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## Discounting for Long-Lived Water Resource Investments


U.S. Department of the Interior

Bureau of Reclamation Technical Service Center Denver, Colorado

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## Executive Summary

Exponential (classic) discounting has been taught to engineers, economists and finance specialists, and routinely employed for decades. When exponential discounting is employed, costs and benefits occurring in the distant future have practically no influence on the investment decision. A number of newly emergent discounting approaches have been described in recent years. Arguably, these new discounting approaches may better represent future economic uncertainty, regional and intergeneration equity, and sustainability considerations. As a group, these new discounting techniques may be better suited for the analysis of longlived infrastructure and environmental investments. A subset of these new discounting approaches is described in this scoping study. These include; Ramsey discounting, hyperbolic and quasi-hyperbolic discounting, gamma discounting, Weibull discounting, Green Book discounting and intergenerational discounting. Many of these new discounting approaches result in declining discount rates (DDRs) over time. DDRs are inconsistent with the underlying tenants of economic theory and can result in suboptimal choice reversals. Even so, DDR approaches have been adopted for official use elsewhere, including France and the United Kingdom. Exploratory use of these new discounting approaches for cost benefit analysis has demonstrated they can have a profound influence on the outcome of an analysis, and hence any policy prescriptions which may follow. This is equally true for environmental programs and for traditional construction and resource extraction projects. At this date, these newly emergent discounting approaches have not been professionally accepted nor sanctioned for official use in the United States. Nonetheless, they appear to hold some promise for the analysis of long-term water resource investments.

## Focus of this Scoping Study

The purpose of this effort is to identify and describe emergent discounting approaches which could be used for long-lived water resource investment analyses. Many factors can influence the outcome of a cost benefit analysis. A subset of these include the choice of discount rate, the systematic and responsible quantification of all costs and benefits associated with the action, as well as deliberate or inadvertent errors, omissions and commissions and explicit representation of uncertainty inherent in the analysis. While these and many other factors are indisputably critical to such analyses, this scoping study will explore only one component of the cost benefit analysis process-newly identified discounting procedures.

## Intergenerational Equity

Intergenerational equity is a concept closely linked to that of sustainability, a topic which will be described subsequently. At its basic level, intergenerational equity holds that each generation should be treated "fairly" and in a similar fashion as other generations. The MIT Dictionary of Modern Economics offers the following definition:
"Fairness in the use of natural resources over time by different generations. The use of a finite resource (unless it can be recycled) or of a renewable resource, at a rate greater than its regenerative, capacity denies the benefits of the use of the resource to a future generation, so that there is an opportunity cost, which is not usually considered in the costs of the current use of the resource" MIT Dictionary (1997 p. 211).

Some of the more influential research on this subject is attributed to John Hartwick and Robert Solow. Using a mostly mathematical argument, Solow (1974) helped illustrate the relationships between economic growth, natural resource use and intergenerational equity. He explicitly cautioned there were limits to substitutability between natural capital and constructed capital. Following his work, Hartwick (1977) formulated the so called, "Hartwick Rule" linking economic growth with the optimal exploitation of natural resources. Under the Hartwick Rule, the gains that society enjoys from current optimal depletion of an exhaustible resource must be optimally reinvested in other forms of capital to preserve consumption levels over time. He assumed there was a high degree of substitutability between natural resources and human built capital resources and went on to conclude natural resource depletion is justified, in an intergenerational context, so long as the increase in capital at least offsets the value of the exploited natural capital. His view, like those of many economists that followed, would later be termed, "weak sustainability.

As can be discerned from these research efforts, there is considerable overlap between the concepts of intergenerational equity and sustainability. While there are many writings on these two topics and their intersection, the work by Padilla (2002) and Stavins, Wagner and Wagner (2003) are among the more accessible. Stavins, Wagner and Wagner (2003) argue that dynamic efficiency, or optimality across time, is a necessary but not sufficient condition for sustainability. They assert that in addition to dynamic efficiency, welfare must be non-decreasing over time, to assure intergenerational equity.

## Sustainability

The concept of sustainability is frequently, if not universally, invoked in discourse about appropriate methods for discounting of long-lived phenomenon.

Sustainability and sustainable approaches are in the forefront of philosophical, ethical and ecological discussions on this topic. Interestingly enough, the term "sustainability" did not originate in these disciplines but instead was coined by Gro Harlem Brundtland, Prime Minister of Norway and the architect of the report, Our Common Future (Brundtland 1987) to describe self-perpetuating project development and investments in lesser-developed countries. This is quite possibly the antithesis of what some in the environmental field might have imagined. Brundtland defined sustainable development as, "development that meets the needs of the present without compromising the ability of future generations to meet their own needs."

The achievement of sustainability is hampered by the many different notions of what the term means. In cases where agreed upon definitions of sustainability exist, they differ markedly across disciplines. Notably, the definition of sustainability and the indicators of sustainability vary considerably between ecologists and economists.

The American Society of Civil Engineers (ASCE) represents the preponderance of professional engineers involved in the development and management of water resources. In 1996 the ASCE revised its Code of Ethics to make the principles of sustainable development part of their canon of civil engineering practice. The following definition of sustainability has been adopted by the Society (ASCE 2014):
"A set of environmental, economic and social conditions in which all of society has the capacity and opportunity to maintain and improve its quality of life indefinitely without degrading the quantity, quality or availability of natural, economic, and social resources."

In terms of practical applications, economists have the majority of the experience. Economic definitions of sustainability and conceptual approaches to the application of this concept are much more richly developed. The definition of sustainability used in Hackett's (2011) ubiquitous environmental economics textbook was developed by Viederman (1996, p. 46), who states:
"Sustainability is a community's control and prudent use of all forms of capital-nature's capital, human capital, [constructed] capital, social capital and cultural capital - to endure, to the degree possible, that present and future generations can attain a high degree of economic security and achieve democracy while maintaining the integrity of the ecological systems upon which all life and production depends."

Viederman's definition is based on five capitals of sustainable development that shape, and are shaped by human society (Hacket 2011).

As described by Hackett (2011), two general branches of sustainability theory are routinely encountered in the literature. These are the theory of weak sustainability and the theory of strong sustainability. Typically, ecologists and conservation biologists are proponents of strong sustainability, while economists by and large are champions of weak sustainability.

## Weak Sustainability

Weak sustainability is built around the concept of wealth, which can be represented by the sum of the five capitals of Viederman (1996). Weak sustainability is said to occur when total or per-capita wealth does not decrease over time. Development activity that increases one form of capital, for example constructed capital, but depletes natural capital, satisfies the principle of weak sustainability, if the total wealth is not reduced. A central tenant of weak sustainability theory is the assumption that one type of capital can readily be substituted for another. In this case, it is presumed constructed capital can effectively replace natural capital and the attendant services provided by ecological systems.

Paraphrasing from Hacket (2011), an important implication of weak sustainability is that it allows for the mitigation of lost natural capital. For example, land conversion that eliminates an acre of wetland may be mitigated with a number of acres of constructed wetlands. Likewise, under weak sustainability, the loss of natural runs of salmon may be offset by the development of fish hatcheries or fish farms.

Hartwick (1977) was responsible for some of the more influential research on this subject. He formulated the so called, "Hartwick Rule" linking economic growth with weak sustainability. Under the Hartwick Rule, the gains that society enjoys from current depletion of an exhaustible resource must be reinvested in other forms of capital to preserve consumption levels over time. Such depletion is justified so long as the increase in capital at least offsets the value of the exploited natural capital.

## Strong Sustainability

Strong sustainability theory springs from the disciplines of ecology and conservation biology. The emphasis of strong sustainability is on the preservation of existing natural capital stocks and preservation of the ecological services they provide. Proponents of strong sustainability espouse there is very little substitutability between natural capital stocks and the flows of goods and services they provide, and other forms of capital. Strong sustainability is premised on the ecological viewpoint, with discontinuities, discreteness and thresholds rather than the smooth and continuous relationships often postulated by other disciplines.

Whereas weak sustainability is focused on maintaining a non-diminishing sum of human, constructed and natural capital, strong sustainability calls for maintaining each of these forms of capital separately (Costanza and Daly 1992).

