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PHOTOELASTIC STRESS ANALYSIS OF BRIDGE PIERS

By H. B. Phillips and C. N. Zangar

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Engineering Monograph

No. 15

**PHOTOELASTIC STRESS
ANALYSIS OF BRIDGE PIERS**

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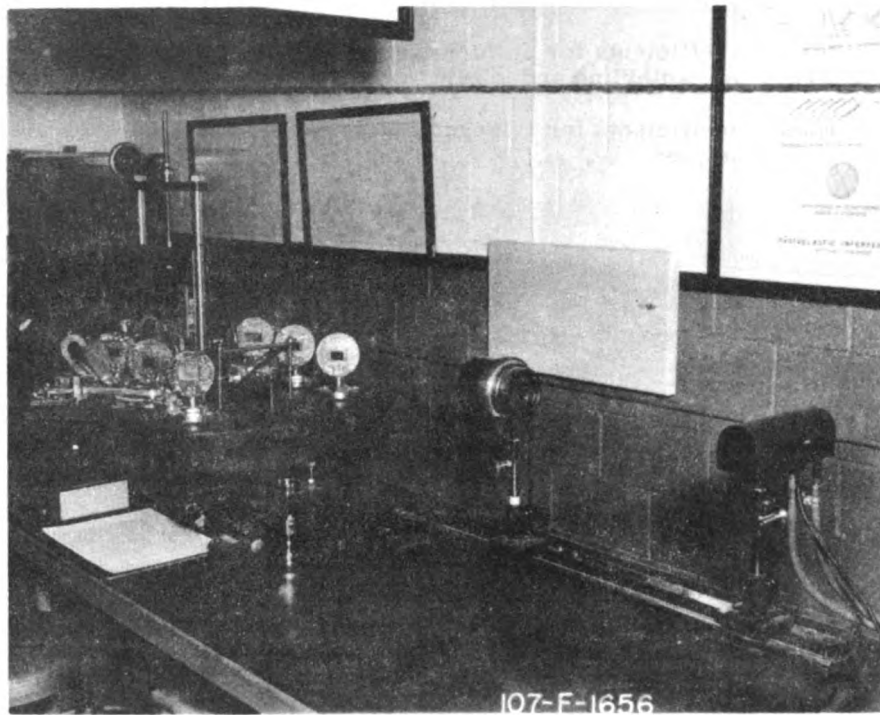


FIGURE 1 - The photoelastic interferometer.

INTRODUCTION

This monograph presents the results of a photoelastic analysis of stress distribution in bridge piers of three different width-to-height ratios, each subjected to 12 different loading conditions. The study considers the pier to have a finite width with the load acting over a definite part of the pier width. The effect of the ratio of pier width to height has also been considered in the analysis so that the bending effect present in shallow piers is evaluated.

The three ratios of bridge pier width to height studied were:

1. Width and height of pier equal
2. Width equal to twice the height
3. Width equal to four times the height

The twelve loading conditions applied to each of these pier sizes were further divided into single and double loadings.

In the single loadings three uniform loads, symmetrical about the vertical center line and differing only in the length of load appli-

cation, were used. The three lengths were 0.9, 0.7, and 0.5 times the width of the pier.

In the double loadings two variables, the spacing between centers and the length of load, were introduced. The three center line spacings were 0.4, 0.6, and 0.8 times the width of the pier. The load lengths were 0.05, 0.1, and 0.2 times the width of the pier for each load.

For each condition the principal stresses, σ_1 and σ_2 , and their direction, α , for a load of unit intensity were determined experimentally at points along certain lines traversing the pier, by use of the photoelastic interferometer. Then these were resolved to give the stresses normal to the reference lines, σ_n , the stresses tangential to the reference lines, σ_t , and the shear stresses along the reference lines, τ_{nt} .¹

Acknowledgements

H. J. Kahm, I. E. Allen, W. J. Brown, Norman S. Johnson, N. F. Forell, and H. E. Willmann assisted in the collection of the data, the computations, and the preparation of the drawings.

DETAILS OF THE STUDY

The photoelastic interferometer² (Figure 1) can be used to measure changes in the refractive index of isotropic transparent materials as they undergo stress changes. In Figure 2, a highly simplified representation, L_1 and L_4 are partially-reflecting, partially-transmitting optical flats; L_2 and L_3 are fully-reflecting optical flats. Polarized monochromatic light from the point source S is split at L_1 , travels along the inside

and outside paths, and is recombined at L_4 , where it is observed in the eyepiece. At a certain very slight angular difference between the two beams of light as they leave L_4 the eye sees in the combined beam a series of alternately bright and dark bands where the waves of the two beams first reinforce, then cancel, each other (Figure 3). These interference fringes shift with a change in stress within the model, and the change

¹Timoshenko, S., and MacCullough, G. H. Elements of Strength of Materials, D. Van Nostrand, New York, 1935, pp. 42-43.

²Only a brief description of the basic principle of operation of the interferometer is given here. For a more detailed description of the instrument used in these studies see Appendix of Engineering Monograph No. 12, Photoelastic Interferometer Determination of Stresses Around an Inspection Gallery

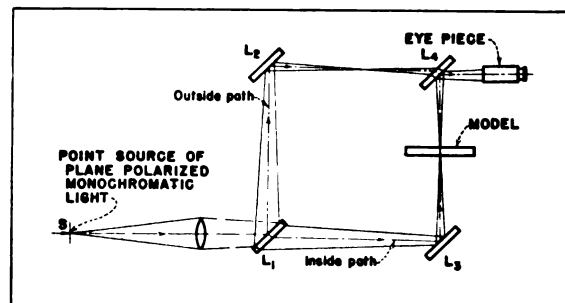


FIGURE 2 - The photoelastic interferometer, simplified diagram.

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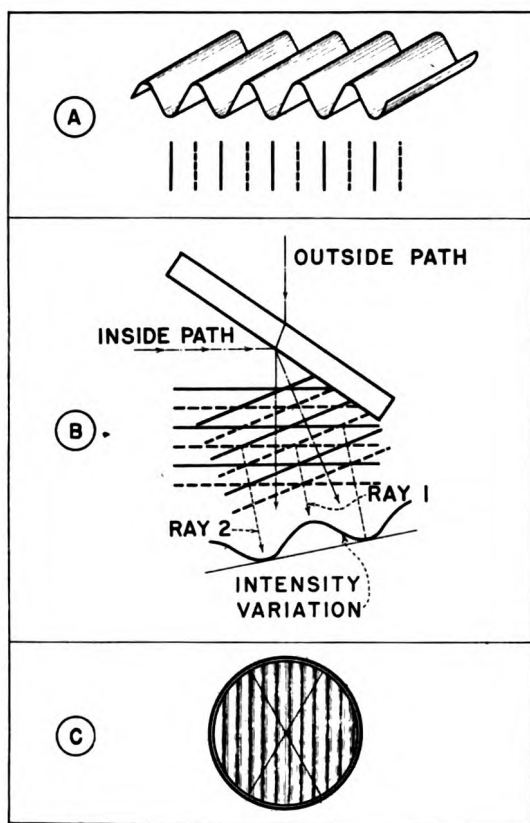


FIGURE 3 - Interference fringes. (Polarized light is represented here by alternate full and broken lines corresponding to troughs and peaks, respectively, as at A. B shows how the light from the inside and outside paths combines in different rays to give a resultant intensity variation. C shows the actual appearance of these "interference fringes" in the eyepiece. Sizes and quantities in A and B are greatly exaggerated.)

in stress is linearly related to the number of fringes that cross a fixed point in the field of the eyepiece.

Thus, with suitably elaborate apparatus, methods of calibration, and operating procedures, and with the use of the equations relating changes in refractive index to principal stresses, this basic idea may be used to determine stresses in scale models of various structures.

In this study both pier and foundation model material were Columbia Resin, CR-

39. Figure 4 is a photograph of the model, mounted in the loading frame, for a double loading condition where pier width is four times pier height. It was assumed that no bond existed between pier and foundation, and that the modulus of elasticity of the pier was equal to that of the foundation.

As often occurs in such studies, difficulty was encountered in obtaining a uniform loading over the entire loaded area, especially in applying the single uniform loading to the piers of w/h ratios two and four. In the first attempt a single solid shoe of the proper length was used. The bending of the model was of sufficient magnitude to cause a wide deviation from uniform loading. A more successful method was the use of multiple shoes on the loaded area; four shoes of the proper length gave satisfactory results. As can be seen from some of the tabulated values, however, particularly in those cases where the loaded area was large, the load applied was not entirely uniform. In most cases the non-uniformity was small and such non-uniformity might more closely approach actual conditions.

Checks made on the results of the study by use of Boussinesq's equations and comparison with parts of similar experimental studies showed consistent values between studies.³ Equilibrium checks were made across horizontal lines in all cases and were found to be satisfactory.

³ Brahtz, J. H. A., "The Stress Function and Photoelasticity Applied to Dams," Transactions ASCE. Vol. 101, 1936, pp. 1251-1253

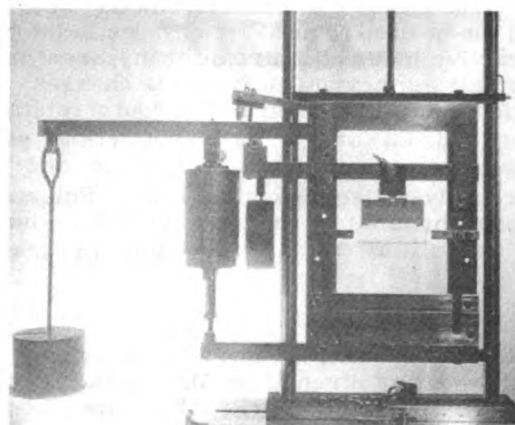


FIGURE 4 - Loading frame and model.

