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Report DSO-08-05

Determining Erosion Indices of Cohesive Soils with the Hole Erosion Test and Jet Erosion Test



Dam Safety Technology Development Program



U.S. Department of the Interior
Bureau of Reclamation
Technical Service Center
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14. ABSTRACT <p>Two methods of soil erodibility testing, the hole erosion test (HET) and submerged jet erosion test (JET), were investigated to determine the correlation between their results and to evaluate and improve them for potential application to the modeling of embankment dam erosion and breach processes. Basic assumptions regarding the behavior of the friction factor for flow through the predrilled hole in the HET were investigated, and it was found that the friction factor was best correlated with the hole diameter rather than the test time as previous investigators had assumed. This finding and others were used to develop improved HET testing and data analysis procedures including a method that does not require measurement of the final eroded hole diameter. The HET and JET methods were compared to one another by using them to determine erodibility parameters of identically prepared remolded soil specimens. The JET method indicated much greater erodibility in a direct comparison of quantitative results, which indicates that results of each test should be interpreted using criteria adapted to each particular test. Differences in erodibility were one or more orders of magnitude in erosion rate and two or more orders of magnitude in critical shear stress. The JET method seemed to be more sensitive to variations in soil fabric, and specimens with a coarse and nonuniform soil structure seemed to produce the greatest differences between HET and JET results. Differences between HET and JET results are also thought to include simplified stress descriptions in each test environment and fundamental differences in the mechanisms of erosion exploited by each test. The JET proved to be a more easily applied test method, with a higher ratio of successful tests and a greater ability to successfully test soils of widely varying erodibility. The JET also has the advantage of being suitable for <i>in situ</i> field testing wherever a soil surface of interest can be exposed. Ultimately, selection of a test for any particular purpose should be made primarily based on the application and the erosion mechanisms of importance.</p>					
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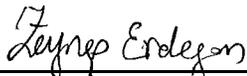
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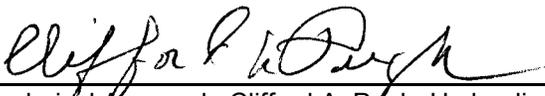
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Contents

	Page
Acknowledgments.....	iii
Electronic Distribution.....	iii
Disclaimer.....	iii
Executive Summary.....	1
Background.....	2
Hole Erosion Test.....	2
Submerged Jet Erosion Test.....	4
Interpretation of HET Data.....	6
Issues Affecting Interpretation of HET Data.....	9
Interpretation of JET Data.....	12
Issues Affecting Interpretation of JET Data.....	14
Experimental Objectives and Approach.....	14
Alternative Method for Interpreting HET Data.....	18
Results and Discussion.....	19
HET Friction Factor.....	19
Alternative Method for Interpreting HET.....	21
HET vs. JET Results.....	25
Paired Samples.....	25
HET and JET Erodibility versus Moisture Content.....	29
Discussion of HET and JET Differences.....	30
Practical Considerations.....	38
Erodibility Classifications and Scope of Test Capabilities.....	39
Applications for the HET and JET.....	41
Conclusions.....	42
References.....	43

Appendix—Hole Erosion Test Procedures Used by the Bureau of Reclamation

Tables

No.		Page
1	Qualitative description of rates of progression of internal erosion or piping for soils with specific erosion rate indices.....	3
2	Qualitative description of rates of progression of internal erosion or piping for soils with specific erosion rate indices.....	6

3	Properties of tested soils. Detailed gradation analyses were not available for some soils	17
4	Friction factors computed for hole erosion tests on soil 55T-160	20
5	Results of alternative methods for interpreting HET data	23
6	Summary of erosion indices and critical stresses determined by HET and JET methods.....	26
7	Erodibility test results for P2 and P3 soils over a range of compaction moisture contents	29

Figures

No.		Page
1	HET apparatus.	4
2	Laboratory JET apparatus.	5
3	Proposed erodibility classifications for streambank soils.....	7
4	Typical results of a hole erosion test. Chart (a) shows the time history of the test; (b) shows computed hole diameters and a third order polynomial curve fit that models the evolution of the hole diameter through time; (c) shows computed erosion rates and stresses over time, and (d) shows erosion rate versus shear stress. Flow condition was turbulent throughout this test (Re = 8,000 to 11,000).	9
5	An HET conducted at a test head that caused immediate progressive erosion. Flow was turbulent throughout this test (Re = 6,250 to 17,000). ..	11
6	Plaster casts of eroded holes illustrate difficulties in measuring final hole diameter following an HET.	12
7	Schematic of circular submerged jet with stress distribution and parameter definitions	13
8	Example charts illustrating interpretation of JET data.	15
9	Turbulent flow friction factor variation during HETs.	22
10	Comparison of I_{HET} values computed by alternative methods for interpreting hole erosion test data.	24
11	Comparison of computed values of I_{HET} and τ_c using different methods for interpreting HET data. Dashed lines connect pairs of data markers for each individual test. Chart (A) shows all tests, but values of $\tau_c = 0$ are not shown due to the use of a logarithmic scale. Chart (B) shows the subset of the tests in which critical stresses were less than 25 Pa; an arithmetic scale is used to allow plotting values of $\tau_c = 0$	24
12	Erosion rate index values obtained by HET and JET methods, ranked subjectively from most rapid to least rapid erosion rate.	27
13	Critical shear stresses obtained from HET and JET methods, ranked subjectively from most rapid to least rapid erosion rate (same ranking as fig. 12 above).....	27

14	Comparison of erosion rate indices determined by HET and JET methods, and the relationship found by Lim (2006) relating HET and rotating cylinder test (RCT) results for nondispersive soils.....	28
15	Comparison of critical stresses determined by HET and JET methods.....	28
16	Results of erodibility tests on soil P2 at compaction moisture contents ranging from about 4 percent dry of optimum to 4 percent wet of optimum.....	31
17	Results of erodibility tests on soil P3 at compaction moisture contents ranging from about 3 percent dry of optimum to 5.5 percent wet of optimum.....	32
18	Variation of erodibility for soils P2 and P3 as a function of compaction moisture content.....	33
19	Differences in soil fabric of P2 (a,b) and P3 (c,d,e) JET specimens.....	35
20	Example of nonlinear relation between erosion rate and applied stress.....	36
21	Variation of I_{HET} as a function of the duration of the progressive erosion phase of each test.....	37
22	Variation of critical stress obtained from HETs as a function of the duration of the progressive erosion phase of each test.....	37
23	HET and JET data collected by the Bureau of Reclamation in field and laboratory tests since 2007.....	40

Executive Summary

An extensive investigation was undertaken of two methods for quantifying erodibility of cohesive soils. The hole erosion test (HET) and submerged jet erosion test (JET) are two methods for determining soil erodibility that are promising for application to the field of embankment dam erosion and breach modeling. The HET measures the enlargement of a predrilled hole in a soil specimen subjected to flow through the hole under a controlled hydraulic head. The JET measures the depth of scour produced beneath a submerged jet impinging on an exposed soil surface.

For the HET, basic assumptions regarding the behavior of the friction factor for flow through the predrilled hole were investigated, and it was found that the friction factor was best correlated with the hole diameter rather than the test time as previous investigators had assumed. This finding was also confirmed in a study whose results became known to the principal investigator during the latter stages of this project. Criteria for discriminating between laminar and turbulent flow during HET data analysis were also revised, and an alternative data analysis method was investigated that does not require measurement of the final diameter of the eroded hole. The combined test procedure and data analysis improvements and the construction of a new high-head HET facility have greatly improved the Bureau of Reclamation's capability to perform hole erosion tests.

The HET and JET methods were compared to one another by using them to determine erodibility parameters of identically prepared soil specimen pairs. The JET method indicated much greater erodibility in a direct comparison of quantitative results, which indicates that results of each test should be interpreted using criteria adapted to each particular test. Differences of one order of magnitude or more were observed in determined erosion rate coefficients and differences of two or more orders of magnitude were observed in the critical shear stress needed to initiate erosion. The JET method also seemed to be more sensitive to variations in soil fabric; for specimens with a coarse and nonuniform soil structure, the differences between HET and JET results seemed most pronounced. The reasons for differences between HET and JET results are also thought to include simplified stress descriptions in each test environment and fundamental differences between the important erosion mechanisms in each test.

Given the differences in test results, the application of the erodibility data should be the primary criteria for choosing a test. The HET probably is the best test when one is trying to understand erosion through small holes or confined cracks, whereas the JET is probably best for studying erosion in larger, developing erosion pipes and overtopping flow. Identifying the transition between best applicability of the two tests is still a subject for further research.

The experience gained through this research has shown that the JET is a more easily applied test method, with a higher ratio of successful tests and a greater ability to successfully test soils of widely varying erodibility. Beyond these practical advantages, the JET is also suitable for *in situ* field testing wherever a soil surface of interest can be exposed. In the laboratory, both methods are suitable for testing of remolded samples or undisturbed tube samples.

Background

The hole erosion test (Wan and Fell 2004) and the jet erosion test (Hanson and Cook 2004) are two of several available methods for evaluating the erodibility of cohesive soils. The hole erosion test (HET) utilizes an internal flow through a hole predrilled in the specimen, similar to that occurring during piping erosion of embankment dams, while the jet erosion test (JET) utilizes a submerged jet to produce scouring erosion, similar to that which might occur at a headcut or a free overfall. As presently performed and interpreted, both tests determine a critical shear stress needed to initiate erosion and a coefficient that defines the rate of erosion per unit of applied excess stress. The similarities and differences between the tests have prompted this investigation into the methods for performing the tests and a comparison of their results. These two tests are attractive because of the relative simplicity of each apparatus and the fact that the equipment and procedures are fully described in the literature and can be duplicated freely. Both tests have been performed widely in recent years, and the erosion parameters measured with them are being utilized in numerous applications, including numerical models for overtopping erosion and tool boxes for estimating risks related to internal erosion of embankment dams. Prior to this study, no detailed comparison of these two tests had been made.

Hole Erosion Test

The hole erosion test is conducted in the laboratory using an undisturbed tube sample or a soil specimen compacted into a Standard Proctor mold. A 6-mm diameter hole is predrilled through the centerline axis, and the specimen is then installed into a test apparatus in which water flows through the hole under a constant hydraulic head that is increased incrementally until progressive erosion is produced. (Lefebvre et al. [1984] described a hole erosion test performed with a constant flow rate under varying head, with a more detailed data collection and analysis procedure). Once erosion is observed, the test is continued at a constant hydraulic head for up to 45 minutes, or as long as flow can be maintained. Measurements of the increasing flow rate during the test and the initial and final diameter of the erosion hole are used to compute applied hydraulic stress and the

erosion rate. As presently conducted, significant post-test work is performed to obtain the measurement of the final hole diameter.

HET data are analyzed to determine two parameters of a basic detachment-driven erosion equation describing the growth of the erosion hole:

$$\dot{m} = C_e(\tau - \tau_c)$$

where,

\dot{m} = the rate of mass removal per unit of surface area (kg/s/m²)

τ and τ_c = the applied shear stress and threshold shear stress for soil detachment, respectively

C_e = a proportionality constant, often called the coefficient of soil erosion

The equation applies only for $\tau > \tau_c$; otherwise, the erosion rate is zero. Values of C_e in S.I. units are kg/s/m²/Pa, which reduces to seconds per meter (s/m). The hole erosion test (HET) and a companion slot erosion test (SET) have been developed and refined in Australia (Wan and Fell 2004), and the HET has been studied further at the Bureau of Reclamation (Reclamation). The coefficient of soil erosion varies over several orders of magnitude in soils of engineering interest. For convenience, a second parameter, the erosion rate index (I_{HET}) is often computed:

$$I_{HET} = -\log_{10} C_e$$

with C_e in units of s/m. Typical values of this index range from 1 to just above 6, with larger values indicating decreasing erosion rate or increasing erosion resistance. The fractional part of the index is often dropped and the test result reported as a simple integer group number for erosion resistance. Soils with group numbers less than 2 are usually so erodible that they cannot be effectively tested in the HET device. Table 1 shows proposed descriptive terms associated with the I_{HET} index.

Table 1.—Qualitative description of rates of progression of internal erosion or piping for soils with specific erosion rate indices

Group number	Erosion rate index, I_{HET}	Description
1	< 2	Extremely rapid
2	2-3	Very rapid
3	3-4	Moderately rapid
4	4-5	Moderately slow
5	5-6	Very slow
6	> 6	Extremely slow

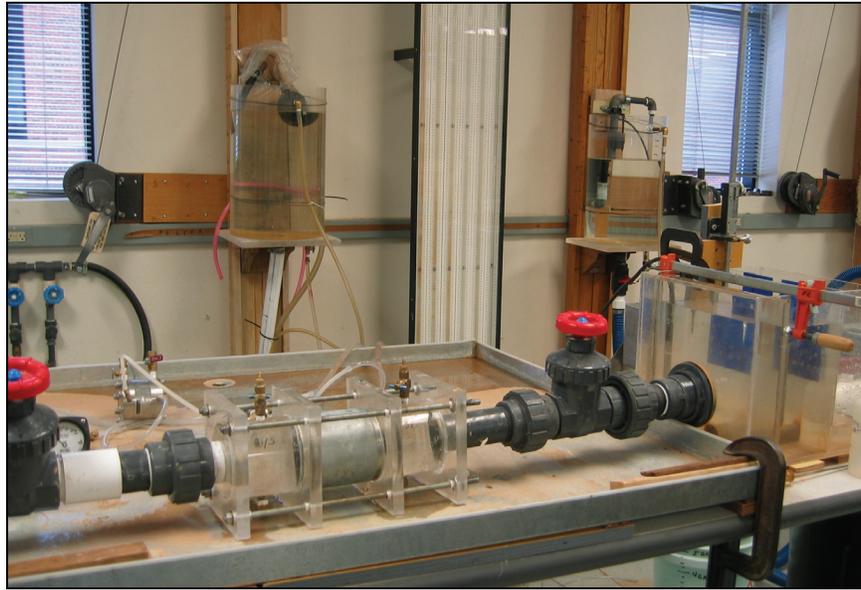


Figure 1.—HET apparatus.

Figure 1 shows the HET apparatus installed in the Bureau of Reclamation soils laboratory in Denver, Colorado. Flow rate through the specimen is measured by a custom V-notch weir on the downstream side of the apparatus. The weir is calibrated in place by volumetric methods (stopwatch and graduated cylinder). Measurements of differential head across the specimen and head on the weir are automated using pressure transducers and a computerized data acquisition system that records data at 5-second intervals throughout a test. In the course of this project, the apparatus and data collection procedures were substantially improved, especially the flow measurement method, which previously utilized a weigh tank with a periodic digital output that provided insufficient measurement precision. The maximum head that can be applied in the apparatus shown in figure 1 is about 1,600 mm. During the course of this study, a new high-head HET facility was constructed in Reclamation's hydraulics laboratory, where a higher ceiling makes it possible to produce test heads up to about 5,400 mm. Both facilities operate with water originating from the tap. Water in the hydraulics lab sump is treated by an ozonator and stays in residence for long periods of time, making it essentially chlorine free.

Submerged Jet Erosion Test

The submerged jet erosion test was developed at the Agricultural Research Service Hydraulic Engineering Research Unit, Stillwater, Oklahoma (Hanson and Cook 2004). This test can be performed *in situ*, or in the laboratory using tube samples or remolded samples in compaction molds (Hanson and Hunt 2006) (fig. 2). Testing has been successfully carried out on specimens as small as

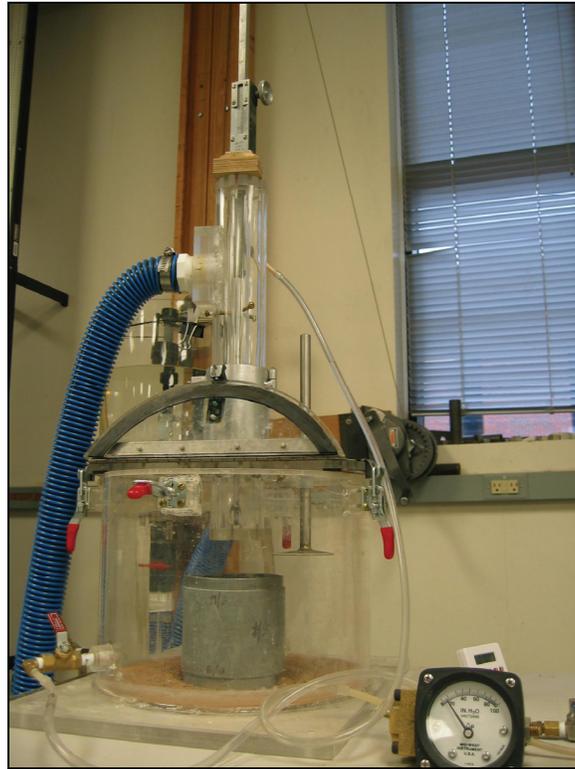


Figure 2.—Laboratory JET apparatus.

75 mm (3 inches) in diameter, but a minimum specimen size has not been firmly established. The test is described in ASTM standard D5852.

The JET apparatus is designed to attack the soil surface with a submerged jet, which is produced by a 6.35-mm ($\frac{1}{4}$ -inch) diameter nozzle initially positioned between 6 and 30 nozzle diameters from the soil surface. The starting nozzle position and test head may be adjusted to vary the stress applied to the soil sample, although once a test head is selected it is usually held constant for the duration of a test. Scour of the soil surface beneath the jet is measured over time (usually up to 2 hours) using a point gauge aligned with the axis of the jet. No post-test handling or processing of the specimen is needed. The jet is typically vertical, but can also be positioned at an angle when performing an *in situ* test of an inclined soil surface (Hanson et al. 2002). Data from the JET have typically been analyzed using a volumetric form of the same erosion model used to analyze HET data:

$$\dot{\epsilon} = k_d (\tau - \tau_c)$$

where,

$\dot{\epsilon}$ = the volume of material removed per unit surface area per unit time
($\text{m}^3/\text{s}/\text{m}^2$, or m/s)

k_d = a detachment rate coefficient

Typical units for k_d are $\text{m}^3/\text{s}/\text{m}^2/\text{Pa}$, which reduces to $\text{m}/\text{s}/\text{Pa}$ or $\text{m}^3/\text{N}\cdot\text{s}$ in S.I. units; k_d is also commonly reported in $\text{cm}^3/\text{N}\cdot\text{s}$, or when working in U.S. customary units, k_d is usually expressed in $\text{ft}/\text{hr}/\text{psf}$ ($1 \text{ cm}^3/\text{N}\cdot\text{s} = 0.5655 \text{ ft}/\text{hr}/\text{psf} = 10^{-6} \text{ m}^3/\text{N}\cdot\text{s}$). Values of C_e and k_d can be compared by recognizing that $C_e = k_d \cdot \rho_d$, where ρ_d = dry density of the soil.

