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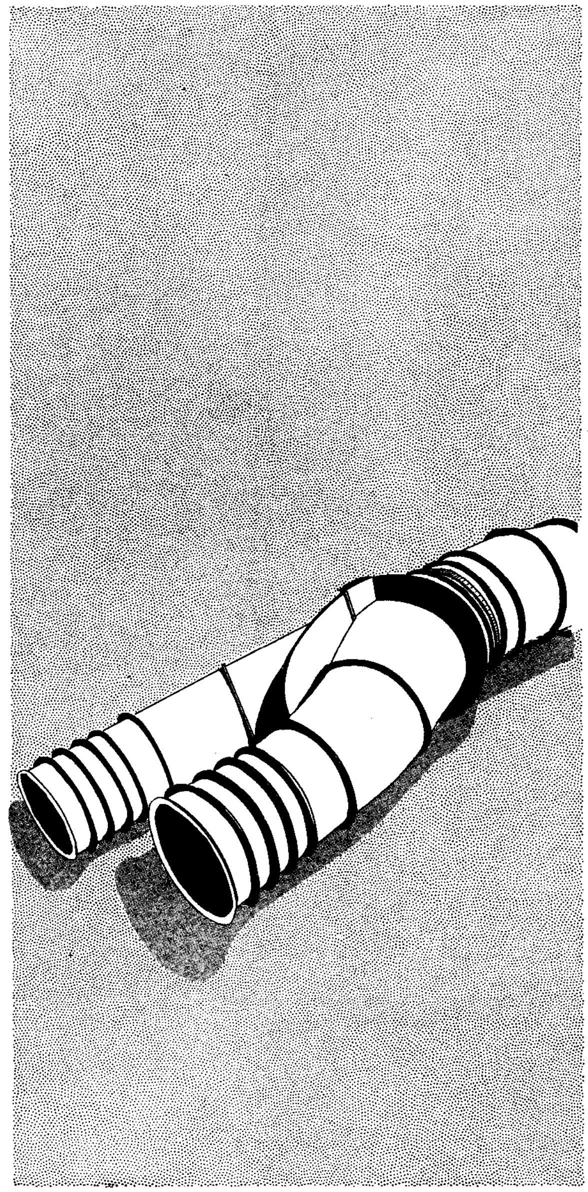

Stress Analysis of Wye Branches

UNITED STATES DEPARTMENT OF THE INTERIOR

BUREAU OF RECLAMATION

A Water Resources Technical Publication
Engineering Monograph No. 32

Stress Analysis of Wye Branches

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United States Department of the Interior • BUREAU OF RECLAMATION

In its assigned function as the Nation's principal natural resource agency, the Department of the Interior bears a special obligation to assure that our expendable resources are conserved, that renewable resources are managed to produce optimum yields, and that all resources contribute their full measure to the progress, prosperity, and security of America, now and in the future.

ENGINEERING MONOGRAPHS are published in limited editions for the technical staff of the Bureau of Reclamation and interested technical circles in Government and private agencies. Their purpose is to record developments, innovations, and progress in the engineering and scientific techniques and practices that are employed in the planning, design, construction, and operation of Reclamation structures and equipment.

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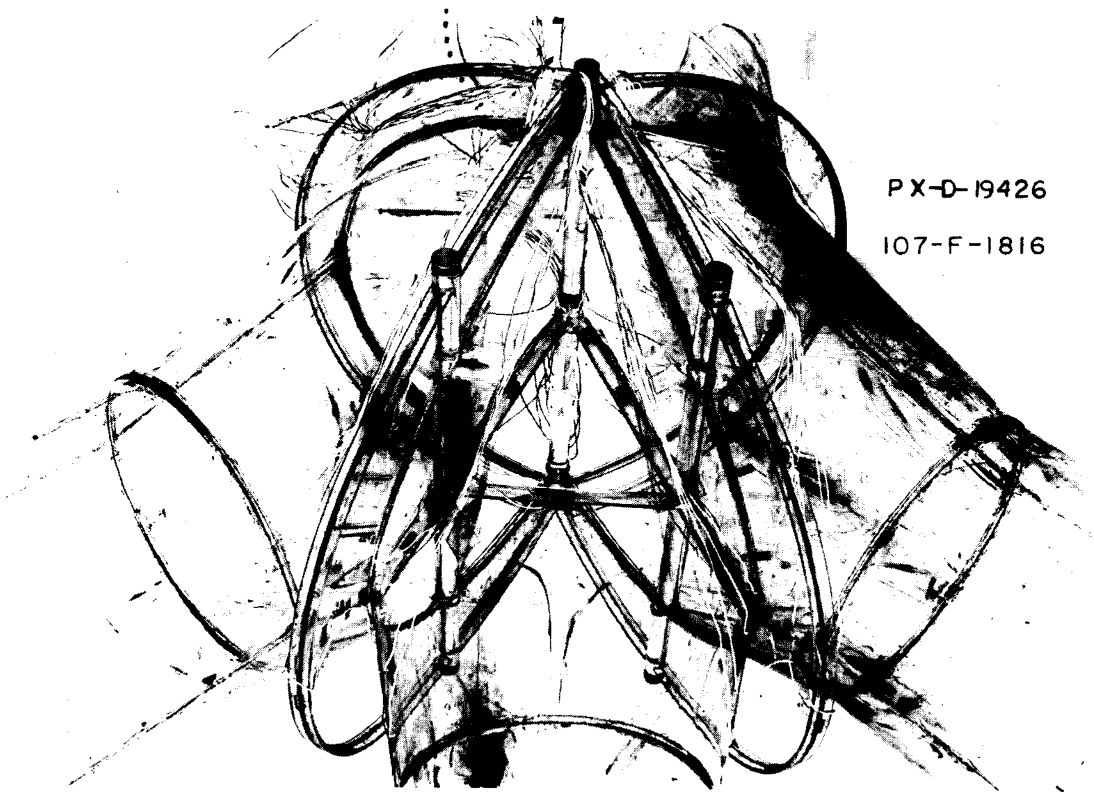
NOMENCLATURE

A	area
E	modulus of elasticity (tension)
G	modulus of elasticity (shear)
I	moment of inertia
K	slope of load line, curvature factor
L	length
M	moment
R	radius
T	tension
V	shear
β	angle between vertical and a line perpendicular to elastic axis
Δ	deflection, increment
θ	angle between stiffener and pipe centerline
ϕ	angle of conicity of outlet pipe
ψ	angle of rotation of end of beam
ν	Poisson's ratio
c	web thickness
m	unit moment
p	internal pressure
r	radius
s	distance along elastic axis
t	unit tension, thickness
v	unit shear
w	effective flange width
x	distance
y	ordinate of $\frac{M}{EI}$ diagram, distance



Frontispiece--Experimental Analysis of Wye Branch Models

Preface

FOR MANY YEARS the Bureau of Reclamation has been engaged in the design and construction of penstock branch connections, or wye branches, of various types. As a result of these studies, methods of analysis have been developed which incorporate a number of improvements on methods that were available before those described in this monograph were devised.

The standard procedure presented in the monograph systematizes and condenses the computing processes. Tabular forms for numerical integration and solution of the deflection equations and for stress computations have been completed with illustrative examples and are included. By using these forms, procedural mistakes and numerical errors will be reduced to a minimum.

While the procedure is designed specifically for use in the analysis of particular structures, other wye branches of similar form may be analyzed and the results obtained from a different set of continuity equations.

Rib shortening and shear deflection of the stiffener beams have been introduced into the method, as well as a variable flange width. The effects of end loads and conicity of the outlet pipes has been neglected as being small in comparison to the vertical load on the beams. Illustrative examples are given of each type of wye branch analyzed.

Introduction

A penstock branch connection is a complicated structure, usually having several stiffening beams to resist the loads applied by the shell of the pipe, and often having internal tension members called tie rods. The purpose of the tie rods is to assist the stiffening beams in carrying the applied loads.

In order to analyze the branch connection, many simplifications and approximations must be utilized. The localized effect of structural discontinuities, restraints of the stiffening beams, methods of support and dead load of the filled pipe have been neglected.

Structural analysis of the pipe branch connection consists in general of four parts:

a. Determination of the part of the structure which resists the unbalanced load.

b. Determination of the load imposed on the resisting members.

c. Analysis of the loaded structure.

d. Interpretation of the findings of the analysis.

The parts of the branch connection resisting the unbalanced pressure load are assumed to consist of the external stiffening beams and rings, the internal tie rods, and the portion of the pipe shell adjacent to the stiffener acting integrally as an effective flange.

The stiffener beams are assumed to carry the vertical component of the membrane girth stress resultant at the line of attachment of the shell to the stiffener. This load varies linearly from zero at the top centerline of the pipe to a maximum at the horizontal centerline of the pipe.

The intersecting beams and tie rods are analyzed as a statically indeterminate structure by the virtual work method, utilizing the conditions of continuity at the junctions of the beams and rods to determine the moments and shears at the ends of the individual beams and rods.

Interpretation of the stresses obtained in any structure is done by appraisal of the general acceptability of the assumptions made in the method of structural action, the applied loading, and the accuracy of the analysis. For the conditions given, the methods presented herein are considered to represent the best currently available solution for determination of stresses in wye branches.

Appendixes I and II present model studies and prototype results compared to the computed stresses.

Symmetrical Trifurcation

Members

In the symmetrical trifurcation shown in Figure 1, and on Drawing No. 1, the structures requiring analysis are the primary load carrying members, which are the reinforcing rings 'OA' and 'OB', and the tie rods at 'O' and 'C'. The applied loading on the structure will be carried by bending, shear, and tension of the reinforcing beams, assisted by the tie rods.

Loads

Consider the large elliptical beam 'OB'. It is assumed to be loaded by vertical forces varying linearly from zero at $x = 0$ to $p(r_1 \cos \theta_1 + r_2 \cos \theta_2)$ at $x = x_8$ (where p is the internal pressure), by the forces V_1 and V_2 due to tie rod tensions at 'O' and 'C' (in the plan view on Drawing No. 1), and by the end moment M_1. The linearly varying portion of the load represents the vertical component of the circumferential stress resultant of a cylindrical shell. The horizontal component of this resultant is reacted by an equal and opposite load from the adjacent shell.

In the case of conical outlet pipes, it may be determined that the vertical loading given by the above formula is somewhat below the actual value. For a typical conical shell ($\theta_2 = 35°$, $\phi_2 = 12°$), the total load applied to the beam by the shell will be approximately 12 percent more than the assumed load given here.

Effective Flange Width

From the shape of an assumed moment diagram we may approximate the amount of the shell acting as an effective flange width (see References d and e). The moment diagram is divided into parts, each part fitting a shape for which the flange width is known. The effective flange width is assumed to be a continuous function, and an approximation of the flange width is made at points along

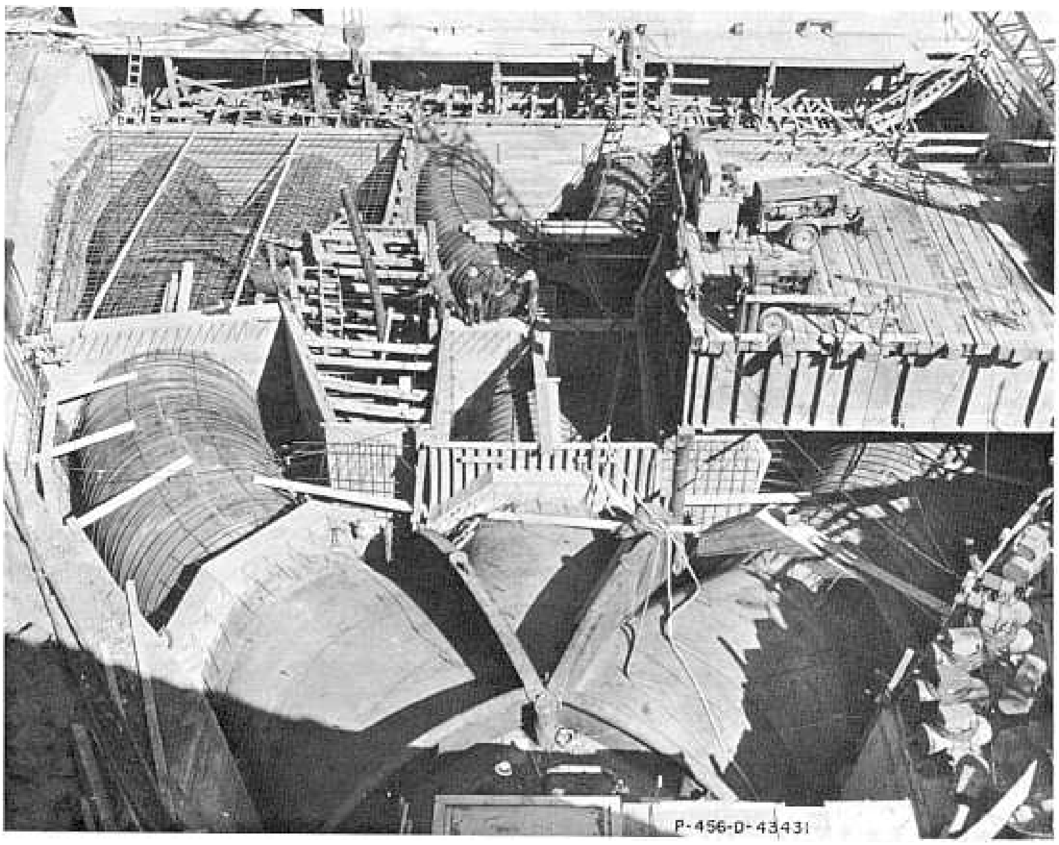


Figure 1. --Symmetrical Trifurcation

the elastic axis of the beam from the shape of the moment diagram at these points. The angle at which the shell intercepts the beam is considered inconsequential, since the flange effect is obtained by shear of the pipe walls.

The way in which the effective flange width is chosen is largely a matter of judgment and experience (see References b, c, d, and e). However, previous analyses show that some latitude may be tolerated in choosing an effective flange width without seriously affecting the final results. The assumed elastic axis is divided into four equal parts in each interval. The centroid of the beam is located, using the effective flange width at each point. The revised elastic axis is plotted through the centroids, and divided into four equal segments in each interval as before. The moments of inertia of the beam at the various points are then computed including the effective flange widths.

Equations for Moment, Shear, and Tension

The elastic axis of the beam is in three regions of loading, each of which is divided into four equal parts. Writing the expressions for the moment, shear, and tension in the beam, we have for the region $0 < x < x_4$,

$$M = M_1 + V_1 x + \frac{Kx^3}{6},$$

$$V = \left(V_1 + \frac{Kx^2}{2}\right) \cos\beta,$$

$$T = \left(V_1 + \frac{Kx^2}{2}\right) \sin\beta,$$

where

$$K = \frac{p(r_1 \cos\theta_1 + r_2 \cos\theta_2)}{x_8}$$

and β is the angle between a vertical line and a line perpendicular to the elastic axis, as shown on the drawing.

For the region $x_4 < x < x_8$,

$$M = M_1 + V_1 x + V_2(x - x_4) + \frac{Kx^3}{6},$$

$$V = \left(V_1 + V_2 + \frac{Kx^2}{2}\right) \cos\beta,$$

$$T = \left(V_1 + V_2 + \frac{Kx^2}{2}\right) \sin\beta,$$

and for the region $x_8 < x < x_{12}$,

$$M = M_1 + V_1 x + V_2(x - x_4) + \frac{Kx_8^2}{2}\left(x - \frac{2}{3}x_8\right),$$

$$V = \left(V_1 + V_2 + \frac{Kx_8^2}{2}\right) \cos\beta,$$

$$T = \left(V_1 + V_2 + \frac{Kx_8^2}{2}\right) \sin\beta.$$

Deflection and Rotation of Members

The equations for the deflection and rotation of beam 'OB' at Point 'O' are (including the effects of shear deflection and rib shortening):

$$\Delta = \int_{x_0}^{x_{12}} \frac{Mmds}{EI} + \int_{x_0}^{x_{12}} \frac{Vvds}{GA} + \int_{x_0}^{x_{12}} \frac{Ttds}{EA},$$

$$m = x, \quad v = \cos\beta, \quad t = \sin\beta,$$

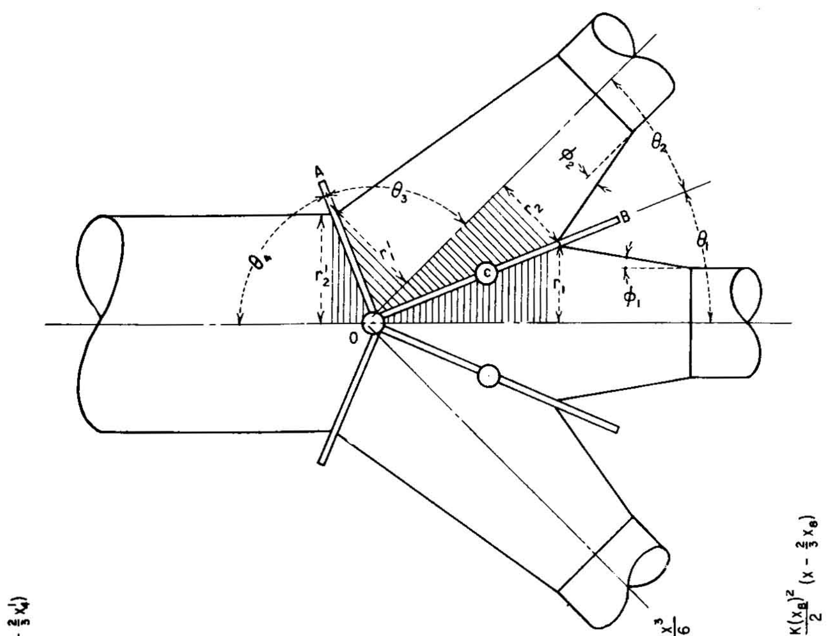
$$\psi = \int_{x_0}^{x_{12}} \frac{Mmds}{EI}, \quad m = 1, \quad v = 0, \quad t = 0.$$

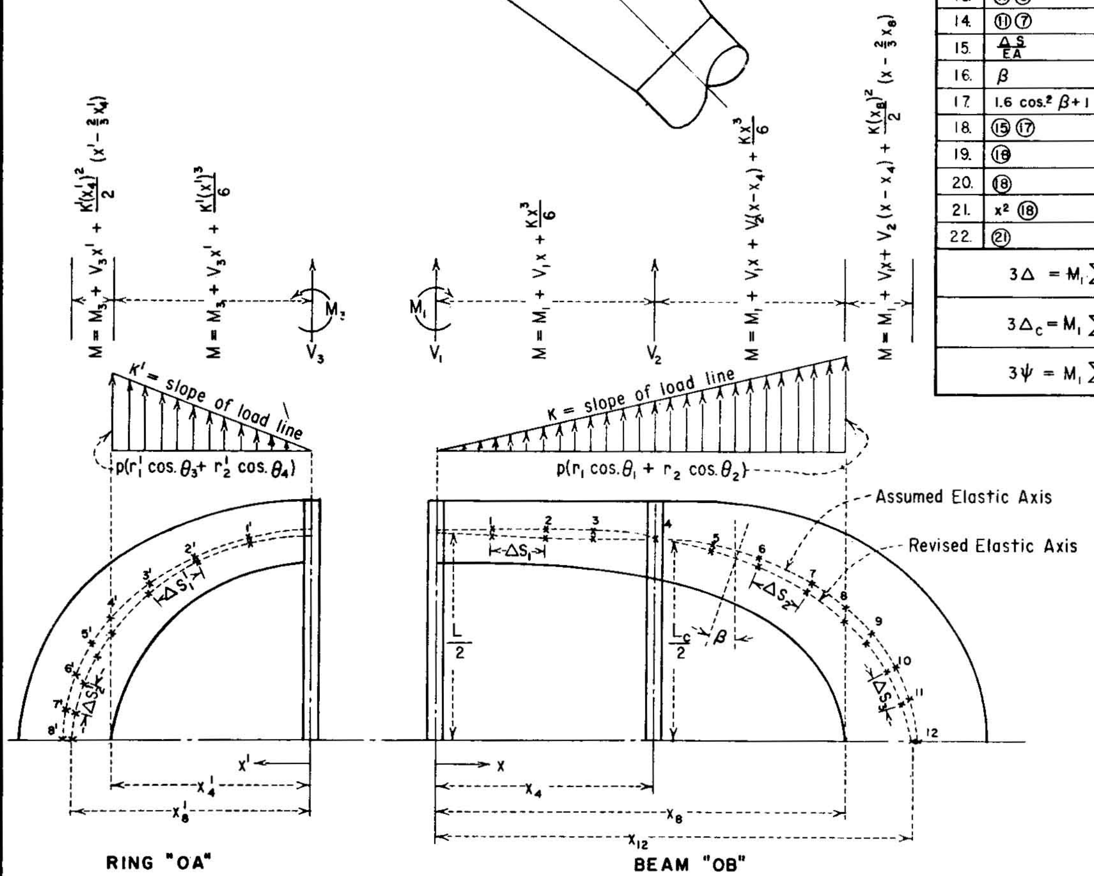
The equation for the deflection of the beam at Point 'C' is

