DISCUSSION / DISCUSSION

Discussion of "Three-dimensional slope stability based on stresses from a stress-deformation analysis"

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We have read this paper with interest because of our involvement in slope stability analysis of embankment dams and natural slopes. This discussion is motivated primarily from the results of example problems included in the paper for which the proposed procedure to calculate the factor of safety (FS) shows a large sensitivity to Poisson's ratio (υ).

Intuitively, we were expecting to see no or very little change in computed FS results over a commonly used range of υ values (say from 0.30 to 0.45). To check our intuitions, we re-analyzed each of the four example problems included in the paper using the continuum-mechanics-based procedure implemented in the Fast Lagrangian Analysis of Continua in 3 Dimensions (FLAC3D) computer program (Itasca Consulting Group 2002). Results of these analyses form the basis of comments included in this discussion. For comparison purposes, the example problems were also analyzed in plane strain mode using the continuum-mechanics-based procedure implemented in the Fast Lagrangian Analysis of Continua (FLAC) computer program (Itasca Consulting Group 2006). Size adequacy of the continuum models was verified by analyzing them via the computer program CLARA-W (O. Hungr Geotechnical Research 2010) and comparing the results with the ones included in the paper. FLAC, FLAC3D, and CLARA-W are commercially available computer programs and their adoption for re-analysis of the example problems was for convenience.

Continuum-mechanics-based procedures using the elastoplastic constitutive model with a Mohr–Coulomb yield condition and a flow rule require elastic constants (two for an isotropic material) and plasticity parameters (cohesion c, angle of internal friction φ, and dilation angle ψ). The elastic constants used in the paper are Young's modulus (E) and υ. FLAC and FLAC3D require data for bulk modulus (K) and shear modulus (G). Values for G and K were calculated from the E and υ values using the relations $G = E/[2(1 + \upsilon)]$ and $K = E/[3(1 - 2\upsilon)]$.

Two alternatives commonly used in comparing relative merits of different slope stability analysis methods are to compare (*i*) relative values of FS calculated for a specified shear surface and (*ii*) relative values of FS corresponding to critical shear surfaces determined by the different methods. The authors have used alternative (*i*). However, in this discussion, alternative (*ii*) is adopted — the expectation is that the trends in FS versus υ observed herein are applicable for comparison with the trends presented in the paper.

The objectives of this discussion are to (*i*) assess the effects of Poisson's ratio on computed FS results and (*ii*) observe the geometry of three-dimensional (3-D) critical shear surfaces based on FLAC3D results. FLAC and CLARA-W model results are included for comparison purposes.

In the continuum model analysis, the critical shear surface is determined as a part of the solution and is along the path of velocity discontinuity. For the continuum model results included herein, the path of velocity discontinuity is taken to be the velocity contour of lowest value. For the limit equilibrium model results, the critical shear surface is assumed to be of ellipsoidal, spherical or cylindrical shape. The comparisons of critical shear surface geometry from continuum and limit equilibrium analyses are visual only, i.e., no mathematical expressions for critical shear surfaces based on continuum models are included.

The following features were kept consistent in the 3-D models for each of the four example problems: (*i*) geometry is referenced in an x, y, z coordinate system that follows the right-hand rule with a two-dimensional (2-D) cross section in the x–z plane; (*ii*) u, v, w refer to displacements in the x, y, z coordinate directions, respectively; (*iii*) the length of the model in the y-direction is 50 m; (*iv*) the 2-D cross section is taken midway in the y-direction; (*v*) gravity turn-on is used to simulate initial stresses in the model; (*vi*) G and K values correspond to the specified E and υ = 0.0, 0.1, 0.2, 0.3, 0.4, 0.42, 0.45, and 0.49; (*vii*) ψ is set equal to zero, i.e., the flow rule is nonassociative; and (*viii*) each set of material property values is treated as a new problem and solved as such. CLARA-W models are similar to the FLAC3D models, and are analyzed using aspect ratios (AR) of 1 and 1000. The discretizations of numerical models were selected by inspection and the same model was used for all combinations of E, υ, and loading condition. For example prob-

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lems 1 and 3, the numerical models were assigned dimensions and material properties that are likely to be encountered in the field.

