ENGINEERING MONOGRAPHS No. 6

United States Department of the Interior
BUREAU OF RECLAMATION

STRESS ANALYSIS of CONCRETE PIPE

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Denver, Colorado
October 1950

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Engineering Monographs

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INTRODUCTION

Stress analysis of concrete pipe is in part based on the assumed distribution of earth pressures. Most commonly it has been assumed that the vertical earth load is uniformly distributed over the horizontal width of the pipe and that the lateral load is exerted so that the force diagram is trapezoidal in shape, applied on both sides, and extending the full height of the pipe. Further, it has been assumed that the reaction due to these loads is uniformly distributed over the full width of that portion of the bottom of the pipe in contact with the supporting surface. That these assumptions are rather arbitrary has long been recognized and efforts have been made to develop assumptions that would agree more closely with tests and with the general fund of knowledge about soil mechanics.

In 1930, the late Dr. Anson Marston, director of the Iowa Engineering Experiment Station at Iowa State College, advanced the theory that earth pressures on a rigid conduit and the reactions to those pressures would be exerted in such a manner that the force diagram representing the pressures would have characteristic bulb-like shapes above and below the outline of the pipe. Dr. Marston's theory was confirmed and extended by an investigation carried out at the Iowa Engineering Experiment Station and reported by M. G. Spangler of the Station staff.

While the design of the concrete pipe for the first section of the Salt Lake Aqueduct was being developed in 1938, W. A. Larsen and H. W. Birkeland, Bureau of Reclamation engineers under the direction of C. P. Vetter, who was in charge of the design, developed an analysis based on the assumption of bulb-like distribution of earth loads and soil reactions. This assumption closely approximates actual conditions as disclosed by tests. Since 1938, a number of other pipe lines have been designed, and the original analysis has been expanded so that various widths of bedding of the pipe may be analyzed.

In the original analysis the least work method by summation was used, but in subsequent studies a mathematical solution using integral equations was developed and is described in this monograph.

STRESS ANALYSIS OF CONCRETE PIPE

General

Under ordinary conditions a pipe placed under a fill is subjected to loads due to earth pressure around the pipe, the dead load of the pipe itself, and the internal hydrostatic pressure. For convenience of design the hydrostatic pressure is divided into two parts: (a) that part producing a uniform internal pressure, of which the head H is measured from the hydraulic gradient to the top of the inside of the pipe; and (b) the remaining part of the pressure of which the head is measured from the top to the bottom of the inside of the pipe.

The uniform internal pressure, part (a), produces only uniform tension around the pipe and is equal to $62.4\ Hr_o$, where r_o is the inside radius of the pipe in feet, and H is the head in feet measured as stated above.

The remaining part of the hydrostatic pressure produces bending, direct, and shear stresses in the pipe. This part is designated as water load and will be discussed in combination with earth load and dead load.

Earth Load, Dead Load, and Water Load

The assumptions for the distribution of earth pressures are based on results obtained from tests as summarized in Bulletin 112 of the Iowa Engineering Experiment Station. Figure 1 is developed from data in this bulletin and shows the distribution of earth pressures around a rigid pipe placed on compacted backfill, representing the ordinary condition of a pipe placed under fill or in a wide trench. The

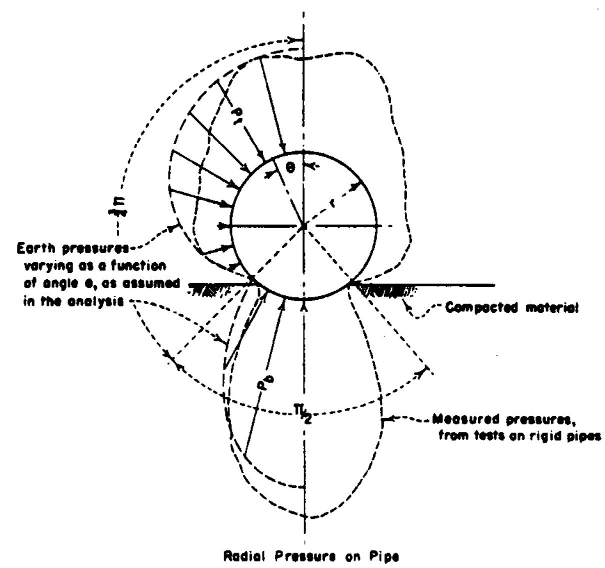


FIGURE 1 - Earth pressures around a rigid pipe placed on compacted backfill exhibit characteristic bulb-shaped patterns of distribution.

reaction resisting external loads is assumed to be distributed over the bottom of the pipe, limited by a definite central angle and varying as some function of the angle θ, see Figure 4. The limits of the central angle will depend on the bedding of the pipe, see Figure 2. Generally a 90° angle is used, but the analysis of the pipe has been prepared here for central angles

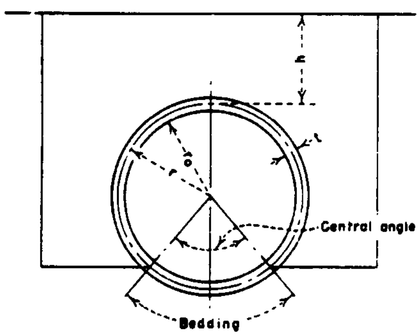


FIGURE 2 - Stress analysis of rigid pipe requires that these basic dimensions be known.

of 45°, 90°, 120°, and 180°. In the analysis, loads and pressure distribution around the pipe are assumed to be symmetrical about the vertical centerline.

Nomenclature

P_t = unit external pressure from earth. It is a function of angle θ and the total downward force is equal to the weight of earth on the pipe.

P_b = unit external pressure on the bottom of the pipe. It is a function of angle θ and equals the reaction of the earth on the pipe, from earth load, water load, and dead load.

P_i = unit internal water pressure from part (b) of hydrostatic pressure mentioned previously.

$= 62.4 r_o(1-\cos\theta)$

h = height of earth fill above pipe in feet. See Figure 2.

r = radius of centerline of pipe shell in feet.

r_o = radius of inside of pipe in feet.

W_e = unit weight of effective earth cover in pounds per cubic foot.

A = total weight of earth on pipe $= 2rW_eh$

W = total weight of water in pipe per foot of pipe $= 62.4\pi r_o^2$.

t = thickness of pipe shell in feet.

D = total weight of pipe $= 2\pi r 150 t = 300\pi rt$.

M'_θ, P'_θ, V'_θ = moment, thrust, and shear respectively on section at angle θ when pipe is assumed to be cut at top. See Figure 3b.

M_θ, P_θ, V_θ = final moment, thrust, and shear on section at angle θ.

