Movement of Finite Amplitude Sediment Accumulations

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Abstract: The movement of finite amplitude sediment accumulations is studied using a simple advection-diffusion relation derived from the sediment continuity equation and using some heuristic reasoning. The movement of a finite amplitude sediment accumulation is found to be strongly diffusive with a small advection component due to the increase in transport rate of the sediment accumulation relative to the transport rate of the original bed material. A semianalytical solution to the advection-diffusion equation is found and the equation is applied to two laboratory experiments. The equation is found to predict the general movement of finite amplitude sediment accumulations with a minimal number of parameters.

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Introduction

Sediment accumulations can occur in a river system as the result of landslides, debris flows, or man-made dams. Sediment accumulations are defined here as any sediment that is placed or deposited on a previously stable riverbed. To predict the impacts associated with the movement of such accumulations, a model of the system needs to be constructed. The complexity of the model applied to the system should be consistent with the data and resources available. Most often, the prediction of the movement of these accumulations is accomplished by using a one-dimensional hydraulic model coupled with a sediment transport model (MBH Software 2001; Stillwater Sciences 2002; Reclamation 2001). However, such models can be complex and require large amounts of input data. A simple method would be beneficial in providing initial estimates and for cases where complex models are not necessary. One such method was developed by Soni et al. (1980) to model aggradation due to overloading. In this model Soni et al. (1980) used the steady flow equations, a flow resistance relation, sediment continuity, and a sediment transport function to develop a diffusive wave model. Soni et al. (1980) then developed an analytical solution for the diffusive wave model for the case of a sudden and permanent increase in sediment concentration in a previously stable reach. Jain (1981) improved the analytical solution by using more appropriate boundary conditions. The model of Soni et al. (1980) and Jain (1981) was applicable to the case of a constant over loading of single sized sediment. Begin et al. (1980) applied a diffusive wave model to the upstream migration of a knickpoint. These models were applicable to single-sized sediment, but in the case of sediment accumulations, the accumulated sediment may be much finer than the original bed material. This paper describes the necessary additions to the diffusive wave model so that the effect of this change in sediment transport capacity is captured.

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Derivation of Model

The idealized problem is shown in Fig. 1 as a sediment accumulation placed on top of a river bed with uniform slope. The sediment accumulation is composed of uniformly sized sediment and the original bed is composed of uniformly sized sediment. It is assumed that the sediment size in the accumulation is the same or smaller than the sediment size of the original bed. Therefore, all the sediment present in the river system is assumed to fall into one of two classes, that of the sediment accumulation or that of the original bed material. The following equations apply to the system.

Water continuity for steady flow in one-dimension

$$h\frac{\partial U}{\partial x} + U\frac{\partial h}{\partial x} = 0 \tag{1}$$

Steady flow energy conservation in one-dimension

$$\frac{\partial z_b}{\partial x} + \frac{\partial h}{\partial x} + \frac{U}{g}\frac{\partial U}{\partial x} = -S_f \tag{2}$$

Flow resistance

$$U = C\sqrt{hS_f} \tag{3}$$

Sediment continuity in one-dimension

$$\frac{\partial z_b}{\partial t} + \frac{1}{1-\lambda}\left(\frac{\partial G_d}{\partial x} + \frac{\partial G_0}{\partial x}\right) = 0 \tag{4}$$

