stays constant, but, at the moment of its passage through a place where it encounters another phenomenon, whatever it may be, the dimensions u and i at this place change because of the meeting, not in a haphazard manner but change linearly with respect to the magnitudes which have been established previously or which will be established later by the wave under consideration.

If the phenomenon encountered is another wave f circulating in an inverse sense, it imposes an identical law and the dimensions u and i will be given, at the moment and place of encounter, by the point of intersection of the two straight lines, one referring to wave F and the other to wave f because the two waves, being coexistent at the same place, must have there the same regimen u, i. Then, the two waves separate and continue their courses, always remaining constant and always testifying that the dimensions u and i are on their respective lines but at other points which depend on their new adventures. If, for example, the wave F does not encounter anything more, the dimensions u and i at the passing points take on again the values which characterized the starting point, which shows clearly the perpetuity of this wave, subject to all the modifications of regimen provoked by incidents along the route.

If, finally, the wave F encounters a limit, it will always be defined by a function of two dimensions u, i. If it is an explicit func-

tion $u = Y(i)$, the regimen u_Y, i_Y at the instant of the arrival of the wave at the limit will be given by the intersection of the lines referring to the wave F and the function Y. But that is the end of the wave F, or, more properly, it breaks into one or more waves which pass through the limit, one of which returns (partial reflection) or even returns completely (total reflection). In any case it would be useless to calculate what becomes of it because, the point u_Y, i_Y being known, the lines referring to new waves before all of them pass by this point will be perfectly determined since their angular coefficient $\frac{1}{K}$ depends only on the medium of propagation and is not influenced by the limiting condition Y.

One can now follow these waves anew and, without knowing them but knowing their characteristic line, can find directly the regimen u, i at every place where they pass, as in the preceding case.

This is the principle of the graphical method. The phenomenon taken at its known origin u_o, i_o will be followed by one or more observers who travel through the medium being considered in the same time as the waves and see with their own eyes what happens to the regimen u, i in terms of time and space without ever computing the magnitude of the waves that they are following because the line referring to these waves will always be known to them from the regimen u_Y, i_Y, determined at the last limit left by the observer.

Although a calculation permits him to also find, with the same equations, the regimen u, i one can see that the method whose principles have just been sketched differs greatly from a computation. This utilization of a physical law,[1] universal for all mediums where a phenomenon is

[1] We excuse ourselves for perhaps abusing the word physical, but it is mainly the property of the method being proposed to avoid every abstraction and to follow the events in their reality and not by the aid of mathematical symbols.

propagated, is applicable for a mobile observer who travels with the velocity of propagation and serves much better than a method of calculation because it is a veritable real experience that the observer carries out and that he in-

terprets by proportion with rule and square. The operation has a concrete reality for him which excludes every possibility of error and renders it practical for every engineer, even the nonspecialist. It is eminently a method for engineers and not for savants, even though it is absolutely rigorous and permits a resolution of problems of a complexity before which the greatest calculators would be obliged to declare themselves forfeit.

Let us add that the trace of the diagrams by the repeated application of the same law does in a way assist experience itself and makes it so that no detail of the phenomenon passes without notice. This develops intuition so well that after having resolved several varied cases, the engineer can foresee with great sureness the effects resulting from such and such modification to the system; his faculty for research or invention is enriched by it.

We will now develop this method for several fields of mechanics by establishing it each time through a simple demonstration, using only the fundamental theorems and the elements of mechanics.

CHAPTER V

13. Pressure waves along a pipe full of liquid, otherwise called water hammer.[1]

[1] It is in this field that the graphical method appeared. Löwy, who gave it in his book, Druckschwankung in Druckrohrleitung (1928) attributed it to Kreitner; then O. Schnyder developed it and applied it to centrifugal pumps in an article in Wasserkraft und Wasserwirtschaft in 1931, and it is there that we knew it. Since then, O. Schnyder has first published his extension to conduits with multiple characteristics, that is, variable section, or with branches, and rendered account of head losses in Wasser u. Wasserwat, Nos. 5 and 6 (1932) and No. 12 (1935). However, we have discovered these extensions on our own without knowing of the later works of Schnyder and we have given demonstrations and a different presentation, more physical than algebraic, in one memoire to the Societé Hydrotechnique de France, Rev. Gen. de l' Hyd, Nos. 1 and 2, and these demonstrations have led us little by little to the generalization for all the fields of mechanics that we are presenting today under a form which has arrived, we hope, at its definitive simplicity.

Let figure 25 be a section of pipe XY of cross-sectional area S and of constant thickness e full of a fluid under a pressure of h_o meters of water and flowing with an initial steady velocity C_o. If, due to any cause, the velocity at X is made to change from C_o to C_1, the pressure will change from h_o to $h_o + F$ and this disturbance will propagate itself toward Y at the speed a. It is shown on the figure at AB over a length at' and a second afterwards appears a meters farther at $A'B'$. It is the same case as paragraph 4, at each second a cylinder of length a, of section S, of mass $\frac{\omega}{g} a S$, changes from the velocity C_o to the velocity C_1. The variation per second in the momentum of the water column is $\frac{\omega}{g} a S (C_o - C_1)$, and the magnitude is equal to the sum of the external forces acting on the column, viz.,

$$\omega(h_o + F) S - \omega h_o S = \omega F S$$

Simplifying this, we get

$$\frac{a}{g} (C_o - C_1) = F \quad \text{.....} (48)$$

In the most general case, another cause at Y changes the velocity from C_1 to C and the pressure from $h_o + F$ to $h_o + F + f$, and this change is propagated towards X at the velocity a, the change in momentum per second is $\frac{\omega}{g} a S (C_1 - C)$, and this is equal to the sum of the external forces $\left[\omega(h_o + F) S - \omega(h_o + F + f)\right] = (-\omega f s)$ which gives:

$$\frac{a}{g}(C_1 - C) = -f \quad \text{.....(49)}$$

Adding (48) and (49)

