

AMERICAN WATER RESOURCES ASSOCIATION

# STAGE-DISCHARGE RELATIONSHIPS FOR U-, A-, AND W-WEIRS IN UN-SUBMERGED FLOW CONDITIONS<sup>1</sup>

Christopher I. Thornton, Anthony M. Meneghetti, Kent Collins, Steven R. Abt, and S. Michael Scurlock<sup>2</sup>

ABSTRACT: Instream rock weirs are routinely placed into stream systems to provide grade control, reduce streambank erosion, provide energy dissipation, and allow fish passage. However, design and performance criteria for site specific applications are often anecdotal or qualitative in nature, and based upon the experience of the design team. A study was conducted to develop generic state-discharge relationships for U-, A-, and W-weirs. A laboratory testing program was performed in which scaled, near-prototype U-, A-, and W-rock weir structures were constructed in 11 configurations. Each configuration encompassed a unique weir shape, bed material, and/or bed slope. Thirty-one tests were conducted in which each structure was subjected to a sequence of predetermined discharges that minimally included the equivalent of 1/3 bankfull, 2/3 bankfull, and bankfull conditions. All tests were performed in subcritical, un-submerged flow conditions. Stage-discharge relationships were developed using multivariant, power regression techniques for each of the U-, A-, and W-rock weirs as a function of the effective weir length, flow depth, mean weir height, rock size, and discharge coefficient. Unique coefficient expressions were developed for each weir shape, and a single discharge coefficient was proposed applicable to the weirs for determining the channel stage-discharge rating.

(KEY TERMS: rock weirs; stage-discharge ratings; streamflow measurement; stream restoration structures; weir.)

Thornton, Christopher I., Anthony M. Meneghetti, Kent Collins, Steven R. Abt, and S. Michael Scurlock, 2011. Stage-Discharge Relationships for U-, A-, and W-Weirs in Un-submerged Flow Conditions. *Journal of the American Water Resources Association* (JAWRA) 47(1):169-178. DOI: 10.1111/j.1752-1688.2010.00501.x

## INTRODUCTION

Stream bed and bank stability are essential aspects enabling a watershed to approach and sustain dynamic equilibrium. A stable stream system provides predictability in flood control capacity, water quality, habitat and riparian sustainment, and property boundary continuity. The use of instream structures such as weirs, sills, vanes, dikes, drops, and spurs, and their predecessors have been utilized in the United States (U.S.) since the 1880s and earlier in Europe (Thompson and Stull, 2002) to provide grade control, reduce streambank erosion/degradation, enhance energy dissipation, increase aquatic habitat, and allow fish passage. The art and/or science of

<sup>1</sup>Paper No. JAWRA-10-0058-P of the *Journal of the American Water Resources Association* (JAWRA). Received April 21, 2010; accepted October 5, 2010. © 2011 American Water Resources Association. **Discussions are open until six months from print publication**.

<sup>2</sup>Respectively, Assistant Professor, Director of Hydraulic Laboratory, Director of Engineering Research Center (Thornton), Professor (Abt), Graduate Research Assistant (Scurlock), Department of Civil and Environmental Engineering, Colorado State University, Fort Collins, Colorado 80523; Project Manager (Meneghetti), Colorado Department of Transportation, Reg. 4, 2207 East Highway 402, Loveland, Colorado 80537; and Hydraulic Engineer (Collins), Sedimentation and River Hydraulics Group, Technical Services Center, U.S. Bureau of Reclamation, Denver, Colorado (E-Mail/Thornton: thornton@engr.colostate.edu).



FIGURE 1. Instream U-Weir (Reclamation, 2007).

developing design and performance criteria for each of these structures tend/tends to be site specific, experiential based, and often anecdotal or qualitative in nature. However, instream structures have become critical components to channel stability in the era of stream restoration and/or rehabilitation.

