VII-3. Seismic Failure of Spillway Radial (Tainter) Gates

Radial Gate Arrangement

Introduction

Radial gates (sometimes referred to as Tainter gates) consist of a cylindrical skinplate reinforced by vertical or horizontal support ribs, horizontal or vertical girders, and the radial arm struts that transfer the hydraulic and seismically induced hydrodynamic loads to the gate trunnions. Radial gates rotate about their horizontal trunnion axis during opening/ closing operations. This chapter addresses potential failure modes related to radial gates during seismic load conditions. This includes conditions when a radial gate is in closed position and the reservoir water surface (RWS) is at or below the normal reservoir level.

In general, two types of radial gates can be identified at dams: surface gates (spillway crest-, canal-, or navigation radial gates) and top sealing gates. Radial gates come in all sizes from only a few feet wide up to 110-feet (or even wider) for navigation structures. Similarly, the height of the gate may reach 50 feet or even more. Radial gates are operated by hydraulic cylinders or by wire ropes or chain winches (ref. Figure **VII-1**-1).

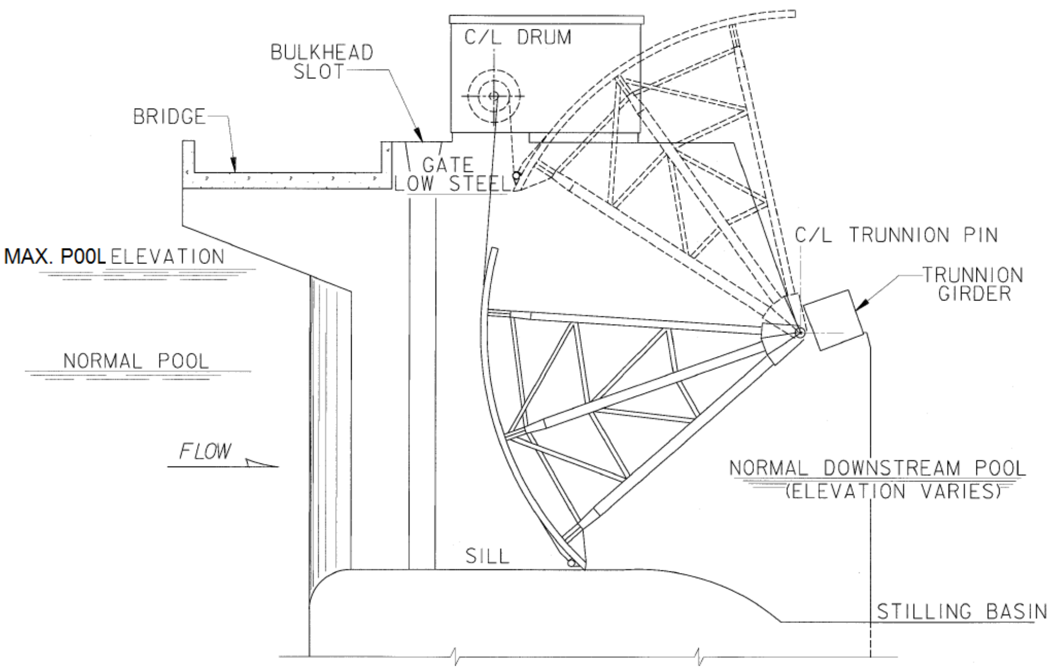


Figure VII-3-1 – Section through a radial gate [USACE]

Radial Gate Components

The primary components of a radial gate structure (Figure VII-3-2) are:

- Skin plate assembly (gate leaf) consisting of the cylindrical skin plate reinforced by vertical or horizontal ribs,
- Vertical or horizontal girders,
- Gate arms (end frames) consisting of radially spaced struts strengthen by brace members,
- Gate arm trunnion hubs,
- Gate support assembly (trunnion pins, bushings, and yokes). The yokes could be installed on trunnion girders or directly embedded in the concrete piers.

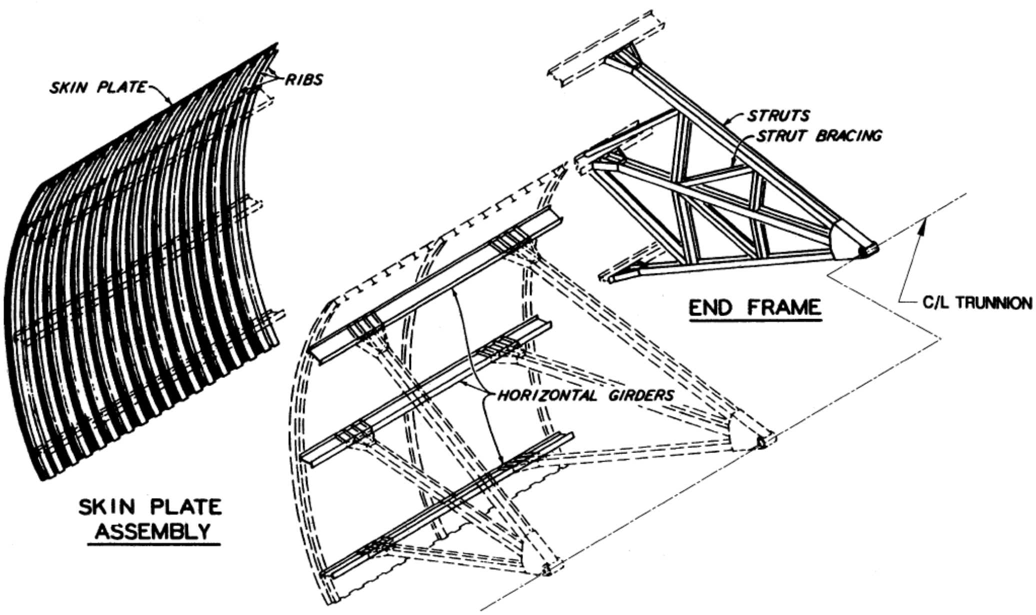


Figure VII-3-2 – Primary radial gate components [USACE]

Load Carrying Mechanism

Loads that are applicable to radial gate analysis during earthquake include gravity loads, hydrostatic loads, and earthquake loads. Radial gates transfer the hydrostatic and seismically induced hydrodynamic pressure from the reservoir into the gate skin plate then the load is conveyed in to the trunnion supports through gate girders and gate arms (Figure **VII-3**-2).

In the structural analysis of radial gates, three principal operational conditions are considered:

Gate closed during earthquake includes the combination of *the hydrostatic and the seismically induced hydrodynamic loads, and the self-weight of the gate structure and the weight of the installed equipment*.

- Hydrostatic pressure from the reservoir is the primary load acting on the gate in the closed position. In the analysis for seismic conditions, the hydrostatic pressure corresponding to normal RWS is combined with the hydrodynamic effects.

- Hydrodynamic loads are induced at the spillway gate as the dam is moved during earthquake.

- Self-weight of the entire gate and of any installed equipment and the corresponding inertia forces are considered in the seismic analysis of the gate structure.

Gate closed for normal RWS conditions are discussed in Chapter VII-1 of the Best Practice Manual.

Gate operated for normal RWS conditions are discussed in Chapter VII-1 of the Best Practice Manual.

Critical Structural Components of Radial Gates

- Gate arms struts – the arrangement of the gate geometry and imperfections of gate arms may introduce second order forces that can lead to:

  Out-of-plane buckling of arm struts – deformation of gate girders may bend the arm struts in the lateral direction. The eccentricity will be magnified by the compression forces in the arm struts increasing the bending moment in the members. The second order bending moment will be larger for an arm strut with large imperfections.

  In-plane bending of arm struts – imperfections in the assembly of the gate structure together with deflection of the arm struts caused by the self-weight may result in eccentricities of the arm struts. These eccentricities will lead to increased second order bending moment in the struts due to the axial compressive load from the reservoir.

- Gate arm braces and their connections – bending of the gate arms may result in a failure of the brace members or their connections leading to increased unsupported length of the struts and consequently bucking and failure of the gate arms.

