STRESS ANALYSIS of CONCRETE PIPE

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ENGINEERING MONOGRAPHS are published in limited editions for the technical staff of the Bureau of Reclamation and interested technical circles in government and private agencies. Their purpose is to record developments, innovations, and progress in the engineering and scientific techniques and practices that are employed in the planning, design, construction, and operation of Reclamation structures and equipment. Copies may be obtained from the Bureau of Reclamation, Denver Federal Center, Denver, Colorado, and Washington, D.C.
INTRODUCTION

Stress analysis of concrete pipe is in part based on the assumed distribution of earth pressures. Most commonly it has been assumed that the vertical earth load is uniformly distributed over the horizontal width of the pipe and that the lateral load is exerted so that the force diagram is trapezoidal in shape, applied on both sides, and extending the full height of the pipe. Further, it has been assumed that the reaction due to these loads is uniformly distributed over the full width of that portion of the bottom of the pipe in contact with the supporting surface. That these assumptions are rather arbitrary has long been recognized and efforts have been made to develop assumptions that would agree more closely with tests and with the general fund of knowledge about soil mechanics.

In 1930, the late Dr. Anson Marston, director of the Iowa Engineering Experiment Station at Iowa State College, advanced the theory that earth pressures on a rigid conduit and the reactions to those pressures would be exerted in such a manner that the force diagram representing the pressures would have characteristic bulb-like shapes above and below the outline of the pipe. Dr. Marston's theory was confirmed and extended by an investigation carried out at the Iowa Engineering Experiment Station and reported by M. G. Spangler of the Station staff.

While the design of the concrete pipe for the first section of the Salt Lake Aqueduct was being developed in 1938, W. A. Larsen and H. W. Birkeland, Bureau of Reclamation engineers under the direction of C. P. Vetter, who was in charge of the design, developed an analysis based on the assumption of bulb-like distribution of earth loads and soil reactions. This assumption closely approximates actual conditions as disclosed by tests. Since 1938, a number of other pipe lines have been designed, and the original analysis has been expanded so that various widths of bedding of the pipe may be analyzed.

In the original analysis the least work method by summation was used, but in subsequent studies a mathematical solution using integral equations was developed and is described in this monograph.

STRESS ANALYSIS OF CONCRETE PIPE

General

Under ordinary conditions a pipe placed under a fill is subjected to loads due to earth pressure around the pipe, the dead load of the pipe itself, and the internal hydrostatic pressure. For convenience of design the hydrostatic pressure is divided into two parts: (a) that part producing a uniform internal pressure, of which the head \( H \) is measured from the hydraulic gradient to the top of the inside of the pipe; and (b) the remaining part of the pressure of which the head is measured from the top to the bottom of the inside of the pipe.

The uniform internal pressure, part (a), produces only uniform tension around the pipe and is equal to \( 62.4 \, H r_0 \), where \( r_0 \) is the inside radius of the pipe in feet, and \( H \) is the head in feet measured as stated above.

The remaining part of the hydrostatic pressure produces bending, direct, and shear stresses in the pipe. This part is designated as water load and will be discussed in combination with earth load and dead load.

Earth Load, Dead Load, and Water Load

The assumptions for the distribution of earth pressures are based on results obtained from tests as summarized in Bulletin 112 of the Iowa Engineering Experiment Station. Figure 1 is developed from data in this bulletin and shows the distribution of earth pressures around a rigid pipe placed on compacted backfill, representing the ordinary condition of a pipe placed under fill or in a wide trench. The
reaction resisting external loads is assumed to be distributed over the bottom of the pipe, limited by a definite central angle and varying as some function of the angle $\theta$, see Figure 4. The limits of the central angle will depend on the bedding of the pipe, see Figure 2. Generally a 90$^\circ$ angle is used, but the analysis of the pipe has been prepared here for central angles of 45$^\circ$, 90$^\circ$, 120$^\circ$, and 180$^\circ$. In the analysis, loads and pressure distribution around the pipe are assumed to be symmetrical about the vertical centerline.

**Nomenclature**

- $P_t =$ unit external pressure from earth. It is a function of angle $\theta$ and the total downward force is equal to the weight of earth on the pipe.

- $P_b =$ unit external pressure on the bottom of the pipe. It is a function of angle $\theta$ and equals the reaction of the earth on the pipe, from earth load, water load, and dead load.

- $P_i =$ unit internal water pressure from part (b) of hydrostatic pressure mentioned previously.

  \[ = 62.4r_0(1 - \cos \theta) \]

- $h =$ height of earth fill above pipe in feet. See Figure 2.

