RECLAMATION

Managing Water in the West

Value Attributes in Pump-Generation Plants

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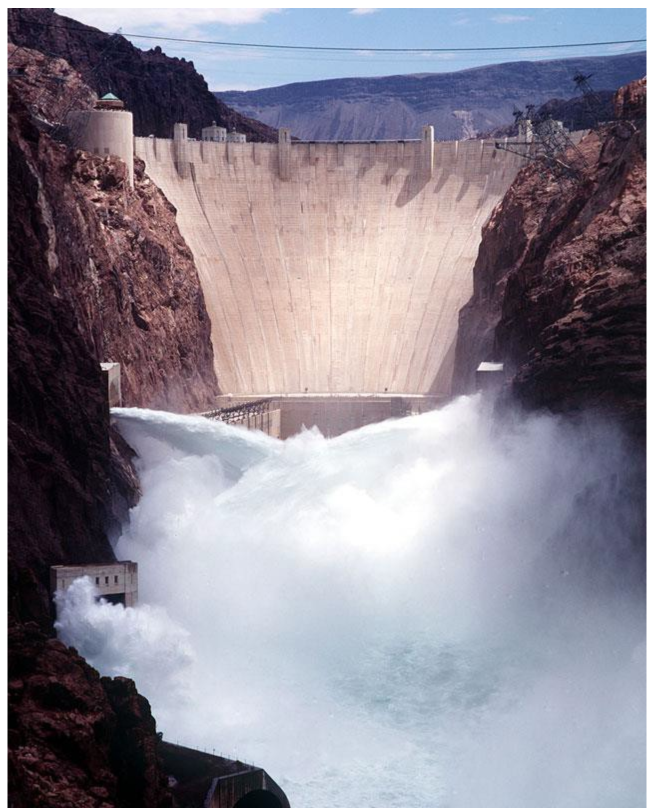

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Hoover Dam

Celebration of Reclamation's 100th anniversary at Hoover Dam.

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Any errors are the sole responsibility of the authors.

Acronyms and Abbreviations

ANL	Argonne National Laboratory
CAISO	California Independent System Operator
CHEERS	Conventional Hydropower Energy and Environmental Systems
DAM	day-ahead market
DOE	U.S. Department of Energy
EPRI	Electric Power Research Institute
FERC	Federal Energy Regulatory Commission
GAMS	Generalized Algebraic Modeling System
GTMax	generation and transmission maximization
HDR	HDR Engineering, Inc.
ISOS	Independent System Operators
LMPs	locational marginal prices
MW	megawatt
MWH	Montgomery Watson Harza
NERC	North American Electric Reliability Corporation (formerly North American Electric Reliability Council)
NGCCCT	natural gas combined cycle combustion turbine
NGCT	natural gas combustion turbine
NR	nonspinning reserve
NREL	National Renewable Energy Laboratory
NWPCC	Northwest Power and Conservation Council
PG	pump generation
RD	regulation down
Reclamation	Bureau of Reclamation
RPSs	renewable portfolio standards
RU	regulation up
SR	spinning reserve
WECC	Western Electricity Coordinating Council
Western	Western Area Power Administration

Symbols

%	percent

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EXECUTIVE SUMMARY

Attributes such as unit design, gross head, forebay (upper) reservoir volume and unit size shape the economic benefits produced by a pump-generation power plant. This study estimates the independent contribution of each of these determinants to net economic benefits. These value estimates can be used to assess the importance of these determinants, informing planners and decision-makers as they examine potential locations for siting pump-generation plants.

Harpman, Kubitschek and Wittler (2014) describe a pump-generation plant sited adjacent to an existing Bureau of Reclamation reservoir. Concept 5, as they termed it, has been used in several related analyses including Bureau of Reclamation (2013a, 2013b) and HDR-CDM Joint Venture (2014).

Detailed mathematical optimization models of single speed (SS) and variable speed (VS) pump-generation units were crafted to represent Concept 5 pump-generation plants. These hourly models operate over a 1-week (168 hours) period. Some research license was employed in developing these models. This allows the size of the pump-generation unit, the forebay (upper) reservoir storage volume and the gross head to be independently changed. These abstractions from the real-world enable estimation of the economic benefits of these determinants, independent of all other factors.

The pump-generation models simulate the hourly production and sale of ancillary services as well as energy. The ancillary services provided by the pump-generation plants are up-regulation (UR), down-regulation (DR), spinning reserves (SR) and non-spinning reserves (NR). Publically available 2014 hourly price data were obtained from the California Independent System Operator (CAISO) and used to characterize hourly energy and ancillary service prices in the study.

Mathematical representations of pump-generation models are inherently nonlinear constrained optimization problems. This class of problems is notoriously difficult to solve. The underlying nonlinear specifications were reformulated as more tractable piece-wise linear approximations. The resulting mixed integer linear programming (MILP) models were then solved using the special ordered sets of type 2 (SOS2) approach.

A set of reference operating conditions formed the basis for all comparisons in this research effort. These reference conditions featured a single 100 MW unit, 600 feet of gross head, a (live) forebay (upper) reservoir storage volume of 8 hours.

For a typical week, each of the following factors is systematically varied and the model resolved; (a) design of the pump generation plant [fixed speed or variable speed pumps], (b) gross head, (c) unit size, (d) number of units and (e) forebay storage volume. By comparing the results of each of these experiments to the reference case, the independent contribution of each of these factors to the economic value of the plant is estimated.

The SS and VS models were used to explore the potential differences in the economic benefits which could be produced by these two plant designs. Both models were used to simulate the behavior of pump generation units at the reference operating conditions. Results from this analysis indicate VS units may produce approximately 20% greater net revenues (gross revenues minus pumping costs) relative to SS units. If the price data utilized were different, the results reported here would differ, perhaps markedly. In particular, if the spread between AS and energy prices were larger, gross revenue would be higher and both plants would be expected to produce more ancillary service and less energy revenues. Potentially, higher AS prices might further advantage the VS unit in this comparison.

Like almost all published studies, this research assumes the producer/operator is a price taker facing a fixed price which does not vary with the output quantity produced. In the case of energy, which is traded over a wide area, this approach seems quite plausible. For the large-scale provision of ancillary services, the extent of the market is limited and this assumption is more difficult to support.

Conceptually, the coordinated operation of multiple pump-generation units could yield economic benefits which exceed those of a single unit multiplied by the number of units installed. The SS and VS pump-generation models were used to investigate this possibility. For hydraulically independent units operating under the price taker assumption no additional economic benefit was observed.

Arbitrage is the creation of net revenues by purchasing a good when its price is low, and then selling it when its price is high(er). Pump-generation units can be used strategically to exploit the difference or spread between low and high energy prices. Producers can employ their pump-generation resources for arbitrage by purchasing and using energy for pumping when the price is low, typically during off-peak hours, and then generating and selling energy when the price is high, generally during the on-peak hours.

For purposes of this research project, the live forebay (upper) reservoir storage volume is measured by the number of hours of storage necessary to provide generation at the maximum output level. The analysis shows 99% of the net economic storage benefits are captured in a range of 6.31 to 7.23 hours of storage. This level of forebay reservoir storage is considerably less than commonly used rules of thumb would suggest. For energy arbitrage only plants, higher forebay storage capabilities may be prudent.

The SS and VS pump-generation models were used to explore the incremental net economic benefits of gross head, independent of all other factors. Analysis at the reference conditions indicates the predominant fraction of the net economic benefits can be captured at levels of gross head in the neighborhood of 500 feet. Suitable topographic conditions in this head range can be found adjacent to many Bureau of Reclamation facilities.

Relative to the "without" ancillary service case, the net economic benefits produced by SS and VS 1-unit plants are 63% and 56% greater respectively, when AS are provided and sold. These results were obtained at the reference operating conditions. The reported numerical results are very sensitive to the specific price data used in the analysis. If ancillary service prices are higher and the spread between the AS and energy prices greater, net revenues increase and the share of gross revenues derived from ancillary services is much higher. Exploration reveals energy arbitrage is the predominant source of revenue for both SS and VS plant designs, regardless of the price set employed for analysis.

This study compliments several preceding efforts and contributes some new insights. It yields a richer understanding of the relationships between each attribute and its incremental contribution to value, over a wide range of attribute values. These attributes are common to all pump generation plants and the findings reported here will help inform future site selection and design decisions.

Contributions of this Study

This research effort builds upon a number of recent studies. It contributes to the literature and informs decision-making in three principle ways. First, this study carefully explores the incremental contribution of economic value derived from the provision of four ancillary services; up-regulation, down-regulation, spinning reserve and non-spinning reserve. Second, this research effort provides an estimate of the additional economic value which might be obtained by constructing a variable speed pump generator, rather than a single or fixed speed pump generator. Finally, this research explores the independent value of selected site characteristics such as head and forebay (upper) reservoir storage, thereby contributing to informed design and site selection decisions.

Pump Generation versus Pumped Storage

In this document, the term "pump generation" is employed rather than the more commonly encountered phrase, "pumped storage." The term pump generation is used here to clearly distinguish between pump generation for the purpose of energy storage and pumped storage, which can also include purposes, in some cases solely, for water storage.

Closed-Loop versus Open-Loop Systems

Many pump-generation plants currently in service are comprised of a closed-loop system. In a closed-loop system, water is pumped from a lower off-stream reservoir (the afterbay) to an upper off-stream reservoir (the forebay). Water is then released from the upper reservoir to generate electricity and returned to the lower reservoir. This cycle of pumping and generation continues, and the preponderance of the available water volume is used repeatedly. In such a closed-loop system there is some recharge from a nearby water source with little or no outflow from the lower reservoir, and losses from the system are a minimal proportion of the total, perhaps being restricted mainly to evaporation losses.

The focus of this investigation, however, is on the potential for utilizing existing Reclamation facilities for pump-generation. By virtue of their location and the necessity of meeting existing downstream water deliveries, these are inherently open-loop systems. As these configurations are envisioned, some, possibly only a relatively small fraction, of the total annual inflow may be stored for subsequent reuse.

Relation to Previous Studies

This research effort is built upon and complements a number of recent studies. Technological innovations, concerns about climate change and carbon emissions and increases in relative energy prices have given rise to much of the recent interest in energy storage technologies. Pumped storage hydropower is a technologically proven energy storage approach and there has been a resurgence of interest and recent exploration of this subject.

A large subset of this research has focused on the ability of pump generation plants to compliment and facilitate the use of variable and renewable energy sources, such as solar and wind. A selection of recent studies on this topic might include; Acker (2011), Loose (2011), Yang and Jackson (2011), Botterud, Levin and Koritarov (2014) and EPRI (2012).

As is the case with many energy storage options, only certain locations are suitable for the installation of pump generation hydropower plants. Considerable effort has been devoted to the identification of potential sites which may be both economically favorable and practical. In the Western United States, a comprehensive exploration of sites with existing "auxiliary" reservoirs has been completed by Hall and Lee (2014). More pertinent to this research effort are studies by the Bureau of Reclamation (2013) and HDR-CDM Joint Venture (2014). These two studies focused on identifying potentially suitable sites where existing Reclamation reservoirs could serve as an afterbay (lower) reservoir, and the surrounding topography allowed for construction of a new forebay (upper) reservoir. In this document, this configuration is described as Concept 5 following the work of Harpman, Kubitschek and Wittler (2014).

Increased penetration of variable renewable generation sources is almost universally projected to increase the need for ancillary services in the interconnected electricity system. The capability of pump generation plants to produce these services is widely touted. The economic value of these contributions, as traded in existing markets, is the subject of considerable ongoing research. Several recent studies provide some important insights into the economic and financial value of pump-generation plants, with particular emphasis on their role in the provision of ancillary services. Among the most recent of these studies are those by EPRI (2012) and Koritarov et al (2015).

Mathematical characterization of advanced pump generation plants and solution of the resulting optimization model are daunting tasks. In addition, information on the engineering characteristics of variable speed pump generation plants is not widely available. This study would not have been possible without the generous assistance of Argonne National Laboratory staff and their expertise in this type of analysis. This study utilizes much of the approach and engineering design data common to studies by Gasper et al (2013), Koritarov et al (2015) and Mahalk et al (2012). As might be anticipated, the mathematical underpinnings of this study have many similarities to the latter two studies.

Concept 5

This research effort builds upon the pump-generation concept previously defined as Concept 5 – Pump Generation Expansion. As described in Harpman, Kubitschek and Wittler (2014), Concept 5 is considered the most promising of five retrofit concepts initially identified and which could potentially be applied at existing Reclamation facilities. It essentially consists of constructing a PG facility that utilizes an existing Reclamation reservoir. Coincidently, this is similar to concepts described in several recent FERC filings (Symbiotics, LLC [2008, 2009]). Figure 1 illustrates Concept 5 which utilizes an existing reservoir as the afterbay for the PG plant and requires construction of new (upper) reservoir to serve as the forebay. Concept 5 can employ multiple units and either conventional or variable speed equipment could be considered.

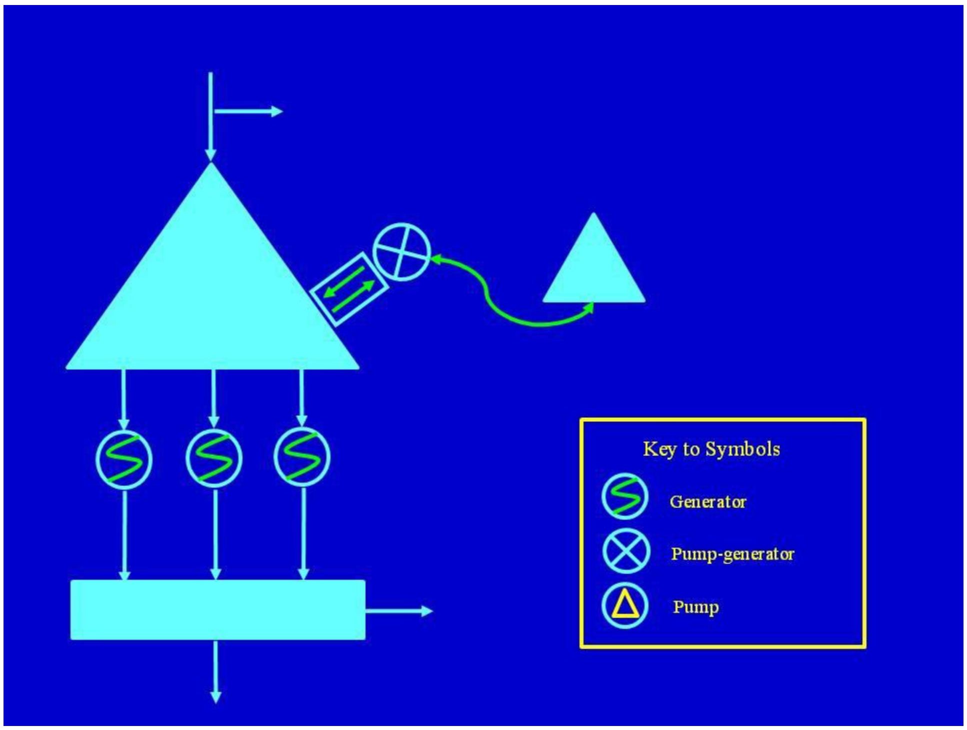

Figure 1.—Concept 5 plant schematic.

The practicality of this concept primarily involves the potential for minimum impacts to existing operations. Other features include, but are not limited to, forebay sizing that can be tailored to the PG capacity needs, sufficient tailbay depth for pumping operation, proximity to existing power generation infrastructure, and some flexibility in locating the PG plant. For these and other strategic reasons, this follow-on project utilizes a Concept 5 type pump-generation facility throughout.

In conjunction with this effort, Reclamation completed a study of sites with suitable characteristics similar to Concept 5 (Bureau of Reclamation 2013a, 2013b) and subsequently funded more comprehensive analyses of what were considered the most promising Reclamation sites. The reconnaissance level study by HDR-CDM Joint Venture (2014) identified and ranked 60 potential Reclamation projects having storage capacities greater than 100,000 acre-ft. Concurrent with that effort, five sites linked to Yellowtail, Seminoe, and Trinity facilities were further evaluated for technical and economic viability. These sites were chosen due to their existing infrastructure including existing upper and lower storage reservoirs and powerplant infrastructure. However, it was concluded that none of the sites have positive net benefits under the base case scenario.

The Phase 2 Final Report recommended:

Reclamation should continue to monitor ancillary service markets and prices to understand how they might further develop or change in the future. *An initial screening effort could focus on these topics to try to narrow down a broad list of sites to a smaller list for more detailed evaluation:*

Identify sites with a suitable L/H (operating head over water conductor length) ratio. *Finding a site that has a large elevation change at a short distance from the existing forebay reservoir would help reduce costs of the facilities.*

Use large existing reservoirs. *The operations model found that pumped storage operations result in very small changes to water levels or volumes in the existing forebay reservoirs because the reservoirs are much larger than the proposed new reservoirs. Using a smaller reservoir would increase the likelihood that pumped storage operations could affect water supply for downstream environmental needs and water users.*

Focus on sites in the Pacific Northwest, California, or Arizona. *These areas have a widespread transmission system that would help reduce the high transmission costs associated with the sites in this study. Additionally, ancillary service markets are likely to be established in these areas because of the large demand centers and focus on renewable energy.*

Locate projects in areas with high potential for wind power development. *Pumped storage projects have maximum benefits when they can integrate with other renewable resources, such as wind power.*

As a complement to parallel studies, this research effort has focused on analyzing optimal operations of Concept 5 given historical market prices for energy and ancillary services to further assess economic factors influencing the potential value of pump generation.

Ancillary Services and CAISO

Ancillary Services

The electric power system is designed and operated to maintain a real-time balance between generation and load and to adjust the output of generators in order to manage power flows through the transmission system. The services needed to meet these requirements are known as ancillary services. FERC (1995) defined ancillary services as "those services necessary to support the transmission of electric power from seller to the purchaser given the obligations of control areas and transmitting utilities within those control areas to maintain reliable operations of the interconnected transmission system." FERC identified six ancillary services: reactive power and voltage control (regulation), loss compensation (contingency), scheduling and dispatch, load following, system protection, and energy imbalance. Of particular relevance to this research are regulating reserves and contingency reserves.

Regulating reserves are employed to keep the system voltage and frequency within narrow limits and to provide energy on the timescale of one to several seconds. Regulating reserves are typically furnished by hydropower plants and other plants that have automatic generation control systems. Where markets exist, regulation is often differentiated into up-regulation (UR) and down-regulation (DR) services.

Contingency reserves are needed to restore the generation and load balance in the event of a contingency event such as the unexpected outage of a generator or transmission line. In many markets two types of reserves are traded; spinning reserves (SR) and non-spinning reserves (NR).

Up and Down-Regulation

Regulation is the amount of operating reserve capacity required by the control area to respond to automatic generation control to ensure the Area Control Error (ACE) remains within the performance standards described in North American Electricity Reliability Council (NERC) (2011). ACE is the instantaneous difference between a Balancing Authority's net actual and scheduled interchange, taking into account the effects of frequency bias and correction for meter error.

Units designated for providing down-regulation services must operate at sufficiently high output levels such that sudden decreases in load will not reduce generation below their technical or regulatory minimum output levels. To provide up-regulation services, unit generation levels must be sufficiently low such that a power plant can respond to instantaneous increases in grid loads without exceeding their maximum output capability.

The portion of units designated to meet regulation (up and down) requirements is unavailable for real power dispatch.

Spinning Reserve

Spinning reserves are defined as a designated block of unloaded generation, connected to an output bus, synchronized to the electric system, and ready to take immediate load. When a generator supplies spinning reserve services, it will increase output in response to an outage situation. When a unit is fully or partially designated to spinning reserve, its output level is reduced so it can meet the spinning reserve obligation without exceeding the maximum capability of the generator.

Units, or portions of units, designated to meet spinning reserve requirements are unavailable for real power dispatch.

Non-Spinning Reserve

Non-spinning reserves are defined as generating capacity that is unloaded, connected to an output bus, and not synchronized to the electricity system. Depending on balancing area regulations, these resources must be capable of being brought online in 10 minutes if it is offline, and which is capable being operated for at least two hours. When a hydropower unit is designated for non-spinning reserves it is often dewatered and idle.

Units designated to meet non-spinning reserve requirements are unavailable for real power dispatch (they are on reserve).

CAISO

Within the last two decades entities known as Independent System Operators (ISOs) have formed to dispatch and manage some parts of the wholesale electricity system. These ISOs have arisen following the issuance of FERC Orders 888 and 889 (FERC 1995). At least in theory, ISOs can more efficiently market wholesale electric power over large(r) geographic expanses, efficiently utilizing the available transmission system, exploiting system diversity and economy of scale opportunities, and thereby reducing overall system operation costs.

The California Independent System Operator (CAISO) is perhaps the largest of the existing ISOs in terms of geographic scope and influence. In some ways its name is misleading since it operates in primarily in California and Nevada.

Fortuitously, the CAISO footprint either encompasses, or is adjacent to, many WECC marketing areas of particular interest to Reclamation. CAISO is required to publically report a variety of system operation metrics at different timescales. These data are monitored closely by both regulators and market participants. Among these data are load, energy, and ancillary service values for different locations within the ISO service area.

Ancillary Service Provision

Both variable speed (VS) and single speed (SS) pump generation units can provide ancillary services. SS units are capable of providing reserves and regulation in generation mode, but can only provide reserves in pump mode. VS units can be used to produce reserves and regulations in both generation and pump mode.

This section of the document discusses some of the practical details of AS provision and provides some insights into their effect on energy generation, energy use, water release and pumping.

Setpoint

The planned output level for an operating unit in generation mode is known as the generation setpoint. Assuming the unit does not produce ancillary services or suffer an outage during that hour, the setpoint and the actual level of energy generation are identical. However, when ancillary services are provided, in addition to energy, there is often a divergence between the setpoint and actual generation during the hour.

The planned output level for a unit in pump mode is known as the pump setpoint. Assuming the unit does not produce ancillary services or suffer an outage during that hour, the setpoint and the actual level of pumping are identical. However, when ancillary services are provided by the pump, there is often a divergence between the setpoint and actual amount of pumping during the hour.

Spinning Reserve

Spinning reserves can be provided by VS and SS pump generation units in both generation and pumping modes. Spinning reserves allocated to these units must be synchronized and ready for near instantaneous deployment. Units in generation mode can produce spinning reserves by deploying an unallocated block of capacity. Units in pumping mode can produce spinning reserves by pumping less (VS units) or by ceasing to pump (VS and SS units).

The amount of spinning reserve allocated to a SS or VS unit in generation mode, and the probability of its provision determine how much energy above the setpoint is generated, and how much more water is released in the process.

The amount of spinning reserve allocated to a SS or VS pump unit, and the probability of its provision, determine how much less energy below the pump setpoint is used, and hence how much less water is pumped.

Producers commit to providing a block of spinning reserve to fulfill reserve sharing obligation or when market prices dictate. Marketing arrangements for the provision of spinning reserves are frequent and routine. However instances where producers actually have to deliver spinning reserves are relatively rare and typically are related to outage situations.

The probability of a unit having to deliver spinning reserves is not well understood but is an important consideration both in real-life and for modeling. For expositional purposes, assume that a release of 1000 cfs would be required to produce 100 MW of spinning reserve. The plant must maintain this amount of water in the forebay in order to provide the reserves in generator mode, when required to do so. And, when a reserve call occurred, additional water over and above the setpoint would need to be released. Similarly, the plant would have to pump less water back into the forebay in order to deliver spinning reserves in pump mode. Both cases effect plant dispatch and scheduling plans. The operative question is how often do spinning reserve calls occur? If such calls happened frequently, this would have an influence on the producer's decision to offer this ancillary service, at a given market price.

Non-Spinning Reserve

Non-spinning reserves can also be provided by VS and SS pump generation units in both generation and pumping modes. Non-spinning reserves must be available for grid support in 10-minutes, or less. Units in generation mode can produce non-spinning reserves by deploying an unallocated block of capacity. If a unit can be brought online in 10-minutes or less, it too can provide non-spinning reserve. Units in pumping mode can produce spinning reserves by pumping less (VS units) or by ceasing to pump (VS and SS units).

The amount of non-spinning reserve allocated to a SS or VS unit in generation mode, and the probability of its provision determine how much energy above the setpoint is generated, and how much more water is released in the process.

The amount of non-spinning reserve allocated to a SS or VS pump unit, and the probability of its provision, determine how much less energy below the pump setpoint is used, and hence how much less water is pumped.

Producers commit to providing a block of non-spinning reserve to fulfill reserve sharing obligation or when market prices dictate. Producer obligations to provide non-spinning reserves are frequent and routine. However instances where producers actually have to deliver spinning reserves are very rare and typically are related to large-scale outage situations.

The probability a unit will have to deliver non-spinning reserves is not well established but is an important consideration both in real-life and for modeling. The operative question is how often do non-spinning reserve calls occur? Although this is currently an open question, the probability of such calls has an influence on the producer's decision to offer this ancillary service, at a given market price.

Up and Down-Regulation

Up-regulation and down-regulation can be furnished second-by-second and minute-by-minute by both SS and VS units in generation mode and VS units in pump mode. The primary purpose of regulation is to maintain system frequency and hence voltage, within established limits. By nature, regulation is constantly varying between zero (the setpoint) and the amount of regulation allocated to the unit. The active provision of ancillary services results in deviations between the setpoint and the actual levels of delivery in both generation mode and pumping mode (VS units only).

The amount of up-regulation allocated to a SS or VS unit in generation mode, and the probability of its provision determine how much energy above the setpoint is generated, and how much more water is released in the process. The amount of down-regulation allocated to a SS or VS hydropower generation unit, and the probability of its provision determine how much energy less than the setpoint is generated, and how much less water than planned is released in the process.

SS units in pump mode cannot produce regulation. The amount of up-regulation allocated to a VS pump unit, and the probability of its provision, determine how much less input power below the pump setpoint is used, and hence how much less water is pumped. The amount of down-regulation allocated to a VS pump unit, and the probability of its provision, determine how much energy above the pump setpoint is used, and hence how much more water is pumped.

To provide some further insight into these concepts, up-regulation is used as the focus. The concepts and explanations which are introduced apply equally well to down-regulation. Figure 2 helps illustrate some of the operational nuances of up-regulation provision and is employed to help explain some important concepts further.

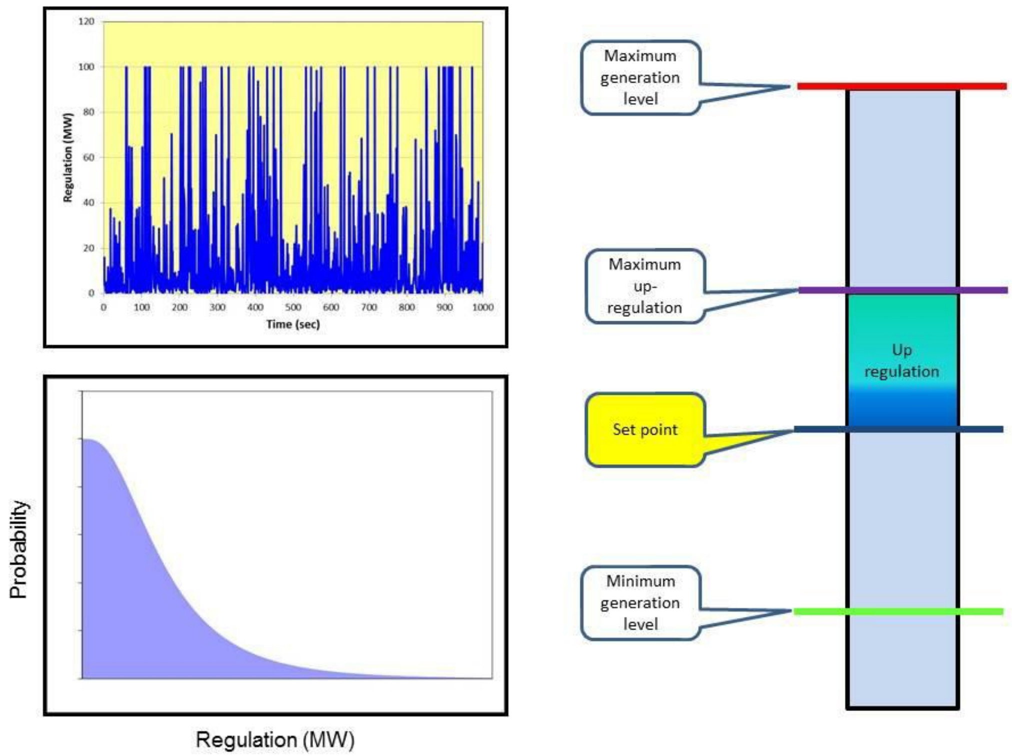

Figure 2.—Regulation concepts.

The right-hand most panel in this figure illustrates a hypothetical unit in generation mode. As shown, part of the unit is allocated to the provision of up-regulation. The up-regulation block is differentially shaded to indicate that during actual operations, the generator is dispatched above the setpoint only some of the time.

The upper left-hand panel in Figure 2 provides a closer look (1-second time step) at 100 MW of up-regulation over a 1000-second period. As shown here, the amount of up-regulation furnished by the plant is quite variable. The plant is very seldom dispatched to the full extent of the up-regulation block but instead operates somewhere in the range from the setpoint (zero) to the top of the 100 MW up-regulation block. If the information shown in this plot is sorted, a frequency distribution similar to the one shown in the lower left-hand corner can be constructed. While a variety of statistical measures can be calculated from this kind of data, the mean or expected value is a particularly useful measure. Using these data we could, for instance, calculate an expected value. For our purposes, we will say the expected value is equal to 20 MW. This measure indicates that, on the average, 20 megawatts out of the committed 100 MW of up-regulation are being provided by the plant. This is 20 percent of the committed amount, or 0.20 when expressed as a decimal fraction.

