Coupling a Two-Dimensional Model with a Deterministic Bank Stability Model

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Abstract

Stream bank erosion can be an important form of channel adjustment in unstable alluvial environments and hence should be accounted for in geomorphic studies, river restoration, dam removal, and channel maintenance projects. Recently, one-dimensional and two-dimensional simulation models have become useful tools for predicting channel responses; but most either ignore bank failure mechanisms or implement only simple *ad hoc* methods. In this study, a two-dimensional model (SRH-2D) is coupled with a deterministic bank stability and toe erosion model (BSTEM) to predict channel adjustment and planform development. Herein, the proposed coupling approach is described, along with numerical aspects of the procedures. For test and verification purposes, the coupled model is used to predict bank retreat of Goodwin Creek in Mississippi. A comparison of the model results with the measured data is presented and discussed.

Introduction

Stream bank erosion is a natural geomorphic process occurring in all alluvial channels. It is an important mechanism by which a channel adjusts its size, shape, and slope to convey the water and sediment supplied to it. In recent years, both one-dimensional (1D) and two-dimensional (2D) numerical models have used for predicting channel responses, but most either ignore bank failure mechanisms or implement only simple *ad hoc* methods. Not accounting for stream bank failures in mobile-bed simulations may result in biased or erroneous predictions of degradation, aggradation, equilibrium channel geometry, and sediment loadings.

In this study, a 2D mobile bed model, SRH-2D, is coupled with a deterministic bank stability and toe erosion model, BSTEM, to improve predictions of channel adjustment and planform development. SRH-2D, developed at U.S. Bureau of Reclamation (Lai, 2008), is a flexible, accurate, and robust model; it has been applied to many river engineering projects (e.g., Lai and Randle, 2007; Lai and Greimann, 2008, 2010; Lai, 2010; 2011; Lai et al., 2011a). SRH-2D has mobile bed capabilities to predict stream bed change and track multi-size, non-equilibrium sediment transport for suspended, mixed, or bed load for both cohesive and non-cohesive materials. The effects of gravity and secondary flows are accounted for by displacing the direction of the sediment transport vector from that of the local depth-averaged flow vector. BSTEM, developed at USDA Agricultural Research Service (Simon et al., 1999, 2000, 2006, 2011; Pollen-Bankhead et al., 2009), has similarly been used successfully in diverse alluvial environments in both static and dynamic modes, the latter using daily stage data for simulations of up to 100 years. As incorporated in this study, BSTEM utilizes the depth-averaged boundary shear stress provided by SRH-2D and distributes it vertically along the submerged part of the bank face to provide enhanced resolution of the shear stress acting on the bank. The geotechnical sub-model evaluates the force-equilibrium factor of safety of either a planar or cantilever shear failure in a layered stream bank. The resisting forces are comprised of the sum of the cohesive and frictional strengths of the soil, forces due to positive and negative pore-water pressures, and the hydrostatic confining force afforded by water in the channel. The driving forces comprise the weight of the failure block reduced by a component of the hydrostatic confining force. A global search algorithm is employed to search for the minimum factor of safety.

Description of the Coupled 2D and Bank Erosion Model

Two processes are considered: fluvial processes acting on the bank surface and mass failure processes acting on the bank mass. The fluvial hydraulic process involves both shear stress and erosion prediction. Fluvial erosion is often a precursor to gravitational mass failures; it is also responsible for transport of sediment deposits produced by bank failures. There are three types of hydraulically-induced erosion: channel bed degradation (vertical), basal erosion (bank undercutting), and basal cleanout. In this study, bed degradation is predicted with SRH-2D. Basal erosion refers to the fluvial entrainment of bank material by flow-induced forces acting on wetted bank surfaces. A combination of SRH-2D and BSTEM methods is used for this prediction. Basal cleanout of failed bank material is a complex process that has often been ignored by previous studies. The process is included in this study and will be discussed later. Mass failure occurs when basal erosion and/or bed degradation proceeds to a point that gravitational forces exceed the shear strength of the bank material. The mass failure algorithm of BSTEM, which is applicable to multi-layer cohesive banks, is adopted.