- $r =$ radius of centerline of pipe shell in feet.

- $r_0 =$ radius of inside of pipe in feet.

- $W_e =$ unit weight of effective earth cover in pounds per cubic foot.

- $A =$ total weight of earth on pipe

  \[ = 2rW_eh \]

- $W =$ total weight of water in pipe per foot of pipe

  \[ = 62.4\pi r_0^2 \]

- $t =$ thickness of pipe shell in feet.

- $D =$ total weight of pipe

  \[ = 2\pi 150t \]

  \[ = 300\pi t \]

- $M_e, P_e, V_e =$ moment, thrust, and shear respectively on section at angle $\theta$ when pipe is assumed to be cut at top. See Figure 3b.

- $M_e, P_e, V_e =$ final moment, thrust, and shear on section at angle $\theta$.

**Pipe Analysis**

The stress analysis is made by the least work method, taking advantage of certain known facts about the final deflected shape of the pipe ring, resulting from the symmetrical loads assumed. Due to the assumed loads the pipe must deflect symmetrically about the vertical centerline, and the two points on this centerline, top and bottom, will neither rotate nor deflect horizontally. The pipe then is assumed to be cut in half and each half considered as a curved cantilever beam free at the top and fixed at the bottom. With the pipe cut in this manner, earth, water, and dead loads will cause the top to deflect. Thrust $H_0$ and Moment $M_0$, see Figure 3, are then calculated to restore the free end to its final position so that its horizontal deflection and rotation are zero. Shear $V_0$ at the top is zero.

The solution for a pipe bearing over a central angle equal to $\pi/2$ will be derived...
in detail, for earth load, water load, and
dead load. The solutions for other assumed
bearing areas are similar and only those
equations which are different will be given
as they appear.

FIGURE 4 — With a central angle of \(\pi/2\),
earth pressures and reactions exhibit
the characteristic bulb shapes.

1. Analysis for Reaction over Central
Angle \(\pi/2\).

A. Earth Load. See Figure 4.

assume \(P_t\) to vary as \(\cos^2 \theta/3\), or \(P_t = P_0 \cos^2 \theta/3\)
then \(2 \int_0^{\frac{3\pi}{4}} P_0 \cos^2 \theta \cos \theta \cos \theta d\theta = A\)
from which \(P_0 = A/1.697\)
and \(P_t = A \cos^2 \theta/3\) (1)

assume \(P_b\) to vary as \(\cos 2\theta\), or \(P_b = P_n \cos 2\theta\)
then \(2 \int_0^{\frac{\pi}{2}} P_n \cos 2\theta \cos \theta d\theta = A\)
\(P_n = A/0.943\), \(P_b = A/0.943 \cos 2\theta\) (2)

The forces on the cut section at \(\theta\), are
as follows:

From \(P_t\), where \(0 \leq \theta \leq \pi/2\)
\(M_e = 1.061A (\cos^2 \theta - \cos \theta)\) (3)
\(P_e = 1.061A (\cos^2 \theta - \cos \theta)\) (4)
\(V_e = 1.061A (\sin \theta - 2/3 \sin 2\theta)\) (5)

Where \(\pi/4 \leq \theta \leq \pi\)
\(M_e = 0.750A (0.667 \sin \theta - 0.748 \cos \theta)\) (6)
\(P_e = -A/2 (\sin \theta - 1.122 \cos \theta)\) (7)
\(V_e = A/2 (1.122 \sin \theta + \cos \theta)\) (8)

From \(P_b\), where \(\pi/4 \leq \theta \leq \pi\)
\(M_e = -A/2 (0.707 \cos 2\theta + \sin \theta + \cos \theta)\) (9)
\(P_e = -A/2 (0.707 \cos 2\theta + \sin \theta + \cos \theta)\) (10)
\(V_e = A/2 (1.414 \sin 2\theta + \sin \theta - \cos \theta)\) (11)

From \(P_t + P_b\), where \(\pi/4 \leq \theta \leq \pi\)
\(M_e = -1.061A (1/3 \cos 2\theta + \cos \theta)\) (12)
\(P_e = -A/2 (2.122 \cos \theta + 0.707 \cos 2\theta)\) (13)
\(V_e = A/2 (2.122 \sin \theta + 1.414 \sin 2\theta)\) (14)

From \(M_o\), where \(0 \leq \theta \leq \pi/2\)
\(M_e = M_o\) (15)