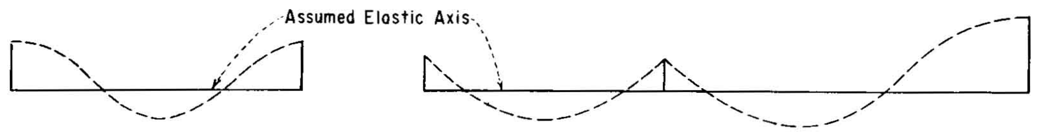
$$\Delta_c = \int_{x_4}^{x_{12}} \frac{Mmds}{EI} + \int_{x_4}^{x_{12}} \frac{Vvds}{GA} + \int_{x_4}^{x_{12}} \frac{Ttds}{EA},$$

$$m = (x - x_4), \quad v = \cos\beta, \quad t = \sin\beta.$$

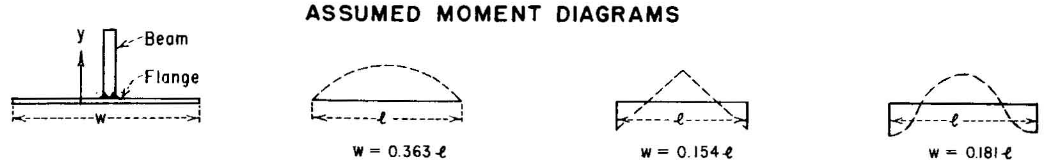
These equations are now integrated using Simpson's Rule and the accompanying tabular form. Applying the rule to our present problem, we have:







ASSUMED MOMENT DIAGRAMS



EFFECTIVE FLANGE WIDTHS

FOR BEAM: K = 0.7072 $\qquad \frac{\Delta S_1}{E} = 1.5666\times10^{-6} \qquad \frac{\Delta S_2}{E} = 0.90833\times10^{-6} \qquad \frac{\Delta S_3}{E} = 0.7125\times10^{-6}$

	POINT	0	1	2	3	4L	4R	5
	FACTOR	1	4	2	4	1	1	4
1.	x	0	46	92.3	138	183	183	207.5
2.	$\frac{\Delta S}{EI}$	57.5985×10^{-12}	28.6935×10^{-12}	19.6078×10^{-12}	16.6844×10^{-12}	15.1223×10^{-12}	8.7677×10^{-12}	7.9192×10^{-12}
3.	x ②	0	1.3199×10^{-9}	1.8098×10^{-9}	2.3024×10^{-9}	2.7674×10^{-9}	1.6045×10^{-9}	1.6432×10^{-9}
4.	x^2 ②	0	6.0715×10^{-8}	16.7045×10^{-8}	31.7738×10^{-8}	50.6431×10^{-8}	29.3621×10^{-8}	34.0969×10^{-8}
5.	x^3 ②	0	2.79291×10^{-6}	15.4182×10^{-6}	43.8478×10^{-6}	92.6770×10^{-6}	53.7328×10^{-6}	70.7517×10^{-6}
6.	x^4 ②	0	128.4739×10^{-6}	1423.101×10^{-6}	6050.998×10^{-6}	16959.811×10^{-6}	9833.063×10^{-6}	14680.937×10^{-6}
7.	$(x - x_4)$ ②	—	—	—	—	—	0	0.1940×10^{-9}
8.	$(x - x_4)^2$ ②	—	—	—	—	—	0	4.7535×10^{-9}
9.	x ⑦	—	—	—	—	—	0	4.0255×10^{-8}
10.	x^3 ⑦	—	—	—	—	—	0	1.7332×10^{-3}
11.	$x - \frac{2}{3}x_8$	—	—	—	—	—	—	—
12.	⑪②	—	—	—	—	—	—	—
13.	⑪③	—	—	—	—	—	—	—
14.	⑪⑦	—	—	—	—	—	—	—
15.	$\frac{\Delta S}{EA}$	10.8091×10^{-9}	8.28617×10^{-9}	7.6587×10^{-9}	7.7153×10^{-9}	7.7627×10^{-9}	4.5007×10^{-9}	4.3215×10^{-9}
16.	β	0°	8.3°	10°	13.6°	22.2°	22.2°	29.0°
17.	$1.6 \cos^2\beta + 1$	2.6000	2.5665	2.5517	2.5117	2.3717	2.3717	2.2238
18.	⑮⑰	28.1037×10^{-9}	21.2665×10^{-9}	19.5427×10^{-9}	19.3785×10^{-9}	18.4108×10^{-9}	10.6743×10^{-9}	9.6102×10^{-9}
19.	⑱	—	—	—	—	—	10.6743×10^{-9}	9.6102×10^{-9}
20.	⑱	—	—	—	—	—	—	—
21.	x^2 ⑱	0	44.99391×10^{-6}	166.4899×10^{-6}	369.04415×10^{-6}	616.5593×10^{-6}	357.4716×10^{-6}	413.7768×10^{-6}
22.	㉑	—	—	—	—	—	357.4716×10^{-6}	413.7768×10^{-6}

	POINT	6	7	8L	8R	9	10	11	12	Σ	
	FACTOR	2	4	1	1	4	2	4	1	—	
1.	x	230.4	253	275.4	275.4	289	300.5	308.5	311.5	—	1.
2.	$\frac{\Delta S}{EI}$	6.4788×10^{-12}	6.25143×10^{-12}	6.2215×10^{-12}	4.8801×10^{-12}	4.6266×10^{-12}	4.4727×10^{-12}	4.2639×10^{-12}	4.2639×10^{-12}	4.31729×10^{-10}	2.
3.	x ②	1.4927×10^{-9}	1.5816×10^{-9}	1.7134×10^{-9}	1.3440×10^{-9}	1.3371×10^{-9}	1.3440×10^{-9}	1.3154×10^{-9}	1.3154×10^{-9}	56.0489×10^{-9}	3.
4.	x^2 ②	34.3921×10^{-8}	40.0148×10^{-8}	47.1870×10^{-8}	37.0131×10^{-8}	38.6418×10^{-8}	40.3885×10^{-8}	40.5804×10^{-8}	41.3736×10^{-8}	11.53266×10^{-6}	4.
5.	x^3 ②	79.2396×10^{-6}	101.2375×10^{-6}	129.9534×10^{-6}	—	—	—	—	—	1.3401984×10^{-3}	5.
6.	x^4 ②	18256.805×10^{-6}	25613.059×10^{-6}	35789.179×10^{-6}	—	—	—	—	—	287.835744×10^{-3}	6.
7.	$(x - x_4)$ ②	0.3071×10^{-9}	0.4376×10^{-9}	0.5749×10^{-9}	0.45092×10^{-9}	0.49042×10^{-9}	0.52554×10^{-9}	0.53512×10^{-9}	0.54791×10^{-9}	9.86757×10^{-9}	7.
8.	$(x - x_4)^2$ ②	14.5563×10^{-9}	30.6320×10^{-9}	53.1177×10^{-9}	41.6651×10^{-9}	51.9845×10^{-9}	61.7512×10^{-9}	67.1575×10^{-9}	70.40658×10^{-9}	935.9144×10^{-9}	8.
9.	x ⑦	7.0756×10^{-8}	11.0713×10^{-8}	15.8327×10^{-8}	12.4183×10^{-8}	14.1731×10^{-8}	15.7925×10^{-8}	16.5085×10^{-8}	17.0674×10^{-8}	274.1682×10^{-8}	9.
10.	x^3 ⑦	3.7560×10^{-3}	7.0866×10^{-3}	12.0084×10^{-3}	—	—	—	—	—	54.7996×10^{-3}	10.
11.	$x - \frac{2}{3}x_8$	—	—	—	91.800	105.4	116.9	124.9	127.9	—	11.
12.	⑪②	—	—	—	0.4480×10^{-9}	0.4876×10^{-9}	0.52286×10^{-9}	0.53256×10^{-9}	0.54535×10^{-9}	6.11971×10^{-9}	12.
13.	⑪③	—	—	—	0.12338×10^{-6}	0.14093×10^{-6}	0.15711×10^{-6}	0.16429×10^{-6}	0.16824×10^{-6}	1.82672×10^{-6}	13.
14.	⑪⑦	—	—	—	41.3945×10^{-9}	51.6903×10^{-9}	61.4356×10^{-9}	66.8365×10^{-9}	70.0777×10^{-9}	708.4506×10^{-9}	14.
15.	$\frac{\Delta S}{EA}$	3.9120×10^{-9}	3.8745×10^{-9}	3.96756×10^{-9}	3.1122×10^{-9}	3.0670×10^{-9}	3.0554×10^{-9}	3.02946×10^{-9}	3.02946×10^{-9}	—	15.
16.	β	34.7°	33.9°	45.1°	45.1°	52.5°	62.4°	74.5°	90°	—	16.
17.	$1.6 \cos^2\beta + 1$	2.0813	2.1022	1.7973	1.7973	1.5930	1.3434	1.1424	1.000	—	17.
18.	⑮⑰	8.1420×10^{-9}	8.1450×10^{-9}	7.1309×10^{-9}	5.5936×10^{-9}	4.8857×10^{-9}	4.1046×10^{-9}	3.46086×10^{-9}	3.02946×10^{-9}	403.5084×10^{-9}	18.
19.	⑱	8.1420×10^{-9}	8.1450×10^{-9}	7.1309×10^{-9}	5.5936×10^{-9}	4.8857×10^{-9}	4.1046×10^{-9}	3.46086×10^{-9}	3.02946×10^{-9}	155.3285×10^{-9}	19.
20.	⑱	—	—	—	5.5936×10^{-9}	4.8857×10^{-9}	4.1046×10^{-9}	3.46086×10^{-9}	3.02946×10^{-9}	50.2185×10^{-9}	20.
21.	x^2 ⑱	432.2099×10^{-6}	521.3533×10^{-6}	540.8431×10^{-6}	—	—	—	—	—	8.108970×10^{-3}	21.
22.	㉑	432.2099×10^{-6}	521.3533×10^{-6}	540.8431×10^{-6}	—	—	—	—	—	5.503255×10^{-3}	22.

$$3\Delta = M_1\Sum ③ + V_1\left(\Sum ④ + \Sum ⑱\right) + \frac{K}{6}\left(\Sum ⑥ + 3\Sum ㉑\right) + V_2\left(\Sum ⑨ + \Sum ⑲\right) + \frac{Kx_8^2}{2}\left(\Sum ⑬ + \Sum ⑳\right)$$

$$\Delta = 18.6830\times10^{-9}M_1 + 3.9787\times10^{-6}V_1 + 0.96567\times10^{-6}V_2 + 29.04363\times10^{-3}$$

$$3\Delta_c = M_1\Sum ⑦ + V_1\left(\Sum ⑨ + \Sum ⑲\right) + V_2\left(\Sum ⑧ + \Sum ⑲\right) + \frac{K}{6}\left(\Sum ⑩ + 3\Sum ㉒\right) + \frac{Kx_8^2}{2}\left(\Sum ⑭ + \Sum ⑳\right)$$

$$\Delta_c = 3.28919\times10^{-9}M_1 + 0.96567\times10^{-6}V_1 + 0.36375\times10^{-6}V_2 + 9.583877\times10^{-3}$$

$$3\psi = M_1\Sum ② + V_1\Sum ③ + \frac{K}{6}\Sum ⑤ + V_2\Sum ⑦ + \frac{Kx_8^2}{2}\Sum ⑫$$

$$\psi = 0.14391\times10^{-7}M_1 + 18.6787\times10^{-9}V_1 + 3.28919\times10^{-9}V_2 + 107.36277\times10^{-6}$$

REVISION OF ELASTIC AXIS AND COMPUTATION OF I

POINT	ℓ	w	ΣA	ΣAy	$\bar{y}$	I
0		30	144.94	2448.80	17.17	27200
1		50	189.07	3522.83	18.63	54600
2		48	204.56	4628.28	22.63	79900
3		36	203.06	5543.65	27.30	93900
4		26	201.82	6368.34	31.55	103 600
5		30	210.19	6632.18	31.55	114 700
6		44	232.19	6998.62	30.14	140 200
7		44	234.44	7179.75	30.625	145 300
8		36	228.94	7542.50	32.95	146 000
9		36	232.31	7825.15	33.68	154 000
10		34	233.19	8111.31	34.78	159 300
11		32	235.19	8501.25	36.15	167100
12		32	235.19	8501.25	36.15	167100
0'		50	177.3	2874	16.21	40500
1'		50	181.9	3120.	17.15	45700
2'		50	183.0	3181.	17.38	47100
3'		50	180.3	3035	16.83	43900
4'		50	176.7	2845	16.10	39 900
5'		50	174.4	2729	15.65	37 500
6'		50	172.6	2633	15.29	35 600
7'		50	172.2	2616	15.19	35 200
8'		50	172.0	2605	15.15	34 900

$L = 346.4$ $\qquad A = 176.715$

$L_c = 262.21$ $\qquad A_c = 176.715$

$\theta_1 = 22° 30'$ $\qquad E = 3\times10^7$

$\theta_2 = 22° 30'$

$\theta_3 = 67.76°$

$\theta_4 = 67.76°$

FOR RING: K' = 0.68707 #/in² $\qquad \frac{\Delta S_1'}{E} = 1.6875\times10^{-6} \qquad \frac{\Delta S_2'}{E} = 0.5750\times10^{-6}$

	POINT	0'	1'	2'	3'	4'R	4'L	5'	6'	7'	8'	Σ	
	FACTOR	1	4	2	4	1	1	4	2	4	1	—	
1.	x'	0	50	96.5	135.5	162.375	162.375	169	174	176.5	177.5	—	1.
2.	$\frac{\Delta S'}{EI}$ [redacted]	41.6667×10^{-12}	36.9256×10^{-12}	35.8280×10^{-12}	38.4396×10^{-12}	42.2932×10^{-12}	14.4110×10^{-12}	15.3333×10^{-12}	16.1517×10^{-12}	16.3352×10^{-12}	16.4756×10^{-12}	646.9407×10^{-12}	2.
3.	x' ②	0	1.84628×10^{-9}	3.4574×10^{-9}	5.20856×10^{-9}	6.86736×10^{-9}	2.3400×10^{-9}	2.59133×10^{-9}	2.8104×10^{-9}	2.88316×10^{-9}	2.9244×10^{-9}	74.7847×10^{-9}	3.
4.	$(x')^2$ ②	0	9.2314×10^{-8}	33.3639×10^{-8}	70.5760×10^{-8}	111.5087×10^{-8}	37.9955×10^{-8}	43.7935×10^{-8}	48.9009×10^{-8}	50.8878×10^{-8}	51.9084×10^{-8}	10.63897×10^{-6}	4.
5.	$(x')^3$ ②	0	4.6157×10^{-6}	32.1962×10^{-6}	95.6305×10^{-6}	181.0623×10^{-6}	—	—	—	—	—	646.4338×10^{-6}	5.
6.	$(x')^4$ ②	0	0.230785×10^{-3}	3.10693×10^{-3}	12.95794×10^{-3}	29.4000×10^{-3}	—	—	—	—	—	88.36876×10^{-3}	6.
7.	$x' - \frac{2}{3}x_4'$	—	—	—	—	—	54.125	60.75	65.75	68.25	69.25	—	7.
8.	⑦②	—	—	—	—	—	0.78000×10^{-9}	0.9315×10^{-9}	1.06197×10^{-9}	1.11488×10^{-9}	1.14093×10^{-9}	12.2304×10^{-9}	8.
9.	⑦③	—	—	—	—	—	0.12665×10^{-6}	0.15742×10^{-6}	0.18478×10^{-6}	0.19677×10^{-6}	0.20251×10^{-6}	2.1155×10^{-6}	9.
10.	$\frac{\Delta S'}{EA}$ [redacted]	9.51777×10^{-9}	9.27707×10^{-9}	9.2213×10^{-9}	9.3594×10^{-9}	8.5501×10^{-9}	3.254×10^{-9}	3.29702×10^{-9}	3.3314×10^{-9}	3.3391×10^{-9}	3.3430×10^{-9}	—	10.
11.	β	0	13.8°	31.0°	47.8°	64.5°	64.5°	71.0°	77.5°	83.7°	90°	—	11.
12.	$(1.6\cos^2\beta + 1)$	2.600	2.5088	2.1757	1.7219	1.2965	1.2965	1.1696	1.0749	1.0192	1.000	—	12.
13.	⑩⑫	24.7462×10^{-9}	23.2743×10^{-9}	20.0628×10^{-9}	16.1159×10^{-9}	12.3817×10^{-9}	4.2189×10^{-9}	3.8562×10^{-9}	3.5809×10^{-9}	3.4032×10^{-9}	3.3430×10^{-9}	278.575×10^{-9}	13.
14.	⑩⑫	—	—	—	—	—	4.2189×10^{-9}	3.8562×10^{-9}	3.5809×10^{-9}	3.4032×10^{-9}	3.3430×10^{-9}	43.7613×10^{-9}	14.
15.	$(x')^2$ ⑬	0	58.18575×10^{-6}	186.8298×10^{-6}	295.8929×10^{-6}	326.4514×10^{-6}	—	—	—	—	—	2.164256×10^{-3}	15.

$$3\Delta' = M_3\Sum ③ + V_3\left(\Sum ④ + \Sum ⑬\right) + \frac{K'}{6}\left(\Sum ⑥ + 3\Sum ⑮\right) + \frac{K'(x_4')^2}{2}\left(\Sum ⑨ + \Sum ⑭\right)$$

$$\Delta' = 3.63918\times10^{-6}V_3 + 24.92823\times10^{-9}M_3 + 10.134618\times10^{-3}$$

$$3\psi' = M_3\Sum ② + V_3\Sum ③ + \frac{K'}{6}\Sum ⑤ + \frac{K'(x_4')^2}{2}\Sum ⑧$$

$$\psi' = 24.92823\times10^{-9}V_3 + 0.21565\times10^{-9}M_3 + 61.600663\times10^{-6}$$

FINAL BASIC EQUATIONS

$$\frac{-\Sigma V_1 L}{2AE} = \Delta \qquad \psi \cos\theta_4 = -\psi' \cos\theta_1$$

$$\frac{-\Sigma V_1 L}{2AE} = \Delta' \qquad M_1 \cos\theta_1 = M_3 \cos\theta_4$$

$$-\frac{V_2 L_c}{2A_c E} = \Delta_c$$

STRESS ANALYSIS OF WYE BRANCHES

SYMMETRICAL TRIFURCATIONS
DEFLECTIONS AND ROTATIONS

Palisades Wye W-I

DRAWING I

$$\Delta = \sum_{x_0}^{x_{12}} \frac{Mm\Delta s}{EI} + \sum_{x_0}^{x_{12}} \frac{Vv\Delta s}{GA} + \sum_{x_0}^{x_{12}} \frac{Tt\Delta s}{EA}$$

$$= \frac{\Delta s_1}{3} (y_0 + 4y_1 + 2y_2 + 4y_3 + y_4)$$

$$+ \frac{\Delta s_2}{3} (y_4 + 4y_5 + 2y_6 + 4y_7 + y_8)$$

$$+ \frac{\Delta s_3}{3} (y_8 + 4y_9 + 2y_{10} + 4y_{11} + y_{12}),$$

where $y_0 = \frac{Mm}{EI} + \frac{Vv}{GA} + \frac{Tt}{EA}$ at Point 'O',

$y_1 = \frac{Mn}{EI} + \frac{Vv}{GA} + \frac{Tt}{EA}$ at Point 1, etc.

