I-8. Combining and Portraying Risks

Key Concepts

After all potential failure modes have been identified and described, and their risks have been evaluated, the results need to be combined and portrayed so that the technical reviewers and decision makers can understand and act upon them. This requires attention to detail, and if not undertaken properly, could result in an incorrect portrayal of the risk. This chapter describes some of the details needed to properly do the job.

A risk analysis, whether by a team or by an individual, produces estimates of risk for individual potential failure modes. These estimates might include probability or risk values for different loading conditions, loading ranges, spatial segments, or other situations. The risks from individual potential failure modes are often combined in some way to express their collective effect.

In practice, the most common problems encountered during risk analyses are related to systems, correlations, common-cause loading, and combining risks. Although the methods to evaluate these issues can become complex, some simplifications can be applied to situations commonly seen when evaluating risks for dams and levees.

System Considerations

For purposes of dam and levee safety risk analysis, a dam or levee system is typically defined to include all components of the project that are intended to retain the reservoir (dams) or exclude water from the leveed area. In both cases, the system considers those components that can affect a common consequence center(s) within the associated floodplain.

Mutually Exclusive Events

The probability of the union of two or more mutually exclusive events is equal to the sum of their probabilities. Similarly, risks associated with mutually exclusive events can be summed. In an event tree, all branches originating from the same node must be mutually exclusive. This allows the AFPs and the ALLs at the end branches of an event tree to be summed to obtain the total AFP or total risk for an individual potential failure mode or group of potential failure modes. The total AFP and total risk can be obtained by summing all of the end branches of the event tree. Similar event tree summations can be performed to obtain the probability of failure (or risk) associated with a particular loading range (e.g. floods having a return period more frequent than a 100 years), a physical feature (e.g. spillway), a consequence center location, or a particular breach mechanism. It is common practice to model floods and earthquakes as mutually exclusive events (even though they are not). In most cases this is a reasonable assumption because the probability of a coincident earthquake and flood is remote enough to not significantly influence the risk estimate. As a result, floods and earthquakes can be evaluated using separate event trees and the results summed to obtain an estimate of the total risk.

Collectively Exhaustive Events

Events that are collectively exhaustive include all possible outcomes. In an event tree, all branches originating from the same node must be collectively exhaustive. The probabilities associated with all branches originating from a particular node must sum to 1.0. This can be used as a check to help validate that the event tree has been properly constructed.

Statistically Independent Events

Two or more events are statistically independent if the occurrence of one event does not affect the probability for occurrence of the other event(s). The definition of statistical independence means that the probability of one event can be estimated without explicitly considering whether the other event(s) has occurred or not. In practice, a probability of failure associated with a particular potential failure mode is often estimated under the assumption that the PFM is statistically independent of other potential failure modes. Correlation and/or common cause effects, if they are relevant, can be accounted for by adjusting the probability estimates and/or modifying the event tree structure.

Conditional Probability

Two or more events are statistically dependent if the occurrence of one event affects the probability for occurrence of the other event(s). For such events, the occurrence of the conditioning event must be considered when estimating the probability of the conditional event. Probabilities of failure are often conditional on the magnitude of the load because a greater load will typically result in a greater probability of failure. Event trees are constructed such that conditional probabilities are shown to the right of the events on which their probabilities are conditioned.

Correlated Events

Correlation is the degree to which the probabilities for two or more events are linearly related. For correlated events, the occurrence of one event is an indication that the other event is also likely to occur (positive correlation) or likely to not occur (negative correlation). Probabilities associated with dam/levee components of similar character (e.g. spillway gates, concrete monoliths) might be correlated. If one spillway gate fails to operate, then it may be likely that additional spillway gates will also fail to operate. Correlation can be quantitatively accounted for in the risk analysis using correlation matrices or more qualitatively accounted for by applying expert judgment to the estimated probabilities associated with the responses of groups of similar components.

Total System Probability

The uni-modal bounds theorem (Ang and Tang, 1984) states that for 'n' positively correlated events ($E_1, E_2, E_3, \ldots, E_n$) with corresponding probabilities [$P(E_1)$, $P(E_2)$, $P(E_3)$, …, $P(E_n)$], the total probability for the union of the events [$P(E) = P(E_1 \cup E_2 \cup E_3 \ldots \cup E_n)$] lies between an upper and lower bound, as follows:

$$\max_i P(E_i) \leq P(E) \leq 1 - \prod_{i=1}^{n}[1 - P(E_i)]$$

The uni-modal bounds theorem can be used to obtain the total probability of failure for a dam or levee system from a set of failure probabilities associated with individual potential failure modes. The lower bound is obtained if the potential failure modes are perfectly correlated. The upper bound is obtained if the potential failure modes are statistically independent. In practice, the upper bound is often used in dam and levee safety risk analysis unless specific knowledge of the degree of positive correlation is available.

If one potential failure mode is dominant (i.e. has a probability significantly greater than that of all other failure modes), then the upper bound and lower bound obtained from the equation will be approximately equal to each other.

If probabilities for individual potential failure modes are small, the upper bound can be approximated by summing the individual failure mode probabilities. The maximum error in this approximation will be less than about 5% if the sum of the individual probabilities is less than about 0.1. As the sum increases, the error in the approximation also increases. When the sum becomes greater than 1.0, the approximation violates the axioms of probability.

Consider a dam with the following three seismic potential failure modes: A) sliding within the foundation of a concrete gravity monolith, B) buckling of a spillway gate arm, and C) liquefaction of the embankment foundation leading to crest deformation and overtopping. The probability of failure for each of these potential failure modes has been estimated independently.

$$P(A) = 0.3$$
$$P(B) = 0.1$$
$$P(C) = 0.2$$

Applying the uni-modal bounds theorem, the total probability of failure for the dam is estimated to be between 0.3 and 0.496.

$$\max\{0.3, 0.1, 0.2\} \leq P(\text{Fail}) \leq 1 - (1 - 0.3)(1 - 0.1)(1 - 0.2)$$

$$0.3 \leq P(\text{Fail}) \leq 0.496$$

These values place bounds on the probability that the dam will fail due to any combination of one or more of the individual potential failure modes. In contrast, a summation of the individual probabilities of failure would result in an estimate of 0.6. Using the summation approximation would in this case overestimate the upper bound for the total probability of failure by about 20%.

The upper bound estimate is represented by the shaded area on the Venn diagram in Figure I-8-1. The estimate includes the total area enclosed by all of the circles, each representing an individual potential failure mode. The uni-modal bounds equation calculates a range of total areas using the limiting cases of circle overlap. The summation approximation, which does not consider the intersection area, is subject to double counting error (i.e. the overlapping areas on the Venn diagram are counted more than once). The double counting of the overlapping area is the source of the 20% error in the example.