From \(H_o\), where \(0 \leq \theta \leq \pi/2\)
\(M_e = -H_o (1 - \cos \theta)\) (16)
\(P_e = H_o \cos \theta\) (17)
\(V_e = -H_o \sin \theta\) (18)
The rotations $\Delta \phi$ of the top ($\theta = 0$) of the cut section are as follows:

From $P_t$ and $P_b$,
\[
\Delta \phi = \frac{1.061 A r}{E I} \left\{ \int_0^{\frac{3}{4}} \left( \cos \frac{2}{3} \theta - \cos \theta \right) r d\theta + \frac{1}{2} \cos 2\theta \cos \theta - \cos \theta \right\} - \frac{1.414 A r^2}{E I}
\]

From $M_o$,
\[
\frac{\Delta \phi}{E I} = \frac{M_o}{E I} \int_0^\theta \left( \cos \frac{2}{3} \theta - \cos \theta \right) r d\theta = -\frac{\pi r^2 M_o}{E I}
\]

Horizontal deflections $\Delta h$ are as follows:

From $P_t$ and $P_b$,
\[
\Delta h = \frac{1.061 A r}{E I} \left\{ \int_0^{\frac{3}{4}} \left( \cos \frac{2}{3} \theta - \cos \theta \right) r d\theta + \frac{1}{2} \cos 2\theta \cos \theta - \cos \theta \right\} - \frac{2.013 A r^3}{E I}
\]

From $M_o$,
\[
\Delta h = -\frac{\pi r^2 M_o}{E I}
\]

From $H_o$,
\[
\Delta h = \frac{H_o}{E I} \int_0^\theta r^2 \left( 1 - \cos \theta \right)^2 r d\theta = \frac{3}{2} \pi r^2 H_o
\]

Since the rotation and the horizontal deflection at the top are zero, the following equations can be written:

\[
E I \Delta \phi = 1.414 A r^2 + \pi r M_o - \pi r^2 H_o = 0
\]

From which
\[
H_o = +0.382 A r M_o = -0.068 A r
\]

The final equations then for earth loads on section $\theta$ are:

Where $0 \leq \theta \leq \frac{3}{4} \pi$

\[
M_e = A r \left( 1.061 \cos \frac{2}{3} \theta - -0.450 \right)
\]

\[
P_e = A \left( 1.061 \cos \frac{2}{3} \theta - -0.450 \right)
\]

\[
V_e = A \left( 0.678 \sin \theta + -0.707 \sin \frac{2}{3} \theta \right)
\]

\[
\text{Note: } V_e = \frac{1}{r} \frac{dM_e}{d\theta}
\]

Where $\frac{3}{4} \pi \leq \theta \leq \pi$

\[
M_e = A \left( -0.354 \cos 2\theta + -0.78 \cos \theta - -0.450 \right)
\]

\[
P_e = A \left( -0.354 \cos 2\theta + -0.78 \cos \theta - -0.450 \right)
\]

\[
V_e = A \left( -0.707 \sin 2\theta + -0.678 \sin \theta \right)
\]

Forces indicated are positive.

FIGURE 5 - Stresses within the pipe shell are in accordance with these assumptions. Positive direction of forces is indicated.

B. Water Load. See Figure 6.

\[
P_i = -62.4 r_o \left( 1 - \cos \theta \right) = \frac{-W}{\pi r_o} \left( 1 - \cos \theta \right)
\]

For simplification let $r_o = r$, then

\[
P_i = \frac{-W}{\pi r} \left( 1 - \cos \theta \right)
\]

\[
P_b = \frac{W}{0.943 r} \cos 2\theta
\]
The forces on the cut section at $\theta$ are as follows:

From $P_i$, where $0 \leq \theta \leq \pi$

$$M_\theta = \frac{-W_r}{\pi} (1 - \cos \theta - \frac{\theta}{2} \sin \theta)$$

$$P_\theta = \frac{-W}{\pi} (1 - \cos \theta - \frac{\theta}{2} \sin \theta)$$

$$V_\theta = \frac{-W}{\pi} \left( \frac{1}{2} \sin \theta - \frac{\theta}{2} \cos \theta \right)$$

The forces from $P_B$ are the same as given under earth loads with $W$ substituted for $A$. See Equations 9, 10, and 11.

