ENGINEERING MONOGRAPH No. 13

United States Department of the Interior
BUREAU OF RECLAMATION

ESTIMATING FOUNDATION SETTLEMENT BY ONE-DIMENSIONAL CONSOLIDATION TESTS

Denver, Colorado
March 1953

Engineering Monograph

No. 13

ESTIMATING FOUNDATION SETTLEMENT BY ONE-DIMENSIONAL CONSOLIDATION TESTS

by Harold J. Gibbs
Engineering Laboratories Branch
Design and Construction Division

Technical Information Office
Denver Federal Center
Denver, Colorado

ENGINEERING MONOGRAPHS are published in limited editions for the technical staff of the Bureau of Reclamation and interested technical circles in government and private agencies. Their purpose is to record developments, innovations, and progress in the engineering and scientific techniques and practices that are employed in the planning, design, construction, and operation of Reclamation structures and equipment. Copies may be obtained from the Bureau of Reclamation, Denver Federal Center, Denver, Colorado, and Washington, D. C.

CONTENTS

LIST OF FIGURES

LIST OF TABLES

INTRODUCTION

This monograph demonstrates the application of one-dimensional consolidation test data to a foundation settlement analysis. Soil samples are tested in the laboratory to determine the settlement characteristics of the soil under load. These characteristics are used to estimate the amount of settlement of a structure which would result from the consolidation of its earth foundation because of the structure load. The test is also used to determine the settlements that will occur within dams and earth embankments.

The consolidation characteristics of a soil mass are influenced by numerous factors. Some of these are size and shape of the soil particles, moisture content, permeability, initial density, and physical and chemical properties of the soil. Because these factors are so numerous it is usually not possible to describe the consolidation characteristics with a high degree of confidence by means of judgment and simple index values. For structures that are critical regarding settlement and those whose cost would justify such tests, it is advisable to analyze settlement from consolidation tests on the actual foundation material.

The one-dimensional consolidation testing equipment and procedures used in the Bureau of Reclamation laboratories are similar to those developed by Casagrande.[1] The testing procedures now conform quite consistently with the procedures presented in popular soil mechanics publications.[2,3,4] As conducted by the Bureau the standard test[5] provides four main items of information:

1. Magnitude of consolidation for various loads

2. Rate of consolidation

3. Influence of saturation on consolidation

4 Permeability of the material while under load.

In addition to giving information on these items, the testing equipment has been applied to such specialized problems as soil-expansion studies and the estimating of pore-pressure development. The main purpose of the test and the reason for its development, however, are to permit rational estimates of structure settlement through the determination of consolidation characteristics.

The discussions of consolidation and settlement in this monograph include the description of the standard data obtained by the Bureau's method of test, and general applications of the results to settlement problems. Much of the information has been obtained from a research study of many publications dealing with consolidation problems in the design of various structures; and throughout this paper footnote references are given for the purpose of further study in this subject by the reader when desired. These references are to publications issued prior to August 1951, when the manuscript for this publication was completed.

The reader should bear in mind that, while this monograph is principally concerned with one-dimensional test data, the possibility of shear failure must not be left unobserved. Thus in the design of any foundation it is equally important that (1) the bearing capacity or criterion of shear failure and (2) the settlement be studied.

---

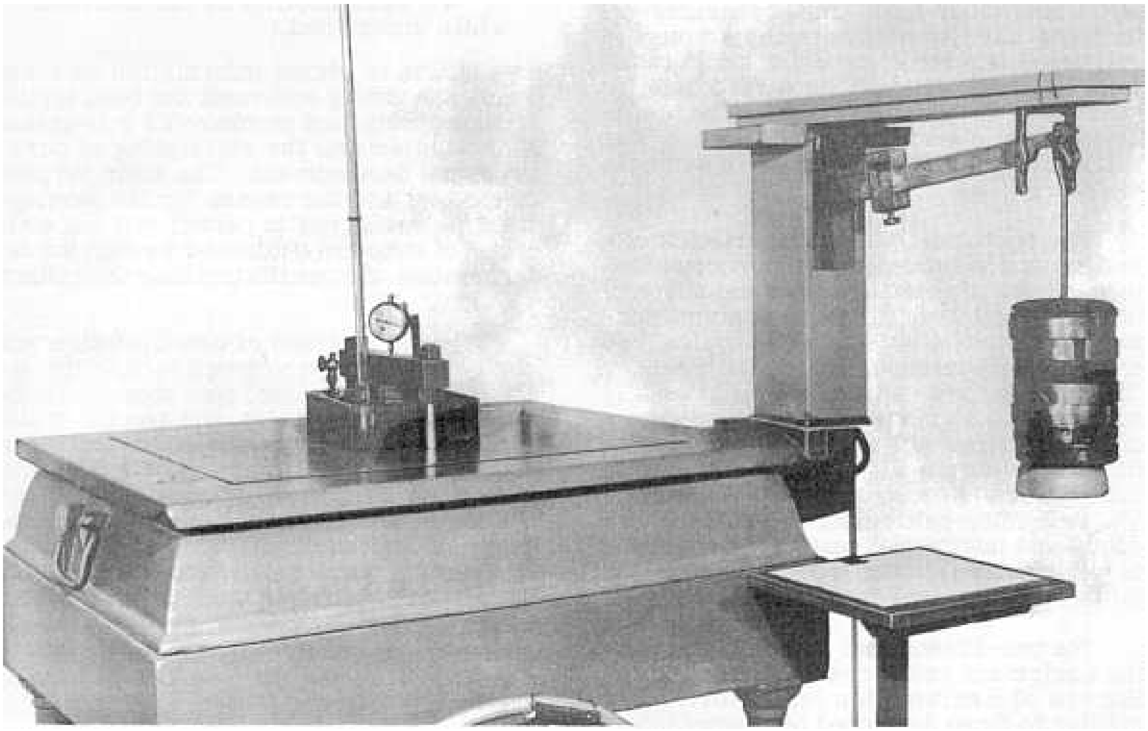
[1] Casagrande, A., "The Structure of Clay and Its Importance in Foundation Engineering," Journal of Boston Society of Civil Engineers, Volume XIX, 1932, pp. 168-209. (Reprinted in Contributions to Soil Mechanics, 1925-1940, Boston Society of Civil Engineers 1940.)

[2] Casagrande, A., and Fadum, R. E., Notes on Soil Testing for Engineering Purposes, Graduate School of Engineering, Harvard University, 1940, pp. 37-49

[3] Taylor, D. W., Fundamentals of Soil Mechanics, John Wiley & Sons, New York, 1948, pp. 212-215.

[4] Lambe, T. W., Soil Testing for Engineers, John Wiley & Sons, New York, 1951, pp. 74-87.

[5] "One-dimensional Consolidation Test Designation E-13," Earth Manual, Bureau of Reclamation, Denver, 1951.



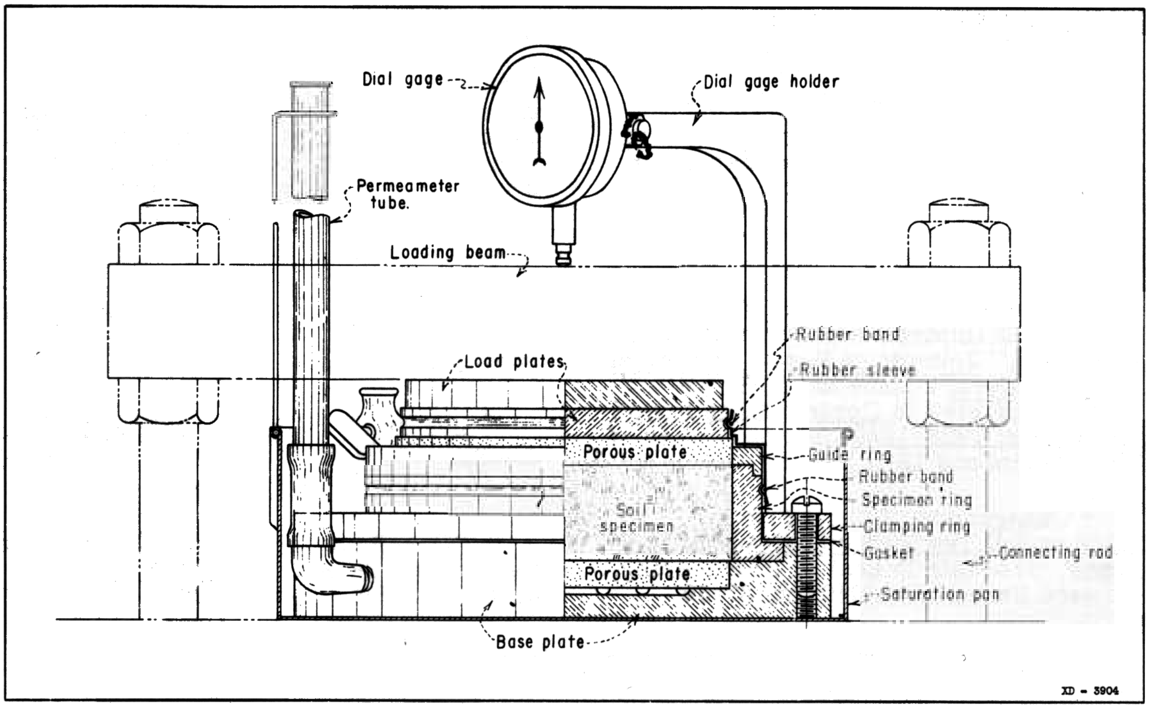


FIGURE 1 - The one-dimensional consolidometer.

DESCRIPTION OF EQUIPMENT, PROCEDURE, AND DATA

Equipment and Procedure

A photograph and an elevation drawing of the standard consolidation test equipment (one-dimensional consolidometer) are shown in Figure 1. The soil specimen is confined laterally by a rigid ring 4-1/4 inches inside diameter by 1-1/4 inches in depth, and is loaded and drained in the vertical direction. Porous plates at the top and bottom allow moisture and air movements into or out of the specimen. The top porous plate is free to move downward when a load is applied, and the amount of settlement of the specimen is read on a dial gage graduated to 1/10,000 of an inch.

The load is applied in a series of four or more increments, usually 12.5, 25, 50, and 100 percent of the maximum load. Increments of 1.5, 3, 6, 12.5, 25, 50, 100, and 200 percent are recommended when a greater number of increments are desired. The intensities of load to be used depend on the weight of the structure and the overburden pressures that occur in the material, and should be of such values as to include the maximum anticipated pressure on the foundation. Loading the test specimen is performed expeditiously and as accurately as possible to secure readings at such early time intervals as 4, 10, and 20 seconds. The rate of consolidation is obtained by observing the amount of movement at frequent time intervals until consolidation is complete. The specimen is allowed to consolidate fully under each increment of load so that a final magnitude of consolidation may be observed. (This generally requires from 5 to 24 hours.)

A permeameter tube, attached to the base of the container and leading directly into the bottom porous plate, is provided to saturate the specimen and measure its permeability. The head of water in the tube provides the pressure that causes flow through the soil, and the amount of water flowing through the specimen in any given time interval is measured by the drop in head in the tube. From these data the coefficient of permeability of the soil may be computed for the density or void ratio condition at the time of the test.

Information Obtained From the Test

The general method of plotting test results is shown in Figures 2 and 3. The curves in these figures are plotted on the basis of consolidation in percent of initial volume. They show accurately the consolidation of the test specimen, and give a general indication of the magnitude and rate of settlement which may be expected in the foundation material represented by the specimen.