Performing this integration, we have for the deflection of the beam 'OB' at Point 'O' (assuming $G = \frac{E}{2(1+v)}$, where $v = 0.3$),

$$\Delta = \sum_{x_0}^{x_{12}} \left[(M_1 + V_1 x)(x) \frac{\Delta s}{EI} + V_1 (1.6 \cos^2 \beta + 1) \frac{\Delta s}{EA} \right]$$

$$+ \sum_{x_0}^{x_8} \left[\frac{Kx^3}{6} (x) \frac{\Delta s}{EI} + \frac{Kx^2}{2} (1.6 \cos^2 \beta + 1) \frac{\Delta s}{EA} \right]$$

$$+ \sum_{x_4}^{x_{12}} \left[V_2 (x - x_4)(x) \frac{\Delta s}{EI} + V_2 (1.6 \cos^2 \beta + 1) \frac{\Delta s}{EA} \right]$$

$$+ \sum_{x_8}^{x_{12}} \left[\frac{Kx_8^2}{2} (x - \tfrac{2}{3} x_8)(x) \frac{\Delta s}{EI} + \frac{Kx_8^2}{2} (1.6 \cos^2 \beta + 1) \frac{\Delta s}{EA} \right]$$

and for the deflection at Point 'C',

$$\Delta_c = \sum_{x_4}^{x_{12}} \Big[[M_1 + V_1 x + V_2 (x - x_4)] (x - x_4) \frac{\Delta s}{EI}$$

$$+ (V_1 + V_2)(1.6 \cos^2 \beta + 1) \frac{\Delta s}{EA} \Big]$$

$$+ \sum_{x_4}^{x_8} \frac{Kx^3}{6} (x - x_4) \frac{\Delta s}{EI} + \frac{Kx^2}{2} (1.6 \cos^2 \beta + 1) \frac{\Delta s}{EA} \Big]$$

$$+ \sum_{x_8}^{x_{12}} \Big[\frac{Kx_8^3}{2} (x - \tfrac{2}{3} x_8)(x - x_4) \frac{\Delta s}{EI}$$

$$+ \frac{Kx_8^2}{2} (1.6 \cos^2 \beta + 1) \frac{\Delta s}{EA} \Big]$$

For the rotation of the beam at Point 'O',

$$\psi = \sum_{x_0}^{x_{12}} (M_1 + V_1 x) \frac{\Delta s}{EI} + \sum_{x_0}^{x_8} \frac{Kx^3}{6} \frac{\Delta s}{EI} +$$

$$\sum_{x_4}^{x_{12}} V_2 (x - x_4) \frac{\Delta s}{EI} + \sum_{x_8}^{x_{12}} \frac{Kx_8^2}{2} (x - \tfrac{2}{3} x_8) \frac{\Delta s}{EI}$$

Turning our attention now to the ring designated 'OA', a similar procedure may be followed and the deflection and rotation of the end of the ring computed. For the ring 'OA', the equation for the deflection of the ring at Point 'O' is:

$$\Delta' = \int_{x_0'}^{x_8'} \frac{Mmds}{EI} + \int_{x_0'}^{x_8'} \frac{Vvds}{GA} + \int_{x_0'}^{x_8'} \frac{Ttds}{EA},$$

where

$$m = x', \quad v = \cos\beta, \quad t = \sin\beta,$$

$$\text{and} \quad K' = \frac{p(r_1' \cos\theta_3 + r_2' \cos\theta_4)}{x_4'}.$$

This becomes:

$$\Delta' = \sum_{x_0'}^{x_8'} \Big[(M_3 + V_3 x')(x') \frac{\Delta s'}{EI}$$

$$+ V_3(1.6\cos^2\beta + 1)\frac{\Delta s'}{EA}\Big] + \sum_{x_0'}^{x_4'} \Big[\frac{K'(x')^3}{6}$$

$$(x')\frac{\Delta s'}{EI} + \frac{K'(x')^2}{2}(1.6\cos^2\beta + 1)\frac{\Delta s'}{EA}\Big]$$

$$+ \sum_{x_4'}^{x_8'} \Big[\frac{K'(x_4')^2}{2}(x' - \tfrac{2}{3}x_4')(x')\frac{\Delta s'}{EI}$$

$$+ \frac{K'(x_4')^2}{2}(1.6\cos^2\beta + 1)\frac{\Delta s'}{EA}\Big].$$

Also, for the rotation of the ring at 'O' we have:

$$\psi' = \int_{x_0'}^{x_8'} \frac{Mmds}{EI}$$

$$\text{where} \quad m = 1, \quad v = 0, \quad \text{and} \quad t = 0.$$

This becomes:

$$\psi' = \sum_{x_0'}^{x_8'} (M_3 + V_3 x')\frac{\Delta s'}{EI} + \sum_{x_0'}^{x_4'} \frac{K'(x')^3}{6}\frac{\Delta s'}{EI}$$

$$+ \sum_{x_4'}^{x_8'} \frac{K'(x_4')^2}{2}(x' - \tfrac{2}{3}x_4')\frac{\Delta s'}{EI}.$$

Final Equations

The deflection of beam 'OB' at Point 'C' is set equal to the elongation of the tie rod at 'C'. The deflection of the beam 'OB' at Point 'O' is equal to the elongation of the tie rod at Point 'O'. Also the deflection of the ring 'OA' at Point 'O' is equal to the elongation of the tie rod at 'O'.

The rotation of the end of beam 'OB' multiplied by the cosine of the angle between the ring and the penstock centerline is equal and opposite to the rotation of the end of the ring 'OA' multiplied by the cosine of the angle between the beam and the penstock centerline. Also, the components of the end moments along the axis of the penstock are equal, from vector considerations.

The above procedure yields the basic continuity equations for the symmetrical trifurcation, which are:

At 'O',

$\Sigma\Delta = 0$ (2 equations),

$\Sigma\psi = 0$,

$\Sigma M = 0$,

and at 'C',

$\Sigma\Delta = 0$.

These equations become:

$$-\frac{\Sigma V}{AE}\frac{L}{2} = \Delta, \quad -\frac{\Sigma V}{AE}\frac{L}{2} = \Delta',$$

$$-\frac{V_2 L_c}{2A_c E} = \Delta_c,$$

where $\Sigma V = 2(V_1 + V_3)$ for the symmetrical trifurcation,

$$\psi\cos\theta_4 = -\psi'\cos\theta_1, \text{ and}$$

$$M_1 \cos \theta_1 = M_3 \cos \theta_4 ,$$

where

L , A = length and area of rod at 'O', and

L_C, A_C = length and area of rod at 'C',

(If no tie rods are provided, V_2 becomes zero and Δ_C is eliminated. Then $\Delta = \Delta$ ' and $V_1 + V_3 = 0$ are the deflection and shear equations.)

These are our five equations in five unknowns. Solving for the unknowns and resubstituting their values into the original equations enables us to determine the moment, shear, and tension at the various points along the elastic axis of the beam and ring (see Drawing No. 2). Moment, shear, and tension diagrams may then be plotted. The compatibility of the actual values of rotation and deflection obtained from the foregoing equations will comprise one effective check on the computations.

Computation of Stresses

From the values of moment and tension, the stress may be computed at the different locations in the beam and ring on Drawing No. 2.

An appropriate curvature factor may be applied to the bending stress. The illustrative examples in the appendixes show the results of model studies and field tests on two structures.

The values of stress found at the various points in the structure should then be compared with the allowable working stress of the material. At the inside edge of the beam at the horizontal centerline, critical stresses are likely to be found. Also, highly stressed regions are likely to occur in regions adjacent to the tie rods. Based on judgment, the stresses at these points might be accepted at values higher than the usual allowable working stress.

In the example shown, stresses have been computed for an internal pressure of 1 psi.

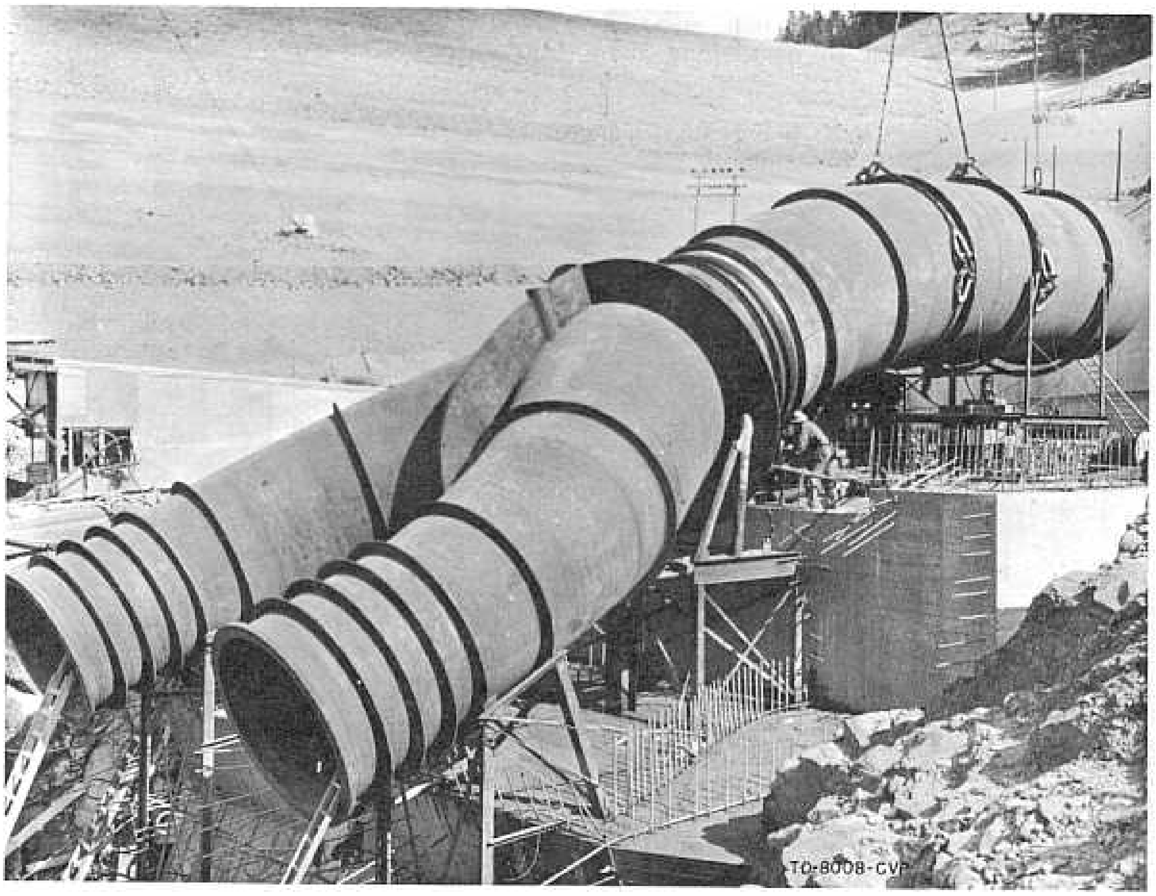


Figure 2. --Symmetrical Bifurcation

Symmetrical Bifurcation

Deflection and Rotation of Members

For the symmetrical bifurcation with one tie rod (see Figure 2), Drawing No. 3 shows the equations for deflection and rotation of the ends of the members.

Final Equations

Five equations in the five unknowns are: The sum of the moments is zero, the sum of the vertical shears is zero, the deflections of the ends of the beam and ring are equal, the sum of the rotations is zero, and the deflection of the beam at the tie rod is equal to the elongation of the rod. (If no tie rod is provided, V_2 becomes zero, and the equation for Δ_C is eliminated. If two tie rods are provided, the deflections of the ends of the beam and ring are equated to the tie-rod elongation.)

Computation of Stresses

Stresses in the symmetrical bifurcation may be computed on Drawing No. 4. A typical example is shown with an internal pressure of 1 psi.

Unsymmetrical Bifurcation

Equations

The analysis of the unsymmetrical bifurcation (see Figure 3) is shown on Drawing No. 5. A procedure similar to that already described is followed in developing the equations for deflection and rotation of the ring 'OA' and the beam 'OB' at Point 'O'. These equations are identical with those given for the symmetrical trifurcation.

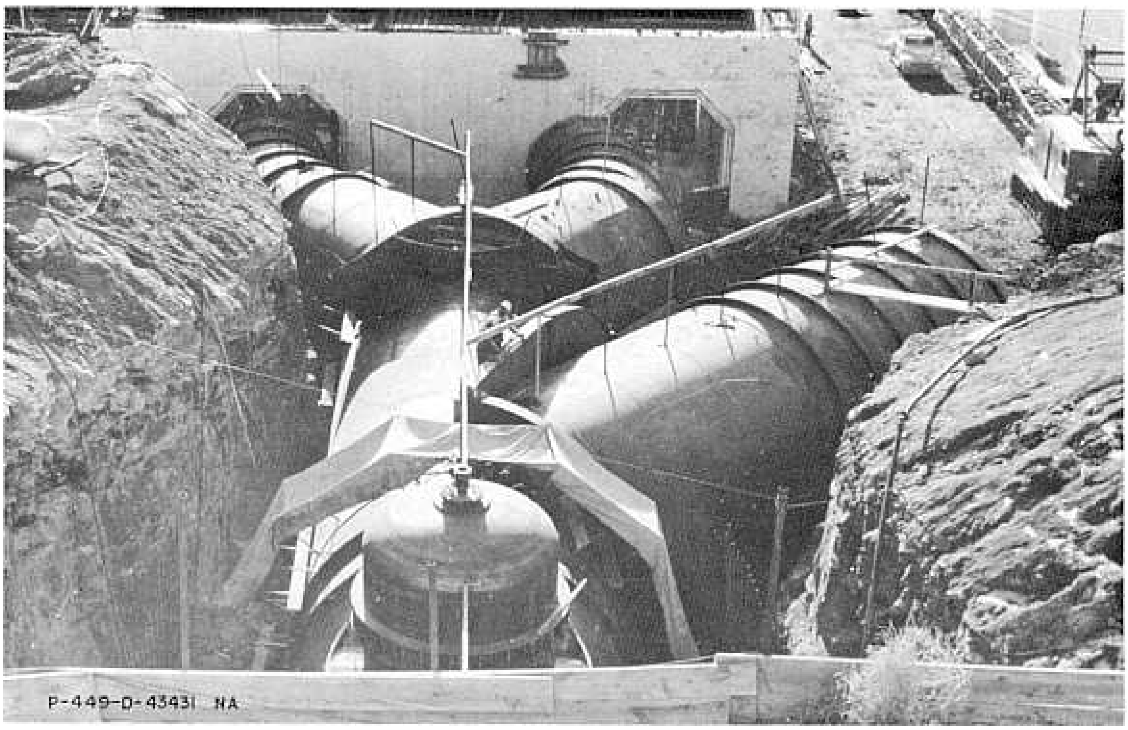


Figure 3.--Unsymmetrical Bifurcation

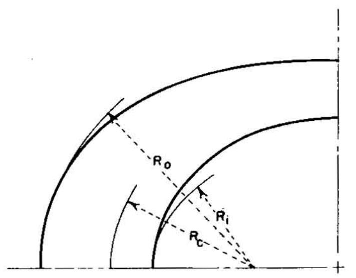
FINAL EQUATIONS:

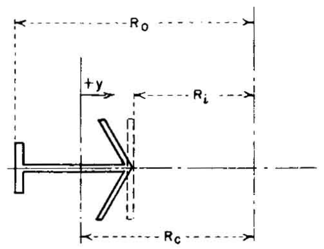
M_1	M_3	V_1	V_3	V_2	= C	CHECK
0.018679	0	4.044041	0.065341	0.96567	-29,043.633	-29,038.536
0.	0.024928	0.065341	3.704521	0	-10,134.618	-10,130.823
0.032892	0	9.6567	0	3.88480	-95,838.77	-95,825.125
0.055680	0.199234	7.226976	23.030693	1.272621	-98,451.350	-98,419.565
0.92388	-0.38691	0	0	0	0	+0.53697

AUXILIARY EQUATIONS

M_1	M_3	V_1	V_3	V_2	C	CHECK
0.018679	0	216.502008	3.498099	51.69816	-1,554,881.417	-1,554,608.705
0	0.024928	2.621189	148.608833	0	-406,555.600	-406,403.369
0.032892	0	2.535516	-0.045379	0.861499	-17,627.78875	-17,625.9729
0.055680	0.199234	-5.350086	-7.014795	-0.428118	3,590.41731	3590.98945
0.92388	-0.38691	-199.007711	45.235649	143.048244	-16,716.4396	-16,715.444

SOLUTION		CHECK
V_2	-16,716.44	
V_3	-3,566.19	
V_1	-3,388.42	
M_3	132,293.57	
M_1	55,402.40	





FOR BEAM:

	POINT	0	1	2	3	4	5	6	7	8	9	10	11	12
1	M_1	55,402	55,402	55,402	55,402	55,402	55,402	55,402	55,402	55,402	55,402	55,402	55,402	55,402
2	$V_1 x$	0	-155,867	-312,751	-467,602	-620,081	-703,097	-780,692	-857,270	-933,171	-979,253	-1,018,220	-1,045,328	-1,055,493
3	$V_2 (x-x_4)$						-409,553	-792,359	-1,170,151	-1,544,599	-1,771,943	-1,964,182	-2,097,913	-2,148,063
4	$Kx^3/6$													
5	$\frac{1}{2}K x_8^2 (x-\frac{2}{3}x_8)$										2,826,707	3,135,123	3,349,674	3,430,131
6	Σ Moments	55,402	-88,922	-164,667	-102,438	157,665	-4,204	-76,069	-63,254	39,607	130,913	208,123	261,835	281,977
7	I	27,200	54,600	79,900	93,900	103,600	114,700	140,200	145,300	146,000	154,000	159,300	167,100	167,100
8	y_o	-19.0	-26.3	-29.5	-31.4	-32.0	-33.5	-36.9	-37.6	-36.9	-38.0	-38.2	-38.9	-38.9
9	y_i	17.2	18.6	22.6	27.3	31.5	31.5	30.1	30.6	32.9	33.7	34.8	36.2	36.2
10	My_o/I	-38.70	42.87	60.80	34.26	-48.70	1.23	20.02	16.37	-10.01	-32.30	-49.91	-60.95	-65.64
11	My_i/I	35.03	-30.32	-46.58	-29.78	47.94	-1.16	-16.33	-13.32	8.92	28.65	45.47	56.72	61.09
12	β	0°	8.3°	10°	13.6°	22.2°	29°	34.7°	33.9°	45.1°	52.5°	62.4°	74.5°	90°
13	Sin β	0	0.14436	0.1736	0.2351	0.3778	0.4848	0.5693	0.5577	0.7083	0.7934	0.8862	0.9636	1.0
14	$(V_1 + Kx^2/2)$ Sin β	0	-381.140	-65.273	786.536	3,193.653								
15	$(V_1 + V_2 + Kx^2/2)$ Sin β						-2365.949	-759.650	1410.268	4755.478				
16	$(V_1 + V_2 + Kx_8^2/2)$ Sin β										5326.88	5949.938	6469.601	6713.99
17	Σ Tensile Force	0	-381.140	-65.273	786.536	3193.653	-2365.949	-759.650	1410.268	4755.478	5326.88	5949.938	6469.601	6713.99
18	A	144.94	189.07	204.56	203.06	201.82	210.19	232.19	234.44	228.94	232.31	233.19	235.19	235.19
19	T/A	0	-2.02	-0.32	3.87	15.82	-11.26	-3.27	6.02	20.77	22.93	25.52	27.51	28.55
20	K_o	1	1	1	1	1	1	1	1	1	1	0.452	0.444	0.450
21	K_i	1	1	1	1	1	1	1	1	1	1	1.705	1.700	1.691
22	$T/A + K_o My_o/I$	-38.7	40.9	60.5	38.1	-32.9	-10.0	16.7	22.4	10.8	-9.4	3.0	0.4	-1.0
23	$T/A + K_i My_i/I$	35.0	-32.3	-46.9	-25.9	63.7	-12.4	-19.6	-7.3	29.7	51.6	103.0	123.9	131.8