- Gate supports - equally important to carrying the load are the gate supports and the anchorage. The trunnion anchorage typically carries the load in tension. Trunnion anchors can be configured in a variety of ways. The anchorage can be post tensioned or a passive anchor. Post tensioned anchors have been known to fail due to the post tensioned load, which may or may not be apparent from visual inspection. This chapter focuses on the buckling of radial gate arms during seismic loading. The capacity of the trunnion anchorage should be evaluated for seismic loadings and if the anchorage is inadequate this failure mechanism should be incorporated into the risk analysis. This could be done within the event tree outlined in this chapter or estimated as a separate event tree.

Key Concepts and Factors Affecting Risk

Reservoir Water Level

Reservoir water level on the gates is a key parameter in the risk analysis since it affects the level of loading on the gates and influences the consequences of the potential gate failure (due to the effect on the breach outflow).

For seismic considerations, the reservoir level is typically evaluated in ranges from the normal RWS elevations to a threshold level. The likelihood of various reservoir levels can typically be estimated from the historic reservoir exceedance curves. Flood conditions or wave effect is usually not combined with the seismic loads induced during earthquake but would be based on the probability of the coincident events.

Seismic Hazard

Most radial gates will have some reserve capacity beyond the stress levels created by full reservoir static loads. However, the level of seismic loading in combination with the reservoir level at the time of loading will determine whether the gate arms are overstressed, and if so to what level.

Number of Gates

Multiple spillway gates on a given project will typically increase the probability of a gate failure (with the outcome varying from a single gate failing to all the gates failing). Multiple spillway gate failures also create the potential for a large breach outflow and higher potential loss of life.

Maintenance of Spillway Gates

Gates that are well maintained can usually be relied upon to have their original design capacity at the time of an earthquake. If gates are not maintained and the gate members corrode, the original design capacity may be reduced. An examination is usually needed to determine the actual condition of the gates.

Analyses of the Gate Structure

Structural analyses of the gate structure are performed to evaluate the stability and the stresses levels in the gate members under combined reservoir and seismic loadings. The analysis for the strength and stability should include axial, shear, and bending deformation in the structural members and their connections. In general, it is required that stability is maintained for the gate as a whole and for each component of the structure. The type of analysis used to evaluate the structural performance of the gate will be dependent of the level of risk assessment being performed.

- Unless the spillway gates have a simple arm frame arrangement, where hand calculations might be performed, Reclamation will perform finite element (FE) analysis when evaluating gate performance. The level of the FE model complexity can be adjusted to the level of the analysis. For preliminary analyses, only the arm struts may be modeled. For higher level analyses, all gate members including the gate leaf, girders, and gate arms would be modeled.
- USACE will evaluate gates through less rigorous analysis and existing information when performing Periodic Assessments or Semi-Quantitative Risk Assessments. If the gates are found to be above tolerable risk guidelines through these somewhat

conservative assessments, more rigorous approaches such as FE analysis should be considered.

When analyzing the gate structure for seismic conditions, the following should be considered either numerically or qualitatively:

a. Initial imperfections of the gate assembly – imperfections considered at the points of intersection of arm struts and their brace members
b. Initial deformation of the gate arms due to the gravity load
c. Stiffness reduction due to inelasticity
d. Deformation of gate arms by the bending moment generated by deflection of the gate girders
e. Defects in the gate members and their connections

In order to incorporate some of the above items, an inspection will be required to capture the current condition of the gates. The structural analysis of the gate should incorporate second-order effects (P-Δ and P-δ effects for braced gate arm struts and P-δ effects only when arm struts are unbraced). In general, the rigorous analysis method of second-order analysis is acceptable for all gate arm arrangements considering conditions of the structure geometry (listed as items a. through e. above). Alternatively, an approximate second-order analysis can be utilized by amplifying the required strength determined in a first-order analysis. The multipliers to account for P-Δ and P-δ effects, not discussed in this document, should be determined as provided in Appendix 8 of AISC 360-10.

Seismic Induced Loads on Spillway Gates

The dynamic reservoir loads developed during an earthquake are of importance in the structural evaluation of the spillway gates. The ground acceleration at the base of a dam during an earthquake can be considerably amplified at the top of the dam. Spillway gates may be subject to this amplified acceleration. This acceleration at the spillway gates could be several times greater than that measured on rock at the abutment or the base of the dam, depending on the response of the dam structure and the location of the spillway gate. The flexibility of the gate structure, actual water head on the gate, and whether the transverse, longitudinal or vertical acceleration is considered will affect the load on the gate [Reclamation, DSO-11-06].

However, for "rigid dams", defined here as a structure with a natural period of below 0.2 sec., a simplified Westergaard's approach could be used for computing hydrodynamic loads on spillway gates as described in the section below. For "flexible dams" (dams with the natural period of vibration above 0.2 sec.) amplifications of the spillway acceleration needs to be included in computations of the hydrodynamic loads on spillway gates or a full FE analysis performed, that includes interaction between the reservoir, dam structure, and the spillway gate.

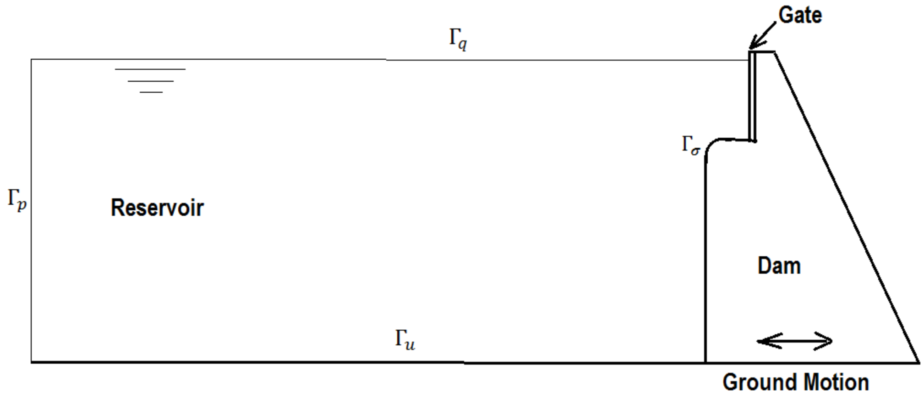


Figure **VII-3**-3 – Model of the Dam-Gate-Reservoir System.

Westergaard’s exact solution

The seismic motion of a straight rigid dam of height *h* with an infinite, in length, reservoir was mathematically expressed in terms of the theory of elasticity of solids (see Figure VII-3-3). The solution given by Westergaard (1931) in the form of the maximum pressure distribution at the upstream face of the dam is expressed by Eq. **VII-3-1**.

$$p = \frac{8\alpha w h}{\pi^2} \sum_{1,3,5,\ldots}^{n} \frac{1}{n^2 c_n} \sin \frac{n\pi y}{2h} \qquad \text{(Eq. VII-3-1)}$$

$$c_n = \sqrt{1 - \frac{16\, w h^2}{n^2 g k T^2}} = \sqrt{1 - \frac{0.71889}{n^2}\left(\frac{h \text{ sec.}}{1\,000\, T \text{ ft.}}\right)^2}$$

where:

x, y = the axis of x is at the surface of the water directed upstream and the axis y is vertical downward (Figures VII-3-4 and VII-3-5),
w = weight of water per unit volume (w = 62.4 lb/ft^3),
g = acceleration due to gravity (g = 32.2 ft/sec^2)
α = maximum horizontal acceleration of foundation divided by g,
T = period of horizontal vibration of the foundation,
t = time,
k = modulus of elasticity of water (assumed k = 300,000 lb/in^2),

The solution expressed by Eq. **VII-3-1** was derived with the following assumptions:

- The dam upstream face is straight and vertical,
- The dam does not deform or displace and is considered to be a rigid block,
- Dam sinusoidal oscillations are horizontal,
- Small motions are assumed during earthquake,
- The problem is defined in 2-D space,
- Period of free vibration of the reservoir, T_0, needs to be significantly smaller than the period of vibration, T, of the earthquake (resonance is not expected),
- The effect of water compressibility was found to be small in the range of the frequencies that are supposed to occur in the oscillations due to earthquake.