Example problems 2, 3, and 4 were also analyzed using the associative flow rule, $\psi = \varphi$, to verify if the flow rule could possibly affect the sensitivity of FS versus υ. Example problem 1 was not included because of the $\varphi = 0$ characterization of the material strength for which the flow rule does not apply.

FLAC3D and FLAC models are identified as such; CLARA-W models are identified using the abbreviation CLW. CLW3D refers to the 3-D CLARA-W model, and CLW2D refers to the 2-D CLARA-W model. Only details relevant to the objectives of this discussion are included herein to conserve space; additional details can be obtained from the writer on request.

Verification example No. 1

Figure D1 shows the layout of the numerical model used for re-analysis of a 10 m high, 2H:1V (H denotes horizontal and V denotes vertical) homogeneous slope (simulating an approximately 9 m thick infinite slope). Table D1 lists the results of FS versus υ for example 1.

The CLW model results are

- CLW3D: AR = 1, FS = 1.20; AR = 1000, FS = 1.05.
- CLW2D: FS = 1.05.

The CLW model was made 60 m long in the y-direction to accommodate the critical shear surface.

Critical shear surfaces from FLAC3D, FLAC, and CLW models are included in Fig. D1: FLAC3D and FLAC shear surfaces shown correspond to υ = 0.40 and are typical of those associated with other discrete values of υ shown in Table D1, and the CLW model results correspond to AR = 1.

It should be noted that for the selected c = 45 kPa and unit weight $\gamma = 18.84 \times 10^3$ kg/m^3, a radius R equal to 23.89 m satisfies the dimensionless parameter $c/\gamma R = 0.1$. For a spherical shear surface with the center at x = 19.66 m, y = 30.00 m, z = 33.18 m, the tangent plane elevation = 9.29 and AR = 1, CLW3D FS = 1.40. This compares well with the closed-form solution of 1.402 included in Table 1 of the paper. The slope was made 13.5 m high to accommodate the specified shear surface.

Verification example No. 2

Figure D2 shows the layout of the numerical model used for re-analysis of the 12.2 m high slope overlying a weak layer. In FLAC3D and FLAC models, the weak layer is modeled as a 5 m thick layer, G and K correspond to E = 5000 kPa and varying υ values, and a shear strength of φ' = 10°, c' = 0 (where the prime symbol denotes effective strength). In the CLW model, the weak layer is modeled as a discontinuity located 1 m below the toe of the slope (i.e., z = –1) with φ' = 10° and c' = 0. Table D2 lists the FS versus υ results for example problem 2.

The CLW model results are

- CLW3D, without water table: AR = 1, FS = 1.65; AR = 1000, FS = 1.42.

Fig. D1. Verification example No. 1; continuum model results correspond to υ = 0.4 (typical). (*a*) FLAC3D model; (*b*) FLAC3D critical shear surface; (*c*) CLW 3D critical shear surface; (*d*) FLAC model; (*e*) FLAC critical shear surface; (*f*) CLW 2D Bishop's simplified critical shear surface. All dimensions in metres. b.c., boundary condition; g, gravitational acceleration (= 9.81 m/s^2); γ, unit weight; ρ, mass density.

Example No. 1 material properties

Material ID	$\rho \times 10^3$ (kg/m^3)	c (kPa)	φ (°)	G (kPa)	K (kPa)
Slope	1.92	45	0	Correspond to E = 1.0×10^4 and selected value of υ	
Base	4.40	1×10^4	45	1.0×10^7	2.0×10^8