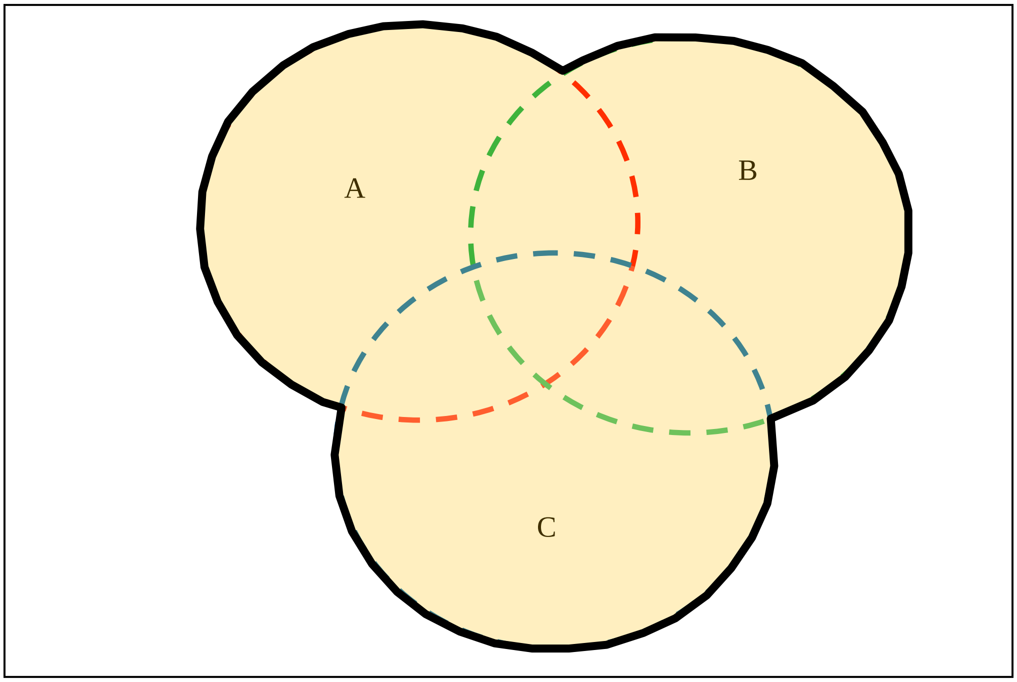

Figure I-8-1. Venn Diagram for Upper Bound Estimate

Common Cause Adjustment

In typical dam and levee safety risk analyses, intersection events representing the simultaneous occurrence of two or more potential failure modes are not explicitly evaluated in the event trees. This is usually, but not always, a reasonable simplification. If the probabilities of the intersection events are small relative to the probabilities of each potential failure mode, then the intersection event probabilities can be ignored. This allows the potential failure mode probabilities to be summed to obtain a reasonable approximation of the total probability of failure. When the intersection probabilities are not small, the probabilities for each potential failure mode may need to be adjusted to account for the intersection probability so that the correct total probability of failure for the dam or levee can be obtained. The common cause adjustment is one of the methods available to make this correction.

The Annualized Failure Probability (AFP) associated with a simple adverse event chain (e.g., A happens, B happens, C happens, D happens ...) is calculated as the probability of the intersection event P(ABCD...). For a dam susceptible to multiple adverse event chains (i.e., Potential Failure Modes), the total probability of failure is calculated as the probability of the union event. For example, given Potential Failure Modes (PFMs) 1 and 2 with calculated Annualized Failure Probabilities AFP1 and AFP2, the total AFP is calculated as:

$$P(\text{PFM 1 U PFM 2}) = P(\text{PFM 1}) + P(\text{PFM 2}) - P(\text{PFM 1} \cap \text{PFM 2})$$

$= \text{AFP1} + \text{AFP 2} - \epsilon$

where ϵ is a number between zero and the smaller of (AFP1, AFP2). When PFMs 1 and 2 are SI, ϵ is usually small enough to be ignored without inflating the total risk estimate (<u>not</u> because SI events have trivial intersections, but because P[PFM 1 ∩ PFM 2] reduces to the product of two small numbers for SI events). However, when PFMs 1 and 2 are not SI (for example, when both PFMs involve the occurrence of a flood or earthquake), it may not be possible to simply assume that ϵ is negligible.

Consider a set of structural response probabilities estimated by a risk team using engineering judgment. Given a 50,000-year earthquake (Event Q), the probability of breach due to embankment liquefaction (Event A) is estimated as 0.7 (= P[A|Q]) and the probability of breach due to the gravity section sliding (Event B) as 0.8 (= P[A|Q]). Since the rules of probability theory still apply within a reconditioned sample space, the fact that the conditional probabilities of A and B sum to greater than 1 implies that within the reconditioned sample space, there is intersection between the events. Note that this does not imply there is anything "wrong" with the team's conditional probability estimates.

Figure I-8-2 shows the Venn diagram for the above example, both before (left) and after (right) the sample space transformation associated with the occurrence of the 50,000-year earthquake. Prior to the occurrence of the quake, the "area" of S occupied by Q is relatively small (since the earthquake has only about a 1/50,000 chance of occurring). However, once it is known that the earthquake has occurred, the sample space changes from all of S to only the region bounded by Q. Although the "size" of AB given Q is not obvious, the conditional probability of A U B can be estimated by assuming statistical independence between A and B <u>within</u> the reconditioned sample space. This assumption forms the basis of the Common Cause Adjustment or CCA.

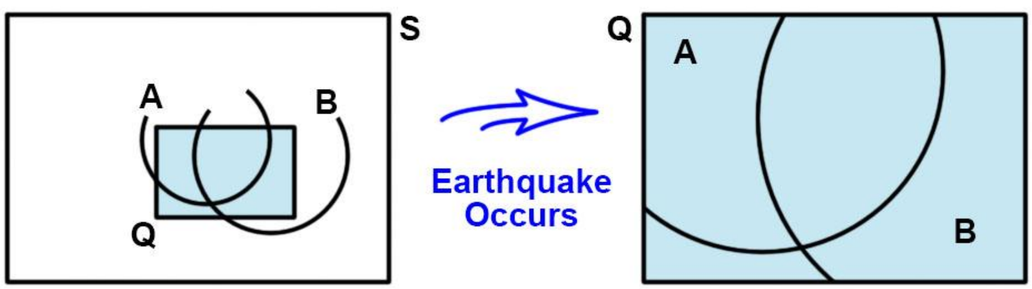

Figure I-8-2. The probabilities of events A (e.g., breach due to liquefaction mechanism) and B (e.g., breach due to gravity section sliding mechanism) within the overall sample space (left panel) and the reconditioned sample space associated with the occurrence of event Q, a 50,000-year earthquake (right panel).