For expositional purposes, assume that a release of 1000 cfs would be required to produce 100 MW of additional generation. In the absence of any scheduled down-regulation, on the average, 20 MW of energy, over and above the setpoint will be produced. We can also infer that, on the average, 200 cfs of additional release, over and above the setpoint release level, would be required to support this amount of up-regulation.

Probability of Provision

As described here, the probability of an ancillary service call is an important indicator both in real-life and for modeling purposes. In both real-life and simulated operations, the probability of having to furnish ancillary services, plays a role in the amount of energy generated, the amount of energy used for pumping and the amount of water released or pumped. In addition, it plays and important role in the revenues which can be obtained from the sale of ancillary services, and thus the owner/operator's desire to commit to their provision.

The probability of having to deliver an ancillary service is dependent on a variety of factors. These may include the season of the year, the availability of transmission, the mix of installed capacity and the collective reliability of the operating generation resources in the system.

Table 1 illustrates the expected values, or mean probability levels, used in this analysis. The assumptions for these important values are also shown in the table.

Table 1.—Probability of AS provision

AS	1-hour prob.	Notes
Up-regulation (UR)	0.20	Assumed
Down-regulation (DR)	0.20	Assumed
Spinning reserve (SR)	0.01	≈ 1hr/4 days
Non-spinning reserve (NR)	0.0005	≈ 4.5 hr/year

These numeric values shown in the table are critically important to the outcome of this analysis. Unfortunately, there is little available information on their specific magnitudes or even their range. The values of these probabilities used in this research effort are plausible, but not informed by data or experience. Hence, they represent a significant source of uncertainty in this analysis.

Revenue Streams

For traditional arbitrage-only pump generation units, the sole source of revenue is quite simple—the sale of energy. With the increased penetration of variable generation resources such as solar and wind, pump generation plants have shifted their marketing strategies to take advantage of not only their capabilities for energy arbitrage, but more importantly for their ability to produce and sell up-regulation, down-regulation, spinning reserves and non-spinning reserves. Where ancillary services are involved, correctly accounting for revenues becomes more complex. This section describes the revenue streams in generation and pump mode.

Generation Mode

Energy

In the past, pump generation plants have been used primarily for energy arbitrage. They used energy when the market price was low for pumping and produced energy for sale when the market price was high. The greater the price spread between low (off-peak) prices and high (on-peak) prices, the greater the opportunities for energy arbitrage and the greater the revenues which could be realized. As shown in Table 2, revenues from the sale of energy are simply payments made for the delivery of this product.

Table 2.—Generation mode revenue streams

Product	Payment Rule
Energy	Payment for delivery.
Up-regulation	Payment for ability to deliver, with separate energy payment.
Down-regulation	Payment for ability to deliver, with associated energy losses.
Spinning reserve	Payment for ability to deliver, with separate payment for energy when delivered.
Non-spinning reserve	Payment for ability to deliver, with separate payment for energy when delivered.

Up-Regulation

Depending on the market structure and price, modern pump generation plants may also elect to provide up-regulation in generation mode. From the provider's perspective, production of up-regulation can be advantageous. First, the price of this ancillary service is typically high relative to other ancillary services. Second, providing this service does not require as much water as does energy production. Although market rules differ, the revenue stream for this product generally has two components; a payment made for the ability to deliver and a payment made for the energy produced, over and above the setpoint. Because the additional energy generated depends on the probability of furnishing up-regulation, the gains in energy production are only a fraction of the amount of up-regulation provided.

Down-Regulation

Depending on the market price, pump generation plants in generation mode may elect to provide down-regulation to the interconnected system. The provision of down-regulation may not always be desirable since the price of this ancillary service is typically relatively low. Under some situations however, providing this service may become attractive since some water is saved for generation at a future date. Although market rules differ by location, the revenue stream for this product has two components; a payment for the ability to deliver and the costs incurred by foregoing some energy production. Potential energy losses are reduced by the probability of furnishing down-regulation and these losses are typically only a fraction of the amount of down-regulation provided.

Spinning Reserves

Depending on the market price, pump generation plants in generation mode may be used to provide spinning reserves. From the provider's perspective, production of spinning reserve can often be financially rewarding. First, the price of spinning reserve is relatively high compared to other ancillary service products. Second, providing this service requires very little water and has a minimal effect on generation, on the average. Although market rules differ depending on location, the revenue stream for this product generally has two components; a payment is made for the ability to deliver spinning reserves, and, payments are made for the additional energy produced, if any. The probability of a spinning reserve call is relatively low. Because the additional energy generated depends on the probability of actually furnishing spinning reserve, on average the gains in energy production are only a fraction of the amount of the spinning reserves provided.

Non-Spinning Reserves

Depending on the market price, units in generation mode may be used to provide non-spinning reserves to the interconnected system. From the provider's perspective, production of non-spinning reserve may or may not be financially rewarding. The price of non-spinning reserve is typically the lowest of the market traded ancillary service products. However, providing this service requires very little water and, on the average, has little to no effect on generation. Although market rules differ depending on location, the revenue stream for this product generally has two components; a payment for the ability to deliver non-spinning reserves, and, payments made for the additional energy produced, if any. The probability of a non-spinning reserve call is quite low. Because the additional energy generated depends on the probability of actually furnishing non-spinning reserve, the average gains in energy production are only a fraction of the amount of non-spinning reserves provided. These concepts are summarized in Table 2

Pump Mode

Up-Regulation

Depending on the market structure and price, VS pump generation plants may also elect to provide up-regulation in pump mode. SS units cannot provide regulation in pump mode. From the provider's perspective, production of up-regulation may be financially attractive. First, the price of this ancillary service is typically relatively high compared to other ancillary services. Second, providing this service does not require as much energy as pumping. Although market rules differ by location, the revenue stream for this product generally has two components; a payment is made for the ability to deliver the product, and, there is a reduction in the amount of energy used for pumping. Because reduced energy use depends on the probability of furnishing up-regulation, on the average, reductions in energy use are only a fraction of the amount of up-regulation provided. Table 3 summarizes this and other salient points.

Table 3.—Pump mode revenue streams

Product	Payment Rule
Up-regulation (VS units only)	Payment for ability to deliver, with some associated energy savings from reduced pumping.
Down-regulation (VS units only)	Payment for ability to deliver, with some additional energy required for increased pumping.
Spinning reserve	Payment for ability to deliver, with associated energy savings from reduced pumping
Non-spinning reserve	Payment for ability to deliver, with associated energy savings from reduced pumping.

Down-Regulation

Depending on the market price, VS units in pump mode may elect to provide down-regulation to the interconnected system. SS units cannot provide down-regulation in pump mode. The provision of down-regulation may not always be desirable since the price of this ancillary service is often relatively low and some increase in energy use is required. Under some situations however, providing this service may become attractive. During the provision of down-regulation, some additional pumping must occur which can reduce the amount of pumping required at a future date. Although market rules differ, the revenue stream for this product has two components; a payment for the ability to deliver, and, the energy costs incurred by additional pumping (VS units only). Because the amount of increased energy use is reduced by the probability of furnishing down-regulation, on the average, the increase in energy use is a small fraction of the amount of down-regulation provided

Spinning Reserves

Depending on the market price, both SS and VS units in pump mode may be used to provide spinning reserve. In pump mode, this requires a unit to reduce or cease pumping operations. From the provider's perspective, production of spinning reserve can often be financially and economically desirable. The price of spinning reserve is relatively high compared to other ancillary service products. On the average, providing this service requires has a minimal effect on pumping. Although market rules differ depending on location, the revenue stream for this product generally has two components; payment for the ability to deliver spinning reserves, and, a reduction in the energy used for pumping, if any. The probability of a spinning reserve call is relatively low. Because the amount of energy saved depends on the probability of actually furnishing spinning reserve, on the average energy savings are only a fraction of the amount of spinning reserves provided.

Non-Spinning Reserves

Depending on the market price, pump generation plants in pump mode may be used to provide non-spinning reserves to the interconnected system. Both SS and VS units may provide spinning reserve in pump mode. Provision of non-spinning reserves requires the unit to reduce or cease pumping operations. From the provider's perspective, production of non-spinning reserve may or may not be commercially attractive. The price of non-spinning reserve is typically the lowest of the market traded ancillary service products. However, on the average providing this service has little to no effect on pumping. Although market rules differ depending on location, the revenue stream for this product generally has two components; payment for the ability to deliver non-spinning reserves, and, the potential for reduced energy use. The probability of a non-spinning reserve call is quite low. Because energy savings depend on the probability of actually furnishing non-spinning reserve, average reductions in energy use are only a fraction of the amount of non-spinning reserves provided.

Unit Dispatch with AS

The electric power system is designed and operated to maintain a real-time balance between generation and load and to adjust the output of generators in order to manage power flows through the transmission system. The services needed to meet these requirements are known as ancillary services. Depending on the design of the pump generation units, they can provide some or all of these ancillary services in pump mode or generation mode. This section of the document reviews the capabilities of these units.

SS and VS Units – Generation Mode

In generation mode, the capabilities of SS and VS units are nearly identical. In generation mode, SS and VS pump generation units can generate energy and can provide spinning reserve, non-spinning reserve, up-regulation and down-regulation services. Depending on system requirements, some, or all of these services may be provided on a single unit or a set of units. Figure 3 illustrates the hypothetical dispatch of a single SS or VS unit in generation mode.

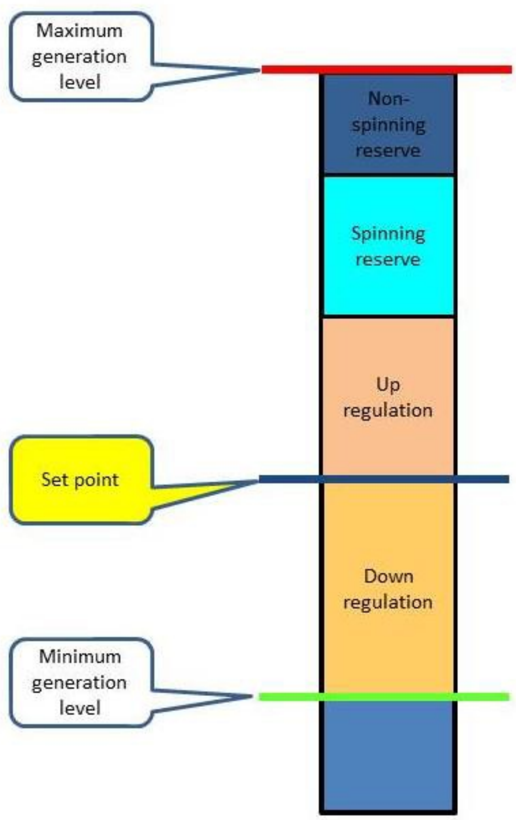

Figure 3.—SS and VS units in generation mode.

As shown in this figure, the unit must be scheduled in such a way that its generation setpoint is greater than the minimum generation level and high enough so that any scheduled down-regulation does not stray below the minimum generation constraint. Up-regulation must be allocated so that it does not exceed the maximum generation capability of the unit plus the scheduled amount of spinning reserve and non-spinning reserve, if any. The scheduled spinning reserve cannot exceed the amount of unloaded capacity less the non-spinning reserve scheduled, if any. In this figure, non-spinning reserve is shown as a dedicated block of unloaded capacity, in addition to the other energy and ancillary services scheduled on the unit. Non-spinning reserve is typically assigned to units which are offline, or motoring, and can be brought online within 10-minutes or less. However, it can be scheduled on operational units, should system requirements and market conditions dictate.

SS Units Pump Mode

In pumping mode, the capabilities of single speed (SS) and variable speed (VS) units are considerably different. SS pump generation units use energy for pumping and can provide spinning reserves and non-spinning reserves by suspending pumping operations (i.e.; turning the pump “off”). Unlike VS units, SS units cannot provide regulation in pump mode because they are either pumping (“on”) or not pumping (“off”) and the pump’s output cannot vary outside of those two states. Depending on system requirements, SS units in pump mode can provide both spinning and non-spinning reserves. Figure 4 illustrates the hypothetical dispatch of a single SS unit in pump mode.

As shown in this figure, a SS unit must be scheduled in such a way that its pumping setpoint is at the maximum pumping capacity level. For a SS unit, this is also the minimum pumping level. Scheduled spinning reserve cannot reduce pumping capacity to less than zero, or the amount of non-spinning reserve scheduled on the pumping unit, if any. In this figure, non-spinning reserve is shown as a block of loaded pumping capacity. Non-spinning reserve can be scheduled on SS units in pump mode, should system requirements and market conditions dictate. If called upon to deliver spinning or non-spinning reserves at a particular time, a SS unit must cease pumping. The SS unit will then resume pumping at some other time.

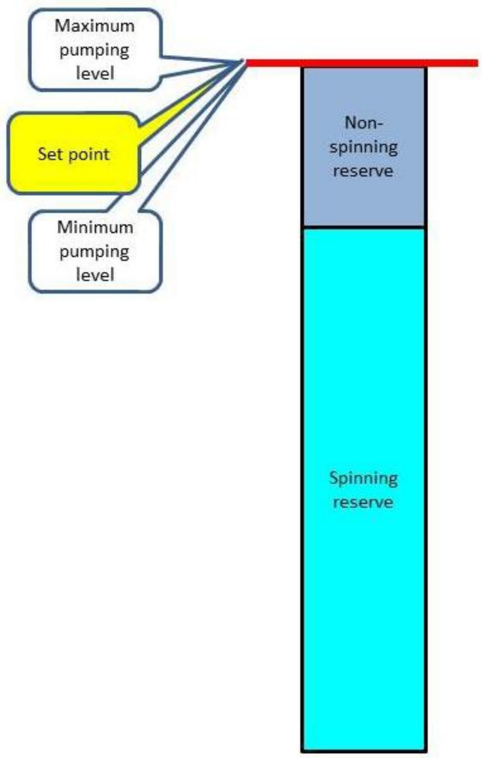

Figure 4.—SS unit in pump mode.

VS Units—Pump Mode

In pumping mode, the capabilities of variable speed (VS) units are more diverse than are SS units. In pump mode, VS pump generation units have a wider range of operation and can very quickly change their pumping level between approximately 70 to 100 percent of pumping capacity. This greater degree of flexibility in the pumping mode allows VS units to use energy for pumping and to provide spinning reserves and non-spinning reserves by suspending pumping operations (i.e.; turning the pump “off”) or by pumping less. Most importantly, within their range of variable operations they can provide up-regulation and down-regulation. Up-regulation can be provided by pumping less and down-regulation can be provided by pumping more. Figure 5 illustrates the hypothetical dispatch of a single VS unit in pump mode.

As shown in this figure, a VS unit must be scheduled in such a way that its pumping setpoint is between 70% and 100% of the maximum pumping capacity level. Up-regulation, produced by pumping less, can be scheduled for the range between the setpoint and 70% of capacity. Down-regulation, produced by pumping more, can be produced between the setpoint and 100% pumping capability. Scheduled spinning reserve cannot reduce pumping capacity to less

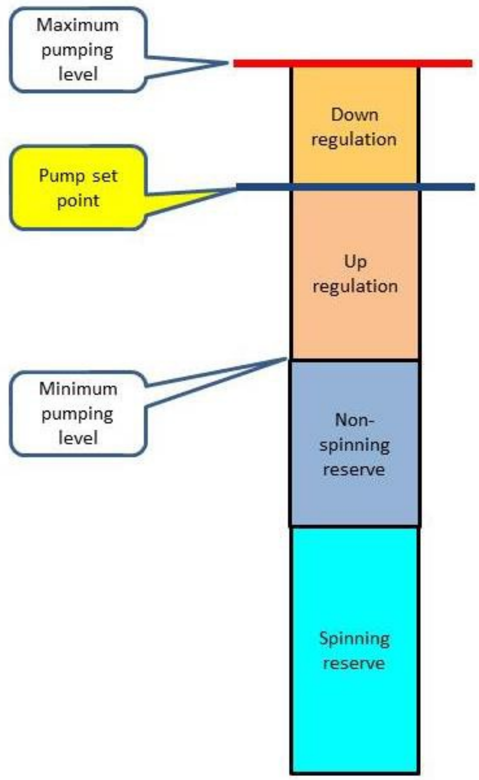

Figure 5.—VS unit in pump mode.

than zero, or the amount of non-spinning reserve scheduled on the pumping unit, if any. In this figure, non-spinning reserve is shown as a block of loaded pumping capacity. If called upon to deliver spinning or non-spinning reserves at a particular time, a VS unit can cease pumping, or, if the setpoint is high enough, could reduce pumping down to 70% pumping capacity. In both cases, the VS unit will presumably pump more or pump longer at some other point in time.

Summary of Unit Capabilities

In this research project, two different designs of pump generator units are considered. These are a conventional, fixed or single speed (SS) pump generator unit and the newer and more advanced variable speed (VS) pump generator units. In general, the VS units are said to be slightly more capable machines and are touted primarily for their increased ability to provide ancillary services in pump mode.

Table 4 summarizes the capabilities of both the SS and VS pump generation units examined in this research effort. In generation mode, there are no differences in

Table 4.—Summary of unit capabilities

Unit Type	Mode	Energy	Regulation Services		Reserves	
			Up	Down	Spinning	Non-spinning
Single speed	generation	☆	☆	☆	☆	☆
	pumping		🚫	🚫	☆	☆
Adjustable speed	generation	☆	☆	☆	☆	☆
	pumping		☆	☆	☆	☆

capabilities between these two unit designs. Both SS and VS pump generator units can generate energy and provide up-regulation, down-regulation, spinning and nonspinning reserves. In pump mode, the SS units can consume energy and can provide spinning and nonspinning reserves. However, SS speed units cannot provide regulation in pump mode because they can operate at only a single output level. In contrast, VS units can consume energy, provide up and down-regulation and can provide spinning and nonspinning reserves. Their enhanced capability to provide regulation services arises because they can operate over a range of pumping output levels.

Unit Limitations

Both variable speed (VS) and single speed (SS) pump generation units have characteristic engineering and design limitations on their range of generation levels. Typically, these include minimum generation levels and prohibited operation zones.

Rough Zones

Prohibited operating zones are production, output, or operational regions which create excessive vibrations of the plant equipment (also known as, “rough zones”) or output zones which might result in hydraulic cavitation. Most, if not all, generator/turbine units have rough zones, sometimes more than one. These prohibited operating zones may vary with head, further complicating this problem. Units of different designs and wear-status typically have rough zones in different regions of their output surfaces. Sustained operation of a unit within a rough zone is not permitted under normal operating conditions. It is also advisable to change output levels quickly and move through prohibited operating zones as rapidly as feasible.

Prohibited operating zones place additional constraints on the output levels of generator/turbine units. Since continuous generation within these zones is not allowed under ordinary circumstances, rough zones divide the feasible generation space into smaller and discontinuous regions. These smaller and discontinuous regions limit the ability of the unit to provide energy as well as reserves and regulation in certain ranges. Figure 6 illustrates the generation space for a generator with a minimum generation level and a single rough zone. As shown in this figure, the rough zone limits the feasible output levels of the generator.

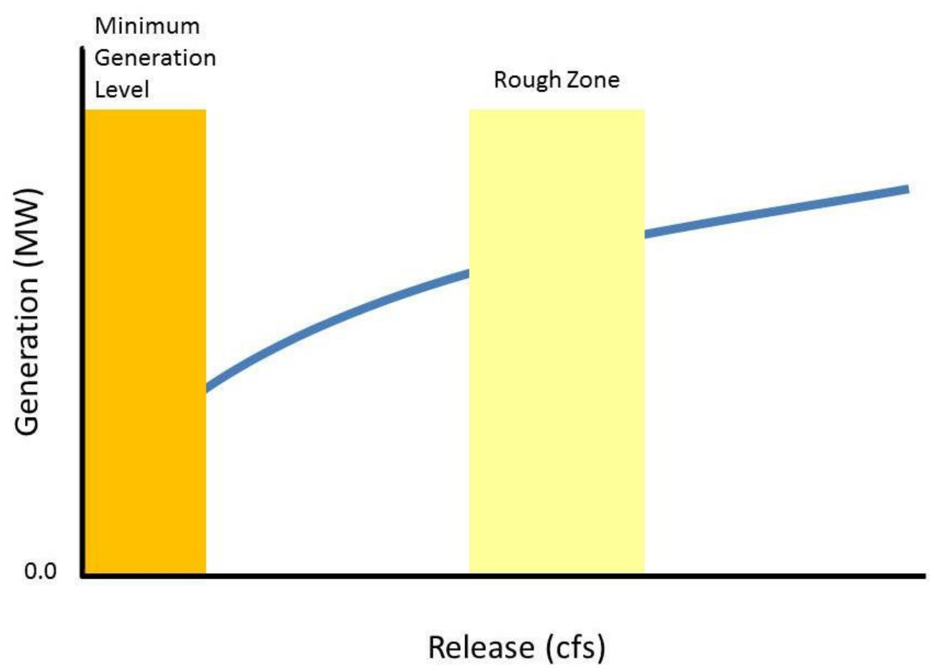

Figure 6.—Unit with minimum generation level and rough zone.

Representation of Rough Zones

For purposes of this research project, each pump generation unit must be characterized in a way that meets specific requirements. In particular, the capacity of each unit must be represented in a manner such that it can be altered, independent of all other factors including the, head, forebay (upper) and afterbay (lower) reservoir elevations. To do so in a plausible fashion, the rough zones are represented as a single fixed region within the generation space. The extent and location of this region is related, on a percentage basis, to the capacity of the unit.

Drawing upon Reclamation's corporate knowledge of pump generator units, a single rough zone in a generic single speed unit is assumed to lie in the range of

40% to 60% of the unit capability. The lower limit (LL) of the rough zone is assumed to occur at 40% of unit capacity and the upper limit (UL) of a SS unit is assumed to lie at 60% of unit capacity. This rough zone is assumed to be fixed in size and fixed with respect to head.

Reclamation has no experience with and little corporate knowledge of variable speed pump generation units. The position and extent of the rough zones in this type of plant is the subject of some speculation. Anecdotal evidence suggests the rough zone in variable speed units is smaller than it is for single speed units. For purposes of this research project, it is assumed the rough zone for variable speed units lies in the range of 45% to 55% of the unit capacity. The lower limit (LL) of the rough zone for a variable speed unit is assumed to occur at 45% of capacity and the upper limit (UL) of the rough zone for a variable speed unit is assumed to occur at 55% of the unit's capacity. This rough zone is assumed to be fixed in size and invariant with respect to head.

In real-life, there may be more than one rough zone and the position of these rough zones may well vary with head. For a specific site and design, these operational details would be established. For purposes of this research project, which utilizes a generic unit, this level of specificity cannot be characterized in a practical manner.

Minimum Generation Levels

For VS and SS pump generation units there is some minimum level of release necessary before generation can commence, the unit dispatched to meet load and energy and AS provision begins. In this document this is termed the, "minimum generation level." At release levels below the minimum generation level, water is released but generation is insufficient to allow for dispatch. For any given VS or SS unit, this minimum generation level may vary with head, and other factors.

Consideration of a minimum generation level places additional constraints on the feasible output region for generator/turbine units. Since generation between zero and the minimum operating level is not allowed, a minimum generating level creates an exclusion zone which segments the feasible generation space into smaller and discontinuous region. Figure 6 illustrates the generation space for a generator with a minimum generation level and a single rough zone. As shown in this figure, the minimum generation level limits the feasible output levels of the generator. For purposes of this research project, this minimum generation level is not influenced by head.

Representation of Minimum Generation Levels

For purposes of this research project, each pump generation unit must be characterized in a way that meets specific requirements. In particular, the capacity of each unit must be represented in a manner such that it can be altered, independent of all other factors including the, head, forebay (upper) and afterbay (lower) reservoir elevations. To do so in a plausible fashion, the minimum generation levels are represented as a lower limit below which generation cannot occur. This minimum generation level is related, on a percentage basis, to the capacity of the unit. For both SS and VS units, the minimum generation level is assumed to be 20% of the unit's maximum generation capacity.

Summary of Unit Limitations

Table 5 summarizes the salient limits for VS and SS pump generation units in generation mode. As shown in this table, for both VS and SS units, the minimum generation level is assumed to lie at 20% of the unit capacity. For SS units in generation mode, a single rough zone is assumed to span the range of 40% to 60% of the unit's generation capacity. VS pump generation units are thought to have a narrower rough zone. For VS pump generation units in generation mode, a single rough zone is assumed to lie in the range from 45% to 55% of the unit capacity.

Table 5.—Unit limits

	Minimum generation (%)	Rough zone lower limit (%)	Rough zone upper limit (%)	Maximum generation (%)
SS unit	20	40	60	100
VS unit	20	45	55	100

The values shown in this table are the percentage of maximum generation capability measured in megawatts.

General Description of the Model

The pump-generation model constructed for this research is coded in Lingo Systems Inc., LINGO 14 and relies on that optimization framework for solution. The LINGO 14 script language is described in Lingo Systems, Inc. (2013) and Schrage (2013). A wide range of further examples are available from the LINDO Systems website (www.lindo.com). Figure 7 provides a high-level overview of

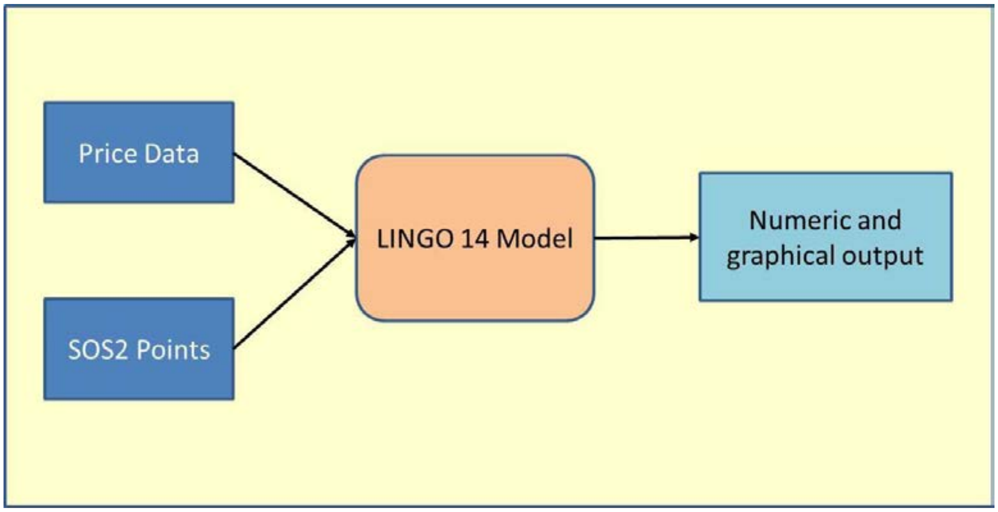

Figure 7.—Conceptual overview of the model.

the models used in this analysis. As shown in this figure, two separate Excel data worksheets are used by the LINGO program. One of these worksheets contains the July 2014 typical week price data. These data include; the locational marginal price (LMP), the up-regulation price, the down-regulation price, the spinning reserve price and the non-spinning reserve price. All prices are reported in units of $/MWh. The other Excel data worksheet computes the special ordered sets of type 2 (SOS2) points for the pump-generator unit, given the size of the unit (MW) and the gross head (feet). Both of these data files are read by the LINGO program using the object linking and embedding (OLE) approach, available in the Microsoft Windows ® environment.

The LINGO 14 development environment compiles the LINGO 14 code, reads these two data files, solves the pump-generation optimization problem and then writes a subset of the output to a third Excel worksheet, keyed to the name of the code file. The summary numeric output is used by the Excel worksheet to create useful graphical outputs which greatly facilitate understanding of the results.

Mathematical Overview of the Model

The objective function of the pump generation model is naturally represented as a nonlinear function. For numerical tractability, it is re-specified as a piece-wise linear representation of the underlying nonlinear relationship. The piece-wise linear objective function is formulated in a specific manner, known as special ordered set of type 2 (SOS2), which allows for an efficient solution using mixed linear programming (MILP). The majority of the constraints are linear. Detailed mathematical specifications for the pump-generation models can be found in the Appendices.

Forebay Reservoir Modeling

Owing to the nature of this modeling exercise, the forebay reservoir is a generic and nonspecific characterization of an upper reservoir sited above an existing, but unspecified, Reclamation facility. For purposes of this research project the forebay (upper) reservoir must meet certain specific requirements. First, the forebay (upper) reservoir represented in the model must be devised in a manner which will allow for changes in live storage volume, without affecting other physical and engineering parameters in the model, including the gross head. Second, the forebay reservoir must be constructed in a manner which is independent of other site characteristics, such that the gross head can be altered, independent of the reservoir storage volume.

Real-life forebay reservoirs have relatively complex shapes and volume characteristics which necessarily depend on the site topography. However, it is difficult to conceive of and operationalize a complex shapely shaped reservoir with these desired properties. In a like manner, it is difficult to fathom, in real-life, how a reservoir can be sited or placed, so that it does not affect the storage volume.

For purposes of this research effort, the forebay reservoir is assumed to be rectangular in shape with a fixed (live) crest height of 100 feet. By making this assumption we abstract from the constraints of real-life complexity, instead substituting a geometrically shaped forebay (upper) reservoir with a readily computable live volume. By holding the (crest) height of the rectangle constant, the live volume can be altered for analysis purposes, independently of the head. We further assume this rectangular reservoir can be moved up and down in elevation, without regard to the site topography, allowing the head to be altered for analysis purposes, independent of the storage volume.