<u>Hydraulics and Channel Bed Deformation</u>: Flow hydraulics and vertical channel-bed change are predicted with SRH-2D. The relevant governing equations, numerical methods, and sample applications may be found in Lai (2008, 2010), Lai and Greimann (2010) and Lai et al. (2011).

<u>Basal Erosion</u>: Users designate a “toe point” on each modeled bank profile and the location of this point is tracked throughout the simulation. The bank is landward of this point and the bank-toe zone and channel bed are streamward of this point. Each segment of the cross-section may be of arbitrary slope. Erosion of the toe point is computed first; it consists of a vertical movement (ω_V) and a lateral movement (ω_L). The vertical component is computed with SRH-2D, and the lateral component is computed with the following equation (Darby et al., 2002):

$$\omega_L = k(\tau - \tau_c) \quad (1)$$

where ω_L is lateral erosion rate (ms^{-1}), k is erodibility ($m^3N^{-1}s^{-1}$), τ is bed shear stress (Pa), and τ_c is critical shear stress (Pa). For granular material, critical shear stress may be estimated with the Shields diagram and k may be calibrated with available data or from existing relations (Simon et al., 2010; 2011). For cohesive banks, shear stress and erodibility may be obtained through *in-situ* jet tests (Hanson and Simon, 2001). A submerged jet test device has been developed by Hanson (1990) to conduct soil erodibility tests *in situ*. Utilizing this device, Hanson and Simon (2001) developed the following relation between τ_c and k for cohesive silts, silt-clays, and clays:

$$k = 2 \times 10^{-7} \tau_c^{-0.5} \tag{2}$$

and it was updated using more than 800 tests to (Simon et al., 2010; 2011):

$$k = 1.62 \times 10^{-6} \tau_c^{-0.838} \tag{3}$$

The use of such a regression relation reduces the two parameters of equation (1) to one.

Erosion of the wetted bank face is also computed with equation (1). In the present approach, the boundary shear stress exerted by the flow on each bank node, *i*, is estimated by dividing the flow area at a cross-section into segments that are affected only by the roughness of the bank or the bed and then further subdividing to determine the flow area affected by the roughness on each node. This procedure is similar to Langendoen and Simon (2008).

<u>Basal Cleanout</u>: Bank material added to the channel following geotechnical failure is generally not transported immediately by the water in the channel. Instead, it is often deposited near the bank toe and provides temporary "protection" from further bank erosion. To account for basal cleanout, a simple approach is adopted in this study. The failed materials are placed in an invisible "tank" that is available for hydraulic erosion (*cf*. Langendoen, 2000). That is, the 2D flow and sediment transport modeling is not directly impacted by the existence of the failed materials. When the bank erosion module is executed, the material in the tank is preferentially eroded and erosion of the bank face only occurs when the material in the tank is completely removed. This approach explicitly accounts for the protection afforded by the failed bank materials. However, the erosion rate of the failed materials is assumed to be the same as that of the source.

<u>Mechanistic Bank Geotechnical Failure Model</u>: The geotechnical failure model is based on that implemented in BSTEM. The basic theory was initially described by Simon et al. (2000). Later, improvements were proposed by Langendoen and Simon (2008). It is assumed that a bank consists of either a single layer or multiple layers with each layer having its own hydraulic resistance and geotechnical properties. In addition to the bank geometry, other inputs include (a) layer elevations; (b) ground water level; and (c) material parameters for each bank layer such as porosity, saturated unit weight, friction angle, ϕ^b-parameter, effective cohesion, and sediment composition. Geotechnical analysis is carried out using the horizontal layer approach for steep failure planes and cantilever failures (Simon et al., 2000) and the vertical slice approach with tension cracks for shallower failure planes (Langendoen and Simon, 2008). Both methods account for the strength of multiple soil layers, the effects of both positive and negative pore-water pressure, and the confining pressure due to water in the channel abutting the bank face. At each failure plane base elevation, a modified version of the Brent function is used to first bracket and then isolate the minimum factor of safety. The global minimum factor of safety is then selected from all the local minima. Further details of the factor of safety equations and their numerical implementation are omitted due to space limitations and the interested reader may refer to Simon et al. (2011) and Langendoen and Simon (2008), respectively.

