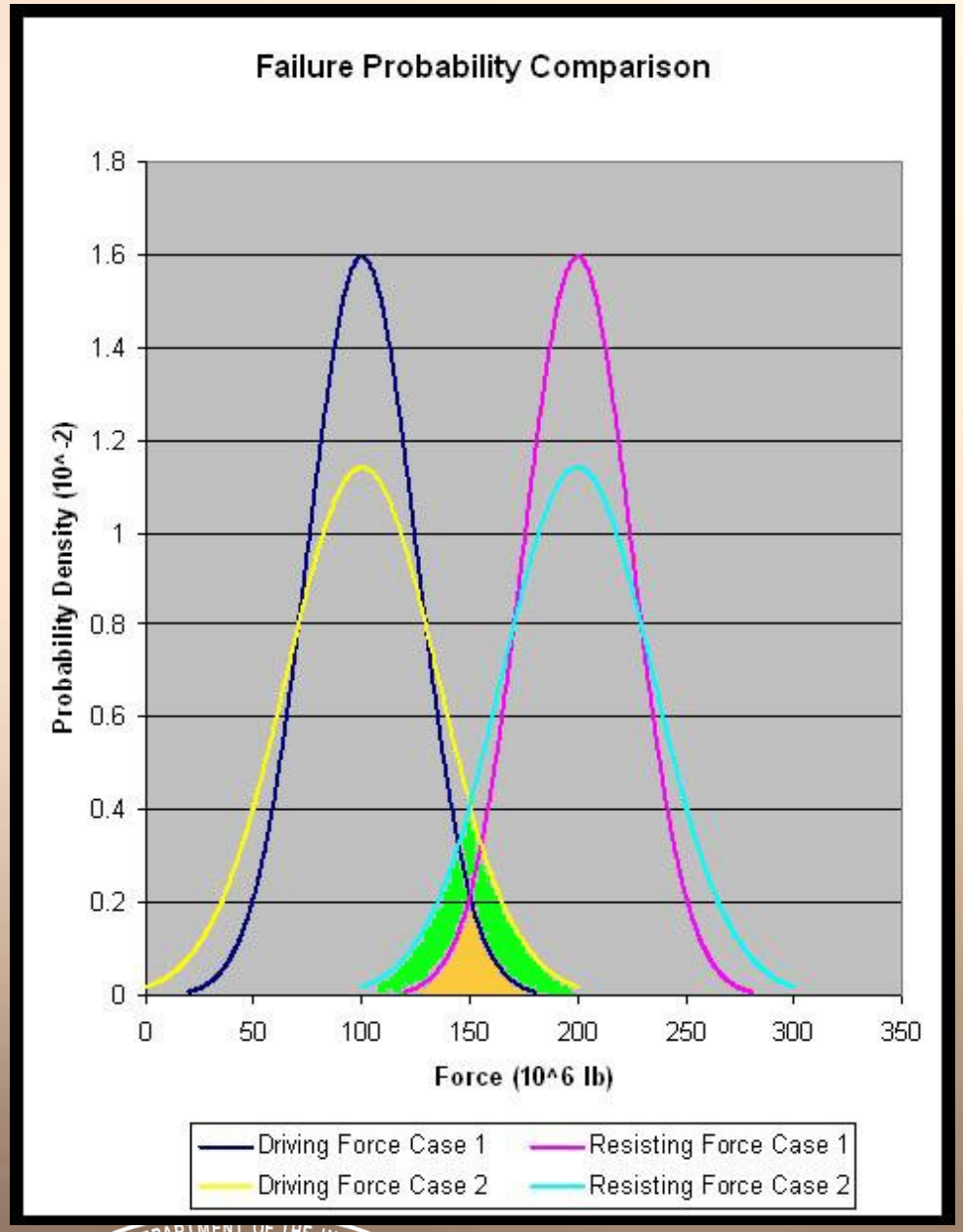
Best Practices in Dam and Levee Safety Risk Analysis

I-7. Probabilistic Stability Analysis

19 March 2015

RECLAMATION
Managing Water in the West

US Army Corps of Engineers
BUILDING STRONG®



Failure Probability Comparison

- Case 1 and 2 have the same mean FS = 2
- Failure probability for case 2 (green + orange areas) is more than twice that of case 1 (orange area only)
- The difference is the higher “dispersion” for case 2

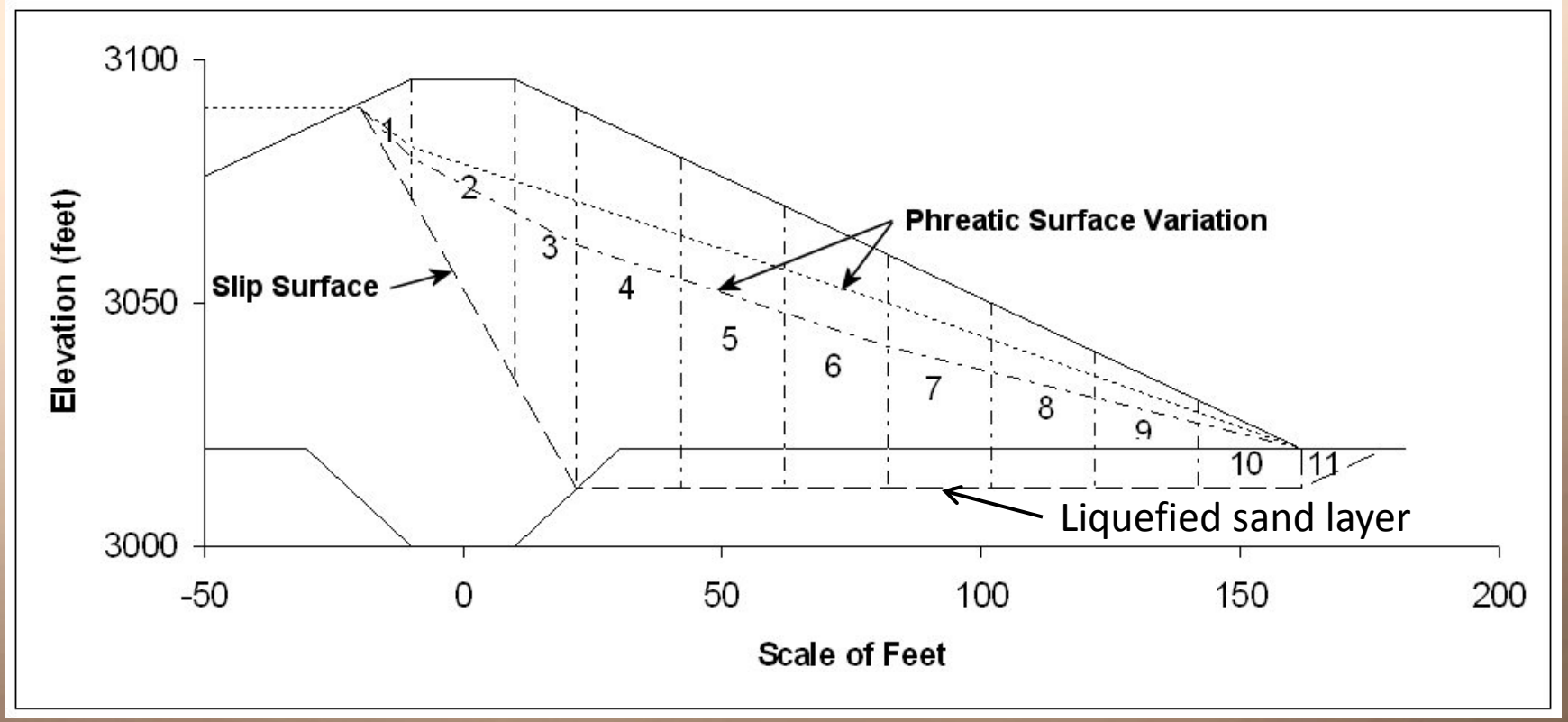
Analysis Methodology

- Program “deterministic” analysis in Microsoft Excel
- Use @Risk – commercially available Macro add-in
- Instead of defining input parameters as single values, define them as distributions
- Perform “Monte-Carlo” analysis using @Risk to calculate many factors of safety by sampling input distributions
- Use the output distribution of safety factor to determine the probability of F.S. <1.0 (or other value representing unsatisfactory performance)
- Prob F.S.<1.0 = (Number of F.S. hits<1.0) / (Total No. Trials)

