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HYDRODYNAMIC PRESSURES ON DAMS DUE TO HORIZONTAL EARTHQUAKE EFFECTS

by C. N. Zangar

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HYDRODYNAMIC PRESSURES ON DAMS DUE TO HORIZONTAL EARTHQUAKE EFFECTS

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INTRODUCTION

This monograph presents a rapid, inexpensive, and accurate method for determining the increase in water pressure on dams, or on vessels of any shape, due to horizontal earthquakes and gives the magnitude of these pressures for a number of cases. Although earlier papers[1]* have shown that the increase in water pressure on dams due to earthquake is not excessively large, it is an important factor in their design. It has been recognized [2] that water pressures due to earthquake diminish with decrease in the upstream slope of a dam, but to the writer's knowledge data do not exist giving these pressures as a function of slope. Mathematical methods may be used to compute these pressures, but they are complicated and time-consuming.

If water is assumed to be incompressible,[3] an electric analog may be used to determine the magnitude and distribution of the water pressure increases caused by a horizontal earthquake on a dam of any profile. Although this assumption is not conservative, a comparison with Westergaard's [1] analytical results for dams with vertical upstream faces shows that for dams under 400 feet in height the error is exceedingly small, and that it is not excessive for dams as high as 800 feet.

The electric analog method consists of constructing a tray geometrically similar to the dam and reservoir area. A linearly varying electric potential is placed along the boundary representing the upstream face of the dam, and a constant electric potential is placed along the boundary representing the bottom of the reservoir. The tray is then filled with an electrolyte and the streamlines are surveyed by means of a modified Wheatstone bridge. The distribution and magnitude of pressures on the face of the dam are obtained from the equipotential lines that are constructed from the streamlines. The procedure is explained later in detail.

The increase in water pressure, P_e, caused by an earthquake is given by the equation

$$P_e = C\alpha wh \quad \ldots\ldots\ldots\ldots\ldots (1)$$

As shown under Notation, w is the unit weight of water, h the depth of the reservoir at the section being studied, and α the horizontal earthquake intensity. C, the unknown quantity, defines the magnitude and distribution of pressures which are determined by the equipotential lines in the flow net. C is a function of the shape of the dam and reservoir and is unaffected by the intensity of the quake. The designer need only select a reasonable value for α and use the proper C values given herein to determine the water pressures on any dam due to a horizontal earthquake. With the water pressures known the stresses in the dam can be computed by statical methods.

*Superscripts refer to similarly numbered references in bibliography.

NOTATION

a = acceleration due to earthquake

C = coefficient giving the distribution and magnitude of pressures (dimensionless)

C_m = maximum value of C for constant slopes

E = bulk modulus of water

g = acceleration due to gravity

h = depth of reservoir at section being studied

$K = \sqrt{\frac{gE}{w}}$ = velocity of sound in water

M_e = moment of the pressure P_e above y and about y

P_e = increase in water pressure at point y due to the horizontal earthquake

T = period of the earthquake vibration

t = time

V_e = total horizontal shear at y due to P_e

w = unit weight of water

u,v,s = three orthogonal displacements

x,y,z = rectangular coordinates

α = horizontal earthquake intensity $= \frac{a}{g}$

ε_0 = displacement of ground

ϕ = potential

θ = angle between a vertical and the upstream face of dam

THEORY

When the compressibility of water is considered in the hydrodynamic effect of a horizontal earthquake, it is convenient to assume that the earthquake manifests itself in a harmonic motion. Analytic solutions are also based upon the assumption that the dam is a rigid wall that moves as a unit with the foundation. The displacements are assumed to be small and may be determined from the equation

$$\varepsilon_o = -\frac{\alpha g T^2}{4\pi^2} \cos\left[\frac{2\pi t}{T}\right] \dots\dots \quad (2)$$

By assuming that the displacements of the water body are small, the differential equations in rectangular coordinates expressing the relationship of pressure (P_e), time (t), and the three orthogonal displacements u, v, and s are:

$$\left.\begin{aligned} \frac{\partial P_e}{\partial x} &= \frac{w}{g}\frac{\partial^2 u}{\partial t^2} \\ \frac{\partial P_e}{\partial y} &= \frac{w}{g}\frac{\partial^2 v}{\partial t^2} \\ \frac{\partial P_e}{\partial z} &= \frac{w}{g}\frac{\partial^2 s}{\partial t^2} \end{aligned}\right\} \dots\dots\dots\dots \quad (3)$$

With these assumptions for a compressible fluid, the conditions of continuity are given by the equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial s}{\partial z} = \frac{P_e}{'E} \quad \dots\dots\dots \quad (4)$$

Using equations (3) and (4), the following differential equation for the pressure in three-dimensional flow is obtained

$$\frac{\partial^2 P_e}{\partial x^2} + \frac{\partial^2 P_e}{\partial y^2} + \frac{\partial^2 P_e}{\partial z^2} = \frac{1}{K^2}\frac{\partial^2 P_e}{\partial t^2} \dots \quad (5)$$

For two-dimensional flow the equation becomes

$$\frac{\partial^2 P_e}{\partial x^2} + \frac{\partial^2 P_e}{\partial y^2} = \frac{1}{K^2}\frac{\partial^2 P_e}{\partial t^2} \dots\dots \quad (6)$$

Analytically the problem resolves itself into determining solutions for the differential equations (5) or (6) which also satisfy the boundary conditions. The general conditions to be met at any boundary may be written after consideration of Figure 1. For the two-dimensional case, the displacements at the face are:

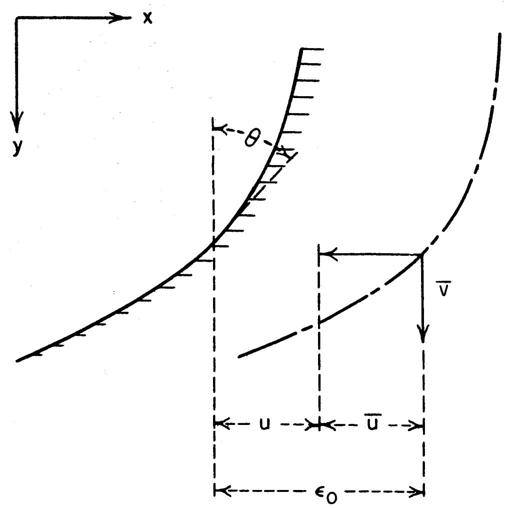

FIGURE 1 - Displacement of fluid relative to face of dam.

$$u = \varepsilon_o - \bar{u} \dots\dots\dots\dots\dots \quad (7)$$

$$v = \bar{v} \dots\dots\dots\dots\dots\dots \quad (8)$$

The top indices refer to the movement of the water relative to the dam. The displacement component perpendicular to any point on the face of the dam must be zero since the face is a streamline. Therefore, the following equation may be written:

$$u + v \tan\theta = \varepsilon_o \dots\dots\dots\dots \quad (9)$$

Now, if water is considered as incompressible, E and hence K become infinite. With K infinite the right sides of equations (5) and (6) become zero. For two-dimensional flow (only two-dimensional flow is considered hereafter) and an incompressible fluid, equation (6) then becomes:

$$\frac{\partial^2 P_e}{\partial x^2} + \frac{\partial^2 P_e}{\partial y^2} = 0 \dots\dots\dots \quad (10)$$

This is Laplace's equation, which also governs the steady state flow of electricity. Therefore, the electric analogy tray apparatus may be used to obtain flow nets for studying horizontal earthquake effects on dams of various upstream shapes. The flow net is an orthogonal system which consists of two sets of curves, one representing streamlines and the other equipotential lines.

