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STRESSES AROUND A GALLERY — determined by the photoelastic interferometer

By H. B. Phillips and C. N. Zangar

Denver, Colorado
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United States Department of the Interior

OSCAR L. CHAPMAN, Secretary

Bureau of Reclamation

MICHAEL W. STRAUS, Commissioner
L. N. McCLELLAN, Chief Engineer

Engineering Monograph

No. 12

STRESSES AROUND A GALLERY —

determined by the photoelastic interferometer

by H. B. Phillips and C. N. Zangar
Photoelastic Unit
Dams Branch
Design and Construction Division

Technical Information Office
Denver Federal Center
Denver, Colorado

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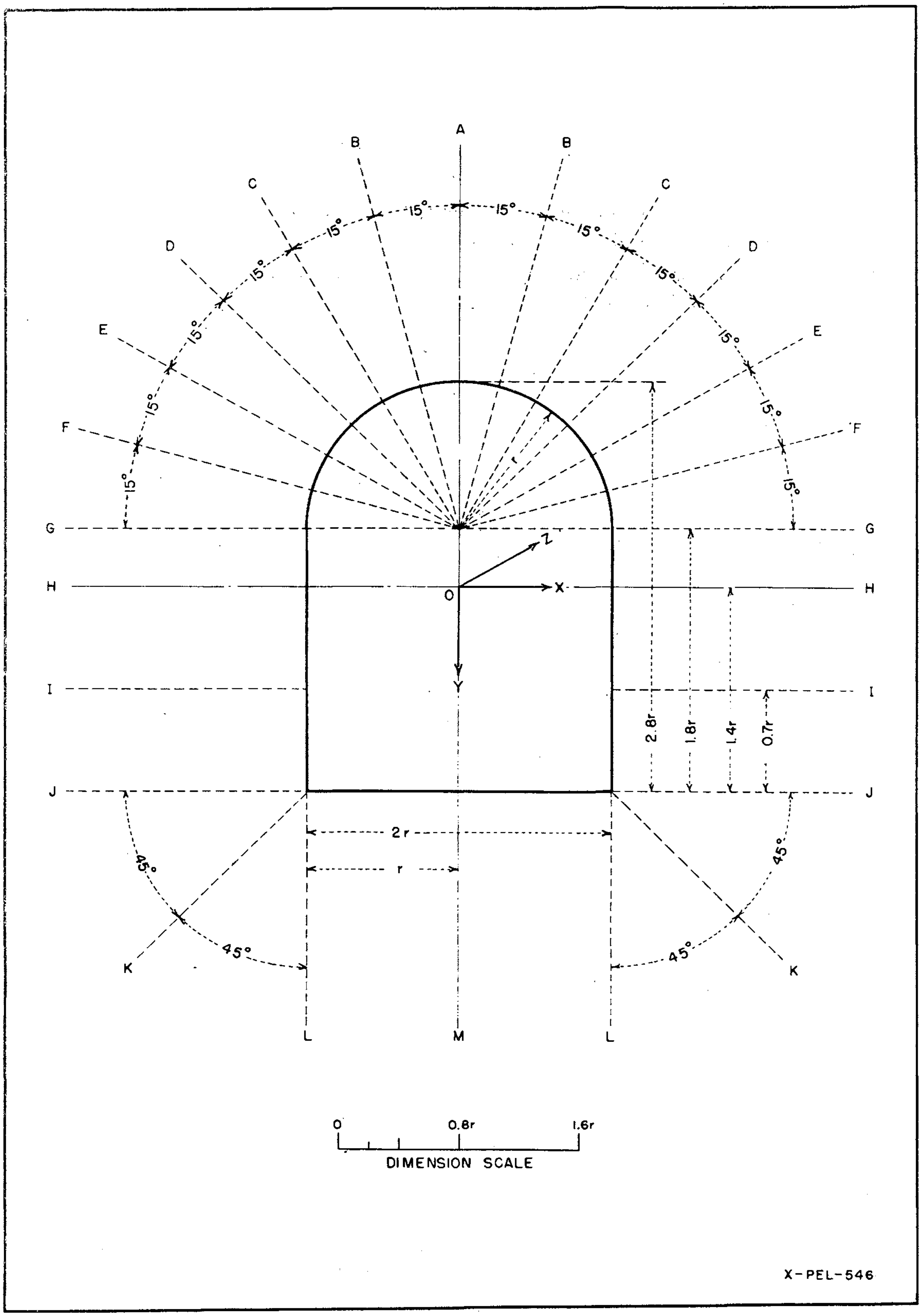


FIGURE 1 - Dimensions of gallery and location of reference lines.

INTRODUCTION

The stresses that exist around galleries and other openings are of great importance in the design of concrete dams. The quantity and location of steel reinforcement placed around these openings are dependent upon the magnitude and distribution of the stresses. Several analytical solutions[1,2,3,4,5]* give the stress distribution around various shaped openings in an infinite elastic medium, but none apply exactly to the gallery shape used in concrete dams. For this reason a photoelastic interferometer[6] study was undertaken to determine the stresses around the standard five-by seven-foot inspection gallery used in Bureau of Reclamation dams.

Stress coefficients were determined in the study at several points on various lines radiating from the gallery (Figure 1). These coefficients are expressed in terms of unit loads acting on the external boundaries of the model. Dimensions are expressed in terms of the crown radius of the gallery. These procedures permit the data to be used repeatedly on geometrically similar problems. The designer can determine the stresses around a gallery in a matter of minutes.

It is difficult, if not impossible, to determine the true effect of placing reinforcement steel in mass concrete. This paper is not concerned with that problem; however, a method is given for determining the approximate quantity of steel reinforcement needed.

No attempt has been made here to evaluate the effects of plastic flow, although boundary stresses determined photoelastically may be somewhat higher than actual stresses because of relief of stresses by plastic flow.[7]

DETAILS OF THE STUDY

Procedure

The gallery with its center at point O (Figure 1) was considered to be situated in an infinite elastic medium and subjected to a uniform stress field composed of the stresses initially present at point O. A coordinate system was selected (Figure 1) with the X-axis horizontal, the Y-axis vertical downward, and the Z-axis horizontal and parallel with the axis of the gallery. Since a slice of material of unit width normal to the Z-axis was used, the study is one of plane stress.

A gallery-shaped opening 0.714 inch wide and one inch high was milled in a plate of Allite (CR-39) five inches square and one-fourth inch thick. This model was placed in the interferometer** (Figures 2 and 3) and subjected to a uniform vertical load of intensity p. Principal stresses and their directions were read at several points on various lines radiating from the gallery, then resolved into normal, tangential, and shear stresses on the radiating lines as shown in Figures 6, 7, 8, and 9. Next, the gallery model was subjected to a uniform horizontal load of intensity p and again stresses were determined on the radiating lines as shown in Figures 10, 11, 12, and 13.

The gallery was then subjected to a uniform load of intensity p acting at 45 degrees with the vertical axis. Principal stresses and their directions were read at the same points on the radiating lines. The stresses existing around the gallery due to pure shear, initially acting at the gallery center, were determined by considering the load acting at 45 degrees on each side of the vertical axis of the gallery, with the applied load considered compressive on one side, tensile on the other. The sum of these two effects gave the pure shear situation. The results of this part of the study are shown in Figures 14, 15, 16, and 17.

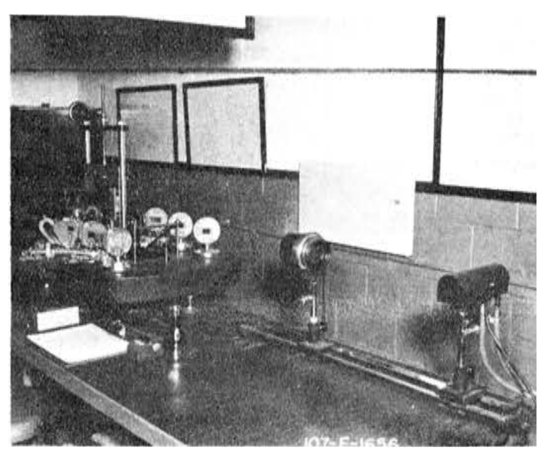


FIGURE 2 - Photoelastic interferometer.

*Superscripts refer to numbers in Bibliography.

**See Appendix for description of the instrument used in this study.

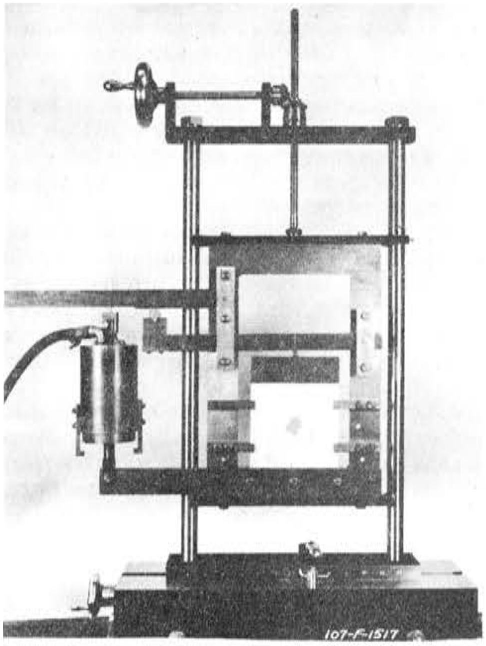


FIGURE 3 - Model and loading mechanism.