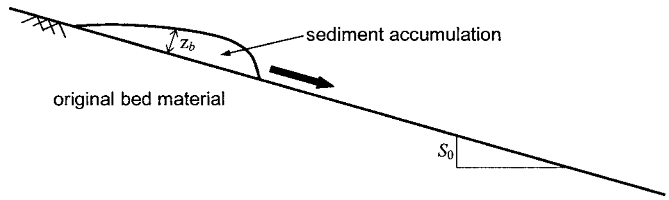

Fig. 1. Schematic of idealized problem

Sediment transport capacities

$$G_d = p_d a_d U^{b_d}, \quad G_0 = p_0 a_0 U^{b_0} \tag{5}$$

where a, b=sediment transport coefficients; g=acceleration of gravity; h=depth of flow; p_d=fraction of the accumulation sediment class in surface bed material; p_0=fraction of the original sediment class in surface bed material; t=time; x=stream-wise distance; z_b=depth of sediment above the original river bed; λ=porosity; C=Chezy coefficient of hydraulic friction; G_d=sediment transport rate per unit width of the sediment accumulation; G_0=sediment transport rate per unit width of the original bed material; U=cross-sectional average velocity of flow; and S_f=friction slope.

The sediment accumulation is subscripted with a d, whereas the original bed is subscripted with a 0. The sediment transport rate of a particular sediment type is a function of the fraction of the type in the bed, the properties of the sediment, and the hydraulic properties. In writing Eq. (5) it was assumed that the sediment transport rate of a particular sediment type is linearly related to the fraction of that size class in the bed and nonlinearly to the flow velocity. The parameter b is generally bounded between 4 and 6 (Chien and Wan, 1999).

Similar to Soni et al. (1980), steady uniform flow is assumed, so that the following can be written:

$$\frac{\partial z_b}{\partial x} = -S_f, \quad U\frac{\partial U}{\partial x} = -g\frac{\partial h}{\partial x} \tag{6}$$

The derivative of the velocity is therefore

$$\frac{\partial U}{\partial x} = \frac{\partial}{\partial x}(C\sqrt{hS_f}) = -\frac{U}{3S_f}\frac{\partial^2 z_b}{\partial x^2} \tag{7}$$

Using Eq. (5) and the requirement that $p_0+p_d=1$ (all sediment either bed or accumulation material), allows the sediment continuity equation (4) to be rewritten as

$$\frac{\partial z_b}{\partial t} + \frac{(G_d^* - G_0^*)}{(1-\lambda)}\frac{\partial p_d}{\partial x} - \frac{(b_d p_d G_d^* + b_0 p_0 G_0^*)}{3S_f(1-\lambda)}\frac{\partial^2 z_b}{\partial x^2} = 0 \tag{8}$$

where $G_d^* = a_d U^{b_d}$ and $G_0^* = a_0 U^{b_0}$.

The following assumptions are used to transform Eq. (8) into a simple advection-diffusion equation with constant coefficients. First, the velocity and friction slope (U and S_f) are assumed constant in space and time so that $S_f = S_0$, where S_0=original bed slope. It follows that G_d^* and G_0^* are then also constant. If the accumulation is too large with respect to the flow depth this assumption is severely violated. Further analytical and experimental work is necessary to determine the quantitative limits of applicability of this method. It may be that a separate model of the eroding sediment in the area upstream of the dam may be necessary for large accumulations.

Second, it is assumed that the fraction of the accumulation sediment class in the bed is linearly related to relative depth of the deposition, or specifically

$$p_d = \frac{z_b}{h_d} \tag{9}$$

where h_d=maximum depth of the sediment accumulation. The physical basis for this assumption is that as the depth of the deposition increases, more of the accumulation sediment class is present. It is certainly possible, however, that the transition between p_0 and p_d occurs as a step function. The model, therefore, must be validated against experimental data to assure that the assumption made in Eq. (9) does not cause excessive error.

The third and final assumption is that the coefficient of the last term in Eq. (8), the diffusive term, is constant. Using Eq. (9), the value of the diffusion coefficient at middepth in the accumulated sediment is

$$K_d = \frac{(b_d G_d^* + b_0 G_0^*)}{6S_0(1-\lambda)} \tag{10}$$

Comparison with experimental data shows that the previous assumptions produce reasonable results. Further work, however, should be performed to verify the applicability of these assumptions for more general cases. Using the above-mentioned assumptions Eq. (8) may be written as a familiar advection-diffusion equation

$$\frac{\partial z_b}{\partial t} + u_d\frac{\partial z_b}{\partial x} = K_d\frac{\partial^2 z_b}{\partial x^2} \tag{11}$$

where u_d=velocity of accumulation translation, defined as

$$u_d = \frac{(G_d^* - G_0^*)}{h_d(1-\lambda)} \tag{12}$$

and K_d=accumulation diffusion coefficient, given by Eq. (10).

