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The mission of the Bureau of Reclamation is to manage, develop, and protect water and related resources in an environmentally and economically sound manner in the interest of the American public.
**T4. TITLE AND SUBTITLE**
Hydro-Economic Model Completion and Technology Transfer

**14. ABSTRACT (Maximum 200 words):**
Since 2006, Reclamation and the University of Idaho have been collaborating on the development of a hydrologic and economic modeling methodology that can be used to quantify benefits that may result from water management planning alternatives. Though the concept had been proven and documented in both Reclamation and peer reviewed journal publications, the tool still needed to be improved for usability by other Reclamation hydrologists and economists. This project resulted in the development of a hydro-economic tool called HydroSense, which is a C# based code that reads input and sends output to an Excel spreadsheet. A user’s manual for the hydro-economic methodology was also developed. Lastly, the methodology was applied in the Henrys Fork basin in eastern Idaho to demonstrate that it can be transferred to other basins.

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Executive Summary

Since 2006, Reclamation and the University of Idaho have been collaborating on the development of a hydrologic and economic modeling methodology that can be used to quantify benefits that may result from water management planning alternatives. Though the concept had been proven and documented in both Reclamation and peer reviewed journal publications, the tool still needed to be improved for usability by other Reclamation hydrologists and economists.

This project resulted in the development of a hydro-economic tool called HydroSense, which is a C# based code that reads input and sends output to an Excel spreadsheet. A user’s manual for the hydro-economic methodology was also developed. Lastly, the methodology was applied in the Henrys Fork basin in eastern Idaho to demonstrate that it can be transferred to other basins.
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- Appendix C – HydroSense Code ReadMe
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Project Summary

Reclamation water management planning studies often require the calculation of net benefits (benefits minus costs) or benefit cost ratios (net benefits divided by net costs) for each alternative that is being considered. Previous methods used to quantify benefits utilized a supply management approach, which ignores the elasticity of demand and leads to an incomplete economic valuation of water. Hydro-economic modeling provides a framework for integrating physical, ecological, economic, and social/cultural systems into a single valuation that more completely explains the costs and benefits of water management alternatives.

Background

Reclamation and University of Idaho hydrologists and economists have been collaborating on the development of a combined hydrologic and economic modeling methodology since 2006. Early work focused on developing the theoretical approach and equations used to combine hydrologic and economic models. The approach was tested using both surface and groundwater models and using data in the Boise basin. The conceptual approach and test cases were documented in two Reclamation reports (Reclamation 2009, Reclamation 2010), an Idaho Water Resources Research Institute report (IWRRI 2013), and a journal article (Taylor et al. 2014).

Project Goals and Products

This study was the final phase of the hydro-economic model development project where the major goals were geared towards preparing the methodology for wider distribution and use within Reclamation.

To that end, a HydroSense tool was developed in C# that reads input from and sends output to an Excel spreadsheet. The tool is publicly available on github.com at https://github.com/usbr/hydrosense. A readme file is attached as Appendix C and is also included on github.com. In addition to making the code easily available to future users of HydroSense, providing the code on github allows for continued development of the code by future users and developers.

A user’s manual for the hydro-economic methodology was also developed and was published as an IWRRI publication (see Appendix A). Finally, in an effort to prove the concept was useful in a basin other than the Boise basin, the methodology was applied in the Henrys Fork basin in eastern Idaho. Documentation of that application is found in the IWRRI publication in the Appendix B.
Partnerships

This project would not have been possible without a strong collaboration between researchers in the Agricultural Economics Department at the University of Idaho, Garth Taylor, Leroy Stodick, and Bryce Contor, IWRRI, John Tracy and R.D. Schmidt, and Reclamation, Bob Lounsbury, Jennifer Cuhaciyan, and Jennifer Johnson. The combined economic, hydrologic, agricultural, modeling, and programming expertise was required to make this project successful.
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Appendix A

An Approach to Hydro-Economic Modeling Using Partial Equilibrium Optimization
October 2014

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Part 1 – The Hydro-Economic Approach to Water Resources Management

Why Use Hydro-Economic Modeling for Water Resources Planning?

“Managing water as an economic good is an important way of achieving efficient and equitable use, and encouraging conservation and protection of water resources.” - U.N. Dublin Statement on Water and Sustainable Development, 1992

Conventional, economics-based water planning approaches often fail to adequately evaluate the economic efficacy of water projects by ignoring the dynamic relationship that exists between water supply and demand. More specifically, under the conventional approach to water management, which can be referred to as the supply management approach, the value of water is based upon the amount of compensation necessary to recover distribution costs (O&M, infrastructure, construction, etc.) with water demand forecasts assumed to be static and not affected by the cost of the supplied water (Howitt and Lund, 1999). The demand management approach, on the other hand assumes that the costs associated with developing and delivering water supplies is invariant, and focuses on the value of water relative to the amount of water demand and controlling factors such as regulation, conservation, and availability of infrastructure. By ignoring how the demand for water changes as a result of changes in price and ignoring the change in cost associated with supplying greater amounts of water (referred to as the price/cost elasticity), both of these conventional approaches to water management fall short in their ability to adequately inform the development of effective water management strategies.

Hydro-economic analysis presents an alternative to the Demand Management and Supply Management approaches. This type of analysis utilizes economic concepts to understand how the supply and demand for water are affected by changes in the cost of developing and delivering water supplies and how the demand for these water supplies is based on the value that can be derived from the water by the water users (ie crop value). This approach moves away from a static view with a fixed and invariant water demand, to a view where the demand for water is related to the economic concept of “value”. Use

1 Quoted in J.J. Harou et al. 2009
of an economic approach in water management and planning, particularly under conditions where water is a scarce resource, enhances the ability to develop management alternatives that are based on an efficient and equitable use of water, thereby reducing wasteful practices at both the individual and institutional scale (Harou et al., 2009).

Given that the value of water changes with both quantity and type of use, understanding the economic costs and benefits associated with meeting the demand for water resources allows for a more effective comparison of water management alternatives. Hydro-economic analysis provides a framework for incorporating multiple-, and often competing-, objectives (i.e., water supply, flood control, hydropower, recreation, ecosystem requirements, etc.) into a single analysis. By translating the value of each objective (or hydro-service) into its respective economic benefit, hydro-economic analysis allows for a direct evaluation of the economic efficacy of competing water management alternatives. Such an approach allows for a more holistic evaluation of water resource management actions, resulting in the development of more effective and sustainable water management strategies, and in turn reducing the likelihood of undesirable outcomes or unsustainable plan.

**Basics of Hydro-Economic Models**

Hydro-economic modeling can be traced back to the use of water demand curves developed in the 1960s and 1970s by Jacob Bear and others (1964, 1966, 1967, and 1970) for optimization of water resource systems in arid regions of Israel and the south-western United States. Researchers since then have used different names to refer to applications and extensions of this integrated systems approach to hydrologic, engineering, and economic water modeling including: hydrologic–economic (Gisser and Mercado, 1972), hydroeconomic (Noel and Howitt, 1982), institutional (Booker and Young, 1994), demand and supply (Griffin, 2006) analysis approaches, among others.

Hydro-economic models have the ability to represent physical, environmental, and economic aspects of basin-scale water resource systems in an integrated framework that accounts for the value of water in terms of the services or benefits it generates for users (Harou et al., 2009; Brouwer and Hofkes, 2008). There are two basic forms for hydro-economic models. The more holistic configuration combines hydrology and
economic optimization into a single model, while the modular configuration (illustrated in Figure 1) involves a transfer of supply and demand information from an independent hydrologic model to an economic optimization model. For basin-scale studies, the modular approach is generally preferred because it allows for more robust and realistic representation of basin hydrology and more efficient optimization of a basin-wide network of water supply and demand nodes (Brouwer and Hofkes, 2008).

![Diagram of Basin-wide Hydro-economic Modeling](image)

**Figure 1: Basin-wide hydro-economic modeling, modular components**

Most hydro-economic models share basic elements including spatial representation of hydrologic flows and water supply infrastructure, supply costs and constraints, economic demands, and operating rules affecting water allocations. Basin-wide hydro-economic model application involves five basic steps:

1. Develop a basin-wide network consisting of nodes (representing locations where water can be supplied or demanded) and links (representing the conveyance system that is responsible for delivering water from supply nodes to demand nodes).

2. Develop relationships that describe the marginal cost for supplying water from each supply node, the marginal benefits accrued through the use of water by each node, and the economic demands for water at each demand node.
demand node, the cost of conveying water between each water supply node and each demand node, and the loss of water through each part of the conveyance system.

3. Calibrate the parameters for the basin-wide model relative to available hydrologic and water-budget data in the basin.

4. Develop alternative water infrastructure and management scenarios and predict changes in the physical and cost relationships between water supplies, demands, conveyance costs, and conveyance losses.

5. Perform a Cost Benefit Analysis (CBA) comparing the various water infrastructure and management scenarios to determine the most cost effective water management scenario(s).

### Water Economic Valuation

The economic valuation of water can occur from a supply or demand perspective and produces a supply-cost function or a demand-price function. For water suppliers, the economic value of water is determined by the fixed costs of infrastructure and the operating costs associated with supplying water to users. When calculated by engineering economists, a water supply-cost curve is often simplified into a block rate structure (illustrated in Figure 2) with price steps reflecting the increasing capital and operating costs associated with the addition of new supplies.

![Figure 2: A block rate supply-cost function](image)
From the demand perspective, water is an input into a production process (such as irrigation, hydropower generation, or recreation) and water demand is therefore derived from the demand for the final product produced. Price elasticity is also an important component in the valuation of water from the demand perspective and represents the variation in willingness-to-pay for water with respect to varying quantity of water provided.

Demand price elasticity varies with type of water use (agricultural, municipal, industrial, recreational etc.) and with hydrologic condition (e.g. dry year, normal year, wet year, etc). A steeply sloping demand curve implies a water use that is more price responsive (has low price-elasticity) and a valuation that is more sensitive to water availability. Meanwhile, a demand curve that is gently sloped implies a water use that is less price responsive (high price-elasticity) and a valuation that is less sensitive to availability. Figure 3a illustrates a situation where the demand for water is inelastic, in other words there is no change in the demand for water with respect to price. Such a relationship would represent a situation where there is a “requirement” to provide a specified amount of water to meet the demand, no matter what the cost. Figure 3b illustrates a situation where the demand for water is elastic, in other words the demand for water does change with respect to its price.

![Figure 3: A requirements demand function and a constant elasticity demand function.](image-url)
Elements of Partial Equilibrium Modeling

The mathematical goal in hydro-economic modeling is to determine the point where a market equilibrium exists between the marginal costs of supplying water and the marginal benefits that can be accrued by the use of the water at demand nodes to produce other economic goods (e.g. crops, ecosystem services, hydropower, etc.). This equilibrium is referred to as a Partial Equilibrium (PE) and is the point where the maximum economic net benefit can be accrued by optimally distributing water between the supply nodes and demand nodes. The concept of marginality, which expresses the supply-cost or demand-price associated with one additional unit of water (at the margin), is central in PE modeling. The microeconomic equi-marginal principle states that in an optimal allocation of water, each water user derives the same value (or utility) from the last unit of water allocated (Harou et al, 2009).

PE modeling is not equivalent to advocating water marketing, nor does it assume all water resources are private goods. Constraints on private allocations, and on demands for public goods such as river system eco-services, are readily included in hydro-economic models.

Part 2 - Methodology and Application

Generally speaking, hydro-economic modeling follows a four step process (illustrated in Figure 4). The first three steps define the water supply, demand, and delivery relationships as mathematical functions for input into a PE solver. The fourth step involves using a PE solver to find the equilibrium solution given the mathematical functions developed in the first three steps. There is essentially no limit to the number or form of the mathematical functions used in the PE model. The only requirement is that they define relationships in terms of price (or cost) and quantity alone. These steps are described in more detail below, and provide examples of their application.
Step 1 – Developing Marginal Cost Curves for Water Supply

Water valuation from the supply perspective results in a supply-cost curve that represents the unit change in price for a unit change in quantity supplied, taking into account the costs and constraints associated with:

- development of the water supply infrastructure, such as the construction of groundwater wells, the building of water storage facilities, and the development of water conveyance structures;
- operation of the infrastructure, such as the energy and maintenance required to operate pumps, and the maintenance required to ensure the efficient and safe operation of conveyance systems such as canal and pipe system;
- and regulatory considerations associated with water rights administration and environmental legislation and policies.
The purpose of this step is to define the cost of supplying water to each demand node with mathematical functions which can then be input into the partial equilibrium optimization model. In cases where a demand node has multiple supply sources, a separate function can be developed to represent the cost of delivering water from each source. As stated previously, there is essentially no limit to the number or form of these mathematical functions as long they calculate cost in terms of a quantity supplied (e.g. \( y = f(x) \), where \( y \) is cost and \( x \) is a quantity of water). Figure 5 provides a schematic representation of the analysis elements that must be completed in this step.

**Step 1: Elements for Developing Marginal Cost Relationships:**

- Develop an integrated model that predicts the physical movement of water through the surface water and ground water components of the study area that accounts for:
  - changes in climate conditions;
  - water resource management practices; and
  - water resources management infrastructure.

Changes in water resource management practices include:
- Changes in monitoring of water diversions and use;
- Changes in water application efficiency;

Changes in water resource management infrastructure include:
- Development of water storage facilities;
- Reducing losses from water conveyance structures;
- Development of groundwater resources.

The form of the supply-price function will depend upon the nature of the diversion and the supply and the factors influencing the cost to deliver water from the supply to the diversion. It is up to the modeler to identify the level of detail required for a particular study and which factors should be considered in the development of these supply-cost functions. Figure 6 depicts the general shape of the marginal cost curves that would be expected for various types of water supplies within a study area. The
following sections provide a more detailed discussion of the development of supply-cost functions for various types of supplies, namely: canal irrigation (surface water) supply, groundwater and drain water irrigation supply, flood control storage supply, and instream flow supply.

**Canal Irrigation Supply Costs**

Water supply costs for canal irrigators typically have a stepped block-rate structure. The lowest step typically represents the cost associated with the delivery of natural flows that simply pass through reservoirs (determined by water rights associated with the demand). Higher steps typically reflect the added cost of water delivery from reservoir storage (determined by operation and maintenance of existing infrastructure and/or repayment costs for the construction of new facilities). Figure 6 shows an example of the block-rate cost structure for irrigation water delivered to the head (river point of diversion) of four different canal systems within the Boise Project. The quantity
of natural flow and storage water available for delivery to each canal system is constrained by the water rights and storage account space owned by each system. In this example, supply costs range from $2.60 to $7.20 per acre-foot (AF) for natural flow (reflecting the various canal system O&M costs), while the delivery of storage water adds an additional $1.60 per AF (reflecting reservoir O&M costs).

Conveyance losses associated with delivery of irrigation water to end users (at farm head-gates further down the canal) should also be considered in the supply-price function and are dependent upon the characteristics of the canal infrastructure and basin hydrology. The effective supply cost, or the cost to deliver water to a particular farm head-gate, will often be higher than the cost to deliver water to the head of the canal system due to conveyance losses that occur between the head of the canal and the farm head-gate. There are two options to account for conveyance losses in a PE model. Given a conveyance cost that can be represented by the following equation:

$$Conveyance \ cost = river \ point \ of \ diversion \ supply \ cost \times \% \ seepage \ loss$$

The cost to deliver the water to the farm head-gate could be computed as

$$effective \ supply \ cost = river \ point \ of \ diversion \ supply \ cost \times (1 + \% \ seepage \ loss).$$

Alternatively, the amount of water made available to the demand nodes can be adjusted to account for the loss of water through the conveyance system, such that

$$Available \ water \ for \ Demand = Water \ from \ Supply \ Node - Conveyance \ Losses$$
Groundwater and Drain Water Irrigation Supply Costs

Supply costs for groundwater and drain water irrigators can often largely be determined by pump operating costs. For both groundwater and drain water irrigators, supply-cost curves (representing the unit change in cost per unit change in diversion or pumping rate) can be estimated from power costs, pump characteristics (efficiency, etc.), and pumping lift. For drain water irrigators, pumping lift is fixed and supply cost is a function of pumping rate alone. However, for groundwater irrigators, pumping lift is not only dependent upon the general depth to groundwater (DTW), but is also influenced directly by pumping rate.

Taking this into account and incorporating any costs associated with the delivery of irrigation water from the well-head to the field (which are likely fixed costs), the marginal supply-cost function for groundwater irrigators can be expressed as

\[ \text{groundwater supply cost} = G_1 + G_2 \cdot \text{pumping lift}, \]  

(3)

Where \( G_1 \) is the cost of delivering one AF of irrigation water from the well head to the field and \( G_2 \) is the cost of lifting one AF of water one foot in the well bore. For drain water irrigators, where water supply costs depend only on the fixed costs associated with pumping and delivering one AF of water from the drain to the field, the marginal supply cost function can be expressed as a constant rate

\[ \text{drain water supply cost} = G_1, \]  

(4)

regardless of how much water is diverted from the drain.

While the cost of diversion (per AF) may be constant for drain water diverters, these entities have no control over the availability of drain return flow, which is subject to other factors such as canal seepage and groundwater pumping rates. Similarly, groundwater irrigators have little control over changes in depth to groundwater and the associated changes in cost of diversion. In situations where groundwater elevations and drain water return flows are influenced by canal seepage, decreases in canal seepage will result in increased depth to groundwater (and therefore increased pumping lift) and decreased drain return flow availability. In such cases the cost of diversion is dependent on (or constrained by) groundwater processes in addition to the quantity of diversion and
the use of groundwater models to generate response functions can help reduce the function into terms of diversion quantity alone (as is necessary for input into the PE trading model).

The groundwater and drain flow response functions for various locations can be estimated by performing a series of hydrologic modeling runs with incremental changes in a particular stressor of interest (e.g. groundwater pumping rate or canal seepage rate). The output from these model runs provides a series of points along a curve that relate the depth to groundwater (or drain flow response) to incremental changes in the stressor. Where more than one stressor must be considered, the entire series of model runs can be repeated for each incremental change in the additional stressor. The data points provided by the multiple model runs can then be used to fit analytic response functions that define depth to groundwater or drain flow response in terms of the particular stressor.

Figures 5 and 6 illustrate an example where the groundwater and drain flow responses to canal seepage and groundwater pumping rates were evaluated using a series of model runs. As can be seen in Figure 5, decreases in canal seepage result in increasing depths to groundwater, which in turn increase the cost of using groundwater as a water supply. Figure 6 illustrates the reduction in drain flows that occurs as a result of decreases in canal seepage, thereby making less drain flow water available for use within the study area. In this example the response function for pumping lift (Equation 1) at a particular location has a non-linear form representing the nature of a shallow aquifer that transitions from confined to unconfined as canal seepage is reduced:

\[
pumping\ lift = C_1 e^{(C_2 \text{canal seepage} + C_3 \text{groundwater pumping rate})}
\]  
\[\text{(1)}\]

Meanwhile, the response function for drain flow is assumed to have the form:

\[
drain\ flow = D_1 \cdot e^{(D_2 \text{canal seepage})} + D_3 \cdot \text{groundwater pumping rate}.
\]  
\[\text{(2)}\]

Values for the coefficients \(C_1, C_2 \text{ and } C_3\) and \(D_1, D_2 \text{ and } D_3\) can be obtained using a non-linear least squares regression procedure.
Figure 5: Fitted DTW response to Boise Project canal seepage, for five groundwater pumping rates.

Figure 6: Fitted drain return flow response to Boise Project canal seepage, for five groundwater pumping rates.

Taken together, the relationships represented by Equations 1 and 2, and the response functions represented by Equations 3 and 4, can be used to generate supply-cost
functions that are defined in terms of diversion rate alone. Figure 7 shows the marginal supply cost for groundwater in one particular groundwater response zone as a function of pumping rate and canal seepage rate. Figure 8 shows the marginal supply cost functions for drain water in one particular drain water response zone. Since the cost per AF is fixed for drain water irrigators, canal seepage and ground water pumping affects only the quantity of drain water available and not the supply-cost. In the example of varying canal seepage, multiple PE model runs will be required, one for each canal seepage rate. The results from the multiple PE model runs can then be compared to one another to evaluate the impact of canal seepage on the system.

Figure 7: Upward shifts in groundwater irrigator’s supply cost due to reduction in Boise Project canal seepage.
Flood Control Storage Supply Costs

Supply costs for new flood control storage typically take on a form that is analogous to the stepped block-rate structure shown previously in the section on calculating supply costs for canal irrigation. The lowest step represents the supply cost of current flood control storage and subsequent steps represent the supply costs associated with delivery of new reservoir storage for additional flood control. The supply-cost functions associated with new reservoir storage will likely vary between individual reservoir storage options.

For example, based on the options for new reservoir storage outlined in the Army Corp of Engineers Boise Basin Water Storage feasibility study (USACOE, 2010), construction of a larger dam at Arrowrock reservoir would provide 317,000 AF of new storage at an estimated construction cost of $2,700/AF, and a new Twin Springs dam and reservoir would provide 304,000 AF of new storage at an estimated construction cost of $3,600/AF. Assuming that a new dam and a new reservoir would have 100-year life spans, the annualized per AF reservoir construction costs would be $27/AF/year for
additional Arrowrock reservoir storage and $37/AF/year for new Twin Springs reservoir storage.

The resulting flood storage supply-cost function, illustrated in Figure 9, incorporates existing flood control storage (assumed by USACOE to be 987,000 AF) and possible future flood control storage options. The curve starts at $1.60/AF/year, representing the cost of existing flood control storage (assumed to be equivalent to the current O&M charge for irrigation storage and assumed by USACOE to total approximately 987,000 AF). The cost of supply then rises to $28.60/AF/year with the construction of a new Arrowrock dam and then to $38.60/AF/year with the addition of Twin Springs reservoir. Note that the shape of this curve is dependent upon the order in which the new storage projects are added. This particular curve assumes that the least expensive option, in terms of annualized cost, would be implemented first. Other factors may influence this order.

![Figure 9: New reservoir storage marginal supply-cost function.](image)

**Instream Flow Supply Costs**

The meaning of the term “instream flow” has evolved over the years, but usually describes the quantity of water set aside to sustain river ecology and river eco-services. An instream flow regime may be a single-value minimum flow recommendation, but
more often it describes a range of natural flow conditions that vary according to the time of year, the river reach, and the type of eco-services provided (fisheries, recreation etc).

In situations where instream flow demands can be used by downstream irrigation and/or hydropower demands (defined as a non-rival demand), the supply cost of instream flows is borne by these entities. In situations where instream flow demands cannot be used by other consumptive uses (rival demands), supply costs may be derived from water acquisition costs (e.g. rental pool rates); or if instream flows are “requirements” through Reclamation O&M, the costs borne by Reclamation (a federal agency) are passed along to the public through taxes.

In the Henrys Fork (HF) river basin, nearly all instream flows are non-rival with irrigation demands of the Freemont Madison Irrigation District (FMID) (Van Kirk et al, 2011), thus instream flow supply costs are borne mainly by HF irrigation entities. Depending on canal O&M costs, the resulting FMID supply cost for irrigation water ranges from $0.29/AF to $0.59/AF for natural flow and an additional $3.00/AF for water released from Island Park storage.
Step 2 – Determining the Water Flux Relationships

Step 2 - Elements for Determining Water Flux Relationships:

Utilizing the integrated water resources model, determine the availability of water for each of the demand nodes for incremental changes in water supply availability for each of the supply nodes.

Example: For reducing canal conveyance losses, model simulations can be run for 10% reduction in canal seepage, 20% reduction, 30% reduction, etc. to determine the change in water that will be available for each demand node.

Example: For increasing water storage, model simulations can be run using incremental changes in reservoir storage that are considered technical feasible to determine the change in water that will be available for each demand node.

In order to link the marginal cost associated with providing water from the supply nodes to the benefits associated with utilizing water at the demand nodes, a model (or mathematical relationships) must be developed to simulate the movement of water between all of the supply and demand nodes within the study area. The model used to complete this step can be as simple as a water budget, to something as complex as a physically-based model that simulates the behavior of water movement throughout the study area. The type of model developed will depend on the available resources, hydrologic data, and modeling expertise associated with the project, as well as the types of simulations that are needed to develop the relationships to complete the PE economic optimization analysis.