As a check on boundary stresses around the gallery opening, the model was placed in the photoelastic polariscope and the isochromatic stress pattern was photographed. Figure 4 is the stress pattern for the gallery with the load applied parallel to its vertical axis. Checks on the accuracy of the interferometer were also made by checking horizontal and vertical equilibrium from the measured stresses. Errors were found to be generally less than five percent and in many cases less than one percent.

The accuracy of the interferometer may also be checked, for problems to which both analytic and interferometer methods are applicable, by comparison of the two methods. This is shown in Figure 5, where analytic [1,2] and interferometer solutions are plotted together for the stresses around a circular opening contained in an infinite elastic medium subjected to a uniform stress field.

Conclusions

Stresses around a gallery resemble somewhat the stresses around a circular opening located in an infinite elastic medium subjected to a uniform stress field. They are, however, different in magnitude and distribution.

When the gallery is subjected to a vertical compressive stress field of intensity p, then (Figure 6):

(a) The boundary tensions at the top and bottom of the gallery are equal to -1.00 p.

(b) The boundary compressive stress near the spring line of the top arch is equal to 2.75 p, while the vertical boundary compressive stress at the center of the gallery side is equal to 2.18 p.

(c) Corner stresses are approximately equal to 3.30 p.

When the gallery is subjected to a uniform horizontal compressive stress field of intensity p then (Figure 10):

(a) The boundary tensions at the center point of the sides are equal to -1.10 p.

(b) The boundary compression at the top of the gallery is equal to 4.00 p, and the boundary compression at the midpoint of the base of the gallery is equal to 2.72 p.

(c) Corner stresses are approximately equal to 3.30 p.

When the gallery is subjected to a pure shear of intensity p then (Figure 14):

(a) A maximum boundary tension of -6.12 p occurs at a point midway between the crown and the spring line of the gallery and a tension of -7.00 p occurs at the sharp reentrant corner.

(b) Maximum boundary compressions are the same magnitude as the tensions, and occur at corresponding points on the opposite side of the vertical centerline of the gallery.

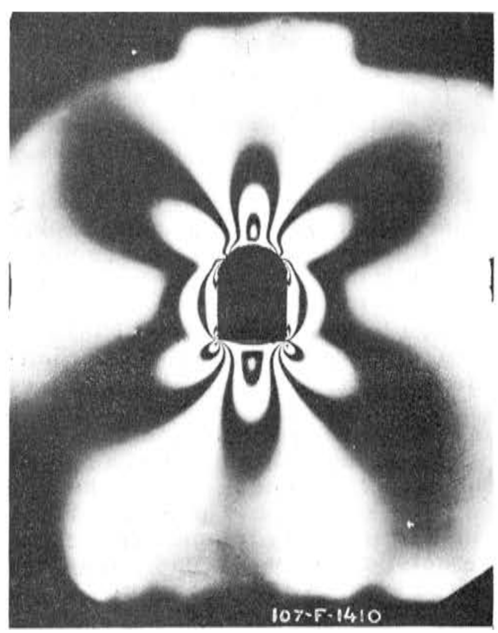


FIGURE 4 - Isochromatic photograph of uniform load applied parallel to vertical axis of gallery.

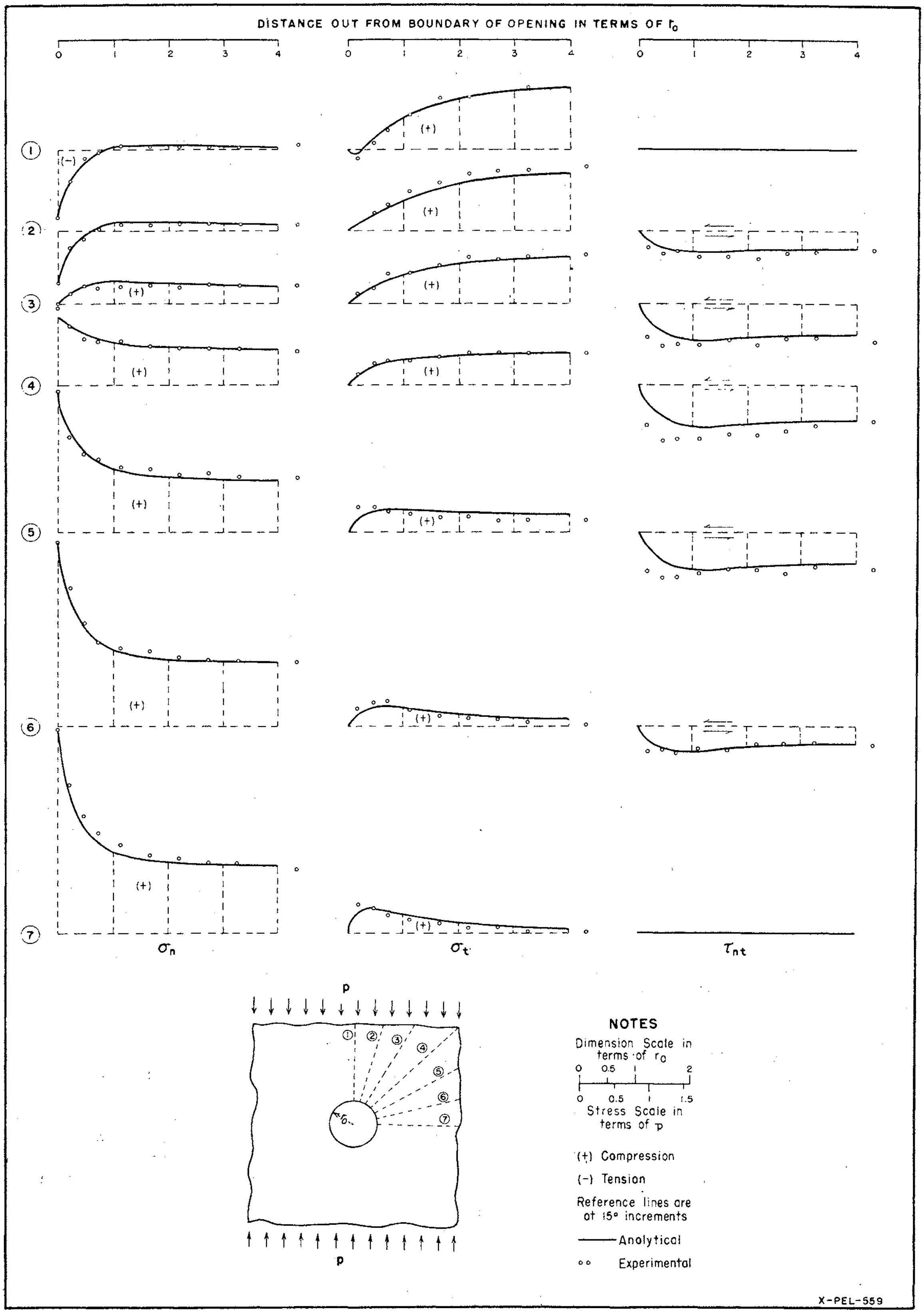


FIGURE 5 - Comparison of analytical solution with experimental results, for stress distribution around a circular opening in an infinite elastic plate.

NOTATION

p = magnitude of stress field initially acting at the center of the gallery in pounds per square inch

A, B. . . M = designation of lines radiating from the gallery face along which have been determined normal, tangential, and shear stresses.

σ_n = stress normal to designated lines

σ_t = stress tangential to designated lines

τ_{nt} = shear stress along designated lines

r = crown radius of gallery

σ_x = horizontal stress initially acting at the gallery center

σ_y = vertical stress initially acting at the gallery center

τ_{xy} = shear stress initially acting at the gallery center

σ_1 = maximum principal stress

σ_2 = minimum principal stress

α = direction of principal stress

+ = compression stress

− = tensile stress

APPLICATION OF THE DATA

Since both unit loads and unit dimensions have been used, and all dimensions of the gallery are expressed in terms of the crown radius r, the study presented here has very general application to similar problems. The data may be used directly to determine the complete state of stress around a gallery if the normal and shear stresses σ_x, σ_y and τ_{nt} are given at the point in the structure where the gallery is to be located[8]. If the principal stresses and their directions, σ_1, σ_2, and α only are given, they must be rotated into equivalent normal and shear stresses. In either case the designer need only multiply the stress coefficients given on the figures by the stresses existing initially at the center of the gallery and sum the separate stress effects. Example:

Assume a standard five-by seven-foot gallery with a crown radius of 2-1/2 feet with initial stresses at the gallery center of--

$$\sigma_x = +\ 75 \text{ psi}$$

$$\sigma_y = +\ 200 \text{ psi}$$

$$\tau_{xy} = +\ 40 \text{ psi}$$

Compute the stresses on Lines A, D, and H on the left side of the gallery.