Some discussion of the general behavior of the advection-diffusion equation is warranted. For advection-diffusion equations, it is possible to compute the nondimensional ratio of advection processes to diffusion processes. The nondimensional ratio is called the Péclet number (P) and can be defined as

$$\mathsf{P} \equiv \frac{u_d h_d}{K_d} \tag{13}$$

If it is assumed that the velocity component of the sediment transport relationship for the bed and accumulation sediment is constant, $b=b_d=b_0$, the following can be written

$$\mathsf{P} = \frac{6S_0(\Gamma - 1)}{b(\Gamma + 1)} \tag{14}$$

where $\Gamma = G_d^*/G_0^*$=ratio of the accumulation transport rate to the original bed material transport rate. It is useful to note the limiting behavior of P with respect to Γ. For $\Gamma \gg 1$, $\mathsf{P}=6S_0/b$ and for $\Gamma=1$, $\mathsf{P}=0$. Because b is constrained between 4 and 6 and S_0 is much smaller than 1 in natural streams, the Péclet number of the sediment accumulation is bounded and its maximum value is constrained by the river bed slope. Therefore, the diffusion processes dominate the advection processes for sediment accumulations.

Eq. (11) is amenable to analytical solution if appropriate initial and boundary conditions are defined. It is assumed that sediment is not allowed to travel upstream of the sediment accumulation, which starts at $x=0$. Therefore, a reflective boundary is placed at the upstream end, meaning that the first derivative of z_b with respect to x is set to zero at $x=0$. The initial condition is the depth of the sediment accumulation above the natural bed. The solution to Eq. (11) with the initial depth of the sediment accumulation represented by z_1 is

$$z_b(x,\tau) = \int_0^{\infty} \frac{z_{1i}}{\sqrt{4\pi K_d t}} \left[\exp\left(\frac{-(x-u_d t-\xi)^2}{4K_d t} \right) + \exp\left(\frac{-(x+u_d t+\xi)^4}{4K_d t} \right) \right] d\xi \tag{15}$$

The nondimensional form of Eq. (15) can be written as

$$\eta(\chi,\tau) = \int_0^{\infty} \frac{\eta_{1i}}{\sqrt{4\pi\tau}} \left[\exp\left(\frac{-(\chi-\tau\mathsf{P}-\xi)^2}{4\tau} \right) + \exp\left(\frac{-(\chi+\tau\mathsf{P}+\xi)^2}{4\tau} \right) \right] d\xi \tag{16}$$

where $\eta = z_b/h_d$; $\tau = tK_d/h_d^2$; and $\chi = x/h_d$.

The second term in Eqs. (15) and (16) in the integral is due to the reflection of the boundary at $x=0$, where it is assumed that the sediment deposit begins at $x=0$. The integral in Eq. (15) can be numerically approximated by dividing the stream into N segments and assuming a constant depth of the sediment accumulation over each segment

$$z_b(x,t) = \sum_{i=1}^{N-1} \frac{(z_{1i}+z_{1i+1})}{4} \left[\mathrm{erf}\left(\frac{x-u_d t-x_i}{2\sqrt{K_d t}} \right) - \mathrm{erf}\left(\frac{x-u_d t-x_{i+1}}{2\sqrt{K_d t}} \right) - \mathrm{erf}\left(\frac{x+u_d t+x_i}{2\sqrt{K_d t}} \right) + \mathrm{erf}\left(\frac{x+u_d t+x_{i+1}}{2\sqrt{K_d t}} \right) \right] \tag{17}$$

where "erf" denotes the error function. Application requires some trial and error to determine appropriate distances between stream segments. A general consideration is that there should be enough segments so that the total volumes of the initial deposit and resulting bed profiles are accurately represented.