Hill et al (2003) have proposed a simplified approach for adjusting the system response probabilities for each potential failure mode. The method redistributes the overlapping area in Figure I-8-1 to each individual failure mode (or breach mechanism, in the case of a reconditioned sample space). The magnitude of the redistribution is proportional to the estimated probability of failure for each potential failure mode (or the conditional probability of failure for a breach mechanism). Events with larger probabilities of failure receive a larger portion of the overlapping area. The approach is implemented using the following equation, where p_j is the unadjusted probability of failure for potential failure mode (or breach mechanism) j and p_j' is the adjusted probability of failure.

$$p_j' = p_j \frac{1 - \prod_{i=1}^{n}[1 - p_i]}{\sum_{i=1}^{n} p_i}$$

For the previous example, the adjusted probabilities of failure are

$$p_A' = 0.3 \frac{1 - (1 - 0.3)(1 - 0.1)(1 - 0.2)}{0.3 + 0.1 + 0.2} = 0.248$$

$$p_B' = 0.1 \frac{1 - (1 - 0.3)(1 - 0.1)(1 - 0.2)}{0.3 + 0.1 + 0.2} = 0.083$$

$$p_C' = 0.2 \frac{1 - (1 - 0.3)(1 - 0.1)(1 - 0.2)}{0.3 + 0.1 + 0.2} = 0.165$$

The adjusted failure probabilities could now be used directly in an fN chart, or inserted back into the event tree in place of the existing conditional probability estimates (e.g., to replace the originally estimated values of P(A|Q) and P(B|Q) in the Figure I-8-2 example). The sum of the adjusted probabilities is now also equal to the total probability of failure obtained using the upper bound from the uni-modal bounds theorem.

$$0.248 + 0.083 + 0.165 = 0.496$$

Additional details regarding the common cause adjustment are provided in Chapter I-5 – Event Trees.

Length Effects

Dam and levee systems may be comprised of significant lengths of constructed embankments or walls extending thousands of feet to hundreds of miles. This may result in considerable uncertainty about the loadings, performance, and consequences for sections within the system.

Systems fail at locations where loads are high and strengths are low. If these critical locations are identified ahead of time, traditional methods can be used to analyze stability and estimate probabilities of failure. In such situations, the overall length of the system is immaterial, because the weakest spots have been identified and dealt with, and the performance of the system depends on the probability of failure for the weak spots. The more common situation is that the system is not characterized with enough detail for the risk analyst to know unambiguously where the weakest spots are. In this case, any section of the system has some probability of experiencing higher than average loads and/or lower than average strengths. Since these locations cannot be uniquely identified before a failure occurs, a longer system length results in a greater probability of a failure somewhere.

Long systems can be divided into reaches for which the engineering properties and loading conditions are similar enough to assume they are statistically independent. A characteristic length is chosen to be the maximum length for which performance of adjacent lengths can be assumed to be statistically independent.

USACE suggests the following approach to consider length effects in risk analysis. The approach is illustrated in Figure I-8-3:

1) Divide the length into reaches having homogeneous properties of H&H loading, structural profile, geotechnical subsurface conditions, and consequences of failure. Such reaches may be 100's of feet to several miles in length.
2) For the engineering properties of each reach, develop a system response curve relating loading conditions to the probability of failure of a "unit length".
3) Subdivide long reaches into characteristic lengths which reflect sections that are thought to behave as if statistically independent. Characteristic length could be based on a statistical analysis of spatial correlation, by analogy to earlier projects, or by expert judgment.
4) Apply the DeMorgan rule ($P=1-(1-p)^n$) to approximate the probability of system failure, where p is the probability associated with the 2D system response curve, and n is the number of characteristic lengths.

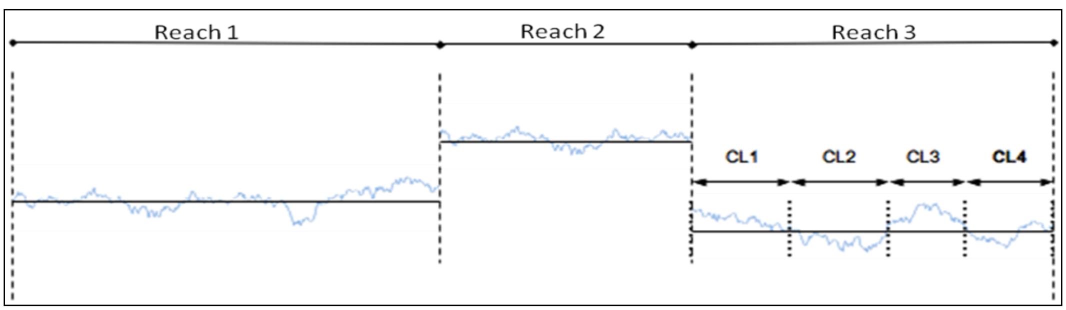

Figure I-8-3. System Divided into Reaches and Characteristic Lengths

Cascading Failures

Individual dams and levees are often part of larger infrastructure systems. Within these watershed systems, Reclamation and USACE generally attribute risk to the specific infrastructure that is the source of the risk. This includes cascading impacts in the 'downstream' direction. If failure of the dam or levee being assessed would result in overtopping and subsequent breach of downstream dams and/or levees, then the risk associated with these cascading failures would be attributed back to the dam or levee being assessed. Risks generated by failure of 'upstream' infrastructure are usually not considered. If failure of an upstream dam would result in overtopping and breach of the dam or levee being assessed, then increases in the magnitude and frequency of loading caused by failure of the upstream dam would typically not be included in the risk estimate. Similarly, a decrease in load magnitude and frequency due to failure of an upstream levee would not be considered. In general, the potential failure of an upstream dam or levee is not used as justification to take action (or not take action) on the dam or levee being assessed.

To support portfolio prioritization decisions or to communicate the flood risk from multiple flooding sources, there may be a benefit in estimating the risk from a river systems perspective. These analyses can support improved prioritization decisions within the larger watershed to obtain more efficient and effective total risk reduction across the portfolio. In these situations, it may be appropriate to evaluate the cascading impacts of failure in both the 'upstream' and 'downstream' directions.

Intervention

Use of Intervention in Risk Assessment – USACE Approach

The USACE approach is to evaluate and communicate the potential risk reduction that can be achieved with intervention while at the same time to not mask the seriousness of a potential dam or levee safety issue by relying on intervention to manage the risk.

Portraying risk analysis results with consideration for intervention is useful in supporting prioritizing actions across a portfolio of dams and/or levees. The potential for successful intervention could be a discriminating factor in the recommended risk management actions at a particular dam or levee. It can also be a discriminating factor in prioritizing actions between two otherwise similar dams or levees. Including intervention in the risk analysis provides a more realistic picture of the risk based on the fact that intervention actions will be taken and will have some chance of being successful.

Portraying risk analysis results without consideration for intervention is useful in supporting decisions related to tolerability of risk for a particular dam or levee. USACE does not consider a high likelihood of successful intervention as justification to characterize the risks at a particular dam or levee as being tolerable. Significant reliance on intervention, is not considered by USACE to be an effective long term permanent risk management strategy. The potential for successful intervention can be used as an effective short term interim risk management strategy.

Current USACE policy is to evaluate and communicate risk for two scenarios. The first scenario includes consideration of the possibility for successful intervention. The second scenario includes an assumption that no significant intervention actions will be taken. These scenarios can be evaluated quantitatively or qualitatively in the risk analysis.