Table I gives the tabulation for σ_n.

Table II gives the tabulation for σ_t.

Table III gives the tabulation for τ_{nt}.

The required reinforcement steel can be found by determining the tension area on a particular line, then dividing the total tensile force so determined by the allowable steel stress.

Acknowledgement

The assistance of Ira E. Allen and Norman S. Johnson in the experimental work and the preparation of the drawings is acknowledged.

TABLE I

Tabulation for σ_n on lines A, D, and H on left side of gallery (in Example)

Line	Distance out in feet	Stress due to σ_x	Stress due to σ_y	Stress due to τ_{xy}	σ_n
A	0	+300	-200	0	+100
	0.5	+212	- 68	0	+144
	1.0	+165	- 20	0	+145
	2.0	+114	0	0	+114
	3.0	+ 99	+ 4	0	+103
	4.0	+ 92	+ 10	0	+102
D	0	+ 91	+174	-245	+ 20
	0.5	+ 82	+158	-106	+134
	1.0	+ 74	+146	- 87	+133
	2.0	+ 64	+134	- 65	+133
	3.0	+ 58	+132	- 52	+138
	4.0	+ 54	+126	- 41	+139
H	0	- 81	+436	- 18	+337
	0.5	- 32	+378	+ 4	+350
	1.0	- 15	+336	+ 10	+331
	2.0	+ 7	+286	+ 13	+306
	3.0	+ 20	+258	+ 11	+289
	4.0	+ 26	+234	+ 9	+269

TABLE II

Tabulation of σ_t on lines A, D, and H on left side of gallery (in Example)

Line	Distance out in feet	Stress due to σ_x	Stress due to σ_y	Stress due to τ_{xy}	σ_t
A	0	0	0	0	0
	0.5	+29	+ 4	0	+ 33
	1.0	+38	+ 26	0	+64
	2.0	+36	+ 76	0	+112
	3.0	+24	+118	0	+142
	4.0	+17	+142	0	+159
D	0	0	0	0	0
	0.5	+17	+ 28	-15	+ 30
	1.0	+23	+ 46	-12	+ 57
	2.0	+28	+ 64	- 1	+ 91
	3.0	+31	+ 76	+ 6	+113
	4.0	+32	+ 82	+12	+126
H	0	0	0	0	0
	0.5	- 6	+ 18	-25	- 13
	1.0	0	+ 22	-26	- 4
	2.0	+12	+ 28	-26	+ 14
	3.0	+25	+ 34	-23	+ 36
	4.0	+35	+ 38	-17	+ 56

TABLE III

Tabulation of τ_{nt} on lines A, D, and H on left side of gallery (in Example)

Line	Distance out in feet	Stress due to σ_x	Stress due to σ_y	Stress due to τ_{xy}	τ_{nt}
A	0	0	0	0	0
	0.5	0	0	- 8	- 8
	1.0	0	0	-15	-15
	2.0	0	0	-30	-30
	3.0	0	0	-42	-42
	4.0	0	0	-55	-55
D	0	0	0	0	0
	0.5	-45	+ 72	+ 2	+29
	1.0	-53	+120	- 2	+65
	2.0	-57	+132	- 6	+69
	3.0	-57	+124	- 4	+63
	4.0	-57	+116	- 3	+56
H	0	0	0	0	0
	0.5	- 4	0	+12	+ 8
	1.0	- 5	0	+22	+17
	2.0	- 5	0	+37	+32
	3.0	- 5	0	+48	+43
	4.0	- 5	0	+55	+50

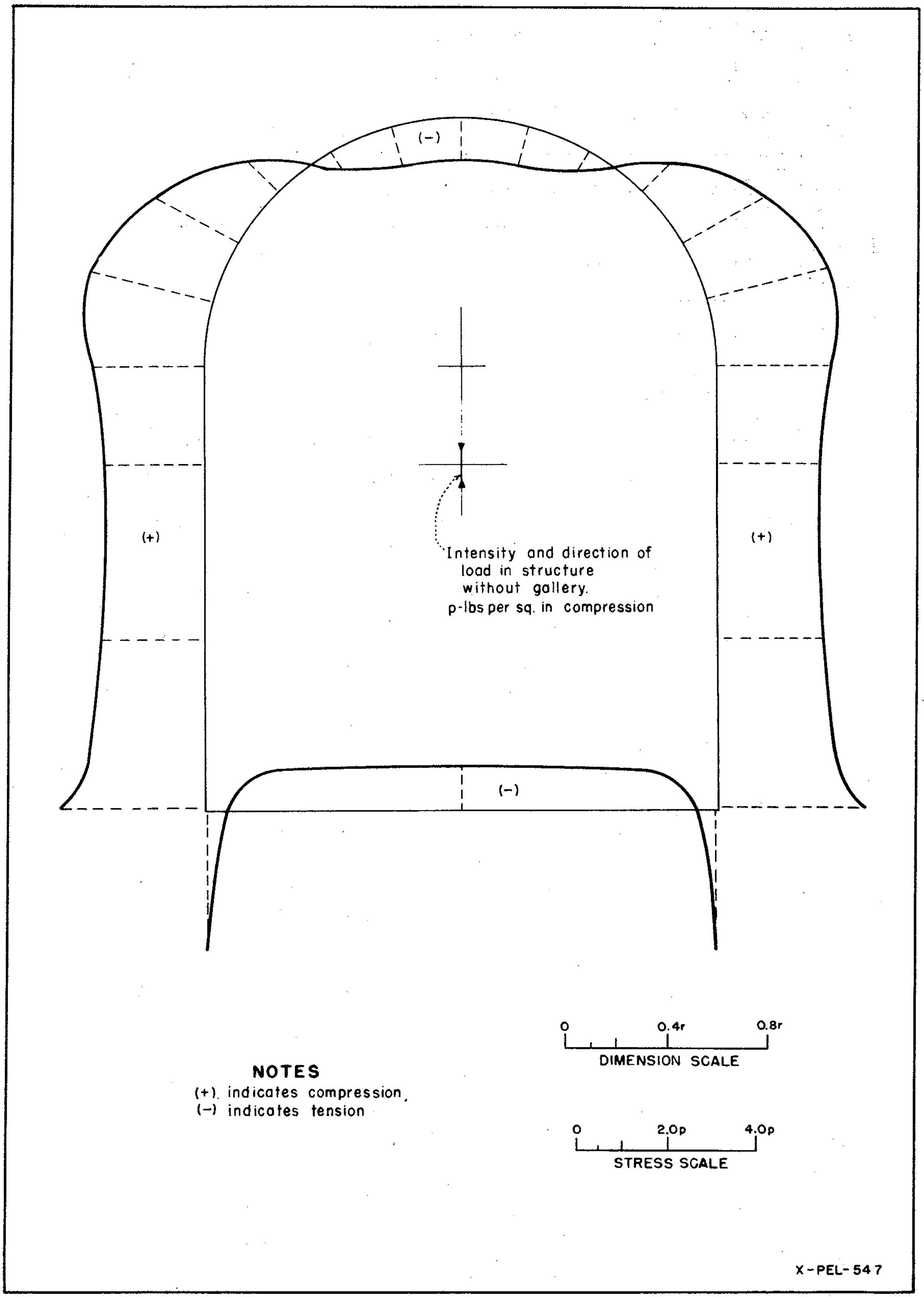
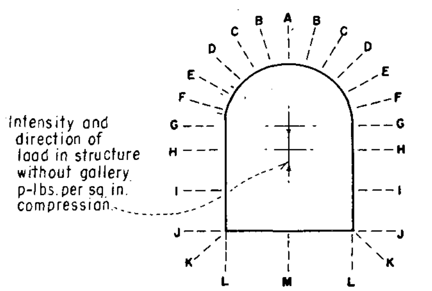


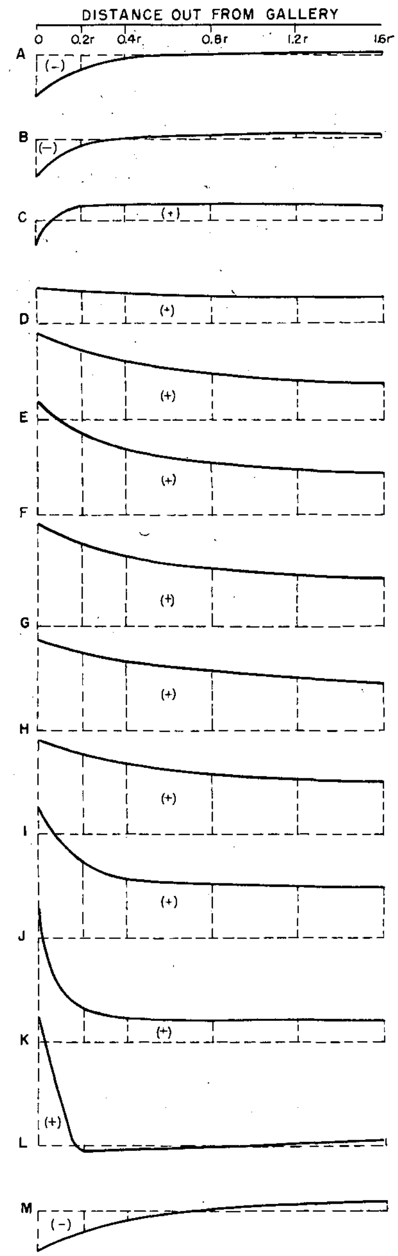
FIGURE 6 - Uniform load applied parallel to vertical axis of gallery, principal stress at boundary of gallery.