$$\frac{a}{g}(C_o - C) = F - f \quad \text{.....(50)}$$

and since the total pressure h is equal to $h_o + F + f$

$$h - h_o = F + f \quad \text{.....(51)}$$

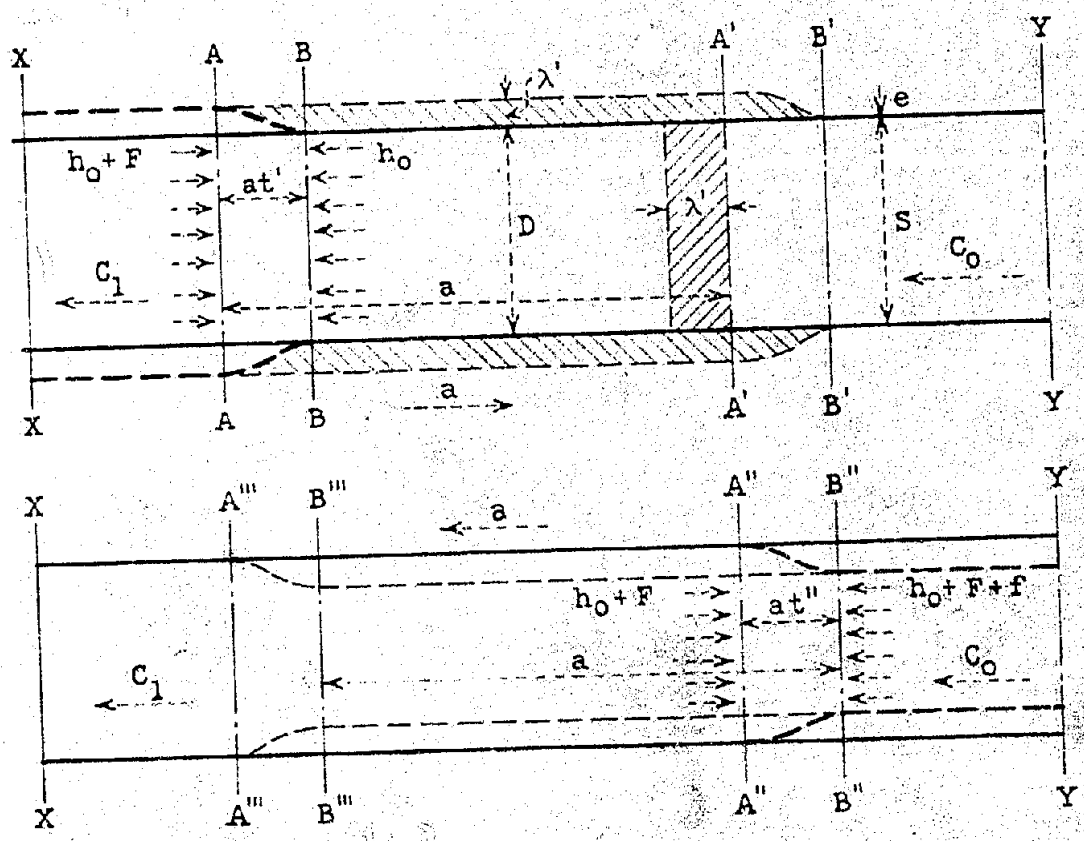


Figure 25

We again have the two general equations, but for convenience in drawing we will insert the discharge $q = CS$ in equation (50) which is written

$$\frac{a}{gS}(q_o - q) = (F - f) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (52)$$

Now take any point M (fig. 26) where the regimen is h_i, q_i, at time i, represented by the point i_M of ordinate h_i and abscissa q_i. This regimen results from the passage of two waves F_i and f_i which cross each other at M.

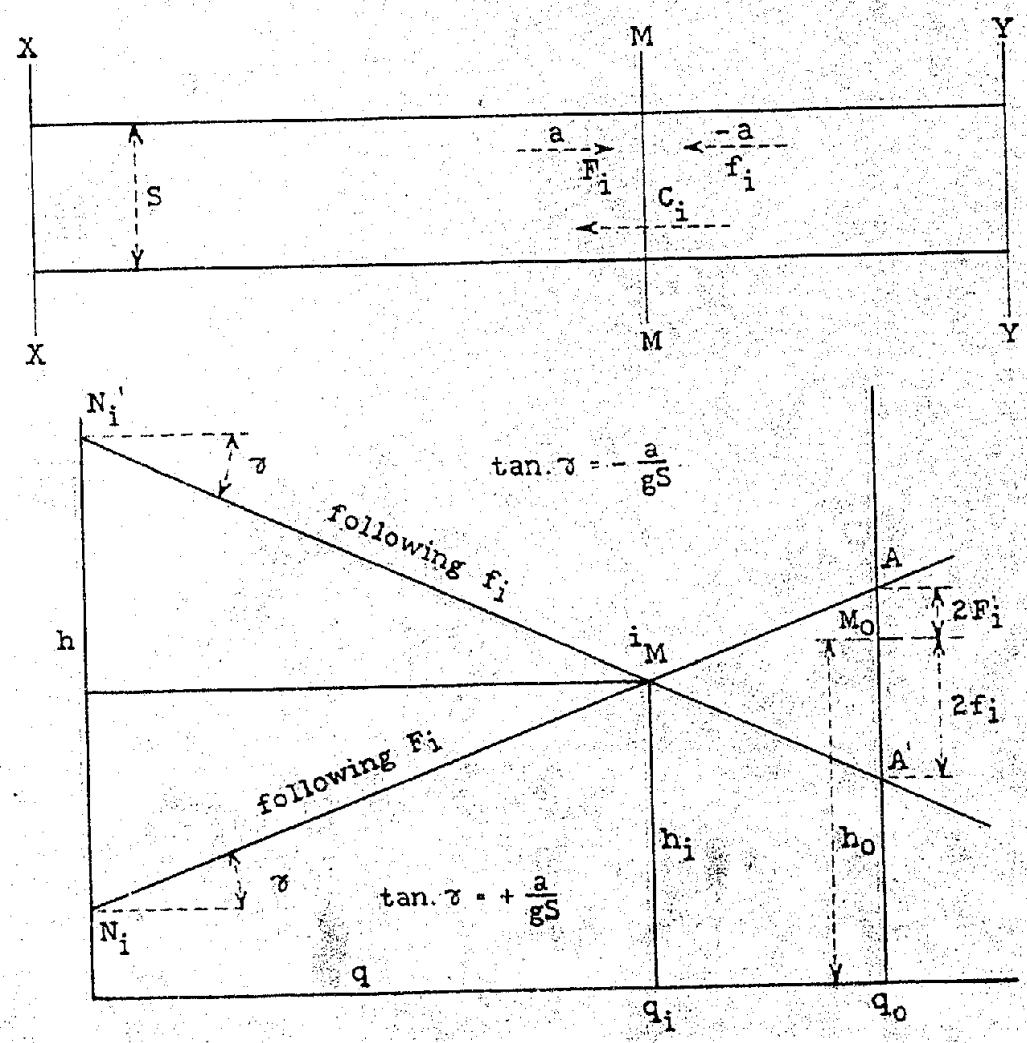


Figure 26

By applying equations (50) and (52) at this place at time i, for an observer who follows the wave F_i one finds, in operating as for equations (5) to (8),

$$h_i - h = \frac{a}{gS}(q_i - q) \quad \text{.....(53)}$$

which is the law relating h and q for this observer. This law is represented by the line AN_i with a slope $+\frac{a}{gS}$ and passes through the point i_M of the regimen at the time and place of the departure of the observer.

In like manner, for another observer leaving M at time i, but following the wave f_i, one finds the law

$$h_i - h = \frac{a}{gS}(q - q_i) \quad \text{.....(54)}$$

represented by the line $A'N_i'$ with a slope $-\frac{a}{gS}$ passing through the same point i_M, that is, symmetrical to the preceding one with respect to a horizontal through point i_M.

It will be noted that the sense of the velocity C_1 was chosen as positive; so the slope of the line is negative for the observer who travels in the direction of the velocity and positive for the one who travels in the opposite direction. The positive values of h correspond to a pressure rise.

14. Calculation of the velocity a.

Let us return to figure 25 and consider the cylinder $AAA'A'$. In one second a quantity of C_1S flows out across the section AA and a quantity of C_oS enters through the section $A'A'$. The difference $(C_1 - C_o)S$ is then accumulated in the cylinder. But the surface $A'A'$ advances during this time by the compression λ' of the water whose pressure increases by F over all the length a and this permits the cylinder to receive a volume $\lambda'S$ of supplementary water. On the other hand, under the pressure rise F the pipe expands by λ'' which permits it to receive a volume $\pi D\lambda''a$ of supplementary water. These are the two volumes which must absorb the excess discharge and one has:

$$\lambda' S + \pi D \lambda'' a = S(C_o - C) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (55)$$

From the law of elasticity, $\lambda' = \frac{(\omega F)\, a}{\varepsilon}$ where ε is the coefficient of elasticity of the fluid. In order to find λ'' let us recall that the stress in the metal of the conduit corresponding to a pressure rise ωF is $\phi = \frac{(\omega F)\, D}{2e}$, and that this stress produces an elongation of the radius $\frac{D}{2}$ equal, according to the laws of elasticity, to $\frac{\phi \frac{D}{2}}{E}$ where E is the coefficient of elasticity of the metal comprising the wall of the pipe. Inserting these quantities in (55) and taking into account (48), one has, after simplification

$$a = \sqrt{\frac{g}{\omega\left(\frac{1}{\varepsilon} + \frac{D}{Ee}\right)}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (56)$$

15. Applications

We have already given a large number of examples of all kinds in the "Revue General d' Electricite" of May 14, 1932; in Nos. 1 and 2 of the "Revue Generale de l'Hydraulique," in the "Technique Moderne" of Mar. 1, 1935, and in the "Technique Moderne" of January and February 1936. Some cases, such as equalizing stacks and air accumulators, deserved to be treated by the method that we have indicated for a mass falling on a chord and for flywheels on a revolving cylinder, a method that we had not completed at the time. It solves also the problem of the accumulator with weights or with springs.