APPLICATION OF THE DATA

Results are summarized in the accompanying figures 5 through 17. Figure 5 gives dimensions of piers and locations of reference lines. Each of the remaining figures gives stress coefficients for all three ratios of pier width to pier height, loading condition used, and the locations of the reference lines along which the stresses are given. The stress coefficients are expressed in terms of an intensity of loading p where p represents the average uniform pressure applied over the

loaded area. All dimensions are given in terms of the pier width, w .

The appendix gives additional data for a pier with width and height equal for three different loadings. These were a uniform vertical load applied over 0.25 width of pier (Figure 18); a uniform vertical load applied over 0.125 width of pier (Figure 19); and a triangular vertical load applied over 0.25 width of pier (Figure 20).

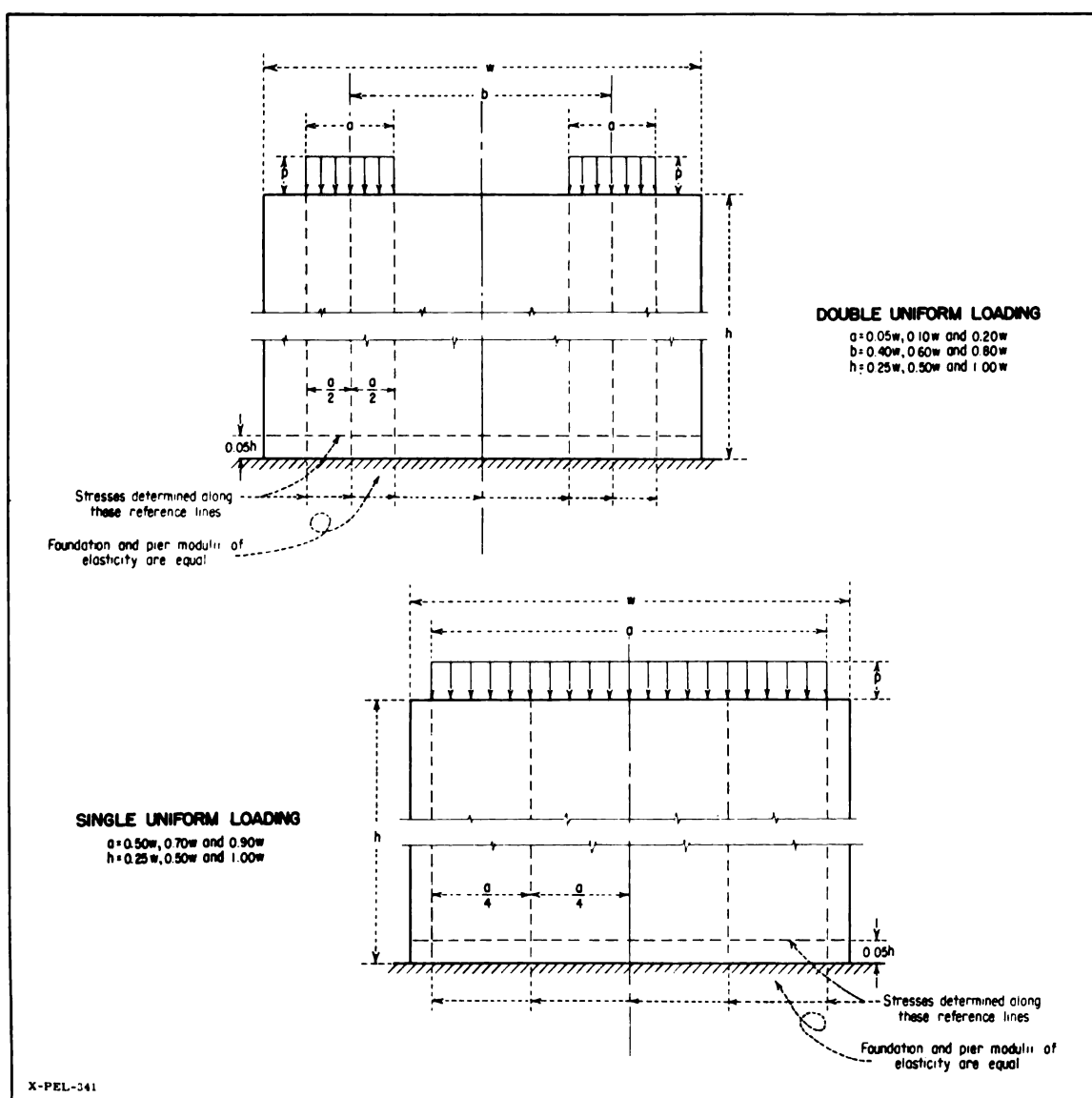
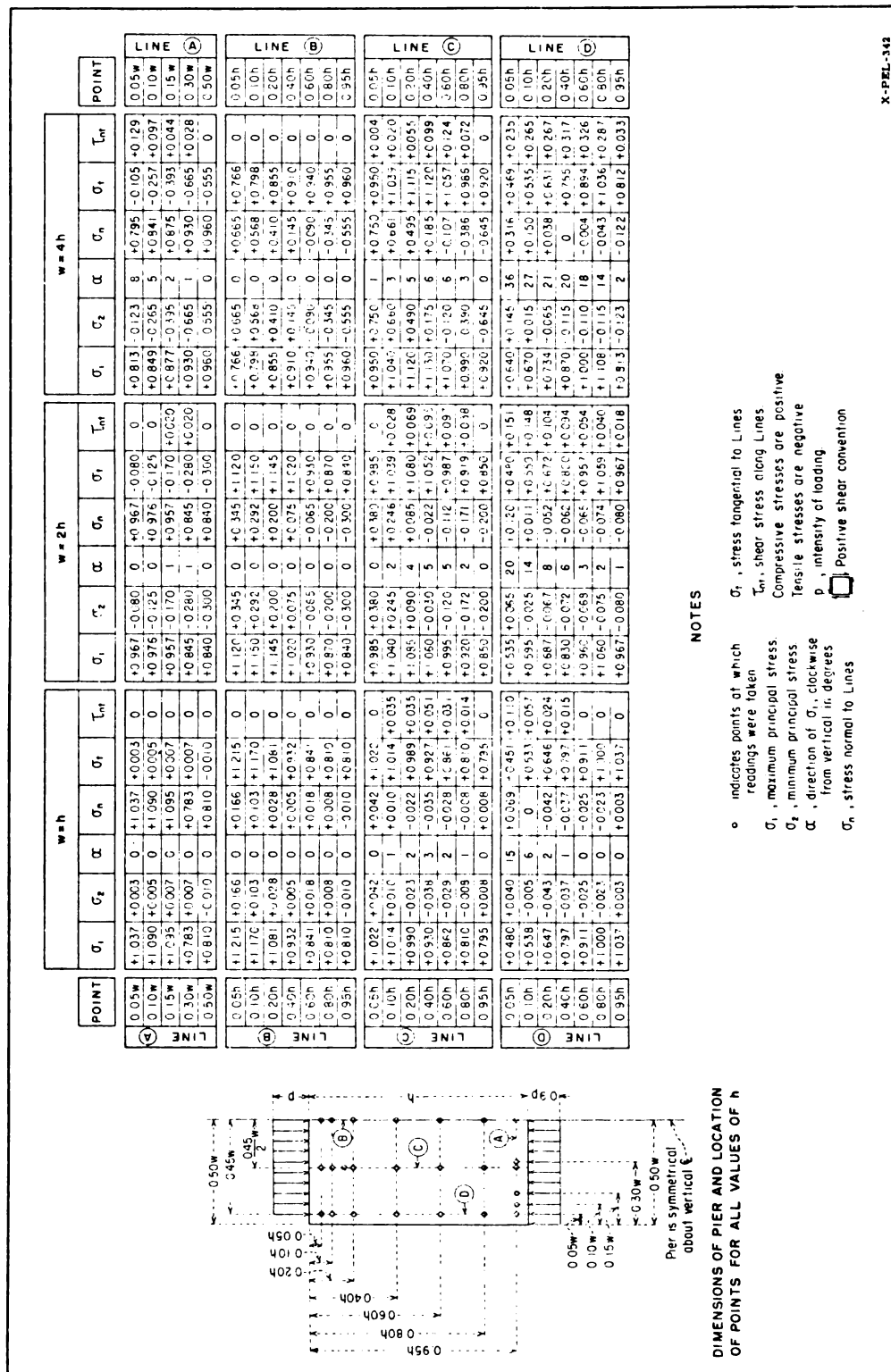


FIGURE 5 - Dimensions of piers and locations of reference lines.



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FIGURE 6 - Stress coefficients for uniform vertical load symmetrical about vertical centerline and applied over 0.90 width of pier.

