Title page

Discussion of "Three-dimensional slope stability based on stresses from a stress-deformation analysis" by J. R. Stianson, D. G. Fredlund, and D. Chan. Canadian Geotechnical Journal, **48** (6): 891-904, 2011.

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Three-dimensional slope stability based on stresses from a stress-deformation analysis by J. R. Stianson, D. G. Fredlund, and D. Chan. Canadian Geotechnical Journal, **48** (6): 891-904, 2011.

Discussion by
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We have read this paper with interest because of our involvement in slope stability analysis of embankment dams and natural slopes. This discussion is motivated primarily from the results of example problems included in the paper for which the proposed procedure to calculate the factor of safety (FS) shows large sensitivity to Poisson's ratio (υ).

Intuitively, we were expecting to see no or very little change in computed FS results over a commonly used range of υ values (say from 0.30 to 0.45). In order to check our intuitions, we re-analyzed each of the four example problems included in the paper using the continuum-mechanics-based procedure implemented in the computer program FLAC3D (Fast Lagrangian Analysis of Continua in 3 Dimensions), Itasca Consulting Group (2002). Results of these analyses form the basis of comments included in this discussion. For comparison purposes, the example problems were also analyzed in plane strain mode using the continuum mechanics based procedure implemented in the computer program FLAC (Fast Lagrangian Analysis of Continua), Itasca Consulting Group (2006). Size adequacy of the continuum models was verified by analyzing them via the computer program CLARA-W, O. Hungr Geotechnical Research (2010), and comparing the results with the ones included in the paper. FLAC, FLAC3D, and CLARA-W are commercially available computer programs and their adoption for re-analysis of the example problems was for convenience.

Continuum-mechanics based procedures using elasto-plastic constitutive model with Mohr-Coulomb yield condition and a flow rule require elastic constants (two for an isotropic material) and plasticity parameters (cohesion c, angle of internal friction φ, and dilation angle ψ). The elastic constants used in the paper are Young's modulus (E), and υ. FLAC and FLAC3D require data for bulk modulus (K) and shear modulus (G). Values for G and K were calculated from the E and υ values using the relations: $G = \frac{E}{2(1+\upsilon)}; \quad K = \frac{E}{3(1-2\upsilon)}$.

Two alternatives commonly used in comparing relative merits of different slope stability analysis methods are to compare: (i) relative values of FS calculated for a specified shear surface, and (ii) relative values of FS corresponding to critical shear surfaces determined by the different methods. The authors have used alternative (i). However, in this discussion, alternative (ii) is adopted – the expectation is that the trends in FS versus υ observed herein are applicable for comparison with the trends presented in the paper.

The objectives of this discussion are to: (i) assess the effects of Poisson's ratio on computed FS results, and (ii) observe the geometry of 3-D critical shear surfaces based on FLAC3D results. FLAC and CLARA-W model results are included for comparison purposes.

In the continuum model analysis, critical shear surface is determined as a part of the solution and is along the path of velocity discontinuity. For the continuum model results included herein, the path of velocity discontinuity is taken to be the velocity contour of lowest value. For the limit equilibrium model results, the critical shear surface is assumed to be of ellipsoidal, spherical or cylindrical shape. The comparisons of critical shear surface geometry from continuum and limit equilibrium analyses are visual only, i.e. no mathematical expressions for critical shear surfaces based on continuum models are included.

The following features were kept consistent in the three-dimensional (3-D) models for each of the four example problems: (i) geometry is referenced in an x, y, z coordinate system which follows the right-hand rule with two-dimensional (2-D) cross section in x-z plane; (ii) u, v, w refer to displacements in the x, y, z coordinate directions, respectively; (iii) length of the model in y-direction is 50 m; (iv) 2-D cross section is taken midway in the y-direction; (v) gravity turn-on is used to simulate initial stresses in the model; (vi) G and K values correspond to the specified E and υ = 0.0, 0.1, 0.2, 0.3, 0.4, 0.42, 0.45, and 0.49; (vii) ψ is set equal to zero, i.e. flow rule is non-associative; and (viii) each set of material property values is treated as a new problem and solved as such. CLARA-W models are similar to the FLAC3D models, and are analyzed using aspect ratios (AR) of 1 and 1000. The discretizations of numerical models were selected by inspection and same model was used for all combinations of E, υ, and loading condition. For example problems 1 and 3, the numerical models were assigned dimensions and material properties that are likely to be encountered in the field.

Example problems 2, 3, and 4 were also analyzed using the associative flow rule, $\psi = \varphi$, in order to verify if the flow rule could possibly affect the sensitivity of FS versus υ. Example problem 1 was not included because of the $\varphi = 0$ characterization of the material strength for which the flow rule does not apply.

FLAC3D and FLAC models are identified as such; CLARA-W models are identified using the abbreviation CLW. CLW3D refers to the 3-D CLARA-W model, and CLW2D refers to the 2-D CLARA-W model. Only details relevant to the objectives of this discussion are included herein to conserve space; additional details can be obtained from the writer on request.

Verification example No. 1:

Figure A-1 shows the layout of the numerical model used for re-analysis of a 10 m high, 2H:1V homogeneous slope (simulating an approximately 9 m thick infinite slope). Table A-1 lists the results of FS versus υ for example 1.

Table A-1. Verification example No. 1

Poisson's ratio, υ	Computed factor of safety (FS)	
	FLAC3D	FLAC
0.0	1.00	0.94
0.10	1.01	0.94
0.20	1.00	0.92
0.30	1.01	0.93
0.40	1.00	0.92
0.42	1.00	0.92
0.45	1.00	0.92
0.49	1.00	0.92

The CLW model results are: CLW3D ─ AR = 1, FS = 1.20; AR = 1000, FS = 1.05. CLW2D ─ FS = 1.05. The CLW model was made 60 m long in the y-direction to accommodate the critical shear surface.

Critical shear surfaces from FLAC3D, FLAC, and CLW models are included in Fig. A-1; FLAC3D and FLAC shear surfaces shown correspond to υ = 0.40 and are typical of those associated with other discrete values of υ shown in Table A-1; and CLW3D model results correspond to AR = 1.

It should be noted that for the selected c = 45 kPa and $\gamma = 18.84 \times 10^3$ kg/m^3, a radius R equal to 23.89 m satisfies the dimensionless parameter $\frac{c}{\gamma R} = 0.1$. For a spherical shear surface with the center at x = 19.66 m, y = 30.00 m, z = 33.18 m, tangent plane elevation = 9.29, and AR = 1, CLW3D FS = 1.40. This compares well with the closed form solution of 1.402 included in Table 1 of the paper. The slope was made 13.5 m high to accommodate the specified shear surface.