In the present study we will confine ourselves to giving two new examples, introducing first the phenomenon of cavitation, which is peculiar to hydraulics, and second, energy loss which can be applied in all the other fields.

(a) Abrupt closure of an orifice which sets up cavitation on the counterstroke. Shown in figure 27 is a conduit AB of length ℓ which discharges into atmosphere at A with a discharge q_o under a constant head h_o.

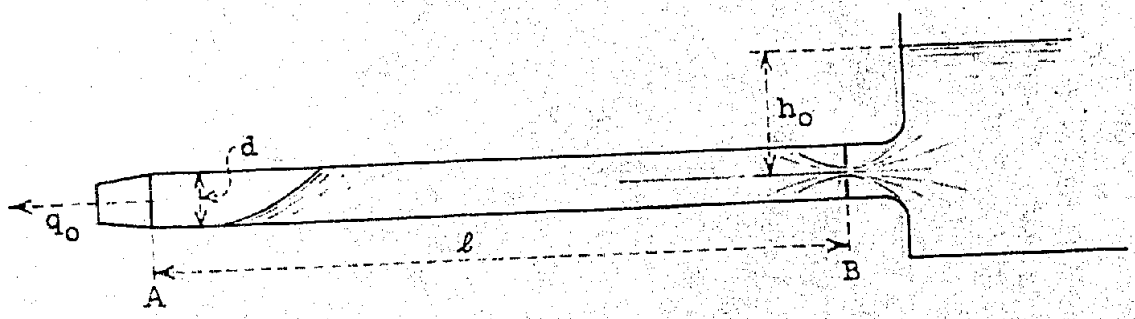


Figure 27

We will take the case where the loss of head due to friction on the pipe A B is important. The effect of the loss of head is a drop in pressure proportional to the square of the discharge, taking place linearly over the entire length A B. One can conceive of it as due to an infinite number of discontinuities, scattered regularly along A B, each one of which imposes a drop $\Delta h = kq^2$ from one side to the other of the discontinuity. One will obtain an approximation by substituting for this infinite number of discontinuities a limited number which produce the same total loss.

The simplest scheme would be a single discontinuity creating the total loss, for example, a unique diaphragm placed at B. In this case the setup is too coarse to restore the reality of the phenomenon along the entire length A B, but it will allow us to find with accuracy the effect of the loss on damping, and this will suffice for the first example. The reservoir and the diaphragm constitute a discontinuity whose equation is

$$h = h_o \pm kq^2$$

where h is the pressure in the conduit against the diaphragm and the positive sign + refers to the sense of the flow from B toward A while the negative sign - refers to flow from A toward B. This function is represented on the graph (fig. 28a) by two parabolas N' M and M N.

The orifice A is a discontinuity with the equation

$$h = Kq^2$$

where h is the pressure in the conduit at the last section of diameter d

in front of the orifice. It is represented by the parabola Y_o. The initial permanent regimen is given by point O_A, the intersection of these two curves.

For an observer who arrives at A coming from B the characteristic line will be O_AP which passes through O_A and which has a slope of $-\frac{a}{gS}$. If this observer arrives at just the moment of the closure, he will see a zero discharge, that is, the regimen given by point O_A' on the axis of the ordinates. The ordinate of this point fixes the pressure rise from sudden closure. For the observer returning to B the characteristic line passes through O_A' and its slope is $+\frac{a}{gS}$. Upon his arrival at B in time $\frac{l}{a}$, chosen as unity, the characteristic curve of the discontinuity is $N'M$; the figurative point for the regimen will be the intersection $1'_B$ of the line and the curve.

Returning to A, the observer will have as the characteristic line a line parallel to O_AP passing through $1'_B$. Having arrived at A at time 2, he will find the orifice closed, but the abscissa of the characteristic line corresponds to an inverse discharge which is not opposed by the closed orifice. This inverse discharge having taken place, the pressure will decrease and will have as a limit zero pressure, with the result that the characteristic curve of the discontinuity at A is now the horizontal of ordinate $-h_a = -10$ m, 33 (33.9 ft.) or rather the vapor tension of the fluid. The figurative point of the regimen will be the point $2'_A$. The intersection of these two lines and the value of the inverse discharge at A will be given by the abscissa of this point. At this discharge a cavity begins to form, as indicated in figure 27.

From this moment the goings and successive returnings of the observer correspond to the trace of the broken line $2'_A$, $3'_B$, $4'_A$, $5'_B$, $6'_A$, whose peaks are alternately on $N'MN$ and on the horizontal $-h_a$.

The cavity at A continues to increase up to time 10 at which time the discharge changes suddenly from inverse discharge 10_A to direct discharge $10'_A$. The apostrophe indicates that the regimen exists at time

$10 + \varepsilon$, the absence of the apostrophe that it exists at time $10 - \varepsilon$. Let us trace on figure 29 the curve of discharge as a function of time. The cavity at A will be given by the area of this curve. It increases from time 0 to time 10, then diminishes by the appearance of the positive portion corresponding to direct discharge. In computing it by proportion, which is easy since the intervals of time are equal, one finds that it is zero at time 19.3, that is, the cavity closes up at this instant.

At time $19.3-\varepsilon$ the figurative point was 19.3A on the horizontal $-h_a$ and on the characteristic line $17'_B\ 18'_A$, which is valid for time 18. This latter is still the characteristic line for an observer who arrives at A at time $19.3 + \varepsilon$, but at this instant, the cavity being closed, the discharge is zero and the regimen becomes suddenly that of point $19.3'_A$ on the axis of the ordinates, but this regimen will last only until time 20. At time $20 + \varepsilon$, returning to A, the first observer who left A at time $18 + \varepsilon$, point $18'_A$, arrived at B at time $19 + \varepsilon$, point $19'_B$, and finding the discharge to be zero at his return sees the regimen of point $20'_A$. There results from this a tooth on the pressure curve (fig. 29).

The diagram continues in figure 28b by now having a second observer who leaves A at time 19.3, whereas the first one leaves there at time 20, and to each one there corresponds a broken line (fig. 28b). the cavity recommences and closes up in this case at time 35, which gives the pressure rise of point $35'_A$ and requires the presence of a third observer to continue the diagram. A new cycle starts which finishes at time 48.5 (fig. 28c) with the pressure rise $48.5'_A$ and requires a fourth observer. Figure 28d gives the fourth cycle, which ends at time 60.6 and one could continue it. Figure 29 represents the pressures and discharges as a function of time, and it can be seen that the durations of the pressure rises are always $\frac{2l}{a}$, but the duration of the pressure drops decreases in geometric progression, and this is due to the loss of head which cancels, each time, the same proportion of the kinetic energy placed in action by contraction of the pipe. The mean ordinate of the pressure rises decreases

following the same geometric progression, and for the same reason. However, during the duration of a pressure rise there appears some notching which arises from the fact that the duration of the cavity is not a multiple of $\frac{2\ell}{a}$ and this, in each cycle, increases by unity the number of observers and that of notches.

This reconstruction, obtained so simply, of a phenomenon so complicated, is very instructively confirmed in the oscillograms picked up by M. A. Langevin, with the aid of piezoelectric quartz.[1] Figure 30 represents one

[1]See A. Langevin, Bulletin de l' Union technique du Bâtiment (1928).

of these oscillograms. The analogy is striking even in the notches of the zones of pressure rise, which indicates the explanation of one of their causes.