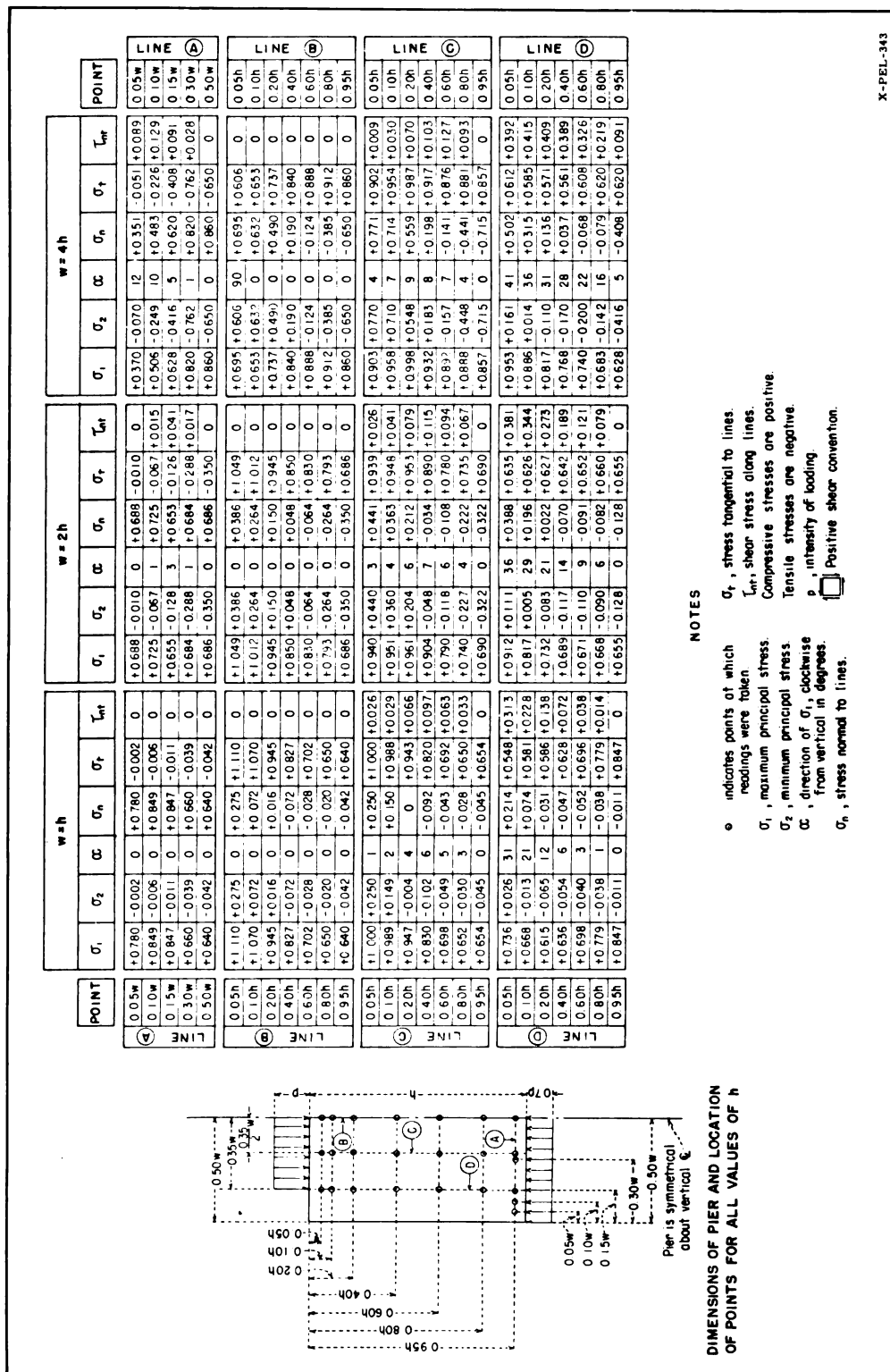


FIGURE 7 - Stress coefficients for uniform vertical load symmetrical about vertical centerline and applied over 0.70 width of pier.