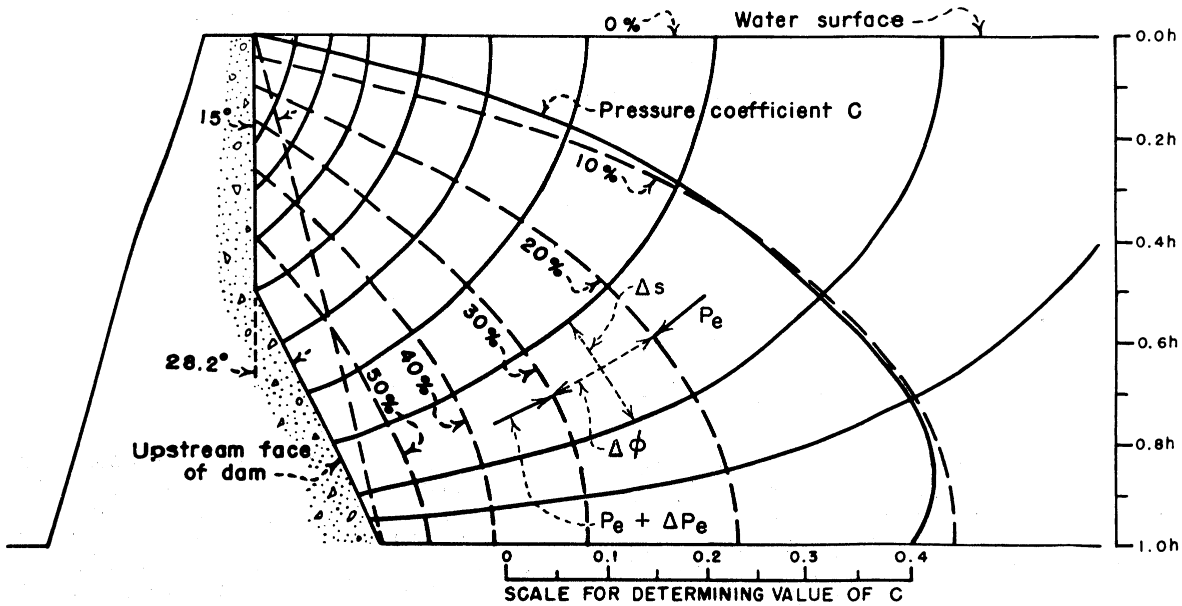

FIGURE 2 - Typical flow net.

Once the flow net is obtained, the proper scale of the pressures in the net must be determined. The pressure scale is easily determined by the following considerations:

a. Divide the reservoir depth h into n equal parts

b. Assume the dam is rigid, then the same quantity of water must flow through each element

c. No water can flow across a streamline at any point

d. Apply the equations of motion and continuity to a square in the flow net (see Figure 2). Then

$$\Delta P_e = \frac{\alpha w h}{n}\left[\frac{\Delta\phi}{\Delta s}\cos\frac{2\pi t}{T}\right] \quad \ldots \quad (11)$$

Equation (11) determines the scale of the pressure. The equation can be further simplified if the flow net is made into squares so that $\Delta\phi = \Delta s$. Only the maximum pressure increase is important, which occurs when $t = T$. And so equation (11) becomes

$$\Delta P_e = \frac{1}{n}\alpha w h \quad \ldots\ldots\ldots\ldots \quad (12)$$

The pressure coefficient C becomes the $1/n$ value which is determined directly from the nominal value of the equipotential line intersecting the face of the dam.

The pressure distribution and magnitude are shown in the attached figures for several upstream slopes of dams. The pressures are given by the equation

$$P_e = C\alpha w h \quad \ldots\ldots\ldots\ldots \quad (13)$$

which is equivalent to equation (12).

EARTHQUAKE INTENSITIES

In order to determine the total horizontal force due to an earthquake, it is necessary to know the acceleration of the quake or the earthquake intensity. The use of earthquake spectra[4,5] derived from recorded accelerographs is suggested for determining the intensity. Biot's[4] proposed standard spectrum may be used if a damage scale is applied since it does not include damping. A joint committee of the ASCE and Structural Engineer's Association of California[6] has applied a damage factor to Biot's proposed spectrum and has suggested that the maximum value of α be 0.10 and the minimum value 0.03 for other than frame structures. The Bureau of Reclamation has consistently used a horizontal intensity, α, 0.10 on dams, along with a vertical intensity of about equal or smaller magnitude. Kosi Dam and Bhakra Dam in India, however, were analyzed for a horizontal earthquake intensity of 0.15.

Resonance in dams is not apt to occur for several reasons. The fundamental period of vibration of the usual concrete or earth gravity dam will be from 0.08 to about 1.00 second[1,7,8] while the maximum

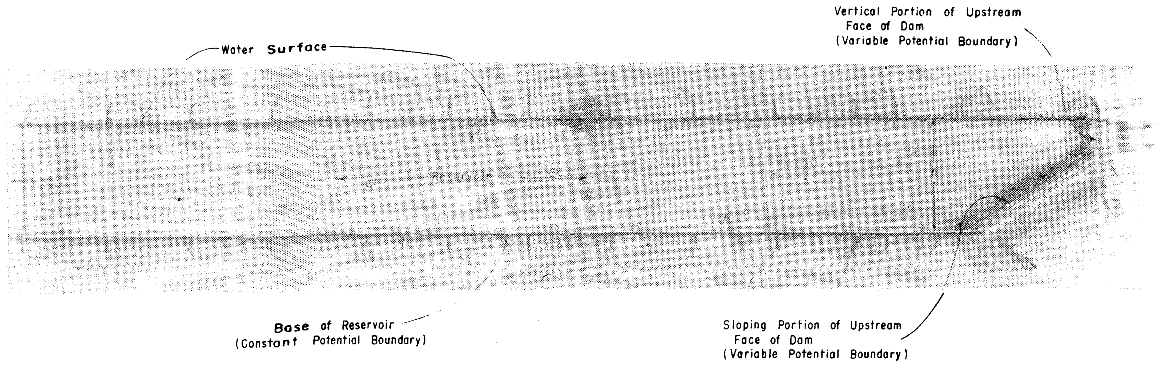

FIGURE 3 - Electric analogy tray model.

energy of the earthquake appears in most spectra[4,5,6] at a period of approximately 0.2 second. Resonance with the foundation is not apt to occur since studies of the fundamental ground periods[9] show values of 0.03 to 0.05 seconds. Although earthquakes are experimentally and analytically treated as harmonic, recorded ground motions do not appear to be harmonic in the destructive zone of the quake, and a steady state response of the structures is usually not established. Also, many forms of damping that are difficult to evaluate act to prevent resonance.

At the present time, the choice of earthquake intensity to apply to a structure must be based upon experience in conjunction with available seismic records. Earthquake spectra including the effects of damping need to be determined for structures having a wide range of fundamental periods. The spectra should be obtained by subjecting the structure with damping to actual recorded accelerograms of destructive earthquakes such as the Helena, Montana quake of 1935, the Ferndale, California quake of 1938, and the El Centro, California quake of 1940.