<u>Treatment of Bank Retreat</u>: SRH-2D is modified to accommodate bank movement. Two methods may be used: the fixed mesh method and the moving mesh method. With the first, a fixed mesh

is used when the bank retreats. This has the advantage that no additional mesh movement and associated interpolations are needed. However, the banklines do not then align with the mesh faces, leading to a significant challenge: the accuracy representing the bank retreat is severely degraded unless a very fine mesh is used. With the moving mesh approach, the mesh lines are aligned with the bankline initially, and this alignment is enforced continuously throughout the bank retreat process. The advantage of the moving mesh approach is that bank retreat may be computed and represented more accurately. The disadvantage is that the mesh has to be "moved" every time the bank retreats. As a result, the mesh needs to be readjusted and variables need to be updated or interpolated. In this study, the moving mesh approach if Lai and Przekwas (1994) is adopted as its advantages outweigh its drawbacks. An advantage of the adopted moving mesh approach is that all flow and sediment variables represented by the mesh cell are automatically computed and there is no need for additional interpolation. The only variable that needs to be interpolated after mesh movement is the bed topography.

Coupling of SRH-2D and BSTEM: Separate input files are used for the coupled SRH-2D and BSTEM modeling. The input for SRH-2D is the same as the non bank-erosion modeling case. An initial solution domain and a 2D mesh are developed that represents the initial channel bathymetry and topography, along with the regular boundary conditions specified. With bank erosion modeling, a number of "bank zones" may be created and specified in the mesh; they represent the number of banks on which bank erosion is to be performed with BSTEM. As an illustration, Figure 1 shows a stream section with the right bank being modeled for bank retreat using the coupled model. The right bank is divided into two bank zones and if retreat is predicted by BSTEM all mesh points in the bank zones are moved. Two mesh lines are defined for each bank zone: one is along the top of the bank toe while another represents the top of the bank.

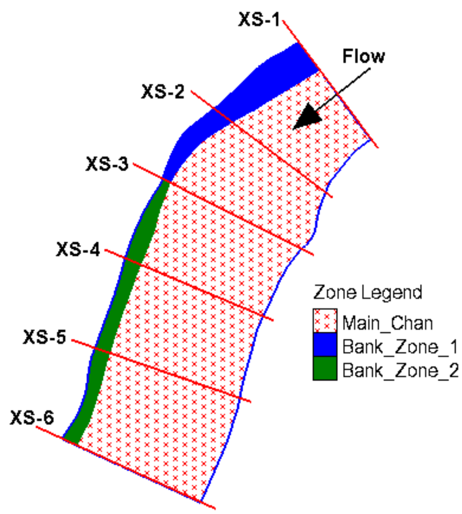


Figure 1. Illustration of a stream with the right bank being modeled by BSTEM

A separate input file is prepared for running BSTEM. An arbitrarily selected number of bank profiles (or cross-sections) may be used to simulate bank retreat. Take the example in Figure 1, six cross-sections are selected to model bank retreat. Cross-sections 1 through 3 belong to bank zone 1 while cross-sections 3 through 6 belong to bank zone 2. The input file provides the following information for each bank profile: the geometry (lateral distance versus bank elevation), ground water elevation, bank stratigraphy, erosion and geotechnical properties, and sediment composition. The only requirement between the bank geometry and the 2D mesh is that the toe and top points in the bank profile coincide with the toe and top nodes in the 2D mesh. The bank profile between the top of the toe and the top of the bank does not have to be the same between BSTEM profiles and the 2D mesh.