STRESS COEFFICIENTS IN TERMS OF p

LINES	DISTANCE OUT FROM GALLERY					
	0	0.2r	0.4r	0.8r	1.2r	1.6r
A	-1.00	-0.34	-0.10	0	+0.02	+0.05
B	-0.94	-0.11	+0.01	+0.08	+0.11	+0.12
C	-0.55	+0.34	+0.39	+0.41	+0.39	+0.36
D	+0.87	+0.79	+0.73	+0.67	+0.66	+0.63
E	+2.10	+1.62	+1.38	+1.10	+0.97	+0.89
F	+2.75	+1.96	+1.58	+1.27	+1.10	+1.04
G	+2.50	+2.01	+1.70	+1.42	+1.28	+1.20
H	+2.18	+1.89	+1.68	+1.43	+1.29	+1.17
I	+2.26	+1.92	+1.72	+1.46	+1.31	+1.26
J	+3.20	+1.82	+1.44	+1.32	+1.22	+1.22
K	+3.20	+0.80	+0.59	+0.53	+0.52	+0.52
L	+3.20	-0.12	-0.10	-0.04	+0.04	+0.13
M	-1.00	-0.53	-0.22	0	+0.15	+0.21



LOCATION OF REFERENCE LINES



NOTES

(+) indicates compression
(-) indicates tension

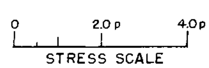


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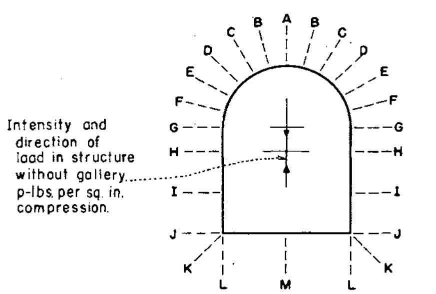
FIGURE 7 - Uniform load applied parallel to vertical axis of gallery, stress normal to reference lines.

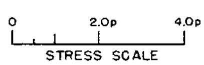
STRESS COEFFICIENTS IN TERMS OF p

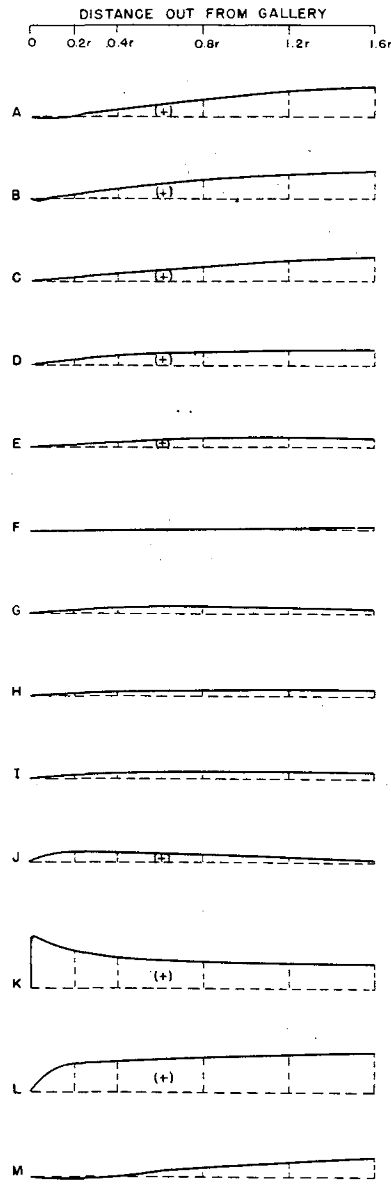
LINES	DISTANCE OUT FROM GALLERY					
	0	0.2r	0.4r	0.8r	1.2r	1.6r
A	0	+0.02	+0.13	+0.38	+0.59	+0.71
B	0	+0.14	+0.27	+0.45	+0.59	+0.67
C	0	+0.10	+0.20	+0.37	+0.50	+0.59
D	0	+0.14	+0.23	+0.32	+0.38	+0.41
E	0	+0.11	+0.17	+0.22	+0.24	+0.25
F	0	0	+0.01	+0.03	+0.04	+0.06
G	0	+0.14	+0.19	+0.21	+0.18	+0.14
H	0	+0.09	+0.11	+0.14	+0.17	+0.19
I	0	+0.09	+0.14	+0.18	+0.18	+0.18
J	0	+0.26	+0.24	+0.21	+0.12	+0.10
K	0	+0.92	+0.73	+0.68	+0.64	+0.61
L	0	+0.72	+0.78	+0.88	+0.92	+1.00
M	0	-0.03	0	+0.21	+0.37	+0.47



LOCATION OF REFERENCE LINES

NOTES
(+) indicates compression
(−) indicates tension



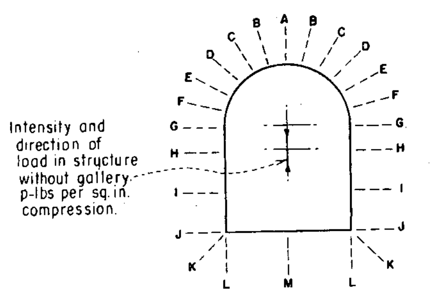


X-PEL-549

FIGURE 8 - Uniform load applied parallel to vertical axis of gallery, stress tangential to reference lines.

STRESS COEFFICIENTS
IN TERMS OF p

LINES	DISTANCE OUT FROM GALLERY					
	0	0.2r	0.4r	0.8r	1.2r	1.6r
A	0	0	0	0	0	0
B	0	+0.38	+0.40	+0.36	+0.32	+0.29
C	0	+0.51	+0.55	+0.56	+0.57	+0.58
D	0	+0.36	+0.60	+0.66	+0.62	+0.58
E	0	+0.28	+0.42	+0.56	+0.59	+0.54
F	0	+0.24	+0.32	+0.37	+0.38	+0.39
G	0	+0.22	+0.18	+0.07	+0.02	0
H	0	0	0	0	0	0
I	0	0	0	0	0	0
J	0	+0.05	+0.04	+0.04	+0.04	+0.04
K	0	-0.72	-0.70	-0.68	-0.66	-0.64
L	0	-0.15	-0.27	-0.35	-0.27	0
M	0	0	0	0	0	0



LOCATION OF REFERENCE LINES

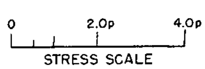
NOTES

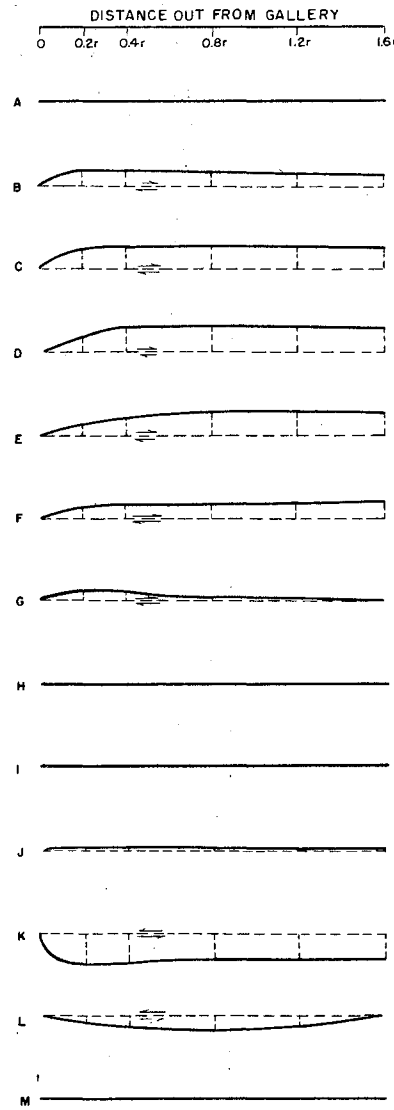
Sign convention for left half.