An important implication of strong sustainability is that it does not allow for the mitigation of lost natural capital. For example, land conversion that eliminates an acre of wetland cannot simply be mitigated with a number of acres of constructed wetlands, as it can under the precepts of weak sustainability. Likewise, under strong sustainability, the loss of natural runs of salmon may not simply be offset by investments in fish hatcheries or fish farms.

Safe minimum standards and the precautionary principle are prominent features of policies consistent with strong sustainability. Safe minimum standards seek to ensure a viable population or habitat area so as to ensure the continuing survival of an at-risk population or habitat niche. The goal of the precautionary principle is to assure the continued existence and minimal functional integrity of the population or habitat area.

## Sustainability Practice

Clearly, there is considerable overlap between the concepts of intergenerational equity and sustainability and there are numerous discussions which overlap both concepts. The vast majority of these discussions are entirely conceptual in nature (for example, see Padilla 2002, Elliot 2005, Howarth 2007, Voinov and Farley 2007 and Baumgartner and Quaas 2010 ). Of the articles reviewed, there are very few examples which contained any practical guidance. In Stavins, Wagner and Wagner (2003), the authors argue (clearly and directly, but again conceptually) that dynamic efficiency is a necessary but not sufficient condition for sustainability. They assert that in addition to dynamic efficiency welfare must be non-decreasing over time, to assure intergenerational equity. Although there may be other examples in the literature, perhaps the best practical treatment can be found the text by Jon Conrad. Conrad (2010, chapter 7) gives a high-level overview of these concepts and provides several concrete and straightforward numerical examples, which helps to illustrate the difficulties in operationalizing them.

## Discounting and CBA

## Discounting

Given two identical rewards, humans typically show a preference for the one that is delivered sooner rather than later. Humans are said to discount, or intuitively
weight, the value of the later reward, by a factor that increases with the length of the delay. As reviewed in Dasgupta (2008), there are many motivations for discounting. Among the more commonly cited are impatience and the role of human mortality.

Discounting is a mathematical procedure employed to make rewards (costs and benefits), which occur at different points in time, temporally equivalent. The costs and benefits of most water resource investments are incurred at different times over what are frequently long time horizons. A fundamental concept in engineering, finance and economics is the timing of benefits and costs makes a difference in the attractiveness of an investment. All other things being equal, one would prefer to receive the benefits of an investment as soon as possible and to incur the costs as far out in the future as possible. Given the choice between receiving $\$ 100$ today or $\$ 100$ a year from now, most people would prefer $\$ 100$ today. Alternatively, if given the choice between paying out $\$ 50$ today or one year from now, most of us would prefer the latter.

Because the timing of these costs and benefits differs across alternative potential investment, responsible choice requires the use of appropriate techniques to allow for commensurate comparisons. Typically, the present value of the future stream of costs and benefits for each investment alternative is computed and the results arrayed for decision-makers.

To reiterate, discounting is the methodology used for identifying the present value of a cost or benefit that occurs at some time in the future. The process of "discounting" is used to make costs or benefits which occur at different points in time commensurate with each other.

## Role of Discounting in CBA

Discounting is a relatively small part of undertaking a cost-benefit analysis (CBA). Members of the general public sometimes confuse discounting with CBA. Cost-benefit analysis is a systematic process for identifying, quantifying, calculating and comparing the benefits and costs of a proposed management alternative, or project. CBA is typically described as having two purposes. The first purpose, and some would say the most important, is to determine if the proposed project represents a sound investment decision. Second, CBA provides a framework for comparing alternative projects or investment decisions. CBA requires comparing the total expected cost of each option against the total expected benefits, to assess whether the benefits outweigh the costs, and by how much. Discounting is a small and mathematically straightforward aspect of the CBA process. There are quite a large number of text and reference books describing the CBA process including those by Mishan (1988), Shaner (1979), James and Lee (1971), DeGarmo, Sullivan and Bontadelli (1993) and Sullivan, Wicks and Koelling (2011). There are likewise an impressive list of documents providing guidance for CBA in a variety of contexts and disciplines. Examples of
those include publications by the U.S. Army Corps of Engineers (2009), the U.S. Environmental Protection Agency (2010), the U.S. Water Resource Council (1983), the California Department of Water Resources (2008), Millennium Challenge Corporation (2009) and Her Majesty's Treasury (2003). A quick perusal of the abundant resources on CBA will further solidify one important point-- discounting is but one small component of undertaking a rigorous, systematic and responsible CBA.

## Discounting and Long-Lived Investments

The mathematical process of discounting has been described as "controversial" (Carson and Tran 2009, Goulder and Stavins 2002) and a "tyranny" (Pearce et al 2003). Conventional cost benefit analysis, using the exponential discounting approach, favors projects with benefits which accrue in the near-term and costs which are incurred in future. Costs or benefits which occur very far out in the future have little practical importance to the investment decision.

## The Problem of Discounting

Numerous authors from a wide array of disciplines have asserted that exponential (classic) discounting is the antithesis of intergenerational equity and sustainability. They argue that the process of discounting ignores the well-being of future generations and contributes to the near-term and unsustainable exploitation of exhaustible natural resources.

A simple numeric example can yield considerable insight into these allegations. Consider, for example, a net benefit of $\$ 100$ which occurs 100 years from now. Using the exponential (classic) discounting approach, with a discount rate of 8 -percent ( 0.08 ), a benefit of $\$ 100$ accruing in year 100 would be worth about $\$ 0.045$ in present value terms [taken from an example described later in this document]. If the time period is extended out to 200 years, using the same discount rate, a benefit of $\$ 100$ accruing in year 200 would be worth about $\$ 0.00000021$ in present value terms. This simple example nicely illustrates the fundamental issue- when exponential discounting is employed, costs and benefits occurring in the distant future, have practically no influence on the investment decision.

There are numerous justifications both theoretical and practical (Belzer 2000) for some form of discounting. There are likewise arguments against discounting on the basis of moral and ethical grounds (Heinzerling and Ackerman 2002, Verchick 2005). Particularly rancorous discussions are often associated with the evaluation of long-lived projects. As described in Padilla (2002, p. 70),
"Conventional cost-benefit analysis discounts all future impacts by applying the time preference as if they happened to present individuals. This procedure ignores the fact that society is composed of mortal individuals of different generations." Padilla (2002 p. 70) also provides a particularly colorful quote attributed to Ramsey (1928 p. 543), a Cambridge mathematician, who said that discounting the consumption of future generations, "is ethically indefensible and arises merely from weakness of the imagination."

## Religious Objections to Discounting

Although sometimes overlooked by modern engineers and economists, there have always been, and continue to be, strident objections against discounting on religious grounds. As related by Belzer (2000), for more than 1500 years the legitimacy of compound interest, and by extension, discounting has been questioned. Usury is the lending of money at exorbitantly high rates of interest. Typically these loans are made to financially disadvantaged individuals, essentially enriching wealthy lenders at the expense of the poor borrowers. "The Church of Rome and the civil authorities it commanded forbade the practice of lending at interest as a deadly sin and punished "usurers" as heretics, thieves and murderers (Belzer 2000 p. 780). The practice of lending with interest was a capital crime in England circa 1550 with some relaxation occurring during the Reformation.

Contemporary opposition to lending at interest is more restrained, but visible nonetheless. Belzer (2000, p. 781) cites arguments made against "illegal lending" and "excessive rates." Many states and the U.S. Federal Government regulate so-called "pay day loans"-short-term loans made in advance of receiving a pay check. These loans are particularly egregious since they are often marketed to less advantaged and less financially savvy members of society, with little other recourse. A recent and interesting account of lending and borrowing by traditional religious parties in America can be found in Frangos (2003).

## Why Not Zero Discount Rates?

Although the discounting dilemma would seem to be readily solved simply by not discounting, it turns out the solution is not nearly so straightforward. Making a conscious decision not to discount is formally equivalent to discounting at a discount rate of zero (0.000). A discount rate of zero implies the weight ascribed to future benefits and costs is identical to the weight given to benefits and costs which occur in the present. Essentially, benefits and costs are equal now, and at any time during the future.