Once developed, the model is used to determine the relationships between the extraction of water from a supply node and the amount of water delivered to a demand node within the study area. These relationships must be determined for the conveyance
of water between each supply and demand node and for each water management scenario being considered. For example, if the potential water management scenario being considered is the lining of canals within the Boise Project area, simulations would be performed to determine the reduction in canal seepage that would be associated with lining a certain percentage of the canals. The simulation model developed in Schmidt et al. (2013) was used to determine the reduction in canal seepage, and hence the increase in water available to the demand nodes relying on delivery of water through the canal system and the impacts on depth to groundwater and drain flow, for a range of canal lining options within the Boise Project Study Area (see Figures 5, 6, 7 and 8). The results of each of these simulations was then used to determine changes to the marginal costs associated with providing water from each supply node, and the amount of water that can be delivered to each demand node.

In terms of the changes to the marginal cost relationships, it was assumed in the study that there would be no changes to the natural inflow to the reservoirs and storage supply node costs. However, by lining a portion of the canal, the marginal cost relationships for the ground water supplies are changed. The marginal costs for groundwater change as a result of increased depth to groundwater and the associated increase in pumping cost for ground water users (see Figure 5). Supply constraints for the drain water were also altered by canal lining. While the marginal costs for drainage water do not change, the availability of drain water is lowered due to the reduction in seepage losses in the canal that occur when the canal is lined (see Figures 6 and 8).

This analysis must be performed for each water management scenario under consideration. Such scenarios might include canal lining, increasing on-farm irrigation efficiency, or the development of new storage facilities. Examples of the types of analyses that must be performed can be found in the study by Schmidt et al. (2013), which evaluated how changes in water management conditions would impact water use in the Boise River Basin from a hydro-economic perspective.
Step 3 in the hydro-economic analysis procedure requires the development of relationships representing the marginal benefits associated with increased use of water for each water demand in the study area. Two broad approaches are available to model water demand (Kindler and Russell, 1984) and develop demand functions: inductive techniques, which rely on econometric- or statistical-analysis of observed data to estimate price-response and deductive methods which can be viewed as more of a modeling approach using production functions and mathematical programming.

The inductive method is commonly used for determining hydropower and instream flow water demands. Demand prices for hydropower flows are often calculated using alternative-cost techniques, where the cost of hydropower is compared to the next less expensive alternative (Gibbons, 1986; Booker and Young, 1994). Demand prices for instream flows (discussed in more detail later in this chapter) can be calculated based on recreation travel costs or user surveys.
Deductive methods are more commonly used for determining agricultural water demand (Tsur et al., 2004; Young, 2005). Irrigation demand prices are typically developed using deductive modeling approaches the employ the use of crop production models, commodity prices and crop acreages to determine the relationship between the amount of irrigation water used and the value of the crop produced (Martin et al., 1984).

**Irrigation Water Demand Prices**

Demand-price relationships can be developed using the *Irrigation Water Demand from Evapotranspiration Production Function* (IDEP) calculator (IWRRI, 2008). This calculator (described in greater detail in Appendix B) uses commodity prices and the evapotranspiration (ET) production function of Martin and Supalla (1989) to derive static, short-term demand for irrigation water for a particular crop. This is accomplished by transforming the ET production function into an irrigation water production function through the use of an exponent related to crop irrigation efficiency. The IDEP calculator can derive these exponents for up to six crops using basin-specific production and agronomic inputs. The calculator assumes that market mechanisms have already maximized crop acreages and the mix of crops and therefore all existing constraints on crop distribution are assumed to be fully reflected in the status-quo allocation of crops to lands. The IDEP calculator also assumes that limited water supplies will be optimally delivered when most needed and does not consider seasonal demand for irrigation water (only full-season volume delivered).

Water demand for a mix of crops is calculated by horizontally summing the demands of individual crops at every marginal price, thus ensuring crops are allocated water on an equal-marginal basis (Figure 10). Although crop mix is fixed in the horizontal summation, lower value crops may drop out of production at higher prices. The IDEP summation of marginal water demand quantities for high value cash crops and for low value field crops plots as a series of steps, indicating the price points at which different crop lands are taken in or out of production as the price of irrigation water decreases or increases.
These plots of IDEP price and quantity data can be translated into demand-price functions for high value and low value crop irrigation is accomplished by performing a regression analysis and fitting the data to analytic functions of the form

\[ demand\ price = B_0 \cdot (1 - B_1 \cdot demand\ quantity^{B_2}) \]  

(5)

Where \( B_0, B_1, \) and \( B_2 \) are calibrated parameters estimated using a least squares regression analysis approach. Figures 11 and 12 illustrate the example where fitted irrigation water demand-price functions were developed using the IDEP calculator for a groundwater irrigated zone and a drain water irrigated zone based on given crop distributions, acreages and irrigation efficiencies.
Figure 11: Marginal water demand-price data for high value and low value crops in a groundwater irrigated zone.

Figure 12: Marginal water demand-price data for high value and low value crops in drain flow irrigated zone.

Flood Control Storage Marginal Demand Prices

The demand price estimation for flood control storage depends on a variety of factors including the recurrence interval for flood flows, the expected duration of peak flood flows, and the expected flood damages within a 100 year or 500 year flood plain (IWRRI, 2013). Considered together, this information enables the formulation of demand-price curves, defined in terms of storage volume, that can then be incorporated into a PE optimization model.

A recent Corp of Engineers (USACOE) Lower Boise River Reconnaissance Study (USACOE, 1995) used a frequency-curve averaging technique to estimate the recurrence interval of various unregulated flow in the Lower Boise River. The same study estimated damages within the 500 year flood plain of the Boise River as a function of unregulated flow. Annually expected damage due to flooding was obtained by multiplying the exceedence probability of flood flow by the damage (cost) associated with those flood flows. Figure 13 shows the relationship between flood flow and annually expected damage.
damage after applying a multiplier of 2.5 to account for population growth and inflation since 1994.

![Graph showing expected damage as a function of peak flow at Glenwood Bridge](image)

**Figure 13:** Annually expected Boise basin flood damage as a function of unregulated flow at Glenwood Bridge

A separate USACOE Boise River water storage feasibility study (USACOE, 2010) calculated that, for adequate flood control, 60 days of storage would be required for each 1-cfs of peak flow. Such information defines the relationship between peak unregulated flow and required reservoir storage space and allows the expected damage to be translated from terms of flow into terms of reservoir storage as is shown in Figure 14. The reduction in annually expected flood damage with increasing flood storage space can then be represented by a fitted utility curve that has the form of a power function

\[
\text{flood storage utility} = F_1 \cdot (1 - F_2 \cdot \text{storage}^{F_3}),
\]

where \(F_1\) is the expected damage in the absence of all flood storage, and \(F_2\) and \(F_3\) are parameters which define the reduction in damage that results from the availability of flood storage. For the Boise River, the fitted flood storage utility function for the 60 day storage equivalent of unregulated flows of 16,600 cfs (a one in 100 year event) is,

\[
\text{flood storage utility} = 10^7 \cdot (1 - 0.030754 \cdot \text{storage}^{0.23972}).
\]
The fitted Boise basin utility function (Figure 14) is downward sloping because of the inverse relationship between downstream flood flows and the availability of flood storage space. In other words, increasing storage would correspond to a decrease in downstream flood flows and therefore a decrease in damages.

A backward extension of the utility curve produces an estimate of the utility of existing flood storage space. For example, in the absence of all flood storage, the annually expected damage due to flooding is estimated to be about $7.9 million. Assuming currently available flood storage is 987,000 AF, annually expected flood damage is reduced to about $1.6 million. The annual utility of current storage (i.e. the reduction in annually expected damages due to flooding) is therefore about $6.3 million.

![Figure 14: Utility function for Boise basin flood storage.](image)

The marginal utility of flood control storage is defined as the reduction in annually expected flood damage resulting from the availability of each additional AF of flood storage space. The flood storage marginal utility function, which is the derivative of (6), is then

\[
\text{flood storage marginal utility} = -F_s \cdot F_s^{\text{storage}^{-1}},
\]

and the fitted Boise basin marginal utility function is,

\[
\text{flood storage marginal utility} = -0.030754 \cdot (0.23972)^{\text{storage}^{-0.7021}}.
\]

\(^2\)Defining marginal utility in terms of an AF of flood control storage space is equivalent to defining marginal utility in terms of an AF of regulated flood release made to create an AF of storage space.
Equation 9 yields a demand price for each additional AF of flood storage (Figure 15). For example, given 5,000 AF of available flood storage, the demand price for one additional AF is $112.00, and given the currently available quantity of storage (987,000 AF) the demand price of one additional AF is $3.63. The marginal utility of flood storage decreases as storage space increases due to the fact that each additional AF reduces the annual expected damage from flooding.

![Figure 15: Marginal utility (demand-price) function for Boise basin flood storage.](image)

Current allocation of Boise River/reservoir system flood control storage space is based on rule curve operations. Rule curve requirements for flood control and irrigation storage are determined by runoff forecasts, carryover from the previous year, and snowpack (USBR, 2008). Rule-curve operations provide assurances that Boise River flows do not reach flood stage and that reservoirs refill to meet subsequent irrigation demand (USACOE, 1985). Assuming accurate forecasting, reservoir rule curve operations mean that demands for irrigation and flood control allocations of existing reservoir storage are mostly non-rival.

An increase in flood probability increases the marginal utility of flood control storage which is represented by an outward shift in the marginal demand-price function for flood storage. Shifts in demand representing 5-, 10-, and 20-fold increases in flood flow probability (Figure 16) approximate recent projections of increased flood potential in the Boise basin due to climate change (WCRP, 2012). Outward shifts in the marginal
Demand-price function translates to an increased willingness-to-pay for flood control storage, making flood control increasingly rival with irrigation. This increased willingness-to-pay applies not only to new rival storage, but to existing storage as well.

**Figure 16**: Shifts in the marginal utility function for flood control storage due to increased flood probability.

### Instream Flow Marginal Demand Prices

Demand for instream flows can be inferred from the willingness to pay for these hydro-services as public goods, with different individuals and groups expressing differing values for these flows. Estimation of the value of the benefits associated with instream flows cannot be estimated directly, and thus must be derived through either implicit analysis methods (e.g. travel cost or hedonic pricing analyses), or surveys of users of instream flow ecoservices which can include fishermen, boaters and wildlife viewers (Young, 2005 and Loomis, 2006). Because eco-service public goods tend to be non-consumptive, and thus non-competitive, the total demand-price is obtained by summing the individual demand prices (i.e. willingness-to-pay for fisheries, boating recreation, wildlife viewing etc.), which is necessary to accurately value the public goods, and for accurate CBA of water projects that affect river system eco-services (Figure 17). Nevertheless, since no one can be excluded from using public goods, their value is prone
to under valuing (the free-rider effect) and, as a consequence, public goods are likely to be under produced (Nicholson and Snyder, 2008).

Figure 17: Vertical summation of water demand-prices for two non-rival eco-services

The main eco-service generated by instream flows in the Henrys Fork basin is trout fishing. Two reaches of the Henrys Fork attract recreational anglers, the upper reach, located just below Island Park dam; and the lower reach, located just above St Anthony. Empirically derived equations (Van Kirk, 2012) describe fishable trout populations in both reaches as a function of instream flow. The marginal increase in trout population per AF of instream flow is obtained by calculating the derivatives of these equations. For the upper reach the marginal increase in the fishable trout population is given by
\[
\frac{dN_i}{dx}(\text{Island Park utility}) = 8.5603 \cdot 0.5276 \cdot \sum_{j=0}^{4} 0.4^j (x_{i-j-1})^{0.5276-1.0} 
\]  
(10)

where \(N_i\) is the fishable trout population in year \(i\), and \(x_{i-j-1}\) is instream flow (in AF) during three months following spawning (Dec, Jan, & Feb) for each of the previous five years. For the lower reach, which has a different spawning habitat, the marginal increase in fishable trout is,

\[
\frac{dN_i}{dx}(\text{St Anthony utility}) = 4.109 \cdot 0.5276 \cdot \sum_{j=0}^{4} 0.4^j (x_{i-j-1})^{0.5276-1.0} 
\]  
(11)

Inductive methods of valuation (revealed and stated preferences) indicate that Henrys Fork angler’s willingness-to-pay to catch one additional Cutthroat trout averages about $22.45 (Loomis, 2005). Marginal demand-price functions for instream flows to sustain this trout species is then obtained by multiplying (8) and (9) by this valuation of catching a single trout. For the upper reach this equates to

\[
\text{Island Park instream flow marginal utility} = 22.45 \cdot \frac{dN_i}{dx}(\text{Island Park utility}) . 
\]  
(12)

and for the lower reach to

\[
\text{St Anthony instream flow marginal utility} = 22.45 \cdot \frac{dN_i}{dx}(\text{St Anthony utility}) . 
\]  
(13)

The specific source of water supply determines whether the demands for instream flows in the two reaches are rival or non-rival with irrigation. If the source of supply is a storage release for irrigation that flows through the upper reach but is diverted before reaching the lower reach, irrigation demand is non-rival with instream flow demand in the upper reach, but rival with instream flow demand in the lower reach. If the source of supply is irrigation return flow that enters the river below the upper reach but above the lower reach, irrigation demand is non-rival with instream flow demand in the lower reach but rival with instream flow demand in the upper reach. If storage water is being released for operational purposes, or for downstream aquifer recharge, then the instream flow demands in both reaches are non-rival. Only when instream demands are non-rival is the total willingness-to-pay for instream flow equal to the vertical sum of the two marginal demand prices (Figure 18).
Step 4 – Solving the Integrated Problem

Once the marginal cost relationships have been developed for all of the water supply nodes (Step 1), the simulation of the conveyance of water between all of the supply and demand nodes has been developed (Step 2), and the marginal benefit relationships have been developed for all of the water demand nodes (Step 3), the Partial-Equilibrium, or optimal solution (Step 4), is defined as the point where the amount of water delivered from each supply node to each demand node maximizes the Consumer Surplus plus the Producer Surplus (CSPS). This can be represented mathematically as:

\[
\text{maximize } OF[q(i,j)]
\]

\[\text{s.t. } q(i,j) < \text{flow availability and limits}\]

\[q(i,j) < \text{minimum flow requirements}\]

\[\sum q(i,j) = Q_S - Q_L - Q_D\]

In which

\(q(i,j) = \text{the amount of water provide from supply node } j \text{ to demand node } i;\)

\[\sum q(i,j) = Q_S - Q_L - Q_D = \text{the water balance constraint, where:}\]

\(Q_S = \text{the amount of water provided by the supply nodes;}\)

\(Q_L = \text{the amount of water lost through conveyance from supply to demand nodes;}\)

\(Q_D = \text{the amount of water used by the demand nodes;}\)
\( OF = \) the Objective Function, defined as:

\[
OF[q(i,j)] = \sum_i B[q(i,j)] - \sum_i C[q(i,j)] - \sum_i \sum_j T_c[q(i,j)]; \text{ where:}
\]

\( B[q(i,j)] = \) the total benefits derived from using all water delivered to demand node \( i; \)

\( C[q(i,j)] = \) the total costs associated with providing water from supply node \( i; \)

\( T_c[q(i,j)] = \) the total cost of conveying water from supply node \( j \) to demand node \( i; \)

The fully developed Partial-Equilibrium Optimization problem can be solved utilizing a number of tools. For simple problems, the optimization utilities available in commercial spreadsheet analysis tools (e.g. Excel©) can be used to determine the allocation of water between supply and demand nodes that maximizes the objective function described above. For more complex problems, the solution of the PE Optimization problem may require specialized computer software that is specifically designed to solve optimization problems. One class of software that can be used are generic modeling systems, such as the GAMS© model, which links equations written in algebraic notation to commercial solvers that implement linear, integer, or non-linear optimization. These systems are flexible, transparent, self-documenting, and provide a simple link between model formulation and the solver solution. These characteristics have resulted in the early and widespread use of generic modeling systems by both economists and engineers in implementing hydro-economic models.

Another option is to develop computer software that is designed to specifically solve the PE Optimization problem for hydro-economic models. One example of this was the development of the HydroSense tool, a simple optimization solver written in the C# language that was designed to solve the PE Optimization problem. In brief, the HydroSense solver employs a Gradient Descent search method that utilizes numerical approximations of the first and second derivatives of the Objective Function with respect to the decision variables. The solution proceeds by developing an initial guess for the optimal decision variables which is then used to estimate the first and second derivatives of the Objective Function with respect to the array of decision variables. The decision variables are then updated by solving the linear system of equations as:

\[
\{dv^i\} = \{dv^{i-1}\} - \left[ \frac{\Delta^2 OF}{\Delta dv^2} \right]^{-1} \{\Delta \}
\]
Where:
\( d^i_v \) = the updated array of the estimated optimal decision variables for iteration \( i \) of the solution;
\( \Delta d^i v \) = the incremental change in the decision variable used to calculate the numerical estimates of the first and second derivatives of the Objective Function. This value is set to 0.01 within the Hydro$ense program.

\( \frac{\Delta \text{OF}}{\Delta d^i v} \) = the numerical estimates of the first derivative of the Objective Function (OF) with respect to the estimate of the optimal decision variables at iteration \( i-1 \); and
\( \left[ \frac{\Delta^2 \text{OF}}{\Delta d^i v^2} \right]^{-1} \) = the inverse of the matrix containing the numerical estimates of the second derivatives of the Objective Function (OF) with respect to the estimate of the optimal decision variables at iteration \( i-1 \).

At the end of each iteration, the updated optimal solution is checked to make sure that all of the problem constraints are met. If an updated decision variable falls outside of its constraint, the decision variable is set to equal its constraint limit and is then used in the optimal set of decision variables for the next iteration in the solution.

To aid in converging towards a stable solution, an adjustment to the diagonal values of the matrix (representing the second derivatives of the Objective Function with respect to the decision variables) is performed utilizing a Marquardt adjustment, defined as:

\[
1 - e^{(i-500) \times \Delta d^i v}
\]

The optimization solver will iterate towards the optimal solution using the procedure described above until the change in the values of the Objective Function and decision variables meet a user defined convergence tolerance, or the user defined maximum number of iterations is reached.

**Simplified PE Model Applications**

PE modeling of water policy alternatives using an economic objective function that is subject to physical and management constraints provides insights regarding benefits and efficiencies that are essential for Cost Benefit Analysis. To demonstrate this, one qualitative example explaining the use of PE modeling using the GAMS© program is provided, along with two simplified PE models that are solved using the
GAMS© modeling program. The first model provides a conceptual understanding of the hydro-economic PE modeling using a mixed complementary programming approach. The second model evaluates three alternatives for managing hydrologic externalities resulting from irrigation and canal seepage. The third model evaluates two alternatives for managing rival and non-rival water demands for instream flow public goods. The models are highly simplified representations of the Lower Boise basin and the Henrys Fork basin water management and planning alternatives, and the results presented here are for illustration purposes only.

Appendix B contains the annotated GAMS code for the simplified PE model with hydrologic externalities, and Appendix C contains the GAMS data file for this application. Appendix D contains the annotated code for the simplified PE model with rival and non-rival instream flow demands, and appendix E contains the GAMS data file for this application. The changes necessary for each application are described in the code along with the changes described in the Appendix C and E data files. Text annotations are indicated by a * in the first column.

Example 1: PE Modeling using Mixed Complementary Programming

When Takayama and Judge (1971) published their book, numerical optimization techniques were well understood, but mixed complementary programming (MCP) was in its infancy. With the advent of GAMS (Brooke et al. 1988) and accompanying solvers, it is now possible to formulate PE problems as complementary slackness equations in a mixed complementary problem and solve them directly. Five sets of complementary slackness equations, provided in Appendix B, define economic equilibrium conditions in the presence of hydrologic externalities:

- Equation 1 states that, at equilibrium, if the quantity of surface water demanded is greater than zero, demand price must equal marginal benefit from irrigation;

---

3 Although the current GAMS model does not incorporate a graphical user-interface a utility exists for developing GAMS model GUIs. (http://www.gams.com/dd/docs/tools/ask.pdf)

4 The operand denotes complementary slackness. Thus $x \ f(y)$ means $x \geq 0$, $f(y) \geq 0$ and $xf(y)=0$.
• Equation 2 states that, at equilibrium, if the quantity of groundwater supplied is greater than zero, supply price must equal marginal cost at equilibrium plus the externality marginal cost;

• Equation 3 states that, at equilibrium, if the quantity of surface water traded is greater than zero, the sum of supply price and transportation cost (i.e. cost of surface water seepage losses) must equal demand price;

• Equation 4 states that, at equilibrium, if the demand price is greater than zero, quantity of water demanded must equal the sum of all deliveries from supply nodes less seepage losses;

And equation 5 states that, at equilibrium, if supply price is greater than zero, quantity of water delivered must equal quantity of water produced at each supply node.

By solving these equations together, the PE model solution describes an allocation of water quantities and prices that is Pareto efficient, meaning that no other water allocation can provide further gain in total benefit without simultaneously creating an equivalent loss. PE models are capable of representing both aggregate Pareto efficiency, which maximizes the net benefits of a system irrespective of the allocation of water between demand nodes, and neutral Pareto efficiency, which incorporates social preferences in the efficiency objective (e.g. the valuation of public goods such as river system eco-services).

Figure 19 illustrates the PE model Pareto optimal equilibrium solution for a single water supply and demand node with a non-binding supply constraint (i.e. the supply is more than sufficient to satisfy the demand, with the optimal solution occurring where the supply and demand cost curves intersect). Consumer surplus (or net benefit) is defined as the difference between what the demand nodes are willing to pay (characterized by the demand function) and what they are required to pay (i.e. the equilibrium price generated in the PE model solution) for a particular quantity of water.
Figure 19: PE model equilibrium solution with non-binding supply constraint.

Figure 20 illustrates the PE model solution for a single water supply and demand node that is not Pareto optimal because of a binding supply constraint (i.e. the supply is not sufficient to satisfy demand and the supply and demand cost curves do not intersect). Relative to the equilibrium solution in Figure 19, consumer surplus (net benefit) is reduced due to the binding constraint. When a supply constraint is binding, the PE model calculates the constraint cost (or shadow price), which in the illustration is the willingness-to-pay for one more AF of water in order to relax the binding constraint. Constraint costs are important model results that can reveal the marginal value of eliminating infrastructure bottlenecks such as new reservoir storage for irrigation or flood control. Shadow prices can also reveal the opportunity cost to society resulting from restricted public goods such as instream flows.
Example 2: Managing Hydrologic Externalities in the Lower Boise Basin

The PE model application incorporating hydrologic externalities is demonstrated using a much simplified model comprised of just three nodes, a reservoir supply node and two irrigation demand nodes representing a canal user and a groundwater pumper (Figure 21).

Jointness-of-production occurs as a result of canal seepage losses which hydrologically link the canal irrigator’s reservoir supply to the groundwater irrigator’s aquifer supply. A hydrologic externality arises due to the fact that the canal seepage contribution to the groundwater supply is un-priced. In this example, three alternatives for dealing with the hydrologic externality are modeled. They include eliminating the externality (eliminating...
seepage), pricing the externality, or a tax/subsidy scheme whereby negative externalities are taxed and positive externalities are subsidized (Taylor et al., 2014).

**Base-case Scenario**

The base-case is the “without” scenario, as is required in a “with-versus-without” CBA. In the base-case, the aquifer is connected to the leaky canal for some portion of the canal length. The groundwater pumper receives a positive externality of reduced pumping lift due to a decrease in depth to groundwater caused by canal seepage and pumping inflicts a negative externality upon canal users by inducing additional seepage.