Pipe Analysis

The stress analysis is made by the least work method, taking advantage of certain known facts about the final deflected shape of the pipe ring, resulting from the symmetrical loads assumed. Due to the assumed loads the pipe must deflect symmetrically about the vertical centerline, and the two points on this centerline, top and bottom, will neither rotate nor deflect horizontally. The pipe then is assumed to be cut in half and each half considered as a curved cantilever beam free at the top and fixed at the bottom. With the pipe cut in this manner, earth, water, and dead loads will cause the top to deflect. Thrust H_o and Moment M_o, see Figure 3, are then calculated to restore the free end to its final position so that its horizontal deflection and rotation are zero. Shear V_o at the top is zero.

The solution for a pipe bearing over a central angle equal to $\pi/2$ will be derived

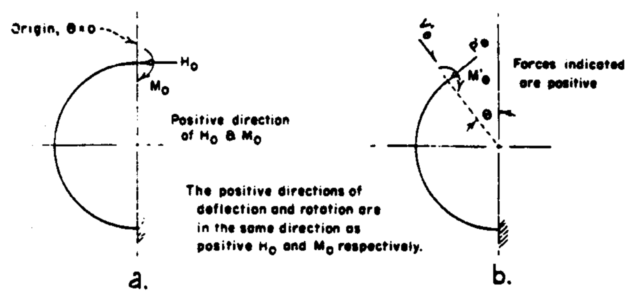


FIGURE 3 - Moment, shear, and thrust are determined assuming that the pipe is cut at the top and is free to deflect. At a, the forces are assumed on the vertical centerline; and at b, at any angle θ to that centerline.

in detail, for earth load, water load, and dead load. The solutions for other assumed bearing areas are similar and only those equations which are different will be given as they appear.

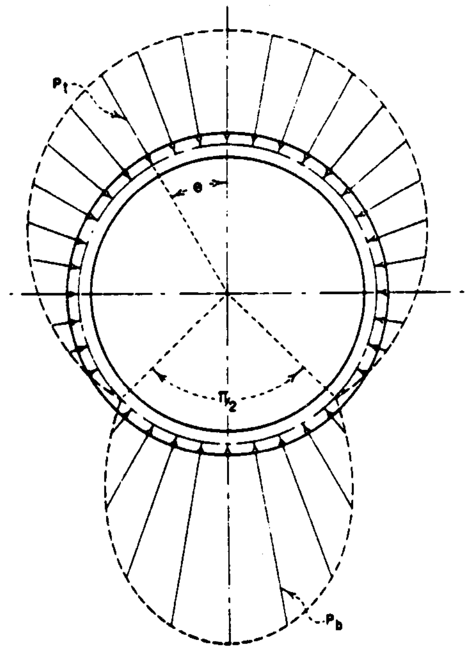


FIGURE 4 - With a central angle of $\pi/2$, earth pressures and reactions exhibit the characteristic bulb shapes.

1. Analysis for Reaction over Central Angle $\pi/2$.

A. Earth Load. See Figure 4.

assume P_t to vary as $\cos\frac{2}{3}\theta$, or $P_t = P_o\cos\frac{2}{3}\theta$

then $2\int_0^{\frac{3}{4}\pi} P_o\cos\frac{2}{3}\theta\cos\theta r d\theta = A$

from which $P_o = \dfrac{A}{1.697r}$

and $P_t = \dfrac{A}{1.697r}\cos\frac{2}{3}\theta$ (1)

---

assume P_b to vary as $\cos 2\theta$, or $P_b = P_\pi\cos 2\theta$

then $2\int_{\frac{3}{4}\pi}^{\pi} P_\pi\cos 2\theta\cos\theta r d\theta = A$

$P_\pi = \dfrac{A}{.943r}$, $\quad P_b = \dfrac{A}{.943r}\cos 2\theta$ (2)

The forces on the cut section at θ, are as follows:

From P_t, where $0 \leqq \theta \leqq \frac{3}{4}\pi$

$$M'_\theta = 1.061Ar(\cos\tfrac{2}{3}\theta - \cos\theta) \tag{3}$$

$$P'_\theta = 1.061A(\cos\tfrac{2}{3}\theta - \cos\theta) \tag{4}$$

$$V'_\theta = 1.061A(\sin\theta - \tfrac{2}{3}\sin\tfrac{2}{3}\theta) \tag{5}$$

Where $\frac{3}{4}\pi \leqq \theta \leqq \pi$

$$M'_\theta = 0.750Ar(0.667\sin\theta - 0.748\cos\theta) \tag{6}$$

$$P'_\theta = \frac{A}{2}(\sin\theta - 1.122\cos\theta) \tag{7}$$

$$V'_\theta = \frac{A}{2}(1.122\sin\theta + \cos\theta) \tag{8}$$

From P_b, where $\frac{3}{4}\pi \leqq \theta \leqq \pi$

$$M'_\theta = \frac{-Ar}{2}(0.707\cos 2\theta + \sin\theta + \cos\theta) \tag{9}$$

$$P'_\theta = \frac{-A}{2}(0.707\cos 2\theta + \sin\theta + \cos\theta) \tag{10}$$

$$V'_\theta = \frac{A}{2}(1.414\sin 2\theta + \sin\theta - \cos\theta) \tag{11}$$

From $P_t + P_b$, where $\frac{3}{4}\pi \leqq \theta \leqq \pi$

$$M'_\theta = -1.061Ar\,(\tfrac{1}{3}\cos 2\theta + \cos\theta) \tag{12}$$

$$P'_\theta = \frac{-A}{2}(2.122\cos\theta + 0.707\cos 2\theta) \tag{13}$$

$$V'_\theta = \frac{A}{2}(2.122\sin\theta + 1.414\sin 2\theta) \tag{14}$$

From M_o, where $0 \leqq \theta \leqq \pi$

$$M'_\theta = M_o \tag{15}$$

From H_o, where $0 \leqq \theta \leqq \pi$

$$M'_\theta = -H_o r\,(1 - \cos\theta) \tag{16}$$

$$P'_\theta = H_o\cos\theta \tag{17}$$

$$V'_\theta = -H_o\sin\theta \tag{18}$$

The rotations $\Delta\phi$ of the top ($\theta = 0$) of the cut section are as follows:

From P_t and P_b,

$$\Delta_\phi = \frac{1.061\ Ar}{EI}\left\{\int_0^{\frac{3}{4}\pi} (\cos\tfrac{2}{3}\theta - \cos\theta)\, r d\theta - \int_{\frac{3}{4}\pi}^{\pi} (\tfrac{1}{3}\cos 2\theta + \cos\theta)\, r d\theta\right\}$$

$$= \frac{1.414\ Ar^2}{EI}$$

From M_o,

$$\Delta_\phi = \frac{M_o}{EI}\int_0^{\pi} r d\theta = \frac{\pi r}{EI} M_o$$

From H_o,

$$\Delta_\phi = \frac{-H}{EI}\int_0^{\pi} r(1-\cos\theta) r d\theta = \frac{-\pi r^2 H_o}{EI}$$