	POINT	10	11	12	8'
1	A	233.19	235.19	235.19	172.0
2	I	159,300	167,100	167,100	34,900
3	R_c	91.03	92.90	93.9	170.212
4	R_o	129.25	131.75	132.75	192.0
5	R_i	56.25	56.75	57.75	155.06
6	y_o	-38.22	-38.85	-38.85	-21.79
7	y_i	34.78	36.15	36.15	15.15
8	$\Sigma \frac{y^2}{R-y} \Delta A$	1,900.7	1,978.5	1,951.9	199.743
9	$Z = \frac{⑧}{①③}$	0.089540	0.090554	0.088386	0.006823
10	K_o	0.452	0.444	0.450	0.855
11	K_i	1.705	1.700	1.691	1.205

$$K_o = \frac{I}{AR_c}\left(\frac{1}{R_o Z} + \frac{1}{y_o}\right)$$

$$K_i = \frac{I}{AR_c}\left(\frac{1}{R_i Z} + \frac{1}{y_i}\right)$$

FOR RING:

	POINT	0'	1'	2'	3'	4'	5'	6'	7'	8'
1	M_3	132,294	132,294	132,294	132,294	132,294	132,294	132,294	132,294	132,294
2	$V_3 x'$	0	-178,310	-344,137	-483,219	-579,060	-602,686	-620,517	-629,433	-632,999
3	$K'\frac{(x')^3}{6}$	0	14,314	102,904	284,884	490,237				
4	$\frac{1}{2}K'(x_4')^2(x'-\frac{2}{3}x_4')$						550,243	595,531	618,175	627,233
5	Σ Moments	132,294	-31,702	-108,939	-66,041	43,471	79,857	107,308	121,036	126,528
6	I	40,500	45,700	47,100	43,900	39,900	37,500	35,600	35,200	34,900
7	y_o	-23.04	-24.15	-24.42	-23.77	-22.90	-22.35	-21.91	-21.81	-21.79
8	y_i	16.21	17.15	17.38	16.83	16.10	15.65	15.29	15.19	15.15
9	My_o/I	-75.3	16.8	56.5	35.8	-25.0	-47.6	-66.0	-75.0	-79.0
10	My_i/I	53.0	-11.9	-40.2	-25.3	17.5	33.3	46.1	52.2	54.9
11	β	0	13.8°	31.0°	47.8°	64.5°	71.0°	77.5°	83.7°	90°
12	Sin β	0	0.2385	0.5150	0.7408	0.9026	0.9455	0.9763	0.9940	1.0
13	$(V_3 + K'(x')^2/2)$ Sin β	0	-645.704	-189.06	2,030.679	4956.46				
14	$(V_3 + K'(x_4')^2/2)$ Sin β						5,192.039	5,361.172	5,458.368	5,491.316
15	Σ Tensile Force	0	-645.704	-189.06	2,030.679	4,956.46	5,192.039	5,361.172	5,458.368	5,491.316
16	A	177.3	181.9	183.0	180.3	176.7	174.4	172.6	172.2	172.0
17	T/A	0	-3.55	-1.03	11.26	28.05	29.77	31.06	31.70	31.93
18	K_o	1	1	1	1	1	1	1	1	0.855
19	K_i	1	1	1	1	1	1	1	1	1.205
20	$T/A + K_o My_o/I$	-75.3	13.2	55.4	47.0	3.1	-17.8	-35.0	-43.3	-35.6
21	$T/A + K_i My_i/I$	53.0	-15.4	-41.2	-14.0	45.6	63.0	77.1	83.9	98.1

FOR TIE RODS:

At 'O'	
ΣV	-13,909
A	176.715
$-\Sigma V/A$	78.7
At 'C'	
V_2	-16,716
A_c	176.715
$-V_2/A_c$	94.6

NOTE

Positive sign denotes tensile stresses

Stresses are for 1 psi pressure

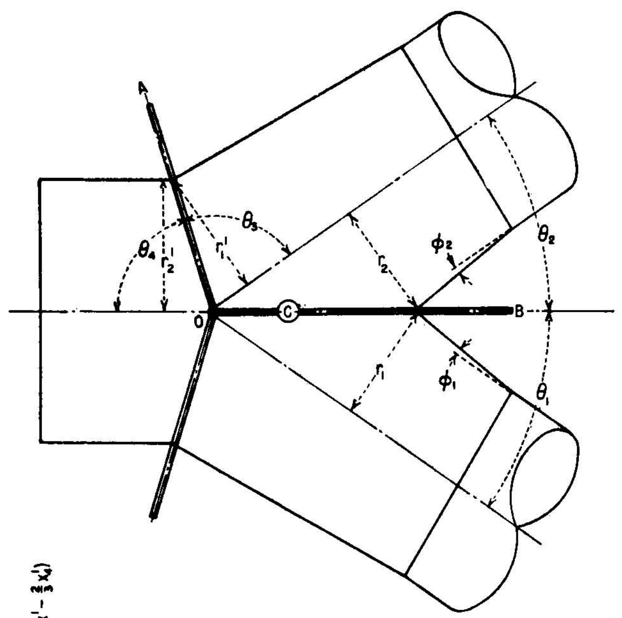
Design pressure is 110 psi

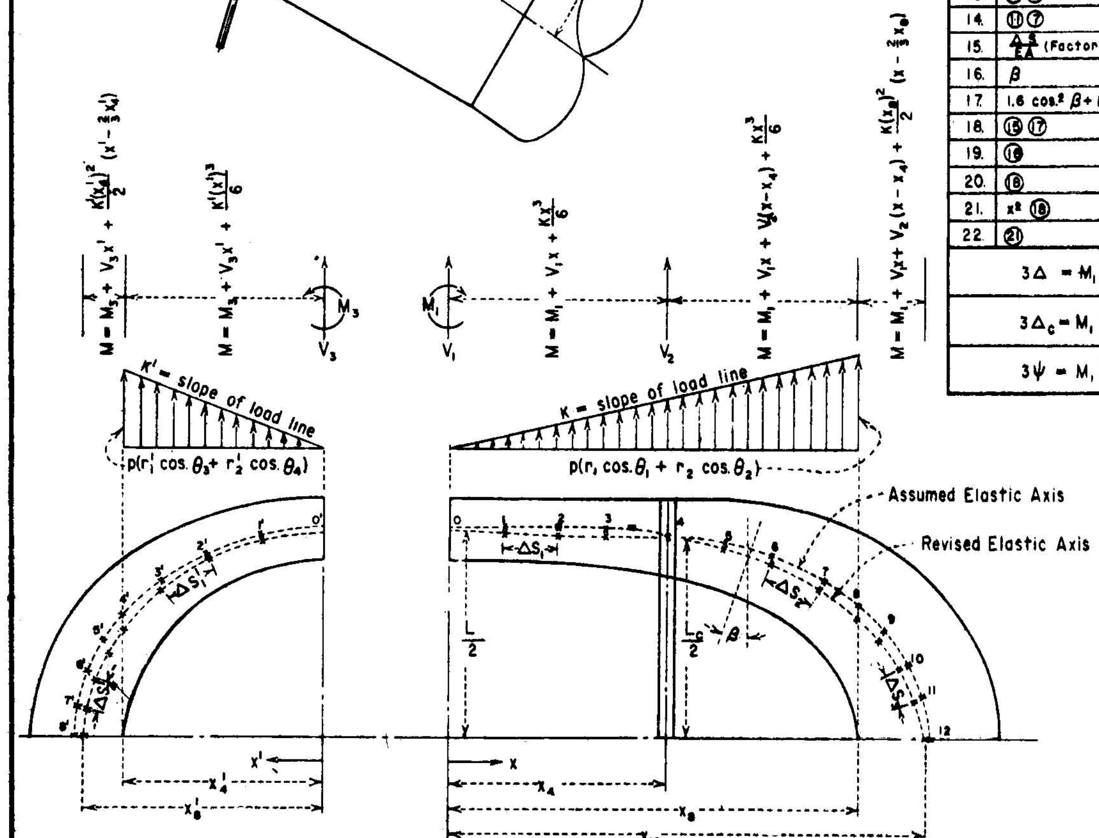
STRESS ANALYSIS OF WYE BRANCHES

STRESSES IN SYMMETRICAL TRIFURCATIONS

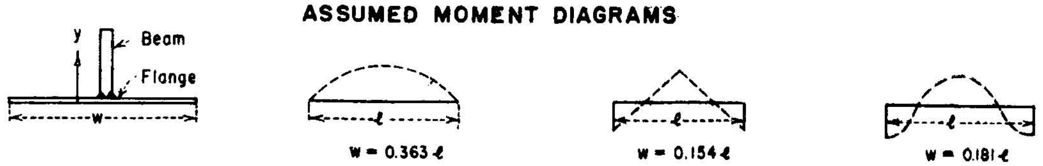
Palisades Wye W-1

DRAWING 2





ASSUMED MOMENT DIAGRAMS



EFFECTIVE FLANGE WIDTHS

FOR BEAM: $K = 0.9500$ $\frac{\Delta S_1}{E} = 1.024 \times 10^{-6}$ $\frac{\Delta S_2}{E} = 0.7740 \times 10^{-6}$ $\frac{\Delta S_3}{E} = 0.5552 \times 10^{-6}$

	POINT	0	1	2	3	4L	4R	5	6	7	8L	8R	9	10	11	12	Σ	
	FACTOR	1	4	2	4	1	1	4	2	4	1	1	4	2	4	1	—	
1.	x	0	30.84	61.32	91.68	120	120	141.0	159.96	175.92	191.04	191.04	201.6	208.92	213.0	214.2	—	1.
2.	$\frac{\Delta S}{EI}$ (Factor)	0.1416x10⁻⁹	0.4270x10⁻⁹	0.1753x10⁻⁹	0.3022x10⁻⁹	0.0639x10⁻⁹	0.0464x10⁻⁹	0.1612x10⁻⁹	0.0677x10⁻⁹	0.0964x10⁻⁹	0.0207x10⁻⁹	0.0155x10⁻⁹	0.0565x10⁻⁹	0.0260x10⁻⁹	0.0486x10⁻⁹	0.0175x10⁻⁹	1.1608x10⁻⁹	2.
3.	x ②	0	13.168x10⁻⁹	10.749x10⁻⁹	27.703x10⁻⁹	7.667x10⁻⁹	5.571x10⁻⁹	22.737x10⁻⁹	10.843x10⁻⁹	16.960x10⁻⁹	3.970x10⁻⁹	2.963x10⁻⁹	11.394x10⁻⁹	5.435x10⁻⁹	10.355x10⁻⁹	2.513x10⁻⁹	152.028x10⁻⁹	3.
4.	x² ②	0	0.4061x10⁻⁶	0.6592x10⁻⁶	2.5398x10⁻⁶	0.9205x10⁻⁶	0.6685x10⁻⁶	3.2060x10⁻⁶	1.7344x10⁻⁶	2.9836x10⁻⁶	0.7586x10⁻⁶	0.5660x10⁻⁶	2.2970x10⁻⁶	1.1355x10⁻⁶	2.2057x10⁻⁶	0.5383x10⁻⁶	20.619x10⁻⁶	4.
5.	x³ ②	0	0.0125x10⁻³	0.0404x10⁻³	0.2328x10⁻³	0.1104x10⁻³	0.0802x10⁻³	0.4520x10⁻³	0.2775x10⁻³	0.5249x10⁻³	0.1449x10⁻³	—	—	—	—	—	1.8757x10⁻³	5.
6.	x⁴ ②	0	0.3871x10⁻³	2.4780x10⁻³	21.3477x10⁻³	13.2486x10⁻³	9.6267x10⁻³	63.739x10⁻³	44.3785x10⁻³	92.339x10⁻³	27.6826x10⁻³	—	—	—	—	—	0.275225	6.
7.	(x - x₄) ②	—	—	—	—	—	0	3.386x10⁻⁹	2.709x10⁻⁹	5.391x10⁻⁹	1.476x10⁻⁹	1.102x10⁻⁹	4.612x10⁻⁹	2.313x10⁻⁹	4.521x10⁻⁹	1.105x10⁻⁹	26.615x10⁻⁹	7.
8.	(x - x₄)² ②	—	—	—	—	—	0	0.07112x10⁻⁶	0.10825x10⁻⁶	0.301464x10⁻⁶	0.104852x10⁻⁶	0.078288x10⁻⁶	0.376344x10⁻⁶	0.205668x10⁻⁶	0.420456x10⁻⁶	0.104088x10⁻⁶	1.770526x10⁻⁶	8.
9.	x ⑦	—	—	—	—	—	0	0.47743x10⁻⁶	0.43333x10⁻⁶	0.94838x10⁻⁶	0.281976x10⁻⁶	0.210528x10⁻⁶	0.929784x10⁻⁶	0.483228x10⁻⁶	0.962976x10⁻⁶	0.236688x10⁻⁶	4.964320x10⁻⁶	9.
10.	x³ ⑦	—	—	—	—	—	0	0.009492	0.011087	0.029350	0.010290	—	—	—	—	—	0.060219	10.
11.	x - ⅔ x₈	—	—	—	—	—	—	—	—	—	—	63.68	73.7	81.56	85.64	86.84	—	11.
12.	⑪②	—	—	—	—	—	—	—	—	—	—	0.987x10⁻⁹	4.164x10⁻⁹	2.120x10⁻⁹	4.162x10⁻⁹	1.020x10⁻⁹	12.453x10⁻⁹	12.
13.	⑪③	—	—	—	—	—	—	—	—	—	—	0.188684x10⁻⁶	0.739738x10⁻⁶	0.443279x10⁻⁶	0.886902x10⁻⁶	0.218229x10⁻⁶	2.576132x10⁻⁶	13.
14.	⑪⑦	—	—	—	—	—	—	—	—	—	—	0.070175x10⁻⁶	0.306739x10⁻⁶	0.188648x10⁻⁶	0.387178x10⁻⁶	0.095958x10⁻⁶	1.048698x10⁻⁶	14.
15.	$\frac{\Delta S}{EA}$ (Factor)	11.58x10⁻⁹	43.62x10⁻⁹	20.88x10⁻⁹	39.94x10⁻⁹	9.624x10⁻⁹	6.496x10⁻⁹	26.928x10⁻⁹	12.960x10⁻⁹	22.656x10⁻⁹	5.436x10⁻⁹	4.056x10⁻⁹	15.84x10⁻⁹	7.704x10⁻⁹	15.216x10⁻⁹	3.780x10⁻⁹	—	15.
16.	β	2.2°	6.8°	11.0°	16.6°	23.6°	23.6°	28.0°	37.6°	41.5°	48.8°	48.8°	58.0°	67.2°	80.0°	90°	—	16.
17.	1.6 cos² β + 1	2.5976	2.5776	2.5418	2.4694	2.3435	2.3435	2.2474	2.0044	1.8975	1.6942	1.6942	1.4493	1.2403	1.0482	1.000	—	17.
18.	⑮⑰	30.084x10⁻⁹	112.464x10⁻⁹	53.064x10⁻⁹	98.616x10⁻⁹	22.548x10⁻⁹	16.392x10⁻⁹	60.516x10⁻⁹	25.98x10⁻⁹	42.984x10⁻⁹	9.204x10⁻⁹	6.876x10⁻⁹	22.956x10⁻⁹	9.552x10⁻⁹	15.948x10⁻⁹	3.78x10⁻⁹	0.530964x10⁻⁶	18.
19.	⑱	—	—	—	—	—	16.392x10⁻⁹	60.516x10⁻⁹	25.98x10⁻⁹	42.984x10⁻⁹	9.204x10⁻⁹	6.876x10⁻⁹	22.956x10⁻⁹	9.552x10⁻⁹	15.948x10⁻⁹	3.78x10⁻⁹	0.214188x10⁻⁶	19.
20.	⑱	—	—	—	—	—	—	—	—	—	—	6.876x10⁻⁹	22.956x10⁻⁹	9.552x10⁻⁹	15.948x10⁻⁹	3.78x10⁻⁹	0.059112x10⁻⁶	20.
21.	x² ⑱	0	0.106765x10⁻³	0.199529x10⁻³	0.828889x10⁻³	0.324671x10⁻³	0.236045x10⁻³	1.203118x10⁻³	0.664755x10⁻³	1.330263x10⁻³	0.335911x10⁻³	—	—	—	—	—	5.230116x10⁻³	21.
22.	㉑	—	—	—	—	—	0.236045x10⁻³	1.203118x10⁻³	0.164755x10⁻³	1.330263x10⁻³	0.335911x10⁻³	—	—	—	—	—	3.770092x10⁻³	22.

$$3\Delta = M_1\sum③ + V_1\left(\sum④ + \sum⑱\right) + \frac{K}{6}\left(\sum⑥ + 3\sum㉑\right) + V_2\left(\sum⑨ + \sum⑲\right) + \frac{Kx_8^2}{2}\left(\sum⑬ + \sum⑳\right) \quad 3\Delta = 0.152029\times10^{-6}M_1 + 21.147\times10^{-6}V_1 + 5.178596\times10^{-6}V_2 + 0.091753$$

$$3\Delta_c = M_1\sum⑦ + V_1\left(\sum⑨ + \sum⑲\right) + V_2\left(\sum⑧ + \sum⑲\right) + \frac{K}{6}\left(\sum⑩ + 3\sum㉒\right) + \frac{Kx_8^2}{2}\left(\sum⑭ + \sum⑳\right) \quad 3\Delta_c = 0.026615\times10^{-6}M_1 + 5.178596\times10^{-6}V_1 + 1.984716\times10^{-6}V_2 + 0.030529$$

$$3\psi = M_1\sum② + V_1\sum③ + \frac{K}{6}\sum⑤ + V_2\sum⑦ + \frac{Kx_8^2}{2}\sum⑫ \quad 3\psi = 0.001661\times10^{-6}M_1 + 0.152029\times10^{-6}V_1 + 0.026615\times10^{-6}V_2 + 512.525\times10^{-6}$$

REVISION OF ELASTIC AXIS AND COMPUTATION OF I

POINT	ℓ	w	ΣA	ΣAy	ȳ	I
0	122.64	18.8	88.47	1162.9	13.15	7230
1	"	"	93.89	1361.7	14.50	9590
2	"	"	98.06	1550.0	15.81	11,680
3	"	"	102.53	1943.6	17.01	13,560
4	"	"	106.42	1919.8	18.04	16,030
5	"	"	110.45	2099.5	19.01	18,460
6	"	"	114.77	2325.9	20.26	21,950
7	175.2	31.7	131.33	2647.8	20.16	30,870
8	"	"	136.80	2927.2	21.40	35,790
9	"	"	140.26	3122.5	22.26	39,300
10	"	"	144.29	3328.1	23.07	42,690
11	"	"	145.73	3454.3	23.70	45,680
12	"	"	146.88	3518.2	23.95	47,320
0'	219.5	39.77	105.41	1185.4	11.25	9,940
1'	"	"	"	"	"	"
2'	"	"	"	"	"	"
3'	"	"	"	"	"	"
4'	"	"	"	"	"	"
5'	"	"	"	"	"	"
6'	"	"	"	"	"	"
7'	"	"	"	"	"	"
8'	"	"	"	"	"	"

$L = 272.4$
$L_c = 225$
$\theta_1 = 35°$
$\theta_2 = 35°$
$\theta_3 = 72.5314°$
$\theta_4 = 72.4686°$

$A = 113.097$
$A_c = 113.097$
$E = 3 \times 10^7$

FOR RING: $K' = 0.5736$ $\frac{\Delta S_1'}{E} = 1.375 \times 10^{-6}$ $\frac{\Delta S_2'}{E} = 0.459 \times 10^{-6}$