Perhaps, one of the most formable instream structures developed, refined, and applied as part of the stream restoration/rehabilitation process is the rock weir as shown in Figure 1. Weir-type structures enhance diverse flow conditions and supply additional roughness to river systems (Rosgen, 2006). Weirs also establish a hydraulic control within the channel that provides both vertical and lateral stability. Rock weirs redirect streamflow, often to the center of the stream channel, and disrupt velocity gradients in the near-bank region. Instream rock weir structures routinely take the shape of linear (broad-crested), J-, U-, A-, and W-forms.

Weirs have also been used to provide channel flow measurements since the 1800s (Schmidt, 2002) through the development of stream stage-discharge relationships. Stage-discharge relationships, or discharge rating curves, define a unique relationship between water-surface stage and the corresponding discharge. Direct measurement of flow in streams is often cumbersome and time consuming. Stagedischarge rating relationships allow for an expedient means of determining channel flow. The broadcrested weir is one of the most commonly applied structures used for flow determination as it horizontally spans the stream from bank to bank with sufficient length to allow a fully developed flow regime (Brater and King, 1976). Stage-discharge relations have also been established for the broad-crested weir, which reliably estimate discharge.

However, stage-discharge relationships have not been fully developed to measure reliably streamflow in channels in which U-, A-, and W-rock weir structures have been placed. The objective of this study is: (1) to construct and test a series of scaled, rock weir structures, with varying shape, bed slope and bed material sizes, in the laboratory; and (2) to develop empirically a first generation expression(s), with appropriate coefficients, to predict reliably the stage-discharge relationship of a U-, A-, or W-rock weir structure placed in a stable stream under un-submerged conditions.

#### STAGE-DISCHARGE EXPRESSIONS

Stage-discharge relationships define a unique relationship between water-surface stage (or flow depth) and the corresponding discharge in the channel. Rating curves are established by obtaining concurrent measurements of stage and discharge yielding a rating curve, usually applicable to a particular structure shape and locale. Stage-discharge reliability and accuracy are dependent upon a structure placed in a stable section of the channel where a critical control depth may be maintained over a spectrum of discharges.

Perhaps, the most generally applied expression for determining the stage-discharge relationship for a sharp-edged, broad-crested weir is

$$Q = CLH^{3/2},\tag{1}$$

where Q is the discharge, C is the discharge coefficient, L is the effective length of the weir crest, and H is the measured head above the crest (Chow, 1959). The effectiveness of Equation (1) is dependent upon the coefficient of discharge, C, which incorporates the effects of relative depth and relative width of the approach channel (Reclamation, 1997), and the roughness of the crest (Novak *et al.*, 1996). Equation (1) was developed for steady, uniform flow conditions, but has been applied to estimate gradually varied flows.

The more commonly utilized stage-discharge ratings address discharge as a unique function of stage. Herschy (1999) indicates that ratings are generally in the form of a power curve as:

$$Q = C_{\rm d}(a+h_1)^n,\tag{2}$$

where  $C_d$  is the discharge coefficient, a is the gageheight of zero flow or offset (ft),  $h_1$  is the flow depth measured relative to weir crest (ft), and n is a constant dependent on channel geometry. The "gage-height of zero flow" or "offset" is defined as the gage-height at which flow over the control ceases and is determined by subtracting the depth of water over the lowest point on the control from the stage indicated by the gage reading. Herschy (1995) indicated that n is 1.5 for a rectangular channel section, 2 for a concave section of parabolic shape, and 2.5 for a triangular or semicircular section. Equation (2) accounts for a constant slope and flow resistance.

Schmidt (2002) presented a stage-discharge expression for flow over a weir, referenced as the control for the rating, as:

$$Q = C_{\rm d} L_{\rm w} (h_{\rm l} - a)^n, \tag{3}$$

where  $L_{\rm w}$  is the length of the weir (ft) and *n* is theoretically equal to 1.5, all other variables are defined as for Equation (2). Equation (3) incorporates the weir length not addressed in Equation (2).