Figure 2 shows the laboratory JET apparatus installed in the Bureau of Reclamation soils laboratory in Denver, Colorado. Data are collected manually during a test using the procedures described by Hanson and Cook (2004). The top portion of the device (jet tube and lid) can also be installed onto a metal submergence tank for field use.

Hanson and Simon (2001) have proposed a qualitative classification of the erodibility of soils, similar to that suggested by Wan and Fell (2004) for the HET. Their classification scheme identifies five erodibility groupings, illustrated in figure 3. It uses both the k_d and τ_c value of the soil, in contrast to Wan and Fell's approach of using just the erosion rate index to classify soils in terms of the rate at which an internal erosion or piping event might progress.

Hanson (personal communication) has also suggested a six-tier classification system shown in table 2, which is based only on the k_d value expressed in units of $\text{ft}/\text{hr}/\text{psf}$. The conversion to $\text{cm}^3/(\text{N}\cdot\text{s})$ is of the order of 2, and since the classifications are based on order of magnitude ranges of k_d , one could argue that a classification system using similar numerical divisions would also be appropriate for k_d values expressed in $\text{cm}^3/(\text{N}\cdot\text{s})$.

Table 2.—Qualitative description of rates of progression of internal erosion or piping for soils with specific erosion rate indices

k_d , (ft/hr)/(lb/ft ²)	Description
> 10	Extremely erodible
1–10	Very erodible
0.1–1	Moderately erodible
0.01–0.1	Moderately resistant
0.001–0.01	Very resistant
<0.001	Extremely resistant

Interpretation of HET Data

The hole erosion test developed by Wan and Fell (2004) is performed by starting flow through the predrilled hole at a low test head (usually 50 mm of water). At Reclamation, if erosion is not observed, standard procedure has been to repeatedly double the test head to 100, 200, 400, 800, and 1,600 mm of water until

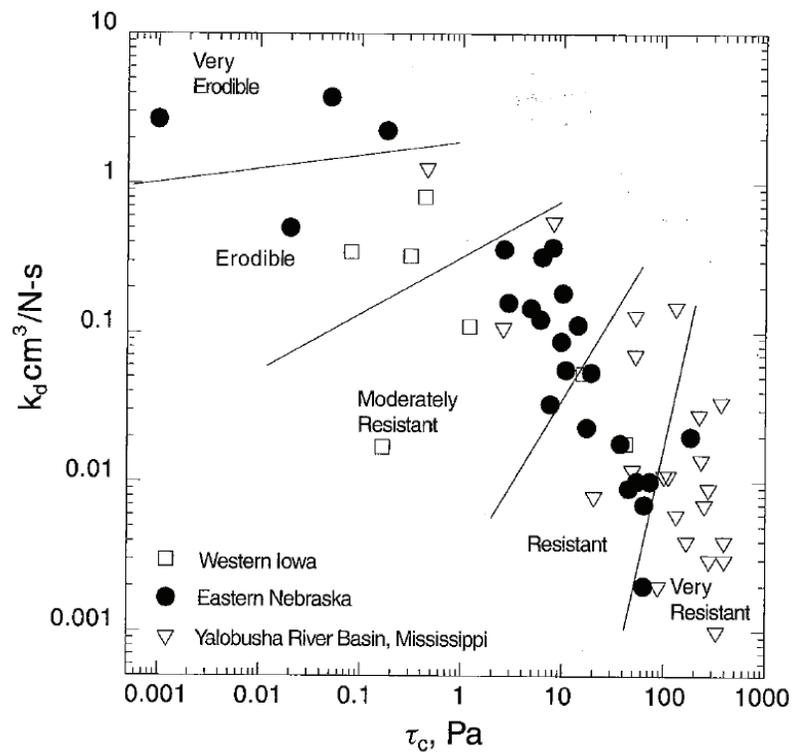


Figure 3.—Proposed erodibility classifications for streambank soils (Hanson and Simon 2001).

accelerating erosion occurs in the predrilled hole. Once erosion is observed, the test head is maintained for as long as possible up to 45 minutes while the hole enlarges. Flow rates and hydraulic gradients are monitored continuously during a test, and the initial and final hole diameters are also measured. Measurement of the final hole diameter is often complicated by irregularity of the hole, especially around the entrance and exit where the soil often caves and spalls off from the face of the specimen or scours due to eddies at the entrance and exit. These problems are more common when testing weaker soils. End plates with an orifice opening of 15 or 25 mm are sometimes helpful to reduce these problems. A number of methods have been tested in an effort to bring consistency to this part of the data analysis, including averaging of hole diameters measured by calipers from dried and cut specimens, measurement of diameters from plaster (hydrostone) castings of eroded holes, and measurements of water displacement of plaster castings.

The procedures developed by Wan and Fell (2004) analyze data from hole erosion tests in a deterministic way. Using the hole diameters and flow rates at the start and end of the test, friction factors for laminar and turbulent flow are computed, and these friction factors are then assumed to vary linearly with time during the course of the test. Once the friction factors are known, the flow rates and differential heads measured during the test are used to compute hole diameters at

Determining Erosion Indices of Cohesive Soils with the Hole Erosion Test and Jet Erosion Test

intermediate times. The appendix provides details of the equations used to make these calculations. Using the computed time series of hole diameters, a polynomial function relating the hole diameter to time is determined, and its time derivative defines the rate of erosion, $\dot{\epsilon}$. Shear stresses along the walls of the eroding hole can also be computed once the hole diameters have been determined. As the hole diameter increases, the shear stress increases if the hydraulic head is held constant. This leads to an accelerating flow rate, indicating a progressive erosion process. Plotting the erosion rate versus the computed shear stress *during the period of progressive erosion* produces a chart that graphically shows the coefficient of soil erosion and the critical shear stress. Figure 4 shows results from a typical successful test. It is important to emphasize that the determination of erodibility parameters should consider only the data collected during the period of progressive erosion. Past practice at Reclamation was to include all data (even those collected at low head during periods of no erosion) in the curve-fitting analysis, which often produced erroneous results.

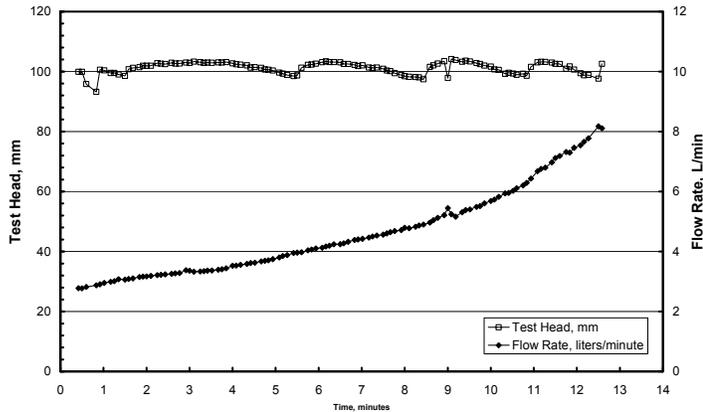
During a typical HET, several flow and erosion regimes can occur, depending on how the test is run. If the test is begun at a low differential head that applies a stress less than the critical shear stress of the intact soil, little or no erosion occurs initially. The erosion that does occur is localized around the hole entrance or is the removal of material disturbed during the drilling of the hole; in this phase of the test, the rate of erosion drops with time as the disturbed material is removed and the hole stabilizes. This can be seen in the data plotted in figure 4(d) at shear stresses below 24 N/m^2 , where the erosion rate decreases with time while the stress is increasing slowly with time (since the hole diameter is increasing slowly). This is opposite to the expected result that erosion rate should increase as shear stress increases.

To enter a progressive (accelerating) erosion phase, one of three things must occur. First, if the initial head produces only slightly less than the required critical shear stress, the low initial “cleanout” erosion may slowly increase the hole diameter, causing an increase in hydraulic stress so that it eventually exceeds the erosion threshold. Second, the influence of time and gradual saturation of the surface of the hole might lead to progressive erosion, especially if the soil is dispersive. Finally, the hydraulic gradient can be increased enough to immediately raise the shear stress above the critical value. In the test shown in figure 4, the test head was held constant, so one of the first two situations must have occurred. Some tests are begun at a test head that is sufficiently high to entirely skip the cleanout phase of the test.

In the progressive erosion phase of the test, the critical shear of the material is exceeded and erosion occurs, causing an increase in flow rate. Both the increasing hole diameter and increasing flow rate cause the shear stress to increase further, producing accelerating erosion. The slope of the erosion versus shear stress curve in this phase defines the coefficient of soil erosion, and the x-intercept of the line through these data points indicates the critical shear stress

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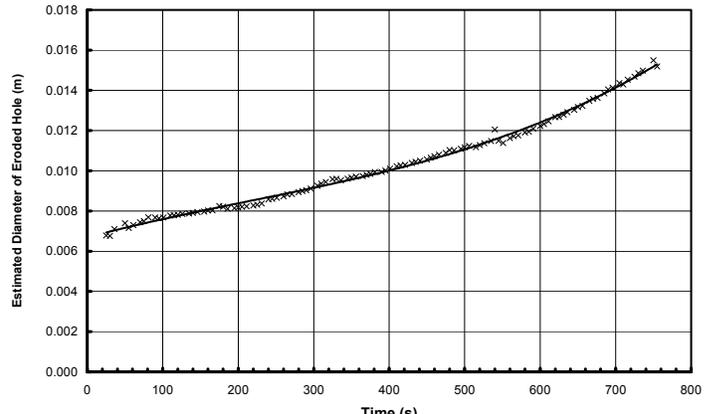
HET Test Record



(a)

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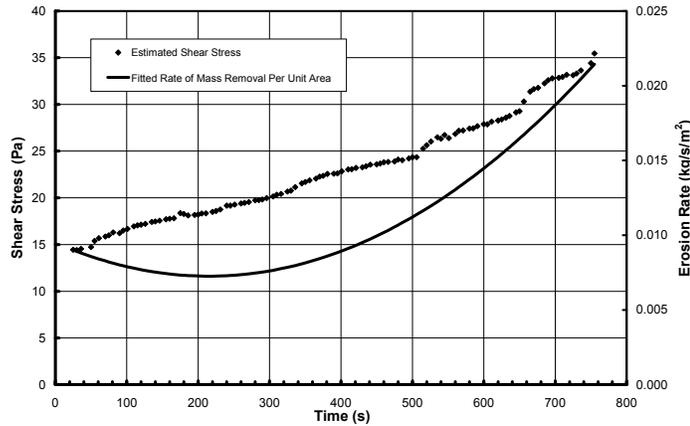
COMPUTED DIAMETER OF ERODED HOLE



(b)

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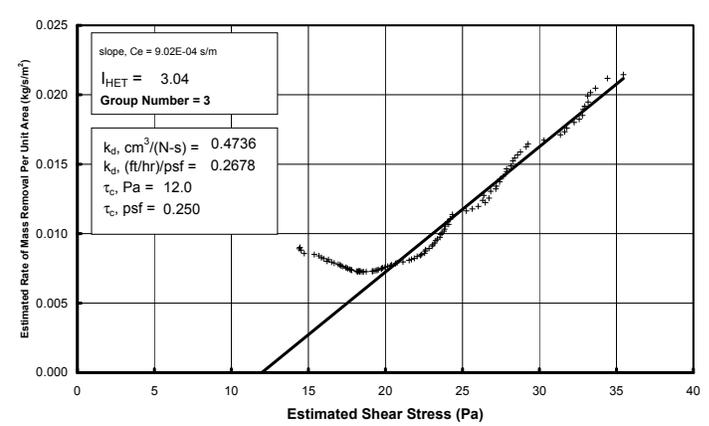
EROSION RATE AND SHEAR STRESS VS. TIME



(c)

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EROSION RATE VS. SHEAR STRESS



(d)

Figure 4.—Typical results of a hole erosion test. Chart (a) shows the time history of the test; (b) shows computed hole diameters and a third order polynomial curve fit that models the evolution of the hole diameter through time; (c) shows computed erosion rates and stresses over time, and (d) shows erosion rate versus shear stress. Flow condition was turbulent throughout this test ($Re = 8,000$ to $11,000$).

[fig. 4(d)]. In some cases, the x-intercept is negative, and the critical shear stress is then reported as zero. It should be emphasized that the HET is a test that begins with a stress that is low and increases with time.

Issues Affecting Interpretation of HET Data

Several issues make the interpretation of HET data problematic. The most important of these are:

Determining Erosion Indices of Cohesive Soils with the Hole Erosion Test and Jet Erosion Test

- Curve-fitting procedures
- Identification of erosion regimes
- Laminar versus turbulent flow
- Variation of the friction factor
- Determination of final hole diameter

Curve-fitting procedures

For a test such as that shown in figure 4 where the initial head is insufficient to cause immediate progressive erosion but progressive erosion begins without any further increase in test head, it is usually most effective to model the evolution of hole diameter over time using a third order polynomial function. This produces a second order polynomial (parabolic) erosion rate over time, and the characteristic v-shaped erosion rate versus shear stress plot [fig. 4(d)]. When the initial head produces immediate progressive erosion, a second order polynomial usually produces a better and more realistic model of the hole diameter versus time, which causes the erosion rate to be a first order (linear) function of time. Figure 5 shows the results from such a test. The need to adjust curve-fitting procedures to fit the manner in which a test is conducted is a solvable problem, but adds complexity to the overall test and analysis procedure.

Identifying erosion regimes

When a test is performed with an initially low head that is increased until accelerating erosion is observed, the early part of the test and a significant portion of the data are collected during times of no erosion, or during the phase of “cleanout” erosion of material disturbed by the hole-drilling process. These data may include several different head conditions if a test was started at a very low head. Tests performed at the Bureau of Reclamation prior to this project often utilized these data for the analysis. These data should, in fact, be neglected; only the data collected during the initiation and continuation of progressive erosion are useful for defining the relation between erosion rate and shear stress of the intact soil. In tests in which the accelerating erosion phase is not reached or is of inadequate duration, it may be impossible to obtain a meaningful test result.

Laminar versus turbulent flow

Distinguishing between laminar and turbulent flow is important for the deterministic analysis method. In the procedure developed by Wan and Fell (2004), both laminar and turbulent friction factors are computed at the start and end of the test, even though flow conditions at each point can be in only one state or the other. Interpolation is performed through time using these “virtual” friction factors (virtual because they are hypothetical and never physically existed), a practice that is difficult to justify. Also, Wan and Fell (2004) assume the transition to turbulent flow occurs at a Reynolds number of 5,000; a value of 1,000 to 2,000 is recognized as the traditional transition range in most fluid

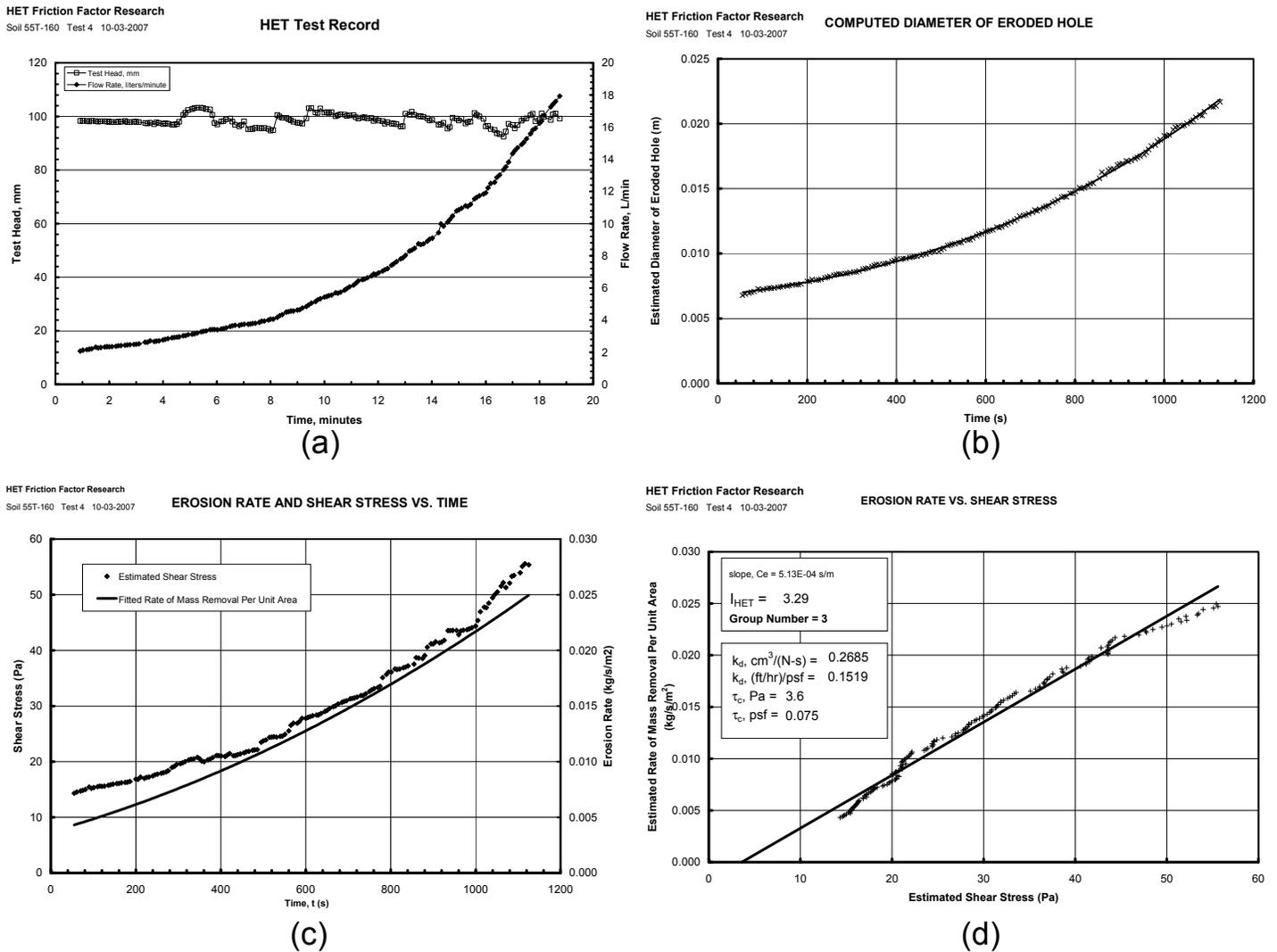


Figure 5.—An HET conducted at a test head that caused immediate progressive erosion. Flow was turbulent throughout this test ($Re = 6,250$ to $17,000$).

mechanics texts (e.g., Roberson and Crowe 1985). The value of 5,000 may have been selected because in the original hole erosion test performed at constant flow rate, friction losses through the hole were modeled with the Colebrook-White equation, which applies to turbulent flows at Reynolds numbers greater than 5,000 (Rohan et al. 1986). For all tests described in this report, turbulent flow was deemed to exist when the Reynolds number exceeded 2,000.