The rotations $\Delta \phi$ at the top are as follows:

From $P_i$,

$$\Delta \phi = \frac{-W_r}{E \pi} \int_0^\pi (1 - \cos \theta - \frac{\theta}{2} \sin \theta) \, \mathrm{d}\theta$$

$$= \frac{-W_r^2}{2E}$$

From $P_B$,

$$\Delta \phi = \frac{-W}{2E} \int_0^{\pi/4} (0.707 \cos \theta + \sin \theta + \cos \theta) \, \mathrm{d}\theta = -\frac{0.319 \, W_r}{EI}$$

From $P_1 + P_B$,

$$\Delta \phi = -\frac{470 \, W_r^2}{EI}$$

The horizontal deflections $\Delta h$ at the top are:

From $P_i$,

$$\Delta h = \frac{W_r}{E \pi} \int_0^\pi \left( 1 - \cos \theta \right)$$

$$\frac{-\theta}{2} \sin \theta \left( 1 - \cos \theta \right) \, \mathrm{d}\theta = +\frac{7 \, W_r^3}{8 \, EI}$$

From $P_B$,

$$\Delta h = \frac{W}{2E} \int_0^{\pi/4} (0.707 \cos \theta + \sin \theta + \cos \theta) \, \mathrm{d}\theta = -\frac{0.06 \, W_r^3}{EI}$$

From $P_1 + P_B$,

$$\Delta h = -\frac{815 \, W_r^3}{EI}$$

Then by the same reasoning as for Equations 19 and 20, we write:

$$E I \Delta \phi = -0.470 W_r^2 + \pi r M_o$$

$$-\pi r^2 H_o = 0$$

$$E I h = +0.315 W_r^3 - \pi r^2 M_o$$

$$+\frac{3}{2} \pi r^3 H_o = 0$$

from which

$$H_o = -0.220 W$$

and $$M_o = -0.070 W$$

The final equations for water loads are:

Where $0 \leq \theta \leq \frac{3}{4}\pi$

$$M_\theta = W_r \left( 0.099 \cos \theta + 0.159 \theta \sin \theta - 0.169 \right)$$

$$P_\theta = W \left( 0.099 \cos \theta + 0.159 \theta \sin \theta - 0.319 \right)$$

$$V_\theta = W \left( 0.051 \sin \theta + 0.159 \theta \cos \theta \right)$$
Where \( \frac{3}{4} \leq \theta \leq \pi \)

\[ M_\theta = W r (1.1599 \sin \theta - 0.500 \sin \theta - 0.401 \cos \theta - 0.354 \cos 2\theta - 0.169) \] (37)

\[ P_\theta = W (1.1599 \sin \theta - 0.500 \sin \theta - 0.401 \cos \theta - 0.354 \cos 2\theta - 0.319) \] (38)

\[ V_\theta = W (0.561 \sin \theta + 1.1599 \cos \theta - 0.500 \cos \theta + 0.707 \sin 2\theta) \] (39)

C. Dead Load. See Figure 7.

Total dead load = \( D = 300 \pi r t \)
\[ w = 150 \pi r d \theta - \frac{D}{2\pi} d \theta \] (40)

\[ P_b = \frac{D}{0.943} \cos 2\theta \] (41)

The forces on the cut section from dead load are:

Where \( 0 \leq \theta \leq \pi \)

\[ M_\theta = \frac{D r}{2\pi} (\theta \sin \theta + \cos \theta - 1) \] (42)

\[ P_\theta = \frac{D}{2\pi} \theta \sin \theta \] (43)

\[ V_\theta = \frac{D}{2\pi} \theta \cos \theta \] (44)

The forces from \( P_b \) are the same as for earth loads with \( D \) substituted for \( A \). See Equations 9, 10, and 11.

The rotations at the top are as follows:

From \( D \),
\[ \Delta \phi = \frac{D r}{2\pi EI} \int_0^\pi (\theta \sin \theta + \cos \theta - 1) \theta d\theta = 0 \]

From \( P_b \),
\[ \Delta \phi = \frac{0.030 D r^2}{EI} \]

The horizontal deflections are:

From \( D \),
\[ \Delta h = \frac{D r}{2\pi EI} \int_0^\pi (\theta \sin \theta + \cos \theta - 1) \theta (1 - \cos \theta) r d\theta = \frac{D r^3}{8 EI} \]

From \( P_b \),
\[ \Delta h = \frac{-0.060 D r^3}{EI} \]

From \( D + P_b \),
\[ \Delta h = \frac{0.065 D r^3}{EI} \]