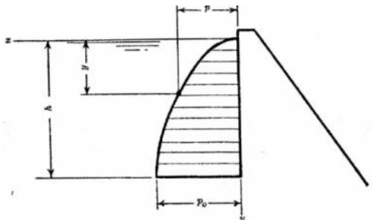


Figure **VII-3**-4 – Pressure distribution on dam for exact solution (Eq. **VII-3-**2)

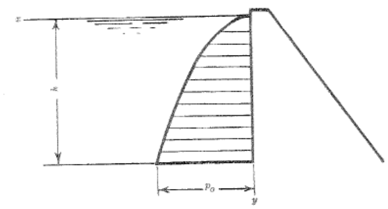


Figure **VII-3**-5 – Pressure distribution on dam for approximate solution (Eq. **VII-3-**3)

Westergaard's approximate solution

A parabola (Figure VII-3-4) represents the hydrodynamic water pressure *p* on the dam expressed by Eq. **VII-3-2** and it is a result of simplification of Eq. **VII-3-1**.

$$p = 0.875\ \mathrm{w}\ \alpha\ \ (h\ y)^{0.5} \qquad \text{(Eq. VII-3-2)}$$

This formula (Eq. **VII-3-2**) is widely used by the industry in the preliminary calculations of the hydrodynamic pressure on dams and very often on the spillway gates.

Comparison between Westergaard's exact and approximate solutions

Significant difference in the seismic induced pressure distribution on the dam face could be observed between the exact and the approximate Westergaard's solutions. In general, the largest differences in the hydrodynamic pressure exist at the top and bottom portion of the dam (see Figure **VII-3**-6).

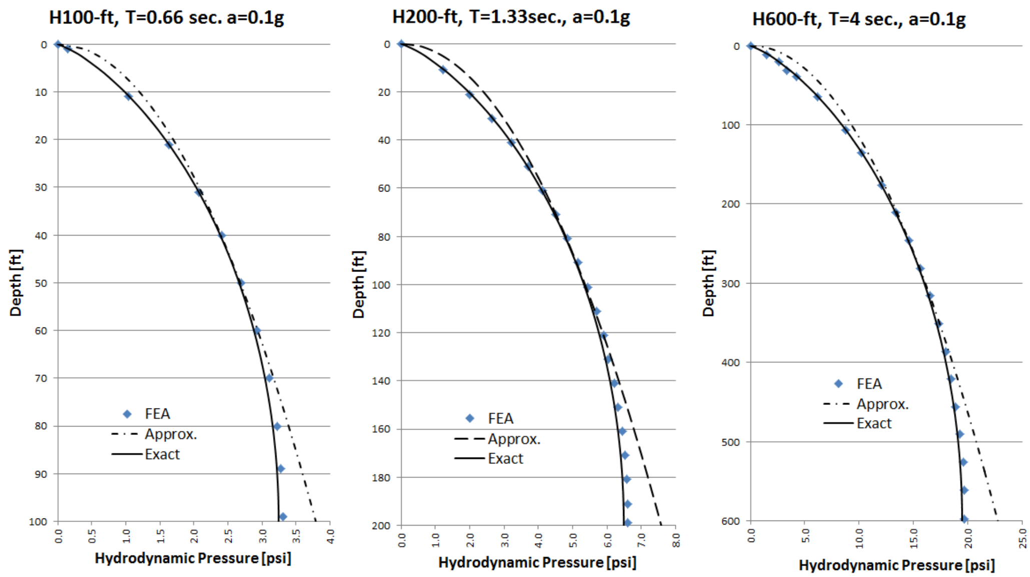


Figure **VII-3**-6 – Comparison of hydrodynamic pressures calculated according to Westergaard's exact and approximate formulas and the FE results for ground acceleration of 0.1g [DSO-11-06].

Gate Set-back Effect

The results of the FE analysis performed by Salamon (2015) and Nakayama et al. (2007) experiments showed that during earthquakes, the spillway gates installed at "rigid dams" experience significantly lower loads from the reservoir when the gates are set back from the upstream face of the dam (see Figure VII-3-7).

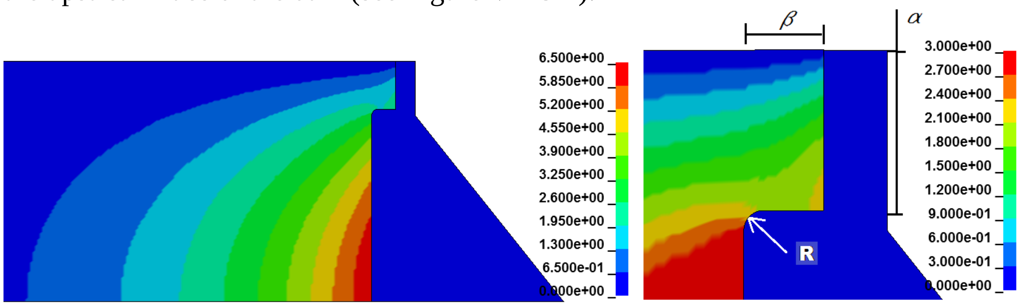


Figure **VII-3-**7 – Hydrodynamic pressure distribution [psi] for 200-feet deep reservoir with α=0.2, β=0.5 and ground acceleration of 0.1g [Salamon]

The effect of the gate set-back effect was expressed by Salamon (2015) by a formula

$$P_{Total}(h) = P_{Westergaard} \cdot (1.1 - 0.45\beta) \qquad (\beta \leqq 0.7) \qquad \text{(Eq. \textbf{VII-3-3})}$$

where:

$P_{Westergaard}$ is the hydrodynamic pressure distribution at the face of vertical dam obtained from the exact Westergaard's solution

$\beta = d / h_1$ is the gate set-back ratio defined as a distance of the gate location to the face of the dam divided the gate height

Spillway Gates at "Flexible Dams"

Dynamic stability analyses of spillway gates installed at "flexible dams" could be grossly incorrect based on the use of simplified methods for calculations of the hydrodynamic and inertia forces on the dam and the spillway gates. The problem arises with the use of the Westergaard formulation when the flexibility of the gates, accurate calculation of the amplification of the ground motion acceleration up through the dam, and the three dimensional effects when the gates are set back from the face of the dam needs to be considered. The effects of skinplate curvature on hydrodynamic loading for radial gates is currently being investigated at Reclamation. Information on this effect will be added to this chapter in the future.

Current Reclamation's Practice

A significant number of seismic analyses of radial gates have been conducted by the Reclamation based on a two stage dynamic analysis. First, the dam without the gate is analyzed for the specified ground motions. In most cases, the added mass approach is used to approximate the dynamic behavior of the dam and reservoir. The acceleration obtained from this analysis, at the location of the gate, is then applied to a separate FE model of the gate only. The reservoir associated with the gate is approximated by an added-mass calculated using the total depth of the reservoir. The model of the gate, with the reservoir added mass, is then subjected to the acceleration history (or the corresponding response spectra) calculated from the dam analysis model.

Strength Evaluation of Gate Arms

In the structural based evaluation of the radial gate, a limit state approach is used to determine conditions in which the gate has reached its ultimate loading capacity (Strength Limit State). In general, limit states take the form:

$$Demand \leq Capacity$$

Required strength or demand is the internal force in a gate member derived from the structural analysis. The available strength or capacity is the predicted capacity of these members. Uncertainties in the loading and variability of material should be considered during the risk assessment through sensitivity analysis or probabilistic analysis. The interaction of compression and flexure in doubly symmetric members of the gate is expressed by the Interaction Ratio (*IR*) in equation Eq. **VII-3-4**.

$$IR = \frac{P_u}{P_n} + \frac{8}{9}\left(\frac{M_{ux}}{M_{nx}} + \frac{M_{uy}}{M_{ny}}\right) \qquad for\ \frac{P_u}{P_n} \geq 0.2$$

$$IR = \frac{P_u}{2P_n} + \left(\frac{M_{ux}}{M_{nx}} + \frac{M_{uy}}{M_{ny}}\right) \qquad for\ \frac{P_u}{P_n} < 0.2 \qquad \text{Eq. VII-3-4}$$

where: P_u – required axial strength

P_n - the available axial strength equals the nominal compressive strength

M_u – required flexural strength

M_n -the available flexural strength equals the nominal flexural strength

subscript x and y relating to strong and weak axis bending, respectively

The required strength (axial forces and moments) includes second-order effects in the interaction equation (Eq. **VII-3-4**). Second order effects are calculated in the analysis, not in the interaction equation as it was done previously in 2009 Best Practice Manual. This is the <u>key change</u> in the approach implemented in the current version of the Best Practice Manual when compared with the previous editions. For less rigorous analysis, where second order effects are not quantified directly, an approximate second order analysis can be utilized by amplifying the required strength determined in a first order analysis, as described previously.