Variation of the friction factor

Behavior of the friction factor during an HET is poorly understood, but has important implications in the deterministic analysis. Friction factors tend to increase during most tests, and the assumption that they increase linearly with time causes one to compute erosion in the early stages of a test, even though other factors (steady flow rate under a steady test head, no visible turbidity in outflow)

Determining Erosion Indices of Cohesive Soils with the Hole Erosion Test and Jet Erosion Test

often suggest that none is occurring (see also, Lim 2006). Inaccurate modeling of the friction factor variation can also lead to unrealistic “jumps” in the computed hole diameter when the hydraulic gradient is increased, or when the flow states changes from laminar to turbulent or vice versa. These jumps are evidence that the friction factor is being incorrectly modeled.

Determination of final hole diameter

Figure 6 shows plaster castings of erosion holes produced in several different HETs conducted on a variety of soils. Some holes are relatively uniform in size throughout their length while others exhibit dramatic variation. Some of the castings also show that there is increased loss of material at the entrances and exits of the holes, which may not be due to erosion caused simply by hydraulic shear stress, but may include gravitational effects that lead to sloughing and spalling of soil from the faces of the specimens. These factors make objective determination of the final hole diameter difficult in some cases. Efforts to minimize the loss of material at the entrance and exit of the hole have been generally unsuccessful at the Bureau of Reclamation. Protecting the sample with upstream and downstream plates with predrilled orifices larger than the initial hole diameter has been marginally effective, but trial and error is often needed to determine an orifice size that is large enough to allow unimpeded progressive erosion but small enough to effectively protect the faces of the sample.

Interpretation of JET Data

The interpretation of data from the jet erosion test is described by Hanson and Cook (2004) (fig. 7). It begins with a description of the hydraulic stress produced by the jet. In a potential core close to the jet (less than 6 diameters from the nozzle), the jet velocity is uniform, and the stress is at a maximum. Beyond



Figure 6.—Plaster casts of eroded holes illustrate difficulties in measuring final hole diameter following an HET.

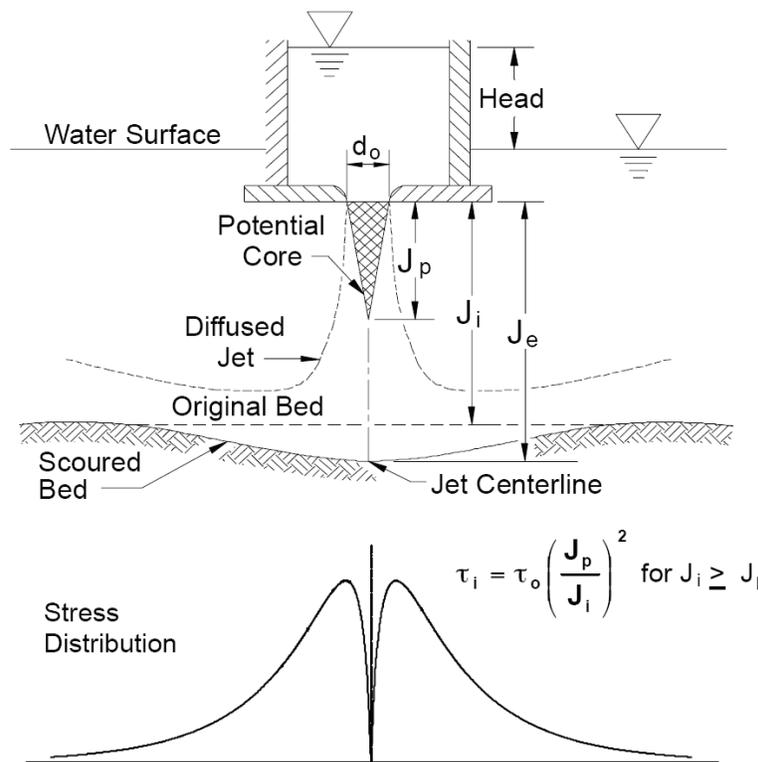


Figure 7.—Schematic of circular submerged jet with stress distribution and parameter definitions (from Hanson and Cook 2004).

6 diameters from the nozzle, the jet no longer retains a potential core, and the peak jet velocity and stress decrease in proportion to the square of the distance from the nozzle. In theory, the shear stress at the center of the jet is zero, and the peak stress occurs just off of the center of jet. In practice, maximum scour usually occurs directly beneath the jet, so it is assumed that the theoretical peak stress applies also to the centerline of the jet, which is where the scour is measured during a test.

The solution method utilizes an Excel spreadsheet and the Solver utility, a goal-seeking optimization tool. Two parameters are determined with the Solver to determine the critical shear stress and the detachment rate coefficient, analogous to the coefficient of soil erosion discussed previously for the HET. The solution procedure begins by fitting the measured scour and time data to an asymptotic function that predicts the equilibrium depth of scour that would occur at $t = \infty$. Once this distance is determined, the corresponding stress that would be produced by the jet at this scour distance is defined as the critical shear stress. Note that the stress on the soil surface decreases as scour occurs, so the test is progressing from a condition of high stress to one of low stress, opposite from the HET. The critical shear stress condition is not actually reached in most tests. The applied stress always exceeds the critical stress, and erosion never ceases.

The second part of the analysis determines the value of the detachment rate coefficient that produces a best fit of the dimensionless scour and dimensionless time to a function derived from solution of the ordinary differential equation describing the erosion of the soil surface caused by the varying stress that occurs as the scour increases (Hanson and Cook 2004). Example charts illustrating the analysis are shown in figure 8. In the upper chart, scour depth is represented by the dashed line fit through the observed data points. Scour increases with time, asymptotically approaching an equilibrium depth represented by the upper diagonal line. The data are presented in this diagonal orientation to facilitate fitting them to the equation of a hyperbola. The lower chart shows the dimensionless scour and dimensionless time, fitted to the theoretical model by optimizing the value of the detachment rate coefficient.

Issues Affecting Interpretation of JET Data

The interpretation of jet erosion test data is generally straightforward. The most common issue affecting a test is nonuniform erosion in time or space.

Nonuniformity in time is handled by the use of curve-fitting to the integrated, cumulative scour function, rather than direct calculation of differential scour and resulting erosion rates, which would also be possible with the collected data.

Spatially nonuniform erosion can lead to a condition in which the maximum depth of scour occurs away from the centerline of the specimen and is thus not measured. This fact can sometimes be detected during the test if the technician uses a finger to lightly feel the eroded soil surface, but there is no remedy if the condition is detected. With highly erodible materials, use of the finger to locate the soil surface is also helpful to avoid plunging the point gauge probe into a soft soil surface, since the water within the submergence tank is usually very turbid during a test.

The JET can be performed on a relatively wide range of cohesive soils, but is not well suited to soils that include fractions of larger gravel, as the gravel may fail to wash out of the scour hole and can gradually armor the surface. This problem may be overcome in some cases by performing the test with the jet in a nonvertical orientation so that gravity helps to remove material from the scour hole.

Experimental Objectives and Approach

The preceding background discussion has illustrated several areas in which there could be valuable investigations of issues related to the analysis and interpretation of HET data and the correlation between the HET and JET methods for quantifying erodibility of cohesive soils. Three primary objectives were established for this project:

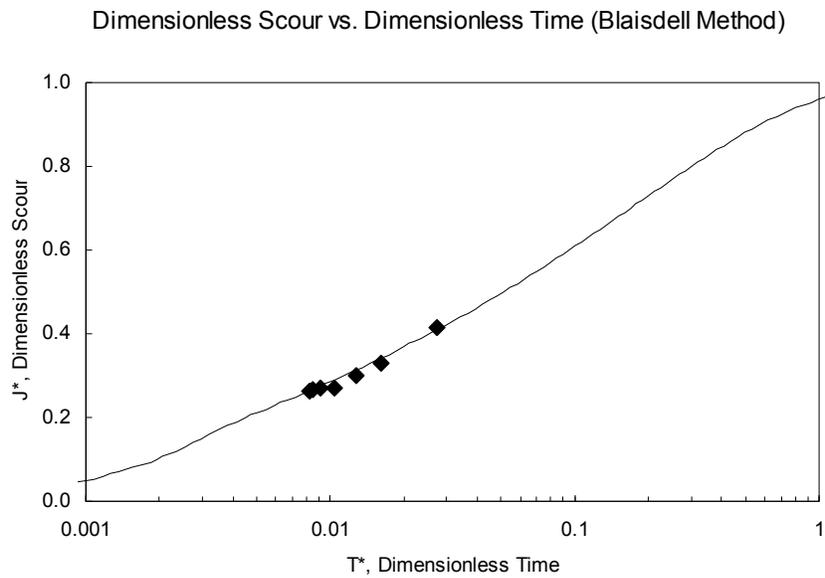
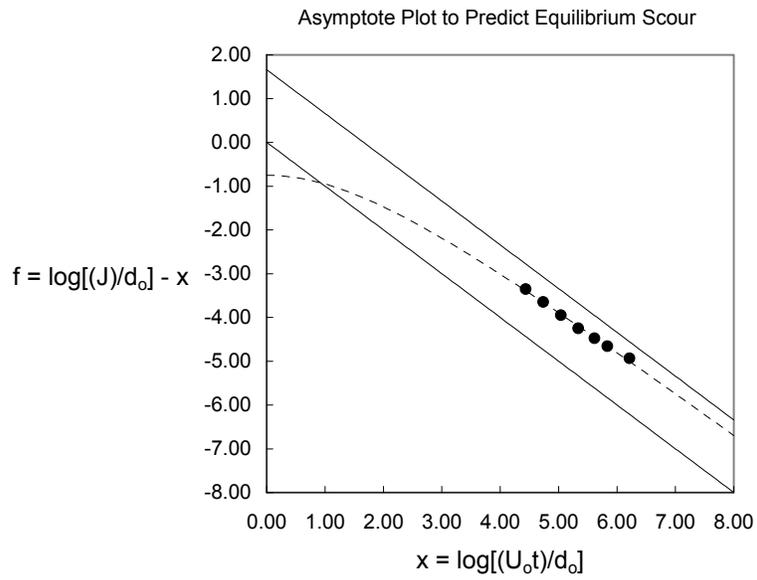


Figure 8.—Example charts illustrating interpretation of JET data.

1. Improve our understanding of the variation of the friction factor through the eroding hole during the HET.
2. Investigate alternative methods for analyzing and interpreting HET data, with the objective of simplifying the test and making data interpretation more robust.
3. Compare the erodibility parameters obtained with the HET and JET methods.

To achieve these objectives, the HET and JET methods were used to measure erodibility of specimens of several different soils in the laboratories of the Bureau of Reclamation at Denver, Colorado (table 3). These included:

- Soil 55T-160, a sandy lean clay, s(CL). This is a research and “earth school” soil used at the Bureau of Reclamation. This soil was used to conduct a series of hole erosion tests in which multiple specimens were prepared at similar moisture conditions and compaction effort and then tested for varying lengths of time to evaluate the variation of the friction factor during the HET. Companion jet erosion tests were also performed to explore correlation between the HET and JET methods. This soil was selected because it was expected to be well behaved and easy to work with in the HET.
- Two undisturbed Shelby tube samples of lean clays recovered from Reclamation’s recently constructed Ridges Basin Dam were tested in the HET.
- Four soils tested by both the HET and JET methods during the spring of 2007 by a visiting student (Regazzoni 2007). Specimens of each soil were prepared in the laboratory in parallel under similar moisture conditions (about 1 percent dry of optimum) with Standard Proctor compaction effort and then tested by the HET and JET methods. Data from these tests were reanalyzed for this report using $Re > 2,000$ as the criteria for turbulent flow and to ensure that the erodibility parameters were computed using only the data from the progressive erosion phase of each test. After reanalysis, some of the tests performed by Regazzoni were excluded from this study because the tests did not reach a progressive erosion state, or because the observed changes in flow rate were erratic and difficult to analyze (believed to be a result of clogging and/or localized entrance/exit scour that is inconsistent with the HET analysis method).
- Samples of three soils used in large-scale, piping-initiated embankment breach tests conducted by the Agricultural Research Service (ARS) in Stillwater, Oklahoma (Hunt et al. 2007). Soil samples delivered to Reclamation in November 2007 were used to study HET and JET specimens prepared in two different compaction states: (1) optimum moisture content and 95 percent of maximum density (Standard Proctor), and (2) at conditions similar to the ARS breach tests. For these tests, paired samples were created with essentially identical compaction moisture and effort. A second shipment of two of the ARS soils was provided to Reclamation in July 2008 and was used to perform a third series of tests across a range of compaction moisture contents. This third series of tests did not utilize paired samples, but the curves of erodibility versus compaction moisture content could be compared. Detailed results of the tests of the ARS soils are provided by Wahl and Erdogan (2008).

Table 3.—Properties of tested soils. Detailed gradation analyses were not available for some soils

Source	Designation	USCS	Fines					LL	PI	w _{opt} %	
			Gravel	Sand	Silt		Clay				Total fines
			> 4.76 mm	0.075-4.76 mm	0.005-0.075 mm	< 0.005 mm	< 0.075 mm				
			%	%	%	%	%				
Earth School	55T-160	s(CL)	0	37	32	31	63	34	23	12	
Ridges Basin Dam	59L-354	CL						45	25	--	
Ridges Basin Dam	59L-355	CL						37	20	--	
Teton	TE	CL-ML	0	16	70	14	84	29	4	17	
Many Farms	MF	CL						47	34	17	
Mountain Park	MP	CH/CL						54	31	20	
Tracy Fish Facility	TF	CH						55	40	18	
ARS Piping Test P1	P1	SM	0	76	19	5	24	NP	NP	12.5	
ARS Piping Test P2	P2	s(CL)	0	31	50	19	69	25	9	12.2	
ARS Piping Test P3	P3	(CL)s	0	20	50	30	80	36	24	14.2	
ARS P2 July 2008	P2	s(CL)	0	31	49	20	69	26	9	11.8	
ARS P3 July 2008	P3	(CL)s	0	21	47	32	79	33	19	12.3	

Alternative Method for Interpreting HET Data

The issues discussed earlier regarding analysis of HET data prompted an investigation into ways to improve and simplify the hole erosion test data collection and analysis procedures. Bonelli et al. (2006) proposed a universal model for piping erosion, applicable to the hole erosion test. They showed that the change in dimensionless hole radius is an exponential function of the dimensionless test time and the initial and critical shear stresses:

$$\frac{R(t)}{R_0} = 1 + \left(1 - \frac{\tau_c}{\tau_0}\right) \left(e^{t/t_{er}} - 1\right)$$

where,

- $R(t)$ = radius at any time t
- R_0 = the initial radius at time zero
- τ_c = critical shear stress
- τ_0 = shear stress at time zero
- t = test time
- t_{er} = characteristic erosion time scale for each test

$$t_{er} = \frac{2L}{k_d \gamma_w \Delta h} = \frac{2L \gamma_d}{C_e \gamma_w \Delta h}$$

where,

- L = length of the hole
- γ_w = unit weight of water ($\rho_w g$)
- Δh = head differential across the hole

Their model assumes turbulent flow conditions and neglects any variation of the friction factor, the test head, or the length of the eroded hole. The method also presumes that the test data are collected entirely during the period of accelerating erosion. Bonelli et al. (2006) showed that the proposed model fit the observed hole radius data computed from 17 hole erosion tests performed by Wan and Fell (2002) using 9 different soils. This model seemed promising, but as presented was not fully developed for practical, simple application.

Recognizing that dimensionless discharge, Q^* , is proportional to the 2.5 power of the dimensionless radius (again neglecting effects of any change in the friction factor during a test), the Bonelli model was modified to provide an equation that predicts the variation of discharge as a function of time:

$$Q^* = \frac{Q(t)}{Q_0} = \left(\frac{R(t)}{R_0}\right)^{5/2} = \left[1 + \left(1 - \frac{\tau_c}{\tau_0}\right) \left(e^{t/t_{er}} - 1\right)\right]^{5/2}$$

Bonelli and Brivois (2007) proposed a similar modification of the model that became known to the authors after the laboratory studies described in this report had been completed.

Since flow rates are measured throughout a test and the initial shear stress is known from the starting hole diameter and flow rate, this model has only two unknown parameters, the erosion time scale, t_{er} , and the critical shear stress, τ_c . Using a nonlinear optimization tool such as the Excel Solver, one can optimize these two parameters to obtain a best fit of the observed dimensionless values of discharge to predicted values computed for each dimensionless test time, t/t_{er} . The coefficient of soil erosion or the detachment rate coefficient can then be determined from the fitted value of the time scale factor, t_{er} . The significant advantages of the method are the fact that the final hole diameter does not need to be measured, and the curve-fitting procedure minimizes the influence of short-term anomalies in erosion behavior during a test.