(b) Sudden inflow and sudden suppression of flow in a canal. The flow in this case occurs in free air, which implies very different basic conditions from those which define flow in a pipe. Nevertheless, one has the same equations, subject to the following reservations, which constitute an approximation which is sufficiently close for problems generally met in practice: (1) the height of the wave is small compared to the depth H of the canal and (2) the velocity of flow c of the water in the canal is small compared to the velocity of the waves.

A canal of width b shown on figure 31, in which the velocity changes from C_o to C_1 in AB, the surface rises by distance F. A second later this change will be reproduced at A'B' at a distance of a meters, a being the velocity of the wave.[2]

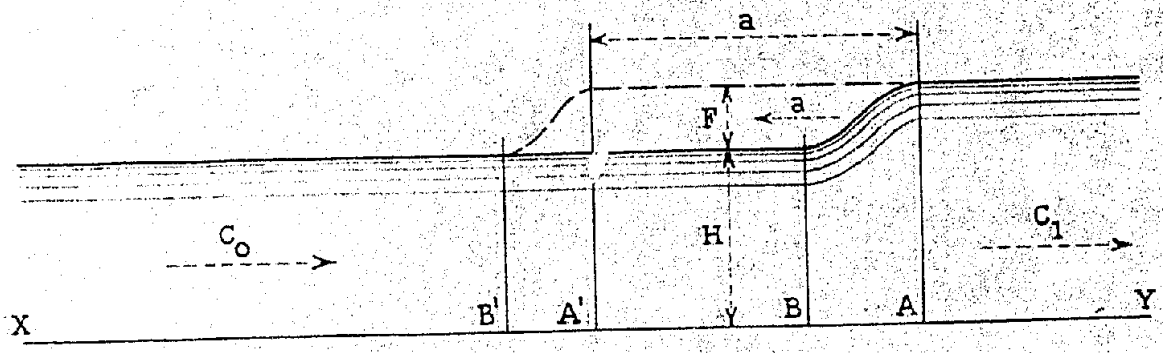
[2]Because one neglects C_1, compared to wave velocity.

The variation in momentum per second $(\frac{\omega}{g} a b H)(C_o - C_1)$ will be in equilibrium with the force $(\omega F)(b H)$ due to the rise in water surface,[3]

[3]This value is obtained by neglecting the force exerted on the height F of the wave.

and from this results

$$F = \frac{a(C_o - C_1)}{g} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (57)$$



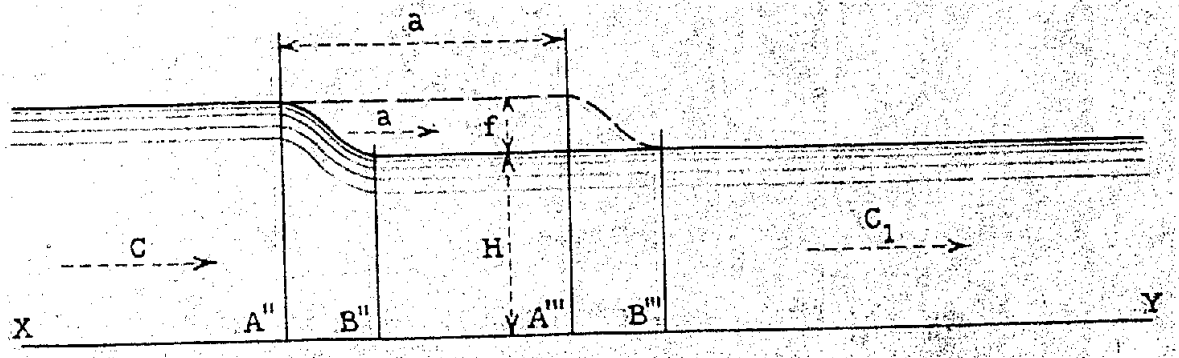


Figure 31

In like manner, if the velocity changes from C_1 to C in $A''\ B''$, the water surface rises by amount f and this variation is reproduced a second later at $A'''\ B'''$ and, since the force now is $(-\omega f)\, b H$, one will have:

$$-f = \frac{a(C_1 - C)}{g} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (58)$$

and adding (57) to (58)

$$\frac{a}{g}(C_o - C) = F - f$$

or

$$\frac{a}{gS}(Q_o - Q) = F - f \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (59)$$

where S is the cross-sectional area b H of the canal, and for the total height h,

$$h - H = F + f \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (60)$$

which are the same equations as the preceding ones.

On the other hand, the volume $(C_o - C)(bH)$ [1] due to the difference

[1] Obtained by neglecting F compared to H in section B.

in discharge at B' and at B having gone into the rise F over a length a one has,

$$(C_o - C_1)bH = a \cdot b \cdot F$$

and by replacing F by (57)

$$a = \sqrt{gH} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (61)$$

We can now apply in this case exactly the same construction we found for conduits. Let us study now the case shown schematically in figure 32a. At the extremity a of a canal ax one suddenly admits a discharge Q_o which remains constant for a certain time T, then is interrupted suddenly (take the starting and stopping of a hydroelectric plant, for example) and one wishes to find the profile of the surface of the canal, or river, as a function of time. In order to account for loss of head we will suppose for each length δ a constriction equivalent to the friction loss in this length and we shall obtain a curve in steps for which the real curve is the limit. We will take as a unit of time $\tau = \frac{\delta}{a}$, the time of passage be-

tween two constrictions.

The initial regimen is $(Q = 0)$, $(h = H)$ for which the figurative point is 0 and the characteristic line for all observers leaving from b, c, d . . . . m, at the moment when this regimen exists is $0\,0_A$ with slope $+\frac{a}{gS}$ where S is the cross section of the canal taken as rectangular and $a = \sqrt{gH}$. If the observers leap over the diaphragm at which they arrive, the height increases by $k\,q^2$, that is, the figurative point is then on the parabola 0 P, drawn with broken line from the ordinate $Y_o = k\,Q_o^2$ for abscissa Q_o. For each regimen at b, c, d . . . . m, one will have two points with the same abscissa; one on the characteristic line which is valid up to the downstream side of the diaphragm, the other on the parabola which is valid up to the upstream side. The first will be denoted on the diagram by the index ' and the second by the index ''.

The reader who is familiar with the construction of the diagrams from the preceding studies will be able to follow this in the table No. 32.

We will find only the points at time intervals of 2τ. The discharge Q_o starts at (a) at time 0 and stops at time 8.

We will designate by Y_o the vertical at abscissa Q_o which is the characteristic curve at (a) from 0 to time 8, inclusive, and by Y_{10} the vertical of abscissa zero which is the characteristic curve at (a) from time $10 + \varepsilon$ and beyond. The diagram is followed best in the order indicated by the table. It is stopped at time 10 at (a) but the construction repeats itself in identical form and the reader can continue it without any trouble.

By taking all the points with the same index, one can trace (fig. 32b) the curves of the height of the wave as a function of time at a, b, c, d, ... m, which shows clearly the effect of the head losses on the form of the wave with respect to time and its damping effect with respect to distance.

CHAPTER VI (Last Paragraph Only)

19. Conclusion

For the majority of engineers a graph is more eloquent than a computation and this is sufficient to arouse an interest in the method which has been demonstrated. If a person does not have a terror of errors in sign and of the nervous tension which leads to these errors, he has never made computations. A drawing avoids all this in an absolute manner because an error in location of the lines is literally impossible without introducing into the drawing that which would be apparent to even lesser engineers.

The examples which have been given are sufficient to prove that the application of this method does not require any special knowledge of mathematics but only a knowledge of the principles of mechanics in order to correctly establish conditions at the limits. It is here that the possibility of error exists, but the same is true of any method. It is therefore within the range of every nonspecialized engineer and not monopolized by the mathematicians. This derives from the fact that the law relating the two variables, which define the state of the medium, has the simple form of a straight line with a slope depending only on the medium, for an observer moving with the wave velocity, and the repeated application of the method leads to the construction of a drawing which is nothing more than the execution at a slower pace of the experience being studied.