It should be noted that radial gates typically include bracing to reduce the unsupported length of the gate struts in weak axis bending. The analysis may indicate that a bracing member or its connection is a critical component in the stability of the gate arm, and a judgment will be needed as to the likelihood that the bracing would fail under the loading range evaluated, leading to a greater unsupported length of the gate strut arms. If a bracing member is judged likely to fail through FE modeling, the bracing member should be removed from the model and the analysis rerun. As a result, the members that are considered as fracture critical members (members whose failure will lead to the failure of the whole gate structure) need to be identified.

For additional discussion on the evaluation of spillway gate arms, including some examples of analysis results for different gate arrangements, see Chapter VII-1.

Risk Analysis

Failure of Radial Gates under Seismic Load Conditions

The radial gate potential failure mode during earthquake is broken into the following component events:

- ↳1. Reservoir load ranges
  - ↳2. Seismic load ranges
    - ↳ 3. Reduction factor due to gate arrangement/structural condition
      - ↳ 4. Arm struts buckle and gate fails
        - ↳ 5. Unsuccessful Intervention

The following is an example potential failure mode description for the failure of a radial gate under normal operational conditions:

> *Due to increased hydrodynamic load induced during earthquake the strength of the gate members reaches a level where the bending stresses combined with the axial stresses from a full reservoir causes the arm brace member to fail and the struts to buckle. This causes a rapid progressive failure of the gate structure, resulting in a release of the reservoir through a not-restricted or partially-restricted spillway.*

Event Tree

A relatively simple example event tree is shown in Figure **VII-3-**8, and typical event nodes that might be used in a risk analysis. Each branch consists of five events – a reservoir elevation range, a seismic load range, an event that considers the gate arrangement/structural conditions, the conditional probability of gate failure given the load probabilities and an event that considers intervention (with the associated consequences of gate failure). If the gates are loaded to the point of overstressing the radial gate arms, the gate arms can buckle and fail, leading to gate collapse and reservoir release without additional steps in the event sequence. Refer also to the Chapter I-5 on Event Trees for other event tree considerations.

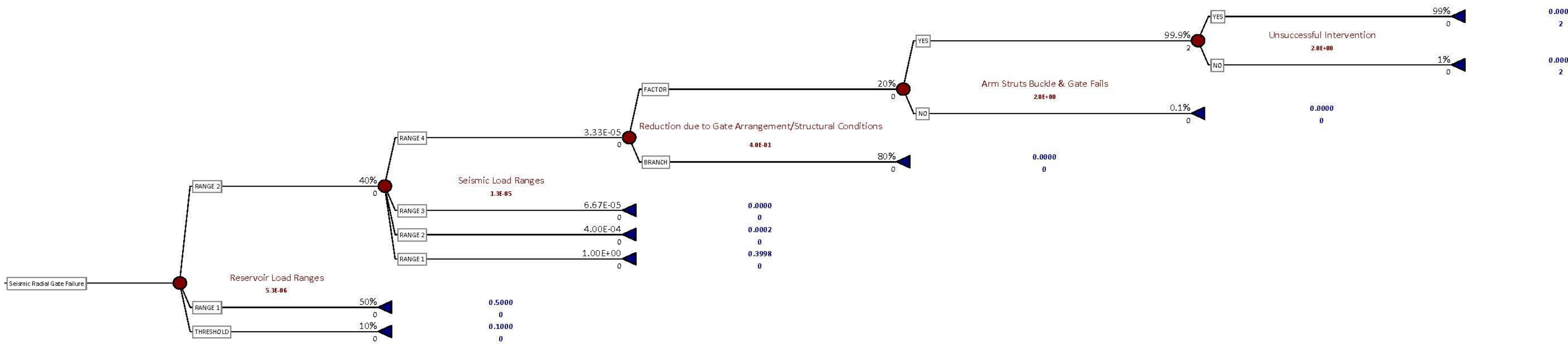


Figure VII-3-8 – Example of an Event Tree for Seismic potential Failure of Tainter Gates

Reservoir Load Ranges

Reservoir load ranges are typically chosen to represent a reasonable breakdown of the larger reservoir range from the normal water surface (typically at or near the top of the gates in the closed position) and an elevation in the lower half of the gate in which stresses in the gate members are not a concern. The number of load ranges depends on the variation in failure probability, and should be chosen as much as possible to avoid large differences in failure probability at the top and bottom of the range. Historical reservoir elevation data can be used to generate the probability of the reservoir being within the chosen reservoir ranges, as described in the sections on Reservoir Level Exceedance Curves and Event Trees.

Seismic Load Ranges

Seismic load ranges are typically chosen to provide a reasonable breakdown of the earthquake loads, again taking into account the variation in failure probability to avoid large differences between the top and bottom of each range. The total range should include loading from the threshold level (load at which the risk team determines the failure becomes possible) at the lower end, to the level at which failure is nearly certain, or to the level at which the load probability multiplied by the maximum gate failure consequences is still below tolerable risk guidelines (the latter of which assumes a conditional gate failure probability of 1.0). Seismic hazard curves are used to generate the probability distributions for the seismic load ranges, as described in the sections on Seismic Hazard Analysis and Event Trees.

Gate Arrangement/Structural Conditions

The third node in the event tree is a reduction factor to account for the gate arrangement and the structural conditions that could affect the failure of the gate given a calculated IR. A factor between 0.1 (for very favorable conditions) and 1.0 (for adverse conditions) can be used and the risk team should evaluate the conditions and determine a factor to be used.

Some of the conditions that could influence the team in the selection of the reduction factor are included in Table VII-3-1 below. The extent that a condition applies and the number of conditions that are applicable should be considered when selecting the appropriate value.

Table VII-3-1 Reduction Factor Considerations Related to Gate Arrangement/Condition	
Condition	**Considerations**
Age of Gate and Frequency of Gate Operations	Older gates (more than 50 years old) will be more vulnerable to failure given potential fatigue in the gate structure members during operational life of the structure.
Complexity of the Gate Arm Frame Assembly	Gates with more members may be more vulnerable to failure due to an increased number of connections and the increased potential for one or more of the critical members to have defects which could lead to the failure of the whole gate structure.
Fracture Critical Members	Fracture critical members are defined as members whose failure would lead to a catastrophic failure of the gate. Gates with multiple fracture critical members are more vulnerable to catastrophic failure.
Fatigue of the Gate Members	Cyclic loading of the gates members during the operational life of the gate combined with loading during an earthquake may lead to fatigue of the fracture critical members or their connections. Gates with multiple fracture critical members and with longer operational life and higher operational frequency, or that have a history of vibration during operation are more vulnerable to failure of their members.
Welded Connections	Welded connections can be more vulnerable to undetected cracking, during earthquakes.
Age of Coatings	Coatings that are older are more likely to have localized failures that could lead to corrosion and loss of material.

Arm Struts Buckle and Gate Fails

The fourth event in the event tree is the conditional failure probability that is based on the calculated interaction ratio of the gate arms. If the gates are loaded to the point of overstressing the radial gate arms, the strut arms can buckle and fail, leading to gate collapse and reservoir release without additional steps in the event sequence.

Table VII-3-2 - Gate Failure Response Curve

Interaction Ratio	Probability of Failure (1 gate)
< 0.5	0.0001
0.5 to 0.6	0.0001 to 0.001
0.6 to 0.7	0.001 to 0.01
0.7 to 0.8	0.01 to 0.1
0.8 to 0.9	0.1 to 0.9
0.9 to 1.0	0.9 to 0.99
> 1.0	0.9 to 0.999

With the interaction ratio curves as a guide (see Figure VII-3-9 and Table VII-3-2), estimates can be made for the probability of a single gate failing under the seismic conditions analyzed. These estimates are made based on the highest interaction ratio calculated for the gate arms from the structural analyses.