$\gamma = \rho \times g$

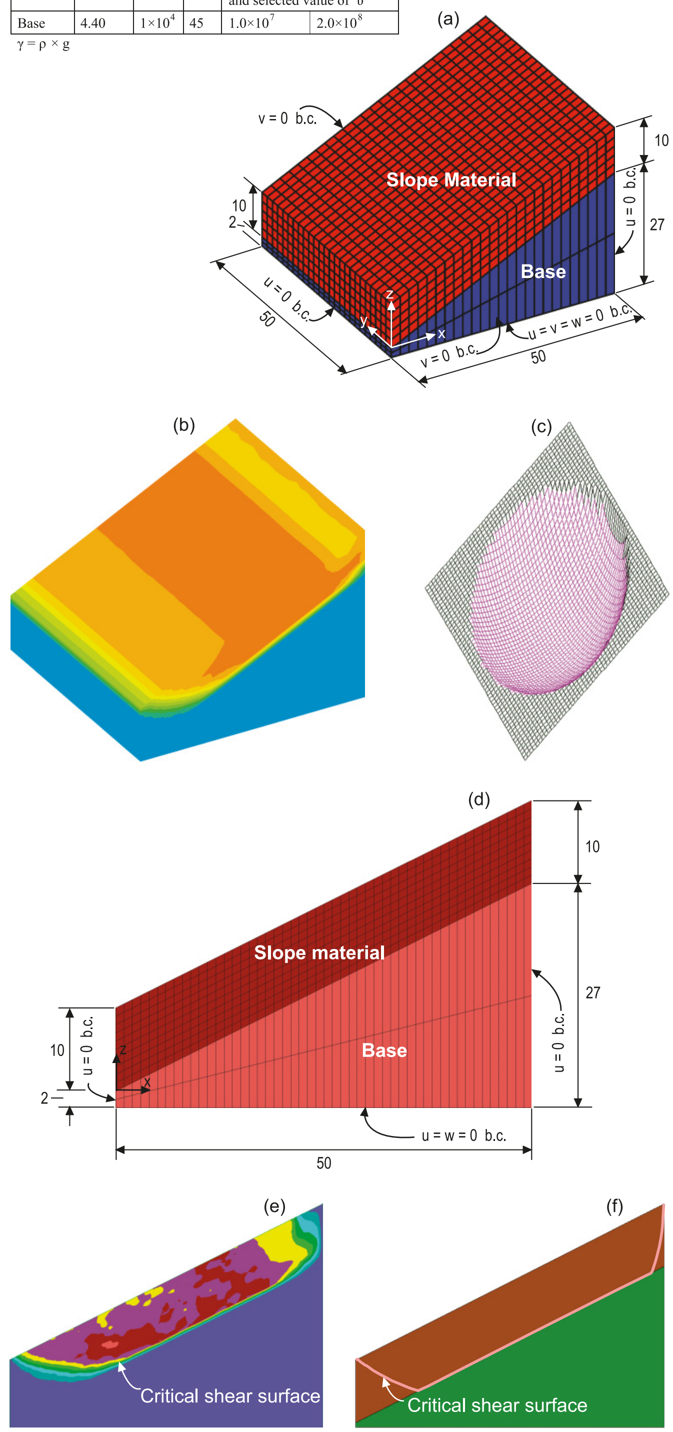


Table D1. Verification example No. 1; two- and three-dimensional continuum model results.

	Computed factor of safety (FS)	
Poisson's ratio, υ	FLAC3D	FLAC
0.0	1.00	0.94
0.10	1.01	0.94
0.20	1.00	0.92
0.30	1.01	0.93
0.40	1.00	0.92
0.42	1.00	0.92
0.45	1.00	0.92
0.49	1.00	0.92

- CLW3D, with water table: AR = 1, FS = 1.44; AR = 1000, FS = 1.21.
- CLW2D, without water table: FS = 1.39.
- CLW2D, with water table: FS = 1.19.

The CLW model in the y-direction was made 100 m to accommodate shear surfaces associated with the search.

Critical shear surfaces from FLAC3D, FLAC, and CLW models are included in Fig. D2 — FLAC3D and FLAC shear surfaces shown correspond to a $\upsilon = 0.40$ and $\psi = 0$ combination and are typical of those associated with other values of υ and ψ shown in Table D2. Also, critical shear surfaces shown in Fig. D2 are for the no-water-table loading condition; those for the with-water-table loading condition are similar and are not included herein to conserve space.

It should be noted that for AR = 1, CLW3D FS = 1.65 for the no-water-table and 1.44 with-water-table loading conditions — the volume of material involved is about 14 125 m^3 in both cases. The corresponding values for FS included in Table 2 of the paper are 1.62 and 1.54, respectively, and the associated volumes of material listed are 13 000 and 16 000 m^3, respectively.