For these reasons the graphical method is called upon to render to all wave problems the same service that graphical statics renders to problems in the resistance of materials. We will be content if our presentation contributes toward promotion of its use in research offices where a number of engineers will be able to use it to handle problems which would otherwise confound them. It is fitting to recognize the men who have promoted this method for use in hydraulics; the Austrian engineer Löwy[1] and the Swiss engineer O. Schnyder whom we have already cited.

[1] Löwy makes reference in his book to the previous work of Kreitner whom we have already recognized.

TABLE
To be used with Figure 32A

Upon his arrival at place	At time	Observer leaves from place	At time	States slope point is on	Passing through point in leaving	Increased by head loss	On following curve	Intersection	Which gives in addition the point
a	zero	b	-1	+ tan γ	0		Y_o	0_a	
b	1	a	zero	- tan γ	0_a			1_b	$1_b''$
		c	zero	+ tan γ	0	Z_o			
a	2	b	1	+ tan γ	$1_b'$		Y_o	2_a	
c	2	b	1	- tan γ	$1_b''$				
		d	1	+ tan γ	0	Z_o		$2_c'$	$2_c''$
b	3	a	2	- tan γ	2_a				
		c	2	+ tan γ	2_c	Z_o		$3_b'$	$3_b''$
a	4	b	3	+ tan γ	$3_b'$		Y_o	4_a	
d	3	c	2	- tan γ	$2_c''$				
		e	2	+ tan γ	0	Z_o		$3_d'$	$3_d''$
c	4	b	3	- tan γ	$3_b''$				
		d	3	+ tan γ	$3_d'$	Z_o		$4_c'$	$4_c''$
b	5	a	4	- tan γ	4_a				
		c	4	+ tan γ	$4_c'$	Z_o		$5_b'$	$5_b''$
a	6	b	5	+ tan γ	$5_b'$		Y_o	6_a	
e	4	d	3	- tan γ	$3_d''$				
		f	3	+ tan γ	0	Z_o		$4_e'$	$4_e''$
d	5	c	4	- tan γ	$4_c''$				
		e	4	+ tan γ	$4_e'$	Z_o		$5_d'$	$5_d''$
c	6	b	5	- tan γ	$5_b'$				
		d	5	+ tan γ	$5_d'$	Z_o		$6_c'$	$6_c''$
b	7	a	6	- tan γ	6_a				
		c	6	+ tan γ	6_c	Z_o		$7_b'$	$7_b''$
a	8	b	7	+ tan γ	$7_b'$		Y_o	8_a	
f	5	e	4	- tan γ	$4_e'$				
		g	4	+ tan γ	0	Z_o		$5_f'$	$5_f''$
e	6	d	5	- tan γ	$5_d''$				
		f	5	+ tan γ	$5_f'$	Z_o		$6_e'$	$6_e''$
d	7	c	6	- tan γ	$6_c''$				
		e	6	+ tan γ	$6_e'$	Z_o		$7_d'$	$7_d''$
c	8	b	7	- tan γ	$7_b''$				
		d	7	+ tan γ	$7_d'$	Z_o		$8_c'$	$8_c''$
b	9	a	8	- tan γ	8_a				
		c	8	+ tan γ	8_c	Z_o		$9_b'$	$9_b''$
a	10	b	9	+ tan γ	$9_b'$		Y_o	10_a	

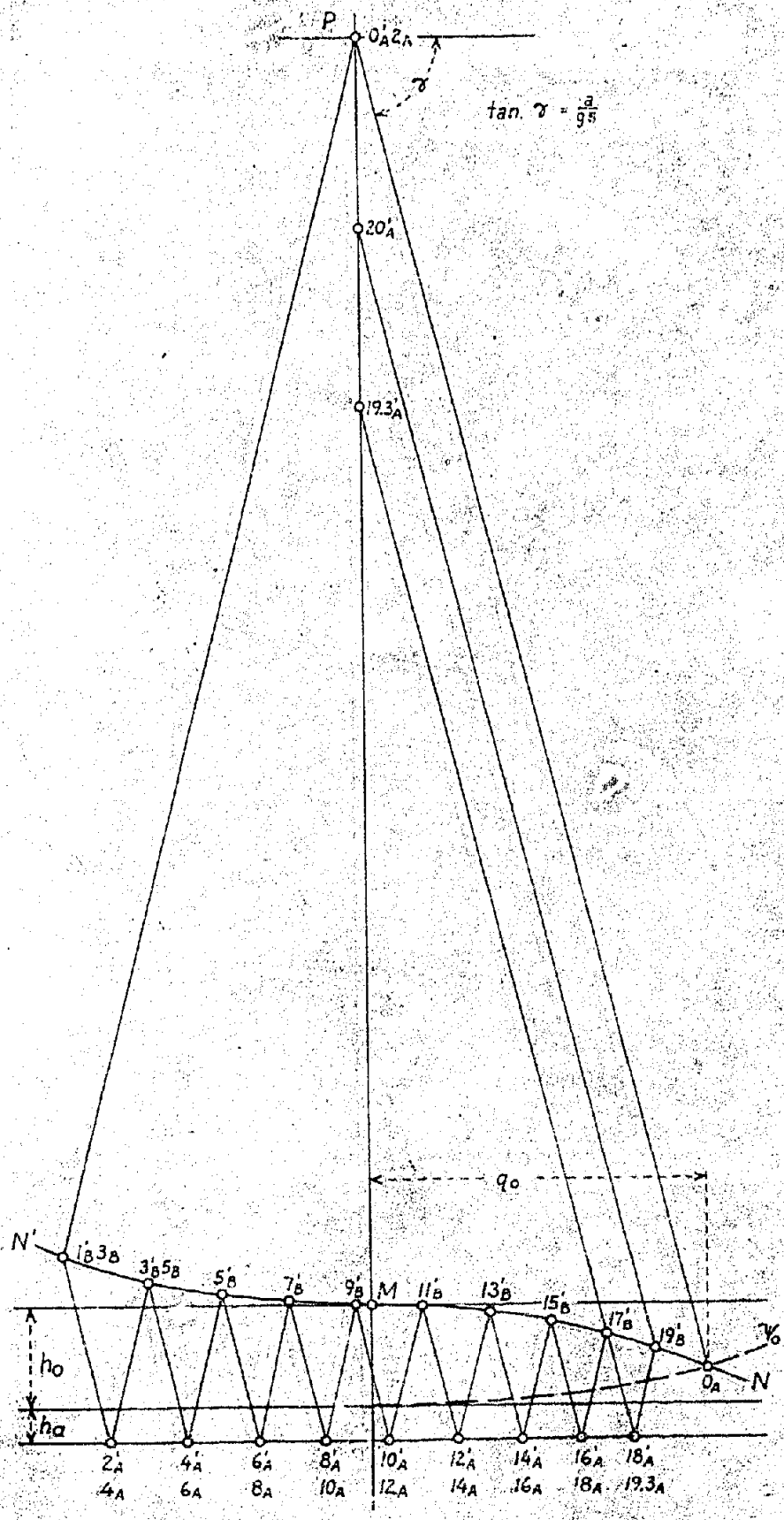

P
$0'_A 2_A$
γ
$\tan.\ \gamma = \frac{a}{g s}$
$20'_A$
$19.3'_A$
q_0
N'
$1'_B 3_B$
$3'_B 5_B$
$5'_B$
$7'_B$
$9'_B$
M
$11'_B$
$13'_B$
$15'_B$
$17'_B$
$19'_B$
γ_0
0_A
N
h_0
h_a
$2'_A$ 4_A
$4'_A$ 6_A
$6'_A$ 8_A
$8'_A$ 10_A
$10'_A$ 12_A
$12'_A$ 14_A
$14'_A$ 16_A
$16'_A$ 18_A
$18'_A$ 19.3_A