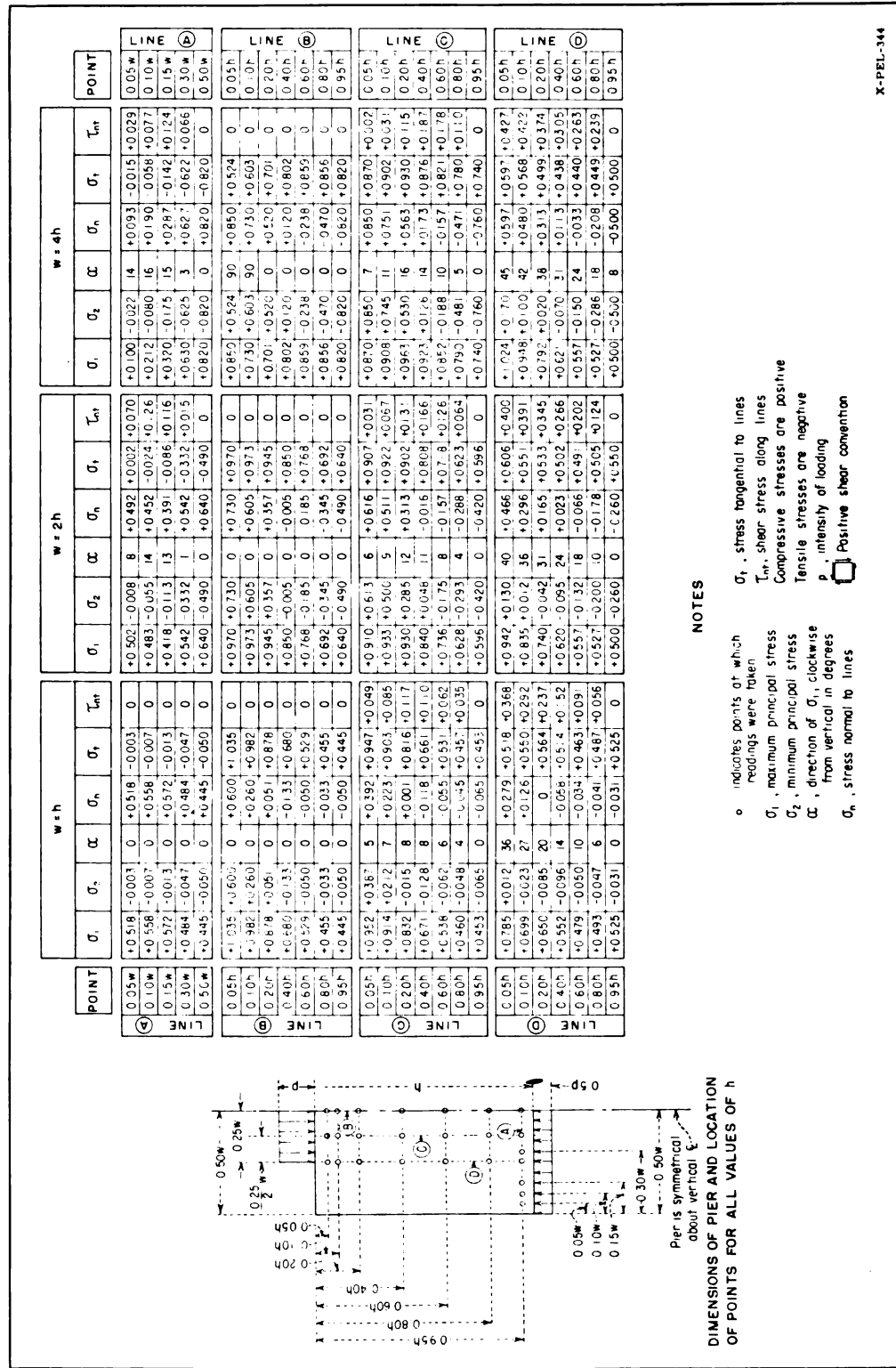
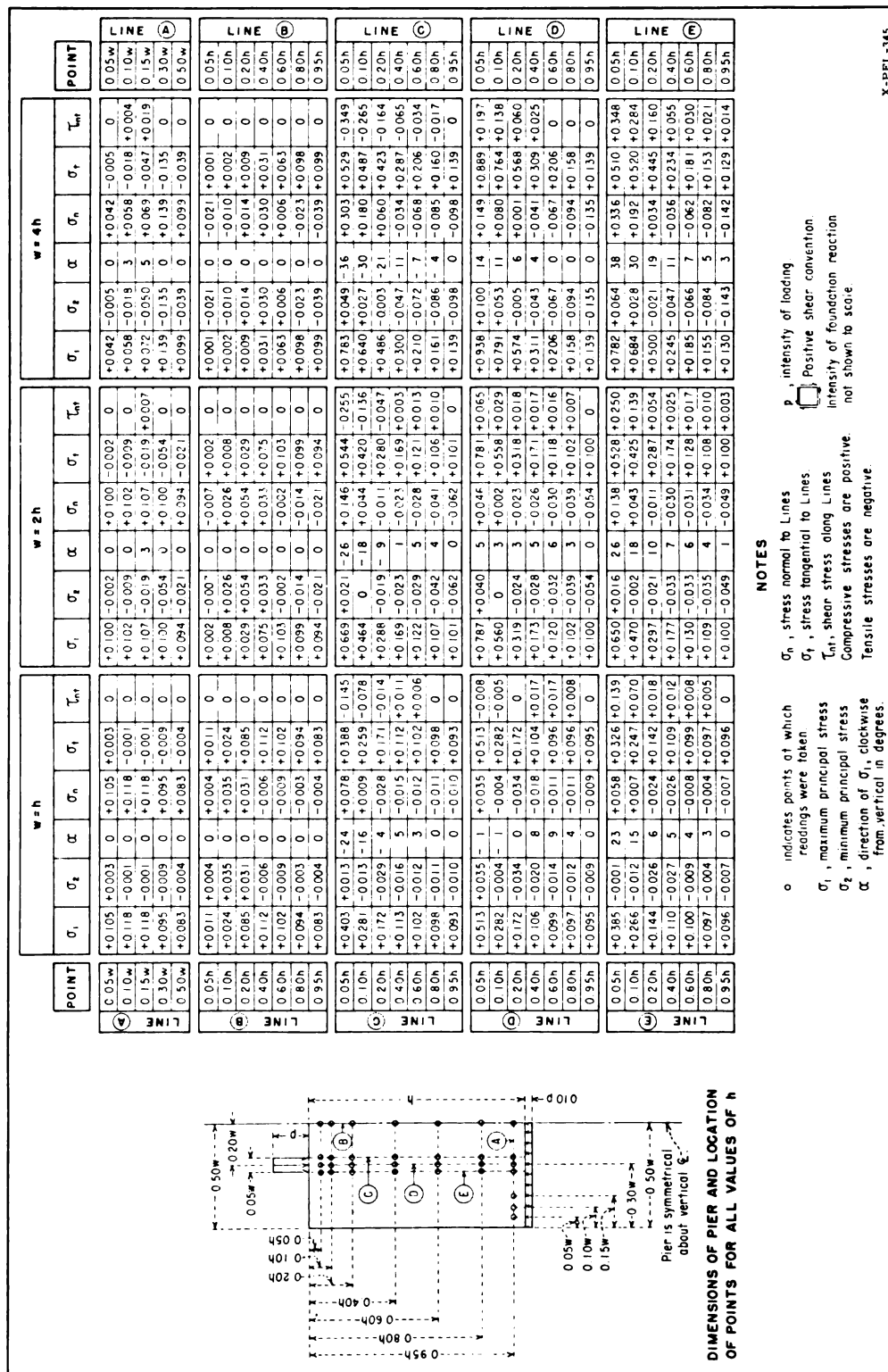
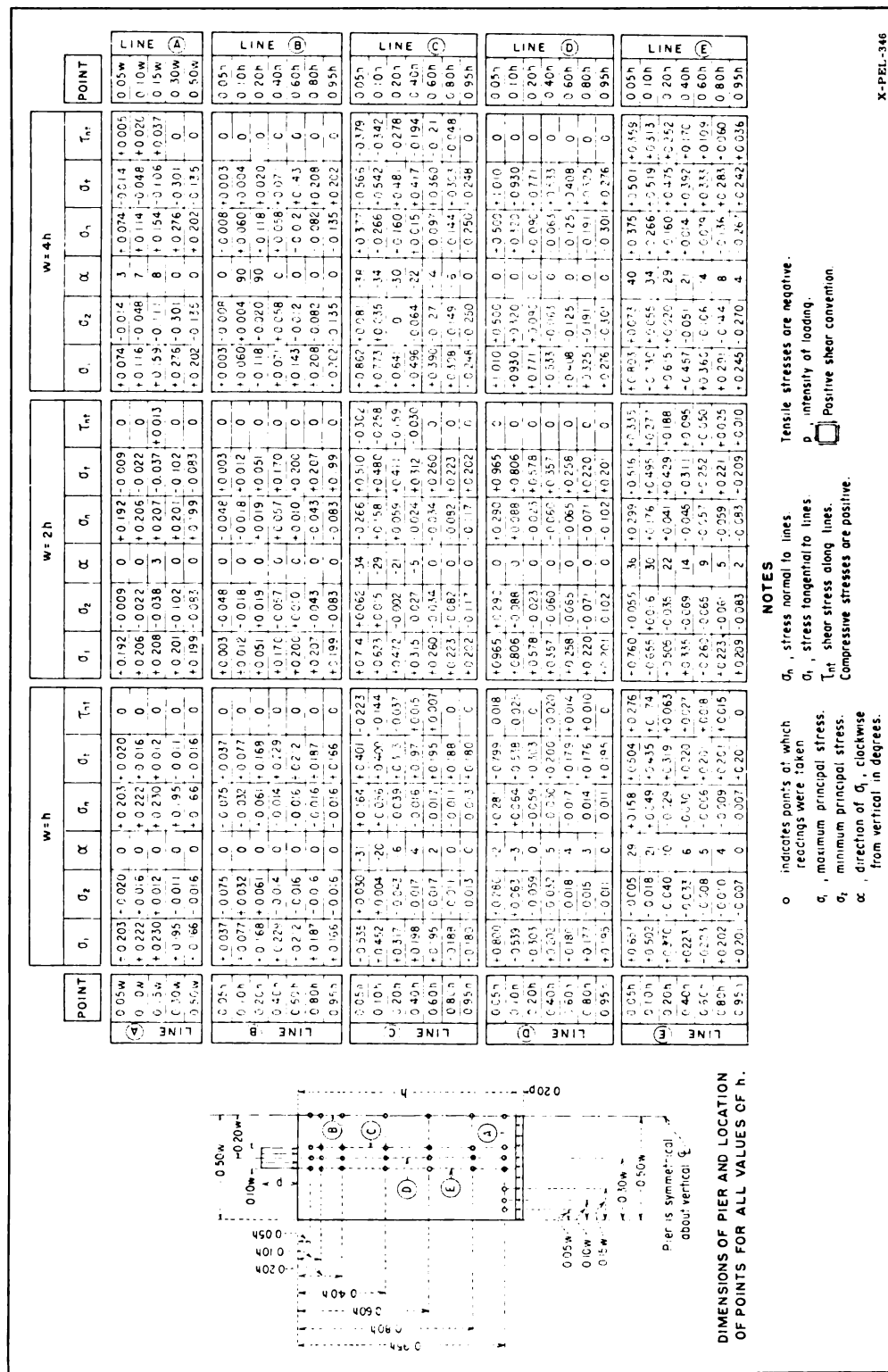


FIGURE 8 - Stress coefficients for uniform vertical load symmetrical about vertical centerline and applied over 0.50 width of pier.





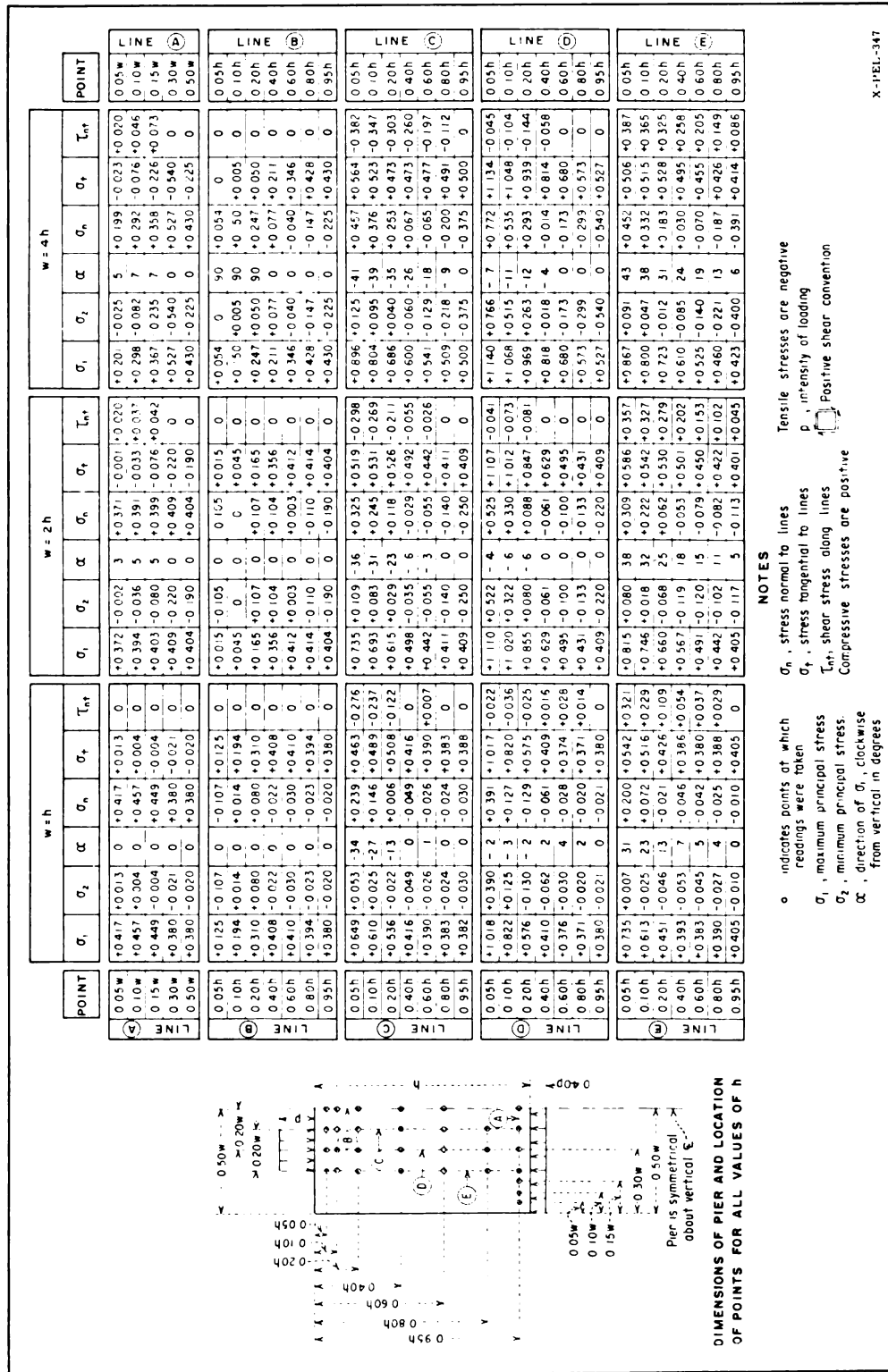


FIGURE 11 - Stress coefficients for two uniform vertical loads symmetrical about vertical centerline of pier with each load 0.20 width of pier and load centerlines 0.40 width of pier apart.