There are two logical implications of using zero discount rates. First, using a discount rate of zero means that we care as much for someone who lives one hundred years from now as we do for someone alive now. But it also means we care as much for an individual who lives one thousand years from now, or even one million years from now. Observations of human behavior strongly suggest this is not the case (Dasgupta 2008). Second, zero discounting implies current generations should reduce their incomes in order to benefit future generations. The effect of lowering the discount rate towards zero is to increase the amount of saving the current generation should undertake. The lower the discount rate, the more future consumption matters, and hence more savings and investment should take place in the current generation. Thus, while lowering the discount rate favors the well-being of future generations, it implies bigger and bigger sacrifices must be made by the current population. By extension, the logical implication of zero discounting is the impoverishment of current generations (Pearce, Groom, Hepburn and Koundouri 2003). A corresponding reduction in productive investment by the current generation will have deleterious effects on future generations. In short, not discounting (discounting at a zero discount rate) is not the solution to the discounting dilemma.

## The Problem of Time Inconsistency

Heal (1998) has shown all types of declining discount rate approaches, except for logistic discounting, result in time inconsistency. "Time inconsistency or incongruence, refers to a situation where plans that are made at one point in time are contradicted by later behavior" (Pearce et al 2003, p. 132). Time inconsistency or incongruence manifests itself as preference reversals.

Frederick, Loewenstein and O'Donoghue (2002) provide a short and very comprehensible explanation of preference reversals at the individual level. Paraphrasing, they report that with DDRs, individual preferences between two delayed rewards can reverse in favor of the more proximate (near-term) reward as the time to both rewards diminishes. For example, an individual may prefer $\$ 110$ in 31 days over $\$ 100$ in 30 days, but with declining discount rates may prefer $\$ 100$ now over $\$ 110$ tomorrow.

From the project analysis standpoint, the effect of DDRs is profound. Time consistency requires that when faced with making a decision, generation A will choose a particular policy. This policy will reflect the optimal choice over a given time horizon. Generation B will then act in accordance with this policy, recognizes it to be in their best interest and does not revise it. If generation A's plan is revised by generation $B$, then generation $A$ will not have chosen the
optimal plan. In the latter case, a preference reversal will have occurred. In the presence of DDRs, the selection of the optimal plan is no longer constant across time and generations, but instead is relative to when the evaluation is made. As Solow (1999) wryly observes, this "sounds like a poor way to run a railroad."

## Selected Discounting Paradigms

## Treatment of Time

The preponderance of publications in the theoretical economic literature represents time as a continuous phenomenon (which it is, of course). All of their important theoretical findings are illustrated using continuous time mathematics, namely differential and integral calculus. In contrast, most practicing engineers, finance specialists and economists use discrete time for analysis purposes. Since, this report is primarily aimed at practitioners, discrete time conventions are used consistently throughout this document.

## Nomenclature

This section of the document introduces selected discounting paradigms using a standardized approach and nomenclature. For purposes of this document, the net benefit occurring in any particular time period (t) will be described as the value $\left(\mathrm{V}_{\mathrm{t}}\right)$ at time ( t ).
(1) $V_{t}=B_{t}-C_{t}$
where: $V_{\mathrm{t}}$ is the value or net benefit at time $(\mathrm{t})$
$B_{\mathrm{t}}$ is the benefit at time ( t )
$C_{\mathrm{t}}$ is the cost at time ( t )
$t$ is a time period index
Consistent with discrete time discounting convention, we employ $\mathrm{t}=0$ to indicate the present time. A discount weight $\left(\mathrm{W}_{\mathrm{t}}\right)$ is a factor equilibrating a value occurring at time ( t ) to a value occurring in the present, or time $(\mathrm{t}=0)$.

The generalized expression for the net present value (NPV) of a series of values occurring over the time horizon between the present $(t=0)$ and the end of the analysis period ( $\mathrm{t}=\mathrm{T}$ ) is illustrated in equation (2).

$$
\begin{equation*}
N P V=\sum_{t=0}^{t=T} V_{t} * W_{t} \tag{2}
\end{equation*}
$$

where: $N P V$ is the net present value
$V_{\mathrm{t}}$ is the value at time ( t )
$W_{t}$ is the discount factor or weight at time ( t )
$t$ is a time period index

## Exponential (Classic) Discounting

Exponential or classic discounting is used ubiquitously in business, engineering, finance and economics and is currently taught to all students in these disciplines. A small sample of the available text books on these subjects includes DeGarmo, Sullivan and Bontadelli (1993), Shaner (1979), Sullivan, Wicks and Koelling (2011), Block, Hirt and Danielsen (2009), Mishan (1988), and James and Lee (1978). Indeed, exponential discounting is the sole discounting approach described in the vast majority of pertinent State and Federal Guidelines including those promulgated by the U.S. Water Resources Council (1983), Army Corps of Engineers (2009), Office of Management and Budget $(1992,2003)$ and the California Department of Water Resources (2008).

For exponential discounting, the discrete ${ }^{1}$ discount weight $\left(W_{t}\right)$ is defined as shown in equation (3), where the discount rate (r) is a time invariant constant.

$$
\begin{equation*}
W_{t}=\left(\frac{1}{1+r}\right)^{t} \tag{3}
\end{equation*}
$$

where: $W_{t}$ is the discount factor or weight at time ( t )
$r$ is the (constant) discount rate
$t$ is a time period index
In this approach, the discount weight $\left(W_{t}\right)$ exponentially decreases as time ( t ) increases, giving rise to this moniker. Figure 1 illustrates the value of the exponential discount weight for 100 years, when the discount rate is 0.080 . As shown, under exponential discounting, the discount weights in years 50 , and beyond, are quite small. As a result, net values (V) which occur in late in the analysis period, would be expected to have little importance in the Net Present Value (NPV) calculation (equation 2).

Exponential discounting is said to be a rational, time consistent discounting approach since decisions made at any one point in time are identical to those which would be made at any other point in time.

[^0]

Figure 1. Exponential Discount Weights.

## Ramsey Discounting

Frank Ramsey (1903 - 1930) a mathematician at Cambridge University made a number of fundamental contributions to economics, among them the Ramsey (1928) formula shown in equation (4). The Ramsey rule, as it is often called, "... provides a useful conceptual framework for examining intergenerational discounting issues." (Arrow et al 2012).

$$
\begin{equation*}
r=\rho+\eta g_{t} \tag{4}
\end{equation*}
$$

Where: $r$ is discount factor
$\rho$ is the pure rate of time preference
$\eta$ is the elasticity of marginal utility of consumption
$g_{t}$ is the growth of consumption
$t$ is a time period index
The Ramsey rule expresses the discount rate (r) as a function of three factors; $\rho$ the pure rate of time preference, $\eta$ the elasticity of marginal utility of consumption and $g_{t}$ the growth of consumption. The pure rate of time preference $(\rho)$ is the rate at which our society discounts benefits (or costs) which occur in the future, a reflection of people's impatience. The absolute value of the marginal utility of consumption $(\eta)$ changes as consumption levels increase and thus can affect the discount rate. In a larger intergenerational context, $\eta$ can be viewed as a measure of inequality aversion. Higher values of $\eta$ imply that wealthier people should forgo more income to make the less-fortunate better off. Depending on the values of these parameters, the Ramsey rule discount rate (r) can be positive, zero or even negative.

Ramsey discounting requires the estimation of two parameters, $\rho$ and $\eta$, as well as a forecast of the rate of growth in consumption $\left(\mathrm{g}_{\mathrm{t}}\right)$. Estimation of these parameter values is the subject of considerable debate. There are two generally recognized approaches for estimating these parameters; the prescriptive approach and the investment or descriptive approach. Using the prescriptive approach, researchers have inferred values for $\rho$ and $\eta$ from income tax schedules and using stated preference techniques (Carson and Tran 2009). Practitioners of the investment approach have estimated values for $\rho$ and $\eta$ from financial market data, albeit with the drawback that financial markets reflect short-term rather than intergenerational decisions. The return on risk-free investments is often suggested to approximate the consumption rate of discount ( $\rho$ ). Considerable advice is available in the applied literature on this topic, some of it is practical in nature (see Moore et al 2004 and EPA 2010 chapter 6). An expert panel convened in 2011 by the Environmental Protection Agency concludes that future consumption growth rates $\left(\mathrm{g}_{\mathrm{t}}\right)$ can be estimated econometrically, but estimation of the remaining parameters remains challenging (Arrow et al 2012). In applied work, a range of literature values is frequently used, the same course of action followed in this document.