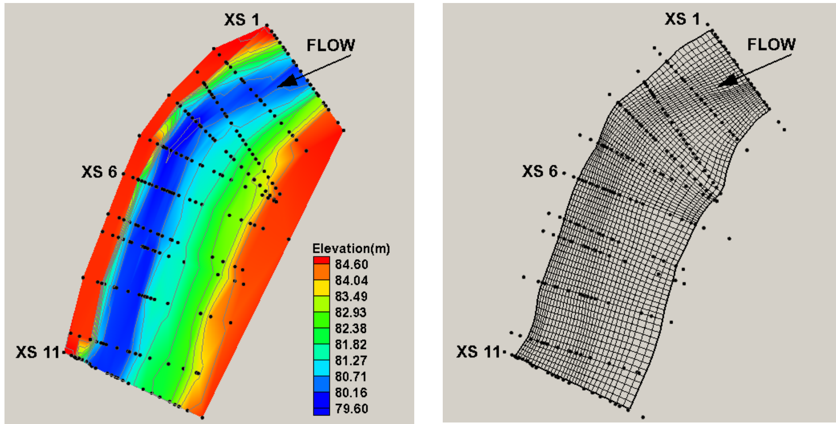
The time step used for BSTEM is independent of the time step for SRH-2D. In general, the time step of BSTEM is much larger than that of SRH-2D since the time scale of bank retreat is large. In a typical modeling cycle, flow and sediment transport in the main channel are simulated first with SRH-2D, assuming the bank is fixed, until BSTEM is activated. The shear stress and water elevation at the toe point are obtained through averaging SRH-2D results over the time step of BSTEM. These toe variables are used by BSTEM to predict fluvial erosion and mass failure. The estimated distances retreated by the toe and top points are then passed to SRH-2D to move the mesh. The materials removed from the bank during the fluvial erosion process are added to the stream (by particle-size class) for immediate transport by the flow and sediment transport modules of SRH-2D, while the materials due to mass failure are added to the “tank” for future basal erosion.

Results and Discussion of the Coupled Modeling

The coupled SRH-2D and BSTEM model presented above has gone through several steps of testing and verification. First, a FORTRAN version of a portion of the original BSTEM model (in Visual Basic form) was developed and tested. Second, the converted BSTEM model was modified to simulate erosion of a bank profile in a dynamic mode. One cross-section at Goodwin Creek, Mississippi, was used for the test using the time series results produced by Langendoen and Simon (2008) were used. Third, the coupled SRH-2D and BSTEM model was used to simulate the same cross-section of Goodwin Creek, but using the results predicted by SRH-2D. The purpose is to ensure that the coupling procedures and their numerical implementations work properly. Finally, the coupled model was applied to all measured cross-sections at a bend on Goodwin Creek. Evaluation of the model performance was made by comparing model results with measured data. The results of the first three steps can be found in Lai et al. (2011b). Selected results of the final step are presented herein.

The coupled model was applied to a bend on Goodwin Creek, Mississippi. A long term stream bank failure study has been carried out at the site since 1996 (Simon et al., 2000). Bank retreat data are available over a number of years. These data provides a good test case to verify a bank erosion model for multiple-layer cohesive banks. This site has been modeled previously with an earlier version of BSTEM (Simon et al., 2000), and with CONCEPTS, a 1D flow and sediment transport model coupled to a bank erosion model similar to BSTEM (Langendoen and Simon, 2008).

The initial topography of the bend is based on eleven surveyed cross-sections in March, 1996 (Figure 2a) which serves as the initial condition. The initial mesh is developed (Figure 2b) to represent the channel topography. Bank profiles on the right bank of all cross-sections are selected for BSTEM bank retreat modeling. Coupled simulations were carried out for five years, from March, 1996 to March, 2001. Since the channel bed is relatively stable with negligible vertical change during the study period, no vertical bed change is modeled with SRH-2D. Inputs for SRH-2D are as follows: at the upstream boundary (XS 1), time series discharge data obtained from the flow record is imposed; at the downstream boundary (XS 11), a stage-discharge rating curve developed by Simon et al. (2000) and Langendoen and Simon (2008) is used to obtain the water stage; the Manning's roughness coefficient is 0.032 for the bend, based on an analysis of the stage records at the upstream and downstream boundaries (Langendoen and Simon, 2008).



(a) Topography (b) Mesh for SRH-2D

Figure 2. Initial topography in March, 1996 and the 2D mesh used.