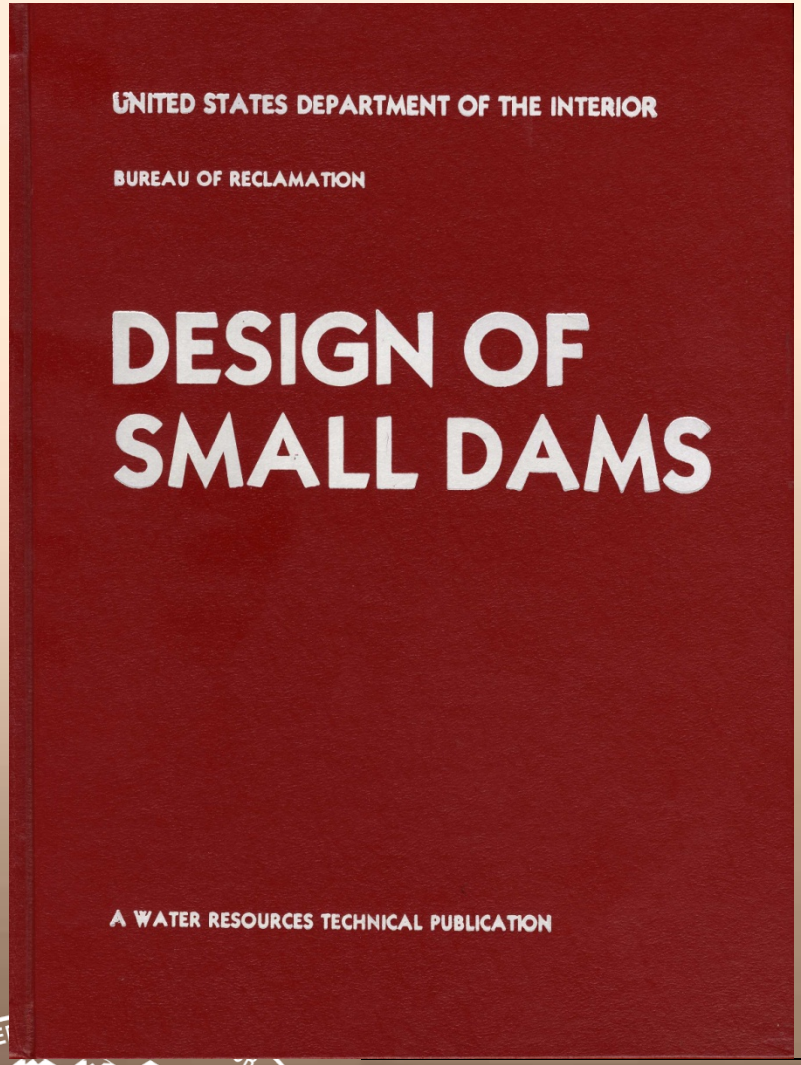
Example 1 – Post Liquefaction Stability of an Embankment Dam

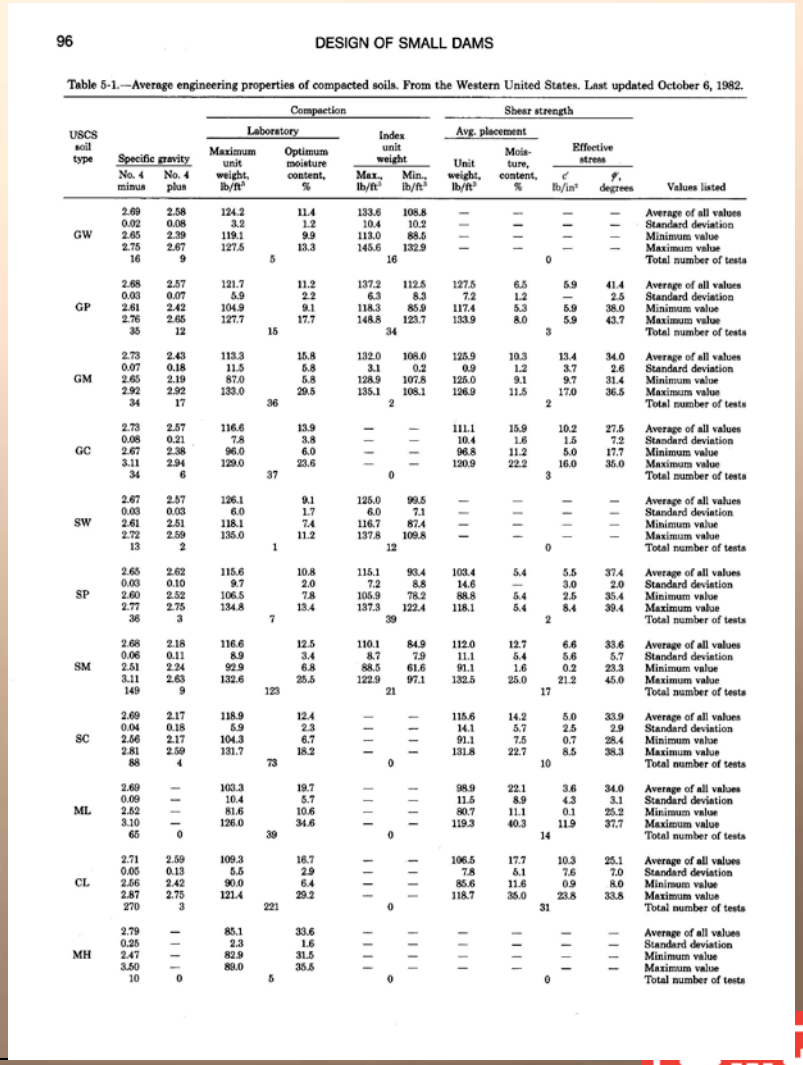
- 76-foot-high homogeneous dike
- Constructed in late 1940's
- SC material compacted with sheepsfoot roller
- Cutoff trench through 20 feet of alluvium to rock
- 3 borings through d/s shell into foundation alluvium
- Continuous clean sand layer 4' to 6' thick @ 8' below alluvium contact - $(N_1)_{60}$ = 13 to 15
- Wet area at toe of dike
- Seismically active area
- Piezometers in embankment show high phreatic surface - varies by 9 feet depending on boring

Embankment Geometry



Embankment Properties





96 DESIGN OF SMALL DAMS

Table 5-1.—Average engineering properties of compacted soils. From the Western United States. Last updated October 6, 1982.