It should be emphasized that the formulation of the Bonelli model requires the fitted value of the critical shear stress τ_c to be less than the initial stress, τ_0 ; otherwise, the quantity $(1-\tau_c/\tau_0)$ is negative. This means that tests must be conducted at a stress level that exceeds the critical stress and produces immediate progressive erosion, or one must customize the analysis to only examine the portion of the test in which the shear stress exceeds τ_c . If a test began at a gradient that was slightly lower than the value needed to initiate progressive erosion, but the stress then increased during the cleanout phase as described earlier, the only way to accurately determine the critical stress would be to estimate the increase in hole diameter and shear stress that takes place leading up to the progressive erosion phase, then start the Bonelli analysis.

Results and Discussion

HET Friction Factor

Fourteen tests were performed with soil 55T-160 to investigate the variation of the friction factor during the hole erosion test. Specimens were mixed with water and stored for at least 48 hours. They were then compacted into standard 102-mm (4-in) diameter by 116-mm (4 5/8 -in) long compaction molds at 12 percent water content, approximately optimum for this soil. Specimens were manually compacted using Standard Proctor procedures in three layers of approximately equal thickness. Each layer was compacted by 25 blows from a 50.8-mm diameter, 2.49-kg hammer dropped freely a distance of 0.305 m. Following compaction, specimens were stored overnight in plastic bags to allow for curing.

Determining Erosion Indices of Cohesive Soils with the Hole Erosion Test and Jet Erosion Test

Initial plans were to perform 12 tests in sets of 3 tests each, with each set of tests run for a different amount of time, representing 25, 50, 75, and 100 percent of the normal test time for this soil. The first test was used to determine the head needed to initiate progressive erosion (100 mm), and the first 3 tests were run for the maximum possible time that the test head and flow could be maintained, limited by the test facility. These tests established the 100-percent test time target for the soil as 25 minutes.

Among the first 12 tests conducted, two tests (11 and 12) were unsuccessful due to a failure of the flow measurement system, and four tests (5, 6, 8, and 10) were unsuccessful because 100 mm of head did not produce progressive erosion. Two additional tests were added to replace some of the lost data. Test 13 was performed, but discarded from the data set because the computed final friction factor was significantly out of bounds compared to the other tests; this is believed to be due to clogging of the erosion hole.

For most of the tests, the head was set to 100 mm, with two exceptions. For test 9 the head was initially set to 100 mm but was increased during the test to 200 mm to initiate erosion, and for test 14 the head was initially set to 50 mm and then increased to 100 mm. For all other tests, the head was maintained at 100 mm throughout the test.

Table 4 summarizes the friction factors computed at the start and end of each of the useful tests (see the appendix for equations used). Although tests 5, 6, 8, and 10 did not produce progressive erosion, the initial and final friction factors could be computed and included in the analysis. Turbulent flow conditions ($Re > 2000$) prevailed throughout all of the tests.

Table 4.—Friction factors computed for hole erosion tests on soil 55T-160

Test	Total test time (min)	Progressive erosion time (min)	Turbulent friction factor, $\text{Pa}/(\text{m/s})^2$		Hole diameter, mm	
			Initial	Final	Initial	Final
Test 1	26.67	26.67	10.3	71.2	6.35	27.35
Test 2	30.60	30.60	11.2	65.3	6.35	24.52
Test 3	24.93	24.93	8.5	71.6	6.35	24.52
Test 4	18.75	18.75	13.6	84.5	6.35	21.68
Test 5	12.50	0.00	9.7	23.4	6.35	7.35
Test 6	20.50	0.00	11.1	43.5	6.35	12.38
Test 7	12.58	8.92	6.9	63.7	6.35	15.19
Test 8	19.83	0.00	9.2	31.7	6.35	8.08
Test 9	18.75	13.25	9.4	59.4	6.35	14.98
Test 10	7.50	0.00	9.0	36.5	6.35	8.81
Test 14	24.92	18.40	29.7	12.9	6.35	11.00

Figure 9 shows the initial and final friction factors as a function of the total test time, progressive erosion time, and hole diameter. The progressive erosion time was evaluated subjectively by visually determining the point at which the flow rate began to accelerate during each test.

The heavy solid line is a linear regression trend line through the initial and final friction factor values. Although the trends versus time are in approximate agreement with the variation assumed by Wan and Fell (2004), the R^2 values for the trends indicate only weak relationships to time. The relationship with hole diameter is the most significant but is still not dramatic. Based on this result, it seems more reasonable to relate the variation of the friction factor to the hole diameter than the test time. This especially seems more justified for tests in which one or more low head settings are used that produce no erosion. Lim (2006) obtained a similar result from a similar series of tests conducted with three soils (SC, CL, and CH) that were nondispersive in tap water. This work was discovered after the completion of the Reclamation tests.

A practical difficulty encountered in applying this result to the analysis is that the hole diameter is not known until the data analysis is completed. An iterative solution method could be used, but testing with real data sets showed occasional problems with obtaining convergence. Instead, it is proposed that a similar result can be obtained by relating the friction factor to the variation of $(Q/S)^{1/3}$ and $(Q/S)^{1/5}$ for the laminar and turbulent cases, respectively, which are each approximately proportional to the hole diameter (see the appendix).

Alternative Method for Interpreting HET

Three groups of test data were used to evaluate the alternative method for interpreting HET data. Twelve tests of soil 55T-160, two tests of Ridges Basin Dam soils, and 14 tests performed by Regazzoni (2007) were analyzed using the methods of Wan and Fell (2004) and the new method based on the model of Bonelli et al. (2006). Tests were individually analyzed to ensure that only the progressive erosion phase of each test was being used to determine I_{HET} and τ_c . Many of these tests started at low test heads that caused significant cleanout erosion, but did not enter the progressive erosion phase until the hole diameter had increased significantly. For these tests, the Wan and Fell analysis was used to estimate the starting hole diameter for the Bonelli analysis, which considered only the progressive erosion phase. Ideally, if one were using the Bonelli analysis procedure exclusively, tests would be started at hydraulic gradients high enough to cause immediate progressive erosion. This would allow one to use the predrilled hole diameter as the initial condition in the Bonelli analysis, avoiding the need to also perform the Wan and Fell analysis (which requires measurement of the final hole diameter in order to compute intermediate hole diameters).

Determining Erosion Indices of Cohesive Soils with the Hole Erosion Test and Jet Erosion Test

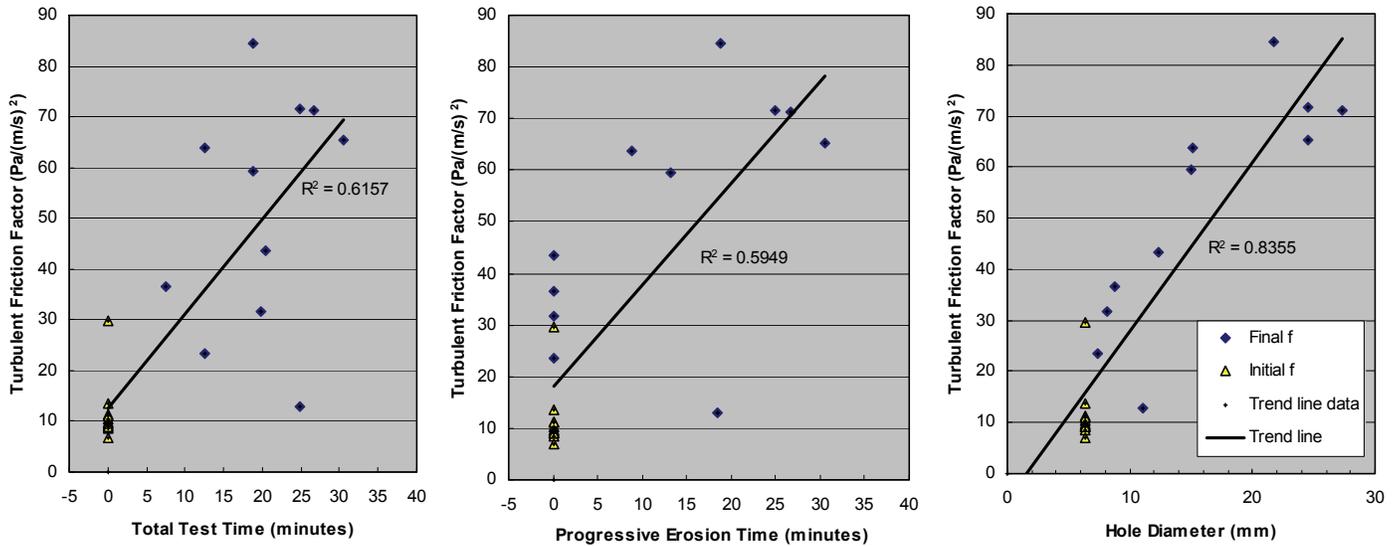


Figure 9.—Turbulent flow friction factor variation during HETs.

As expected, soil 55T-160 was relatively easy to work with and produced many successful tests. Erosion of the predrilled hole tended to be relatively uniform, and there was little or no additional loss of material at the upstream and downstream ends of the erosion hole. Some of the soils tested by Regazzoni proved to be more difficult. Tests of the fat clay from Tracy Fish Facility (TF) were often affected by clogging of the erosion hole as chunks of material broke free from the interior walls of the hole but were too large to be transported or became jammed in the hole. This completely prevented the analysis of some tests and required careful interpretation of others. One test of the CH/CL soil from Mountain Park (MP-3) also exhibited some clogging, but it did not prevent a successful interpretation of the test.

The silty clay soil from Teton Dam (TE) was also difficult to test because it was highly erodible, with large amounts of material removed near the entrance and exit of the erosion hole. This was accounted for in both the Wan and Fell and Bonelli methods by estimating the effective length of the constricted portion of the erosion hole at the end of the test, and then using a linear variation of the hole length with time (Wan and Fell) or an average length (Bonelli) to perform the analysis. Additionally, for many of the tests on this soil, accelerating erosion could not be sustained for more than a few minutes. It was necessary to analyze just the first few minutes of most of these tests, as analysis of longer time periods led to the conclusion that erosion rate decreased with increasing stress (a negative coefficient of soil erosion C_e).

Table 5 shows values of the erosion rate index I_{HET} and critical stress τ_c computed by the two methods. Figure 10 shows a graphic comparison of the I_{HET} values computed by the two methods, and figure 11 compares both the I_{HET} and τ_c values

determined by the two methods. The left-hand chart in figure 11 shows the subset of the data having critical stresses less than 25 Pa, using an arithmetic scale to show zero values; the right-hand chart shows all data, using a logarithmic scale.

Table 5. —Results of alternative methods for interpreting HET data

Test	Wan and Fell		Bonelli	
	I_{HET}	τ_c	I_{HET}	τ_c
55T-160-1	3.48	0.0	3.52	0.0
55T-160-2	3.51	0.0	3.56	2.2
55T-160-3	3.34	0.9	3.33	8.0
55T-160-4	3.29	3.6	3.27	8.1
55T-160-7	3.04	12.0	3.16	10.8
55T-160-9	3.36	10.8	3.28	23.0
55T-160-14	3.20	10.6	3.07	11.5
TF-5	4.86	164.6	4.67	195.7
MF-3	4.03	0.0	3.89	33.7
MF-5	3.08	7.0	3.03	8.7
MF-6	3.16	5.7	3.07	7.2
MF-7	3.00	6.4	3.08	6.0
MP-4	5.01	241.3	5.31	148.7
TE-1	2.68	5.5	2.85	3.9
TE-2	3.12	7.3	3.14	9.0
TE-3	2.47	6.6	2.45	10.1
TE-5	2.94	4.2	2.93	7.6
Ridges Basin 59L-354	4.73	250.0	5.19	244.2
Ridges Basin 59L-355	3.61	110.2	3.82	107.4

Agreement between the two methods is good for all soils investigated, with perhaps a slight bias toward lower I_{HET} values from the Bonelli analysis method for the more erosion-resistant soils. There is some significant variation of the τ_c value for individual tests, but considering all of the tests together, the two methods yield similar results.

Determining Erosion Indices of Cohesive Soils with the Hole Erosion Test and Jet Erosion Test

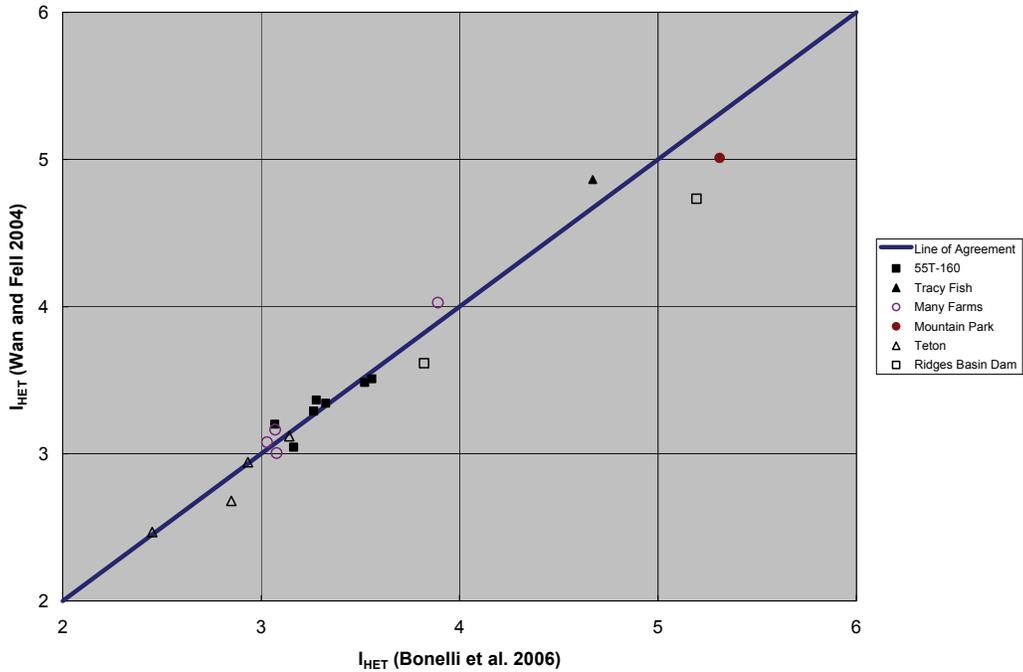


Figure 10.—Comparison of I_{HET} values computed by alternative methods for interpreting hole erosion test data.

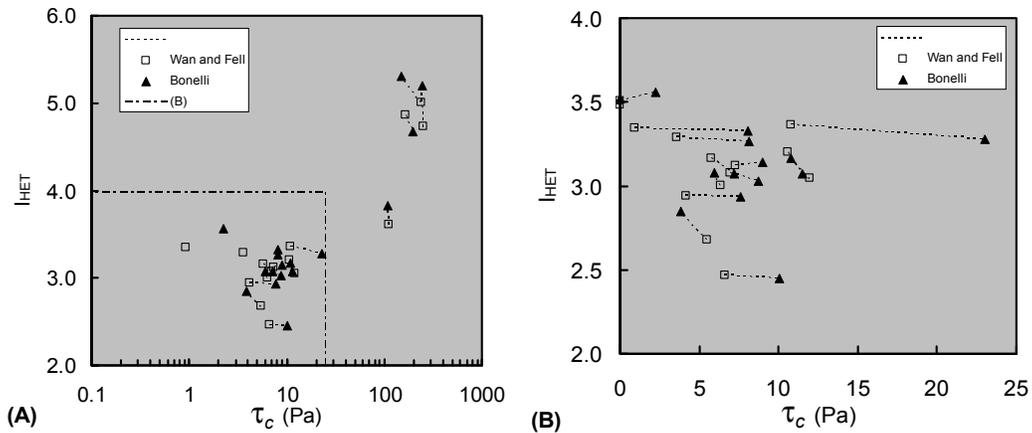


Figure 11.—Comparison of computed values of I_{HET} and τ_c using different methods for interpreting HET data. Dashed lines connect pairs of data markers for each individual test. Chart (A) shows all tests, but values of $\tau_c = 0$ are not shown due to the use of a logarithmic scale. Chart (B) shows the subset of the tests in which critical stresses were less than 25 Pa; an arithmetic scale is used to allow plotting values of $\tau_c = 0$.

HET vs. JET Results

Paired Samples

A comparison of HET and JET results was made using the HET data previously presented, JET data collected by Regazzoni (2007), two jet erosion tests of soil 55T-160, and the two series of paired HET and JET tests of the ARS soils described previously (Wahl and Erdogan 2008). All jet tests were conducted with the specimens inverted so that the jet attacked the bottom surface of the first layer.

The JET is typically analyzed to obtain the detachment rate coefficient k_d , which is expressed volumetrically, as opposed to the HET which yields C_e and I_{HET} , with C_e expressed in terms of mass and $I_{HET} = -\log_{10}(C_e)$. To allow a convenient comparison, a similar index for the jet erosion test was computed, $I_{JET} = -\log_{10}(k_d \rho_d)$. Table 6 summarizes the data, and figures 12 and 13 provide graphical comparisons for each tested soil. Figures 14 and 15 provide a comparison across the range of the tested soils. Where multiple tests were performed, error bars in figures 14 and 15 indicate the full range of measured values for each particular soil type and/or compaction condition.

Clearly, there is significant difference between the erosion indices and critical shear stresses obtained with the two tests. The rankings of soils from least to most erodible were similar for most cases, but the quantitative differences between soils appear to be more pronounced with the HET, and the HET indicates greater erosion resistance for all of the soils. Differences between the HET and JET results seem to be greater for the fat clays (Mountain Park = CH/CL and Tracy Fish Facility = CH), and smaller for the leaner clays and silts [55T-160=s(CL), Many Farms = CL, Teton = CL-ML, P2 = s(CL)].