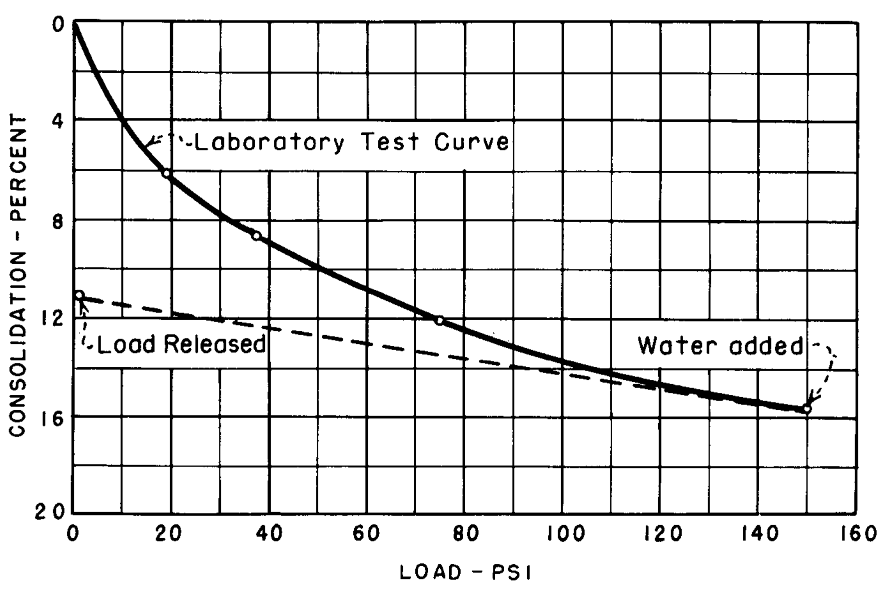
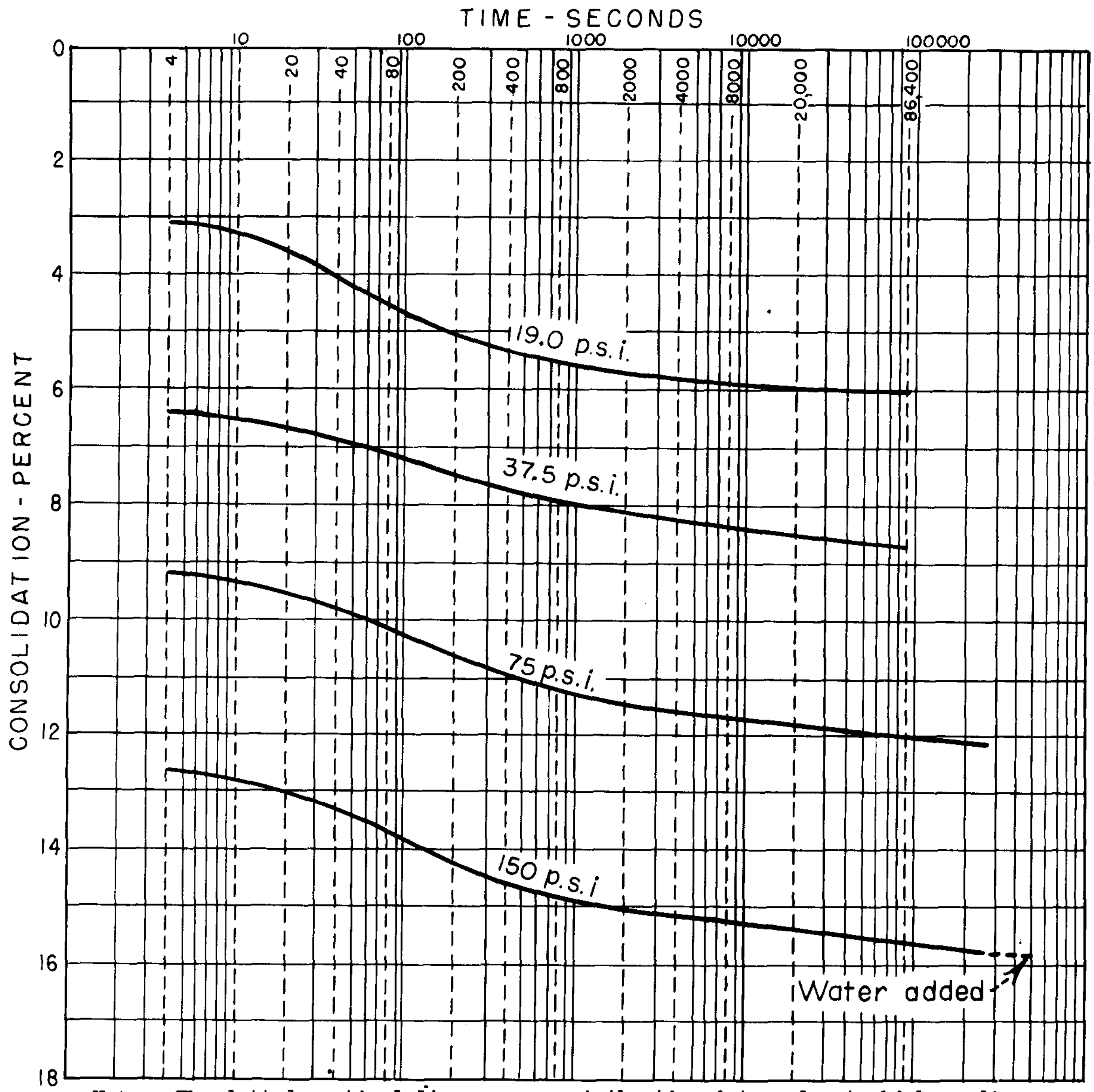


FIGURE 2 - Load-consolidation test curve for a moist clay. Addition of water after application of final load does not affect consolidation.



Note: The dotted vertical lines represent the time intervals at which readings are usually made for standard tests.

FIGURE 3 - Time--Consolidation test data for each increment of load application.

These curves may show several other characteristics of soil volume change. A sudden downward bend may indicate a breakdown of soil structure at a particular loading, whereas normally the shape of the consolidation curve is concave upward. Figure 2 (load-consolidation curve for a moist clay) shows that the addition of water after application of the final load does not affect consolidation. Yet some soils, such as those tested when they are initially quite dry, may show effects due to saturation that will be indicated by a change in settlement at the time water is added. This feature is frequently important in arid regions where ordinarily dry soils will eventually become wetted through the operation of hydraulic structures. Another characteristic may be obtained from the load release data. The position of the load release point indicates the amount of the elastic rebound. For an ordinary soil, it will, in general, be only a portion of the total settlement. On the other hand an expansive characteristic is seen in a specimen which rebounds to almost its initial volume or beyond it. Many more soil characteristics may be derived from this curve as the analyzer becomes familiar with its various shapes.

Figure 3 shows the standard method of presenting the time-consolidation data. These curves are obtained from specimen consolidation readings taken at frequent intervals, and are shown for each increment of load. A general indication of the rate of consolidation may be obtained by visual examination of these curves. The curves of a rapid-consolidating soil will show that practically all of the settlement occurs in a very short time, sometimes in less than four seconds. The delay in settlement of a slow-consolidating soil is indicated by a sloping curve at later time intervals. Figure 3 is an example of the curves for a moderately slow-consolidating clay; the sloping part of the curves indicates that a major part of the consolidation for the test specimen occurred between 10 and 800 seconds. If this soil were rapid-consolidating the curves would be quite flat or gently sloping within this time interval; the major portion of the consolidation for each increment of load would have occurred near the beginning of the curve or before the 4-second reading.

Information describing the initial and final conditions and the permeability of the test specimens is shown in tabular form as Table 1.

TABLE 1

SUMMARY OF ONE-DIMENSIONAL CONSOLIDATION TEST RESULTS

Sample identification				Specific gravity	Specimen data--initial			Percent consolidation						Specimen data--maximum load and saturated			
Laboratory sample No. 5L	Field sample No.	Excavation No.	Depth (feet)		Dry density (pcf)	Moisture content (%)	Degree of saturation (%)	19.0 psi	37.5 psi	75.0 psi	150.0 psi	150.0 psi saturated	After rebound from max. load	Dry density (pcf)	Moisture content (%)	Degree of saturation (%)	Percolation rate (fpy)
58	3a	TP-5	41.0-42.0	2.670	90.8	30.2	96.5	6.1	8.7	12.2	15.8	15.8	11.0	107.8	20.5	100.0	0.06

THEORETICAL INTERPRETATION FOR THE APPLICATION OF TEST DATA

The consolidation-load and consolidation-time data may be studied in greater detail by further analysis of the test curves. A convenient way to study consolidation-load data is to plot void ratio against pressure. This curve may be plotted by arithmetic scales or with the pressure values to a logarithmic scale, depending on the type of material. The usual method is to use a semilogarithmic plot sheet, as in Figures 4 and 5. When using such a plot for clayey soils the recognized theories[6,7] related to this plotting method are very often helpful.

Load Consolidation

The shape of the consolidation curve for a natural clay soil, initially deposited in a very loose condition and gradually loaded with increasing overburden and structural pressures (referred to as normally-loaded soil), has been found to be an approximately straight line on a semilogarithmic plot. It may be represented by the empirical equation,

$$e = e_o - C_c \operatorname{Log}_{10} \frac{p_o + \Delta p}{p_o} \quad . \; . \; . \; . \quad (1)$$

where

C_c = compression index

e_o = initial void ratio

p_o = initial load pressure

Δp = structural pressure

e = final void ratio

[6] Terzaghi, K., and Peck, R. B., Soil Mechanics in Engineering Practice, John Wiley & Sons, New York, 1948, pp. 61-65.

[7] Taylor, op. cit., pp. 217-219.

The value of C_c indicates the slope of the curve. This straight line is called the field compression or the virgin compression curve.

The method of calculating C_c from a one-dimensional consolidation test curve is described and illustrated in Figure 4. The consolidation curve for a test specimen will always be a recompression curve with the first part having a flatter slope and which curves into the virgin compression curve at a value of pressure approximately equal to a previous maximum loading. (In a normally loaded soil, as described above, this previous maximum loading will be very nearly equal to the present overburden. When soil has been highly preconsolidated by desiccation or by past ice or depositional loads which have since been removed, the previous maximum loading will be larger than the present overburden. Such a soil is spoken of as being "highly preconsolidated" in contrast to a "normally loaded" soil.) The application of such a plot to the typical example is shown in Figure 5. The virgin compression curve has been drawn and the compression index is given at the bottom of the figure.

The settlement of a soil stratum may be calculated in terms of the change in void ratio with the equation

$$S = \frac{e_o - e}{1 + e_o} H \ldots\ldots\ldots\ldots\ldots \quad (2)$$

where

S = settlement

e_o = initial void ratio

e = final void ratio

H = depth of the stratum

By combining this equation with the equation of the virgin compression curve (Equation 1), the settlement of a normally loaded soil may be calculated in terms of the initial void ratio, the compression index, and the change in soil pressure from the present overburden to the overburden plus the structural loading. The combined equation is

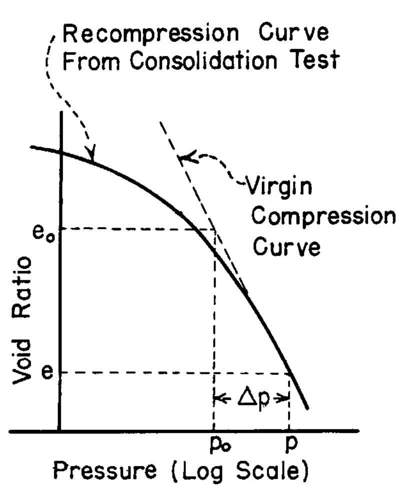
$$S = \frac{H}{1 + e_o} C_c \operatorname{Log}_{10} \frac{p_o + \Delta p}{p_o} \ldots (3)$$

where

p_o = overburden pressure

Δp = structural pressure

Equation 2 is applicable to any soil structure in which the initial and final void ratios can be estimated from one-dimensional consolidation test results. Equation 3 is applicable only to a soil stratum that has a consolidation characteristic showing the structural pressures to be in a range described by the virgin compression curve (normally loaded soils). That is, the maximum previous pressure is equal to the present overburden. Soils preconsolidated by greater pressure than the present overburden cannot be analyzed with the compression index and may best be analyzed in terms of estimated initial and final void ratios. Frequently these soils are so firm and dense that the settlement problem is not of sufficient importance to warrant a detailed analysis.



The Virgin Compression Curve, or the Field Consolidation Curve, for ordinary clays, appears in a semi-logarithmic diagram as a straight line as shown at left. This line can be represented by the equation

$$e = e_o - C_c \log_{10} \frac{p_o + \Delta p}{p_o}$$

in which C_c (dimensionless) is the Compression Index.

The Virgin Compression Curve is established by extending the straight-line part of the Recompression Curve. By selecting two points (e_o, p_o) and (e, p) and substituting in the above equation, C_c can be determined.

$$C_c = \frac{e_o - e}{\log_{10} \frac{p_o + \Delta p}{p_o}}$$

FIGURE 4 - Method of determining the Compression Index, C_c.

Time Consolidation

The time-consolidation data may be studied in greater detail by means of the Terzaghi theory,[8] which was advanced about 25 years ago and is still quite widely accepted. This theory is based on the time required for the escape of pore water. The most important assumptions for its true application are:

1. The soil is completely saturated.

2. The water and solid constituents of the soil are incompressible.

3. Darcy's law is valid and the coefficient of permeability is constant during a particular loading.

4. The time lag of consolidation is due entirely to the low permeability of the soil.

[8] Terzaghi, K., Theoretical Soil Mechanics, John Wiley & Sons, New York, 1943, pp. 265-290.

In many of our studies these assumptions will generally be acceptable. Actual applications will most commonly deviate from these in assumptions 1 and 4. That is, natural soils may not be 100 percent saturated and consolidation may be somewhat delayed for reasons other than permeability, such as plastic lag.[9] The phenomenon of plastic lag is noticeable in the gradual slope of the latter part of the time-consolidation curve (Figure 3). This is referred to as secondary consolidation. The portion of the consolidation which complies with the Terzaghi theory is that represented by the steeper slope and the reverse curvature in Figure 3 and is called the primary consolidation. A large part of the consolidation delay may in most cases be explained by the Terzaghi theory, which permits at least rough estimates of the speed at which settlement will take place. Although the secondary consolidation may appear to be large in the laboratory test on a small specimen, it may not be of serious consequence in the foundation of the structure. The greater time required for primary consolidation in a deep soil stratum of the structure foundation will

[9] Taylor, op. cit., pp. 243-247.

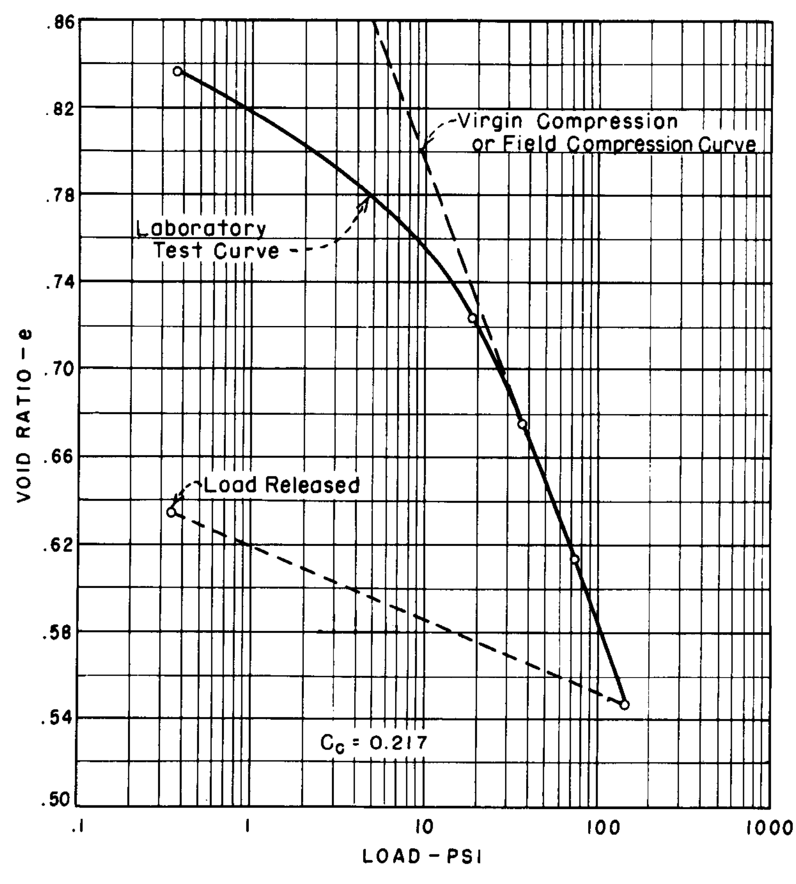
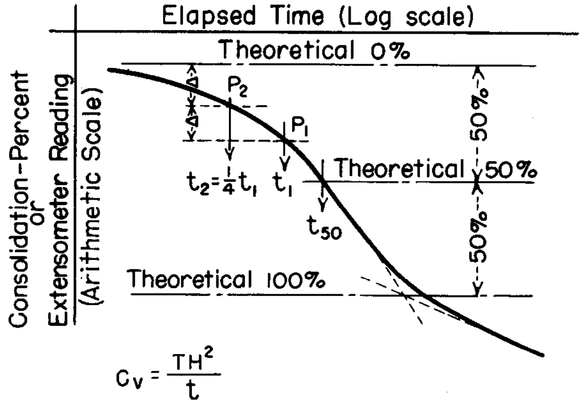


FIGURE 5 - Determination of the Compression Index for the typical example.