Incremental Risk

Use of Incremental Risk in Risk Assessment – USACE Approach

Current USACE practice is to estimate the incremental risk resulting from failure of the infrastructure. This is done to distinguish between the risks imposed by failure of the dam or levee and the inherent flood risks that exist in the floodplain. The incremental risk is obtained by estimating the incremental consequences associated with each failure pathway in the event tree. For a given scenario defined by a specific pathway, the incremental consequences are computed as the difference between the consequences assuming the failure occurs and the consequences assuming the failure does not occur. The incremental consequences are then multiplied by the probability for the pathway to obtain the incremental risk for the pathway. The results for multiple pathways can be aggregated and summed to estimate the incremental risk for a failure mode or the total incremental risk for the dam or levee.

Non-Breach Risk

Use of Non-Breach Risk in Risk Assessment – USACE Approach

Current USACE practice is to assess and communicate residual inundation risk which is essentially the risk that remains even if the infrastructure performs its intended function without failing. This risk is not necessarily caused by the presence of the dam or levee although one could argue that the dam or levee facilitates more development of the floodplain. Nonetheless, the residual risk can still be high and should be communicated to affected parties. Most of the information needed to estimate risk for non-breach scenarios is readily available because it is already needed to build the event tree and estimate the incremental risk. Estimating the non-breach risk is often a simple exercise involving minor edits to the risk model and a few additional model runs. The non-breach risk is estimated for the non failure pathways in the event tree by multiplying the consequences for the pathway and the probability for the pathway. In the calculation, the probabilities associated with failure branches are modified to assume that breach will not occur (i.e. probability of breach is zero, probability of non-breach is one). Results can be aggregated by summing the results across multiple non failure pathways.

Flood Scenarios

Use of Flood Scenarios in Risk Assessment – USACE Approach

Risks may be attributed to one of the four scenarios illustrated in Figure I-8-4: 1) breach prior to overtopping; 2) overtopping with breach; 3) malfunction of system components or misoperation; and 4) spillway flow without breach or overtopping without breach. Attribution of the risk provides decision makers with information on the source of the risk and provides insight into the types of actions available to reduce or manage the risk. For example, risks associated with scenarios 1 and 3 might be reduced by improving performance of the existing system. Risks associated with scenario 2 might be reduced by improving resiliency of the system or increasing the capacity of the system. Risks associated with scenario 4 might be managed by improving warning and evacuation effectiveness.

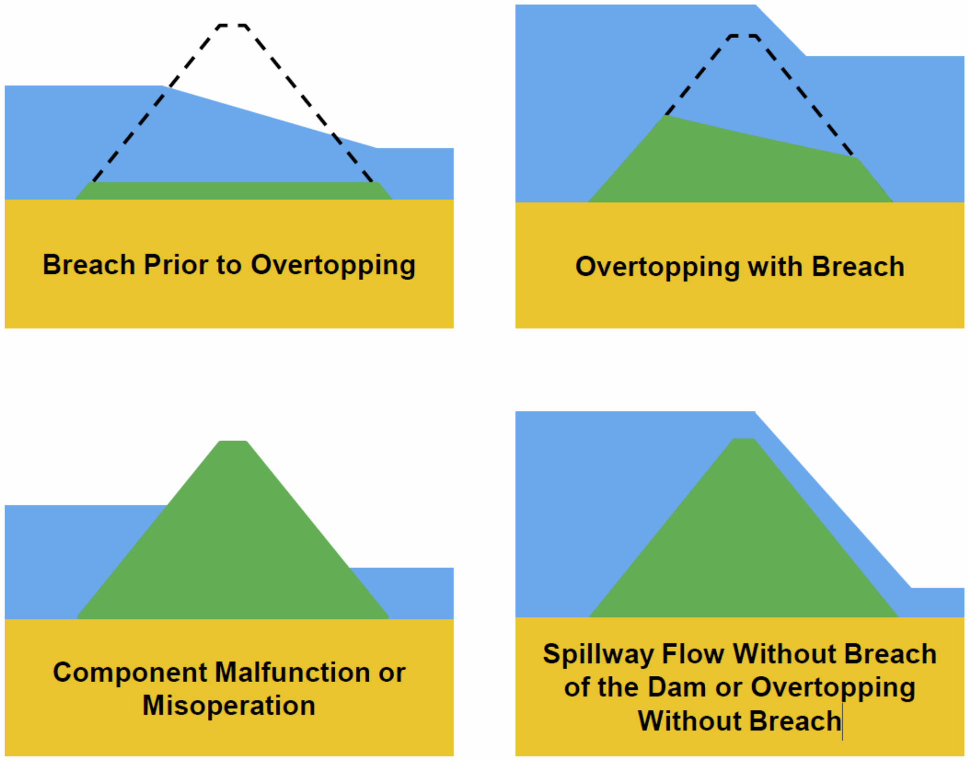

Figure I-8-4. Scenarios

System Response Curves

In dam and levee risk analysis, probability of failure estimates associated with system response curves are often interpreted as the products of conditional breach probabilities and specific load probabilities. This is a simplification that is made to facilitate event tree analysis and event tree calculations. This approach assumes that breach will occur at the maximum loading during a flood or earthquake event. It is important to recognize that a system response curve is actually a cumulative distribution function for the capacity or strength of the dam or levee. In practice, this means that the system response curve gives the probability that a failure or breach will occur at a load that is less than or equal to the specified load. This means that breach can occur at a loading that is less than the maximum loading experienced during a specific flood or earthquake event. Refer to Chapter I-5 – Event Trees for more details on the development of system response curves.

Characterizing the system response curve as a cumulative distribution function provides an important and powerful tool for simulation based risk models. For each simulation, the capacity of the system can be randomly sampled from the system response curve prior to the start of the simulation. During the simulation, the modeled failures or breaches occur when the demand reaches the sampled capacity. In this framework, it is not necessary to assume that failure or breach always occurs at the peak demand. Not making this assumption allows for explicit consideration of the temporal aspects of loading events such as floods and explicit analysis of scenarios where a failure or breach can occur at a water level that is less than the peak water level.

The branches in an event tree do not explicitly consider the temporal aspects of events like floods, and it is often assumed that failure or breach will occur at the peak load. The risk analyst should keep in mind that failure or breach can also occur at loads less than the peak load for the event. This simplification is usually not a significant issue when estimating or portraying the total annual probability of failure or breach. It can sometimes be an issue when estimating or portraying the total annualized life loss. In certain situations, failure at a load less than the peak load could result in different consequences and different risks. In these situations, the risk analyst should consider whether or not further refinement of the risk model is needed to obtain a more accurate portrayal of the risk for decision makers. A similar issue can arise in event tree risk models when attributing the risk to various loading categories such as static and flood or, for example, to the scenarios shown in Figure I-8-5. In these situations, the event tree risk model may also need to be refined to ensure that the risk is attributed to the appropriate scenarios. The following simplified example is used to illustrate the concept. Figure I-8-5 provides a levee scenario having a flood hazard defined by the annual chance exceedance for a peak flood stage at the toe (ACE_{Toe}) and at the top (ACE_{Top}) of the levee. The system performance is defined by a zero chance of breach at the toe of the levee, a chance of breach of Pf_{Top} given a water level at the top of the levee, and a chance of breach of $Pf_{Overtop}$ given an overtopping water level.