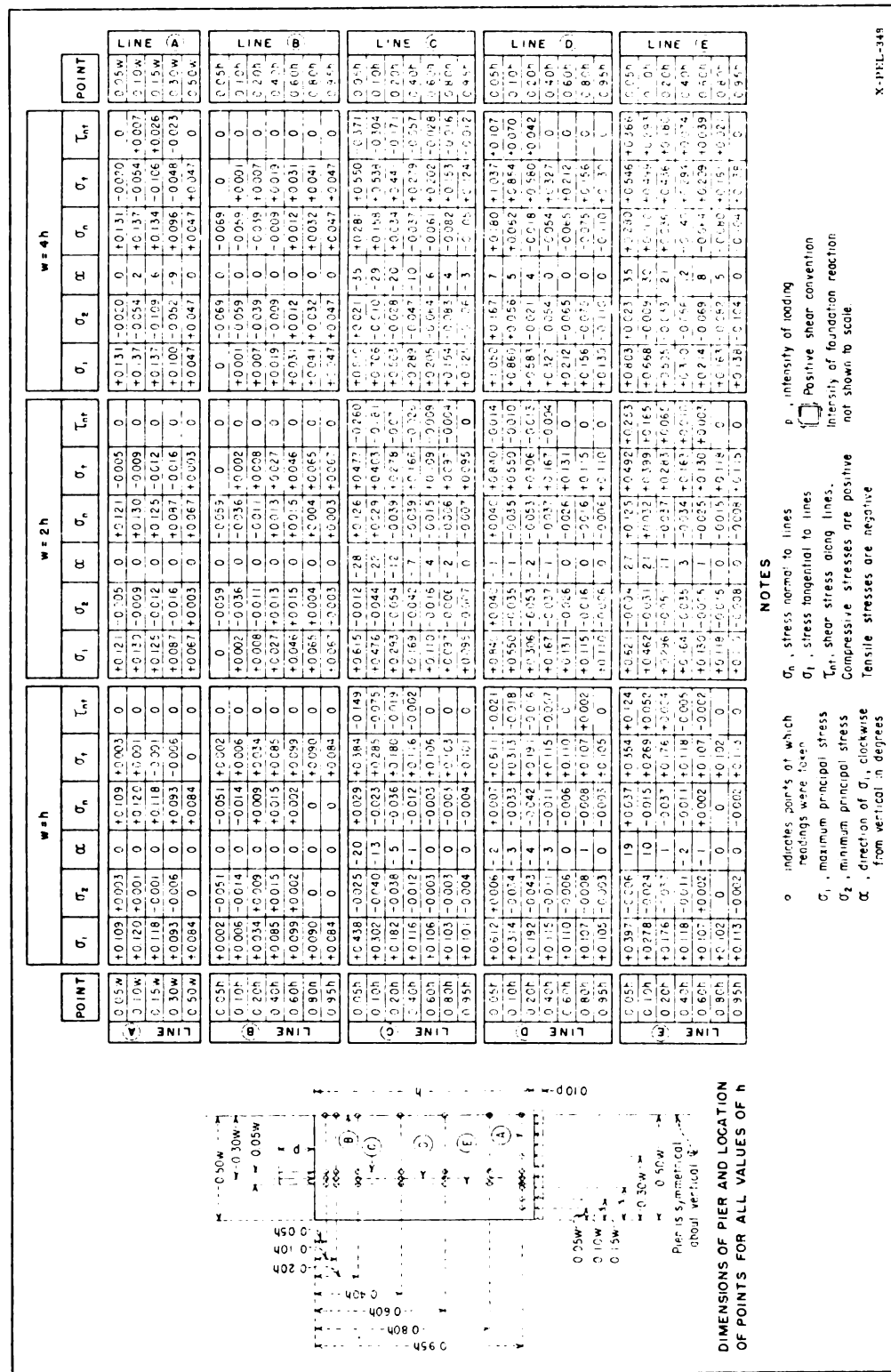


FIGURE 12 - Stress coefficients for two uniform vertical loads symmetrical about vertical centerline of pier with each load 0.05 width of pier and load centerlines 0.60 width of pier apart.

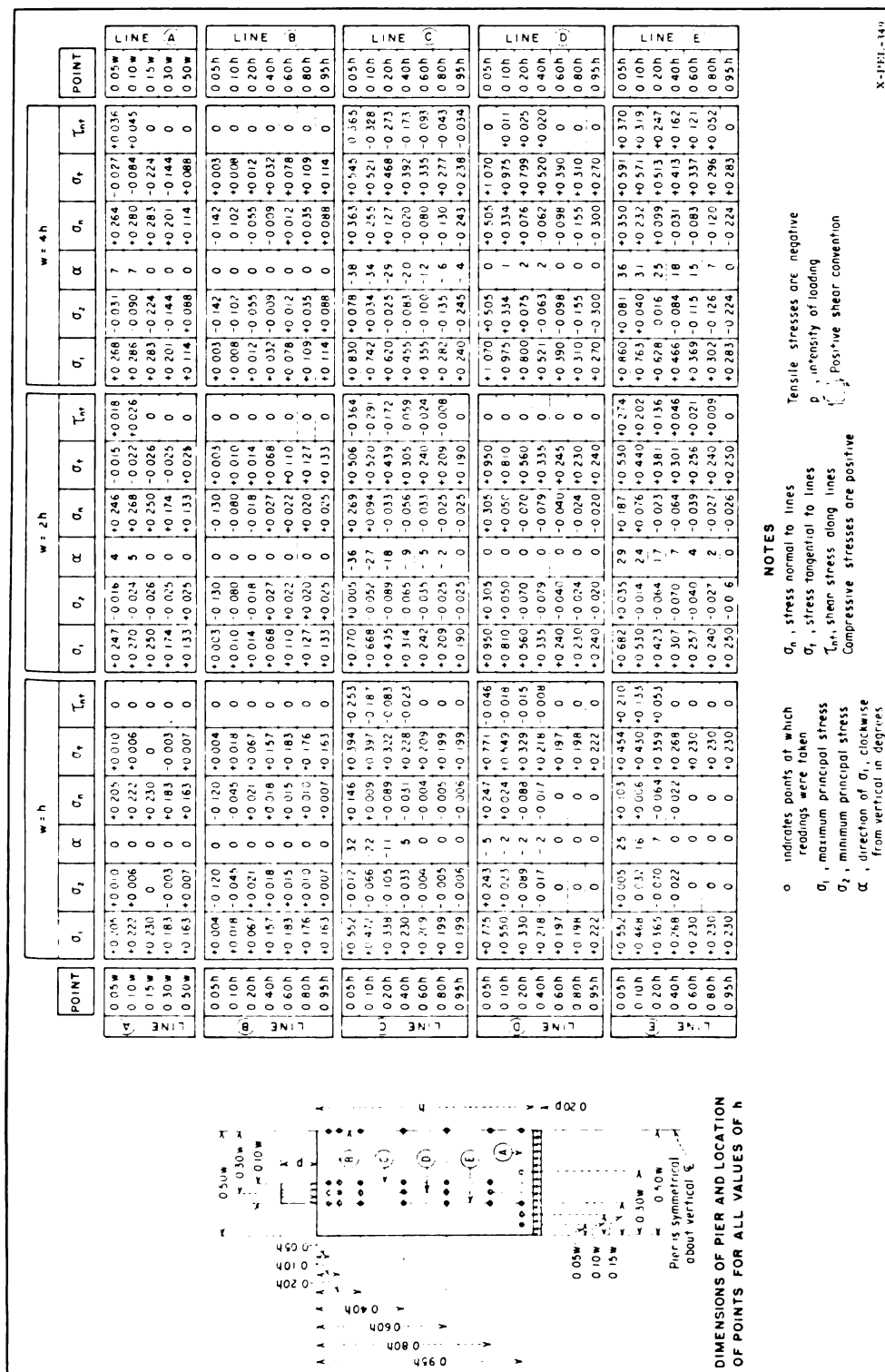


FIGURE 13 - Stress coefficients for two uniform vertical loads symmetrical about vertical centerline of pier with each load 0.10 width of pier and load centerlines 0.60 width of pier apart.