For Ramsey discounting, the discrete discount weight $\left(W_{t}\right)$ is defined as shown in equation (5).

$$
\begin{equation*}
W_{t}=\left(\frac{1}{1+\rho+\eta g_{t}}\right)^{t} \tag{5}
\end{equation*}
$$

where: $W_{t}$ is the discount factor or weight at time ( t )
$\rho$ is the pure rate of time preference
$\eta$ is the elasticity of marginal utility
$g_{t}$ is the growth of consumption
$t$ is a time period index
As can be deduced by inspection, Ramsey discounting and exponential discounting are mathematically very similar. Figure 2 compares the Ramsey discount weight and the exponential (classic) discount weight for 100 years. In this figure, the Ramsey discount weight is computed with a $\rho=0.015$ (following Nordhaus 2011), $\eta=1.0$ (as used by Johnson and Hope 2012, among others) and $\mathrm{g}_{\mathrm{t}}=0.025$. With these parameter values, the Ramsey discount rate (r) is equal to 0.040 . The exponential discount weight, calculated using a discount rate of $\mathrm{r}=0.08$, is also shown for comparison purposes. With the parameter values described, the Ramsey discount weight $\left(W_{t}\right)$ decreases over time ( t ) in approximately the same fashion as the exponential (classic) discount factor. Potentially, the two approaches can be made to coincide, with judicious choices of $\rho, \eta, g_{t}$ and r . Like exponential discounting, with Ramsey discounting the net values (V) which occur in late in the analysis period, are relatively small and have relatively less importance in the calculation of Net Present Value (NPV) (refer to equation 2 ).


Figure 2. Ramsey Discount Weights.

The basic Ramsey discounting approach described here is a time consistent approach, as its proponents are quick to point out. A time-dependent decision made today, will be identical to such a decision made at a different point in time.

The original concept of Ramsey discounting has been extended in an impressive variety of contexts and is currently the focus of an extensive and ongoing research. Some of this research is aimed at improved estimates of the two parameters, $\rho$ and $\eta$ and some is designed to explore the role of uncertainty in estimates of these parameters. Typically, this work is mathematically and statistically complex. A small subset of these investigations is typified by Guo et al (2006), Newell and Pizer (2001), Gollier and Weitzman (2010), Goulder and Williams (2012), Weitzman (2007) and Tol (2013). Extensions of Ramsey discounting to intergenerational and multi-regional applications are numerous and include Johnson and Hope (2012) and others. The Ramsey rule has also been extended to include declining discount rate schedules (see Gollier 2013 for a survey of these applications).

As described in Johnson and Hope (2012), in the context of climate change analyses, various equity weighting schemes are commonly employed. These cross-sectional equity weights are applied to wealthier or poorer regions. The Appendix in Johnson and Hope (2012) contains a nice summary of this approach. Conceptually at least, a similar approach could also be used to weight the time series of net benefits estimated for a particular water resource investment project. Such a weighting scheme might be used to address the intergenerational nature of the stream of net benefits associated with the investment.

## Hyperbolic Discounting

Hyperbolic discounting ${ }^{2}$ is an alternative approach to characterizing the timepreference for delay which has been observed in experimental trials involving both humans and animals (Groom et al 2005). In hyperbolic discounting, valuations fall very rapidly for small delay periods, but then fall much more slowly for longer delay periods. This contrasts with exponential discounting, in which valuation falls by a constant factor per unit of delay, regardless of the total length of the delay.

Hyperbolic discounting refers to the tendency for people to choose a smaller reward obtained sooner in time, rather than a larger reward later in time. Hyperbolic discounting has been applied to a wide range of phenomena. These include lapses in willpower, health outcomes, individual consumption choice over time and personal finance decisions (Laibson 1997, Groom et al 2005, Frederick, Loewenstein and O'Donoghue 2002). It should be noted a growing body of recent work typified by Read (2001), Benhabib, Bisin and Schotter (2010), and others, has called into question the interpretations reached in some of these earlier research efforts. It is as yet unknown if there will be a widespread refutation of this previous body of work.

Several different characterizations of hyperbolic discount weights have appeared in the literature (see the treatment by Cropper and Laibson 1999, especially footnote 11). The form of the hyperbolic discount weight employed in Loewenstein and Prelec (1992) is described here. For this hyperbolic discount weight, $\left(W_{t}\right)$ is shown in equation (6) with the time invariant (constant) parameters $k$ and $h$.

$$
\begin{equation*}
W_{t}=\left(\frac{1}{1+k t}\right)^{h / k} \tag{6}
\end{equation*}
$$

where: $W_{t}$ is the discount factor or weight at time ( t )
with $k$ and $h>0$
$t$ is a time period index
The parameter $h$ controls the effect of time perception. As the parameter $h$ tends towards infinity, time is not perceived to pass at all. If $h$ is 0 , time is perceived as passing extremely rapidly. The parameter $k$ influences the degree to which the hyperbolic discount weight differs from the exponential discount weight. As the parameter $k$ approaches zero, the hyperbolic discount weight approaches the exponential case.

[^1]In this approach, the discount weight $\left(W_{t}\right)$ decreases approximately hyperbolically as time ( t ) increases. Figure 3 illustrates the value of the hyperbolic discount weight for 100 years, when $k=0.08$ and $h=0.40$ and compares it with the exponential discount weight $(\mathrm{r}=0.08)$. As shown in this figure, in hyperbolic discounting with this combination of parameter values, after the initial period the discount weights are everywhere higher than the exponential discount weights. As a result, net values ( V ) which occur after present $(\mathrm{t}=0$ ), would be expected to have more importance in the Net Present Value (NPV) calculation (equation 2).


Figure 3. Hyperbolic Discount Weights.

Hyperbolic discounting is said to be a time inconsistent discounting approach since decisions made at one point in time may differ from those which are made at another point in time.

## Quasi-Hyperbolic Discounting

A simpler version of hyperbolic discounting, first attributed to Phelps and Pollak (1968), has been popularized by the economist David Laibson (1997). He has accounted for several phenomena using a particularly simple form of "quasihyperbolic" discounting. Here, future rewards are discounted by a factor of $\beta^{*} \delta^{t}$ where $0<\beta^{*} \delta \leq 1.0$. This implies that people discount future rewards by a constant factor reflecting the presence of a delay. Although not truly hyperbolic, this simpler formulation still captures many of the basic aspects of hyperbolic discounting including short-term impatience.

For quasi-hyperbolic discounting, the discount weight $\left(W_{t}\right)$ is defined as shown in equation (7), where the parameters $\beta$ and $\delta$ are time invariant constants and $t$ can take on only discrete values.

$$
W_{t}=\left\{\begin{array}{ll}
1, & \text { for } t=0  \tag{7}\\
\beta \delta^{t}, & \text { for } t>0
\end{array}\right\}
$$

where: $W_{t}$ is the discount factor or weight at time ( t )
$0<\beta<1.0$
$0<\delta<1.0$
$t$ is a discrete time period index
In the quasi-hyperbolic approach, the discount weight $\left(W_{t}\right)$ decreases approximately hyperbolically as time ( t ) increases. Since $t$ can take on only discrete values, the set of discount weights generated by equation (7) is given by $W_{t}=\left(1.0, \beta^{*} \delta^{1}, \beta^{*} \delta^{2}, \beta^{*} \delta^{3}, \ldots \beta^{*} \delta^{\mathrm{T}}\right)$ where ( $T$ ) is the terminal period.

