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Discharge Curves and Equations for Yellowtail Afterbay Dam

Missouri River Basin Project, Montana

U.S. Department of the Interior
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The three gated outlet systems at Yellowtail Afterbay Dam were analyzed to develop discharge curves and equations needed for an automated gate control system intended to regulate flow rates in the Bighorn River and the Bighorn Canal. Existing and previously developed discharge curves and equations were reviewed, and new gate-controlled discharge curves were developed by analytical methods.

The overflow weir structure is equipped with 5 large radial gates mounted on an ogee crest spillway. Existing rating curves were compared to curves developed using the analytical method described in U.S Army Corps of Engineers Hydraulic Design Criteria Charts 311-1 to 311-5 and *Design of Small Dams*. The new analytically developed curves indicate slightly larger discharges, especially for large gate openings. Free weir flow occurs very seldom and the existing free flow rating curves are all similar to one another and adequate for discharge calculation if they are ever needed. The new gate-controlled flow curves use a complex definition of the effective gate opening that is not practical for field use, so they were further analyzed to convert them to a simpler form based on the vertical gate opening.

The river sluiceway and canal sluiceway systems contain vertical slide gates mounted on horizontal sills leading to sloped downstream aprons and stilling basins. Existing discharge curves and equations were compared to new analytical equations developed using a computer program, WinGate, designed to calibrate canal check gates for discharge measurement.

Previously developed discharge curves for the river sluiceway have assumed that the gates operate in free flow, but the WinGate analysis showed that submerged flow can occur about 80% of the time. The effects of submergence are small for most cases, but the data generated analytically were used to develop simplified equations that allow the submergence effects to be accounted for. The newly developed equations indicate similar, but somewhat smaller discharges than the existing curves and equations for gate openings up to 4 ft. For larger gate openings the new curves indicate much lower discharges than the older curves.

The canal sluiceway always operates in a submerged flow condition. The equations previously proposed for these gates were reviewed and physically unrealistic behavior of the equations was demonstrated. New discharge curves were developed using WinGate and the discharge coefficients needed to calculate flow with a basic orifice equation were related to the relative gate opening and relative submergence.

Two specific actions are recommended to enable implementation and further refinement of the newly developed discharge equations:

1. Validate the field calibration of the gate position sensors for the radial gates on the overflow weir structure. The sensors measure cable take-up in the gate hoist system, and this must be converted to a physical gate opening that is consistent with the equations presented in this report. Discharge equations in this report are provided in terms of both vertical gate opening (from gate lip to apex of weir crest) and minimum gate opening (distance from gate lip to a tangent to the weir crest profile). The non-linear relationship between cable take-up and one of these gate openings must be known to apply the discharge equations.

2. Once the gate opening sensor calibration is confirmed, future data (gate opening, reservoir level, river level and flow rate) can be used to validate the discharge equations. The most valuable data would be those associated with steady state conditions and times when current-meter measurements are being made to confirm the river gage rating relation, and/or times when either the overflow weir or river sluiceway discharges are known to be zero (so that flow contributions from only one outlet system could be analyzed separately). Previously recorded data could also be useful, if historic gate opening records can be accurately related back to the original physical gate openings.

Introduction

Yellowtail Afterbay Dam is a 72-ft high concrete gravity dam with embankment wings located on the Bighorn River in southern Montana, 2.2 miles downstream from Yellowtail Dam near St. Xavier, MT. The dam provides reregulation storage below Yellowtail Dam and Powerplant to stabilize flows in the Bighorn River.

Three gated outlet systems release water from the dam into two waterways:

- An **overflow weir** section equipped with five 30-ft-wide by 13.5-ft-tall radial gates releases water into the Bighorn River and serves as the spillway for the dam. Flows through this structure can be gate-controlled, or the gates can be raised out of the water to allow free weir flow over the dam crest, which is at elev. 3179.50 ft. Maximum discharge past the overflow weir is approximately 25,000 ft^3/s at reservoir elevation 3192.0 ft.

- The **river sluiceway** is a set of three 10-ft-wide by 8-ft-tall vertical slide gates that also release water to the Bighorn River at the right end of the overflow weir. The gate sill elevation is 3157.00 ft, and the maximum discharge capacity is 8,100 ft^3/s. Flow through the river sluiceway is always gate-controlled.

- The **canal sluiceway** contains two 10-ft-wide by 8-ft-tall vertical slide gates that release water into the Bighorn Canal, also designated as the BIA Canal. The gate sill elevation is 3167.00 ft, and the maximum discharge capacity of the canal sluiceway is 750 ft^3/s. Flow is normally gate-controlled, but for unusually low reservoir elevations, free weir flow is possible.

Flows in the Bighorn River below the dam and in the Bighorn Canal are measured at river gage sites operated by the Bureau of Reclamation and incorporated into the U.S. Geological Survey gaging network. Gage 06287000 (Bighorn River near St. Xavier, MT) is located in the Bighorn River about 780 ft downstream from the Afterbay Dam. This gage has provided daily discharge data since October 1, 1934. A second gage is located in the Bighorn Canal approximately 720 ft downstream from the end of the canal sluiceway stilling basin. Reclamation's Hydromet system provides daily average flows at both gages as well as reservoir elevations for the Afterbay Dam pool from October 1, 1985 to present.

The original operational scheme for Yellowtail Afterbay Dam called for the dam outlets to be operated to maintain a steady water surface elevation in the river downstream from the dam, with the intention that this would yield steady discharges in the river. However, seasonal growth of algae in the river and other factors cause the river gage rating to shift significantly over time. The result is that maintaining steady water levels in the river produces varying river discharges. To compensate, it has become necessary to make frequent changes to the water level set point in the river.

There is also a history of high total dissolved gas levels in the Bighorn River below the Afterbay Dam. The river sluiceway is the primary generator of the high dissolved gas concentrations. To control total dissolved gas levels, it is desirable to operate the dam outlets to generally maintain a 25% / 75% split of flows released through the river outlet works and the overflow weir, respectively, although this flow split ratio is subject to adjustment.

These two issues led to the development of a new Afterbay Automated Gate Control System (AAGCS) that was designed to control river flows based upon calculated flows through the gated outlets. A target flow set point is entered and the control system regulates to the set point. The system also attempts to maintain the desired flow split for control of total dissolved gas in the river. This system would allow the river elevation to vary throughout the season as the river gage rating varies, while maintaining the total discharge at the set point. Unfortunately, during commissioning of the new system, fluctuations in the river elevation exceeded allowable limits during power peaking, and gates operated excessively while attempting to maintain the set point.

As a result, the system was turned off, and the Technical Service Center was asked to review the control system and make recommendations for modifications that would improve system performance. A preliminary investigation (Fabbri and Clair 2012) by the Hydropower Technical Services Group concluded that one issue that needed to be resolved was the accuracy of the discharge equations being used in the control system. This report investigates the algorithms and equations used to compute discharge through each of the gated outlets.

Scope of Investigation

For each of the three outlet systems, polynomial equations have previously been developed for use in the automated control system. These equations compute discharge as a function of gate settings and relevant water levels. In addition, the Standing Operating Procedures for the dam provide graphical rating curves for the overflow weir and river sluiceway, based in part on the original 1:24-scale physical model study of the dam performed in 1965 (Arris 1965; Report No. HYD-523). A second 1:24-scale physical hydraulic model study of the river sluiceway (Young 1982; Report No. GR-82-5) also provides an alternative rating curve for the river sluiceway gates.

For each outlet system, this investigation reviewed and compared the polynomial equations developed for the automated control system, the graphical rating curves in the SOP, and other available rating curves. The engineering basis for each alternative method of determining discharge was reviewed. In addition, new equations or rating curves were developed analytically where appropriate. The report recommends methods for future determination of discharges through each outlet system, including calibration and maintenance of sensors that provide data needed for the flow calculation process. The report also makes recommendations for continued logging of operational data to support future reviews, calibration, and refinement of the flow control system.

Operational Records (Hydromet Data)

The Bureau of Reclamation Hydromet system provides operational data for the site using station name BHSX (Bighorn River near St. Xavier, MT). The system can be accessed on the Internet at http://www.usbr.gov/gp/hydromet/. Daily values of several useful parameters are available from October 1, 1985 to present. Some variable names are the same for both real-time and historic daily values, while others differ as indicated in Table 1.

Table 1. — Available Hydromet data for Yellowtail Afterbay Dam.

	Variable names	
Parameter	Real-time data	Historic daily values
Forebay elevation	FB	FB
River gage height	GH	GD
River gage shift	HH	HH
River discharge	QR	QRD
Total discharge (river + canal)	Q	QD
Canal gage height	CH	GJ
Canal gage height shift	HJ	HJ
Canal discharge	QC	QJ

Overflow Weir

Existing Methods for Determining Discharge

Rating Curves

The Standing Operating Procedures for Yellowtail Afterbay Dam (SOP) provides a rating curve (drawing 459-D-2146) dated July 1966 (Figure 1), showing both free weir flow and gate-controlled flow.

The curve shown for discharge over the free crest of the weir (gates out of the water) is similar to a curve provided by Arris (1965) from a 1:24-scale physical hydraulic model study that included the weir crest, but did not include functional gates (Figure 2). The Arris curve can be reproduced mathematically using $Q=5(30)[0.001131(H)^2+0.0406(H)+3.077](\mathrm{H})^{1.5}$, where H is the reservoir head above the crest in feet and Q is the discharge in ft^3/s. The reservoir head is calculated from $H = RWSE - 3179.5$, where $RWSE$ is the reservoir elevation in feet above mean sea level. The polynomial expression in brackets is the discharge coefficient.

The source of the curves in Figure 1 for gate-controlled flow is unknown, but it is likely that discharges for specific conditions were determined through some analytical procedure and curves were hand-drawn. The drawing specifically notes that the gate openings used for flow computation should be the <u>vertical</u> opening of the gate (difference in elevation between the gate bottom and the weir crest).

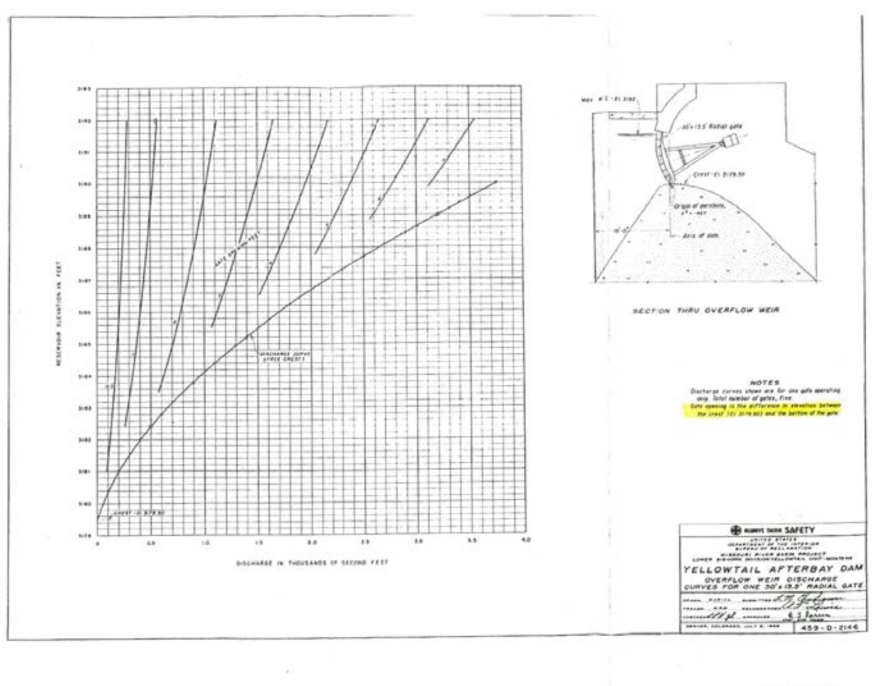

Figure 1. — Discharge curves for overflow weir, from Standing Operating Procedures.