Figure 8 illustrates the conceptual approach used to model the forebay (upper) reservoir in the presence of an existing afterbay (lower) reservoir. Only the live storage volume of the forebay is depicted in this graphic. As shown in this figure, the forebay is modeled as a rectangle with a fixed height of 100 feet. The length and width dimensions are set equal to each other for computational ease. In aggregate, these simplifications allow the storage volume to be readily altered, while maintaining a specified gross head. Similarly, the entire reservoir can be moved up or down in elevation so the gross head can be changed, independently of the live storage volume.

By making these assumptions, we abstract from the real-life complexities which might apply to a specific site and design. In aggregate, these assumptions facilitate the analyses described in this document and allow for the incremental assessment of economic value determinants, independent of other factors.

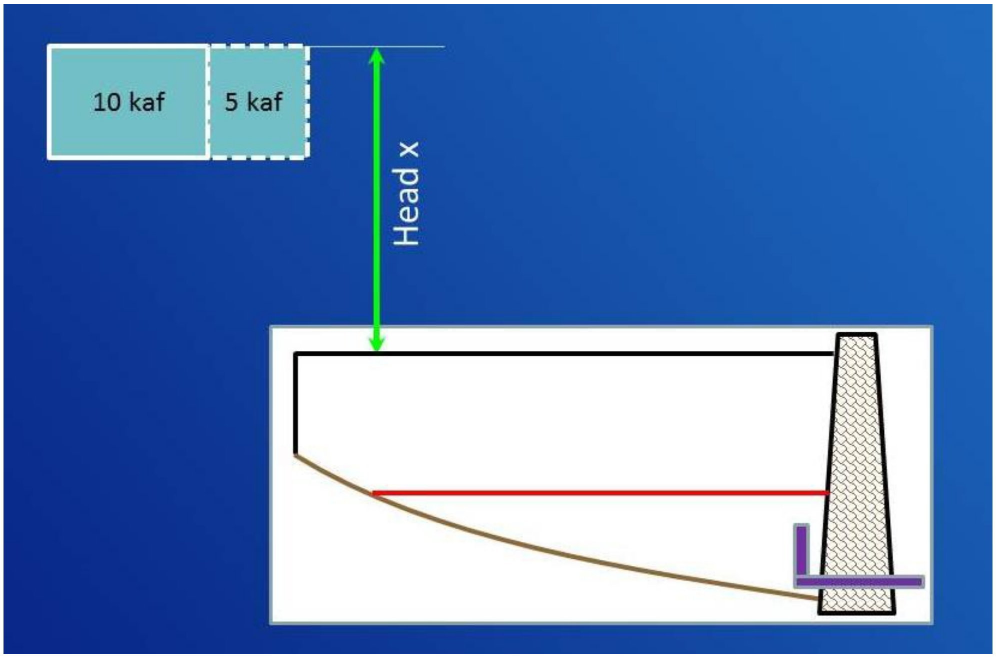

Figure 8.—Conceptual approach to modeling the Forebay Reservoir.

Modeling Changes in Head

For purposes of this research project, the gross head must be characterized in a way that meets specific requirements. In particular, the gross head must be represented in a manner which allows the head to be altered, independent of the forebay (upper) and afterbay (lower) reservoir storage contents.

In practice, the gross head is determined by the location of the forebay (upper) reservoir relative to the afterbay (lower) reservoir and their respective water surface elevations. Topographic characteristics typically determine the location of both reservoirs, while prior operations and hydrologic factors dictate their contents, and hence elevations.

As we have described earlier, the forebay reservoir is assumed to be rectangular in shape with a fixed height of 100 feet (live storage). We further assume the rectangular reservoir can be moved up and down in elevation, without regard to the site topography, allowing the head to be altered for analysis purposes, independent of the forebay storage volume. A previous study of potential Concept 5 installations by the Bureau of Reclamation (2013) found the elevation of the afterbay (lower reservoir) is primarily controlled by inflows, downstream delivery requirements and conventional hydropower operations. As a result, Concept 5 pump generation plant operations would have a negligible effect on the elevation of the afterbay (lower) reservoir. Consistent with these findings, we assume the elevation of the afterbay (lower) reservoir is fixed.

Figure 9 illustrates the conceptual approach used to model the gross head. As shown in this figure, the forebay is modeled as a rectangle with a fixed crest height of 100 feet. As depicted, the entire forebay (upper) reservoir can be moved up or down in elevation so the gross head can be changed, independently of the live storage volume.

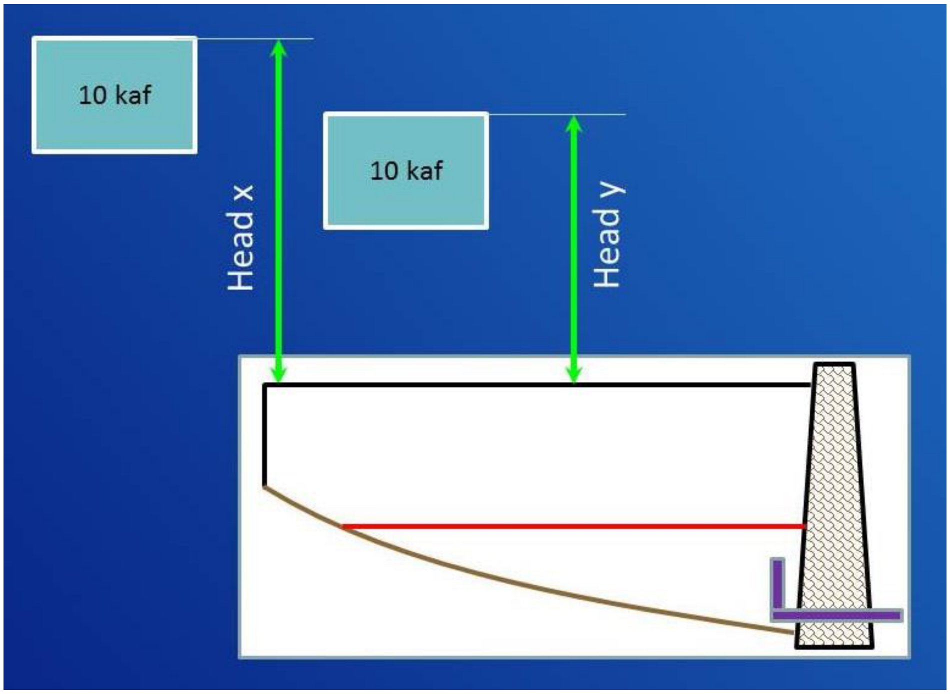

Figure 9.—Conceptual approach to modeling gross head.

By making these assumptions, we abstract from the real-life complexities, which might apply to a specific site and project design. In aggregate, these assumptions facilitate the analyses described in this document and allow for an assessment of the economic value of specific determinants, independent of all other factors.

Modeling Changes in Capacity

For purposes of this research project, the capacity of each pump generation unit must be characterized in a way that meets specific requirements. In particular, the capacity must be represented so that it can be altered, independent of all other factors, including the, head, forebay (upper) and afterbay (lower) reservoir elevations.

Typically, the capacity of a pump generation unit is the subject of considerable study and careful planning. The selection of a unit's capacity is determined, in large part, by the available head and the volume of the forebay (upper) reservoir relative to the afterbay (lower). Selection of a pump generation unit's capacity also depends on some practical considerations including the sizes of the units which are commercially available.

In this study, the size or capacity of the unit is the subject of analysis. For this reason, it is assumed the unit capacity can be varied over a substantial range, while holding the head, forebay reservoir volume, and all other factors, unchanged at the reference operating conditions.

By making this assumption, we abstract from the complexities which might apply to a specific site and engineering design. In aggregate, these assumptions facilitate the analyses described in this document and allow for an assessment of the economic value of specific determinants, independent of all other factors.

Solving Pump-Generation Models

Many of the physical and engineering relationships which are part of a pump-generation powerplant are nonlinear in nature. The relationship between release, head and generation is nonlinear, as is the relationship between power input, head and water pumped. In most applications, generator and pump efficiencies vary in a nonlinear manner with release and head. The water surface elevation in both the forebay (upper) reservoir and the afterbay (lower) reservoir are typically a nonlinear function of storage volume.

In the early phases of this research project, the SS and VS pump-generation units were represented in a nonlinear formulation. From the formulation standpoint, this had two benefits. First, it characterized the underlying physical and engineering features in a natural and technically correct fashion. Second, the nonlinear formulation approach allows for a much more compact and straightforward coding effort.

Nonlinear models are inherently difficult to solve (Press et al, 1989, Judd 1999, Miranda and Fackler 2006) and the literature is filled with writings on this topic. Nonlinear models with discontinuities, integer logic and numerous constraints are even more problematic. Unfortunately for this research project, even straightforward models of pump-generation units fall into the latter category.

Initial formulations of pump-generation units were dimensionally small and had a limited number of constraints. These early nonlinear models solved rapidly in the LINGO 14 solver framework giving hope that larger dimension models with all of the necessary constraints would also be handily solved.

Unfortunately, as the size of the problem increased and additional constraints for minimums and rough zones were added, the complexity of these models grew. It then proved impossible to achieve an optimal solution for these models. Considerable effort was then expended reformulating the models in different manners and sequentially adding constraints, and combinations of constraints, to identify problematic formulations. This proved to be an intensive and extensive undertaking. After expending large amounts of time, efforts to employ a nonlinear formulation were abandoned.

Staff from Argonne National Laboratory had previously advised using an approach known as special ordered sets of type 2 (SOS2). With their gracious and patient assistance, nonlinear versions of the pump-generation models were reformulated as linear piece-wise approximations, which were amenable to solution using the SOS2 approach.

SOS2 Approach

For a variety of reasons, a detailed and realistic mathematical characterization of a pump-generation unit is inherently nonlinear in nature. When formulated in a natural manner, these types of relationships give rise to mixed integer nonlinear programming (MINLP) model. Such models can be very difficult if not impossible to solve.

To facilitate the practical solution of this type of model, they are often re-formulated as piecewise linear approximations. Piecewise linear models are, as the name implies, made up of linear segments which approximate the underlying nonlinear function. A piecewise linear approximation to a nonlinear function is illustrated in Figure 10.

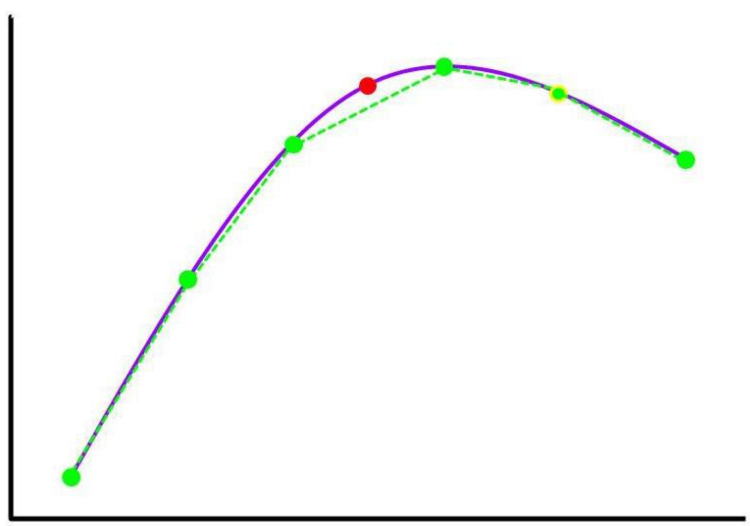

Figure 10.—Piecewise linear approximation.

As shown in this figure, a number of linear segments are employed to represent the underlying nonlinear function. In intervals where the slope of the nonlinear function does not change a great deal, there is good correspondence between the linear segment and the nonlinear function. In regions where there is more curvature, there is more error between the linear approximation and the nonlinear function. For example, in the region where the red point is located, there is a poor correspondence between the nonlinear approximation and the function. This might suggest two, or more, additional segments be used in this interval instead of a single linear segment. In general as the number of segments is increased, each segment can be made smaller, and the error between the linear approximation and the nonlinear function is decreased. For a finite number of segments, there is an inherent tradeoff between the accuracy of the approximation and the number and locations of the segments. The location of the segment breakpoints, the number of segments, and the length of each segment, should be strategically and logically identified.

As can be surmised by examining this figure, there is some professional skill and even art required to construct a good linear approximation to an arbitrary nonlinear function. Researchers at Argonne National Laboratory have recently published an approach which can greatly assist practitioners in this endeavor (Goldberg, et al 2014).

With the nonlinear function reformulated as a piecewise linear approximation, the problem becomes soluble as a mixed integer linear programming (MILP) model. Such models are considerably more tractable.

MILP models can be formulated directly using a combination of integer constraints to isolate the active segment of the piecewise linear function. A description of this approach can be found in Bisschop (2009, p. 77). While certainly feasible, this approach is more tedious and error-prone than other options. This is particularly evident as the number of linear segments is increased. For coding purposes and computational efficiency reasons, the analyst may prudently elect to specify the model as a special ordered set of type 2 (SOS2) model.

The SOS2 approach was developed by Beale and Tomlin (1970) and is aptly described in Tomlin (1984) and in LINDO Systems, Inc. (2010, p. 100). SOS2 models are specified as a naturally ordered set of points. Some of the salient features of the SOS2 approach are summarized in Text Box 1. For example, the set of points (X, Y values) in Figure 10 should be specified (ordered) from smallest to largest X-value. SOS2 points have some particularly appealing technical properties. First, at most two points are "active" or have a nonzero weight value. The weights of the remaining points must be exactly equal to zero. The active points must be adjacent to each other and the two points specify the ends of a line segment in X, Y space. The weights for the active points must sum to 1.0, which facilitates the use of linear interpolation for any value along the active segment.

The SOS2 approach is operationalized in many modern linear programming (LP) solvers, including LINGO 14. This greatly simplifies the analysts coding effort and provides an efficient and reliable technique for solving MILP problems.

Text Box 1.—Features of the SOS2 approach

- Ordered set of points
- At most, 2 points are active
- Active points are adjacent
- Their weights must sum to 1.0
- Embedded in many solvers, including LINGO 14
- Computationally efficient
- Other advantages

The SOS2 approach has some other advantages. In the present context, the use of the SOS2 approach greatly eases incorporation of the minimum generation constraint. With a minimum generation constraint, generation becomes what is known as a semi-continuous variable. The value of the generation variable can either be 0.00 or it can take on some continuous non-negative value between the minimum generation and the maximum generation levels. Semi-continuous variables are traditionally coded using the approach described in Bisschop (2009, p. 77), although LINGO 14 has a built-in function for characterizing these variables which provides some relief. Use of the SOS2 approach however obviates the need to account for and code these variables separately.

Constant Unit Efficiencies

The universal power equation (Appendix 1) with a variable generation efficiency is mathematically nonlinear. The nature of this variable generation efficiency is discussed in detail in Appendix 2. When the generation efficiency is fixed and constant over the range of release, this function becomes linear in release and generation space. Similarly, the universal pumping equation, shown in Appendix 3, is mathematically nonlinear when pumping efficiency varies. When the pumping efficiency is fixed and constant over the range of release, the pumping equation becomes linear. To linearize both of these nonlinear functions, the generation efficiency and the pumping efficiency are held fixed and constant over the analysis period. The fixed, constant values of these efficiencies for both types of units are shown in Table 6.

Table 6.—Efficiency values used in the analysis

Mode	SS unit (%)	VS unit (%)
Generation	84.00	86.00
Pumping	86.00	88.55

As shown in Table 6, the generation and pumping efficiencies used for the VS unit are slightly higher than the values used for a SS unit. Pump-generation units are typically more efficient while pumping than they are in generation mode. The table reflects this efficiency relationship between operation modes.

Characterization of the SS Unit

For purposes of this study, the single speed (SS) pump-generation unit is represented by the universal power equation (in generation mode) and the universal pumping equation (in pump mode). These relationships are described in considerable detail in Appendices 1 and 3. As further discussed in Appendix 4, these relationships are nonlinear in nature.

When the SS pump-generation unit is expressed in its natural or nonlinear form and appropriate constraints are employed, a constrained nonlinear mathematical optimization problem results. To make the SS pump-generation model amenable to solution, it was reformulated as a piece-wise linear approximation to the underlying nonlinear problem. This section of the document describes the piece-wise linear model used in the study and relates some of its important features.

Generation Mode

In generation mode the relationship between generation, head and release for the SS pump-generation unit was initially characterized by the universal power equation described in Appendix 1. The addition of a minimum generation level and a rough zone divides the feasible generation space into discrete segments as described earlier. For purposes of this study the nonlinear segments of this function were reformulated as five distinct linear segments. The linear piece-wise SS generation function is illustrated in Figure 11 with the summary details described in Table 7.

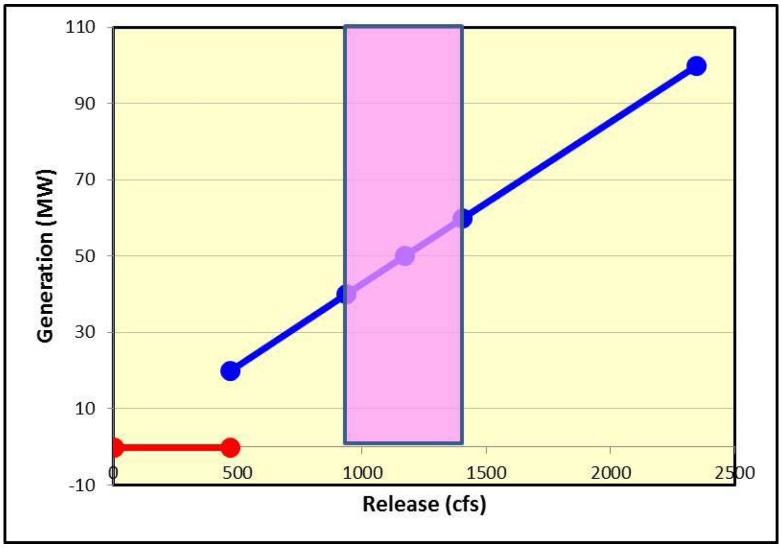

Figure 11.—SS generation function.

Table 7.—Points on the SS generation function

Point	Cap (%)	Gen (MW)	Q (cfs)	Penalty	Notation
1	0.00%	0.00	0.0000	0	Unit is off, no release no generation
2	19.999%	0.00	468.8318	-99999	Unit is off, release no generation
3	20.00%	20.00	468.8552	0	Unit is on, minimum generation level
4	40.00%	40.00	937.7104	0	Start of rough zone
5	50.00%	50.00	1172.1380	-89898	Mid-pt rough zone
6	60.00%	60.00	1406.5656	0	End of rough zone
7	100.00%	100.00	2344.2760	0	Unit is at maximum capacity

Seven points were used to delineate the five line segments which make up the piece-wise linear SS generation function. The line segment shown in red represents the region from a release of zero to release at (very near) the minimum generation level. In this range, generation is zero. There is a discontinuity between the red segment and the blue line segments. The line segments shown in blue represent the potential range of generation. The pink shaded region in the central portion of this figure is the rough zone or prohibited operational zone. The rough zone divides the remaining portion of the generation function into two feasible generation ranges; the range from the minimum generation level to the lower limit of the rough zone, and the range from the upper limit of the rough zone to the maximum generation level.

Comparing the shaded rough zones shown in Figure 11 (SS unit) with those shown in Figure 13 (VS unit) is useful. As discussed in a previous section, SS units have a wider rough zone than do VS units. Consequently the feasible generation range for a VS unit is larger than for a SS unit.

Table 7 provides some important details for each point on the generation function. The notation column is very helpful in understanding the role of these points in the analysis. Point 2 is strategically crafted as discussed subsequently. With the exception of point 2, all these points were generated at the reference operating conditions using the universal power equation and the efficiency levels reported previously. Seven points define the five segments on the piece-wise linear generation function. Points 1 and 2 define a segment within which there is no positive generation. This is the region below the minimum generation level. Although releases can occur in this range, no energy generation occurs in this range. Since there is no energy output along this line segment, no revenue can be realized and it would be unwise to operate the pump-generator in this region. Notice the generation at point 2 is very, very close to the minimum generation level value, but by design, is not the same. Point 2 is computed as 19.999% of the maximum unit capability. Point 3 is the minimum generation level (20% of generation capacity) and the associated release at this generation level. Point 4 defines the lower limit of the rough zone (40% of generation capacity). Point 5 is located at the mid-point of the rough zone (50% of generation capacity) and is artificially introduced to take advantage of the mathematical properties of the SOS2 approach, a topic discussed shortly. Point 6 demarcates the upper limit of the rough zone (60% of generation capacity) and point 7 corresponds to the maximum (100%) generation level.

One column in the table is labeled “Penalty.” This column contains an arbitrarily large (negative) value which is used to mathematically penalize potential solutions which include particular points. Notice that only two points in this column have nonzero penalty values. The remaining points have values of 0.0 (or no penalty). Recall that for SOS2 sets, only two points can be active (non-zero), the active points must be adjacent, and the sum of the SOS2 weights must equal 1.0. The penalty values in this table are used in tandem with these SOS2 properties to obtain a feasible and optimal solution to the piece-wise linear optimization problem. For example, if a potential generation level were selected which falls at the half-way point on the first (red) segment, points 1 and 2 would be active and the SOS2 weights might well be 0.50 and 0.50. The penalty associated with point 2 would be then assessed (revenue time penalty value times 0.50). Operations at this point along the red line segment are permitted but the solution will be assessed a large negative penalty. Consequently, the resulting solution would be undesirable from a maximization standpoint. A useful insight is than an optimal solution could result in 0.0 generation in a particular hour. Such a solution would also occur on the red line segment but the active SOS2 point weights would then be 1.0 and 0.0. This solution would avoid incurring the penalty (penalty value times 0).

An easily overlooked but substantive advantage of this approach is that it avoids introducing additional binary variables into the formulation. Had the SOS2 with penalty approach not been employed, binary variables would have been required to represent semi-continuous operations of the powerplant which is either operating at zero output level (off) or (on) at some value greater than or equal to the minimum generation level. Introduction of binary variables would have resulted in a nonlinear formulation, which may or may not have been amenable to solution.

Point 5 is an artificially introduced point representing the midpoint of the rough zone. A penalty is also associated with this point. Consequently, operations within the rough zone would be heavily penalized and, while not precluded, are highly undesirable outcomes in a maximization problem.

By exploiting the properties of the SOS2 approach, the piece-wise linear approximation to the underlying nonlinear model can be quickly and efficiently solved. It would be useful to understand any approximation error introduced by this approach. Unfortunately, the team has been unable to solve the nonlinear model and hence cannot offer any insights on the magnitude or direction of approximation error.

Pump Mode

For a SS unit in pump mode the relationship between power input, head and water pumped was initially characterized by the universal pumping equation described in Appendix 3. Using the fixed and constant pumping efficiency for a SS unit in pump mode (see Table 8), the pumping function was linearized. A single speed unit can operate only at one speed however and the pump is either “on” or “off.” The input power is normally set at 100% of unit capacity.

For the SS unit in pump mode, the linear functional relationship is reduced to a single feasible operation point which is marked with a red star in Figure 12. The summary details for this graphic are described in Table 8.

Table 8 provides some important details for each point on the SS pumping function. With the exception of point 2, all these points were generated at the reference operating conditions using the universal power equation and the efficiency levels reported previously. The four points shown in the table delineate two line segments. These line segments are shown here primarily as an aid to understanding the computational approach. To reiterate, there is only a single

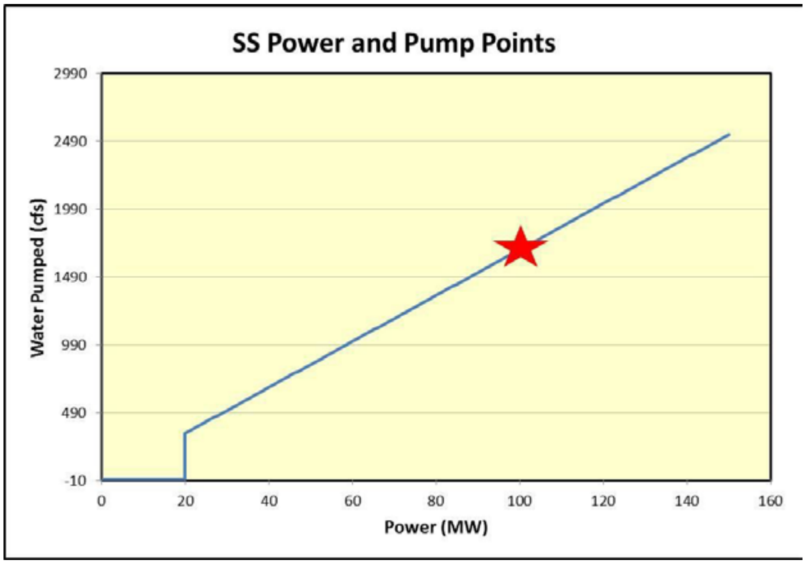

Figure 12.—SS pump operation point.

Table 8.—Points on the SS pumping function

Point	Cap (%)	Input power (MW)	Water pumped (cfs)	Penalty	Notation
1	0.00%	0.00	0.0000	0	Pump is off, no power input, no water is pumped
2	19.9999%	19.9999	0.0000	-99999	Pump is off, power input below min, no release
3	20.00%	20.00	338.7010	0	Pump is off, minimum power input ;level
4	100.00%	100.00	1693.5050	0	Pump is on maximum power input level

A single speed (SS) pump is either "on" or "off". Consequently, there is only a single relevant point on the power input and water pumped function. It is located at the 100% power input level (where the red star is). The SOS2 points shown here were constructed primarily for consistency with the VS pumping function.

relevant point on the SS unit pumping function. Explicit constraints in the LINGO code ensure the SS unit in pump mode will operate at this point (shown by the red star), if it is "on," or at 0.0, if it is "off."

Characterization of the VS Unit

When the VS pump-generation unit is expressed in its natural or nonlinear form and appropriate constraints are employed, a constrained nonlinear mathematical optimization problem results. To make the VS pump-generation model amenable to solution, it was reformulated as a piece-wise linear approximation to the underlying nonlinear problem. This section of the document describes the piece-wise linear model used in the study and relates some of its important features.

Generation Mode

In generation mode the relationship between generation, head and release for the VS pump-generation unit was initially characterized by the universal power equation described in Appendix 1. The addition of a minimum generation level and a rough zone divides the feasible generation space into discrete segments as described earlier. For purposes of this study the nonlinear segments of this function were reformulated as five distinct linear segments. The linear piece-wise VS generation function is illustrated in Figure 13 with the summary details described in Table 9.

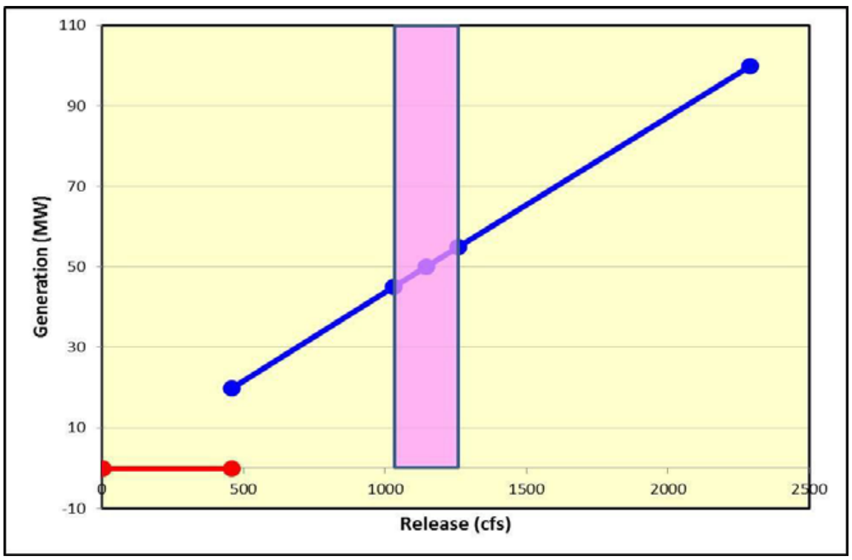

Figure 13.—VS generation function.

Seven points were used to delineate the five line segments which make up the piece-wise linear VS generation function. The line segment shown in red represents the region from a release of zero to release at (very near) the minimum generation level. In this range, generation is zero. There is a discontinuity between the red segment and the blue line segments. The line segments shown in blue represent the potential range of generation. The pink shaded region in the central portion of this figure is the rough zone or prohibited operational zone. The rough zone divides the remaining portion of the generation function into two feasible generation ranges; the range from the minimum generation level to the lower limit of the rough zone, and the range from the upper limit of the rough zone to the maximum generation level.

Table 9.—VS unit SOS2 points

Point	Cap (%)	Gen (MW)	Q (cfs)	Penalty	Notation
1	0.00%	0.00	0.0000	0	Unit is off, no release no generation
2	19.999%	0.00	457.9287	-99999	Unit is off, release no generation
3	20.00%	20.00	457.9516	0	Unit is on, minimum generation level
4	45.00%	45.00	1030.3911	0	Start of rough zone
5	50.00%	50.00	1144.8790	-89898	Mid-pt rough zone
6	55.00%	55.00	1259.3669	0	End of rough zone
7	100.00%	100.00	2289.7579	0	Unit is at maximum capacity

Comparing the shaded rough zones shown in Figure 13 (VS unit) with those shown in Figure 11 (SS unit) is useful. As discussed in a previous section, SS units have a wider rough zone than do VS units. Consequently the feasible generation range for a VS unit is larger than for a SS unit.