Verification example No. 3

Figure D3 shows the layout of the numerical model used for re-analysis of the approximately 9.25 m high embankment (for $c = 20.2$ kPa and $\gamma = 18.83 \times 10^3$ kg/m^3, using $c/\gamma H = 0.116$ gives $H \approx 9.25$ m). Table D3 lists the FS versus υ results for example problem 3.

The CLW model results are

- CLW3D: AR = 1, FS = 1.15; AR = 1000, FS = 0.97.
- CLW2D: FS = 0.97.

Critical shear surfaces from FLAC3D, FLAC, and CLW models are included in Fig. D3 — FLAC3D and FLAC shear surfaces shown correspond to a $\upsilon = 0.40$ and $\psi = 0$ combination and are typical of those associated with other values of υ and ψ shown in Table D3.

It should be mentioned that for AR = 0.66, CLW3D FS = 1.25. This compares with the FS value of 1.23 credited to Hungr et al. (1989) in Fig. 13 of the paper.

Verification example No. 4

Figure D4 shows the layout of the numerical model used for re-analysis of the 9 m high slope with a surcharge load (q) of 55 kPa over a 5 m × 4 m area located 1 m from the edge and centered in the middle of the slope. In each of the numerical models, q was applied as an external force. For comparison purposes, this example was also analyzed for $q = 0$. Table D4 lists the FS versus υ results for example problem 4.

The CLW model results are

- CLW3D, $q = 0$: AR = 1, FS = 1.65; AR = 1000, FS = 1.43.
- CLW3D, $q = 55$ kPa: AR = 1, FS = 1.56; AR = 1000, FS = 1.42.
- CLW2D, $q = 0$: FS = 1.45.

Critical shear surfaces from FLAC3D, FLAC, and CLW models are included in Fig. D4 — FLAC3D and FLAC shear surfaces shown correspond to a $\upsilon = 0.40$ and $\psi = 0$ combination, and are typical of those associated with other values of υ and ψ shown in Table D4. In addition, the FLAC3D critical shear surface for $q = 550$ kPa is included in Fig. D4*i* and the associated FS using $\psi = 0$ is 1.02; the corresponding value of FS for $\psi = \varphi$ is 1.08. Similar analyses using the CLW model were not performed because of uncertainty in selecting an appropriate value for AR.

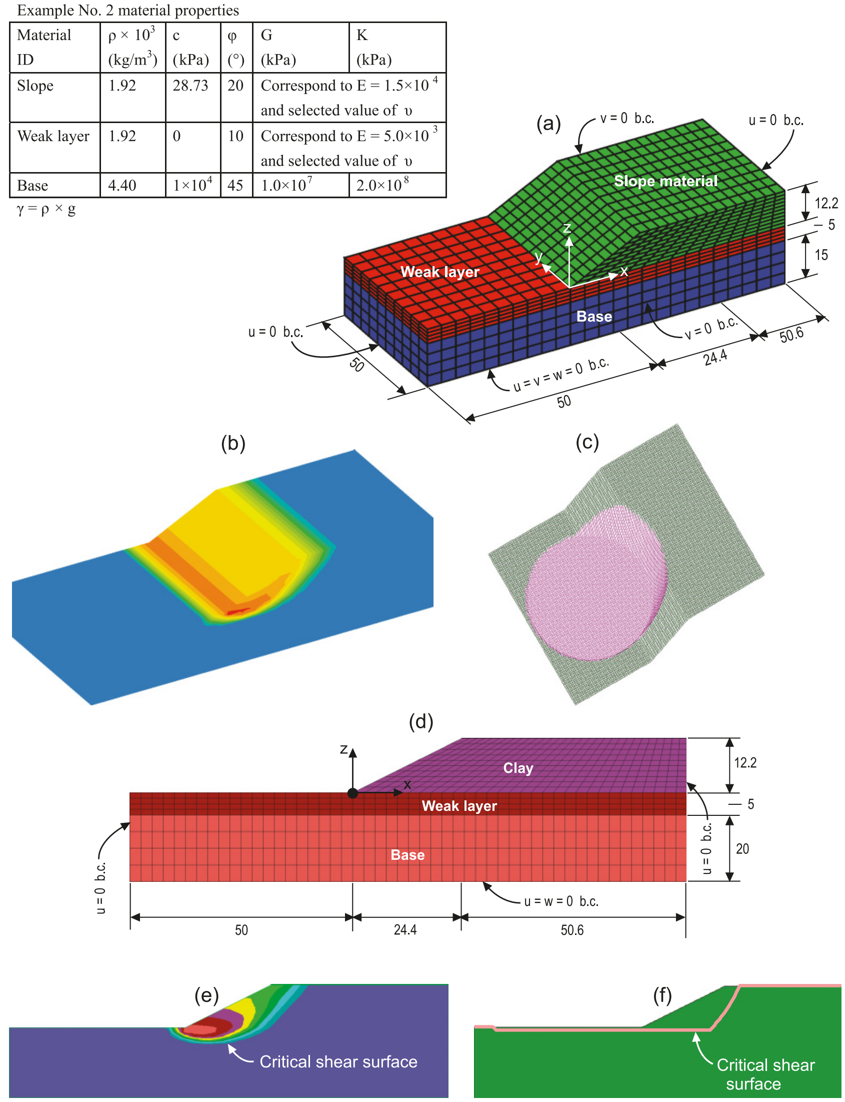
It should be noted that for $q = 55$ kN/m^2 and AR = 1.0, CLW3D FS = 1.56; this compares well with the value of 1.58 credited to Hungr et al. (1989) in Fig. 15 of the paper. Also, for $q = 550$ kPa, the FLAC3D FS of 1.02 compares favorably with the statement in the paper that for $q > 600$ kPa, the 3-D FS decreases below 1.0.