Fig. 28 a

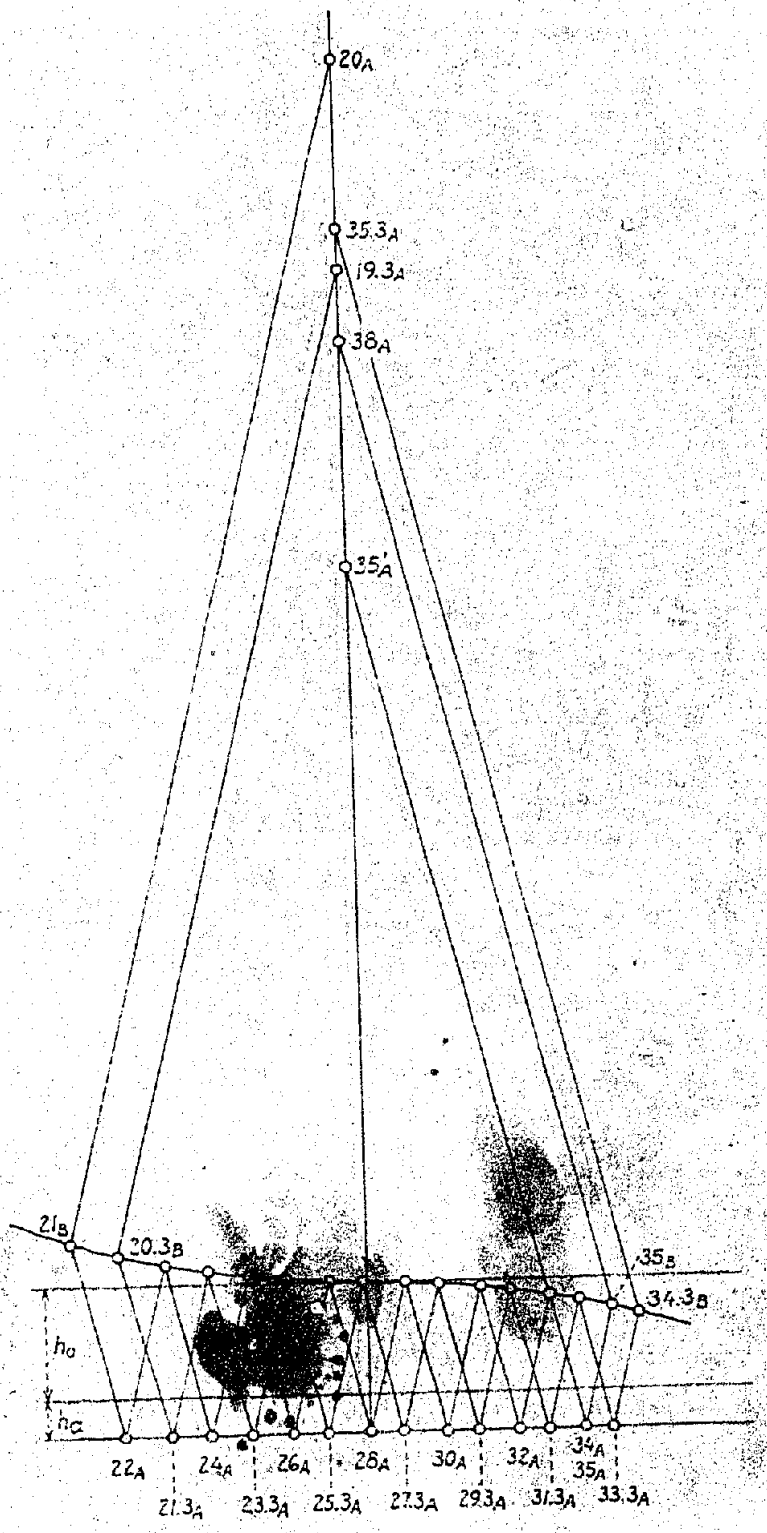

20A
35.3A
19.3A
38A
35'A
21B
20.3B
35B
34.3B
hb
hc
22A
24A
26A
28A
30A
32A
34A
35A
21.3A
23.3A
25.3A
27.3A
29.3A
31.3A
33.3A

Fig. 28 b

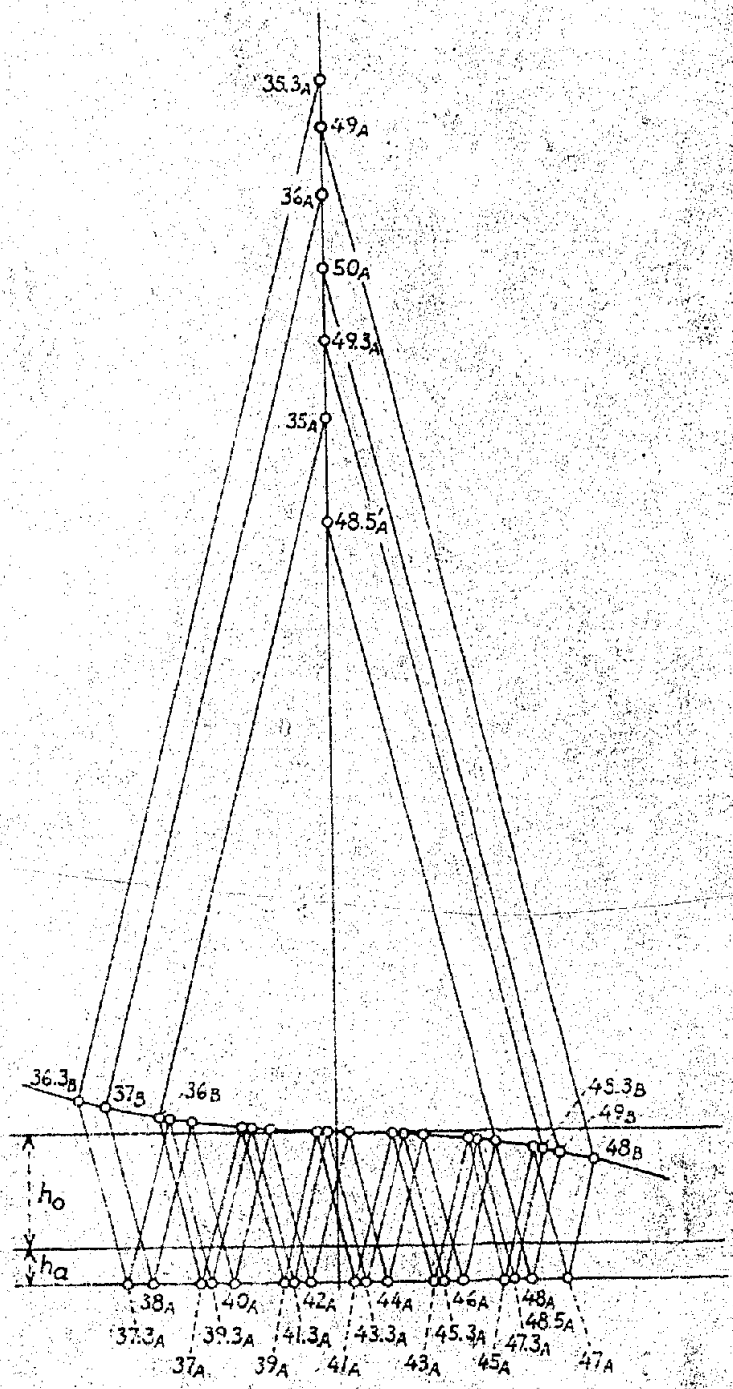

35.3A
49A
36A
50A
49.3A
35A
48.5'A
36.3B
37B
36B
48.3B
49B
48B
ho
ha
38A
40A
42A
44A
46A
48A
48.5A
37.3A
39.3A
41.3A
43.3A
45.3A
47.3A
37A
39A
41A
43A
45A
47A