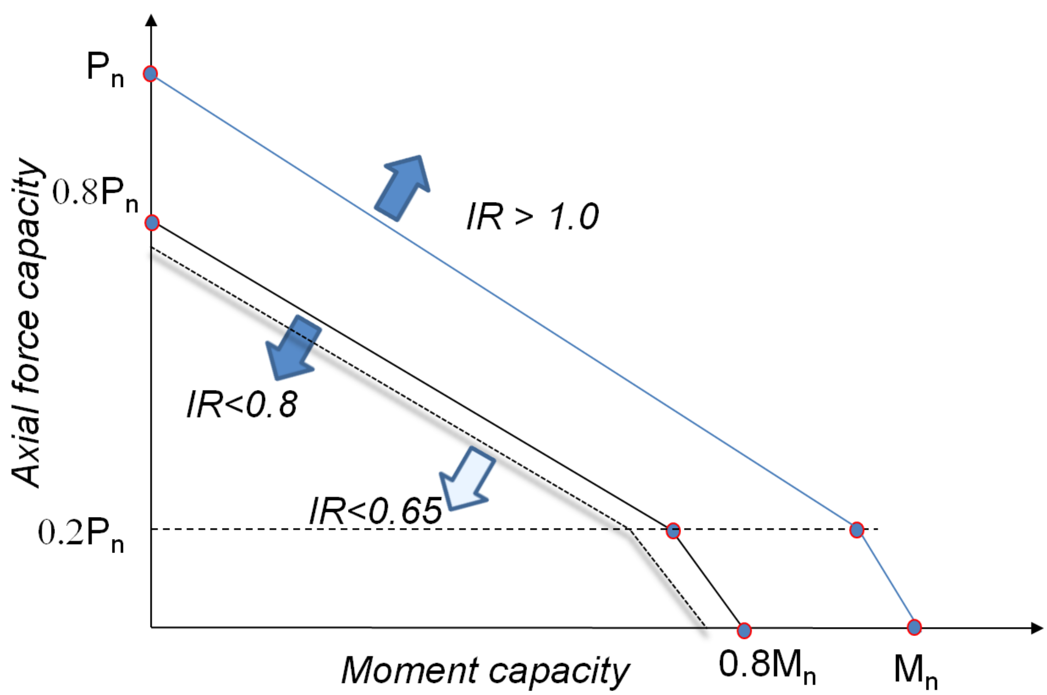


Figure VII-3-9 – Illustration of interaction ratios for radial gate.

With the fragility curve as a guide, estimates can be made for the probability of a single gate failing under different combinations of reservoir loads and earthquake loads. These estimates are made based on the highest interaction ratio for the gate arms from the structural analyses. Table VII-3-3 shows the interaction stress ratios for an example gate analysis study. For this example study, a number of gate analyses were performed for different combinations of reservoir water elevations and seismic loadings. Total gate loads were estimated for all load combinations. Analyses were performed for some of the load combinations and the critical interaction ratios for those load combinations are shown in Table VII-3-3, and this information was used to estimate conditional failure probabilities, using Table VII-3-2. Using the information from the analyzed cases, failure probabilities were projected for all load combinations.

Table VII-3-3 – Single Gate Failure Probability

Res WS El	Acceleration at Trunnion Pin			
	0.25g	0.5g	1.0g	2.0g
466	4590 0.005	5650 0.05	8300 0.95 0.95	13800 1.4 0.999
458	3320 -	4200 0.001	6400 0.81 0.20	10200 1.1 0.999
450	2054 -	2530 -	3720 0.6 .001	6100 0.9 0.9
434	600 -	760 -	1200 -	2000 .001

Notes:
Gate load in kips
Combined stress ratio
Estimated failure probability of single gate

Unsuccessful Intervention

The fifth event in the event tree allows for termination of this potential failure mode if intervention can succeed in stopping or significantly reducing flow in a reasonable period of time (before significant downstream consequences are incurred). In most cases, it will be likely to virtually certain that intervention will be unsuccessful. In order to be successful there will need to be an upstream gate or a bulkhead (either of which would have to be able to be installed under unbalanced conditions) that could be closed to stop flow through the failed gate.

Statistical Considerations for Multiple Gates

Spillways with multiple gates can have a variety of potential gate failure outcomes, ranging from one gate failing to all the gates failing. Multiple gates can fail due to failure of the gate body during a seismic event however; gate failure could also result from a seismic failure of the gate anchorage or trunnion pin. The focus on this chapter is on the seismic failure of radial gates due to buckling of the gate arms. Trunnion anchorage is not specifically addressed, but if this is identified as an issue, the following approach can be used to evaluate the total probability of the specific failure mode. Once individual probabilities for each failure mode have been evaluated, common cause adjustments can be made using DeMorgan's rule or other statistical methods to account for multiple failure modes leading to the same breach. Pascal's triangle provides the number of combinations of each outcome for a given number of gates. Figure **VII-3-**10 shows the Pascal's triangle coefficients.

> For a spillway that has six radial gates, the Pascal's triangle coefficients are highlighted in yellow. The coefficients represent the number of combinations of each outcome, as follows:0 gates failing – 1 combination
> 1 gate failing – 6 combinations
> 2 gates failing – 15 combinations
> 3 gates failing – 20 combinations
> 4 gates failing – 15 combinations
> 5 gates failing – 6 combinations
> 6 gates failing – 1 combination

It can be noted that the triangle is constructed with "1's" along the sides (representing the number of combinations of zero gates failing and of all gates failing). The number in each cell is then filled in by adding the two numbers diagonally above the cell. These numbers are used as coefficients in the probability equations. For example, Table **VII-3-**3 provides the equations for various failure outcomes (from zero to eight gates failing) based a spillway with eight gates (see far left column). The total at the bottom is the probability of one or more gates failing (i.e. is the sum of from 1 to 8 gates failing and does not include the probability of zero gates failing).

The generic form of the equation for a failure outcome (the outcome represents the number of gates that fail) is as follows:

$$P_v = C\left(\left(P_f\right)^y\right)\left(\left(1-P_f\right)^{n-y}\right) \qquad \text{Eq. VII-3-5}$$

where: P_y = probability of failure outcome, where y represents the number of gates failing for a specific outcome.
C = coefficient from Pascal's triangle representing the number of combinations of a given failure outcome (see Figure VII-3-10)
P_f = probability of a single gate failure
n = the total number of spillway gates

For use in Excel, Equation VII-3-6 can be used

P_v = BINOMDIST(y, n, P_f, FALSE) Eq. VII-3-6

The portion of the equation represented by $(P_f)^y$ accounts for all the gates that fail. The portion of the equation represented by $(1- P_f)^{n-y}$ accounts for all the gates that do not fail.

It should be noted that this approach assumes that the failure probability of each gate is independent of the failure probabilities of other gates. This is not necessarily the case. It holds true if there is an unknown defect that is unique to each gate which controls its failure probability. On the other hand, if it were known that one gate was near failing (not necessarily related to a unique defect), then this would affect the failure probabilities for the other gates. However, the Pascal triangle approach seems reasonable, in that if the failure probability of a single gate is small, the failure probability of multiple gates is also small; whereas, if the probability of a single gate is high, the failure probability of multiple gates is also high, as illustrated in Table **VII-3**-4. The most likely outcome (number of gates that will fail based on the probability estimate of a single gate failing) can be predicted by multiplying the total number of gates by the estimate of a single gate failing. From Table **VII-3**-4, for a single gate failure probability of 0.16, the most likely outcome is 8 x 0.16 = 1.28 or close to 1 gate failing. This is supported by the results in the table.

Typically, the combination of lower seismic load and lower reservoir elevation will have a significantly greater likelihood than higher seismic load and higher reservoir elevation, in each load range. Therefore, assigning equal weight to the boundary failure probabilities for a load range is generally conservative. This is especially true when there is a large range of failure probabilities at the boundaries of the load range (in which case it may be appropriate to look at smaller load ranges). Thus, the tree is often run using conditional failure probabilities that represent both the average of the ends of the ranges, and the lower ends of the ranges. If there is a large difference in the results, then additional refinement or weighting is probably needed (see also the section on Event Trees).