	POINT	0'	1'	2'	3'	4'R	4'L	5'	6'	7'	8'	Σ	
	FACTOR	1	4	2	4	1	1	4	2	4	1	—	
1.	x'	0	40.92	78.00	109.32	131.28	131.28	136.32	140.16	142.32	142.80	—	1.
2.	$\frac{\Delta S'}{EI}$ (Factor)	0.1382x10⁻⁹	0.5530x10⁻⁹	0.2765x10⁻⁹	0.5534x10⁻⁹	0.1382x10⁻⁹	0.0461x10⁻⁹	0.1845x10⁻⁹	0.0923x10⁻⁹	0.1846x10⁻⁹	0.0461x10⁻⁹	2.2131x10⁻⁹	2.
3.	x' ②	0	22.629x10⁻⁹	21.567x10⁻⁹	60.45x10⁻⁹	18.149x10⁻⁹	6.061x10⁻⁹	25.174x10⁻⁹	12.941x10⁻⁹	26.28x10⁻⁹	6.593x10⁻⁹	199.854x10⁻⁹	3.
4.	(x')² ②	0	0.92597x10⁻⁶	1.6822x10⁻⁶	6.6088x10⁻⁶	2.3827x10⁻⁶	0.79566x10⁻⁶	3.4317x10⁻⁶	1.8139x10⁻⁶	3.7404x10⁻⁶	0.9414x10⁻⁶	22.323x10⁻⁶	4.
5.	(x')³ ②	0	37.891x10⁻⁶	131.214x10⁻⁶	722.448x10⁻⁶	312.768x10⁻⁶	—	—	—	—	—	1.2043x10⁻³	5.
6.	(x')⁴ ②	0	1.5505x10⁻³	10.2349x10⁻³	78.9817x10⁻³	41.0642x10⁻³	—	—	—	—	—	131.831x10⁻³	6.
7.	x' - ⅔ x'₄	—	—	—	—	—	43.76	48.24	52.08	54.24	54.72	—	7.
8.	⑦②	—	—	—	—	—	2.020x10⁻⁹	8.900x10⁻⁹	4.809x10⁻⁹	10.016x10⁻⁹	2.522x10⁻⁹	28.379x10⁻⁹	8.
9.	⑦③	—	—	—	—	—	0.26520x10⁻⁶	1.2144x10⁻⁶	0.6740x10⁻⁶	1.4255x10⁻⁶	0.3608x10⁻⁶	3.9399x10⁻⁶	9.
10.	$\frac{\Delta S'}{EA}$ (Factor)	0.09058x10⁻⁹	0.3623x10⁻⁹	0.1811x10⁻⁹	0.3623x10⁻⁹	0.09058x10⁻⁹	0.03025x10⁻⁹	0.1210x10⁻⁹	0.0605x10⁻⁹	0.1210x10⁻⁹	0.03025x10⁻⁹	—	10.
11.	β	0	15.6°	33.6°	47.0°	64.6°	64.6°	72.5°	76.0°	84.1°	90°	—	11.
12.	(1.6 cos²β+1)	2.600	2.4872	2.1100	1.7219	1.2944	1.2944	1.1478	1.0936	1.0169	1.0	—	12.
13.	⑩⑫	0.2355x10⁻⁹	0.9012x10⁻⁹	0.3823x10⁻⁹	0.6239x10⁻⁹	0.1172x10⁻⁹	0.0391x10⁻⁹	0.1389x10⁻⁹	0.0661x10⁻⁹	0.1230x10⁻⁹	0.03025x10⁻⁹	2.6576x10⁻⁹	13.
14.	⑩⑫	—	—	—	—	—	0.0391x10⁻⁹	0.1389x10⁻⁹	0.0661x10⁻⁹	0.1230x10⁻⁹	0.03025x10⁻⁹	0.3947x10⁻⁹	14.
15.	(x')² ⑬	0	1.5090x10⁻⁶	2.3261x10⁻⁶	7.4573x10⁻⁶	2.0207x10⁻⁶	—	—	—	—	—	13.318x10⁻⁶	15.

$$3\Delta' = M_3\sum③ + V_3\left(\sum④ + \sum⑬\right) + \frac{K'}{6}\left(\sum⑥ + 3\sum⑮\right) + \frac{K'(x_4')^2}{2}\left(\sum⑨ + \sum⑭\right) \quad 3\Delta' = 0.199850\times10^{-6}M_3 + 22.323\times10^{-6}V_3 + 0.132083$$

$$3\psi' = M_3\sum② + V_3\sum③ + \frac{K'}{6}\sum⑤ + \frac{K'(x_4')^2}{2}\sum⑧ \quad 3\psi' = 0.002213\times10^{-6}M_3 + 0.199850\times10^{-6}V_3 + 25[illegible]\times10^{-6}$$

FINAL BASIC EQUATIONS

(Tie rod at Point 'O')

$$\Delta = -\frac{(V_1 + 2V_3)L}{2AE}$$
$$\Delta' = -\frac{(V_1 + 2V_3)L}{2AE}$$

$\Delta' = \Delta$

$V_1 + 2V_3 = 0$

$\psi \cos\theta_4 = -\psi'$

$M_1 = 2M_3 \cos\theta_4$

$-\frac{V_2 L_c}{2A_c E} = \Delta_c$

STRESS ANALYSIS OF WYE BRANCHES

SYMMETRICAL BIFURCATIONS
DEFLECTIONS AND ROTATIONS

Glendo Outlet Wye

DRAWING 3

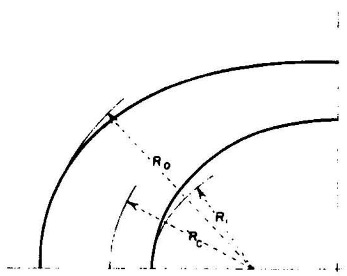
FINAL EQUATIONS:

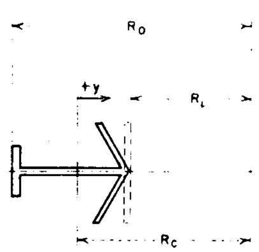
M_1	M_3	V_1	V_3	V_2	$= C \times 10^{-5}$	CHECK
0.152029	—	21.267	0.240854	5.178596	-0.91753	25.920949
—	0.199850	0.120427	22.564	—	-0.32083	22.563447
0.026615	—	5.178596	—	2.084184	-0.30529	6.984105
0.5004	2.213	45.800	199.850	8.018	-4.09280	252.2886
1	-0.60252	—	—	—	—	0.39748

AUXILIARY EQUATIONS

M_1	M_3	V_1	V_3	V_2	C	CHECK
1	0	139.887784	1.584264	34.063212	-6.035230	170.500029
0	0.199850	0.602587	112.904679	0	-1.105354	112.901911
0.026615	0	1.455483	-0.028970	0.909073	-0.099392	1.680711
0.500400	2.213000	-25.533373	-51.540523	-0.225670	0.001124	0.775454
1	-0.602520	-139.524714	62.401032	92.904508	-0.095472	0.904528

SOLUTION		CHECK
$V_2 \times 10^{-5}$	-0.095472	0.904528
$V_3 \times 10^{-5}$	-0.020422	0.979578
$V_1 \times 10^{-5}$	-0.022740	0.977260
$M_3 \times 10^{-5}$	0.714088	1.714087
$M_1 \times 10^{-5}$	0.430255	1.430254





FOR BEAM:

	POINT	0	1	2	3	4	5	6	7	8	9	10	11	12
1	M_1	43,030	43,030	43,030	43,030	43,030	43,030	43,030	43,030	43,030	43,030	43,030	43,030	43,030
2	$V_1 x$	0	-70,130	-139,440	-208,480	-272,880	-320,630	-363,750	-400,040	-434,420	-458,440	-475,080	-484,360	-487,090
3	$V_2 (x - x_4$						-200,490	-381,500	-533,870	-678,220	-779,040	-848,920	-887,870	-899,330
4	$Kx^3/6$	0	4,640	36,510	122,010	273,600	443,840	648,050	862,020	1,103,940				
5	$\frac{1}{2}K x_8^2 (x - \frac{2}{3} x_8$										1,277,640	1,413,900	1,484,630	1,505,440
6	Σ Moments	43,030	-22,460	-59,900	-43,440	43,750	-34,250	-54,170	-28,860	34,330	83,190	132,930	155,430	162,050
7	I	7230	9590	11,680	13,560	16,030	18,460	21,950	30,870	35,790	39,300	42,690	45,680	47,320
8	y_o	-11.4	-12.96	-14.16	-14.64	-15.96	-16.56	-18.48	-21.0	-22.2	-22.8	-23.64	-24.12	-25.44
9	y_i	13.15	14.50	15.81	17.01	18.04	19.01	20.26	20.16	21.40	22.26	23.07	23.70	23.95
10	$M y_o / I$	-67.8	30.4	72.6	46.9	-43.6	30.7	45.6	19.6	-21.3	-48.3	-73.6	-82.1	-87.1
11	$M y_i / I$	78.3	-34.0	-81.1	-54.5	49.2	-35.3	-50.0	-18.8	20.5	47.1	71.8	80.6	82.0
12	β	2.2°	6.8°	11.0°	16.6°	23.6°	28.0°	37.6°	41.5°	48.8°	58°	67.2°	80°	90°
13	$\sin \beta$	0.03839	0.11840	0.19081	0.28569	0.40035	0.46947	0.61015	0.66262	0.75241	0.84805	0.92186	0.98481	1.0
14	$(V_1 + Kx^2/2) \sin \beta$	-87	-216	-93	491	1,828								
15	$(V_1 + V_2 + Kx^2/2) \sin \beta$						-1,116	202	1907	4,149				
16	$(V_1 + V_2 + Kx_8^2/2) \sin \beta$										4,676	5,083	5,430	5,514
17	Σ Tensile Force	-87	-216	-93	491	1828	-1116	202	1907	4149	4676	5083	5430	5514
18	A	88.42	93.89	98.06	102.53	106.42	110.45	114.77	131.33	136.80	140.26	144.29	145.73	146.88
19	T/A	-1.0	-2.3	-0.9	4.8	17.2	-10.1	1.8	14.5	30.3	33.3	35.2	37.3	37.5
20	K_o	1	1	1	1	1	1	1	1	1	1	0.64	0.51	0.54
21	K_i	1	1	1	1	1	1	1	1	1	1	1.50	1.49	1.67
22	$T/A + K_o M y_o / I$	-68.8	28.1	71.7	51.7	-26.4	20.6	47.4	34.1	9.0	-15.0	-11.9	-4.57	-9.5
23	$T/A + K_i M y_i / I$	77.3	-36.3	-82.0	-49.7	66.4	-45.4	-48.2	-4.3	50.8	80.4	142.9	157.4	174.4

	POINT	10	11	12	8'
1	A	144.29	145.73	146.88	105.41
2	I	42,690	45,680	47,320	9940
3	R_c	87.12	73.80	70.08	131.28
4	R_o	110.76	97.92	94.08	144.12
5	R_i	64.08	50.04	46.08	120.12
6	y_o	-23.64	-24.12	-25.44	-12.84
7	y_i	23.04	23.76	24.00	11.16
8	$\Sigma \frac{y^2}{R-y} \Delta A$	493.52	634.69	695.00	61.86
9	$Z = \frac{⑧}{①③}$	0.0393	0.0590	0.0675	0.0052
10	K_o	0.64	0.51	0.54	0.9
11	K_i	1.50	1.49	1.67	1.2

$$K_o = \frac{I}{AR_c} \left(\frac{1}{R_o Z} + \frac{1}{y_o} \right)$$

$$K_i = \frac{I}{AR_c} \left(\frac{1}{R_i Z} + \frac{1}{y_i} \right)$$

FOR RING:

	POINT	0'	1'	2'	3'	4'	5'	6'	7'	8'
1	M_3	71,410	71,410	71,410	71,410	71,410	71,410	71,410	71,410	71,410
2	$V_3 x'$	0	-83,560	-159,280	-223,230	-268,070	-278,370	-286,210	-290,620	-291,600
3	$K' \frac{(x')^3}{6}$	0	6,550	45,370	124,900	216,290				
4	$\frac{1}{2}K' (x_4')^2 (x' - \frac{2}{3}x_4')$						238,440	257,420	268,100	270,470
5	Σ Moments	71,410	-5,600	-42,500	-26,900	19,600	31,500	42,600	48,900	50,300
6	I	9940	9940	9940	9940	9940	9940	9940	9940	9940
7	y_o	-12.84	-12.84	-12.84	-12.84	-12.84	-12.84	-12.84	-12.84	-12.84
8	y_i	11.25	11.25	11.25	11.25	11.25	11.25	11.25	11.25	11.25
9	$M y_o / I$	-92.3	7.2	54.9	34.8	-25.4	-40.7	-55.1	-63.2	-65.0
10	$M y_i / I$	80.8	-6.3	-48.1	-30.5	22.2	35.6	48.2	55.3	56.9
11	β	0	15.4°	33.6°	47.8°	64.6°	72.5°	76°	84.1°	90°
12	$\sin \beta$	0	0.26556	0.55339	0.74080	0.90334	0.95372	0.97030	0.99470	1.0
13	$(V_3 + K'(x')^2/2) \sin \beta$	0	-415	-165	1026	2620				
14	$(V_3 + K'(x_4')^2/2) \sin \beta$						2770	2810	2890	2900
15	Σ Tensile Force	0	-415	-165	1026	2620	2770	2810	2890	2900
16	A	105.41	105.41	105.41	105.41	105.41	105.41	105.41	105.41	105.41
17	T/A	0	-3.9	-1.6	9.7	24.9	26.3	26.7	27.4	27.5
18	K_o	1	1	1	1	1	1	1	1	0.9
19	K_i	1	1	1	1	1	1	1	1	1.2
20	$T/A + K_o M y_o / I$	-92.3	3.3	53.3	44.5	-0.5	-14.4	-28.4	-35.8	-31.0
21	$T/A + K_i M y_i / I$	80.8	-10.2	-49.7	-20.8	47.1	61.9	74.9	82.7	95.8

FOR TIE RODS:

At 'O'	
ΣV	-6358
A	113.097
$-\Sigma V / A$	56.2
At 'C'	
V_2	-9547
A_c	113.097
$-V_2 / A_c$	84.4

NOTE

Positive sign denotes tensile stresses

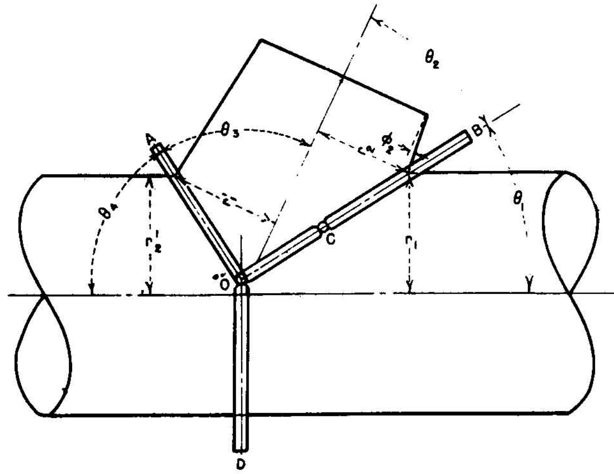
Stresses are for 1 psi pressure

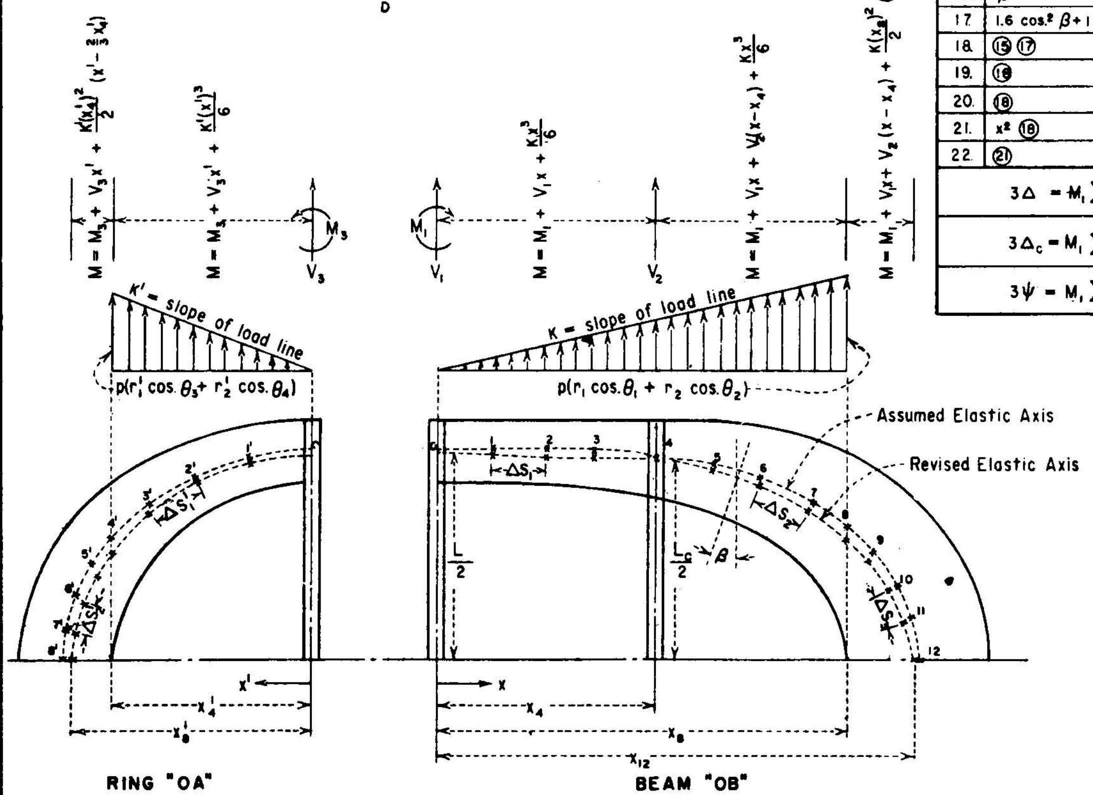
Design pressure is 85 psi

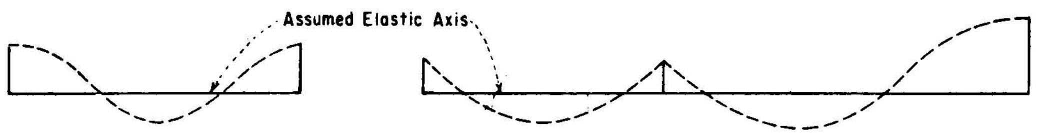
STRESS ANALYSIS OF WYE BRANCHES

STRESSES IN SYMMETRICAL BIFURCATIONS

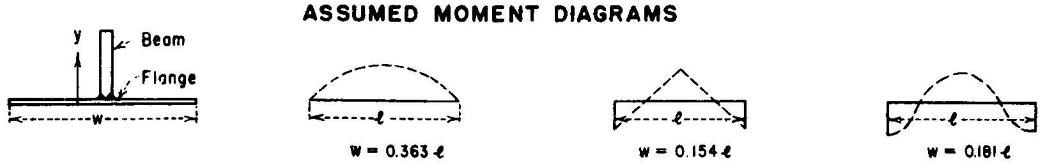
Glendo Outlet Wye **DRAWING 4**







ASSUMED MOMENT DIAGRAMS



EFFECTIVE FLANGE WIDTHS

FOR BEAM: K = 0.7071 $\qquad \frac{\Delta S_1}{E} = 171 \times 10^{-8} \qquad \frac{\Delta S_2}{E} = 115.3 \times 10^{-8} \qquad \frac{\Delta S_3}{E} = 66.8 \times 10^{-8}$