In the surface water market, node S1 supplies 3,097 AF priced at $15/AF, of which 2,130 AF reaches node X2, who is willing-to-pay $21.65/AF for the water delivered at the canal end. Node X2 makes a payment to node S1 in the amount of $46,406 ($15/AF × 3,097 AF), which includes $14,505 ($15/AF × (3,097 AF - 2,130 AF)) for water not received, but lost to canal seepage. In the groundwater market, node X3,3 pumps 1,283 AF (286 AF of induce seepage, plus 681 AF of passive seepage, plus 316 AF from sources other than seepage) for which he pays $30.65/AF in pumping costs. The node X3,3 pumper thus makes a payment to the node S3 supplier (i.e., the power company) in the amount of $39,325 ($30.65/AF × 1,283 AF). The total base-case surplus (benefit) totals $164,087, where $90,900 is node X2 consumer surplus, $64,242 is node X3 consumer surplus, and $8,946 is node S3 producer surplus. The horizontal supply function of node S1 yields no producer surplus because the supply cost is a single block rate, thus there is no marginal increase in the cost with an increasing amount of water supplied from node S1.

**Pigouvian Tax/Subsidy Scenario**

In a competitive equilibrium, the welfare of two agents depends only on consequences of their own choices. An externality creates an asymmetry between social and private prices, while internalization of the externality forces both agents to account for the consequences of their actions on the other’s welfare by aligning prices. Absent this internalization (as in the base-case), the canal water user responds only to the supply
cost of the canal company and the “shrinkage” cost of seepage. In the base-case, the groundwater pumper responds only to changes in the cost of pumping, while the reduction in costs that the pumper receives from canal seepage is ignored, as is the increased cost borne by the canal diverter for pumping-induced seepage.

A Pigouvian tax/subsidy internalizes the externality by aligning marginal supply prices of canal diverters and groundwater pumpers. The price alignment creates a “signal” to decrease production of the negative externality (pumping induced seepage) and increase production of the positive externality (canal diversions that create seepage). To internalize the hydrologic externality, the canal water supply function is redefined as the marginal cost at node $S_1$, plus the negative externality of seepage in the conveyance of water from node $S_1$ to node $X_2$, plus a Pigouvian subsidy (represented by $\beta$), plus feedback from the pumping tax. The Pigouvian subsidy equals the reduction in marginal cost provided by canal seepage to the node $X_3$ groundwater pumper. Similarly, the groundwater supply function is redefined as the cost of pumping at node $X_3$, plus a Pigouvian tax (represented by $\alpha$) that is equal to the marginal cost of pumping-induced canal seepage for the canal diverter at node $X_2$, plus feedback from the pumping tax (which creates a signal that reduces pumping).

Internalization of the externality through a Pigouvian tax/subsidy increases the welfare of both the groundwater pumper and canal diverter. The tax/subsidy causes a downward shift in the supply-cost curves for the canal irrigators (the supply cost at the end of the canal). This is due to the canal irrigator's marginal supply cost at the end of the canal shifting downward because the Pigouvian subsidy reduces the cost associated with canal seepage. The resulting feedback also causes a downward shift in the supply-cost curves for the groundwater irrigators. This occurs as a result of the feedback from the Pigouvian tax which reduces groundwater pumping and pumping-induced canal seepage. In this scenario, the equilibrium price of groundwater fell from $30.65/AF in the base-case to $25.72/AF and equilibrium pumping increased from 1,283 AF in the base-case to 1,306 AF. In a with-versus-without CBA comparison, benefit in the groundwater market increases by 10% and benefit in the surface water market increases by 34%, while total irrigator benefit increases by 21% relative to the base-case.
Eliminating Seepage

In this scenario, the hydrologic externality is eliminated through lining of the leaky canals. When the canals are lined, the cost of seepage is no longer imposed upon the canal diverter and the quantity of water supplied at node X₁ equals the quantity demanded at node X₂. Absent canal seepage, pumping costs in this scenario increase from $30.65 (base-case scenario) to $95.24/AF due to increased depth to groundwater, which in turn results in a decrease in groundwater pumping. The consumer surplus of the surface water market increases by 16% as a result of the increased canal efficiency, but is offset by a 64% decrease in consumer and producer surplus in the groundwater market. With-versus-without CBA comparison reveals a 67% decline in total surplus as a result of canal lining. Note that, in this example, construction costs are ignored and the sole beneficiary of canal lining is the short run increase in irrigation intensity of the existing crop mix and acreage of the canal water user.

Aquifer Recharge Payment

In contrast to the external Pigouvian tax/subsidy scenario, payments for aquifer recharge are internal, that is the groundwater irrigator pays to receive the benefit of aquifer recharge from surface water. The canal diversion is priced via a payment from node X₃ to node X₂ that matches the decrease in total pumping cost that is attributable to canal seepage.

The marginal cost of pumping with respect to diversion is calculated by integrating the groundwater pumpers’ marginal cost function with respect to pumping yield and then differentiating with respect to canal diversion. This produces a function that represents the cost of groundwater pumping, plus the cost of the water that seeps from the canal into the groundwater, priced as if it were a canal diversion. By definition, groundwater pumpers maximize their benefits by paying the canal diverter this amount for each acre-foot of water diverted down the canal.

Table 2 summarizes the equilibrium results from the aquifer recharge scenario along with the results from the other scenarios discussed here. In the groundwater market, the transfer payment from node X₃ to node X₂ increases the marginal cost for node X₃ to
$51.68 per/AF, relative to the base-case scenario cost of $30.65 per AF. As a consequence, groundwater pumping decreases relative to the base-case from 1,283 AF to 1,124 AF. In the surface water market, the transfer payment reduces node X2 marginal cost relative to the base-case from $21.65 per AF to $9.79 per AF, and as a result, the quantity of water delivered from node X1 to node X2 increases and the total payment made by node X3 increases to $29,445. Consumer surplus increases by 30% in the surface water market (node X2) and decreases by 40% in the groundwater market (node X3). Although the managed recharge scenario does not penalize pumping-induced seepage, the CBA total surplus from this scenario exceeds that of the base-case. In contrast to the tax/subsidy remedy, which corrects both sides of the reciprocal externality, the recharge payment sustains only the positive externality of seepage.

**Example 3: Managing Rival and Non-Rival Water Demands in the Henry’s Fork Basin**

The PE model application with rival and non-rival demands for instream flow public goods is demonstrated using a model comprised of just four nodes, a reservoir supply node an irrigation demand node, and two spatially distributed demands nodes for instream flow to support fisheries (Figure 22). The application consists of three scenarios:

- The base-case scenario calculates instream flow allocations and benefits assuming the two instream flow demands and irrigation demand are all rival.
- The second scenario assumes the two instream flows are non-rival with one another in meeting fisheries demands, but rival with irrigation demand.
- The third scenario assumes that the two non-rival instream flow demands are also non-rival with specific irrigation demands (i.e. irrigation water storage releases made during winter months as part of reservoir operations or for aquifer recharge).

In the base-case scenario, the net benefit is determined by summation of the instream flow demand quantities (along the horizontal axis) for the two reaches. In the second, the net benefit is determined by summation of instream flow demand prices (along the vertical axis), as these flow demands are not in competition with each other. In the third
scenario, the net benefit is determined by the summation of instream flow for fisheries and irrigation demand prices (along the vertical axis), as the flows for fisheries and irrigation are not in competition with one another. Once again, the base-case is the “without” scenario, as required for conducting a with-versus-without CBA.

\[
\begin{align*}
\text{reservoir supply } X_i &= X_1 \\
\text{river flow } X_{ij} &= X_{1,2} \\
\text{irrigation demand } X_j &= X_2 \\
\text{fisheries demand } X_j &= X_3
\end{align*}
\]

Figure 22: Schematic of four node PE model with rival and non-rival water demands.

Results from the base-case PE model scenario (which assumes that the sources of supply for instream flows in both reaches will result in rival instream flow demand conditions) generates the lowest total surplus for fisheries ($5,539). The second PE model scenario (which assumes sources of supply for the two HF reaches result in non-rival instream flow demand conditions) generates a total surplus for fisheries that is greater than the base-case by a factor of four ($21,104). Finally, the third PE model scenario (which assumes that instream flow demands are non-rival with specific irrigation demands) generates total fisheries surplus that is nearly two orders of magnitude greater than the base-case (all rival) scenario ($584,178).

Of the three scenarios, the third scenario is the closest to approximating the actual management of instream flows for fisheries in the Henrys Fork (HFAG/JPC, 2005). The difference between the total benefits for scenario 3 and scenario 2 ($584,178-$21,104) is therefore closest to representing the value of instream flows to Henrys Fork fisheries that can be derived from the use of Henrys Fork reservoir storage being managed for both irrigation and fisheries.
References


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methods. Resources for the Future, Washington, DC.
Appendix A  IDEP Demand Function Calculator

The underlying production function developed by Martin and others (Evaluation of Irrigation Planning Decisions. Journal of Irrigation and Drainage Engineering. Vol. 115, No. 1, February 1989, 58-77) is expressed in equation (1) with altered notation:

\[
Y = Y_m - Y_d - \frac{ET_m - ET_d}{I_m} \cdot \frac{I}{m} \Rightarrow (1)
\]

where

- \( Y \) = crop yield (yield units/area)
- \( Y_m \) = crop yield at full irrigation (same units as \( Y \))
- \( Y_d \) = non-irrigated (dry land) crop yield (same units as \( Y \))
- \( I \) = irrigation depth (length)
- \( I_m \) = irrigation depth at full irrigation (same units as \( I \))
- \( ET_m \) = evapotranspiration at \( Y_m \) (same units as \( I \))
- \( ET_d \) = evapotranspiration at \( Y_d \) (same units as \( I \))
- \( B = \frac{ET_m - ET_d}{I_m} \) (unitless) \[1\]

For the spreadsheet tool, "\( I_m \)" is assumed to include any leaching requirement. \[2\]

Substituting "\( a \)" for \( \frac{1}{B} \), equation (1) can be rearranged as:

\[
Y = Y_m - Y_d - \frac{ET_m - ET_d}{I_m} \cdot \frac{I}{m} \Rightarrow (2)
\]

Multiplying yield by irrigated area (\( A \)) and commodity price \[3\] (\( P_c \)) gives the gross revenue (\( R \)):

\[
R = AP \cdot Y_m - AP \cdot Y_d - \frac{AP 
(ET_m - ET_d)}{I_m} \Rightarrow (3)
\]

The derivative of revenue with respect to irrigation depth (\( I \)) is:

\[
\frac{dR}{dI} = \frac{1}{m} \cdot aAP \cdot \frac{I}{m} \cdot \frac{I}{m} \Rightarrow (4)
\]

The derivative "\( dR/dI \)" is the marginal production value of water \[4\] and may be considered the willingness-to-pay for irrigation water, or the water-depth demand price "\( Pwd. \)". Solving equation (4) for irrigation depth, the depth of irrigation water demanded as a function of price is:

\[
m^{\frac{1}{(a-1)}} \left( \frac{mBP_{wd}}{AP_c} \right) \Rightarrow (5)
\]
Equation (5) gives a relationship between depth of irrigation demanded and price per depth of irrigation. The units of Pwd (price per water depth) are (currency units/length). We need price in terms of water volume, and irrigation in terms of volume. Pwv (price per water volume) has units (currency/length^3), so Pwd = Pwv times area (currency/length^3 x length^2 = currency/length). Substituting Pwv * A for Pwd, and multiplying all of equation (5) times depth to obtain volume, gives equation (6), the volume of irrigation demanded as a function of the price per volume:

\[ V = A \ln \left( \frac{m \cdot \text{BP}_{wv}}{\text{Pc}(m^d)} \right)^{1/(a-1)} \]  

(6)

This equation will give a nonsensical result of negative volumes of water at high prices; therefore, the spreadsheet uses equation (7) which includes a conditional test:

\[ V = \max \left( 0, A \ln \left( \frac{m \cdot \text{BP}_{wv}}{\text{Pc}(Y_m - Y_d)} \right)^{1/(a-1)} \right) \]  

(7)

If the contemplated use of the composite demand function can accommodate multiple conditional tests, then the composite demand for the farm or region in question is simply the horizontal summation of all individual crop demands:

\[ V = \sum \left( 0, A_i \ln \left( \frac{m \cdot \text{BP}_{wv}}{\text{Pc}(m^d)} \right)^{1/(a-1)} \right) \]  

(8)

Where subscript "i" denotes an individual crop, with its unique acreage and other parameters.

For uses where the contemplated use of the demand function cannot accommodate conditional statements for each component of the summation, the spreadsheet tool offers an opportunity to manually calibrate two approximations of the composite demand function:

\[ V = b_0 + \frac{b_1}{(P_{wv} - b_3)} + b_2 (P_{wv} - b_3) \]  

(9)

\[ = b_4 (P_{wv} + b_5)^{b_6} + b_7 \]  

(10)

where

- \( b_7 \) = empirical parameter.
Values from the crop worksheet may also be used in regression equations to estimate demand equations. All these approximations will give nonsensical results beyond the price-axis and quantity-axis intercepts. Therefore, if any of the equations are to be used in further computer processing, steps must be taken to limit calculations to an appropriate reasonable range of values.

End Notes
[1] Parameter "B" is closely related to irrigation efficiency at full irrigation depth, depending on the particular definition of efficiency.
[2] See leaching requirement worksheet for assumptions regarding leaching requirements.
[3] "Pe" is the net price after deducting per-unit harvest costs such as hay twine or drying.
[4] This derivative depends on the important assumptions that commodity prices are perfectly competitive (i.e. independent of local production quantity) and that allocation of crop acres is fully constrained by considerations besides water supply.

EXPLORATION OF PRODUCTION FUNCTION EQUATION

Not all the parameters of equation (1) are physically or conceptually independent. In the spreadsheet tool, the following parameters are variables that the user may input:

- $I_m$  Irrigation depth at full yield
- $ET_m$  Evapotranspiration depth at full yield
- $Y_m$  Yield at full irrigation
- $Y_d$  Dryland Yield
- $P_c$  Price of commodity (net of per-unit harvest costs)

Guidance worksheets aid in selecting these parameters. The remaining parameters are calculated by the spreadsheet:

\[
ET_d = \left( \frac{d}{m} \right) ET_m
\]

\[ a = \frac{1}{B} \text{ no italics} \]

\[ K = \frac{1}{I_m} \text{ no italics} \]

The calculation of $ET_d$ depends on an assumption that the yield/evapotranspiration relationship is approximately linear with an intercept near zero (see FAO56 and FAO33). Martin and others (1989) defined the calculation of B.
Figure 1 shows the relationship between the yield curves generated by equation (1) using three pairs of values for the interrelated parameters Im and B. The other parameters are:

- \( ET_m = 2 \) feet
- \( Y_m = 5 \) tons
- \( Y_d = 1 \) ton

Theoretical

![Graph of Figure 1. Yield/Irrigation relationship from production function equation.](image)

In theory, the yield would begin to decline at application depths beyond "full" irrigation, as illustrated by the "theoretical" curve in Figure 1. However, except when parameter "I/B" happens to be an even integer, equation (1) gives a spreadsheet error when depth of irrigation is greater than or equal to full-yield irrigation. This is not a serious limitation; for most economic studies, this range of the production function is not of interest, since rational producers will not enter this region.

ECONOMIC DEMAND FOR IRRIGATION WATER

The production value and hence willingness-to-pay (i.e. demand price) are derived from the slope of the production function. The \( B = 0.99 \) curve illustrates that at very high irrigation efficiency, the slope is nearly constant, up to full production. The low-efficiency curve shows a marked decline in slope as depth of irrigation increases. These characteristics affect the calculation of production value of various depths of irrigation.
water (using equation (7)), as shown in Figure 2. The figure is consistent with expectations from examining Figure 1. A commodity price of $100/ton unit was used, with 100 acres of crop.

![Figure 2. Demand for irrigation water at different values of B.](image)

At first glance, Figure 2 may not match intuitive expectations. However, comparison of the high-efficiency curves with the low-efficiency curves actually makes sense. For instance, at $200/acre foot, the 80%-efficiency user is able to profitably utilize up to 118 acre feet, but the low-efficiency user cannot extract as much economic value and therefore is only willing to use 46 acre feet. Once the price drops to $100/acre foot, the 80%-efficiency user purchases an essentially full supply, so that any further price reduction does not entice meaningful further purchases. However, the low efficiency user can still extract some marginal benefit of additional water even up to 600 acre feet, if the price is low enough. The price intercept of individual demand curves is defined by the value of the crop. These curves represent the same crop; they all have very similar price intercepts because physically, at very low application depths, nearly all of the water is used for crop production (irrigation efficiency begins to approach 100% for any application method). In the production-function equation, this characteristic is achieved by entering \((1/B)\) as an exponent. The quantity intercept is defined by the crop acreage. In Figure 3, both curves have identical parameters, except that one curve is for 100 acres and the other is for 200 acres.
EXPLORATION OF HORIZONTAL SUMMATION

The standard construction of aggregate demand is to horizontally sum individual demands. The summation process can produce a convex-to-the origin aggregate demand curve even when individual demand curves may be knee shaped, as shown in Figure 4. One can imagine that if this were an aggregation of hundreds or thousands of individual demand curves, the aggregate demand could indeed become a smooth curve.
DERIVATION OF EQUATIONS WITH INDEPENDENT VARIABLES

Equation (6) from above is repeated:

\[
= \text{Al}_m \cdot \text{Al}_m \left( \frac{m_{m\text{BP}}{m\text{BP}}}{P_c(m - d)} \right)^{1/(a-1)}
\]

Equation (6) is defined using readily-available input data, but these data are not independent. Therefore, marginal analyses using partial derivatives of equation (6), or iterative exploration by varying one input value at a time will not be valid. To derive equations of only independent exogenous variables, the following simplifications and assumptions are relied upon:

1. The relationship between yield and evapotranspiration is linear (this is implicit in the form of equation (6)). This leads to the following relationships:

   \[ Y_m = K_1 \ ET_m \]  
   \[ Y_d = K_1 \ ET_d \]  

   Where \( K_1 \) is a crop-specific yield coefficient.

2. ET at the dry-land yield equals effective precipitation (Re). This leads to two additional relationships:

   \[ ET_d = Re \]  
   \[ Y_d = K_1 \ Re \]  

3. The relationship that defines \( B \) is a function of irrigation system, crop agronomy and management. It will be essentially unaffected by the range of climate changes for which these simplifications are appropriate. This leads to:

   \[ I_m = a(ET_m - Re) \]  

   Note that if effective precipitation exceeds \( ET_m \), \( I_m \) will be negative. This is simply an indication that irrigation is not required; the magnitude of \( I_m \) is the depth by which effective rainfall could decrease without affecting yield (assuming appropriate temporal distribution of rainfall).

Substituting these simplifications into equation (6) gives equation (20):

\[
= Aa \left( ET_m^{-} \right)^{-} - Aa \left( ET_m^{-} \right)^{\left( \frac{P_{BW}}{P_c} \right)^{1/(a-1)}}
\]  

(20)
Implicit in these simplifications is an assumption that (K1) and (a) are independent of climate change. If one further assumes that (Pc) is independent of (Pwv) and climate, the following rates of change can be derived from equation (20):

\[
\frac{\partial V}{\partial P_{wv}} = \frac{-1}{(a-1)} Aa \left( ET_m \cdot Re \right) \left( \frac{1}{P_{c1}} \right)^{\left( \frac{1}{(a-1)} \right)} \left( P_{wv} \right)^{\left( \frac{1}{(a-1)} \cdot 1 \right)}
\]  

(21)

\[
\frac{\partial V}{\partial ET_m} = Aa \left( \frac{P_{wv}}{P_{c1}} \right)^{\left( \frac{1}{(a-1)} \right)}
\]  

(22)

\[
\frac{\partial V}{\partial Re} = -Aa \left( \frac{P_{wv}}{P_{c1}} \right)^{\left( \frac{1}{(a-1)} \right)}
\]  

(23)

\[
\frac{\partial V}{\partial P_c} = \frac{1}{a-1} Aa \left( ET_m \cdot Re \right) \left( \frac{P_{wv}}{P_{c1}} \right)^{\left( \frac{1}{(a-1)} \right)} \left( P_{c} \right)^{\left( \frac{-1}{(a-1)} \cdot 1 \right)}
\]  

(24)
Appendix B  GAMS PE Model Code with Hydrologic Externalities

$ONTEXT
Partial Spatial Equilibrium Water Distribution Model
Version 14.0

Active model parameters in this file represent base-case scenario conditions

$OFFTEXT
* set path to data set
$SETGLOBAL PROGPATH "C:\watermodel\Denver_folder\"
$INCLUDE "$PROGPATH%base_data_final.gms"
$SETGLOBAL TEXTNAME test

* allow empty data sets to be initialized
$ONEMPTY

* choose solvers: PATH is mixed complementary program solver
  *    MINOS is non-linear programming optimization solver
  * The MINOS NLP solution is used here as a check on PATH MCP solution
OPTION MCP = PATH;
OPTION NLP = MINOS;

*print formatting
OPTION LIMCOL = 3, LIMROW = 3;

* QS3LIMIT is a constraint on gw pumping = quantity pumped in the
  managed recharge (direct payment) scenario.
* Also used as a water right constraint.
SCALAR QS3LIMIT
  /1124.,
99999999.0
/;

* list all variables in the model
VARIABLES

* node 1 is canal diverter at river point of diversion
* node 2 is canal diverter at head gate
* node 3 is groundwater pumper
* WELFARE is the maximized NLP value of objective function
QD2 quantity demanded at node 2
QD3 quantity demanded at node 3
QS1 quantity supplied at node 1
QS3 quantity supplied at node 3
X12 quantity transported from node 1 to node 2
X33 quantity transported from node 3 to node 3
RHOS1 supply price at node 1
RHOS3 supply price at node 3
RHOD2 demand price at node 2
RHOD3          demand price at node 3
BETA            marginal benefits received from seepage at node 3
ALPHA           marginal cost with respect to QS3 of induced
seepage
RHOCON          price of constraint

POSITIVE VARIABLES QD2, QD3, QS1, QS2, X12, X33, RHOD2, RHOD3, RHOS1, RHOS3, RHOCON;

* list of all equations in the model
EQUATIONS
  EQ1C
  EQ2C
  EQ3C
  EQ4C
  EQ5C
  EQ6C
  EQ7C
  EQ8C
  EQ9C
  EQ10C
  ALPHACALC
  BETACALC
  QCONSTR

;  
* Marginal demand-price functions for canal diveters, node 2. All demand
prices are GE 0.
  EQ1C..
  * Marginal demand-price function, P=f(Q), NOT compatible with IDEP
demand calculator coefficients.
  * RHOD2 = B20*(1-(B21*QD2)**B22) =G= 0

* Inverse of marginal demand-price function, Q=f(P), Compatible with
IDEP demand calculator coefficients.
  RHOD2-(1/B21*(-(QD2-B30)/B20)**(1/B22)) =G= 0
  
* Marginal demand-price functions for gw pumpers, node 3. All demand
prices are GE 0.
  EQ2C..
  * Marginal value function  P=f(Q), NOT compatible with IDEP demand
calculator coefficients.
  * RHOD3 = B30*(1-(B31*QD3)**B32) =G= 0

* inverse of marginal demand-price function, Q=f(P) Compatible with
IDEP demand calculator coefficients.
  RHOD3-(1/B31*(-(QD3-B30)/B30)**(1/B32)) =G= 0

* Canal supply price at river (node 1) is ge 0.
  EQ3C..
  A10 - RHOS1 =G= 0

;
Pigouvian Tax-Subsidy insertion

Supply-cost functions with and without Pigouvian tax on gw pumpers and Pigouvian subsidy to canal diverters are inserted here.

ALPHA is the pigouvian tax that groundwater pumpers pay for the damage done to canal diverters. ALPHA is added to the per AF supply cost of groundwater pumping RHOS3.

EQ4C.

* Groundwater supply-cost function with pigouvian tax
* Groundwater supply cost (RHOS3) is Groundwater supply cost with seepage (function) + tax paid by gw pumpers to the state (ALPHA) + pumping constraint cost (RHOCON) if any.

A30 + A31*[A32*EXP(A33*QS3-A34*X12)] + ALPHA - RHOS3 + RHOCON =G= 0

Groundwater supply-cost function without pigouvian tax.