USCS soil type	Specific gravity: No. 4 minus	Specific gravity: No. 4 plus	Compaction: Laboratory: Maximum unit weight, lb/ft³	Compaction: Laboratory: Optimum moisture content, %	Compaction: Index unit weight: Max., lb/ft³	Compaction: Index unit weight: Min., lb/ft³	Shear strength: Avg. placement: Unit weight, lb/ft³	Shear strength: Avg. placement: Moisture, content, %	Shear strength: Effective stress: c′ lb/in²	Shear strength: Effective stress: φ′, degrees	Values listed
GW	2.69	2.58	124.2	11.4	133.6	108.8	—	—	—	—	Average of all values
	0.02	0.08	3.2	1.2	10.4	10.2	—	—	—	—	Standard deviation
	2.65	2.39	119.1	9.9	113.0	88.5	—	—	—	—	Minimum value
	2.75	2.67	127.5	13.3	145.6	132.9	—	—	—	—	Maximum value
	16	9	5		16				0		Total number of tests
GP	2.68	2.57	121.7	11.2	137.2	112.5	127.5	6.5	5.9	41.4	Average of all values
	0.03	0.07	5.9	2.2	6.3	8.3	7.2	1.2	—	2.5	Standard deviation
	2.61	2.42	104.9	9.1	118.3	85.9	117.4	5.3	5.9	38.0	Minimum value
	2.76	2.65	127.7	17.7	148.8	123.7	133.9	8.0	5.9	43.7	Maximum value
	35	12	15		34				3		Total number of tests
GM	2.73	2.43	113.3	15.8	132.0	108.0	125.9	10.3	13.4	34.0	Average of all values
	0.07	0.18	11.5	5.8	3.1	0.2	0.9	1.2	3.7	2.6	Standard deviation
	2.65	2.19	87.0	5.8	128.9	107.8	125.0	9.1	9.7	31.4	Minimum value
	2.92	2.92	133.0	29.5	135.1	108.1	126.9	11.5	17.0	36.5	Maximum value
	34	17	36		2				2		Total number of tests
GC	2.73	2.57	116.6	13.9	—	—	111.1	15.9	10.2	27.5	Average of all values
	0.08	0.21	7.8	3.8	—	—	10.4	1.6	1.5	7.2	Standard deviation
	2.67	2.38	96.0	6.0	—	—	96.8	11.2	5.0	17.7	Minimum value
	3.11	2.94	129.0	23.6	—	—	120.9	22.2	16.0	35.0	Maximum value
	34	6	37		0				3		Total number of tests
SW	2.67	2.57	126.1	9.1	125.0	99.5	—	—	—	—	Average of all values
	0.03	0.03	6.0	1.7	6.0	7.1	—	—	—	—	Standard deviation
	2.61	2.51	118.1	7.4	116.7	87.4	—	—	—	—	Minimum value
	2.72	2.59	135.0	11.2	137.8	109.8	—	—	—	—	Maximum value
	13	2	1		12				0		Total number of tests
SP	2.65	2.62	115.6	10.8	115.1	93.4	103.4	5.4	5.5	37.4	Average of all values
	0.03	0.10	9.7	2.0	7.2	8.8	14.6	—	3.0	2.0	Standard deviation
	2.60	2.52	106.5	7.8	105.9	78.2	88.8	5.4	2.5	35.4	Minimum value
	2.77	2.75	134.8	13.4	137.3	122.4	118.1	5.4	8.4	39.4	Maximum value
	36	3	7		39				2		Total number of tests
SM	2.68	2.18	116.6	12.5	110.1	84.9	112.0	12.7	6.6	33.6	Average of all values
	0.06	0.11	8.9	3.4	8.7	7.9	11.1	5.4	5.6	5.7	Standard deviation
	2.51	2.24	92.9	6.8	88.5	61.6	91.1	1.6	0.2	23.3	Minimum value
	3.11	2.63	132.6	25.5	122.9	97.1	132.5	25.0	21.2	45.0	Maximum value
	149	9	123		21				17		Total number of tests
SC	2.69	2.17	118.9	12.4	—	—	115.6	14.2	5.0	33.9	Average of all values
	0.04	0.18	5.9	2.3	—	—	14.1	5.7	2.5	2.9	Standard deviation
	2.56	2.17	104.3	6.7	—	—	91.1	7.5	0.7	28.4	Minimum value
	2.81	2.59	131.7	18.2	—	—	131.8	22.7	8.5	38.3	Maximum value
	88	4	73		0				10		Total number of tests
ML	2.69	—	103.3	19.7	—	—	98.9	22.1	3.6	34.0	Average of all values
	0.09	—	10.4	5.7	—	—	11.5	8.9	4.3	3.1	Standard deviation
	2.52	—	81.6	10.6	—	—	80.7	11.1	0.1	25.2	Minimum value
	3.10	—	126.0	34.6	—	—	119.3	40.3	11.9	37.7	Maximum value
	65	0	39		0				14		Total number of tests
CL	2.71	2.59	109.3	16.7	—	—	106.5	17.7	10.3	25.1	Average of all values
	0.05	0.13	5.5	2.9	—	—	7.8	5.1	7.6	7.0	Standard deviation
	2.56	2.42	90.0	6.4	—	—	85.6	11.6	0.9	8.0	Minimum value
	2.87	2.75	121.4	29.2	—	—	118.7	35.0	23.8	33.8	Maximum value
	270	3	221		0				31		Total number of tests
MH	2.79	—	85.1	33.6	—	—	—	—	—	—	Average of all values
	0.25	—	2.3	1.6	—	—	—	—	—	—	Standard deviation
	2.47	—	82.9	31.5	—	—	—	—	—	—	Minimum value
	3.50	—	89.0	35.5	—	—	—	—	—	—	Maximum value
	10	0	5		0				0		Total number of tests