The error of this method is potentially large because of the simplifying assumptions made during development. A partial list of assumptions follows:

- Assumes accumulation depth is not large compared to flow depth;
- Assumes a rectangular cross section;
- Assumes constant bed slope;
- Assumes the flow rate, sediment transport rate, and roughness are constant in space and time;
- Is not applicable upstream of the sediment accumulation;
- Assumes accumulation can be represented by a single size class; and
- Assumes accumulation travels as bed load. Ignores sediment sizes that travel as pure suspended load.

The analytical solution to Eq. (11) holds promise as a simple assessment tool to determine impacts associated with aggradation downstream of sediment accumulations. The solution, Eq. (15), requires a minimal number of input parameters and requires a fraction of the time required to complete a more complicated numerical model. The parameters that need to be estimated to use the model are listed in Table 1. All the parameters except for b_d are physical quantities that can be measured. The parameter b_d is the exponent in the sediment transport relation and based on results from several researchers is generally bounded between 4 and 6 (Chien and Wan 1999). The model requires the estimation of the sediment transport rate of the sediment accumulation, G_d^*, or equivalently, the estimation of a and b in the sediment transport relation.

Table 1. Description of Parameters Necessary to Use Proposed Model

Parameter	Dimension	Range of values or method of obtaining value
S_0	—	Average natural stream slope, measured from topographic maps
G_d^*	L^2/T	Transport capacity of sediment accumulation in units of volume per unit width
G_0^*	L^2/T	Transport capacity of bed material in units of volume per unit width
λ	—	Sediment porosity, usually between 0.3 and 0.5
b_d	—	Exponent in sediment transport relation, usually between 4 and 6
h_d	L	Maximum depth of sediment accumulation; estimated from field surveys

Extensions to Nonuniform Flow

Regarding the assumption of uniform flow, Ribbernik and Van Der Sande (1985) and Gill (1988) investigated the effect of including terms accounting for nonuniform flow effects. Ribbernik and Van Der Sande defined the nondimensional time parameter, $\tilde{t}$

$$\tilde{t} = \frac{6bG_d^* S_0}{(1-\mathsf{F}_0^2)^2 h_0^2} t \tag{18}$$

where F_0 and h_0=Froude number and depth, respectively, of the flow over the undisturbed bed. They found that if $\tilde{t}>25$ the aggradation equations derived assuming uniform flow are valid. This limits the accuracy of the method for areas close to the accumulation and it is suggested that Eq. (15) be applied only when $\tilde{t}>25$ is true. If deposition results are desired where $\tilde{t}\leq 25$, terms accounting for nonuniform flow can be included to give

$$\frac{\partial z_b}{\partial t} + u_d \frac{\partial z_b}{\partial x} + \frac{K_d}{c_b} \frac{\partial^2 z_b}{\partial x \partial t} = K_d \frac{\partial^2 z_b}{\partial x^2} \tag{19}$$

where

$$c_b = \frac{U_0}{h_0(1-\mathsf{F}_0^2)} \tag{20}$$

where U_0=flow velocity over the undisturbed bed. Eq. (19) is not analyzed in this paper and only Eq. (15) is used to predict the movement of sediment accumulations. It should be recognized that for $\mathsf{F}=1$, Eq. (19) is equivalent to Eq. (15). Eq. (19) generally requires a numerical solution, though complex analytical solutions are possible for simple boundary conditions (Gill 1988). All model results in this paper are based on Eq. (15).