Inputs for BSTEM are as follows: at XS 1, the bank consists of a uniform cohesive layer with porosity = 0.38, saturated unit weight = 19.4 kN m^{-3}, friction angle = 28.6°, ϕ^b = 10.4°, and effective cohesion = 4.5 kPa; at XS 6, the bank consists of multi-layer cohesive materials. The initial bank profile in March, 1996 and its layering are shown in Figure 3. Relevant measured bank properties used for BSTEM modeling are listed in Table 1. The remainder of the bank profiles (XS 2 through XS 11) was assigned the same properties as XS 6. In addition, all banks were assigned a constant groundwater elevation of 82.0 meters. This elevation represents an average over the simulation period, and no variations due to precipitation were considered. Most of the above inputs are the same as the study of Langendoen and Simon (2008). No changes of the above parameters were attempted for calibration purposes.

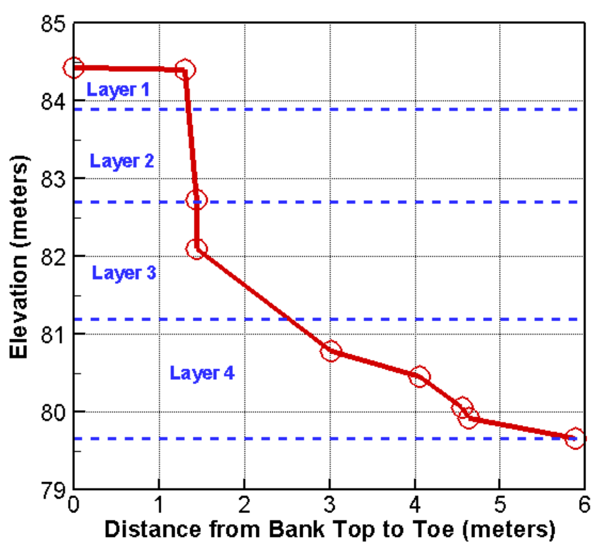


Figure 3. Bank profile of XS 6 and locations of four layers.

Table 1. Bank stratigraphy and geotechnical properties at XS 6 used for simulation.

Layer	Depth below surface	Porosity	Saturated Unit Weight (kN m^{-3})	Friction Angle (°)	Angle ϕ^b (°)	Cohesion (kPa)
1	0–0.5	.489	16.9	33.1	17.0	1.41
2	0.5-1.7	.489	19.3	28.1	10.2	2.70
3	1.7-3.2	.380	19.9	27.0	17.0	6.30
4	>3.2	.320	21.0	35.0	17.0	1.0

Two more model parameters need to be determined: critical shear stress and erodibility for basal erosion in equation (1). Different critical shear stress values were used for different cross-sections in the study of Langendoen and Simon (2008). For example, the critical shear stress (in Pa) was 8, 8, 4, 4, 2, 1, 1, 1, 4, 8, 8, respectively, for XS 1 through XS 11, and the erodibility was computed by $1\text{x}10^{-7}\ \tau_c^{-0.5}$. We recommend to select erodibility as the primary calibration parameter and critical shear stress as a secondary parameter. The critical shear stress may be estimated first based on either survey data or other reliable sources. In this study, a critical shear stress of 2.0 Pa was used for all banks based on an average of the measured data reported by Langendoen and Simon (2008). The erodibility coefficient was then calibrated; the final values are listed in Table 2 for all cross-sections. Of note is that much less variation of the erodibility at different banks was obtained with this coupled modeling than the study of Langendoen and Simon (2008). This shows that the use of a 2D model for hydraulics and basal erosion may be more advantageous than a 1D model. Inaccurate prediction of near-bank fluvial hydraulics is likely a dominant contributor to the large variation of the calibration parameters since uncertainties have to be included in the calibration parameters. Coupled 2D and bank erosion models, such as the one developed in this study, may hold promise to be applicable to the field as determination of the calibration parameters can be reach-based, instead of bank profile-based.

Table 2. Basal erosion parameters used for final simulation for all cross sections.

Cross-Section	1	2	3	4	5	6	7	8	9	10	11
τ_c (Pa)	2	2	2	2	2	2	2	2	2	2	2
$k \times 10^7$ m^{3}N^{-1}s^{-1}	1.4	1.4	1.4	1.4	1.4	2.4	2.4	2.4	1.4	1.4	1.4

The coupled simulation was carried out from March, 1996 to March, 2001. The time step for SRH-2D was 5 seconds. The time step for BSTEM was variable and was the time needed for a given amount of water to flow through the bend; in this study, a volume of 4,000 m^3 was used. It was found that model results were insensitive to these time steps.