Note: The value of C_c is conveniently obtained by taking the difference in the values of void ratio for one complete logarithmic cycle on the virgin or field compression curve. By doing this, the denominator in the equation in Figure 4 becomes equal to one.

FIGURE 6 - Procedure for determining the Coefficient of Consolidation, C_v.



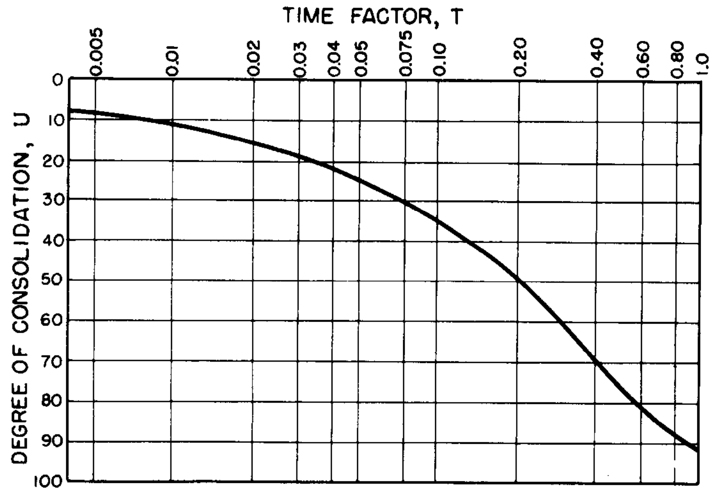
a. (left) Analysis of the consolidation curve for the calculation of the Coefficient of Consolidation C_v. (Reference: Casagrande, A., and Fadum, R. E., "Application of Soil Mechanics in Designing Building Foundations," Transactions ASCE, No. 109, 1944, p. 473.)

Using 50% consolidation point:

T_{50} = Time factor = 0.20 (see below)

H_{50} = 1/2 depth at 50% consolidation

$$C_v = \frac{0.20\ H_{50}^2}{t_{50}}$$



b. (right) Theoretical consolidation curve (after Terzaghi) for obtaining the time factor, T. This curve is for free drainage at the top and bottom of the layer, and therefore is applicable to the one-dimensional consolidation test.

cause the primary consolidation to greatly overshadow the secondary consolidation.

The application of the Terzaghi theory involves the fitting of a theoretical consolidation curve[10] to the laboratory test curve. Based on the fitting of these curves, a coefficient of consolidation, C_v, is obtained which identifies the characteristics of the rate of consolidation for the laboratory test specimen. The theoretical consolidation curve and procedure for fitting it to the laboratory test curve are described in Figure 6. The equation,

$$C_v = \frac{TH^2}{t} \qquad (4)$$

where

T = time factor (for rectangular-shaped pressure distribution and drainage at the top and bottom)

H = greatest distance for pore water to flow for drainage (one-half the specimen height)

t = time for consolidation to take place,

[10] The theoretical consolidation curve is a plot of degree of consolidation against a pure number called the time factor, T. The value of T is dependent only on the conditions of loading (shape of the vertical-pressure distribution curve) and the conditions of drainage. Its shape and position on the plot therefore depend on whether the pressure distribution is rectangular, triangular, or trapezoidal, and whether free drainage takes place at both sides of the soil layer or at just one side. The value of T has been developed for these various conditions and is included in the form of curves for convenient use in the time-of-consolidation equations.

has been developed from the consolidation theory and is used for calculating the value of C_v from the laboratory test results. The application of this equation to the typical example of time-consolidation test curves is shown in Figure 7. Calculations of C_v for the different loading increments are made directly on the standard laboratory plot sheet.

This equation may be applied to the time of settlement in the field in the form

$$t = \frac{TH^2}{C_v} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

In this case

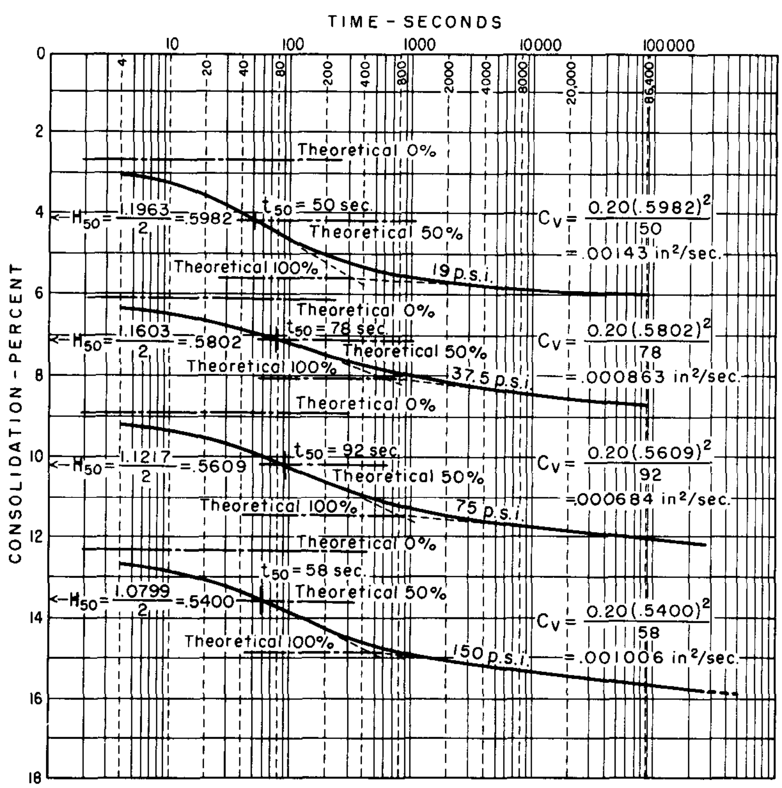
- H = greatest distance for pore water to flow for drainage
- C_v = coefficient of consolidation as obtained from the consolidation test
- T = time factor (dependent on the drainage conditions and the shape of the pressure distribution curve caused by the structure)
- t = time required for settlement.

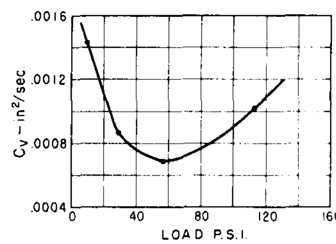
The reader should note that the time factor or the theoretical curve used for the test specimen is for the special case of uniform pressure (or rectangular distribution of pressure throughout the specimen), complete lateral restraint, and free drainage at the top and the bottom of the specimen. This curve is called Case No. 1, and it applies to several types of pressure distribution for the condition of free drainage at both the top and bottom. When drainage is only on one side, the rectangular pressure distribution is the only one which applies to Case No. 1. The theoretical time-factor curve for Case No. 1 and the various types of pressure distribution that apply to it are shown in Figure 8.

In many cases the conditions of the structure itself will compare to the conditions of the test specimen. The values of C_v and the time factor, T, will be the same for the structure as for the laboratory test, and Equation 4 indicates that the following relation exists:

$$\frac{H_f^2}{t_f} = \frac{H_s^2}{t_s} \quad \ldots\ldots\ldots\ldots\ldots \quad (6)$$

where





NOTES:
Equation (4) is used for determining C_v.

$$C_v = \frac{TH^2}{t}$$

The values of C_v are plotted for the average pressure between increments.

FIGURE 7 - Determination of the Coefficient of Consolidation, C_v, for the typical example.

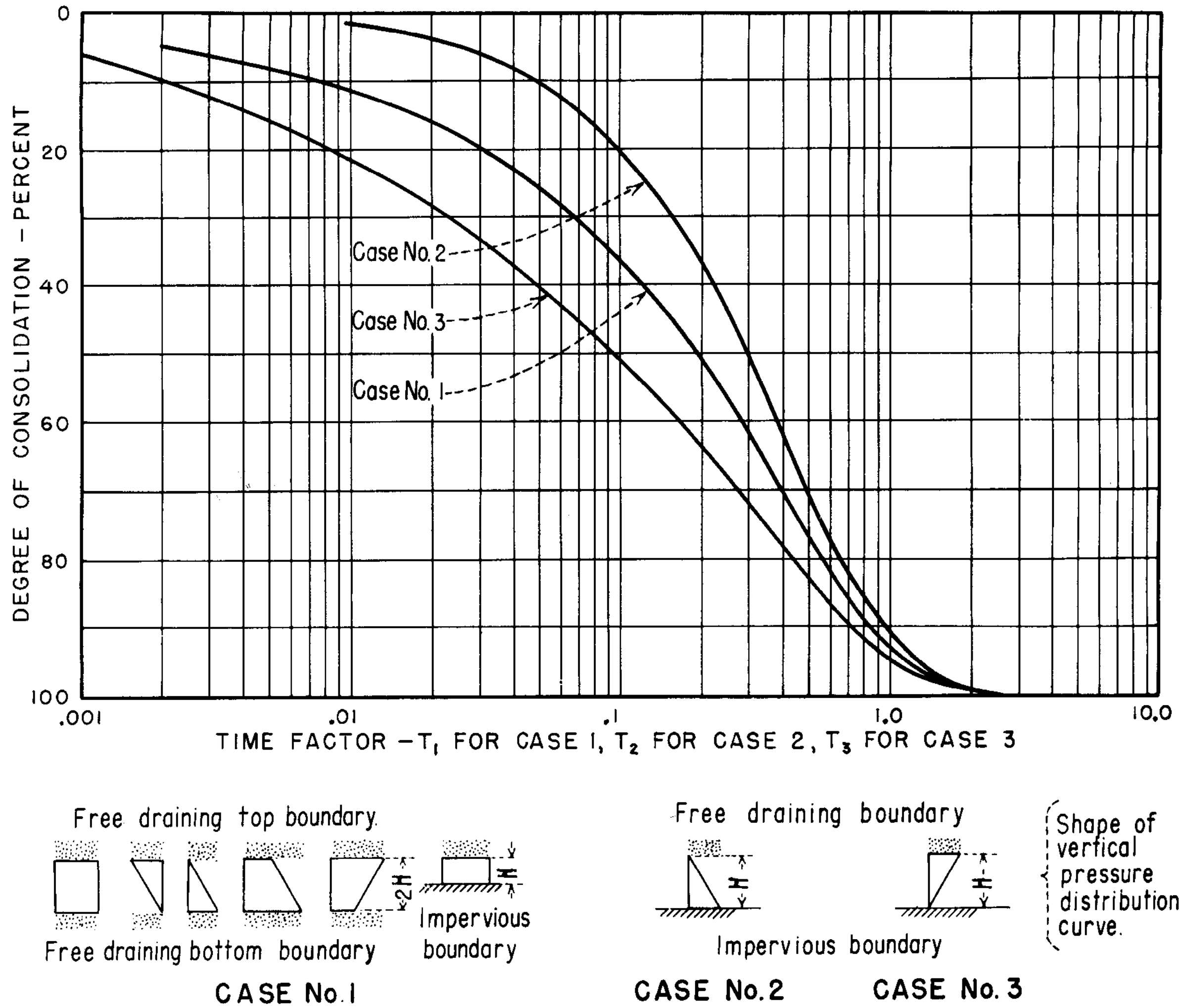


FIGURE 8 - Time factor curves for Cases No. 1, 2, and 3.

H_f = thickness (height) of the stratum in the field

t_f = time of settlement in the field

H_s = thickness (height) of laboratory specimen

t_s = time of settlement of laboratory specimen.