Test of Formulation Using Data from St. Anthony Falls Laboratory Experiments

Results from laboratory tests at St. Anthony Falls Laboratory (SAFL) were reported in Cui et al. (2006). In order to perform the experiments the following procedure was adopted: (1) the flume was allowed to reach a prepulse mobile-bed equilibrium; (2) The flow was temporarily halted to allow for a pulse of sediment to be installed toward the upstream end of the flume; and (3) The flow

Table 2. Problem Parameters for Simulations of Runs 2, 3, and 4b of SAFL Experiments

Parameter	Value for Run 2	Value for Run 3	Value for Run 4b
S_0	0.0108	0.0108	0.0108
G_d^* (m^2/s)	5.0×10^{-5}	2.3×10^{-6}	1.27×10^{-4}
G_0^* (m^2/s)	5.7×10^{-7}	5.7×10^{-7}	5.7×10^{-7}
λ	0.4	0.4	0.4
b_d	5	5	5
h_d (m)	0.04	0.04	0.044

was then recommenced, and the flume was allowed to equilibrate over time as the sediment pulse deformed. Five runs, Runs 1, 2, 3, 4a, and 4b were conducted. Runs 2, 3, and 4b were selected for comparison because they represent a range of sediment sizes in the sediment accumulation.

In these experiments, a mobile-bed equilibrium was created by feeding in a sediment mix that was half gravel and half sand, so that the median feed size was 2 mm and the maximum size was 8 mm. The flume was 0.5 m wide and 40 m long with an approximate slope of 0.0108. The water flow rate was 9 L/s (0.009 m^3/s) and the sediment feed rate was 45 g/min for all runs, which in terms of volume transport rate per unit width is 5.7×10^{-7} m^2/s. The undisturbed flow depth was approximately 0.0325 m and the mean flow velocity was 0.55 m/s, giving a Froude number of 0.98. It should be noted that the critical nondimensional time parameter defined by Eq. (18) is 16, 200, and 6 s for Runs 2, 3, and 4b, respectively. The applicability condition stated in Eq. (18) is therefore satisfied for all the comparison cases presented (see Table 2.).

A sediment accumulation was placed by hand and had an approximate thickness of 4 cm. In Run 2, the sediment deposit had a similar composition to the incoming sediment load. The sediment deposit in Run 3 was of similar composition to the original bed material. In Run 4b, the sediment was much finer than the incoming sediment feed. The sediment transport rates were reported in Cui et al. (2006), but because the sediment samplers were not calibrated, Cui et al. recommended that the transport rates only be interpreted as order of magnitude estimates. The maximum transport rate reported in Run 2 was 2,000 g/min, which gives a volume transport per unit width of 2.6×10^{-5} m^2/s. The maximum transport rate of Run 3 was approximately 800 g/min (or 1.0×10^{-5} m^2/s on a per unit width basis) immediately after the flow began to erode the placed sediment. At 1 h, it had decreased to approximately 100 g/min (or 1.3×10^{-6} m^2/s) near the placed gravel. For Run 4, the maximum transport rate was over 1,000 g/min (1.3×10^{-5} m^2/s). It is expected that the transport rate in Run 4b should have been significantly greater than in Run 2 because the placed sediment in Run 4b was significantly finer than for Run 2.

The initial conditions for model comparison were taken from the measured experimental accumulation profiles. Model transport rates were also taken from the measured experimental values when possible. It is difficult, however, to measure the sediment transport rate of the accumulation and the sediment transport rates can vary significantly in time and space. Initially, the sediment transport rate is quite high at the face of the accumulation because of the steep front at its face. The front quickly dissipates and the sediment transport rate decreases, but can remain significantly higher than that of the original bed material. For the purposes of this paper, the transport rate of the accumulation (G_d^*) was adjusted to obtain the best fit with the experimental data. The final