Figure 14 also shows the relationship found by Lim (2006) for erosion rate indices of nondispersive clay soils determined by the hole erosion test and rotating cylinder test (RCT). Moore and Masch (1962) originally developed the rotating cylinder test, which uses a soil block suspended and submerged inside of a rotating cylindrical chamber. Rotation of the cylinder induces a flow around the specimen, which causes erosion. Torque applied to the specimen and erosion rates are measured and used to estimate applied stresses and erodibility parameters. The test apparatus is very expensive, and the test is difficult to perform, but it gives an excellent measure of erodibility, with good correlation to flume experiments of flow across erodible surfaces. Figure 14 shows that the JET and RCT both produce higher erosion rates (lower values of I_{JET} and I_{RCT}) than the HET, and the differences are of a similar order of magnitude, although not in perfect agreement. Notably, Lim (2006) found that the HET and RCT have very similar results for dispersive soils, with the RCT producing only slightly higher erosion rates (lower values of I_{RCT}).

Determining Erosion Indices of Cohesive Soils with the Hole Erosion Test and Jet Erosion Test

Table 6.—Summary of erosion indices and critical stresses determined by HET and JET methods

USCS*	Soil ID	HET				JET			
		wc %	ρ_d (Mg/m ³)	I_{HET}	τ_c	wc %	ρ_d (Mg/m ³)	I_{JET}	τ_c
s(CL)	55T-160	11.7	1.924	3.52	0.0	11.9	1.922	2.71	0.45
		11.4	1.905	3.56	2.2	11.9	1.922	2.38	0.71
		11.9	1.922	3.33	8.0				
		11.6	1.909	3.27	8.1				
		11.6	1.905	3.16	10.8				
		11.6	1.908	3.28	23.0				
		11.6	1.908	3.07	11.5				
CH	TF	17.7	1.507	4.67	196.	17.4	1.659	2.84	5.4
						17.7	1.664	2.20	0.08
						17.2	1.587	3.16	0.22
						17.8	1.583	3.21	1.80
CL	MF	14.7	1.776	3.89	33.7	15.2	1.757	2.71	0.13
		14.8	1.808	3.03	8.7	14.9	1.776	2.99	0.43
		14.2	1.802	3.07	7.2	14.4	1.783	2.97	2.3
		14.1	1.789	3.08	6.0	14.8	1.780	2.71	0.44
						14.2	1.776	2.57	0.11
				14.1	1.783	2.46	0.27		
CH/CL	TE	18.6	1.666	5.31	149.	17.8	1.653	3.31	7.7
						17.0	1.674	3.57	9.2
						19.0	1.682	3.58	8.2
						18.6	1.655	3.57	7.2
CL-ML	TE	15.6	1.703	2.85	3.9	15.6	1.700	2.55	0.65
		16.2	1.695	3.14	9.0	16.2	1.696	2.74	0.90
		16.5	1.695	2.45	10.1	16.5	1.701	2.65	0.66
		16.5	1.692	2.93	7.6	16.3	1.698	2.51	0.33
S(CL)	P2 (95/owc)	12.0	1.758	4.33	200.	12.4	1.766	3.09	0.23
		12.4	1.783	4.37	103.	12.8	1.811	3.43	0.95
(CL)s	P3 (95/owc)	14.2	1.739	4.77	207.	14.2	1.696	2.53	0.18
		14.2	1.706	4.71	402.	14.2	1.747	2.67	0.22
s(CL)	P2 (~ARS breach test)	12.4	1.749	4.20	231.	12.5	1.732	3.17	0.91
		12.0	1.731	3.42	357.	12.5	1.752	3.47	0.76
(CL)2	P3 (~ARS breach test)	15.1	1.768	4.90	346.	15.3	1.765	3.48	1.62
		16.2	1.744	4.80	132.	15.4	1.775	4.05	18.82

* Unified soil classification system

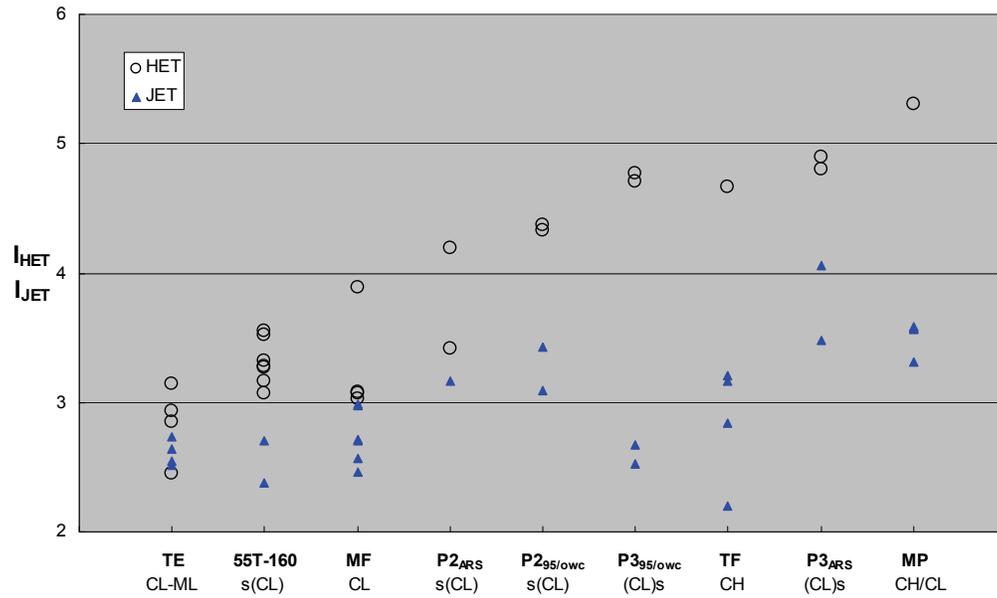


Figure 12.—Erosion rate index values obtained by HET and JET methods, ranked subjectively from most rapid to least rapid erosion rate.

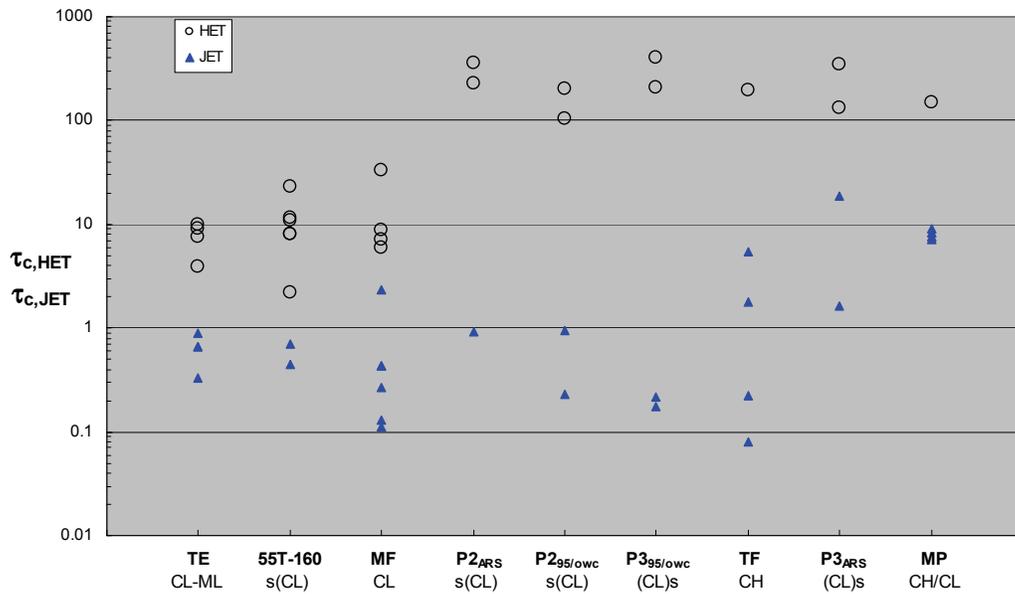


Figure 13.—Critical shear stresses obtained from HET and JET methods, ranked subjectively from most rapid to least rapid erosion rate (same ranking as fig. 12 above).

Determining Erosion Indices of Cohesive Soils with the Hole Erosion Test and Jet Erosion Test

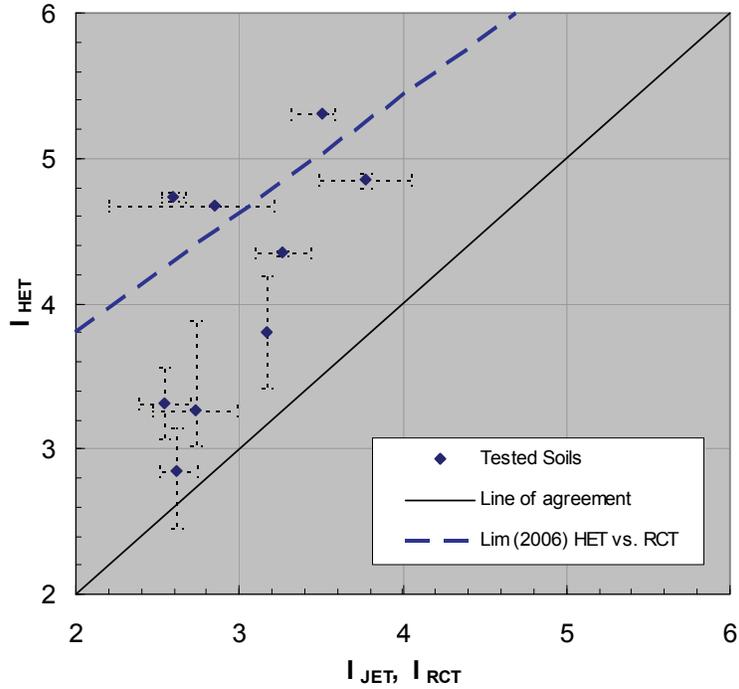


Figure 14.—Comparison of erosion rate indices determined by HET and JET methods, and the relationship found by Lim (2006) relating HET and rotating cylinder test (RCT) results for nondispersive soils.

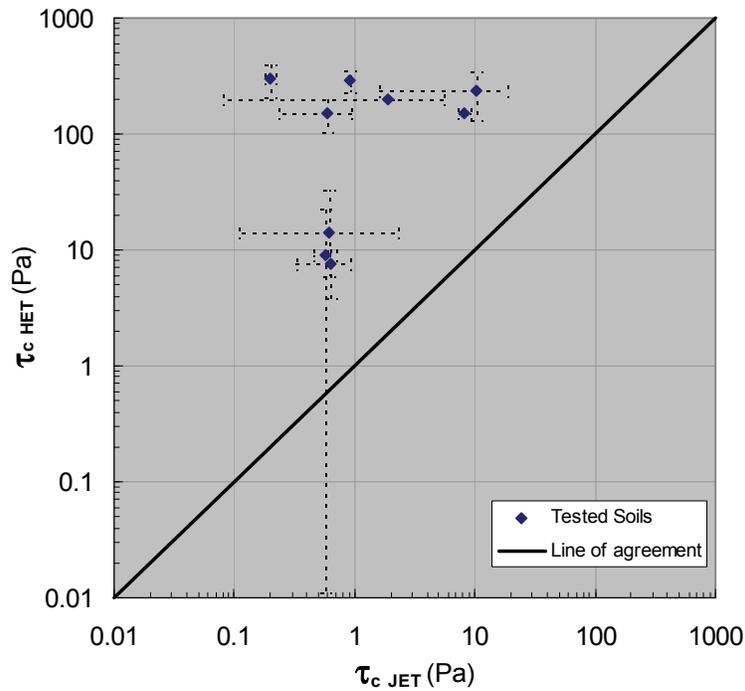


Figure 15.—Comparison of critical stresses determined by HET and JET methods.

HET and JET Erodibility versus Moisture Content

Two of the ARS soils, P2 and P3, were chosen for a more intensive study of the variation of HET and JET results as a function of compaction moisture content. Specimens were prepared using Standard Proctor compaction at a range of compaction moisture contents from about 4 percent dry of the presumed optimum to 4 percent wet of optimum at 2-percent increments; actual optimum moisture content for the tested soils was established after the fact from the test data (see table 3).

Table 7 shows the test results, including subjectivity indices for the HETs. The subjectivity index was developed late in the course of this research as a means of quantifying the level of subjectivity required to analyze each hole erosion test. A value of 0 indicates little subjective judgment was needed; larger values indicate the use of more subjective judgment and corresponding increased uncertainty in test results (see the appendix for details). There were three tests with a subjectivity index of 2, indicating poor confidence in the test result, and three additional tests of soil P3 (not shown in the table) were excluded entirely because analyses could not be completed. All of the jet tests were fully successful.

Table 7.—Erodibility test results for P2 and P3 soils over a range of compaction moisture contents

Soil	Test type	Compaction conditions		Results		
		Moisture content, %	Dry density, g/cm ³	τ_c , Pa	k_d , cm ³ /(N·s)	HET subjectivity index
P2	HET	7.51	1.795	65.	0.217	2
		9.36	1.853	958.	0.0578	2
		11.56	1.895	856.	0.0311	0
		13.59	1.872	242.	0.0547	1
		15.65	1.795	133.	0.0372	1
	JET	7.55	1.785	0.062	1.39	-
		9.27	1.847	0.168	0.688	-
		11.57	1.929	7.58	0.0410	-
		13.43	1.872	0.081	0.188	-
		15.49	1.794	0.558	0.203	-
P3	HET	11.73	1.877	622	0.00420	0
		12.56	1.913	510	0.00266	1
		13.75	1.884	378	0.00253	2
		13.96	1.884	731	0.0122	1
		14.45	1.875	968	0.00524	0
		15.82	1.827	656	0.0131	1
		17.55	1.768	385	0.0205	1
	JET	10.18	1.848	0.456	0.508	-
		11.48	1.918	20.4	0.0329	-
		13.78	1.888	43.8	0.0234	-
		14.02	1.892	49.8	0.0493	-
		14.06	1.897	60.7	0.0198	-
		14.49	1.869	28.6	0.0124	-
		15.67	1.839	23.2	0.0303	-
17.82	1.773	15.1	0.0568	-		

Figures 16, 17, and 18 show the results graphically, first for the individual soils (figs. 16 and 17), and then for both soils together (fig. 18). The tests confirm that in general the P3 soil is less erodible than P2, but the erodibility of the P3 soil is more sensitive to moisture content differences on the dry side of optimum. This effect is sufficient to cause the JET results for P3 to indicate more erodibility than P2 when compaction moisture contents of both soils are below about 10 percent. The HET results in this range of moisture contents are incomplete because the HET on the driest P3 specimen was unsuccessful, but the trend in the data appears to be similar.

Differences between HET and JET results for soil P2 were relatively consistent across the range of tested moisture contents. The JET yielded detachment rate coefficients about 0.75 to 1 order of magnitude greater than those obtained from the HET. Critical shear stresses were about 2 to 3 orders of magnitude lower in the JET than in the HET.

Differences between the tests for soil P3 appear to be somewhat sensitive to the compaction moisture content. The detachment rate coefficients were only about 0.5 orders of magnitude different on the wet side of optimum, and about 1 order of magnitude different on the dry side, although there was not a successful HET test at the 4-percent dry condition to completely illustrate the effect. Critical shear stresses were consistently about 1.5 orders of magnitude different in the range for which a comparison could be made. The sensitivity of the JET results (both the detachment rate coefficient and the critical shear stress) to changes in moisture content on the dry side was greater for soil P3 than for P2. The unsuccessful HET performed on soil P3 at the nominally 4-percent dry condition experienced excessive local scour at the entrance and exit and erratic variations in flow during the test, making analysis impossible; this probably indicates a material with high erodibility, so the HET may have been as sensitive as the JET to the effect of dry compaction of this soil. Unfortunately, performing a successful test becomes difficult with the HET as the soil becomes more erodible.

Discussion of HET and JET Differences

A host of factors probably contributes to the differences in erodibility parameters obtained from the two tests. The most important of these are probably the inherent differences in the nature of the hydraulic attack upon the eroding surface in each test, the way that the flow exploits different weaknesses in the soil structure, and differences in the geometry of the exposed soil surface.

Briaud (2008) has suggested that soil erodibility may depend fundamentally on three different types of stress: pure shear, turbulent fluctuations of shear stress, and turbulent fluctuations of normal stress. Adequately controlling, describing, and utilizing these different stresses for the analysis of soil erodibility in these test environments is beyond our present capability. To some degree, in each individual test environment, the pure shear stress and turbulent stress factors are probably somewhat correlated, so it may not be necessary to isolate the effects of

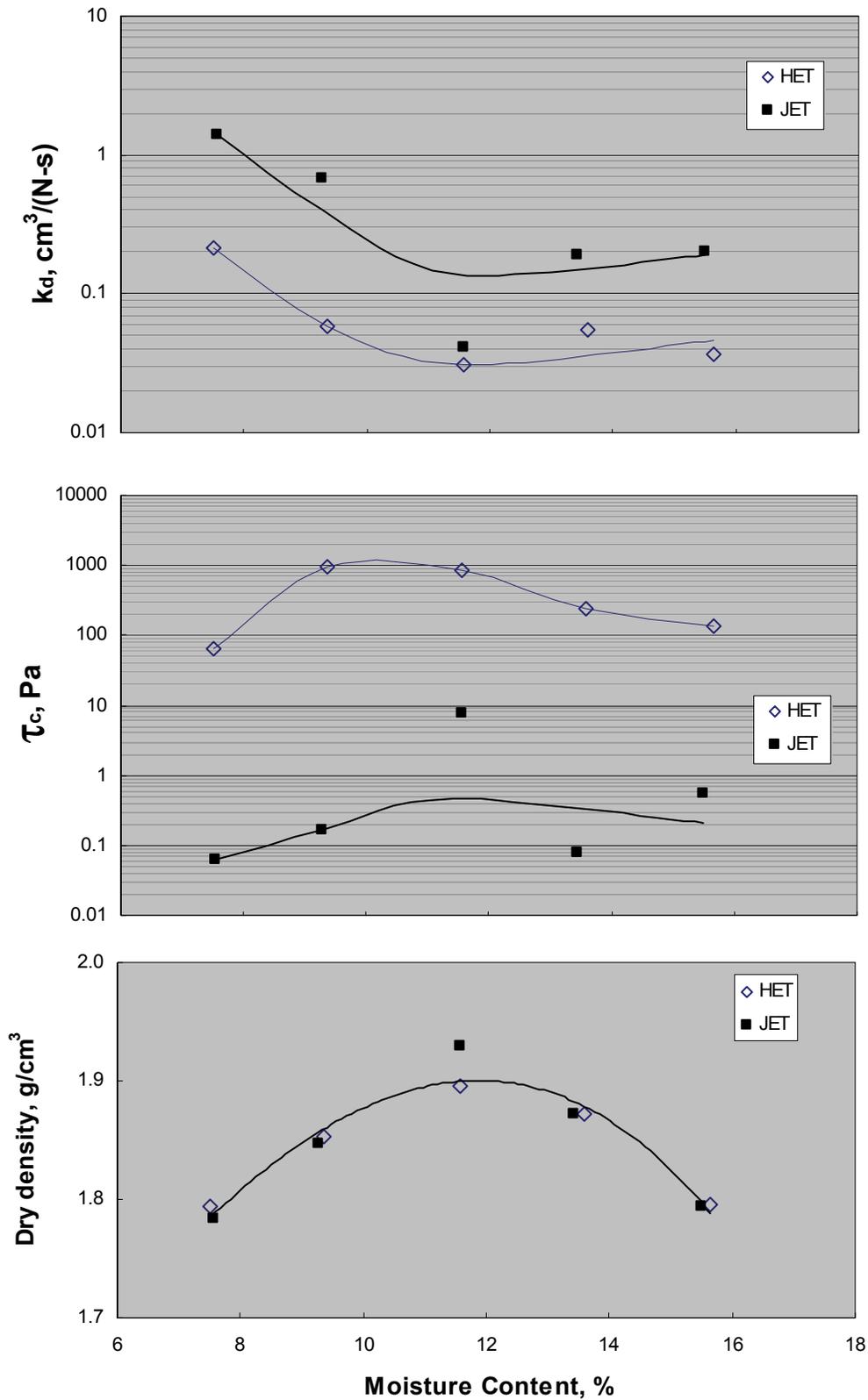


Figure 16.—Results of erodibility tests on soil P2 at compaction moisture contents ranging from about 4 percent dry of optimum to 4 percent wet of optimum.