ELECTRIC ANALOG PROCEDURE

The electric analogy tray experiments (see Figure 3) were conducted by first constructing a tank of sheet plastic 2 inches deep, 32 inches long, and 4 inches wide. The plastic boundary at one end of the tank was shaped to represent the upstream face of the dam being studied (see Figure 4), while the plastic boundary at the opposite end of the tank is merely installed in a plane. Any shape for this latter boundary could be used if it is placed upstream a distance greater

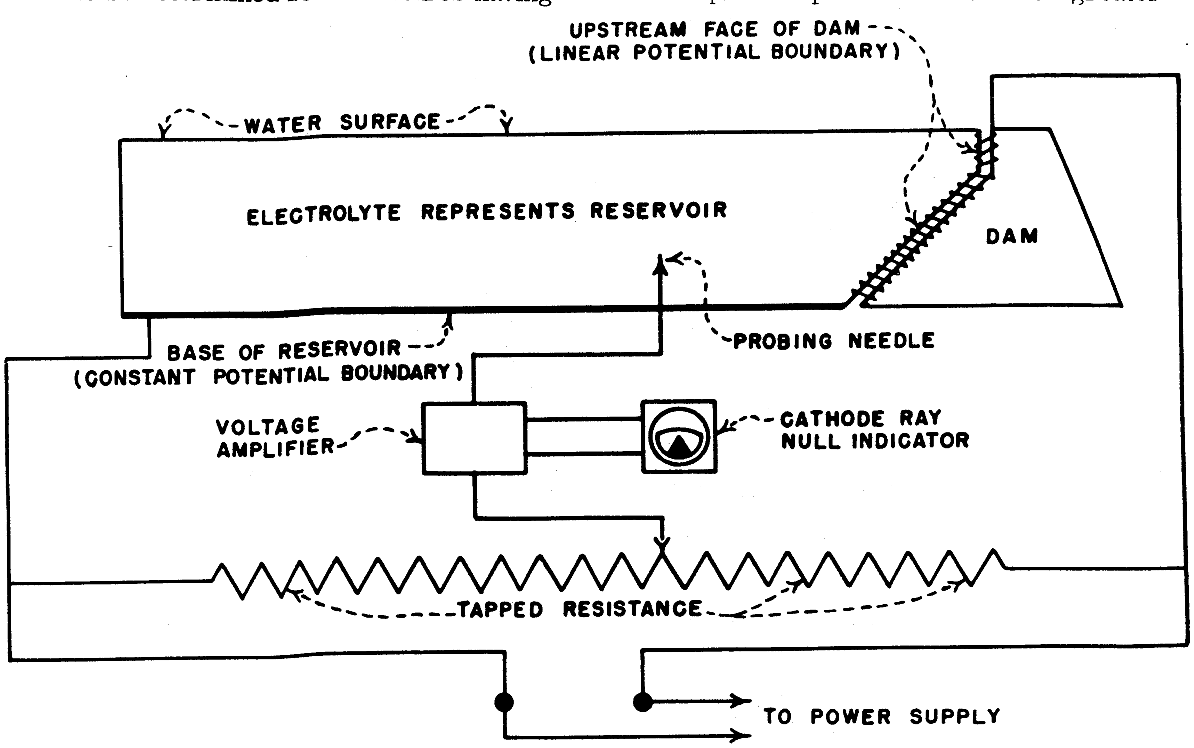

FIGURE 4 - Diagrammatic layout of electric analogy tray.

than three times the height of the dam. In these experiments the distance was made 8h.

Since the theory assumes that the dam moves as a rigid body into (or away from) the reservoir water, the quantity of water displaced for any elemental height of dam will be equal to that quantity displaced at any other element of height. This boundary condition can be met in the analogy by establishing a linearly varying potential along the plastic boundary representing the upstream face of the dam. Nichrome wire was wound around this boundary to bring about the linear drop in potential. All streamlines at the face of the dam (see Figure 2) will then have the same vertical spacing h/n. Since the bottom of the reservoir is a streamline, a constant potential electrode represented by a copper strip is placed along this boundary. Naturally the potential at the base of the dam must be the same as the potential along the reservoir bottom. Note that in this analogy the electric potentials represent the streamlines of the prototyped problem.

Proper boundary potentials having been established, the tray is filled with an electrolyte. Experience has shown that ordinary tap water is a satisfactory electrolyte. The model or tray is connected to a modified Wheatstone bridge and to the power supply as shown in Figure 4. The bridge is set to read a constant potential, say 10 percent, and several points in the tray at this potential are determined, plotted on coordinate paper, and connected by a smooth curve. This process is repeated for bridge settings at 10 percent intervals from 20 through 90 percent. The plot of the electric potential gives the streamline spacing in the prototype. The potentials in the prototype are now drawn perpendicular to the streamlines forming a system of squares as illustrated in the example of Figure 2. The zero potential is the water surface. Proceeding into the fluid along a streamline, the potential lines in the square net become in succession the 10 percent, 20 percent, 30 percent, etc. A potential line gives the value of the pressure coefficient, C. Therefore, the pressure coefficient at the face of the dam is the value of the potential line at its intersection with the face of the dam.

APPLICATION OF DATA

In order to make this study of general value to designers, pressures due to earthquake were determined for several shapes of dams. Dams studied were those with constant upstream slopes θ of 0, 15, 30, 45, 60, and 75 degrees. The pressure at the base of the dam and the maximum pressure on the slope are shown in Figure 5. The

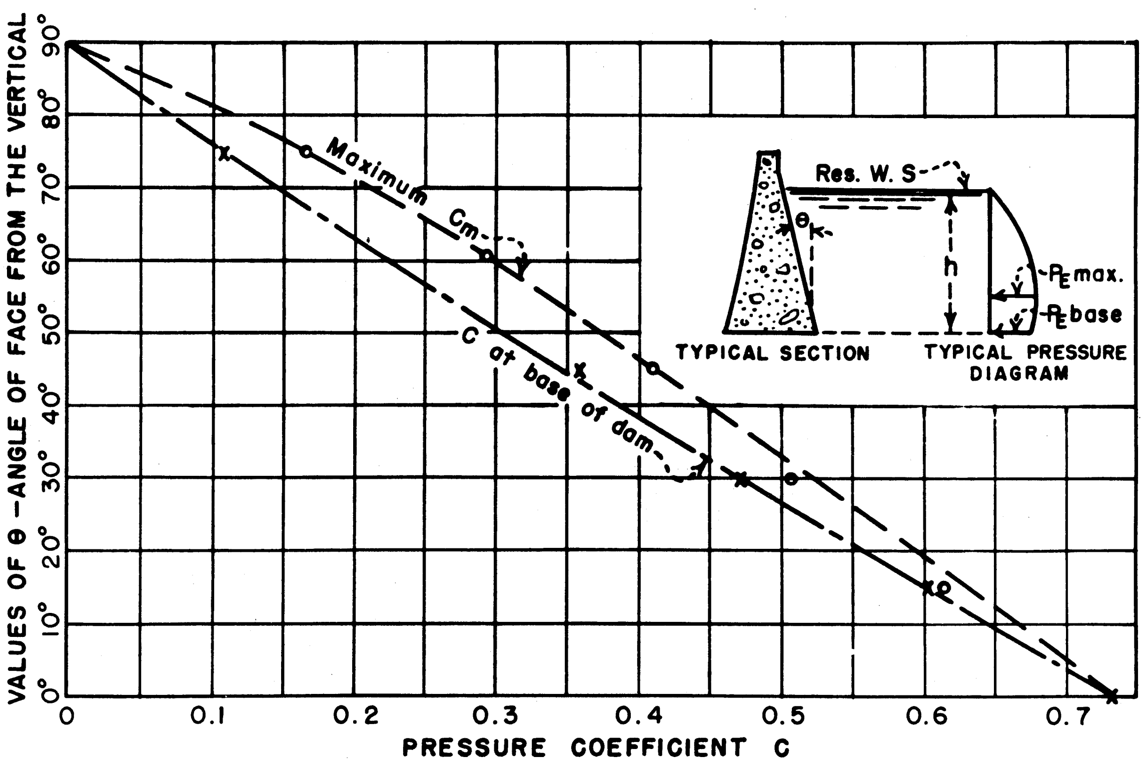

FIGURE 5 - Pressure coefficients for constant sloping faces.