Horizontal deflections Δ_h are as follows:

From P_t and P_b,

$$\Delta_h = \frac{1.061 Ar}{EI}\left\{-\int_0^{\frac{3}{4}\pi} (\cos\tfrac{2}{3}\theta - \cos\theta) r(1 - \cos\theta) r d\theta + \int_{\frac{3}{4}\pi}^{\pi} (\tfrac{1}{3}\cos 2\theta + \cos\theta) r(1 - \cos\theta) r d\theta\right\} = \frac{-2.013}{EI} Ar^3$$

From M_o,

$$\Delta_h = \frac{-\pi r^2 M_o}{EI}$$

From H_o,

$$\Delta_h = \frac{H_o}{EI}\int_0^{\pi} r^2 (1-\cos\theta)^2 r d\theta = \frac{\frac{3}{2}\pi r^3 H_o}{EI}$$

Since the rotation and the horizontal deflection at the top are zero, the following equations can be written:

$$EI\Delta_\phi = 1.414\ Ar^2 + \pi r M_o - \pi r^2 H_o = 0 \qquad (19)$$

$$EI\Delta_h = -2.013\ Ar^3 - \pi r^2 M_o + \tfrac{3}{2}\pi r^3 H_o = 0 \qquad (20)$$

from which

$$H_o = +.382A \text{ and } M_o = -.068Ar$$

The final equations then for earth loads on section θ are:

Where $0 \leqq \theta \leqq \frac{3}{4}\pi$

$$M_\theta = Ar\,(1.061 \cos\tfrac{2}{3}\theta - .678\cos\theta - .450) \qquad (21)$$

$$P_\theta = A\,(1.061\cos\tfrac{2}{3}\theta - .678\cos\theta) \qquad (22)$$

$$V_\theta = A\,(.678\sin\theta - .707\sin\tfrac{2}{3}\theta) \qquad (23)$$

<u>Note:</u> $V_\theta = \frac{1}{r}\frac{dM_\theta}{d\theta}$

Where $\frac{3}{4}\pi \leqq \theta \leqq \pi$

$$M_\theta = Ar\,(-.354\cos 2\theta - .678\cos\theta - .450) \qquad (24)$$

$$P_\theta = A\,(-.354\cos 2\theta - .678\cos\theta) \qquad (25)$$

$$V_\theta = A\,(.707\sin 2\theta + .678\sin\theta) \qquad (26)$$

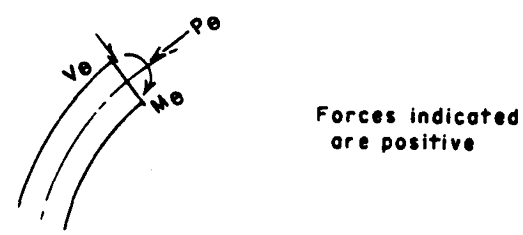


FIGURE 5 - Stresses within the pipe shell are in accordance with these assumptions. Positive direction of forces is indicated.

B. Water Load. See Figure 6.

$$P_i = -62.4 r_o\,(1-\cos\theta) = \frac{-W}{\pi r_o}(1-\cos\theta)$$

For simplification let $r_o = r$, then

$$P_i = \frac{-W}{\pi r}(1-\cos\theta) \qquad (27)$$

$$P_b = \frac{W}{.943r}\cos 2\theta \qquad (28)$$

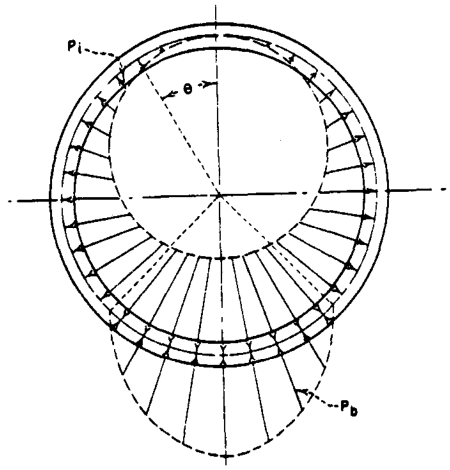


FIGURE 6 - Assumed pressure distribution of water load is in accordance with this diagram.

The forces on the cut section at θ are as follows:

From P_i, where $0 \leqq \theta \leqq \pi$

$$M_\theta^{\prime} = \frac{-Wr}{\pi}\left(1-\cos\theta - \frac{\theta}{2}\sin\theta\right) \quad (29)$$

$$P_\theta^{\prime} = \frac{-W}{\pi}\left(1-\cos\theta - \frac{\theta}{2}\sin\theta\right) \quad (30)$$

$$V_\theta^{\prime} = \frac{-W}{\pi}\left(\frac{1}{2}\sin\theta - \frac{\theta}{2}\cos\theta\right) \quad (31)$$

The forces from P_b are the same as given under earth loads with W substituted for A. See Equations 9, 10, and 11.

The rotations $\Delta\phi$ at the top are as follows:

From P_i,

$$\Delta\phi = \frac{-Wr}{\pi EI}\int_0^{\pi}\left(1-\cos\theta - \frac{\theta}{2}\sin\theta\right) r d\theta$$

$$= \frac{-Wr^2}{2EI}$$

From P_b,

$$\Delta\phi = \frac{-Wr}{2EI}\int_{\frac{3}{4}\pi}^{\pi}(.707\cos 2\theta + \sin\theta$$

$$+ \cos\theta)\, r d\theta = \frac{-.030\, Wr^2}{EI}$$

From $P_i + P_b$,

$$\Delta_\phi = \frac{-.470\, Wr^2}{EI}$$

The horizontal deflections Δ_h at the top are:

From P_i,

$$\Delta_h = \frac{Wr}{\pi EI}\int_0^{\pi}(1-\cos\theta$$

$$- \frac{\theta}{2}\sin\theta)\, r\,(1-\cos\theta)\, r d\theta = +\frac{7}{8}\frac{Wr^3}{EI}$$

From P_b,

$$\Delta_h = \frac{Wr}{2EI}\int_{\frac{3}{4}\pi}^{\pi}(.707\cos 2\theta + \sin\theta$$

$$+ \cos\theta)\, r\,(1-\cos\theta)\, r d\theta = \frac{-.060\, Wr^3}{EI}$$

From $P_t + P_b$,

$$\Delta_h = \frac{+.815\, Wr^3}{EI}$$

Then by the same reasoning as for Equations 19 and 20, we write:

$$EI\Delta\phi = -.470Wr^2 + \pi r M_o - \pi r^2 H_o = 0 \quad (32)$$

$$EI\Delta_h = +.815Wr^3 - \pi r^2 M_o + \frac{3}{2}\pi r^3 H_o = 0 \quad (33)$$

from which

$H_o = -.220W$ and $M_o = -.070Wr$

The final equations for water loads are:

Where $0 \leqq \theta \leqq \frac{3}{4}\pi$

$$M_\theta = Wr\,(.099\cos\theta + .159\,\theta\sin\theta - .169) \quad (34)$$

$$P_\theta = W\,(.099\cos\theta + .159\,\theta\sin\theta - .319) \quad (35)$$

$$V_\theta = W\,(.061\sin\theta + .159\,\theta\cos\theta) \quad (36)$$

Where $\frac{3}{4}\pi \leq \theta \leq \pi$

$$M_\theta = Wr\,(.159\theta \sin\theta - .500 \sin\theta - .401 \cos\theta - .354 \cos 2\theta - .169) \quad (37)$$

$$P_\theta = W\,(.159\theta \sin\theta - .500 \sin\theta - .401 \cos\theta - .354 \cos 2\theta - .319) \quad (38)$$

$$V_\theta = W\,(.561 \sin\theta + .159\theta \cos\theta - .500 \cos\theta + .707 \sin 2\theta) \quad (39)$$

C. Dead Load. See Figure 7.

Total dead load = $D = 300\pi rt$

$$w = 150 trd\,\theta = \frac{D}{2\pi}\,d\theta \quad (40)$$

$$P_b = \frac{D}{.943r} \cos 2\theta \quad (41)$$

The forces on the cut section from dead load are:

Where $0 \leq \theta \leq \pi$

$$M'_\theta = \frac{Dr}{2\pi}(\theta \sin\theta + \cos\theta - 1) \quad (42)$$

$$P'_\theta = \frac{D}{2\pi}\,\theta \sin\theta \quad (43)$$

$$V'_\theta = \frac{D}{2\pi}\,\theta \cos\theta \quad (44)$$

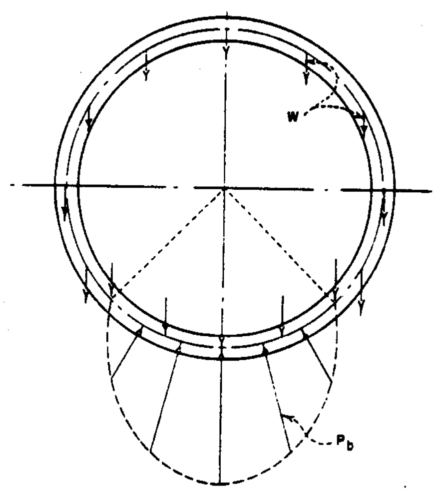


FIGURE 7 - Dead load due to the weight of the pipe itself is assumed in this manner.

The forces from P_b are the same as for earth loads with D substituted for A. See Equations 9, 10, and 11.

The rotations at the top are as follows:

From D,

$$\Delta\phi = \frac{Dr}{2\pi EI}\int_0^\pi (\theta \sin\theta + \cos\theta - 1)\,r d\theta = 0$$

From P_b,

$$\Delta\phi = \frac{.030}{EI} Dr^2$$

The horizontal deflections are:

From D,

$$\Delta_h = \frac{-Dr}{2\pi EI}\int_0^\pi (\theta \sin\theta + \cos\theta - 1) r (1 - \cos\theta) r d\theta = \frac{Dr^3}{8EI}$$

From P_b,

$$\Delta_h = \frac{-.060 Dr^3}{EI}$$

From $D + P_b$,

$$\Delta_h = \frac{.065 Dr^3}{EI}$$

We may then write:

$$EI\Delta\phi = .030 Dr^2 + \pi r M_o - \pi r^2 H_o = 0 \quad (45)$$

$$EI\Delta_h = .065 Dr^3 - \pi r^2 M_o + \frac{3}{2}\pi r^3 H_o = 0 \quad (46)$$

from which

$$H_o = -.061D, \quad M_o = -.070Dr$$

The final equations for dead load are:

Where $0 \leq \theta \leq \frac{3}{4}\pi$

$$M_\theta = Dr(.159\theta \sin\theta + .099 \cos\theta - .169) \quad (47)$$

$$P_\theta = D(.159\theta \sin\theta - .061 \cos\theta) \quad (48)$$

$$V_\theta = D(.159\theta \cos\theta + .061 \sin\theta) \quad (49)$$

Where $\frac{3}{4}\pi \leqq \theta \leqq \pi$

$$M_\theta = Dr(.159\theta \sin\theta - .500 \sin\theta - .401 \cos\theta - .354 \cos 2\theta - .169) \quad (50)$$

$$P_\theta = D(.159\theta \sin\theta - .561 \cos\theta - .500 \sin\theta - .354 \cos 2\theta) \quad (51)$$

$$V_\theta = D(.159\theta \cos\theta - .500 \cos\theta + .561 \sin\theta + .707 \sin 2\theta) \quad (52)$$

<u>NOTE</u>: The formulas for M_θ and V_θ are the same as for water load with D substituted for W. This holds also for bearing areas other than $\frac{\pi}{2}$ and in following derivations for dead load this fact will be used.

2. Analysis for Reaction over Central Angle $\frac{\pi}{4}$.

A. Earth Load.

$$P_t = \frac{A \cos\frac{4}{7}\theta}{1.568r} \quad (53)$$

$$P_b = \frac{A \cos 4\theta}{.493r} \quad (54)$$

Where $0 \leqq \theta \leqq \frac{7}{8}\pi$

$$M_\theta' = .947\ Ar(\cos\frac{4}{7}\theta - \cos\theta) \quad (55)$$

Where $\frac{7}{8}\pi \leqq \theta \leqq \pi$

$$M_\theta' = .947\ Ar(-\frac{1}{7}\cos 4\theta - \cos\theta) \quad (56)$$

$P_\theta' = \frac{1}{r} M_\theta'$ and $V_\theta' = \frac{1}{r}\frac{dM_\theta'}{d\theta}$ for all conditions of loading except for dead load where $P_\theta' \neq \frac{1}{r} M_\theta'$. See Equation 43. Hereafter the equations for P_θ' and M_θ' will not be given.

$$EI\Delta\phi = 1.623\ Ar^2 + \pi r M_o - \pi r^2 H_o = 0 \quad (57)$$

$$EI\Delta h = -2.335\ Ar^3 - \pi r^2 M_o + \frac{3}{2}\pi r^3 H_o = 0 \quad (58)$$