Chin (2006) expressed the stage-discharge relationship for free discharge over a fully suppressed flow condition for a sharp-crested weir as:

$$Q = \frac{2}{3}C_{\rm d}(2g)^{0.5}bh_{\rm l}^{2/3},\tag{4}$$

where g is the acceleration of gravity  $(ft/s^2)$  and b is the width of the weir crest (ft), all other variables as previously defined. The term "a" has been eliminated from the expression. The discharge coefficient ( $C_d$ ) expressed in Equation (4) is computed by

$$C_{\rm d} = 0.611 + 0.075 \{ h_{\rm l}/P \},\tag{5}$$

as P is the height of the weir (ft).

Chin (2006) defined a broad-crested weir as where the ratio of the head measured above the weir divided by the width of the weir crest is between 0.08 and 0.33. The free flow rating over a broad-crested weir in a rectangular channel can be expressed as:

$$Q = C_{\rm d}(g)^{0.5} b \left\{ \frac{2}{3} H_{\rm l} \right\}^{2/3},\tag{6}$$

as  $H_1 = h_1 + (v_1^2/2g)$  and  $v_1$  is the average channel velocity. The discharge coefficient is determined as:

$$C_{\rm d} = \frac{0.65}{[1+H_{\rm l}/P]}.$$
(7)

The U.S. Reclamation (1997) presented a rating expression for broad-crested, rectangular weirs as:

$$Q = C_{\rm d} C_{\rm v} L_{\rm b} \frac{2}{3} \left(\frac{2}{3}g\right)^{0.5} h_{\rm l}^{-3/2},\tag{8}$$

where  $C_{\rm v}$  is a velocity coefficient and  $L_{\rm b}$  is the channel width.

Equations (1-8) were developed to compute the discharge over weir structures extending bank to bank, perpendicular to the stream. Tullis *et al.* (1995) presented a stage-discharge rating expression applicable to a labyrinth weir without side contractions and with normal approach flow conditions. Their general equation is

$$Q = \frac{2}{3}C_{\rm d}L(2g)^{0.5}H_{\rm l}^{1.5}, \qquad (9)$$

as  $H_1$  is the elevation difference between the upstream reservoir and the weir crest and L is the effective length of the weir. The crest coefficient,  $C_d$ , is dependent upon the  $H_1/P$  ratio, the wall thickness, crest configuration (i.e., weir arm angles), and nappe aeration.

The stage-discharge rating expressions presented are by no means a comprehensive collection of the relationships provided within the body of knowledge. However, these relations portray the significant variables that influence the development of a reasonably accurate rating relationship for weir structures independent of the geometry. It is apparent that effective weir length, flow depth above the weir crest, and a dimensionless coefficient depicting the geometry of the weir are prevalent in developing a rating expression for each structure type.

It must be noted that most weir stage-discharge expressions were empirically developed and dimensionally dependent upon a particular system of units. Therefore, in some cases the relationship may not be dimensionally correct. Although the crest coefficients are dimensionless, the stage-discharge relationships may or may not dimensionally conform to the dependant variable. Therefore, the user must be cognizant of the units of applicability of any specific expression.

#### MODEL AND TEST PROGRAM

A hydraulic testing program was conducted by Meneghetti (2009) and Scurlock (2009) in which scaled, U-, A-, and W-shaped rock weir structures were constructed and evaluated at the Engineering Research Center Hydraulics Laboratory at Colorado State University. The prototype stream is located in the upper Rio Grande, New Mexico and a model was constructed using a Froude scaling approach with prototype to model ratio of 1:5.

Eleven rock weir configurations (3 A-weirs, 5 U-weirs, and 3 W-weirs) were subjected to 31 live-bed test conditions to determine weir stage-discharge ratings. Each configuration encompassed a unique combination of weir shape, bed material size, and/or bed slope. All tests were performed in subcritical, un-submerged flow conditions.

## TEST FACILITY AND INSTRUMENTATION

The scaled, near-prototype U-, A-, and W-rock weir structures were constructed in a rectangular flume 16 ft (4.88 m) wide, 50 ft (15.24 m) long, and 4 ft (1.22 m) deep as schematically presented in Figure 2. The flume comprised an intake manifold, flow baffle, head box/transition section, test section incorporating the weir structures, outlet section with tail water control, and tail box. Granular bed materials were placed throughout the flume extending from the entrance of the test section to the tail water control. Pumps supplied flow to the flume capable of providing discharges up to 40 cf/s (1.13 m<sup>3</sup>/s). Tail water control was provided with stop logs.