Table VII-3-4 – Example Pascal's Triangle Failure Probability Estimates

Probability for Single Gate → Failure		0.001	0.05	0.16	0.94
No. of Gates Failing	Equation for "x" Gates Failing	Probability for "x" Gates Failing	Probability for "x" Gates Failing	Probability for "x" Gates Failing	Probability for "x" Gates Failing
0	$1P^0(1-P)^8$	0.99	0.66	0.25	1.7E-10
1	$8P^1(1-P)^7$	0.0079	0.28	0.38	2.1E-08
2	$28P^2(1-P)^6$	2.8E-05	0.051	0.25	1.2E-06
3	$56P^3(1-P)^5$	5.6E-08	0.0054	0.096	3.6E-05
4	$70P^4(1-P)^4$	7.0E-11	0.00036	0.023	0.00071
5	$56P^5(1-P)^3$	5.6E-14	1.5E-05	0.0035	0.0089
6	$28P^6(1-P)^2$	2.8E-17	3.9E-07	0.00033	0.070
7	$8P^7(1-P)^1$	8.0E-21	5.9E-09	1.8E-05	0.31
8	$1P^8(1-P)^1$	1.0E-24	3.9E-11	4.3E-07	0.61
Total		0.0080	0.34	0.75	1.00

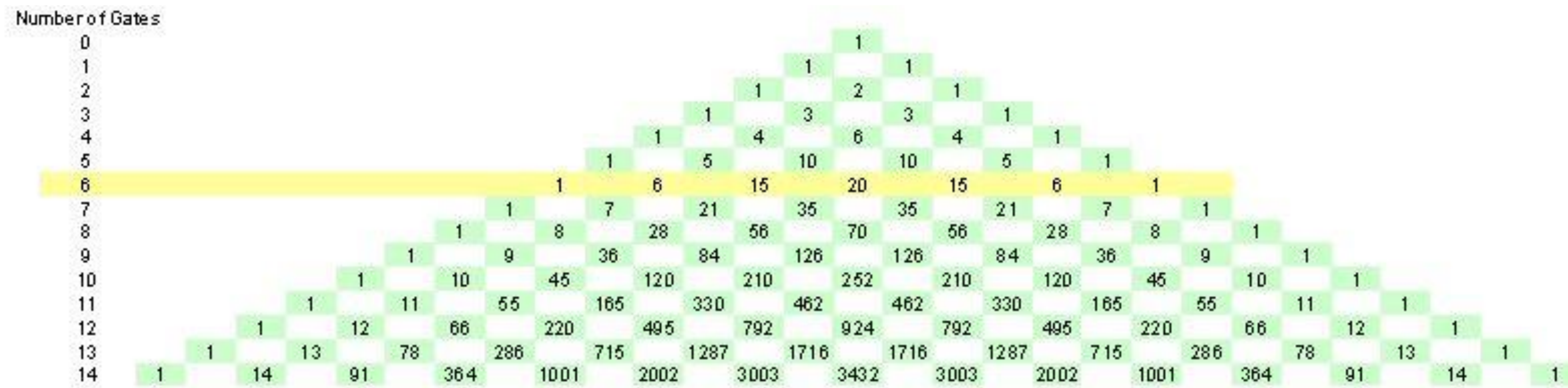


Figure VII-3-10 – Pascal's Triangle for Multiple Gate Failure Probability Coefficients

Consequences

Consequences are a function of the number of gates that fail and the reservoir level at the time of failure (or the breach outflow). It is usually assumed that failure will result in a completely unrestricted spillway bay (the gate fails and washes away). This may not always be the case and the gate may not be completely removed, which could limit discharge for a failed gate to something less than that represented by a free-flow discharge (no restriction through bay). In this example, at least 4 gates need to fail to exceed the safe channel capacity of 160,000 ft^3/s. However, smaller flows from fewer gate failures could impact recreationists adjacent to the river. Loss of life can be estimated from these breach flows and the estimated population at risk that would be exposed to the breach outflows using the procedures outlined in the section on Consequences of Dam Failure. To estimate a weighted loss of life for each seismic load and reservoir elevation range, the estimated loss of life associated with various gate failure outcomes (i.e. number of gates that fail) is multiplied by the conditional failure probability for the corresponding outcomes. The total (sum) conditional loss of life estimate is then divided by the total (sum) conditional failure probability estimate to arrive at the weighted average loss of life value. Example calculations for weighted loss of life are shown in Table **VII-3**-5, for a given reservoir elevation and single gate failure probability.

Table VII-3-5 – Weighted Average Loss of Life – Single Gate Failure Probability (P) = 0.16, RWS El 458

Number of Gates Failing	Probability of Failure Equations	Probability (P_x) of (x) Gates Failing	Expected Value Loss of Life	Loss of Life for (x) Gates Failing x (P_x)
1	$P_1 = 8(P)^1(1-P)^7$	0.38	8*	3.0
2	$P_2 = 28(P)^2(1-P)^6$	0.25	16*	4.0
3	$P_3 = 56(P)^3(1-P)^5$	0.096	23*	2.2
4	$P_4 = 70(P)^4(1-P)^4$	0.023	30*	0.69
5	$P_5 = 56(P)^5(1-P)^3$	0.0035	147	0.51
6	$P_6 = 28(P)^6(1-P)^2$	0.00033	164	0.054
7	$P_7 = 8(P)^7(1-P)^1$	1.8E-05	181	0.0033
8	$P_8 = 1(P)^8(1-P)^0$	4.3E-07	201	8.6E-05
Totals		0.75		10.5

* Loss of life due to recreational activity only

For this case, the Weighted Average Loss of Life = 10.51/0.75 = 14. The consequences for each seismic and reservoir load range are considered in the same way as the conditional failure probability. If the average of the load range boundaries produces risks that are considerably different than using the low value for the load range boundaries, additional refinement or weighting should be considered.

Results

The complete event tree for the example described here is shown in Table **VII-3-**6. Due to the large number of load ranges, it is usually easier to enter the event tree as rows and columns in a spreadsheet than to use Precision Tree. If Precision Tree is used, the resulting tree will take up several pages. It is important to review the results and isolate the major risk contributors. In this case, the risk is fairly evenly distributed between the seismic load ranges, with the lower load range contributing the least risk, and the middle load ranges contributing the most. The upper few reservoir ranges contribute the most risk.

Accounting for Uncertainty

The method of accounting for uncertainty in the seismic loading is described in the section on Event Trees. Typically, the reservoir elevation exceedance probabilities are taken directly from the historical reservoir operations data, which do not account for uncertainty. Uncertainty in the failure probability and consequences are accounted for by entering the estimates as distributions (as describe above) rather than single point values. A "Monte-Carlo" simulation is not practical for this failure mode, given the complexity of the calculations. Parametric studies should be considered however, to establish a reasonable range for the estimates.

Consequences of gate failure may also have uncertainty related to the breach outflow that will occur and the estimated loss of life due to the additional outflow. While it is usually assumed that the gate is completely removed and that free-flow conditions exist, this may not always be the case. It may be appropriate to consider breach outflow based on a range of conditions – from free-flow conditions to restricted flow due to the gates partially blocking the spillway bay.