Table 9 provides some important details for each point on the VS generation function. The notation column is very helpful in understanding the role of these points in the analysis. Point 2 is strategically crafted as discussed subsequently. With the exception of point 2, all these points were generated at the reference operating conditions using the universal power equation and the efficiency levels reported previously. Seven points define the five segments on the piece-wise linear generation function. Points 1 and 2 define a segment within which there is no positive generation. This is the region below the minimum generation level. Although releases can occur in this range, no energy generation results. Since there is no energy output along this line segment, no revenue can be realized and it would be unwise to operate the pump-generator in this region. Notice the generation at point 2 is very, very close to the minimum generation level value, but by design, is not the same. Point 2 is computed as 19.999% of the maximum unit capability. Point 3 is the minimum generation level (20% of generation capacity) and the associated release at this generation level. Point 4 defines the lower limit of the rough zone (45% of generation capacity). Point 5 is located at the mid-point of the rough zone (50% of generation capacity) and is artificially introduced to take advantage of the mathematical properties of the SOS2 approach, a topic discussed shortly. Point 6 demarcates the upper limit of the rough zone (55% of generation capacity) and point 7 corresponds to the maximum (100%) generation level.

One column in the table is labeled “Penalty.” This column contains an arbitrarily large (negative) value which is used to mathematically penalize potential solutions which include particular points. Notice that only two points in this column have nonzero penalty values. The remaining points have values of 0.0 (or no penalty). Recall that for SOS2 sets, only two points can be active (non-zero), the active points must be adjacent, and the sum of the SOS2 weights must equal 1.0. The penalty values in this table are used in tandem with these SOS2 properties to obtain a feasible and optimal solution to the piece-wise linear optimization problem. For example, if a potential generation level were selected which falls at the half-way point on the first (red) segment, points 1 and 2 would be active and the SOS2 weights might well be 0.50 and 0.50. The penalty associated with point 2 would be then assessed (revenue time penalty value times 0.50). Operations at this point along the red line segment are permitted but the solution will be assessed a large negative penalty. Consequently, the resulting solution would be undesirable from a maximization standpoint. A useful insight is than an optimal solution could result in 0.0 generation in a particular hour. Such a solution would also occur on the red line segment but the active SOS2 point weights would then be 1.0 and 0.0. This solution would avoid incurring the penalty (revenue value times penalty value times 0).

An easily overlooked but substantive advantage of this approach is that it avoids introducing additional binary variables into the formulation. Had the SOS2 with penalty approach not been employed, binary variables would have been required to represent semi-continuous operations of the powerplant which is either operating at zero output level (off) or (on) at some value greater than or equal to the minimum generation level. Introduction of binary variables would have resulted in a nonlinear formulation, which may or may not have been amenable to solution.

Point 5 is an artificially introduced point representing the midpoint of the rough zone. A penalty is also associated with this point. Consequently, operations within the rough zone would be heavily penalized and, while not precluded, are highly undesirable outcomes in a maximization problem.

By exploiting the properties of the SOS2 approach, the piece-wise linear approximation to the underlying nonlinear model can be quickly and efficiently solved. It would be useful to understand any approximation error introduced by this approach. Unfortunately, the team has been unable to solve the nonlinear model and hence cannot offer any insights on the magnitude or direction of approximation error, if any.

Pump Mode

For a VS unit in pump mode the relationship between power input, head and water pumped was initially characterized by the universal pumping equation described in Appendix 3. Using the fixed and constant pumping efficiency for a VS unit in pump mode (see Table 6), the pumping function was linearized. A VS pump-generation unit can pump in a range of approximately 70% to 100% of the maximum capability and can rapidly change its output within this range. This capability enables a VS unit to provide up-regulation and down-regulation in addition to reserves.

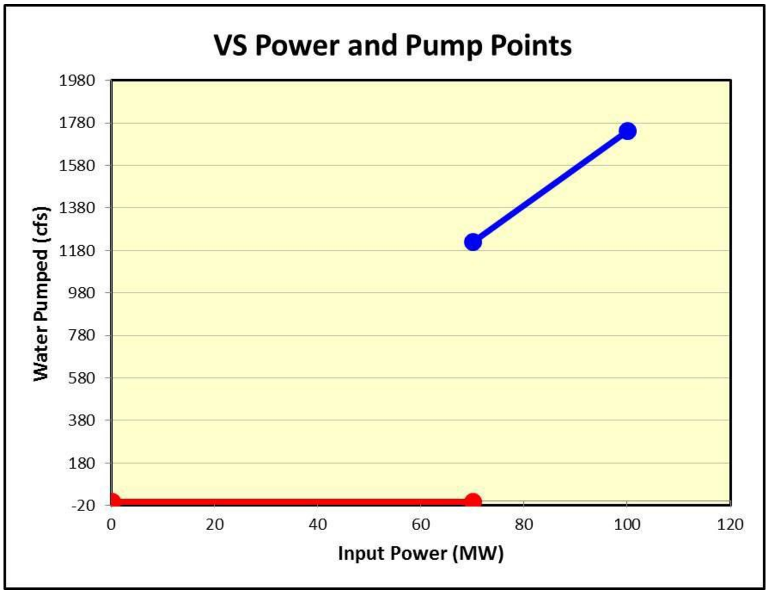

Figure 14.—VS pump operation point.

Four points were used to delineate the two line segments which make up the piece-wise linear VS pumping function. The line segment shown in red represents the region from a power input of zero to a power input at (very near) the minimum input level. In this range pumping is zero. There is a discontinuity between the red segment and the blue line segments. The line segment shown in blue represents the feasible pumping region.

Table 10 provides some important details for each point on the VS pumping function. With the exception of point 2, all these points were generated at the reference operating conditions using the universal power equation and the efficiency levels reported previously. The four points shown in the table delineate two line segments.

Points 1 and 2 define a segment within which there is no positive pumping. This is the region below the minimum pump operation level. Although power input can occur in this range, no pumping will occur. Since there is no pumping along this line segment, energy is expended but no water is pumped and it would be unwise to operate the pump-generator in this region. Notice the power input at point 2 is very, very close to the minimum pump power input level value, but by

Table 10.—VS unit pump SOS2 points

Point	Cap (%)	Power (MW)	Water pumped (cfs)	Penalty	Notation
1	0.00%	0.00	0.0000	0	Pump is off, no water is pumped
2	69.9999%	69.9999	0.0000	-99999	Power below minimum, no water is pumped
3	70.00%	70.00	1220.6035	0	Pump is on, 70% minimum power input ;level
4	100.00%	100.00	1743.7194	0	Pump is on, maximum power input level

design, is not the same. Point 2 is computed as 69.9999% of the maximum unit capability. Point 3 is the minimum pumping power input level (70% of generation capacity) and the associated amount of water pumped. Point 4 corresponds to the maximum pumping power input (100%) level.

The “Penalty” column contains an arbitrarily large (negative) value which is used to mathematically penalize potential solutions which include particular points. Notice that only two points in this column have nonzero penalty values. The remaining points have values of 0.0 (or no penalty). Recall that for SOS2 sets, only two points can be active (non-zero), the active points must be adjacent, and the sum of the SOS2 weights must equal 1.0. The penalty values in this table are used in coordination with these SOS2 properties to obtain a feasible and optimal solutions to the piece-wise linear optimization problem. For example, if a potential generation level were selected which falls at the half-way point on the first (red) segment, points 1 and 2 would be active and the SOS2 weights might well be 0.50 and 0.50. The penalty associated with point 2 would be then assessed (revenue times penalty value times 0.50). Operations at this point along the red line segment are permitted, but the solution will be assessed a large negative penalty. Consequently, the resulting solution would be undesirable from a maximization standpoint. A useful insight is than an optimal solution could result in 0.0 pumping in a particular hour. Such a solution would also occur on the red line segment but the active SOS2 point weights would then be 1.0 and 0.0. This solution would avoid incurring the penalty (penalty value times 0).

An easily overlooked but substantive advantage of this approach is that it avoids introducing additional binary variables into the formulation. Had the SOS2 with penalty approach not been employed, binary variables would have been required to represent semi-continuous operations of the pump which is either operating at zero output level (off) or (on) at some value greater than or equal to the minimum pumping level. Introduction of binary variables would have resulted in a nonlinear formulation, which may or may not have been amenable to solution.

Price Data

The economic value of operating a pump generation plant is critically dependent on the value of both the energy and ancillary services it can provide. Some authors have asserted the value of the ancillary services which could be provided is quite large and greatly exceeds the energy value. They assert, in fact, the value of the ancillary services alone could justify the construction and operation of such a plant. Notably, several recent studies have failed to validate these assertions. Rather than debate the merits of these differing views, it may be more instructive to explore the currently available information on the economic values of both energy and ancillary services.

In the past, publically observable information on hourly energy and ancillary service values were exceedingly rare. In more recent years, entities known as Independent System Operators (ISOs) have formed to dispatch and manage some parts of the wholesale electricity system. These ISOs were formed following the issuance of FERC orders 888 and 889 (FERC 1996). At least in theory, ISOs can more efficiently market wholesale electric power over large(r) geographic expanses, efficiently utilizing the available transmission system, exploiting system diversity, taking advantage of economy of scale opportunities and reducing overall system operation costs. The California Independent System Operator (CAISO) is perhaps the largest of the existing ISOs in terms of geographic scope and influence.

Fortuitously, the CAISO footprint either encompasses, or is adjacent to, many Western Electricity Coordinating Council (WECC) marketing areas of particular interest to Reclamation. CAISO is required to publically report a variety of system operation metrics at different time-scales. These data are monitored closely by both regulators and market participants. Among these data are load, energy and ancillary service values for different locations within the ISO service area.

Peak electricity demand in Palo Verde region and the Western United States in general occurs in the summer, often in July. Owing to the high costs of obtaining, preparing, formatting and analyzing hourly price data sets, this analysis is focused on the month of July, typically the peak demand period during the year.

For this effort, July 2014 day ahead market (DAM) hourly locational marginal prices (LMPs) for the Palo Verde Node, located near Phoenix, Arizona, were extracted from the CAISO website[1]. This location is a relatively large market hub which is also tracked by Dow Jones, Inc., and other firms. Day ahead market (DAM) hourly ancillary service (AS) values for the CAISO exchange

[1] The gracious assistance and advice of Dr. Ziad K. Shawwash, Department of Civil Engineering, University of British Columbia in acquiring these data is gratefully acknowledged.

(CAISO_EXP) were also obtained for 2014. The DAM AS price sets include hourly prices for spinning reserve (SR), non-spinning reserve (NR), up-regulation (UR) and down-regulation (DR).

For analysis purposes, these five series of July 2014 hourly data were aggregated using a typical week approach. Each observation in the typical week approach is the mean value of all like observations in the monthly data set. For example, Monday at 1 am in a typical week, is the mean of all other Monday 1 a.m. observations contained in the monthly data set. This aggregation scheme is frequently used because it allows the n = 744 × 5 data points to be addressed in a more tractable fashion, while maintaining the time-series patterns inherent in the data set.

For some of the observations in these DAM data sets, energy and AS prices are sign negative. These negative values arise due to congestion on the transmission system. For the hours when these values are negative, additional provision of these services at that particular location imposes a cost on the interconnected electric power system.

The energy and ancillary service prices for a typical week in July 2014 are plotted in Figure 15. In this figure, Sunday is the first day plotted, followed by Monday through Saturday. Within the WECC Region, the weekday (Monday – Saturday) hours from 7 a.m. through 11 p.m. inclusive are considered to be on-peak, with all other hours during the week considered to be off-peak. All Sunday hours are considered to be off-peak.

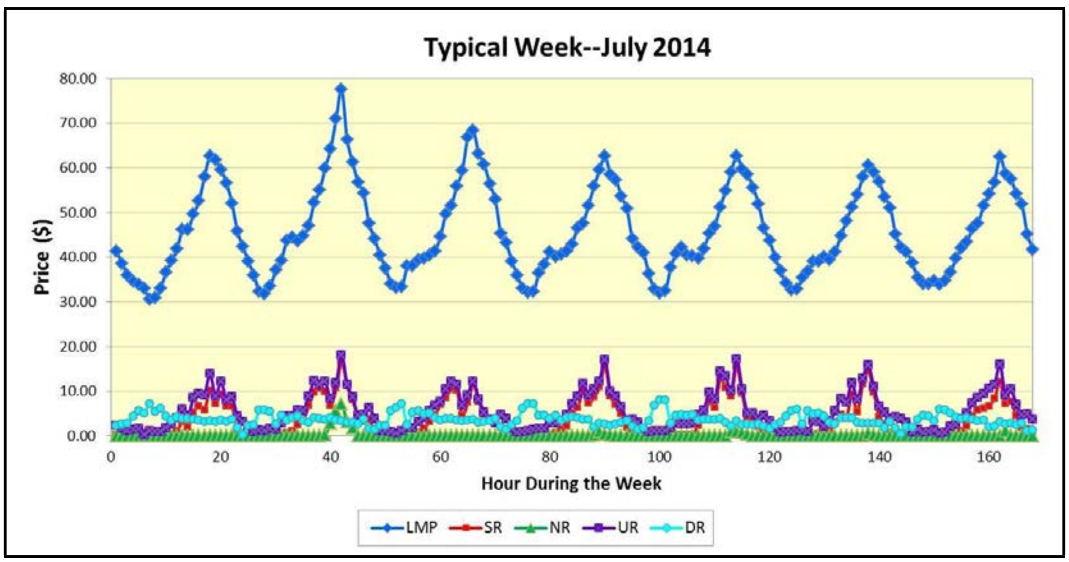

Figure 15.—Typical week energy and AS prices, July 2014.

The hourly energy price (LMP) pattern for July reflects the typical pattern of summer demand in the WECC region. The time series pattern of SR, NR, and RU prices coincides with that of the LMPs (e.g., their peaks are coincident with the LMPs). In contrast, regulation down (RD) is typically the most valuable when the output of other online generators has been reduced during off-peak hours. The pattern RD of prices is the mirror image of the other AS prices.

Figure 16 shows the July 2014 typical week data in terms of a box and whisker plot. Appendix 10 contains an explanation of box and whisker plots and how to interpret them. This plot illustrates the absolute and relative magnitudes of hourly energy, spinning, non-spinning, up-regulation and down-regulation services as well as information at-a-glance about their mean, median, upper and lower quartile and 90 percent value range. As shown in this figure, the hourly LMP price is always higher than the hourly AS prices, often considerably higher. Of the ancillary services, up-regulation (UR) is typically the most valuable, followed by spinning reserve (SR), down-regulation (DR) and then non-spinning reserves (NR). In light of the assertions made by some, it is noteworthy that AS prices are everywhere lower (at least for this data set) than the hourly energy prices (LMPs).

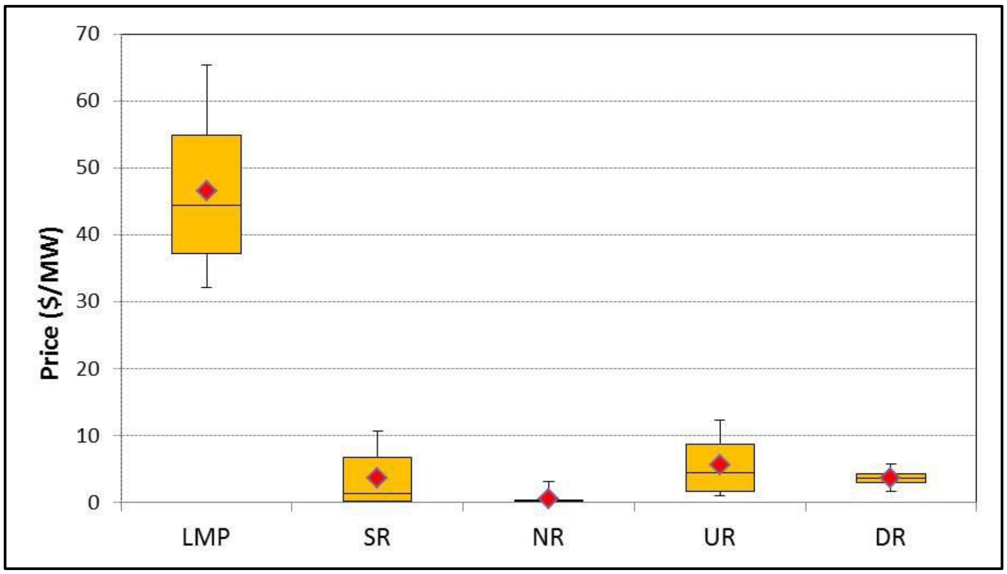

Figure 16.—July 2014 typical week box and whisker plot.

Table 11 reports the descriptive statistics for these typical week data. Additional context can be found in Appendix 11 which contains some time series plots of July LMP and AS prices in recent years. Although LMP prices have risen, hourly AS prices have fallen in recent years. Some informed observers have suggested that due to market imperfections, current AS prices do not reflect the true value of ancillary services. This topic has been the subject of some scholarly debate. AS prices are loosely related to energy prices, which are likewise quite low relative to previous years. It is almost certainly the case that current low prices for electric

Table 11.—Descriptive statistics typical week in July 2014

	LMP	SR	NR	UR	DR
Nobs	168	168	168	168	168
Mean	46.49	3.62	0.50	5.49	3.64
Std. dev	11.53	4.17	1.37	4.34	1.29
Minimum	30.55	0.13	0.09	0.30	0.22
05th perc	32.03	0.16	0.09	1.01	1.68
25th perc	37.22	0.24	0.09	1.60	2.88
Median	44.35	1.27	0.09	4.37	3.54
75th perc	54.86	6.73	0.10	8.75	4.25
95th perc	65.43	10.72	3.07	12.33	5.80
Maximum	77.54	16.69	7.16	18.08	7.16

energy reflect the relatively low demand for energy in general, during this period of macroeconomic downturn, and are surely influenced as well by the current historically low prices for natural gas.

The Price Taker Assumption

This research effort, like the vast majority of other published works, employs the standard competitive market assumption that producers are price takers. This assumption has important implications for the results reported here. Under this assumption, the owner/operator of the pump generation hydropower plant faces a fixed market determined price for both energy and ancillary services. The owner/operator is able to sell all of the energy and ancillary services they produce into the market without having any effect on the (fixed) market price. Under the price taker assumption, the demand function faced by any single market participant, plotted in price (P) and quantity (Q) space is shown in Figure 17.

As shown in this figure, a single market participant faces a flat (or as economists would say, perfectly elastic) demand function. If a producer attempted to increase revenues by increasing the price of their output, they would lose market share to other producers. Any single market participant can sell an arbitrarily large quantity (Q) of their products while leaving the market price (P) unchanged.

For large organized markets where each market participant supplies a small share of the market demand, the price taker assumption is a valid and often employed approach. For situations where markets are relatively thinly traded and the production of one or more market participants constitutes a substantial share of the market production—this assumption is not supportable.

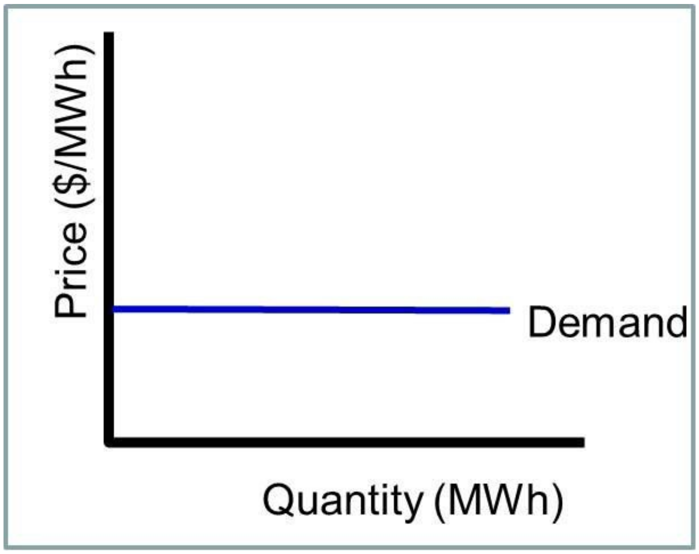

Figure 17.—Demand under the price taker assumption.

Some new electricity marketing mechanisms have evolved in the last two decades. In the Western United States, Independent System Operators (ISO's) and Regional Transmission Organizations (RTO's), have become much more prominent, and some would say, the dominant market entities. Especially prominent is the California ISO (CAISO), which has largely supplanted the traditional role of BA's in California.

Within the CAISO, the market for electric energy is relatively large in scope and scale. Every hour, large quantities of electric energy are bought and sold. In this context, the price taker assumption seems appropriate and supportable. However, the quantities of ancillary service products (up-regulation, down-regulation, spinning reserve and non-spinning reserve) which are traded, relative to system requirements, are relatively small. Furthermore, the distance over which ancillary service products can be transmitted is relatively limited, leading to a more localized market, compared to the electrical energy. In the case of ancillary services, the price taker assumption seems less supportable.

In the context of this research effort, the price taker assumption is pivotal to the results reported. This is particularly evident in cases where large quantities of ancillary services are presumed to be produced and sold in the market.

Figure 18 illustrates the more commonly encountered situation in which the quantity of the good demanded is sensitive to the price.

The demand schedule shown in Figure 18 is downward sloping. This implies that as the quantity supplied increases, the price of the good will decrease, and *vice versa.* If the price assumptions employed in this research effort followed this

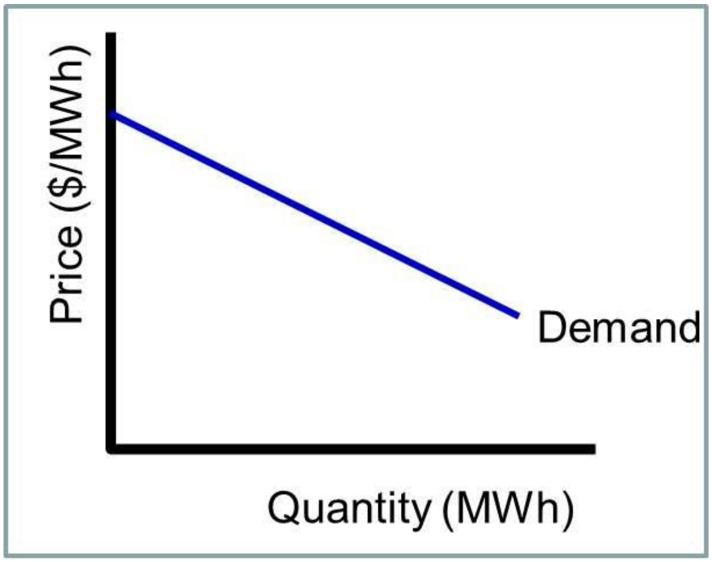

Figure 18.—Typical downward sloping demand function.

more general behavior, the results reported in this study would be considerably different. For instance, let's presume the demand for up-regulation was characterized by Figure 18. If that were the case, when substantive amounts of up-regulation were provided, the price would be expected to fall. This would have concomitant effects on the total revenue realized from the sale of up-regulation. Relative to the case where the price taker assumption is employed, if the demand schedule were downward sloping, the revenue received would be less.

The influence of the price taker assumption on the results reported here will be further described at pertinent points in this document.

Reference Conditions

For purposes of this research effort, the term "reference conditions" is used to describe the set of base case parameter values which specify the design, configuration and shape the operations of a pump-generation unit. These reference conditions jointly provide the point of comparison, against which all alternative parameter variations are compared. The reference operating conditions are summarized in Table 12.

As described in Table 12, two plant types; a variable speed unit and a single or fixed speed unit were examined. For each of these two plant designs, a single unit with a maximum generation capability (capacity) of 100 MW serves as the reference condition. The forebay (upper) reservoir in the reference conditions is

Table 12.—Reference operating conditions

Parameter setting	Value	Units
Plant type	• Variable speed • Fixed speed	Engineering design of plant
Number of units	1	Count
Unit capacity	100	MW/Unit
Forebay (upper) reservoir volume	8	Hours of storage at maximum output level
Gross head	600	Feet
Price set	Typical week in July 2014	$/MWh
Time-step	1	hour
Analysis period	168 hr	1-week

sized to provide 8 hours of continuous release at 100 MW. The elevation difference between the forebay (upper) reservoir and the afterbay (lower) reservoir, also known as the gross head, is set at 600 feet. The afterbay (lower) reservoir is assumed to be very large, relative to the forebay (upper) reservoir. The water surface elevation of the afterbay (lower) reservoir is assumed to be controlled by factors such as inflows, conventional hydropower operations and downstream deliveries. Releases from the forebay (upper) reservoir are assumed to have a negligible effect on the gross head.

The shaded rows in Table 12 were the subject of extensive experimentation whereas the unshaded rows in the reference conditions were held fixed for all analyses undertaken.

Example 24-Hour Results

The pump-generation model constructed for this research project produces an extensive output file. The numerical output is incredibly detailed, voluminous and requires considerable experience to understand. This section of the document provides a summary view of the variable speed (VS) pump-generation model results for a 24-hour period in July 2014, along with an overview of those results. The shorter time period summarization and visualization provides some insights into the larger body of results presented in this document.

Figure 19 illustrates a selection of the hourly results over a 24-hour period representing a typical Wednesday in July of 2014.

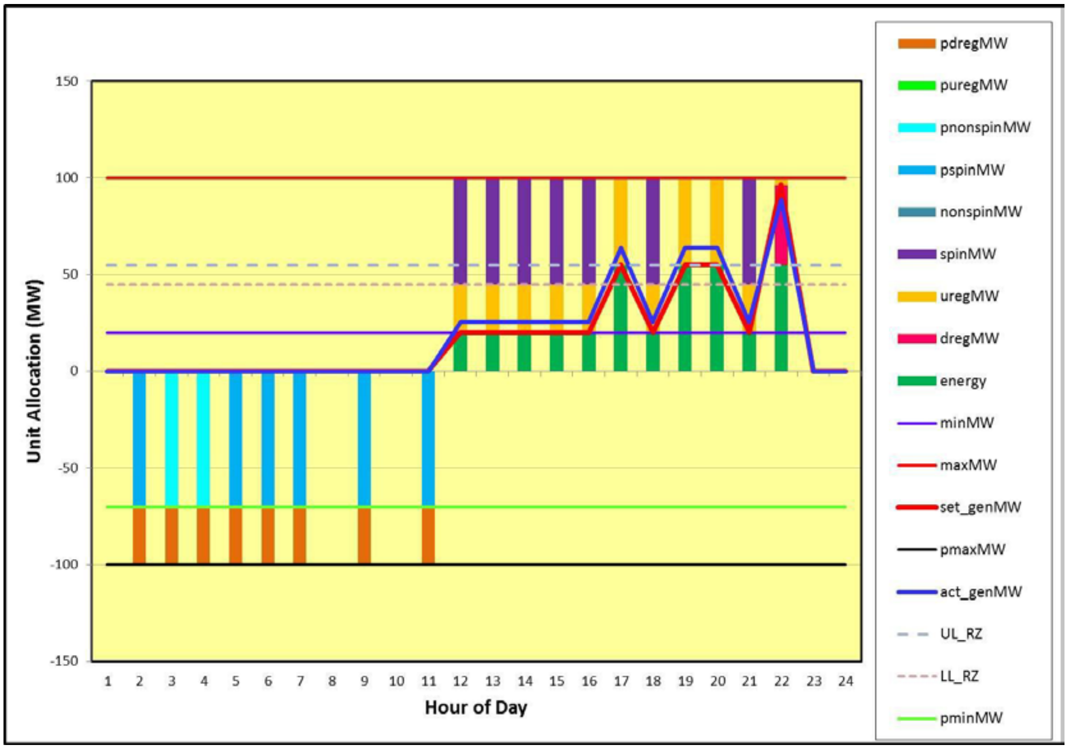

Figure 19.—Optimal dispatch on a Wednesday in July 2014.

This figure illustrates VS unit dispatch for a single 24-hour period and shows many of the modeled results which may occur in a simulation. During this simulation, the plant operator must decide whether to pump, generate or do neither during each hour. Once the decision is made to pump or generate, the operator must decide how to produce generation or pumping services in a profit maximizing manner. In generation mode, the variable speed plant can operate between the minimum generation level (minMW) and the maximum generation level (maxMW) while avoiding the operation range between the lower rough zone limit (LL_RZ) and the upper rough zone limit (UL_RZ). It can produce energy (energy), up-regulation (uregMW), down-regulation (dregMW), spinning reserve (spinMW) and/or non-spinning reserve (nonspinMW). Aancillary services are produced in the generation mode, the generator set point (set_genMW) will differ from the actual generation level (act_genMW). In pump mode, the pump must be used in the range between the minimum pumping level (pminMW) and the maximum pumping level (pmaxMW). A variable speed plant can use energy to pump, and/or can also be used to produce pump up-regulation (puregMW), pump down-regulation (pdregMW), pump spinning reserve (pspinMW), and pump non-spinning reserve (pnonspinMW) services. Similarly, when ancillary services are produced in the pump mode, the pumping set point will differ from the actual pumping level.

Referring to Figure 19, in hours 1, 8, 10, 23 and 24 the plant is idle; it is not generating or pumping. In hours 2 through 7, 9 and 11 the plant is in pump mode.

Looking closely at hour 4, shows the pump set point is 70 MW. In this hour the pump is simultaneously producing 70 MW of non-spinning reserve and furnishing an additional 30 MW of pump mode down-regulation between the upper and lower pump operation limits of 70 to 100 MW. There are different pump mode unit dispatch allocations evident in other hours.

This variable speed 100 MW plant is operating in generation mode during hours 12 through 22. A careful look at hour 17 shows the generation set point is at the upper rough zone limit (55 MW) and the actual level of generation is higher than this. There is a divergence between the set point level of generation and the actual level of generation because ancillary services (up-regulation) are provided in this hour. The 100 MW variable speed generator is producing energy up to the upper rough zone limit (55 MW) and is providing up-regulation between the upper limit of the rough zone (55 MW) and the maximum generation level (100 MW). There are different unit dispatch allocations evident in other hours.