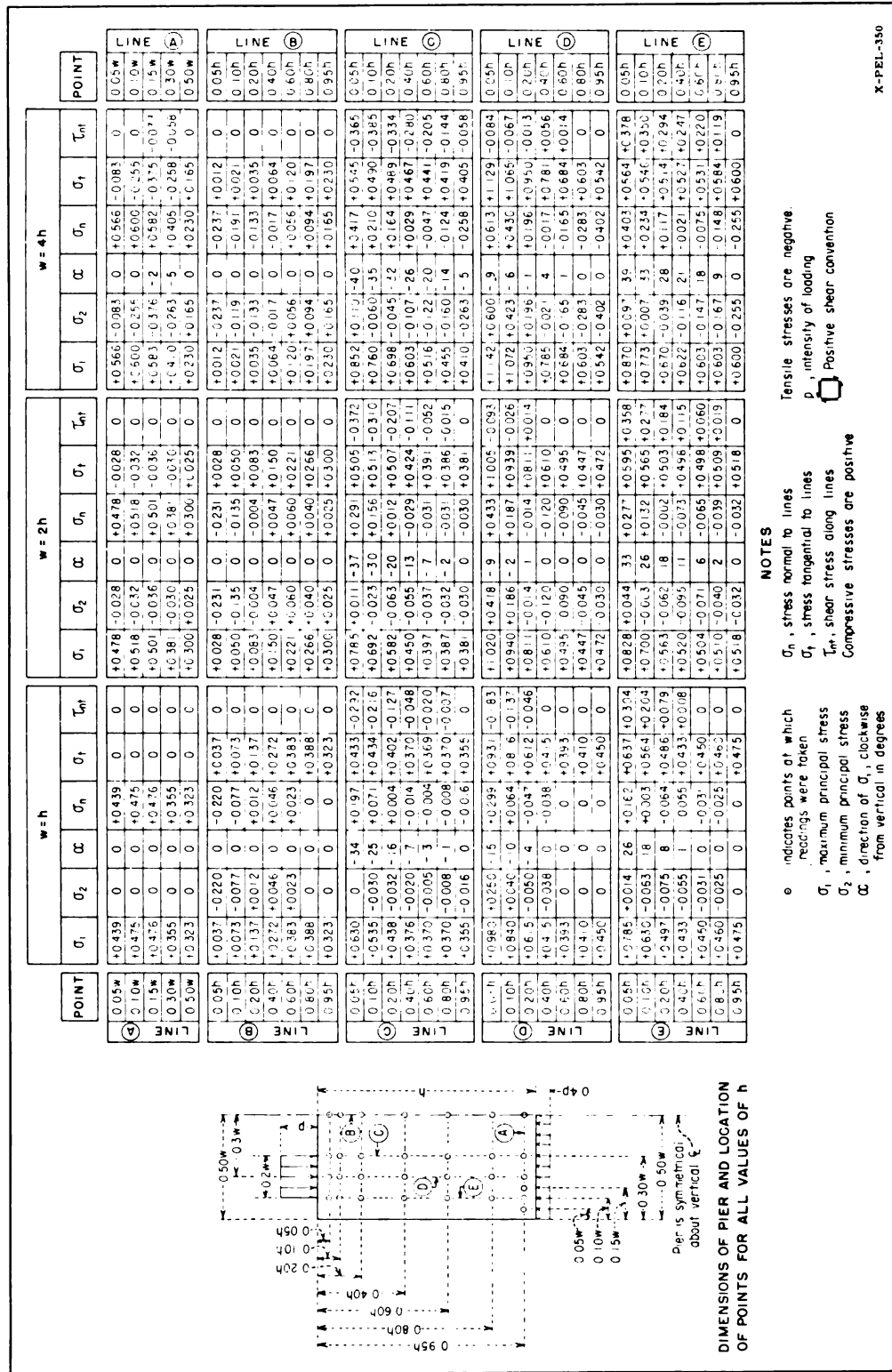


FIGURE 14 - Stress coefficients for two uniform vertical loads symmetrical about vertical centerline of pier with each load 0.20 width of pier and load centerlines 0.60 width of pier apart.

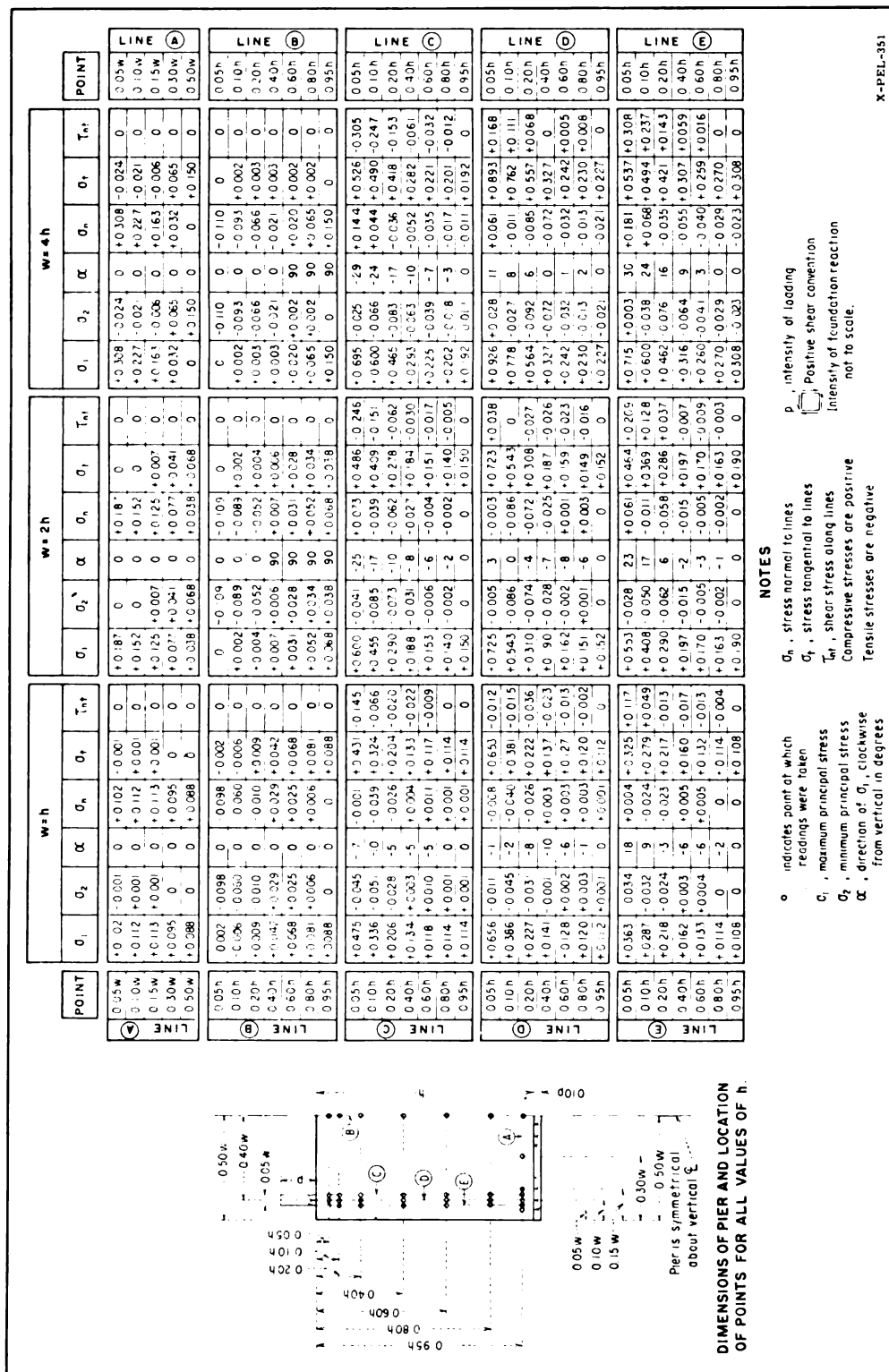


FIGURE 15 - Stress coefficients for two uniform vertical loads symmetrical about vertical centerline of pier with each load 0.05 width of pier and load centerlines 0.80 width of pier apart.