A30 + A31*[A32*EXP(A33*QS3-A34*X12)] - RHOS3 + RHOCON =G= 0;

BETA is the subsidy that canal diverters get for the seepage benefit provided to gw pumpers. NOTE: THE CALCULATION OF BETA ASSUMES THAT THE CANAL IS UNLINED. BETA WILL BE NON ZERO EVEN WHEN THE CANAL IS LINED.

EQ5C.

* Canal demand-price at node 2 (end of canal) with pigouvian subsidy. Demand-price at the end of the canal is GE canal supply-cost at the head of the canal (RHOS1) + cost of canal seepage - subsidy from state

RHOS1 - RHOD2 + RHOD2*(C0*C1*EXP(-C1*X12) + C2*(1-EXP(-C3*QS3))) - BETA =G= 0

Canal demand-price at node 2 without pigouvian subsidy.

RHOS1 - RHOD2 + RHOD2*(C0*C1*EXP(-C1*X12) + C2*(1-EXP(-C3*QS3))) =G= 0;

Quantity supplied at node 3 (gw pumper) is GE quantity demanded at node 3

EQ6C.

RHOS3 - RHOD3 =G= 0;

Quantity transported from node 1 to node 2 - canal seepage (via seepage function) = quantity demanded at node 2.

EQ7C.

X12 - C0*(1-EXP(-C1*X12)) - C2*X12*(1-EXP(-C3*QS3)) - QD2 =G= 0;
* Quantity of groundwater pumped at node 3 (i.e. transported) = quantity groundwater demanded at node 3.
EQ8C..
    X33 - QD3 =G= 0
;
* Quantity of canal water supplied at node 1 is GE to the quantity transported from node 1 to node 2.
EQ9C..
    QS1 - X12 =G= 0
;
* Quantity of groundwater pumped at node 3 is GE to the quantity transported from node 3 to node 3.
EQ10C..
    QS3 - X33 =G= 0
;
* The groundwater pumping constraint at node 3 is GE to the quantity pumped at node 3.
QCONSTR..
    QS3LIMIT - QS3 =G= 0
;
********************************* ALPHA and BETA calculation***************************************************
BETACALC..
*       Calculation of BETA, the subsidy received by canal diverters.
*       BETA is calculated by integrating the marginal pumping cost function between pumping rate 0 and QS3,
*       yielding the total cost of pumping QS3 AF of groundwater. The derivative
*       with respect to canal diversion X12 then yields QS3 pumping cost per AF of canal diversion.

    BETA =E= A31*A32*A34*EXP(-A34*X12)*(EXP(A33*QS3)-1)/A33

* CANAL MUST BE UNLINED IF BETA SUBSIDY EQUATION IS INCLUDED IN EQ5C.
;
ALPHACALC..
* Calculation of ALPHA, the tax paid by gw pumpers for (induced) seepage damage to canal diverter.
*    ALPHA is calculated by integrating the seepage cost function between diversions 0 and X12, yielding
*    the total cost of seepage for X12 AF of diversion. The derivative with respect to QS3 then yields
*    the damage per AF of groundwater pumped.

    ALPHA =E= RHOD2*C2*C3*X12*EXP(-C3*QS3)

* For Managed Aquifer Recharge or Coase Scenario
* ALPHA*QS3 is set equal to BETA*X12
* In the Coase scenario gw pumpers make a direct payment to canal diverters equal to the
* benefit they derive from canal seepage.

*    ALPHA*QS3 =E= BETA*X12
;
* A model called EXTMODEL is defined by the following equations with specification of variable results to be displayed.

MODEL EXTMODEL
/
EQ1C.QD2
EQ2C.QD3
EQ3C.QS1
EQ4C.QS3
EQ5C.X12
EQ6C.X33
EQ7C.RHOD2
EQ8C.RHOD3
EQ9C.RHOS1
EQ10C.RHOS3
ALPHACALC
BETACALC
QCONSTR.RHOCON
/
;
* ALPHA AND BETA ARE solved for by the model. Initial values are required.
ALPHA.L = 0;
BETA.L = 12;

* The EXTMODEL uses a mixed complementary programming solver. The MCP solver is PATH.
* The EXTMODEL is solved twice for greater accuracy. the second uses results from first
* as starting values. Only the results from the second solution are displayed.
SOLVE EXTMODEL USING MCP;

ALPHA.L = 0;
BETA.L = 12;

SOLVE EXTMODEL USING MCP;

* Seepage is calculated using EXTMODEL results
SCALAR SEEPAGE;
SEEPAGE = C0*(1-EXP(-C1*X12.L)) + C2*X12.L*(1-EXP(-C3*QS3.L));

* EXTMODEL results are displayed
DISPLAY
PAGE,ALPHA.L,BETA.L;

* Consumer surpluses are also calculated using EXTMODEL results.
* The calculation depends on the form of the demand function used, whether from the IDEP calculator or not.
SCALAR CONSUP2, CONSUP3, PROSUP3, tt1, tt2, tt3, TOTSUP;

* Consumer surplus with canal diverter marginal value function \( P = f(Q) \)
* \( \text{CONSUP2} = B20*QD2.L - (B20*B21*QD2.L**(B22+1))/(B22+1) - QD2.L*RHOD2.L; \)

* Consumer surplus with inverse canal diverter marginal value function \( Q = f(P) \) coming from IDEP calculator.
  \( \text{CONSUP2} = \left(-\frac{B20}{B21}\right) \left(\frac{B22}{1+B22}\right) \left(-\frac{QD2.L-B20}{B20}\right)^{\left(\frac{1+B22}{B22}\right)} - \left(-\frac{B20}{B21}\right) \left(\frac{B22}{1+B22}\right) - QD2.L*RHOD2.L; \)

* Consumer surplus with gw pumper marginal value function \( P = f(Q) \)
* \( \text{CONSUP3} = B30*QD3.L - (B30*B31*QD3.L**(B32+1))/(B32+1) - QD3.L*RHOD3.L; \)

* Consumer surplus with inverse gw pumper marginal value function \( Q = f(P) \) coming from IDEP calculator.
  \( \text{CONSUP3} = \left(-\frac{B30}{B31}\right) \left(\frac{B32}{1+B32}\right) \left(-\frac{QD3.L-B30}{B30}\right)^{\left(\frac{1+B32}{B32}\right)} - \left(-\frac{B30}{B31}\right) \left(\frac{B32}{1+B32}\right) - QD3.L*RHOD3.L; \)

******************************************************************************
* Groundwater pumper producer surplus
******************************************************************************
* \( tt1 \) and \( tt2 \) are the reductions in gw producer surplus due to gw pumping
  \( tt1 = A31*A32*\exp(-A34*X12.L)/A33; \)
  \( tt2 = \exp(A33*QS3.L)-1; \)

* \( tt3 \) is the contribution of canal seepage to the gw producer surplus.
  \( tt3 = -RHOD2.L*C2*X12.L*\exp(-C3*QS3.L)-1; \)
  \( \text{PROSUP3} = QS3.L*RHOS3.L - A30*QS3.L - tt1*tt2+tt3; \)

******************************************************************************

* Total consumer surplus
\( \text{TOTSUP} = \text{CONSUP2} + \text{CONSUP3} + \text{PROSUP3}; \)
DISPLAY CONSUP2, CONSUP3, PROSUP3, TOTSUP, RHOCON.L;

* calculate total payment by groundwater pumpers for canal seepage, either tax or damages
SCALAR DSEEP, VSEEP, TPAY, ESEEP;

DSEEP = RHOD2.L*C2*C3*X12.L*\exp(-C3*QS3.L);
VSEEP = DSEEP*RHOD2.L;
TPAY = VSEEP*QS3.L;
ESEEP = TPAY/X12.L;
DISPLAY DSEEP, VSEEP, TPAY, ESEEP;

* Display groundwater pumper producer surplus
DISPLAY tt3;
* \( \text{PROSUP3} = QS3.L*RHOS3.L - A30*QS3.L - tt1*tt2+tt3; \)
DISPLAY PROSUP3, tt1, tt2;

* \( \text{qpay} \) is the subsidy/AF of canal diversion at the river multiplied by quantity diverted
SCALAR QPAY total benefits of water in canal for pumper;
* xpay is the tax/AF of pumping multiplied by quantity pumped
SCALAR XPAY total damages caused by pumping;

XPAY = ALPHA.L*QS3.L;
QPAY = BETA.L*X12.L;
DISPLAY QPAY,XPAY;
$EXIT
Appendix C  GAMS PE Model Data for Hydrologic Externalities

*Active variables in this file represent base-case scenario conditions

SCALAR A10 parameter for the supply function for node 1
  * 13.27
   15.00
  /;

SCALAR A30 first parameter for the supply function for node 3
  * 9.46
   9.5
  /;

SCALAR A31 second parameter for the supply function for node 3
  0.08
  /;

SCALAR A32 third parameter for the supply function for node 3
  * 132.27
   * 900
   1000
   /;

SCALAR A33 fourth parameter for the supply function for node 3
  * 1.6959E-4
   * 3.0E-4
   1.7e-4
  /;

SCALAR A34 fifth parameter for the supply function for node 3
  * Along with canal seepage coefficients, A34 must be set to zero when canal is lined.
  * This is so the marginal cost function for the gw pumper does not include the benefit of canal diversion when the canal is lined.
  * (below)
  * A30 + A31*[A32*EXP(A33*QS3-A34*X12)]- RHOS3 + RHOCON =G= 0
  * Increasing the value of A34 increases the effect that canal water has upon
  * the pumper’s marginal cost and increases the value of BETA.
  * Reducing this number can be used for partial canal lining scenarios.
SCALAR B20 first parameter for the demand function for node 2
   2888.

SCALAR B21 second parameter for the demand function for node 2
   .009

SCALAR B22 third parameter for the demand function for node 2
   0.818181818

SCALAR B30 first parameter for the demand function for node 3
   1350.

SCALAR B31 second parameter for the demand function for node 3
   0.009

SCALAR B32 third parameter for the demand function for node 3
   2.33333333

SCALAR C0 first parameter for the seepage function
   15000
* Note: A34 must also be set to zero when the canal is lined and
seepage = 0

SCALAR C1 second parameter for the seepage function
   1.5E-5

SCALAR C2 third parameter for the seepage function
   / 
   0.10 * 0
   / 
;

SCALAR C3 fourth parameter for the seepage function
   / 
   0.002 * 0
   / 
;
Appendix D  GAMS PE Model Code with Rival and Non-Rival Demands

$ONTEXT
Partial Spatial Equilibrium Water Distribution Model

*  Henrys Fork 9/23/2013 RDS

By Leroy Stodick
16 June 2011

$OFFTEXT

$SETGLOBAL PROGPATH C:\watermodel\Henrys Fork folder\rival and non rival HF\Rival and non rival fisheries\n
$SETGLOBAL TEXTNAME 16June2011

$ONEMPTY

*

* 

OPTION MCP = PATH;
OPTION LIMCOL = 3, LIMROW = 3;

* base-case models (no rentals)
$INCLUDE "%PROGPATH%HF_FMID_base_non_rival_irrigation.gms"
*$INCLUDE "%PROGPATH%HF_FMID_base_RNR4.gms"

FILE KDATA3 / "%PROGPATH%DEMANDFUNC2.csv" /
KDATA3.pw = 900;
FILE KDATA2 / "%PROGPATH%ALL_SUP&DEM.csv" /
KDATA2.pw = 900;
PUT KDATA2;
PUT "QSOUT";//
PUT ",";

"EGIN_BENCH_BARLEY,EGIN_BENCH_WHEAT,EGIN_BENCH_POTATOES,EGIN_BENCH_ALFALFA,"
"L_WATERSHED_BARLEY,L_WATERSHED_WHEAT,L_WATERSHED_POTATOES,L_WATERSHED_ALFALFA,"
"N_FREEMONT_BARLEY,N_FREEMONT_WHEAT,N_FREEMONT_POTATOES,N_FREEMONT_ALFALFA,"
"ST_ANTHONY_FISH,ISLAND_PARK_FISH,EGIN_BENCH_RECHARGE,L_WATERSHED_RECHARGE,N_FREEMONT_RECHARGE,"
"PUMPERS_BARLEY,PUMPERS_WHEAT,PUMPERS_POTATOES,PUMPERS_ALFALFA,"
"SUP_CON$_EGIN_BENCH_IRR_N,CON$_N_FREEMONT_IRR_N,CON$_L_WATERSHED_IRR_N,CON$_EGIN_BENCH_IRR_S,"
"CON$_N_FREEMONT_IRR_S,CON$_L_WATERSHED_IRR_S,CON$_EGIN_BENCH_NON_N,CON$_N_FREEMONT_NON_N,CON$_L_WATERSHED_NON_N,"
"CON$_EGIN_BENCH_DRAIN,SCON$_L_WATERSHED_DRAIN,"/;

VARIABLES

WELFARE value of objective function
QD(DEM) quantity demanded
QS(SUP) quantity supplied
X(SUP,DEM) quantity transported from node I to node J
RHOS(SUP) supply prices
RHOD(DEM) demand prices
* RHOG(SUP) COST OF GROUNDWATER CONSTRAINT
RHOM(SUP) cost of drain water constraint
RHOF(SUP) cost of fixed drain constraint
RHOC(SUP) cost of canal constraint
SEEPAGE total seepage from canal
RECH_SEEP recharge seepage
RECHDPR(SUP) demand price for recharge water per acre foot of water pumped
;

POSITIVE VARIABLES QD,QS,X,RHOD,RHOS,RHOM,RHOC,RHOF;

EQUATIONS

OBJ objective function
*Kuhn Tucker conditions complementary slackness equations
* 1
DEMCONS(I) demand must be met at all nodes
* 2
SUPCONS(I) cannot ship more than is produced
DEMPRIN(I) marginal utility equal to demand price inverse demand function
DEMPR(I) marginal utility equal to demand price forward demand function
SUPPR(I) marginal cost equal to supply price

* SUPPRB(I) marginal cost equal to supply price (base model)
PRLINKB(I,J) price linkage equation (base model)
*
DEMCONS(DEM)..
  SUM(SUP,X(SUP,DEM)) - QD(DEM) -
  SUM(CANAL,S0(CANAL,DEM)*X(CANAL,DEM))
  - SUM(RECHNODES,RECH_S0(RECHNODES,DEM)*X(RECHNODES,DEM))
  =G= 0
;

SUPCONS(SUP)..
  QS(SUP) - SUM(DEM,X(SUP,DEM)) =G= 0
;

************************************************************************
***********************
DEMPRIN(DEM1)..
* Inverse of marginal demand-price function, Q=f(P), Compatible with IDEP demand
calculator coefficients.
  RHOD(DEM1)-(1/B1(DEM1)*(-(QD(DEM1)-
  B0(DEM1))/B0(DEM1))**(1/B2(DEM1))) =G= 0
;
************************************************************************
***********************
* forward demand price function
* second term (B3, B4 & B5)represents non rival demand
DEMPR(DEM2)..
  RHOD(DEM2) - B0(DEM2)*(1 - B1(DEM2)*(QD(DEM2)**B2(DEM2))) =G= 0
  RHOD(DEM2) - B0(DEM2)*((1 - B1(DEM2)*(QD(DEM2)**B2(DEM2)))-
  B3(DEM2)*(1 - B4(DEM2)*(QD(DEM2)**B5(DEM2)))) =G= 0
  RHOD(DEM2) - B3(DEM2)*(1 - B4(DEM2)*(QD(DEM2)**B5(DEM2))) =G= 0
;
************************************************************************
***********************
SUPPR(SUP)..
  A0(SUP)
  * + A1(SUP)*A2(SUP)*EXP[A3(SUP)*QS(SUP)-
  A4(SUP)*(SEEPAGE+RECH_SEEP)]
- RHOS(SUP) + RHOC(SUP) + RHOF(SUP) + RHOM(SUP)
  * SUM(AGDRN,RHOM(AGDRN)*C1(AGDRN)*C3(AGDRN)*EXP[C2(AGDRN)*SEE
  PAGE - C3(AGDRN)*SUM(PUMP,QS(PUMP))]$PUMP(SUP)
    =G= 0;

SUPPRB(SUP).
  A0(SUP) + A1(SUP)*A2(SUP)*EXP[A3(SUP)*QS(SUP)-A4(SUP)*(SEEPAGE+RECH_SEEP)]
  - RHOS(SUP)
    + RHOM(SUP) + RHOC(SUP) + RHOF(SUP)
    + RECHDPR(SUP)$PUMP(SUP)
    =G= 0;

************************************************************************
********************************
PRLINKB(SUP,DEM)$ARCS(SUP,DEM).
  RHOS(SUP) - RHOD(DEM) + T(SUP,DEM) + RHOD(DEM)*S0(SUP,DEM)
    =G= 0

**************************************************************************
* Seepage is proportional to diversion
* Drain return supply is also proportional to diversion (drain return is partly seepage)
* Drain constraint multiplier x the seepage proportion (table S0) = the proportion of diversion that is drain return.
* e.g if seepage proportion of diversion is .25 and the drain return multiplier of seepage is 0.1, then
* the drain return portion of diversion, QS(AGDRN), is 0.025. C0 (below) is the drain
  constraint multiplier

DRNCONS(AGDRN).
  C0(AGDRN)*SEEPAGE - QS(AGDRN) =G= 0

DRNFIXED(AGDRN).
  CFIXED(AGDRN) - QS(AGDRN) =G= 0

CANALCONS(CANAL).
  D0(CANAL) - QS(CANAL) =G= 0

*GWCONS(PUMP)..
* E0(PUMP) - QS(PUMP) =G= 0
*
*$\text{CALCSEEP.}$
 SEEPAGE - SUM((CANAL,DEM),X(CANAL,DEM)*S0(CANAL,DEM)) =E= 0
;

CALCRECH..
 RECH_SEEP -
 SUM((RECHNODES,DEM),RECH_S0(RECHNODES,DEM)*X(RECHNODES,DEM))
 =E= 0
;

CALCDPR(PUMP)..
 RECHDPR(PUMP) -

[SUM((RECHNODES,DEM),X(RECHNODES,DEM)*RECH_S0(RECHNODES,DEM))]
*A1(PUMP)*A2(PUMP)*A4(PUMP)/A3(PUMP)
 * *[EXP(A3(PUMP)*QS(PUMP))-1]*EXP(-
 A4(PUMP)*(SEEPAGE+RECH_SEEP))]/QS(PUMP)
 =E= 0
;

X.FX(SUP,DEM)$NO_ARCS(SUP,DEM) = 0;
RHOM.FX(NONAGDRN) = 0;
RHOC.FX(NONCANAL) = 0;
RHOF.FX(NONAGDRN) = 0;
RECHDPR.FX(NONPUMP) = 0;

** Third solution using MCP and Path solver with externaites.
MODEL BASEMODEL
 /
 DEMCONS.RHOD
 SUPCONS.RHOS
 DRNCONS.RHOM
 DRNFIXED.RHOF
 CANALCONS.RHOC
 DEMPR.QD
 DEMPRIN.QD
 SUPPRB.QS
 PRLINKB.X
 CALCSEEP
 CALCRECH
 CALCDPR

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SET MNAMES names of models


PARAMETER QDOUT(DEM, MNAMES) quantity demanded;
PARAMETER QSOUT(SUP, *) quantity supplied;
PARAMETER RHOSOUT(SUP, *) supply price;
PARAMETER RHODOUT(DEM, *) demand price;
PARAMETER RHOMOUT(SUP, *) marginal cost of variable constraint for drain water users;
PARAMETER RHOCOUT(SUP, *) marginal cost of canal constraints;
PARAMETER RHOFOUT(SUP, *) marginal cost of fixed constraint for drain water users;
PARAMETER XOUT(*, SUP, DEM) quantity supplied from node SUP to node DEM;
PARAMETER CANSEEP(*, SUP, DEM) seepage in canal from node SUP to node DEM;
PARAMETER SEEPOUT(*) Total seepage;
PARAMETER RECHOUT(*) recharge seepage;
PARAMETER PROSUP(*, SUP) Producer surplus;
PARAMETER CONSUP(*, DEM) Consumer surplus;
PARAMETER TOTCONSUP(*) TOTAL CONSUMER SURPLUS
PARAMETER TOTSUP(*) total surplus;

OPTION QDOUT : 0
OPTION QSOUT : 0
OPTION RHOSOUT : 2
OPTION RHODOUT : 2
OPTION RHOMOUT : 2
OPTION RHOCOUT : 2
OPTION RHOFOUT : 2
OPTION XOUT : 0
OPTION CANSEEP : 0

SET LNUM1/LN1*LN1/;
PARAMETER SUP_PRICE1;
PARAMETER SUP_PRICE2;
PARAMETER SUP_PRICE3;
PARAMETER SUP_PRICE4;
PARAMETER SUP_PRICE5;
PARAMETER SUP_PRICE6;

PARAMETER SUP_PRICES2;
PARAMETER SUP_PRICES3;
PARAMETER SUP_PRICES4;
PARAMETER SUP_PRICES5;
PARAMETER SUP_PRICES6;

PARAMETER SUP_QUAN1;
PARAMETER SUP_QUAN2;
PARAMETER SUP_QUAN3;
PARAMETER SUP_QUAN4;
PARAMETER SUP_QUAN5;
PARAMETER SUP_QUAN6;

PARAMETER XEB1;
PARAMETER XEB2;
PARAMETER XEB3;
PARAMETER XEB4;
PARAMETER XEB5;
PARAMETER XEB6;
PARAMETER XEB7;

PARAMETER DEM_QUAN1;
PARAMETER DEM QUAN2;
PARAMETER DEM QUAN3;
PARAMETER DEM QUAN4;
PARAMETER DEM QUAN5;
PARAMETER DEM QUAN6;

PARAMETER SUP_CONSTRN1;
PARAMETER SUP_CONSTRN2;
PARAMETER SUP_CONSTRN3;
PARAMETER SUP_CONSTRN4;

PARAMETER SUP_CONSTRDW1;
PARAMETER SUP_CONSTRDW2;
PARAMETER SUP_CONSTRDW3;
PARAMETER SUP_CONSTRDW4;

PARAMETER CONSUP1;
PARAMETER CONSUP2;
PARAMETER CONSUP3;
PARAMETER CONSUP4;
PARAMETER CONSUP5;
PARAMETER CONSUP6;

*Below is the starting value for quantity demanded for MCP solver. In some cases with inverse demand
* functions it must be set to a fairly large number to avoid division by zero and achieve solution
* convergence. In the absence of inverse demand functions it can still cause problems. Although
* setting to zero is the default value.