If the pressure distribution in the field is rectangular but drainage occurs in one direction only, the relation becomes

$$\frac{4H_f^2}{t_f} = \frac{H_s^2}{t_s} \quad \ldots\ldots\ldots\ldots\ldots \quad (7)$$

The many cases where the field stratum does not have drainage at both top and bottom, and pressure distributions are not rectangular, must be represented by other theoretical time-factor curves. These curves have been developed for straight-line variations in the vertical distribution of pressure; some are triangular and some are trapezoidal in shape. They include Cases No. 2 to 5 and are shown in Figures 8 and 9. Case No. 2, Figure 8, is for zero pressure at the side of the stratum having free drainage, and a distribution of pressure varying in a triangular shape to the side of the stratum having no drainage. Case No. 3, Figure 8, is for zero pressure at the side of the stratum having no drainage, and a distribution of pressure varying in triangular shape to the side of the stratum having free drainage. Case No. 4, Figure 9, is for trapezoidal pressure distribution with the smallest pressure at the side of the stratum having free drainage and the largest pressure at the side having no drainage. Such a condition combines Cases No. 1 and 2. Case No. 5, Figure 9, is for a trapezoidal

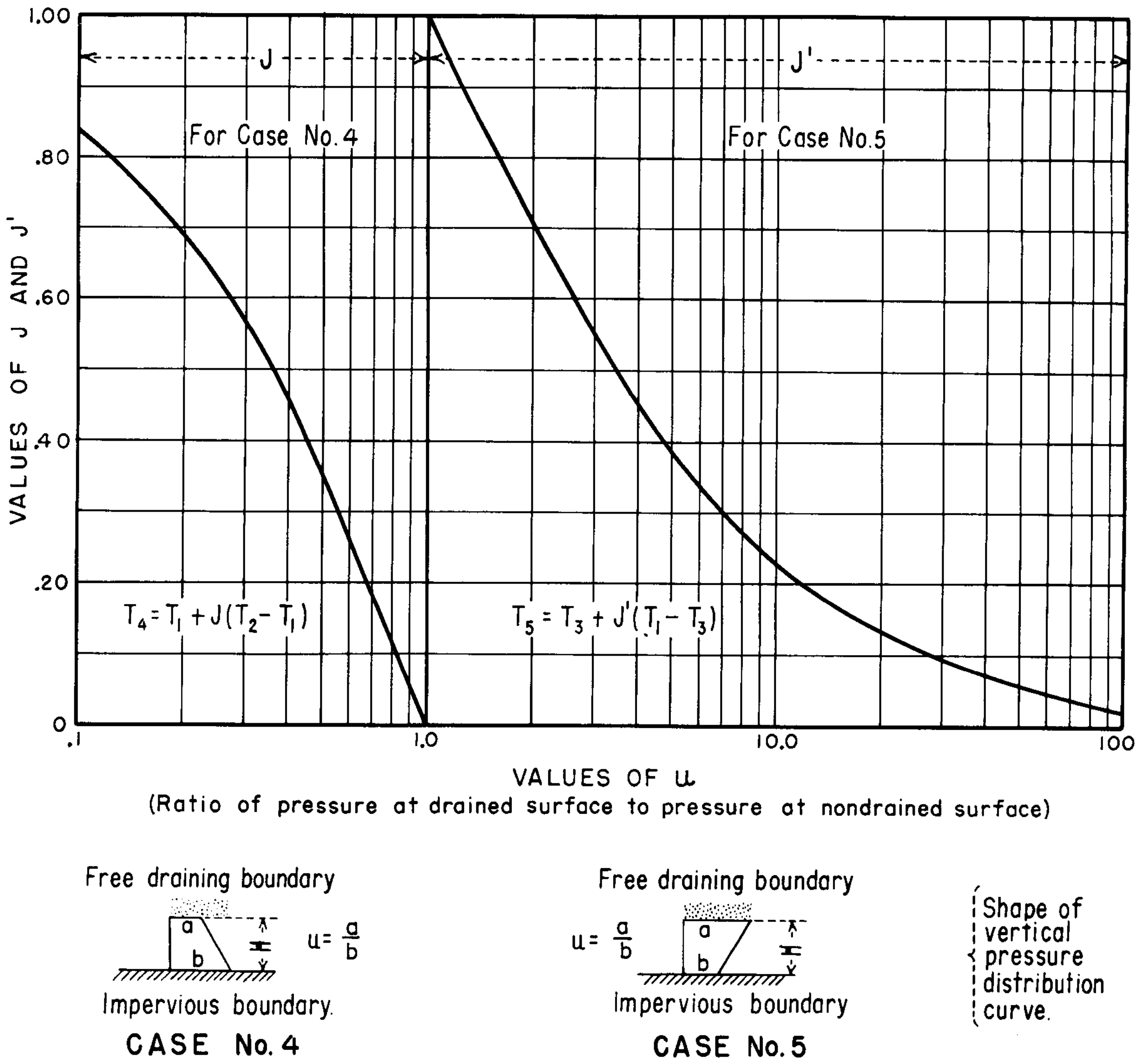


FIGURE 9 - Time factor curves for Cases No. 4 and 5.

pressure distribution with the smallest pressure near the side of the stratum having no drainage and the largest pressure near the side having the free drainage. Such a condition combines Cases No. 1 and 3.

The portion of the consolidation theory involving time of consolidation contains the most cumbersome mathematical derivations of the entire theory. These derivations are fully carried out in many soil mechanics texts and articles on consolidation.[11,12,13] It is intended here to show only the theoretical consolidation data in curve form for the purpose of making practical applications to settlement studies.

---

11 Terzaghi, op. cit., pp. 265-290.

12 Taylor, op. cit., pp. 220-234.

13 Palmer, L. A., and Barber, E. S., "The Theory of Soil Consolidation and Testing of Foundation Soils," Public Roads, Volume 18, No. 1, March 1937, pp. 1-20.

LIMITATIONS OF THE ONE-DIMENSIONAL CONSOLIDATION TEST

As seen in the descriptions of the apparatus and the testing procedure, the one-dimensional consolidation test represents the settlement of a soil structure that has total lateral restraint, and in which there is drainage only in the vertical direction. It is quite apparent that these conditions are not truly comparable to the conditions found in most foundations. The degree of reliance to be placed on settlement studies based on this type of test depends on how nearly the foundation conditions will approach those of the test specimen. In any event, sound reasoning is necessary to make the best application of the data. In general, it is felt that the actual structural loading most comparable to the laboratory test loading is that exerted on a compressible stratum at relatively great depth and of fine material of finite thickness, and which is bounded above and below by dense free-draining materials. In order for the consolidating load to be uniform over a reasonably large portion of the stratum, the structural loading would have to cover a rather large surface area.

The laboratory testing equipment, primarily intended for use in the study of the consolidation of clays, limits the grain size to minus No. 4 (4.76 mm diameter). Actually, the maximum grain size should be considerably smaller than No. 4 for best results in estimating settlement.

It has been found by experiment that gravelly material reduces consolidation.[14] Not only do the gravel particles replace compressible soil, but there is a definite indication that particle interference of the gravel reduces the consolidation of the fine material. This reduction in consolidation becomes more pronounced as the rock content becomes greater. Although this effect does occur with small rock contents, in general it is believed that the effect is only slight for rock contents less than 25 percent.

In the case of a settlement study for a stratum near the surface and for a small loaded area, lateral bulging may be of considerable importance. Under these conditions the soil would not have complete lateral confinement and much settlement may be attributed to the shifting of material and not to consolidation. Figure 10 is a diagrammatic sketch that illustrates the action of the settlement of a loaded area.[15] The solid lines below the footing represent an idealized pressure bulb or zone within which appreciable stresses are caused by the structural loading on the footing. The displaced positions of these lines are shown by the dashed lines with the magnitude of change considerably exaggerated. If the settlement is caused principally by the squeezing out of the soil from under the loaded area, the zone and the element shown in the center of the zone are distorted with little change in

---

[14] Gibbs, Harold J., "The Effect of Rock Content and Placement Density on Consolidation and Related Pore Pressure in Embankment Construction," Proceedings ASTM, Volume 50, 1950, pp. 1343-1360.

[15] Taylor, op. cit., p. 570.

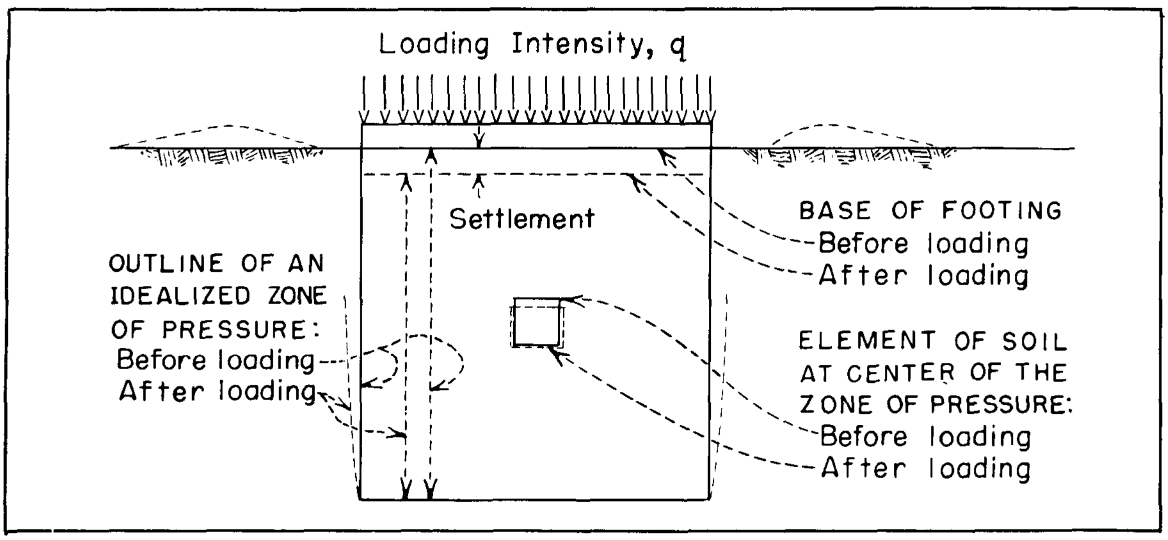


FIGURE 10 - Movements caused by loading.

volume. But if the settlement is due mainly to the consolidation of the soil, the changes in position of horizontal lines would be those of settling, while the shifting of the vertical lines would be considerably less.

The shearing resistance of the material largely governs the lateral bulging property of a foundation. Factors that may contribute to lateral shifting of material include footings at shallow depths, footings resting on material of low shearing resistance, and footings of small area. The design criteria for such conditions are generally governed by shear values and may be analyzed with "bearing capacity" equations.[16] On the other hand, structures having deep footings, or structures having extensive loaded areas, or both, are less likely to fail in shear and are more likely to have consolidation as the governing factor. To such structures the consolidation test data are applicable. The data are also applicable when the compressible stratum is at greater depth, but still within the effect of pressure from the loading.

[16] Terzaghi, op. cit., pp. 118-136.

DETERMINATION OF THE PRESSURE DISTRIBUTION BELOW A LOADED AREA

As a first step toward applying the one-dimensional consolidation test data to a settlement analysis, it is necessary to estimate the pressures in the foundation caused by the proposed structural loading and the present overburden. Several theories have been developed for obtaining pressure distribution due to structural loading. A theory that has shown fairly reliable results and has been given perhaps the greatest recognition in soil mechanics literature is that of Boussinesq. The original Boussinesq equations[17] describe the stress condition below the horizontal surface of a semi-infinite elastic solid under a point load at the surface. The development of these equations, although long and involved, is based on the fundamental theories of elasticity. To apply them to a foundation study it is necessary to assume that the condition of a soil foundation material is that of a semi-infinite elastic solid. This assumption is difficult to conceive for a material such as soil, but a number of experiments by such investigators[18,19] as Kogler, Scheidig, Enger, and Faber, indicate that the elastic theory can at least be used for estimating soil pressures.

The elastic theories given by the Boussinesq equations are most applicable to clay materials. For more sandy materials, soil pressures become more concentrated, causing larger pressures at greater depth. An attempt has been made to adjust the Boussinesq equations empirically to fit the cases of varying types of material. This approach has been discussed by Cummings, and references to the work of Frohlich and others are given in his paper.[20] The theory involves an adjustment in the Boussinesq formula by changing the value of a constant called the "concentration factor." An example of how this factor is applied is as follows:

The Boussinesq equation for the vertical pressures caused by a concentrated load at the surface of a semi-infinite elastic solid is

$$\sigma_Z = \frac{NP}{2\pi} \frac{Z^N}{R^{(N+2)}} \quad \ldots\ldots\ldots \quad (8)$$

where

σ_Z = vertical pressure at the point in question

P = concentrated load at the surface

Z = depth of the point in question

[17] Boussinesq, J., "Application des Potentials a l'Etude de l'Equilibre et du Mouvement des Solides Elastiques," Gauthier-Billard, Paris, 1885. (The derivation is given on pp. 328-331 of Theory of Elasticity by S. Timoshenko, McGraw-Hill, New York, 1934.)

[18] "Soil Mechanics Fact Finding Survey," Progress Report, Triaxial Shear Research and Pressure Distribution Studies on Soils, Waterways Experiment Station. Vicksburg, Mississippi, April 1947.

[19] Cummings, A. E., "Distribution of Stresses under a Foundation," Transactions ASCE, Vol. 101, 1936, p. 1072.

[20] Ibid.

R = distance of the point in question from the location of the concentrated load

N = concentration factor

When the value of N is taken as 3, the formula becomes the original Boussinesq equation and is applicable to a clayey type of material. A value of 6 is recommended when the material is a sand. The idea of a concentration factor is not frequently used, probably due to the complexity of handling, but recent literature has indicated that use of such a factor may increase in the future.

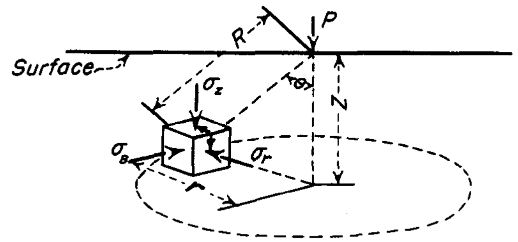
The Boussinesq equations have been developed for both horizontal and vertical stresses. The vertical-stress equations are the only ones used, since the horizontal-stress equations include the elastic constant of Poisson's ratio and are not recommended for soils. These equations have been developed by Newmark into tables and charts[21,22] for convenient use. These charts are based on a concentration factor of 3. The Waterways Experiment Station[23] has prepared charts for other concentration factors similar to those shown by Newmark for a factor of 3.