Fig. 28c

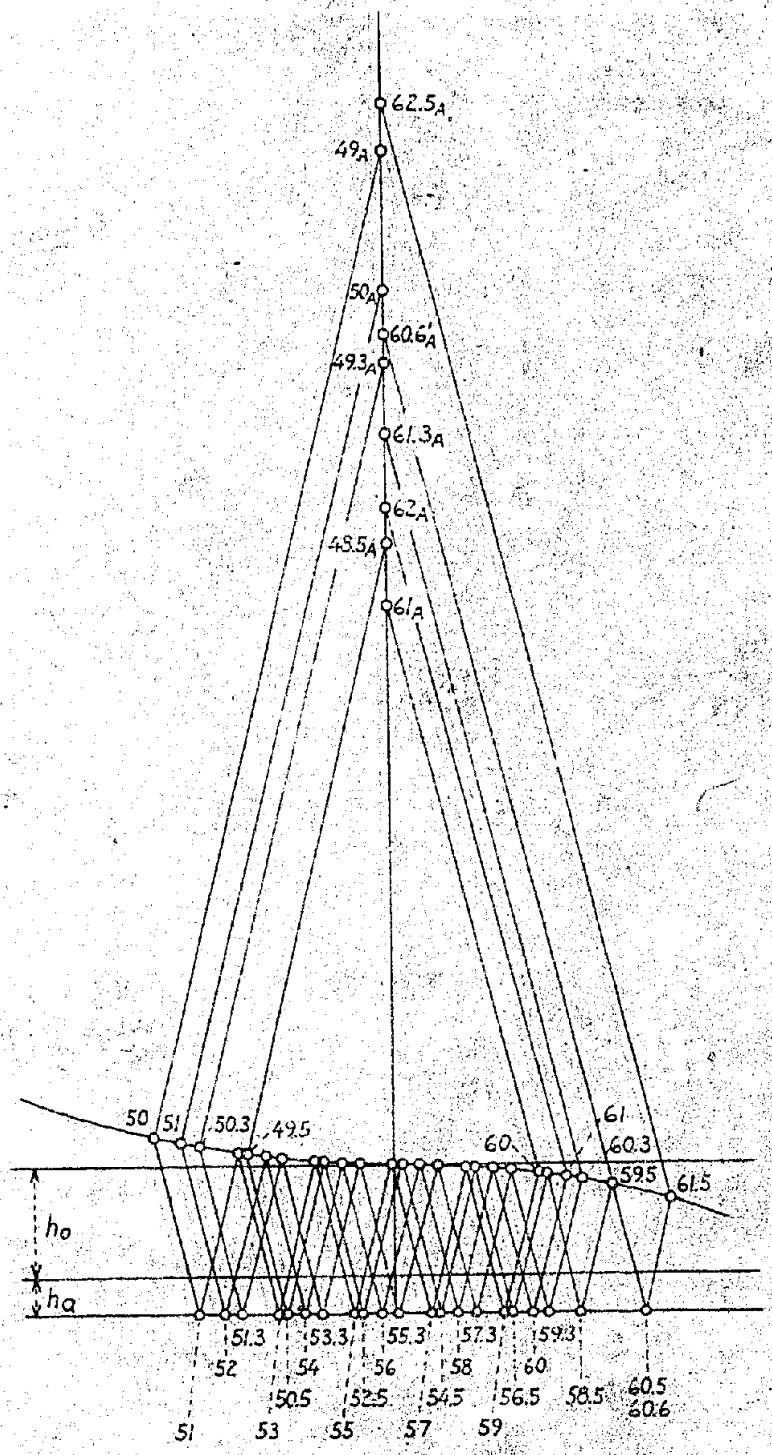

62.5A
49A
50A
60.6'A
49.3A
61.3A
62A
48.5A
61A
50
51
50.3
49.5
60
61
60.3
59.5
61.5
ho
ha
51.3
53.3
55.3
57.3
59.3
52
54
56
58
60
50.5
52.5
54.5
56.5
58.5
60.5
60.6
51
53
55
57
59

Fig. 28 d

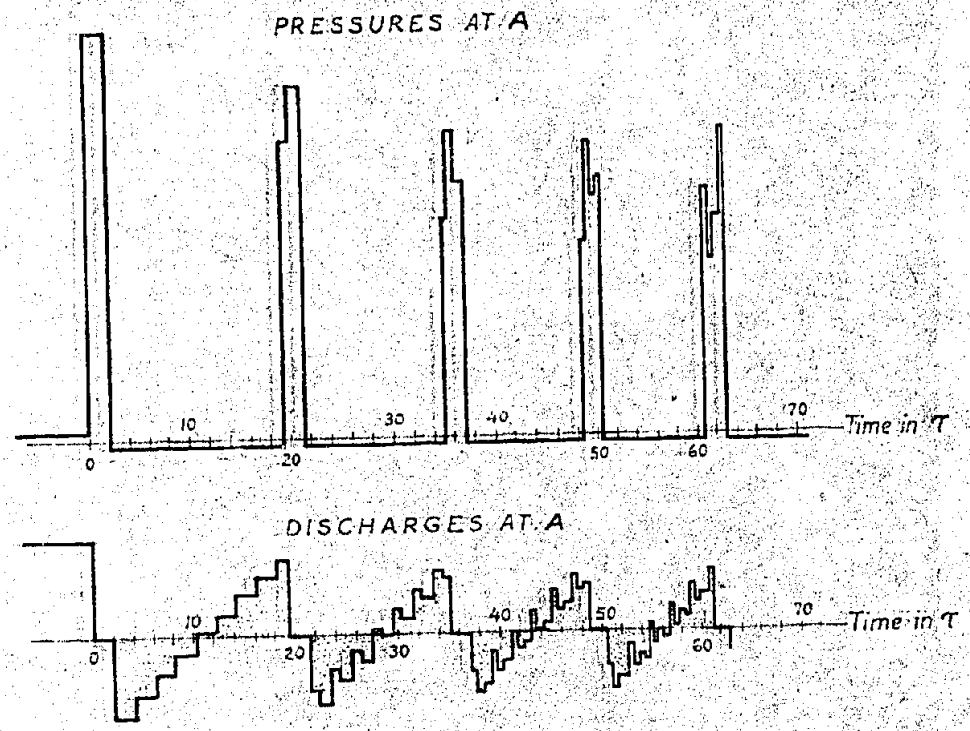

PRESSURES AT A
0
10
20
30
40
50
60
70
Time in τ
DISCHARGES AT A
0
10
20
30
40
50
60
70
Time in τ