Verification example No. 2

Figure A-2 shows the layout of the numerical model used for re-analysis of the 12.2 m high slope overlying a weak layer. In FLAC3D and FLAC models, the weak layer is modeled as a 5 m thick layer, G and K corresponding to E = 5000 kPa and varying υ values, and shear strengths of φ' = 10° and c' = 0. In the CLW model, the weak layer is modeled as a discontinuity located 1 m below the toe of the slope (i.e., z = -1) with φ' = 10° and c' = 0. Table A-2 lists the FS versus υ results for example problem 2.

Table A-2. Verification example No. 2

Poisson's ratio, υ	Computed factor of safety (FS)							
	Flow rule							
	Non-associative (ψ = 0)				Associative (ψ = ϕ)			
	Without water table		With water table		Without water table		With water table	
	FLAC3D	FLAC	FLAC3D	FLAC	FLAC3D	FLAC	FLAC3D	FLAC
0.0	1.01	0.94	0.75	0.68	1.03	0.95	0.77	0.69
0.10	1.01	0.94	0.75	0.68	1.03	0.95	0.77	0.69
0.20	1.01	0.94	0.75	0.68	1.03	0.95	0.77	0.69
0.30	1.01	0.94	0.75	0.68	1.03	0.95	0.77	0.69
0.40	1.01	0.94	0.75	0.68	1.03	0.95	0.77	0.69
0.42	1.01	0.94	0.75	0.68	1.03	0.95	0.77	0.69
0.45	1.01	0.93	0.75	0.68	1.03	0.95	0.76	0.69
0.49	1.01	0.93	0.75	0.67	1.02	0.92*	0.76	0.67!

* for ψ = 0.5 × φ; ! for ψ = 0.75 × φ

The CLW model results are: CLW3D ─ without water table ─ AR = 1, FS = 1.65; AR = 1000, FS = 1.42. CLW3D ─ with water table ─ AR = 1, FS = 1.44; AR = 1000, FS = 1.21. CLW2D ─ without water table ─ FS = 1.39. CLW2D ─ with water table ─ FS = 1.19. The CLW model in the y-direction was made 100 m to accommodate shear surfaces associated with the search.

Critical shear surfaces from FLAC3D, FLAC, and CLW models are included in Fig. A-2 ─ FLAC3D and FLAC shear surfaces shown correspond to υ = 0.40 and ψ = 0 combination and are typical of those associated with other values of υ and ψ shown in Table A-2. Also, critical shear surfaces shown in Fig. A-2 are for the no-water-table loading condition; those for the with-water-table loading condition are similar and are not included herein to conserve space.

It should be noted that for AR = 1, CLW3D FS = 1.65 for the no-water-table and 1.44 with-water-table loading condition – the volume of material involved is about 14,125 m^3 in both cases. The corresponding values for FS included in Table 2 of the paper are 1.62 and 1.54, respectively, and the associated volumes of material listed are 13,000 m^3 and 16,000 m^3, respectively.

Verification example No. 3

Figure A-3 shows the layout of the numerical model used for re-analysis of the approximately 9.25 m high embankment (for c = 20.2 kPa and $\gamma = 18.83 \times 10^3$ kg/m^3, using $\frac{c}{\gamma H} = 0.116$ gives H ≈ 9.25 m). Table A-3 lists the FS versus υ results for example problem 3.

Table A-3. Verification example No. 3

Poisson's ratio, υ	Computed factor of safety (FS)			
	Flow rule			
	Non-associative (ψ = 0)		Associative (ψ = ϕ)	
	FLAC3D	FLAC	FLAC3D	FLAC
0.0	1.07	1.03	1.09	1.05
0.10	1.07	1.03	1.09	1.05
0.20	1.07	1.03	1.09	1.05
0.30	1.07	1.03	1.09	1.05
0.40	1.07	1.03	1.09	1.04
0.42	1.07	1.03	1.09	1.04
0.45	1.08	1.03	1.09	1.03
0.49	1.08	1.03	1.09	0.99*

* for $\psi = 0.5 \times \varphi$

The CLW model results are: CLW3D ─ AR = 1, FS = 1.15; AR = 1000, FS = 0.97. CLW2D ─ FS = 0.97.

Critical shear surfaces from FLAC3D, FLAC, and CLW models are included in Fig. A-3 - FLAC3D and FLAC shear surfaces shown correspond to υ = 0.40 and ψ = 0 combination and are typical of those associated with other values of υ and ψ shown in Table A-3.

It should be mentioned that for AR = 0.66, CLW3D FS = 1.25. This compares with the FS value of 1.23 credited to Hungr in Figure 13 of the paper.

Verification example No. 4:

Figure A-4 shows the layout of the numerical model used for re-analysis of the 9 m high slope with a surcharge load (q) of 55 kPa over a 5 m × 4 m area located 1 m from the edge and centered in the middle of the slope. In each of the numerical models, q was applied as an external force. For comparison purposes, this example was also analyzed for q = 0. Table A-4 lists the FS versus υ results for example problem 4.

Table A-4. Verification example No. 4

Poisson's ratio, υ	Computed factor of safety (FS)					
	Flow rule					
	Non-associative ($\psi = 0$)			Associative ($\psi = \phi$)		
	Surcharge load (q)			Surcharge load (q)		
	q = 0		q = 55 kPa	q = 0		q = 55 kPa
	FLAC3D	FLAC	FLAC3D	FLAC3D	FLAC	FLAC3D
0.0	1.51	1.45	1.49	1.55	1.50	1.53
0.10	1.51	1.45	1.49	1.55	1.49	1.53
0.20	1.51	1.45	1.49	1.55	1.49	1.53
0.30	1.51	1.45	1.49	1.55	1.48	1.53
0.40	1.51	1.44	1.49	1.55	1.49[*]	1.53
0.42	1.51	1.44	1.49	1.55	1.49[*]	1.53
0.45	1.51	1.44	1.49	1.55	1.48[*]	1.53
0.49	1.50	1.44	1.49	1.54	1.44[!]	1.53

[*] for $\psi = 0.5 \times \varphi$; [!] for $\psi = 0$

The CLW model results are: CLW3D — q = 0 — AR = 1, FS = 1.65; AR = 1000, FS = 1.43. CLW3D — q = 55 kPa — AR = 1, FS = 1.56; AR = 1000, FS = 1.42. CLW2D — q = 0 — FS = 1.45.

