Objective

• To develop an understanding of methods to evaluate erosion caused by river currents and waves.

Key Concepts

• The rate of erosion is a function of the erodibility of the bank material and the hydraulic shear stress exerted on the bank from the flow and waves.

• Soil, vegetation, channel curvature, and armoring all affect the resistance to erosion.
Note:

Chapter D-4 Riverine Erosion in Best Practices for Risk Assessment is under development and will include information from this presentation.
Presentation Outline

• Soil Erosion Model / Theory
• Water-Induced Shear Loads
• Levee & Foundation Modeled Factors and Soil Parameters
• Use of Erosion Spreadsheet
Soil Erosion Model

Based on *Soil, Water, Levee, Armor and Vegetation Model* used in the USACE Erosion Toolbox

Total Erosion (ft) = Erosion Rate (ft/s) x Time (s)

\[ L_e = \dot{e} \times T \]
10.3 Evaluation of Surficial Current and Wind/Wave Action Erosion

10.3.1 Several erosion studies have been performed that focus on identifying the erosion parameters and correlating those parameters to formulate an expression (a physical model) for erosion rates (Hanson and Temple, 2001; Hanson and Cook, 2004). The governing equation for this model is:

\[
\dot{e} = k_d (\tau - \tau_c)
\]

where

- \( \dot{e} \) = erosion rate
- \( k_d \) = erodibility coefficient or detachment rate coefficient (ft^3/lb-hr)
- \( \tau \) = effective hydraulic stress on the soil boundary (lb/ft^2)
- \( \tau_c \) = critical shear stress (lb/ft^2), i.e. the shear stress at which erosion starts

10.3.2 The erosion rate (\( \dot{e} \)) is a function of both hydraulic (\( \tau \)) and geotechnical (\( k_d, \tau_c \)) parameters. Effective hydraulic stress (\( \tau \)) mainly depends on characteristics of water-soil boundary, current/stream velocity and/or wind wave height and period. Both \( k_d \) and \( \tau_c \) are functions of the engineering properties of the levee and the foundation materials. The following sections describe the hydraulic and geotechnical parameters in the above model.
Presentation Outline

• Soil Erosion Model / Theory
• Water-Induced Shear Stresses
• Levee & Foundation Modeled Factors and Soil Parameters
• Use of Erosion Spreadsheet
Soil Erosion Model: The Water Part

\[ \frac{\partial \varepsilon}{\partial t} = \left( k_d (\tau - \tau_c) \right) \]

- Waves
- Flow Current
- Soil Properties

Briaud et al., 2001
Current-Induced Shear Stress

Shear stress exerted on the boundary (i.e., bed, banks, vegetation) by the flow.

\[ \tau_s = \frac{1}{2} \rho f_c V^2 \]  

\( \rho = \text{mass density of water} \) (slugs/ft\(^3\))

\( f_c = \text{current friction factor} \) (dimensionless)

\[ = 2(2.5(\ln(30h/k_b) - 1))^{-2} \]

\( h = \text{water depth (ft)} \)

\( k_b = \text{bed roughness (ft)} \)

\( V = \text{current speed (ft/s)} \)

\[ = V_{\text{ave}}, \text{average current speed for straight channels} \]

\[ = V_{\text{ss}}, \text{maximum velocity in the bend if on the outside of a channel bend (levee)} \]

Where bed roughness is a function of grain size of the bed material (D\(_{90}\) X3) and is determined by selecting the roughness of the material using the chart below and considering the dominate bed grain size.

<table>
<thead>
<tr>
<th>Typical Levee Surface Material (Unified Soil Classification System)</th>
<th>Associated General Erosion Resistance</th>
<th>Recommended Values for k(_b) (feet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cobble (should be the dominant surface material)</td>
<td>Very Resistant</td>
<td>0.33</td>
</tr>
<tr>
<td>Gravel (should be the dominant surface material, D(_{90})=16mm)</td>
<td>Resistant</td>
<td>0.157</td>
</tr>
<tr>
<td>Coarse sand (D(_{90})=2mm)</td>
<td>Erodible</td>
<td>0.0197</td>
</tr>
<tr>
<td>Fine sand (D(_{90})=0.250 mm)</td>
<td>Erodible to Very erodible</td>
<td>0.00082</td>
</tr>
<tr>
<td>Silt (D(_{90})=62.5(\mu)m)</td>
<td>Very erodible</td>
<td>0.00020</td>
</tr>
<tr>
<td>Clay (D(_{90})=4(\mu)m)</td>
<td>Moderately Resistant</td>
<td>0.000013</td>
</tr>
</tbody>
</table>

Table 11: Recommended Values for Levee and Foundation Slope Roughness, k\(_b\)
Current-Induced Shear Stress

Shear stress should only be calculated using the above equation in relatively straight channels. Shear stress exerted on the bank around a bend should be quantified using an alternate procedure.

\[ \tau_s = \frac{1}{2} \rho f_c V^2 \]

Roberts, 2003
Current-Induced Shear Stress – Around a bend

Nomographs used to relate the maximum velocity around a bend to average velocity as a function of bend geometry and water depth (Maylord, 1995).