Fig. 29

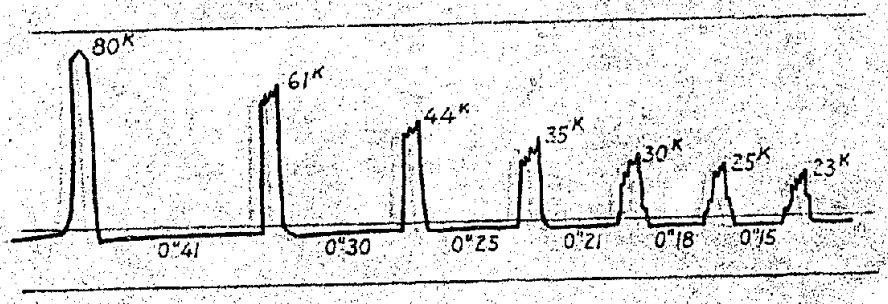

80K
61K
44K
35K
30K
25K
23K
0"41
0"30
0"25
0"21
0"18
0"15

Fig. 30

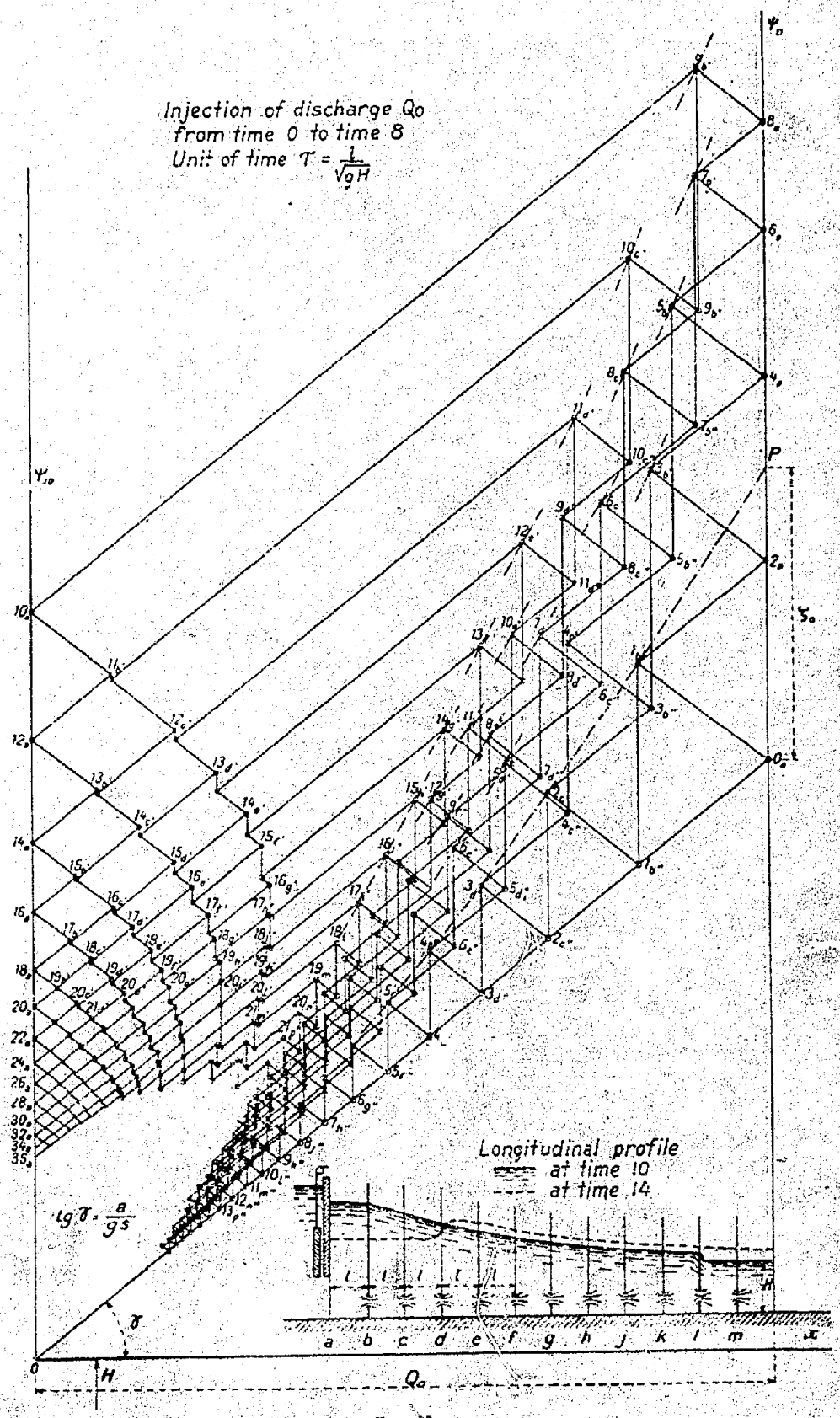

Injection of discharge Qo
from time 0 to time 8
Unit of time $\tau = \frac{1}{\sqrt{gH}}$
Longitudinal profile
at time 10
at time 14
$tg\,\gamma = \frac{a}{gs}$
ψ_0
ψ_{10}
P
ζ_0
H
Q_0
a b c d e f g h j k l m x

FIG. 32 a

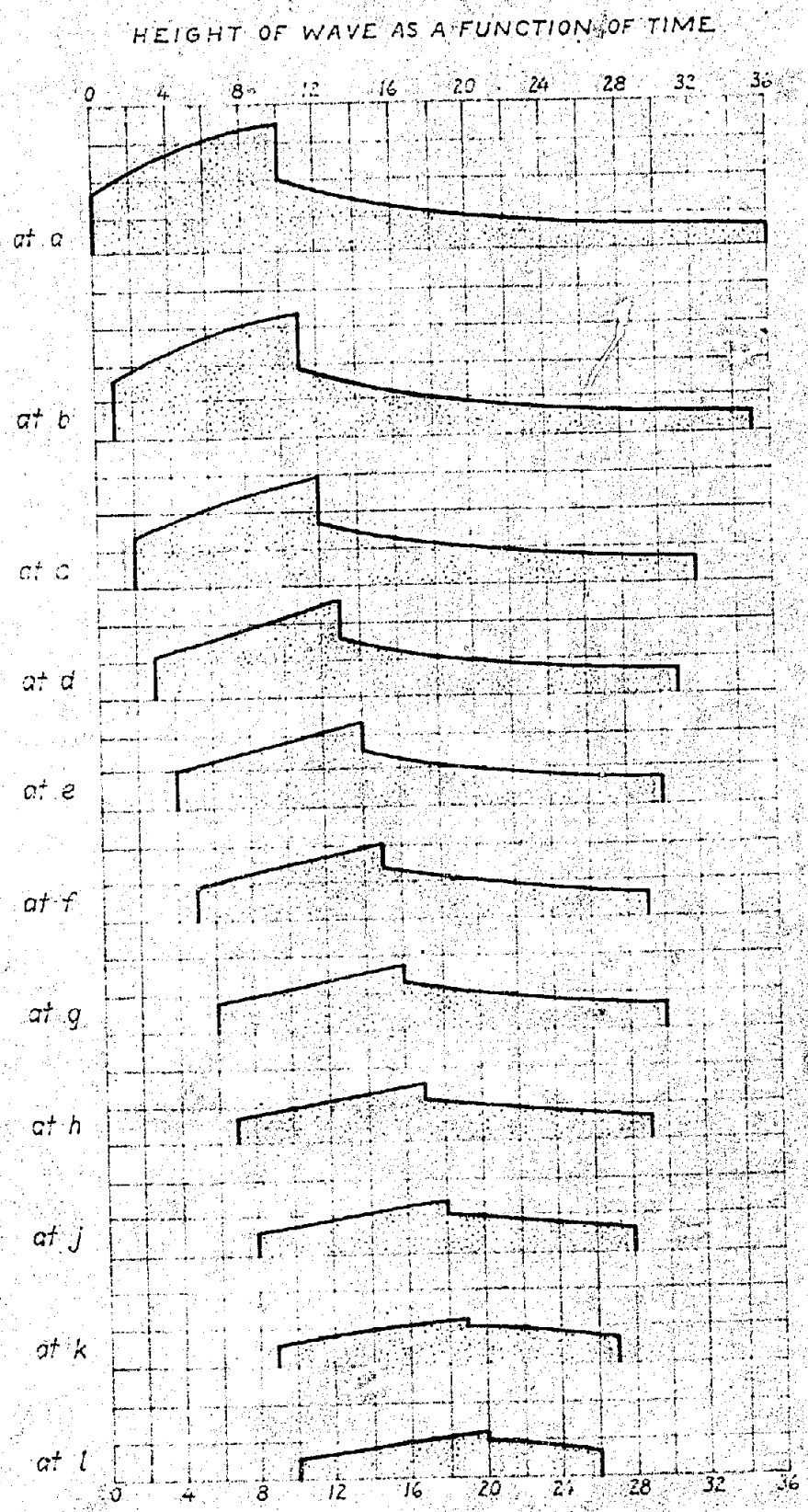

HEIGHT OF WAVE AS A FUNCTION OF TIME
0
4
8
12
16
20
24
28
32
36
at a
at b
at c
at d
at e
at f
at g
at h
at j
at k
at l

Fig. 32 b