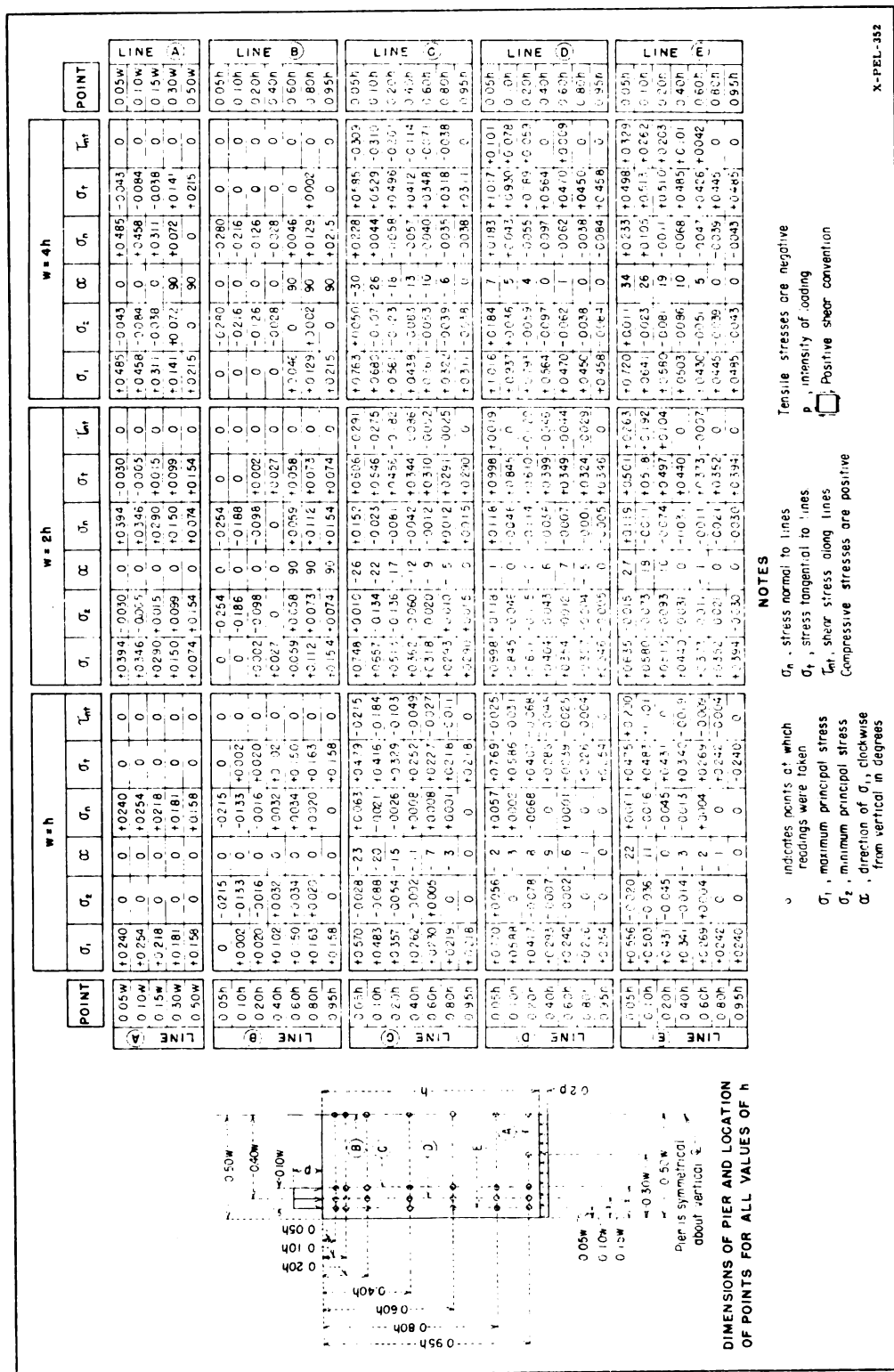
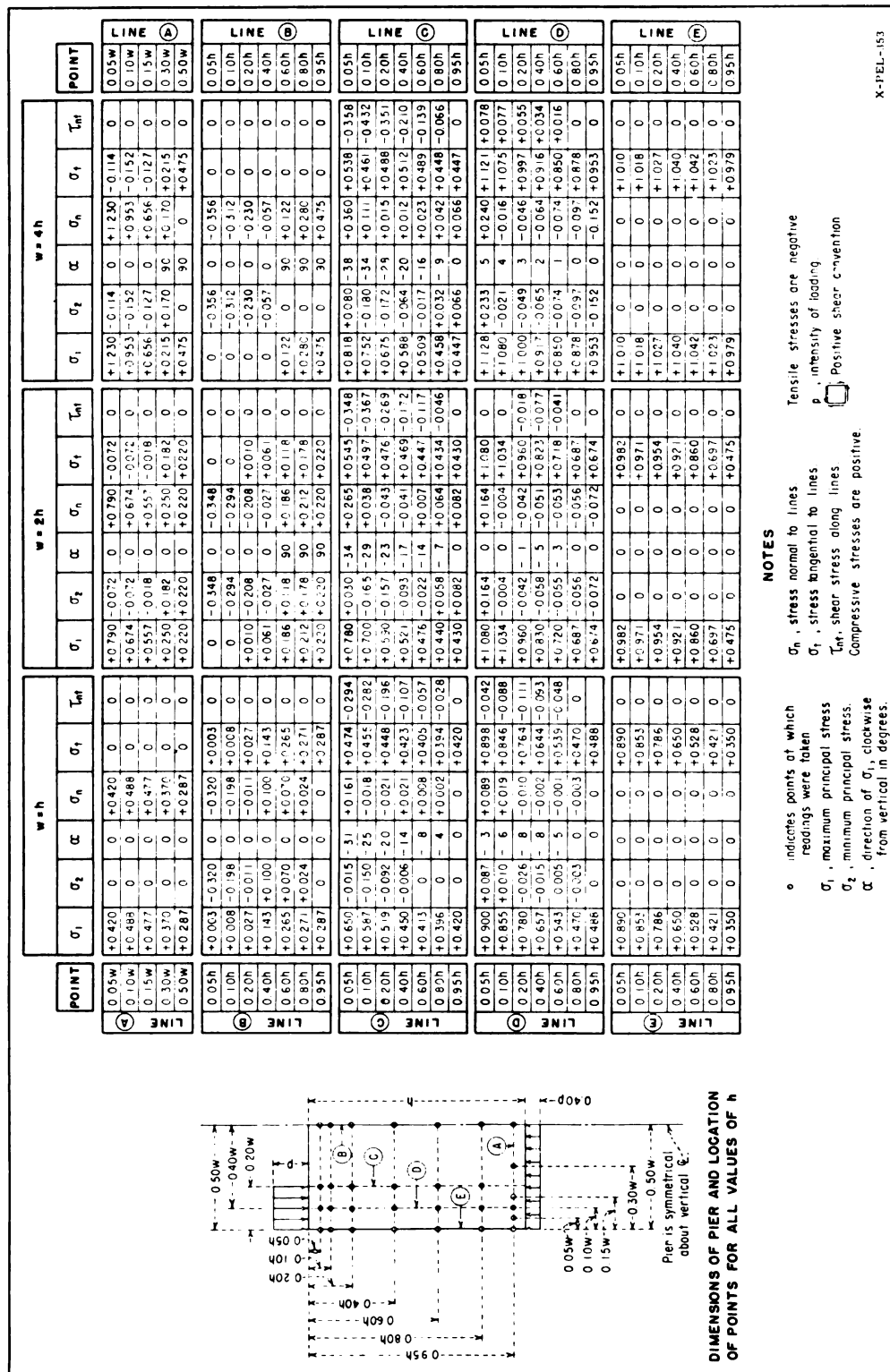
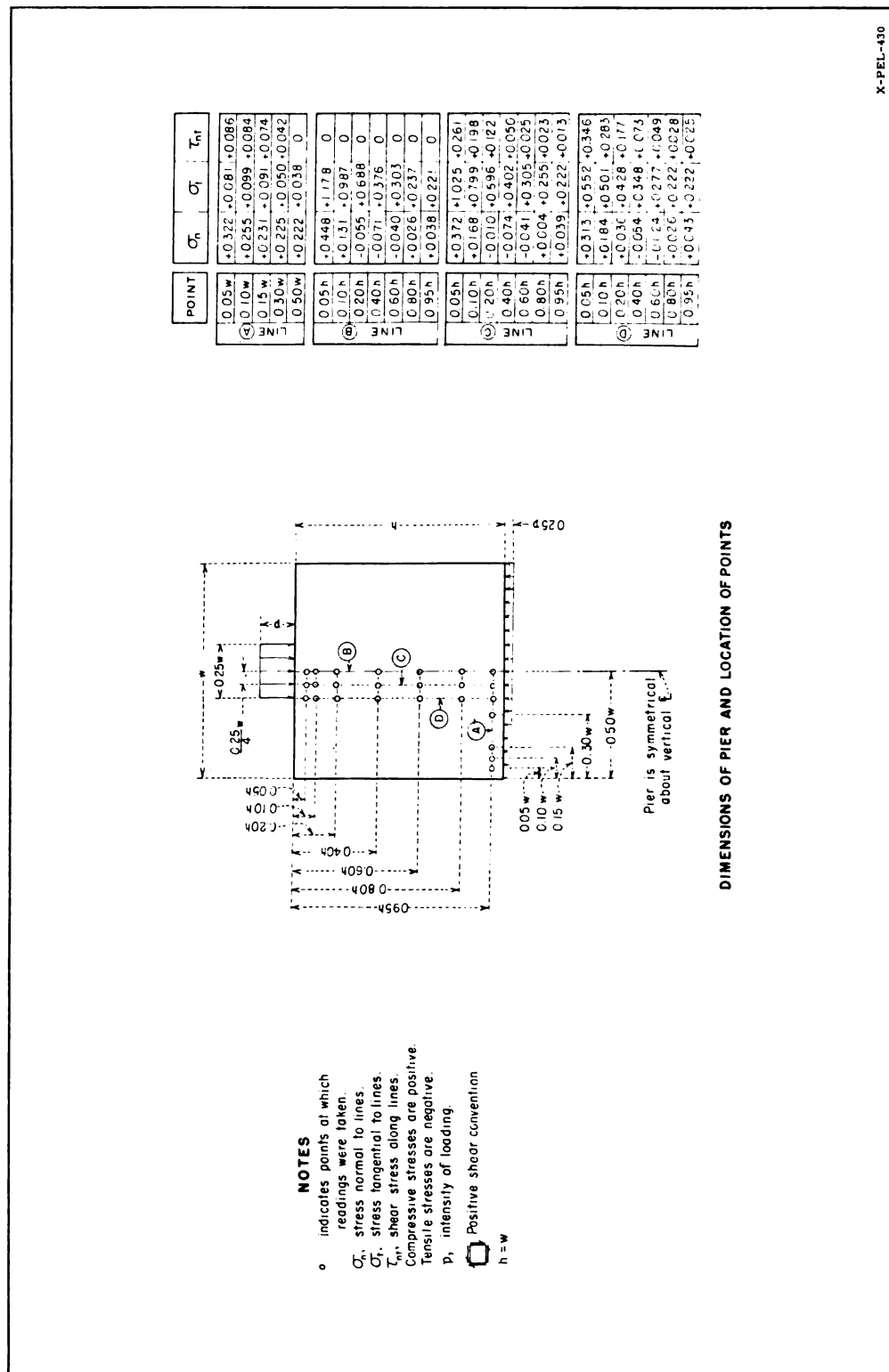


FIGURE 16 - Stress coefficients for two uniform vertical loads symmetrical about vertical centerline of pier with each load 0.10 width of pier and load centerlines 0.80 width of pier apart.