*QD.L(DEM)=100.0;
QD.L(DEM)=0.0;
SOLVE BASEMODEL USING MCP;
    SUP_PRICE1=A0("FMID_IRRIGATE_NFL");
    SUP_PRICE2=A0("FMID_IRRIGATE_STO");
    SUP_PRICE3=A0("FMID_NON_IRRIGATE_STO");
    SUP_PRICE4=A0("ST_ANTHONY_RETURN_FLOW");
    SUP_PRICE5=A0("MUD_LAKE_GROUNDWATER");
    SUP_PRICE6=A0("FMID_CANAL_SEEPAGE");

    SUP_QUAN1=QS.L("FMID_IRRIGATE_NFL");
    SUP_QUAN2=QS.L("FMID_IRRIGATE_STO");
    SUP_QUAN3=QS.L("FMID_NON_IRRIGATE_STO");
    SUP_QUAN4=QS.L("ST_ANTHONY_RETURN_FLOW");
    SUP_QUAN5=QS.L("MUD_LAKE_GROUNDWATER");
    SUP_QUAN6=QS.L("FMID_CANAL_SEEPAGE");

    XEB1=X.L("FMID_IRRIGATE_NFL","FMID_IRRIGATION");
    XEB2=X.L("FMID_IRRIGATE_STO","FMID_IRRIGATION");
    XEB3=X.L("FMID_IRRIGATE_NFL","ST_ANTHONY_FISHERIES");
    XEB4=X.L("FMID_IRRIGATE_STO","ST_ANTHONY_FISHERIES");
    XEB5=X.L("FMID_NON_IRRIGATE_STO","FMID_IRRIGATION");
    XEB6=X.L("FMID_NON_IRRIGATE_STO","FMID_CARRYOVER");
    XEB7=X.L("FMID_NON_IRRIGATE_STO","ISLAND_PARK_FISHERIES");

    DEM_QUAN1=QD.L("FMID_IRRIGATION");
    DEM_QUAN2=QD.L("MUD_LAKE_IRRIGATION");
    DEM_QUAN3=QD.L("ST_ANTHONY_FISHERIES");
    DEM_QUAN4=QD.L("ISLAND_PARK_FISHERIES");
    DEM_QUAN5=QD.L("FMID_CARRYOVER");
    DEM_QUAN6=QD.L("FMID_IS_PARK_FISH");

    QDOUT(DEM,"BASE") = QD.L(DEM);
    QSOUT(SUP,"BASE") = QS.L(SUP);
    RHOSOUT(SUP,"BASE") = RHOS.L(SUP);

    RHODOUT(DEM,"BASE") = RHOD.L(DEM);
    XOUT("BASE",SUP,DEM) = X.L(SUP,DEM);
    CANSEEP("BASE",SUP,DEM) = S0(SUP,DEM)*X.L(SUP,DEM);

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RHOMOUT(SUP,"BASE") = RHOM.L(SUP);
RHOCOUT(SUP,"BASE") = RHOC.L(SUP);
RHOFOUT(SUP,"BASE") = RHOF.L(SUP);
SEEPOUT("BASE") = SEEPAGE.L;
RECHOUT("BASE") = RECH_SEEP.L;

* STORAGE CONSTRAINT COSTS FOR PRINTING
SUP_CONSTRN1=RHOCOUT("FMID_IRRIGATE_NFL","BASE");
SUP_CONSTRN2=RHOCOUT("FMID_IRRIGATE_STO","BASE");
SUP_CONSTRN3=RHOCOUT("ST_ANTHONY_RETURN_FLOW","BASE");
SUP_CONSTRN4=RHOCOUT("FMID_NON_IRRIGATE_STO","BASE");

SUP_CONSTRDW1= RHOM.L("FMID_IRRIGATE_NFL");
SUP_CONSTRDW2= RHOM.L("FMID_IRRIGATE_STO");
SUP_CONSTRDW3= RHOM.L("ST_ANTHONY_RETURN_FLOW");
SUP_CONSTRDW4= RHOM.L("FMID_NON_IRRIGATE_STO");

* RECHPX is the value of an acre foot of water in the recharge canal to the
  groundwater pumper. It is The integral of marginal pumping cost with respect to his
  pumping rate
  then the derivative of this integral (total pumping cost) with respect to canal seepage
  This gives change in his total pumping cost per unit of canal seepage
  which is the value of seepage in terms of reduced pumping cost

*RECHPX(RECHNODES,DEM) =
SUM(PUMP,[RECH_S0(RECHNODES,DEM)*A1(PUMP)*A2(PUMP)*A4(PUMP)/A
3(PUMP)]*[EXP(A3(PUMP)*QS.L(PUMP))-1]
* EXP(-A4(PUMP)*(SEEPAGE.L+RECH_SEEP.L)));

PROSUP("BASE",PUMP) = - A0(PUMP)*QS.L(PUMP);

*consumer surplus from demands represented by forward demand function
CONSUP("BASE",DEM2) = B0(DEM2)*QD.L(DEM2) - (B0(DEM2)*B1(DEM2)/(B2(DEM2)+1))*QD.L(DEM2)**(B2(DEM2)+1) -
QD.L(DEM2)*RHOD.L(DEM2);  
*consumer surplus from demands represented by inverse demand function
CONSUP("BASE",DEM1) =(-B0(DEM1)/B1(DEM1))*B2(DEM1)/(1+B2(DEM1)))*(-
QD.L(DEM1)-B0(DEM1))/B0(DEM1))*(1+B2(DEM1)))/(1+B2(DEM1)))*(-
B0(DEM1)/B1(DEM1))*B2(DEM1)/(1+B2(DEM1))) -QD.L(DEM1)*RHOD.L(DEM1);  
* total consumer surplus
TOTCONSUP("BASE") = SUM(DEM2,CONSUP("BASE",DEM2)) +
SUM(DEM1,CONSUP("BASE",DEM1));

TOTSUP("BASE") = SUM(SUP,PROSUP("BASE",SUP)) +
SUM(DEM,CONSUP("BASE",DEM));
DISPLAY
QDOUT,QSOUT,RHOSOUT,RHODOUT,RHOCOUT,RHOFOUT,RHOMOUT,RECH
DPR.L,SEEPOUT,RECHOUT,XOUT,CANSEEP,PROSUP,CONSUP,TOTCONSUP,T
OTSUP;

CONSUP1=CONSUP("BASE","FMID_IRRIGATION");
CONSUP2=CONSUP("BASE","MUD_LAKE_IRRIGATION");
CONSUP3=CONSUP("BASE","ISLAND_PARK_FISHERIES");
CONSUP4=CONSUP("BASE","ST_ANTHONY_FISHERIES");
CONSUP5=CONSUP("BASE","FMID_CARRYOVER");
CONSUP6=CONSUP("BASE","FMID_IS_PARK_FISH");

*Generate excel file supply and demand prices quantities and consumer surpluses

FILE KDATA1 / "%PROGPATH%DEMANDFUNC1.csv" /;
KDATA1.pw = 900;
PUT KDATA1;

PUT "FMID nat flow price, FMID nat flow supplied, FMID storage price, FMID storage supplied" /;
PUT SUP_PRICE1","SUP_QUAN1","SUP_PRICE2","SUP_QUAN2 /;

PUT "FMID Non-Irr price, FMID Non-Irr supplied" /;
PUT SUP_PRICE3","SUP_QUAN3 /;

PUT "St Anthony drain water price, St Anthony drain water supplied" /;
PUT SUP_PRICE4","SUP_QUAN4 /;

PUT "Mud Lake gw supply price, Mud Lake gw quantity supplied," /;
PUT SUP_PRICE5","SUP_QUAN5 /;

PUT "FMID canal seepage supply price, FMID canal seepage quantity supplied," /;
PUT SUP_PRICE6","SUP_QUAN6 /;

PUT "FMID irrigation nat. flow constraint, FMID irrigation storage constraint" /;
PUT SUP_CONSTRN1","SUP_CONSTRN2 /;

PUT "FMID irrigation nat. flow constraint cost, FMID irrigation storage constraint cost" /;
PUT SUP_CONSTRN1","SUP_CONSTRN2 /;

PUT "St Anthony return flow supply constraint, St Anthoy return flow supply constraint cost" /;
PUT SUP_CONSTRN3","SUP_CONSTRDW3 /;
PUT"FMID non-irrigation supply constraint, FMID non-irrigation supply constraint cost"
/;
PUT SUP_CONSTRN4","SUP_CONSTRDW4/;

PUT"FMID irrigation demand quantity" /
PUT DEM_QUAN1/

PUT"Mud Lake irrigation demand quantity" /
PUT DEM_QUAN2/

PUT"St Anthony fisheries demand quantity" /
PUT DEM_QUAN3/

PUT"Island Park fisheries demand quantity" /
PUT DEM_QUAN4/

PUT"FMID carryover demand quantity" /
PUT DEM_QUAN5/

PUT"Mud Lake irrigation & Island Park fisheries demand quantity" /
PUT DEM_QUAN6/

PUT"FMID_IRRIGATE_NFL to FMID_IRRIGATION, FMID_IRRIGATE_STO to FMID_IRRIGATION"/
PUT XEB1"," XEB2/

PUT"FMID_IRRIGATE_NFL to ST_ANTHONY_FISHERIES, FMID_IRRIGATE_STO to ST_ANTHONY_FISHERIES"/
PUT XEB3"," XEB4/

PUT"FMID_NON_IRRIGATE_STO to FMID_IRRIGATION, FMID_NON_IRRIGATE_STO to FMID_CARRYOVER, FMID_NON_IRRIGATE_STO to ISLAND_PARK_FISHERIES"/
PUT XEB5"," XEB6"," XEB7/

PUT"FMID irrigation consumer surplus"/
PUT CONSUP1/

PUT"Mud Lake irrigation consumer surplus"/
PUT CONSUP2/

PUT"Island Park fisheries consumer surplus"/
PUT CONSUP3/

PUT"ST Anthony fisheries consumer surplus"/;
PUT CONSUP4/

PUT"FMID carryover consumer surplus"/
PUT CONSUP5/

PUT"Mud Lake irrigation & Island Park fish consumer surplus"/
PUT CONSUP6/

PUTCLOSE KDATA1 /
$EXIT
Appendix E  GAMS PE Model Data for Rival and Non-Rival Demands

$SETGLOBAL TITLENAME "FMID Scenarios 26 August 2013"
* Average year Automation model
* revised demand functions "new_demands4.xls"
* base-case nat flow and storage constraints are average year diversions from nat. flow and storage
* no rental storage to B-unit
* P =adjusted potato demand function TC =adjusted transportation cost
* Updated irrigation and non-irrigation rental storage.

*THIS DATA SET IS UPDATED WITH IRRIGATION AND NON-IRRIGATION RENTAL CONSTRAINTS FOR AVERAGE AND DRY YEARS
*THIS DATA SET ALSO HAS MOST UPDATED COMMENTS 12/2/13 9:30 AM
*zero trib flow 12/4/2013
* eliminated the IS_PARK_NON_RELEASE_LR demand and supply nodes because St Anthony demand is Jul-Sep., not winter months 12/4/2013

SET I index of the nodes
/
* supply nodes
  FMID_IRRIGATE_NFL,
  FMID_IRRIGATE_STO,
  FMID_NON_IRRIGATE_STO,
  ST_ANTHONY_RETURN_FLOW,
  MUD_LAKE_GROUNDWATER,
  FMID_CANAL_SEEPAGE,
/

* demand nodes
  FMID_IRRIGATION,
  ST_ANTHONY_FISHERIES,
  ISLAND_PARK_FISHERIES,
  MUD_LAKE_IRRIGATION,
  FMID_CARRYOVER,
  FMID_IS_PARK_FISH
/
;

ALIAS (I,J);

SET DEM(I) index of demand nodes
/
  FMID_IRRIGATION,
  ST_ANTHONY_FISHERIES,
  ISLAND_PARK_FISHERIES,
  MUD_LAKE_IRRIGATION,
FMID_CARRYOVER,
FMID_IS_PARK_FISH
/
;

SET DEM1(DEM) INDEX OF MARGINAL DEMAND FNS. QTY=F(PRICE)
/
*   NONE
/
;

SET DEM2(DEM) INDEX OF MARGINAL UTILITY FNS. PRICE=F(QTY)
/
   FMID_IRRIGATION,
   ST_ANTHONY_FISHERIES,
   ISLAND_PARK_FISHERIES,
   MUD_LAKE_IRRIGATION,
   FMID_CARRYOVER,
   FMID_IS_PARK_FISH
/
;

SET SUP(I) index of supply nodes (n=naturalflow s=storage)
/
   FMID_IRRIGATE_NFL,
   FMID_IRRIGATE_STO,
   FMID_NON_IRRIGATE_STO,
   ST_ANTHONY_RETURN_FLOW,
   FMID_CANAL_SEEPAGE,
   MUD_LAKE_GROUNDWATER
/
;

SET CANAL(SUP) index of canal nodes
/
   FMID_IRRIGATE_NFL,
   FMID_IRRIGATE_STO,
   FMID_NON_IRRIGATE_STO
/
;

SET PUMP(SUP) index of groundwater supply nodes
/
   MUD_LAKE_GROUNDWATER
/
;

SET AGDRN(SUP) index of drainwater supply nodes
ST_ANTHONY_RETURN_FLOW

SET RECHNODES(SUP) index of recharge water supply nodes

*  NONE

SET NONPUMP(SUP) index of supply nodes other than groundwater;

NONPUMP(SUP) = NOT PUMP(SUP);

SET NONAGDRN(SUP) index of supply nodes other than drain water;

NONAGDRN(SUP) = NOT AGDRN(SUP);

SET NONCANAL(SUP) index of supply nodes other than canal nodes;

NONCANAL(SUP) = NOT CANAL(SUP);

SET ARCS(SUP,DEM) all possible arcs

FMID_IRRIGATE_NFL.FMID_IRRIGATION,
FMID_IRRIGATE_NFL.ST_ANTHONY_FISHERIES,
FMID_IRRIGATE_STO.FMID_IRRIGATION,
FMID_IRRIGATE_STO.ST_ANTHONY_FISHERIES,
*  FMID_NON_IRRIGATE_STO.ISLAND_PARK_FISHERIES,
ST_ANTHONY_RETURN_FLOW.ST_ANTHONY_FISHERIES,
MUD_LAKE_GROUNDWATER.MUD_LAKE_IRRIGATION,
FMID_NON_IRRIGATE_STO.FMID_CARRYOVER,
FMID_NON_IRRIGATE_STO.FMID_IS_PARK_FISH

SET NO_ARCS(SUP,DEM) arcs which are not possible;

NO_ARCS(SUP,DEM) = NOT ARCS(SUP,DEM);

PARAMETER B0(DEM) First parameter for the marginal utility functions

FMID_IRRIGATION 27
FMID_CARRYOVER 27
*fitted for marginal demand price/fish =$22.45
* Non-rival demands
ST_ANTHONY_FISHERIES 750
ISLAND_PARK_FISHERIES  1600
MUD_LAKE_IRRIGATION    27

* Vertical addition of Mud Lake Irrigation and Island Park fisheries
* This B0 is first parameter for Mud Lake irrigation
   FMID_IS_PARK_FISH    27

PARAMETER B1(DEM) Second parameter for the marginal utility functions
    / FMID_IRRIGATION .00095
    FMID_CARRYOVER   .00095
    ST_ANTHONY_FISHERIES .9948
    ISLAND_PARK_FISHERIES .9949
    MUD_LAKE_IRRIGATION .0009

* Vertical addition of Mud Lake Irrigation and Island Park fisheries
* This B1 is the second parameter for Mud Lake irrigation
   FMID_IS_PARK_FISH .00095

PARAMETER B2(DEM) Third parameter for the marginal utility functions
    / FMID_IRRIGATION  .612
    FMID_CARRYOVER   .612
    ST_ANTHONY_FISHERIES .00043
    ISLAND_PARK_FISHERIES .0004
    MUD_LAKE_IRRIGATION  .613

* Vertical addition of Mud Lake Irrigation and Island Park fisheries
* This B2 is the third parameter for Mud Lake irrigation
   FMID_IS_PARK_FISH .612

PARAMETER B3(DEM) First parameter for the non-rival marginal utility functions
    / FMID_IRRIGATION  0
    FMID_CARRYOVER   0
    ST_ANTHONY_FISHERIES  0
    ISLAND_PARK_FISHERIES  0
    MUD_LAKE_IRRIGATION  0

* Vertical addition of Mud Lake Irrigation and Island Park fisheries
* This B3 is the first parameter for non-rival Island Park fisheries
FMID_IS_PARK_FISH 1600
/
;

PARAMETER B4(DEM) Second parameter for the non-rival marginal utility functions
/
FMID_IRRIGATION 0
FMID_CARRYOVER 0
ST_ANTHONY_FISHERIES 0
ISLAND_PARK_FISHERIES 0
MUD_LAKE_IRRIGATION 0
* Vertical addition of Mud Lake Irrigation and Island Park fisheries
* This B4 is the second parameter for non-rival Island Park fisheries
  FMID_IS_PARK_FISH .9949
/
;

PARAMETER B5(DEM) Third parameter for the non-rival marginal utility functions
/
FMID_IRRIGATION 0
FMID_CARRYOVER 0
ST_ANTHONY_FISHERIES 0
ISLAND_PARK_FISHERIES 0
MUD_LAKE_IRRIGATION 0
* Vertical addition of Mud Lake Irrigation and Island Park fisheries
* This B5 is the third parameter for non-rival Island Park fisheries
  FMID_IS_PARK_FISH .0004
/
;

* Marginal supply cost for irrigation water is cost of natural flow and storage water.
There is added transportation cost for this water
* due to return flow, the magnitude of which are indicated in the following three tables
*(Trans. cost, seepage pct. and return multiplier). Natural flow supply costs are what IDs
charge irrigators for water delivered to the canal
* diversion point, not to the headgates.
PARAMETER A0(SUP) First parameter for the marginal cost functions
/
FMID_IRRIGATE_NFL .46
FMID_IRRIGATE_STO 3.46
FMID_NON_IRRIGATE_STO 3.46
ST_ANTHONY_RETURN_FLOW .01
FMID_CANAL_SEEPAGE .01
MUD_LAKE_GROUNDWATER 10.00
/
* O&M transportation costs are the IDs costs for delivery of water from the canal diversion point to the headgate.
* They are applied to all diversions including seepage losses and return flows as well as to water consumptively used by irrigators.
* Seepage costs are associated with the supply cost of water that seeps from the canal and never reaches the farm headgate.
* O&M transportation costs are separate from supply costs.

**TABLE T(SUP,DEM) per unit conveyance cost from Node SUP to Node DEM O&M charge (per AF charge)**

<table>
<thead>
<tr>
<th>FMID_IRRIGATION</th>
<th>1.37</th>
</tr>
</thead>
<tbody>
<tr>
<td>FMID_IRRIGATE_NFL</td>
<td>1.37</td>
</tr>
<tr>
<td>FMID_IRRIGATE_STO</td>
<td>0.0</td>
</tr>
</tbody>
</table>

**TABLE S0(SUP,DEM) First parameter for the canal seepage functions**

<table>
<thead>
<tr>
<th>FMID_IRRIGATION</th>
<th>0.66</th>
</tr>
</thead>
<tbody>
<tr>
<td>FMID_IRRIGATE_NFL</td>
<td>0.66</td>
</tr>
</tbody>
</table>

**TABLE RECH_S0(SUP,DEM) first parameter for the (not incidental) recharge seepage function**

<table>
<thead>
<tr>
<th>RECH_DEM</th>
<th>0.5</th>
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</thead>
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* The drain return multiplier determines the percentage of seepage loss that is drain return.
* Automation scenario drain return is zeroed out

**PARAMETER C0(SUP) first parameter for drain return constraint multiplier**

| ST_ANTHONY_RETURN_FLOW | 0.12 |

**PARAMETER G0(SUP) first parameter for GROUNDWATER constraint multiplier**

| MUD_LAKE_GROUNDWATER   | 0.88 |

**PARAMETER CFIXED(SUP) fixed constraint for drain water supply**
/ST_ANTHONY_RETURN_FLOW 1.0E10
/
;

PARAMETER D0(SUP) RHS for canal constraints(natural flow and storage constraints)
/
* average year natural flow useage (constraint)
  FMID_IRRIGATE_NFL  760140

* total available irrigation season storage (average year)
  FMID_IRRIGATE_STO  191227

* Total storage available for irrigation carryover (average year) (measured at the end of the irrigation season)
  * = baseline irrigation season storage - baseline FMID irrigation season diversions from storage.
  FMID_NON_IRRIGATE_STO  136977
/
;
Appendix B

Henrys Fork Hydro-Economic Modeling
October 3, 2014

AUTHORS:

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Principal Investigator

John Tracy

Idaho Water Resources Research Institute

Technical Completion Report 201401

University of Idaho
Boise
Henrys Fork Hydro-Economic Modeling

Robert Schmidt, Garth Taylor and Leroy Stodick

University of Idaho
Water Resources Research Institute

September 2014
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Introduction

Hydro-economic models represent the hydrologic, engineering, environmental and economic aspects of basin scale water resource systems in an integrated framework that accounts for the economic value of water services generated. Hydro-economic modeling can be traced back to the use of water demand curves developed in the 1960s and 1970s by Jacob Bear and others (1964, 1966, 1967, 1970). Most hydro-economic models share basic elements including spatial representation of hydrologic flows, water supply infrastructure, supply costs and constraints, economic demands, and operating rules affecting water allocations. Basin-wide hydro-economic model application involves five basic steps:

1. Development of a node-arc model framework incorporating water suppliers and demanders as nodes, and supply and demand linkages as arcs.
2. Development of marginal water supply-cost and demand-price functions for supply and demand nodes, and conveyance-cost functions for model arcs.
3. Calibration of the baseline model using basin hydrologic and water budget data.
4. Development of model scenarios by modifying baseline model variables to represent alternative water resources plans.
5. Evaluation of scenario results to generate policy insights and reveal opportunities for improved water resource planning.

Two basic approaches exist for hydro-economic modeling. The holistic approach combines hydrology and economic optimization into a single model. The modular approach (figure 1) involves a transfer of exogenous supply and demand information from an independent hydrologic model to an economic optimization model. For basin scale studies, the modular approach is generally preferred because it allows for more robust and realistic representation of basin hydrology and more efficient optimization of a basin-wide network of water supply and demand nodes (Brouwer and Hofkes, 2008).
**Henry Fork Basin Hydrologic Setting**

The Henrys Fork (HF) River flows for 120 miles in the eastern part of Idaho, joining the upper Snake River from the north near Rexburg, Idaho (Figure 2). The HF basin encompasses approximately 3,300 square miles bound by high desert areas of the Eastern Snake Plain on the west and on the north by the Continental Divide along the Centennial and Henry's Lake mountains. The Yellowstone Plateau and Teton Mountains form the eastern boundary and the southern boundary is marked by the Snake River. Originating at the northern part of the basin, the main stem of the Henrys Fork River flows generally southward, supplemented by water from tributaries flowing from the mountains to the east. The HF watershed has three major storage reservoirs, and multiple irrigation diversions ranging from small pumps to large canal headworks which regulate the flows in the basin. In the early 1900s, farmers took advantage of an abundant river water supply to sub-irrigate lands. The resulting watertable rise led to greatly expanded groundwater irrigation. Basin soils are highly productive and produce primarily grain, alfalfa, and potato crops.
The total basin water supply, computed as the mean annual rainfall over the total watershed area (30-year average) is about 4.9 million AF. Almost half (2.3 million AF) is lost to evaporation and deep groundwater, and a little more than half (2.5 million AF) is measured as surface water supply (Van Kirk et al., 2011).

The Island Park Dam was constructed by the Bureau of Reclamation in 1935 as part of the Upper Snake River Division of the Minidoka Project and the Freemont Madison Irrigation District (FMID) was formed from numerous small irrigation companies across Fremont, Madison, and Teton Counties. FMID provides water to about 1,500 water users who irrigate over 285,000 acres. Most of the water in the HF basin is appropriated, and water is available for use only to the extent that flows exceed the demands of FMID irrigators with priority water rights. Figure 3 shows the three sub-basins of the Henrys Fork (North Freemont, Egin Bench and Lower Watershed) which make up the Freemont Madison Irrigation District.

As part of the Greater Yellowstone Ecosystem, the HF basin provides habitat for a variety of large and small mammals and birds. National Forest lands in the basin provide both summer and winter outdoor recreational opportunities which draw tourists from all over the world. The HF has a reputation for world-class fly fishing and the basin supports wild populations of native Yellowstone cutthroat trout and nonnative rainbow and brown trout. However water storage and irrigation deliveries have significantly altered river and stream hydrology in the HF basin (Van Kirk and Jenkins, 2005). Stream flow alterations are greatest during drought years and as a result rainbow trout have largely displaced native Yellowstone cutthroat trout throughout most of the watershed (Van Kirk and Jenkins 2005).

Minimum stream flows necessary to preserve desired stream values have been recommended by the Idaho Department of Fish and Game (IDFG), however except for the high flows of spring runoff, the 30-year average flow in the river is consistently lower than the IDFG flow recommendations to benefit aquatic life. Federal and State agencies, FMID, and the Henrys Fork Foundation (HFF) have worked cooperatively to set the timing and quantity of winter releases from Island Park reservoir in order to promote fish habitat while maintaining the primacy of irrigation demands (Van Kirk 2011).
The HF watershed exhibits a high degree of surface water and groundwater interaction both spatially and temporally. Canal seepage losses account for about 25% of total diversions from the river (Van Kirk, 2011). Seepage from irrigation canals is the primary source of aquifer recharge. Aquifer recharge also occurs by direct delivery of water to managed recharge sites in the basin. Groundwater discharge to agricultural drains is the primary source of instream flows during winter months.