	POINT	0	1	2	3	4L	4R	5	6	7	8L	8R	9	10	11	12	Σ	
	FACTOR	1	4	2	4	1	1	4	2	4	1	1	4	2	4	1	—	
1.	x	0	51.36	102.36	153.48	204.0	204.0	237.60	269.64	300.60	329.25	329.25	341.52	352.68	357.76	363.24	—	1.
2.	$\frac{\Delta S}{EI}$ (Factor)	5.3122×10^{-11}	19.7112×10^{-11}	7.9572×10^{-11}	12.9349×10^{-11}	3.1290×10^{-11}	3.1290×10^{-11}	7.1559×10^{-11}	2.9290×10^{-11}	4.8887×10^{-11}	1.0444×10^{-11}	1.0444×10^{-11}	2.1474×10^{-11}	1.0073×10^{-11}	1.8169×10^{-11}	0.43713×10^{-11}	74.6453×10^{-11}	2.
3.	x ②	0	1.0214×10^{-8}	0.8145×10^{-8}	1.9352×10^{-8}	[illegible]	0.6383×10^{-8}	1.7002×10^{-8}	0.7798×10^{-8}	1.4695×10^{-8}	0.3439×10^{-8}	0.3439×10^{-8}	0.7334×10^{-8}	0.3553×10^{-8}	0.6537×10^{-8}	0.1588×10^{-8}	11.6372×10^{-8}	3.
4.	x^2 ②	0	51.4970×10^{-8}	83.3723×10^{-8}	304.6476×10^{-8}	130.2200×10^{-8}	130.2200×10^{-8}	403.9753×10^{-8}	212.9526×10^{-8}	441.7416×10^{-8}	113.2180×10^{-8}	113.2180×10^{-8}	250.4593×10^{-8}	125.2953×10^{-8}	235.1630×10^{-8}	57.6827×10^{-8}	26.5421×10^{-6}	4.
5.	x^3 ②	0	2.6706×10^{-5}	8.5340×10^{-5}	46.7652×10^{-5}	26.5649×10^{-5}	26.5649×10^{-5}	95.9847×10^{-5}	57.4206×10^{-5}	132.7876×10^{-5}	37.2771×10^{-5}	—	—	—	—	—	4.34570×10^{-3}	5.
6.	x^4 ②	0	1.3716×10^{-3}	8.7354×10^{-3}	71.7752×10^{-3}	54.1923×10^{-3}	54.1923×10^{-3}	228.0593×10^{-3}	154.8287×10^{-3}	399.1593×10^{-3}	122.7347×10^{-3}	—	—	—	—	—	1.095050	6.
7.	$(x - x_4)$ ②	—	—	—	—	—	0	240.4366×10^{-11}	192.2596×10^{-11}	472.2483×10^{-11}	130.8110×10^{-11}	130.8110×10^{-11}	295.3103×10^{-11}	149.7653×10^{-11}	283.0056×10^{-11}	69.6090×10^{-11}	1.96426×10^{-8}	7.
8.	$(x - x_4)^2$ ②	—	—	—	—	—	0	8.0788×10^{-8}	12.6199×10^{-8}	45.6193×10^{-8}	16.3841×10^{-8}	16.3841×10^{-8}	40.6111×10^{-8}	22.2671×10^{-8}	44.0809×10^{-8}	11.0846×10^{-8}	2.1713×10^{-6}	8.
9.	x ⑦	—	—	—	—	—	0	5.7128×10^{-7}	5.12410×10^{-7}	14.1958×10^{-7}	4.3069×10^{-7}	4.3069×10^{-7}	10.0854×10^{-7}	5.2819×10^{-7}	10.1864×10^{-7}	2.5285×10^{-7}	6.1784×10^{-6}	9.
10.	x^3 ⑦	—	—	—	—	—	0	32.2508×10^{-3}	37.6913×10^{-3}	128.2736×10^{-3}	46.6897×10^{-3}	—	—	—	—	—	2.4491×10^{-1}	10.
11.	$x - \frac{2}{3}x_8$	—	—	—	—	—	—	—	—	—	—	109.75	122.02	133.18	140.26	143.74	—	11.
12.	⑪②	—	—	—	—	—	—	—	—	—	—	1.1462×10^{-9}	2.6203×10^{-9}	1.3415×10^{-9}	2.5484×10^{-9}	0.62834×10^{-9}	8.2848×10^{-9}	12.
13.	⑪③	—	—	—	—	—	—	—	—	—	—	3.7739×10^{-7}	8.9485×10^{-7}	4.7314×10^{-7}	9.1683×10^{-7}	2.2826×10^{-7}	2.8905×10^{-6}	13.
14.	⑪⑦	—	—	—	—	—	—	—	—	—	—	1.4356×10^{-7}	3.6034×10^{-7}	1.9946×10^{-7}	3.9694×10^{-7}	1.0006×10^{-7}	1.2004×10^{-6}	14.
15.	$\frac{\Delta S}{EA}$ (Factor)	0.9928×10^{-8}	6.2500×10^{-8}	1.9545×10^{-8}	3.9583×10^{-8}	1.0169×10^{-8}	1.0169×10^{-8}	2.6017×10^{-8}	1.2075×10^{-8}	2.2578×10^{-8}	0.5333×10^{-8}	0.5333×10^{-8}	1.1916×10^{-8}	0.5807×10^{-8}	1.1203×10^{-8}	0.2769×10^{-8}	—	15.
16.	β	0	0	2°30'	8°	14°	14°	21°15'	26°	31°30'	44°	44°	49°30'	62°	74°30'	90°	—	16.
17.	$1.6\cos^2\beta + 1$	2.60	2.60	2.59696	2.56901	2.50636	2.50636	2.38982	2.29253	2.16319	1.82792	1.82792	1.67485	1.35264	1.11427	1.0000	—	17.
18.	⑮⑰	2.5813×10^{-8}	10.2917×10^{-8}	5.7058×10^{-8}	10.1690×10^{-8}	2.5486×10^{-8}	2.5486×10^{-8}	6.2175×10^{-8}	2.7682×10^{-8}	4.8841×10^{-8}	0.9748×10^{-8}	0.9748×10^{-8}	1.9957×10^{-8}	0.7854×10^{-8}	1.2483×10^{-8}	0.2769×10^{-8}	5.3341×10^{-7}	18.
19.	⑱	—	—	—	—	—	2.5486×10^{-8}	6.2175×10^{-8}	2.7682×10^{-8}	4.8841×10^{-8}	0.9748×10^{-8}	0.9748×10^{-8}	1.9957×10^{-8}	0.7854×10^{-8}	1.2483×10^{-8}	0.2769×10^{-8}	2.2674×10^{-7}	19.
20.	⑱	—	—	—	—	—	—	—	—	—	—	0.9748×10^{-8}	1.9957×10^{-8}	0.7854×10^{-8}	1.2483×10^{-8}	0.2769×10^{-8}	0.5281×10^{-7}	20.
21.	x^2 ⑱	0	2.7148×10^{-4}	5.3182×10^{-4}	23.9542×10^{-4}	10.6064×10^{-4}	10.6064×10^{-4}	35.1001×10^{-4}	20.1262×10^{-4}	44.1326×10^{-4}	10.5677×10^{-4}	—	—	—	—	—	1.6313×10^{-2}	21.
22.	㉑	—	—	—	—	—	10.6064×10^{-4}	35.1001×10^{-4}	20.1262×10^{-4}	44.1326×10^{-4}	10.5677×10^{-4}	—	—	—	—	—	1.2053×10^{-2}	22.

$$3\Delta = M_1\Sigma③ + V_1(\Sigma④ + \Sigma⑱) + \frac{K}{6}(\Sigma⑥ + 3\Sigma㉑) + V_2(\Sigma⑨ + \Sigma⑲) + \frac{Kx_8^2}{2}(\Sigma⑬ + \Sigma⑳)$$
$$3\Delta = 1.16372\times10^{-7}M_1 + 2.7076\times10^{-5}V_1 + 6.4051\times10^{-6}V_2 + 0.247627$$

$$3\Delta_c = M_1\Sigma⑦ + V_1(\Sigma⑨ + \Sigma⑲) + V_2(\Sigma⑧ + \Sigma⑲) + \frac{K}{6}(\Sigma⑩ + 3\Sigma㉒) + \frac{Kx_8^2}{2}(\Sigma⑭ + \Sigma⑳)$$
$$3\Delta_c = 1.9643\times10^{-8}M_1 + 6.4051\times10^{-6}V_1 + 2.3980\times10^{-6}V_2 + 0.081156$$

$$3\psi = M_1\Sigma② + V_1\Sigma③ + \frac{K}{6}\Sigma⑤ + V_2\Sigma⑦ + \frac{Kx_8^2}{2}\Sigma⑫$$
$$3\psi = 0.746453\times10^{-9}M_1 + 0.116372\times10^{-6}V_1 + 0.0196426\times10^{-6}V_2 + 0.00052982$$

REVISION OF ELASTIC AXIS AND COMPUTATION OF I

POINT	ℓ	w	ΣA	ΣAy	ȳ	I
0		75	172.24	2469	14.33	32,190
1		70	172.80	2634	15.24	34,700
2		69	174.98	3192	18.24	42,980
3		35	172.80	4158	24.05	52,890
4		13	168.16	4959	29.41	54,650
5		15	177.27	5430	30.63	64,450
6		26	190.98	5779	30.26	78,130
7		35	204.27	6180	30.26	94,340
8		40	216.20	6748	31.21	110,400
9		45	224.20	7065	31.51	124,430
10		45	230.08	7504	32.61	132,630
11		48	238.51	7965	33.40	147,060
12		49	241.21	8175	33.89	152,810
0'		43	143.60	2324	16.19	24,260
1'		43	150.50	2639	17.54	29,200
2'		43	151.70	2696	17.77	30,220
3'		43	149.30	2623	17.57	28,310
4'		43	146.60	2459	16.77	26,340
5'		43	144.80	2377	16.42	25,090
6'		43	143.90	2337	16.24	24,460
7'		43	143.90	2337	16.24	24,460
8'		43	143.90	2337	16.24	24,460

FOR RING: K' = 0.70707 $\qquad \frac{\Delta S_1'}{E} = 138.6\times10^{-8} \qquad \frac{\Delta S_2'}{E} = 58.9\times10^{-8}$

	POINT	0'	1'	2'	3'	4'R	4'L	5'	6'	7'	8'	Σ	
	FACTOR	1	4	2	4	1	1	4	2	4	1	—	
1.	x'	0	40.56	78.00	111.60	136.38	136.38	143.76	149.04	152.40	153.00	—	1.
2.	$\frac{\Delta S'}{EI}$ (Factor)	5.7131×10^{-11}	18.9813×10^{-11}	9.1827×10^{-11}	19.5832×10^{-11}	5.2620×10^{-11}	5.2620×10^{-11}	9.3939×10^{-11}	4.8160×10^{-11}	9.6321×10^{-11}	2.4080×10^{-11}	9.0229×10^{-10}	2.
3.	x' ②	0	0.7701×10^{-8}	0.7155×10^{-8}	2.1855×10^{-8}	0.7176×10^{-8}	0.7176×10^{-8}	1.3505×10^{-8}	0.7178×10^{-8}	1.4679×10^{-8}	0.3684×10^{-8}	9.0109×10^{-8}	3.
4.	$(x')^2$ ②	0	3.1235×10^{-7}	5.5807×10^{-7}	24.3910×10^{-7}	9.7870×10^{-7}	9.7870×10^{-7}	19.4144×10^{-7}	10.6978×10^{-7}	22.3712×10^{-7}	5.6369×10^{-7}	1.1079×10^{-5}	4.
5.	$(x')^3$ ②	0	1.2669×10^{-5}	4.3529×10^{-5}	27.2193×10^{-5}	13.3475×10^{-5}	—	—	—	—	—	4.6187×10^{-4}	5.
6.	$(x')^4$ ②	0	0.5138×10^{-3}	3.3953×10^{-3}	30.3767×10^{-3}	18.2033×10^{-3}	—	—	—	—	—	5.2489×10^{-2}	6.
7.	$x' - \frac{2}{3}x_4'$	—	—	—	—	—	45.46	52.84	58.12	61.48	62.08	—	7.
8.	⑦②	—	—	—	—	—	2.3921×10^{-9}	4.9638×10^{-9}	2.7991×10^{-9}	5.9218×10^{-9}	1.4949×10^{-9}	1.7572×10^{-8}	8.
9.	⑦③	—	—	—	—	—	3.2664×10^{-7}	7.1357×10^{-7}	4.1717×10^{-7}	9.0249×10^{-7}	2.2872×10^{-7}	2.5882×10^{-6}	9.
10.	$\frac{\Delta S'}{EA}$ (Factor)	0.7652×10^{-8}	3.6837×10^{-8}	1.7273×10^{-8}	3.7133×10^{-8}	0.9454×10^{-8}	0.9454×10^{-8}	1.6271×10^{-8}	0.8186×10^{-8}	1.6372×10^{-8}	0.4093×10^{-8}	—	10.
11.	β	0	14°	27°30'	44°30'	60°30'	60°30'	68°30'	75°30'	82°30'	90°	—	11.
12.	$(1.6\cos^2\beta + 1)$	2.6	2.50636	2.25886	1.81396	1.31797	1.31797	1.21492	1.10030	1.02726	1.0	—	12.
13.	⑩⑫	2.5094×10^{-8}	9.2327×10^{-8}	4.1276×10^{-8}	6.7358×10^{-8}	1.3122×10^{-8}	1.3122×10^{-8}	1.9769×10^{-8}	0.9007×10^{-8}	1.6819×10^{-8}	0.4093×10^{-8}	3.0199×10^{-7}	13.
14.	⑩⑫	—	—	—	—	—	1.3122×10^{-8}	1.9769×10^{-8}	0.9007×10^{-8}	1.6819×10^{-8}	0.4093×10^{-8}	6.2809×10^{-8}	14.
15.	$(x')^2$ ⑬	—	1.5189×10^{-4}	2.5112×10^{-4}	3.3394×10^{-4}	2.4407×10^{-4}	—	—	—	—	—	1.4860×10^{-3}	15.

$$3\Delta' = M_3\Sigma③ + V_3(\Sigma④ + \Sigma⑬) + \frac{K'}{6}(\Sigma⑥ + 3\Sigma⑮) + \frac{K'(x_4')^2}{2}(\Sigma⑨ + \Sigma⑭)$$
$$3\Delta' = 0.90109\times10^{-7}M_3 + 1.1381\times10^{-5}V_3 + 0.024143$$

$$3\psi' = M_3\Sigma② + V_3\Sigma③ + \frac{K'}{6}\Sigma⑤ + \frac{K'(x_4')^2}{2}\Sigma⑧$$
$$3\psi' = 0.90229\times10^{-9}M_3 + 0.90109\times10^{-7}V_3 + 0.00016997$$

FOR RING 'O.D.'

$$\Delta_R = \frac{M_4R^2 + \frac{\pi}{4}V_4R^3}{EI_R} + \frac{2.8274\,V_4R + (1.42\sqrt{rt} + c)\,prR}{EA_R} = 0.312320\times10^{-7}M_4 + 35.4632\times10^{-?}V_4 + 746\times10^{-7}$$

$$\psi_R = \frac{\frac{\pi}{4}M_4R + V_4R^2}{EI_r} = 0.352611\times10^{-9}M_4 + 0.031232\times10^{-6}V_4$$

Where R = radius to centroid of ring = 139.132
r = radius to inside of shell = 123.406
c = web thickness = 2.5

$A = A_c = 176.7$
$\left.\begin{matrix}\theta_1\\ \theta_2\end{matrix}\right\} 22\tfrac{1}{2}°$
$\left.\begin{matrix}\theta_3\\ \theta_4\end{matrix}\right\} 67\tfrac{1}{2}°$
$L = 285$
$L_c = 257.76$
$t = 0.8125$
$I_R = 20{,}660$
$A_R = 128.068$

FINAL BASIC EQUATIONS

$$\frac{-V_2L_c}{2A_cE} = \Delta_c$$
$$\frac{-(V_1 + V_3 + V_4)L}{2AE} = \Delta = \Delta' = \Delta_R$$
$$\psi_R \sin(\theta_4 + \theta_1) + \psi' \cos\theta_1 + \psi \cos\theta_4 = 0$$
$$M_1 \cos\theta_1 = M_3 \cos\theta_4$$
$$M_1 \sin\theta_1 + M_3 \sin\theta_4 = M_4$$

STRESS ANALYSIS OF WYE BRANCHES

UNSYMMETRICAL BIFURCATIONS
DEFLECTIONS AND ROTATIONS
Glendo Dam Penstock Branch

DRAWING 5

In the case where a second ring, 'OD', is used on the connection, the expressions for the moment, shear, and tension in the ring are

$$M = M_4 + V_4 R \sin \varphi$$

$$V = V_4 \cos \varphi$$

$$T = V_4 \sin \varphi$$

where the angle φ is measured from the vertical centerline of the pipe in the plane of the ring.

For the second ring, 'OD', the equations for the deflection and rotation at Point 'O' may be written as

$$\Delta_R = \frac{M_4 R^2 + \frac{\pi}{4} V_4 R^3}{EI_R}$$

$$+ \frac{2.8274\ V_4 R + (1.42 \sqrt{rt} + c) pr\ R}{EA_R}$$

where

r = inside radius of cylindrical shell,

R = radius to center of gravity of ring cross section,

I_R = Moment of inertia of ring cross section, using the effective flange width $w = 1.56 \sqrt{rt} + c$ where c is the web thickness, and t is the shell thickness,

A_R = cross sectional area of ring

and

$$\psi_R = \frac{\frac{\pi}{2} M_4 R + V_4 R^2}{EI_R}$$

where M4 and V4 are the end moment and shear on the ring 'OD'.

Final Equations

We may write the final equations for the deflections, rotations, and moments of the common junction in a manner similar to that described for symmetrical junctions as follows:

$$\Delta_c = \frac{-V_2 L_c}{2 A_c E}$$

$$\Delta = \frac{-(V_1 + V_3 + V_4) L}{2 AE}$$

$$\Delta' = \frac{-(V_1 + V_3 + V_4) L}{2 AE}$$

$$\Delta_R = \frac{-(V_1 + V_3 + V_4) L}{2 AE}$$

$$\psi_R \sin (\theta_4 + \theta_1) + \psi' \cos \theta_1$$

$$+ \psi \cos \theta_4 = 0$$

$$M_1 \cos \theta_1 = M_3 \cos \theta_4$$

$$M_1 \sin \theta_1 + M_3 \sin \theta_4 = M_4$$

(If no tie rods are provided, V_2 becomes zero and Δ_c is eliminated. Then $V_1 + V_3 + V_4 = 0$, $\Delta = \Delta'$, and $\Delta = \Delta R$ are the shear and deflection equations.) These are our seven equations in seven unknowns.

Computation of Stresses

Substitution of these quantities into the expressions for moment, shear, and tension enables one to evaluate the stress in the beams and rings on Drawing No. 6 in the same manner as for the trifurcation previously treated. The stresses in the example are for an internal pressure of 1 psi.

General

Development of Equations for End Rotation

The equation for the sum of the rotations of the ends of the beams and rings is derived as follows (neglecting twisting of the members): A vertical plane of principal rotation is assumed, passing through the common junction. The actual rotations of the ends of the beams are projected upon this plane. The projected rotation of each beam and ring is then equal to the angle of principal rotation. The resulting equations may then be solved for the final equation of vector summation of the beam rotations.

For the case where the ring 'OD' is located at an angle $\theta 5$ from the upstream axis of the penstock, the equation for summation of the rotations is:

$$\psi_R \sin(\theta_1 + \theta_4) + \psi' \sin(\theta_5 - \theta_1)$$

$$+ \psi \sin(\theta_4 + \theta_5) = 0$$

Special Designs

For the case of an unsymmetrical bifurcation without tie rods, we have six unknowns--the shear and moment on the end of each beam. The six equations at the common junction are: The sum of the shears is equal to zero, the sum of the moments is equal to zero (2), the vector sum of the rotations of the ends of the beams is equal to zero, and the sum of the deflections of the ends of the beams is equal to zero (2).

For the case of an unsymmetrical trifurcation, referring to Drawing No. 1, we now have 10 unknowns: Shear and moment on the end of each beam and ring (8), and the two shears on the intermediate tie rods (2). We also have 10 equations: The deflections of the beams at the intermediate tie rods are equal to the tie-rod elongation (2), the deflections of the ends of the beams are equal to the elongation of the tie rod (4), the vector sum of the rotations is zero (2), and the summation of the moments is zero (2).

For any other general case of a wye branch connection of this type, adaption can be made of the general equations and the procedures outlined to obtain a solution to problems similar to those given. For instance, if n beams have a common coplanar junction without tie rods, the 2n unknowns may be obtained by solving the following set of equations: (n-1) equations involving deflections of the ends of the beams, the equation of the sum of the end shears to zero, the sum of the end moments equated to zero about a pair of orthogonal axes, and (n-2) equations of the rotations of the ends of the beams.

In closing, it is considered that the methods provided herein constitute a suitable engineering solution to a very complicated problem. While refinements have been introduced into the method, the fundamental assumptions of loading and structural action determine the accuracy of the solution.

Stresses caused by erection procedures and dead loads have not been considered. The support structures contribute to the prototype stresses, and should be designed with care.

For a more rapid method of preliminary design, the members may be considered as alternately pinned- or fixed-ended. The number of intervals taken for integration may be halved, and the flange widths may be assumed. This will substantially reduce the labor involved.

Acknowledgments

This study was made in the Technical Engineering Analysis Branch under the general supervision of W. T. Moody. Many basic contributions to the method of analysis were made by C. C. Crawford. Model studies of certain designs were made by H. Boyd Phillips and I. E. Allen.