Critical shear surfaces from FLAC3D, FLAC, and CLW models are included in Fig. A-4 - FLAC3D and FLAC shear surfaces shown correspond to $\upsilon = 0.40$ and $\psi = 0$ combination, and are typical of those associated with other values of υ and ψ shown in Table A-4. In addition, FLAC3D critical shear surface for q = 550 kPa is included in Fig. A-4 (i) and the associated FS using $\psi = 0$ is 1.02; the corresponding value of FS for $\psi = \varphi$ is 1.08. Similar analyses using CLW model were not performed because of uncertainty in selecting an appropriate value for AR.

It should be noted that for q = 55 kN/m^2, and AR = 1.0, CLW3D FS = 1.56; this compares well with the value of 1.58 credited to Hungr (1989) in Fig. 15 of the paper. Also, for q = 550 kPa, FLAC3D FS of 1.02 compares favorably with the insert in the paper that for q > 600 kPa, the three dimensional FS decreases below 1.0.

Summary

Significant observations from the results of the re-analyses of the four example problems include:

1. FLAC model results (FS and associated shear surface) for example problem 2, using associative flow rule ($\psi = \varphi$), do not result in identifiable shear surfaces for the with- and

without- water-table loading condition corresponding to υ = 0.49. Similarly, for example problem 3, FLAC model results using the associative flow rule do not result in identifiable shear surfaces for υ = 0.49. For example problem 4, FLAC model results using the associative flow rule do not result in identifiable shear surfaces for the q = 0 loading condition corresponding to υ ≥ 0.4. These discrepancies in FLAC model are attributed to the flow rule (ψ = φ), and not the Poisson's ratio (υ) because, in each case, for the assigned υ values and using 0 < ψ < φ, identifiable shear surfaces do develop and the corresponding FS values are similar to the ones before the numerical discrepancy occurs. FLAC3D models did not encounter this occurrence.

2. Continuum model results show relatively little sensitivity to computed factor of safety due to Poisson's ratio value.

3. The lateral extent in the y-direction for example problems 1 - 3 and example problem 4 without the surcharge load make them appropriate for 2-D plane strain analysis. In this sense, the results included in Tables A.1 to A.4 are useful in comparing 3-D FS to their 2-D counterparts for individually determined critical shear surfaces.

4. Use of non-associative flow rule (ψ = 0) results in lower FS than with the use of associative flow rule (ψ = φ).

5. Continuum model critical shear surfaces have FS which are less than those determined using a mathematically defined shear surface shape in the limit-equilibrium models. However, the CLARA-W model results for this discussion were limited to only one search mode and in that sense, may not be reflective of the true critical shear surface and the associated FS, i.e. other search modes may identify critical shear surfaces with lower FS values.

6. FLAC3D, FLAC, and CLARA-W model results are consistent in themselves, i.e., computed factors of safety degrade as the loading conditions worsen, as in example problems 2 and 4.

7. It will be helpful to know the authors' views on (i) the continuum model results (FS and associated shear surface) for the four example problems, and (ii) their experiences in selecting 3-D shear surface geometry (shape and lateral extent in y-direction) for use in limit equilibrium based analyses.

It should be noted that Wright et al. (1973) and Adriano et al. (2008) used procedures similar to the one presented in the paper and assessed relatively little differences in the computed factors of safety over a range of Poisson's ratio values. Wright et al. models were two dimensional and the FS varied from about 1.93 to 2.05 (scaled values) for discrete values of υ from 0.3 to 0.49; Adriano et al. models were three dimensional and for plane slope, the FS varied from about 1.42 to 1.45 (scaled values) for discrete values of υ from 0 to 0.49.

References:

Adriano, P.R.R., Fernandes, J.H., Gitirana, G.F.N., and Fredlund, M.D. (2008). Influence of ground surface shape and Poisson's ratio on three-dimensional factor of safety. GeoEdmonton,, 2008.

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O. Hungr Geotechnical Research. 2010. User's manual for CLARA-W – Slope stability analysis in two or three dimensions for microcomputers. O. Hungr Geotechnical Research Inc., Vancouver, B.C.

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Figure captions

Fig. A-1. Verification example problem No. 1; continuum model results correspond to $\upsilon = 0.4$ (typical).
(a) FLAC3D model
(b) FLAC3D critical shear surface
(c) CLW 3D critical shear surface
(d) FLAC model
(e) FLAC critical shear surface
(f) CLW 2D Bishop's simplified critical shear surface

Fig. A-2 Verification example problem No. 2; continuum model results correspond to $\upsilon = 0.4$ and $\psi = 0$ combination with no-water-table loading condition (typical).
(a) FLAC3D model
(b) FLAC3D critical shear surface
(c) CLW3D critical shear surface
(d) FLAC model
(e) FLAC critical shear surface
(f) CLW2D critical shear surface

Fig. A-3 Verification example problem No. 3; continuum model results correspond to $\upsilon = 0.4$ and $\psi = 0$ combination (typical).
(a) 3-D model
(b) FLAC3D critical shear surface corresponding to $\upsilon = 0.40$ (typical)
(c) CLW3D critical shear surface
(d) 2-D model
(e) FLAC critical shear surface corresponding to $\upsilon = 0.40$ (typical)
(f) CLW2D critical shear surface

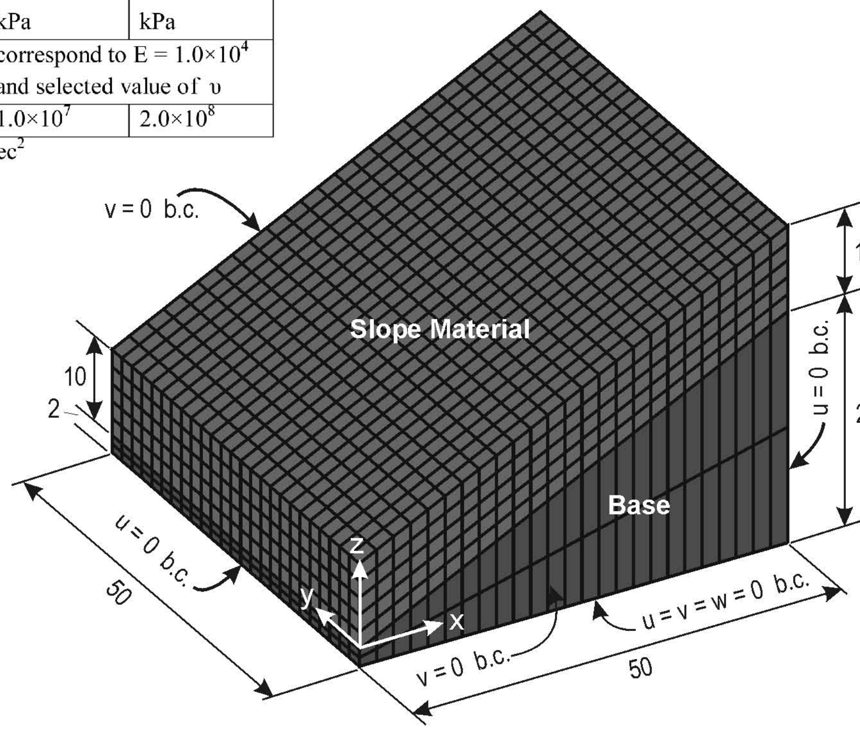
Fig. A-4 Verification example problem No. 4; continuum model results correspond to $\upsilon = 0.4$ and $\psi = 0$ combination (typical) for the marked surcharge load (q) condition.
(a) FLAC3D model
(b) FLAC3D critical shear surface for $q = 0$
(c) CLW3D critical shear surface for $q = 0$
(d) FLAC3D critical shear surface for $q = 55$ kN/m^2
(e) CLW3D critical shear surface for $q = 55$ kN/m^2
(f) FLAC model
(g) FLAC critical shear surface for $q = 0$
(h) CLW2D critical shear surface for $q = 0$
(i) FLAC3D critical shear surface for $q = 550$ kN/m^2