$H_o = .453A$, $M_o = -.064\ Ar$

Where $0 \leqq \theta \leqq \frac{7}{8}\pi$

$$M_\theta = Ar(.947 \cos\frac{4}{7}\theta - .494 \cos\theta - .517) \quad (59)$$

$$P_\theta = A(.947 \cos\frac{4}{7}\theta - .494 \cos\theta) \quad (60)$$

$$V_\theta = A(.494 \sin\theta - .541 \sin\frac{4}{7}\theta) \quad (61)$$

Where $\frac{7}{8}\pi \leqq \theta \leqq \pi$

$$M_\theta = Ar(-.135 \cos 4\theta - .494 \cos\theta - .517) \quad (62)$$

$$P_\theta = A(-.135 \cos 4\theta - .494 \cos\theta) \quad (63)$$

$$V_\theta = A(.541 \sin 4\theta + .494 \sin\theta) \quad (64)$$

B. Water Load.

$$P_b = \frac{W \cos 4\theta}{.493r} \quad (65)$$

Where $\frac{7}{8}\pi \leqq \theta \leqq \pi$

$$M_\theta' \text{ from } P_b = -Wr(.135 \cos 4\theta + .500 \sin\theta + .207 \cos\theta) \quad (66)$$

$$EI\Delta\phi = -.493\ Wr^2 + \pi r M_o - \pi r^2 H_o = 0 \quad (67)$$

$$EI\Delta h = +.860\ Wr^3 - \pi r^2 M_o + \frac{3}{2}\pi r^3 H_o = 0 \quad (68)$$

$H_o = -.234W$, $M_o = -.078Wr$

Where $0 \leqq \theta \leqq \frac{7}{8}\pi$

$$M_\theta = Wr(-.162 + .084 \cos\theta + .159\theta \sin\theta) \quad (69)$$

$$P_\theta = W(-.318 + .084 \cos\theta + .159\theta \sin\theta) \quad (70)$$

$$V_\theta = W(.075 \sin\theta + .159\theta \cos\theta) \quad (71)$$

Where $\frac{7}{8}\pi \leqq \theta \leqq \pi$

$$M_\theta = Wr(-.162 - .500 \sin\theta + .159\theta \sin\theta - .123 \cos\theta - .135 \cos 4\theta) \quad (72)$$

$$P_\theta = W(-.318 - .500 \sin\theta + .159\theta \sin\theta - .123 \cos\theta - .135 \cos 4\theta) \quad (73)$$

$$V_\theta = W(-.500 \cos\theta + .159\,\theta\cos\theta + .282\sin\theta + .541\sin 4\theta) \quad (74)$$

C. Dead Load

$EI\Delta\phi = +.007\,Dr^2$ (This rotation is from P_b only.)

Since $M_o = -.078\,Dr$ we may write the following equation:

$$EI\Delta\phi = +.007\,Dr^2 - \pi r(.078)Dr - \pi r^2 H_o = 0 \quad (75)$$

from which,

$H_o = .075\,D$

Where $0 \leq \theta \leq \frac{7}{8}\pi$

$$M_\theta = Dr(-.162 + .084\cos\theta + .159\,\theta\sin\theta) \quad (76)$$

$$P_\theta = D(.159\,\theta\sin\theta - .075\cos\theta) \quad (77)$$

$$V_\theta = D(.075\sin\theta + .159\,\theta\cos\theta) \quad (78)$$

Where $\frac{7}{8}\pi \leq \theta \leq \pi$

$$M_\theta = Dr(-.162 - .500\sin\theta + .159\,\theta\sin\theta - .123\cos\theta - .135\cos 4\theta) \quad (79)$$

$$P_\theta = D(.159\,\theta\sin\theta - .500\sin\theta - .282\cos\theta - .135\cos 4\theta) \quad (80)$$

$$V_\theta = D(-.500\cos\theta + .159\,\theta\cos\theta + .282\sin\theta + .541\sin 4\theta) \quad (81)$$

3. Analysis for Reaction over Central Angle $\frac{2}{3}\pi$

A. Earth Load

$$P_t = \frac{7}{12}\frac{A}{r}\cos\frac{3}{4}\theta \quad (82)$$

$$P_b = \frac{5}{6}\frac{A}{r}(-\sin\frac{3}{2}\theta) \quad (83)$$

Where $0 \leq \theta \leq \frac{2}{3}\pi$

$$M_\theta' = \frac{4}{3}Ar(\cos\frac{3}{4}\theta - \cos\theta) \quad (84)$$

Where $\frac{2}{3}\pi \leq \theta \leq \pi$

$$M_\theta' = \frac{4}{3}Ar(\frac{1}{2}\sin\frac{3}{2}\theta - \cos\theta) \quad (85)$$

$$EI\Delta\phi = \frac{4}{3}Ar^2 + \pi r M_o - \pi r^2 H_o = 0 \quad (86)$$

$$EI\Delta_h = -1.885\,Ar^3 - \pi r^2 M_o + \frac{3}{2}\pi r^3 H_o = 0 \quad (87)$$

$H_o = .351\,A$, $M_o = -.073\,Ar$

Where $0 \leq \theta \leq \frac{2}{3}\pi$

$$M_\theta = Ar(\frac{4}{3}\cos\frac{3}{4}\theta - .982\cos\theta - .424) \quad (88)$$

$$P_\theta = A(\frac{4}{3}\cos\frac{3}{4}\theta - .982\cos\theta) \quad (89)$$

$$V_\theta = A(-\sin\frac{3}{4}\theta + .982\sin\theta) \quad (90)$$

Where $\frac{2}{3}\pi \leq \theta \leq \pi$

$$M_\theta = Ar(\frac{2}{3}\sin\frac{3}{2}\theta - .982\cos\theta - .424) \quad (91)$$

$$P_\theta = A(\frac{2}{3}\sin\frac{3}{2}\theta - .982\cos\theta) \quad (92)$$

$$V_\theta = A(\cos\frac{3}{2}\theta + .982\sin\theta) \quad (93)$$

B. Water Load

$$P_b = \frac{5}{6}\frac{W}{r}(-\sin\frac{3}{2}\theta) \quad (94)$$

Where $\frac{2}{3}\pi \leq \theta \leq \pi$

$$M_\theta' \text{ from } P_b = Wr(\frac{2}{3}\sin\frac{3}{2}\theta - \frac{1}{2}\sin\theta - .866\cos\theta) \quad (95)$$

$$EI\Delta\phi = -\frac{4}{9}Wr^2 + \pi r M_o - \pi r^2 H_o = 0 \quad (96)$$

$$EI\Delta_h = .766\,Wr^3 - \pi r^2 M_o + \frac{3}{2}\pi r^3 H_o = 0 \quad (97)$$