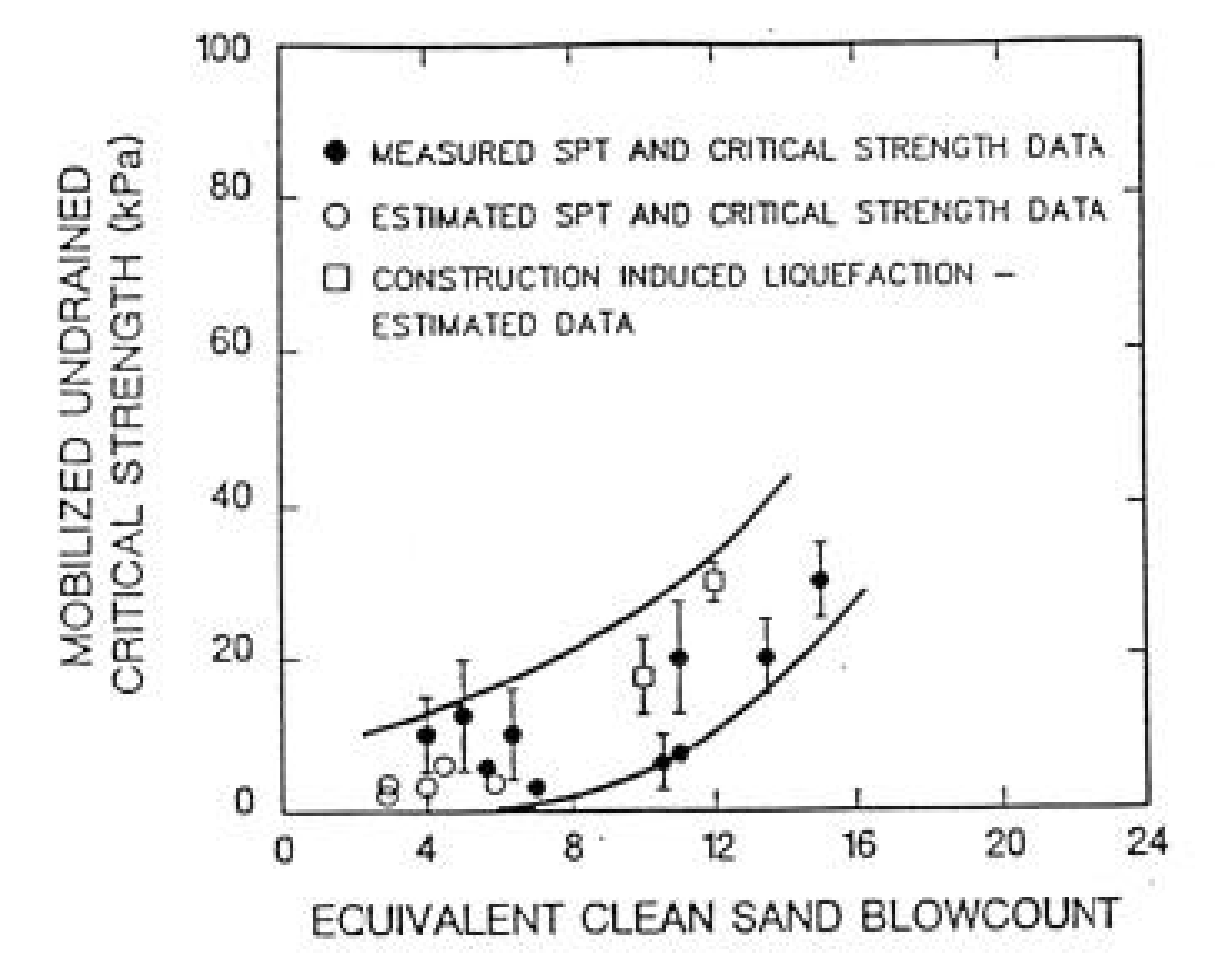
Embankment SC Material Properties

Property	Minimum	Maximum	Mean	Standard Deviation
Unit Weight (lb/ft^3)	91.1	131.8	115.6	14.1
c' (lb/ft^2)	101	1224	720	360
φ' (degrees)	28.4	38.3	33.9	2.9

Friction angles converted to TAN ϕ' for calculations

Be careful of c'

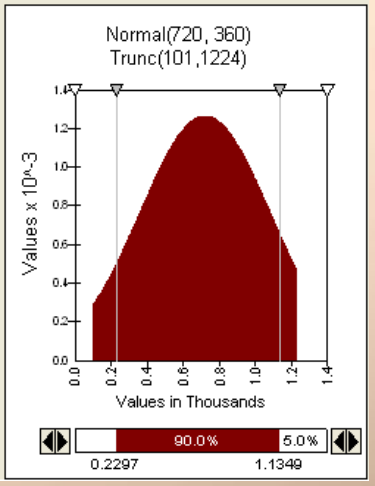
Undrained Residual Shear Strength

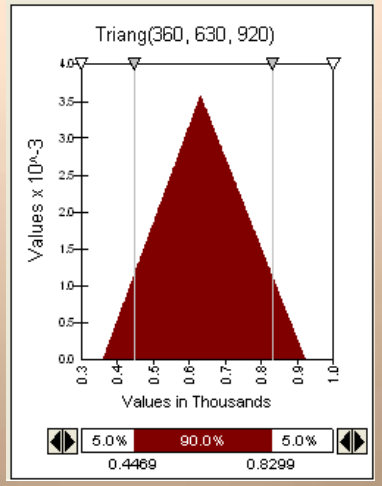


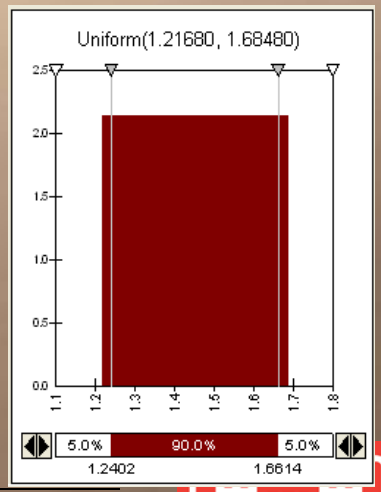
Seed and Harder (1990)

Example Input Distributions

- Embankment c'
  - RiskNormal(720,360,Risk Truncate(101,1224)) psf
- Alluvium undrained residual shear strength
  - RiskTriangle(360,630,920) psf for $(N_1)_{60}$=14
- Pore pressure force at base of embankment slice
  - RiskUniform(1.22,1.68) ksf







Bishop's Method Spreadsheet

Microsoft Excel - BishopsIterationJW3.xls

B4 =RiskNormal(720, 360, RiskTruncate(101, 1224))

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Bishop's Slope Stability Analysis												
2													
3	Drained Strengths			Undrained Strengths			Density						
4	c' =	696	psf	c =	637	psf	γ =	114	pcf				
5	tan Φ' =	0.671		tan Φ =	0								
6	Φ' =	33.9	degrees	Φ =	0								
7													
8													
9	Slice	Δxi	c*Δxi	ui	ui*Δxi	Wi	θi	(Wi-ui*Δxi)*tanΦ	C + H	cos(θi)*[1+(tanθi*tanΦ/FS)]	I / J	Wi*sinθi	
10		(ft)	(kips)	(ksf)	(kips)	(kips)	(deg)	(kips)				(kips)	
11													
12	1	12	8.35	0.28	3.37	17.72	60.6	9.64	17.99	0.92	19.63	15.44	
13	2	20	13.92	1.45	29.02	99.97	60.6	47.63	61.55	0.92	67.18	87.10	
14	3	12	8.35	2.90	34.82	94.06	60.6	39.77	48.12	0.92	52.52	81.95	
15	4	20	12.73	0	0.00	165.86	0.0	0.00	12.73	1.00	12.73	0.00	
16	5	20	12.73	0	0.00	143.14	0.0	0.00	12.73	1.00	12.73	0.00	
17	6	20	12.73	0	0.00	120.42	0.0	0.00	12.73	1.00	12.73	0.00	
18	7	20	12.73	0	0.00	97.70	0.0	0.00	12.73	1.00	12.73	0.00	
19	8	20	12.73	0	0.00	74.98	0.0	0.00	12.73	1.00	12.73	0.00	
20	9	20	12.73	0	0.00	52.26	0.0	0.00	12.73	1.00	12.73	0.00	
21	10	20	12.73	0	0.00	29.54	0.0	0.00	12.73	1.00	12.73	0.00	
22	11	16	11.13	0.19	3.00	7.27	-26.6	2.87	14.00	0.68	20.73	-3.26	
23										Sum ->	249.19	181.23	
24	FS = ΣK/ΣL =	1.37											