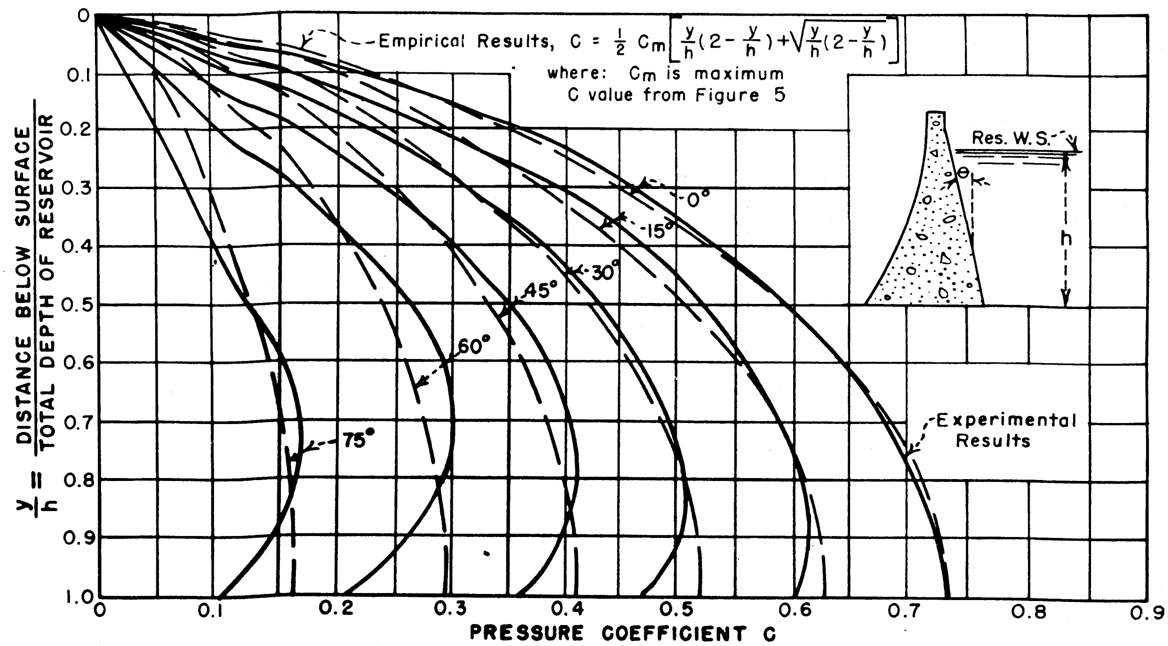

FIGURE 6 - Comparison of experimental and empirical pressure distribution curves.

pressure coefficient, C, varies almost linearly from 0.735 for a dam with vertical face, $\theta = 0$ degrees, to 0.165 for $\theta = 75$ degrees. The distribution of pressure for these constant slopes is shown in Figure 6. To permit rapid use of these data by designers, the experimentally determined pressure curves of Figure 6 are represented by a family of parabolas which closely approximate the experimental curves for constant slopes. The parabolic distribution is given by the equation

$$C = \frac{C_m}{2}\left[\frac{y}{h}\left(2 - \frac{y}{h}\right) + \sqrt{\frac{y}{h}\left(2 - \frac{y}{h}\right)}\right] \quad \ldots\ldots\ldots (14)$$

where C_m is the maximum value of C obtained from Figure 5. So, for dams with constant upstream slopes the increase in pressure due to horizontal earthquake becomes

$$P_e = \tfrac{1}{2}\alpha w h C_m \left[\frac{y}{h}\left(2 - \frac{y}{h}\right) + \sqrt{\frac{y}{h}\left(2 - \frac{y}{h}\right)}\right] \quad \ldots\ldots\ldots (15)$$

The total horizontal force V_e above any elevation y and the total overturning moment M_e above y due to P_e may analytically be shown to be

$$V_e = 0.726\, P_e y \quad \ldots\ldots\ldots (16)$$

and

$$M_e = 0.299\, P_e y^2 \quad \ldots\ldots\ldots (17)$$

If one desires, he may use the experimentally determined results of Figure 6 in preference to the approximations of equations (15), (16), and (17).

Figure 2 is a typical flow net system. The percent value of the equipotential lines gives the magnitude of the pressure coefficient C. Figures 7 through 14 show the magnitude and distribution of pressures for certain combinations of vertical with sloping faces of dams.

The slight errors resulting from the assumption of incompressibility of water can be shown by comparing the experimental values for pressure on a vertical face with Westergaard's exact analytical solution. Westergaard's data are computed for a period T of 4/3 seconds. Table 1 shows percent errors for several heights of dams. Minus signs preceding the percentages indicate values less than those given by Westergaard.

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Table 1

PERCENTAGE ERRORS INTRODUCED BY ASSUMPTION THAT WATER IS INCOMPRESSIBLE

Quantity	Height of Dam				
	100'	200'	400'	600'	800'
P_e	-0.9	-1.9	-5.2	-9.1	-15.7
V_e	-1.7	-2.4	-4.9	-8.8	-14.8
M_e	+1.1	+0.2	-2.1	-5.8	-11.5

These errors are small and are usually negligible compared to the total water force applied to the dam.