We may then write:

\[ EI \Delta \phi = \frac{0.030 D r^2 + \pi r M_0}{-\pi r^2 H_0} = 0 \] (45)

\[ EI \Delta h = \frac{0.065 D r^3 - \pi r^2 M_0}{\frac{3}{2} \pi r^3 H_0} = 0 \] (46)

from which

\[ H_0 = -0.061 D, M_0 = -0.070 D r \]

The final equations for dead load are:

Where \( 0 \leq \theta \leq \frac{3}{4} \pi \)

\[ M_\theta = D r (1.1599 \sin \theta + 0.099 \cos \theta - 0.189) \] (47)

\[ P_\theta = D (1.1599 \sin \theta - 0.061 \cos \theta) \] (48)

\[ V_\theta = D (1.1599 \cos \theta + 0.061 \sin \theta) \] (49)
Where $\frac{3}{4} \pi \leq \theta \leq \pi$

$$M_e = Dr(1.159 \theta \sin \theta - 0.500 \sin \theta - 401 \cos \theta - 354 \cos 2\theta - 169)$$

$$P_e = D(1.159 \theta \sin \theta - 0.561 \cos \theta - 0.500 \sin \theta - 354 \cos 2\theta)$$

$$V_e = D(1.159 \theta \sin \theta - 0.500 \cos \theta + 0.561 \sin \theta + 0.707 \sin 2\theta)$$

**NOTE:** The formulas for $M_e$ and $V_e$ are the same as for water load with $D$ substituted for $W$. This holds also for bearing areas other than $\frac{\pi}{2}$ and in following derivations for dead load this fact will be used.

2. Analysis for Reaction over Central Angle $\frac{\pi}{4}$

**A. Earth Load.**

$$P_t = \frac{A \cos \frac{4}{7} \theta}{1.588 r}$$

$$P_b = \frac{A \cos 4\theta}{0.493 r}$$

Where $0 \leq \theta \leq \frac{7}{8} \pi$

$$M_b = 0.947 A r (\cos \frac{4}{7} \theta - \cos \theta)$$

Where $\frac{7}{8} \pi \leq \theta \leq \pi$

$$M_b = 0.947 A r (-\frac{1}{7} \cos 4\theta - \cos \theta)$$

$$P_b = \frac{1}{r} M_b$$

and $V_b = \frac{1}{r} \frac{dM_b}{d\theta}$ for all conditions of loading except for dead load where $P_b \neq \frac{1}{r} M_b$. See Equation 43. Hereafter the equations for $P_b$ and $M_b$ will not be given.

$$EIA_d = 1.623 A r^2 + \pi r M_0$$

$$EIA_h = -2.335 A r^3 - \pi r^2 M_0$$

$$H_0 = -0.234 A, M_0 = -0.084 A r$$

Where $0 \leq \theta \leq \frac{7}{8} \pi$

$$M_e = A r (0.947 \cos \frac{4}{7} \theta - 0.494 \cos \theta - 0.517)$$

$$P_e = A (0.947 \cos \frac{4}{7} \theta - 0.494 \cos \theta)$$

$$V_e = A (0.494 \sin \theta - 0.541 \sin \frac{4}{7} \theta)$$

Where $\frac{7}{8} \pi \leq \theta \leq \pi$

$$M_e = A(r (-1.35 \cos 4\theta - 0.494 \cos \theta - 0.517))$$

$$P_e = A (-1.35 \cos 4\theta - 0.494 \cos \theta)$$

$$V_e = A (0.541 \sin 4\theta + 0.494 \sin \theta)$$

**B. Water Load.**

$$P_b = \frac{W \cos 4\theta}{0.493 r}$$

Where $\frac{7}{8} \pi \leq \theta \leq \pi$

$$M_b = W r (0.135 \cos 4\theta - 0.494 \cos \theta)$$

$$P_e = W (-0.318 + 0.084 \cos \theta + 0.159 \theta \sin \theta)$$

$$V_e = W (0.075 \sin \theta + 0.159 \theta \cos \theta)$$

Where $\frac{7}{8} \pi \leq \theta \leq \pi$

$$M_e = W r (-1.162 + 0.084 \cos \theta + 0.159 \theta \sin \theta)$$

$$P_e = W (-0.318 - 0.500 \sin \theta + 0.159 \theta \sin \theta - 0.135 \cos 4\theta)$$

$$V_e = W (0.075 \sin \theta + 0.159 \theta \cos \theta)$$
\[ V_e = W(-0.500 \cos \theta + 0.159 \theta \cos \theta + 0.282 \sin \theta + 0.541 \sin 4\theta) \]  