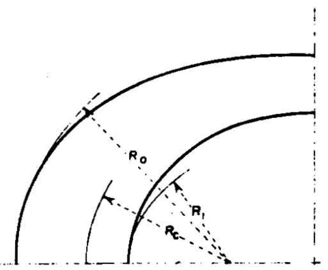
FINAL EQUATIONS:

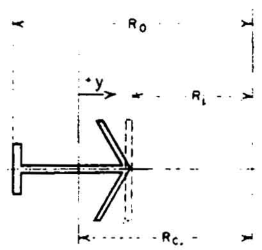
M_1	M_3	M_4	V_1	V_2	V_3	V_4	$= C \times 10^5$	CHECK
1.16372	—	—	271.561	64.051	0.806387	0.806387	-24.7626	313.62628
—	0.90109	—	0.806387	—	114.6144	0.806387	-2.414253	114.7140
—	—	0.93696	0.806387	—	0.806387	116.22956	-0.022390	118.7969
0.196430	—	—	64.051	24.70971	—	—	-8.115360	80.841780
0.382683	0.923880	-1	—	—	—	—	—	0.306563
0.923880	-0.382683	—	—	—	—	—	—	0.541197
0.002857	0.008333	0.010578	0.445336	0.075170	0.832499	0.936960	-0.047454	2.264279

AUXILARY EQUATIONS

M_1	M_3	M_4	V_1	V_2	V_3	V_4	C	CHECK
1.16372	—	—	233.3563	55.039872	0.692939	0.692939	-21.278830	269.50321
—	0.90109	—	0.894902	—	127.195270	0.894902	-2.679258	127.305816
—	—	0.936960	0.860642	—	0.860642	113.419530	-0.023896	116.116917
0.196430	—	—	18.212823	0.763101	-0.007474	-0.007474	-0.216087	1.532067
0.382683	0.923880	-1	-87.267627	47.057392	-2.498755	2.372655	-0.184717	0.689326
0.923880	-0.382683	-1	-215.250749	113.907825	329.804861	-0.821719	-0.020994	0.157287
0.002857	0.008333	0.010578	-0.237925	0.099482	0.008300	-0.503244	-0.006085	0.993912

SOLUTION		CHECK
$V_4 \times 10^{-5}$	-0.006085	0.993912
$V_3 \times 10^{-5}$	-0.025995	0.974003
$V_2 \times 10^{-5}$	-0.235294	0.764712
$V_1 \times 10^{-5}$	-0.036774	0.963223
$M_4 \times 10^{-5}$	0.720283	1.720627
$M_3 \times 10^{-5}$	0.665538	1.665797
$M_1 \times 10^{-5}$	0.275396	1.275759





FOR BEAM:

	POINT	0	1	2	3	4	5	6
1	M_1	27,540	27,540	27,540	27,540	27,540	27,540	27,540
2	$V_1 x$	0	-188,850	-376,380	-564,350	-750,110	-873,660	-991,470
3	$V_2(x-x_4)$						-790,570	-1,544,440
4	$Kx^3/6$	0	15,970	126,390	426,070	1,000,510	1,580,770	2,310,380
5	$\frac{1}{2}K x_8^2 (x-\frac{2}{3}x_8)$							
6	Σ Moments	27,540	-145,340	-222,450	-110,740	277,940	-55,920	-197,990
7	I	32,190	34,700	42,980	52,880	54,650	64,450	78,730
8	y_o	-21.67	-22.20	-23.76	-25.00	-24.95	-26.61	-29.02
9	y_i	14.33	15.24	18.24	24.05	29.41	30.63	30.26
10	My_o/I	-18.54	92.98	122.97	52.35	-126.89	23.09	72.98
11	My_i/I	12.26	-63.83	-94.40	-50.36	149.57	-26.58	-76.10
12	β	0	0	2°30′	8°	14°	21°15′	26°
13	Sin β	0	0	0.043619	0.139173	0.241922	0.362438	0.438371
14	$(V_1 + Kx^2/2)$ Sin β	0	0	1.19	647.33	2669.93		
15	$(V_1 + V_2 + Kx^2/2)$ Sin β						-2626.51	-657.95
16	$(V_1 + V_2 + Kx_8^2/2)$ Sin β							
17	Σ Tensile Force	0	0	1.19	647.33	2669.93	-2626.51	-657.95
18	A	172	173	175	173	168	177	191
19	T/A	0	0	0.01	3.74	15.89	-14.84	-3.44
20	K_o	1	1	1	1	1	1	1
21	K_i	1	1	1	1	1	1	1
22	$T/A + K_o My_o/I$	-18.54	92.98	124.16	56.09	-111.00	8.25	69.54
23	$T/A + K_i My_i/I$	12.26	-63.83	-93.21	-46.62	165.46	-41.42	-79.54

	POINT	7	8	9	10	11	12
1	M_1	27,540	27,540	27,540	27,540	27,540	27,540
2	$V_1 x$	-1,105,310	-1,210,650	-1,255,770	-1,296,800	-1,322,840	-1,335,630
3	$V_2(x-x_4)$	-2,272,900	-2,947,010	-3,235,710	-3,498,290	-3,664,880	-3,746,760
4	$Kx^3/6$	3,201,080	4,206,360				
5	$\frac{1}{2}K x_8^2 (x-\frac{2}{3}x_8)$			4,676,630	5,104,360	5,375,720	5,509,090
6	Σ Moments	-149,590	76,240	212,690	336,810	415,540	454,240
7	I	94,340	110,400	124,430	132,630	147,060	152,810
8	y_o	-31.30	-33.47	-34.85	-36.63	-37.52	-38.11
9	y_i	30.26	31.21	31.51	32.61	33.40	33.89
10	My_o/I	49.63	-23.11	-59.57	-91.50	-106.02	-113.29
11	My_i/I	-47.98	21.55	53.86	82.81	94.38	100.74
12	β	31°30′	44°	49°30′	62°	74°30′	90°
13	Sin β	0.522499	0.694658	0.760406	0.882948	0.963630	1.0
14	$(V_1 + Kx^2/2)$ Sin β						
15	$(V_1 + V_2 + Kx^2/2)$ Sin β	2477.12	7725.14				
16	$(V_1 + V_2 + Kx_8^2/2)$ Sin β			8456.3	9819.08	10,716.32	11,120.79
17	Σ Tensile Force	2477.12	7725.14	8456.3	9819.08	10,716.32	11,120.79
18	A	204	216	224	230	239	241
19	T/A	12.14	35.76	37.75	42.69	44.84	46.14
20	K_o	1	1	1	0.75	0.71	0.70
21	K_i	1	1	1	2.78	2.78	2.84
22	$T/A + K_o My_o/I$	61.77	12.65	-21.82	-25.94	-30.43	-33.16
23	$T/A + K_i My_i/I$	-35.84	57.31	91.61	272.90	307.2	332.24

	POINT	10	11	12	8'
1	A	230	239	241	144
2	I	132,630	147,060	152,810	24,460
3	R_c	74.61	74.48	73.89	142.24
4	R_o	110.63	112	112	162
5	R_i	42	41	40	126
6	y_o	-36.03	-37.52	-38.11	-19.76
7	y_i	32.61	33.48	33.89	16.24
8	$\Sigma \frac{y^2}{R-y} \Delta A$	1240.46	1411.73	1477.20	146.24
9	$Z = \frac{⑧}{①③}$	0.072287	0.079307	0.082954	0.00714
10	K_o	0.75	0.71	0.70	0.97
11	K_i	2.78	2.79	2.84	1.401

$$K_o = \frac{I}{AR_c}\left(\frac{1}{R_o Z} + \frac{1}{y_o}\right)$$

$$K_i = \frac{I}{AR_c}\left(\frac{1}{R_i Z} + \frac{1}{y_i}\right)$$

FOR RING:

	POINT	0'	1'	2'	3'	4'	5'	6'	7'	8'
1	M_3	66,550	66,550	66,550	66,550	66,550	66,550	66,550	66,550	66,550
2	$V_3 x'$	0	-105,420	-202,720	-290,050	-354,450	-373,630	-387,350	-396,090	-397,650
3	$K'\frac{(x')^3}{6}$	0	7,860	55,920	163,800	298,920				
4	$\frac{1}{2}K'(x_4')^2(x'-\frac{2}{3}x_4')$						347,450	382,170	404,270	408,210
5	Σ Moments	66,550	-31,010	-80,250	-59,750	11,020	40,370	61,370	74,730	77,110
6	I	24,260	29,200	33,220	26,310	26,340	25,080	24,460	24,460	24,460
7	y_o	-18.6	-20.02	-20.27	-19.51	-19.23	-18.86	-18.68	-18.68	-18.68
8	y_i	16.19	17.54	17.77	17.57	16.77	16.42	16.24	16.24	16.24
9	My_o/I	-51.02	21.26	53.83	41.18	-8.04	-30.36	-46.87	-57.07	-58.89
10	My_i/I	44.41	-18.63	-47.19	-37.08	7.02	26.43	40.75	49.62	51.20
11	β	0	14°	27½°	44½°	60½°	68½°	75½°	82½°	90°
12	Sin β	0	0.241922	0.461749	0.700909	0.870356	0.930418	0.968148	0.991445	1.0
13	$(V_3 + K'(x')^2/2)$ Sin β	0	-488	-207	1264	3461				
14	$(V_3 + K'(x_4')^2/2)$ Sin β						3699	3849	3942	3976
15	Σ Tensile Force	0	-488	-207	1264	3461	3699	3849	3942	3976
16	A	144	151	152	149	147	145	144	144	144
17	T/A	0	-3.23	-1.36	8.48	23.54	25.51	26.73	27.38	27.61
18	K_o	1	1	1	1	1	1	1	1	0.97
19	K_i	1	1	1	1	1	1	1	1	1.40
20	$T/A + K_o My_o/I$	-51.02	18.03	52.47	49.66	15.50	-4.85	-20.14	-29.69	-29.63
21	$T/A + K_i My_i/I$	44.41	-21.86	-48.55	-28.60	30.56	51.94	67.48	77.00	99.34

FOR TIE RODS:

At 'O'	
ΣV	-6885.4
A	176.7
-ΣV /A	39
At 'C'	
V_2	-23,529
A_c	176.7
$-V_2/A_c$	133

NOTE

Positive sign denotes tensile stresses

Stresses are for 1psi pressure

Design pressure is 85 psi

STRESS ANALYSIS OF WYE BRANCHES

STRESSES IN UNSYMMETRICAL BIFURCATIONS

Glendo Dam Penstock Branch

DRAWING 6

GPO 841-064

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Appendix I

Stress analysis of pipe branch--Glendo Dam
Missouri River Basin Project

Introduction

An experimental study has been made of the stresses existing in the Glendo Dam outlet pipes at the first branch immediately downstream from the surge tank, an unsymmetrical bifurcation.

Four different reinforcement schemes were considered. These were:

a. Two-way reinforcement.

b. Two-way reinforcement with revision of larger U-beam.

c. Three-way reinforcement by addition of third ring to the model in b. above.

d. Three-way reinforcement as in c. above with the outside flange of the largest U-beam doubled in thickness for a distance of approximately 11 feet on each side of the line of symmetry.

A scale model was constructed of sheet plastic. Compressed air was used to apply an internal pressure to the structure. Stresses were determined by use of strain gages.

Results

Stresses have been determined at various points on the U-beams of the two-way reinforcement system, on the two tie rods, and at certain points on the pipe shell. These locations are indicated on Figure 4.

Table 1 gives stress values at the various points. These stresses are for an internal pressure of 85 psi acting in the prototype structure.

TABLE 1--EXPERIMENTAL MODEL STRESSES IN THE UNSYMMETRICAL BIFURCATION

GLENDO DAM--MISSOURI RIVER BASIN PROJECT

Point	Scheme a	Scheme b	Scheme c	Scheme d
1	32, 100	27, 700	26, 900	26, 300
2	- 2, 100	0	0	- 1, 200
3	10, 800	4, 200	7, 900	7, 800
4	13, 900	11, 700	13, 000	12, 700
5	18, 300	12, 100	10, 800	10, 000
6	10, 600	10, 300	10, 700	10, 500
7	10, 000	10, 200	10, 300	10, 100
8	4, 100	3, 900	4, 100	--
9	1, 200	1, 200	1, 600	--
10	1, 900	1, 800	2, 400	--
11	6, 900	6, 400	7, 000	--
12	4, 200	4, 500	3, 700	--
13	--	19, 700	10, 300	--
14	--	22, 200	--	--
15	--	26, 200	--	--
16	--	26, 800	7, 500	--

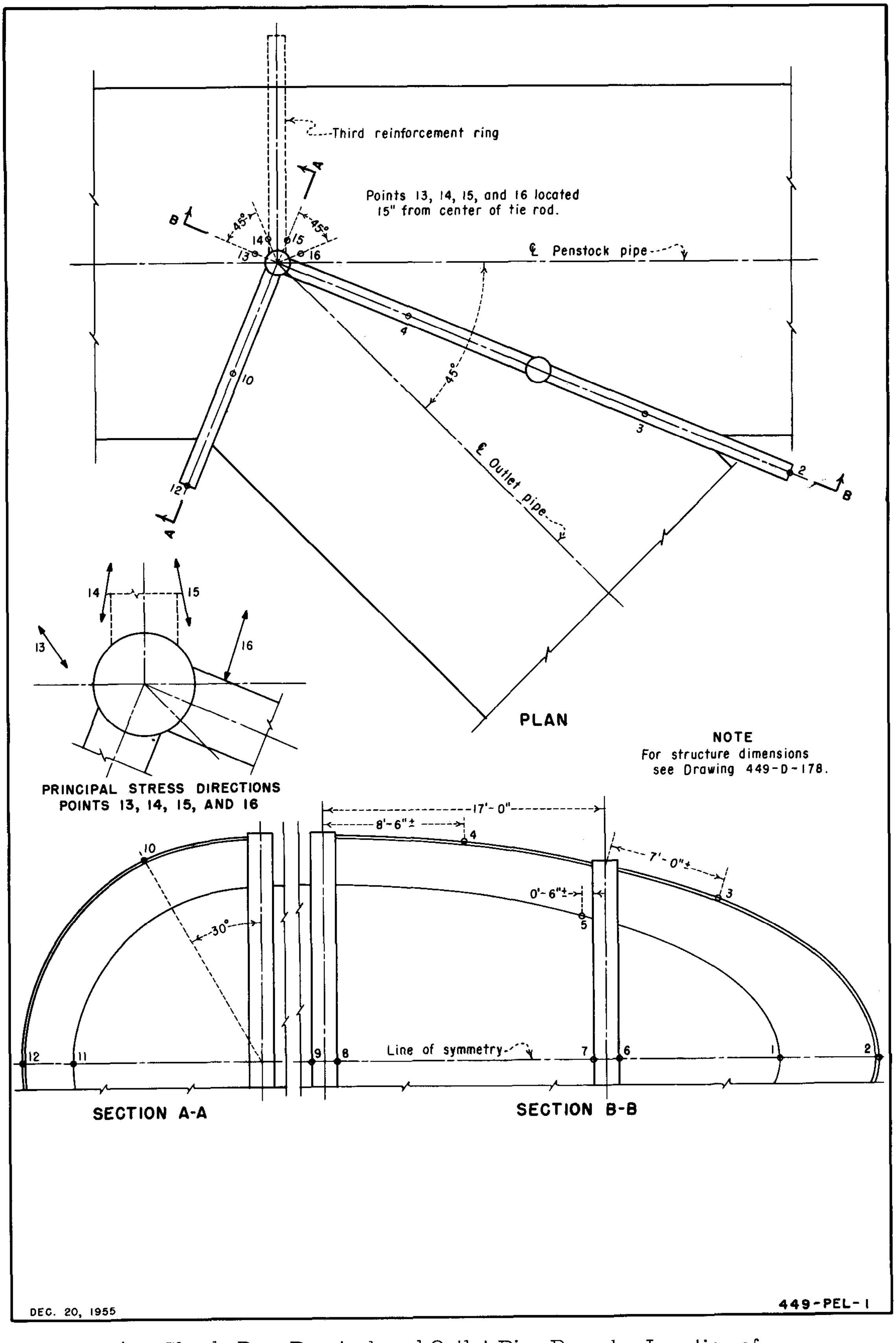

Figure 4.--Glendo Dam Penstock and Outlet Pipe Branch--Location of Stress Points

Conclusions

The regions of high stresses can be seen from a study of Table 1. These high stresses exist in the crotch of the large U-beam (Point 1), in the U-beam where it joins the intermediate tie rod (Point 5), and in the shell in the vicinity of the junction of the U-beams (Points 13, 14, 15, and 16.)

Increasing the depth of the large U-beam (Scheme b) lowered the stress at Point 5 by nearly 50 percent. At Point 1, in the crotch of the large U-beam, the stress decreased by less than 15 percent. Stresses in the pipe shell remained virtually unchanged.

The addition of the third reinforcement ring (Scheme c) had a small effect on stresses in the U-beams. At Point 5 the stress was reduced about 10 percent over Scheme b, while at Point 1 the stress reduction over Scheme b was less than 5 percent. However, stresses in the pipe shell in the vicinity of the junction of the U-beams were reduced 50 to 75 percent, to values which are within the usual allowable limits.

Adding a cover plate to part of the length of the outside flange of the large U-beam (Scheme d) caused insignificant changes in the stresses.

Basic Data

Inside diameter of pipe	21' 0"
Plate thickness of pipe	13/16"
Plate thickness of U-beams	2-1/2"
Diameter of tie rods	15"
Internal water pressure	85 psi

Technical Details

A scale model of the pipe branch was constructed of transparent plastic, cast methyl methacrylate. A shell plate thickness of

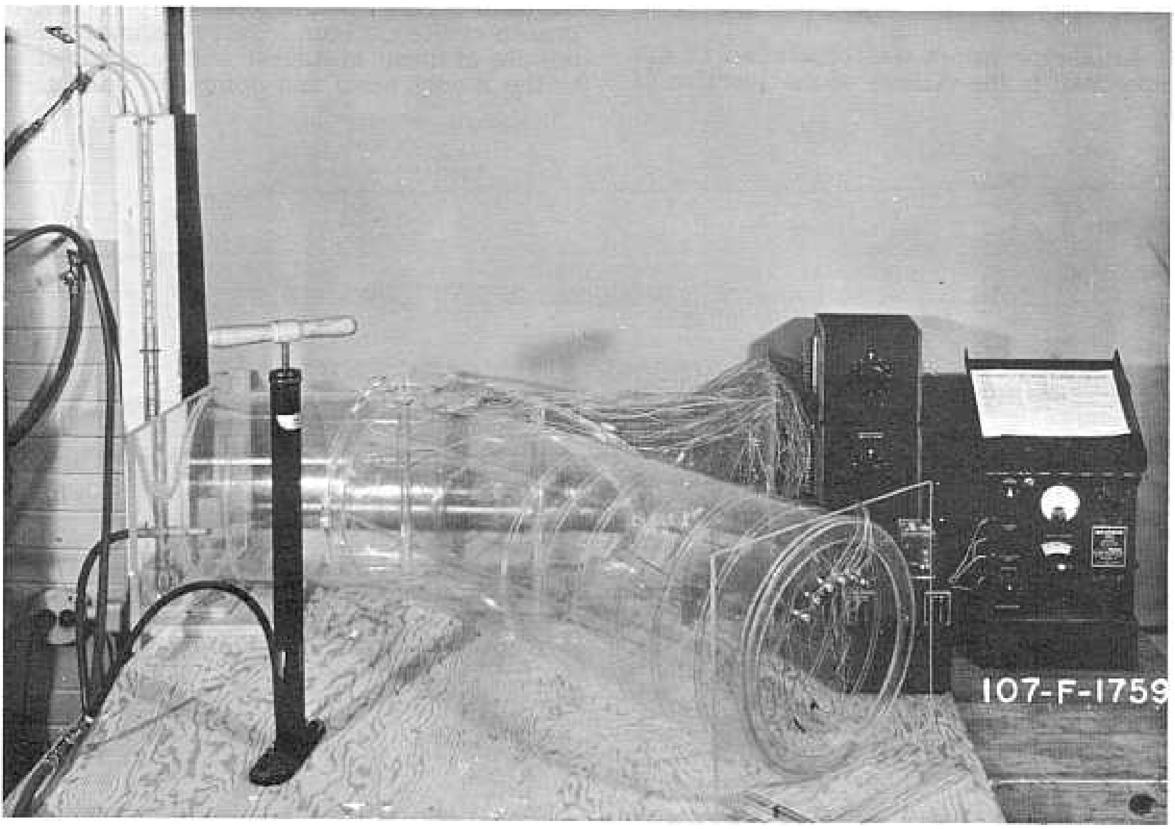

Figure 5.--Glendo Dam Penstock and Outlet Pipe Branch--Model Arrangement

0.04 inch was used. This gave a scale factor of approximately one-twentieth (0.04923). The stiffener rings were fabricated from 0.05-inch-thick plastic, and the webs and flanges of the U-beams were made from material 1/8 inch thick. The penstock pipe extended approximately 1 pipe diameter upstream and 2 diameters downstream from the intersection of the U-beams. The outlet pipe extended approximately 1-1/2 diameters downstream from the intersection.