A planar view of the predicted versus measured bankline retreat from March, 1996 to February, 2001 is displayed in Figure 4. Further comparison of results between model prediction and measured data is shown in Figure 5, in which bank profiles at different times are compared. These results demonstrate that the proposed coupling between SRH-2D and BSTEM works quite well.

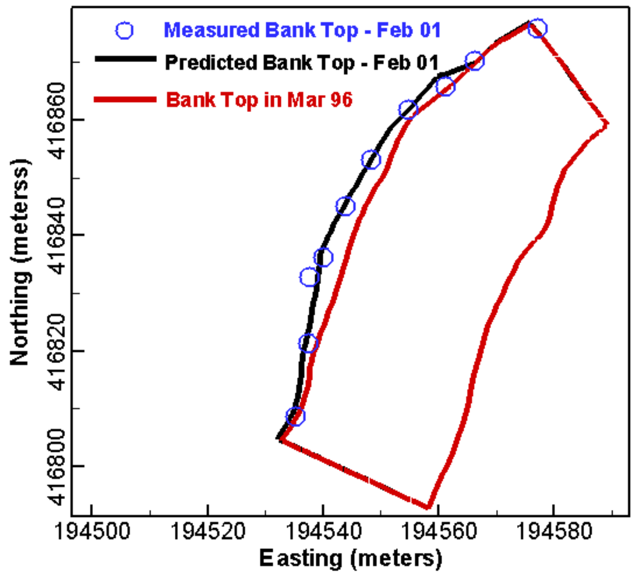
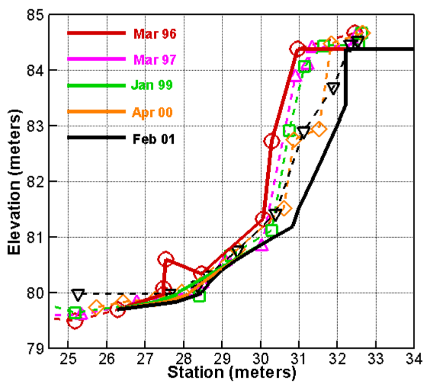
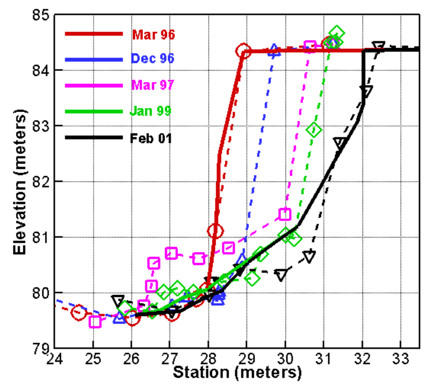


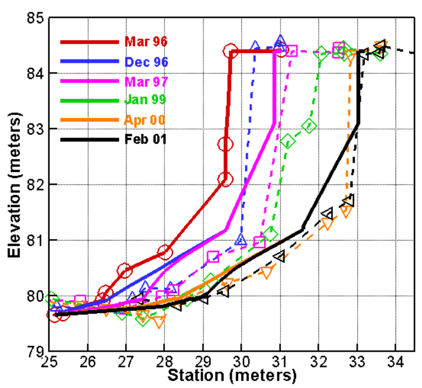
Figure 4. Comparison of predicted and measured top bank retreat.