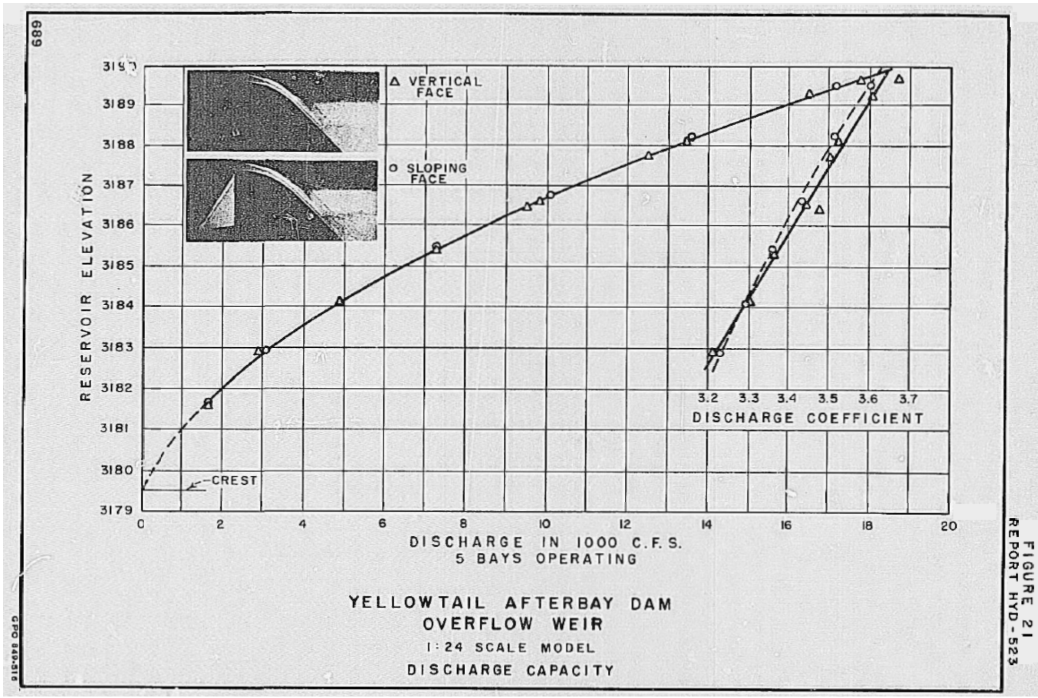

Figure 2. — Discharge curve for overflow weir crest, from 1:24-scale physical model study (Arris 1965).

Polynomial Equations

The automated control system uses complex polynomial equations to compute flow through the overflow weir structure. For free flow over the weir (gates out of the water), flow through 5 bays is computed using:

$$Q = 5(70)(RWSE - 3179.5)^{1.7} \tag{1}$$

For gate-controlled flow, the equation developed for the control system was:

$$Q = 5*(-0.011403 \cdot G^2+0.5238 \cdot G-0.022388)*(0.5731462 \cdot RWSE^3-5484.069 \cdot RWSE^2+17491232.4556 \cdot RWSE-18595929785.25) \tag{2}$$

with *G* being the gate opening in feet. This equation is problematic for use in the control system because the interim values of individual terms of the equation are so large that double-precision math is needed to maintain even marginal computational accuracy. An improved and equivalent equation that does not require double-precision math is:

$$Q = 5*(-0.01140 \cdot G^2 + 0.5238 \cdot G - 0.02239)*(0.5732 \cdot H^3 - 17.11 \cdot H + 221.3 \cdot H - 85.88) \tag{3}$$

where *H* is the reservoir head above the weir crest. The exact definition of the gate opening (vertical or otherwise) is not specified for these equations.

Gate Openings

Gate openings for radial gates on spillway crests can be defined in several different ways. The gate opening measured by instrumentation must be consistent with the gate opening definition used in any discharge curve or equation. The two most common definitions of gate opening are:

- Vertical opening – the difference in elevation between the bottom of the gate lip and the apex of the spillway crest

- Minimum gate opening – the inclined minimum distance from the gate lip to a line tangent to the spillway crest profile.

The Phase I investigation by Fabbri and Clair (2012) reported that gate position measurements for the radial gates are presently made by measuring rotation of the cable take-up drums in the gate hoist systems with a rotary encoder. These measurements have been correlated to the inclined distance between the gate lip and the apex of the spillway crest (gate seat position), and it is this parameter that is recorded and used for calculation purposes. This does not match either of the common definitions of gate position, and yields larger gate opening values than other methods, especially at the smallest gate openings. Furthermore, it is not clear at this time whether a non-linear correlation between inclined opening and cable take-up has been established using data from small intervals of gate opening (0.5 or 1.0 ft increments), or if the relation has been established just at fully closed and fully open positions of the gates (and presumed to be linear for all intermediate openings). This uncertainty makes it difficult to utilize currently available historical data for discharge calibration purposes.

Analytical / Physically-Based Method

Because the engineering basis for the gate-controlled flow equations was not known, a documented analytical method was used to create new curves to allow evaluation of the accuracy of the other curves and equations. The method used is that described in U.S Army Corps of Engineers Hydraulic Design Criteria Charts 311-1 to 311-5. This method is also presented in *Design of Small Dams* (Reclamation, 1987). The method accounts for variation of the gate discharge coefficient as a function of the gate opening, and is specifically intended for use with radial gates mounted on ogee crest spillways. The method does not account for unique approach flow conditions that may exist at the site, and also does not account for interaction between adjacent gates, so it is likely to perform best when the gates are operated in unison so that flow conditions are uniform across the width of the structure. Approach flow effects and interaction between gates are very complex phenomena that can only be evaluated through a physical hydraulic model study or detailed three-dimensional computational fluid dynamics simulation.

The USBR/USACOE method is summarized in Figure 3. The discharge equation is a basic orifice equation with three parameters that must be determined for any specific gate opening. It is very important to note that in this method the gate opening is defined to be the minimum distance from the gate lip to a line tangent to the spillway crest. The head term is defined to be the difference between the reservoir water level and the center of the effective (non-vertical) gate opening. Finally, the discharge coefficient varies as a function of the angle β between the face of the gate leaf and a tangent to the crest at the point of minimum opening. Since an ogee crest spillway is composed of a sequence of curves typically including two circular arcs upstream from the apex of the crest and a power curve function downstream from the apex, determining these three parameters can be a complex geometry problem.

Fortunately, at Yellowtail Afterbay Dam the radial gates on the overflow weir are seated exactly on the apex of the crest, and their geometry and that of the crest is such that the point of minimum gate opening always passes through the first upstream circular arc element of the crest, defined by radius r1 in Figure 4. This simplifies the calculations considerably, although there are still numerous steps needed to compute the gate opening, the β angle, and the effective head term for a known gate position. Details of the computational procedure are given in Appendix A.

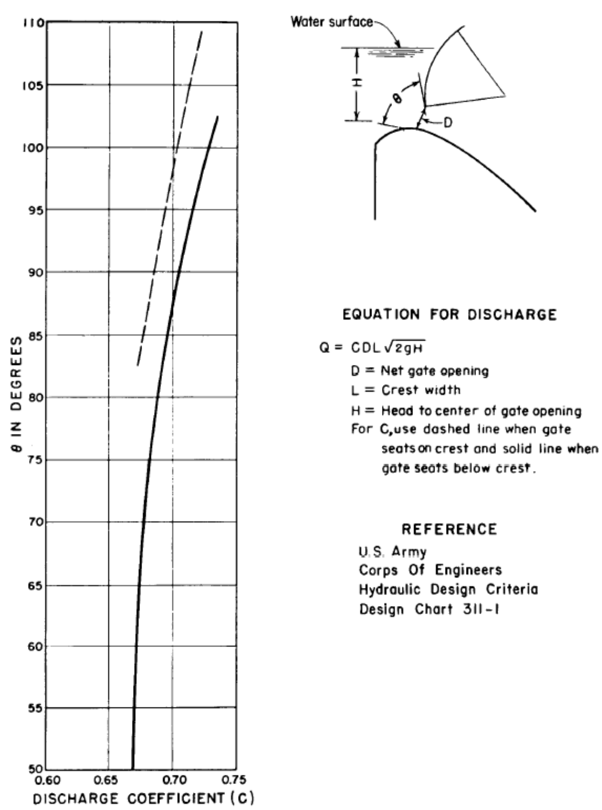

Figure 3. — USBR/USACOE method for computing discharge through radial gates on ogee crest spillways.

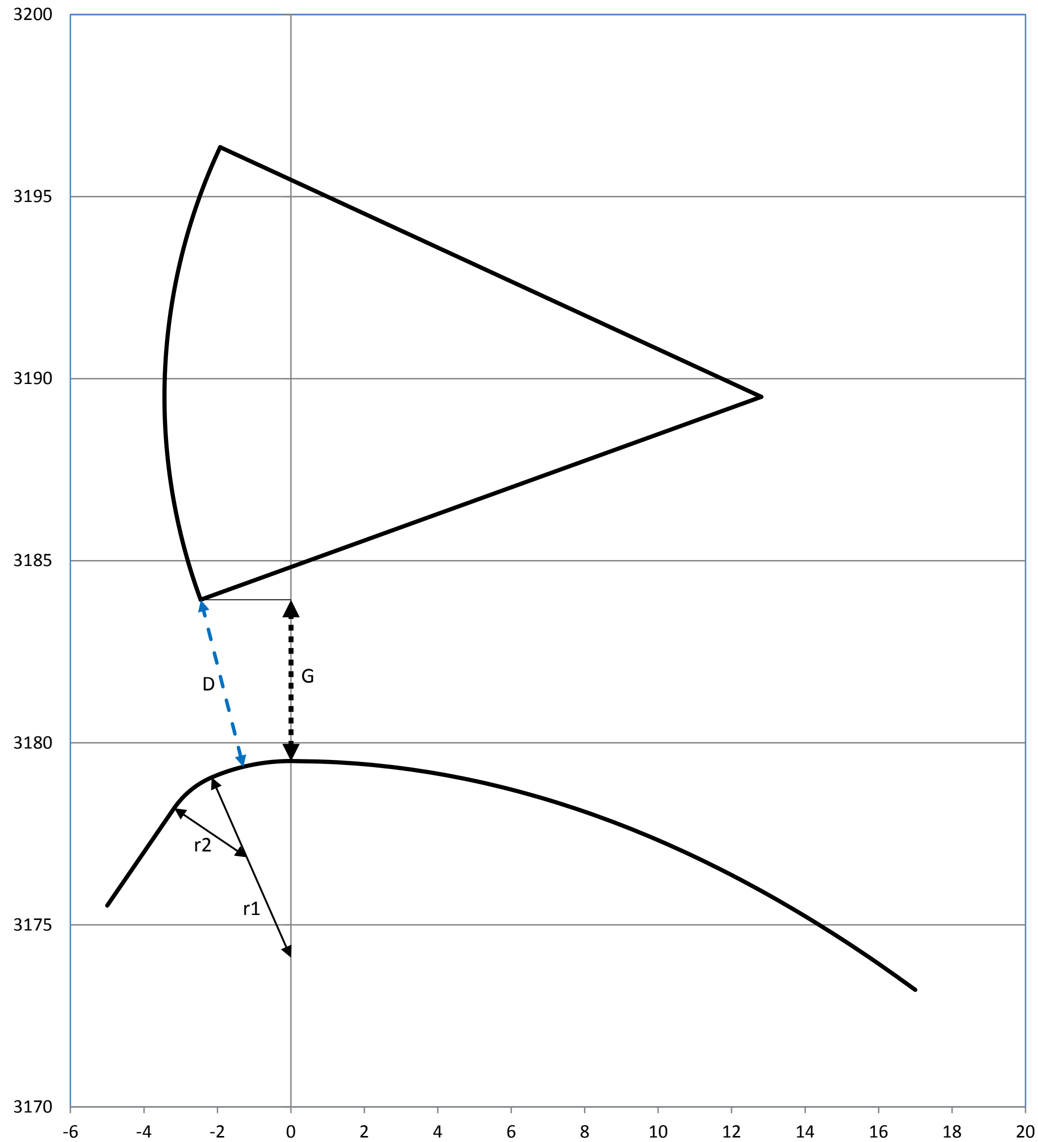

Figure 4. — Illustration of gate parameters for overflow weir gates. The dashed line (D) indicates the minimum distance between the gate lip and the crest needed for the USBR/USACOE method, while the dotted line (G) is the vertical gate opening as defined for the SOP rating curve.

To create discharge curves, a spreadsheet was developed to carry out the necessary calculations. The discharge coefficient curve shown in Figure 3 was fit with a polynomial equation to allow the discharge coefficient to be determined for each gate setting. It is noteworthy that for small gate openings (1 to 4 ft), the β angle is 63° to 83°, which is below the end of the dashed line shown in Figure 3. Thus, extrapolation is needed to estimate discharge coefficients in this range.

Curves were generated for 1 ft increments of the vertical gate opening, as defined for the SOP discharge curves, although net gate openings were used for the actual calculations, as required by the USBR/USACOE method. The curves obtained in this manner are shown in Figure 5, along with the SOP curves and the curves produced by the AAGCS polynomial equations. The three curves for free weir flow are nearly identical. For gate-controlled flow, all methods compare relatively well at low gate openings of 1 and 2 ft. For larger gate settings the SOP curves and the AAGCS polynomial gate equations begin to differ, especially at high reservoir levels, and the USBR/USACOE curves indicate significantly greater discharge than the other two methods.