Example No. 1 Material properties

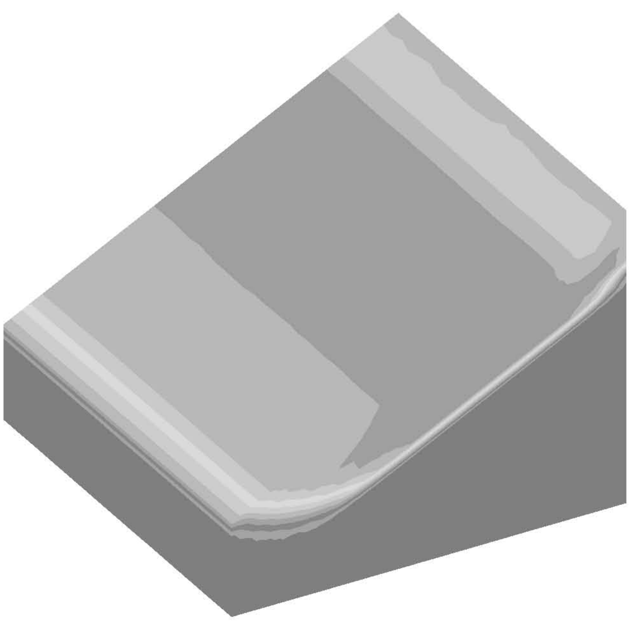
Material ID	$\rho \times 10^3$ kg/m^3	c kPa	φ °	G kPa	K kPa
Clay	1.92	45	0	correspond to $E = 1.0\times10^4$ and selected value of υ	
Base	4.40	1×10^4	45	1.0×10^7	2.0×10^8

Unit weight, $\gamma = \rho \times g$; $g = 9.81$ m/sec^2

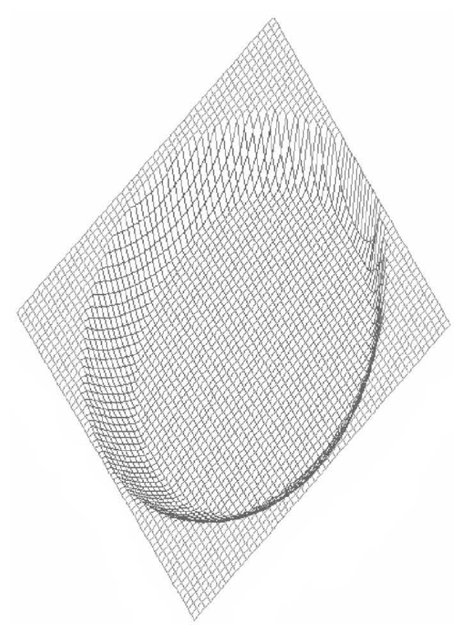
b.c. Boundary condition
All dimensions in *m*