Allow circular reference

Bishop's Method Spreadsheet

Microsoft Excel - BishopsIterationJW3.xls

B24 =RiskOutput() + K23/L23

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Bishop's Slope Stability Analysis												
2													
3	Drained Strengths			Undrained Strengths			Density						
4	c' =	696	psf	c =	637	psf	γ =	114	pcf				
5	tan Φ' =	0.671		tan Φ =	0								
6	Φ' =	33.9	degrees	Φ =	0								
7													
8													
9	Slice	Δxi	c*Δxi	ui	ui*Δxi	Wi	θi	(Wi-ui*Δxi)*tanΦ	C + H	cos(θi)*[1+(tanθi*tanΦ/FS)]	I / J	Wi*sinθi	
10		(ft)	(kips)	(ksf)	(kips)	(kips)	(deg)	(kips)				(kips)	
11													
12	1	12	8.35	0.28	3.37	17.72	60.6	9.64	17.99	0.92	19.63	15.44	
13	2	20	13.92	1.45	29.02	99.97	60.6	47.63	61.55	0.92	67.18	87.10	
14	3	12	8.35	2.90	34.82	94.06	60.6	39.77	48.12	0.92	52.52	81.95	
15	4	20	12.73	0	0.00	165.86	0.0	0.00	12.73	1.00	12.73	0.00	
16	5	20	12.73	0	0.00	143.14	0.0	0.00	12.73	1.00	12.73	0.00	
17	6	20	12.73	0	0.00	120.42	0.0	0.00	12.73	1.00	12.73	0.00	
18	7	20	12.73	0	0.00	97.70	0.0	0.00	12.73	1.00	12.73	0.00	
19	8	20	12.73	0	0.00	74.98	0.0	0.00	12.73	1.00	12.73	0.00	
20	9	20	12.73	0	0.00	52.26	0.0	0.00	12.73	1.00	12.73	0.00	
21	10	20	12.73	0	0.00	29.54	0.0	0.00	12.73	1.00	12.73	0.00	
22	11	16	11.13	0.19	3.00	7.27	-26.6	2.87	14.00	0.68	20.73	-3.26	
23										Sum ->	249.19	181.23	
24	FS = ΣK/ΣL =	1.37											

F.S. Output - 10,000 iterations

@RISK Output Data Report

Output Data

Outputs	FS = ?K/?L = / (ft)
Simulation#	1
Iteration / Cell	B24

1	1.40282166
2	1.400596261
3	1.501422405
4	1.251672149
5	1.42097795
6	1.175672889
7	1.477007627
8	1.59830904
9	1.235151768
10	1.571461201
11	1.303617954
12	1.47099793
13	1.271955848
14	1.104541183
15	1.32243073
16	1.567054033
17	1.259018064
18	1.44313252
19	1.224726915
20	0.953297615
21	1.272130251

Summary Report | Outputs Data Report

	A	B
214		0.995250225
215		0.995401144
216		0.996119738
217		0.996244669
218		0.996596992
219		0.996732235
220		0.996939957
221		0.997377455
222		0.9976843
223		0.997831881
224		0.998167098
225		0.998627961
226		0.998666167
227		0.999451816
228		0.99964112
229		1.000186086
230		1.000269651
231		1.000651717
232		1.000991702
233		1.001196861
234		1.001266599
235		1.001507759
236		1.003768325
237		1.00380218
238		1.004098296
239		1.004418135
240		1.005377531
241		1.005683422
242		1.005916238

Prob. F.S. < 1.0 = 228/10,000 = 0.0228

Sensitivities from @Risk

Rank	Name	Cell	Regression	Correlation
1	c'	B4	0.726344	0.732725132
2	S_U	E4	0.590719261	0.575407848
3	γ	H4	-0.292465055	-0.272376816
4	φ'	B5	0.137192535	0.130003719
5	u slice 2	D13	-0.072522808	-0.070526537
6	u slice 3	D14	-0.052556307	-0.051631753
7	u slice 11	D22	-0.020666858	-0.020920598
8	u slice 1	D12	-0.018467738	-0.004675745