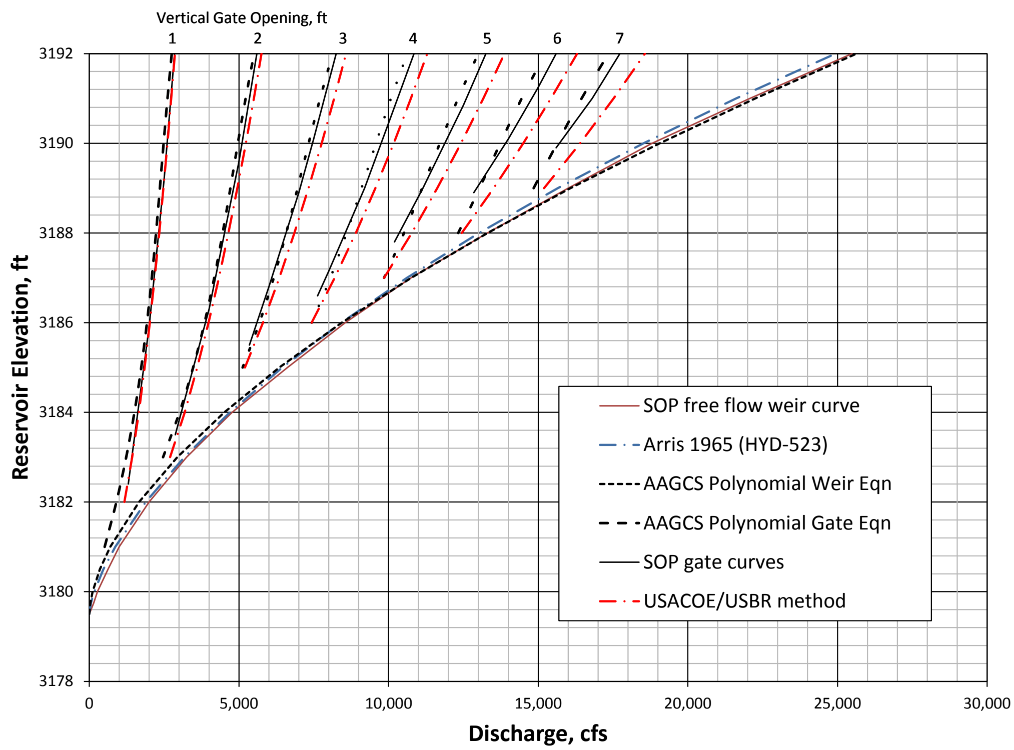

Figure 5. — Comparison of overflow weir discharge curves.

Figure 6 shows Hydromet data from October 1, 1985 to February 3, 2013. The figure shows that most operations take place in the range of 1 to 3 ft gate openings, and free flow over the weir is probably very uncommon. Some data plot below the free weir flow curve because the chart shows the total river flow, which includes flow through the river sluice gates.

Direct calculation of discharges by the USBR/USACOE method requires the gate opening to be defined as the minimum distance from the gate lip to the spillway crest. Although a concise geometric definition of this parameter is possible (see Appendix A), it is impractical to calibrate gate position sensors in the field to provide this parameter. It would be far more practical to calibrate the gate opening sensor system to indicate the vertical gate opening.

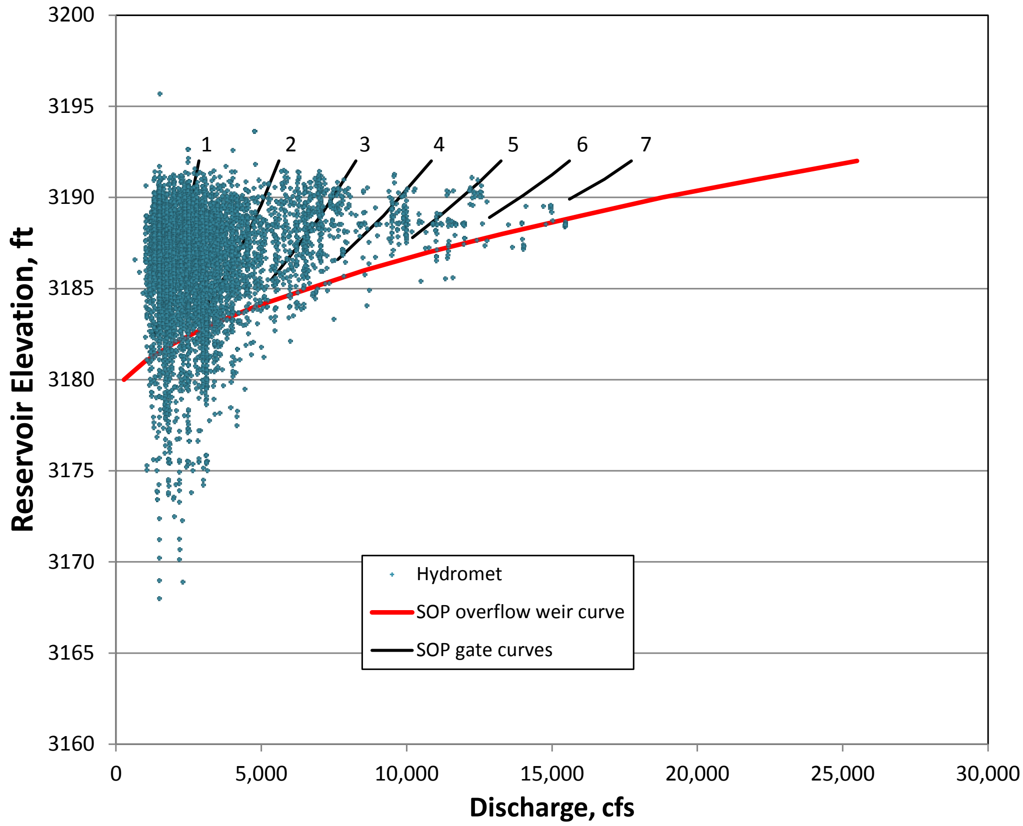

Figure 6. — Hydromet records of daily river discharge and reservoir elevation upstream from the overflow weir

To develop a practical equation for gate-controlled flow that will be useful in the automated control system, the discharge curves generated by the USBR/USACOE method were reanalyzed to determine effective discharge coefficients for use in a basic orifice equation constructed with simpler definitions of the gate opening and head terms:

$$Q = 5(C_d G L \sqrt{2 g H_{CL}}) \tag{4}$$

where: Q = discharge in ft^3/s through 5 gates set to equal gate openings

C_d = effective discharge coefficient

G = vertical gate opening (elevation difference between bottom of gate and top of crest)

L = gate width (30 ft)

g = acceleration due to gravity (32.2 ft/s^2)

H_{CL} = head to centerline of vertical gate opening = $R - 3179.5 - G/2$

Figure 7 shows values of the effective discharge coefficient obtained by solving this equation for C_d. A polynomial curve fitting these data to a function of the gate opening is:

$$C_d = 0.000253(G)^3 - 0.004631(G)^2 + 0.03073(G) + 0.6605 \tag{5}$$

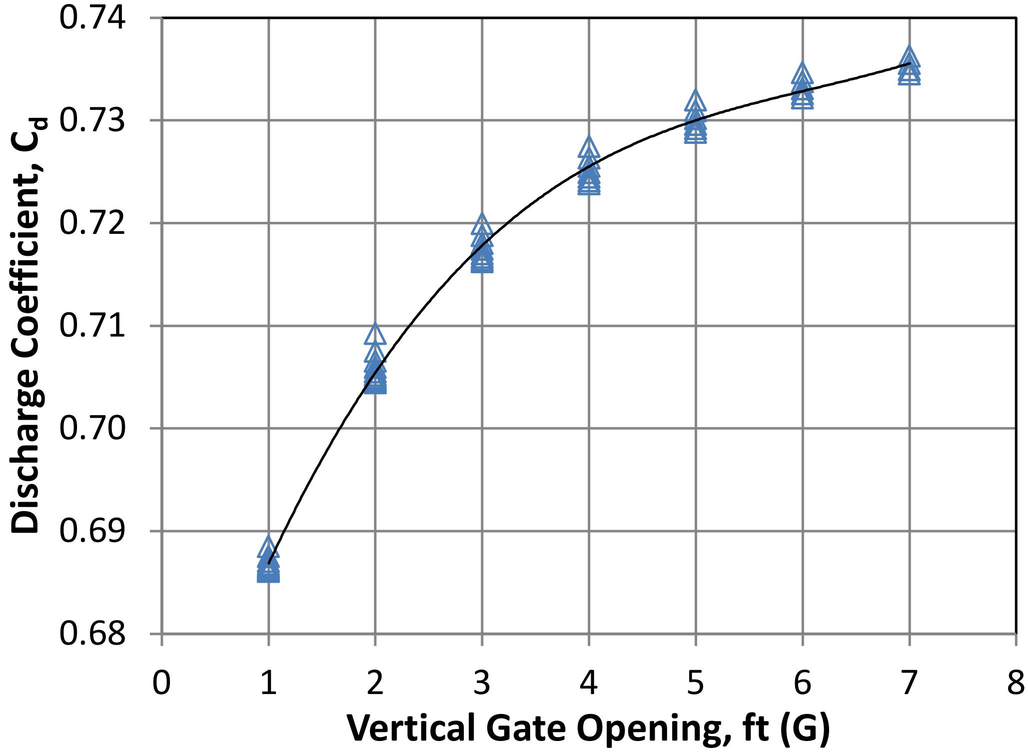

Figure 7. — Variation of discharge coefficient as a function of gate opening.

This enables straightforward calculation of gate-controlled flows for a given vertical gate opening. Note that radial gate discharge coefficients are typically related to the gate lip angle (e.g., Clemmens et al. 2012), but there is a unique relation between the gate opening and gate angle, so the discharge coefficient can also be related directly to the gate opening, which simplifies computational use.

The automated gate control system will also need to determine the desired gate setting to produce a specific discharge. For this calculation, iteration is needed because the gate opening (the unknown parameter) affects the head term and the C_d value, and it is not possible to rearrange the equations algebraically to solve directly for G. To carry out an iterative solution, one should begin by assuming $C_d = 0.7$ and $G = 0$ when computing the head term, H_{CL}., or if the gates are already open, the current gate opening could be used as the starting value. Solve Eq. 4 with the known reservoir elevation and desired discharge to determine the gate opening, G. Now, with this estimate of the gate opening the head term can be recomputed and the value of C_d can be refined. Next, solve for the gate opening a second time. Iterating a third time should produce convergence. Table 2 shows an example calculation. Typically, stabilization of the head term is the primary determinant of the number of iterations needed.

Table 2. — Example calculation of gate setting to achieve a discharge set point.

Iteration No.	C_d	H, ft	G, ft
1	0.7	3190 – 3179.5 – 0.5(0) = 10.5	1.831
2	0.7028	3190 – 3179.5 – 0.5(1.831) = 9.584	1.909
3	0.70405	3190 – 3179.5 – 0.5(1.909) = 9.5455	1.910
4	0.704058	3190 – 3179.5 – 0.5(1.910) = 9.5452	1.910
RWSE=3190.0 ft Discharge set point = 5000 ft^3/s			

Comparison to Field Data

A paucity of field data exists for evaluating the overflow weir rating curves. The overflow weir and river sluiceway usually operate simultaneously, so the total flow at the river gage cannot be divided into the amounts contributed from each discharge source. Steady flow conditions past the dam are also uncommon due to variable operations at Yellowtail Powerplant.

There are no known data points corresponding to free flow over the weir, and only 5 data points are known to exist for gate-controlled flow. These were provided to the author by Don Read, Hydraulic Equipment Group (86-68420) in April 2012. Table 3 shows these data and the discharges calculated using the polynomial equations in the automated control system and the newly developed equations based on the USBR/USACOE method. The data only cover a narrow range of gate openings and reservoir levels. Differences from observed discharges are somewhat greater for the USBR/USACOE method, but are still reasonable, given the uncertainties in the observed discharges which likely come from current-meter measurements or the river gage rating curve.

Table 3. — Field data for discharge through the overflow weir structure.

Reservoir elevation, RWSE, ft	Vertical gate opening, G, ft	Observed discharge, ft^3/s (one gate)	Predicted discharges and % differences from observed	
			AAGCS Polynomial Eqn., ft^3/s	USBR/USACOE Eqn., ft^3/s
3186.15	2.238	880	871 (–1.1%)	889 (+1.0%)
3186.27	1.995	794	787 (–0.9%)	804 (+1.3%)
3188.35	1.685	763	769 (+0.8%)	792 (+3.8%)
3186.41	1.725	688	690 (+0.3%)	706 (+2.6%)
3186.39	1.485	592	594 (+0.3%)	607 (+2.6%)

The newly developed equations based on the USBR/USACOE method are recommended for future use, since they are based on a documented and physically-based method (orifice equation) and can be readily incorporated into the control system. If free weir flow ever occurs, any of the existing equations could be used, with only slight variation in the results.

River Sluiceway

Existing Methods for Determining Discharge

Three methods for computing discharge through the river sluiceway are currently available. The SOP provides a rating curve based on the 1965 physical model study, a second rating curve was developed during the 1982 physical model study, and polynomial equations were developed for the automated gate control system.

SOP Rating Curve

The SOP rating curve is shown in Figure 8. Notes on the drawing indicate that the curve is based on the original hydraulic model study (Arris 1965), which contained a poorly drawn discharge curve that was probably inaccurate for low reservoir elevations. The SOP rating curve was also presumably adjusted to obtain a good fit to the seven field measurements shown on the drawing.