Figure 4 illustrates the value of the quasi- hyperbolic discount weights for 100 years, where $\beta=0.660$ and $\delta=0.990$, and compares it with a plot of the exponential discount weights (with $\mathrm{r}=0.08$ ) and the hyperbolic discount weights with $k=0.08$ and $h=0.40$. As shown in this figure, in quasi-hyperbolic discounting with these parameter values, after the initial period the discount weights are everywhere higher than the exponential discount weights and are similar to those of the hyperbolic discounting approach. Relative to exponential discounting, when quasi-hyperbolic discounting is employed, a large proportion of the net values $(V)$ which occur after the present $(t=0)$, would be expected to have more importance in the Net Present Value (NPV) calculation (equation 2).

Like hyperbolic discounting, quasi-hyperbolic discounting is said to be a time inconsistent discounting approach since decisions made at one point in time may differ from those which are made at another point in time.

## Gamma Discounting

Martin L. Weitzman is well-known figure in the realm of social discounting and policy research. His seminal paper on Gamma discounting (Weitzman 2001) is but one of his contributions to the field. Weitzman $(1998,2001)$ was among good company when he pointed out the appropriate discount rate for the distant future is uncertain. In the face of this uncertainty, he surveyed Ph.D. economists worldwide, to elicit the discount rates they felt should be used to assess climate change damages occurring in future periods. He enjoyed a response rate of approximately 77 percent (which is quite high) and received nearly 2,800 completed responses. He concluded the distribution of responses was


Figure 4. Quasi-Hyperbolic Discount Weights.
best represented by a Gamma function with a mean of mu ( $\mu$ ) and a standard deviation of sigma ( $\sigma$ ), giving rise to the term, gamma discounting. The gamma discount rate at a particular time $\left(\mathrm{r}_{\mathrm{t}}\right)$ could then be characterized by equation (8). As shown in this equation, the gamma discount rate is not a constant, but instead varies systematically with time ( t ).
(8) $\quad r_{t}=\frac{\mu}{1+t \sigma^{2} / \mu}$
where: $r_{t}$ is the certainty equivalent discount rate at time ( t )
$\sigma$ is the standard deviation of the Gamma distribution
$\mu$ is the mean of the Gamma distribution.
$t$ is a time period index
Using the time-varying gamma discount rate equation (8), Gamma discount weights can be characterized as shown in equation (9) where again, the subscript on $r$ indicates the time-varying nature of gamma discount rates.

$$
\begin{equation*}
W_{t}=\left(\frac{1}{1+r_{t}}\right)^{t} \tag{9}
\end{equation*}
$$

where: $W_{t}$ is the gamma discount factor or weight at time ( t )
$r_{t}$ is the gamma discount rate at time ( t ) from equation (8)
$t$ is a time period index

Notice the gamma discount weight in equation (9) is calculated in much the same as the exponential discount weight (refer back to equation 3), except that the gamma discount weight uses a discount rate which varies with each time period (t).

Figure 5 compares the gamma discount weight and the exponential (classic) discount weight for 100 years. In this figure, the gamma discount weight is computed using a discount rate calculated using equation (8) with a $\mu=0.08$ and with $\sigma=0.05$. These values differ from those reported by Weitzman (2001) and were chosen for consistency with the other discount weight approaches illustrated in this document. The exponential discount weight is also shown in this figure for comparison purposes. The exponential discount weight is calculated using a discount rate of $\mathrm{r}=0.08$. In the case of gamma discounting, the discount weight $\left(W_{t}\right)$ decreases over time ( t ) but at a rate which is less than the exponential (classic) discount factor As a consequence, for gamma discounting, the net values ( V ) which occur in late in the analysis period, would be expected to more important in the calculation of Net Present Value (NPV) (refer to equation 2).

Like hyperbolic discounting, gamma discounting is said to be a time inconsistent discounting approach since decisions made at one point in time may differ from those which are made at another point in time.

The original published research on gamma discounting by Weitzman (1998, 2001) has given rise to an important and very extensive line of follow-on research, particularly with regard to the characterization of future uncertainty in discount rates. Some of this work is mathematically complex. A small subset of these investigations is typified by Newell and Pizer (2001), Guo et al (2006), Almansa and Martinez-Paz (2011), Johnson and Hope (2012), and Arrow, et al (2013).

## Weibull Discounting

As described in Anderson et al (2011), Jamison and Jamison (2010), Axtel and McRae (2008) and others, any probability density function $f(t)$ defined on $[0, \infty)$ can form the basis of a discounting function by taking the integral of the function $f(t)$ between $t$ and $\infty$. The Weibull function is one function which is occasionally employed, with applications appearing shortly after the publications by Weitzman (2001).

The time-varying discrete Weibull discount weights $\left(W_{\mathrm{t}}\right)$ can be characterized as shown in equation (10) where again, the subscript indicates the time-varying nature of Weibull discount weight. In this equation, the parameter (s) affects time


Figure 5. Gamma Discount Weights.
perception by slowing down or speeding up the influence of time on the Weibull discount weight. When the parameter $s=1$, the Weibull discount weight collapses to exponential discount weight with the same annual (constant) discount rate. When $\mathrm{s}>1$, time perception slows down and the Weibull weight lies everywhere above the exponential discount weight. If $s<1$, time perception is speeded up and the Weibull weight lies below the plot of the exponential weight.

$$
\begin{equation*}
W_{t}=\left(\frac{1}{1+r}\right)^{t^{1 / s}} \tag{10}
\end{equation*}
$$

where: $W_{t}$ is the Weibull discount factor or weight at time ( t )
$r$ is the constant annual discount rate
s is a parameter affecting time perception
$t$ is a time period index

Figure 6 compares the Weibull discount weight and the exponential (classic) discount weight for 100 years. In this figure, the Weibull discount weight is computed using a constant annual discount rate $\mathrm{r}=0.08$ and a parameter $\mathrm{s}=1.50$. The exponential discount weight is also shown in this figure for comparison purposes and is also calculated using a discount rate of $\mathrm{r}=0.08$. In the case of the Weibull (with $\mathrm{s}=1.50$ ), the discount weight $\left(W_{t}\right)$ decreases over time ( t ) but at a rate which is less than the exponential (classic) discount factor As a


Figure 6. Weibull Discount Weights.
consequence, for Weibull discounting, the net values (V) which occur in late in the analysis period, would be expected to more important in the calculation of Net Present Value (NPV) (refer to equation 2).

Like hyperbolic discounting, Weibull discounting is said to be a time inconsistent discounting approach since decisions made at one point in time may differ from those which are made at another point in time.

## Green Book Discounting

The Green Book: Appraisal and Evaluation in Central Government is published by Her Majesty's (HM) Treasury in the United Kingdom. The 2003 edition is an update of previous versions and provides guidance on public project analyses which are associated with a significant investment of funds (HM Treasury 2003). In this respect, they are similar to guidance promulgated by the Environmental Protection Agency (EPA 2010), the U.S. Water Resources Council (1983), the U.S. Army Corps of Engineers (USACE 2009) and other American entities. A notable feature of the 2003 edition of the Green Book is the incorporation of a declining schedule of discount rates. This was introduced primarily for the evaluation of potential projects with long-term effects. The Green Book (2003) discount rate at a particular time ( $\mathrm{r}_{\mathrm{t}}$ ) is illustrated in Table (1). As shown in this table, the Green Book discount rate is not a constant, but instead varies in a stepwise and declining fashion with time ( t ).

Table 1. Green Book (2003) Discount Rates

| Period of Years | Discount Rate (\%) |
| :---: | :---: |
| $0-30$ | 3.5 |
| $31-75$ | 3.0 |
| $76-125$ | 2.5 |
| $126-200$ | 2.0 |
| $201-300$ | 1.5 |
| $301+$ | 1.0 |

Using the time-varying Green Book discount rates shown in Table (1), Green Book discount weights can be characterized as shown in equation (11) where again, the subscript on $r$ indicates the time-varying nature of Green Booki discount rates.

$$
\begin{equation*}
W_{t}=\left(\frac{1}{1+r_{t}}\right)^{t} \tag{11}
\end{equation*}
$$

where: $W_{t}$ is the Green Book discount factor or weight at time ( t ) $r_{t}$ is the Green Book discount rate at time (t) from Table (1) $t$ is a time period index

Notice the Green Book discount weight in equation (11) is calculated in much the same manner as the exponential discount weight (refer back to equation 3), except the Green Book discount weight uses a discount rate which varies with each time period ( t ).