Equation 8 above is the Boussinesq equation as it is applied to soil foundations. For a concentration factor of N = 3, which is considered applicable to clays but less applicable to sands, this equation becomes

$$\sigma_Z = \frac{3P}{2\pi}\frac{Z^3}{R^5}$$

$$= \frac{3P}{2\pi Z^2}\,\frac{1}{\left[1+\left(\frac{r}{Z}\right)^2\right]^{5/2}} \quad \ldots . (9)$$

The coordinate system for illustrating this equation is shown below:



Since nearly all loads in practical problems are not point loads but are spread over an area, this equation must be converted to a system of analysis applicable to loaded areas. This may be done by dividing a loaded area into small rectangles (usually of a size such that the ratio of the depth considered to the width of the loaded area is greater than 2) and summarizing the results of all areas by treating them as individual concentrated loads.

A more convenient method of determining pressure distribution under loaded areas is with charts[24] and tables[24,25] prepared for application to uniform loads. These charts and tables are the basis for estimating pressures in the examples shown in this monograph. Charts are generally more convenient for irregularly shaped areas, tables more convenient for simple and regularly shaped areas.

Because of space limitations, such other stress distribution theories as those developed by Westergaard,[26,27] Picket,[28] and Burmister[29] cannot be discussed in detail

---

21 Newmark, N. M., Simplified Computation of Vertical Pressures in Elastic Foundations, Circular 24, 1935, and Influence Charts for Computation of Stresses in Elastic Foundations, Bulletin 338, 1942, Engineering Experiment Station, University of Illinois.

22 Terzaghi and Peck, op.cit., pp. 201-207.

23 "Soil Mechanics Fact Finding Survey," Progress Report Triaxial Shear Research and Pressure Distribution Studies on Soils, Waterways Experiment Station, Vicksburg, Mississippi, April 1947, p. 198.

24 Newmark, op. cit.

25 Terzaghi, op. cit., Appendix, pp. 484-487.

26 Westergaard, H. M., "A Problem of Elasticity Suggested by A Problem in Soil Mechanics: Soft Material Reinforced by Numerous Strong Horizontal Sheets," Contributions to the Mechanics of Solids, Stephen Timoshenko 60th Anniversary Volume, MacMillan, New York, 1938, pp. 268-277.

27 Taylor, op. cit., pp. 250-266.

28 Pickett, Gerald, "Stress Distribution in a Loaded Soil with Some Rigid Boundaries," Highway Research Board Proceedings, Vol. 18, Part II, 1938, pp. 35-48.

29 Burmister, D. M., "The Theory of Stresses and Displacements in Layered Systems and Application to Design of Airport Runways," Highway Research Board Proceedings, Vol. 23, 1943, pp. 126-148.

here. Almost all are based on the theory of elasticity or some related theory. The Boussinesq equations are reviewed in greater detail because convenient charts and tables based on these equations are readily available in literature well known to foundation engineers.

The following analyses demonstrate the use of the Boussinesq equations in calculating stresses below a loaded area for a simple example of an area 40 feet square under a uniform load of 50 pounds per square inch (7,200 pounds per square foot):

Pressure distribution using the Boussinesq equation--Figure 11 (page 19).
Pressure distribution using Newmark's chart--Figure 12 (page 20).
Pressure distribution using Newmark's tables--Figure 13 (page 21).

EXAMPLE OF SETTLEMENT ANALYSIS

A prediction of the amount of settlement can be obtained after the soil characteristics have been determined and the pressure distribution below the loaded area resulting from the structural loading has been estimated.

For a simple example of a foundation condition, let it be assumed that the foundation and loaded area are as shown in the sketch at the bottom of the page.

In this example it is assumed that the upper dense, firm, sandy clay and the lower dense sand are sufficiently incompressible to be unimportant in contributing to the settlement. For this reason the examples of settlement analyses which follow are confined to the compressible clay stratum.

There are two general methods of settlement calculations:

1. Equation 2 (shown previously)

$$S = \frac{e_o - e}{1 + e_o} H$$

2. Equation 3 (shown previously)

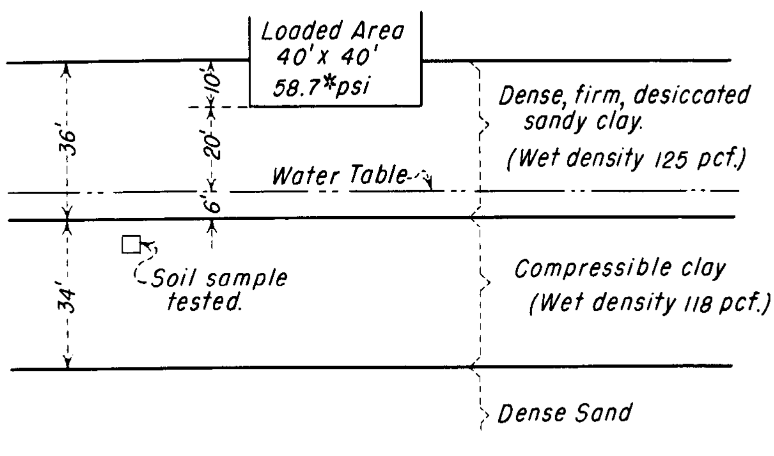
$$S = \frac{H}{1 + e_o} C_c \mathrm{Log}_{10} \frac{p_o + \Delta p}{p_o}$$

The first method is shown in Figure 14 (page 22) and the second method is shown in Figure 15 (page 23).

For the purpose of a demonstration problem the area is considered to have a loading of 58.7 psi. It will be noted in the sketch below that this loaded area is placed at a depth of 10 feet. Thus, when the excavation was made, a loading of 10 feet of overburden was removed. For a wet density of 125 pcf this excavated load amounts to 125 x 10/144 = 8.7 psi.

When excavations are made the removal of overburden must be taken into account. If the removed overburden load is relatively small in comparison with the structural load it may be accounted for by subtraction from the structural load to be placed back on the foundation. In many cases it may be very large in relation to the structural load.

***Note: The loading of 58.7 psi was used for this example so that the resulting load will be 50 psi (same as in pressure examples) after accounting for the required excavation.**



The removal of overburden has often been used as a means of reducing the potential settlement by excavating large basements so that the overburden load removed is equal to or greater than the structural load put back on the soil.[30] This is called "floating" the structure. If an analysis of settlement is made for such a case the study will be confined to the recompression portion of the test curve and the situation becomes similar to that of placing a structure on a preconsolidated soil. As the excavation occurs the soil will have a slight rebound or expansion and subsequently will have a slight recompression when the structural load is replaced. In many instances where this condition is encountered a detailed analysis may not be warranted for the same reason that preconsolidated soils frequently do not warrant such an analysis.

In the illustrated example the removal of overburden is considered to be relatively small in comparison with the structural loading, and is merely subtracted from the structural loading. This leaves an effective structural loading of 50 psi, which is the load used in the previously discussed pressure distribution demonstrations.

Another important consideration in the interpretation of overburden pressures is the buoyant effect on material below the water table. A good discussion of this is given by Terzaghi and Peck.[31] It is referred to as hydrostatic uplift and submerged unit weight. The relationship for submerged unit weight is

$$\gamma' = \gamma - \gamma_w$$

where

γ' = submerged unit weight

γ = wet density of the material

γ_w = unit weight of water (62.4 pcf)

In calculating the overburden pressure distribution, the pressure increases with depth according to the relation γz (where z equals change in depth). When the water table is reached, the relation becomes $(\gamma - \gamma_w) z$. As different strata are reached and densities change, the value of γ changes accordingly. The calculated overburden pressures are shown at right center in Figure 14 for the assumed conditions of the example.

By plotting the structural pressure (the average structural pressure in the example) as an added pressure to the overburden, as shown at right center in Figure 14, the working pressures for a settlement analysis are obtained. When the pressure distribution is curved, the general practice is to divide the compressible stratum into sufficient increments to permit fairly accurate settlement estimates for each increment, and then to make a summation of these for the total settlement.

Figures 14 and 15 are settlement calculations by different methods but for the same conditions of loading and soil characteristics as determined from the laboratory tests. Figure 14 demonstrates the use of Equation 2 and Figure 15 the use of Equation 3. For simplicity in demonstrating the calculations, the results of only one laboratory test are used in each example. In an actual problem it is advisable to have several tests at varying depths; the additional tests improve the accuracy of the final estimate. Only when the material is normally loaded can the results of a single test give a reliable settlement estimate.

In the example in Figure 14, estimates of the initial and final void ratios for each increment of depth are obtained directly from the laboratory test curve; in Figure 15 the data are obtained from the virgin compression curve. While appearing to be representative of a normally loaded soil, the laboratory test curve is slightly lower than the virgin compression curve in the range of structural loading. As a result the estimate of settlement in Figure 14 is slightly lower in value than the estimate in Figure 15.

[30] Casagrande, A., and Fadum, R. E., "Application of Soil Mechanics in Designing Building Foundations," Transactions ASCE, Vol. 109, 1944, pp. 383-416.

[31] Terzaghi and Peck, op. cit., pp. 51-55.

EXAMPLE OF TIME-OF-CONSOLIDATION ANALYSIS

Figure 16 (page 24) demonstrates the time-of-consolidation calculation for the illustrated example. In the upper left corner of the figure the laboratory test data from Figure 7 are shown, and in the upper right corner the overburden and structural pressure distribution from Figure 14 are shown. Although the pressure distribution is slightly curved, it can be considered trapezoidal in shape with an average pressure of 42 psi. At this pressure the laboratory test data give a coefficient of consolidation, C_V, of 0.00075 in^2/sec.

Since the stratum above the compressible stratum is a sandy clay and the stratum below is a dense sand, it is first assumed that drainage takes place on each side of the compressible fat-clay stratum. This is the situation of Case No. 1. It is also assumed that the load is applied rapidly in relation to the time required to reach total consolidation. The solution then becomes that of Equation 5.

$$t\,(hr) = \frac{T_1 H^2}{C_V}$$

where

T_1 = time factor for Case No. 1 (various values obtained from Figure 8)

H = one-half depth of stratum in inches (or the maximum distance for drainage)

C_V = coefficient of consolidation in in^2/hr.

A demonstration of Equation 6 is of interest, since drainage on both sides of the stratum is the condition of Case No. 1 and compares to the action of the laboratory test specimen. In Figure 7 the nearest laboratory test curve that compares to the structural loading is that for the 37.5 psi increment. In this increment the laboratory test curve reached 50 percent consolidation in 78 seconds and the depth of the laboratory specimen is 1.1603 inches. Then, by Equation 6,

$$\frac{(34)^2}{t_f} = \frac{(1.1603/12)^2}{78/3600}$$

t_f = time of settlement in the field = 2665 hours

This compares to 3082 hours shown in the table in Figure 16 for Case No. 1. The difference is due to the fact that the average pressure used in Figure 16 was 42 psi whereas the nearest laboratory test curve was for 37.5 psi pressure.

As a second demonstration in Figure 16, assume that the upper stratum is of a different material than that previously shown and does not permit drainage at the top of the compressible stratum. Drainage is therefore permitted only into the dense sand below and the situation becomes that of Case No. 4, a combination of Cases No. 1 and 2. The time factor, T_4, is solved with the curves in Figures 8 and 9, and the equation,

$$T_4 = T_1 + J(T_2 - T_1)$$

where

T_4 = time factor for Case No. 4

T_2 = time factor for Case No. 2

T_1 = time factor for Case No. 1

J = factor obtained from Figure 9.

The time of consolidation is obtained from Equation 5,

$$t = \frac{T_4 H^2}{C_V}$$

where

H = the total depth since drainage is only in one direction.

The solution is shown at the bottom in Figure 16.

A comparison of the time of settlement for the structure with the time of settlement as determined in the laboratory test cannot be made in this case with Equation 6 or 7, because the structural load distribution is trapezoidal and the time factors are not the same in the two cases.

CLOSING DISCUSSION

The examples presented in this monograph are considerably simplified and are intended only to demonstrate the tools which are available for making settlement estimates. More frequently than not, actual structures will be more complicated and the application of these tools will be more complex.

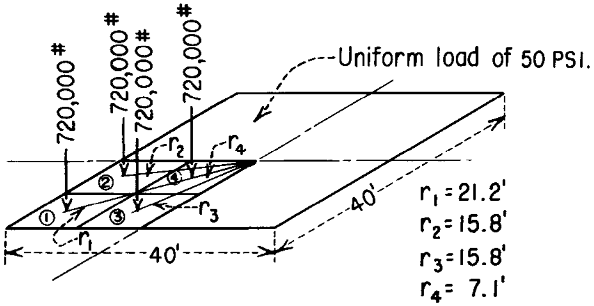
A structure will frequently have odd-shaped foundations and loads that are not evenly distributed. In such cases it may be desirable to analyze pressure and settlement at various points under the structure instead of for an over-all average. The strata contributing to settlement may vary in thickness, a further reason for making analyses at various points. Such conditions contribute to differences in settlement throughout the structure (called differential settlement) which may be far more serious than the total average settlement. Differential settlement is the cause of cracking and unexpected stresses in structures and changes in alinement of moving machinery; but uniform settlement, even though substantial, may not seriously harm a structure.

A foundation is frequently made up of a series of footings so closely spaced that the pressure effect of one footing overlaps those of adjoining footings, and the pressures of all footings should be considered. This is easily handled by dealing with scale drawings of all footings and using the pressure chart as in Figure 12. Close footings under a structure of large area may have a pressure effect on a deep stratum similar to that of the entire building acting as a single spread footing. In this case it is advisable to analyze the structure as a whole instead of each footing separately.

In an analysis of laboratory data it is always important to consider the laboratory test as a recompression of the undisturbed material. When the laboratory specimen was removed from the ground the overburden pressures were removed from it. Thus the percentage of consolidation occurring in the laboratory specimen is not the same as that occurring in the foundation itself. The theoretical interpretation of consolidation presented in this monograph is a tool by which a laboratory test may be used in making an estimate of the amount of settlement in the foundation. The method of analyzing the effect of present and past overburden pressures is common practice among most soil mechanics authorities.