(74)

C. Dead Load

\[ EI\phi = +0.007 Dr^2 \text{ (This rotation is from Pb only.)} \]

Since \( M_o = 0.078 Dr \) we may write the following equation:

\[ EI\phi = +0.007 Dr^2 - \pi r(0.078)Dr \]

\[ - \pi r^2 H_o = 0 \]  

(75)

from which,

\[ H_o = 0.075 D \]

Where \( 0 \leq \theta \leq \frac{7}{3} \pi \)

\[ M_o = Dr(-0.162 + 0.084 \cos \theta + 0.159 \theta \sin \theta - 0.135 \cos 4\theta) \]  

(76)

\[ P_o = D(0.159 \theta \sin \theta - 0.075 \cos \theta) \]  

(77)

\[ V_e = D(0.075 \sin \theta + 0.159 \theta \cos \theta) \]  

(78)

Where \( \frac{7}{3} \pi \leq \theta \leq \pi \)

\[ M_o = Dr(-0.162 - 0.500 \sin \theta + 0.159 \theta \sin \theta - 0.123 \cos \theta - 0.135 \cos 4\theta) \]  

(79)

\[ P_o = D(0.159 \theta \sin \theta - 0.500 \sin \theta - 0.282 \cos \theta - 0.135 \cos 4\theta) \]  

(80)

\[ V_e = D(-0.500 \cos \theta + 0.159 \theta \cos \theta + 0.282 \sin \theta + 0.541 \sin 4\theta) \]  

(81)

3. Analysis for Reaction over Central Angle \( \frac{2}{3} \pi \)

A. Earth Load

\[ P_t = \frac{7}{12} \frac{A}{r} \cos \frac{3}{4} \theta \]  

(82)

\[ P_b = \frac{5}{6} \frac{A}{r} (- \sin \frac{3}{2} \theta) \]  

(83)

Where \( 0 \leq \theta \leq \frac{2}{3} \pi \)

\[ M_o = \frac{4}{3} Ar(\cos \frac{3}{4} \theta - \cos \theta) \]  

(84)

Where \( \frac{2}{3} \pi \leq \theta \leq \pi \)

\[ M_o = \frac{4}{3} Ar(\frac{1}{2} \sin \frac{3}{2} \theta - \cos \theta) \]  

(85)

\[ EI\phi = \frac{4}{3} Ar^2 + \pi r M_o \]

\[ - \pi r^2 H_o = 0 \]  

(86)

\[ EI\phi = -1.885 Ar^3 - \pi r^2 M_o \]

\[ + \frac{3}{2} \pi r^3 H_o = 0 \]  

(87)

\[ H_o = 0.351 A, M_o = -0.073 Ar \]

Where \( 0 \leq \theta \leq \frac{2}{3} \pi \)

\[ M_o = Ar(\frac{2}{3} \sin \frac{3}{2} \theta - 0.982 \cos \theta) \]  

(88)

\[ P_o = Ar(\frac{2}{3} \sin \frac{3}{2} \theta - 0.982 \cos \theta) \]  

(89)

\[ V_e = A(- \sin \frac{3}{4} \theta + 0.982 \sin \theta) \]  

(90)

Where \( \frac{2}{3} \pi \leq \theta \leq \pi \)

\[ M_o = Ar(\frac{2}{3} \sin \frac{3}{2} \theta - 0.982 \cos \theta - 0.424) \]  

(91)

\[ P_o = Ar(\frac{2}{3} \sin \frac{3}{2} \theta - 0.982 \cos \theta) \]  

(92)

\[ V_e = A(\cos \frac{3}{2} \theta + 0.982 \sin \theta) \]  

(93)

B. Water Load.

\[ P_b = \frac{5}{6} \frac{W}{r} (- \sin \frac{3}{2} \theta) \]  

(94)

Where \( \frac{2}{3} \pi \leq \theta \leq \pi \)

\[ M_o \text{ from } P_b = Wr(\frac{2}{3} \sin \frac{3}{2} \theta - \frac{1}{2} \sin \theta - 0.866 \cos \theta) \]  

(95)

\[ EI\phi = -\frac{4}{9} Wr^2 + \pi r M_o - \pi r^2 H_o = 0 \]  

(96)

\[ EI\phi = 0.766 Wr^3 - \pi r^2 M_o \]

\[ + \frac{3}{2} \pi r^3 H_o = 0 \]  