The model was loaded internally with air pressure. The air was introduced through a pressure valve by using a tire pump. The applied pressure was measured with a mercury U-tube manometer.

The arrangement of the model, the tire pump, and the manometer can be seen in Figure 5.

Strain gages were installed at the various points at which the stresses were desired. Two types of linear gages were used.

Rosette-type gages were installed on the pipe shell in the vicinity of the junction of the U-beams and readings taken for reinforcement Scheme a. The results were rather high and the calculated directions of maximum stress inconsistent with what might be expected and also inconsistent between different points. The gage measures the average strain over an area covered by the three legs of the gage. Since the strain changes very rapidly in the vicinity of the U-beam intersection, the spacing between legs of the strain gage is significant. For Schemes b and c the procedure was modified. Linear-type gages were installed and read successively at the same point but rotated 45° each time to get the data required to compute the principal stresses and their directions.

Where possible, duplicate gages were installed at symmetrical points on the structure. The stresses given are the mean values for such points.

Field Data

Strain measurements were conducted in the field during installation of the penstock branch at Glendo Dam. Table 2 shows the results of these measurements compared to the model tests and computed values.

TABLE 2--COMPARATIVE STRESSES IN THE UNSYMMETRICAL BIFURCATION

GLENDO DAM--MISSOURI RIVER BASIN PROJECT

Point	Field test	Model test	Computed
1	27,400 psi	26,900	28,200
2	--	0	- 2,800
3	--	7,900	6,200
4	11,200	13,000	10,300
5	27,000	10,800	14,900
6	9,100	10,700	11,300
7	14,600	10,300	11,300
8	3,700	4,100	3,500
9	1,900	1,600	3,500
10	--	2,400	3,800
11	7,100	7,000	8,800
12	2,600	3,700	- 2,400

Appendix II

Experimental stress study of outlet pipe manifold--WYE W1
Palisades Dam and Powerplant
Palisades Project

Introduction

A stress study was made of the Palisades Dam outlet pipe manifold, using a plastic model with strain gages. This study considered the stresses caused by internal pressure only and concentrated on determination of the stress condition primarily in the region of the horizontal centerline of the elliptical beam.

Results

The prototype stresses shown in Figure 6 were for a uniform internal pressure of 110 psi. The maximum stress (12,500 psi) occurred in the center pipe branch on the horizontal centerline near the U-beam crotch. The crotch stress in the large U-beam was 12, 200 psi and in the small U-beam 11, 500 psi. Strain gages were also installed on the vertical and horizontal centerlines of the branch pipes. A maximum stress of about 10, 500 psi occurred on the horizontal centerline of the center pipe branch and on the outer side of the outside branches. The stress in the horizontal A-frame was approximately 2,000 psi in the legs, and 4, 500 psi in the cross member. Cutting the legs free from the U-beam crotch had no significant effect on the stresses. By observing the cut sections as the load was applied, it was found that the large U-beam crotch moved slightly away from the longitudinal centerline.

Conclusions

Since the stresses of this study were well below the maximum allowable for an internal pressure of 110 psi, no attempt was made to lower them by altering the model.

Basic Data

Inside diameter of main pipe	26'0"
Inside diameter of center branch straight pipe	13'0"
Inside diameter of outside branch straight pipes	16'0"
Plate thickness of main pipe and conical branch pipes	1-1/4"
Plate thickness of center branch straight pipe	5/8"
Plate thickness of outside branch straight pipes	13/16"
Plate thickness of U-beams	2-1/4"
Diameter of tie rods	15"
Internal water pressure	110 psi

TABLE 3--COMPARATIVE STRESSES IN THE SYMMETRICAL TRIFURCATION

PALISADES DAM--PALISADES PROJECT

Point	Field test	Model test	Computed
Horizontal ℄ of beam			
(Inside)	17, 000	12, 200 psi	14, 500 psi
(Outside)	- 1, 600	500	- 110
Horizontal ℄ of Ring			
(Inside)	7, 000	11, 500	10, 800
(Outside)	9, 000	2, 500	- 3, 900
Long tie rod	4, 000	2, 500	8, 700
Short tie rod	8, 000	7, 200	10, 400

Technical Details

A one-twentieth scale model of the pipe manifold was constructed of transparent plastic, cast methyl methacrylate. A straight main pipe extending an equivalent of 36 feet upstream from the center of the main tie rod was used. Straight pipes were attached to the ends of the conical pipes extending an equivalent of 25 feet downstream on the outside branches and 17 feet on the center branch. The branch pipes were sealed with a 1/4-inch-thick plastic plate, and the main pipe with a 1/2-inch-thick plate. Details of the model are shown in Figures 7 through 10.

The model was loaded internally with air pressure, introduced by a tire pump. The pressure was measured with a mercury U-tube manometer.

Strain readings were taken for model loads of 4, 6, and 8 inches of mercury.

Field Data

A comparison of the stresses obtained by the three methods is shown in Table 3.

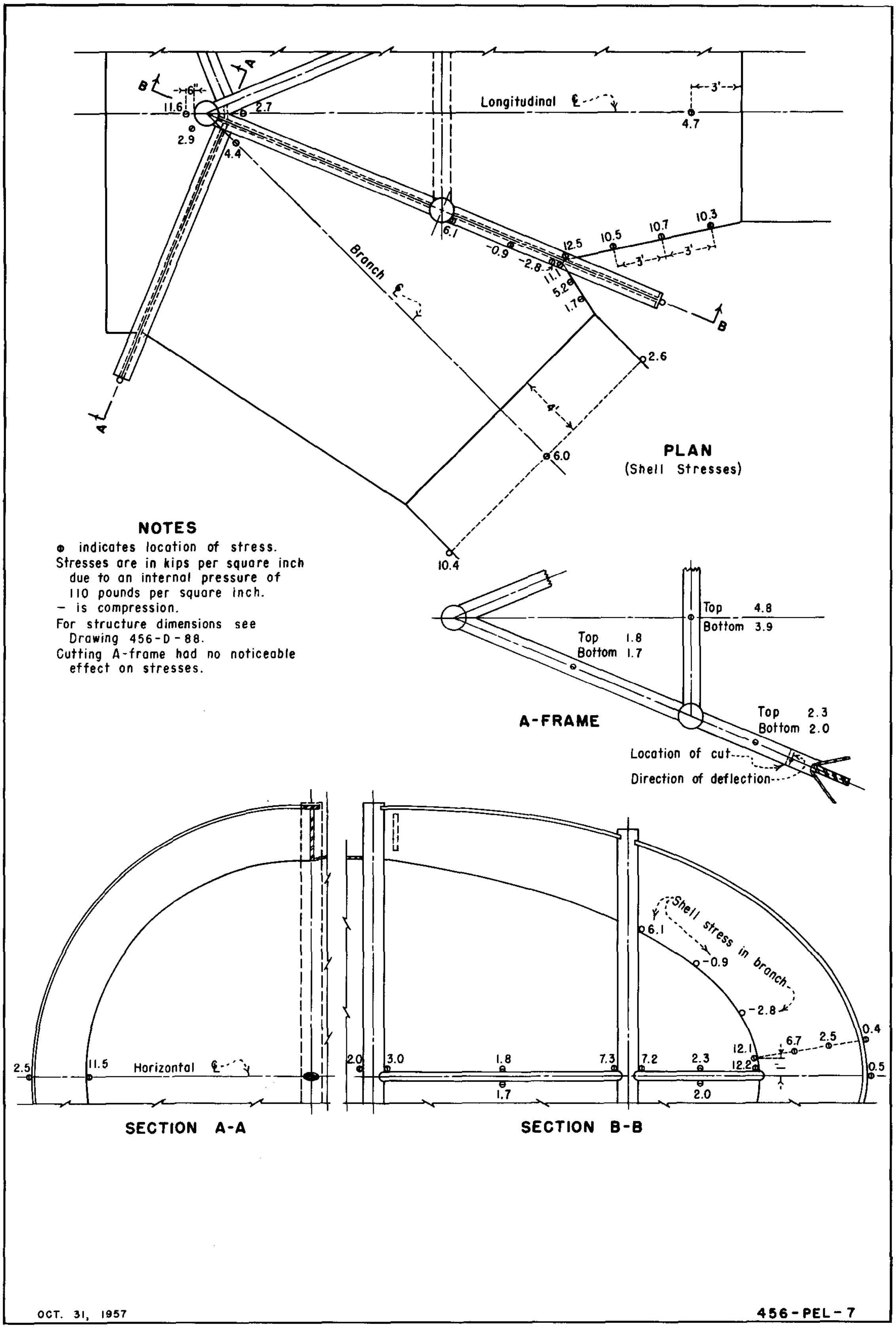

Figure 6. --Palisades Dam and Powerplant Outlet Pipe Manifold--Wye W1

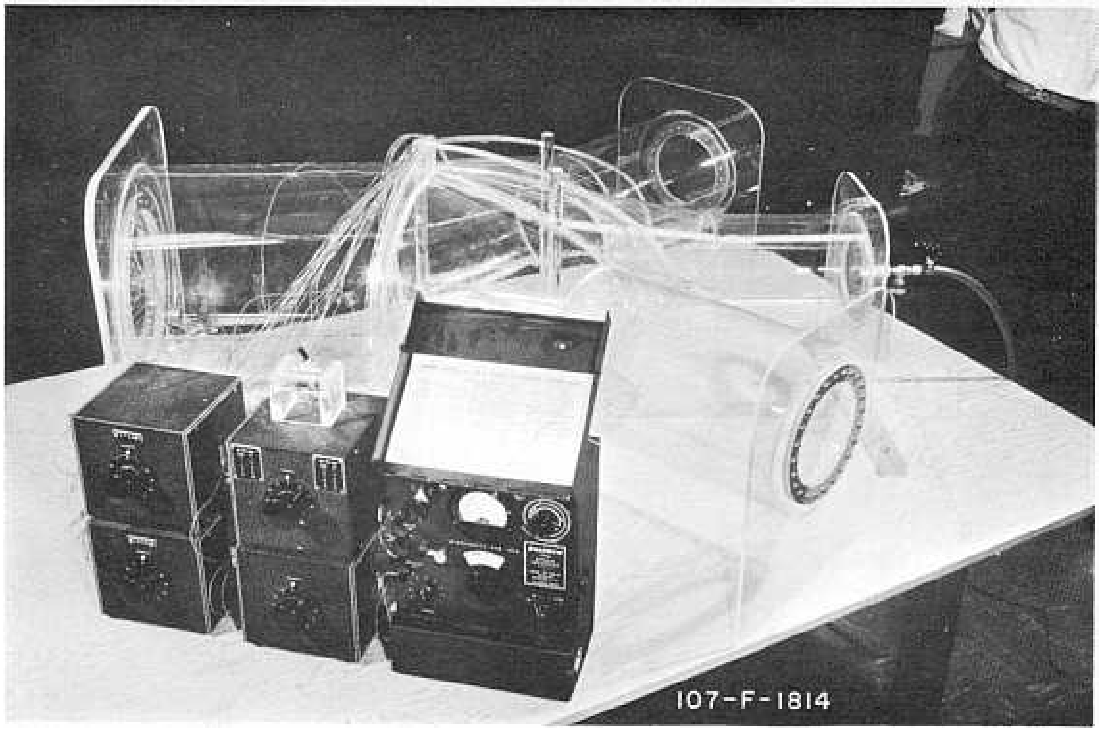

Figure 7.--Palisades Dam and Powerplant Outlet Pipe Manifold--Wye W1--
Test Arrangement

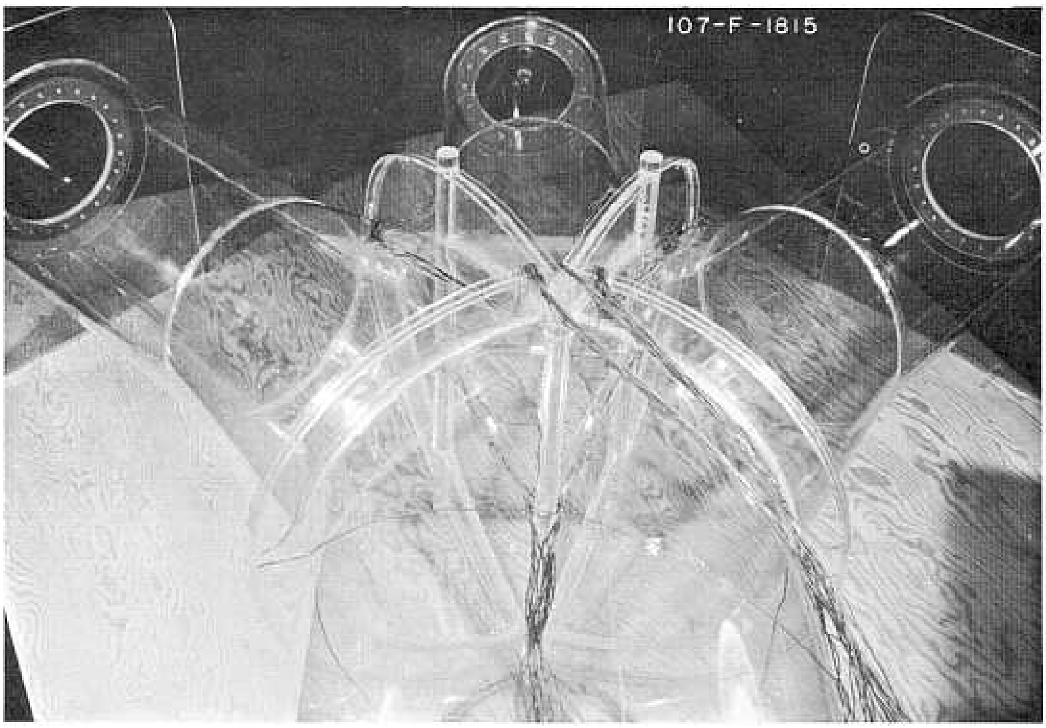

Figure 8.--Palisades Dam and Powerplant Outlet Pipe Manifold--Wye W1--
Looking Downstream at Model

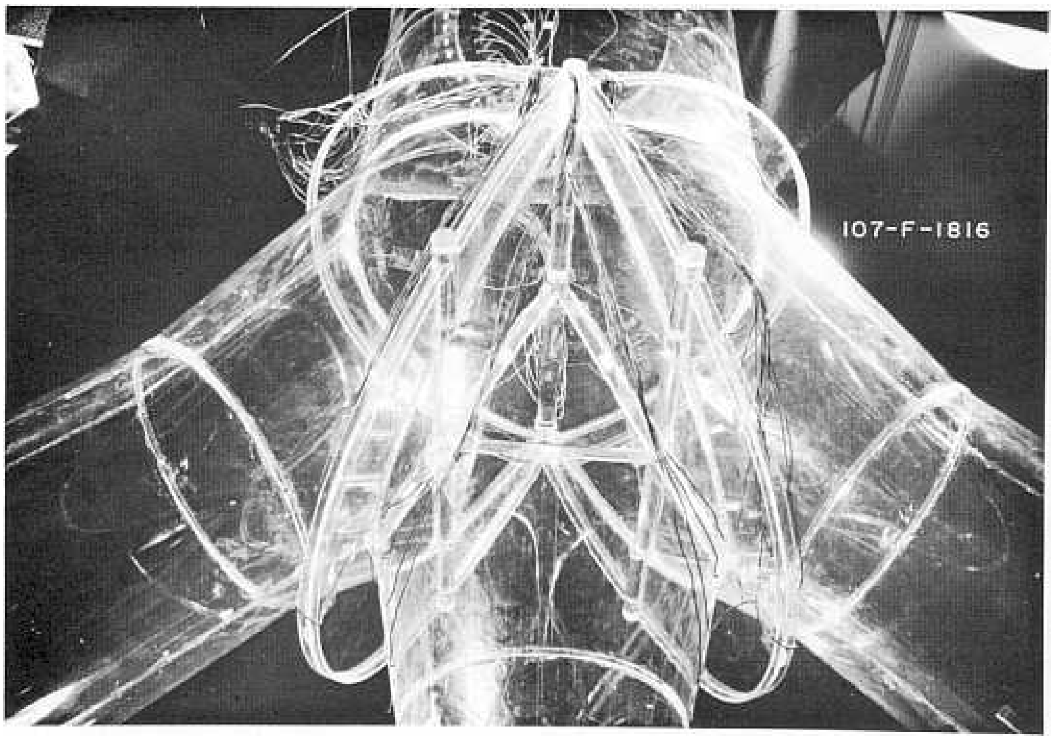

Figure 9. --Palisades Dam and Powerplant Outlet Pipe Manifold--Wye W1--
Looking Upstream at Model

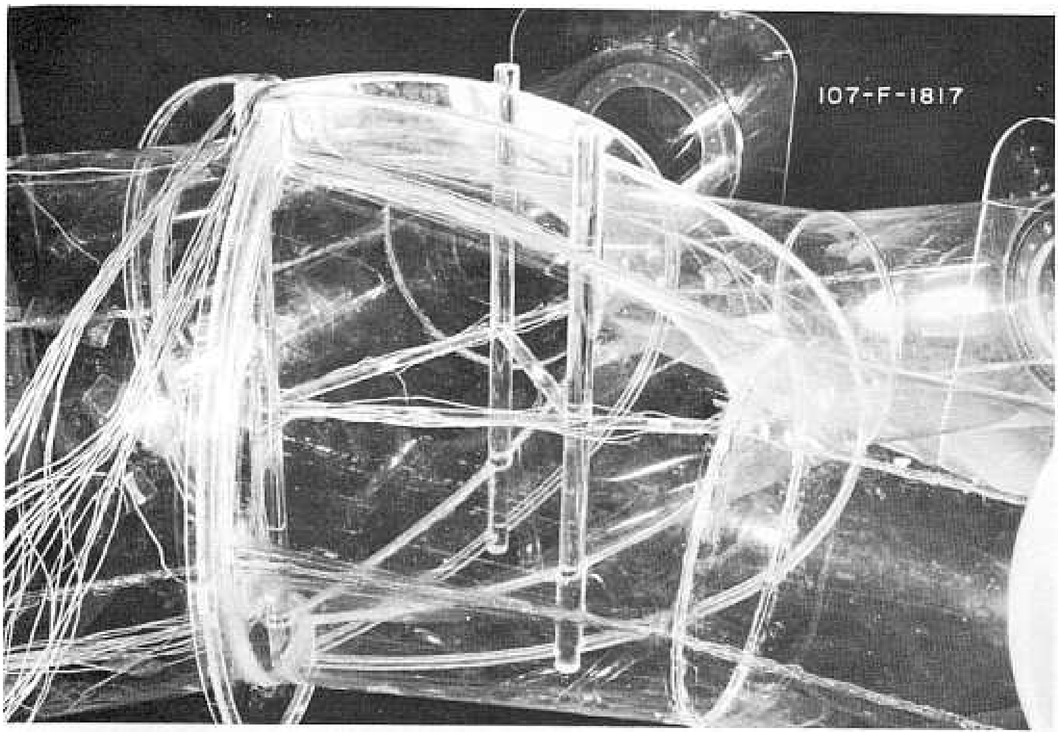

Figure 10. --Palisades Dam and Powerplant Outlet Pipe Manifold--Wye W1-
A Frame