Figure 2: Henrys Fork watershed basin in Eastern Idaho.
Partial Equilibrium (PE) Modeling

The mathematical link between hydrology and economics in hydro-economic modeling is economic optimization. Partial equilibrium (PE) optimization models examine the conditions of market equilibrium that exist when dealing with a single economic commodity (in our case water), all other factors of production are held fixed.

PE economic optimization was introduced in the water literature by Flinn and Guise (1970), who adopted the Takayama and Judge (1964) concept of an interregional trade model. In the hydrologic context, PE modeling generates an optimal allocation of water quantities which maximize basin-wide economic benefit from water use. Individual water quantities and prices vary among demanders because of differences in supply costs and demand prices.

Hydrologic and water engineering features are represented in a PE model by a node-arc network, in which water suppliers and demanders are represented by nodes and arcs denote opportunities for water transfers between nodes. The node-arc network
thereby accommodates both the physical and economic distribution of water supply and demand in a watershed system.

PE modeling is not equivalent to advocating water marketing, nor does it assume all water resources are private goods. Constraints on private allocations and demands for public goods such as river system eco-services are readily included in hydro-economic models. PE models also differ from economy-wide general equilibrium models in that hydro-economic PE models focus on how economics affect water resource management rather than on how water resource management affects the entire economic system (Harou, 2009).

The concept of marginality is central in PE modeling to express the supply-cost or demand-price of one additional unit of water (at the margin). The microeconomic marginal principle states that in an optimal allocation of water, each water user derives the same value (or utility) from the last unit of water allocated (Harou et al, 2009).

**Jointness-of-Production and Hydrologic Externalities**

Jointness-of-production occurs when the economic activity of one entity impacts the production possibilities of another, either positively or negatively. Externalities arise when the impacts of jointness-of-production are not fully accounted for (via pricing) in economic decisions (Mishan, 1971; Baumol and Oates, 1988). The result is a divergence between private and social benefit or cost, with price institutions failing to sustain desirable activities or to curtail undesirable activities (Bator, 1958).

Market failures resulting from hydrologic externalities most commonly take the form of the underproduction of a positive externality. In the HF basin the market failure is the under production of instream flows to sustain river system eco-services, including fisheries.

Instream flows in the HF which sustain trout fisheries and other eco-services are largely dependent on FMID irrigation demands, and since instream flows are public goods there is no direct compensation by users of HF eco-services for the benefit they derive from these services.
PE models have traditionally been cast as optimization problems in which a quasi-welfare or net social payoff function is maximized subject to constraints. Water allocations which maximize the objective function were then assumed to be the supply and demand equilibrium conditions. The presence of hydrologic externalities means that the traditional method of calculating supply and demand equilibrium conditions is no longer appropriate, since an objective functions exists only with the elimination of externalities.

**Calculating Net Benefits with Externalities**

When Takayama and Judge (1971) published their book, numerical optimization techniques were well understood, but mixed complementary programming (MCP) was in its infancy. With the advent of generic modeling systems such as GAMS (Brooke et al., 1988) and the accompanying PATH solver (Ferris and Munson, 1999) PE equilibrium equations containing externalities can be solved directly using MCP wherein certain equality constraints in the optimization problem are replaced by inequality constraints containing Lagrange multipliers (Kjeldsen, 2000).

Absent non-convexities and assuming a unique solution, six sets of complementary slackness\(^1\) equations define economic equilibrium conditions in the presence of hydrologic externalities.
1. \( p_i - p_i \leq 0 \) and \( q_i (p_i - p_i) = 0 \) for \( q_i \geq 0 \)

2. \( q^i - p^i \leq 0 \) and \( q^i (q^i - p^i) = 0 \) for \( q^i \geq 0 \)

3. \( \sum_j x_{ji} + \sum_k x_{kj} = q_i \geq 0 \) and \( p_i (\sum_j x_{ji} + \sum_k x_{kj}) = 0 \) for \( p_i \geq 0 \)

4. \( q^i - \sum_j x_{ji} + \sum_k x_{kj} \geq 0 \) and \( q^i (q^i - \sum_j x_{ji} + \sum_k x_{kj}) = 0 \) for \( q^i \geq 0 \)

5. \( q^i - p^i - t_{ij} + \sum_k c_{ijk} \frac{\partial F_{ijk}(x_{ij})}{\partial x_{ij}} \leq 0 \)

6. \( F_{ijk}(x_{ij}) - EX_{ijk} = 0 \)

Equations 1 and 2 insure that if quantity of water transported is greater than zero, then equilibrium demand and supply prices are points that lie, respectively, on the demand and supply curves. Equation 3 insures that no excess water demand exists. Equation 4 allows for an excess water supply. Equation 5 is the price linkage equation, i.e. the difference between the equilibrium water demand price and the equilibrium water supply price is the cost of the externality. Equation 6 insures that the quantity of externality produced is equal to the quantity of externality delivered.

With externalities, the equilibrium equations of Takayama and Judge include a new exogenous function, \( F_{ijk}(x_{ij}) \), which relates the quantity of un-priced (externalized) water supplied to another quantity of priced (internalized) water that is delivered. The new endogenous variable, \( EX_{ijk} \), is then the quantity of un-priced water that is delivered.

The above conditions are solved for equilibrium water prices and quantities using GAMS and the PATH solver. Consumer surpluses, which are the measure of net benefits used in PE model applications, are obtained using equilibrium prices and quantities as limits of integration (figure 4).
Figure 4: Calculation of net benefits (consumer surpluses) using equilibrium water prices and quantities as limits of integration.

**FMID Irrigation Supply Cost Functions**

Water valuation from the supply perspective results in a supply-cost curves which for canal irrigators typically have a block rate structure. FMID irrigation water supply costs are represented by step functions in which the first step is the per AF cost of natural flow and the second step is the per AF cost of storage water. Currently there are just two steps in the FMID average year and dry year water supply functions (figures 5 and 6). Additional steps would be added if new reservoir storage became available at a higher cost. The average year constraint on natural flow and storage supplies is the 30 year (1978-2008) average, and the dry year constraint is the average of a three year dry period between 2003 and 2005.
Natural flow supply costs for FMID irrigation water vary among the three HF sub basins because canal operation and maintenance (O&M) costs vary. The per AF charges for irrigation water are based on total acreage and total diversions in each sub basin. The Egin Bench natural flow O&M cost is the lowest at $0.29/AF. The North Freemont charge is $0.50/AF, and the Lower Watershed O&M charge is $0.59/AF. An additional $3.00/AF is added for water that is released from HF storage. Supply costs do not include additional conveyance costs associated with canal seepage losses and return flows.

Figure 5: FMID irrigation supply costs and constraints.

Figure 6: Dry year (2003-2005 average) FMID irrigation season supply costs and constraints.
Since natural flow is the sole source of supply for aquifer recharge during winter months, average year and dry year non-irrigation season water supply costs and constraints are represented by functions with a single-step (figures 7 and 8). The much constrained dry year non-irrigation season supply of natural flow results from most HF flows being held in carryover for the next irrigation season.

![Figure 7: FMID 30 year average non-irrigation season supply cost functions.](image)

The supply of instream flows for HF fisheries, which is critical during the non-irrigation season, is largely dependent on natural flows and reservoir releases made in support of irrigation activities. This includes winter-time aquifer recharge, operational reservoir releases and irrigation return flows. Instream flows that are dependent on

![Figure 8: FMID dry year (2003-2005) average non-irrigation season supply cost functions.](image)
irrigation demands are non-rival and un-priced. However reservoir releases that are exclusively for instream flows (and rival with irrigation) are allowed in model scenarios and are arbitrary priced at $3.00 per AF, the same as irrigation releases.

**Rival and Non-Rival Water Demands**

Recently updated principles and requirements for federal water resource planning (P&R, 2013) place increased emphasis on commensurate valuations of watershed costs and benefits including, where possible, the monetizing of currently un-priced or under-priced river system eco-services. Capturing the value of these services (e.g. boating, fishing, ecological diversity) in river systems that are being managed for irrigation and reservoir storage requires hydro-economic modeling of a mix of both private and public goods.

In contrast to private goods such as irrigation water diversions which are excludable and rival, public goods such as river system eco-services are non-excludable and non-rival (Myles 1995). A good is non-exclusive if others cannot be excluded from its use and non-rival if its consumption by one agent does not diminish the amount available to others.

Each person benefiting from a public good pays a price which depends on a personal evaluation of the worth of the good. Since no one can be excluded from using a public good, its valuation is prone to under reporting, and as a consequence the marginal benefit of public goods are under produced, creating a negative externality.

Accurate CBA of water projects that affect river systems being managed for multiple rival and non-rival water uses depends in large measure on the correct valuation of demands for both private and public goods. Rival water demand for a mix of crops is calculated by horizontally summing the demand quantities of individual crops at every marginal price, thus private goods are allocated water on the basis of an equal-marginal price (figure 9). Water demands for eco-services that are non-rival in consumption are calculated by vertically summing the demand prices of individual services at every marginal quantity, thus non-rival public goods are allocated on the basis of their total marginal price (figure 10).
Figure 9: Horizontal summation of water demand quantities for two rival irrigated crops.

Figure 10: Vertical summation of water demand-prices for two non-rival instream flow eco-services.
As a private good, irrigation may be rival or non-rival with river system eco-
services, depending the location and timing of irrigation demands in relation to the
instream flows needed to sustain eco-services. If a storage release for irrigation flows
through the Island Park reach but is diverted before reaching the St. Anthony reach, then
irrigation demand is non-rival with instream flow demand in the Island Park reach but
rival with instream flow in the St. Anthony reach. Similarly, if irrigation return flow
enters the river below the Island Park reach but above the St Anthony reach, irrigation
demand is non-rival with instream flow demand in the St Anthony reach but rival with
instream flow demand in the Island Park reach.

**FMID Irrigation Demand Price Functions**

Two broad approaches are available to model water demand (Kindler and Russell,
1984) and develop demand functions for irrigation and river system eco-services.
Inductive techniques rely on econometric or statistical analysis of observed data to
estimate price-response. Deductive methods involve production functions and
mathematical programming.

A spreadsheet demand function calculator is used to develop the irrigation
demand price functions (IWRRI, 2008) (Martin et al., 1984). Crop and production
function inputs to the calculator, including commodity prices, crop acreages and
evapotranspiration (ET) production functions are obtained from a variety of agricultural
and statistical data bases maintained by the USDA, Idaho Dept of Water Resources and
the University of Idaho.

Demand price functions are developed for principal crops grown in the FMID, the
B-Unit of the A&B district and groundwater pumpers near Thousand Springs (figure 2).
Aquifer recharge demand functions are also developed for crops grown in groundwater
irrigated areas of the HF (figure 3).

The demand function calculator assumes that market mechanisms have already
maximized crop acreages and the mix of crops. Therefore all existing constraints on crop
distribution are assumed to be fully reflected in the status-quo allocation of crops to lands.
Although crop mix is fixed, lower value crops may drop out of production at higher prices. Limited water supplies are assumed to be optimally delivered when most needed.

The multiple demand curves developed for the four principal FMID crops (figure 11) illustrate the range of "best fitting" demand data regressions possible. However to insure a unique PE model solution, demand price elasticity is represented only by convex functions.

![Image of demand-price functions for four crops with varying price elasticities.](image)

Figure 11: FMID Irrigation demand-price functions for four crops with varying price elasticities.

**Henry's Fork Instream Flow Demand Price Functions**

A number of inductive methods have been developed for measuring willingness to pay for environmentally-related public goods. Revealed preference methods rely on actual expenditure mainly travel costs, made by consumers (Young, 2005). Stated preference methods involve asking people directly about the values placed on environmental services. Both approaches have been used to infer the willingness to pay for recreational trout fishing in Eastern Idaho Rivers (Loomis, 2005).
Recreational fishing is one example of a river system eco-service that is can be considered a public good, it is non-rival as long as one angler’s catch does not measurably diminish the stocks available to others. While “free riders” acting in their own self interest are unwilling to pay anything for river services such as recreational fishing, others who value the experience of HF wilderness and wildlife are willing to pay a considerable sum. Somewhere in between are recreational anglers whose willingness to pay depends on the quality of the fishing experience. For some it is the opportunity to catch additional fish of a common species (e.g. Rainbow trout), for others it is the opportunity to catch even one of a much less common species (e.g. Cutthroat trout).

Flows critical for maintaining Rainbow trout populations in the HF occur in two reaches of the river, the upper HF reach below Island Park dam and the lower reach just above St Anthon. Flows are critical during a three month interval (December–February) in the fry stage of development. Trout fry survival during this period is the key determinant of fishable trout population in subsequent years (Van Kirk, 2013).

Empirically derived equations by Van Kirk (2013) describe fishable trout populations $N(i)$, in both reaches as functions of the previous five years of HF instream flows $x_{i-j-1}$ during this three month period (figure 12). The first pair of equations applies to the HF reach below Island Park and the second pair applies to the reach below St. Anthony. The two functions plot as upward sloping curves (figures 13 and 14) so that increasing instream flow results in an increasing population of fishable trout.

---

2 In order to make the equations compatible with the irrigation water supply and demand units of the PE model, instream flow $x_{i-j-1}$ is converted from cfs to AF over a three month interval.
Island Park reach, trout vs cfs (Dec-Feb) (VanKirk, eq 3)

\[ N(i) = 132.01 \sum_{j=0}^{4} 0.4^j (x_{i-j-1})^{0.5276} \]

Island Park reach, trout vs AF/3month (Dec-Feb)

\[ N(i) = 8.5603 \sum_{j=0}^{4} 0.4^j (x_{i-j-1})^{0.5276} \]

St Anthony reach, trout vs cfs (Dec-Feb) (VanKirk, eq 4)

\[ N(i) = 132.01 \sum_{j=0}^{4} 0.4^j (x_{i-j-2})^{0.5276} \]

St Anthony reach, trout vs AF/3month (Dec-Feb)

\[ N(i) = 4.109 \sum_{j=0}^{4} 0.4^j (x_{i-j-1})^{0.5276} \]

Figure 12: Fishable trout population and flow in two Henrys Fork reaches (Van Kirk, 2012)

![Graph showing the relationship between fishable trout and AF flow in Island Park reach.](image)

Figure 13: Island Park reach fishable trout vs AF flow during 3 month period (Dec-Feb) of 5 previous years.
Figure 14: St Anthony reach fishable trout vs AF flow during 3 month period (Dec-Feb) of 5 previous years.

The derivatives of these two functions (figures 15 and 16) with respect to AF of flow yields the marginal rate of increase in fishable trout per AF of flow in each reach, during the critical three month period. For example, in the Island Park reach if flow during the critical period is 60,000 AF, one additional AF would increase the fishable trout population by about 0.04 fish. In the St Anthony reach, if flow is 60,000 AF one additional AF would increase the population by about 0.02 fish.

Figure 15: Marginal rate of increase in Island Park trout per AF of flow during 3 month period (Dec-Feb) of 5 previous years.
Figure 16: Marginal rate of increase in St Anthony trout per AF of flow during 3 month period (Dec-Feb) of 5 previous years

As noted previously, the marginal value of trout to HF anglers is based on a contingent valuation survey of Snake River anglers (Loomis, 2005). The survey results indicated that a HF angler's willingness-to-pay to catch one additional trout was, on average, $22.45. The marginal value of instream flow for HF trout can then be determined by multiplying the marginal rate of increase in trout population in the Island Park and St Anthony reaches per AF of instream flow by $22.45 (figures 17 and 18).

This is not the same however as anglers willingness to pay to catch an additional trout, making it necessary to calculate a relationship between the willingness to pay for trout in the river and willingness to pay for trout caught by anglers. 3 Nevertheless, in the absence of reliable valuations for other river system eco-services, including boating

3 The average daily catch in HF reaches is 8.2 trout (Loomis, 2005), so one additional trout represents a 12 % increase in catch. Assuming catch is directly proportional to fishable trout population, the population of fishable trout in the river would also have to increase by 12% in order for anglers to catch one additional fish. In Dec-Feb of a dry year (2001-2005) Island Park reach flow averages 195 cfs (35,000 AF in the three month interval). Based on the equations of Van Kirk, the Island Park trout population should therefore be about 3527. Since one additional trout caught by anglers requires a 12% increase in trout population (i.e. 423 trout), the total population needed to enable anglers to catch one additional trout is 3,950. The marginal economic value of an additional trout in the river (to anglers) is therefore $22.45/423, about $0.05.
recreation, wildlife viewing etc, an instream flow marginal demand price based on a trout valuation of $22.45 is more reasonable than one based strictly on willingness to pay for successful angling. These marginal demand-price functions for instream flow are therefore used to represent the willingness to pay for the full range of HF eco-services, including a sustainable population of Rainbow trout in the Island Park and St Anthony river reaches.

Figure 17: Marginal demand-price function for instream flow to sustain fisheries in the Island Park Reach.
The two previous demand functions for instream flow in the Island Park reach and the St. Anthony reach are appropriate for valuing instream flows that are rival with each other (but may be non-rival with irrigation). However, if a reservoir release is made exclusively for instream flows then the two instream flow demands are non-rival with each other (but rival with irrigation) and the total willingness-to-pay for instream flow in the two reaches is the vertical summation of the two instream flow marginal demand prices (figure 19).

![Figure 19: Vertical summation of Island Park and St Anthony reach demand-prices for instream flow.](image)

### Henrys Fork PE Model Nodes and Arcs

Supply nodes in the Henrys Fork PE model node-arc network (figure 20) consist of natural flows, storage water and return flows from irrigation. A further seasonal breakdown of supply nodes depends on whether supplies are available during the irrigation season or the non-irrigation season.

Demand nodes in the network include irrigators in the FMID, groundwater irrigators in the HF using aquifer recharge, irrigators in the lower basin, and HF instream flows for fisheries and other river system eco-services in the two reaches.
Transportation (conveyance) costs associated with arcs are the costs of canal seepage losses and return flows and the added charges for storage water delivered to irrigators outside the HF basin.

![Figure 20: Henrys Fork GAMS partial equilibrium model nodes and arcs (T denotes added transportation cost).](image)

**Henry's Fork Hydro-Economic Model Applications**

HF hydro-economic modeling consists of two separate applications. The first evaluates rival demands and benefits for irrigation, aquifer recharge and instream flows under the conditions of two proposed FMID water management alternatives; canal automation and new reservoir storage. The second hydro-economic modeling application evaluates the relative basin-wide net benefits associated with rival and non-rival management of HF reservoir water supplies for irrigation and river system eco-services.

1. **Two FMID Water Management Alternatives**

Three PE model scenarios are developed to evaluate the proposed management alternatives. The first is a calibrated HF baseline scenario with two sets of supply
constraints representing average year and dry year conditions. The second is a canal automation scenario for average and dry years wherein canal seepage losses are reduced from the baseline by one third and drain returns are reduced from the baseline by one half. The third is a new storage scenario for average and dry years, wherein new storage is added with and without out-basin water transfers. Basin-wide net benefits (consumer surpluses) of each management scenario are presented relative to those of the baseline scenario.

**Baseline Scenario**

The baseline equilibration of HF supply and demand is subject to existing water rights and is constrained either by 30 year average water availability (1978-2008) or by the average water availability during three dry years (2003-2005). Baseline scenario water allocations for irrigation, aquifer recharge, and instream flow are calibrated using historical records of diversions and river gaging during average and dry years.

In an average year, baseline HF model natural flow and storage water supply totals about 1.1 million AF. Irrigation season diversions by FMID account for about 74 percent of total annual supply. Aquifer recharge deliveries during the non-irrigation season account for another 6.6%. Instream flows through the Island Park and St Anthony reaches during Dec, Jan and Feb are dependent upon FMID drain returns and HF operational releases made to maintain adequate storage space for projected spring runoff. About 8.4 percent of total annual supply flows through the Island Park and St Anthony reaches during the non-irrigation season. A little more than 11% of the initial HF water supply is carried over in storage to the next year (figure 21).

In a dry year, baseline HF water supply is reduced about 9 percent to about 1.0 million AF. Natural flow and storage diversion by FMID increases to about 78 percent of available supply. Deliveries of aquifer recharge decline to about 6 percent and instream flows through the Island Park and St Anthony reaches are down to about 7.4 percent. Carryover in storage is reduced to about 8.6 percent of the initial supply (figure 21).
Equilibrated baseline results are broken down further in figure 22. Irrigation supply and demand is split among the four major FMID crops. Fisheries flows are split between the two HF reaches\(^4\), and diversions for aquifer recharge allocated to the three HF sub-basins.

---

\(^4\) In an average year, HF tributaries (Warm River, Conant Creek, Teton River, Falls River, and Moody Creek) contribute about 156,000 AF to Dec-Feb flows in the St Anthony reach, in dry years these flows drop to about 87,000 AF. (Only the changes in the HF contribution to Island Park and St Anthony spawning flows are presented in PE model results.)
Consumer surplus, which is the summed difference between willingness to pay for water and the equilibrium water price is the measure of net benefits used in PE modeling. Consumer surplus calculation is subject to binding constraints on water supply (figure 4). For fisheries, the constraint on supply of instream flow depends on required minimum flows, HF tributary flows, irrigation returns and reservoir operational releases. While operational releases are common in average water years they are mostly absent in dry years.

During average years FMID net benefit from water use (consumer surplus) is just over $3.8 million (figure 23). During dry years it declines to about $3.65 million. The net benefit from recreational fishing is much smaller, about $15,000 for the Island Park reach and $58,000 for the St Anthony reach during average years. During dry years, the net benefit from recreational fishing in the Island Park reach drops to just over $6,000, and net benefit from the St Anthony reach drops to $51,000.

PE model results show that in average water years, constraints on non-irrigation season flow for fisheries in the St Anthony reach are binding, and the users of St. Anthony reach public goods would be willing to pay, on average, $0.73 per AF for

---

5 Constraint cost is the difference between the equilibrium supply price and the constrained supply (or shadow) price.

6 Minimum winter time flow from Island Park reservoir during December January and February is generally about 50 cfs.
additional flow. Constraints on flow for fisheries in the Island Park reach are also
binding. In average water years users of the Island Park reach public goods would be
willing to pay about $1.74 per AF for additional flows. Recall that the PE model supply
price for instream flows which are rival with irrigation was set at $3.00 per AF.

Naturally, constraints on fisheries flows are also binding in dry years. St Anthony
instream users would be willing to pay slightly more, $0.75 per AF, for additional flow in
dry years, and Island Park users would be willing to pay $2.36 per AF for additional flow
in dry years (still below the $3.00 per AF that FMID irrigators pay for HF storage water).

![Figure 23: Baseline average year and dry year net benefits (consumer surpluses).](chart)

### Canal Automation and New Reservoir Storage Scenarios

Canal automation and new reservoir storage represent FMID demand
management and supply management alternatives. Demand management alternatives
aim to reduce shortages by curbing demand, supply management alternatives aim to
accomplish the same by increasing supply.

Canal automation and new reservoir storage scenarios which permit out-basin
transfers allow them from either existing or new HF reservoir storage. Groundwater
irrigators in the B-Unit (of A&B Irrigation District) and in the Thousand Springs area
(figure 24) are then included as supply and demand nodes in the PE model (figure 20).
The introduction of new canal automation reduces FMID canal seepage losses and drain returns and thereby the need for storage during the irrigation season. Carryover storage during average water years is increased as a result. Baseline FMID water deliveries from natural flow are unaffected. As a consequence of reduced drain returns however, the supply of (non-rival) HF instream flows for fisheries is cutback by more than 70 percent (figure 25).