Figure 7 compares the Green Book discount weights and the exponential (classic) discount weights for 100 years. In this figure, the Green Book discount weight is computed using the discount rates from Table 1. The exponential discount weight is also shown in this figure for comparison purposes. The exponential discount weight is calculated using a discount rate of $\mathrm{r}=0.08$. In the case of Green Book discounting, the discount weight $\left(W_{t}\right)$ starts at a higher level and decreases over time ( t ) with obvious "steps." It is everywhere higher than the exponential (classic) discount factor As a consequence, for Green Book discounting, the net values ( V ) which occur in late in the analysis period, would be expected to more important in the calculation of Net Present Value (NPV) (refer to equation 2).


Figure 7. Green Book Discount Weights.

Like hyperbolic discounting, Green Book discounting is a time inconsistent discounting approach since decisions made at one point in time may differ from those which are made at another point in time.

## Intergenerational Discounting

Intergenerational discounting was first proposed by Sumaila and Walters (2005, 2007) for potential use in the analysis of sustainable fisheries management programs. The authors state this approach, '...explicitly incorporates the perspectives of both the current and future generations...". With the contributions made by Praeger and Shertzer (2006), the approach is mathematically consistent, although admittedly $a d$ hoc.

Mechanically, intergenerational discounting requires two different discount rates and an assumed length of a generation (G). For intergenerational discounting, the discount weight $\left(W_{t}\right)$ is defined as shown in equation (12) where the notation introduced by Praeger and Shertzer (2006) is employed. In this expression, the present generation annual discount rate $\left(r_{a}\right)$ and the future generation annual discount rate $\left(\mathrm{r}_{\mathrm{fg}}\right)$ are both constant over time.

$$
\begin{equation*}
W_{t}=\left(\frac{1}{1+r}\right)^{t}+\frac{\left(\frac{1}{1+r_{f g}}\right)\left(\frac{1}{1+r_{a}}\right)^{t-1}}{G}\left[\frac{1-\Delta^{t}}{1-\Delta}\right] \tag{12}
\end{equation*}
$$

where: $W_{t}$ is the discount factor or weight at time ( t )
$r_{a}$ is the present generation annual discount rate,
$r_{f g}$ is the future generation annual discount rate,
$G$ is the assumed length of a generation
$\Delta$ is $(1 /(1+\mathrm{r})) /\left(1 /\left(1+\mathrm{r}_{\mathrm{fg}}\right)\right)$
$t$ is a time period index
Praeger and Shertzer (2006) reported they were unable to reproduce Figure 1 in Sumaila and Walters (2005) and concluded an error in the use of the future generation discount rate was responsible. Praeger and Shertzer (2006) introduce the notation $\left(\mathrm{r}_{\mathrm{fg}}\right)$ to represent the future generation annual discount rate and show $\mathrm{r}_{\mathrm{fg}}$ is correctly calculated by equation (13).

$$
\begin{equation*}
r_{f g}=\frac{\left(1-\sqrt[G]{\frac{1}{1+R_{f g}}}\right)}{\sqrt[G]{\frac{1}{\left(1+R_{f g}\right)}}} \tag{13}
\end{equation*}
$$

where: $r_{f g}$ is the future generation annual discount rate,
$R_{f g}$ is the generational discount rate
$G$ is the assumed length of a generation
Figure 8 compares the intergenerational discount weight and the exponential (classic) discount weight for 100 years. In this figure, the present annual discount rate $r_{a}=0.08$, the future generation annual discount rate $r_{f g}=0.003855465$ (consistent with equation 12 above and a generation discount rate; $\mathrm{R}_{\mathrm{fg}}=0.08$ ) and with the length of a generation assumed to be 20 years. .In the case of intergenerational discounting, the discount weight $\left(W_{t}\right)$ decreases over time ( t ) but at a rate which is less than the exponential (classic) discount factor ${ }^{3 .}$ As a consequence, for intergenerational discounting, the net values (V) which occur in late in the analysis period, would be expected to more important in the calculation of Net Present Value (NPV) (refer to equation 2).

No published assessments of the consistency of intergenerational discounting approach with economic theory have been identified at this juncture. It might be hypothesized however, that since the rate of change in the discount weight with respect to time differs, it is likely this approach is time inconsistent.

[^2]

Figure 8. Intergenerational Discount Weights.

## Comparison of Discount Weights

Table 2 reports the numeric value of the discount weights in selected time periods for each of the discounting approaches examined previously. These weights are computed using the discount rates and parameters described in the text. Comparison of these weights can be instructive. As reported here, the discount weights produced by all of these newly described approaches are typically more favorable for long-term investments with benefits accruing in the distant future, than are the exponential (classic) discounting approach commonly used.

Table 2. Comparison of Numeric Weights ( $W_{t}$ )

| Approach | Time (t) |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | $\mathbf{t = 2 5}$ | $\mathbf{t = 5 0}$ | $\mathbf{t = 7 5}$ | $\mathbf{t = 1 0 0}$ |
| Exponential | 0.14602 | 0.02132 | 0.00311 | 0.00045 |
| Ramsey | 0.37512 | 0.14071 | 0.05278 | 0.01980 |
| Hyperbolic | 0.57735 | 0.44721 | 0.37796 | 0.33333 |
| Quasi-Hyperbolic | 0.51336 | 0.51336 | 0.31059 | 0.24158 |
| Gamma | 0.33343 | 0.21500 | 0.16978 | 0.14649 |
| Weibull | 0.51788 | 0.35186 | 0.25443 | 0.19051 |
| Green Book | 0.42315 | 0.22811 | 0.10895 | 0.08465 |
| Intergenerational | 0.68660 | 0.59125 | 0.53230 | 0.48279 |

For the exponential (classic) discounting approach, these result show a $\$ 100$ benefit accruing in year 100 would be worth $\$ 0.045$ in present value terms. In contrast, if intergenerational discounting approach were employed, with the discount rates and parameters described in the text, a $\$ 100$ benefit accruing in year 100 would be worth $\$ 48.279$ in present value terms. Based on these results, it seems clear that anticipated project investments with net benefits accruing in the far-distant future might be differentially advantaged by the choice of discounting technique.

## Other Discounting Approaches

The discounting techniques reviewed here represent a relatively small subset of the approaches which have been described in the professional literature. While a comprehensive review is outside the scope of this effort, some of the more "common" approaches should be noted. These include the application of dual discount rate discounting approaches (a lower rate for future environmental benefits) by Kula and Evans (2011) ${ }^{4}$ including the related exposition by Gollier (2010). The use of econometrically estimated long-run discount rates, such as those identified using the autoregressive fourth order (AR(4)) approach, which uses discount rates observed in the previous four periods as explanatory variables (Hepburn and Koundouri 2007). Proposed regime switching discounting approaches, in which the discount weight function shifts due to changes in exogenous factors, are discussed by Groom et al (2004), Guo et al (2006) and Hepburn and Koundouri (2007). Finally, the manuscript by Axtell and McRae (2008) contains an amazingly lengthy exposition of mathematically suitable discounting techniques based on the Levy, three parameter Gamma and four parameter Dagum distributions, to mention but a few.

## The Case for Declining Discount Rates

As articulated in Pearce et al (2003), Groom et al (2005), Hepburn and Koundouri (2007), Arrow et al $(2012,2013)$, and elsewhere, economic theory provides a compelling argument for the use of declining discount rates. As described in these and other examples, declining discount rates (DDRs) can be shown to arise from various sources. Among these factors are the following; uncertainty about the social rate of return, uncertainty about growth rate of the economy and uncertainty about the growth rate of consumption. Illustrating the manner in which they might cause declining discount rates is, by and large, mathematically challenging and rather tedious. An exception is a logic based example used by Guo et al (2006), Cropper (2013), Arrow et al $(2012,2013)$ and others.

[^3]Cropper (2013) uses a relatively straightforward example to illustrate that uncertainty about future discount rates can lead to a declining discount rate schedule. The example shown here is reproduced in its entirety from Cropper (2013) except the values in Table 3 are recomputed using the discrete exponential (rather than the continuous exponential) discounting formula Cropper employed for consistency with the remainder of this document.