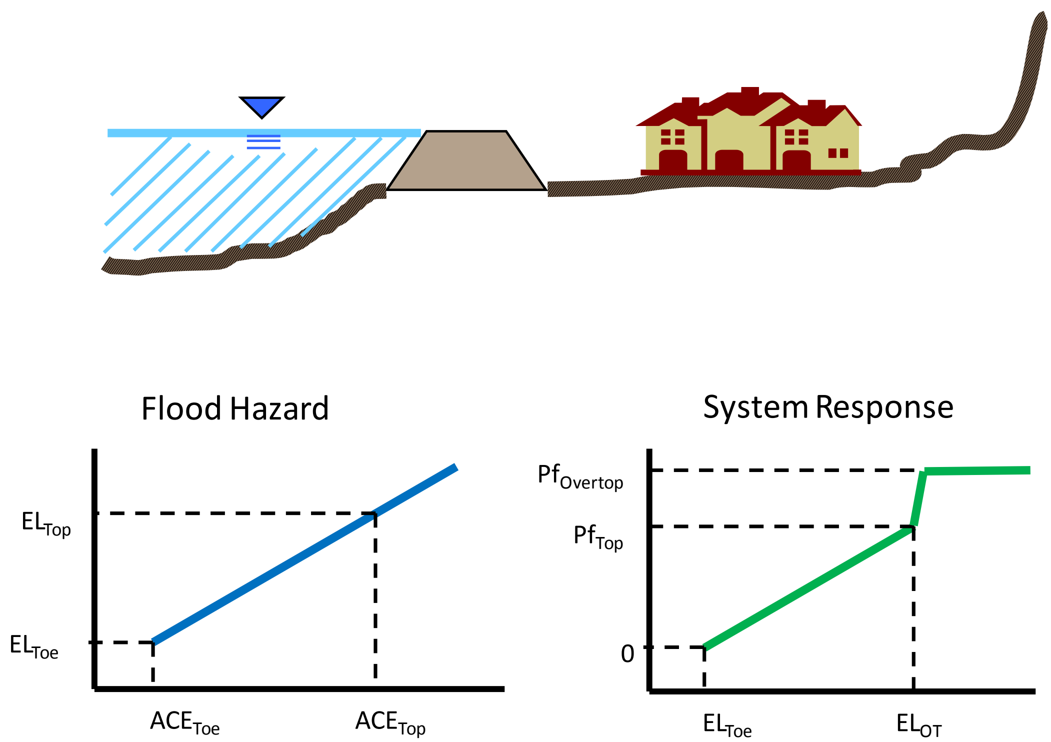

Figure I-8-5

An initial estimate of the probability of breach for the “prior to overtopping” and “overtopping” scenarios is made using the event tree model shown in Figure I-8-6. This event tree model is based on the simplifying assumption that breach occurs at the peak flood stage. The annual probability of failure is estimated using the event tree by numerically combining and integrating the area under the flood hazard and system response curves.

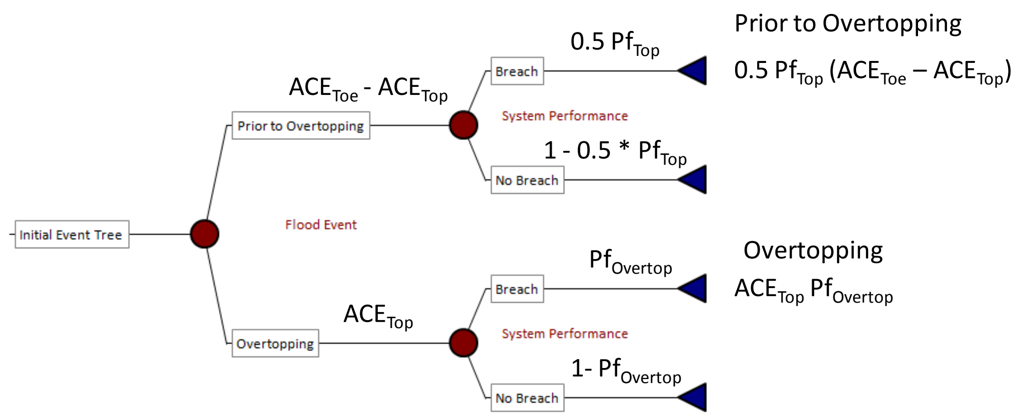

Figure I-8-6

The event tree calculations could be modified to account for the fact that a breach could occur at a stage that is less than the peak flood stage. The modified event tree model is shown in Figure I-8-7. The modified event tree includes an additional branch for the event where a large flood overtops the levee; however, for this branch the levee breaches during the rising limb of the flood hydrograph before overtopping actually occurs. The annual probability of failure is estimated using the event tree by numerically combining and integrating the area under the flood hazard and system response curves. In the modified event tree, the probability associated with a failure occurring before overtopping occurs is attributed to the "prior to overtopping" risk scenario. This modified event tree may result in an improved sense of the attribution of the risk and a clearer picture for decision makers.

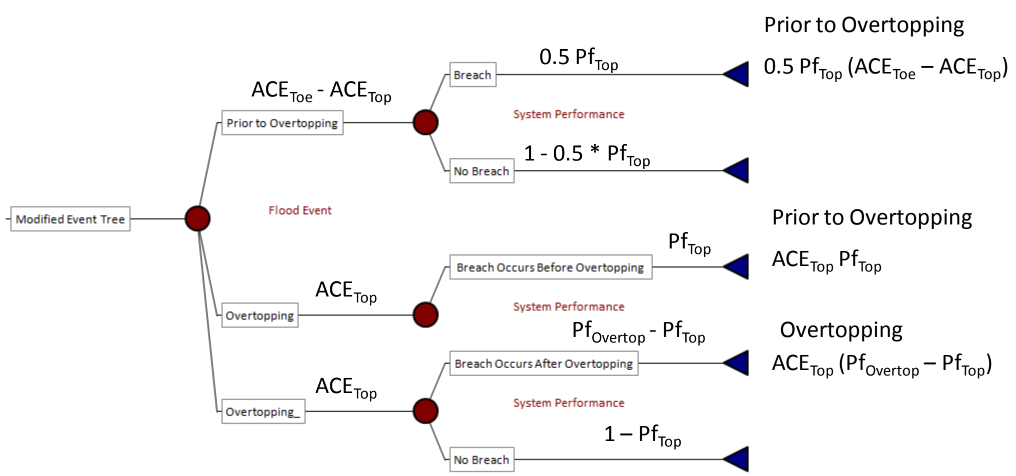

Figure I-8-7

Risk Plots

Risk analysis results are typically portrayed on an f-N or F-N chart. The usual format for an f-N plot features an annual failure probability on the vertical axis and the expected value of consequences given failure on the horizontal axis. Note that the expected value of consequences given failure is not the same as the expected annual consequences. The F-N plot features an annual probability of N or more consequences on the vertical axis and the magnitude of consequences on the horizontal axis. In both plots, the axes are shown with a log scale.