Table 13 reports a summary, of the revenues provided during this particular day. As shown in this table, a preponderance of the revenues is derived from energy production. It is also clear that pumping represents a significant cost, although it does provide some ancillary service benefits as well. In the generation mode, spinning reserves and up-regulation provide the bulk of the ancillary service benefits. In the pump mode, down-regulation constitutes the majority of the ancillary service benefits. Experiments conducted using alternate price sets suggest these results are quite sensitive to the specific LMP and AS prices employed.

Table 13.—Summary of revenues provided

	Value	Fraction Gross
gross generation value =	24967.47	0.7561
gen spin value =	3196.60	0.0968
gen nonspin value =	0.00	0.0000
gen ureg value =	3183.51	0.0964
gen dreg value =	139.49	0.0042
pump spin value =	267.40	0.0081
pump nonspin value =	12.60	0.0004
pump ureg value =	0.00	0.0000
pump dreg value =	1254.00	0.0380
Gross value all products =	33021.06	1.0000
pumping cost =	21866.70	
Net value =	11154.36	

Figure 20 encapsulates the contribution to gross value provided by energy sales and ancillary service provision in both generator and pump mode. As shown in this figure, approximately 76 percent of the gross value is derived from energy sales and 24 percent of the value is derived from the provision of ancillary services. Experiments conducted using alternate price sets suggest these fractions are highly sensitive to the specific LMP and AS prices employed.

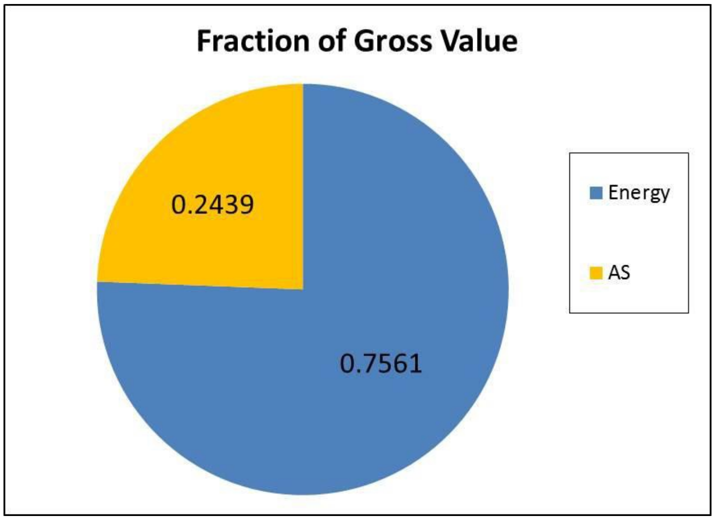

Figure 20.—Fraction of gross value from energy and AS provision.

The detailed results also contain some useful energy balance information. During this 24-hour period, approximately 459.76 MWh of energy were generated and 603.73 MWh of energy was consumed for pumping purposes. This represents a net energy loss of 149.97 MWh. During this period, the round trip efficiency of this variable speed plant was calculated to be 76 percent.

Unit Type Experiments

The chief advantage of variable speed (VS) pump generation units is their ability to produce regulation in the pump mode. Single speed (SS) pump generation units are currently in use at a number of locations in the United States. Variable speed (VS) pump-generation units are a more recent innovation and are more commonly encountered abroad. At least one such unit is planned for use in the U.S. Increasing penetration by renewable variable generation resources suggests greater and greater amounts of ancillary services are required in the interconnected system. This simple fact would appear to favor VS units. However, VS units are larger and more costly to construct. Given their additional cost, relative to a SS unit, how much additional revenue can a VS unit produce and is that sufficient to offset their incremental cost? In this section of the document, the SS and VS models developed in this research effort are used to address the benefits side of this question.

Approach

The SS and VS models detailed earlier in this document are used to explore the potential differences in the economic benefits which could be produced by a SS pump generation unit and a VS pump generation unit. Both models were used to simulate the behavior of the respective pump generation units at the reference operating conditions. The SS and VS plants were both the same size with identical gross heads. Both plants used the same price set and had forebay (upper) reservoirs with identical volumes and physical characteristics. Careful adherence to the reference operating conditions allows for a straightforward comparison of these two engineering designs.

Results

Although the units are of equal size, the VS unit has a narrower rough zone and slightly higher generation efficiency. The VS unit has a wider operational range in pump mode. As shown in Table 14, the VS unit can be dispatched to use these characteristics advantageously. In generation mode the VS unit is able to generate more revenue from the sale of energy, down-regulation and up-regulation than the SS unit. As shown in the table, in generation mode, the SS unit allocates more capacity to spinning reserve than does the VS unit. In pump mode, the SS unit is not able to provide regulation services but does produce some spinning and non-spinning reserve revenues. In contrast, the VS unit in pump mode produces substantive amounts of up-regulation and down-regulation, thereby deriving considerable revenues. The VS unit also derives some revenue from the provision of spinning and non-spinning reserve.

As shown in this table the VS unit produces approximately 24% more gross revenue, at the reference conditions. For purposes of this research, net revenue is defined as gross revenue minus pumping costs. The VS unit produces approximately 20% more net revenue than does the SS unit, at the reference conditions. The table provides a detailed accounting of the revenues derived from the sale of each product in pump mode and generation mode. Figure 21 summarizes the aggregate ancillary service and energy components of gross revenues from both operational modes. As shown in the figure, the bulk of the revenues for both types of units are derived from energy sales (arbitrage) with lesser amounts being obtained from the sale of ancillary services. More ancillary service revenues are produced by the VS unit relative to the SS unit. It should be noted the ancillary service revenues for both plants are not hugely different.

Table 14.—SS and VS revenues for a typical week in July 2014

Mode	Product	SS unit revenue ($)	VS unit revenue ($)
Generation	Energy	176867.64	222734.78
	Spinning reserve	25893.20	17867.70
	Non-spinning reserve	0.00	0.00
	Up-regulation	14100.60	17641.62
	Down-regulation	1325.43	2272.28
Pumping	Spinning reserve	985.60	2452.00
	Non-spinning reserve	50.40	62.10
	Up-regulation	0.00	1031.10
	Down-regulation	0.00	7400.76
Total gross value ($)		219222.87	271462.34
Pumping cost ($)		-148101.74	-185818.36
Net value ($)		71121.13	85643.98

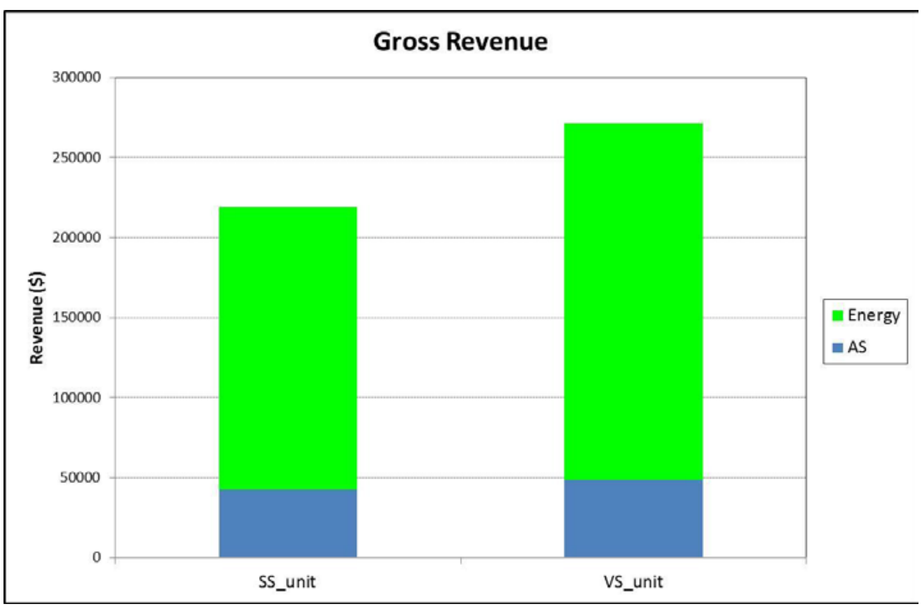

Figure 21.—Revenue components for SS and VS units.

Insights

This analysis provides information that is useful in assessing the revenue tradeoff between a single speed pump generation unit and a variable speed pump generation unit. The results described here are for the same size units at the reference operating conditions described earlier. This approach allows the economic merits of the two types of units to be compared, holding all other factors constant. The results of this analysis indicate the VS unit may produce approximately 20% greater net revenues (gross revenues minus pumping costs) relative to the SS unit.

The analysis results described here are very sensitive to the price set employed; in this case the July 2014 typical week prices. If the price data utilized were different, the results reported here would be different, perhaps very different. In particular, if the spread between AS and energy prices were larger, gross revenue would be higher and both plants would be expected to produce more ancillary services and less energy revenues. Potentially, higher AS prices might further advantage the VS unit in this comparison.

Multiple Unit Experiments

This section of the document examines the economic benefits which might be realized from the installation of more than one pump-generation unit at the same plant. Pump generation units are available in discrete sizes (such as 100 MW) from various manufacturers and are custom made for a given application. Some sites may be suitable for the installation of more than a single unit. For example, at the Mead 2D site described HDR-CDM Joint Venture (2014), six identical 291-MW units are planned. Selection of the number and size of those units are important design questions. A brief exploration is described here.

In this research effort a simple generic unit is portrayed and no site or plant specific information is available. For a planned plant, considerable design and site information would be available. This would include all of the parameters explored in this research effort, as well as engineering and design details about reservoir and penstock configurations.

Conceptually at least, the installation of more than one pump-generation unit could produce economic benefits which might exceed those estimated by simply multiplying the number of units times the benefits of a single unit. Potentially, multiple units could be dispatched more efficiently than a single unit. Additionally, their coordinated operations could be quite different from the dispatch of a single unit. It could be envisioned, for example, that one or more units at a multiple unit plant could be operated in generation mode, while other units are simultaneously operated in pump mode.

Although the questions about multiple unit benefits and scaling would appear to be straightforward, there is considerable technological and conceptual debate on this topic. To help address this issue, the economic merits of a single unit are compared to those produced by a set of two hydraulically independent units.

Approach

The SS and VS models detailed earlier in this document are used to explore the economic benefits of multiple plants. The modeled results for a single unit are compared to the results obtained from the simulated operations of a plant with two identical units.

For this exploration, 2-unit models of SS and VS pump-generation were developed and coded in LINGO 14. These models are identical in every respect to the single unit models described previously, save for the fact 2-units were characterized. These 2-unit models explicitly assume each unit is hydraulically independent. Specifically, the operation of one unit has no physical or hydraulic effect on the operations of any other unit. All units are fed by and discharge from their own penstock. In the 2-unit models, both units share a common forebay (upper) reservoir. The size of the forebay reservoir is exactly 2-times the size of the forebay portrayed in the 1-unit models. The afterbay (lower) reservoir is unaffected by operations of either the 1-unit or 2-unit models.

Results

Although development of the 2-unit models used for this part of the analysis was tedious and time-consuming, the results were, in retrospect, quite predictable. In the 2-unit modeled plant, the units are identical to each other and are hydraulically independent. Furthermore, they face the same hourly price set and, perhaps most importantly, these prices are unaffected by their joint level of output (a.k.a: the price taker assumption). Given these conditions, each unit in the 2-unit plant behaves identically. If one unit is generating at X-MW, so is the other unit. If one unit is pumping at a particular pump set-point, so is the other unit. None of the conceptual gains from coordinated operations were observed in this modeling effort.

As shown in this table the SS 1-unit plant produces approximately $219222.87 in gross revenue and $71121.13 in net revenue, at the reference conditions. At the reference operating conditions, the SS 2-unit unit plant produces approximately $438445.28 in gross revenue and $142245.18 in net revenue. The economic benefits produced by the SS 2-unit plant are almost exactly the same as 2 times

Table 15.—SS revenues by source

Mode	Product	SS 1-unit revenue ($)	SS 2-units revenue ($)
Generation	Energy	176867.64	353734.75
	Spinning reserve	25893.20	51787.04
	Non-spinning reserve	0.00	0.00
	Up-regulation	14100.60	28200.10
	Down-regulation	1325.43	2649.99
Pumping	Spinning reserve	985.60	1972.60
	Non-spinning reserve	50.40	100.80
	Up-regulation	0.00	0.00
	Down-regulation	0.00	0,00
Total gross value ($)		219222.87	438445.28
Pumping cost ($)		-148101.74	-296200.10
Net value ($)		71121.13	142245.18

the benefits of the SS 1-unit plant. It should be noted the results for both the 1-unit and 2-unit plants are a function of the convergence tolerance used in the optimal solution (they are not exact). Hence there is no discernable difference between the results obtained for the 2-unit model and 2-times the 1-unit results.

The nature of the outcomes for both the SS and VS 2-unit models are inherently the same and only the results for the SS models are reported in this document.

Insights

At the reference operating conditions the economic benefits produced by the SS 2-unit plant are almost exactly the same as 2-times the estimated benefits of a SS 1-unit plant. Although not reported here, conclusions drawn from a comparison of 1-unit and 2-unit VS plant results are the same.

For purposes of this investigation, the price set employed was the same, the pump-generation units were identical and assumed to be hydraulically independent. The size of the forebay (upper) reservoir was scaled to fit the number of units and there is no operational effect on the afterbay (lower) reservoir. Under the price taker assumption used in this analysis, prices are given, fixed and invariant with the level of output. In aggregate, these factors shape the results of this experiment. Upon reflection, the results obtained here are entirely predictable, consistent with the experimental design and unremarkable.

With and Without AS Experiments

This section of the document examines the economic benefits which can be realized from units which produce and sell ancillary services, in addition to energy. To reiterate, arbitrage is the creation of net revenues by purchasing a good when its price is low, and then selling it when its price is high(er). Pump-generation units can be used strategically to exploit the difference or spread between low and high energy prices. A savvy producer can employ their pump-generation resources for arbitrage by purchasing and using energy for pumping when the price is low, typically during off-peak hours, and then generating and selling energy when the price is high, generally during the on-peak hours. In the not-so-distant past, ancillary services were bundled with the provision of energy and were not a separately provided product. In those times, energy arbitrage was generally the sole source of revenue for pump-generation units.

Recent changes in market structures have engendered “un-bundling” ancillary services from the provision of energy. Increased penetration by variable generation resources has increased the need for ancillary services in the interconnected electricity system and markets have developed for their exchange. Some observers have opined the sale of ancillary services could produce more revenue for the owners of pump-generation units than energy sales. This analysis provides some insight into the relative magnitudes of the revenues derived from ancillary services and compares them with energy arbitrage (only) revenues.

Approach

The SS and VS models detailed earlier in this document were used to explore the economic benefits of producing ancillary services. A plant which produces AS products and energy is dispatched differently than a plant which produces energy only. A technically accurate assessment of the potential difference in revenues must employ what is known as a “with” and “without” comparison. To facilitate this comparison, the modeled results for a unit which jointly produces energy and ancillary services must be compared to a unit which can only produce energy. This examination required development of a separate “without ancillary services” LINGO models for the SS and VS pump-generation units. All of comparisons were undertaken at the reference conditions described earlier in this document.

Results

Table 16 illustrates the revenues for a single speed (SS) pump-generation unit “with” and “without” ancillary service provision. This analysis was undertaken for a 1-unit SS plant at the reference operating conditions described earlier in this

Table 16.—SS unit with and without AS

Mode	Product	SS with AS revenue ($)	SS without AS revenue ($)
Generation	Energy	176867.64	224636.00
	Spinning reserve	25893.20	0.00
	Non-spinning reserve	0.00	0.00
	Up-regulation	14100.60	0.00
	Down-regulation	1325.43	0.00
Pumping	Spinning reserve	985.60	0.00
	Non-spinning reserve	50.40	0.00
	Up-regulation	0.00	0.00
	Down-regulation	0.00	0.00
Total gross value ($)		219222.87	224636.00
Pumping cost ($)		-148101.74	-181130.30
Net value ($)		71121.13	43505.70

document. In generation mode, a SS unit can produce energy and the full range of ancillary services. In pump mode, the SS unit can produce reserves, but not regulation. As shown in the table, net revenues approximately 63% higher with the production of ancillary services, at reference conditions.

Table 17 illustrates the revenues for a variable speed (VS) pump-generation unit "with" and "without" ancillary service provision. In generation mode, a VS unit can produce energy and the full range of ancillary services. In pump mode, the VS unit can produce reserves and regulation. This analysis was undertaken for a 1-unit VS plant at the reference operating conditions described earlier in this document. As shown in the table, net revenues are approximately 56% higher with the production of ancillary services, at reference conditions.

For both the SS and VS 1-unit models, revenues are enhanced when ancillary services are produced, relative to the "without" case. However, energy arbitrage contributes the bulk of the revenues.

Table 17.—VS unit with and without AS

Mode	Product	VS with AS revenue ($)	VS without AS revenue ($)
Generation	Energy	222734.78	264670.34
	Spinning reserve	17867.70	0.00
	Non-spinning reserve	0.00	0.00
	Up-regulation	17641.62	0.00
	Down-regulation	2272.28	0.00
Pumping	Spinning reserve	2452.00	0.00
	Non-spinning reserve	62.10	0.00
	Up-regulation	1031.10	0.00
	Down-regulation	7400.76	0.00
Total gross value ($)		271462.34	264670.34
Pumping cost ($)		-185818.36	-209694.48
Net value ($)		85643.98	54975.87

Insights

Relative to the “without” ancillary service case, the net economic benefits produced by SS and VS 1-unit plants are 63% and 56% greater respectively, when AS are provided and sold. These results were obtained at the reference operating conditions. Energy arbitrage remains the predominant source of revenue for both plant designs. Although AS revenues are certainly significant in both instances, their magnitudes do not overshadow the revenue derived from energy arbitrage.

Live Volume Experiments

This section of the document describes the contribution of forebay (upper) reservoir volume to economic value. It provides some insights pertinent to the design and sizing of forebays for pump-generation plants. The forebay (upper) reservoir utilized in this research project was devised in a manner which allows the live storage volume to be altered, independently of the head. Admittedly, this is an abstraction from the real-world. However, this conceptual and mathematical design enables this analysis of the independent contribution of the forebay volume to economic benefits.

Approach

The SS and VS pump-generation models detailed earlier in this document were used to explore the economic benefits of live forebay storage volume. For purposes of this research project, the live forebay storage volume is measured by the number of hours of storage necessary to provide generation at the maximum output level. For this analysis, the number of hours (of storage) was systematically varied, and the models were run and rerun. The net economic benefits at each level of storage were then recorded. All of the model runs employed the reference conditions described earlier in this document, unless stated otherwise.

Results

Table 18 illustrates the fraction of net revenues for a single speed (SS) pump-generation unit and a variable speed (VS) pump generation units, which are achieved at different levels of forebay storage volume, for two different price sets; 2014 and 2011. In this table, net revenue is normalized by the maximum net revenue value estimated for the specific unit and price set used. This normalized measure of net revenue is labeled, "Fraction of Net Benefits." This normalization procedure allows relevant comparisons across unit types and price sets.

Table 18.—Fraction of net benefits by volume

	SS unit		VS unit	
	2014 prices	2011 prices	2014 prices	2011 prices
Fraction of net benefit	Hours of storage required	Hours of storage required	Hours of storage required	Hours of storage required
0.50	0.93	1.70	1.04	1.70
0.55	1.05	1.92	1.23	1.92
0.60	1.23	2.17	1.42	2.16
0.65	1.41	2.44	1.61	2.43
0.70	1.59	2.72	1.80	2.70
0.75	1.76	3.00	1.99	2.97
0.80	1.94	3.37	2.38	3.32
0.85	2.32	3.75	2.79	3.69
0.90	2.80	4.23	3.37	4.14
0.95	3.71	4.91	4.34	4.89
0.99	6.31	7.23	6.91	6.97
1.00	10.00	14.00	12.00	15.00

As this table reveals, large initial gains in net revenue (here expressed as a fraction of the total possible revenue) accrue at relatively low levels of storage. Additions to the live storage volume result in increased net economic benefits, but the rate of revenue capture declines markedly with increasing reservoir size. At the reference conditions, 50 percent of the net economic benefits are achieved with a forebay storage volume of between 0.93 and 1.70 hours of storage, across both unit types and price sets. As shown in the table, 75% of the net economic benefits for both types of plants and across the two price sets can be achieved with a forebay storage volume of 1.76 to 3 hours. In this table the row reporting the net benefits level of 99% is highlighted in yellow. We focus our attention on this row for several reasons. Modeled results indicate that to achieve 99% of the possible benefits from a pump-generation plant requires a forebay reservoir with from 6.31 to 7.23 hours of live storage, across both unit types and price sets. Interestingly enough, extracting the last 1-percent (e.g., achieving 100%) of the possible benefits necessitates the construction of a much larger reservoir with a live storage volume of between 10 and 15 hours.

For both types of pump-generation plants, net economic revenues increase as live storage volume in the forebay increases. This increase continues until some upper limit is reached. At this volume level other physical and engineering constraints, principally unit size, become binding and no additional revenues can be achieved. At the binding live forebay storage limit, the incremental benefits of further increases in volume are zero and no additional net benefits can be gained by constructing a larger reservoir.

Insights

The results reported in Table 18 illustrate that net economic revenue increases with storage volume, but not continuously. At some point, other physical and engineering constraints preclude further gains in revenue from increases in the size of the forebay storage reservoir. Moving beyond a reservoir size which captures 99% of the benefits, a substantial increase in reservoir size is required to achieve 100% of the potential economic benefits. All other factors remaining the same, 99% of the net economic benefits of storage are captured at from 6.31 to 7.23 hours of storage. Relative to a forebay reservoir sized to capture 99% of the benefits, a substantial increase in reservoir size is required to achieve 100% of the potential economic benefits. For a SS unit facing 2011 prices, relative to the 99% sized reservoir, the reservoir size would have to be nearly doubled to capture 100% of the economic benefits.

Further reflection on the yellow shaded 99% row is instructive. In this study the reservoir size which captures 99% of the potential economic benefits ranges from 6.31 to 7.23 hours, depending on the unit type and price set employed. This range is substantially below the forebay sizing “rules of thumb” which have been used for other analysis. For example, a recent study by HDR-CDM Joint Venture

(2014) uniformly employs a forebay reservoir sized to allow 10 hours of generation at the maximum generation capability. Potentially, the logic and implications of these rules of thumb should be further examined. Some additional analysis pertinent to this topic is provided in a later section of this document.

The results reported here do vary somewhat with the specific price data used in the analysis Relative to the reference conditions July 2014 price set, the July 2011 price set has lower energy values and higher ancillary service prices. When the 2011 price set is used for analysis, there are some discernable differences in the results. However, at the forebay live storage level which captures 99% of the economic benefits, the nature of the results do not change markedly.

Live Volume Experiments With and Without AS

This section of the document describes the contribution of forebay (upper) reservoir volume to economic value and the influence of ancillary services (AS) on that contribution. It provides some insights pertinent to the design and sizing of forebay reservoirs for pump-generation plants when the provision of AS is anticipated. The forebay (upper) reservoir utilized in this research project was devised in a manner which allows the live storage volume to be altered, independently of the head. Simply put, this is an abstraction from the real-world. However, this abstract design enables the analysis which follows.

Approach

The SS and VS pump-generation models detailed earlier in this document were used to explore the economic benefits of live forebay storage volume when there is ancillary service provision. A technically accurate assessment of the potential effects of ancillary services must employ what is known as a “with” and “without” comparison. To facilitate this comparison, the modeled results for a unit which jointly produces energy and ancillary services (the “with” AS case) must be compared to a unit which can only produce energy (the “without” AS case). This examination required development of separate “without ancillary services” LINGO models for the SS and VS pump-generation units.

For purposes of this research project, the live forebay storage volume is measured by the number of hours of storage necessary to provide generation at the maximum output level. For this analysis, the number of hours (of storage) was systematically varied, and the “with” and “without” AS models were run and rerun. The net economic benefits at each level of storage, with and without AS provision, were then recorded. All of the model runs employed the reference conditions described earlier in this document.

Results

Table 19 illustrates the fraction of net revenues for a single speed (SS) pump-generation unit and a variable speed (VS) pump generation units, with and without AS provision, which are achieved at different levels of forebay storage volume. In this table, net revenue is normalized by the maximum net revenue estimated for the specific unit. This normalized measure of net revenue is labeled, “Fraction of Net Benefits.” The normalization procedure allows relevant comparisons across unit types.

Table 19.—Live volume with and without AS

	SS unit		VS unit	
	With AS	Without AS	With AS	Without AS
Fraction of net benefit	Hours of storage required	Hours of storage required	Hours of storage required	Hours of storage required
0.50	0.93	1.68	1.04	1.87
0.55	1.05	1.89	1.23	2.11
0.60	1.23	2.12	1.42	2.40
0.65	1.41	2.38	1.61	2.69
0.70	1.59	2.63	1.80	2.98
0.75	1.76	2.89	1.99	3.35
0.80	1.94	3.23	2.38	3.73
0.85	2.32	3.64	2.79	4.18
0.90	2.80	4.14	3.37	4.81
0.95	3.71	5.26	4.34	5.88
0.99	6.31	8.21	6.91	9.09
1.00	10.00	12.00	12.00	13.00

As this table reveals, large initial gains in net revenue (here expressed as a fraction of the total possible revenue) accrue at relatively low levels of storage. Additions to the live storage volume result in increased net economic benefits, but the rate of revenue capture declines rapidly with increasing reservoir size. In this table, the row reporting 99% of the net benefits is highlighted in yellow. We focus our attention on this row.

For both types of pump-generation plants, net economic revenues increase as live storage volume increases until some upper limit is reached. At this point, other physical and engineering constraints, principally unit size, become binding and no additional revenues can be achieved. At the binding live forebay storage limit, the incremental benefits of further increases in volume are zero and no additional net benefits can be gained by constructing a larger reservoir.

For both types of pump-generation plants a comparison of the “with” and “without” AS results reveals an interesting pattern. For both the VS and SS pump-generation units, the size of the reservoir required to produce 99% of the economic benefits is smaller when AS are provided. As shown in the yellow highlighted row, for the SS unit 99% of the economic benefits are captured with a reservoir sized at 6.31 hours of storage with AS provision and 8.21 hours without AS provision. For the VS unit, the pattern is the same—the size of the forebay reservoir required to produce 99% of the economic benefits is considerably smaller when ancillary services are provided by the unit.

Insights

The results reported in Table 19 illustrate that net economic revenue increases with storage volume, but not continuously. At some point, other physical and engineering constraints preclude further gains in revenue from increases in the size of the forebay storage reservoir. Relative to the reservoir size which captures 99% of the benefits, a substantial increase in reservoir size is required to achieve 100% of the potential economic benefits.

All other factors being the same, the results of this analysis suggest the forebay (upper) reservoir for a unit providing ancillary services should be smaller when ancillary service provision is expected. Further reflection on the yellow shaded 99% row is very useful. In this study the forebay reservoir size which captures 99% of the potential economic benefits for a VS unit providing ancillary services is 6.91 hours of storage compared to 9.09 hours of storage for a unit which does not provide AS. This reservoir size is substantially below the forebay sizing “rules of thumb” which have been used for other analysis. For example, a recent study by HDR-CDM Joint Venture (2014) uniformly employs a forebay reservoir sized to allow 10 hours of generation at the maximum generation capability. Possibly, this rule of thumb may be more useful when sizing traditional pump-generation plants designed for energy arbitrage (only). In light of the results produced in this study, this rule of thumb would appear to exaggerate the size of forebay reservoirs needed for plants expected to produce ancillary services. It seems clear that additional investigation of this topic is warranted.

Head Experiments

This section of the document describes the contribution of gross head to net economic benefits. It provides some insights on the design and selection of sites for pump-generation facilities. As described earlier, the forebay (upper) reservoir utilized in this research project was devised in a manner which allows the gross head to be altered independently of the live storage volume. Without any question, this is an abstraction from the real-world. However, this abstract design enables the analysis which follows.

Approach

The SS and VS pump-generation models detailed earlier in this document were used to explore the incremental net economic benefits of gross head, independent of all other factors. For purposes of this research project, the gross head is defined as the difference between the water surface elevation in the forebay (upper) reservoir and the water surface elevation in the afterbay (lower) reservoir. It is assumed pump-generation operations have no effect on the elevation of the afterbay. For this analysis, the gross head was systematically varied in 50-foot increments and the pump-generation models were run and rerun. The modeled changes in gross head simulate placement of the forebay (upper) reservoir up and down in elevation on the topography, while keeping all other factors the same. The net economic benefits at each level of gross head were recorded. With the exception of gross head, all of the model runs employed the reference conditions described earlier in this document.

Results

Table 20 illustrates the fractions of net revenues for SS pump-generation and VS pump generation units which are achieved at different levels of gross head. In this table, net revenue is normalized by the maximum net revenue estimated for the specific type of unit. This normalized measure of net revenue is labeled, “Fraction of Net Benefits.” The normalization procedure allows relevant comparisons across unit types.

As this table reveals, large initial gains in net revenue (here expressed as a fraction of the total revenue) accrue at relatively low levels of gross head. Additions to the gross head result in increased net economic benefits, but the rate of revenue capture declines rapidly with increasing head. The LINGO 14 VS model was unable to converge at gross heads lower than 150 feet. Consequently, some observations of net revenue are labeled as not available, or “na.” In this table, the row reporting 99% of the net benefits is highlighted in yellow. For expository reasons we focus our attention on this row.