(+)

(-)

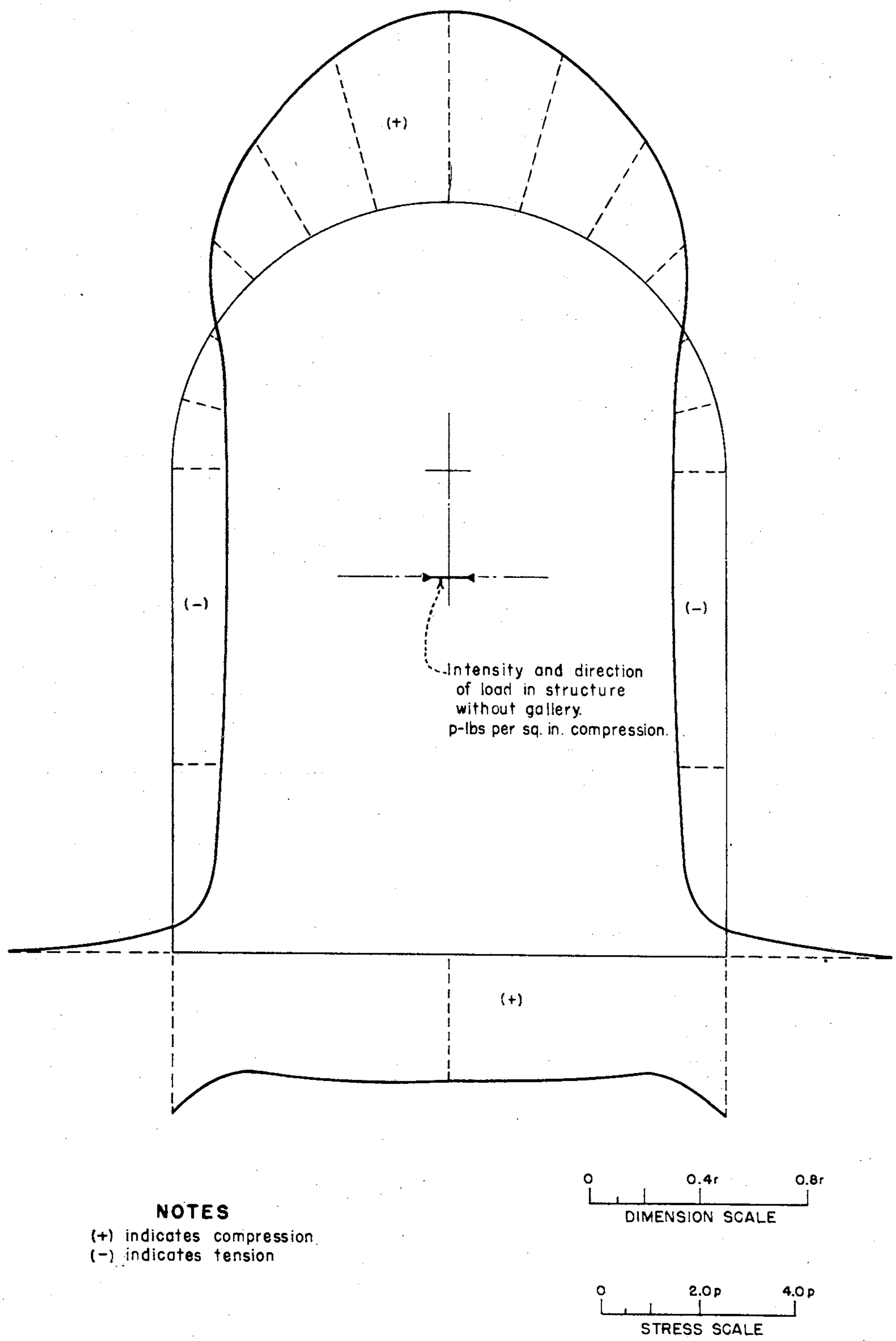
Shear stresses in right half are equal in magnitude to those in left half, but opposite in sign.





X-PEL-550

FIGURE 9 - Uniform load applied parallel to vertical axis of gallery, shear stress along reference lines.



NOTES

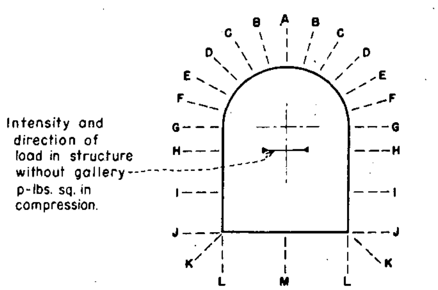
(+) indicates compression
(-) indicates tension

X-PEL-551

FIGURE 10 - Uniform load applied normal to vertical axis of gallery, principal stress at boundary of gallery.

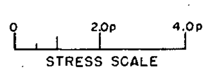
STRESS COEFFICIENTS IN TERMS OF p

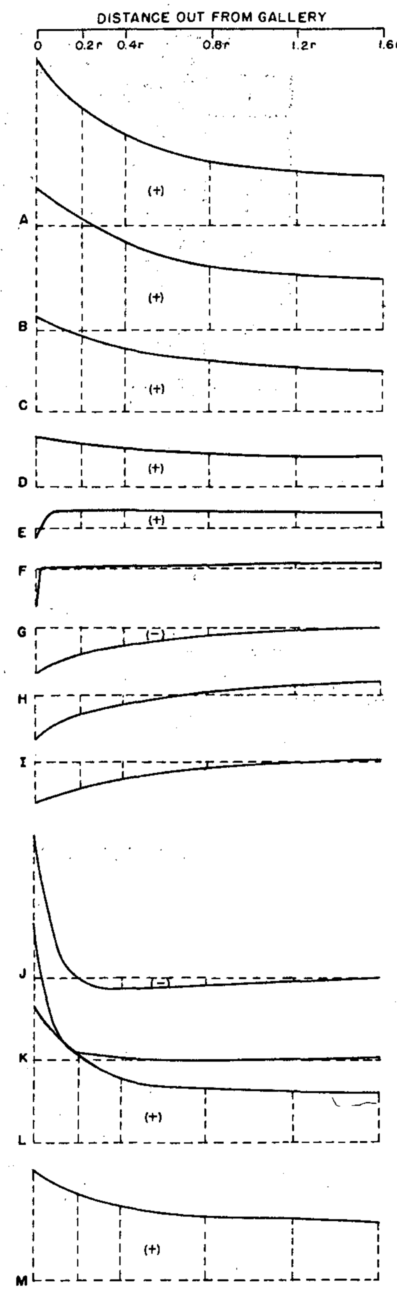
LINES	DISTANCE OUT FROM GALLERY					
	0	0.2r	0.4r	0.8r	1.2r	1.6r
A	+4.00	+2.83	+2.20	+1.52	+1.32	+1.23
B	+3.42	+2.68	+2.13	+1.52	+1.33	+1.21
C	+2.33	+1.85	+1.53	+1.22	+1.08	+0.99
D	+1.21	+1.09	+0.99	+0.85	+0.77	+0.72
E	-0.24	+0.41	+0.40	+0.39	+0.38	+0.37
F	-0.91	+0.06	+0.07	+0.09	+0.11	+0.12
G	-1.10	-0.60	-0.40	-0.17	-0.04	+0.03
H	-1.08	-0.43	-0.20	+0.09	+0.26	+0.34
I	-0.98	-0.64	-0.39	-0.11	0	+0.04
J	+3.40	-0.05	-0.22	-0.18	-0.10	+0.02
K	+3.40	+0.20	+0.09	+0.05	+0.05	+0.05
L	+3.40	+2.20	+1.57	+1.33	+1.29	+1.23
M	+2.72	+2.09	+1.86	+1.57	+1.50	+1.40



LOCATION OF REFERENCE LINES

NOTES
(+) indicates compression
(−) indicates tension



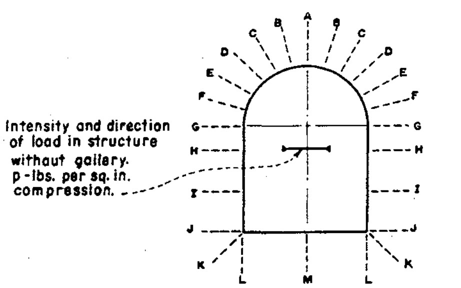


X-PEL-552

FIGURE 11 - Uniform load applied normal to vertical axis of gallery, stress normal to reference lines.