Determining Erosion Indices of Cohesive Soils with the Hole Erosion Test and Jet Erosion Test

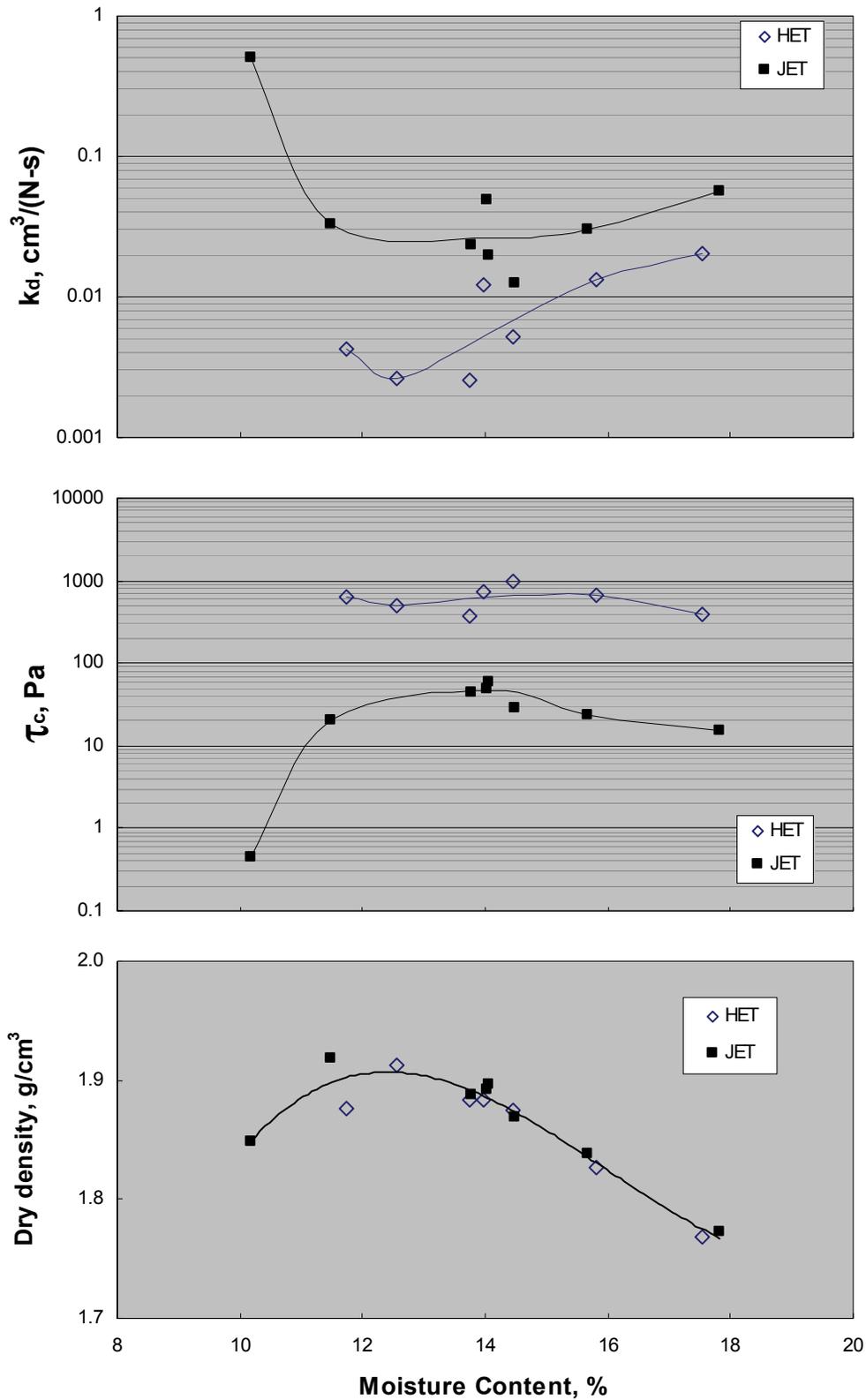


Figure 17.—Results of erodibility tests on soil P3 at compaction moisture contents ranging from about 3 percent dry of optimum to 5.5 percent wet of optimum.

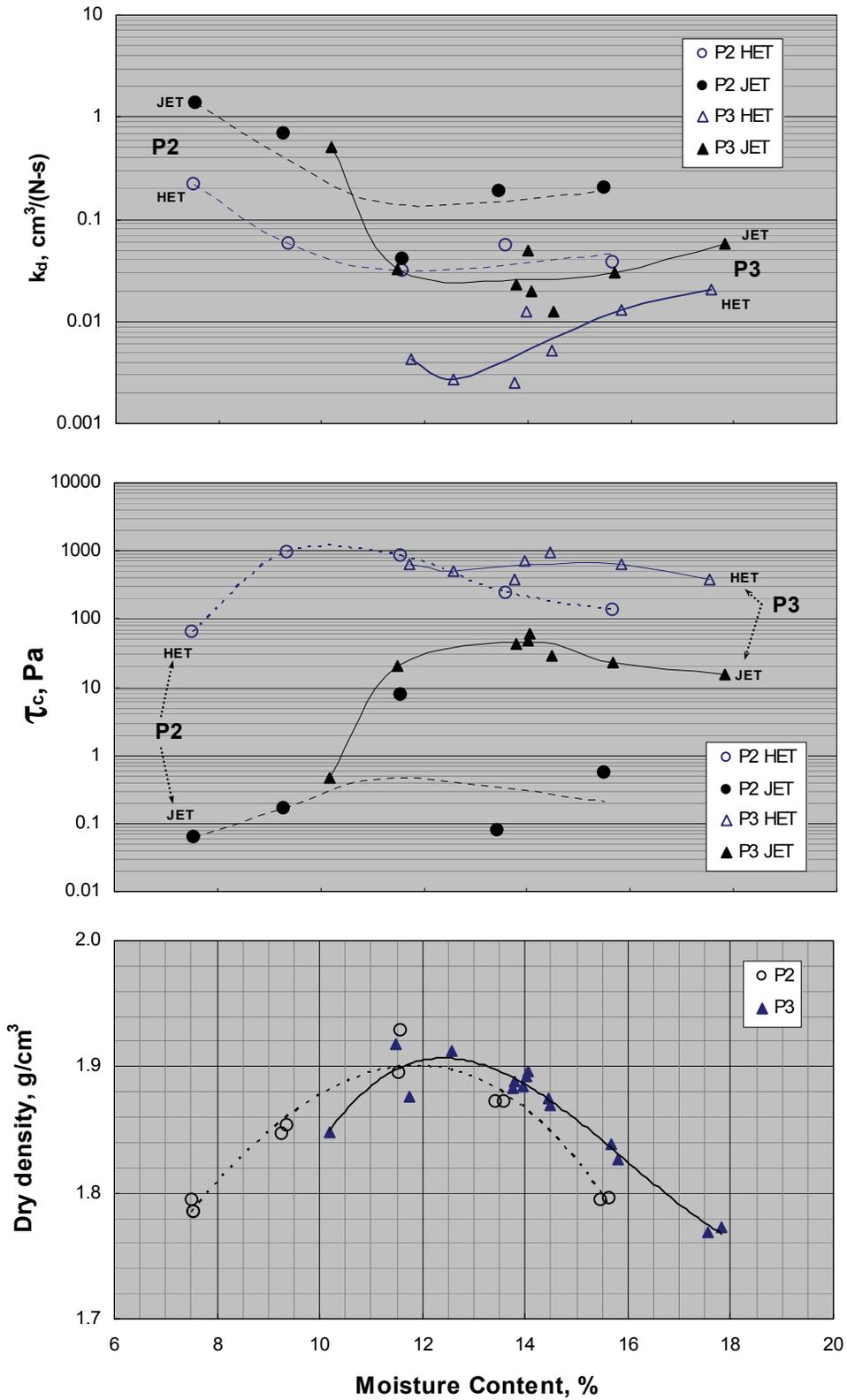


Figure 18.—Variation of erodibility for soils P2 and P3 as a function of compaction moisture content.

each stressor in order to obtain useful results (i.e., we can still simply correlate erodibility to shear stress). However, when comparing the HET and JET methods, it is easy to imagine that the relative influence of the three types of stress may not be the same in the two test environments. This may be the greatest fundamental reason for the differences observed in their results.

Soil fabric is recognized to play an important role in determining erodibility, and these two tests may have different sensitivity to it. Clay materials that are compacted in a dry-of-optimum state have an especially chunky characteristic (clods or peds) in which some soil aggregates remain independent, separating larger masses of conglomerated particles so that the entire soil mass does not mold into one coherent unit. Soils with this type of fabric structure seem especially susceptible to greater erodibility in the JET. The submerged jet is able to attack weaker areas around the top and sides of the stronger chunks, whereas in the HET, only one edge of any given chunk is exposed to stress and erosion. The tests of the ARS P2 and P3 soils are potentially an example of the strong influence of soil fabric. Figure 19 shows several of these specimens. Figures 19(a) and (b) are two specimens of P2 in which water content had little effect on soil fabric. Figures 19(c) and (d) are pre- and post-test views of a JET specimen of P3 which was compacted at 14.2 percent water content and exhibits significant fabric structure. Figure 19(e) is a specimen of P3 compacted at 15.1 percent water content exhibiting a more uniform soil fabric. When tested in the JET, the coarse-fabric specimens were more erodible than the corresponding P2 specimens, but in the HET, the P2 specimens were more erodible. For the specimens compacted at the wetter conditions where fabric differences seemed smaller, both the HET and JET showed the P2 soil to be more erodible. It should be noted that the JET erodibility of the P3 specimen with the coarser fabric was surprisingly high compared to jet tests performed by ARS in their laboratory (Hanson and Hunt 2007), but there were some differences between the soils tested at Reclamation and those tested at ARS (Wahl and Erdogan 2008).

The geometric configuration of the sample and the stressed surfaces seems to be a factor in the differences between computed erodibility parameters in these two tests. In the HET, the circular hole configuration may allow blocks of soil to be locked in place by surrounding material, so that even after they have become somewhat disengaged from surrounding particles, they may still be protected by the proximity of the surrounding particles. In contrast, in the JET the exposed surface is initially a plane, which reduces the degree to which small, coherent blocks of soil can be protected by the integrity of the larger surrounding soil mass. Flakes and thin layers of soil may readily detach and small chunks can be jacked out of place by stagnation pressures that develop in fissures beneath them, whereas they might remain wedged in place inside the confines of the small predrilled hole used in the HET. The geometric configuration of the submerged jet also allows it to apply erosive stress to larger scale soil structures, since the jet spreads when it impinges on the soil surface and attacks an area much larger than the nozzle diameter.

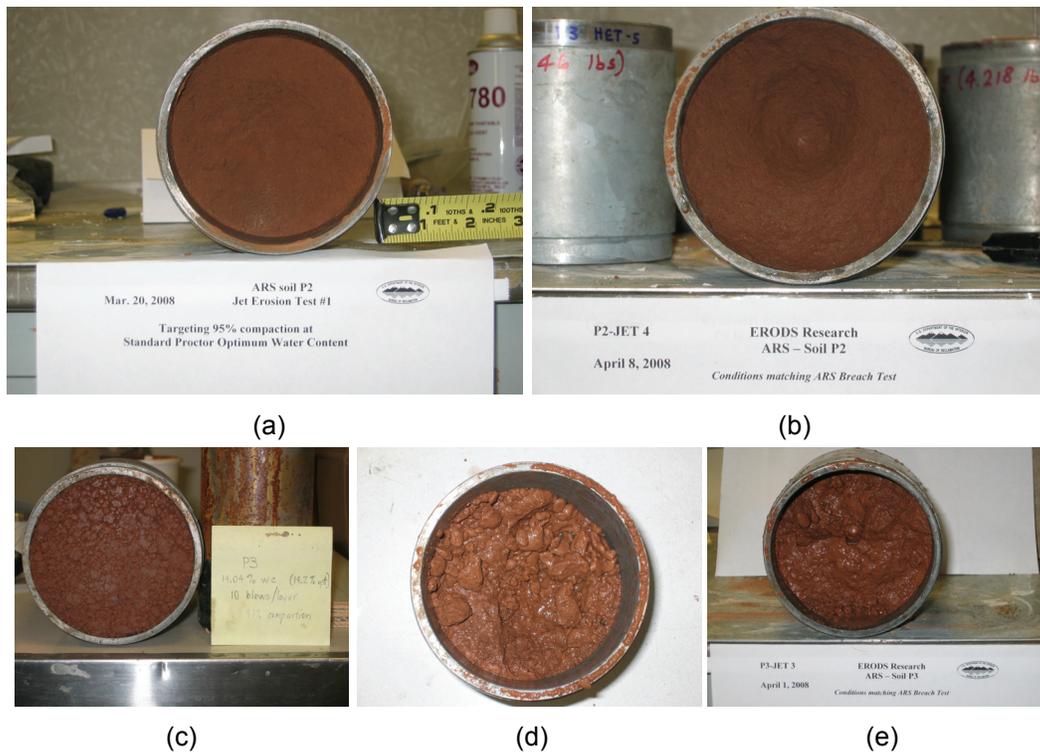


Figure 19.—Differences in soil fabric of P2 (a,b) and P3 (c,d,e) JET specimens.

Other factors that may account for the observed differences include inaccurate descriptions of the shear stress environment at the soil surface in each test. For the HET, the predrilled hole is relatively short, preventing the establishment of fully developed flow, and causing entrance and exit turbulence and associated scour to be significant. The slot erosion test (SET), also developed by Wan and Fell (2004), uses a 1-meter long soil sample to overcome this problem, but is logistically more difficult to perform as a result. For the JET, normal stresses are not considered in the analysis, but may play a significant role; normal stresses are absent or significantly lower in the HET. Also, the shear stress distribution used in the JET analysis was developed for impingement against a planar surface, but this condition is only present at the start of a test, before scour occurs. Both tests may be affected by changes in turbulence intensity that accompany changes in test head and flow rate.

Another factor may be the fact that the tests work opposite to one another, with the HET progressing from a low stress condition toward higher stresses, and the JET beginning with a high stress condition and approaching the low stress condition. If soil erodibility is not truly linear over a range of stresses, then one should expect different results from the two tests for this reason alone. Many of the HETs on soil 55T-160 did exhibit nonlinear relations between erosion rate and applied stress. Figure 20 shows one example. One physical explanation for this effect is that the roughening of the interior surface of the hole as erosion takes

Determining Erosion Indices of Cohesive Soils with the Hole Erosion Test and Jet Erosion Test

HET Friction Factor Research
Soil 55T-160 Test 3 09-26-2007

EROSION RATE VS. SHEAR STRESS

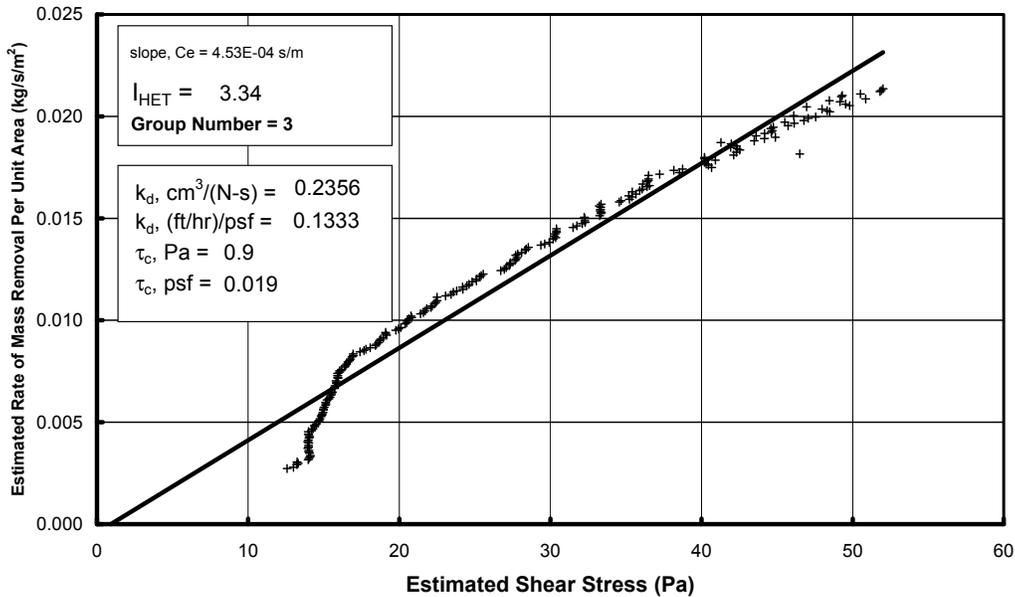


Figure 20.—Example of nonlinear relation between erosion rate and applied stress.

place may create a more pronounced boundary layer in the flow, so that a given computed stress level becomes less effective for causing erosion as a test progresses.