On the f-N plot, potential failure modes are shown individually with a separate f-N pair coordinate points for each potential failure mode. Results can be combined to obtain estimates of the total risk, risk by load ranges, or for any other combination that is needed by the decision makers. The "f" or AFP values are obtained by summing the probabilities for the end branches of relevant event tree pathways. For a given PFM, the N value is obtained by first summing the product of the probability and incremental consequences for the end branches of relevant pathways. The resulting sum (the annualized life loss) is then divided by the total AFP to obtain the expected value of N.

On the F-N plot, the end branch probabilities are accumulated by consequence level irrespective of failure mode. A cumulative curve is developed and plotted showing the probability of N or more lives lost.

Another type of risk plot is the scatter plot (e.g. Figure I-8-8), which uses the same log-scale axes as the fN chart and can also be generated using a standardized template. In order to obtain the data required for a scatter plot, a Monte Carlo simulation must first be performed, as discussed below.

Monte Carlo Simulation

In Dam Safety risk analysis, the intersection formula or "multiplication rule" is used to compute the Annualized Failure Probability (AFP) associated with each PFM. The intersection formula uses conditional probabilities as its primary inputs, and since the probability estimates developed throughout a risk analysis are conditional probability estimates, the AFP (for a given PFM load range) often reduces to a simple product of the probabilities estimated by the team. However, due to the uncertainty inherent in subjective probability estimation, the conditional probability estimates are usually reported as ranges or distributions, rather than as single values. As a result, the probability estimates are themselves random variables, and the AFP obtained from the intersection formula is a product function of the n random variables that comprise the PFM.

In order to obtain an estimate of the mean of a product distribution, it would first be necessary to derive an equation for the PDF of the distribution of the product function. Unfortunately, this is difficult to accomplish unless the input random variables are jointly lognormal. Since this is not typically the case with subjective probability estimates, Monte Carlo simulation (see e.g. Ang and Tang 1984) is used instead to approximate the analytical product distribution. The process can be implemented through the use of specialized software, often available in the form of an add-on to Microsoft Excel, that allows input distributions to be entered in terms of key their parameters. For a given trial, the Monte Carlo simulation process consists of randomly sampling the input probability distributions associated with a PFM, and passing each set of sampled values through the intersection formula to obtain a trial value of AFP. Over thousands of trials, an AFP output distribution, whose mean can be calculated numerically, is built up.

The Monte Carlo generated AFP output distribution is often approximately log-normal (Figure I-8-9), consistent with the predictions of the Central Limit Theorem for product functions of random variables (see e.g. Ang and Tang 1984). However, this may not always be the case, especially if the event tree contains many “short” branches; when summation, rather than multiplication, is the dominant event tree operation, the output distribution tends toward a normal distribution shape. For consequence estimates, the shape of the output distribution can be log-normal (as in the case of an Annualized Life Loss distribution obtained from a simple event tree), normal (e.g. when the Annualized Life Loss is obtained as a sum across many branches), or neither (e.g. when a life loss distribution is back-calculated from the Annualized Life Loss and AFP).

Individual Risk

Individual risk deals with the risk to the most exposed individual. Individual risk guidelines are aimed at providing a level of protection even if the consequences are not large. The USACE uses the concept of individual risk by estimating the probability of exposure. Reclamation uses the annual probability of failure (AFP) to evaluate individual risk.

Informing Decisions

Different information can be presented on different charts depending on the type of decision required or the depth of information required to make an informed decision. Figure I-8-8 is an example of a scatter plot produced from the results of individual Monte Carlo trials for a dam where the slip rate of a local fault was unknown. In this case, a recommendation was made not to investigate the fault any further because its existence would not have a significant effect affect the final decision. The point clouds for both slip rate assumptions (red vs. blue) are are similarly shaped and plot over nearly the same area, suggesting that further refinement of the seismology would not change the decision.