STRESS COEFFICIENTS IN TERMS OF p

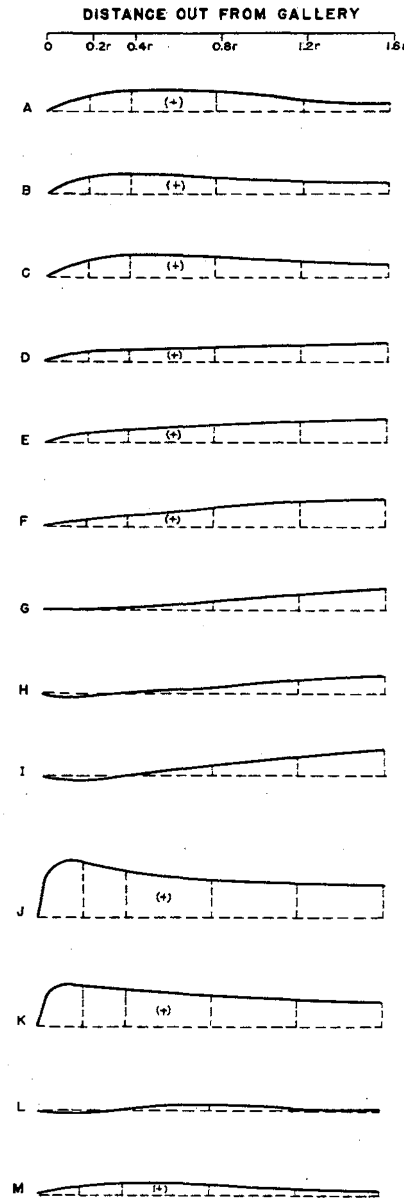
LINES	DISTANCE OUT FROM GALLERY					
	0	0.2r	0.4r	0.8r	1.2r	1.6r
A	0	+0.38	+0.51	+0.48	+0.32	+0.23
B	0	+0.43	+0.52	+0.42	+0.34	+0.29
C	0	+0.43	+0.55	+0.50	+0.40	+0.34
D	0	+0.23	+0.31	+0.37	+0.41	+0.43
E	0	+0.23	+0.32	+0.45	+0.53	+0.59
F	0	+0.19	+0.28	+0.46	+0.61	+0.68
G	0	+0.05	+0.12	+0.27	+0.41	+0.54
H	0	-0.08	0	+0.16	+0.33	+0.47
I	0	-0.11	-0.01	+0.23	+0.44	+0.61
J	0	+1.35	+1.13	+0.95	+0.89	+0.87
K	0	+1.00	+0.92	+0.79	+0.70	+0.66
L	0	-0.04	+0.05	+0.16	+0.10	+0.07
M	0	+0.20	+0.27	+0.25	+0.14	+0.10



LOCATION OF REFERENCE LINES

NOTES
(+) Indicates compression
(−) Indicates tension

STRESS SCALE

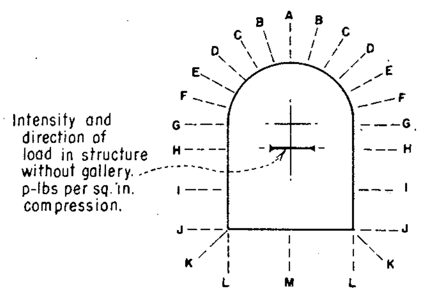


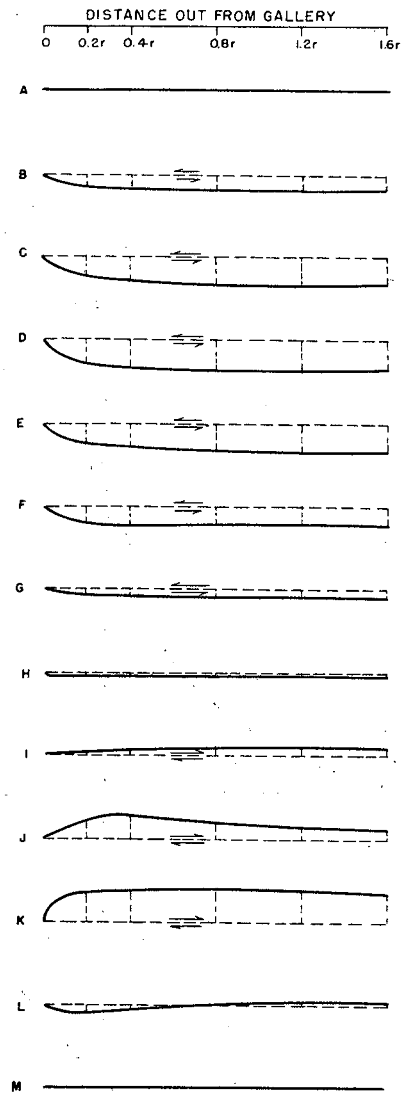
X-PEL-553

FIGURE 12 - Uniform load applied normal to vertical axis of gallery, stress tangential to reference lines.

STRESS COEFFICIENTS IN TERMS OF p

LINES	DISTANCE OUT FROM GALLERY					
	0	0.2r	0.4r	0.8r	1.2r	1.6r
A	0	0	0	0	0	0
B	0	-0.30	-0.32	-0.34	-0.35	-0.36
C	0	-0.47	-0.55	-0.63	-0.66	-0.65
D	0	-0.60	-0.70	-0.76	-0.76	-0.76
E	0	-0.50	-0.57	-0.66	-0.69	-0.70
F	0	-0.42	-0.47	-0.46	-0.44	-0.43
G	0	-0.20	-0.20	-0.20	-0.20	-0.20
H	0	-0.05	-0.06	-0.06	-0.06	-0.06
I	0	+0.09	+0.14	+0.19	+0.19	+0.17
J	0	+0.42	+0.56	+0.42	+0.28	+0.22
K	0	+0.70	+0.79	+0.80	+0.76	+0.70
L	0	-0.18	-0.17	-0.01	+0.03	+0.05
M	0	0	0	0	0	0





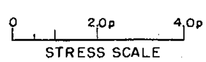
NOTES

Sign convention for left half.

(+)

(-)

Shear stresses in right half are equal in magnitude to those in left half, but opposite in sign.



X-PEL-554

FIGURE 13 - Uniform load applied normal to vertical axis of gallery, shear stress along reference lines.

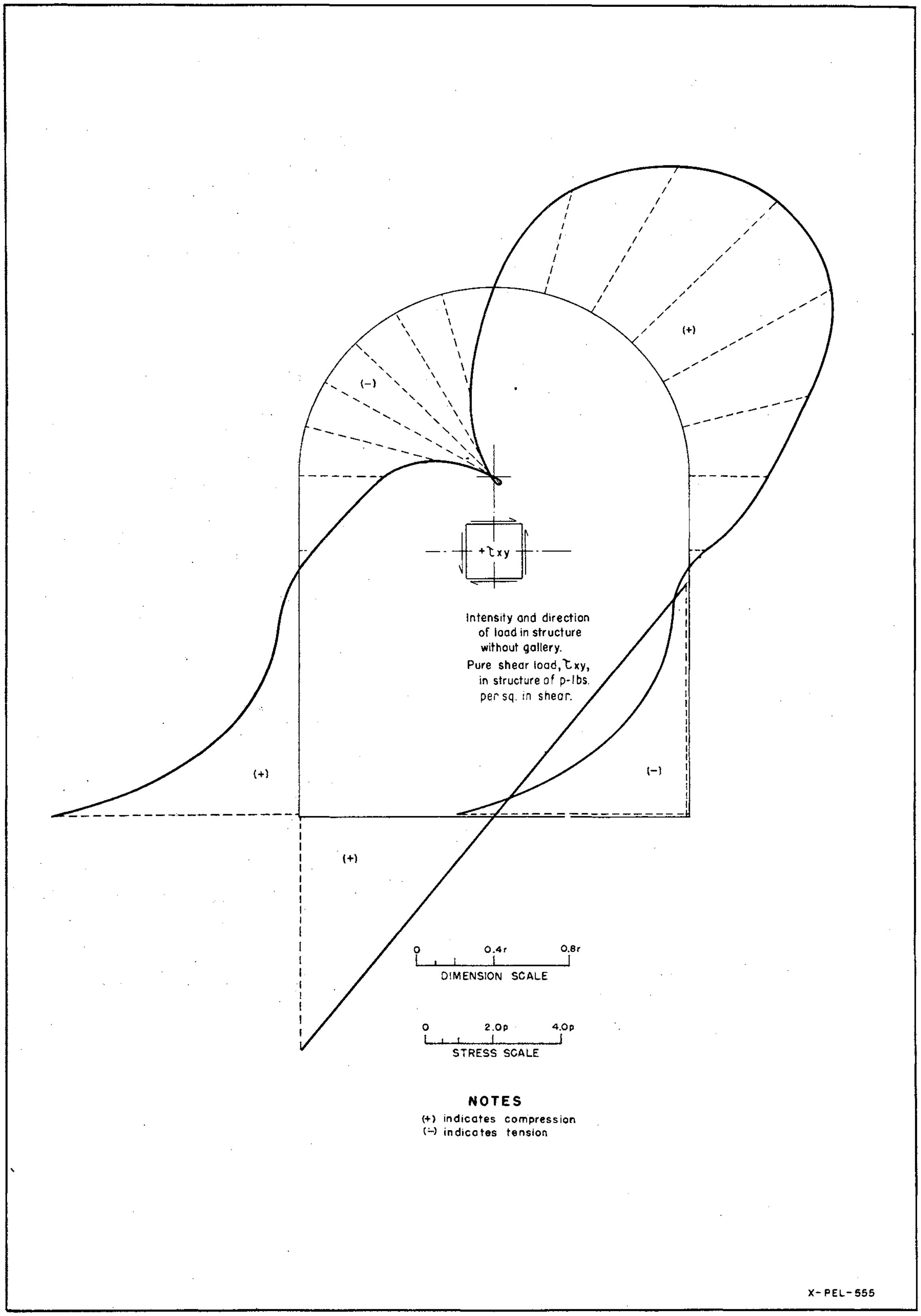
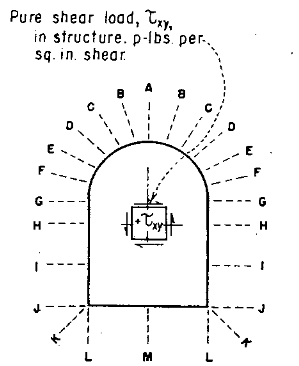