One side effect of fitting a linear model to nonlinear behavior is that the length of the test can change the result. This is shown in figure 21 where the erosion index for soil 55T-160 increases (indicating more erosion resistance) with increasing duration of the progressive erosion phase. It is notable that the shortest duration HETs on the 55T-160 soil have erosion indices that are approaching the values obtained from the two jet tests performed on that material. It is also notable that the two jet tests performed on 55T-160 were conducted in different stress ranges (by adjusting the test head), and the test performed at the lower stress (starting closer to the critical stress condition) produced a lower erosion index. This would be consistent with the shape of the erosion-versus-stress curve shown in figure 20. Finally, figure 22 shows the effect of test duration on the critical stress obtained from the HETs on soil 55T-160. Shorter tests found higher critical stresses, which is again consistent with figure 20.

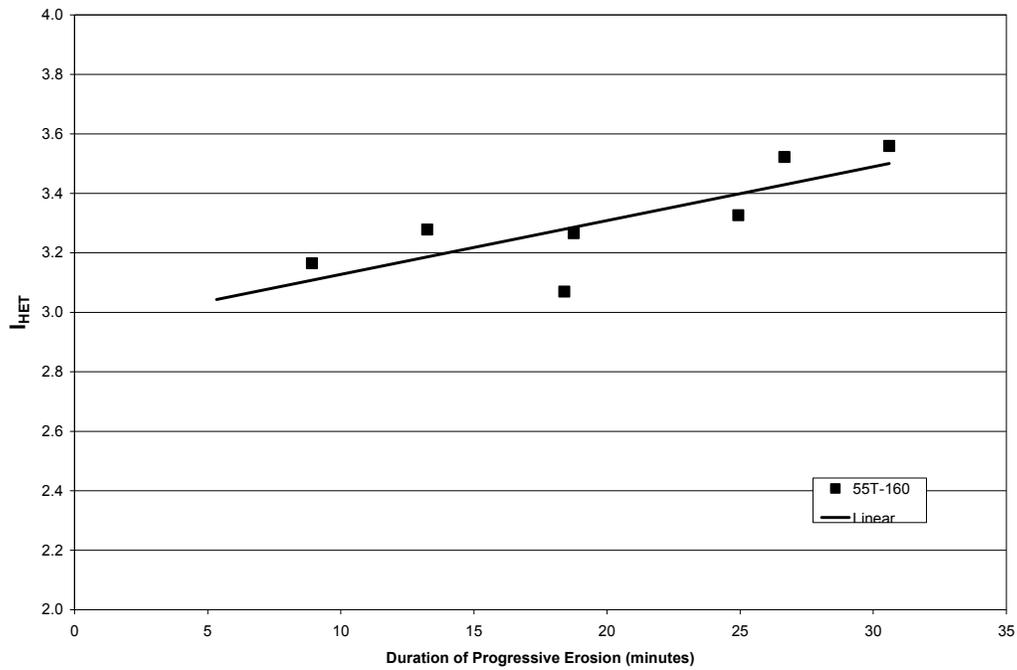


Figure 21.—Variation of I_{HET} as a function of the duration of the progressive erosion phase of each test.

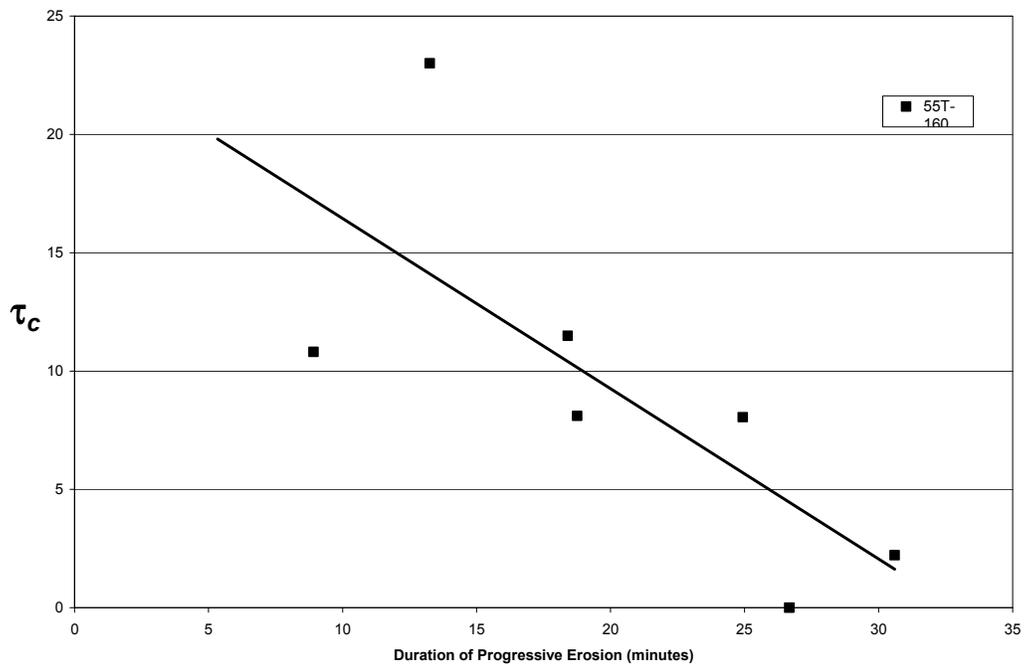


Figure 22.—Variation of critical stress obtained from HETs as a function of the duration of the progressive erosion phase of each test.

Practical Considerations

Experience gained with the soils described in this study was valuable for identifying practical difficulties encountered in the testing of specific soil types.

The HET proved to be the more difficult of the two tests to successfully carry out. It worked well for soils with intermediate strength, but was difficult to conduct with both very weak and very strong soils. Weak soils often collapse during the test or experience scour around the entrance and exit of the hole. Some of this scour seems to be related to gravity. Weak soils often experience slope failures around the entrance and exit holes, either before the test is begun, during the test, or after the test when removing the specimen for examination. Although such “failures” do indicate erodibility to some degree, they confound a quantitative analysis since their mechanism does not fit the model used to analyze HET data. The use of confining upstream and downstream end plates and porous mesh filters to reduce turbulence at the entrance is sometimes helpful but not always successful. End plates can help to prevent slope failures at the entrance and exit but promote scour, especially at the exit when material from the roof of the hole caves in, leaving a cavity larger than the exit orifice of the end plate. This creates recirculation at the exit that leads to further scour. Sometimes, the downstream scour hole advances upstream in a manner similar to a headcut process, even though most of the length of the predrilled hole does not erode. With soils of this type, a successful test can sometimes be conducted by starting at a larger head, one that is sufficient to cause erosion of the hole at the same time that scour of the ends is occurring. Still, one must be careful to complete the test before the upstream and downstream scour holes reach one another and completely breach the specimen. The experience at the Bureau of Reclamation has been that at least two to three trials are often needed of a weak soil in order to produce one successful test.

Very erosion-resistant soils are too strong to test at the heads that can be easily produced in typical laboratory settings. A high-head HET has recently been constructed in the hydraulics laboratory at the Bureau of Reclamation, where the ceiling is over 25 feet high. This facility allows test heads up to 5,350 mm, but even at this head, some fat clay materials have proven to be nonerodible. Wan and Fell (2004) assigned I_{HET} group 6 ratings to soils that would not erode at heads of 1,200 mm, but testing on lean and fat clays in the new high-head HET facility has shown that many materials which erode at heads between 1,600 and 5,300 mm still have rate coefficients high enough to place them in I_{HET} group 4 or 5.

Another problem encountered with some erosion-resistant soils is clogging of the hole during the test. Soils with high clay content and dry of optimum often erode by detachment of clay chunks, which may be large enough to clog the hole. This may be alleviated to some degree by testing at higher heads (capable of pushing the eroded chunks through the hole), or by using a larger predrilled hole, which also increases the applied stress.

These problems cause the HET in general to yield many less-than-ideal tests. The data from these tests can often be salvaged but require significant subjective interpretation. When making such subjective interpretations, it can be helpful to apply both the Wan and Fell deterministic analysis approach and the Bonelli curve-fit approach, separately, or in combination (i.e., using the Wan and Fell method to estimate the starting conditions for the Bonelli analysis).

In contrast to the HET, the JET test is more easily applied to a broader range of weaker and stronger soils. With a vertical jet orientation, gravity works to hold the sample together, rather than cause premature failure by unplanned mechanisms as in the HET. The JET works well with almost all cohesive soils, except those that contain a significant fraction of coarse sand or fine gravel particles that are not easily transported out of the hole. For very weak soils, test durations are usually quite short, but with care, enough data can be obtained to allow for successful analysis. With weak soils, one must be careful to stop data collection when the sides of the deepening scour hole begin to slide down into the bottom of the hole (otherwise, the scour depth may be observed to *decrease* with time). In general, the JET produces usable results more often than the HET and requires less application of subjective judgment to interpret the data. This makes it a more objective method that will produce more repeatable results.

Erodibility Classifications and Scope of Test Capabilities

The developers of the two tests studied here have each suggested erodibility classification schemes utilizing the test results (Hanson and Simon 2001; Wan and Fell 2004). The work reported here and other recent experience with HET and JET testing at the Bureau of Reclamation provide a useful database for examining the capability of each test to successfully test materials across the spectrum of these classification systems.

Figure 23 shows the results of 61 laboratory HETs and 47 laboratory and field JETs performed by the Bureau of Reclamation since 2007. These include the tests reported previously in this report, and other laboratory and field testing of remolded and undisturbed soil and soft-rock (claystone or siltstone) samples. The figure shows that although the HETs in general exhibit lower detachment rate coefficients and higher critical shear stresses, both sets of data generally follow the best-fit line proposed by Hanson and Simon (2001) for JET results. This suggests that both tests are measuring an intrinsic erodibility property of soils, albeit with significant bias between their results, perhaps for some of the reasons previously discussed as well as others.

The HET results shown in figure 23 represent reasonable upper and lower limits on the application of the HET device in its current configuration, with the highest k_d values being for materials that were nearly too weak to be tested and the lowest k_d values corresponding to stiff clay materials that were so erosion resistant that progressive erosion could barely be produced at heads up to 5,400 mm. The highest I_{HET} value obtained from any test in which progressive erosion took place

Determining Erosion Indices of Cohesive Soils with the Hole Erosion Test and Jet Erosion Test

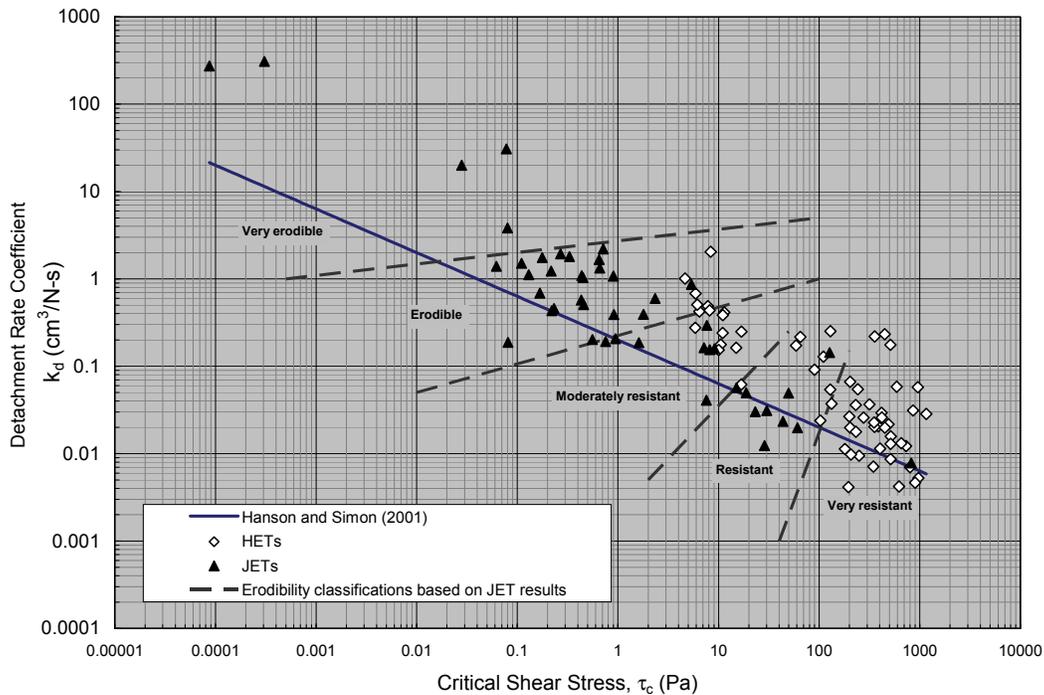


Figure 23.—HET and JET data collected by the Bureau of Reclamation in field and laboratory tests since 2007.

was about 5.2, and the lowest was about 2.5. This corresponds to the nearly 3 orders of magnitude difference in k_d values shown for HETs in figure 23. The test in its current configuration cannot provide a quantitative measure of the erodibility of many materials in groups 1-2 and 5-6.

There is the potential for the HET to be applied successfully to more erosion-resistant soils by modifying the facility to allow a larger maximum head. The use of an elevated head tank is probably not feasible for higher heads, but a pressurized water source with a regulator or flow bypass/waste system could be used. A higher range pressure transducer would also be needed, which would reduce the sensitivity of head measurements during low-head tests. Considering the relation between erosion rate and critical shear stress proposed by Hanson and Simon (2001), an increase in critical shear stress of 2 orders of magnitude is needed for each 1 order of magnitude decrease in erosion rate. Thus, a facility with a 10.5-meter head range (approximately 15 lb/in^2) would probably be capable of testing materials with I_{HET} values up to about 5.35; to cause progressive erosion of materials with an I_{HET} value of 6.0 might require pressures approaching 210 m of head (300 lb/in^2), which would be likely to cause cavitating flow through the predrilled hole. Considering soil and rock erodibility classification schemes of various authors (see Briaud 2008), it seems likely that materials with I_{HET} values of 6 or greater would be rock rather than true soils. Even these modifications would give the test a range of measurable erosion rates

that spans only about 3.5 orders of magnitude. The inability to quantitatively measure erodibility of weaker soils is the real limit on HET applicability.

Other options for testing more erosion-resistant materials with the HET include predrilling a larger hole, or using a shorter test specimen, thereby increasing the hydraulic gradient. Reclamation has used the former approach in a few instances, but significantly larger holes also require much higher flow rates, so the real benefit is limited unless flow capacity of the facility is also greatly increased. As for reducing the specimen length, it is probably already shorter than what is desirable from a hydraulic standpoint, with insufficient length to allow establishment of fully developed flow. An even shorter specimen would probably further exaggerate any existing discrepancies between the real applied stress and the idealized stress description used to analyze the test data.

Figure 23 shows that the JET is capable of performing successful tests across a broader range of materials, and is especially able to test weaker materials that simply disintegrate in the HET. Reclamation's applications have successfully measured detachment rate coefficients varying over about 4.5 orders of magnitude, and, considering the work of Hanson and Hunt (2007) and Hanson and Simon (2001), one finds that 5.5 orders of magnitude can be covered, from k_d values of 0.001 to 300 cm³/N·s. Tests of the most erodible materials must be performed carefully because erosion occurs very quickly, and successful use of the apparatus for the most erosion-resistant materials does require the use of a pressurized water supply. The most erodible data point in figure 23 was a Silty Sand (SM) that was tested for only about 2 minutes before the sample was completely eroded. The most erosion-resistant JET data point in figure 23 was obtained in a test performed *in situ* on a claystone/siltstone material, using a jet pressure of 24 lb/in² (16.9 m of water head), which was able to produce only 0.6 mm of scour in a 1-hour test. The shear stress applied by the JET can also be increased by using a larger nozzle, with a commensurate requirement for increased flow.

Applications for the HET and JET

Given the differences in test results observed here, the selection of which test to use for a specific application should be made primarily on the basis of the desired application of the data. The HET probably is the best test when one is trying to understand erosion through small holes or confined cracks, such as that which probably occurs during the initiation of internal erosion and piping failures of embankments. In contrast, the JET is probably best for studying erosion due to overtopping flow, and may also be best for studying erosion in larger, developing erosion pipes where the size of the flow channel is greater than the size of the soil structural elements that define the soil fabric. Identifying the transition between best applicability of the two tests during the progression from piping initiation to piping-caused breach of an embankment is still a subject for further research.

Conclusions

Variation of the friction factor during the hole erosion test is poorly correlated with time and more strongly correlated with the diameter of the erosion hole. As a result, an analysis method based on the Wan and Fell (2004) approach was developed in which the friction factor was related to surrogate parameters involving the flow rate and hydraulic gradient that are proportional to the diameter of the eroded hole. It was also found that extremely accurate determination of the final eroded diameter is probably not necessary for the Wan and Fell (2004) analysis. The sample-to-sample variability of tested specimens overshadows the uncertainties that result from errors in determination of final diameter.

The model proposed by Bonelli et al. (2006) provides a straightforward method for interpreting the HET. It produces results that are very similar to the method of Wan and Fell (2004) but does not require measurement of the final hole diameter. To best apply the method, the test procedure should be modified so that progressive erosion is initiated immediately and maintained over the full duration of a test. When progressive erosion was not produced immediately at the start of a test, the Bonelli analysis can still produce results similar to the Wan and Fell analysis, but requires one to identify the time at which progressive erosion begins, and then estimate the hole diameter and flow rate at the start of progressive erosion. To obtain these data, one needs to perform the Wan and Fell analysis. Thus, having both analysis methods available is valuable for making a good interpretation of tests that do not proceed exactly as planned.