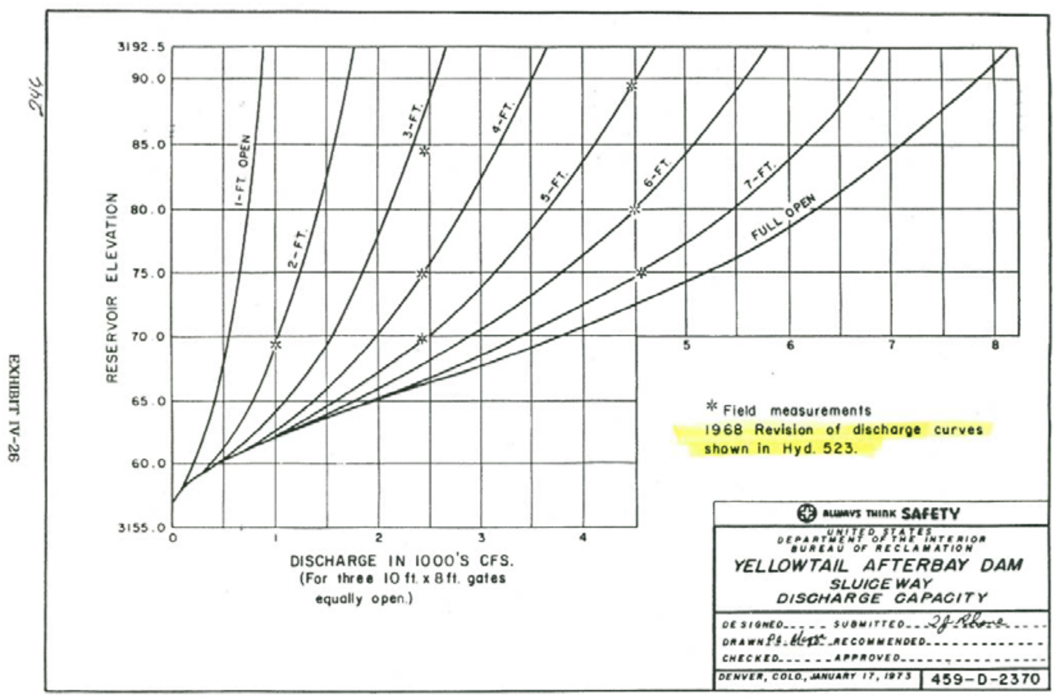

Figure 8. — SOP discharge curve for the river sluiceway.

1982 Hydraulic Model Study (GR-82-5)

A second hydraulic model study of the Afterbay Dam river sluiceway was performed (Young 1982) to determine the best location for installing a flow deflector to reduce gas supersaturation caused by the sluiceway. This study developed a new sluiceway discharge curve (Figure 9) that indicated somewhat lower discharges for a given reservoir elevation. This was attributed to the fact that the second model study included a larger reservoir area than the first, so the velocity head in the first study was higher, and measured reservoir water levels were lower for the same discharge. The use of a larger reservoir area means that the data collected in the second model study should be more accurate, but the discharge curves from the second study have never been incorporated into the SOP. The discharge curves shown in Figure 9 appear to have been hand drawn with a French curve and may have some inaccuracy due to this fact. Note how the curve for the 1 ft gate opening becomes almost perfectly vertical around elevation 3187. This does not accurately represent the behavior of an orifice-controlled outlet.

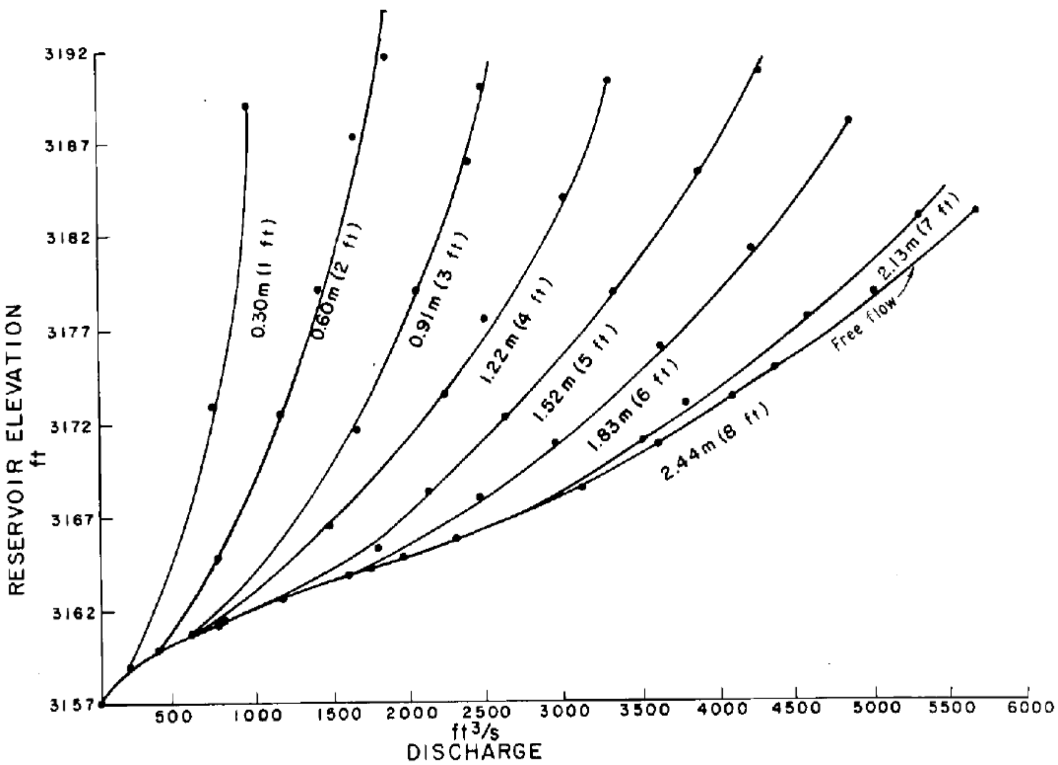

Figure 9. — River sluiceway discharge curve developed by 1982 hydraulic model study. Data points shown on the figure are from the physical model.

Polynomial Equation

The complex polynomial equation used in automated gate control system computes discharge through the sluiceway as follows:

$$Q=G*(43.414+1.21875*((G-4)^2))*(RWSE-3156.79-0.194*G) - 0.02084*((G+4.056)*(RWSE-3156.79-0.194*G))^{(RWSE/1551.707)} \quad (6)$$

This equation gives the total discharge through three gates that are set to the same gate opening, *G*. The equation is purely a curve fit that bears little resemblance to an orifice equation.

Figure 10 shows the SOP discharge curves, the 1982 model study curves, and the curves produced by the AAGCS polynomial equation. For gate openings of 4 ft or less there is good similarity between the SOP and 1982 model study curves, but for higher gate openings the latter model study curves indicate lower discharges, as explained above. The polynomial equations indicate significantly higher discharges than the other two sets of curves, especially at high reservoir levels and large gate openings. There is a huge disparity for gate openings of 6 ft or more. There is no plausible explanation for the polynomial curves indicating that the sluiceway discharge at an 8-ft gate opening is almost double that of a 6-ft opening.

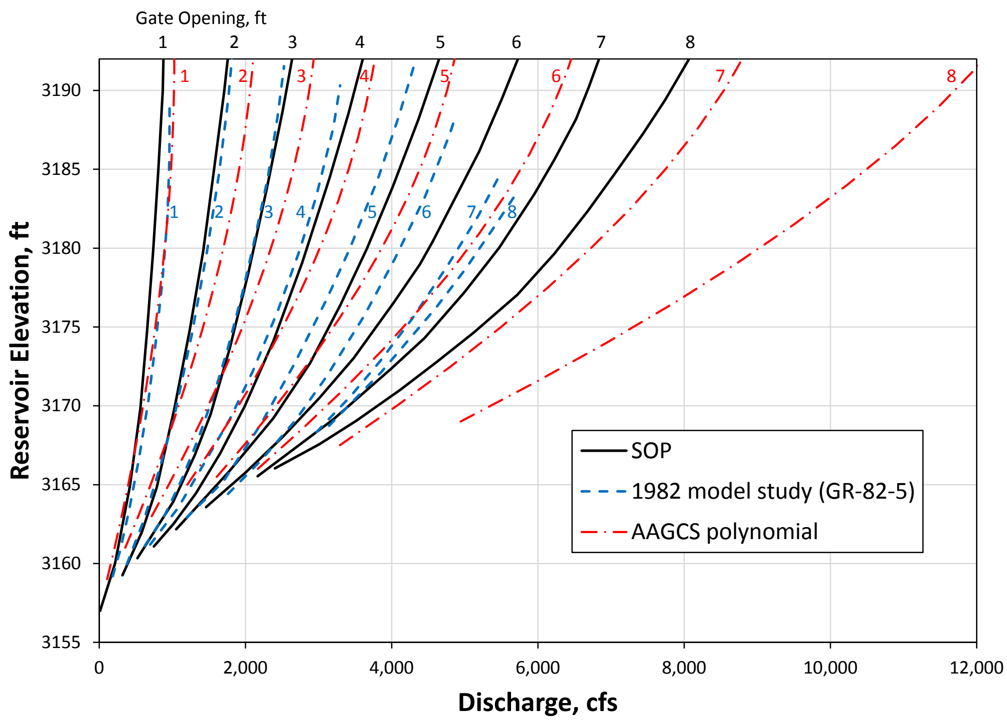

Figure 10. — River sluiceway discharge curves.

Effective Discharge Coefficients

Discharge curves for vertical sluice gates should generally exhibit the behavior of an orifice flow equation:

$$Q = C_d G L \sqrt{2gH} \tag{7}$$

with a discharge coefficient that is relative constant near a value of 0.6. To further evaluate the discharge curves shown in Figure 10, effective discharge coefficients were computed from data points digitized from the original curves. The head term in the orifice equation was computed relative to the center of the gate opening. The variation of these discharge coefficients with relative gate opening is shown in Figure 11.

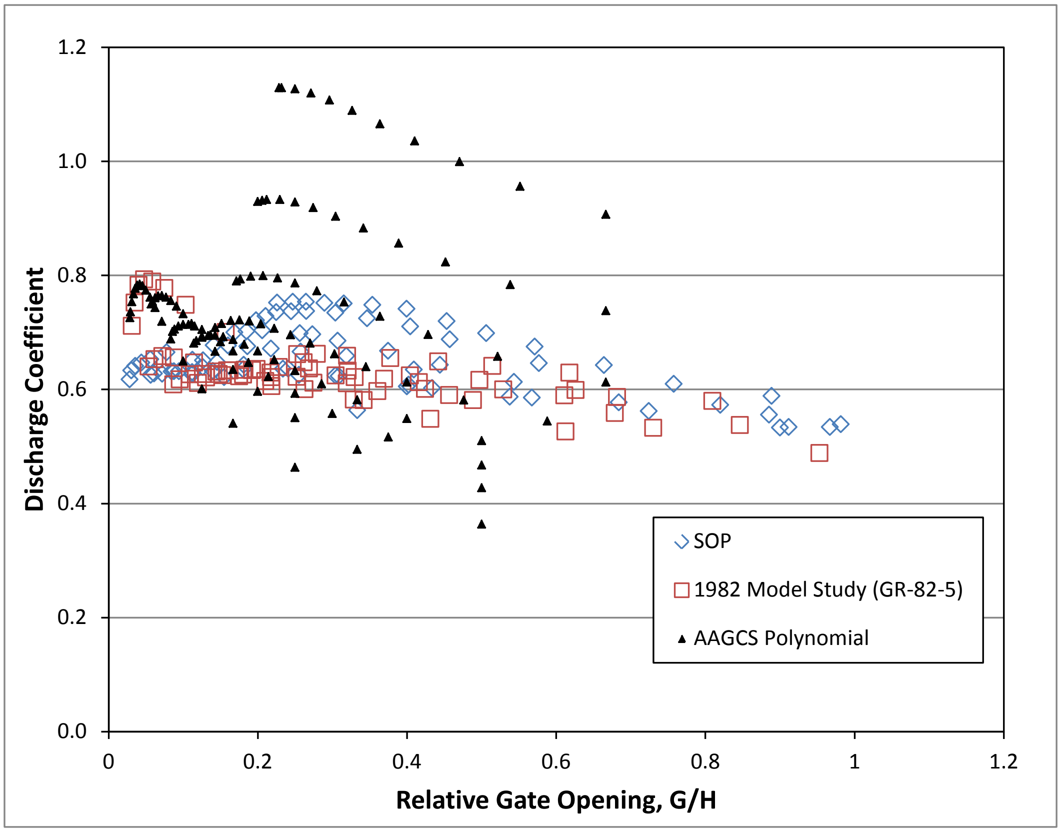

Figure 11. — Effective discharge coefficients for previously established river sluiceway rating curves.

The discharge coefficients for the SOP and 1982 model study curves behave somewhat as expected. The model study data in particular show a steady value close to 0.6, except for the data points associated with the 1 ft gate opening, which was previously noted to have a suspicious appearance. In contrast, the AAGCS polynomial shows dramatic variation of the discharge coefficient and values exceeding 1.0 that are not physically realistic.