Table 20.—Net benefit with gross head

	SS unit	VS unit
Fraction of net benefit	Gross head (ft)	Gross head (ft)
0.50	72.96	na
0.55	82.17	na
0.60	91.38	na
0.65	100.98	na
0.70	116.11	na
0.75	131.25	151.81
0.80	146.39	178.47
0.85	173.98	208.54
0.90	209.82	254.31
0.95	281.30	330.72
0.99	477.16	514.73
1.00	700.00	850.00

Results labeled “na” are not available.

For both types of pump-generation plants, net economic revenues increase as gross head increases until an upper limit is reached. At this point, other physical and engineering constraints, principally unit size and live forebay volume, become binding and no additional revenues can be achieved. At the binding gross head limit, the incremental benefits of further increases in head are zero and no additional net benefits can be gained by placement of the forebay reservoir higher up on the terrain.

The relationships between gross head and net revenue are similar for both types of pump-generation plants. Within the 50% to 99% net benefit range, the differences in response can be attributed to the higher generation efficiencies and smaller rough zones (greater range of generation) inherent in VS plants. The gross head required to produce 99% of the economic benefits is somewhat greater for VS plants than it is for SS plants. As shown in the yellow highlighted row, for the SS unit 99% of the economic benefits are captured with a gross head of 477.16 feet and for the VS plant, 514.73 feet.

Insights

The results reported in Table 20 illustrate that net economic revenue increases with gross head, but not continuously. At some point, other physical and engineering constraints preclude further gains in revenue from increases the gross head. A large proportion of the net revenues are captured at relatively low levels of gross head. At approximately 500 feet of gross head, around 99% of the potential net revenues are realized. Relative to the 99% net benefits level, a substantial increase in gross head is required to achieve 100% of the potential economic benefits.

The results of this analysis indicate the predominant fraction of the net economic benefits can be captured at levels of gross head which can be found adjacent to many Bureau of Reclamation facilities. Early advice suggested pump-generation sites should have gross heads in the neighborhood of 1,500 feet. This advice is not supported by our results and appears to unnecessarily limit the search for potential sites. Given the importance of gross head in identifying suitable pump-generation sites, additional investigation of this topic is surely warranted.

The numerical results reported here could vary somewhat with the data used in the analysis No specific exploration of price effects was undertaken for this study. However, based on our analysis of the effect of price on reservoir size, it might be surmised the nature of the gross head results reported here would not change markedly with price.

Capacity Experiments

This section of the document describes the contribution of unit capacity (size) to net economic benefits. It provides some insights on the sizing of pump-generator units for planned facilities. In the mathematical characterization used in this research project, the size of the pump-generator unit, the forebay (upper) reservoir volume and the gross head can be altered independently of each other. This abstraction from real-world design processes enables the systematic appraisal of alternative pump-generation unit sizes which follows.

Approach

The SS and VS pump-generation models were used to explore the total and incremental net economic benefits of unit capacity (size), independent of all other factors. For purposes of this research project, the gross head is defined as the difference between the water surface elevation in the forebay (upper) reservoir and the water surface elevation in the afterbay (lower) reservoir. It is assumed pump-generation operations have no effect on the elevation of the afterbay. For this analysis, the unit capacity (MW) was systematically varied in 25 MW increments over the range of likely unit sizes. The pump-generation models were run and rerun, at each capacity level. The modeled changes in capacity represent the installation of different size pump-generator units, while keeping all other factors the same. The net economic benefits at each level of unit capacity were recorded. With the exception of the size of the unit, all of the model runs employed the reference conditions described earlier in this document.

Results

Figure 22 illustrates the total and incremental net revenues respectively for single speed (SS) pump-generation and variable speed (VS) pump generation units which are achieved at different unit sizes. As this figure reveals, at much of the range in capacity examined the VS unit realizes a higher level of total and incremental net revenue than does the SS unit. For both types of units, total net revenues increase as capacity increases but the incremental value of capacity additions falls. The LINGO 14 VS model was able to find a feasible but not optimal solution at a capacity of 425 MW. Consequently, the feasible solution was employed.

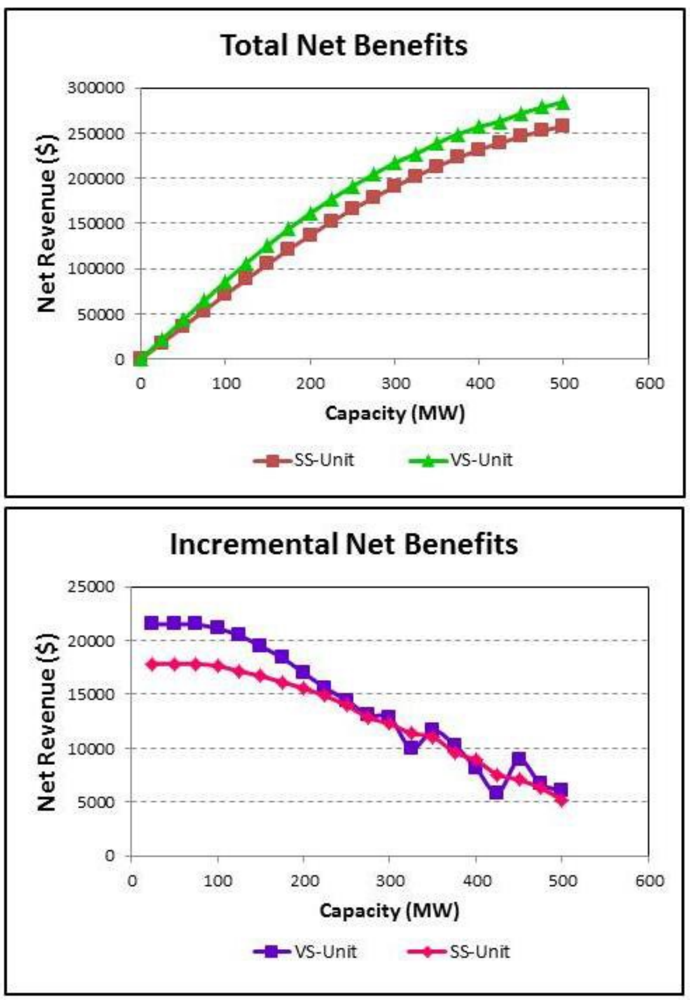

Figure 22.—Capacity benefits.

For both types of pump-generation plants, net economic revenues increase over the range examined. This range represents the sizes of a single pump-generation unit most likely to be installed at a modern pump-generation power plant. The largest unit size examined is 500 MW, which exceeds the size of any known Reclamation pump-generation unit. Potentially, larger pump-generation units could be custom manufactured. Large size individual units would appear to be most suited for use in situations where water is abundant and gross heads were relatively limited. Transportation of the components for these very large units would appear to limit their application.

The relationships between unit size and net revenues are similar for both types of pump-generation plants. For the majority of the range examined, the VS units dominate the SS units. This difference in response can be attributed to the higher generation efficiencies and smaller rough zones (greater range of generation) inherent in VS plants.

Insights

The results reported in Figure 22 confirm that total net economic value increases with the size (capacity) of the installed unit. On reflection, this result is both logical and expected. As unit size increases, the values of all other factors, including live forebay volume and gross head, remain constant at the reference conditions. There are other physical and engineering factors constrain potential operations and the relationship between unit size and revenue capture is not linear. This is most clearly shown in the figure where the incremental net value decreases as the unit capacity (size) increases.

The numerical results reported here could vary somewhat with the price data used in the analysis No specific exploration of price effects was undertaken for this study. Based on the body of results reported previously, it might be surmised the relationship between capacity (size) and revenue capture would not change markedly with price.

Example Model Application

The primary purpose of this research project is to investigate the determinants of economic value and assess their role in shaping the aggregate benefits produced by a pump-generation plant. The models developed here have broader application potential. To provide some insight into the wider utility of the pump-generation models, two example applications are demonstrated. First, the variable speed pump-generation model is applied to a pump-generation plant identified as Mead 2D in HDR-CDM Joint Venture (2014). Second, the variable speed model is employed for an analysis of the Seminoe 5C pump-generation plant described in Bureau of Reclamation (2013).

Both the Mead 2D and the Seminoe 5C plants would have multiple variable speed pump-generation units, 6 units for the former and 2 units for the latter. For modeling purposes, a single VS pump-generation unit is represented with the live forebay volume reduced proportionately. The modeled single unit results are then scaled up by the number of planned units, to represent the revenue created by the plant, during a single (typical) week.

Mead 2D

The Mead 2D plant would have 6 hydraulically independent units, each of which is approximately 291 MW in size. The total plant capacity would be 1745 MW. The gross head for this plant is expected to be 1653 feet. The planned forebay (upper) reservoir would have a live storage volume of 12000 af. The size of the forebay was based on the ability to generate at the maximum capability for 10 hours (HDR-CDM Joint Venture 2014). The column labeled "Plant Value" in Table 21 summarizes this information for the Mead 2D plant.

Table 21.—Plant and unit attributes for Mead 2D

Attribute	Plant value	1-unit value
Gross head (ft)	1653	1653
Capacity (MW)	1745	291
Number of units	6	1
Forebay volume (af)	12000.00	2000.00

This plant is readily characterized using the variable speed (VS) pump-generation model developed for this research project. The single unit model was employed for this purpose. The column labeled "1-Unit Value" illustrates the values of the parameters used in the single unit VS pump-generation model. As shown there, the forebay volume is reduced proportionately to 1998.40 af to represent the amount of water available for discharge and pumping by a single unit. The gross head is common to both the plant and the unit analysis.

The attribute values shown in column 2 of the table are for a single unit at the Mead 2D site. Using these data new SOS2 points describing a variable speed pump-generation unit of this size and head were computed. These new SOS2 points were then used in the LINGO 14 pump-generation model and a solution was obtained. For these parameters the model achieves a global optimum of $250,356.60 as reported in the column labeled "1-Unit Revenues" in Table 23. Again the results reported in Column 2 are for a single unit.

Exploiting the price taker assumption, and the findings for multiple unit plants described earlier, the results for a single 291 MW unit are scaled up to represent the 6-unit, 1745 MW variable speed pump-generation plant planned for this site (HDR-CDM Joint Venture 2014). For the 6-unit plant envisioned at this site, the net revenue in a typical week in July 2014 is estimated to be $1,502,139.60. These results are reported in Table 23 column 1.

These results are quite sensitive to the price set used in this analysis. If other prices were employed, different results would be obtained.

Seminoe 5C

The Mead 5C plant would have 2 hydraulically independent units, each of which is approximately 286 MW in size. The total plant capacity would be 572 MW. The gross head for this plant would be 909 feet. The forebay (upper) would have a live storage volume of 7145 af. The forebay was sized to allow for generation at the maximum plant capability for 10 hours (Reclamation 2013). The column labeled “Plant Value” in Table 22 summarizes this information for the Seminoe 5C plant.

Table 22.—Plant and unit attributes for Seminoe 5C

Attribute	Plant value	1-unit value
Gross head (ft)	909	909
Unit capacity (MW)	572	286
Number of units	2	1
Forebay volume (af)	7145.00	3572.40

This plant is readily characterized using the variable speed (VS) pump-generation model developed for this research project. The single unit VS model was employed for this purpose. The column labeled “1-Unit Value” illustrates the values of the parameters used in the Seminoe 5C single unit VS pump-generation model. As shown there, the forebay volume is reduced proportionately to 3,572.40 acre-feet to represent the amount of water available for discharge and pumping by a single unit. The gross head (909 feet) is common to both the plant and the unit analysis.

Using the attribute values for a single VS unit at the Seminoe 5C plant, new SOS2 points describing the generator and the variable speed pump of this size and head were calculated. These were then used in the LINGO 14 model. The typical week (168 hour) model of July 2014 for a 286 MW unit with a head of 909 feet and the live forebay volume shown in the table, achieves a global optimum of $246,054.90 in just over 1 minute. These results are for one single VS unit at this site.

Exploiting the price taker assumption and the results we obtained for multiple units, both of which were described earlier, these 1-unit results may be scaled up to represent the 2-unit, 572 MW variable speed pump-generation plant planned for this site (HDR-CDM Joint Venture 2014). For the 2-unit 572 MW plant envisioned at this site, the net revenue in a typical week in July 2014 is estimated to be $492,109.80. These results are shown in Table 23.

Table 23.—Unit and plant level net revenues

Plant name	1-unit revenues	Number of units	Plant revenues
Mead 2D	250356.60	6	1502139.60
Seminoe 5C	246054.90	2	492109.80

As explained elsewhere, these results are sensitive to the price set used in this analysis. If other prices were employed, different results would be obtained.

Extensions and Improvements

The plant level results reported in these two example applications are estimated for a single week. Further insights could be obtained from an analysis which employed all 52 weeks in the year for a multi-year time horizon. Obtaining, validating and employing a price set of this size would be a colossal undertaking. While potentially instructive, such an effort will be deferred to a later date when sufficient resources are made available.

Price Sensitivity

The results reported here and the conclusions described are based on July 2014 typical week energy and ancillary service prices. These price sets shape the simulated hourly behavior of the pump-generation units. The available evidence suggests the modeled hourly behavior is quite sensitive to the price set employed for analysis.

AS and energy prices are variable on an hourly, weekly, monthly and annual time-scale, as is the relative difference between them. A limited sensitivity analysis of selected results to price was undertaken and the results are reported here. Figure 23 compares the net and gross revenues for a typical Wednesday in July during 2011 and in 2014. These results were obtained using the 24-hour variable speed pump-generation model.

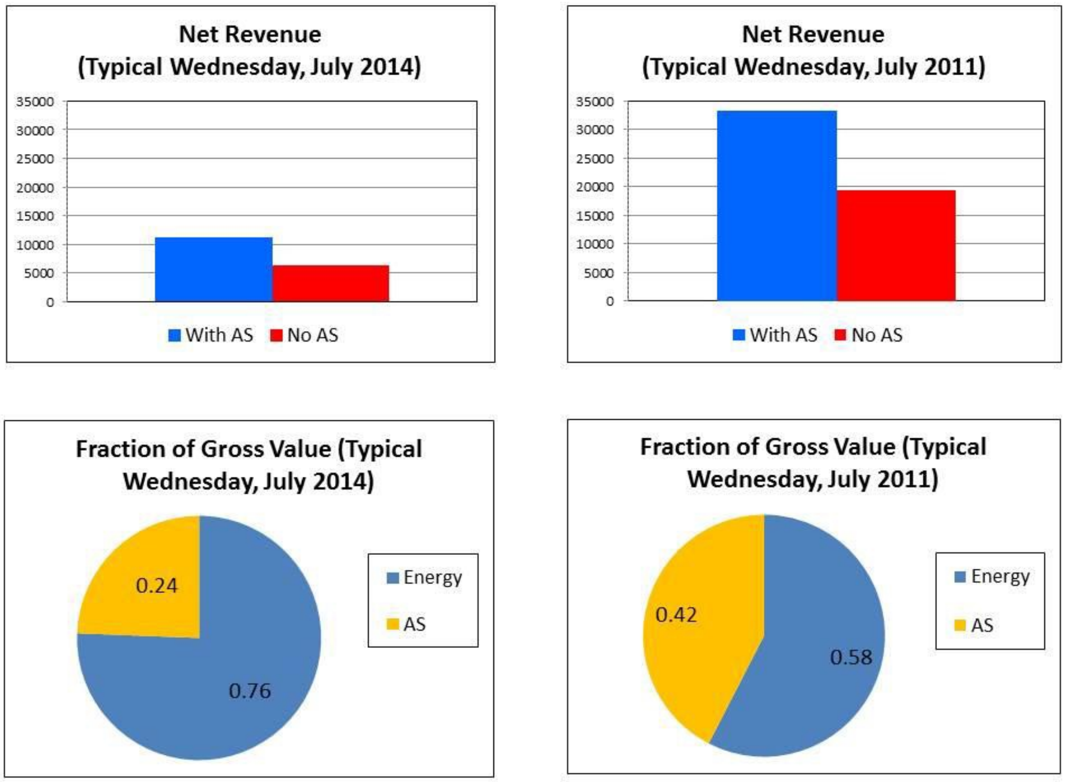

Figure 23.—Price sensitivity analysis.

Compared to July 2014, the July 2011 ancillary service prices are higher and energy prices are lower. As shown in this figure, the modeled net revenue (gross revenue minus pumping costs) is much higher when the 2011 price set is employed. When compared to a typical Wednesday in July 2014, the 2011 gross revenues are nearly 300 percent higher. There is also a marked difference between revenues derived from ancillary service provision and those from energy sales. In 2011, approximately 42 percent of the revenues are from AS provision while in 2014 approximately 24 percent of the revenues come from AS sales.

The price data for July 2014 were used throughout this analysis. These are the most recent AS and energy price data available at the time of this research. As this example demonstrates, the specific price set used in the analysis can have an important influence on the results. Appendix 11 summarizes energy and ancillary service price trends over the last 5 years.

Known Weaknesses

To facilitate feasible mathematical solutions, all of the models described in this document are formulated as linear piece-wise approximations to the underlying nonlinear relationships. The reformulated optimization problem is known as a mixed integer linear programming (MILP) problem. There is necessarily some divergence between the actual (nonlinear) relationships and the linear ones. These MILP models are solved using an approach known as the special ordered sets of type 2 (SOS2) approach. The SOS2 approach allows for a rapid and reliable solution to this class of problem. To accommodate the use of the SOS2 approach, the gross head and efficiency are fixed during all hours of a given simulation. Typically, the head and efficiency varies during generation and pumping operations. Efforts to incorporate variable heads and efficiencies into a solvable model seem warranted. The modeling framework developed for this research effort employs an hourly time-step. The provision of ancillary services, generation and the demand for both is highly variable not only hourly; but on a minute-to-minute and second-to second (or less) time-scale. Conceptually at least, smaller time-steps could be employed if the supporting data (price and other) were more widely available. In the models described here, within hour mode changes from generation to pumping, or vice versa, are not allowed to occur. This limitation is consistent with the hourly time-step employed for modeling purposes. In reality however, some single speed units and many variable speed units are be capable of within-hour mode changes. The assumed probabilities of ancillary service provision used in this analysis are critically important to the reported results. The values are plausible, but not informed by data or experience. They represent a significant source of uncertainty in this analysis.

Conclusions

This research effort explores the incremental economic value derived from energy generation and the provision of four ancillary services; up-regulation (UR), down-regulation (DR), spinning reserve (SR) and non-spinning reserve (NR). It explicitly compares the economic value produced by variable speed (SS) and single speed (SS) pump generator units. Finally, the independent economic value of selected site characteristics such as head, forebay (upper) reservoir storage, and unit size are examined.

Detailed mathematical optimization models of single speed (SS) and variable speed (VS) pump-generation units were crafted to represent Concept 5 (Harpman, Kubitschek and Wittler 2014) open-loop pump-generation plants sited adjacent to existing Bureau of Reclamation facilities. These models simulate the production and sale of ancillary services as well as energy. A set of reference operating conditions forms the basis for all of the comparisons reported. These reference conditions featured a single 100 MW unit, 600 feet of gross head, a (live) forebay (upper) reservoir storage volume of 8-hours and employ July 2014 typical week prices.

A large number of model runs and analyses were undertaken. At the reference conditions, results suggest VS units may produce approximately 20% greater net revenues (gross revenues minus pumping costs) relative to SS units. Relative to the "without" ancillary service case, the net economic benefits produced by SS and VS plants "with" ancillary service provision are 63% and 56% greater respectively. Even when ancillary services are provided, energy arbitrage is the predominant source of revenue for both SS and VS plant designs.

The investigation of forebay live storage volume reveals 99% of the net economic storage benefits are captured in a range of 6.31 to 7.23 hours of storage. This level of forebay reservoir storage is considerably less than commonly used rules of thumb would suggest are necessary. Modeled results suggest plants producing both ancillary services and energy require smaller forebay reservoirs than energy arbitrage only plants.

Analysis at the reference conditions indicates the predominant fraction of the net economic benefits can be captured at levels of gross head in the neighborhood of 500 feet. This level of head is considerably less than some observers have opined. Topographic conditions in this head range can be found adjacent to many existing Bureau of Reclamation reservoirs.

This research effort provides previously unavailable evidence on the relative importance of selected attributes, to the value of pump generation plants. These attributes are common to all pump generation plants. The findings reported here will help inform future site selection and design decisions.

Future Directions

Completion of this research project has required an extensive investment of time and resources. A number of technological challenges have been overcome. Substantive progress has been made towards the development of detailed and useful models of variable speed (VS) and single speed (SS) pump-generation models.

During this intensive process, some potential improvements in this analysis were identified which might be considered at a future date. These include the following:

- Incorporation of a unit startup cost and/or a mode change cost
- Characterization of changes in head during the simulation period
- Use of variable efficiencies during the simulation period
- Use of smaller time-step for modelling
- Investigation of probabilities for up and down-regulation provision
- Investigation of probabilities for spinning and non-spinning reserve
- Relaxation of the price taker assumption
- Estimation of the demand for energy and ancillary services

Collaborators

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APPENDIX 1 – GENERATION, HEAD AND RELEASE

For the single speed (SS) and variable speed (VS) pump-generation unit in generation mode, the relationship between the amount of electricity which can be generated, the head, efficiency and amount of water released is illustrated in equation (A1-1). This equation is known as the universal power equation.

(A1-1) $$p_t = q_t \times \left(\frac{geff_t \times \omega \times ghead_t}{ftptohp} \right) \left(\frac{hptokw}{kwtomw} \right)$$

Where:

p_t	=	real power generation (MW) at time (t)
q_t	=	total volumetric release rate (cfs) at time (t)
ω	=	62.40, weight of water at 50 degrees Fahrenheit (lb/ft^3)
$geff_t$	=	efficiency factor at time (t) (dimensionless)
$ghead_t$	=	gross generation head (ft) at time (t)
ftptohp	=	550, ft-lb/sec to horsepower conversion factor
hptokw	=	0.746, horsepower to kilowatts conversion factor
kwtomw	=	1000, kilowatts to megawatts conversion factor

The generation head and release at time (t) influences the efficiency at time (t). Higher heads increase the amount of electric power which can be generated at any given level of water release.

In generation mode, the efficiency for the pump-generation plant at time (t) is a function of the release and the head at that time. Typically, the generation efficiency declines as the amount of head falls and this effect varies nonlinearly with release. Appendix 2 illustrates one plausible relationship between release and head.

In generation mode, the gross generation head at time (t) is calculated using the simple relationship shown in equation (A1-2).

(A1-2) $$ghead_t = fbelev_t - abelev_t$$

Where:

$ghead_t$	=	gross generation head at time (t)
$fbelev_t$	=	upper (forebay) reservoir elevation at time (t).
$abelev_t$	=	lower (afterbay) reservoir elevation at time (t)

APPENDIX 2 – GENERATION EFFICIENCY

The generation efficiency parameter (*geff*) for the universal power equation described previously in Appendix 1, determines the rate at which falling water is converted into electrical energy. Efficiency is typically measured as a decimal fraction or a percent. The relationship between release, head and generation contained in Appendix 1 utilizes a static value for the efficiency (*geff*) which is constant for all values of head and release. In general however, the efficiency of a Francis turbine varies depending on the head and the release rate and this relationship is unique to the design of each turbine runner and the site where it is installed.

In response to previous review comments, this appendix describes the more general relationship between efficiency, release and head. The generic mathematical relationships developed in this appendix are purposely specified in terms of percent of maximum release and percent of maximum head, to allow for the ease of application in this and future research efforts.

To accommodate generic and nonspecific use, the relationship between total release, head and efficiency becomes slightly more complex, but remains reasonably tractable. A plausible relationship between release and head is the quadratic function shown in equation (A2-1).

(A2-1)
$$E = \frac{-(q - bestQ)^2}{head} + bestE$$

Where:

E = efficiency (dimensionless)
q = total release (cfs)
$bestQ$ = the release yielding the highest value of E
$head$ = gross head (feet)
$bestE$ = the highest value of E which can be attained

The values for bestQ and bestE for a particular research application must be calculated. The maximum value of E which can be obtained at a given head is computed using expression (A2-2).

(A2-2)
$$bestE = \left(A^* \frac{head}{maxhead} + A \right) * 100$$

Where:

bestE	=	the maximum efficiency (dimensionless)
A	=	scalar parameter ($0<A\leq1$).
head	=	gross head (feet)
maxhead	=	the maximum normal gross head at this site

The value of Q which produces the maximum efficiency, for a given head is described by equation (A2-3).

(A2-3) $$bestQ = bestE + B*(MaxE - bestE)$$

Where:

bestQ	=	the release which produces the maximum value of E (cfs)
bestE	=	the maximum efficiency (dimensionless)
B	=	scalar parameter ($0<B\leq1$)
maxE	=	the value of bestE obtained by evaluating equation (26) with the head set equal to the maximum head dimensionless)

For purposes of this exposition, the values of the parameters used are illustrated in Table 24 shown below. Naturally, these parameter values will vary, depending on the details of the specific research application being examined.

Table 24.—Efficiency parameter values

Parameter	Value
Maxhead	400.00
A	0.450
B	0.400

Using the parameter values shown in Table 24 in expressions A2-1 and A2-2, the relationship between release, head and efficiency, described by equation (A2-3) can be plotted for three different levels of gross head as shown in Figure 24.

As illustrated in this figure, the expression for efficiency as a function of release and head (A2-3) provides a very reasonable representation of the relationships between these variables. For instance, at a gross head of 400 feet, the maximum efficiency is 90 percent at a gate opening of 90 percent. At a lower head of 300 feet, the maximum efficiency is 78.75 percent at a gate opening of 83 percent. This relationship closely tracks and is similar to the observed efficiency characteristics at many hydropower facilities where Francis turbines are employed.

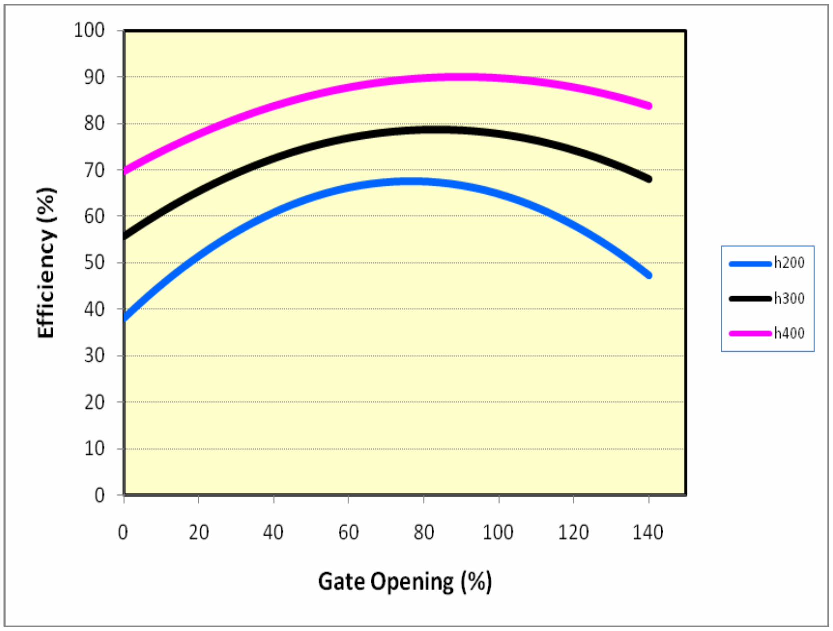


Figure 24.—Example efficiency functions.

APPENDIX 3 – PUMPING, HEAD AND POWER INPUT

For the single speed (SS) and variable speed (VS) pump-generation unit in pump mode, the relationship between the amount of water which can be pumped, the electrical power required for pumping, the pumping efficiency and pumping head is illustrated in equation (A3-1). This is known as the (universal) pumping equation.

(A3-1) $$q_t = p_t * \left(\frac{kwtomw}{hptokw} \right) \left(\frac{peff_t * ftptohp}{\omega * phead_t} \right)$$

Where:

q_t	=	total volumetric pumping rate (cfs) at time (t)
p_t	=	real power input for pumping (MW) at time (t)
ω	=	62.40, specific weight of water at 50 degrees Fahrenheit (lb/ft^3)
$peff_t$	=	pumping efficiency factor at time (t) (dimensionless)
$phead_t$	=	gross pumping head (ft) at time (t)
ftptohp	=	550, ft-lb/sec to horsepower conversion factor
hptokw	=	0.746, horsepower to kilowatts conversion factor
kwtomw	=	1000, kilowatts to megawatts conversion factor

For a single speed unit, the electrical power input level required for pumping is fixed. Typically, the single input level is 100% of capacity. For an adjustable or variable speed unit, the electrical power input can be varied from about 70-percent of capacity to about 100-percent. The head at time (t) influences the efficiency. Higher heads reduce the amount of water which can be pumped at the real power pumping requirement level.

In general, pumping efficiency varies with the pumping head. For purposes of this research effort, pumping efficiency is held constant.

In pump mode, the pumping head at time (t) is calculated using the simple relationship shown in equation (A3-2).

(A3-2) $$phead_t = fbelev_t - abelev_t$$

Where:

$phead_t$	=	gross pumping head at time (t)
$fbelev_t$	=	upper (forebay) reservoir elevation at time (t).
$abelev_t$	=	lower (afterbay) reservoir elevation at time (t)

APPENDIX 4 – LINEAR AND NON-LINEAR GENERATION

For the single speed (SS) and variable speed (VS) pump-generation unit in generation mode, the real power (MW) which can be produced is a nonlinear function of release, efficiency and head. As described earlier and detailed in Appendix 2, typically the efficiency differs with head and release. The gross generation head is the difference between the forebay (upper) reservoir elevation and the afterbay (lower) reservoir elevation. We assume the afterbay is quite large, relative to the forebay and there is little to no effect on its elevation resulting from generation releases. The elevation of the forebay reservoir is determined by antecedent events (previous pumping, release and operations).