a. FLAC3D model

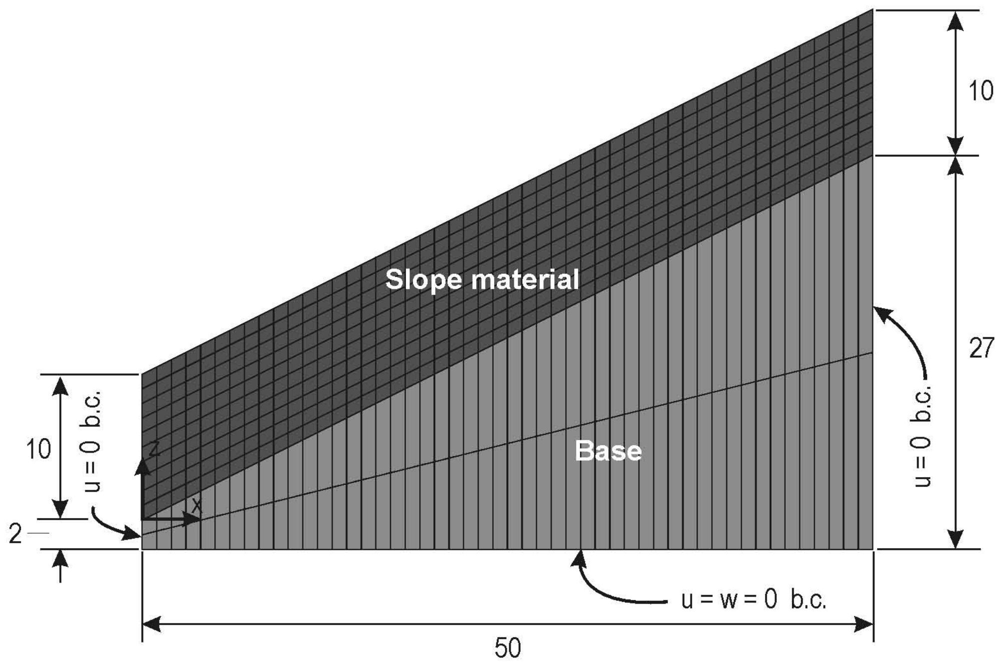


b. FLAC3D – critical shear surface

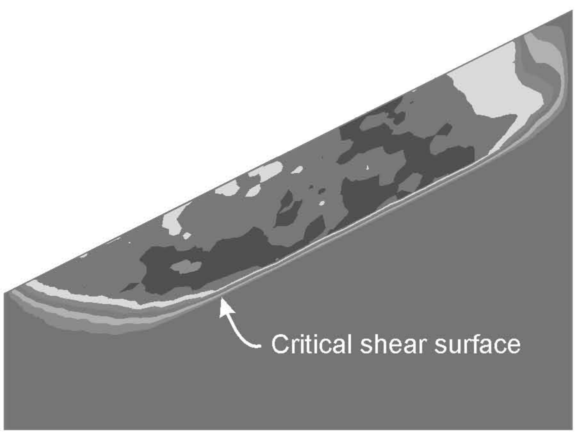


c. CLW3D – critical shear surface

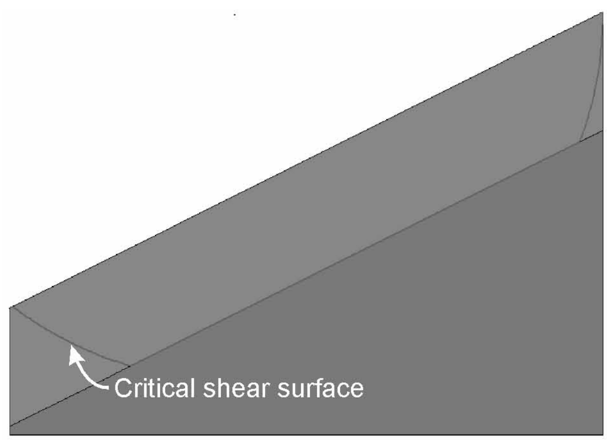
Figure A-1



d. FLAC model



e. FLAC – critical shear surface



f. CLW2D – critical shear surface

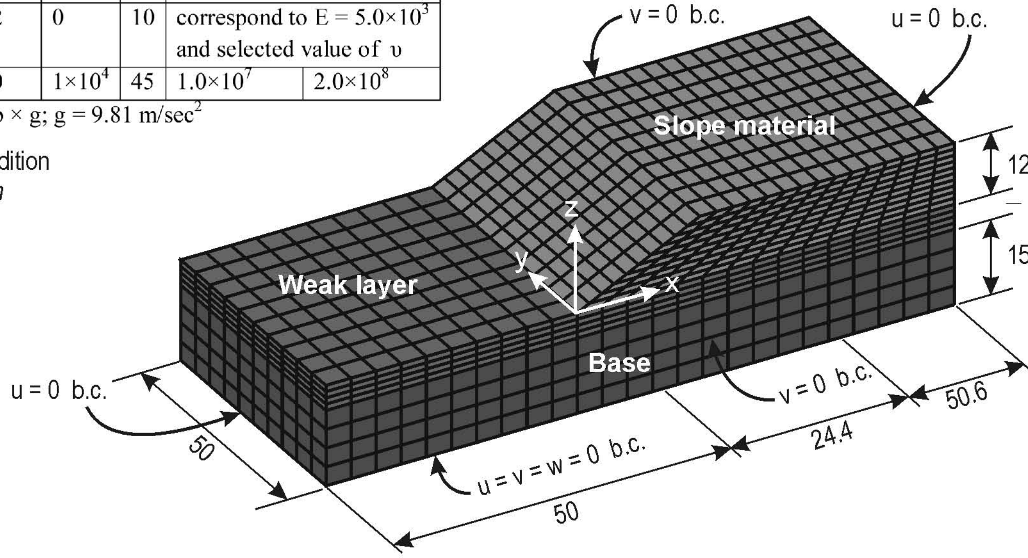
Figure A-1 -continued

Example No. 2 Material properties

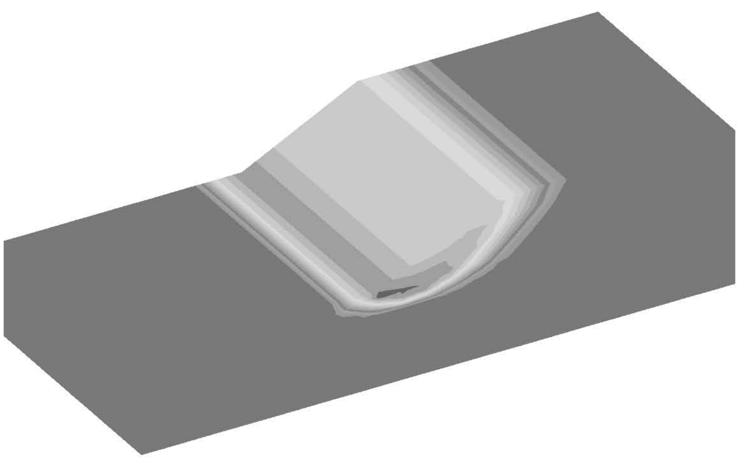
Material ID	$\rho \times 10^3$ kg/m^3	c kPa	φ °	G kPa	K kPa
Slope	1.92	28.73	20	correspond to $E = 1.5\times10^4$ and selected value of υ	
Weak layer	1.92	0	10	correspond to $E = 5.0\times10^3$ and selected value of υ	
Base	4.40	1×10^4	45	1.0×10^7	2.0×10^8

Unit weight, $\gamma = \rho \times g$; $g = 9.81$ m/sec^2

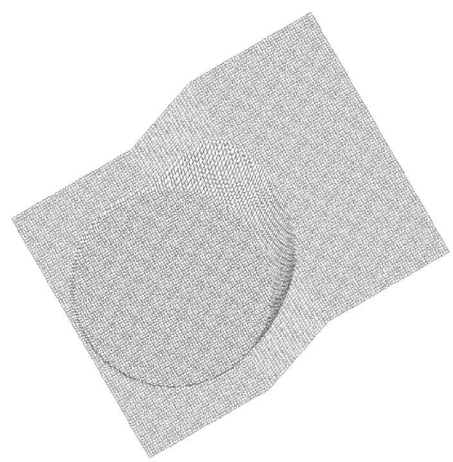
b.c. Boundary condition
All dimensions in *m*