Analytical Discharge Curves

The discharge curves and equations compared in Figure 10 all base the gate discharge on just the gate opening and the upstream reservoir elevation, so they are assuming that free flow occurs through the sluice gates. However, a review of the river gage records and the resulting tailwater conditions below the dam shows that river stages could be high enough to cause submergence of the sluice gates, since the gate seat is at elevation 3157.0 ft and tailwater levels vary from about 3157 to 3165 ft. Although submergence is possible, tailwater levels above the gate seat elevation do not guarantee submerged flow, since the momentum of the flow through the gate opening may be strong enough to sweep the tailwater away from the gate exit and allow the gate to flow free. The most likely condition for gate submergence would be combinations of high discharge through the overflow weir (causing high tailwater in the river) and low discharge through the sluiceway. Future operations will maintain a constant ratio of the sluiceway and overflow weir discharges, and it is not immediately apparent that this will yield submerged flow conditions at the sluiceway.

To evaluate the potential for submergence, flow through the sluiceway gates was analyzed using the new WinGate computer program (Wahl and Clemmens 2012; Clemmens et al. 2012). This program was developed for the purpose of calibrating radial gates and vertical slide gates in canal check structures using the energy-momentum (E-M) method. The program can be applied to these sluice gates since they seat on a horizontal surface, as opposed to an ogee crest. The program solves the energy equation from the upstream pool to the orifice opening beneath the gate and the momentum equation from there to the downstream canal. The combined use of the energy and momentum equations makes the method well suited to accurate modeling of flow in the transition zone from free to submerged flow.

The WinGate analysis made use of historic Hydromet data to model a realistic set of circumstances. Daily values of total river flow, river gage height, and upstream reservoir elevation were obtained from 10/1/1985 to 2/3/2013. These data were filtered by comparing the river flow rates and net river gage height data (gage height plus gage shift) to the river gage rating equation. About 87% of the net gage height values matched the values expected for the corresponding river discharges within ±0.01 to ±0.03 ft, while 13% varied from the expected values by amounts up to ±2 ft. The latter data were excluded from analysis on the assumption that the mismatch to the river gage rating curve indicated that operations on that day were highly transient, and thus average daily values did not accurately represent the conditions.

For the data retained, the total river discharges were used to compute the flow through the overflow weir and sluiceway that would have been set on that date, if the 25% / 75% flow split rule had been applied. WinGate was then used to solve for the sluice gate openings needed to obtain the computed sluiceway flow, using the river gage height as the tailwater elevation just downstream from the sluiceway stilling basin. This neglects the small head loss that occurs in the river reach between the end of the stilling basin and the river gage location, but is a reasonable approximation. The WinGate analysis considered that the reservoir upstream from the sluice gates was only as wide as the gates themselves, so that the velocity heads in the reservoir approaching the sluice gates would be realistic, since WinGate was modeling only the sluiceway and not the simultaneous flow through the overflow weir structure.

The analysis showed that the required sluice gate setting for about 80% of the cases produced a submerged, gate-controlled flow condition, while 20% produced free gate-controlled flow (meaning that the tailwater level was too low to affect the discharge, even though it may have been above the gate seat elevation). When total river discharge was greater than 5225 ft^3/s (sluiceway discharge > 1306 ft^3/s) the sluiceway was always in free flow, and when the river discharge was less than 2475 ft^3/s (sluiceway discharge < 619 ft^3/s) the sluiceway was always submerged. When the river discharge was between these limits, either flow condition was possible. The exact threshold for submergence is a complex function of the gate opening, upstream head, and tailwater elevation and requires a momentum analysis like that performed in WinGate. The situation is also complicated by the fact that the river gage elevation for a given river discharge is not constant due to algae growth in the river and other factors that create the need for adjusting the river gage relation seasonally through the use of shifts. Finally, if flow splits other than the 25% / 75% condition were used, the ranges in which free and submerged flow are possible would vary. It should be emphasized that although this analysis was performed assuming the 25% / 75% flow split, this assumption was made only to obtain a realistic range of operating conditions; the results should be applicable to other flow split ratios.

For the submerged flow cases, WinGate also provides output of the calculated discharge if free flow had existed, and an analysis of these data showed that the effects of submergence were typically small, with the median discharge reduction from the free flow value being only 0.29%. Figure 12 shows the cumulative distribution of errors. The maximum error was 4.73%, and 99.5% of the cases had errors smaller than 2.5%. This indicates that when submergence occurs, the river tailwater levels are typically just above the threshold needed to cause submerged flow.

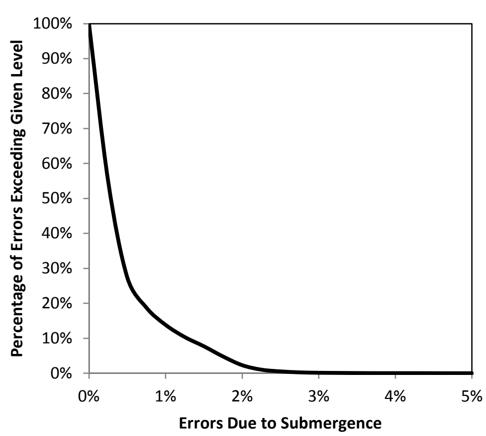

Figure 12. — Cumulative frequency of errors in discharge that will occur if submergence is not considered.

The sluiceway discharges computed with WinGate were used to back-calculate effective discharge coefficients for use in basic orifice equations for free and submerged flow. For free flow the head is measured relative to the center of the gate opening, and for the submerged flow the head is the difference between the reservoir level and the river level at the gaging station. Figure 13 shows the variation of the discharge coefficients as a function of the relative gate opening. Note that the relative gate opening is the ratio of the gate opening to the upstream head *relative to the gate sill elevation* (not the gate centerline). Although the orifice equation for computing free discharge uses head referenced to the center of the gate opening, the discharge coefficient is more closely related to the sill-referenced head.

The total variation of the discharge coefficient for free flow is very slight, in contrast to the effective discharge coefficients computed from the previously discussed rating curves and the AAGCS equations (Figure 11). The nearly constant value of the free flow discharge coefficient is consistent with experimental and numerical simulations of flow through vertical sluice gates (Belaud et al. 2009). The submerged flow discharge coefficient varies more significantly and in a different manner, increasing with relative gate opening. This is partly due to the change in the definition of the head term for submerged flow, and also due to the fact that this coefficient is accounting for the lumped effects of several empirical factors affecting submerged flow (e.g., momentum effects in tailwater channel) that are included in the WinGate analysis.

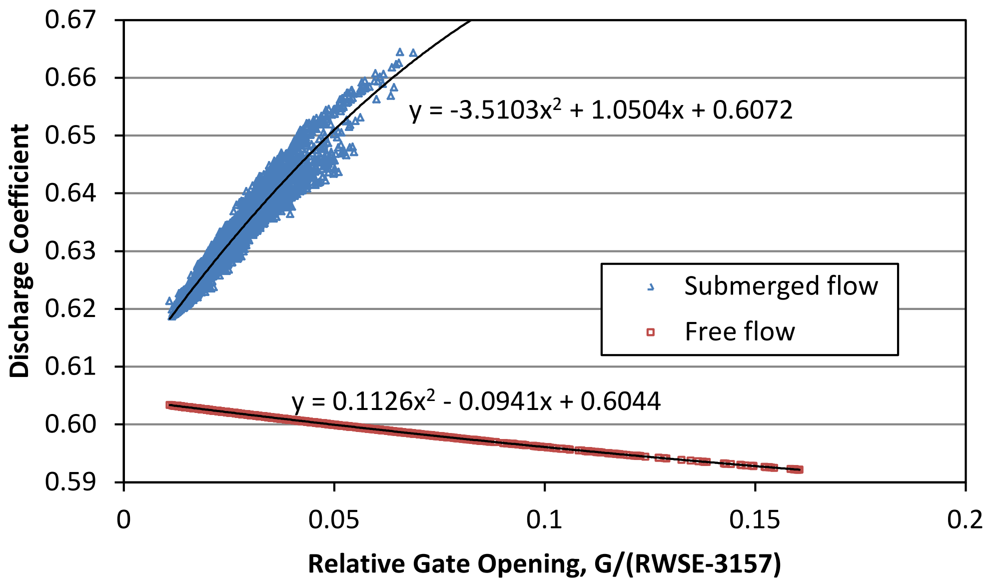

Figure 13. — Discharge coefficients for river sluice gates based on the WinGate analysis.

To compute discharge through the sluiceway using the information in Figure 13, the following steps can be performed:

1. Compute relative gate opening

$$G^* = \frac{G}{RWSE - 3157} \tag{8}$$

2. Compute free-flow discharge as

$$\begin{aligned} C_{d,free} &= 0.6044 - 0.0941(G^*) + 0.1126(G^{*2}) \\ Q &= (3)(C_{d,free})(10)G\sqrt{2g(RWSE - 3157 - G/2)} \end{aligned} \tag{9}$$

3. Compute submerged-flow discharge as

$$\begin{aligned} C_{d,submerged} &= 0.6072 + 1.05(G^*) - 3.51(G^*)^2 \\ Q &= (3)(C_{d,submerged})(10)G\sqrt{2g(RWSE - h_{gage})} \end{aligned} \tag{10}$$

where h_{gage} is the elevation of the water surface at the river gaging station. The final result is the minimum discharge computed by the two methods. Figure 14 shows the resulting discharge prediction errors.

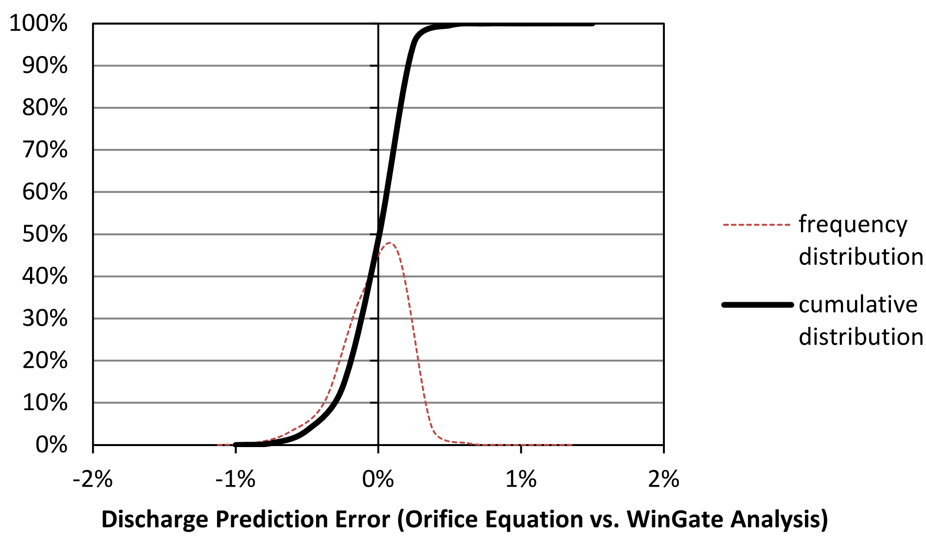

Figure 14. — Discharge prediction errors using minimum predicted from free-flow and submerged-flow orifice equations.

The control system will also need to solve for the gate setting required to obtain a target discharge. For this purpose, free flow conditions should be assumed and the discharge coefficient can be set to 0.6 as an initial value. The gate setting can then be determined from the free-flow orifice equation. Once this has been done, the discharge coefficients for free and submerged flow can be refined and the equation controlling the final result can be determined. Multiple iterations may be needed to reach convergence.

Figure 15 shows a new set of discharge curves that was generated for the sluiceway using the results of the WinGate analysis. Since there is a range of ambiguity for free versus submerged flow and the effect of submergence is slight in the vast majority of cases, the curves were generated using only the free flow equations. The resulting curves most closely match the 1982 model study curves and indicate somewhat lower discharges than all of the previously developed curves. When submerged flow conditions exist, the discharges would be reduced further, but only by a small amount in most cases (0 to 2%).

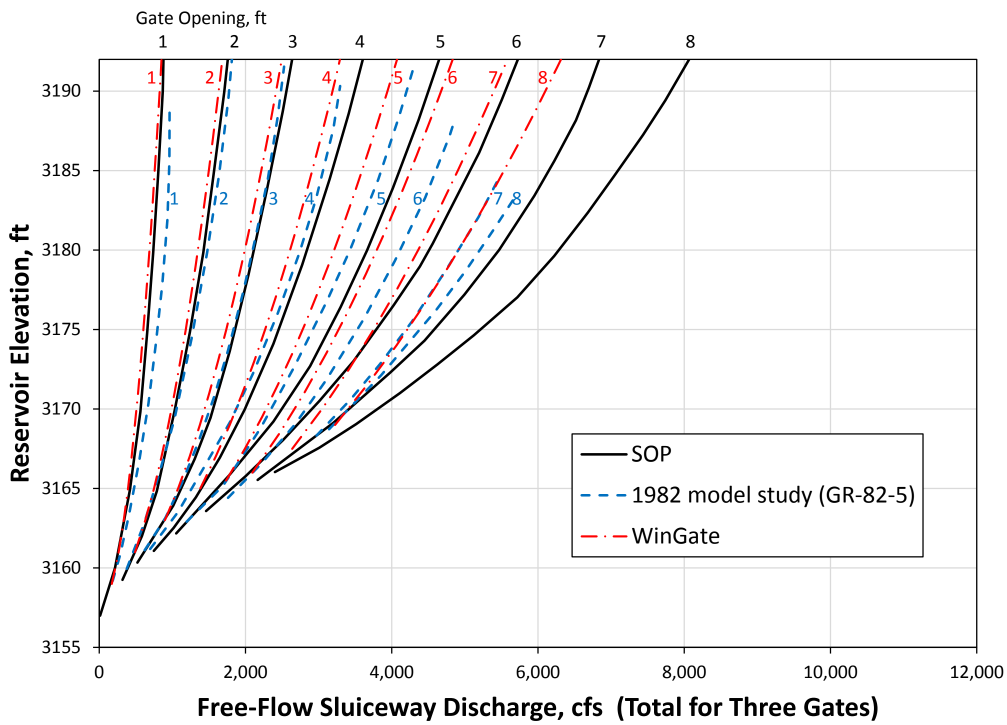

Figure 15. — Discharge curves generated from WinGate analysis.