Figure 25 illustrates the nonlinear nature of both the variable speed and single speed generation functions, plotted in 2-dimensions, percent of maximum release (X-axis) and generation (Y-axis). The specific generation relationship used is reported in Appendix 1 with the variable generation efficiency as described in Appendix 2.

As shown in this figure, the generation functions are indeed nonlinear but are not highly nonlinear. At the scale of this plot, there is no discernable difference between the generation functions for the VS unit and the SS units.

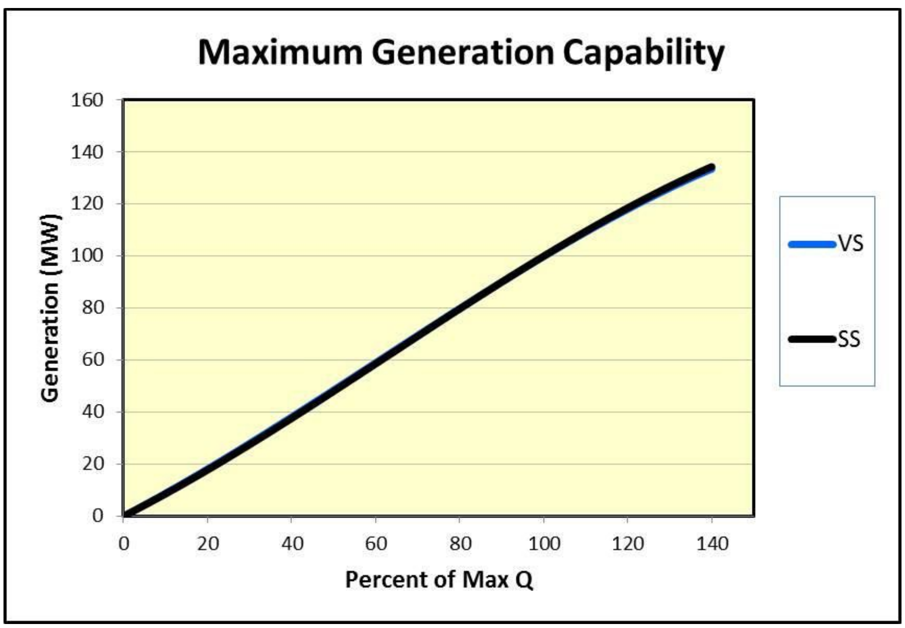


Figure 25.—Nonlinear VS and SS generation functions.

The values of the parameters underlying this plot are illustrated in Table 25. It is presumed operations of the pump generation unit have a negligible effect on gross head and the generation head is held fixed at 600 feet.

Table 25.—Parameters employed in plotting

	Variable speed	Single speed
Capacity (MW)	100	100
Head (ft)	600	600
Generation equation parameters		
gamma	62.40	62.40
fptohp	550.0	550.0
hptokw	0.7460	0.7460
kwtomw	1000.0	1000.0
Efficiency equation parameters		
maxh	850.00	900.00
coeff_A	0.52	0.50
coeff_B	0.10	0.70

To reiterate, the universal power equation is described in some detail in Appendix 1. When the gross generation head and the efficiency are both fixed, the universal power equation becomes linear. Figure 26 compares the linear and nonlinear forms of the single speed (SS) generation function on the same scale. In this figure the efficiency is held fixed at a value of 0.83289. As shown in Figure 26 there is little appreciable difference between the nonlinear generation function, in which there is a variable efficiency, and the linear generation function with a fixed efficiency.

Visually and numerically there is a very close correspondence between the linear and nonlinear generation functions. At 100-percent of the maximum release (Q), the divergence between the two is greatest; 4 MW or approximately 6 percent. The difference between the two functions is smaller over the remaining range.

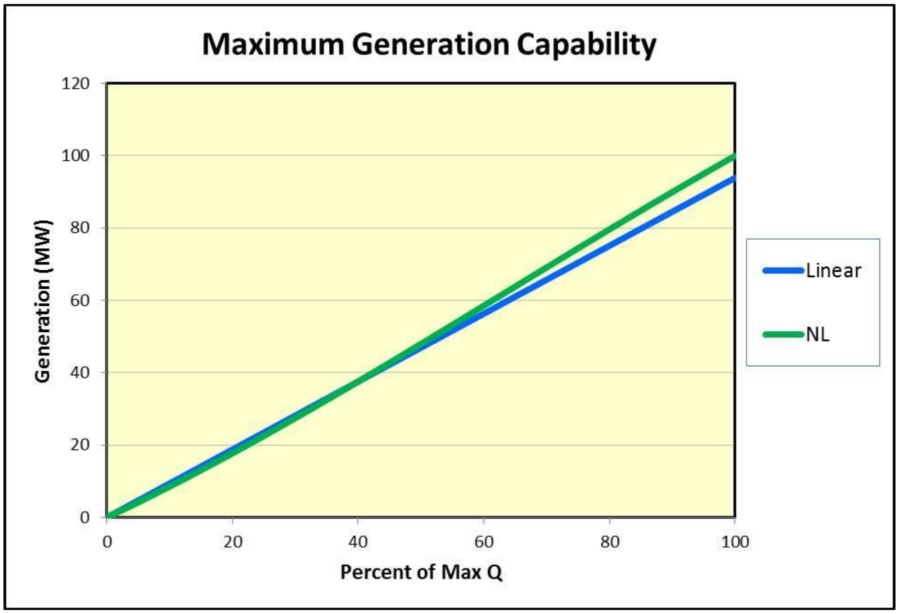


Figure 26.—Linear and nonlinear generation functions.

The SOS2 piece-wise linear approximation approach described earlier could readily be employed on either the nonlinear or the linear version of the generation function. Application of the SOS2 approach to the nonlinear generation function would have some associated approximation error. Although the amount of this error has not been investigated, professional judgement suggests the trade-off in accuracy achieved using the nonlinear approach approximated with SOS2 versus simply using the linear generation function with SOS2, is minimal. Since use of the nonlinear function with SOS2 would unambiguously require more researcher effort, the linear generation function is employed throughout this research project.

APPENDIX 5 – SS UNIT MATHEMATICAL SPECIFICATION

In general, optimal dynamic dispatch of a variable speed (SS) unit able to produce energy and provide up-regulation, down-regulation, spinning reserve and non-spinning reserve in in generation mode and pump mode can and use energy and produce spinning and non-spinning reserves be written in mathematical notation as shown in equations (A5-1) through (A5-21).

(A5-1)

$$Maximize \quad \sum_{t=1}^{T} \begin{bmatrix} act\_genMW_t * eprice_t + urMW_t * urprice_t + drMW_t * drprice_t \\ + srMW_t * srprice_t + nrMW_t * nrprice_t \end{bmatrix} + \sum_{t=1}^{T} \begin{bmatrix} -act\_pumpMW_t * eprice_t \\ + psrMW_t * srprice_t + pnrMW_t * nrprice_t \end{bmatrix}$$

subject to:

(A5-2)

$$set\_genMW_t = act\_genMW_t - urMW_t * urcoef + drMW_t * drcoef - srMW_t * srprob - nrMW_t * nrprob$$

(A5-3)

$$set\_pumpMW_t = act\_pumpMW_t - pdrMW_t * drcoef + purMW_t * urcoef$$

(A5-4)

$$genind_t + pumpind_t \le 1$$

(A5-5)

$$maxurMW_t = \begin{pmatrix} if \ ULrz \le set\_genMW_t, & \text{maxgenMW} - set\_genMW_t \\ if \ set\_genMW_t \le LLrz, & LLrz - set\_genMW_t \end{pmatrix}$$

(A5-6)

$$maxdrMW_t = \begin{pmatrix} if \ ULrz \le set\_genMW_t, & set\_genMW_t - ULrz \\ if \ set\_genMW_t \le LLrz, & set\_genMW_t - \text{mingenMW} \end{pmatrix}$$

(A5-7)

$$urMW_t \le maxurMW_t$$

(A5-8)

$$drMW_t \le maxdrMW_t$$

(A5-9)

$$set\_genMW_t \le \text{maxgenMW}$$

(A5-10) $$set\_genMW_t + urMW_t + srMW_t + nrMW_t \leq \text{maxgenMW}$$

(A5-11) $$set\_genMW_t - drMW_t \geq \text{mingenMW} * genind_t$$

(A5-12) $$set\_pumpMW_t \leq \text{maxpumpMW}$$

(A5-13) $$set\_pumpMW_t - psrMW_t - pnrMW_t \geq 0$$

(A5-14) $$set\_pumpMW_t \geq \text{minpumpMW} * pumpind_t$$

(A5-15) $$act\_genQ_t = gw(act\_genMW_t)$$

(A5-16) $$act\_pumpQ_t = pw(act\_pumpMW_t)$$

(A5-17 $$FBvol_t = FBvol_{t-1} + (act\_pumpQ_t - act\_genQ_t) * cfs\_to\_af$$

(A5-18) $$\text{FBvolmin} \leq FBvol_t \leq \text{FBvolmax}$$

(A5-19) $$Totgenvol = \sum_{t=1}^{T} (act\_genQ_t * cfs\_to\_af)$$

(A5-20) $$Totpumpvol = \sum_{t=1}^{T} (act\_pumpQ_t * cfs\_to\_af)$$

(A5-21) $$Totgenvol \leq Totpumpvol$$

Where:

T	=	terminal time period
t	=	time period index
eprice	=	energy price ($/MWh) at time (t)
urprice	=	up-regulation price ($/MWh) at time (t)
drprice	=	down-regulation price ($/MWh) at time (t)
srprice	=	spinning reserve price ($/MWh) at time (t)
nrprice	=	non-spinning reserve price ($/MWh) at time (t)
act_genMW	=	actual generation (MW) at time (t)
urMW	=	up-regulation (MW) at time (t)
drMW	=	down-regulation (MW) at time (t)
srMW	=	spinning reserve (MW) at time (t)
nrMW	=	non-spinning reserve (MW) at time (t)
act_pumpMW	=	actual pumping input (MW) at time (t)
psrMW	=	spinning reserve (MW) from pump at time (t)

pnrMW	=	non-spinning reserve (MW) from pump at time (t)
set_genMW	=	generator set point (MW) at time (t)
set_pumpMW	=	pump set point (MW) at time (t)
urcoef	=	up-regulation energy coefficient ($0 \leq x \leq 1.0$).
drcoef	=	down-regulation energy coefficient ($0 \leq x \leq 1.0$)
srprob	=	spinning reserve call probability ($0 \leq x \leq 1.0$)
nrprob	=	nonspinning reserve call probability ($0 \leq x \leq 1.0$)
genind	=	generator mode status indicator at time (t) (1=on, 0=off)
pumpind	=	pump mode status indicator at time (t) (1=on, 0=off)
maxurMW	=	maximum up-regulation at time (t)
maxdrMW	=	maximum down-regulation (MW) at time (t)
LLrz	=	lower limit of rough zone (MW)
ULrz	=	upper limit of rough zone (MW)
maxgenMW	=	maximum generation capability (MW)
mingenMW	=	minimum generation level (MW)
maxpumpMW	=	maximum pump input power level (MW)
minpumpMW	=	minimum pump input power level (MW)
act_genQ	=	actual amount of water released (cfs) at time (t)
act_pumpQ	=	actual amount of water pumped (cfs) at time (t)
cfs_to_af	=	cubic feet per second to acre-foot conversion factor.
pw(.)	=	pump power input to water pumped equation (cfs)
gw(.)	=	generation produced to water released equation (cfs).
FBvol	=	forebay live storage volume (af) at time (t).
Totgenvol	=	total volume of water used for generation (af)
Totpumpvol	=	total volume of water pumped (af)

Given a set of prices, the objective the owner/operator is to maximize the total net revenue which can be obtained from operating the SS pump-generation unit, subject to physical and logical constraints on operations. For a SS unit, revenues are obtained by producing energy, up-regulation, down-regulation, spinning reserves and non-spinning reserves in generation mode and using energy and producing spinning and non-spinning reserves in pump mode. Costs must be incurred to pump water back into the forebay (upper) reservoir from the afterbay (lower) reservoir to replace the water released. The starting and ending forebay live storage volumes must be the same.

The objective function shown in equation (A5-1) is split into two parts for convenient explanation and to allow for easy comparison with the VS unit. The first block represents the contributions made in generation mode and the second block specifies those made while in pumping mode.

The first constraint equation (A5-2) defines the generation set point and illustrates its relationship to energy generation and the provision of reserves and regulation. When ancillary services are provided in a particular hour, they add to or subtract from the actual amount of energy generated. For each unit of ancillary services produced, the magnitude of the effect on generation (and hence release) depends on the value of the associated multiplier (coefficient or call probability). This causes a deviation between the generator set point and the actual generation.

Constraint equation (A5-3) defines the pumping set point and illustrates its relationship to pumping energy input as well as the provision of reserves and regulation. When ancillary services are provided in pump mode, they add to or subtract from the actual amount of energy being used by the pump. This causes a deviation between the pump set point and the actual amount of pumping which takes place.

A pump-generation unit has three states. It can be off, it could be operating in pump mode or it could be operating in generation mode. Constraint equation (A5-4) employs binary indicator variables for each mode to ensure the unit is in one, and only one, of these three states at any point in time.

In generation mode, up-regulation services must be constrained so the generator does not continuously cross and re-cross into the rough zone during their provision. Constraint equations (A5-5) and (A5-7) operationalize this logic. The maximum amount of up-regulation which can be provided in generation mode is limited by the location of the generation set point at time (t) relative to the rough zone, the minimum generation level and the maximum generation level. If the set point is below the rough zone, the lower limit of the rough zone creates a ceiling on the maximum amount of up-regulation which can be provided. If the set point is above the rough zone, the maximum generation level creates a ceiling on the amount of up-regulation which can be provided.

In generation mode, down-regulation services must be constrained so the generator does not continuously cross and re-cross into the rough zone during their provision. Constraint equation (A5-6) and (A5-8) operationalize this logic. The maximum amount of down-regulation which can be provided in generation mode is limited by the location of the generation set point at time (t) relative to the rough zone, the minimum generation level and the maximum generation level. If the set point is below the rough zone, the minimum generation level creates a ceiling on the maximum amount of down-regulation which can be provided. If the set point is above the rough zone, the upper rough zone limit creates a ceiling on the amount of down-regulation which can be provided.

The generation set point must be at or below the maximum generation level at all times as shown in equation A5-9.

When in generation mode, the sum of set point generation, up-regulation, spinning reserves and non-spinning reserves must be less than or equal to the maximum generator capability. This is shown in equation A5-10.

Similarly, equation A5-11 ensures that when the unit is “on” and is generating, the generation set point minus the amount of down–regulation provided must equal or exceed the minimum generation level, When the unit is “off”, these quantities must be equal to 0.0.

As required by constraint equation (A5-13) the pumping set point minus the pumping spinning and non-spinning reserves must be greater than or equal to 0.0.

When the unit is in pump mode and is pumping (“On”), constraint equation (A5-14) ensures the pumping set point must equal or exceed the minimum pumping level When the unit is not in pump mode (it is “off”) and these quantities must be equal to 0.0.

Equation (A5-12) ensures the pumping set point must be less than or equal to the maximum pumping capability.

Equation (A5-15) calculates the amount of water which must be released at time (t) to produce the amount of actual generation at time (t).

Equation (A5-16) computes the amount of water which is pumped at time (t) given the actual amount of pumping input power applied at time (t).

The volume of water in the forebay (upper) reservoir at time (t) depends on what the forebay volume was in the previous time period (t-1) plus the actual volume of water pumped into the forebay at time (t), minus the actual amount of water released for generation at time (t). This relationship is required by constraint equation (A5-17).

The amount of water in the forebay reservoir at any point in time must remain within the pre-set minimum and maximum forebay levels. Equation (A5-18) ensures the forebay reservoir will not overflow or be drawn down below allowable limits.

The total amount of water released for generation is required by equation (A5-19) to be equal to the sum of the water used for generation across the analysis period.

The total amount of water used for pumping is exactly equal to the sum of all of the pumping during the analysis period. This constraint is shown in equation (A5-20).

The total amount of water released for generation during the analysis period be less than or equal to the total amount of water pumped. This constraint (A5-21) ensures mass balance and it makes sure the forebay volume at the end of the analysis will be equal to the starting forebay volume.

APPENDIX 6 – VS UNIT MATHEMATICAL SPECIFICATION

In general, optimal dynamic dispatch of a variable speed (VS) unit able to produce and use energy and provide up-regulation, down-regulation, spinning reserve and non-spinning reserve in both generation and pump mode can be written in mathematical notation as shown in equations (A6-1) through (A6-22).

(A6-1)

$$Maximize \quad \sum_{t=1}^{T}\begin{bmatrix} act\_genMW_t * eprice_t + urMW_t * urprice_t + drMW_t * drprice_t \\ + srMW_t * srprice_t + nrMW_t * nrprice_t \end{bmatrix} + \sum_{t=1}^{T}\begin{bmatrix} - act\_pumpMW_t * eprice_t + purMW_t * urprice_t + pdrMW_t * drprice_t \\ + psrMW_t * srprice_t + pnrMW_t * nrprice_t \end{bmatrix}$$

subject to:

(A6-2)

$$set\_genMW_t = act\_genMW_t - urMW_t * urcoef + drMW_t * drcoef - srMW_t * srprob - nrMW_t * nrprob$$

(A6-3)

$$set\_pumpMW_t = act\_pumpMW_t + psrMW_t * srprob + pnrMW_t * nrprob - pdrMW_t * drcoef + purMW_t * urcoef$$

(A6-4) $$genind_t + pumpind_t \le 1$$

(A6-5) $$maxurMW_t = \begin{pmatrix} if \;\; ULrz \le set\_genMW_t, & \mathrm{maxgenMW} - set\_genMW_t \\ if \;\; set\_genMW_t \le LLrz, & LLrz - set\_genMW_t \end{pmatrix}$$

(A6-6) $$maxdrMW_t = \begin{pmatrix} if \;\; ULrz \le set\_genMW_t, & set\_genMW_t - ULrz \\ if \;\; set\_genMW_t \le LLrz, & set\_genMW_t - \mathrm{mingenMW} \end{pmatrix}$$

(A6-7) $$urMW_t \le maxurMW_t$$

(A6-8) $$drMW_t \le maxdrMW_t$$

(A6-9) $$set\_genMW_t \le \mathrm{maxgenMW}$$

(A6-10) $$set\_genMW_t + urMW_t + srMW_t + nrMW_t \leq \mathrm{maxgenMW}$$

(A6-11) $$set\_genMW_t - drMW_t \geq \mathrm{mingenMW} * genind_t$$

(A6-12) $$set\_pumpMW_t \leq \mathrm{maxpumpMW}$$

(A6-13) $$set\_pumpMW_t - psrMW_t - pnrMW_t - purMW_t \geq 0$$

(A6-14) $$set\_pumpMW_t - purMW_t \geq \mathrm{minpumpMW} * pumpind_t$$

(A6-15) $$set\_pumpMW_t + pdrMW_t \leq \mathrm{maxpumpMW}$$

(A6-16) $$act\_genQ_t = gw(act\_genMW_t)$$

(A6-17) $$act\_pumpQ_t = pw(act\_pumpMW_t)$$

(A6-18) $$FBvol_t = FBvol_{t-1} + (act\_pumpQ - act\_genQ_t) * cfs\_to\_af$$

(A6-19) $$\mathrm{FBvolmin} \leq FBvol_t \leq \mathrm{FBvolmax}$$

(A6-20) $$Totgenvol = \sum_{t=1}^{T} (act\_genQ_t * cfs\_to\_af)$$

(A6-20) $$Totpumpvol = \sum_{t=1}^{T} (act\_pumpQ_t * cfs\_to\_af)$$

(A6-21) $$Totgenvol \leq Totpumpvol$$

Where:

T	=	terminal time period
t	=	time period index
eprice	=	energy price ($/MWh) at time (t)
urprice	=	up-regulation price ($/MWh) at time (t)
drprice	=	down-regulation price ($/MWh) at time (t)
srprice	=	spinning reserve price ($/MWh) at time (t)
nrprice	=	non-spinning reserve price ($/MWh) at time (t)
act_genMW	=	actual generation (MW) at time (t)
urMW	=	up-regulation (MW) at time (t)
drMW	=	down-regulation (MW) at time (t)
srMW	=	spinning reserve (MW) at time (t)
nrMW	=	non-spinning reserve (MW) at time (t)

act_pumpMW	=	actual pumping input (MW) at time (t)
purMW	=	up-regulation (MW) from pump at time (t)
pdrMW	=	down-regulation (MW) from pump at time (t)
psrMW	=	spinning reserve (MW) from pump at time (t)
pnrMW	=	non-spinning reserve (MW) from pump at time (t)
set_genMW	=	generator set point (MW) at time (t)
set_pumpMW	=	pump set point (MW) at time (t)
urcoef	=	up-regulation energy coefficient ($0 \le x \le 1.0$).
drcoef	=	down-regulation energy coefficient ($0 \le x \le 1.0$)
srprob	=	spinning reserve call probability ($0 \le x \le 1.0$)
nrprob	=	nonspinning reserve call probability ($0 \le x \le 1.0$)
genind	=	generator mode status indicator at time (t) (1=on, 0=off)
pumpind	=	pump mode status indicator at time (t) (1=on, 0=off)
maxurMW	=	maximum up-regulation at time (t)
maxdrMW	=	maximum down-regulation (MW) at time (t)
LLrz	=	lower limit of rough zone (MW)
ULrz	=	upper limit of rough zone (MW)
maxgenMW	=	maximum generation capability (MW)
mingenMW	=	minimum generation level (MW)
maxpumpMW	=	maximum pump input power level (MW)
minpumpMW	=	minimum pump input power level (MW)
act_genQ	=	actual amount of water released (cfs) at time (t)
act_pumpQ	=	actual amount of water pumped (cfs) at time (t)
cfs_to_af	=	cubic feet per second to acre-foot conversion factor
pw(.)	=	pump power input to water pumped equation (cfs)
gw(.)	=	generation produced to water released equation (cfs)
FBvol	=	forebay live storage volume (af) at time (t)
Totgenvol	=	total volume of water used for generation (af)
Totpumpvol	=	total volume of water pumped (af)

Given a set of prices, the objective the owner/operator is to maximize the total net revenue which can be obtained from operating the VS pump-generation unit, subject to physical and logical constraints on operations. For a VS unit, revenues are obtained by producing energy, up-regulation, down-regulation, spinning reserves and non-spinning reserves in both generation and pump mode. Costs must be incurred to pump water back into the forebay (upper) reservoir from the afterbay (lower) reservoir to replace the water released. The starting and ending forebay live storage volumes must be the same.

The objective function shown in equation (A6-1) is split into two parts for convenient explanation and to allow for easy comparison with the SS unit. The first block represents the contributions made in generation mode and the second block specifies those made while in pumping mode.

The first constraint equation (A6-2) defines the generation set point and illustrates its relationship to energy generation and the provision of reserves and regulation.

When ancillary services are provided in a particular hour, they add to or subtract from the actual amount of energy generated. For each unit of ancillary services produced, the magnitude of the effect on generation (and hence release) depends on the value of the associated multiplier (coefficient or call probability). This causes a deviation between the generator set point and the actual generation.

Constraint equation (A6-3) defines the pumping set point and illustrates its relationship to pumping energy input as well as the provision of reserves and regulation. When ancillary services are provided in pump mode, they add to or subtract from the actual amount of energy being used by the pump. This causes a deviation between the pump set point and the actual amount of pumping which takes place.

A pump-generation unit has three states. It can be off, it could be operating in pump mode or it could be operating in generation mode. Constraint equation (A6-4) employs binary indicator variables for each mode to ensure the unit is in one, and only one, of these three states at any point in time.

In generation mode, up-regulation services must be constrained so the generator does not continuously cross and re-cross into the rough zone during their provision. Constraint equations (A6-5) and (A6-7) operationalize this logic. The maximum amount of up-regulation which can be provided in generation mode is limited by the location of the generation set point at time (t) relative to the rough zone, the minimum generation level and the maximum generation level. If the set point is below the rough zone, the lower limit of the rough zone creates a ceiling on the maximum amount of up-regulation which can be provided. If the set point is above the rough zone, the maximum generation level creates a ceiling on the amount of up-regulation which can be provided.

In generation mode, down-regulation services must be constrained so the generator does not continuously cross and re-cross into the rough zone during their provision. Constraint equation (A6-6) and (A6-8) operationalize this logic. The maximum amount of down-regulation which can be provided in generation mode is limited by the location of the generation set point at time (t) relative to the rough zone, the minimum generation level and the maximum generation level. If the set point is below the rough zone, the minimum generation level creates a ceiling on the maximum amount of down-regulation which can be provided. If the set point is above the rough zone, the upper rough zone limit creates a ceiling on the amount of down-regulation which can be provided.

The generation set point must be at or below the maximum generation level at all times as shown in equation (A6-9).

When in generation mode, the sum of set point generation, up-regulation, spinning reserves and non-spinning reserves must be less than or equal to the maximum generator capability. This is shown in equation (A6-10).

Similarly, equation (A6-11) ensures that when the unit is “on” and is generating, the generation set point minus the amount of down–regulation provided must equal or exceed the minimum generation level. When the unit is “off”, these quantities must be equal to 0.0.

The pumping set point must be less than or equal to the maximum pumping power input level as shown in equation (A6-12).

As required by constraint equation (A6-13) the pumping set point minus the pumping spinning reserve, non-spinning reserve and up-regulation must be greater than or equal to 0.0.

When the unit is in pump mode and is pumping (“On”), constraint equation (A6-14) ensures the pumping set point minus the pumping up–regulation must equal or exceed the minimum pumping level When the unit is not in pump mode (it is “off”) and these quantities must be equal to 0.0.

Equation (A6-15) ensures the pumping set point plus the pumping down-regulation must be less than or equal to the maximum pumping capability.

Equation (A6-16) calculates the amount of water which must be released at time (t) to produce the amount of actual generation at time (t).

Equation (A6-17) computes the amount of water which is pumped at time (t) given the actual amount of pumping input power applied at time (t).

The volume of water in the forebay (upper) reservoir at time (t) depends on what the forebay volume was in the previous time period (t-1) plus the actual volume of water pumped into the forebay at time (t), minus the actual amount of water released for generation at time (t). This relationship is required by constraint equation (A6-18).

The amount of water in the forebay reservoir at any point in time must remain within the pre-set minimum and maximum forebay levels. Equation (A6-19) ensures the forebay reservoir will not overflow or be drawn down below allowable limits.

The total amount of water released for generation is required by equation (A6-20) to be equal to the sum of the water used for generation across the analysis period.

The total amount of water used for pumping is exactly equal to the sum of all of the pumping during the analysis period. This constraint is shown in equation (A6-21).

The total amount of water released for generation during the analysis period be less than or equal to the total amount of water pumped. This constraint (A6-22) ensures mass balance and it makes sure the forebay volume at the end of the analysis will be equal to the starting forebay volume.