Table VII-3-6 – Event Tree Calculations

Seismic Load	Seismic Load Probability	Reservoir Elevation	Reservoir Probability	One or More Gates Fail	Annual Probability	Conseq	Annualized Life Loss
> 2.0g	2.00E-06	462 - 466	0.03	100.00%	6.00E-08	228	1.37E-05
	2.00E-06	458 - 462	0.04	100.00%	8.00E-08	212	1.69E-05
	2.00E-06	454 - 458	0.05	100.00%	1.00E-07	191	1.91E-05
	2.00E-06	450 - 454	0.03	100.00%	6.00E-08	164	9.84E-06
	2.00E-06	442 - 450	0.10	97.10%	1.94E-07	157	3.05E-05
	2.00E-06	434 - 442	0.12	47.50%	1.14E-07	7	7.98E-07
	2.00E-06	426 - 434	0.18	0.40%	1.44E-09	3	3.60E-09
Subtotal					6.10E-07		9.08E-05
1.5g - 2.0g	4.00E-06	462 - 466	0.03	100.00%	1.20E-07	220	2.63E-05
	4.00E-06	458 - 462	0.04	99.98%	1.60E-07	180	2.87E-05
	4.00E-06	454 - 458	0.05	99.88%	2.00E-07	138	2.76E-05
	4.00E-06	450 - 454	0.03	98.45%	1.18E-07	102	1.21E-05
	4.00E-06	442 - 450	0.10	72.30%	2.89E-07	44	1.27E-05
	4.00E-06	434 - 442	0.12	23.95%	1.15E-07	5	5.46E-07
	4.00E-06	426 - 434	0.18	0.20%	1.44E-09	1	1.80E-09
Subtotal					1.00E-06		1.08E-04
1.0g - 1.5g	1.50E-05	462 - 466	0.03	99.90%	4.50E-07	189	8.49E-05
	1.50E-05	458 - 462	0.04	93.68%	5.62E-07	105	5.89E-05
	1.50E-05	454 - 458	0.05	77.10%	5.78E-07	48	2.78E-05
	1.50E-05	450 - 454	0.03	57.08%	2.57E-07	23	5.97E-06
	1.50E-05	442 - 450	0.10	23.95%	3.59E-07	6	2.25E-06
	1.50E-05	434 - 442	0.12	0.20%	3.60E-09	1	4.50E-09
Subtotal					2.21E-06		1.80E-04
0.75g - 1.0g	3.40E-05	462 - 466	0.03	98.35%	1.00E-06	123	1.24E-04
	3.40E-05	458 - 462	0.04	75.68%	1.03E-06	45	4.58E-05
	3.40E-05	454 - 458	0.05	35.85%	6.09E-07	9	5.49E-06
	3.40E-05	450 - 454	0.03	8.83%	9.00E-08	5	4.28E-07
	3.40E-05	442 - 450	0.10	0.23%	7.65E-09	2	1.15E-08
Subtotal					2.74E-06		1.75E-04
0.5g - 0.75g	1.05E-04	462 - 466	0.03	58.80%	1.85E-06	45	8.38E-05
	1.05E-04	458 - 462	0.04	34.10%	1.43E-06	18	2.54E-05
	1.05E-04	454 - 458	0.05	8.83%	4.63E-07	6	2.66E-06
	1.05E-04	450 - 454	0.03	0.20%	6.30E-09	2	9.45E-09
Subtotal					3.75E-06		1.12E-04
0.25g - 0.5g	5.40E-04	462 - 466	0.03	46.10%	7.47E-06	10	7.47E-05
	5.40E-04	458 - 462	0.04	2.33%	5.02E-07	6	3.14E-06
	5.40E-04	454 - 458	0.05	0.20%	5.40E-08	2	1.08E-07
Subtotal					8.02E-06		7.79E-05
Total					**1.83E-05**		**7.44E-04**

What if Gate Failure Probabilities are not Independent?

As noted, the above evaluation assumes the failure probabilities for all gates are independent of each other. In reality, if a gate fails, it would make the potential failure of the remaining gates more suspect. It may be instructive to walk through a scenario such as that presented in Figure **VII-3-**11. In this example, each possible scenario related to potential failure of four gates is evaluated using an "updating" approach. Proceeding from left to right, the following scenarios are evaluated.

- Initially, the estimated probability of gate failure is 0.1. If gate number 1 survives a test, then there is more confidence that gate 2 will survive the test (say, failure probability is reduced to 0.05). Similarly, if gates 3 and 4 survive, additional confidence is gained, and the estimated failure probability reduces in each case.
- If gate number 1 fails the test, then confidence in the initial estimate becomes less. However, there still might be some confidence in the original estimate such that certain failure and the initial estimate are weighted equally at that point (failure probability = 0.55). If then gates 3 and 4 both fail the test, confidence in the original estimate is undermined, and a subsequently higher failure probability is concluded in each case.
- If gate 1 fails the test and gate 2 survives, or gate 1 survives and gate 2 fails, then perhaps the 50% failure rate, weighted equally with the original estimate, would be estimated for gate 3 (or 30% failure probability).
- If one of the first three gates fails the test, then the 1/3 failure rate might be averaged with the initial estimate of 0.1 to arrive at an estimated failure rate of 21.5% for gate 4. If two of the first three gates fail the test, then perhaps the 2/3 failure rate would be averaged with the initial estimate.

Figure **VII-3-**11 shows the difference between the above assessment and Pascal's Triangle assessment discussed previously. It can be seen that the chance of one or more gate failures is higher using Pascal's Triangle, although the chance of 3 or 4 gates failing is actually less. If, in this case, the consequences were to become significantly more severe with 3 or 4 gate failures, it may be important to take this into account in estimating the risks. Figure **VII-3-**12 shows another example of updating, this time with an initial estimated probability of gate failure of 0.5. The results are similar to those shown in Figure **VII-3-**11, with the total probability of failure being greater for the results using Pascal's Triangle, but the chance of 4 gates failing actually being less for the updating approach.

Relevant Case Histories

Although radial gates have failed due to gate arm buckling as a result of trunnion pin friction (see the Best Practice chapter VII-3-3 on Failure of Radial (Tainter) Gates under Normal Operational Conditions), there are no known cases of radial gate failure as a result of earthquake loading.

Considerations for Comprehensive Review/Periodic Assessment

The complete analysis as described in this section is likely too time consuming to be performed during a Comprehensive Review (CR) or a Periodic Assessment (PA). Therefore, simplifications must be made. Fewer load ranges are typically evaluated, and only expected value estimates are entered into the event tree. Uncertainty is typically taken as plus or minus an order of magnitude. Average weighting schemes are typically used for results at the load range boundaries resulting in conservative risk estimates. If results of finite element gate analyses are available, they can be used to help define the load and reservoir ranges to be considered. If no gate analyses are available, searching for results related to similar gates should be undertaken.