Table 3. Present Value of $\mathbf{\$ 1 0 0 0}$ Received after ( $\mathbf{t}$ ) Years

| $(1)$ | $(2)$ | $(3)$ | $(4)$ | $(5)$ | $(6)$ |
| :---: | ---: | ---: | ---: | ---: | ---: |
| Time (t) | Discount <br> rate $=4 \%$ | Discount <br> rate $=1 \%$ | Discount <br> rate $=7 \%$ | Discount <br> rate <br> uncertain* | Certainty <br> equivalent <br> discount <br> rate |
| 1 | 961.5385 | 990.0990 | 934.5794 | 962.3392 | 0.03913 |
| 10 | 675.5642 | 905.2870 | 508.3493 | 706.8181 | 0.03531 |
| 100 | 19.8000 | 369.7112 | 1.1525 | 185.4318 | 0.01699 |
| 200 | 0.3920 | 136.6864 | 0.0013 | 68.3439 | 0.01351 |
|  |  |  |  |  |  |

Source: Cropper (2013) with table values recomputed using the discrete (exponential) discount formula and where column (6) is calculated using the 'average' rather than the marginal approach presumably used by Cropper.
"Consider a case where we know that future discount rates will be constant, but we are uncertain about the correct rate to use. Table 3 contrasts the present value of $\$ 1,000$ received at various future dates using a constant discount rate of 4 percent versus a constant but uncertain discount rate that equals 1 percent and 7 percent with equal probability. When the discount rate is uncertain, governments should use the expected present value (the average of columns 3 and 4 , shown in column 5). The present value computed using the mean discount rate of 4 percent is always smaller than when the discount rate is uncertain. This effect is magnified over time. When the discount rate is uncertain, a certainty-equivalent discount rate will yield the present values in column 5. As shown in column 6, this certainty-equivalent discount rate declines over time" (Cropper 2013).

Arrow et al (2013) note the governments of both France and England (HM Treasury 2003) have adopted declining discount rate schedules for the analysis of long-lived projects. The U.S. Environmental Protection Agency's Science Advisory Board includes several members of the Arrow et al $(2012,2013)$ team. Not surprisingly then, the guidance promulgated by the Environmental Protection Agency (EPA 2010) closely follows the recommendations made in Arrow et al (2012).

## Reported Applications

At this juncture the application of emerging discounting approaches remains preliminary and the majority of examples of their use are perhaps best described as exploratory. Even so, a cursory search reveals applications in the realms of climate change, fisheries, forestry and energy.

The majority of applications of emerging discounting approaches appear in the context of climate change. The Stern Review (2007) on the social cost of carbon has provided the nexus for a staggering number of reviews, comments, replies and subsequent applications of pertinence to this document. A large subset of these pertain discount rate and discounting approaches. The Stern Review's discount rate for climate change damages was approximately $1.4 \%$, a rate lower than used in most previous analyses of climate change. Many criticisms focused on the use of (basic) Ramsey discounting and parameterization of the Ramsey Formula. Stern argued the use of a rate of pure time preference greater than zero is ethically inappropriate. This view was supported by a number of economists who weighed in on this subject. Nordhaus (2007) and others did not support this assertion. There were (and continue to be) debates over the parameter values used in the Ramsey discounting equation, the treatment of uncertainty (see Tol and Yohe 2006), the rate of future consumption growth, the marginal rate of consumption elasticity, technology improvements and the failure of Stern to use market rates of return. A selection of these discussions can be found in Weitzman (2007), Tol and Yohe (2006) and Nordhaus (2007). Pearce et al (2013) recalculate the social cost of carbon using several discounting approaches. They illustrate the sensitivity of existing value estimates to the discounting approaches. Their findings suggest the social cost of carbon is much higher than reported in the Stern Review (2007), indicating a vigorous climate change policy should be implemented.

Considerable time and resources have also been devoted towards estimating the social cost of carbon by the United States Government. One of the more recent efforts was designed to produce a standard set of social carbon values for use for use in regulatory proceedings and cost benefit analyses by all U.S. Government Agencies (Interagency Working Group on Social Cost of Carbon 2010a, 2010b, 2013). The results of this effort are further described in Greenstone, Kopits and Wolverton (2013). Johnson and Hope (2012) provide a very well written and detailed assessment of this study. They criticize the work of Interagency Working Group for failure to incorporate emergent discounting techniques, such as declining discount rates, and to use pertinent extensions to the Ramsey discounting approach. The authors employ these techniques to reevaluate the social cost of carbon. Johnson and Hope (2012) conclude the social cost of carbon, calculated with these improvements in discounting approach, would be 2.6 to over 12 times higher than the values reported by the Interagency Working

Group (2010a). Pearce et al (2013) also recalculate the social cost of carbon using several discounting approaches. They illustrate the sensitivity of value estimates to these discounting approaches and suggest current policy should be formed accordingly.

Intergenerational discounting was first proposed by Sumaila and Walters (2005, 2007) for potential use in the analysis of sustainable fisheries management programs. The authors state this approach, '.. explicitly incorporates the perspectives of both the current and future generations..." and conclude, "... this approach can help policy makers design management solutions for the natural environment that would [help] stop ... overexploitation of environmental and natural resources..."(Sumailia and Walters 2005 p. 140). Subsequent applied work by Duncan, Hepburn and Papachristodouloua (2011) seems to cast some doubt on this assessment. Duncan, Hepburn and Papachristodouloua (2011) identify the optimal harvest pattern for a fishery using declining discount rates. They conclude that, "With a declining discount rate, the planner reduces stock levels in the early stages (when the discount rate is high) and intends to compensate by allowing the stock level to recover later (when the discount rate will be lower). Such a plan may be feasible and optimal, provided that the planner remains committed throughout. However, in practice there is a danger that such plans will be re-optimized and adjusted in the future. It is shown that repeatedly restarting the optimization can drive the stock level down to the point where the optimal policy is to harvest the stock to extinction."

Hepburn and Koundouri (2007) examine the implications of emerging discounting approaches for the practice of forestry. Using short, medium and long rotation examples, they analyze the impacts of two constant discount rates using exponential discounting, as well as four declining discount rate approaches; Green Book discounting (HM Treasury 2007), AR(4) discounting, regime switching and state space discounting. Hepburn and Koundouri (2007) report the use of alternative discounting approaches favor longer rotational approaches, consistent with long-term husbandry practice.

Pearce et al (2013) examine the prospects for nuclear energy development in Europe. They find the case for nuclear energy is buoyed by alternative discounting approaches since the net present value of electricity revenues and the value of carbon emissions avoided are greatly increased, relative to traditional discounting approaches. Ultimately however, they conclude the (net present value of the) costs of decommissioning are so massive that such projects remain unwarranted.

## Applicability to Water Resource Investments


#### Abstract

Many of the newly emergent discounting approaches described here are readily applicable to analyses of water resource investments. Like forestry, fishery and climate change applications, water resource investments are inherently public investment decisions with very long lifespans. Water resource investments commonly involve the use of unique and irreplaceable natural sites, often with potentially irreversible consequences. Application of these new discounting approaches in the water resource field seems quite natural. Although there may be many other examples in the literature, a handful of such cases were identified during this effort. Birol, Koundouri and Kountouris (2010), who are affiliated with the International Food Policy Research Institute (IFRI), describe an exploratory study applied to water resource investments. Another example is an assessment of the Lower Mekong Basin by Costanza et al (2011). Costanza et al (2011) employs traditional (exponential) discounting as the primary discounting approach, but also uses some of the newly described discounting approaches for "sensitivity analysis." The Costanza et al (2011) effort was eventually published as, Kubiszewski et al (2013). There are few mechanical barriers to applications of this nature and it seems very likely there are other examples in the literature. If newly emergent discounting approaches were to become professionally accepted, they appear to hold some promise for use in water resource investment decisionmaking.