Accuracy in estimating the amount of settlement is improved if a large number of samples are tested. Samples at various depths are particularly important, as less dependence will need to be placed on theoretical effects of preconsolidation and existing overburden pressures. Numerous samples at the same elevation but at different locations are not nearly so valuable, although they serve to indicate the consistency of characteristics in a particular stratum.

The time-of-consolidation theories are long and involved and generally consume a major amount of space in most articles on consolidation. Since time analyses are not so frequently required as analyses on the amount of settlement, the space devoted to time studies has been kept to a minimum. In the examples it is assumed that the structure is constructed so rapidly that settlement during construction is small in comparison with that occurring after construction. Very often the construction period may be sufficiently long to allow a considerable amount of settlement to occur as the structure is built. For more precise estimates on the time of settlement, the construction period should be correlated with time of consolidation by considering the load to be built up in periodic stages until construction is complete.



LOADING CONDITION

BOUSSINESQ'S EQUATION

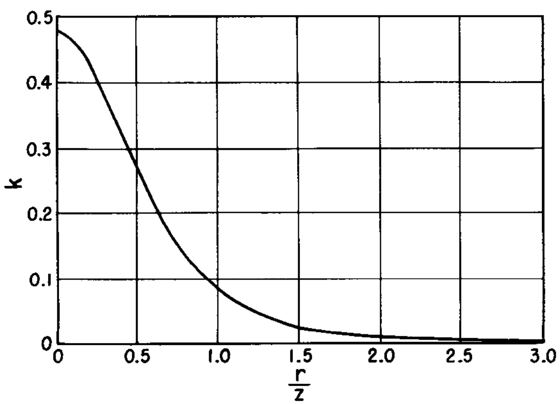
$$\sigma_z = k \frac{P}{z^2}$$

$$k = \frac{3}{2\pi} \frac{1}{\left[1+\left(\frac{r}{z}\right)^2\right]^{5/2}}$$

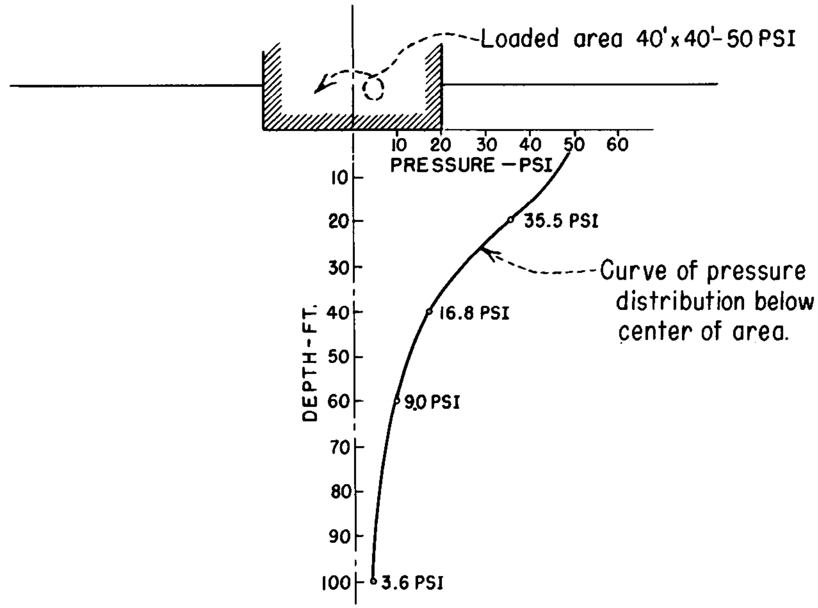
TABLE OF COMPUTATION
FOR PRESSURES BELOW THE CENTER OF THE AREA

DEPTH FT.	AREA ① r/z / K	AREA ② r/z / K	AREA ③ r/z / K	AREA ④ r/z / K	P/z^2	PRESSURES – BY AREAS: AREA ① PSF	AREA ② PSF	AREA ③ PSF	AREA ④ PSF	TOL. σ_z FOR $\frac{1}{4}$ AREA PSF	TOT. σ_z PSF / PSI
0	—	—									50
20	1.06 / 0.07	0.79 / 0.14	0.79 / 0.14	0.35 / 0.36	1800	126	252	252	648	1278	5112 / 35.5
40	0.53 / 0.25	0.40 / 0.33	0.40 / 0.33	0.18 / 0.44	450	112	148	148	198	606	2424 / 16.8
60	0.35 / 0.36	0.26 / 0.40	0.26 / 0.40	0.12 / 0.46	200	72	80	80	92	324	1296 / 9.0
100	0.21 / 0.43	0.16 / 0.45	0.16 / 0.45	0.07 / 0.47	72	31	32	32	34	129	516 / 3.6

NOTE: Similar computations can be made at other locations below the area.



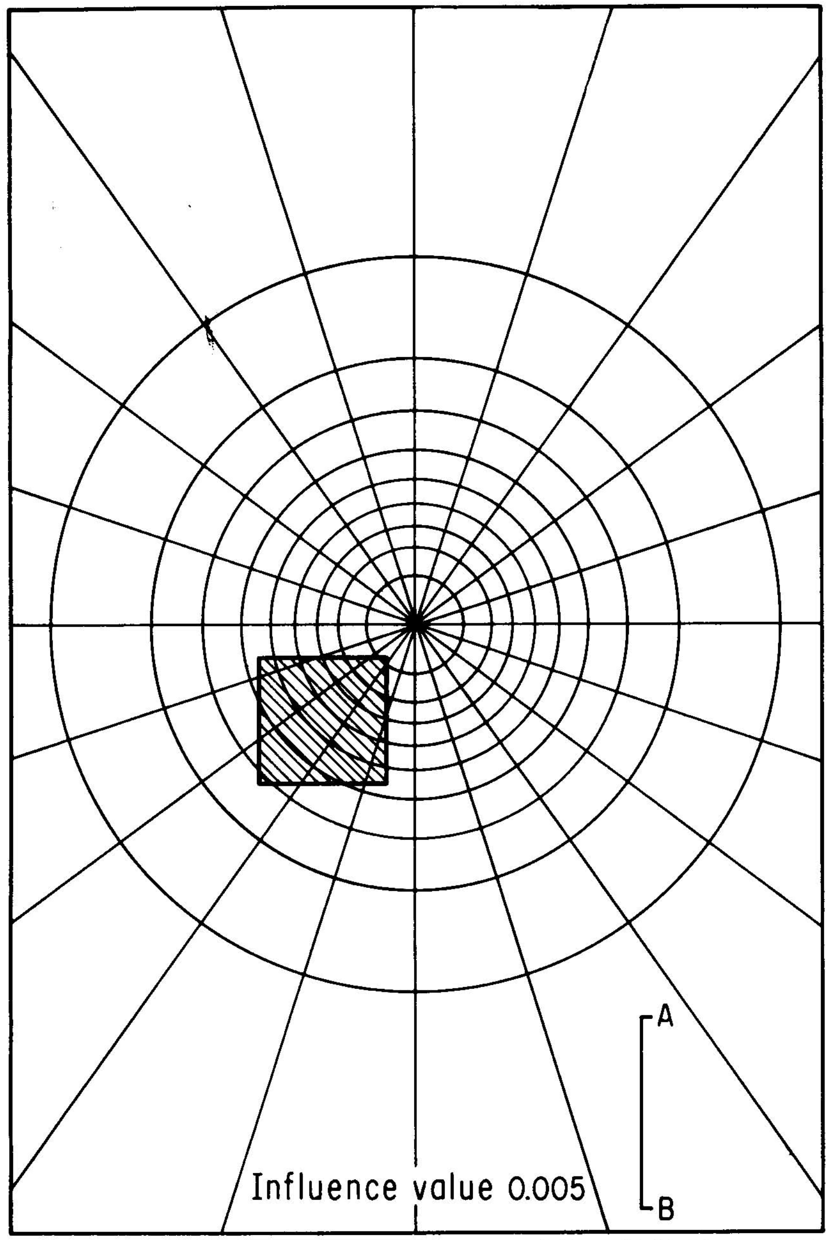
GRAPH FOR DETERMINATION
OF K IN ABOVE EQUATION



VERTICAL PRESSURE DISTRIBUTION
BELOW THE CENTER OF THE AREA

FIGURE 11 - Pressure distribution by Boussinesq's Equation.

NEWMARK'S CHART FOR DETERMINING PRESSURE BELOW A LOADED AREA



After N.M. Newmark

EXAMPLE CALCULATION:
Given: Load of 50 p.s.i. on area 40' x 40'.
Find: Vertical pressure 60' below point D'.
Procedure: Draw area to scale such that distance AB on chart equals 60'. Place area over chart such that point of interest (D') is at center of chart. Count squares covered by area (shaded on chart for this example). Number of squares = 15.3.
Then: Pressure = 15.3 x influence value x 50 p.s.i.
= 15.3 x 0.005 x 50 = 3.82 (ans.)

AREA DRAWN TO SCALE FOR VARIOUS DEPTHS
(See example calculation)

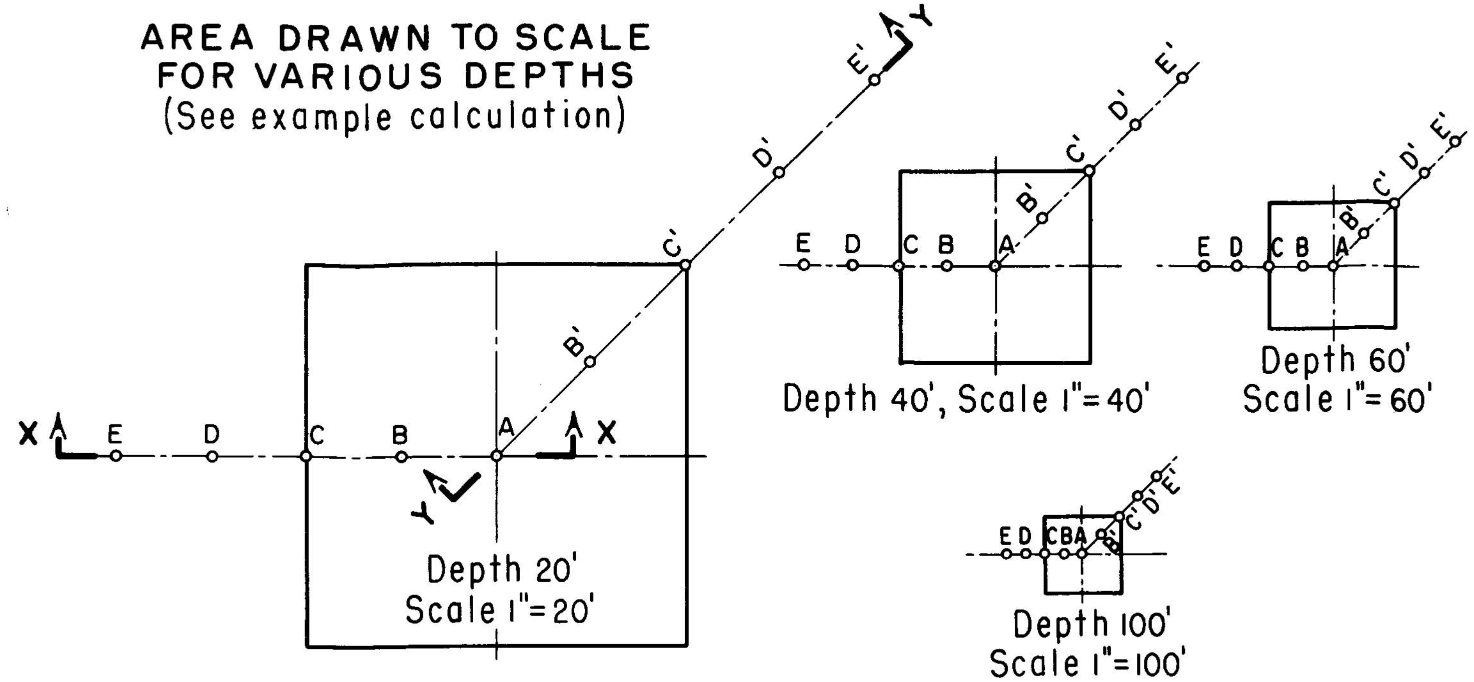
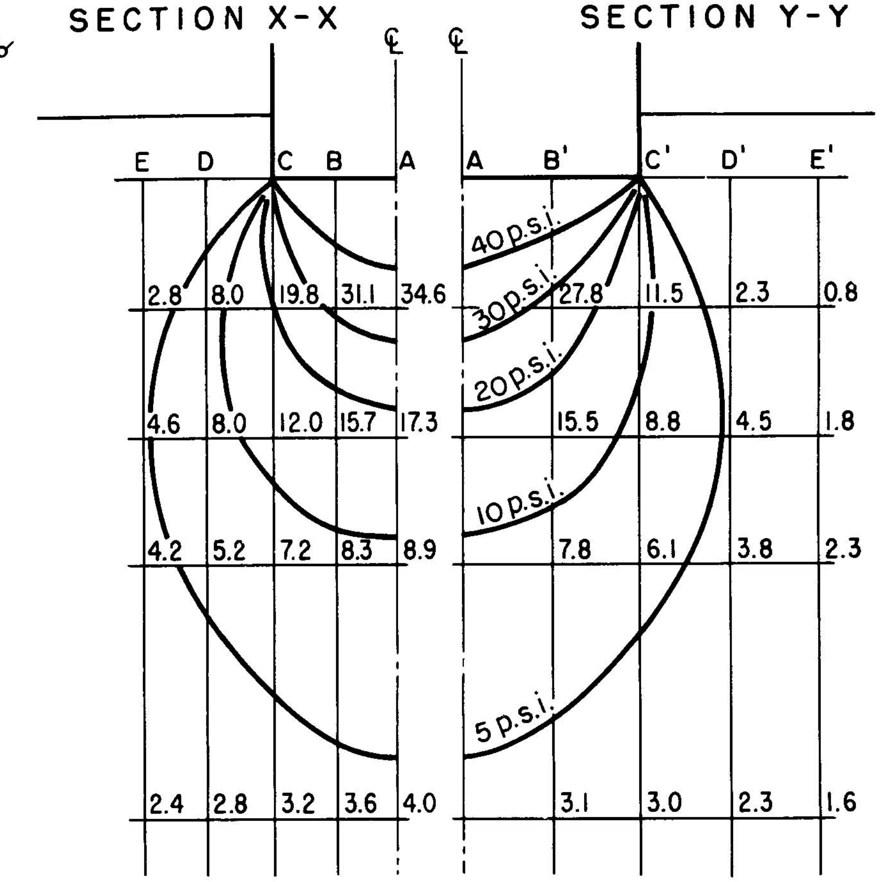