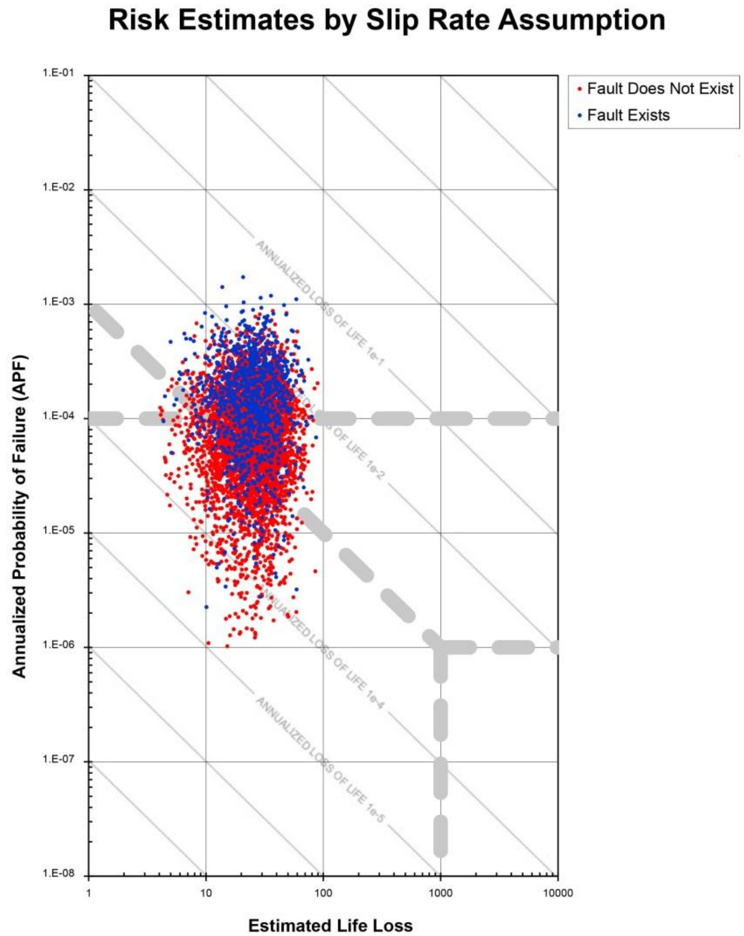

Figure I-8-8. f–N scatter plot Used to Make a Case Against Fault Investigation

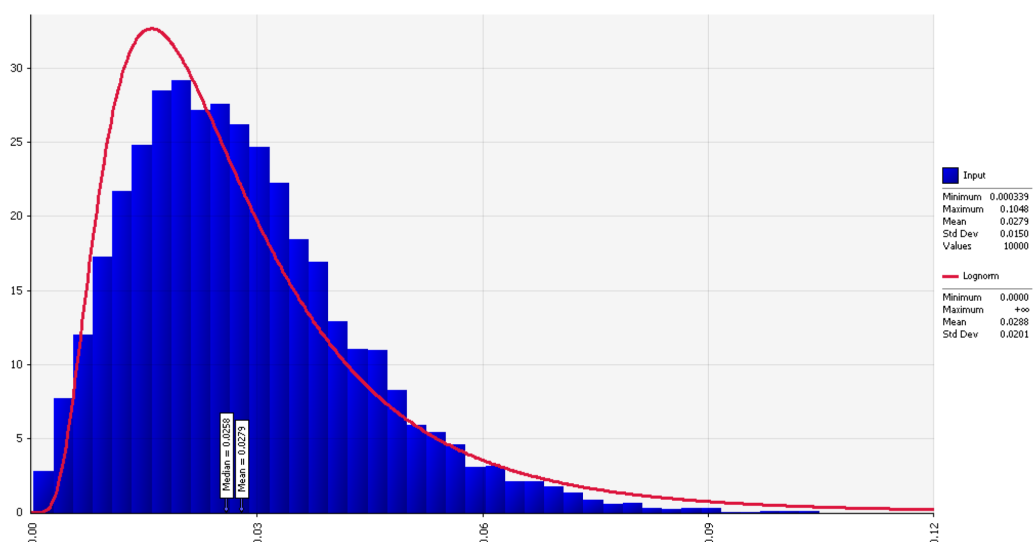

Figure I-8-9. Comparison of a Monte Carlo AFP output distribution (blue) and the PDF of an analytical lognormal distribution (red line)

Many types of charts can be generated depending on the information needing to be conveyed. Some of the common charts used as supplementary information are:

- Charts that show the contribution to risk from each potential failure mode
- Charts that show the contribution to risk from each load range, as in Figure I-8-10 for example)
- Charts that show effects of applying different fault assumptions or flood assumptions (showing the value of additional hazard studies)
- Charts that show the effects of different foundation assumptions (showing the value of additional geologic investigation)

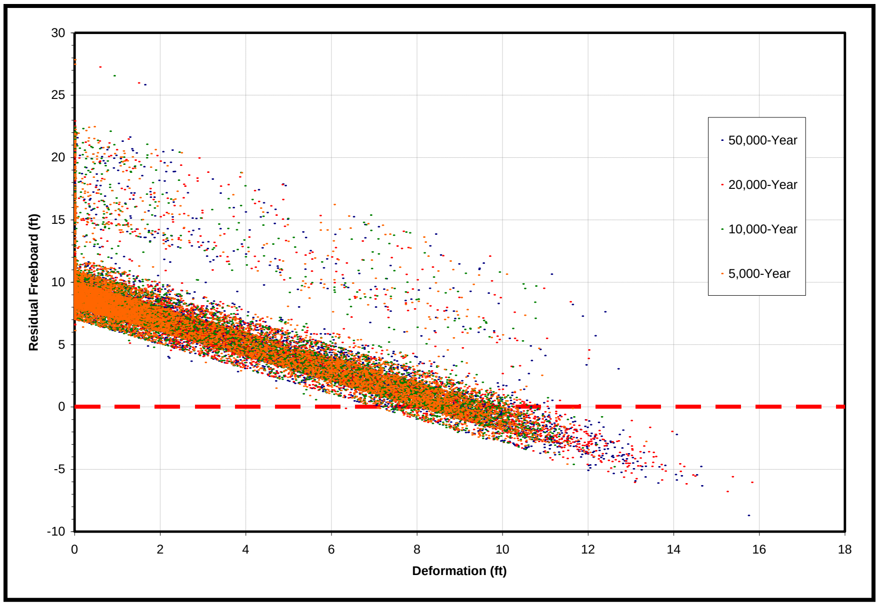

Figure I-8-10. Contributions to Risk by Load Range

Combining Risks

Table I-8-1 provides a summary of methods for combining risks (Vick, 1998).

Table I-8-1. Summary of Probability and Risk Aggregation Techniques

Type of System	Type of Component Failure	Methods for Combining Component Failure Probabilities	Methods for Combining Component Failure Risks
Single dam and PAR	separate loading conditions (flood, seismic)	account for correlations explicitly in event tree(s); add probabilities to determine total failure probability	add end-branch risks to determine expected loss of life from dam failure

<table>
<tr><th>Type of System</th><th>Type of Component Failure</th><th>Methods for Combining Component Failure Probabilities</th><th>Methods for Combining Component Failure Risks</th></tr>
<tr><td rowspan="2">Single dam and PAR</td><td rowspan="2">separate failure modes for any given loading condition</td><td rowspan="2">determine unimodal bounds; if bounds are narrow, add probabilities to determine failure probability over all such failure modes. Otherwise, retain probability bounds</td><td>similar consequences for each failure mode: determine bounds; add applicable end-branch risks if bounds are narrow. Otherwise, retain risk bounds.</td></tr>
<tr><td>different consequences for each failure mode: add applicable end-branch risks to determine risk over all such failure modes</td></tr>
<tr><td>Single dam and PAR</td><td>separate segments of different height and breach conditions (all loading conditions)</td><td>determine unimodal bounds; if bounds are narrow, add probabilities to determine failure probability over all such segments. Otherwise, retain bounds</td><td>add segment risks to determine risk over all such segments</td></tr>
<tr><td rowspan="2">Single dam and PAR</td><td rowspan="2">separate segments with different foundation conditions (static and seismic loading conditions only)</td><td>same geologic origin or process: determine unimodal bounds; if bounds are narrow, add probabilities to determine failure probability over all such segments. Otherwise, retain probability bounds</td><td>same geologic origin or process: determine bounds; if bounds are narrow, add risks to determine risk over all such segments. Otherwise, retain risk bounds</td></tr>
<tr><td>different geologic origin or process: add segment probabilities to determine failure probability over all such segments</td><td>different geologic origin or process: add segment risks to determine risk over all such segments</td></tr>
<tr><td>Multiple dams, same PAR</td><td colspan="3">Determine probabilities of failure separately for each dam. For total risk, treat each dam as for a segment of a single dam.</td></tr>
</table>

Summary f-N Table

The table shown in Figure I-8-11 is a from a spreadsheet used to portray results in decision documents. The purpose of this table and the f-N chart (Figure I-8-12) that accompanies it is to provide a standard presentation format to the decision-makers. The spreadsheet automatically:

- Calculates annualized life loss for each combination of probability of failure and consequences
- Sums annualized failure probabilities and annualized life loss

- Calculates the "average" life loss position of the total risk marker
- Calculates the total annual probability of failure associated with the facility