a. FLAC3D model

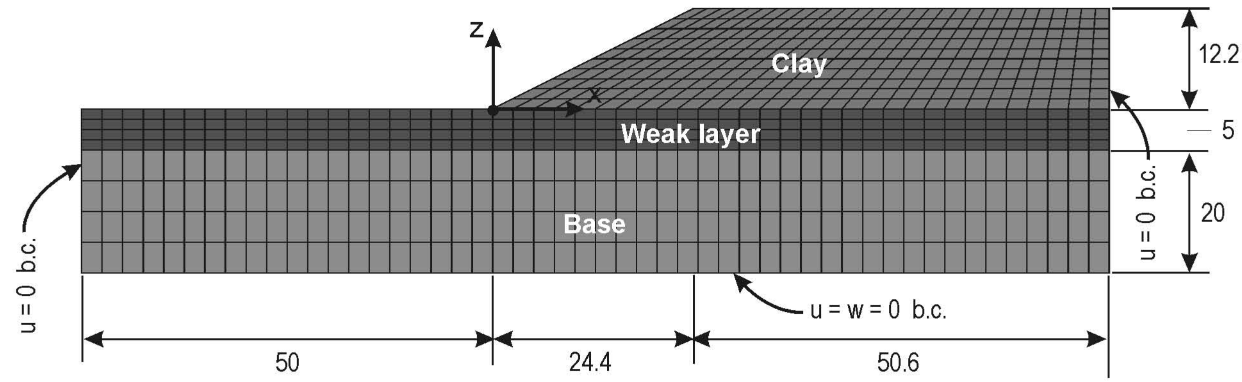


b. FLAC3D – critical shear surface

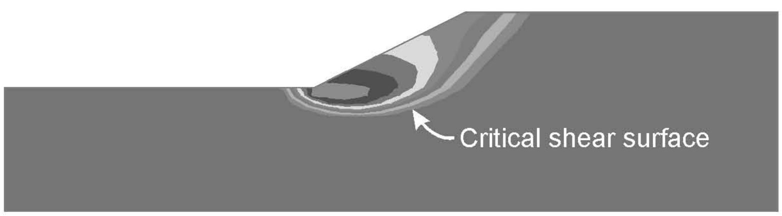


c. CLW3D – critical shear surface

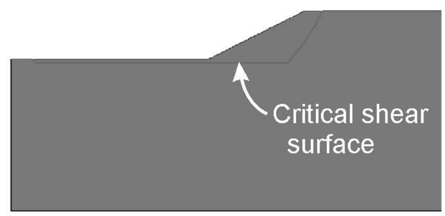
Figure A-2



d. FLAC model



e. FLAC – critical shear surface



f. CLW2D – critical shear surface

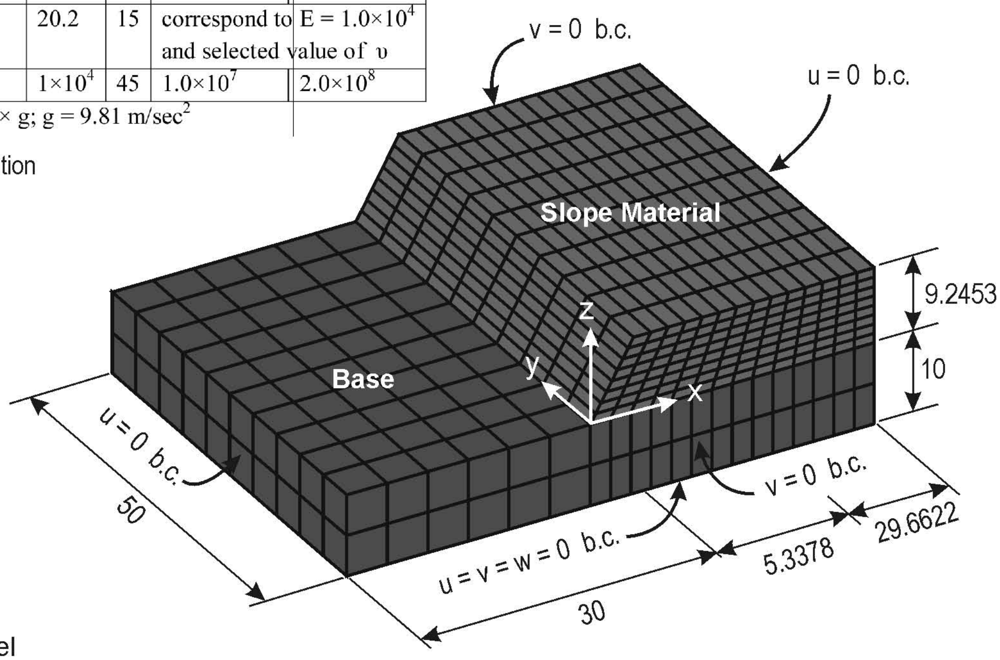
Figure A-2 -continued

Example No. 3 Material properties

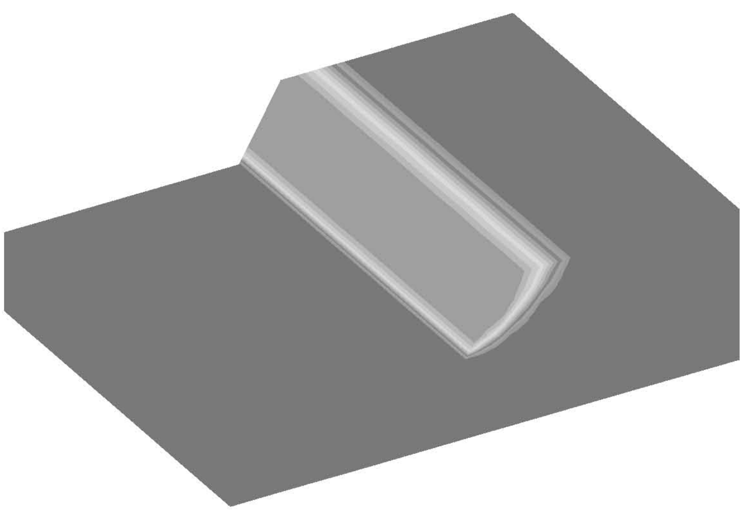
Material ID	ρ× 10^3 kg/m^3	c kPa	φ °	G kPa	K kPa
Slope	1.92	20.2	15	correspond to E = 1.0×10^4 and selected value of υ	
Base	4.40	1×10^4	45	1.0×10^7	2.0×10^8

Unit weight, γ = ρ × g; g = 9.81 m/sec^2

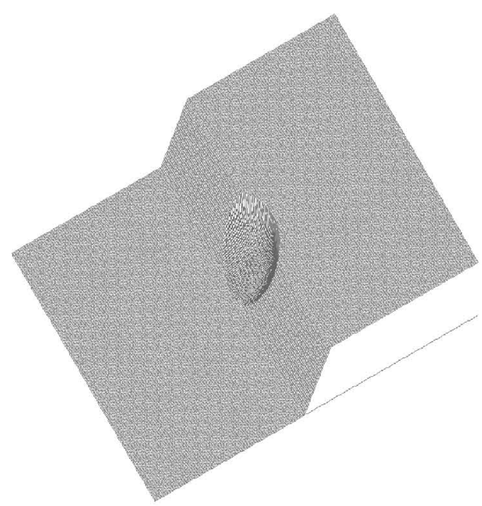
b.c. Boundary condition
All dimensions in *m*