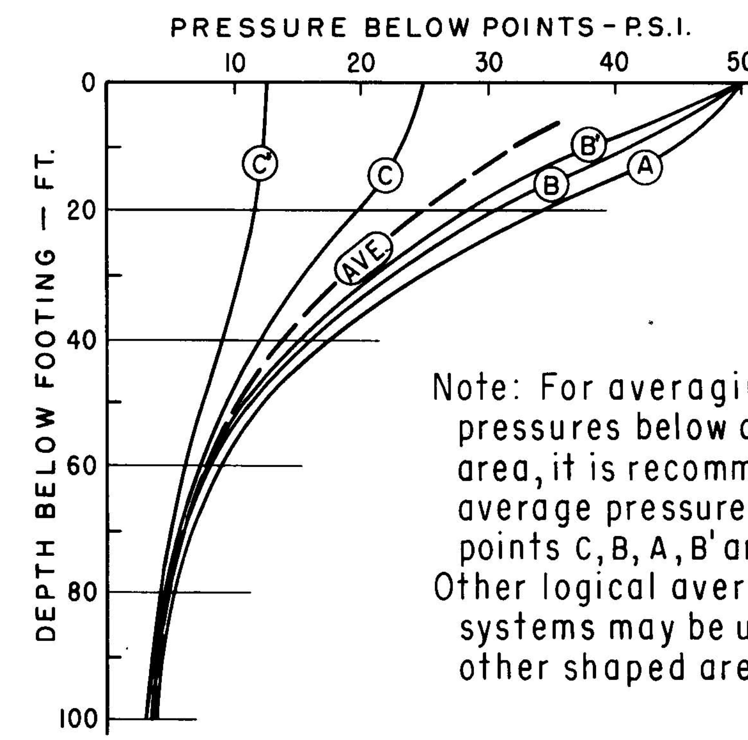
TABLE OF COMPUTATION
FOR PRESSURES BELOW AREA 40'x 40' WITH 50 P.S.I. LOAD

LOCATION OF POINT	DEPTH 20'		DEPTH 40'		DEPTH 60'		DEPTH 100'	
	NO. OF SQUARES	P.S.I. PRESSURE	NO. OF SQUARES	P.S.I. PRESSURE	NO. OF SQUARES	P.S.I. PRESSURE	NO. OF SQUARES	P.S.I. PRESSURE
(A) Center	138.5	34.6	69.3	17.3	35.6	8.9	16.0	4.0
(B) 10' Left	124.6	31.1	63.0	15.7	33.2	8.3	14.2	3.6
(C) 20' Left	79.4	19.8	48.0	12.0	28.6	7.2	12.6	3.2
(D) 30' Left	31.8	8.0	31.8	8.0	20.6	5.2	11.0	2.8
(E) 40' Left	11.0	2.8	18.4	4.6	16.8	4.2	9.6	2.4
(B') 14' Diagonally	111.0	27.8	62.0	15.5	31.0	7.8	12.5	3.1
(C') Corner	46.0	11.5	35.0	8.8	24.5	6.1	12.0	3.0
(D') 42' Diagonally	9.0	2.3	18.0	4.5	15.3	3.8	9.0	2.3
(E') 56' Diagonally	3.0	0.8	7.0	1.8	9.0	2.3	6.5	1.6

PRESSURE CONTOURS BELOW AREA



AVERAGE PRESSURE DISTRIBUTION



Note: For averaging the pressures below a square area, it is recommended to average pressures below points C, B, A, B' and C'. Other logical averaging systems may be used for other shaped areas.

FIGURE 12 - Pressure distribution by Newmark's Chart.

m	n = 0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.2
0.1	0.00470	0.00917	0.01323	0.01678	0.01978	0.02223	0.02420	0.02576	0.02698	0.02794	0.02926
0.2	0.00917	0.01790	0.02585	0.03280	0.03866	0.04348	0.04735	0.05042	0.05283	0.05471	0.05733
0.3	0.01323	0.02585	0.03735	0.04742	0.05593	0.06294	0.06858	0.07308	0.07661	0.07938	0.08323
0.4	0.01678	0.03280	0.04742	0.06024	0.07111	0.08009	0.08734	0.09314	0.09770	0.10129	0.10631
0.5	0.01978	0.03866	0.05593	0.07111	0.08403	0.09473	0.10340	0.11035	0.11584	0.12018	0.12626
0.6	0.02223	0.04348	0.06294	0.08009	0.09473	0.10688	0.11679	0.12474	0.13105	0.13605	0.14309
0.7	0.02420	0.04735	0.06858	0.08734	0.10340	0.11679	0.12772	0.13653	0.14356	0.14914	0.15703
0.8	0.02576	0.05042	0.07308	0.09314	0.11035	0.12474	0.13653	0.14607	0.15371	0.15978	0.16843
0.9	0.02698	0.05283	0.07661	0.09770	0.11584	0.13105	0.14356	0.15371	0.16185	0.16835	0.17766
1.0	0.02794	0.95471	0.07938	0.10129	0.12018	0.13605	0.14914	0.15978	0.16835	0.17522	0.18508
1.2	0.02926	0.05733	0.08323	0.10631	0.12626	0.14309	0.15703	0.16843	0.17766	0.18508	0.19584
1.4	0.03007	0.05894	0.08561	0.10941	0.13003	0.14749	0.16199	0.17389	0.18357	0.19139	0.20278
1.6	0.03058	0.05994	0.08709	0.11135	0.13241	0.15028	0.16515	0.17739	0.18737	0.19546	0.20731
1.8	0.03090	0.06058	0.08804	0.11260	0.13395	0.15207	0.16720	0.17967	0.18986	0.19814	0.21032
2.0	0.03111	0.06100	0.08867	0.11342	0.13496	0.15326	0.16856	0.18119	0.19152	0.19994	0.21235
2.5	0.03138	0.06155	0.08948	0.11450	0.13628	0.15483	0.17036	0.18321	0.19375	0.20236	0.21512
3.0	0.03150	0.06178	0.08982	0.11495	0.13684	0.15550	0.17113	0.18407	0.19470	0.20341	0.21633
4.0	0.03158	0.06194	0.09007	0.11527	0.13724	0.15598	0.17168	0.18469	0.19540	0.20417	0.21722
5.0	0.03160	0.06199	0.09014	0.11537	0.13737	0.15612	0.17185	0.18488	0.19561	0.20440	0.21749
6.0	0.03161	0.06201	0.09017	0.11541	0.13741	0.15617	0.17191	0.16496	0.19569	0.20449	0.21760
8.0	0.03162	0.06202	0.09018	0.11543	0.13744	0.15621	0.17195	0.18500	0.19574	0.20455	0.21767
10.0	0.03162	0.06202	0.09019	0.11544	0.13745	0.15622	0.17196	0.18502	0.19576	0.20457	0.21769
∞	0.03162	0.06202	0.09019	0.11544	0.13745	0.15623	0.17197	0.18502	0.19577	0.20458	0.21770

m	n = 1.4	1.6	1.8	2.0	2.5	3.0	4.0	5.0	6.0	8.0	10.0	∞
0.1	0.03007	0.03058	0.03090	0.03111	0.03138	0.03150	0.03158	0.03160	0.03161	0.03162	0.03162	0.03162
0.2	0.05894	0.05994	0.06058	0.06100	0.06155	0.06178	0.06194	0.06199	0.06201	0.06202	0.06202	0.06202
0.3	0.08561	0.08709	0.08804	0.08867	0.08948	0.08982	0.09007	0.09014	0.09017	0.09018	0.09019	0.09019
0.4	0.10941	0.11135	0.11260	0.11342	0.11450	0.11495	0.11527	0.11537	0.11541	0.11543	0.11544	0.11544
0.5	0.13003	0.13241	0.13395	0.13496	0.13628	0.13684	0.13724	0.13737	0.13741	0.13744	0.13745	0.13745
0.6	0.14749	0.15028	0.15207	0.15326	0.15483	0.15550	0.15598	0.15612	0.15617	0.15621	0.15622	0.15623
0.7	0.16199	0.16515	0.16720	0.16856	0.17036	0.17113	0.17168	0.17185	0.17191	0.17195	0.17196	0.17197
0.8	0.17389	0.17739	0.17967	0.18119	0.18321	0.18407	0.18469	0.18488	0.18496	0.18500	0.18502	0.18502
0.9	0.18357	0.18737	0.18986	0.19152	0.19375	0.19470	0.19540	0.19561	0.19569	0.19574	0.19576	0.19577
1.0	0.19139	0.19546	0.19814	0.19994	0.20236	0.20341	0.20417	0.20440	0.20449	0.20455	0.20457	0.20458
1.2	0.20278	0.20731	0.21032	0.21235	0.21512	0.21633	0.21722	0.21749	0.21760	0.21767	0.21769	0.21770
1.4	0.21020	0.21510	0.21836	0.22058	0.22304	0.22499	0.22600	0.22632	0.22644	0.22652	0.22654	0.22656
1.6	0.21510	0.22025	0.22372	0.22610	0.22940	0.23088	0.23200	0.23236	0.23249	0.23258	0.23261	0.23263
1.8	0.21836	0.22372	0.22736	0.22986	0.23334	0.23495	0.23617	0.23656	0.23671	0.23681	0.23684	0.23686
2.0	0.22058	0.22610	0.22986	0.23247	0.23614	0.23782	0.23912	0.23954	0.23970	0.23981	0.23985	0.23987
2.5	0.22364	0.22940	0.23334	0.23614	0.24010	0.24196	0.24344	0.24392	0.24412	0.24425	0.24429	0.24432
3.0	0.22499	0.23088	0.23495	0.23782	0.24196	0.24394	0.24554	0.24608	0.24630	0.24646	0.24650	0.24654
4.0	0.22600	0.23200	0.23617	0.23912	0.24344	0.24554	0.24729	0.24791	0.24817	0.24836	0.24842	0.24846
5.0	0.22632	0.23236	0.23656	0.23954	0.24392	0.24608	0.24791	0.24857	0.24885	0.24907	0.24914	0.24919
6.0	0.22644	0.23249	0.23671	0.23970	0.24412	0.24630	0.24817	0.24885	0.24916	0.24939	0.24946	0.24952
8.0	0.22652	0.23258	0.23681	0.23981	0.24425	0.24646	0.24836	0.24907	0.24939	0.24964	0.24973	0.24980
10.0	0.22654	0.23261	0.23684	0.23985	0.24429	0.24650	0.24842	0.24914	0.24946	0.24973	0.24981	0.24989
∞	0.22656	0.23263	0.23686	0.23987	0.24432	0.24654	0.24846	0.24919	0.24952	0.24980	0.24989	0.25000

This table gives the pressure below the corner of a rectangular area having a loading of one unit pressure per unit area. The value "m" is the ratio of one side of the area to the depth considered, and "n" is the ratio of the other side to the depth considered.

The pressure below any point in the area may be obtained by dividing the area into rectangles, each having a corner at the point and summing up the pressures due to the rectangles.

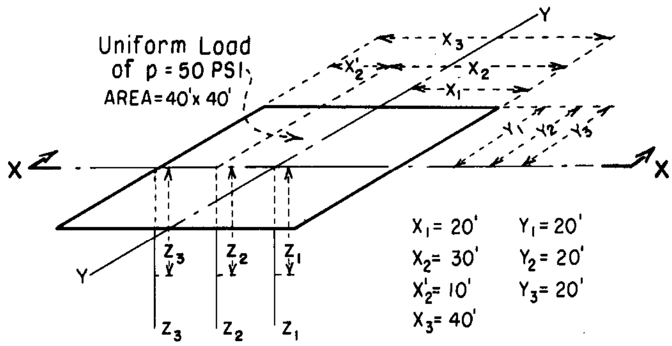
(Reference: Terzaghi, K., Theoretical Soil Mechanics, J. Wiley and Sons, New York, 1943, pages 484-487.)