This relationship was simplified by using a fitted Sigmoid function that relates the ratio of maximum to average bend velocity to the geometry of the bend, described as the “bend factor”.

Figure 10-18. Channel Geometry Parameters.

Figure 10-19. Correction Factor to Estimate Maximum Velocity in a Bend.
Current-Induced Shear Stress – Around a bend

Near bank shear stress around a bend is typically described by the deviation of near bank velocity from mean velocity. Estimate the ratio of maximum velocity to average velocity using the diagram below.

Solve for average velocity using your favorite flow resistance equation to the right.

- Three widely used flow resistance equations:
  1. Chezy equation: \( U = C \cdot (RS)^{1/2} \)
  2. Manning equation: \( U = (1/n) \cdot (R^{0.67} \cdot S^{0.50}) \)
  3. Darcy-Weisbach equation: \( U = \left[ (8gRs)/f_j \right]^{1/2} \)
     - Where \( R \) = hydraulic radius (or depth), \( S \) = slope, and \( C, n, \) and \( f_j \) are indices of flow resistance

Figure 10.19. Correction Factor to Estimate Maximum Velocity in a Bend.
Wave-Induced “Orbital” Stress

\[ \tau_w = \frac{1}{2} \rho f_w U_b^2 \text{ (lb/ft}^2) \]

\( \rho \) = mass density of water (slugs/ft\(^3\))

\( f_w \) = wave friction factor (dimensionless)

\[ f_w = \exp\left(5.213 \left( \frac{a}{k_1} \right)^{-0.194} - 5.977\right) \]

\( k_1 \) = levee slope roughness (ft)

if \( \frac{a}{k_1} \leq 1, f_w = 0.47 \)

\( a \) = horizontal mean wave orbital motion at the bed (ft)

\[ a = \frac{H}{\pi \sinh\left(\frac{2\pi h}{L}\right)} \]

\( L \) = wave length (ft) \( h \) = water depth (ft)

\[ U_b = \frac{2H}{T} \frac{1}{\sinh\left(\frac{2\pi h}{L}\right)} \]

\( H \) = wave height (ft) \( T \) = wave period (s)
Wave-Induced “Orbital” Stress

\[ \tau_w = \frac{1}{2} \rho f_w U_b^2 \text{ (lb/ft}^2) \]

- \( \rho \) = mass density of water (slugs/ft\(^3\))
- \( f_w \) = wave friction factor (dimensionless)
  \[ \begin{align*}
  f_w &= \exp(5.213 \left( \frac{a}{k_1} \right)^{-0.194} - 5.977) \\
  \text{if } k_1 &\leq 1, f_w = 0.47
  \end{align*} \]
- \( k_1 \) = levee slope roughness (ft)

\[ k_1 \text{ based on input soil type} \]

- \( a = \) horizontal mean wave orbital motion at the bed (ft)
  \[ \begin{align*}
  a &= \frac{H}{\pi \sinh(\frac{2\pi h}{L})} \\
  L &= \text{wave length (ft)} \quad h = \text{water depth (ft)}
  \end{align*} \]

- \( L = \frac{gT^2}{2\pi} \tanh \left( \frac{2\pi}{T} \sqrt{\frac{h}{g}} \right) \)
  \( L \) is function of water depth and average period of the wave (T); where T is a function of the wave stress factor and fetch length

\[ \begin{align*}
  U_b &= \text{horizontal mean orbital wave velocity at water – soil interface (ft/s)} \\
  U_b &= \frac{2H}{T} \frac{1}{\sinh(\frac{2\pi h}{L})} \\
  H &= \text{wave height (ft)} \quad T = \text{wave period}
  \end{align*} \]

\( H \) is function of water depth, wave stress factor, and fetch length

**Figure 9.10**
Waves travel because gravity pulls the water in the crests downward. Forced out from beneath the falling crests, the falling water pushes the former troughs upward, and the wave moves to a new position, as indicated. (Notice that the actual motion of the water itself beneath these waves is circular or orbital, which confirms our experience that we are carried up and forward as the wave approaches, and down and back as it passes.)
Wave-Induced “Breaking” Stress - Based on Amount of Energy Dissipated During Wave Breaking

Shear stress applied to the levee by energy dissipated during wave breaking. Most energy is lost to generate turbulence; therefore, the energy dissipated during wave breaking is relatively small compared to the total energy.