Figure 24: FMID, A&B Irrigation District, and junior groundwater pumpers in the vicinity of Thousand Springs along the Snake River.
By reducing canal seepage losses and drain returns, canal automation reduces FMID demand for reservoir storage thereby increasing FMID irrigation consumer surpluses (figure 26). FMID consumer surplus increases because water is being used more efficiently. The St Anthony reach instream fisheries consumer surplus decreases because drain returns to this reach are reduced. Fisheries consumer surplus in the Island Park reach is unaffected by automation because this reach does not rely on drain returns for supply.
Because of the lower cost relative to B-unit groundwater, B-unit will choose to irrigate with existing HF storage water if it is available, resulting in a substantial increase in the B-unit consumer surplus. Canal automation in combination with HF out basin water transfers increases B-unit consumer surplus more than three fold but has no effect on the consumer surplus of Thousand Springs irrigators (figure 27). The difference between B-Unit and Thousand Springs demand price elasticities accounts for this. B-unit groundwater pumpers grow higher valued sugar beet and potato crops than groundwater irrigators in the Thousand Springs area; consequently their willingness-to-pay for HF storage is greater, leaving Thousand Springs irrigators out of the market.

Instream flows for fisheries during the non-irrigation season are unaffected by out-basin transfers which are assumed to occur only during the irrigation season. Out-basin transfers do however reduce HF carryover storage.
Although several HF reservoir sites have been proposed by Reclamation, for modeling purposes the supply of new HF storage is assumed to be located at the proposed Badger Creek reservoir site. The construction cost for a reservoir at this site with 47,000 AF capacity is estimated to be $77,130,000 (Reclamation, 2013). In an average water year the supply constraint for this reservoir is expected to be 39,552 AF. Assuming construction costs are amortized over 50 years, the supply cost to FMID irrigators would then be approximately $39.00 per AF.

FMID demand for reservoir storage increases during dry years when the current storage constraint is binding, nevertheless because of its higher supply price, there is still no HF demand for new storage water (figure 28). Out-basin transfers to B-Unit and Thousand Springs groundwater irrigators during dry years are available exclusively from new storage. However the increased supply price reduces out-basin delivery relative to average water years. In dry years the B-unit irrigation supply is a combination of groundwater and new HF storage. The Thousand Springs irrigation supply is still entirely groundwater however.
2. Rival and Non-Rival Management of HF Water Supplies

The second hydro-economic modeling application represents rival and non-rival demands for instream flow public goods using a PE model comprised of just four nodes; a reservoir supply node, an irrigation demand node, and two spatially distributed demands nodes for instream flow (figure 35).

The application consists of three scenarios. The first scenario calculates instream flow allocations and benefits assuming that the two instream flow demands and the irrigation demand are rival. The second scenario assumes the two instream flows are non-rival in meeting fisheries demands but rival with irrigation demand. The third scenario assumes that the two instream flow demands are also non-rival with irrigation demands during winter months (specifically, with demands for reservoir operational releases and aquifer recharge). It is assumed that these demands are met after flows pass through both fisheries reaches, which means that operational releases are also out-basin transfers, and that aquifer recharge occurs only via canals at or below the Egin Bench.

In the first scenario, net benefits are determined by horizontal summation of all instream flow and irrigation demand quantities. In the second scenario, instream flow net
benefits are determined by vertical summation of fisheries flow demand prices in the two reaches, and irrigation net benefits are determined by horizontal summation of irrigation demand quantities. And in the third scenario, non-irrigation season benefits are determined by vertical summation of fisheries flow demand prices and non-irrigation season demand prices for aquifer recharge and out basin releases. Irrigation season net benefits are determined by horizontal summation of irrigation season demand quantities.

\[
\begin{align*}
\text{reservoir supply } X_i & = X_1 \\
\text{river flow } X_{ij} & = X_{1,2} \\
\text{irrigation demand } X_f & = X_2 \\
\text{fisheries demand } X_f & = X_3 \\
\end{align*}
\]

Figure 29: Schematic of four node PE model with rival and non-rival water demands.

The GAMS LIST file for the three scenarios is displayed in Table 1. Results from the first PE model scenario in which it is assumed that the timing requirements to meet HF instream flow and irrigation demands are such that instream flows in the Island Park and St. Anthony reaches are compelled to be rival with each other and with irrigation demands, generates the lowest total benefit for fisheries ($5,539). The second PE model scenario, in which the timing requirements of instream flow are such that the two HF reaches are non-rival with each other but rival with all irrigation demands generates a total benefit for fisheries that is greater then the first by a factor of four ($21,104). Finally, in the third PE model scenario, the timing requirements are further relaxed so that instream flow demands are assumed non-rival with all irrigation demands that occur during the non-irrigation season. Total fisheries benefit generated is nearly two orders of magnitude greater then the first scenario ($584,178).

Of the three scenarios, the third comes the closest to approximating the actual management practices of instream flows for fisheries in the HF (HFAG/JPC, 2005), (FMID, 2013). The difference between scenario 3 and scenario 2 benefits ($584,178-$21,104) comes closest then to representing the value of HF fisheries and other eco-services that could be realized from mostly non-rival management of HF supplies for both irrigation and instream flows.
Table 1: PE model net benefits and equilibrium prices ($/AF) and quantities (AF) for rival and non-rival demand scenarios.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Equalities</th>
<th>Scenarios</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Rival instream flow &amp; irrigation demands</td>
<td>Part-rival instream flow &amp; irrigation demands</td>
</tr>
<tr>
<td>Benefit (consumer surplus) node 2</td>
<td>$495,204</td>
<td>$495,204</td>
<td>$495,204</td>
</tr>
<tr>
<td>Benefit (consumer surplus) nodes 3 &amp; 4</td>
<td>$5,539</td>
<td>$21,104</td>
<td>$584,178</td>
</tr>
<tr>
<td>Demand price node 2 $\bar{\rho}_2$</td>
<td>$\bar{\rho}_2 = \bar{p}^2$</td>
<td>$3.46$</td>
<td>$3.46$</td>
</tr>
<tr>
<td>Demand price node 3 $\bar{\rho}_3$</td>
<td>$\bar{\rho}_3 = \bar{p}^3$</td>
<td>$3.46$</td>
<td>$3.46$</td>
</tr>
<tr>
<td>Demand price node 4 $\bar{\rho}_4$</td>
<td>$\bar{\rho}_4 = \bar{p}^4$</td>
<td>$3.46$</td>
<td>$3.46$</td>
</tr>
<tr>
<td>Total benefit (total surplus)</td>
<td>$500,743$</td>
<td>$516,308$</td>
<td>$1,079,382$</td>
</tr>
<tr>
<td>Demand quantity node 2 $\bar{q}_2$</td>
<td>$\bar{q}<em>2 = \bar{q}^2 = \bar{X}</em>{12}$</td>
<td>60,338</td>
<td>60,338</td>
</tr>
<tr>
<td>Demand quantity node 3 $\bar{q}_3$</td>
<td>$\bar{q}<em>3 = \bar{q}^3 = \bar{X}</em>{3}$</td>
<td>14,055</td>
<td>14,055</td>
</tr>
<tr>
<td>Demand quantity node 4 $\bar{q}_4$</td>
<td>$\bar{q}<em>4 = \bar{q}^4 = \bar{X}</em>{14}$</td>
<td>1,588</td>
<td>7,822</td>
</tr>
</tbody>
</table>

1 Irrigation demands that are non-rival with instream flow demands are winter time storage releases made as part of reservoir operations or for aquifer recharge.

Appendix A contains the annotated GAMS code for the Henrys Fork PE model with rival and non-rival instream flow demands, and appendix B contains the GAMS data file for this application. The changes necessary for each of the three scenarios are described in the code.

Additional Discussion

Under the prior appropriation doctrine, river flows held in reservoir storage and released only upon irrigation demand are deemed private goods, both excludable and rival. Since ecological and recreational uses of river flows are both non-excludable and non-rival, instream flows which sustain river ecology and recreational usage are deemed public goods. Competitive markets are seldom the sole mechanism used to allocate water in river systems where public goods are involved (Harou, 2009).
Uncertainty associated with the demand-prices for eco-services means that it is not always possible to specify a single Pareto optimal allocation of water for both irrigation and instream flow public goods. A Pareto frontier for allocation of instream flow public goods has been advocated (Griffin, 2005) as a way of maximizing the total benefit from private and public goods subject to a public goods pricing policy that incorporates an array of exogenous demand-price functions representing the full range of revealed and stated preferences for river system eco-services.

Depending on irrigation and canal operational efficiency, canal seepage and drain discharge account for a significant portion of total canal diversions that is not consumptively used, and ultimately return to the river to become public goods. The complexity of hydrologic and economic interactions between rival and non-rival water demands can be a challenge for management of instream public goods, especially when new reservoir storage, new groundwater pumping or new irrigation water conservation measures would alter existing hydrologic dependencies and economic externalities.

Hydro economic modeling to evaluate the relative benefits of rival and non-rival approaches to managing water demands is a first step in developing strategies which would maximize basin-wide benefits from both private and public goods.
References


FMID, 2013 personal communication


Appendix A - GAMS PE Model Code with Rival and Non-Rival Demands

$ONTEXT

Partial Spatial Equilibrium Water Distribution Model

* Henrys Fork 9/23/2013 RDS

By Leroy Stodick

16 June 2011

$OFFTEXT

$SETGLOBAL PROGPATH C:\watermodel\Henrys Fork folder\rival and non rival HF\Rival and non rival fisheries\n
$SETGLOBAL TEXTNAME 16June2011

$ONEMPTY

* base-case models (no rentals)
$INCLUDE "%PROGPATH%HF_FMID_base_non_rival_ irrigation.gms"
*$INCLUDE "%PROGPATH%HF_FMID_base_RNR4.gms"

FILE KDATA3 / "%PROGPATH%DEMANDFUNC2.csv" /
KDATA3.pw = 900;
FILE KDATA2 / "%PROGPATH%ALL_SUP&DEM.csv" /
KDATA2.pw = 900;
PUT KDATA2;
PUT "QSOUT"/;
PUT ","
"EGIN_BENCH_BARLEY,EGIN_BENCH_WHEAT,EGIN_BENCH_POTATOES,EGIN_BENCH_ALFALFA,"
"L_WATERSHED_BARLEY,L_WATERSHED_WHEAT,L_WATERSHED_POTATOES,L_WATERSHED_ALFALFA,"
"N_FREEMONT_BARLEY,N_FREEMONT_WHEAT,N_FREEMONT_POTATOES,N_FREEMONT_ALFALFA,"
"ST_ANTHONY_FISH,ISLAND_PARK_FISH,EGIN_BENCH_RECHARGE,L_WATERSHED_RECHARGE,N_FREEMONT_RECHARGE,"
",PUMPERS_BARLEY,PUMPERS_WHEAT,PUMPERS_POTATOES,PUMPERS_ALFALFA,"
"SUP_CON$EGIN_BENCH_IRR_N,CON$N_FREEMONT_IRR_N,CON$L_WATERSHED_IRR_N,CON$EGIN_BENCH_IRR_S,"
"CON$N_FREEMONT_IRR_S,CON$L_WATERSHED_IRR_S,CON$EGIN_BENCH_NON_N,CON$N_FREEMONT_NON_N,CON$L_WATERSHED_NON_N," 
"CON$EGIN_BENCH_DRAIN,SCON$L_WATERSHED_DRAIN,"

VARIABLES

WELFARE value of objective function
QD(DEM) quantity demanded
QS(SUP) quantity supplied
X(SUP,DEM) quantity transported from node I to node J
RHOS(SUP) supply prices
RHOD(DEM) demand prices
* RHOG(SUP) COST OF GROUNDWATER CONSTRAINT
RHOM(SUP) cost of drain water constraint
RHOF(SUP) cost of fixed drain constraint
RHOC(SUP) cost of canal constraint
SEEPAGE total seepage from canal
RECH_SEEP recharge seepage
RECHDPR(SUP) demand price for recharge water per acre foot of water pumped

POSITIVE VARIABLES QD,QS,X,RHOD,RHOS,RHOM,RHOC,RHOF;

EQUATIONS

OBJ objective function
*Kuhn Tucker conditions complementary slackness equations
* 1
DEMCONS(I) demand must be met at all nodes
* 2
SUPCONS(I) cannot ship more than is produced
DEMPRIN(I) marginal utility equal to demand price inverse demand function
DEMPR(I) marginal utility equal to demand price forward demand function
SUPPR(I)  marginal cost equal to supply price

* SUPPRB(I)  marginal cost equal to supply price (base model)
PRLINKB(I,J)  price linkage equation (base model)

* DRNCONS(I)  right hand side of drain water supply variable constraints
DRNFIXED(I)  right hand side of fixed drain constraints
CANALCONS(I)  canal quantity constraints
CALCSEEP  total seepage
CALCRECH  seepage for the recharge water
CALCDPR(I)  calculate demand price for recharge water

DEMCONS(DEM).

SUM(SUP,X(SUP,DEM)) - QD(DEM) -
SUM(CANAL,S0(CANAL,DEM)*X(CANAL,DEM)) -
SUM(RECHNODES,RECH_S0(RECHNODES,DEM)*X(RECHNODES,DEM))
=G= 0

SUPCONS(SUP).

QS(SUP) - SUM(DEM,X(SUP,DEM)) =G= 0

****************************************************************************************************

DEMPRIN(DEM1).
* Inverse of marginal demand-price function, Q=f(P), Compatible with IDEP demand
calculator coefficients.

RHOD(DEM1)-(1/B1(DEM1)*-(QD(DEM1)-
B0(DEM1))/B0(DEM1)**(1/B2(DEM1)))) =G= 0

****************************************************************************************************

DEMPR(DEM2).
* forward demand price function
* second term (B3, B4 & B5) represents non-rival demand

RHOD(DEM2) - B0(DEM2)*(1 - B1(DEM2)*QD(DEM2)**B2(DEM2)) =G= 0
RHOD(DEM2) - B0(DEM2)*(1 - B1(DEM2)*QD(DEM2)**B2(DEM2)) -
B3(DEM2)*(1 - B4(DEM2)*QD(DEM2)**B5(DEM2)) =G= 0
* RHOD(DEM2)-B3(DEM2)*(1 - B4(DEM2)*QD(DEM2)**B5(DEM2)) =G= 0

43
;******************************************************************************************************************************
SUPPR(SUP).
    A0(SUP)
    * + A1(SUP)*A2(SUP)*EXP[A3(SUP)*QS(SUP)-A4(SUP)*(SEE PAGE+RECH_SEEP)]
    - RHOS(SUP)
        + RHOC(SUP) + RHOF(SUP) + RHOM(SUP)
    * + SUM(AGDRN,RHOM(AGDRN)*C1(AGDRN)*C3(AGDRN)*EXP[C2(AGDRN)*SEE PAGE - C3(AGDRN)*SUM(PUMP,QS(PUMP))])$PUMP(SUP)
    =G= 0;

SUPPRB(SUP).
    A0(SUP)
    * + A1(SUP)*A2(SUP)*EXP[A3(SUP)*QS(SUP)-A4(SUP)*(SEE PAGE+RECH_SEEP)]
    - RHOS(SUP)
        + RHOM(SUP) + RHOC(SUP) + RHOF(SUP)
        + RECHDPR(SUP)$PUMP(SUP)
    =G= 0;

;******************************************************************************************************************************
PRLINKB(SUP,DEM)$ARCS(SUP,DEM).
    RHOS(SUP) - RHOD(DEM) + T(SUP,DEM) + RHOD(DEM)*S0(SUP,DEM)
    =G= 0

;******************************************************************************************************************************
*Seepage is proportional to diversion
*Drain return supply is also proportional to diversion (drain return is partly seepage)
* Drain constraint multiplier x the seepage proportion (table S0) = the proportion of diversion that is drain return.
* e.g if seepage proportion of diversion is .25 and the drain return multiplier of seepage is 0.1, then
* the drain return portion of diversion, QS(AGDRN), is 0.025. C0 (below) is the drain constraint multiplier

DRNCONS(AGDRN).
    C0(AGDRN)*SEE PAGE - QS(AGDRN) =G= 0

;
CFIXED(AGDRN) - QS(AGDRN) = G= 0

; CANALCONS(CANAL).
   D0(CANAL) - QS(CANAL) = G= 0

; *GWCONS(PUMP).
* E0(PUMP) - QS(PUMP) = G= 0
*
**************
CALCSEEP..
   SEEPAGE - SUM((CANAL,DEM), X(CANAL,DEM)*S0(CANAL,DEM)) = E= 0

;

CALCRECH..
   RECH_SEEP -
      SUM((RECHNODES,DEM), RECH_S0(RECHNODES,DEM)*X(RECHNODES,DEM))
      = E= 0

;

CALCDPR(PUMP)..
   RECHDPR(PUMP) -
   [SUM((RECHNODES,DEM), X(RECHNODES,DEM)*RECH_S0(RECHNODES,DEM))]
   *A1(PUMP)*A2(PUMP)*A4(PUMP)/A3(PUMP)
   * [EXP(A3(PUMP)*QS(PUMP))-1]*EXP(-
   A4(PUMP)* (SEEPAGE+RECH_SEEP))]/QS(PUMP)
   = E= 0

;

X.FX(SUP,DEM)$NO_ARCS(SUP,DEM) = 0;
RHOM.FX(NONAGDRN) = 0;
RHOC.FX(NONCANAL) = 0;
RHOF.FX(NONAGDRN) = 0;
RECHDPR.FX(NONPUMP) = 0;

** Third solution using MCP and Path solver with externaities.
MODEL BASEMODEL
   / 
   DEMCONS.RHOD 
   SUPCONS.RHOS 
   DRNCONS.RHOM 
   DRNFIXED.RHOF 
   CANALCONS.RHOC
SET MNAMES names of models

/ BASE

; PARAMETER QDOUT(DEM,MNAMES) quantity demanded;
PARAMETER QSOUT(SUP,*) quantity supplied;
PARAMETER RHOSOUT(SUP,*) supply price;
PARAMETER RHODOUT(DEM,*) demand price;
PARAMETER RHOMOUT(SUP,*) marginal cost of variable constraint for drain water users;
PARAMETER RHOCOUT(SUP,*) marginal cost of canal constraints;
PARAMETER RHOFOUT(SUP,*) marginal cost of fixed constraint for drain water users;
PARAMETER XOUT(*,SUP,DEM) quantity supplied from node SUP to node DEM;
PARAMETER CANSEEP(*,SUP,DEM) seepage in canal from node SUP to node DEM;
PARAMETER SEEPOUT(*) Total seepage;
PARAMETER RECHOUT(*) recharge seepage;
PARAMETER PROSUP(*,SUP) Producer surplus;
PARAMETER CONSUP(*,DEM) Consumer surplus;
PARAMETER TOTCONSUP(*) TOTAL CONSUMER SURPLUS
PARAMETER TOTSUP(*) total surplus;

OPTION QDOUT : 0
OPTION QSOUT : 0
OPTION RHOSOUT : 2
OPTION RHODOUT : 2
OPTION RHOMOUT : 2
OPTION RHOCOUT : 2
OPTION RHOFOUT : 2
OPTION XOUT : 0
OPTION CANSEEP : 0

SET LNUM1/LN1*LN1/;
PARAMETER SUP_PRICE1;
PARAMETER SUP_PRICE2;
PARAMETER SUP_PRICE3;
PARAMETER SUP_PRICE4;
PARAMETER SUP_PRICE5;
PARAMETER SUP_PRICE6;

PARAMETER SUP_PRICES2;
PARAMETER SUP_PRICES3;
PARAMETER SUP_PRICES4;
PARAMETER SUP_PRICES5;
PARAMETER SUP_PRICES6;

PARAMETER SUP_QUAN1;
PARAMETER SUP_QUAN2;
PARAMETER SUP_QUAN3;
PARAMETER SUP_QUAN4;
PARAMETER SUP_QUAN5;
PARAMETER SUP_QUAN6;

PARAMETER XEB1;
PARAMETER XEB2;
PARAMETER XEB3;
PARAMETER XEB4;
PARAMETER XEB5;
PARAMETER XEB6;
PARAMETER XEB7;

PARAMETER DEM_QUAN1;
PARAMETER DEM_QUAN2;
PARAMETER DEM_QUAN3;
PARAMETER DEM_QUAN4;
PARAMETER DEM_QUAN5;
PARAMETER DEM_QUAN6;

PARAMETER SUP_CONSTRN1;
PARAMETER SUP_CONSTRN2;
PARAMETER SUP_CONSTRN3;
PARAMETER SUP_CONSTRN4;

PARAMETER SUP_CONSTRDW1;
PARAMETER SUP_CONSTRDW2;
PARAMETER SUP_CONSTRDW3;
PARAMETER SUP_CONSTRDW4;

PARAMETER CONSUP1;
PARAMETER CONSUP2;
PARAMETER CONSUP3;
PARAMETER CONSUP4;
PARAMETER CONSUP5;
PARAMETER CONSUP6;

*Below is the starting value for quantity demanded for MCP solver. In some cases with inverse demand
* functions it must be set to a fairly large number to avoid division by zero and achieve solution
* convergence. In the absence of inverse demand functions it can still cause problems. Although
* setting to zero is the default value.