X-PEL-153

APPENDIX



X-PEL-430

FIGURE 18 - Stress coefficients for uniform vertical load symmetrical about vertical centerline and applied over 0.25 width of pier.

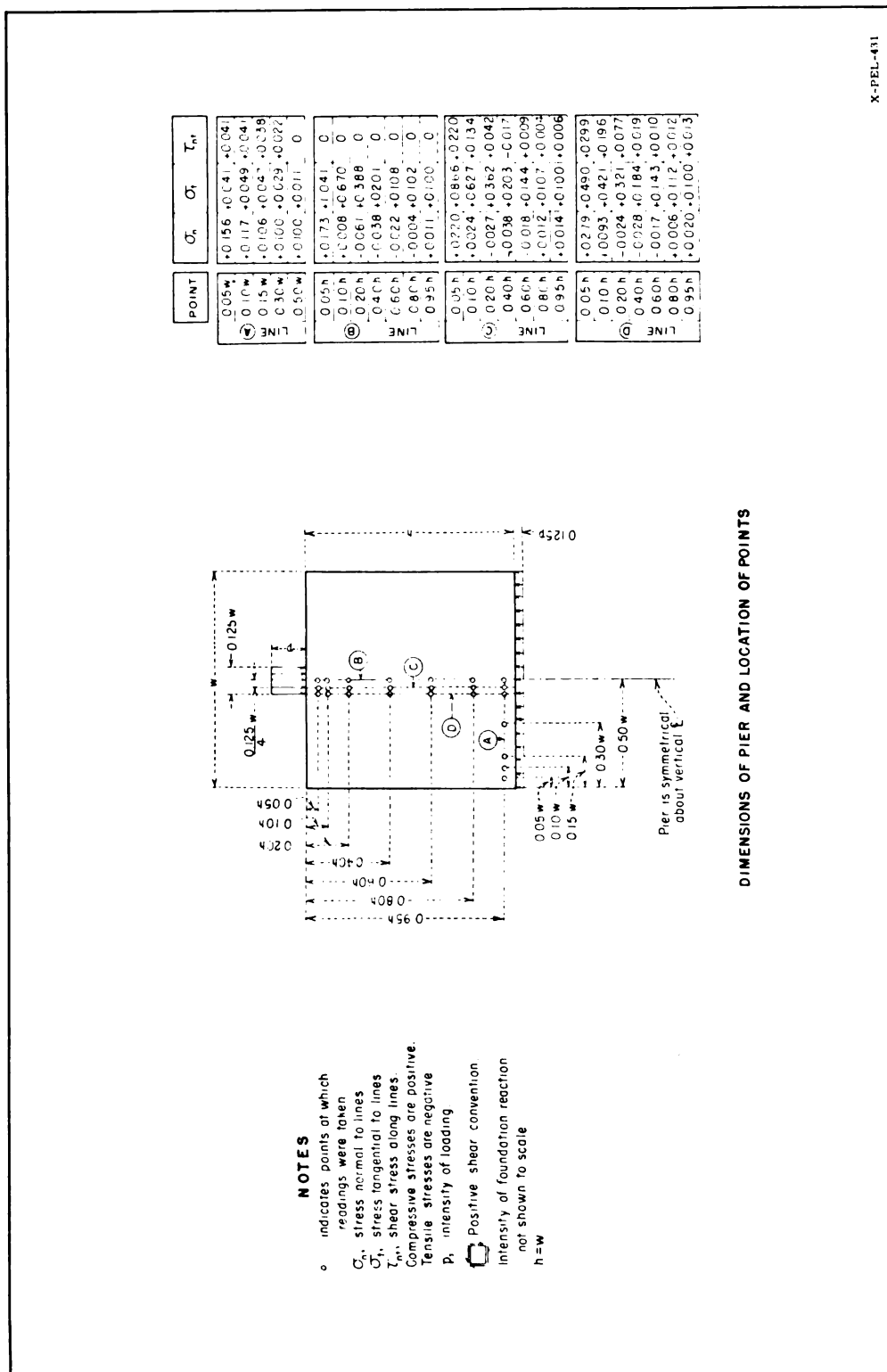


FIGURE 19 - Stress coefficients for uniform vertical load symmetrical about vertical centerline and applied over 0.125 width of pier.

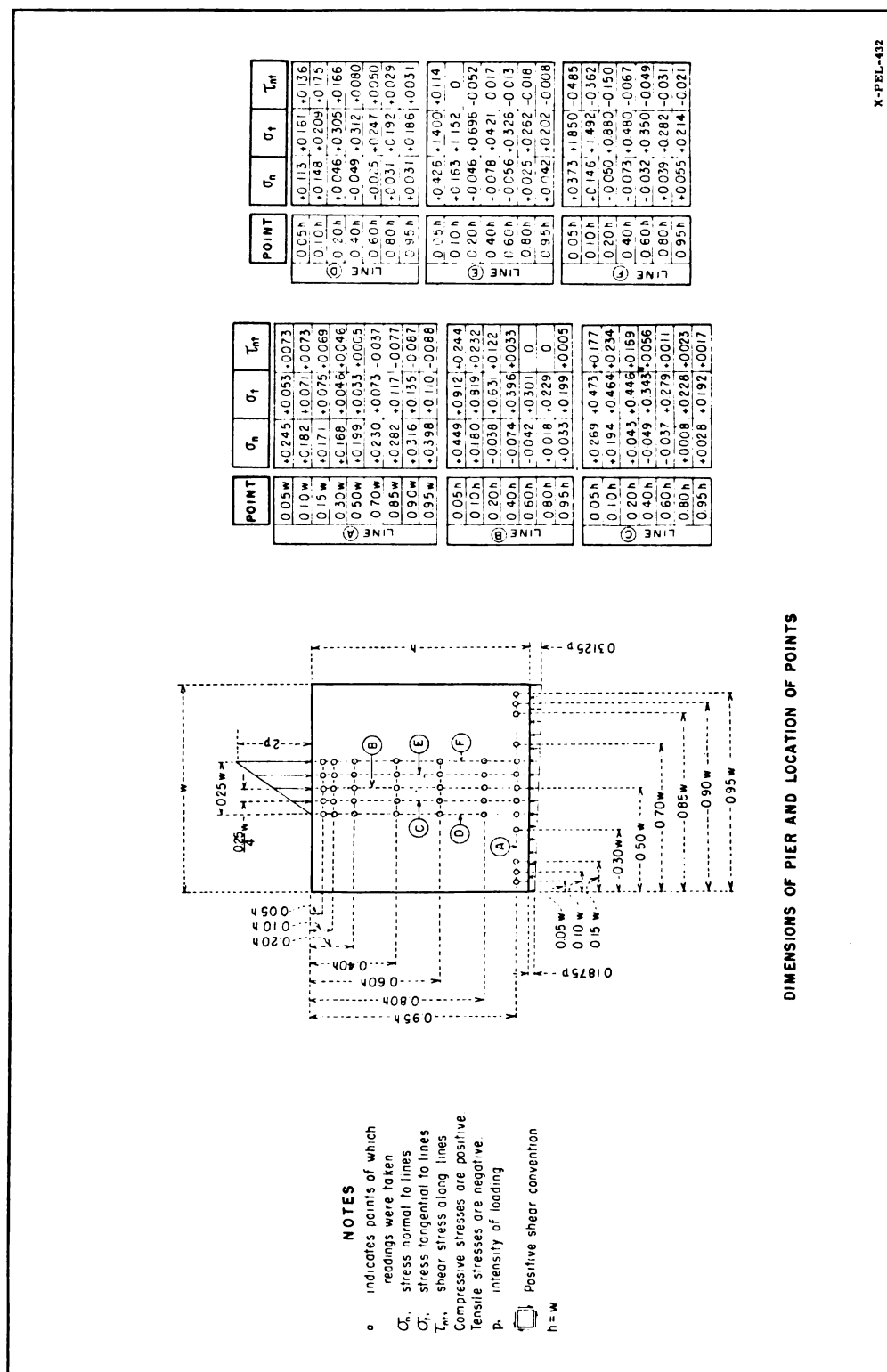


FIGURE 20 - Stress coefficients for triangular vertical load applied over 0.25 width of pier.

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