ACKNOWLEDGMENTS

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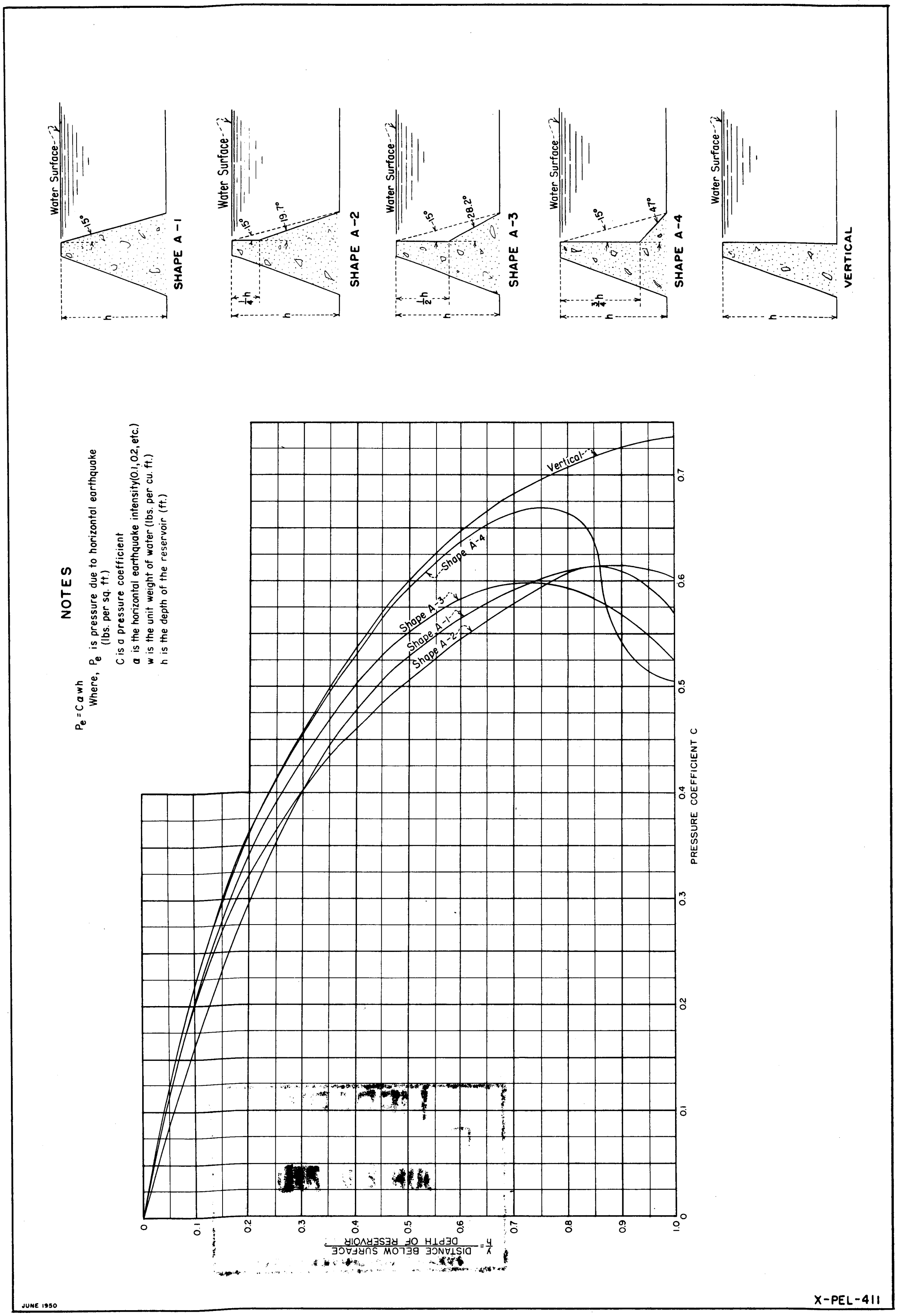

FIGURE 7 - Values of C for combination slopes in which the inclusive angle is 15^{o}, and vertical portion of upstream face is variable.

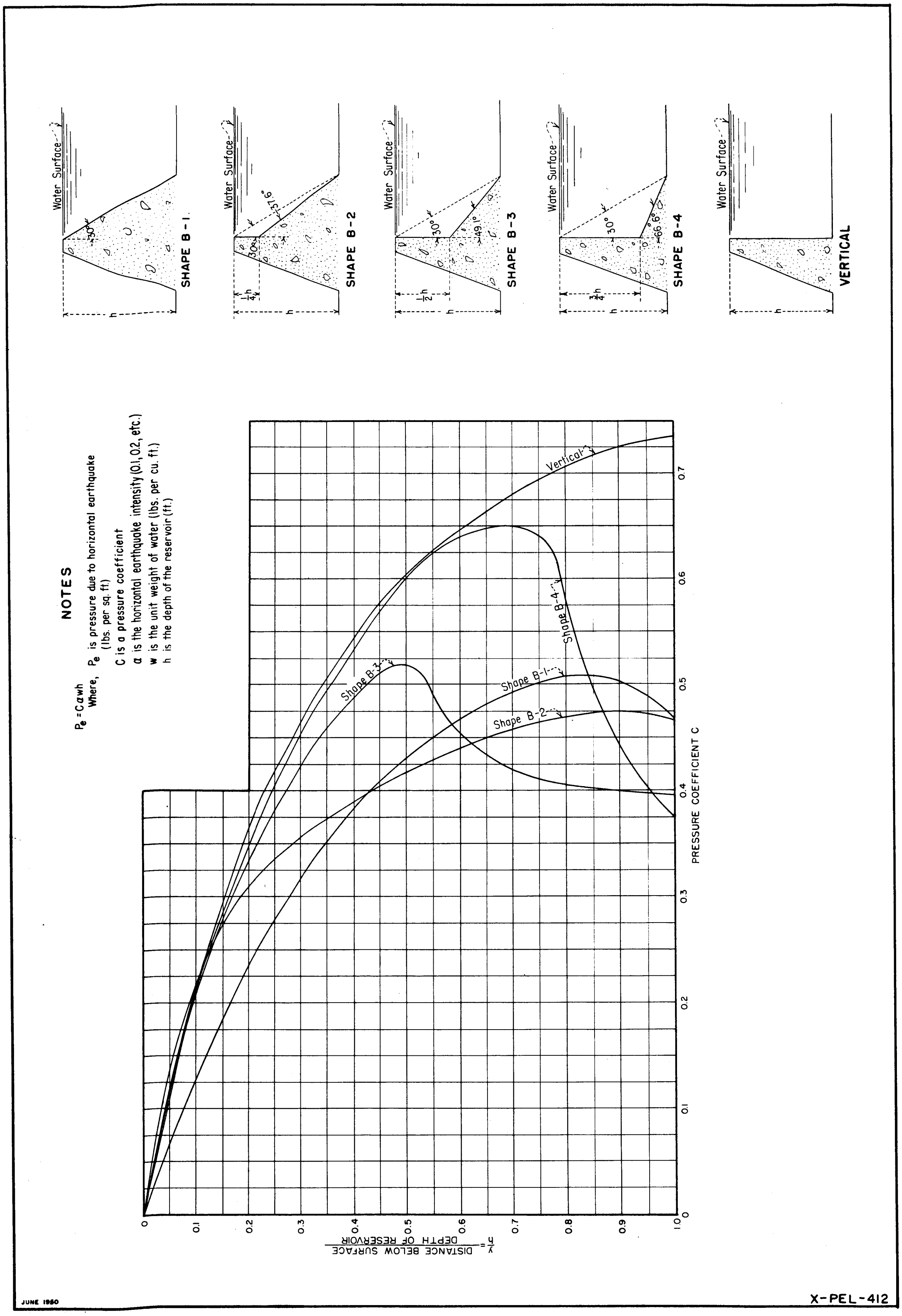

FIGURE 8 - Values of C for combination slopes in which the inclusive angle is 30°, and vertical portion of upstream face is variable.