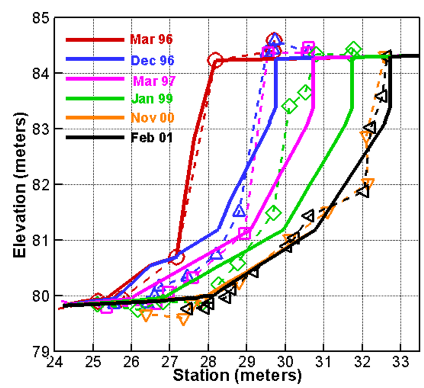
(a) XS 4



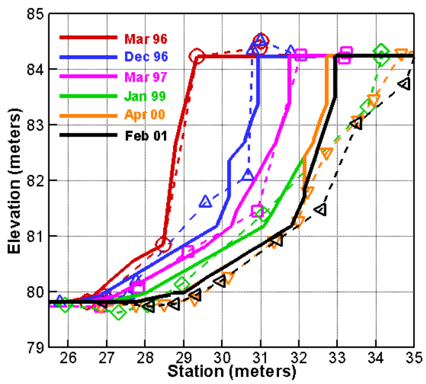
(b) XS 5



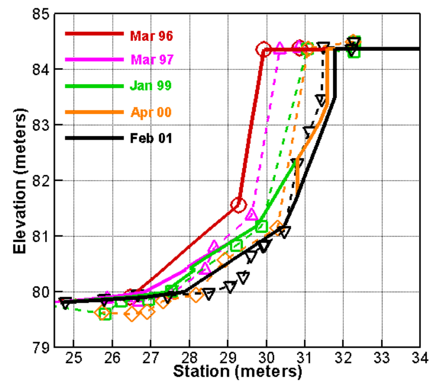
(c) XS 6



(d) XS 7



(e) XS 8



(f) XS 9

Figure 5. Comparison of predicted (solid lines) and measured (dash lines with symbols) bank retreat at cross sections 4 to 9 (the same color corresponds to the same time).

Bank erosion at the Goodwin Creek bend is difficult to model since it consists of multi-layer materials and a relatively tall and steep bank face. Particularly, the structure of the massive silt and meander belt alluvium units promote the development of tension cracks and seepage that seem to dominate the shape of the bank profile. Despite these complexities, the coupled model was seen capable of reproducing the bank profiles with tension cracks relatively well. Despite overall success of the model prediction however, there are still some issues. The location of the tension crack is not well predicted at some cross-sections. At XS 5 and 9, for example, the tension crack is predicted to be too far away from the bank in earlier years, which is contrary to the observed data. A large volume of bank material is predicted to fail very early so that the bank profile is predicted to remain relatively stable for a long time since these materials have to be subsequently cleared through the hydraulic scour process. The exact timing and amount of mass failure are not always in agreement with the observed data. There may be several potential causes of this discrepancy. It may be dueto the impact of a perched water table during rainfall events (Simon et al, 2000; Langendoen and Simon, 2008). The simplification used to model basal cleanout of the failed bank materials may be an important factor. However, the discrepancy is mostly with the short-term bank retreat prediction; the longer-term bank retreat is predicted relatively well by the model.

Different values of critical shear stress, as well as erodibility, were used at different cross-sections by Langendoen and Simon (2008). It was justified because the 1D model cannot predict enhanced near-bank shear stress; a reduction of the critical shear stress at some cross-sections was an attempt to achieve a better calibration. With the SRH-2D and BSTEM coupled model, the same critical shear stress was used for all banks and only a relatively small variation of erodibility values were used. Therefore, the coupled SRH-2D and BSTEM model may hold better prospect for field applications than 1D models since physical processes are better represented and modeled; the current study represents an improvement over the coupled CONCEPTS and BSTEM model of Langendoen and Simon (2008).

Concluding Remarks

A coupled SRH-2D and BSTEM model was developed. SRH-2D is a 2D flow and sediment transport model and BSTEM is a mechanistic basal erosion and mass failure model.

Application of the coupled model to a bend on Goodwin Creek in Mississippi over a five-year period was carried out and model results compared with the measured data. The proposed coupling procedures and the moving grid formulation proved successful. Model results compared reasonably well with measured bank retreat data, showing that the coupled model is capable of predicting bank retreat over a multi-year period. However, the timing of failure and the volume of failed bank materials could not be accurately predicted for some bank profiles. These discrepancies may be attributed to: (a) the presence of seepage processes and tension cracks, (b) a simple model for basal cleanout, and (c) uncertainty in groundwater elevation. It was found that the present 2D coupled model represents an improvement over the previous 1D study by Langendoen and Simon (2008) because it was not necessary to vary both calibration parameters (erodibility and critical shear stress) significantly. This highlights the further need to couple more accurate stream models, e.g., 2D or 3D models, with bank erosion models.

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