There is significant difference between the various discharge curves beginning at gate openings greater than 4 ft, but the practical importance of large gate openings may be small. The WinGate analysis using the 1985-2013 Hydromet data showed that the sluice gate setting to achieve the 25% / 75% flow split objective would be less than 4 ft on 99.5% of the days analyzed (see Figure 16).

Comparison to Field Data

As with the overflow weir, there are few field data available for testing the discharge equations and curves. The SOP discharge curve provides 7 data points, and an additional 6 data points were provided to the author by Don Read, Hydraulic Equipment Group (86-68420) in April 2012. The original source of these latter data is unknown, but they were reportedly used to develop the original AAGCS polynomial equations. Table 4 shows the available data and the discharges computed by different methods. The WinGate discharges are significantly lower than the discharges predicted by the other methods, as expected.

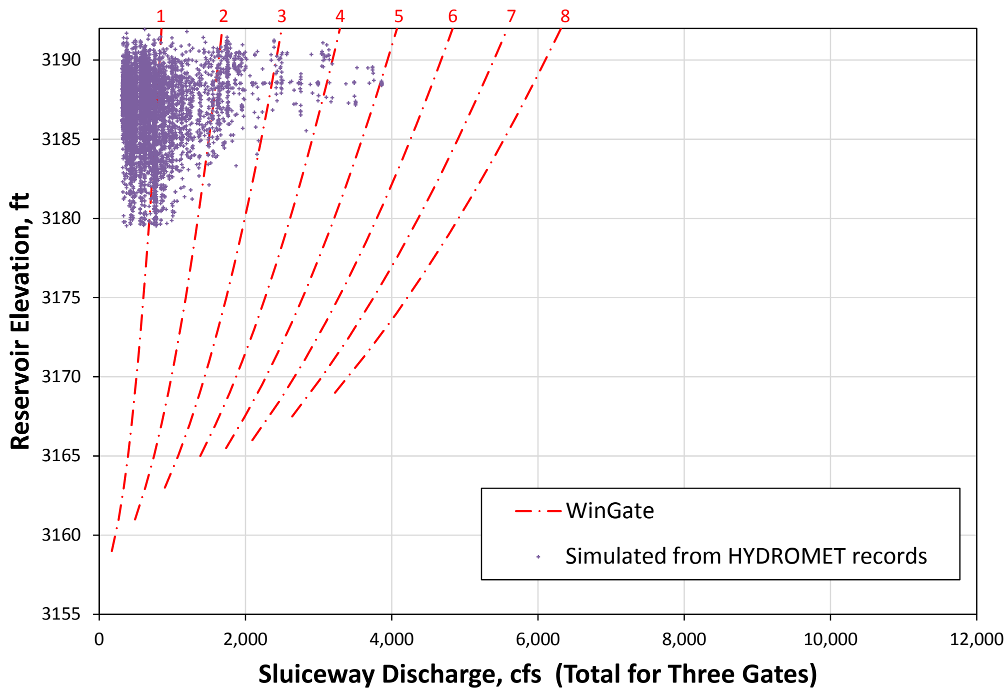

Figure 16. — Sluiceway operating conditions simulated for 1985-2013 period from Hydromet data.

Table 4. — Field data for discharge through the river sluiceway.

Reservoir elevation, RWSE, ft	Gate opening, G, ft	Observed discharge, ft^3/s (three gates)	Predicted discharges and % differences from observed		
			AAGCS Eqn.	WinGate	SOP
Data used to develop AAGCS Equation					
3186.38	0.838	896	807 (-10%)	653 (-27%)	670 (-25%)
3186.38	1.483	1509	1501 (-1%)	1146 (-24%)	1225 (-19%)
3185.99	3.44	2943	3030 (+3%)	2572 (-13%)	2750 (-7%)
3185.33	4.074	3492	3497 (+0%)	2983 (-15%)	3250 (-7%)
3185.80	4.643	4100	4063 (-1%)	3406 (-17%)	3800 (-7%)
3184.00	5.41	4750	4786 (+1%)	3788 (-20%)	4400 (-7%)
Data from SOP discharge curve					
3169.25	2	1013	1032 (+2%)	956 (-6%)	1000 (-1%)
3184.42	3	2445	2623 (+7%)	2189 (-10%)	2300 (-6%)
3174.75	4	2424	2483 (+2%)	2251 (-7%)	2450 (1%)
3169.75	5	2424	2376 (-2%)	2254 (-7%)	2450 (1%)
3189.42	5	4460	4721 (+6%)	3901 (-13%)	4450 (0%)
3179.83	6	4481	5023 (+12%)	3779 (-16%)	4500 (0%)
3174.83	7	4552	5450 (+20%)	3731 (-18%)	4600 (1%)

Despite the large differences from the few field observations, the WinGate discharge curves are believed to offer the best estimate of discharge for the sluiceway. The inconsistent behavior of the discharge coefficients in the other methods has already been discussed and gives good reason for discrediting them. The WinGate curves are physically-based and reflect the most current research on sluice gate discharge characteristics. There are potential sources of error in the WinGate analysis, such as unique site-specific approach flow conditions and head losses that may not be accurately accounted for, but most such factors would tend to reduce the discharges, and WinGate is already predicting lower flows than the other methods. The one factor that could cause the WinGate discharges to be too low for a given reservoir level is not correctly accounting for high velocity head in the reservoir due to the simultaneous operation of the overflow weir with the river sluiceway. If a consistent bias between the new equations and field-measured flows is found in the future, adjustments could be made to the WinGate-based discharge curves.

Bighorn Canal Sluiceway

The canal sluiceway contains two 10-ft-wide by 8-ft-tall vertical slide gates that release water into the Bighorn Canal, also designated as the BIA Canal. The gate sill elevation is 3167.00 ft, and the maximum discharge capacity of the canal sluiceway is 750 ft^3/s. Flow is normally gate-controlled, but for unusually low reservoir elevations, weir flow is possible.

A rating curve for the sluiceway gates is not provided in the SOP for Yellowtail Afterbay Dam. Discharge in the canal is measured at a gaging station located about 720 ft downstream from the start of the canal. Hydromet records define the rating curve at the gaging station, shown in Figure 17, which is elevated about 2 ft above the rating curve that would be expected based on the original design of the canal [trapezoidal, 30-ft base width, 2:1 (H:V) side slopes, S=0.00011, initial invert elev. 3166.30 ft, depth=7.7 ft and normal W.S. El. 3174.0 ft at 750 ft^3/s]. About 90% of the available daily readings fit the rating within ±0.25 ft, with the other 10% scattering widely around the curve defined by the bulk of the data. The reason for the 2 ft difference between the design canal water surface elevation curve and the gage rating curve is unknown.

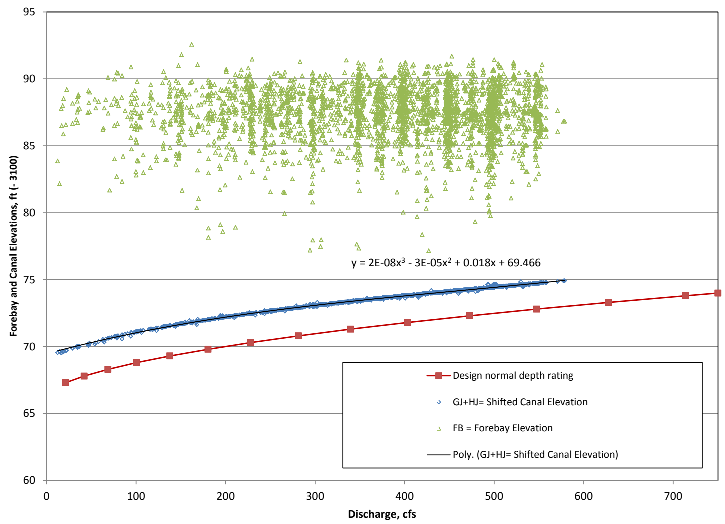

Figure 17. — Hydromet data from 1985 to 2012 showing the rating curve for the Bighorn Canal, the normal depths computed for design conditions, and associated reservoir levels upstream from Yellowtail Afterbay Dam. About 10% of the data for this period are excluded from the plot, because they were scattered widely from the rating curve.

Existing Methods for Determining Discharge

Polynomial Equation

The discharge equation originally developed for the automated gate control system computes the flow through one gate as follows:

$$Q = 7*(\min(G, R - LWS))*(2g(R - LWS))^{1/2} - 50\left[\min\left(1, \frac{R}{\frac{14}{G} + LWS}\right)\right]^{5000} \qquad (11)$$

where:

Q = discharge through **<u>one</u>** gate (ft^3/s)
7 = product of gate width (10 ft) and presumed discharge coefficient (0.7)
G = gate opening, vertical (ft)
g = acceleration due to gravity, 32.2 ft/s^2
R = reservoir water surface elevation
LWS = lower water surface elevation (in downstream canal)

The first part of this equation attempts to implement both a submerged orifice and free weir equation, depending on the reservoir level and gate opening. The last term is an adjustment than can subtract up to 50 ft^3/s from the calculated flow rate.

The term *R-LWS* represents the net head differential across the gate. If the gate opening, *G*, is less than *R-LWS*, then the first part of the equation becomes equivalent to a submerged orifice equation with the head equal to *R-LWS* and a discharge coefficient of 0.7. If *R-LWS* is less than *G*, then the first part of the equation is similar to a weir equation, $Q = CLH^{3/2}$, with $CL = 7(2g)^{0.5} = 56$. Since the gate width is 10 ft, this means the effective discharge coefficient for weir flow is 5.6. This is an unreasonably large coefficient, far exceeding a typical value for even a specially design, high capacity ogee crest (maximum of about 4).

The condition for using the weir-flow option in the equation is also flawed. Weir flow should occur when the depth of flow through the gate is less than the elevation of the gate leaf, but the equation triggers the weir flow option when the gate opening is greater than the differential head across the structure, regardless of the actual flow depth through the gate opening. Flow conditions can be imagined in which this equation is clearly inappropriate. For example, in the case of *R*=3178, *LWS*=3174, *G*=6 ft, the gate lip elevation is 3173 ft, and both the *R* and *LWS* water surfaces are high enough to submerge the gate lip. However, because *R-LWS*=4, which is less than *G*=6, the equation uses the weir formulation.

The last term of the equation reduces the calculated flow by up to 50 ft^3/s, but the value of this adjustment term is discontinuous and there is no justifiable physical basis for the formula.

Figure 18 shows an example set of discharge curves computed with the equation for 1 and 2 ft gate openings. The equation correctly predicts submerged orifice flow for the full range of reservoir elevations shown, but the discontinuity of the curve when the adjustment term takes effect is dramatic and physically unrealistic. This discontinuity will make automatic control of the sluiceway very difficult.

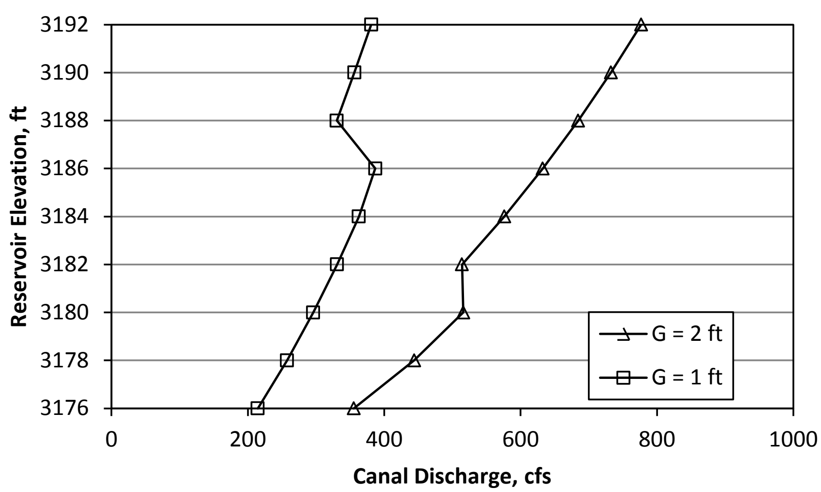

Figure 18. — Example canal sluiceway discharge curves using the AAGCS equation.