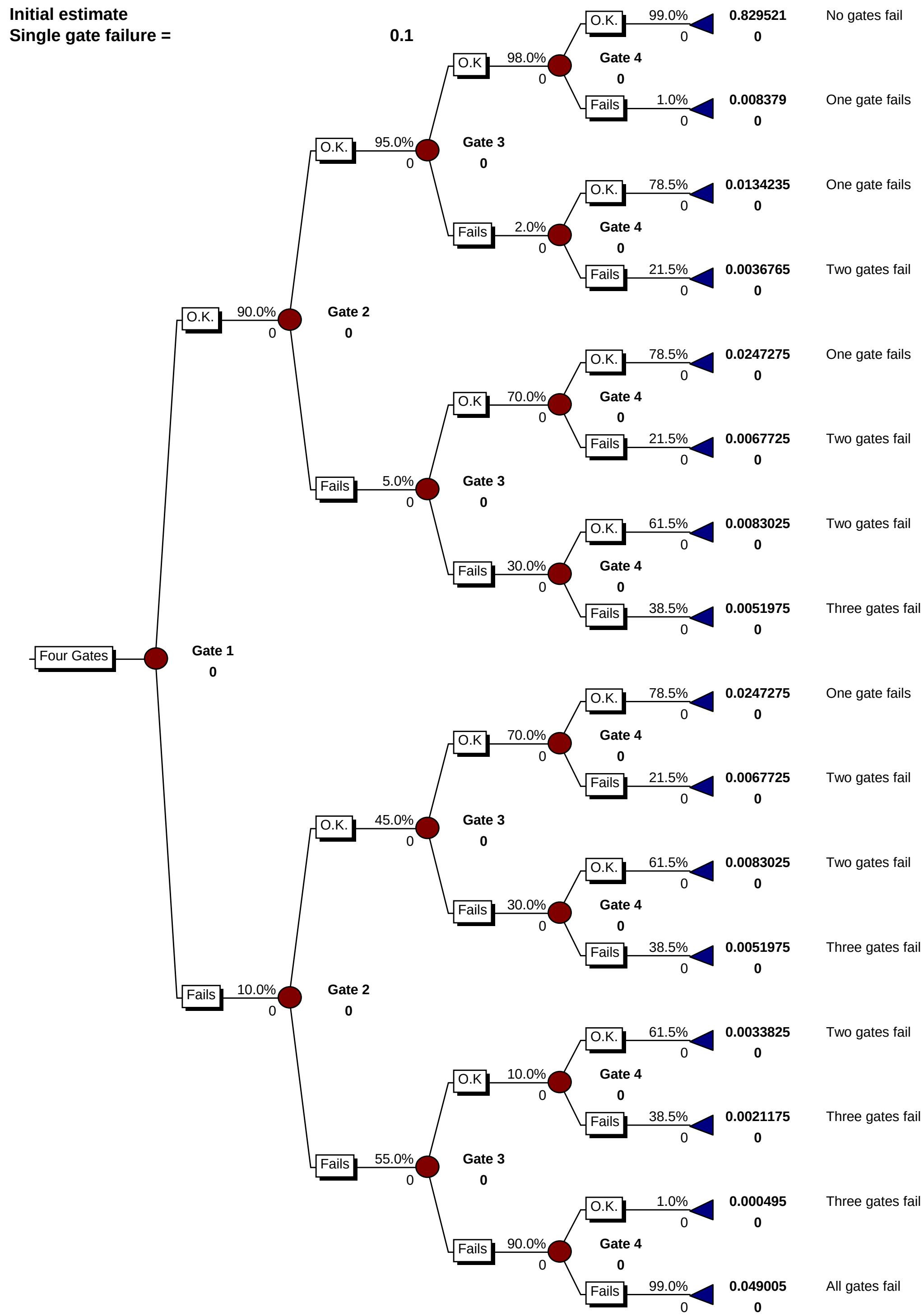
Exercise

Consider a spillway with two radial gates, each 34.5 feet high by 51 feet wide. The outflow through one gate with the reservoir at the top of the gate (when closed; the reservoir is at or above this elevation 10 percent of the time) is 37,500 ft^3/s. The flow through one gate with the reservoir 20 feet up on the gate (the reservoir is at or above this elevation 90 percent of the time) is about 16,500 ft^3/s. Finite element analyses of a gate have been done with the reservoir at both of these elevations, and for peak horizontal ground accelerations of 0.2g (expected value annual exceedance probability = 0.001), and 0.5g (expected value annual exceedance probability = 0.0001) at the trunnion pin. The combined stress ratios for the most highly stressed gate arm are listed in Table **VII-3-**7.

Table VII-3-7 – Interaction Ratios

	0.2g	0.5g
Reservoir at top of gates	0.7	1.0
Reservoir 20’ up on gates	0.6	0.8

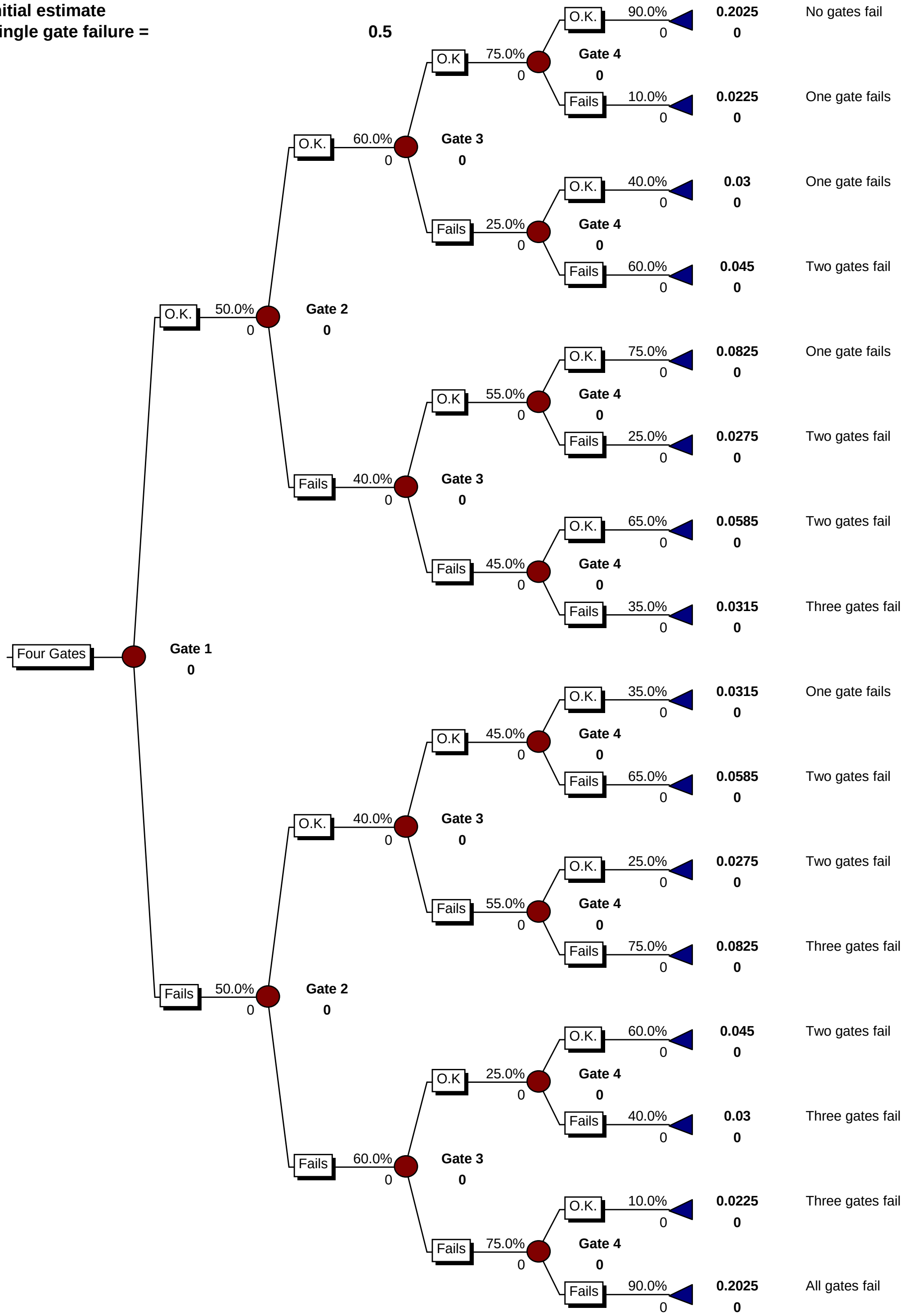
Estimate the expected value annual failure probability for one or more gates failing due to seismic loading.



Pascal's Triangle			Tree	
No gates fail	$1*P^0*(1-P)^4$ =	0.6561	No gates fail =	0.8295
One gate fails	$4*P^1*(1-P)^3$ =	0.2916	One gate fails =	0.0713
Two gates fail	$6*P^2*(1-P)^2$ =	0.0486	Two gates fail =	0.0372
Three gates fail	$4*P^3*(1-P)^1$ =	0.0036	Three gates fail =	0.0130
Four gates fail	$1*P^4*(1-P)^0$ =	0.0001	Four gates fail =	0.0490
One or more		0.3439	One or more =	0.1705

Figure VII-3-8 – "Updating" Event Tree for Four Radial Gates

Initial estimate
Single gate failure = **0.5**



Pascal's Triangle			**Tree**	
No gates fail	$1*P^0*(1-P)^4$ =	**0.0625**	**No gates fail =**	**0.2025**
One gate fails	$4*P^1*(1-P)^3$ =	**0.25**	**One gate fails =**	**0.1665**
Two gates fail	$6*P^2*(1-P)^2$ =	**0.375**	**Two gates fail =**	**0.2620**
Three gates fail	$4*P^3*(1-P)^1$ =	**0.25**	**Three gates fail =**	**0.1665**
Four gates fail	$1*P^4*(1-P)^0$ =	**0.0625**	**Four gates fail =**	**0.2025**
One or more		**0.9375**	**One or more =**	**0.7975**

Figure VII-3-9 – "Updating" Event Tree for Four Radial Gates

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