$H_o = -.205W$, $M_o = -.063\,Wr$

Where $0 \leqq \theta \leqq \frac{2}{3}\pi$

$$M_\theta = Wr(-.177 + .114 \cos\theta + .159\,\theta \sin\theta) \quad (98)$$

$$P_\theta = W(-.318 + .114 \cos\theta + .159\,\theta \sin\theta) \quad (99)$$

$$V_\theta = W(.159\,\theta \cos\theta + .046 \sin\theta) \quad (100)$$

Where $\frac{2}{3}\pi \leqq \theta \leqq \pi$

$$M_\theta = Wr(-.177 - .752 \cos\theta + .159\,\theta \sin\theta - .500 \sin\theta + .667 \sin\tfrac{3}{2}\theta) \quad (101)$$

$$P_\theta = W(-.318 - .752 \cos\theta + .159\,\theta \sin\theta - .500 \sin\theta + .667 \sin\tfrac{3}{2}\theta) \quad (102)$$

$$V_\theta = W(.912 \sin\theta + .159\,\theta \cos\theta - .500 \cos\theta + \cos\tfrac{3}{2}\theta) \quad (103)$$

C. Dead Load.

$$EI\Delta\phi = \frac{1}{18} Dr^2 + \pi r(-.063\,Dr) - \pi r^2 H_o = 0 \quad (104)$$

$$H_o = -.046\,D$$

Where $0 \leqq \theta \leqq \frac{2}{3}\pi$

$$M_\theta = Dr(-.177 + .114 \cos\theta + .159\,\theta \sin\theta) \quad (105)$$

$$P_\theta = D(.159\,\theta \sin\theta - .046 \cos\theta) \quad (106)$$

$$V_\theta = D(.159\,\theta \cos\theta + .046 \sin\theta) \quad (107)$$

Where $\frac{2}{3}\pi \leqq \theta \leqq \pi$

$$M_\theta = Dr(-.177 - .752 \cos\theta + .159\,\theta \sin\theta - .500 \sin\theta + .667 \sin\tfrac{3}{2}\theta) \quad (108)$$

$$P_\theta = D(.159\,\theta \sin\theta - .500 \sin\theta + .667 \sin\tfrac{3}{2}\theta - .912 \cos\theta) \quad (109)$$

$$V_\theta = D(.912 \sin\theta + .159\,\theta \cos\theta - .500 \cos\theta + \cos\tfrac{3}{2}\theta) \quad (110)$$

4. Analysis for Reaction over Central Angle π.

A. Earth Load.

$$P_t = \frac{2A}{\pi r} \cos\theta \quad (111)$$

$$P_b = \frac{-2A}{\pi r} \cos\theta \quad (112)$$

Where $0 \leqq \theta \leqq \frac{\pi}{2}$

$$M'_\theta = \frac{Ar}{\pi}\,\theta \sin\theta \quad (113)$$

Where $\frac{\pi}{2} \leqq \theta \leqq \pi$

$$M'_\theta = Ar(\sin\theta - \frac{\theta}{\pi} \sin\theta - \frac{2}{\pi} \cos\theta) \quad (114)$$

$$EI\Delta\phi = \frac{4}{\pi} Ar^2 + \pi r M_o - \pi r^2 H_o = 0 \quad (115)$$

$$EI\Delta_h = -1.773\,Ar^3 - \pi r^2 M_o + \frac{3}{2}\pi r^3 H_o = 0 \quad (116)$$

$$H_o = .318A,\; M_o = -.087\,Ar$$

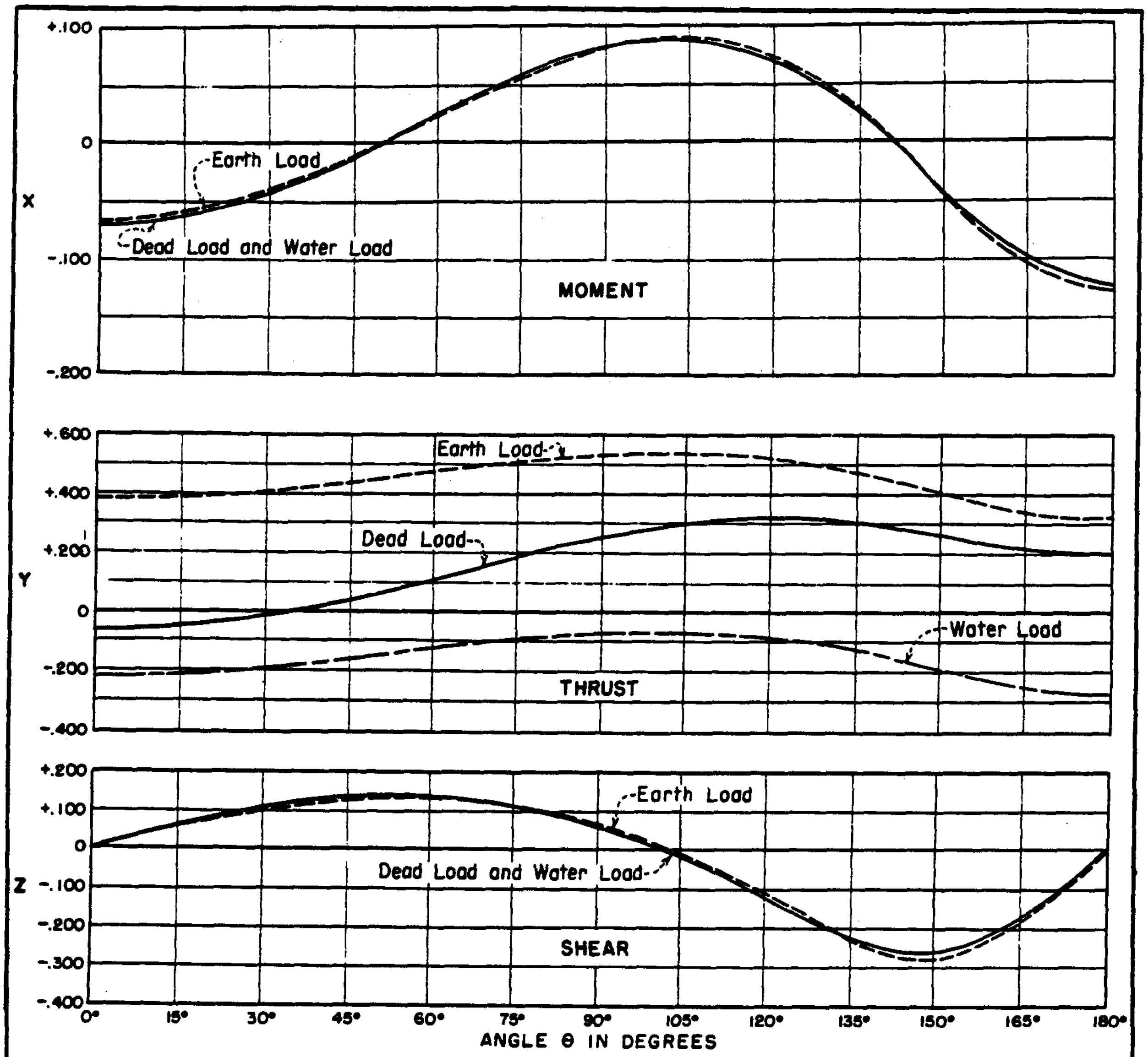
Where $0 \leqq \theta \leqq \frac{\pi}{2}$