The user defines the efficiency of wave breaking to erode sediment, maximum fetch length, and depth of water; from which, the wave generated shear stress is calculated.

**Assumptions:**
1) The rate of energy dissipated by wave breaking is a function of shear stress and velocity.
2) The speed at which energy is propagated is referred to as the group velocity.

Group velocity, \( c_g \)

\[
c_g = 0.5 \sqrt{\frac{g}{k} \tanh \left(2\pi \frac{h}{L}\right) \left(1 + \frac{2kh}{\sinh(2\pi k)}\right)}
\]

If assume \( h/L > \sim 0.5 \) (deep water)

\[
c_{gd}^2 = \frac{gL}{8\pi}
\]

Shear stress

\[
\tau = \varepsilon \Delta / c_g
\]

\( g = \) gravitational acceleration (ft/s\(^2\))
\( h = \) local water depth (ft)
\( k = \) wave number (ft\(^{-1}\)) = \(2\pi/L\)
\( L = \) wave length (ft)
\( \Delta = \) energy dissipation rate (lb-ft/ft\(^2\)s)
\( \varepsilon = \) portion of energy dissipated by wave breaking that is dissipated as bed shear stress (efficiency)
Energy dissipation in surf zone

\[ D = \frac{1}{4} \rho g f \left( \frac{B H_{\text{max}}}{h} \right)^3 \]
Presentation Outline

- Soil Erosion Model / Theory
- Water-Induced Shear Loads
- Levee & Foundation Modeled Factors and Soil Parameters
- Use of Erosion Spreadsheet
Soil Erosion Model

\[ q = (k_d(\tau - \tau_c)) \]

Factors:
- Waves
- Flow Current
- Soil Properties

Briaud et al., 2001
Geotechnical factors that affect soil erosion rates

1) Soil Plasticity
2) Soil Compaction
3) Moisture Content
4) Heterogeneity of Soil
5) Armoring
6) Vegetation
Geotechnical factors that affect soil erosion rates – Armoring

Armoring is the placement of erosion resistant material (i.e., riprap) on the waterside of a levee to prevent erosion. The resistance of the material to erosion is described by a critical velocity, which once reached, mobilizes and strips away the armoring. Once the armoring fails, erosion rates are calculated using the properties of the bare soil underneath.

The armor layer can fail through shear stresses exerted by (1) current forces of the flow and (2) wave action. Both are considered in the soil erosion model.

![Chart used to predict the critical velocity of riprap material based on the diameter of the riprap, and specific weight of the stone and water.](image)

**Source:** USACE EM 1110-2-1601 (1994)
Geotechnical factors that affect soil erosion rates – Vegetation

Vegetation can have a mixed effect on erosion rates. Similar to armoring, a critical velocity should be established for the vegetation cover, above which vegetation is removed and the erosion rate is calculated by the properties of the bare soil underneath. Vegetation can be classified as: 1) helpful; 2) harmful; or 3) neutral/none.

Helpful:
Grass cover, low-lying uniform shrubs, etc. that act to slow down near bank velocities. Effectiveness depends on flow velocity and wave height.

Neutral/None:
Levees with limited to no vegetation.

Harmful:
Large widely spaced trees with or without exposed roots on waterside levee.
Correlation between critical shear and erodibility coefficient

“Hanson” erosion resistance, “Briaud” erodibility, and Levee Erosion Toolbox (URS 2007) default values for $k_d$ and associated $\tau_c$ for the various “Hanson” erosion resistance classifications and Shield’s Diagram $\tau_c$ from Briaud (2001) to be cited as the primary source for analysis parameters in Engineering Manual 1110-2-1913.

User selects an erosion classification descriptor based on the Hanson and Simon (2001) to determine appropriate critical shear and erodibility coefficient.
Soil Erosion Model

Total length of erosion is determined by separately calculating (1) erosion length due to wave and current shear stress, then (2) summing those lengths.

1. Flow Current Erosion Length

\[ L_e({\text{current}}) = \dot{e}_{(\text{current})} \times T \]

Wave Erosion Length

\[ L_e({\text{waves}}) = \dot{e}_{(\text{waves})} \times T \]

Where \( T \) is time, \( \dot{e} \) is the erosion rate for each respective shear stress, and \( L_e \) is the erosion length for each respective shear stress.

2. Total Erosion Length

\[ L_e({\text{total}}) = L_e({\text{current}}) + L_e({\text{wave}}) \]
Presentation Outline

- Physical Model / Theory
- Water-Induced Shear Loads
- Levee & Foundation Modeled Factors and Soil Parameters
How does erosion rate/length relate to risk?
How does erosion rate/length relate to risk?

erosion rate \times \text{event duration compared to the effective width of the levee}

Failure occurs when total erosion > levee effective width
Questions?