APPENDIX 7 – KEY TO LINGO VARIABLES

A consistent variable naming convention was employed for all of the LINGO 14 models. To the extent possible, the same variable names were used in each model. This appendix provides a key to those variables, their units of measure and some additional notes on their application.

filename=Key_to_model_variablesV3.xls
D_Harpman
1/21/2015

Key to Important Variables in Pump-Generation Models (V3)

Name	Definition	Notes
nhrs	length of analysis period (hours)	
ngensos2pts	number of SOS2 points (in each hour) in generation function	
npumpsos2pts	number of S0S2 points (in each hour) in pumping function	
Hvalue	gross value of ALL products produced in the hour ($)	can be positive or negative
genind	binary indicator (1=generation mode, 0=otherwise)	
pumpind	binary indicator (1=if pump mode, 0=otherwise)	
eprice	LMP energy price ($/MWh)	
srprice	spinning reserve price ($/MW)	
nrprice	non-spinning reserve price ($/MW)	
urprice	up-regulation price ($/MW)	
drprice	down-regulation price ($/MW)	
spprob	probability of a spinning reserve call (dec)	
nrprob	probability non-spinning reserve call (dec)	
urcoef	expected value of up-regulation furnished (dec)	
drcoef	expected value of down-regulation furnished (dec)	
FBvollive	Forebay (upper) live reservoir contents (af)	set to x-hours of release at max gen
FBvol	Forebay (upper) reservoir contents at time (t)	
FBvolmin	Forebay (upper) dead pool contents (af)	set at 500 af for all models
FBvolmax	Forebay maximum volume (FBvolmin+FBvollive)*1.10	allows for "free board" in forebay
Targetvol	7 days of release at FBvollive per day (af)	used to limit weekly releases
FBstartvol	Forebay (upper) reservoir starting (live) volume (af)	Forebay starts 1/2 full
Hqgenaf	hourly amount of water released from forebay for generation (af)	
Hqpumpaf	hourly amount of water pumped back to forebay (af)	
qy	SOS2 generation release value (cfs)	raw SOS2 points are calculated
genx	SOS2 generation value (MW)	in spreadsheet and depend
penval	S0S2 generation penalty value	on the head and unit capacity
genweight	S0S2 weight (0<=wt<=1.0), sum of all gen weights == 1.0	weights are computed by the
pumpqy	SOS2 pumped water value (cfs)	optimization routine
pumppowx	SOS2 pump power input value (MW)	
pumppenval	SOS2 pumping penalty value	
pumpweight	S0S2 weight (0<=wt<=1.0), sum of all pump weights == 1.0	
act_genMW	actual generation (MW) at time (t)	nil for pump mode
act_genQ	actual generation release (cfs) at time (t)	nil for pump mode
set_genMW	setpoint generation level (MW)	
uregMW	amount of up-regulation (MW) furnished	
dregMW	amount of down-regulation (MW) furnished	
spinMW	amount of spinning reserves (MW) furnished	
nonspinMW	amount of non-spinning reserves (MW) furnished	
act_pumpMW	actual pump input power (MW) at time (t)	nil for generation mode
act_pumpQ	actual pumping release (cfs) at time (t)	nil for generation mode
set_pumpMW	setpoint pump input power level (MW)	
puregMW	amount of pump up-regulation (MW) furnished	
pdregMW	amount of pump down-regulation (MW) furnished	
pspinMW	amount of pump spinning reserves (MW) furnished	
pnonspinMW	amount of pump non-spinning reserves (MW) furnished	
mingenMW	minimum generation level (MW)	obtained from appropriate SOS2 point
maxgenMW	maximum generation level (MW)	obtained from appropriate SOS2 point
turbminQ	minimum turbine release (cfs)	obtained from appropriate SOS2 point
turbmaxQ	maximum turbine release (cfs)	obtained from appropriate SOS2 point
minpumpMW	minimum pump power input level (MW)	obtained from appropriate SOS2 point
maxpumpMW	maximum pump power input level (MW)	obtained from appropriate SOS2 point
Ulpt	upper generation point of the active SOS2 segment (MW)	used in bounding up-regulation with RZs
maxuregMW	given the setpoint, this is the maximum up-regulation (MW)	used when there are rough zones RZs
Lipt	lower generation point of the active SOS2 segment (MW)	used in bounding down-regulation with RZs
maxdregMW	given the setpoint, this is the maximum down-regulation (MW)	used when there are rough zones RZs

APPENDIX 8 – EXAMPLE LINGO CODE

For all of the LINGO 14 models, the same variable names were employed as was a consistent variable naming convention. While a variety of LINGO models were developed for this research project, this appendix provides an example of the code for these models. In this case, the code for the variable speed (VS) single unit model is shown.

```
Model:
Title VS Pump Storage Powerplant with RZ and AS over 24 hours Version 3.0;
! filename = PS_VS1-UnitV3_24.lng;
! LINGO version 14 with Forebay Reservoir;
! Version 2.0 parameter values, 2014 price set;
! SOS2 piecewise linear formulation;
! see Excel file = Make_sos2pointsV2.xls;
! Variable Speed (VS) generation 1-unit 1-day (24-hr) model Version 3;
! within hour mode changes are NOT permitted;
! ANCILLARY SERVICES;
! Rough zones are characterized;
! no spills are allowed/anticipated;
! WORKING, solves to 11154.36 in about 2 secs (reference conditions);
! 01/20/2015 dh;


Data:
!----------- User selectable parameters/values ---------------------------- ;
  nhrs = 24;
  ngensos2pts = 7;          !number of generator SOS2 points;
  npumpsos2pts = 4;         !number of pump SOS2 points;

  ! Forebay (upper) reservoir specifications;
  FBvollive = 1513.90;       !X hour max release per day volume equivalent (af);
  FBvolmin =  500.00;        !Forebay minimum capacity (af) [dead storage] ;

  ! Ancillary service (AS) parameter values;
  spprob = 0.01;    !probability of a spinning reserve call approx 1hr/4 days;
  nrprob = 0.0005;  !probability of a non-spinning reserve call approx 12hrs/year;
  urcoef = 0.20;    !up-regulation expected release coef ;
  drcoef = 0.20;    !down-regulation expected release coef;

  ! water conversion factors;
  af_to_cfs = 12.10000;    ! conversion factor af to cfs ;
  cfs_to_af = 0.082645;    ! conversion factor cfs to af ;

  ! bounds on feasible generation space (MW);
  fubMW = 999999;          ! upper bound on generation space;
  flbMW = 0.0;             ! lower bound on generation space;

  ! bounds on feasible release space (cfs);
  fubQ = 999999;          ! upper bound on release space;
  flbQ = 0.0;             ! lower bound on release space;

Enddata


! declare variable arrays for use;
sets:
  hourly /1..nhrs/: eprice, srprice, nrprice, urprice, drprice, FBvol,
                    set_genMW, act_genQ, act_genMW, uregMW, dregMW, maxdregMW,
                    maxuregMW, UIpt, LIpt, spinMW, nonspinMW, Hqgenaf,
                    genind,pumpind,set_pumpMW, act_pumpQ, act_pumpMW, pspinMW,
                    nonspinMW, puregMW, pdregMW, Hqpumpaf, Hvalue;
endsets

! declare VS generation SOS2 variable arrays for use;
sets:
  genpts /1..ngensos2pts/: qy, genx, penval;  ! INPUT: gen points for SOS2
  genweight(hourly,genpts): genwt;            ! RESULTS: SOS2 Weights gen;
endsets
! declare VS pump SOS2 variable arrays for use;
sets:
  pumppts /1..npumpsos2pts/: pumpqy, pumppowx, pumppenval; ! INPUT pump SOS2;
  pumpweight(hourly,pumppts): pumpwt;                ! RESULTS: SOS2 Weights pump;
endsets


! read 24-hour (Tuesday) July 2011 price data ($/MWh) file;
! using excel OLE direct import from spreadsheet;
! Dave's laptop path = 'c:\Documents and Settings\dharpman\My Documents\pump
```

```
         generation_research\lingo_models\ ;
Data:
  eprice, urprice, srprice, nrprice, drprice =
!@OLE( 'c:\Documents and Settings\dharpman\My Documents\pump
         generation_research\lingo_models\Tomvs_data24.xls' );
!OLE( 'c:\Documents and Settings\dharpman\My Documents\pump-
         generation_research\lingo_models\July2011data24.xls' );
@OLE( 'c:\Documents and Settings\dharpman\My Documents\pump
        generation_research\lingo_models\July2014data24.xls' );
!@OLE( 'c:\Documents and Settings\dharpman\My Documents\pump
        generation_research\lingo_models\Jan2014data24.xls' );
!@OLE( 'c:\Documents and Settings\dharpman\My Documents\pump
        generation_research\lingo_models\Jan2011data24.xls' );
!@OLE( 'c:\Documents and Settings\dharpman\My Documents\pump-
        generation_research\lingo_models\EXP2011data24.xls' );

Enddata
! set up VS generation SOS2 data points here from Excel filename =
  Make_SOS2pointsV2.xls;
Data:
  qy =     0.00, 457.9287, 457.9516, 1030.39, 1144.88, 1259.37,  2289.76;
 !release Q (cfs) values;
  genx =   0.00,     0.00,    20.00,   45.00,   50.00,   55.00,   100.00;
 !generation (MW) values;
  penval = 0.00,   -99999,     0.00,    0.00,  -89898,    0.00,     0.00;
 !penalty values ;
Enddata
! set up VS pump SOS2 data points here from Excel filename =
  Make_SOS2pointsV2.xls;
Data:
  pumpqy =     0.00,     0.00, 1220.6035,  1743.72;
  !water pumped Q (cfs) values of the 4 breakpoints;
  pumppowx =   0.00,   69.999,     70.00,    100.0;
  !power input (MW) values of the 4 pump breakpoints;
  pumppenval = 0.00,   -99999,      0.00,     0.00;
  !penalty values for the 4 pump breakpoints;
Enddata


!----------obtain VS Operation limits from SOS2 points------;
  mingenMW =   genx(3);      !VS minimum generation (MW);
  turbminQ =  qy(3);         !VS turbine minimum release (cfs);
  maxgenMW = genx(7);
  turbmaxQ = qy(7);
  LL_RZMW = genx(4);         !Lower rough zone limit (MW);
  UL_RZMW = genx(6);         !Upper rough zone limit (MW);
  minpumpMW = pumppowx(3);   !minimum VS pumping power level (MW);
  maxpumpMW = pumppowx(4);   !maximum VS pumping power level (MW);


!-------------declare range of specified variables-----------;
@for(hourly: @free(Hvalue));      ! hourly value ($);

!---------calculations for Forebay (upper) reservoir--------;
calc:
  FBvolmax =  (FBvollive+FBvolmin)*1.10;
 !Forebay maximum capacity (af) is 10% larger than total;
  targetvol = (FBvollive)*(nhrs/24);
 !one week target volume is 7x the one day target release;
  FBstartvol = (FBvollive)*0.50+FBvolmin;
 !Forebay reservoir starts out 1/2 full;
endcalc



!-------------objective function-----------------------------;
!-------------note penalty term at end-----------------------;
[objective] max = @sum(hourly: Hvalue );
@for(hourly : Hvalue = (act_genMW*eprice+ uregMW*urprice+ spinMW*srprice+
                       nonspinMW*nrprice+dregMW*drprice)
                       + @SUM(genpts: genwt*penval)+
```

```
                      (-act_pumpMW*eprice+pspinMW*srprice+pnonspinMW*nrprice+
                       puregMW*urprice+pdregMW*drprice)
                       + @SUM(pumppts: pumpwt*pumppenval));


!---------define SOS2 functions here-------------------------;
@FOR( hourly( i):  @FOR( genpts( j): @SOS2( 'gensos2set_'+hourly( i), genwt(i,
j))));     ! Define gen SOS2 Sets;
@FOR( hourly( i):  @SUM( genpts( j): genwt( i,j)) = 1);
! Weights must sum to 1;
@FOR( hourly( i):  @FOR( pumppts( j): @SOS2( 'pumpsos2set_'+hourly( i), pumpwt(i,
j))));  ! Define pump SOS2 Sets;
@FOR( hourly( i):  @SUM( pumppts( j): pumpwt( i,j)) = 1);
! Weights must sum to 1;


!---------calculate actual generation (MW)-------------------;
!---------using SOS2----------------------------------------;
@for(hourly: act_genMW = @SUM(genpts: genx*genwt));
@for(hourly: @bnd(flbMW, act_genMW, fubMW));

!---------calculate actual pumping powr input (MW)-----------;
!---------using SOS2----------------------------------------;
@for(hourly: act_pumpMW = @SUM(pumppts: pumppowx*pumpwt));
@for(hourly: @bnd(flbMW, act_pumpMW, fubMW));


!---------calc actual amount of water released for generation (cfs)-----------;
!---------using SOS2----------------------------------------;
@for(hourly: act_genQ = @sum(genpts: qy*genwt));
@for(hourly: @bnd(flbQ, act_genQ, fubQ));

!---------calc actual amount of water pumped(cfs)------------;
!---------using SOS2----------------------------------------;
@for(hourly: act_pumpQ = @sum(pumppts: pumpqy*pumpwt));
@for(hourly: @bnd(flbQ, act_pumpQ, fubQ));


!---------calc setpoint gen level (MW)------------------------------;
@for(hourly: set_genMW = act_genMW-uregMW*urcoef-spinMW*spprob-
nonspinMW*nrprob+dregMW*drcoef);

!---------calc VS setpoint pumping power level (MW)------------------------;
@for(hourly: set_pumpMW = act_pumpMW+pspinMW*spprob+pnonspinMW*nrprob-
pdregMW*drcoef+puregMW*urcoef);


!---------Either the Pump or Gen (or both) must be off---------------------;
!---------exploit the properties of SOS2 sets------------------------------;
!---------to construct indicators------------------------------------------;
!construct generation indicator 1=on, 0=off;
@FOR( hourly(i): genind =(genwt(i,3)+genwt(i,4)+genwt(i,6)+genwt(i,7)));
!construct pump indicator 1=on, 0=off;
@FOR( hourly(i): pumpind =(pumpwt(i,3) + pumpwt(i,4))); @for( hourly:
!plant is either pumping, generating or off;
genind+pumpind<=1);

!---------in Generation Mode considering Rough Zones---------;
!---------calculate maximum up/down-regulation (MW)----------;
!---------using properties of the SOS2 sets and--------------;
!---------then constrain regulation appropriately------------;

                     ! identify upper generation limit on the active segment
;
@for(hourly(i): UIpt =
(genwt(i,3)+genwt(i,4))*genx(4)+(genwt(i,6)+genwt(i,7))*genx(7));
                     ! calculate the maximum possible amount of up-regulation,
given the setpoint ;
@for(hourly: maxuregMW = (UIpt-set_genMW));
                     ! constrain up-regulation ;
@for(hourly: uregMW<=maxuregMW);
```

```
                     ! identify lower generation limit on the active segment
;
@for(hourly(i):LIpt =
(genwt(i,3)+genwt(i,4))*genx(3)+(genwt(i,6)+genwt(i,7))*genx(6));
                     ! calculate the maximum possible amount of down-regulation,
given the setpoint ;
@for(hourly: maxdregMW = set_genMW - LIpt);
                     ! constrain down-regulation ;
@for(hourly: dregMW<=maxdregMW);


!------------constrain generator unit loading (MW)-------------;
@for(hourly: set_genMW<=maxgenMW);                             !new;
@for(hourly: (set_genMW+uregMW+spinMW+nonspinMW)<=maxgenMW);  !new;
@for(hourly: set_genMW-dregMW>=mingenMW*genind);               !new-- note this is
a linear constraint;


!------------constrain VS pump loading (MW)-----------------------;
@for(hourly: set_pumpMW<=maxpumpMW);
@for(hourly: (set_pumpMW-pspinMW-pnonspinMW-puregMW)>=0.0);    !OK for reserves;
@for(hourly: (set_pumpMW-puregMW)>=minpumpMW*pumpind);
@for(hourly: (set_pumpMW+pdregMW)<=maxpumpMW);


!-------------forebay (upper) water balance--------------------------;
FBvol(1) = FBstartvol - act_genQ(1)*cfs_to_af+ act_pumpQ(1)*cfs_to_af;
@for(hourly(I) | I #gt# 1: FBvol(I) = FBvol(I-1) - act_genQ(I)*cfs_to_af +
act_pumpQ(I)*cfs_to_af);
@for(hourly: @bnd(FBvolmin, FBvol, FBvolmax));


!-------------forebay (upper) elevation-----------------------------;
!-------------can be calculated from the FB volume-------------------;

!-------------afterbay (lower) elevation----------------------------;
!-------------is fixed in this particular model----------------------;


!-----------constrain turbine release to available volume-----;
@for(hourly: Hqgenaf = (act_genQ*cfs_to_af ));
totgenvol = @sum(hourly: Hqgenaf);
[totvolume] totgenvol<=Targetvol;

!-----------pumping must replace generation release volume-------;
@for(hourly: Hqpumpaf = (act_pumpQ*cfs_to_af ));
totpumpvol = @sum(hourly: Hqpumpaf);
[totpumping] totgenvol<=totpumpvol;


!-----------calculate results for reporting ---------------------;
grossgenvalue = @sum(hourly: act_genMW*eprice);
genspinvalue = @sum(hourly: spinMW*srprice);
gennonspinvalue = @sum(hourly: nonspinMW*nrprice);
genuregvalue =@sum(hourly: uregMW*urprice);
gendregvalue = @sum(hourly: dregMW*drprice);
grosspumpcost = @sum(hourly: act_pumpMW*eprice);
pumpspinvalue = @sum(hourly: pspinMW*srprice);
pumpnonspinvalue = @sum(hourly: pnonspinMW*nrprice);
pumpuregvalue = @sum(hourly: puregMW*urprice);
pumpdregvalue = @sum(hourly: pdregMW*drprice);



!-------------export results to spreadsheet-------------------;
data:
  @OLE( 'c:\Documents and Settings\dharpman\My Documents\pump-
       generation_research\lingo_models\PS_VS1-UnitV3_out24.xls' ) =
       turbminQ, turbmaxQ, targetvol, totgenvol, FBstartvol, FBvol, hqgenaf,
       hqpumpaf, mingenMW, maxgenMW,act_genQ, act_genMW, set_genMW, uregMW,
       dregMW, spinMW, nonspinMW, LL_RZMW, UL_RZMW,set_pumpMW, act_pumpMW,
```

```
        act_pumpQ, pspinMW, puregMW, pdregMW, genind, pumpind, pnonspinMW,
        objective, grossgenvalue, genspinvalue, gennonspinvalue, genuregvalue,
        gendregvalue, minpumpMW, maxpumpMW, grosspumpcost, pumpspinvalue,
        pumpnonspinvalue, pumpuregvalue, pumpdregvalue, Hvalue;
enddata

End
```

APPENDIX 9 – ACQUIRING THE LINGO SOFTWARE

After some deliberation, the study team recommended developing a custom LINDO/LINGO model for estimating the economic benefits of pump-generation plants. The study team reviewed and considered several different modeling approaches that could potentially be used for the reconnaissance-level estimation of economic benefits. This effort was targeted, based on the team's experience and judgment, and was tempered by the resources allocated to this task. This search is described more fully in Harpman, Kubitschek and Wittler (2014). After some deliberation, the team concluded the set of available tools was rather limited, and the subset of those which could be used to complete this research within the time-frame required was quite small.

LINDO/LINGO optimization suite (http://www.lindo.com) and associated solvers are in relatively common standalone use by economists and engineers. The LINDO component is dedicated to the solution of linear programming models, whereas the LINGO component of the suite is a powerful nonlinear optimization solver.

The team concluded it would be most expedient to construct a pump-generation model using the LINDO/LINGO optimization framework. Development of a custom pump-generation model would allow explicit control over the model's features and complexity and allow incorporation of some features, which would not easily be portrayed otherwise. The LINDO/LINGO framework is a commercially available optimization product that has proven to be both reliable and robust. The LINGO architecture includes structures such as objects, vectors, and matrices, and it allocates memory for these at run-time. These features allow for relatively compact coding for large datasets and dynamic problems.

There were two other factors that helped shape this choice. First, the Economics and Resource Planning Team (86-68270) currently owns a license for an earlier version of the LINDO/LINGO optimization suit. Second, members of the team have considerable experience, in general, specifying and solving nonlinear optimization problems using LINGO.

Having made that decision, the team initiated procurement of this software package. Procurement proved to be a huge endeavor. In addition, research accomplishments in Fiscal Year 2014 were greatly delayed due to various circumstances including the Government shutdown and the IT Software Approval Process.

The single greatest impediment to research progress was the team's inability to obtain LINGO 14, the optimization software, in a timely manner. Even though this is a commercially available shrink-wrapped software program, it took about 9 months to physically obtain it. The team completed all of the necessary steps for the IT approval process in August 2014. Even so, permission to acquire this software was not received from the Department of the Interior until February 12, 2014. After that step, the team had to work through the federal acquisitions process to purchase the software. Two copies of LINGO 14 were finally received on May 19, 2014.

While this software acquisition saga was unfolding, the research team did complete some of the preliminary work necessary to proceed with this effort. Nonetheless, the acquisition of this commercially available software program significantly delayed the schedule for completion of this project.

APPENDIX 10 – BOX AND WHISKER PLOTS

Box and whisker plots like the one shown in Figure 27 provide a quick summary of important dataset characteristics including the central tendency, dispersion, asymmetry, and extreme values. These plots are based on descriptive statistics for the underlying empirical distribution and are nonparametric or distribution-free. Consequently, they do not reflect any of the assumptions associated with distributions, such as the normal distribution. They provide an effective way of identifying asymmetrical attributes in datasets. Perhaps most importantly, the graphically compact nature of box and whisker plots facilitates rapid side-by-side comparison of different samples or datasets, which can otherwise be difficult to interpret.

Box and whisker plots were first proposed by statistician John Tukey circa 1970. Their application has become relatively commonplace and standardized. Consumers of technical literature recognize there are some variations in the way these plots are defined. As related by Banacos (2011) the majority of these variations are in the definition of the "whiskers."

For purposes of this research project, the 5th and 95th percentiles are used in this document for the ends of the whiskers. Using this convention, there is a 5% probability a data point will fall beyond the high or low values at the ends of the whiskers. The range between the whiskers encompasses 90% of the empirical distribution.

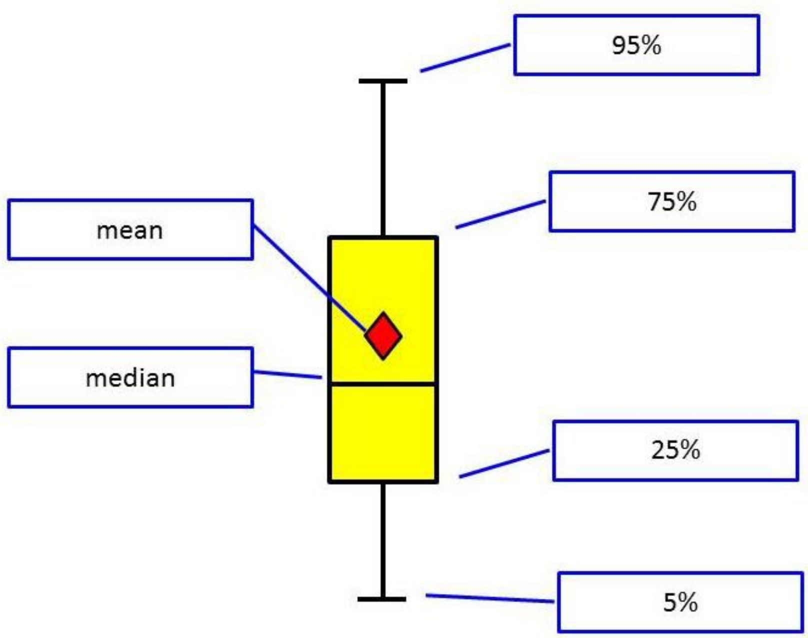


Figure 27.—Interpretation of a box and whisker plot.

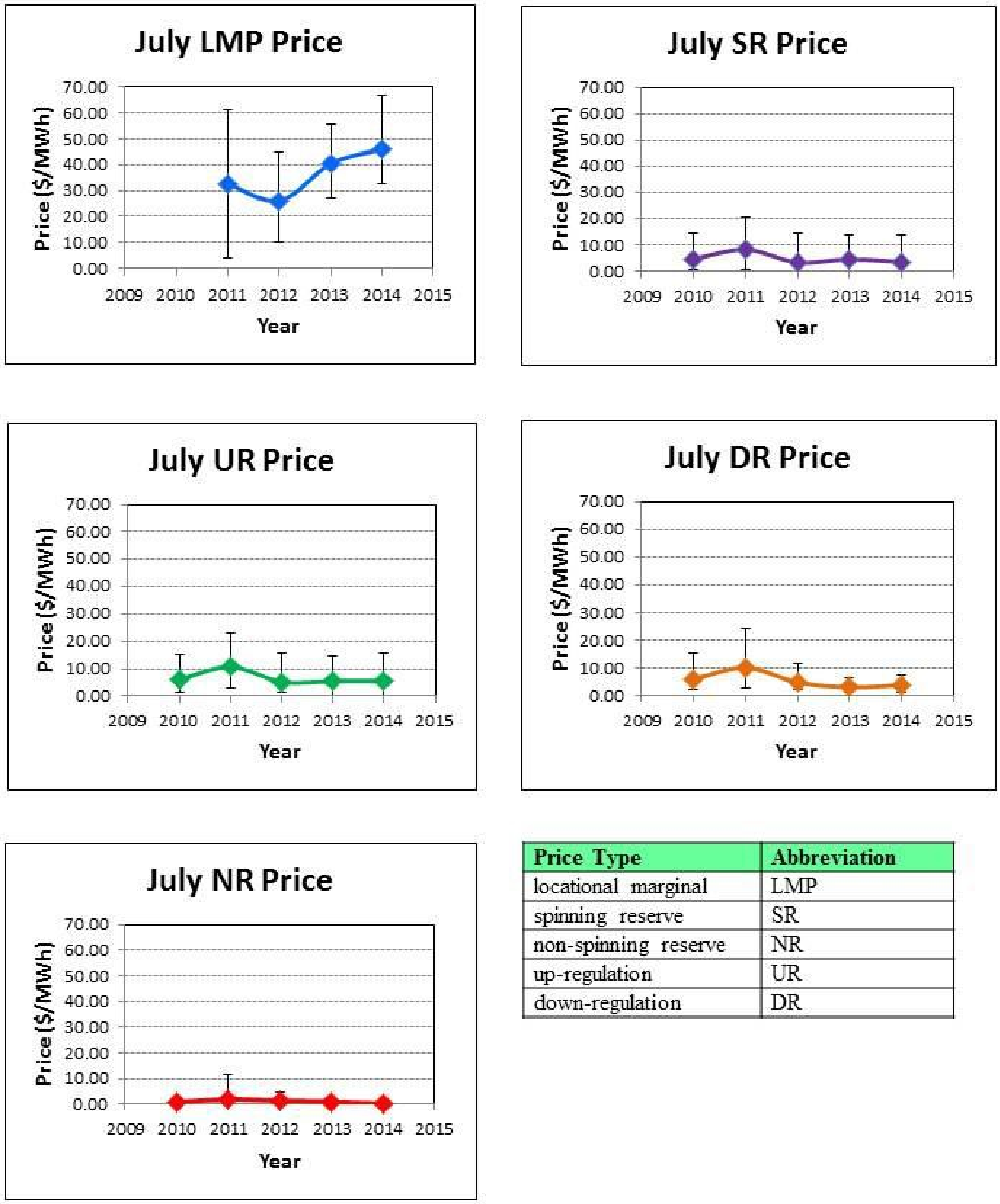
Figure 27 illustrates a representative Box and Whiskers plot as employed in this study. As shown in Figure 27, the median (black horizontal line within the box) illustrates the central tendency measure of the plotted data set. The median is the 50th percentile point, or the point where one-half the data lie above and one-half the data lie below. The "box" illustrates the inter-quartile space between the 75th percentile and the 25th percentile of the empirical data distribution. The size of the box provides an indication of how much variance or dispersion are reflected in the data. The median point may or may not be in the center of the box depending on whether the data is symmetric around the median or is asymmetric (skewed).

In Figure 27 the whiskers extend to the 95th percentile and the 5th percentile of the empirical distribution. The dispersion of the empirical distributions are indicated by the distance between the ends of these whiskers—the greater the distance between them, the more the dispersion and the shorter the distance between the ends of the whiskers, the less the dispersion. If the empirical distribution is symmetric, the whiskers will be the same length. If the distribution is asymmetric, the upper and lower whiskers will be of differing lengths.

There is no built-in functionality for generating a Box and Whisker plot using Microsoft Excel. However, using the tedious and lengthy set of steps described in the very helpful Peltier Tech Blog (www.peltiertech.com), Excel can be used to create these helpful summaries. These box and whisker plots illustrate the characteristics of empirical distributions and facilitate the comparison of different samples.

APPENDIX – 11 PRICE TRENDS

This Appendix contains some time series plots of July LMP and AS prices from 2010 to 2014. The diamond shown in these plots represent the mean of the price data. The whiskers represent the 95^{th} and 5^{th} percentile respectively. The area bounded by the upper and lower whisker demarcates 90% of the range of the empirical data distribution. As shown, the LMP prices have risen over time and hourly AS prices have fallen in recent years. Some informed observers have suggested that due to market imperfections, current AS prices do not reflect the true value of ancillary services. This topic has been the subject of considerable scholarly debate. Even so, a consensus view has not emerged.



Price Type	Abbreviation
locational marginal	LMP
spinning reserve	SR
non-spinning reserve	NR
up-regulation	UR
down-regulation	DR

Figure 28.—Price trends 2010 to 2014.

APPENDIX 12 – DATA DICTIONARY

Filename	Contents	Notes
July2014data168.xls	July 2014 typical week prices	
July2011data168.xls	July 2011 typical week prices	
Jan2014data168.xls	January 2014 typical week prices	
Jan2011data168.xls	January 2011 typical week prices	
July2014data24.xls	Typical day in July 2014 prices	
July2011data24.xls	Typical day in July 2011 prices	
Jan2014data24.xls	Typical day in January 2014 prices	
Jan2011data24.xls	Typical day in January 2011 prices	
SSunit_prod.xls	SS unit relationships	Given head and unit size, computes SOS2 points which are then read by LINGO program
VSunit_prod.xls	VS unit relationships	Given head and unit size, computes SOS2 points which are then read by LINGO program
VS_mead2D.xls	VS unit relationships at Mead Project 2D	Computes SOS2 points for this project
VS_seminoe5C.xls	VS unit relationships at Seminoe Project 5C	Computes SOS2 points for this project
PS_VS1sum_out168.xls	VS 1-week summary output	Minimal summary
PS_SS1sum_out168.xls	SS 1-week summary output	Minimal summary
PS_VS1-UnitV3_out24.xls	VS 1-day output	Very detailed 24-hour output
PS_SS1-UnitV3_out24.xls	SS 1-day output	Very detailed 24-hour output

APPENDIX 13 – CODE DICTIONARY

Filename	Contents	Notes
PS_SS1-UnitProd_168.lg4	SS 1-Unit LINGO code for typical week model	
PS_VS1-UnitProd_168.lg4	VS 1-Unit LINGO code for typical week model	
PS_SS1-UnitProdNOAS_168.lg4	SS 1-Unit LINGO code for typical week model	No ancillary services
PS_VS1-UnitProdNOAS_168.lg4	VS 1-Unit LINGO code for typical week model	No ancillary services
PS_VS_Mead2D_168.lg4	VS LINGO code for Mead Project 2D	
PS_VS_Seminoe5C_168.lg4	VS LINGO code for Seminoe Project 5C	
PS_SS1-UnitV3_24.lg4	SS 1-Unit 24 hour LINGO code	
PS_VS1-UnitV3_24.lg4	VS 1-Unit 24 hour LINGO code	
Excel_InterpolationCode2.txt	Visual Basic code file	Code for interpolation. used to compute some results
CalcRequiredForebayVolume.xls	Forebay reservoir volumes needed for X-hours at maximum output	

Mission Statements

The U.S. Department of the Interior protects America's natural resources and heritage, honors our cultures and tribal communities, and supplies the energy to power our future.

The mission of the Bureau of Reclamation is to manage, develop, and protect water and related resources in an environmentally and economically sound manner in the interest of the American public.