$$M_\theta = Ar(.318\,\theta \sin\theta + .318 \cos\theta - .405) \quad (117)$$

$$P_\theta = A(.318\,\theta \sin\theta + .318 \cos\theta) \quad (118)$$

$$V_\theta = .318\,A\,\theta \cos\theta \quad (119)$$

NOTE: M_θ and P_θ are symmetrical about the horizontal axis. The values of V_θ between $\frac{\pi}{2}$ and π are numerically equal but opposite in sign from those where $0 \leqq \theta \leqq \frac{\pi}{2}$.



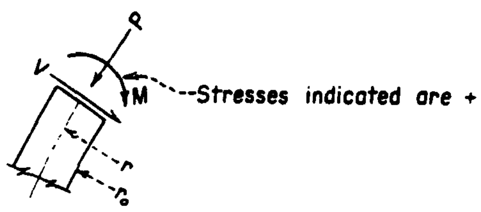
$M = XKr$
$P = YK$
$V = ZK$

For dead load $K = D = 300\pi rt$

For earth load $K = A =$ total weight of earth on pipe.

For water load $K = W = 62.4\pi r_o^2$

		0°	105°	150°	180°
Moment	Dead Load	-.070	+.088		-.122
	Earth Load	-.067	+.089		-.126
	Water Load	-.070	+.088		-.122
Thrust	Dead Load	-.061	+.297		+.207
	Earth Load	+.383	+.539		+.324
	Water Load	-.220	-.062		-.272
Shear	Dead Load	0	-.017	-.259	0
	Earth Load	0	-.010	-.273	0
	Water Load	0	-.017	-.259	0



COEFFICIENTS FOR M_θ, P_θ AND V_θ FOR BEARING AREA WITH CENTRAL ANGLE = $\frac{\pi}{2}$

B. Water Load.

$$P_b = \frac{-2W}{\pi r} \cos \theta \qquad (120)$$

Where $\frac{\pi}{2} \leq \theta \leq \pi$

$$M'_\theta \text{ from } P_b = \frac{Wr}{\pi} \left(\frac{\pi}{2} \sin \theta - \theta \sin \theta - \cos \theta\right) \qquad (121)$$

$$EI\Delta\phi = -.363\ Wr^2 + \pi r M_o - \pi r^2 H_o = 0 \qquad (122)$$

$$EI\Delta_h = .613\ Wr^3 - \pi r^2 M_o + \frac{3}{2}\pi r^3 H_o = 0 \qquad (123)$$

$$H_o = -.159W,\ M_o = -.044\ Wr$$

Where $0 \leq \theta \leq \frac{\pi}{2}$

$$M_\theta = Wr\,(.159 \cos \theta + .159\ \theta \sin \theta - .203) \qquad (124)$$

$$P_\theta = W\,(.159 \cos \theta + .159\ \theta \sin \theta - .318) \qquad (125)$$

$$V_\theta = W\,(.159\ \theta \cos \theta) \qquad (126)$$

Where $\frac{\pi}{2} \leq \theta \leq \pi$

$$M_\theta = Wr\,(-.159 \cos \theta - .159\ \theta \sin \theta + .500 \sin \theta - .203) \qquad (127)$$

$$P_\theta = W\,(-.159 \cos \theta - .159\ \theta \sin \theta + .500 \sin \theta - .318) \qquad (128)$$

$$V_\theta = W\,(.500 \cos \theta - .159\ \theta \cos \theta) \qquad (129)$$

C. Dead Load.

$$P_b = \frac{-2D}{\pi r} \cos \theta \qquad (130)$$

$$EI\Delta\phi = .136\ Dr^2 + \pi r(-.044\ Dr) - \pi r^2 H_o = 0 \qquad (131)$$

$$H_o = 0$$

Where $0 \leq \theta \leq \frac{\pi}{2}$

$$M_\theta = Dr\,(.159 \cos \theta + .159\ \theta \sin \theta - .203) \qquad (132)$$