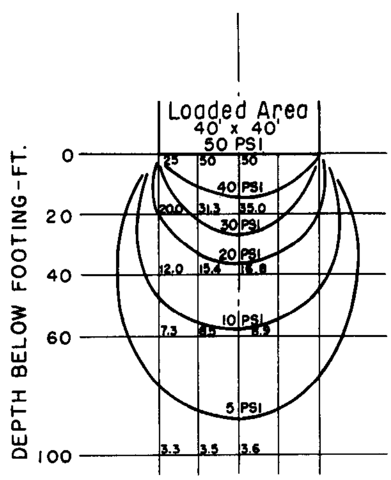
TABLE OF COMPUTATION

Z_1-FT. DEPTH	m X_1/Z_1	n Y_1/Z_1	σ_z/P	σ_z PSI	$4\sigma_z$	
0	∞	∞	0.25	12.5	50.0	
20	1	1	0.17522	8.76	35.0	
40	.5	.5	0.08403	4.20	16.8	
60	.33	.33	0.04427	2.21	8.9	
100	.20	.20	0.01790	0.90	3.6	

Z_2-FT. DEPTH	m X_2/Z_2 / X_2'/Z_2	n Y_2/Z_2 / Y_2'/Z_2	σ_z/P / σ_z'/P	σ_z / σ_z'	$2\sigma_z$ / $2\sigma_z'$	TOTAL
0	∞ / ∞	∞ / ∞	0.25 / 0.25	12.5 / 12.5	25.0 / 25.0	50.0
20	1.5 / 0.5	1.0 / 1.0	.19342 / .12018	9.67 / 6.01	19.3 / 12.0	31.3
40	0.75 / 0.25	0.5 / 0.5	.10688 / .04730	5.34 / 2.36	10.7 / 4.7	15.4
60	0.5 / 0.17	0.33 / 0.33	.06099 / .02404	3.05 / 1.20	6.1 / 2.4	8.5
100	0.3 / 0.1	0.2 / 0.2	.02585 / .00917	1.29 / 0.46	2.6 / 0.9	3.5

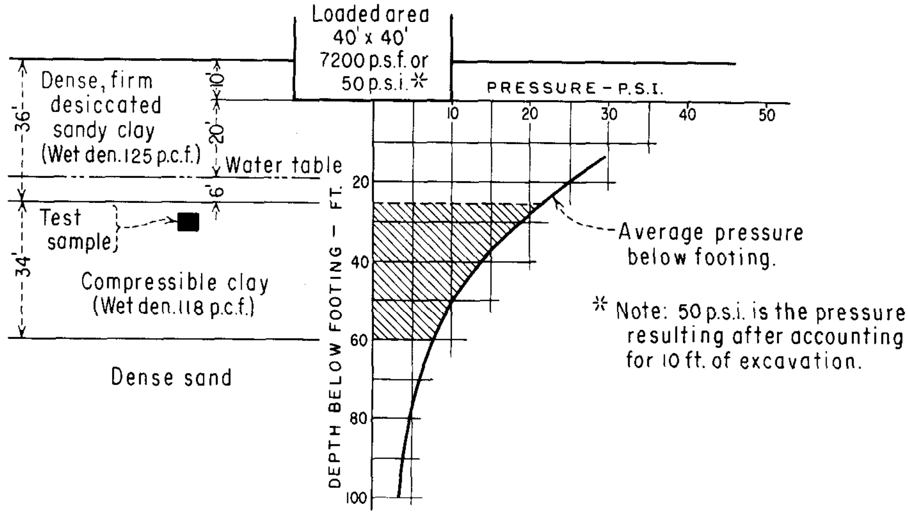
Z_3-FT. DEPTH	m X_3/Z_3	n Y_3/Z_3	σ_z/P	σ_z PSI	$2\sigma_z$
0	∞	∞	0.25	12.5	25.0
20	2.0	1.0	0.19994	10.00	20.0
40	1.0	0.5	0.12018	6.01	12.0
60	0.67	0.33	0.07301	3.65	7.3
100	0.40	0.20	0.03280	1.64	3.3





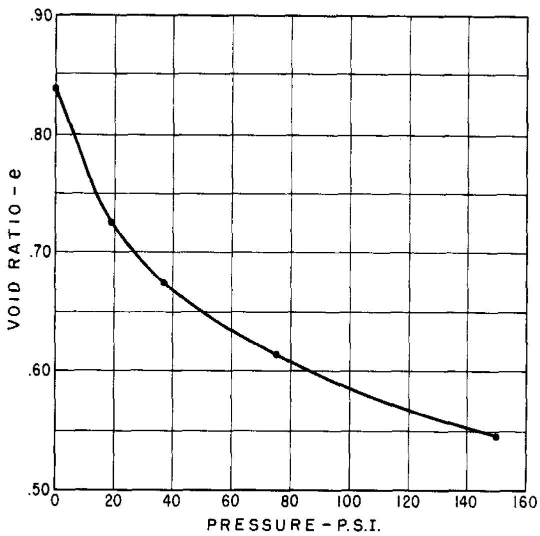
SECTION X-X
PRESSURE CONTOURS
BELOW AREA

FIGURE 13 - Pressure distribution by Newmark's Table.

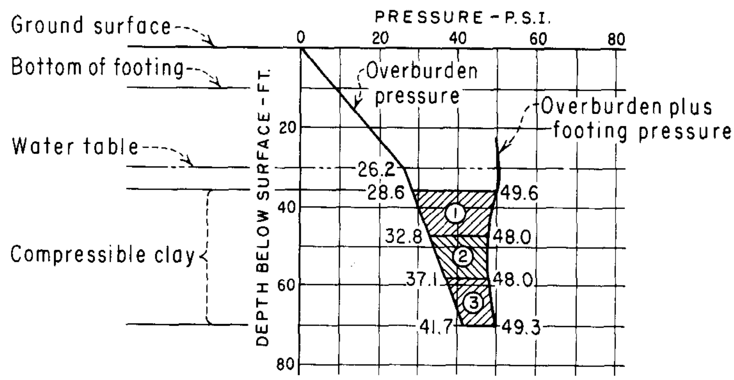


AVERAGE VERTICAL PRESSURE DISTRIBUTION

(FROM FIGURE 12)



LABORATORY TEST CURVE



OVERBURDEN AND STRUCTURAL PRESSURE DISTRIBUTION

(1) Depth 36'-47'

At $p_0 = 28.6$, $e_0 = .694$ At $p = 49.6$, $e = .651$

At $p_0 = 32.8$, $e_0 = .686$ At $p = 48.0$, $e = .652$

Ave. $e_0 = .690$ Ave. $e = .652$

$$S = 11 \frac{.690 - .652}{1.690} = .247 \text{ ft.}$$

(2) Depth 47'-58'

At $p_0 = 32.8$, $e_0 = .686$ At $p = 48.0$, $e = .652$

At $p_0 = 37.1$, $e_0 = .674$ At $p = 48.0$, $e = .652$

Ave $e_0 = .680$ Ave. $e = .652$

$$S = 11 \frac{.680 - .652}{1.680} = .183 \text{ ft.}$$

(3) Depth 58'-70'

At $p_0 = 37.1$, $e_0 = .674$ At $p = 48.0$, $e = .652$

At $p_0 = 41.7$, $e_0 = .666$ At $p = 49.3$, $e = .651$

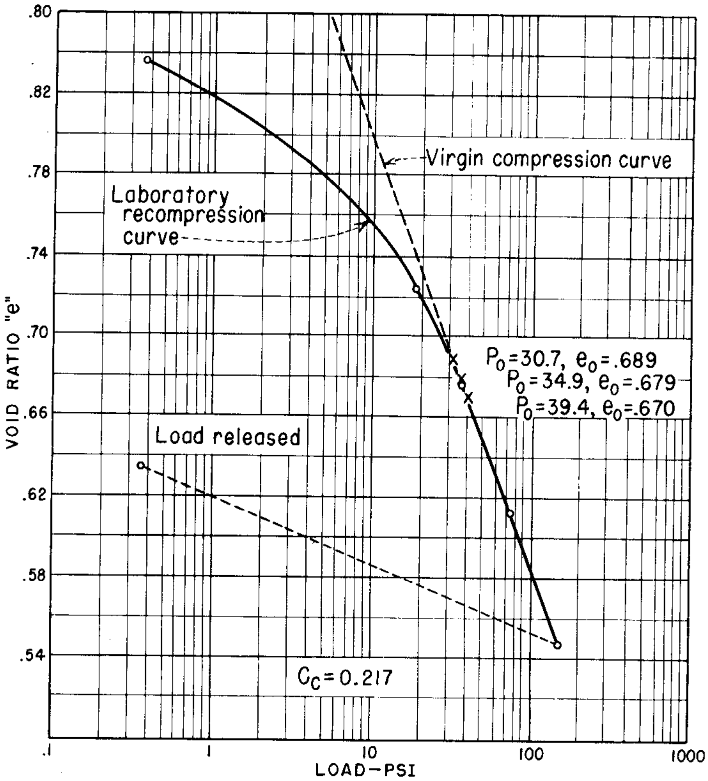
Ave $e_0 = .670$ Ave $e = .652$

$$S = 12 \frac{.670 - .652}{1.670} = .129 \text{ ft.}$$

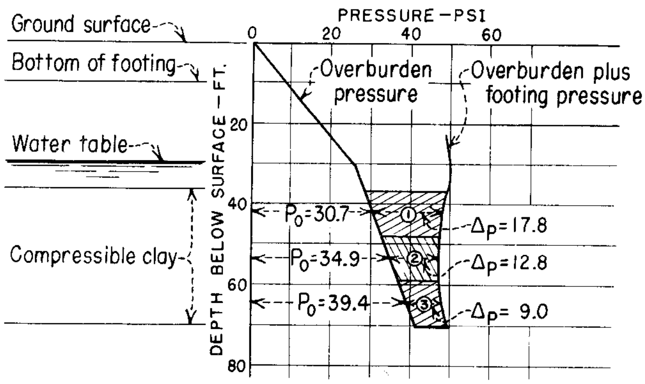
Total settlement = .559 ft.

SETTLEMENT COMPUTATIONS BY INCREMENTS OF DEPTH

FIGURE 14 - Settlement determination by change in void ratio method.



LABORATORY TEST CURVE
(FROM FIGURE 5)



OVERBURDEN AND STRUCTURAL PRESSURE DISTRIBUTION

FORMULA FOR COMPUTATION

$$S = H\,\frac{C_c}{1+e_0} \times \log \frac{P_0+\Delta_P}{P_0}$$

(1) $S = 11\,\frac{0.217}{1+.689} \times \log \frac{30.7+17.8}{30.7} = .281$ ft.

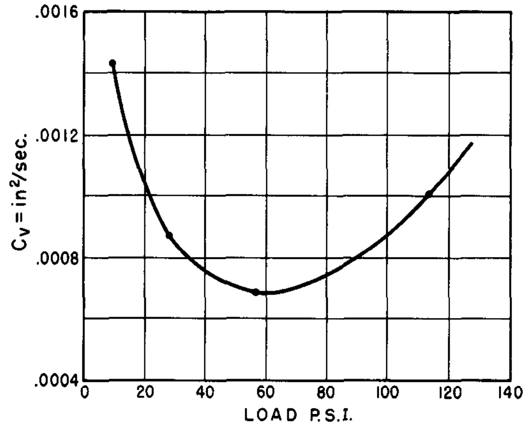
(2) $S = 11\,\frac{0.217}{1+.679} \times \log \frac{34.9+12.8}{34.9} = .193$ ft.

(3) $S = 12\,\frac{0.217}{1+.670} \times \log \frac{39.4+9.0}{39.4} = .139$ ft.

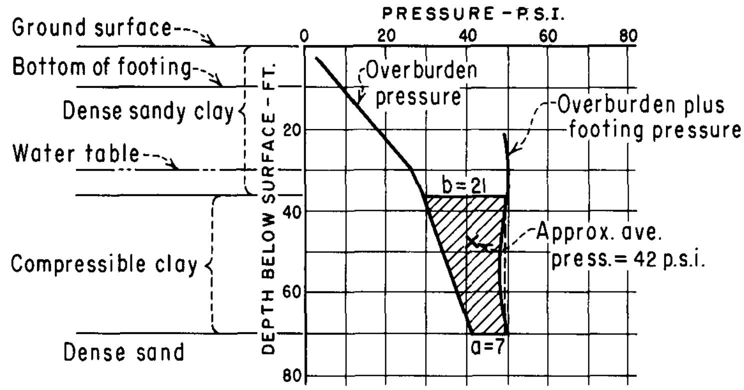
Total settlement = .613 ft.

SETTLEMENT COMPUTATIONS BY INCREMENTS OF DEPTH

FIGURE 15 - Settlement determination by compression index method.



LABORATORY TEST DATA
(FROM FIGURE 7)



For average pressure of 42 p.s.i.
$C_V = .00075\ in^2/sec.$

PRESSURE DISTRIBUTION

CASE No.1 - Drainage at top and bottom of stratum.

$$t\,(hrs.) = \frac{T_1 H^2}{C_V} = \frac{T_1\,(34/2)^2\,(144)}{.00075\,(3600)} = 15410\,T_1$$

% OF COMPLETION	T_1 (FIG. 8)	TIME		
		HOURS	YEARS	MONTHS
10	.0076	117.1		0.2
20	.032	493.1		0.7
30	.069	1063.3		1.5
40	.125	1926.3		2.7
50	.200	3082.0		4.3
60	.290	4468.9		6.2
70	.405	6241.0		8.7
80	.570	8783.7	1.0	
90	.850	13098.5	1.5	

CASE No.4 - Assuming that the upper stratum is of a different material and does not permit drainage. Therefore, drainage is only at the bottom.

$$u = a/b = 7/21 = .33, \text{ then } J\ (\text{Fig.9}) = .53$$

$$T_4 = T_1 + J\,(T_2 - T_1)$$

$$t\,(hrs.) = \frac{T_4 (H)^2}{C_V} = \frac{T_4\,(34)^2\,(144)}{.00075\,(3600)} = 61650\,T_4$$

% OF COMPLETION	T_1	T_2	$T_2 - T_1$	$J(T_2 - T_1)$	T_4	TIME		
						HOURS	YEARS	MONTHS
10	.0076	.049	.0414	.022	.030	1849.5		2.6
20	.032	.099	.067	.036	.068	4192.2		5.8
30	.069	.159	.090	.048	.117	7213.0		10.0
40	.125	.222	.097	.051	.176	10850.4	1.2	
50	.200	.296	.096	.051	.251	15474.2	1.8	
60	.290	.385	.095	.050	.340	20961.0	2.4	
70	.405	.500	.095	.050	.455	28050.7	3.2	
80	.570	.660	.090	.048	.618	38099.7	4.3	
90	.850	.960	.110	.058	.908	55978.2	6.4	

FIGURE 16 - Time of consolidation determination.

☆U.S. GOVERNMENT PRINTING OFFICE: 1993—838-494