Summary

Significant observations from the results of the re-analyses of the four example problems include

1. FLAC model results (FS and associated shear surface) for example problem 2, using associative flow rule ($\psi = \varphi$), do not result in identifiable shear surfaces for the with- and without- water-table loading condition corresponding to $\upsilon = 0.49$. Similarly, for example problem 3, FLAC model results using the associative flow rule do not result in an identifiable shear surface for $\upsilon = 0.49$. For example problem 4, FLAC model results using the associative flow rule do not result in identifiable shear surfaces for the $q = 0$ loading condition corresponding to $\upsilon \geq 0.4$. These discrepancies in the FLAC model are attributed to the flow rule ($\psi = \varphi$), and not the Poisson's ratio (υ) because, in each case, for the assigned υ values and using $0 < \psi < \varphi$, identifiable shear surfaces do develop and the corresponding FS values are similar to the ones before the numerical discrepancy occurs. FLAC3D models did not encounter this occurrence.
2. Continuum model results show relatively little sensitivity to the computed factor of safety due to the value of Poisson's ratio.
3. The lateral extent in the *y*-direction for example problems 1–3 and example problem 4 without the surcharge load make them appropriate for 2-D plane strain analysis. In this sense, the results included in Tables D1 to D4 are useful in comparing 3-D FS to their 2-D counterparts for individually determined critical shear surfaces.

Fig. D2. Verification example No. 2; continuum model results correspond to $\upsilon = 0.4$ and $\psi = 0$ combination with no-water-table loading condition (typical). (*a*) FLAC3D model; (*b*) FLAC3D critical shear surface; (*c*) CLW3D critical shear surface; (*d*) FLAC model; (*e*) FLAC critical shear surface; (*f*) CLW2D critical shear surface. All dimensions in metres.



Material ID	ρ × 10³ (kg/m³)	c (kPa)	φ (°)	G (kPa)	K (kPa)
Slope	1.92	28.73	20	Correspond to E = 1.5×10⁴ and selected value of υ	
Weak layer	1.92	0	10	Correspond to E = 5.0×10³ and selected value of υ	
Base	4.40	1×10⁴	45	1.0×10⁷	2.0×10⁸

γ = ρ × g

Table D2. Verification example No. 2; two- and three-dimensional continuum model results.