(97)

\[ H_o = -0.205W, M_o = -0.063 Wr \]
Where $0 \leq \theta \leq \frac{2\pi}{3}$

\[ M_{0} = W(-.177 + .114 \cos \theta + .159 \theta \sin \theta + .687 \sin \frac{3}{2} \theta) \]  

(98)

\[ P_{e} = W(-.318 + .114 \cos \theta + .159 \theta \sin \theta) \]  

(99)

\[ V_{e} = W(.159 \theta \cos \theta + .046 \sin \theta) \]  

(100)

Where $\frac{2\pi}{3} \leq \theta \leq \pi$

\[ M_{0} = W(-.177 - .752 \cos \theta + .159 \theta \sin \theta + .500 \sin \theta + .687 \sin \frac{3}{2} \theta) \]  

(101)

\[ P_{e} = W(-.318 - .752 \cos \theta + .159 \theta \sin \theta - .500 \sin \theta + .687 \sin \frac{3}{2} \theta) \]  

(102)

\[ V_{e} = W(.912 \sin \theta + .159 \theta \cos \theta - .500 \cos \theta + \cos \frac{3}{2} \theta) \]  

(103)

C. Dead Load.

\[ EI_{\phi} = \frac{1}{18} D r^2 + \pi r (-.063 D r) - \pi r^2 H_0 \]

\[ = 0 \]

\[ H_0 = -.046 D \]  

(104)

Where $0 \leq \theta \leq \frac{2\pi}{3}$

\[ M_{0} = D r (-.177 + .114 \cos \theta + .159 \theta \sin \theta) \]  

(105)

\[ P_{e} = D (.159 \theta \sin \theta - .046 \cos \theta) \]  

(106)

\[ V_{e} = D (.159 \theta \cos \theta + .046 \sin \theta) \]  

(107)

Where $\frac{2\pi}{3} \leq \theta \leq \pi$

\[ M_{0} = D r (-.177 - .752 \cos \theta + .159 \theta \sin \theta + .687 \sin \frac{3}{2} \theta) \]  

(108)

\[ P_{e} = D(.159 \theta \sin \theta - .500 \sin \theta + .687 \sin \frac{3}{2} \theta - .912 \cos \theta) \]  

(109)

\[ V_{e} = D(.912 \sin \theta + .159 \theta \cos \theta - .500 \cos \theta + \cos \frac{3}{2} \theta) \]  

(110)

4. Analysis for Reaction over Central Angle $n$.

A. Earth Load.

\[ P_{t} = \frac{2A}{\text{nr}} \cos \theta \]  

(111)

\[ P_{b} = \frac{-2A}{\text{nr}} \cos \theta \]  

(112)

Where $0 \leq \theta \leq \frac{n}{2}$

\[ M_{b} = A r \theta \sin \theta \]  

(113)

Where $\frac{n}{2} \leq \theta \leq n$

\[ M_{b} = A r (\sin \theta - \frac{\theta}{\pi} \sin \theta - \frac{2}{\pi} \cos \theta) \]  

(114)

\[ EI_{\phi} = \frac{4}{\pi} A r^2 + \pi r M_0 - \pi r^2 H_0 = 0 \]  

(115)

\[ EI_{h} = -1.773 A r^3 - \pi r^2 M_0 + \frac{3}{2} \pi r^3 H_0 = 0 \]  

(116)

\[ H_0 = .318 A, M_0 = -.087 A r \]  

Where $0 \leq \theta \leq \frac{n}{2}$

\[ M_{0} = A r (.318 \theta \sin \theta + .318 \cos \theta - .405) \]  

(117)

\[ P_{e} = A (.318 \theta \sin \theta + .318 \cos \theta) \]  

(118)

\[ V_{e} = .318 A \theta \cos \theta \]  

(119)

**NOTE:** $M_{0}$ and $P_{e}$ are symmetrical about the horizontal axis. The values of $V_{e}$ between $\frac{n}{2}$ and $\pi$ are numerically equal but opposite in sign from those where $0 \leq \theta \leq \frac{n}{2}$. 