WinGate Analysis

Hydromet data for the canal sluiceway were obtained from October 1, 1985 to October 10, 2012. The data set was filtered to retain only those data that fit the rating curve defined by the bulk of the data, with outliers considered to be days on which steady flow did not prevail so daily average values provided a poor representation of real conditions. With the filtered data, WinGate was used to compute sluice gate settings that were required to obtain the recorded value of canal discharge with the given upstream reservoir and downstream canal water levels. As described previously, WinGate performs a momentum analysis that can accurately account for the effects of gate submergence. The results from WinGate were then used to compute effective discharge coefficients for a simplified submerged orifice equation that would be practical for use in the automated gate control system. It is notable that the data set contained no records of conditions for which weir flow was likely.

The WinGate analysis showed that the gates always operate in a submerged flow condition, and the reduction of discharge due to the submergence effect varies from about 5% to 28% of the free-flow discharge.

The WinGate results were used to compute submerged-flow discharge coefficients for a basic orifice equation:

$$Q = C_d G L \sqrt{2g\Delta H} \tag{12}$$

where Q is the discharge in ft^3/s, C_d is the discharge coefficient, G is the gate opening, L is the gate width, g is the acceleration due to gravity, and ΔH is the difference in elevation between the upstream reservoir and downstream canal.

The discharge coefficients were related to both the relative gate opening, G/H_1, and the submergence ratio, H_3/H_1, where H_1 is the upstream head and H_3 is the downstream head, both measured relative to the gate sill elevation. Thus, H_1=$RWSE$-3167 and H_3=Y_{canal}-3167, where $RWSE$ is the upstream reservoir water surface elevation and Y_{canal} is the water surface elevation in the canal. Note that this submergence ratio is a simple parameter that does not perfectly reflect the submergence conditions at the gate itself, since the water level at the back side of the gate leaf will be different from that in the downstream canal, but it is straightforward to compute and useful for prediction purposes. Figure 19 and Figure 20 show the relationships to each parameter. Both relations appear to be promising for predicting discharge coefficients, but an even better relation was found using an equation fitting tool designed for analysis of 3D surface functions, TableCurve 3D. This relation is:

$$C_d = \frac{1}{1.6503 - 1.769\sqrt{G/H_1} + 0.7717\left(H_3/H_1\right)} \tag{13}$$

Figure 21 shows the results when predicting C_d using equations based on G/H_1, H_3/H_1, and both parameters together. The only region where the two-parameter relation performs poorly is when the predicted C_d value is less than 0.62. When this is the case, the equation shown in Figure 19 based on G/H_1 should be used.

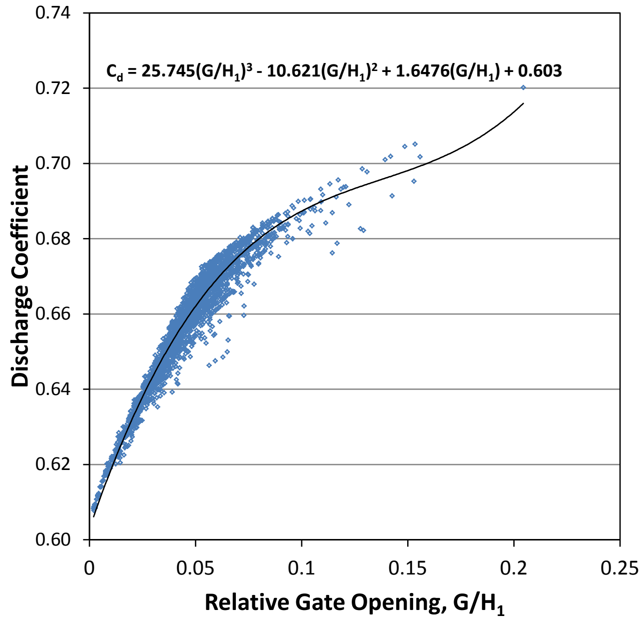

Figure 19. — Relation between canal sluiceway discharge coefficients and relative gate opening.

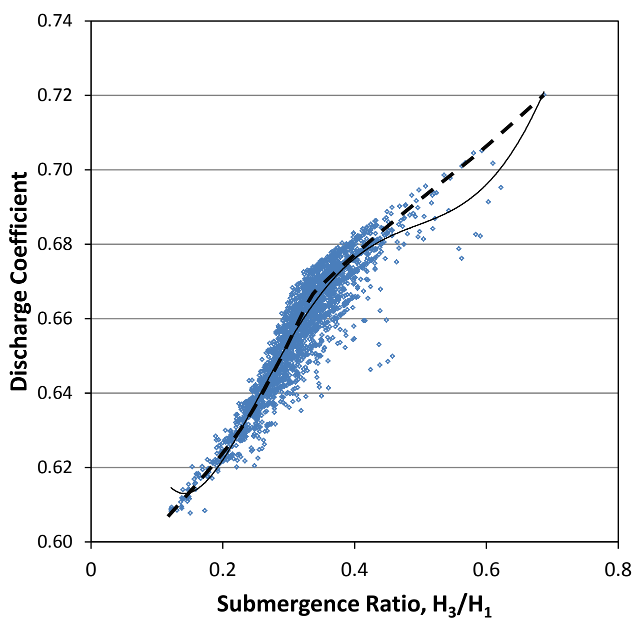

Figure 20. — Relation between canal sluiceway discharge coefficient and submergence ratio. The solid line is a fifth-order polynomial curve fit, and the dashed line is a manual curve fit (by eye).

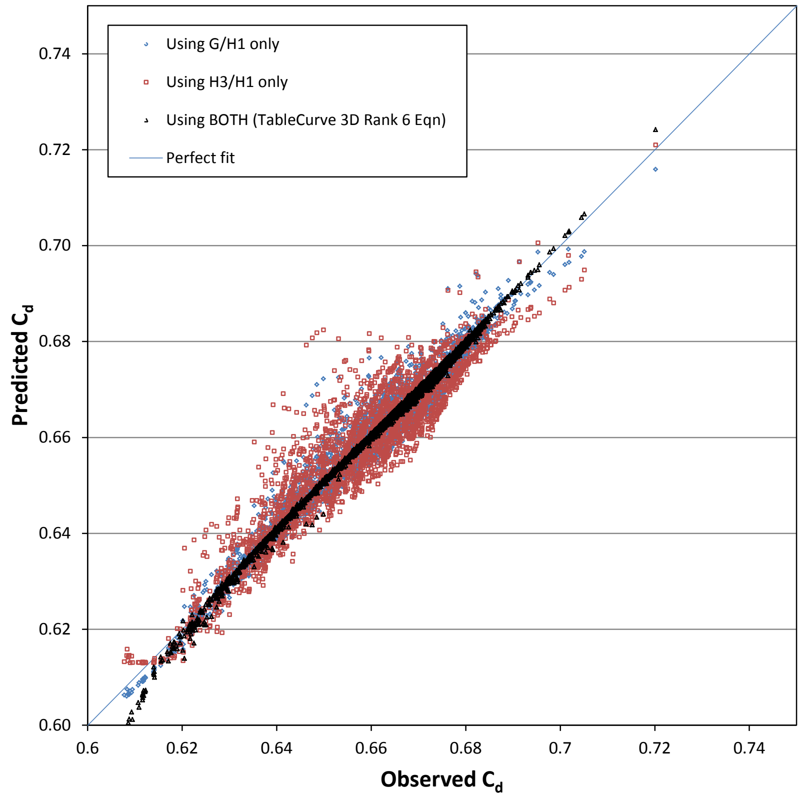

Figure 21. — Performance of several functions that can predict values of discharge coefficients for the canal sluiceway.

Sources of Uncertainty in WinGate Analysis

The WinGate computer program is a physically based model designed to calibrate canal sluice gates for accurate discharge measurement. It applies the energy equation to the upstream side of the gate and includes empirical factors that account for energy loss approaching the gate, assuming a streamlined approach channel. At the gate opening, the program applies empirical relations for estimating the contraction coefficient of the flow through the gate opening. Downstream from the gate, the momentum equation is applied, with empirical relations that estimate the hydrostatic and drag forces on downstream channel boundaries. Each of the empirical relations is a source of uncertainty, as are the assumptions of streamlined approach flow and no interaction with the adjacent river sluiceway and overflow weir. The greatest source of uncertainty in the model results is probably the estimation of flow forces on the sloped apron downstream from the gate, which leads into the stilling basin. The canal check gates that WinGate was designed to analyze typically do not have such sloped aprons or discharge into a stilling basin, so this specific downstream channel configuration has not been studied by researchers involved in the development of WinGate.

If the forces on this surface are not accurately modeling in WinGate, there will be a systematic error in the computed discharge. The relative size of this error should be consistent throughout the operating range, so a calibration adjustment is possible, if future data shows a consistent difference between flow rates predicted from the gate discharge equations and those measured at the canal gaging station. To provide data for checking the accuracy of the proposed equations, all data used in the calculations should be logged for future review (reservoir elevation, canal elevation, canal gaging station discharge, and canal sluice gate openings).

The analysis used to develop the new gate equations made use of the prevailing relationship between canal discharge and canal depth, which establishes the tailwater condition below the sluice gates. It was noted that the prevailing tailwater curve is about 2 ft higher than that which was expected based on the original design parameters of the canal. A second analysis was carried out in which the tailwater curve was set at the design level, and this analysis showed that if the tailwater were lowered, the sluiceway would experience free orifice flow for a significant range of operating conditions. Thus, it should be noted that if tailwater conditions at the site change in the future due to canal maintenance activities or canal rehabilitation projects, that may change the equations needed to accurately predict sluiceway discharges.

Weir Flow

Although the Hydromet records indicate that weir flow through the canal sluiceway is highly unlikely, the potential for it does still exist. If the reservoir level upstream from Yellowtail Afterbay Dam is extremely low, it may be necessary to raise the canal gates out of the water to deliver as much water into the canal as possible. However, a simple estimation of free weir flow ($Q=3.09LH^{1.5}$) through two gates or through one gate (assuming the other gate is closed) shows that in either case the tailwater levels produced in the canal are higher than the reservoir levels needed to obtain a given flow rate. Thus, it is impossible for free weir flow to exist, and the weir will always be submerged by the tailwater if the gates are raised out of the water. In this condition, flow will actually be controlled by the canal cross section and the canal gaging station will offer the best means for estimating the flow rate. The present rating equation for the gaging station was obtained from analysis of the Hydromet data:

$$Q = -0.2182h^3 + 60.91h^2 - 5278.5h + 145915 \tag{14}$$

where h is the net (shifted) canal gage height relative to elevation 3100 ft. (i.e., if the canal water level is at elevation 3170.0 ft, h=70.0).

Recommended Equations for Canal Sluice Gate Discharge

Discharge through the canal sluiceway should be computed using the orifice equation, Eq. (12), with the discharge coefficient computed from Eq. (13), unless the value computed there is less than 0.62. In that case, the discharge coefficient should be computed instead with the equation shown on Figure 19.

If the reservoir elevation and canal elevation are both lower than the gate lip elevation, then the gates will not control the flow and the discharge should be determined at the canal gaging station using Eq. (14).

When it necessary for the control system to compute the gate opening needed to deliver a target discharge into the canal, an iterative calculation is required, since the discharge coefficient is dependent on the gate opening. The current value of the gate opening could be used as an initial guess at the new gate opening and the calculations then proceed as described above. Alternately, the discharge coefficient could be assumed to have a value of 0.66 for the first cycle of calculations (the median of the values obtained from the WinGate analysis).

General Recommendations

Two specific actions are recommended to enable implementation and further refinement of the newly developed discharge equations:

1. Develop new gate position sensor calibrations for the radial gates on the overflow weir structure, or verify existing relations if they are consistent with the parameters needed for the discharge equations developed in this report.
2. Once the gate opening sensor calibration is confirmed, utilize hourly data from the Hydromet system to validate the discharge equations.

Accurate calibration relations for the radial gate position sensors are essential for accurate discharge measurement. The benefit obtained from the development or verification of gate sensor calibrations could be immediate. Even before an automated gate control system is placed back into operation, accurate gate position readings would enable work to continue on validating and refining the discharge equations developed in this report. The most valuable data would be those associated with times when conditions are at steady state and current-meter measurements are being made to confirm the river gage rating (thus minimizing the impact of any time-varying shift in the gage rating), and/or times when either the overflow weir or river sluiceway discharges are known to be zero (so that flow contributions from only one outlet system could be analyzed separately).

The specific data needed include radial gate openings, reservoir level, river level, river flow rate, river sluiceway gate openings, canal sluiceway gate openings, canal level, and canal flow rate. These data are all presently available in the Hydromet system, but the accuracy of gate position data for the radial gate is not presently confirmed. Previously recorded hourly Hydromet data could be used if historic data can be accurately related back to the original physical gate openings.

References

Arris, W.F., 1965. *Hydraulic model studies of the sluiceway and overflow weir – Yellowtail Afterbay Dam, Missouri River Basin Project, Montana*. U.S. Dept. of the Interior, Bureau of Reclamation, Hydraulics Laboratory, Denver, CO. Report No. HYD-523. http://www.usbr.gov/pmts/hydraulics_lab/pubs/HYD/HYD-523.pdf

Belaud, G., L. Cassan, L., and J.-P. Baume, 2009. Calculation of contraction coefficient under sluice gates and application to discharge measurement. *Journal of Hydraulic Engineering*, 135(12), 1086-1091.