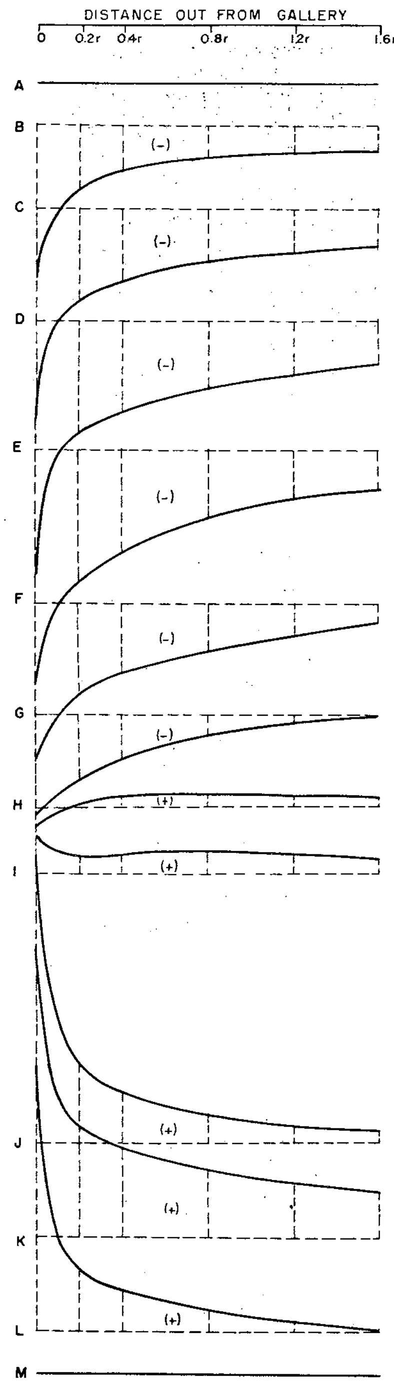
FIGURE 14 - Pure shear load, τ_{xy}, applied to gallery, principal stress at boundary of gallery.

STRESS COEFFICIENTS IN TERMS OF p

LINES	DISTANCE OUT FROM GALLERY					
	0	0.2r	0.4r	0.8r	1.2r	1.6r
A	0	0	0	0	0	0
B	-3.27	-1.51	-1.10	-0.78	-0.68	-0.65
C	-5.18	-2.20	-1.74	-1.27	-1.07	-0.91
D	-6.12	-2.65	-2.17	-1.62	-1.29	-1.03
E	-5.72	-3.18	-2.49	-1.61	-1.13	-0.95
F	-3.77	-2.17	-1.67	-1.11	-0.75	-0.47
G	-2.43	-1.40	-1.08	-0.44	-0.14	-0.02
H	-0.45	+0.10	+0.26	+0.33	+0.28	+0.23
I	+1.04	+0.45	+0.50	+0.52	+0.45	+0.34
J	+7.00	+1.95	+1.29	+0.71	+0.42	+0.29
K	+7.00	+2.68	+2.20	+1.67	+1.36	+1.16
L	+7.00	+1.50	+1.04	+0.56	+0.26	+0.07
M	0	0	0	0	0	0

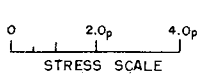


LOCATION OF REFERENCE LINES



NOTES

(+) Indicates compression
(–) Indicates tension
Stresses in right half are equal in magnitude to those in left half but opposite in sign.

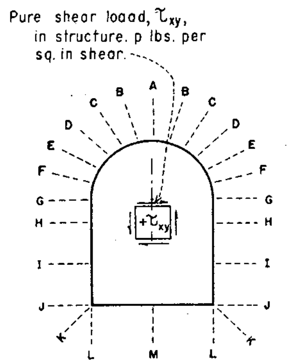


X-PEL-556

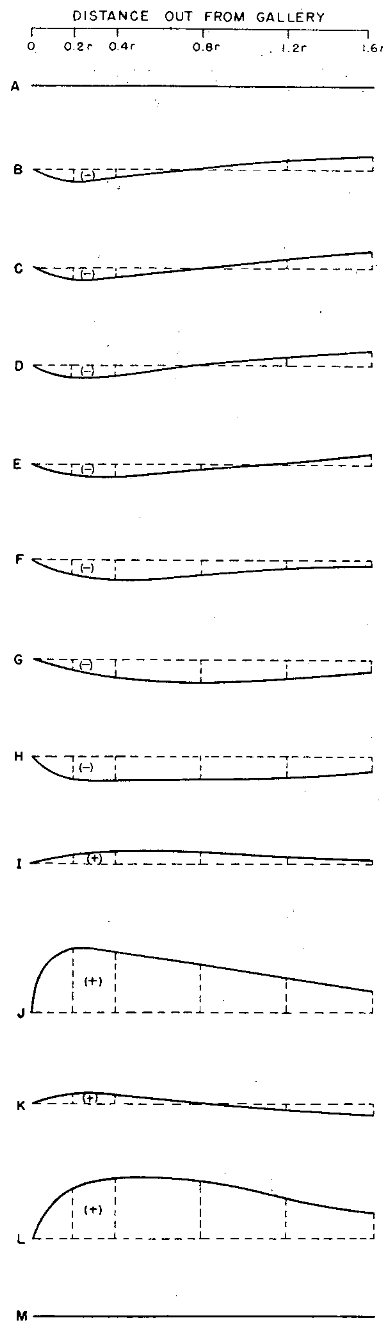
FIGURE 15 - Pure shear load, τ_{xy}, applied to gallery, stress normal to reference lines.

STRESS COEFFICIENTS IN TERMS OF p

LINES	DISTANCE OUT FROM GALLERY					
	0	0.2r	0.4r	0.8r	1.2r	1.6r
A	0	0	0	0	0	0
B	0	-0.33	-0.24	-0.03	+0.15	+0.28
C	0	-0.34	-0.26	-0.06	+0.17	+0.35
D	0	-0.37	-0.31	-0.03	+0.15	+0.30
E	0	-0.31	-0.34	-0.20	-0.02	+0.20
F	0	-0.39	-0.49	-0.39	-0.26	-0.19
G	0	-0.32	-0.49	-0.57	-0.50	-0.35
H	0	-0.63	-0.65	-0.64	-0.57	-0.43
I	0	+0.18	+0.26	+0.25	+0.15	+0.07
J	0	+1.55	+1.46	+1.12	+0.77	+0.47
K	0	+0.21	+0.19	-0.01	-0.19	-0.30
L	0	+1.21	+1.42	+1.35	+0.94	+0.56
M	0	0	0	0	0	0

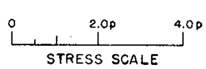


LOCATION OF REFERENCE LINES



NOTES

(+) Indicates compression
(−) Indicates tension
Stresses in right half are equal in magnitude to those in left half, but opposite in sign.

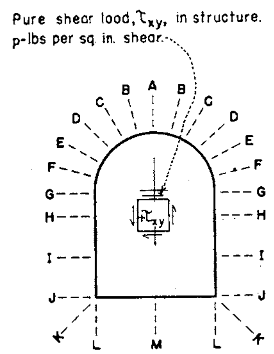


X-PEL-557

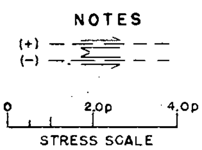
FIGURE 16 - Pure shear load, τ_{xy}, applied to gallery, stress tangential to reference lines.