	Computed factor of safety (FS)							
	Nonassociative flow rule ($\psi = 0$)				Associative flow rule ($\psi = \varphi$)			
	Without water table		With water table		Without water table		With water table	
Poisson's ratio, υ	FLAC3D	FLAC	FLAC3D	FLAC	FLAC3D	FLAC	FLAC3D	FLAC
0.0	1.01	0.94	0.75	0.68	1.03	0.95	0.77	0.69
0.10	1.01	0.94	0.75	0.68	1.03	0.95	0.77	0.69
0.20	1.01	0.94	0.75	0.68	1.03	0.95	0.77	0.69
0.30	1.01	0.94	0.75	0.68	1.03	0.95	0.77	0.69
0.40	1.01	0.94	0.75	0.68	1.03	0.95	0.77	0.69
0.42	1.01	0.94	0.75	0.68	1.03	0.95	0.77	0.69
0.45	1.01	0.93	0.75	0.68	1.03	0.95	0.76	0.69
0.49	1.01	0.93	0.75	0.67	1.02	0.92*	0.76	0.67†

*For $\psi = 0.5\varphi$.
†For $\psi = 0.75\varphi$.

Fig. D3. Verification example No. 3; continuum model results correspond to $\upsilon = 0.4$ and $\psi = 0$ combination (typical). (*a*) 3-D model; (*b*) FLAC3D critical shear surface corresponding to $\upsilon = 0.40$ (typical); (*c*) CLW3D critical shear surface; (*d*) 2-D model; (*e*) FLAC critical shear surface corresponding to $\upsilon = 0.40$ (typical); (*f*) CLW2D critical shear surface. All dimensions in metres.

Example No. 3 material properties

Material ID	$\rho \times 10^3$ (kg/m^3)	c (kPa)	φ (°)	G (kPa)	K (kPa)
Slope	1.92	20.2	15	Correspond to $E = 1.0\times10^4$ and selected value of υ	
Base	4.40	1×10^4	45	1.0×10^7	2.0×10^8