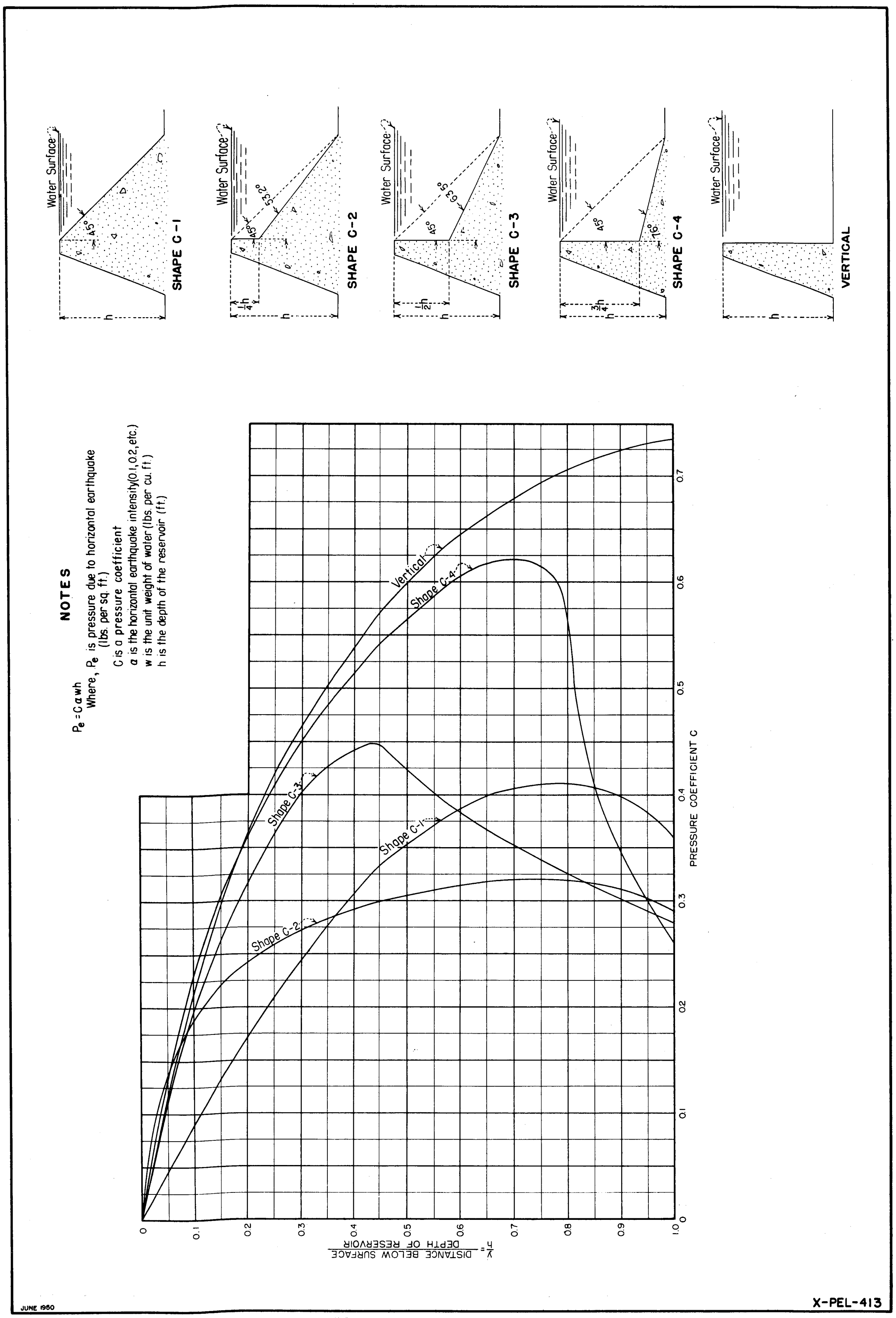

FIGURE 9 - Values of C for combination slopes in which the inclusive angle is 45°, and vertical portion of upstream face is variable.

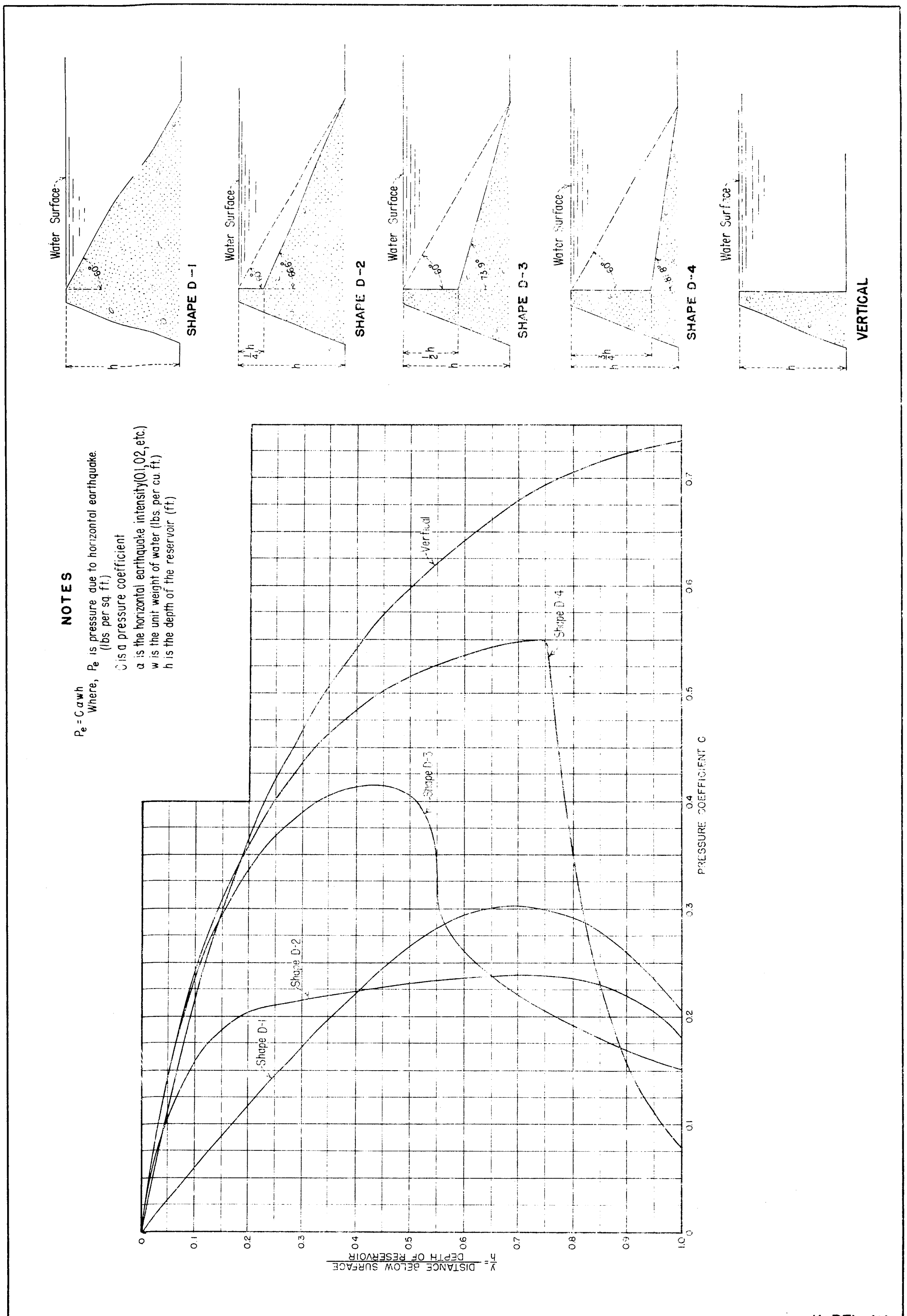

FIGURE 10 - Values of C for combination slopes in which the inclusive angle is 60°, and vertical portion of upstream face is variable.