*QD.L(DEM)=100.0;
QD.L(DEM)=0.0;
SOLVE BASEMODEL USING MCP;
   SUP_PRICE1=AO("FMID_IRRIGATE_NFL");
   SUP_PRICE2=AO("FMID_IRRIGATE_STO");
   SUP_PRICE3=AO("FMID_NON_IRRIGATE_STO");
   SUP_PRICE4=AO("ST_ANTHONY_RETURN_FLOW");
   SUP_PRICE5=AO("MUD LAKE GROUNDWATER");
   SUP_PRICE6=AO("FMID_CANAL_SEEPAGE");

   SUP_QUAN1=QS.L("FMID_IRRIGATE_NFL");
   SUP_QUAN2=QS.L("FMID_IRRIGATE_STO");
   SUP_QUAN3=QS.L("FMID_NON_IRRIGATE_STO");
   SUP_QUAN4=QS.L("ST_ANTHONY_RETURN_FLOW");
   SUP_QUAN5=QS.L("MUD LAKE GROUNDWATER");
   SUP_QUAN6=QS.L("FMID_CANAL_SEEPAGE");

   XEB1=X.L("FMID_IRRIGATE_NFL","FMID_IRRIGATION");
   XEB2=X.L("FMID_IRRIGATE_STO","FMID_IRRIGATION");
   XEB3=X.L("FMID_IRRIGATE_NFL","ST_ANTHONY_FISHERIES");
   XEB4=X.L("FMID_IRRIGATE_STO","ST_ANTHONY_FISHERIES");
   XEB5=X.L("FMID_NON_IRRIGATE_STO","FMID_IRRIGATION");
   XEB6=X.L("FMID_NON_IRRIGATE_STO","FMID_CARRYOVER");
   XEB7=X.L("FMID_NON_IRRIGATE_STO","ISLAND_PARK_FISHERIES");

   DEM_QUAN1=QD.L("FMID_IRRIGATION");
   DEM_QUAN2=QD.L("MUD LAKE Irrigation");
   DEM_QUAN3=QD.L("ST_ANTHONY_FISHERIES");
   DEM_QUAN4=QD.L("ISLAND_PARK_FISHERIES");
   DEM_QUAN5=QD.L("FMID_CARRYOVER");
   DEM_QUAN6=QD.L("FMID_IS_PARK_FISH");

QDOUT(DEM,"BASE") = QD.L(DEM);
QSOUT(SUP,"BASE") = QS.L(SUP);
RHOSOUT(SUP,"BASE") = RHOS.L(SUP);

RHODOUT(DEM,"BASE") = RHOD.L(DEM);
XOUT("BASE",SUP,DEM) = X.L(SUP,DEM);
CANSEEP("BASE",SUP,DEM) = SO(SUP,DEM)*X.L(SUP,DEM);
RHOMOUT(SUP,"BASE") = RHOM.L(SUP);
RHOCOUT(SUP,"BASE") = RHOC.L(SUP);
RHOFOUT(SUP,"BASE") = RHOF.L(SUP);
SEEPOUT("BASE") = SEEPAGE.L;
RECHOUT("BASE") = RECH_SEEP.L;

* STORAGE CONSTRAINT COSTS FOR PRINTING
SUP КонСTRN1=RHOCOUT("FМID_IRRIGATE_NFL","BASE");
SUP КонСTRN2=RHOCOUT("FМID_IRRIGATE_STO","BASE");
SUP КонСTRN3=RHOCOUT("ST_ANTHONY_RETURN_FLOW","BASE");
SUP КонСTRN4=RHOCOUT("FМID_NON_IRRIGATE_STO","BASE");
SUP КонСTRNDW1= RHOM.L("FМID_IRRIGATE_NFL");
SUP КонСTRNDW2= RHOM.L("FМID_IRRIGATE_STO");
SUP КонСTRNDW3= RHOM.L("ST_ANTHONY_RETURN_FLOW");
SUP КонСTRNDW4= RHOM.L("FМID_NON_IRRIGATE_STO");

* RECHPX is the value of an acre foot of water in the recharge canal to the
* groundwater pumper. It is the integral of marginal pumping cost with respect to his
* pumping rate
* then the derivative of this integral (total pumping cost) with respect to canal seepage
* This gives change in his total pumping cost per unit of canal seepage
* which is the value of seepage in terms of reduced pumping cost

*RECHPX(RECHNODES,DEM) =
SUM(PUMP,[RECH_S0(RECHNODES,DEM)*A1(PUMP)*A2(PUMP)*A4(PUMP)/A3(PUMP)]*[EXP(A3(PUMP)*QS.L(PUMP))-1]
* [EXP(-A4(PUMP)*(SEEPAGE.L+RECH_SEEP.L))];

PROSUP("BASE",PUMP) = - A0(PUMP)*QS.L(PUMP);

*consumer surplus from demands represented by forward demand function
CONSUP("BASE",DEM2) = B0(DEM2)*QD.L(DEM2) -
(B0(DEM2)*B1(DEM2)/(B2(DEM2)+1)))*QD.L(DEM2)**(B2(DEM2)+1) -
QD.L(DEM2)*RHOD.L(DEM2);
*consumer surplus from demands represented by inverse demand function
CONSUP("BASE",DEM1) =(-B0(DEM1)/B1(DEM1))*(B2(DEM1)/(1+B2(DEM1)))*(-
(QD.L(DEM1)-B0(DEM1))/B0(DEM1))**((1+B2(DEM1))/B2(DEM1))-(
B0(DEM1)/B1(DEM1)*(B2(DEM1)/(1+B2(DEM1))))-QD.L(DEM1)*RHOD.L(DEM1);
*total consumer surplus

\[
\text{TOTCONSUP("BASE") = SUM(DEM2,CONSUP("BASE",DEM2))} + \text{SUM(DEM1,CONSUP("BASE",DEM1))};
\]

\[
\text{TOTSUP("BASE") = SUM(SUP,PROSUP("BASE",SUP))} + \text{SUM(DEM,CONSUP("BASE",DEM))};
\]

**DISPLAY**

\[
\text{QDOUT, QSOUT, RHOSOUT, RHODOUT, RHOCOUT, RHOFOOUT, RHOOUTH, RECHDPRL, SEEPOUT, RECHOUT, XOUT, CANSEEUP, PROSUP, CONSUP, TOTCONSUP, TOTSUP};
\]

\[
\text{CONSUP1 = CONSUP("BASE", "FMID_IRRIGATION");}
\]

\[
\text{CONSUP2 = CONSUP("BASE", "MUD_LAKE_IRRIGATION");}
\]

\[
\text{CONSUP3 = CONSUP("BASE", "ISLAND_PARK_FISHERIES");}
\]

\[
\text{CONSUP4 = CONSUP("BASE", "ST_ANTHONY_FISHERIES");}
\]

\[
\text{CONSUP5 = CONSUP("BASE", "FMID_CARRYOVER");}
\]

\[
\text{CONSUP6 = CONSUP("BASE", "FMID_IS_PARK_FISH");}
\]

*Generate excel file supply and demand prices quantities and consumer surpluses*

\[
\text{FILE KDATA1 / "%PROGPATH%DEMANDFUNC1.csv"};
\]

\[
\text{KDATA1.pw = 900;}
\]

\[
\text{PUT KDATA1;}
\]

\[
\text{PUT "FMID nat flow price, FMID nat flow supplied, FMID storage price, FMID storage supplied";}
\]

\[
\text{PUT SUP_PRICE1,"",SUP_QUAN1,"",SUP_PRICE2,"",SUP_QUAN2;}
\]

\[
\text{PUT "FMID Non-Irr price, FMID Non-Irr supplied";}
\]

\[
\text{PUT SUP_PRICE3,"",SUP_QUAN3;}
\]

\[
\text{PUT "St Anthony drain water price, St Anthony drain water supplied";}
\]

\[
\text{PUT SUP_PRICE4,"",SUP_QUAN4;}
\]

\[
\text{PUT"Mud Lake gw supply price, Mud Lake gw quantity supplied,";}
\]

\[
\text{PUT SUP_PRICE5,"",SUP_QUAN5;}
\]

\[
\text{PUT"FMID canal seepage supply price, FMID canal seepage quantity supplied,";}
\]

\[
\text{PUT SUP_PRICE6,"",SUP_QUAN6;}
\]

\[
\text{PUT"FMID irrigation nat. flow constraint, FMID irrigation storage constraint";}
\]

\[
\text{PUT SUP_CONSTRN1,"",SUP_CONSTRN2;}
\]
PUT"FMID irrigation nat. flow constraint cost, FMID irrigation storage constraint cost" /
PUT SUP_CONSTRN1,"",SUP_CONSTRN2; 

PUT"St Anthony return flow supply constraint, St Anthoy return flow supply constraint cost" /
PUT SUP_CONSTRN3,"",SUP_CONSTRDW3; 

PUT"FMID non-irrigation supply constraint, FMID non-irrigation supply constraint cost" /
PUT SUP_CONSTRN4,"",SUP_CONSTRDW4; 

PUT"FMID irrigation demand quantity" /
PUT DEM_QUAN1; 

PUT"Mud Lake irrigation demand quantity" /
PUT DEM_QUAN2; 

PUT"St Anthony fisheries demand quantity" /
PUT DEM_QUAN3; 

PUT"Island Park fisheries demand quantity" /
PUT DEM_QUAN4; 

PUT"FMID carryover demand quantity" /
PUT DEM_QUAN5; 

PUT"Mud Lake irrigation & Island Park fisheries demand quantity" /
PUT DEM_QUAN6; 

PUT"FMID_IRRIGATE_NFL to FMID_IRRIGATION, FMID_IRRIGATE_STO to FMID_IRRIGATION"/
PUT XEB1","XEB2; 

PUT"FMID_IRRIGATE_NFL to ST_ANTHONY_FISHERIES, FMID_IRRIGATE_STO to ST_ANTHONY_FISHERIES"/
PUT XEB3","XEB4; 

PUT"FMID_NON_IRRIGATE_STO to FMID_IRRIGATION, FMID_NON_IRRIGATE_STO to FMID_CARRYOVER, FMID_NON_IRRIGATE_STO to ISLAND_PARK_FISHERIES"/
PUT XEB5","XEB6","XEB7; 

PUT"FMID irrigation consumer surplus"/
PUT CONSUP1;
PUT "Mud Lake irrigation consumer surplus";
PUT CONSUP2;

PUT "Island Park fisheries consumer surplus";
PUT CONSUP3;

PUT "ST Anthony fisheries consumer surplus";
PUT CONSUP4;

PUT "FMID carryover consumer surplus";
PUT CONSUP5;

PUT "Mud Lake irrigation & Island Park fish consumer surplus";
PUT CONSUP6;

PUTCLOSE KDATA1 /;
$EXIT
Appendix B - GAMS PE Model Data for Rival and Non-Rival Demands

$SETGLOBAL TITLENAME "FMID Scenarios 26 August 2013"
* Average year Automation model
* revised demand functions "new_demands4.xls"
* base-case nat flow and storage constraints are average year diversions from nat. flow and storage
* no rental storage to B-unit
* P =adjusted potato demand function TC =adjusted transportation cost
* Updated irrigation and non-irrigation rental storage.

*THIS DATA SET IS UPDATED WITH IRRIGATION AND NON-IRRIGATION RENTAL CONSTRAINTS FOR AVERAGE AND DRY YEARS
*THIS DATA SET ALSO HAS MOST UPDATED COMMENTS 12/2/13 9:30 AM
*zero trib flow 12/4/2013
* eliminated the IS_PARK_NON_RELEASE_LR demand and supply nodes because St Anthony demand is Jul-Sep., not winter months 12/4/2013

SET I index of the nodes /
* supply nodes
   FMID_IRRIGATE_NFL,
   FMID_IRRIGATE_STO,
   FMID_NON_IRRIGATE_STO,
   ST_ANTHONY_RETURN_FLOW,
   MUD_LAKE_GROUNDWATER,
   FMID_CANAL_SEEPAGE,

* demand nodes
   FMID_IRRIGATION,
   ST_ANTHONY_FISHERIES,
   ISLAND_PARK_FISHERIES,
   MUD_LAKE_IRRIGATION,
   FMID_CARRYOVER,
   FMID_IS_PARK_FISH
 /
;

ALIAS (I,J);

SET DEM(I) index of demand nodes /
   FMID_IRRIGATION,
   ST_ANTHONY_FISHERIES,
   ISLAND_PARK_FISHERIES,
   MUD_LAKE_IRRIGATION,
FMID_CARRYOVER,
FMID_IS_PARK_FISH
/
;

SET DEM1(DEM) INDEX OF MARGINAL DEMAND FNS. QTY=F(PRICE)
/
* NONE
/
;
SET DEM2(DEM) INDEX OF MARGINAL UTILITY FNS. PRICE=F(QTY)
/
FMID_IRRIGATION,
ST_ANTHONY_FISHERIES,
ISLAND_PARK_FISHERIES,
MUD_LAKE_IRRIGATION,
FMID_CARRYOVER,
FMID_IS_PARK_FISH
/
;

SET SUP(I) index of supply nodes (n=naturalflow s=storage)
/
FMID_IRRIGATE_NFL,
FMID_IRRIGATE_STO,
FMID_NON_IRRIGATE_STO,
ST_ANTHONY_RETURN_FLOW,
FMID_CANAL_SEEPAGE,
MUD_LAKE_GROUNDWATER
/
;

SET CANAL(SUP) index of canal nodes
/
FMID_IRRIGATE_NFL,
FMID_IRRIGATE_STO,
FMID_NON_IRRIGATE_STO
/
;

SET PUMP(SUP) index of groundwater supply nodes
/
MUD_LAKE_GROUNDWATER
/
;

SET AGDRN(SUP) index of drainwater supply nodes
ST_ANTHONY_RETURN_FLOW

; SET RECHNODES(SUP) index of recharge water supply nodes
* NONE

; SET NONPUMP(SUP) index of supply nodes other than groundwater;
NONPUMP(SUP) = NOT PUMP(SUP);

SET NONAGDRN(SUP) index of supply nodes other than drain water;
NONAGDRN(SUP) = NOT AGDRN(SUP);

SET NONCANAL(SUP) index of supply nodes other than canal nodes;
NONCANAL(SUP) = NOT CANAL(SUP);

SET ARCS(SUP,DEM) all possible arcs

FMID_IRRIGATE_NFL,FMID_IRRIGATION,
FMID_IRRIGATE_NFL,ST_ANTHONY_FISHERIES,
FMID_IRRIGATE_STO,FMID_IRRIGATION,
FMID_IRRIGATE_STO,ST_ANTHONY_FISHERIES,
* FMID_NON_IRRIGATE_STO,ISLAND_PARK_FISHERIES,
ST_ANTHONY_RETURN_FLOW,ST_ANTHONY_FISHERIES,
MUD_LAKE_GROUNDWATER,MUD_LAKE_IRRIGATION,
FMID_NON_IRRIGATE_STO,FMID_CARRYOVER,
FMID_NON_IRRIGATE_STO,FMID_IS_PARK_FISH

; SET NO_ARCS(SUP,DEM) arcs which are not possible;
NO_ARCS(SUP,DEM) = NOT ARCS(SUP,DEM);

PARAMETER B0(DEM) First parameter for the marginal utility functions

/ FMID_IRRIGATION 27
FMID_CARRYOVER 27
*fitted for marginal demand price/fish =$22.45
* Non-rival demands
ST_ANTHONY_FISHERIES 750
* Vertical addition of Mud Lake Irrigation and Island Park fisheries
* This B0 is first parameter for Mud Lake irrigation
  FMID_IS_PARK_FISH  27
/
;

PARAMETER B1(DEM) Second parameter for the marginal utility functions
/
  FMID_IRRIGATION .00095
  FMID_CARRYOVER .00095
  ST_ANTHONY_FISHERIES .9948
  ISLAND_PARK_FISHERIES .9949
  MUD_LAKE_IRRIGATION .0009

* Vertical addition of Mud Lake Irrigation and Island Park fisheries
* This B1 is the second parameter for Mud Lake irrigation
  FMID_IS_PARK_FISH .00095
/
;

PARAMETER B2(DEM) Third parameter for the marginal utility functions
/
  FMID_IRRIGATION .612
  FMID_CARRYOVER .612
  ST_ANTHONY_FISHERIES .00043
  ISLAND_PARK_FISHERIES .0004
  MUD_LAKE_IRRIGATION .613

* Vertical addition of Mud Lake Irrigation and Island Park fisheries
* This B2 is the third parameter for Mud Lake irrigation
  FMID_IS_PARK_FISH .612
/
;

PARAMETER B3(DEM) First parameter for the non-rival marginal utility functions
/
  FMID_IRRIGATION 0
  FMID_CARRYOVER 0
  ST_ANTHONY_FISHERIES 0
  ISLAND_PARK_FISHERIES 0
  MUD_LAKE_IRRIGATION 0

* Vertical addition of Mud Lake Irrigation and Island Park fisheries
* This B3 is the first parameter for non-rival Island Park fisheries
FMID_IS_PARK_FISH 1600
/
;

PARAMETER B4(DEM) Second parameter for the non-rival marginal utility functions
/
  FMID_IRRIGATION  0
  FMID_CARRYOVER   0
  ST_ANTHONY_FISHERIES 0
  ISLAND_PARK_FISHERIES 0
  MUD_LAKE_IRRIGATION  0
* Vertical addition of Mud Lake Irrigation and Island Park fisheries
* This B4 is the second parameter for non-rival Island Park fisheries
  FMID_IS_PARK_FISH .9949
/
;

PARAMETER B5(DEM) Third parameter for the non-rival marginal utility functions
/
  FMID_IRRIGATION  0
  FMID_CARRYOVER   0
  ST_ANTHONY_FISHERIES 0
  ISLAND_PARK_FISHERIES 0
  MUD_LAKE_IRRIGATION  0
* Vertical addition of Mud Lake Irrigation and Island Park fisheries
* This B5 is the third parameter for non-rival Island Park fisheries
  FMID_IS_PARK_FISH .0004
/
;
* Marginal supply cost for irrigation water is cost of natural flow and storage water. There is added transportation cost for this water
* due to return flow, the magnitude of which are indicated in the following three tables
*(Trans. cost, seepage pct. and return multiplier). Natural flow supply costs are what IDs charge irrigators for water delivered to the canal
* diversion point, not to the headgates.
PARAMETER A0(SUP) First parameter for the marginal cost functions
/
  FMID_IRRIGATE_NFL  .46
  FMID_IRRIGATE_STO  3.46
  FMID_NON_IRRIGATE_STO 3.46
  ST_ANTHONY_RETURN_FLOW  .01
  FMID_CANAL_SEEPAGE   .01
  MUD_LAKE_GROUNDWATER  10.00
/
* O&M transportation costs are the ID's costs for delivery of water from the canal diversion point to the headgate.
* They are applied to all diversions including seepage losses and return flows as well as to water consumptively used by irrigators.
* Seepage costs are associated with the supply cost of water that seeps from the canal and never reaches the farm headgate.
* O&M transportation costs are separate from supply costs.

TABLE T(SUP,DEM) per unit conveyance cost from Node SUP to Node DEM O&M charge (per AF charge)

<table>
<thead>
<tr>
<th>FMID_IRRIGATION</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>FMID_IRRIGATE_NFL</td>
<td>1.37</td>
</tr>
<tr>
<td>FMID_IRRIGATE_STO</td>
<td>1.37</td>
</tr>
<tr>
<td>FMID_NON_IRRIGATE_STO</td>
<td>0.0</td>
</tr>
</tbody>
</table>

TABLE S0(SUP,DEM) First parameter for the canal seepage functions

<table>
<thead>
<tr>
<th>FMID_IRRIGATION</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>FMID_IRRIGATE_NFL</td>
<td>.66</td>
</tr>
<tr>
<td>FMID_IRRIGATE_STO</td>
<td>.66</td>
</tr>
</tbody>
</table>

TABLE RECH_S0(SUP,DEM) first parameter for the (not incidental) recharge seepage function

* RECH_DEM
* RECH_SUP 0.5

* The drain return multiplier determines the percentage of seepage loss that is drain return.
* Automation scenario drain return is zeroed out

PARAMETER C0(SUP) first parameter for drain return constraint multiplier

/ ST_ANTHONY_RETURN_FLOW .12
/

PARAMETER G0(SUP) first parameter for GROUNDWATER constraint multiplier

/ MUD_LAKE_GROUNDWATER .88
/

PARAMETER CFIXED(SUP) fixed constraint for drain water supply
/ 
ST_ANTHONY_RETURN_FLOW 1.0E10
/
;

PARAMETER D0(SUP) RHS for canal constraints(natural flow and storage constraints)
/
* average year natural flow usage (constraint)
FMID_IRRIGATE_NFL 760140

* total available irrigation season storage (average year)
FMID_IRRIGATE_STO 191227

* Total storage available for irrigation carryover (average year) (measured at the end of the irrigation season)
* = baseline irrigation season storage - baseline FMID irrigation season diversions from storage.
FMID_NON_IRRIGATE_STO 136977
/
;
1. HydroSense Code ReadMe
From: https://github.com/usbr/hydrosense

September 30, 2014
HydroSense is a Hydro-Economic Net Benefit Maximizer. Provided supply and demand for water with the associated economic supply-cost and demand-price, this program solves for the economic partial-equilibrium solution by maximizing the Consumer and Producer Surplus.

The Partial-Equilibrium solution is determined by maximizing the Consumer and Producer Surplus, referred to as the Objective Function, subject to the physical and water management constraints for the Hydro-Economic problem being analyzed. The Objective Function can also be defined as the sum of all the benefits accrued through water use by all of the demanders, minus the costs of providing water from the suppliers and the costs of transporting the water from the suppliers to demanders. The physical constraints include limits on the water available as a supply, and the relationship that defines the loss of water as it is transported from water supplies to water demands, often referred to as the transportation losses. The water management constraints include the limits on the amount of water that demanders are allowed to use as defined by the water rights administration for the problem being analyzed. The optimal (maximum) solution is then determined as the point where the derivative of the Objective Function with respect to the amounts of water provided between the suppliers and demanders (referred to as the decision variables) is equal to zero, as long as this solution is within the physical and management constraints.

The optimal solution must be determined in an iterative fashion utilizing a search algorithm. The search algorithm used in the HydroSense solver employs a Gradient Descent method that utilizes numerical approximations of the first and second derivatives of the Objective Function with respect to the decision variables. The solution proceeds by developing an initial guess for the optimal decision variables which is used to estimate the first and second derivatives of the Objective Function with respect to the array of decision variables. The decision variables are then updated by solving the linear system of equations as:

\[
\{dv^i\} = \{dv^{(i-1)}\} - [(\Delta^2 OF)/\Delta dv^2]^{(-1)} \times \{\Delta OF/\Delta dv\}
\]

Where:

- \(dv^i\) = the updated array of the estimated optimal decision variables for iteration, i, of the solution;
- \(\{\Delta OF/\Delta dv\}\) = the numerical estimates of the first derivative of the Objective Function (OF) with respect to the estimate of the optimal decision variables at iteration, i-1; and
- \([(\Delta^2 OF)/\Delta dv^2]^{(-1)}\) = the inverse of the matrix containing the numerical estimates of the second derivatives of the Objective Function (OF) with respect to the estimate of the optimal decision variables at iteration, i-1.

At the end of each iteration, the updated optimal solution is checked to make sure that all of the problem constraints are met. If an updated decision variable is outside its
constraint, the decision variable is set to its constraint limit, which is then used as the optimal set of decision variables for the next iteration in the solution.
To aid in converging towards a stable solution, an adjustment to the diagonal values of the matrix representing the second derivatives of the Objective Function with respect to the decision variables is performed utilizing a Marquardt adjustment, defined as:

\[ e^{((I-500) \times \Delta dv)} \]

Where:

- \( I \) - the number of iterations;
- \( \Delta dv \) = the incremental change in the decision variable used to calculate the numerical estimates of the first and second derivatives of the Objective Function;

The optimal solver will iterate towards an optimal solution using the procedure described above until the change in the values of the Objective Function and decision variables meet a user defined convergence tolerance, or the user defined maximum number of iterations is reached.

A. Using the software

**Input**
An Excel file, either .xls or .xlsx, containing the following worksheets and data:
- **Supply Curves** - For each supply node two rows of data points representing the marginal cost function. Entered as arrays of flows and marginal costs for each supply node.
- **Demand Curves** - For each demand node two rows of data points representing the marginal price function. Entered as arrays of flows and marginal prices for each demand node.
- **Transportation Losses** - The transportation losses associated with moving water from a supply node to a demand node. Entered as an array of flows from each supply node and an array of flows arriving at each demand node.
- **Transportation Costs** - The transportation costs associated with moving water from a supply node to a demand node. Entered as an array of flows from each supply node and an array of marginal costs to each demand node.
- **Initial Guess** - A matrix guess of the optimal supply to each demand node. Rows representing demand nodes and columns representing supply nodes. If a supply node cannot deliver water to a demand node the guess should be zero.

An example problem (ExampleInput.xlsx) is provided in the software installation directory.

**Model parameters**
The GUI provides access to the following solver parameters:
- **Max Solution Iterations** - The maximum number of iterations the solver will perform. The default is 5000.
- **Convergence Tolerance** - Precision of solver iteration convergence. The default is 0.015.
Advanced parameters

- **Numerical Derivative Increment** - the incremental change in the decision variable used to calculate the numerical estimates of the first and second derivatives of the Objective Function. The default is 0.01.

Output

An excel file, either .xls or .xlsx, the following worksheets will be created or overwritten in the output excel file:

- **Optimal Supply** - Matrix of optimal supply to each demand node. Rows represent demand nodes and columns represent supply nodes.
- **Optimal Deliver** - Matrix of optimal delivery accounting for transportation losses from the supply node to the demand node.
- **Maximum Net Benefit** - The Objective Function value or the sum of the total benefits accrued through water use minus the total costs of water supply and transportation losses.

TO-DO

Setup code tests, using a testing framework like NUnit
Test solver against additional problems with known solutions
Improve solution convergence for a wider range of initial guesses

B. Acknowledgments

This project is based on methodologies developed by the Bureau of Reclamation and University of Idaho - Idaho Water Resources Research Institute, and funded by the Reclamation Research and Development Group.
2. Data Sets that support the final report

Share Drive folder name and path where data are stored:
\IBR1PNRP001\PN6200\Studies\ScienceAndTech\2014.8937.HydroSense.Final

Point of Contact name, email and phone:
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Short description of the data:
There are three folders in this location.

- Code – This folder contains the HydroSense C# code. This is the code that was completed at the time this study was submitted. Any updates to the code can be found on https://github.com/usbr/hydrosense.

- Data – This folder includes the GAMS files that were used to run the Henrys Fork Case study. The files contain the data that supports the study and can be read by GAMS software (http://www.gams.com/) or by a text editor.

- Report – This folder contains PDF files of the two IWRRI publications that were completed for this study: Henrys Fork Hydro-Economic Modeling and An Approach to Hydro-Economic Modeling Using Partial Equilibrium Optimization. A copy of this report is also included in this folder.

Keywords:
Hydro-economic modeling, hydrologic, economic, HydroSense

Approximate total size of all files:
7 MB