$\gamma = \rho \times g$

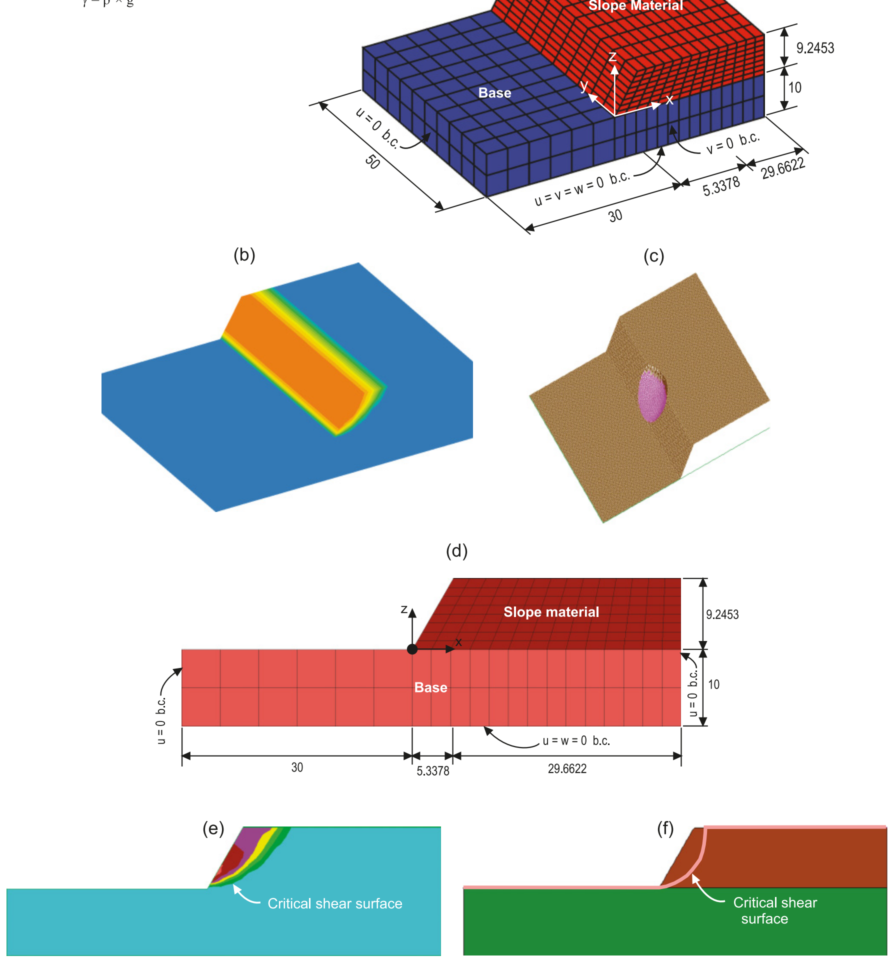
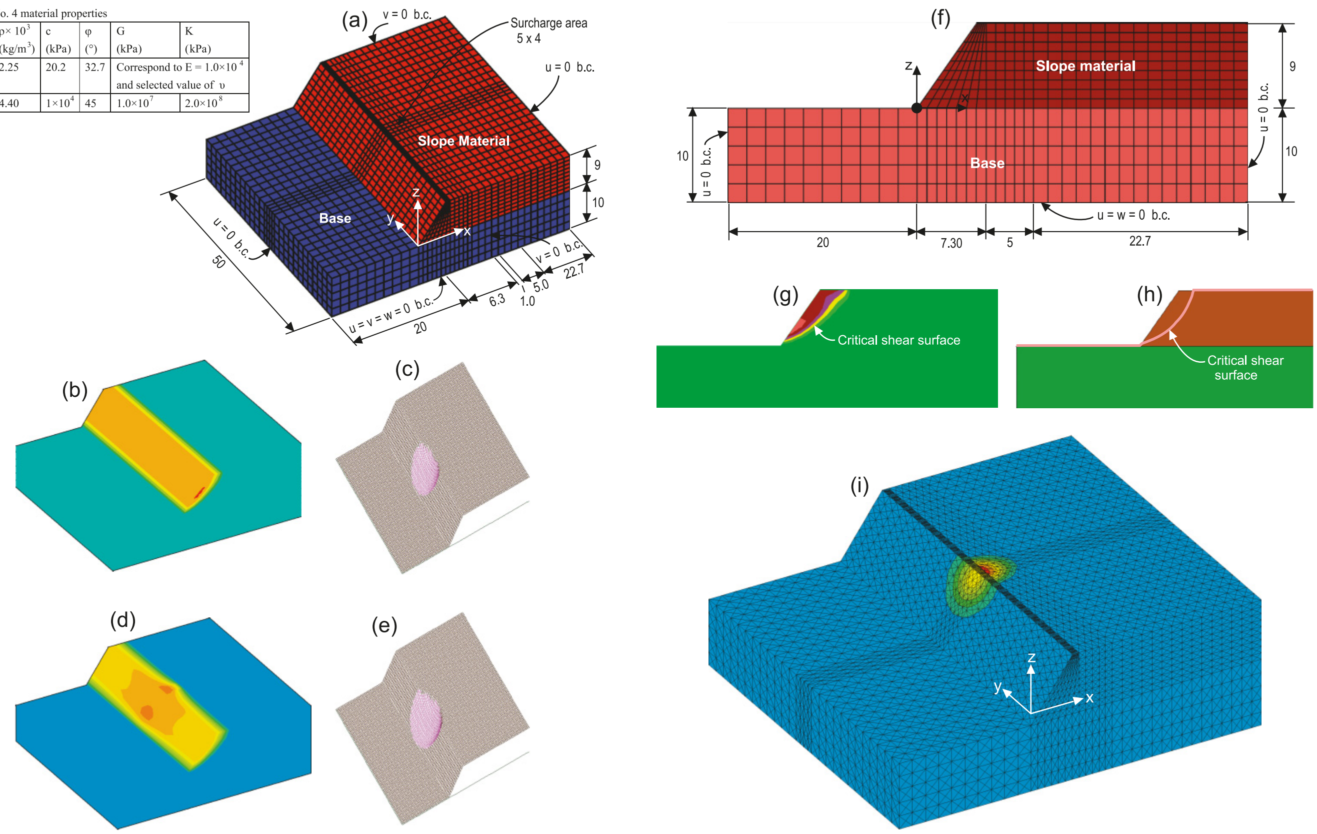


Table D3. Verification example No. 3; two- and three-dimensional continuum model results.