Instructions for the fN chart data table: ***Type only within the red borders*** *; Enter the name and type (static, hydro, seismic...) of the failure mode; Inlcude only the ten most critical failure modes; If there are less than ten failure modes, leave the extra "PFM name and type" fields blank; Enter dam name both on chart and to the right.*									Noname Dam
PFM name and type	**AFP Low**	**AFP mean**	**AFP high**	**Life Loss Low (> 0)**	**Life Loss Mean (≥ 1)**	**Life Loss High**	**Annualized Life Loss Low**	**Annualized Life Loss Mean**	**Annualized Life Loss High**
Static Failure Mode	3.26E-05	1.94E-04	5.40E-04	10	17	25	3.18E-04	3.30E-03	1.36E-02
Hydro Failure Mode	5.00E-08	5.36E-07	5.00E-06	11	72	175	5.50E-07	3.86E-05	8.75E-04
Seismic Failure Mode	2.26E-06	4.76E-06	9.50E-06	229	273	800	5.16E-04	1.30E-03	7.60E-03
							0.00E+00	0.00E+00	0.00E+00
							0.00E+00	0.00E+00	0.00E+00
							0.00E+00	0.00E+00	0.00E+00
							0.00E+00	0.00E+00	0.00E+00
							0.00E+00	0.00E+00	0.00E+00
							0.00E+00	0.00E+00	0.00E+00
							0.00E+00	0.00E+00	0.00E+00
Total Risk and uncertainty bounds	3.49E-05	**1.99E-04**	5.55E-04	(Life Loss weighted mean)	23.26	(Life Loss weighted mean)	8.34E-04	**4.64E-03**	2.21E-02

Figure I-8-11. Summary spreadsheet used to calculate total risks and help portray them for decision documents

The spreadsheet also automatically:

- Plots the estimated risks of each potential failure mode on the fN chart
- Plots the total estimated risk for the facility on the fN chart
- Plots the uncertainty bounds associated with the total risk

Rows can be added to accommodate additional potential failure modes, but including only the highest risk-contributing PFMs is sufficient for most decision documents. For each potential failure mode, the annualized life loss is calculated automatically in the cells to the right of the red line using the formula:

(Probability of Failure) *x* (Consequences) = Annualized Life Loss

The spreadsheet also requires the user to enter plausible ranges for the probabilities of failure and consequences. The lower and upper bounds of these ranges can be estimated using several different approaches, and their use helps communicate the uncertainty of the risk estimates. In each case, the meaning of and reason for the uncertainty bounds used should be explained in the report. Some of the more commonly used lower and upper bounds are:

- +/- 1 standard deviation
- The 5^{th} and 95^{th} percentiles

- The 1st and 99th percentiles
- The bounds of the entire Monte Carlo range (i.e., 0 and 100% confidence)
- +/- one order of magnitude from the mean estimate
- Range over which the mean estimate could reasonably change with additional information or more-refined analysis

The report should explain how and why a particular set of uncertainty bounds was selected. The reporting of uncertainty bounds is always recommended, but they do not by themselves make a compelling case for the risk estimates. Identifying the separate components of risk, discussing the meaning and importance of a particular load range, and describing the sensitivity of the results to a key probability estimate are examples of other things could be done to help build the case for a particular interpretation of risk.

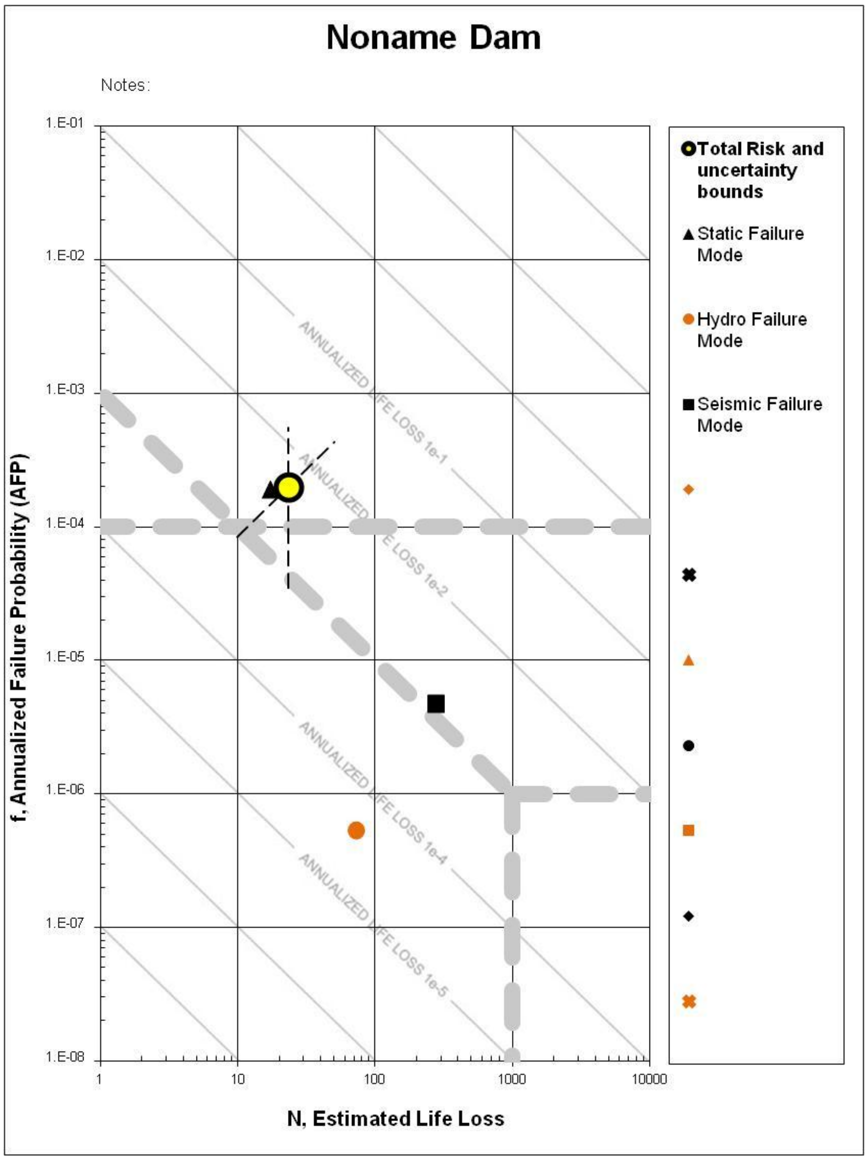

Figure I-8-12. Summary chart used to display risks in decision documents

Exercise

Given the following conditional failure probabilities (Table I-8-2) for five potential failure modes resulting from flood loading that does not overtop the dam, determine the total conditional system failure probability. How much error would be introduced if the probabilities were simply added?

Table I-8-2. Conditional Failure Probabilities

Potential Failure Mode	**Conditional Failure Probability**
Sliding at base of gravity spillway section	0.23
Seepage erosion through embankment wing above core	0.14
Radial gate arm buckling due to trunnion friction	0.06
Spillway wall overtopping erosion and headcutting	0.31
Stilling basin failure and undermining erosion	0.17

References

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Baecher, G. (2007). *Length Effects in Levee System Reliability*, Summary of Discussions from Louisville, KY meeting 18-20 July 2007.

Hill, Peter; Bowles, David; Jordan, Phillip; Nathan, Rory (2003). *Estimating Overall Risk of Dam Failure: Practical Considerations in Combining Failure Probabilities,* ANCOLD.

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