## A Double-Edged Sword

Emerging discounting approaches are a double-edged analysis sword. The choice of discounting approach and the discount rate used in the analysis can have significant effects on cost benefit analyses of all types. Some proponents of emergent discounting approaches suggest their approach will make long-lived environmental programs more advantageous, more readily justify their implementation, and foster sustainability and intergenerational equity (e.g., Sumaila and Walters 2005). However, these emerging approaches can just as readily be employed to justify environmentally damaging projects with construction and resource extractive goals.

In a number of environmental policy debates, reviewers have pointed out the influence the discount rate and discounting approach have on the analysis. Innumerable papers have described the different outcomes (and hence policy
recommendations) which could be obtained by using different discount rates and discounting approaches. The ongoing debate over estimation of the social cost of carbon is a timely and very high-visibility example.

The choice of discount rate and discounting approach have crucially important effects on cost benefit analyses of other types of projects, including resource development, extraction and exploitation projects. Projects which may not have been feasible, or policies which may not have been justified using high discount rates and exponential discounting approaches, become meritorious when lower discount rates and alternative discounting approaches are employed. This has been termed, "the conservationist's dilemma."

On theoretical grounds, Cropper and Laibson (1999) argue against the use of declining discount rate approaches for project analysis. But, as they ruefully note, if declining discount rate approaches were used, they would apply equally well to environmental and traditional investment decisions alike.

## Official Guidance

Federal water resource agencies, such as the Bureau of Reclamation, are required to follow the procedures described in the Economic and Environmental Principles and Guidelines for Water and Related Land Resources Implementation Studies (U.S. Water Resource Council 1983) when undertaking a cost benefit analysis. This document is often fondly referred to as, the "P\&Gs."

A new set of water resource planning guidelines have been announced. These requirements have not been finalized or implemented yet, and are not applicable to this scoping document. In March 2013, the Principles and Requirements for Federal Investments in Water Resources (referred to as the P\&R's) and a set of Draft Interagency Guidelines were released by the President's Council on Environmental Quality. These can be viewed at: (www.whitehouse.gov/sites/default/files/final_principles_and_requirements_marc h_2013.pdf).

Reclamation is participating in an ongoing effort to finalize the Draft Interagency Guidelines and develop Agency Specific Guidelines leading to implementation of the P\&R's. Completion of this effort is slated for some time in 2015. The P\&R's will become effective 180 days after the Final Interagency Guidelines are issued. Until the P\&R's become effective, Reclamation will continue to follow the 1983 P\&G's.

As proscribed in the P\&Gs, the Federal water resource agencies must use an administratively determined discount rate for cost benefit analysis. This rate is known as the Federal discount rate for plan formulation and evaluation. The plan
formulation and evaluation rate is calculated annually by the Secretary of the Treasury, pursuant to 42 United States Code 1962d-1 (See the electronic code of Federal Regulations (http://ecfr.gpoaccess.gov) for a description of the methodology) and then officially transmitted to the water resource agencies. The plan formulation and evaluation rate for fiscal year (FY) 2014 is 3.50 percent (Bureau of Reclamation 2013a, 2013b).

The United States Office of Management and Budget (OMB) Circular A-94 and its Appendices (OMB 1997) serve as guidance to many U.S. Federal Agencies undertaking cost benefit analysis. Circular A-94 is moot on the topic of discounting over long time-horizons or intergenerational discounting. OMB Circular A-4, Regulatory Analysis (OMB 2003) currently requires projects with intra-generational benefits and costs to be discounted at rates of 3 percent and 7 percent. For projects with intergenerational benefits or costs, OMB suggests the analyst, "might consider a further sensitivity analysis using a lower but positive discount rate, in addition to calculating net benefits using discount rates of 3 and 7 percent (OMB 2003 page 36)."

## Professional View of DDR Methods

The prevailing professional viewpoint on declining discount rate approaches is decidedly mixed. Perhaps the greatest single factor is the issue of time inconsistency. All DDR approaches except logistic discounting result in time inconsistency or incongruence (Heal 1998). Time inconsistency is manifested as preference reversals, which are starkly at odds with the long-established tenants of economic theory (see Frederick, Loewenstein and O’Donoghue 2002). Largely on these grounds, Cropper and Laibson (1999) advise against their use for project analysis.

Some recent assessments have been more encouraging. An expert panel convened in 2011 by the Environmental Protection Agency agreed that, "...theory provides compelling arguments for a declining certainty-equivalent discount rate," while acknowledging there are both theoretical and empirical challenges to their use (Arrow et al 2012 page 21). A subsequent 2013 article by the same authors largely restates these findings (Arrow et al 2013).

Hepburn (2004) and others have explored the use of DDR approaches in the optimal exploitation of renewable resources, such as fisheries. They find that naïve or uninformed use of these approaches could lead to over-exploitation and resource collapse. This outcome stems from the time inconsistent nature of DDR approaches.

Outside of academe, DDR approaches are little known and seldom encountered. At this juncture, declining discount rate approaches have not been sanctioned for
use by agencies of the United States Government. Moreover, official guidance promulgated by OMB appears to rule out the use of DDR approaches (OMB 2003 page 36 )."

## Conclusions

Exponential (classic) discounting has been taught to engineers, economists and finance specialists, and routinely employed in cost benefit analyses for decades. A number of new discounting paradigms have emerged. These newly emergent discounting approaches may have some important implications for the practice of cost benefit analysis and potentially for water resource investment analysis and other long-lived phenomenon such as climate change. In the past, most practitioners employed exponential (classic) discounting with a constant discount rate-albeit, one whose value was debatable. A growing body of economic research suggests the discount rate may not be a single (constant) value but may vary in a declining fashion over long time horizons. Declining discount rates (DDRs) are premised on studies of individual behavior, as well as uncertainty arising from various sources, including uncertainty about the future economic conditions. These DDR approaches have been shown to be inconsistent with the underlying tenants of economic theory. In practical terms, when DDRs are employed, a rational economic agent faced with the same choice, may reach different decisions at arbitrary points in time. Applications using these new discounting approaches have demonstrated they can have considerable influence on the outcome of a cost benefit analysis. This is true not only for environmental programs, but for traditional construction and resource extraction projects as well. Although these new discounting approaches have not been officially sanctioned for use in the United States, certain DDR approaches have been officially adopted for the analysis of long-lived investment analyses in France and the United Kingdom. Most practicing engineers and economists are unfamiliar with these new techniques and they are not professionally accepted for analysis purposes. Although it is impossible to foresee the future, these newly emergent discounting approaches could hold some promise for long-term water resource investment analysis.

## Future Directions

There are a surprising number of new and potentially promising discounting approaches. Some of these are (apparently) supported by a cadre of well-known economists working in this arena. Our research team has identified a suite of additional follow-on activities which could be used to explore the applicability of these new discounting methods to water resource investment analysis. We propose to demonstrate these new techniques on one or more textbook type
reservoir management examples. Our intention is to compare and contrast the results obtained using these new discounting approaches with conventional discounting approaches. We will report on their practicality and relevance for decision-making.

## Collaborators

Members of the collaborative team played an important role in the completion of this scoping project. Their generous contributions to this effort are gratefully acknowledged. The collaborative team was comprised of the following individuals, listed in alphabetical order:

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## Mission Statements

The U.S. Department of the Interior protects America's natural resources and heritage, honors our cultures and tribal communities, and supplies the energy to power our future.

The mission of the Bureau of Reclamation is to manage, develop, and protect water and related resources in an environmentally and economically sound manner in the interest of the American public.


[^0]:    ${ }^{1}$ The continuous analog is defined as $\mathrm{e}^{-\mathrm{rt}}$.

[^1]:    ${ }^{2}$ The term "hyperbolic discounting" is used rather loosely in the literature. Some authors use it to refer to all non-exponential approaches. In this document the term is used only to describe approaches based on the hyperbolic function.

[^2]:    ${ }^{3}$ When the future generation annual discount rate $\left(\mathrm{r}_{\mathrm{fg}}\right)$ is calculated correctly, as shown in Praeger and Shertzer (2006), numerical experiments by the author suggest some combinations of present annual discount rates and future generation annual discount rates can produce discount weights which exceed 1.0.

[^3]:    ${ }^{4}$ It is worth noting the use of dual discount rate approach has been described as, "clearly inappropriate" by Arrow et al (2013 p. 8).