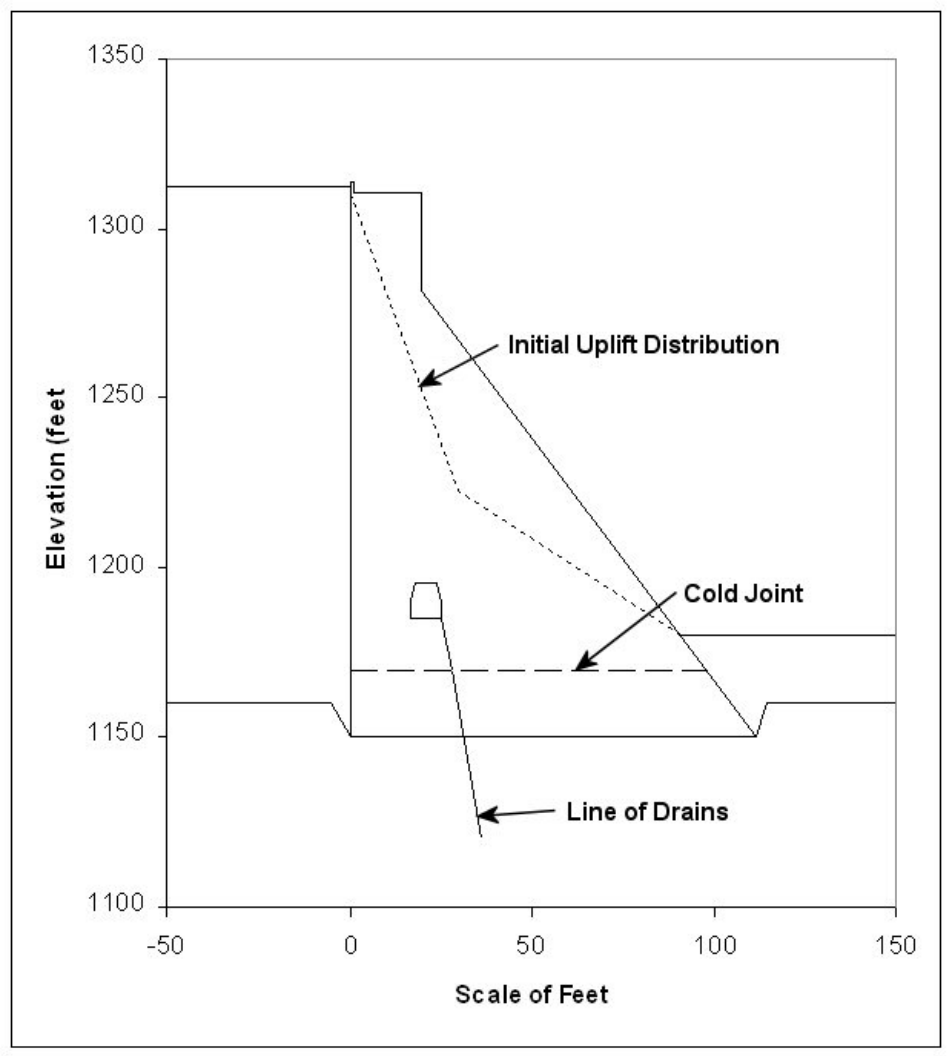
Parametric Analysis Results

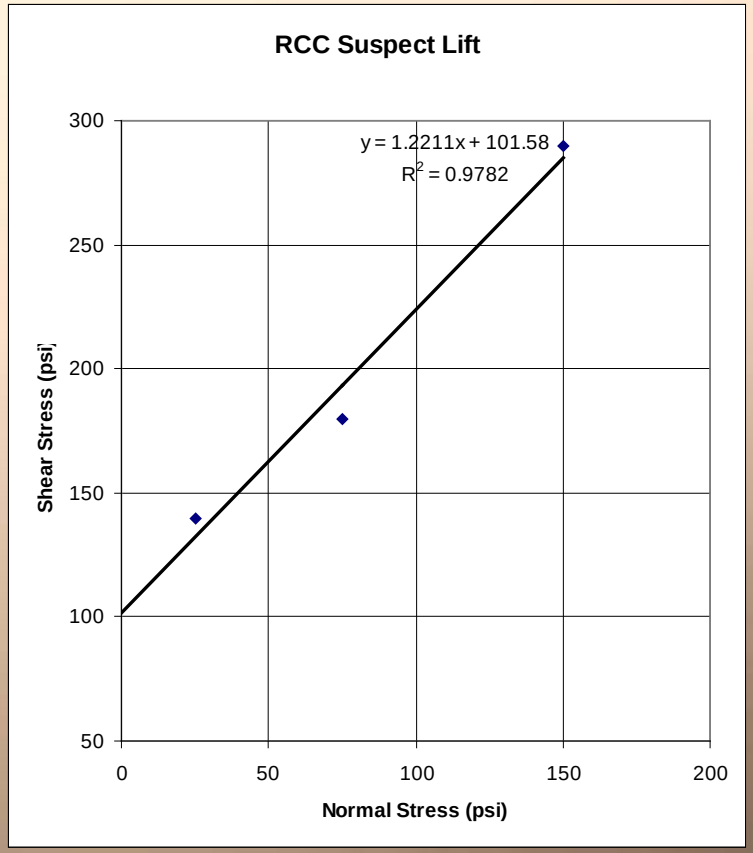
Case	Mean F.S.	Probability F.S.<1.0
Original input distributions	1.38	0.0228
Increase std dev and limits, c' RiskNormal(720,540, RiskTruncate(20,2000))	1.44	0.0345
Undrained residual strength for lower (N1)60 = 13 RiskTriangle(310,560,790)	1.32	0.0605

Example 2 - RCC Gravity Dam

- 160 feet high
- Winter shut-down after first 20 feet of RCC
- Spring - cold joint cleaned, mortar layer, rest of dam
- Gallery in dam with drainage curtain through joint
- Five 6” cores taken through cold joint
  - 3 of 5 were bonded and tested in direct shear
- PMF revised, encroaches 2.3’ on 3.5’ parapet wall

Dam Geometry and Strength Results





c’ and ϕ’ correlated

@Risk Spreadsheet

	A	B
1	Concrete Gravity Dam	
2	2-D Monte Carlo Safety Factor Analysis using @Risk	
3		
4	Coordinate System	
5	> +X (Horizontal) = D/S, +Y (Vertical) = Up	
6	> +M (Moment) = Clockwise	
7		
8	Crest Elevation (ft) =	1310.0
9	Base Elevation (ft) =	1170.0
10	Crest Width (ft) =	20.0
11	D/S Slope of Dam (H/V) =	0.7
12	Reservoir Elevation (ft) =	1312.3
13	Tailwater Elevation (ft) =	1180.0
14	Drain Dist from U/S Face (ft) =	28.0
15	Drain Factor =	0.54
16	TAN Friction Angle (deg) =	1.22
17	Intact Cohesion (psi) =	100.0
18	Percent Intact =	58
19	Concrete Density (pcf) =	149
20		
21	Horiz Length of Slide Plane (ft) =	98.0
22	Base Weight (k/ft) =	-1022.1
23	Crest Weight (k/ft) =	-42.6
24	Total Dam Weight (k/ft) =	-1064.7
25		
26	Pressure Head at Heel (ft) =	142.3
27	Pressure Head at Toe (ft) =	10.0
28	Pressure Head at Drains (ft) =	81.4
29	Total Uplift Force (k/ft) =	395.2

	A	B
30		
31	Horizontal Reservoir Load (k/ft) =	631.8
32	Horizontal Tailwater Load (k/ft) =	-3.1
33	(Vertical tailwater not included)	
34		
35	Stress (+ = tension)	
36	Moment about center of base (k-ft) =	11744
37	Moment of inertia =	78433
38	Total Vert Stress @ D/S Face (psi) =	-126.4
39	Total Vert Stress @ U/S Face (psi) =	-24.5
40	Total Vert Stress @ Drains (psi) =	-97.3
41	Effective Vert Stress @ D/S Face (psi) =	-122.1
42	Effective Vert Stress @ U/S Face (psi) =	37.2
43		
44	Modifications based on Tensile Zone	
45	Drain Factor (Univ. of Colo.) =	0.54
46	Pressure Head at Drains (ft) =	81.4
47	Effective Vert Stress @ Drains (psi) =	-62.0
48	Length of Tension Zone (ft) =	10.5
49	Adjusted Base Length (ft) =	87.5
50	Total Uplift Force (k/ft) =	415.1
51		
52	Total Normal Force (k/ft) =	-649.6
53	Frictional Resistance (k/ft) =	793.4
54	Intact Resistance (k/ft) =	730.8
55	Driving Force (k/ft) =	628.7
56		
57	Factor of Safety =	2.42
58		

Input Distributions

Property	Distribution	Minimum	Peak	Maximum
Initial Drain Factor	Uniform	0.33	n/a	0.75
φ' (degrees)	Triangular	43	50	57
c' (lb/in^2)	Triangular	50	100	150
Percent Intact	Triangular	43	60	71
RCC Unit Weight (lb/ft^3)	Uniform	146	n/a	152