	Computed factor of safety (FS)			
	Nonassociative flow rule ($\psi = 0$)		Associative flow rule ($\psi = \varphi$)	
Poisson's ratio, υ	FLAC3D	FLAC	FLAC3D	FLAC
0.0	1.07	1.03	1.09	1.05
0.10	1.07	1.03	1.09	1.05
0.20	1.07	1.03	1.09	1.05
0.30	1.07	1.03	1.09	1.05
0.40	1.07	1.03	1.09	1.04
0.42	1.07	1.03	1.09	1.04
0.45	1.08	1.03	1.09	1.03
0.49	1.08	1.03	1.09	0.99*

*For $\psi = 0.5\varphi$.

Fig. D4. Verification example No. 4; continuum model results correspond to $\upsilon = 0.4$ and $\psi = 0$ combination (typical) for the marked surcharge (q) load condition. (a) FLAC3D model; (b) FLAC3D critical shear surface for $q = 0$; (c) CLW3D critical shear surface for $q = 0$; (d) FLAC3D critical shear surface for $q = 55$ kN/m^2; (e) CLW3D critical shear surface for $q = 55$ kN/m^2; (f) FLAC model; (g) FLAC critical shear surface for $q = 0$; (h) CLW2D critical shear surface for $q = 0$; (i) FLAC3D critical shear surface for $q = 550$ kN/m^2. All dimensions in metres.



Example No. 4 material properties

Material ID	$\rho \times 10^3$ (kg/m^3)	c (kPa)	φ (°)	G (kPa)	K (kPa)
Slope	2.25	20.2	32.7	Correspond to E = 1.0×10^4 and selected value of υ	
Base	4.40	1×10^4	45	1.0×10^7	2.0×10^8

$\gamma = \rho \times g$

Table D4. Verification example No. 4; two- and three-dimensional continuum model results.

	Computed factor of safety (FS)					
	Nonassociative flow rule ($\psi = 0$)			Associative flow rule ($\psi = \varphi$)		
	$q = 0$		$q = 55$ kPa	$q = 0$		$q = 55$ kPa
Poisson's ratio, υ	FLAC3D	FLAC	FLAC3D	FLAC3D	FLAC	FLAC3D
0.0	1.51	1.45	1.49	1.55	1.50	1.53
0.10	1.51	1.45	1.49	1.55	1.49	1.53
0.20	1.51	1.45	1.49	1.55	1.49	1.53
0.30	1.51	1.45	1.49	1.55	1.48	1.53
0.40	1.51	1.44	1.49	1.55	1.49*	1.53
0.42	1.51	1.44	1.49	1.55	1.49*	1.53
0.45	1.51	1.44	1.49	1.55	1.48*	1.53
0.49	1.50	1.44	1.49	1.54	1.44†	1.53

*For $\psi = 0.5\varphi$.
†For $\psi = 0$.

4. Use of the nonassociative flow rule ($\psi = 0$) results in a lower FS than with the use of the associative flow rule ($\psi = \varphi$).
5. Continuum model critical shear surfaces have FS values that are less than those determined using a mathematically defined shear surface shape in the limit-equilibrium models. However, the CLARA-W model results for this discussion were limited to only one search mode and in that sense, may not be reflective of the true critical shear surface and the associated FS, i.e., other search modes may identify critical shear surfaces with lower FS values.
6. FLAC3D, FLAC, and CLARA-W model results are consistent in themselves, i.e., computed factors of safety degrade as the loading conditions worsen, as in example problems 2 and 4.
7. It would be helpful to know the authors' views on (i) the continuum model results (FS and associated shear surface) for the four example problems and (ii) their experiences in

selecting 3-D shear surface geometry (shape and lateral extent in the *y*-direction) for use in limit-equilibrium-based analyses.

It should be noted that Wright et al. (1973) and Adriano et al. (2008) used procedures similar to the one presented in the paper and assessed relatively little differences in the computed factors of safety over a range of Poisson's ratio values. Wright et al. models were two-dimensional and the FS varied from about 1.93 to 2.05 (scaled values) for discrete values of υ from 0.3 to 0.49; Adriano et al. models were three-dimensional and for a plane slope, the FS varied from about 1.42 to 1.45 (scaled values) for discrete values of υ from 0 to 0.49.

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