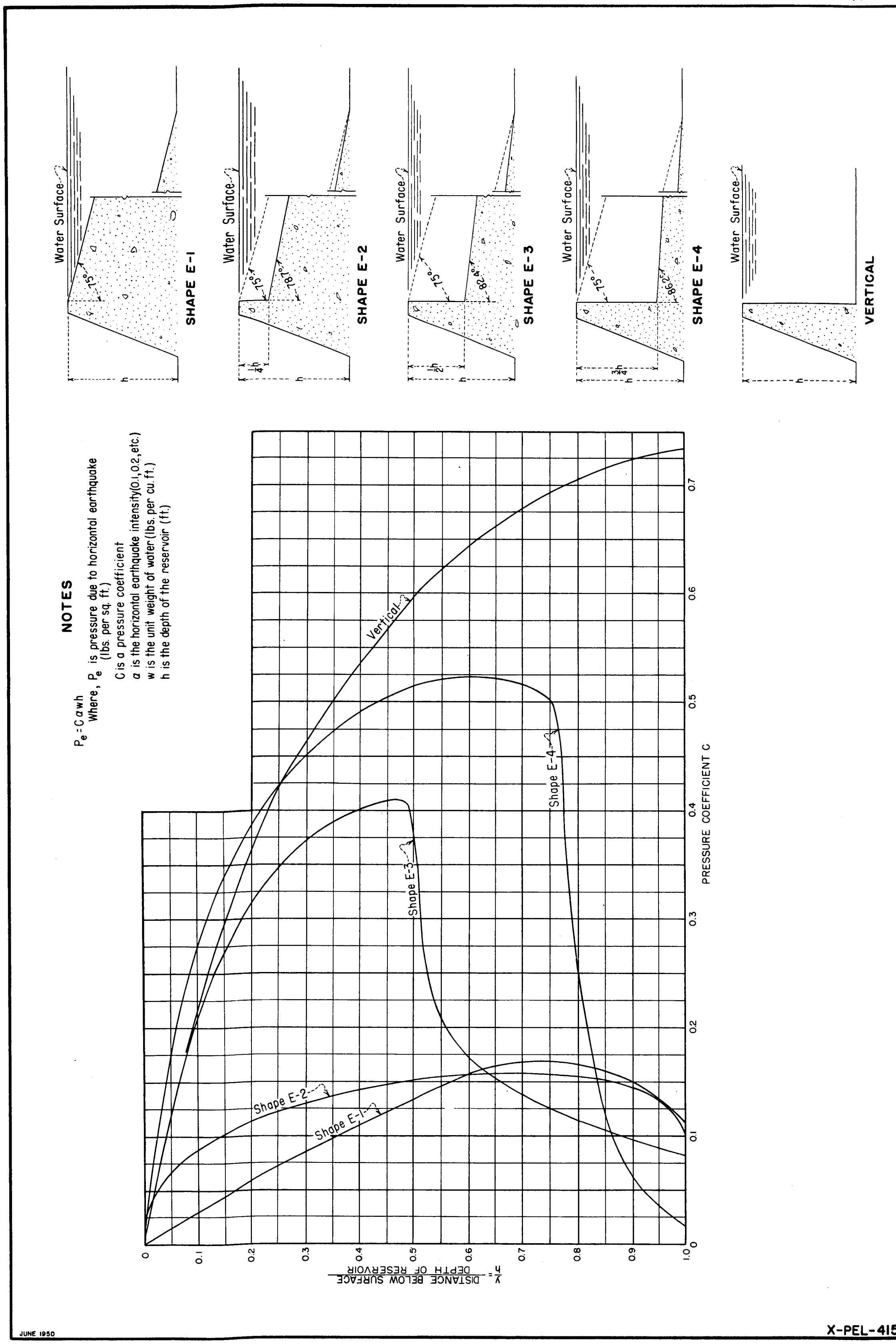

FIGURE 11 - Values of C for combination slopes in which the inclusive angle is 75°, and vertical portion of upstream face is variable.

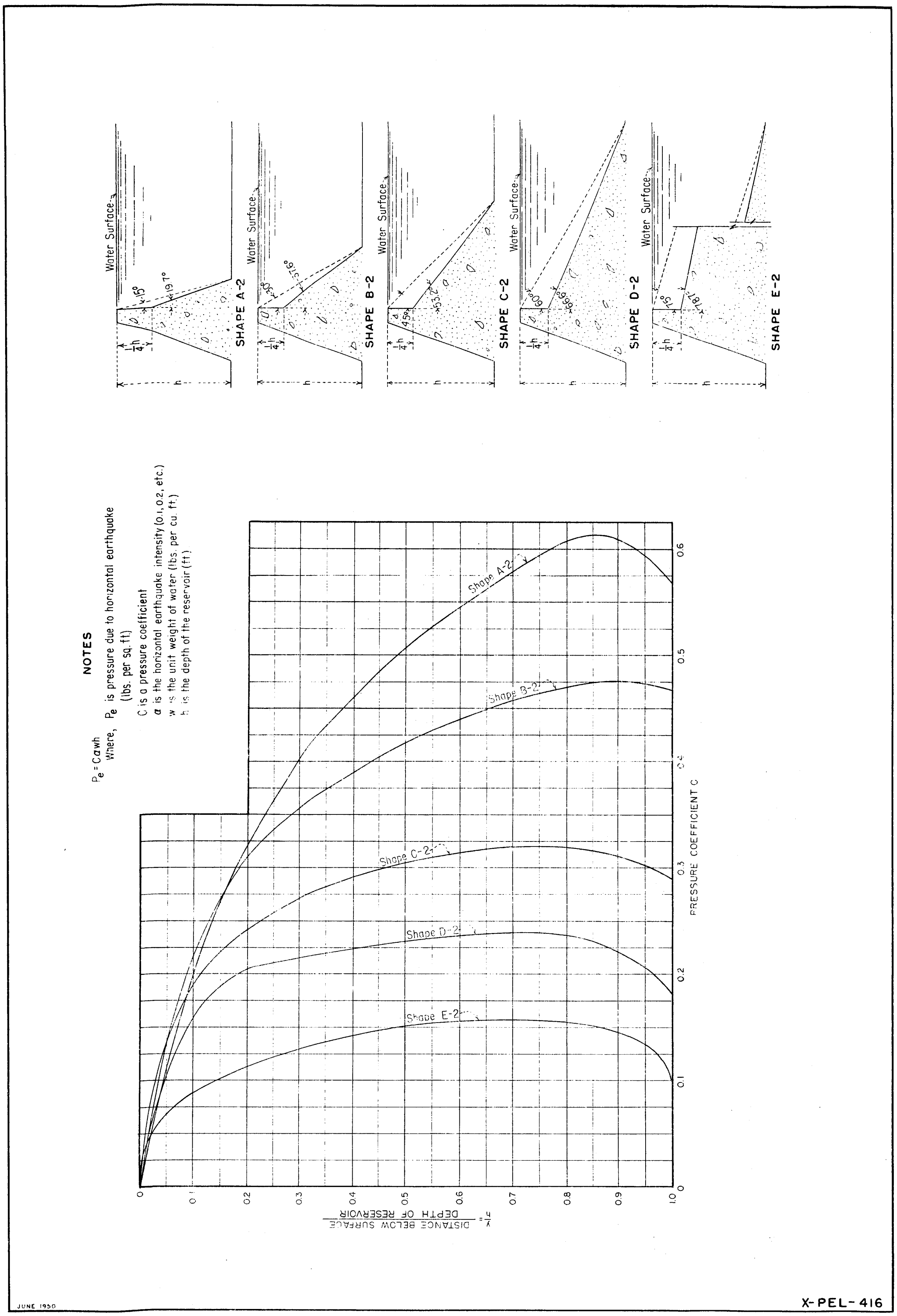

FIGURE 12 - Values of C for variable slopes with vertical portion always h/4.