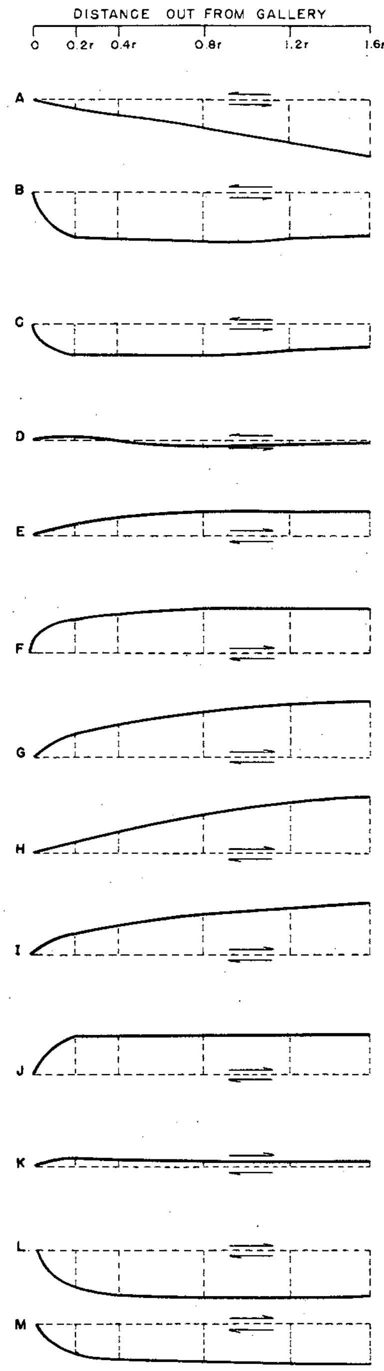
STRESS COEFFICIENTS IN TERMS OF p

LINES	DISTANCE OUT FROM GALLERY					
	0	0.2r	0.4r	0.8r	1.2r	1.6r
A	0	-0.20	-0.38	-0.74	-1.06	-1.38
B	0	-1.15	-1.20	-1.24	-1.21	-1.14
C	0	-0.74	-0.77	-0.77	-0.70	-0.59
D	0	+0.06	-0.06	-0.14	-0.11	-0.08
E	0	+0.25	+0.37	+0.49	+0.51	+0.51
F	0	+0.79	+0.90	+1.00	+1.02	+1.00
G	0	+0.53	+0.78	+1.06	+1.26	+1.34
H	0	+0.30	+0.54	+0.92	+1.21	+1.37
I	0	+0.45	+0.64	+0.88	+1.05	+1.17
J	0	+0.84	+0.91	+0.92	+0.92	+0.91
K	0	+0.14	+0.15	+0.11	+0.06	+0.06
L	0	-0.91	-1.08	-1.15	-1.15	-1.12
M	0	-0.72	-0.82	-0.88	-0.92	-0.96



LOCATION OF REFERENCE LINES





X-PEL-558

FIGURE 17 - Pure shear load, τ_{xy}, applied to gallery, shear stress along reference lines.

Photoelastic stress analysis is ordinarily made by means of the polariscope in conjunction with other experimental or analytical methods. The polariscope alone, however, gives only the boundary principal stresses and the maximum shears at all points in a stressed, transparent plastic model subjected to polarized light. But with the photoelastic interferometer it is possible to measure the magnitudes of the separate principal stresses and their directions at a point in a transparent, stressed, two-dimensional structural model.

Henry Favre [9] brought this instrument to the attention of the scientific world in 1929. Basically the instrument measures the absolute retardation of a beam of light passing through a transparent model subjected to a change in stress. The Bureau's instrument (Figure 2) was patterned after the one built by Favre. This instrument should not be confused with lateral extensometers employing the interferometer principle in their operation.

Figure 18 is a diagrammatic layout of the interferometer. The light source at S is a high-intensity, water-cooled, mercury-vapor lamp. A monochromatic point source of polarized light is obtained by filtering, then passing the light through an aperture 1/128 of an inch in diameter and through a polaroid. The light next passes through a projection lens so located that it brings the light to focus in the model. After passing through the projection lens the light strikes the optical parallel L_1, which has a thin coating of aluminum on one face.

L_1 splits the light into two paths of equal intensity. One path, called the inside path, goes to the full reflecting optical flat L_3, through the stressed model and then to L_6. L_6 is an optical parallel like L_1 which both transmits and reflects light. A second path, called the outside path, goes from L_1 to the fully reflecting optical flat L_4 and then to L_6. The interference phenomenon is developed at L_6 where the two paths converge, and is visible in the eyepiece as a series of alternate light and dark bands called interference fringes. These fringes are entirely different from the familiar isochromatic fringes of the polariscope, particularly in that they are visible both before and after loading the model. The interference fringes are merely shifted by a change in stress within the model, and the change in stress is linearly related to the number of fringes that cross a fixed point in the field of the eyepiece.

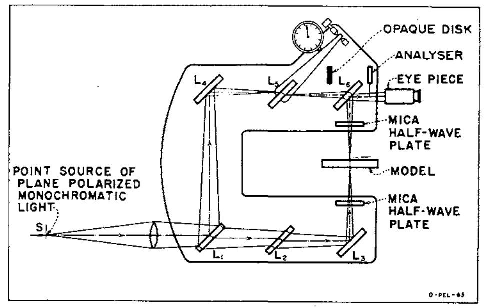


FIGURE 18 - Diagrammatic lay-out of the interferometer.

The fringe movement at a point in a stressed model is, in general, dependent upon the plane of vibration of the polarized light passing through the model. It is therefore essential that the model be penetrated by plane-polarized light vibrating in a principal stress direction. This is accomplished by the use of an auxiliary polarizing disk or analyzer placed in front of the eyepiece. The light from the outside path is cut off at the same time by inserting an opaque disk. The two polaroids are crossed so that the light from the inside path is extinguished. Since the vertically polarized light emitting from the light source will not generally be vibrating in the plane of the principal stresses, the second or auxiliary polaroid set to extinguish vertically polarized light will permit the components of light vibrating in the principal stress planes to pass through into the eyepiece.

The vertically polarized light is made to vibrate in a principal stress plane by the use of a half-wave plate placed in front of the model. A second half-wave plate placed behind the model rotates the light back to the vertical plane. The direction of the principal stress is found by rotating the two half-wave plates in unison and noting the angle of rotation necessary to produce extinction. With the light thus vibrating in a principal stress direction, the analyzing unit (auxiliary polarizing disk) is removed from in front of the eyepiece and the opaque disk is removed from the outside path. The interference fringes are again visible and by slowly removing the applied load acting on the model the fringe movement in the eyepiece is noted and recorded.

The laws relating changes in refractive indices to principal stresses have been investigated by such men as Favre[9], Maxwell[10], Nuemann[11], and Filon[12]. For plane stresses these relations are:

$$\delta_1 = C_1\sigma_2 + C_2\sigma_1 \qquad (1)$$

$$\delta_2 = C_1\sigma_1 + C_2\sigma_2 \qquad (2)$$

where

δ = represents the change in refractive index,

σ = the principal stress, 1 and 2 are subscripts denoting principal stress directions.

C_1 = the transverse stress-optical coefficient, and

C_2 = is the direct stress-optical coefficient.

Solving equations (1) and (2) for the principal stresses gives

$$\sigma_1 = A\delta_1 + B\delta_2 \qquad (3)$$

$$\sigma_2 = B\delta_1 + A\delta_2 \qquad (4)$$

$$A = \frac{C_2}{C_2^2 - C_1^2}$$

$$B = \frac{C_1}{C_1^2 - C_2^2}$$

The constants A and B are determined by a simple calibration described later. When these constants are known the principal stresses σ_1 and σ_2 can be determined by Equations (3) and (4) in terms of the changes in refractive indices which are measured by the number of interference fringes that pass the cross hairs of the eyepiece while the model is undergoing a stress change.

In actual operation the number of fringes that move past the cross hairs is not counted. Instead the outside path is changed in length by an amount equal to the refractive change of the inside path. This is accomplished by using the clear glass optical parallel L_5, which is mounted on a spindle equipped with a tangent arm and tangent motion. Turning the tangent screw changes the effective length of glass through which the outside light path must travel. Thus the change in length of the outside path is made equal to the change of the inside path. A micrometer dial bears against an agate insert on the tangent arm, and changes in length of the outside path are read directly on the micrometer dial. The optical parallel L_2 is necessary only as a compensator for L_5.

Stress measurements are made by first determining the direction of principal stresses and then gradually removing the load acting on the model. As the load is removed the interference fringes move across the field of vision in the eyepiece. The fringe displacement is nullified by turning the tangent screw, and the difference in dial readings before and after loading is a measure of the refractive change δ.

The operation is repeated for the other principal stress direction. Magnitudes of the principal stresses are obtained by substituting the difference in dial readings, δ, into Equations (3) and (4).

Calibration

The constants A and B are determined by a simple calibration experiment. A tensile specimen is cut from the same plate as the experimental model, placed in the interferometer and stressed. The principal stress directions are set in succession on the half-wave plates and the dial readings δ_1 and δ_2 obtained in turn by noting the fringe movement between the position of load and no load on the tensile specimen. In a tensile specimen $\sigma_2 = 0$ and $\sigma_1 = P/A$. By inserting the values of δ_1, δ_2 and σ_1 into Equations (3) and (4) the constants A and B may be determined. These constants are functions of the model material, the model thickness, and the wave length of the light.

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