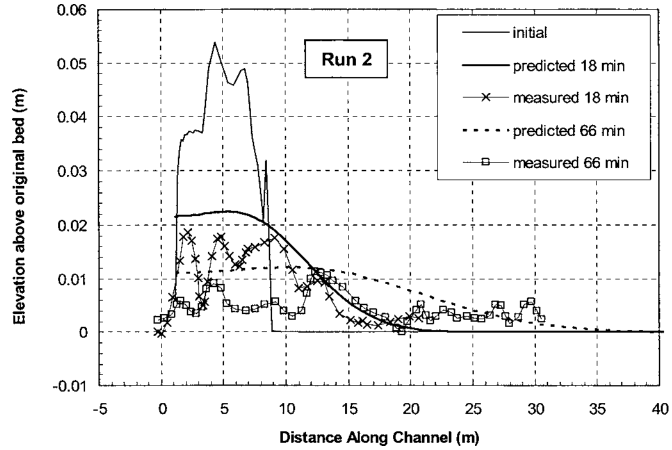

Fig. 2. Comparison between Eq. (17) and Run 2 of the experiments at SAFL

adjusted transport rates of the accumulation used within the model were considered reasonable and within the range of measured values.

A comparison between the experimentally measured and model results using Eq. (17) for Run 2 is shown in Fig. 2. Approximately 50% of the accumulation was gravel and 50% was sand. Based on analysis of the fall velocity to shear velocity ratio, approximately 15% of the load would travel as suspended load. This suspended fraction is ignored and the initial accumulation thicknesses taken from experimental data were reduced by 15%. The transport rate of the accumulation was set equal to 5×10^{-5} m^2/s for Run 2, which was twice the measured value. However, there is a large uncertainty associated with the measured transport rates and they were likely underestimated. The Péclet number computed using Eq. (14) was 0.012.

On average, the predicted deposition downstream of the initial deposit predicts the general behavior of the sediment accumulation of Run 2. Most of the discrepancy between the predicted and measured results can be explained by two phenomena: (1) irregularities in the bed; and (2) alternate bar formations present in the experiment.

Model comparison with the experimentally measured results of Run 3 is shown in Fig. 3. The volume transport rate per unit