9
For dead load \( K = D = xxmt \)
For earth load \( K = A = \text{total weight of earth on pipe} \)
For water load \( K = W = 62.4 Tr_\phi \)

<table>
<thead>
<tr>
<th></th>
<th>0°</th>
<th>105°</th>
<th>150°</th>
<th>180°</th>
</tr>
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<tr>
<td>Moment</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Dead Load</td>
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<td>+0.88</td>
<td>-1.22</td>
<td></td>
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<tr>
<td>Earth Load</td>
<td>-0.067</td>
<td>+0.089</td>
<td>-1.26</td>
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</tr>
<tr>
<td>Water Load</td>
<td>-0.061</td>
<td>+0.287</td>
<td>+0.207</td>
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<tr>
<td>Thrust</td>
<td></td>
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<tr>
<td>Dead Load</td>
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<td>-0.017</td>
<td>-0.259</td>
<td>0</td>
</tr>
<tr>
<td>Earth Load</td>
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<td>-0.273</td>
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</tr>
<tr>
<td>Water Load</td>
<td>0</td>
<td>-0.017</td>
<td>-0.259</td>
<td>0</td>
</tr>
</tbody>
</table>

**COEFFICIENTS FOR M, P, AND V FOR BEARING AREA WITH CENTRAL ANGLE = \( \frac{\pi}{3} \)**

Stresses indicated are +
B. Water Load.

\[ P_b = -\frac{2W}{\pi} \cos \theta \]  

(120)

Where \( \frac{\alpha}{2} \leq \theta \leq \alpha \)

\[ M_e \text{ from } P_b = \frac{Wr}{\pi} (\frac{3}{2} \sin \theta - \theta \sin \theta - \cos \theta) \]  

(121)

\[ EIA_{\phi} = -0.383 W r^2 + \pi r M_0 \]  

(122)

\[ EIA_h = 0.613 W r^3 - \pi^2 r^2 M_0 \]  

\[ + \frac{3}{2} \pi^2 r^2 H_0 = 0 \]  

(123)

\[ H_0 = -0.159 W, M_0 = -0.044 W r \]

Where \( 0 \leq \theta \leq \frac{\alpha}{2} \)

\[ M_e = Wr (0.159 \cos \theta + 0.159 \theta \sin \theta - 0.203) \]  

(124)

\[ P_e = W (0.159 \cos \theta + 0.159 \theta \sin \theta - 0.318) \]  

(125)

\[ V_e = W (0.159 \theta \cos \theta) \]  

(126)

Where \( \frac{\alpha}{2} \leq \theta \leq \alpha \)

\[ M_e = Wr (-0.159 \cos \theta - 0.159 \theta \sin \theta + 0.500 \sin \theta - 0.203) \]  

(127)

\[ P_e = W (-0.159 \cos \theta - 0.159 \theta \sin \theta + 0.500 \sin \theta - 0.318) \]  

(128)

\[ V_e = W (0.500 \cos \theta - 0.159 \theta \cos \theta) \]  

(129)

C. Dead Load.

\[ P_b = -\frac{2D}{\pi} \cos \theta \]  

(130)

\[ EIA_{\phi} = 0.138 D r^2 + \pi r (-0.044 D r) \]  

\[ - \pi^2 r^2 H_0 = 0 \]  

(131)

\[ H_0 = 0 \]

Where \( 0 \leq \theta \leq \frac{\alpha}{2} \)

\[ M_e = Dr (0.159 \cos \theta + 0.159 \theta \sin \theta - 0.203) \]  

(132)

\[ P_e = D (0.159 \theta \sin \theta) \]  

(133)

\[ V_e = D (0.159 \theta \cos \theta) \]  

(134)

Where \( \frac{\alpha}{2} \leq \theta \leq \alpha \)

\[ M_e = Dr (-0.159 \cos \theta - 0.159 \theta \sin \theta + 0.500 \sin \theta - 0.203) \]  

(135)

\[ P_e = D (-0.318 \cos \theta - 0.159 \theta \sin \theta + 0.500 \sin \theta) \]  

(136)

\[ V_e = D (0.500 \cos \theta - 0.159 \theta \cos \theta) \]  

(137)
Note: Coefficients for moment and shear for water load are the same as for dead load.
EXECUTED LOADING: $X_{Dr}$, Earth load $X_{Ar}$, Water load $X_{Wr}$

PO : Dead load $Y_{D}$, Earth load $Y_{A}$, Water load $Y_{W}$

V : Dead load $Z_{D}$, Earth load $Z_{A}$, Water load $Z_{W}$

FORCES INDICATED ARE POSITIVE

COEFFICIENTS FOR $M_{00}$, $P_{0}$ AND $V_{0}$ FOR BEARING AREA WITH CENTRAL ANGLES $= \frac{\pi}{4}$, $\frac{\pi}{2}$, $\frac{2\pi}{3}$ AND $\pi$