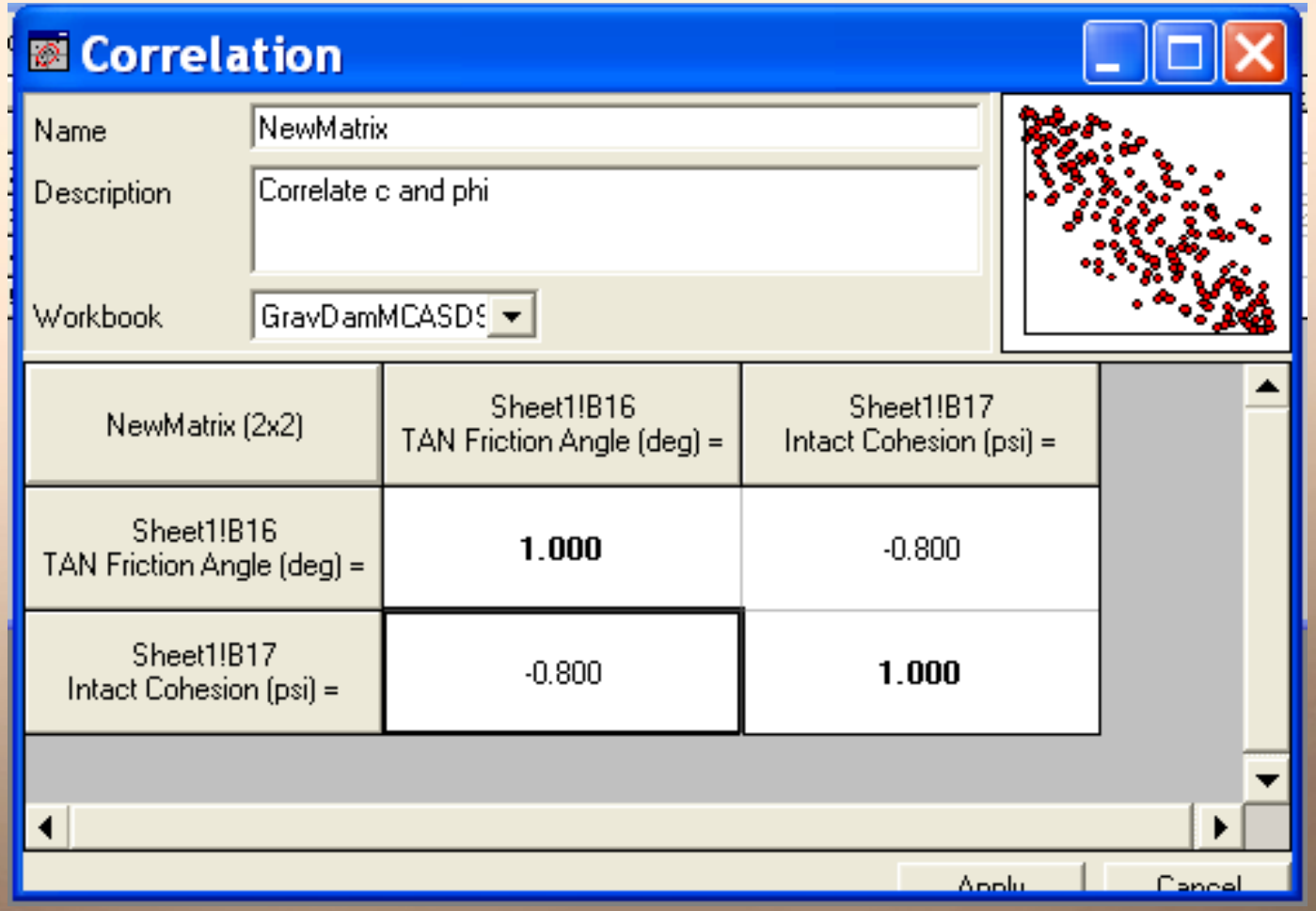
ϕ' converted to TAN ϕ' for calculations

Results and Sensitivities

10,000 iterations: avg. F.S. = 2.42, min F.S. = 1.43

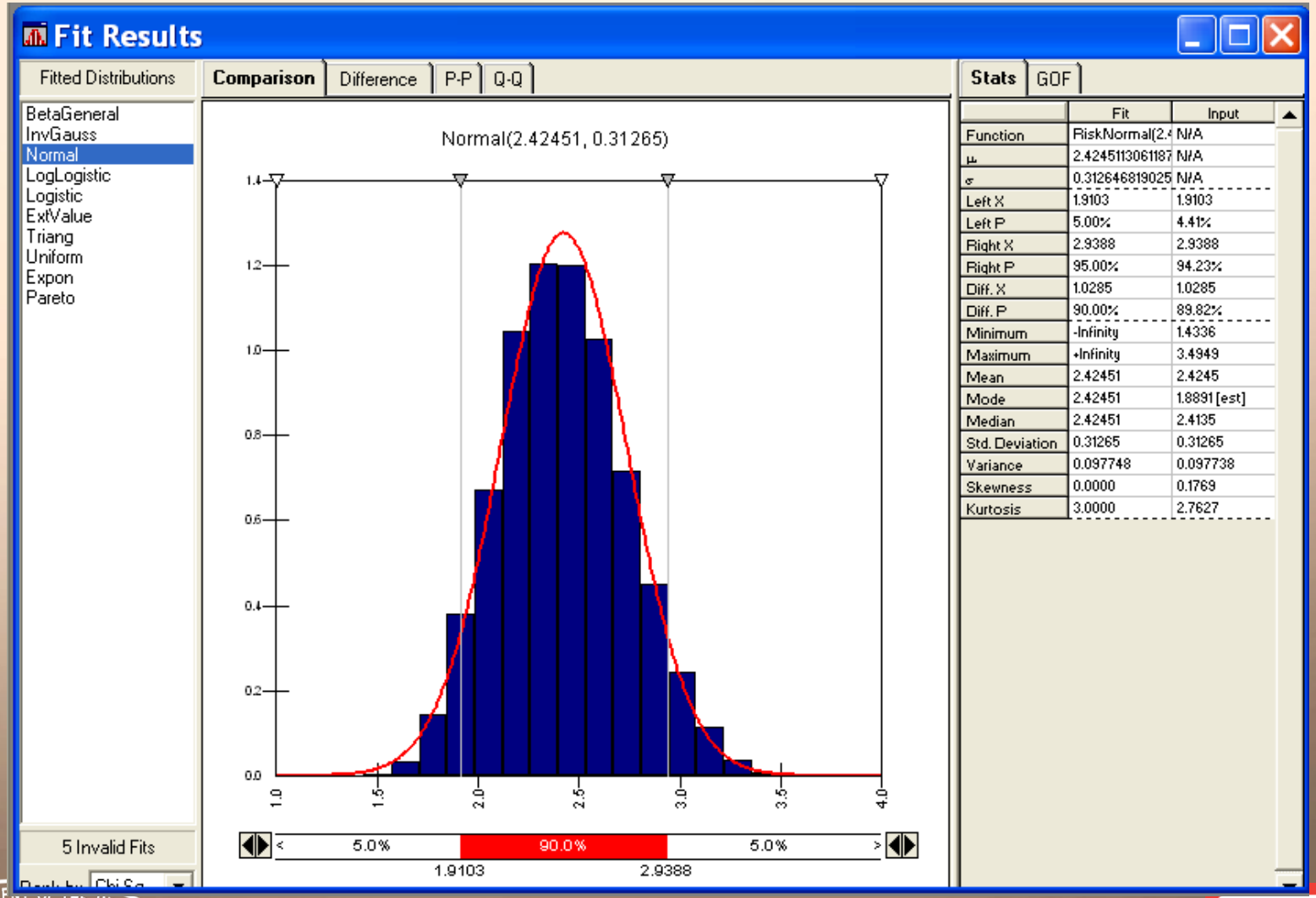
Rank	Name	Cell	Regression	Correlation
1	Intact Cohesion (psi) =	B17	0.759017659	0.759702063
2	TAN Friction Angle (deg) =	B16	0.411501707	0.395787559
3	Percent Intact =	B18	0.368619688	0.349212338
4	Drain Factor =	B15	-0.311968848	-0.314501945
5	Concrete Density (pcf) =	B19	0.09730957	0.085434774

If ϕ’ and c’ Negatively Correlated



Min. F.S. increases to 1.79

F.S. Distribution Fit



Reliability Index, β, and Resulting FS<1.0 Probability

- β= number of "standard deviation units" between the mean value and the value representing failure or unsatisfactory performancce
- For the previous normal distribution:
  - Mean F.S. = 2.42
  - Standard deviation = 0.31
  - Reliability Index, β = (2.42-1.0)/0.31 = 4.56

- Use Excel function for FS<1.0 probability
  - Probability F.S.<1.0 = 1-NORMSDIST(β) = 2.61E-06

Additional Considerations

- FS<1.0 used in examples
- Can use a different value to represent “unsatisfactory performance” (e.g. if a structure is sensitive to deformations, or to approximate deformed shape) and calculate the probability of being less than that value