a. FLAC3D model

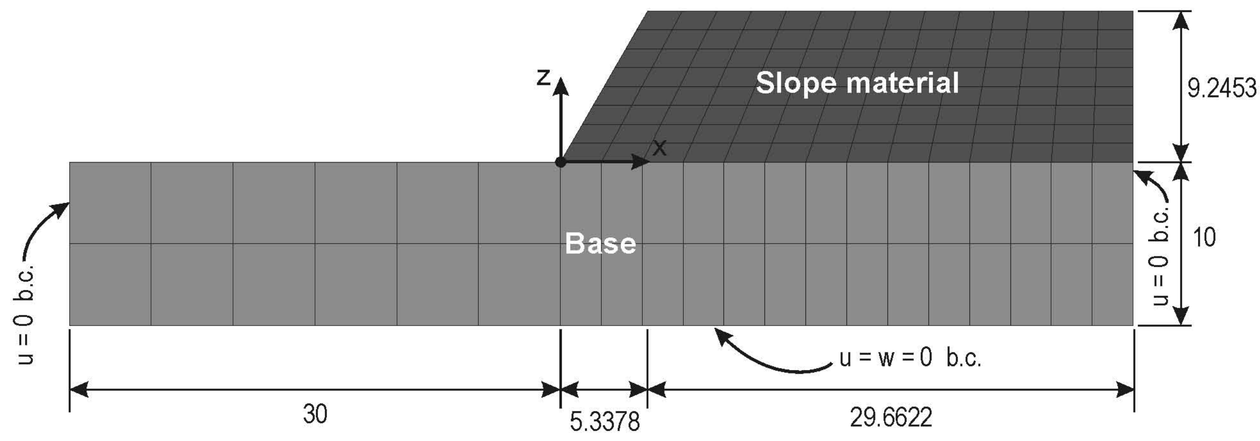


b. FLAC3D – critical shear surface

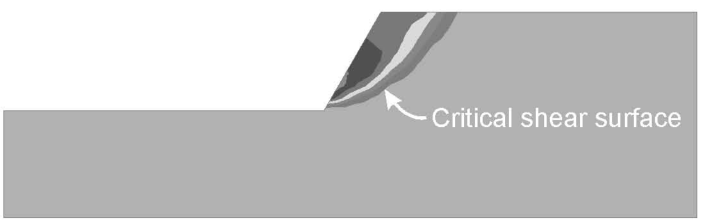


c. CLW3D – critical shear surface

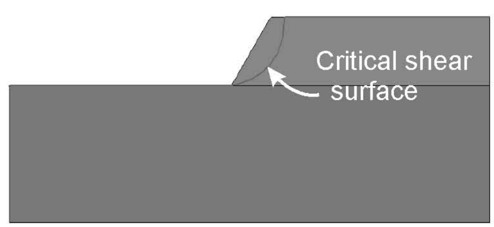
Figure A-3



d. FLAC model



e. FLAC – critical shear surface



f. CLW2D – critical shear surface

Figure A-3 -continued

Example No. 4 Material properties

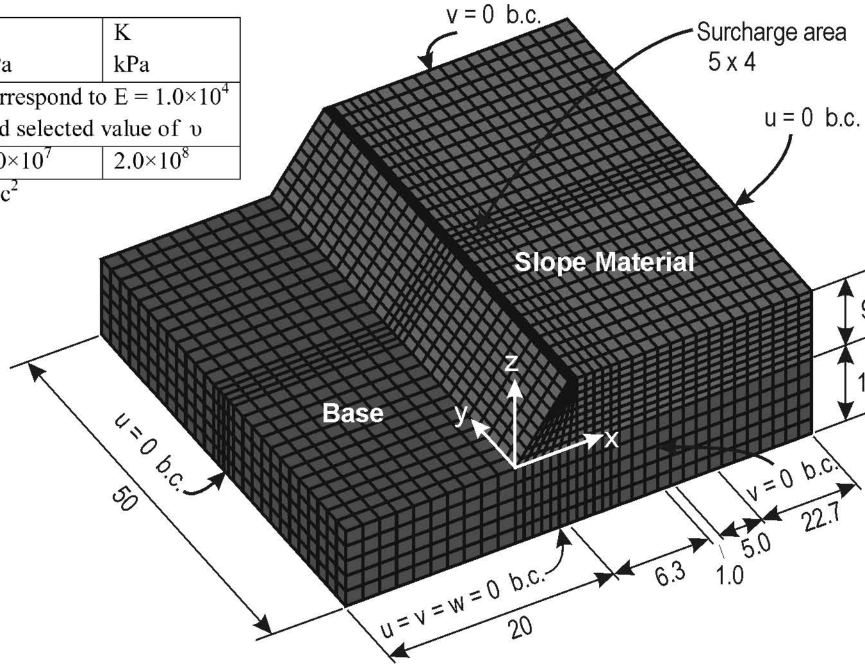
Material ID	ρ× 10^3 kg/m^3	c kPa	φ °	G kPa	K kPa
Slope	2.25	20.2	32.7	correspond to E = 1.0×10^4 and selected value of υ	
Base	4.40	1×10^4	45	1.0×10^7	2.0×10^8