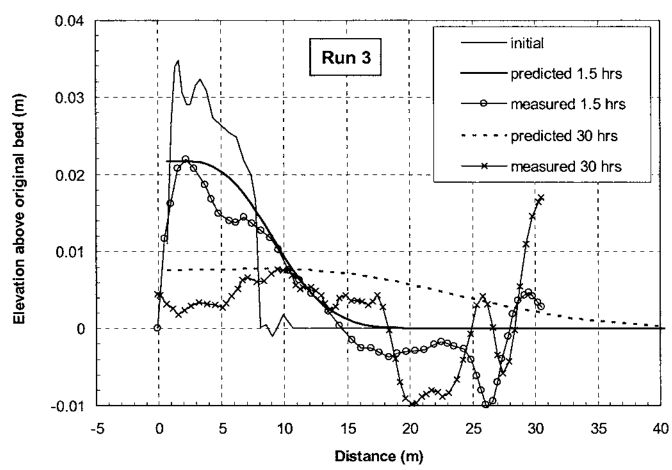

Fig. 3. Comparison between Eq. (17) and Run 3 of the experiments at SAFL

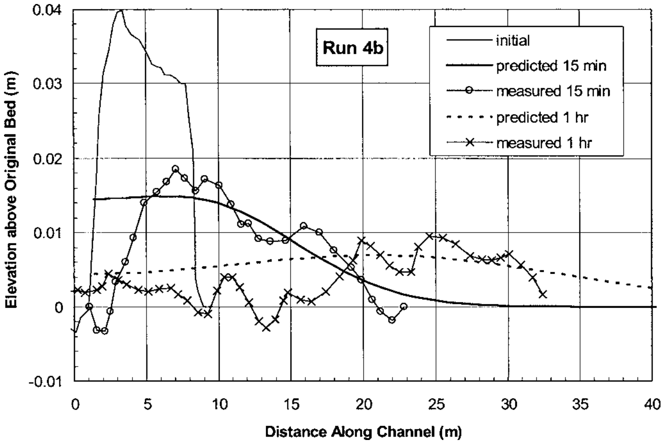

Fig. 4. Comparison between Eq. (17) and Run 4b of the experiments at SAFL

width of the accumulation was set to 2.3×10^{-6} m^2/s, which is between the measured values at the front of the deposit when erosion was initiated and 1 hour later. Similar to Run 2, approximately 10% of the initial deposit was assumed to travel as suspended load. The model predicts the height and location of the peak deposition fairly well. The model seems to have a tendency, however, to overpredict the diffusion of the sediment front.

A comparison between the measured and simulated results for Run 4b is shown in Fig. 4. In this run, the accumulation was almost entirely sand. The volume transport rate per unit width of the accumulation was set equal to 1.27×10^{-4} m^2/s for Run 4. A comparison between the measured maximum deposit and the predicted maximum deposit is shown in Fig. 5. As stated earlier, the model does not capture the variability of the deposition, but the model accurately predicts the average maximum deposition.

Conclusions

Eq. (17), which predicts the movement of sediment accumulations, was derived so that it can be applied to sediment accumulations of finite amplitude with a minimal number of input parameters. The equation is recommended for use as a first estimate of depositional impacts downstream of finite amplitude sediment accumulations. The parameters in the model are few and they can be easily measured or are adequately bounded. The model shows that the diffusion process dominates the advection process.

Eq. (17) was compared against laboratory data and explains the movement of finite amplitude sediment accumulations in laboratory channels. It should be noted, however, that the errors associated with the model are potentially large if applied to field situations because of the simplifying assumptions made. In particular, for accumulations much larger than the flow depth, the assumptions inherit in the derivation may cause significant discrepancy with actual data. A separate model of the eroding sediment upstream of the dam may be necessary for large accumulations.

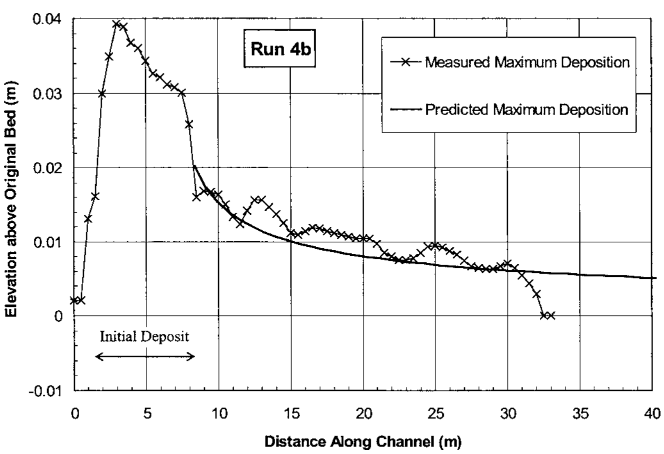

Fig. 5. Comparison between predicted maximum deposition and measured maximum deposition for the duration of Run 4b of SAFL experiments

Notation

The following symbols are used in this paper:

a = sediment transport coefficient;
b = sediment transport coefficient;
C = Chezy's resistance coefficient;
G = sediment transport rate in units of volume per unit width;
g = acceleration of gravity;
h = flow depth;
h_d = maximum depth of sediment accumulation;
K_d = diffusion coefficient of sediment accumulation;
P = Péclet number of sediment accumulation;
p = fraction of sediment in bed;
x = streamwise distance;
S_f = friction slope;
S_0 = original bed slope;
t = time since beginning of motion of sediment accumulation;
U = flow velocity;
u_d = velocity of sediment accumulation;
x = streamwise distance;
z_b = depth of deposition;
z_1 = initial depth of sediment accumulation; and
λ = sediment porosity.

Subscripts

d = refers to sediment accumulation;
m = index indicating size fraction; and
0 = refers to original bed material.

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