$$P_\theta = D\,(.159\ \theta \sin \theta) \qquad (133)$$

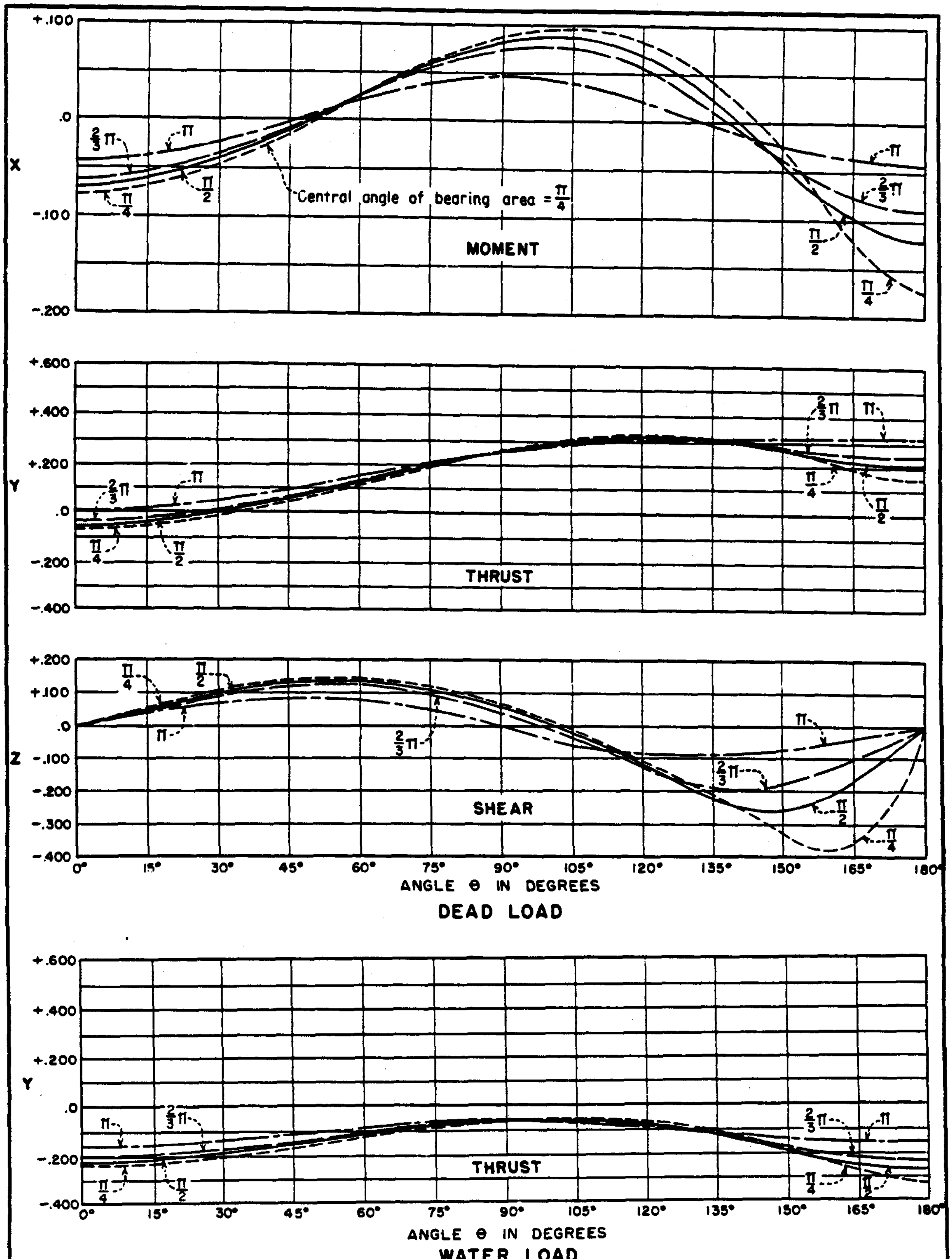
$$V_\theta = D\,(.159\ \theta \cos \theta) \qquad (134)$$

Where $\frac{\pi}{2} \leq \theta \leq \pi$

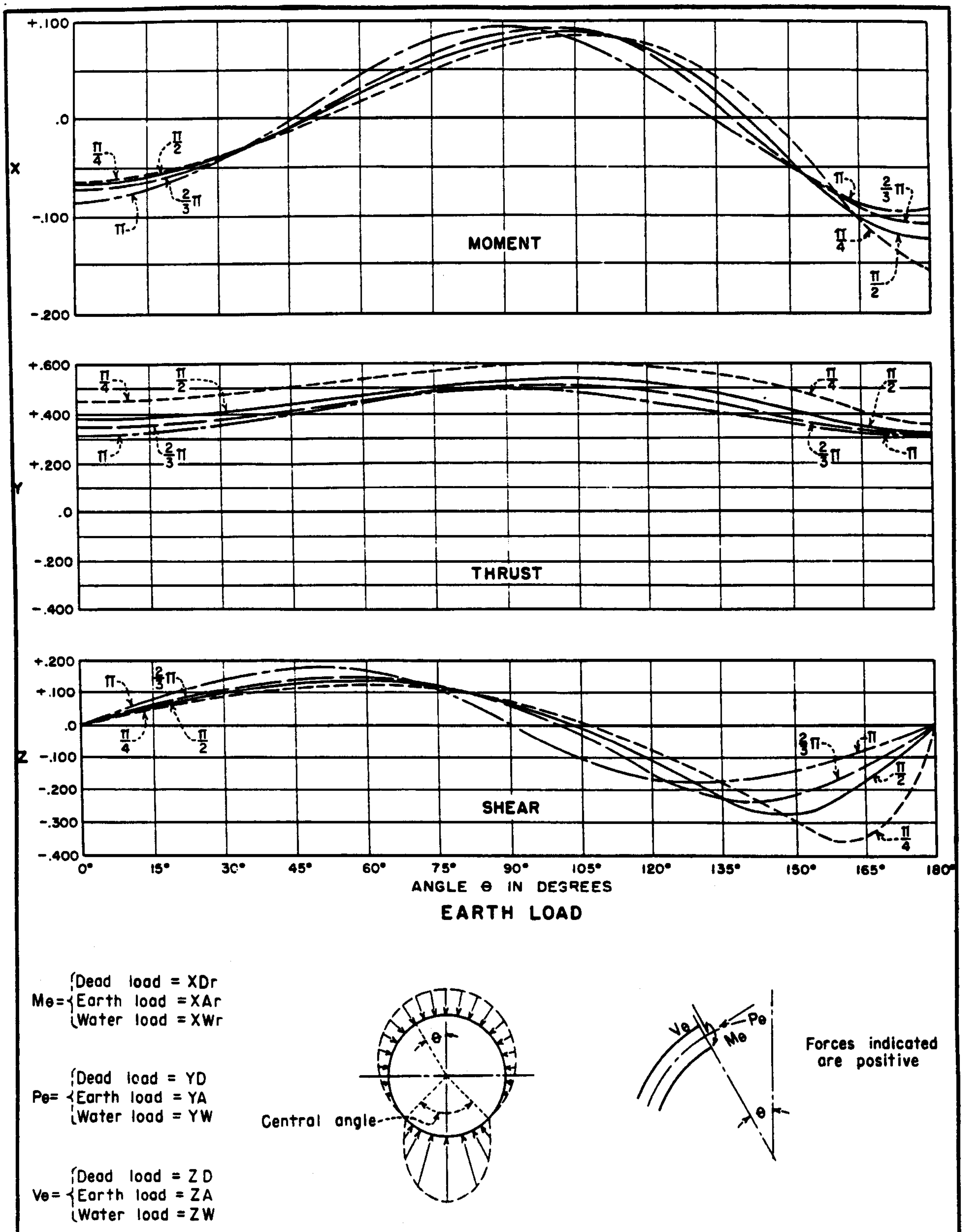
$$M_\theta = Dr\,(-.159 \cos \theta - .159\ \theta \sin \theta + .500 \sin \theta - .203) \qquad (135)$$

$$P_\theta = D\,(-.318 \cos \theta - .159\ \theta \sin \theta + .500 \sin \theta) \qquad (136)$$

$$V_\theta = D\,(.500 \cos \theta - .159\ \theta \cos \theta) \qquad (137)$$



Note: Coefficients for moment and shear for water load are the same as for dead load.



COEFFICIENTS FOR M_θ, P_θ AND V_θ FOR BEARING AREA WITH CENTRAL ANGLES = $\frac{\pi}{4}$, $\frac{\pi}{2}$, $\frac{2}{3}\pi$ AND π