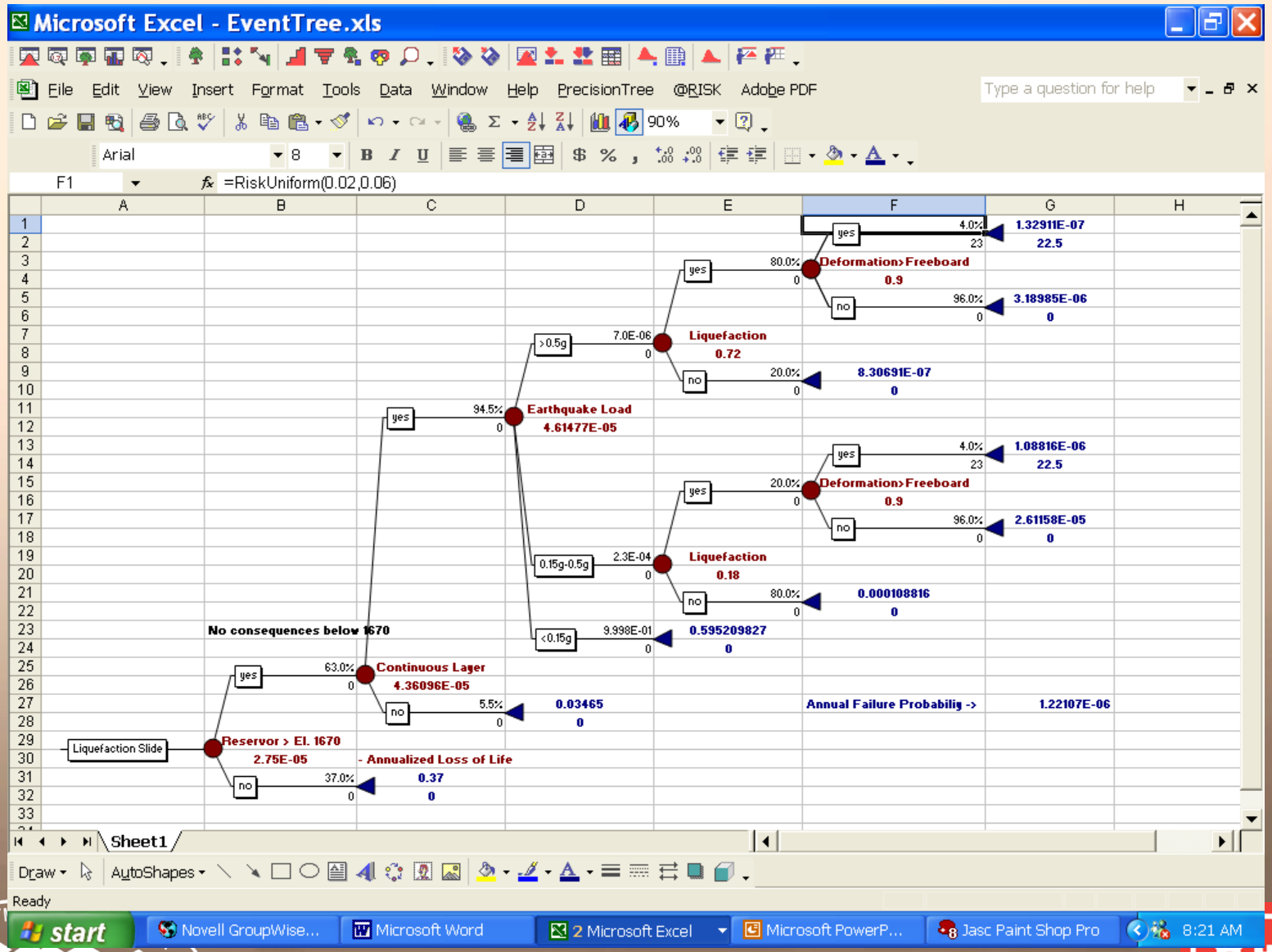
- Slope/W and GRAVDAM have capability, but do not provide sensitivity coefficients, use with caution
- It is not totally clear what FS distributions are assumed when calculating Reliability Index.

Additional Considerations

- If the output distribution is log-normally distributed, different formula is used for β
- Spatial variation – divide failure surface into representative segments and use separate input distributions for each segment

- Preceding discussion focused on “conditional” probabilities - multiply by the probability of the loading (and other relevant PFM events) for total annual failure probability

Event Tree


Microsoft Excel - EventTree.xls
File Edit View Insert Format Tools Data Window Help PrecisionTree @RISK Adobe PDF
Type a question for help
90%
Arial 8
F1
=RiskUniform(0.02,0.06)
Liquefaction Slide
Reservor > El. 1670
2.75E-05
yes
63.0%
0
Continuous Lager
4.36096E-05
no
5.5%
0
0.03465
0
no
37.0%
0
0.37
0
- Annualized Loss of Life
No consequences below 1670
yes
94.5%
0
Earthquake Load
4.61477E-05
>0.5g
7.0E-06
0
yes
80.0%
0
Liquefaction
0.72
yes
4.0%
23
1.32911E-07
22.5
Deformation>Freeboard
0.9
no
96.0%
0
3.18985E-06
0
no
20.0%
0
8.30691E-07
0
0.15g-0.5g
2.3E-04
0
yes
20.0%
0
Liquefaction
0.18
yes
4.0%
23
1.08816E-06
22.5
Deformation>Freeboard
0.9
no
96.0%
0
2.61158E-05
0
no
80.0%
0
0.000108816
0
<0.15g
9.998E-01
0
0.595209827
0
Annual Failure Probabilig ->
1.22107E-06
Sheet1
Draw AutoShapes
Ready
start
Novell GroupWise...
Microsoft Word
2 Microsoft Excel
Microsoft PowerP...
Jasc Paint Shop Pro
8:21 AM

Exercise

Given a mean calculated factor of safety of 1.46 with a standard deviation of 0.26 and assuming a standard normal distribution, what is the probability of failure of the slope (a factor of safety of less than 1)? What would be the probability of failure be with a standard deviation of 0.16?

See the attached chart and Z-table for normal distribution.

Exercise (cont.)

This table gives a probability that a normally distributed random variable outcome is less than Z. This equates to the area of the distribution below Z. Note that Z is equivalent to the Excel function NORMSDIST(β).

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

Possible Exercise Solution

- Given a mean calculated factor of safety of 1.46 with a standard deviation of 0.26 and a standard normal distribution, what is the probability of a factor of safety of less than 1? What is the probability if the standard deviation is only 0.16?
- Solution: How far away from the mean is a factor of safety of 1. The difference between the mean and the value representing poor performance, divided by the standard deviation is the number of standard deviation units from the mean, or the reliability index β. Then use the Z table to look up the portion of the distribution below the value (i.e. NORMSDIST(β)), and subtract from 1 to get the correct value for failure probability (i.e. (1-NORMSDIST(β)).
- 1.46-1/0.26=1.76
- On the Z table look down the left hand column to 1.7, then across under the 0.07 to find the value of 0.9616. To find the percent of the distribution with a factor of safety of less than 1, subtract 0.9608 from 1, and multiply by 100. Answer = 3.8% probability of factor of safety (FS) < 1.0.
- For a standard deviation of 0.16, the answer is 0.2% probability of FS<1.0.