The HET and JET methods yield significantly different estimates of the erosion index (rate coefficient) and critical stress, especially for the more erosion-resistant soils included in this investigation. The HET generally indicates slower rates of erosion and higher critical stresses.

Differences between the HET and JET method seem to be most dramatic in soils having a coarse fabric or structure, such as clays compacted dry of optimum. These soils often have a clumpy structure with seams of independent aggregates between lumps of clay. The JET method seems able to exploit these weak zones more effectively than the HET. Differences between HET and JET results seem reduced (but still very significant) when materials have a more uniform consistency.

Variability of the computed erosion rate coefficients and critical shear stresses is large for both methods, about 1 order of magnitude for the soils tested in this study. This is probably mostly a result of sample-to-sample variability of the compacted materials.

Test procedures may affect the correlation between the HET and JET methods, since some of the data collected here showed that the erosion-shear stress relation

is not linear. This can cause the test duration and the applied stress range to affect the result.

The JET method is more easily and successfully applicable to a wider range of soils. The HET works well with soils of intermediate erodibility that erode with relative ease but have sufficient strength to resist hole collapse and local scour—erosion mechanisms that are inconsistent with the assumptions underlying the HET analysis method. Very weak or strong soils often require multiple attempts before a set of data is produced that can be successfully analyzed. The JET method is more often successful over a broader range of soil erodibilities.

The need for subjective data analysis is generally greater with the HET method, since many tests are affected to some degree by intermittent clogging of the predrilled hole or localized scour erosion at the entrance and exit of the predrilled hole.

Selection of a test for a specific application should be made with consideration for the intended use of the data and the erosion mechanisms that will be most important in the application. Interpretation of the data should be made using techniques developed for the specific test because of the widely differing erosion rates and critical shear stresses indicated by the two tests.

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Determining Erosion Indices of Cohesive Soils with the
Hole Erosion Test and Jet Erosion Test

Appendix—Hole Erosion Test Procedures Used by the Bureau of Reclamation

The hole erosion test (Wan and Fell 2004) is one of several methods for evaluating the erodibility of cohesive soils. The HET utilizes an internal flow, similar to that occurring during piping erosion of embankment dams. A 6-mm or ¼-inch diameter hole is predrilled through a soil specimen and flow is passed through that hole under constant head. The head is increased incrementally until the threshold stress to initiate erosion is exceeded. Once erosion is initiated, the flow rate will accelerate over time, since enlargement of the hole leads to further increases in shear stress and higher rates of erosion. One must reach this “progressive erosion” condition in order to have a successful test.

An ASTM standard for the hole erosion test does not yet exist; in its absence, tests are performed and analyzed using methods consistent with those described by Wan and Fell (2004). Recently, the Bureau of Reclamation and others have also investigated other methods for analyzing the data collected during HETs, focusing on the use of a piping erosion model developed by Bonelli et al. (2006). The data reported here were analyzed using the Wan and Fell (2004) procedures, although they were also checked for consistency using the Bonelli method when applicable. The data analysis procedures are described below.

Test Facilities and Procedures

The hole erosion test facilities at the Bureau of Reclamation are similar to those used by Wan and Fell (2004), except that the maximum head values in our two facilities are approximately 1,600 mm and 5,400 mm. Flow measurement is accomplished using 10° V-notch weirs, and data collection is automated using a computerized data acquisition system that records differential head and flow rate at 5 second intervals. The upstream and downstream chambers are similar to those shown in the schematic diagram. With erosion-resistant soils we have found no need for the 20 mm gravel in the upstream chamber. When testing very erosive soils we have found it helpful to place a plastic geotextile mesh fabric in the upstream chamber and protect the upstream and downstream faces of the compacted soil specimen with end plates. We have a range of end plates available, with orifice openings varying from 10 mm to 25 mm. The orifice size is selected based on the expected erodibility of the sample, with smaller orifices generally used to provide more protection to the faces of weaker specimens. The test operator must consider the orifice size and plan to end the test before the hole enlarges enough to allow the orifice openings to limit the flow rate.

Determining Erosion Indices of Cohesive Soils with the Hole Erosion Test and Jet Erosion Test

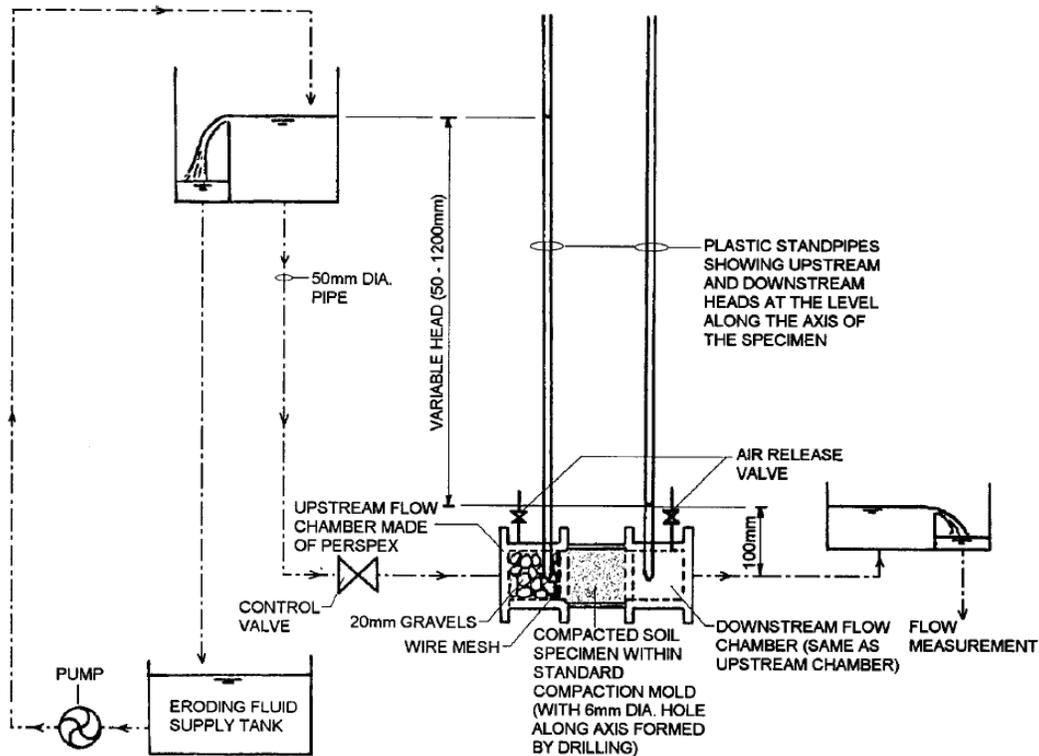


Figure A-1.—Schematic diagram of hole erosion test facilities (Wan and Fell 2004).

The basic test procedure is as follows:

1. Following specimen preparation and compaction, specimens are sealed in plastic bags to prevent moisture loss and cured overnight before testing.
2. After curing, a $\frac{1}{4}$ -inch diameter hole is drilled through the specimen using a drill press and wood auger bit to minimize compaction of the side walls of the hole. Drilling is performed at the slowest possible speed and the bit is advanced slowly and cleaned repeatedly during drilling.
3. The hole is cleaned using a 0.22-inch diameter rifle brush.
4. Specimens are installed into the apparatus with the original top surface (last compacted layer) upstream. If the soil is expected to be highly erodible or susceptible to scour of the upstream and downstream faces, protective end plates are also installed. A plastic geofabric mesh filter is also installed in the upstream chamber to reduce turbulence when specimens are expected to be highly erodible.
5. The test facility is filled slowly with water and all air is bled from piezometer tubes connected to pressure sensors.

6. The water supply head tank is positioned to the desired starting head level. For specimens of unknown erodibility, tests are usually started at 50 mm of head.
7. The downstream weir box tank is filled with water to the level of the horizontal weir that maintains nearly-constant downstream head, and some additional water is then added to produce flow through the V-notch weir at a rate that approximates the expected starting flow rate. This is done in an attempt to have the test start with the weir box system in a state of flow rate equilibrium.
8. The data acquisition system is started and the inlet valve upstream from the test specimen is opened.
9. The flow rate is monitored to determine whether it is increasing or becoming steady. If the flow rate stabilizes at a given head, then the head tank is raised to increase the head. We generally double the head each time, or if we feel that the erosion threshold is near, we will increase the head in somewhat smaller increments.
10. When the flow rate begins to accelerate, the test head is maintained until at least several minutes of accelerating flow is observed. The operator should be aware of the approximate maximum flow increase that can occur if end plates have been installed. For example, if 10 mm end plates have been installed, the ratio of flow rates with a 10 mm hole diameter to the flow through the original 6 mm diameter hole is approximately $(10/6)^2 \approx 3$. Thus, one should stop the test well before the flow rate has tripled from its value at the start of accelerating flow. If the test is allowed to continue too long, the orifice plate opening will begin to limit the flow rate, which will hinder the data analysis.
11. After the test is stopped, the upstream and downstream chambers are drained and the specimen is removed from the test facility. An initial visual estimate of the final hole diameter is made, and the specimen is weighed.
12. Specimens are oven-dried, weighed, and then a hydrostone casting is made of the erosion hole.
13. Hole diameters are determined from the casting, typically at 5 positions spaced approximately equally along the length. The length of the portion of the casting that is of relatively uniform diameter is also recorded. (Large scour holes at the upstream or downstream end are considered to reduce the effective length of the hole, which is taken into account in the data analysis.)

Wan and Fell Analysis Procedure

The deterministic data analysis method described by Wan and Fell (2004) attempts to compute the hole diameter at each time step at which data have been recorded. The computed time series of hole diameters can then be used to estimate the erosion rate and applied shear stress. Microsoft Excel spreadsheets are used to make the computations and present the data graphically.

The analysis begins by considering a cylinder of eroding fluid passing through the predrilled hole in a soil specimen. Assuming that over a short interval of time the flow is at steady state, the equation for force equilibrium is:

$$\tau \cdot P_w \cdot L = \rho_w \cdot g \cdot \Delta h \cdot \frac{\pi d^2}{4}$$

where:

τ = shear stress along the sides of the hole

P_w = perimeter of the hole

L = length of the hole

ρ_w = fluid density

g = acceleration due to gravity

Δh = head difference across the hole from upstream to downstream

d = diameter of the hole

For a laminar flow condition, the shear stress is expected to be proportional to the mean velocity of the flow

$$\tau = f_L \bar{v}$$

where

f_L = friction factor, S.I. units of kg/s/m

\bar{v} = mean velocity of the flow, $Q/(\pi d^2/4)$

Q = flow rate

Combining these equations and solving for the friction factor yields:

$$f_L = \frac{\rho_w g \Delta h \pi d^3}{Q L 16}$$

This equation can be used to solve for the friction factor at the start and end of the test, when the hole diameter, length, head differential and flow rate are all known. This research project has shown that the friction factor is best correlated with the hole diameter, but the hole diameters during the test are not known until the analysis is complete, so the friction factor is instead assumed to vary during the test in proportion to the value of $(Q/\Delta h)^{1/3}$ for laminar flow, and $(Q^2/\Delta h)^{1/5}$ for turbulent flow. These quantities are surrogates for the hole diameter. The length of the erosion hole is assumed to vary linearly with time during the test (although it stays constant in many tests). The quantity $(Q^2/\Delta h)^{1/5}$ is also plotted on the data

acquisition computer during a test to help the operator know when accelerating enlargement of the hole diameter is occurring. Most tests take place with turbulent flow conditions. The onset of turbulence is assumed to occur when the Reynolds number of flow through the hole exceeds 2,000 ($Re=Vd/\nu$, where V is the flow velocity, d is the hole diameter, and ν is the kinematic viscosity).

Denoting friction factors and hole lengths at intermediate times during the test by the subscript t , the same equations can be solved for the hole diameter to allow it to be computed throughout the test from measured values of the flow rate.

$$d = \left(f_{L_t} \frac{Q_t}{\rho_w g} \frac{L_t}{\Delta h_t} \frac{16}{\pi} \right)^{1/3}$$

If the flow is turbulent, the shear stress is proportional the square of the mean velocity and the following equations apply:

$$\begin{aligned} \tau &= f_T \bar{v}^2 \\ f_T &= \frac{\rho_w g}{Q^2} \frac{\Delta h}{L} \frac{\pi^2 d^5}{64} \\ d &= \left(f_{T_t} \frac{Q_t^2}{\rho_w g} \frac{L_t}{\Delta h_t} \frac{64}{\pi^2} \right)^{1/5} \end{aligned}$$

Bonelli Analysis Procedure

Bonelli et al. (2006) proposed a universal model for piping erosion, applicable to analysis of the hole erosion test. They showed that the change in dimensionless hole radius is an exponential function of the dimensionless test time and the initial and critical shear stresses

$$\frac{R(t)}{R_0} = 1 + \left(1 - \frac{\tau_c}{\tau_0} \right) \left(e^{t/t_{er}} - 1 \right)$$

where $R(t)$ =radius at any time t and R_0 =the initial radius at time zero, τ_c =critical shear stress, τ_0 =shear stress at time zero, t =test time, and t_{er} =a characteristic erosion time scale for each test

$$t_{er} = \frac{2L}{k_d \gamma_w \Delta h} = \frac{2L \gamma_d}{C_e \gamma_w \Delta h}$$

where L =length of the hole, γ_w =unit weight of water ($\rho_w g$), Δh =head differential across the hole, γ_d =dry unit weight of soil, C_e =erosion rate coefficient

Determining Erosion Indices of Cohesive Soils with the Hole Erosion Test and Jet Erosion Test

(mass/time/area/stress), and k_d is a volumetric detachment rate coefficient (volume/time/area/stress).

The model assumes turbulent flow conditions and neglects any variation of the friction factor, the test head, or the length of the eroded hole. The method also presumes that the test data are collected entirely during the period of accelerating erosion. Bonelli et al. (2006) showed that the proposed model fit the observed hole radius data computed from 17 HETs performed by Wan and Fell (2002) using 9 different soils. Bonelli and Brivois (2007) have offered further development of the model.

Recognizing that dimensionless discharge, Q^* , is proportional to the 2.5 power of the dimensionless radius (again neglecting effects of any change in the friction factor during a test), one can write

$$Q^* = \frac{Q(t)}{Q_0} = \left(\frac{R(t)}{R_0} \right)^{5/2} = \left[1 + \left(1 - \frac{\tau_c}{\tau_0} \right) \left(e^{t/t_{er}} - 1 \right) \right]^{5/2}$$

Since flow rates are measured throughout a test and the initial shear stress is known from the starting hole diameter and flow rate, this model has only two unknown parameters, the erosion time scale, t_{er} , and the critical shear stress, τ_c . Using a non-linear optimization tool such as the Excel Solver, one can optimize these two parameters to obtain a best fit of the observed dimensionless values of discharge to predicted values computed for each dimensionless test time, t/t_{er} . The coefficient of soil erosion or the detachment rate coefficient can then be determined from the fitted value of the time scale factor, t_{er} . The significant advantages of this analysis method are the fact that the final hole diameter does not need to be measured, and the curve-fitting procedure minimizes the influence of short-term anomalies in erosion behavior during a test.

It should be emphasized that the formulation of the Bonelli model requires the fitted value of the critical shear stress τ_c to be less than the initial stress, τ_0 , otherwise the quantity $(1 - \tau_c/\tau_0)$ is negative. This means that tests must be conducted at a stress level that exceeds the critical stress and produces immediate progressive erosion, or one must customize the analysis to only examine the portion of the test in which the shear stress exceeds τ_c . If a test begins at a stress level that is slightly lower than the value needed to initiate progressive erosion, but the stress then increases due to cleanout erosion of material disturbed during hole drilling, the only way to accurately determine the critical stress would be to estimate the increase in hole diameter and shear stress that takes place leading up to the progressive erosion phase, then start the Bonelli analysis at that point in time. This requires the combined use of both the Wan and Fell and Bonelli analysis procedures.

HET Subjectivity Index

Hole erosion tests do not always proceed according to plan. The ideal erosion mode is a uniform enlargement of the predrilled hole along its full length, producing accelerating flow over the duration of the test, once erosion is initiated. Other erosion modes, such as localized scour at the entrance and exit of the hole can yield data that are difficult or impossible to analyze. To help quantify the potential uncertainty of test results, the table below provides numerical indices for the degree to which subjective judgments were required by the analyst during the processing of HET data.

Subjectivity indices for HETs – These characteristics are offered as guidelines; not every characteristic will be present in any particular case.

- | | |
|-----|---|
| 0 | Start of progressive erosion is definite and progressive erosion and accelerating flow are maintained continuously until end of test. The Wan & Fell (2004) and Bonelli et al. (2006) analysis methods yield nearly identical results. The k_d and τ_c values obtained from the two methods differ by less than 1/10 order of magnitude. |
| 1/2 | Similar to grade 0, except that the two analysis methods yield only similar (not “nearly identical”) results. |
| 1 | Progressive erosion and accelerating flow are not continuously maintained. To get a reasonable result, the analysis must be restricted to a subset of the data following the initiation of erosion. Some judgment is required, but the analyst has good confidence in those judgments. Both analysis methods yield similar results. |
| 2 | Unintended modes of erosion significantly affect the test (e.g., scour at entrance or exit causing hole shortening without significant enlargement, sloughing of roof of pipe, clogging of pipe). Period(s) of progressive erosion and accelerating flow are not continuously maintained and are relatively short. Significantly different test results can be obtained by analyzing different segments of the data, and it is not readily apparent which segment should be used. Only one analysis method yields a result that seems reasonable. Analyst has poor confidence in test result. Analysis indicates $\tau_c \leq 0$, even though there was no erosion observed at low heads (and hence there should be a positive shear stress needed to initiate erosion). |
| 3 | There is no period of progressive erosion that produces continuous hole enlargement with accelerating flow. No reasonable test result can be obtained from either analysis method. |

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