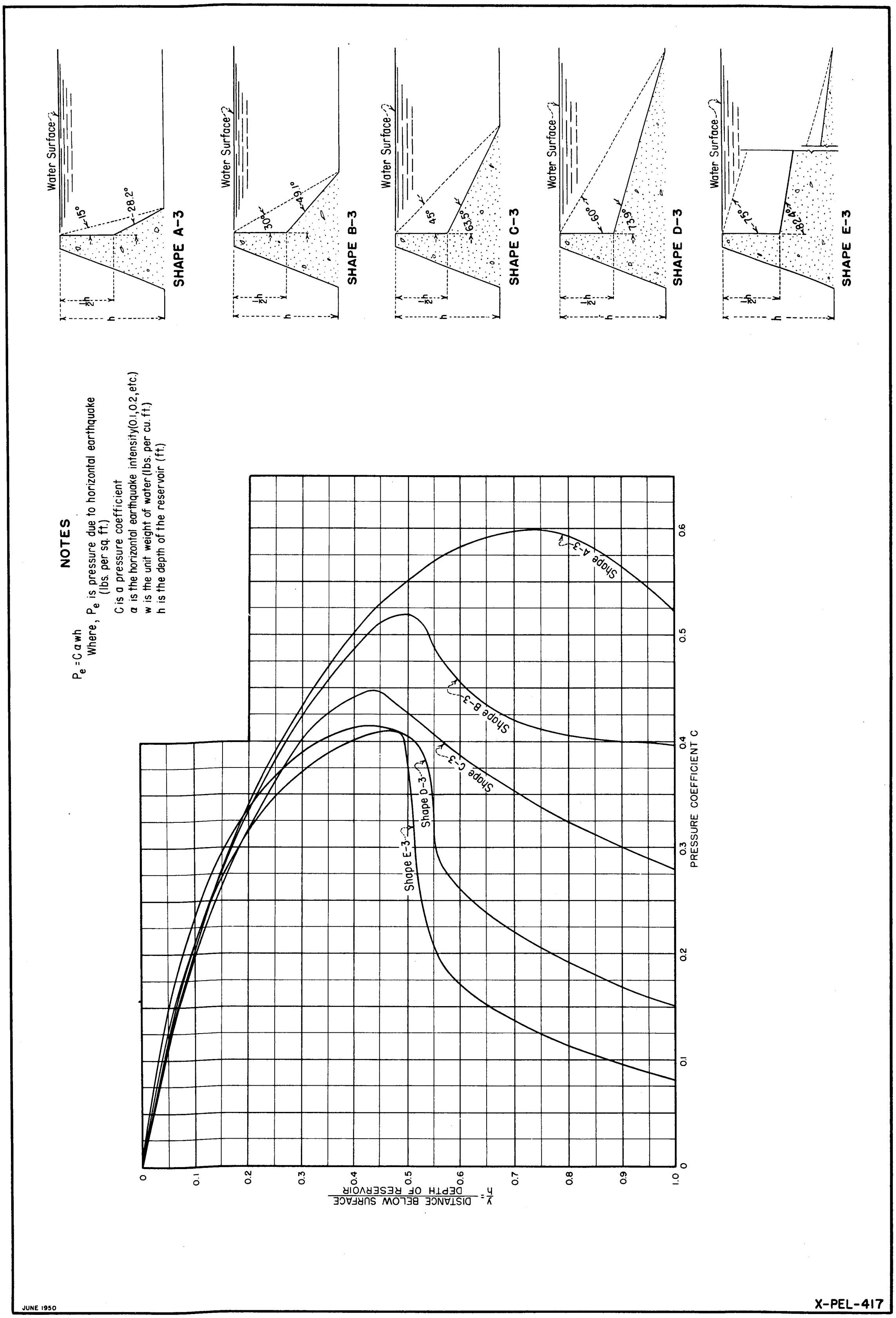

FIGURE 13 - Values of C for variable slopes with vertical portion always h/2.

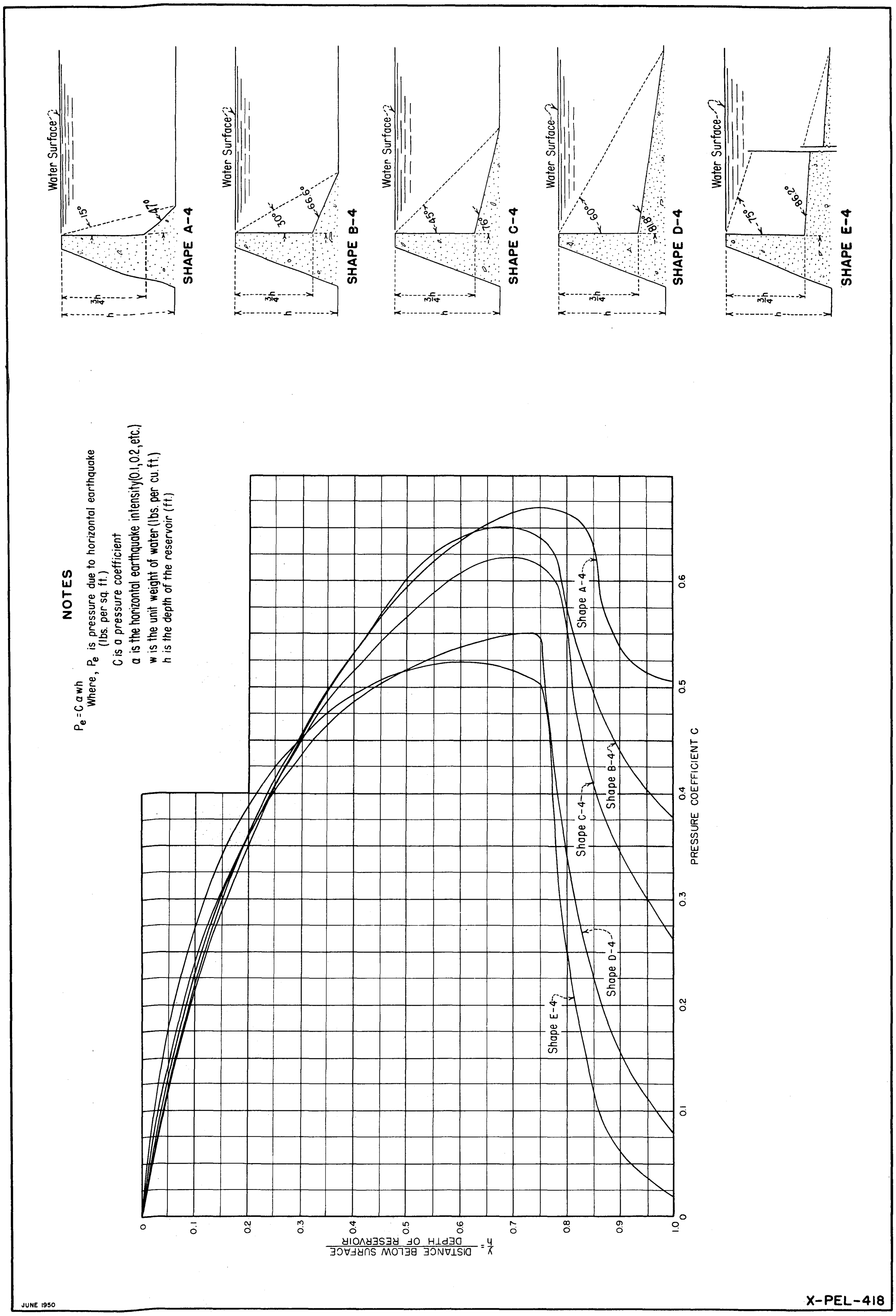

FIGURE 14 - Values of C for variable slopes with vertical portion always (3/4)h.