Unit weight, γ = ρ × g; g = 9.81 m/sec^2

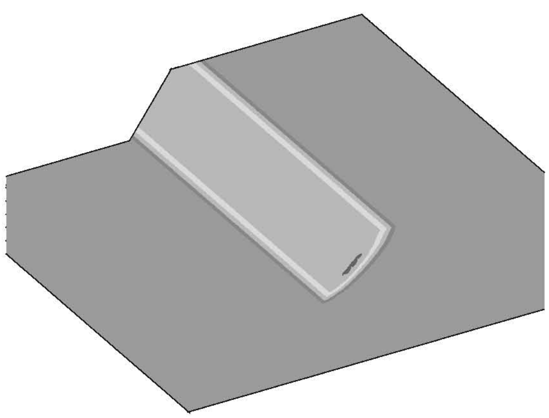
b.c. Boundary condition

q = surcharge load

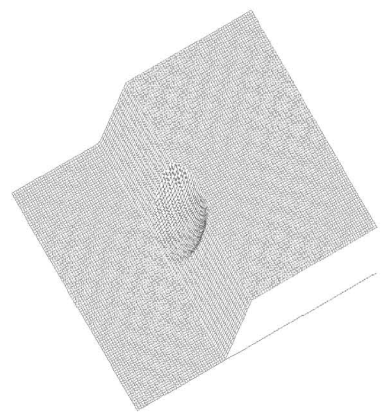
All dimensions in *m*



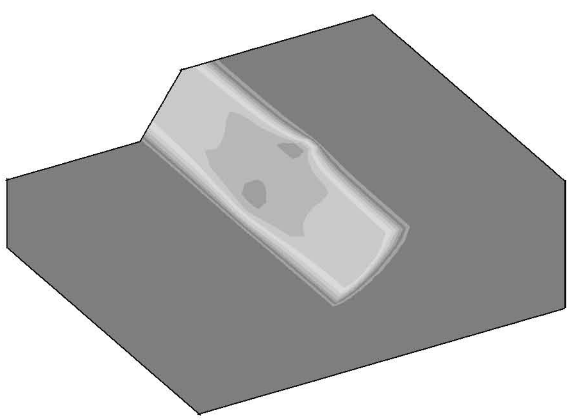
a. FLAC 3D model



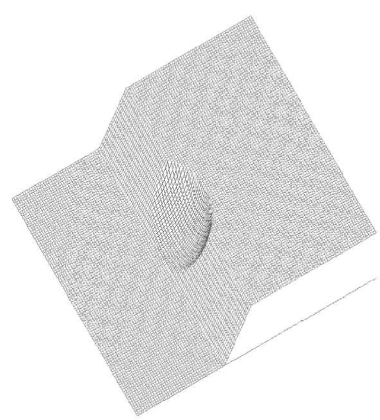
b. FLAC3D - critical shear surface; *q* = 0



c. CLW3D – critical shear surface; *q* = 0

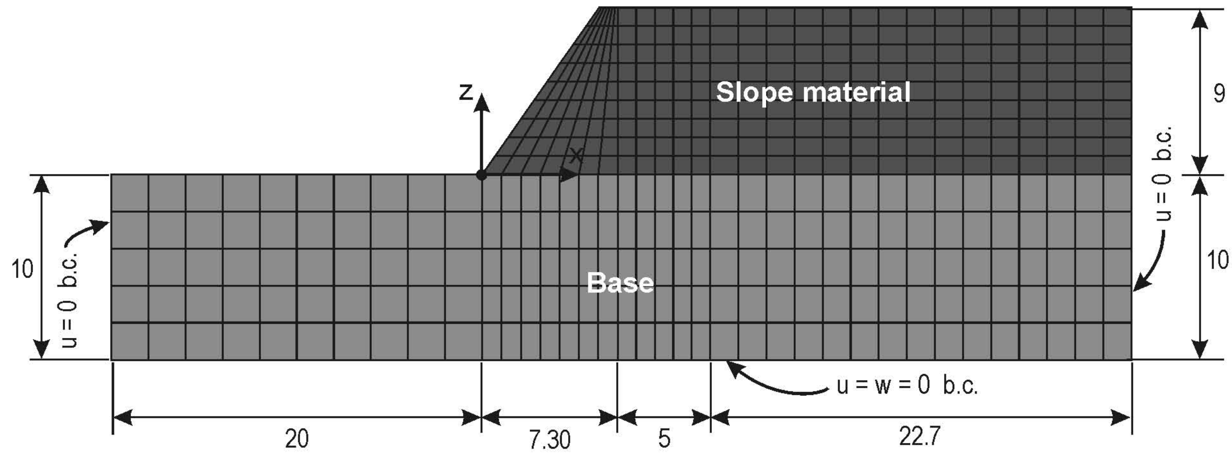


d. FLAC3D – critical shear surface; *q* = 55 kN/m^2

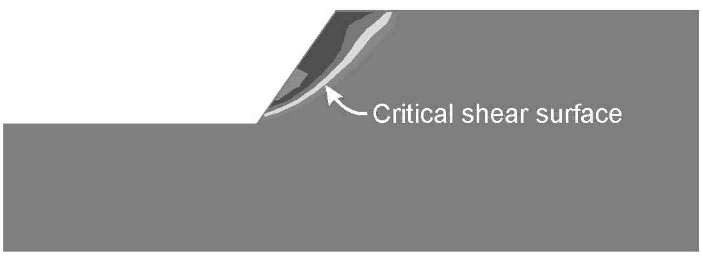


e. CLW3D – critical shear surface; *q* = 55 kN/m^2

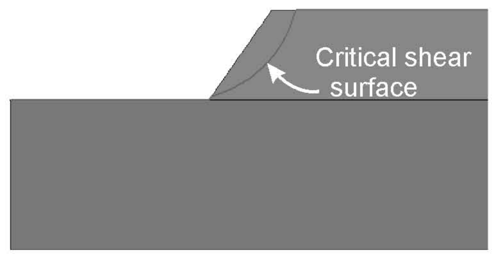
Figure A-4



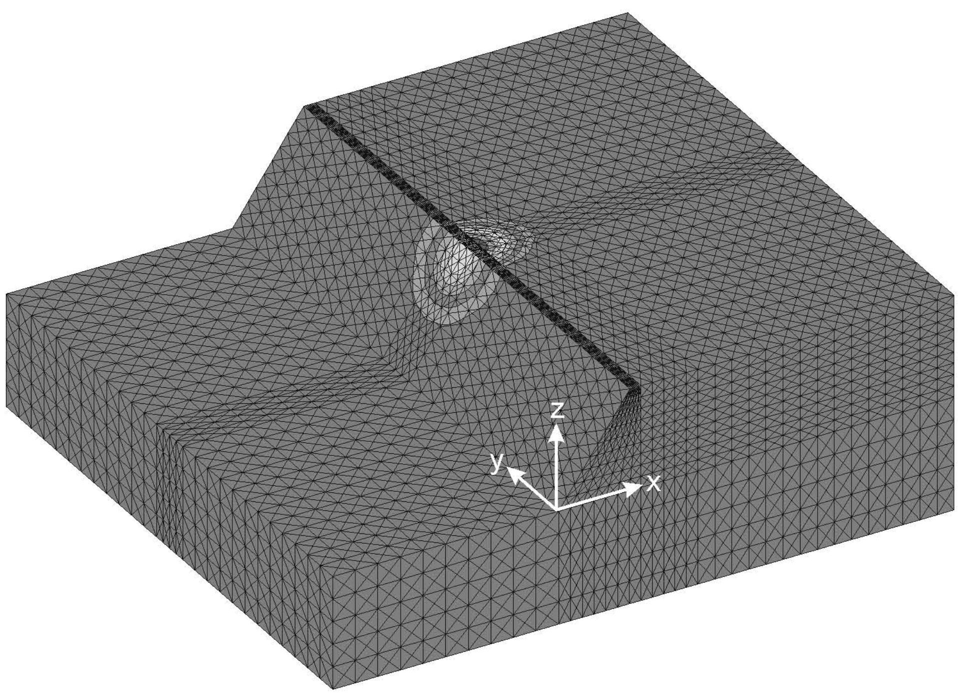
f. FLAC model



g. FLAC – critical shear surface; $q = 0$



h. CLW2D – critical shear surface; $q = 0$



i. FLAC3D – critical shear surface; $q = 550\ kN/m^2$

Figure A-4 -continued