Clemmens, A.J., T.L. Wahl, and G. Belaud, 2012. Contraction coefficients for radial and sluice gates in WinGate software. USCID Water Management Conference, November 13-16, 2012. Reno, NV.

Bureau of Reclamation, 1987. *Design of small dams*, 3rd ed.pp. 376-379. http://www.usbr.gov/pmts/hydraulics_lab/pubs/manuals/SmallDams.pdf

Fabbri, M. and K. Clair, 2012. Yellowtail Afterbay flow control system evaluation. DRAFT Project Technical Notes 8440-2012-xx.

U.S. Army Corps of Engineers, 1987. Tainter gates on spillway crests – discharge coefficients. Hydraulic Design Criteria Sheets 311-1 to 311-5. http://chl.erdc.usace.army.mil/hdc

Wahl, T.L., and A.J. Clemmens, 2012. WinGate software for discharge calibration of gated check structures. 2012 World Environmental and Water Resources Congress, Environmental and Water Resources Institute of the American Society of Civil Engineers, Albuquerque, NM, May 20-24, 2012. http://www.usbr.gov/pmts/hydraulics_lab/pubs/PAP/PAP-1062.pdf

Young, M.F., 1982. *Hydraulic model studies – Yellowtail Afterbay Dam sluiceway*, U.S. Dept. of the Interior, Bureau of Reclamation, Hydraulics Laboratory, Denver, CO. Report No. GR-82-5. http://www.usbr.gov/pmts/hydraulics_lab/pubs/GR/GR-82-05.pdf

Appendix A - USBR/USACOE radial gate discharge calculation

The calculation of flow through radial gates (tainter gates) installed on ogee crest spillways is described in U.S. Army Corps of Engineers Hydraulic Design Criteria Charts 311-1 to 311-5. This method is also presented in *Design of Small Dams* (Reclamation, 1987). The method accounts for variation of the gate discharge coefficient as a function of the gate opening, and is specifically intended for use with radial gates mounted on ogee crest spillways. The method utilizes a basic orifice equation, but defines the effective head and gate opening based on the minimum distance between the gate lip and the spillway crest surface (see Figure 22), even if that distance is not a vertical line.

$$Q = CDL\sqrt{2gH}) \tag{15}$$

where: Q = discharge in ft^3/s through 5 gates set to equal gate openings

C = discharge coefficient

D = vertical gate opening (elevation difference between bottom of gate and top of crest)

L = gate width

g = acceleration due to gravity

H = head to centerline of gate opening

The geometry of the radial gates installed on the Yellowtail Afterbay Dam overflow weir section is illustrated in Figure 22. The discharge coefficient varies as a function of the angle β between a line tangent to the gate leaf at the gate lip and a line tangent to the spillway surface at the point of minimum opening (see Figure 23). Thus, to utilize the discharge equation, one must use trigonometry to determine D, H, and β.

Fortunately, for all gate openings of interest, the minimum distance from the gate lip to the spillway crest lies along a line passing through the first circular arc section upstream from the apex of the crest, defined by radius r1. This line runs from the gate lip to the center of the circular arc section.

To compute the various parameters, define a Cartesian coordinate system in which the apex of the crest is located at x=0, y=3179.5. Several fixed properties of the gate are known or can be computed. The gate trunnion pin is located 10 ft above the crest and the gate radius is R = 16.25 ft. The radius of the circular arc just upstream from the apex of the crest is r1=64.75". The angle $\theta = \sin^{-1}(10/R)$ = 37.98°, and the x coordinate of the trunnion pin is $X_T = (R)\cos(\theta)$ = 12.809 ft.

Now, for a specific vertical opening of the gate, *G*, calculate the *x* and *y* coordinates of the bottom of the gate:

$$
\begin{aligned}
Y_{lip} &= 3179.5 + G \\
X_{lip} &= X_T - \sqrt{R^2 - (Y_T - Y_{lip})^2}
\end{aligned}
\qquad (16)
$$

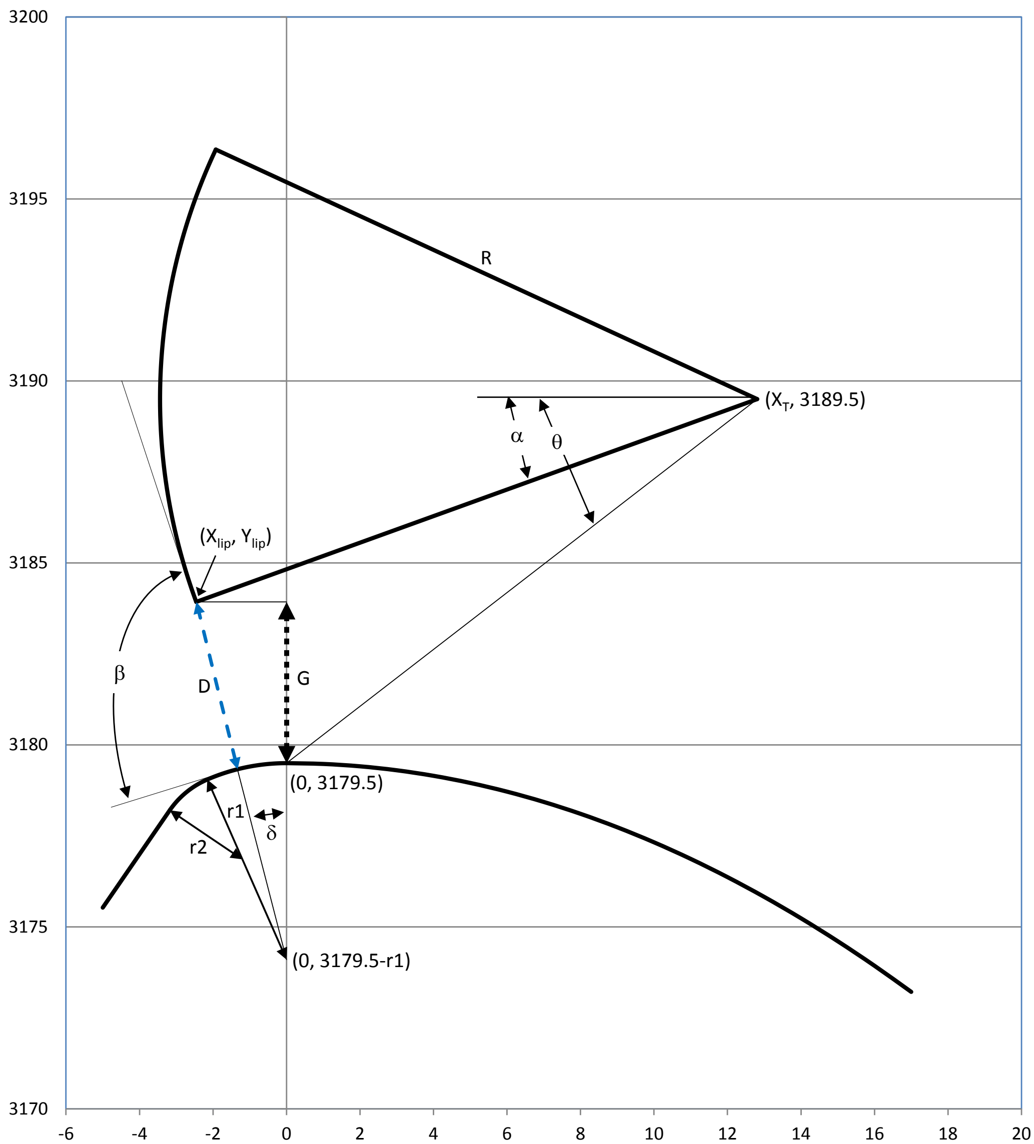

Figure 22. — Key variables needed to define gate openings and other gate parameters.

The next step is to determine the gate opening at the minimum distance from the crest, *D*. This can be computed as follows:

$$\delta = \tan^{-1}\left(\frac{-X_{lip}}{r1+G}\right)$$
$$D = \frac{r1+G}{\cos(\delta)} - r1 \tag{17}$$

Finally, the angle β can be computed from:

$$\alpha = \sin^{-1}\left(\frac{3189.5 - Y_{lip}}{R}\right)$$
$$\beta = 90 + \delta - \alpha \tag{18}$$

The equation for β may not be immediately apparent. Consider the case of the gate being raised so that α=0. At that position, the tangent to the gate leaf at the gate lip is vertical and β = 90 + δ. As the gate is then lowered, the tangent to the gate lip rotates by the angle α, decreasing β.

The head, *H*, above the center of the minimum opening is computed from:

$$H = WSE - [3179.5 - r1 + (r1 + D/2)\cos(\delta)] \tag{19}$$

where *WSE* is the reservoir water surface elevation.

The final parameter needed is the discharge coefficient, which can be read visually from Figure 23, or computed from a curve-fit equation developed during this study:

$$C = 0.00002134 \times \beta^2 - 0.002274 \times \beta + 0.7175 \tag{20}$$

with β given in degrees. Figure 23 highlights the values of C for gate openings of 1 to 7 ft.

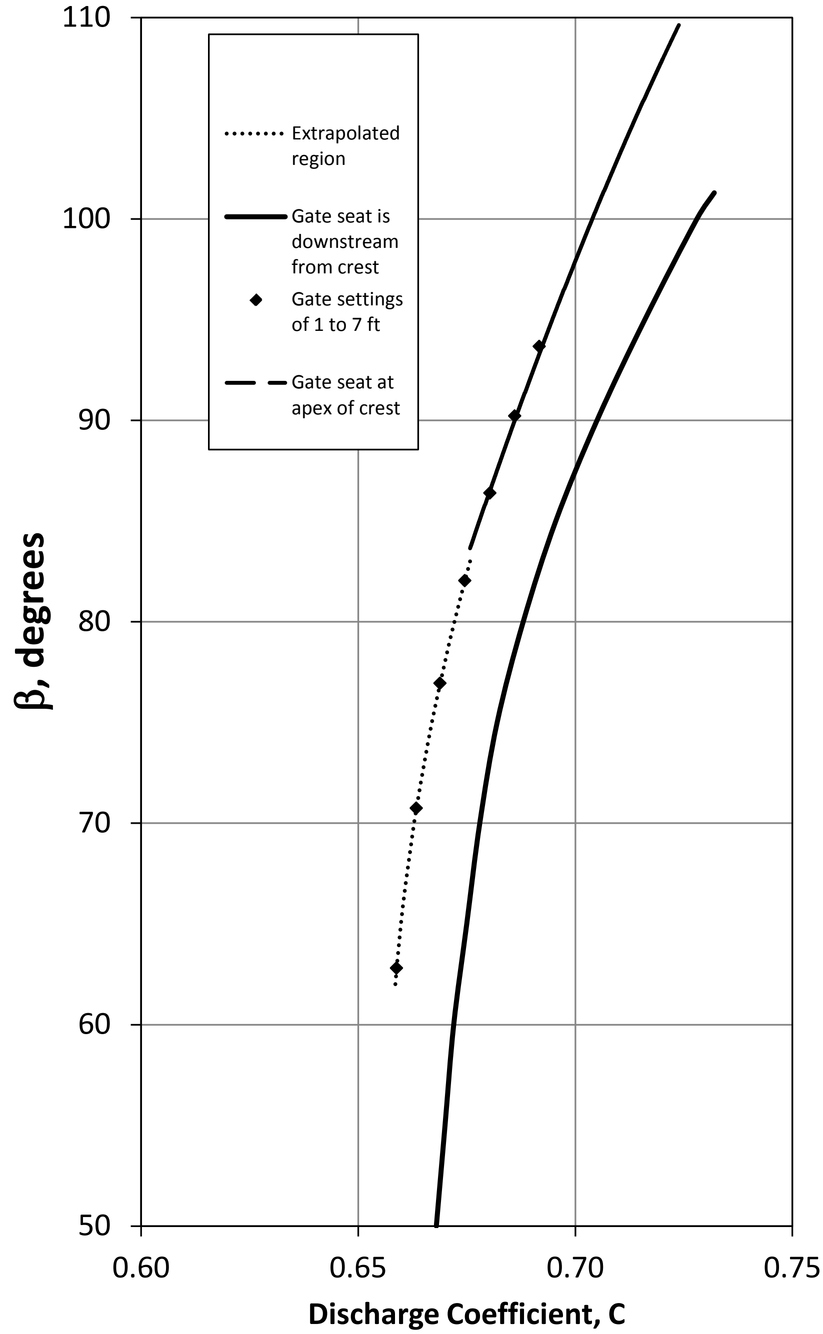

Figure 23. — Discharge coefficient curve for USBR/USACOE method (adapted from *Design of Small Dams*, 3rd ed., 1987). The curve used for this application is the dashed line and the extrapolated section (dotted line). The extrapolation was adjusted manually to maintain approximately parallelism of the two lines in the extrapolated region.