Discharges through the flume were measured with an orifice plate placed in the inflow pipe accurate to  $\pm 3\%$ . Flow depths and rock weir elevations were recorded with a point gauge, accurate to  $\pm 0.001$  ft (0.03 cm), suspended from a data collection platform that spanned the flume.

## ROCK WEIR STRUCTURES

Three distinct rock weir structures were tested: the U-, A-, and W-weirs. The modeled U-weir consisted of a horizontal sill placed perpendicular to the flow, centered in the lateral dimension and spanning one-third of the channel width as illustrated in Figure 3a. The weir arm extended from the horizontal sill to the bank with arm angles and arm slopes as depicted. The modeled A-weir was constructed utilizing the U-weir design and incorporated a second horizontal sill (cross-sill) spanning the weir arms





FIGURE 2. Plan and Profile Schematic of Flume (adapted from Scurlock, 2009).

172



FIGURE 3. (a) U-Weir Conceptual Design (adapted from Reclamation, 2007), (b) A-Weir Conceptual Design (adapted from Reclamation, 2007), and (c) W-Weir Conceptual Design (adapted from Reclamation, 2007).

approximately half way through the structure as shown in Figure 3b. The cross-sill elevation was approximately one-half of the height between the downstream bed elevation and the upstream sill. The W-weir consisted of four sill segments with the center point facing downstream as presented in Figure 3c. The weir arms were placed such that the outer arms tied into the stream bank at the bankfull elevation, while the inner arms attained a one-half bankfull elevation. Also, the downstream structure points were higher than the upstream points and the middle downstream point was lower than the points at bank intersection to concentrate flow away from the banks. The stream width for all rock-weir structures evaluated was 16 ft (4.9 m). The conceptual U-, A-, and W-rock weir structure designs were based upon criteria derived from Reclamation (2007). Structural components of throat width, arm angle, and arm slope were developed from guidelines presented by Rosgen (2001). Structural arms were designed to approach the midpoint of the guideline ranges such that plan angles ranged from 20 to  $30^{\circ}$  (25° targeted) and profile angles ranged from 2 to 7% (4.5% targeted). Weir dimensions and bed material sizes are presented in Table 1.

The grouted rock weir structures were constructed to simulate a field condition where the crest rock was stable and the bed upstream of the rock was aggraded/ silted to an elevation mid to top rock of the structure thereby restricting interstitial flow. The rock weirs

THORNTON.	Meneghetti.	COLLINS.	Abt.	AND	SCURLOCK
			,		

TABLE 1. Test Matrix.

Config.	Test Number	Discharge (cf/s)	Weir Type	Weir Rock Size (in)	Drop (ft)	Grain Size (mm)	Slope (ft/ft)	Arm Length (ft)	Profile Angle (°)	Plan Angle (°)
1	8	13.3	U	6.87	0.14	12.3	0.0047	11.44	3.54	24.99
	9	26.6	U	6.87	0.14	12.3	0.0047	11.44	3.54	24.99
	10	40.0	U	6.87	0.14	12.3	0.0047	11.44	3.54	24.99
2	11	13.3	U	6.87	0.14	12.3	0.0047	11.44	3.54	24.99
	12	26.6	U	6.87	0.14	12.3	0.0047	11.44	3.54	24.99
	13	26.6	U	6.87	0.14	12.3	0.0047	11.44	3.54	24.99
	14	34.0	U	6.87	0.14	12.3	0.0047	11.44	3.54	24.99
3	21	10.0	U	8.80	0.17	9.8	0.0033	9.83	2.51	28.48
	22	20.0	U	8.80	0.17	9.8	0.0033	9.83	2.51	28.48
	23	30.0	U	8.80	0.17	9.8	0.0033	9.83	2.51	28.48
4	24	6.7	U	10.00	0.18	4.2	0.0021	11.24	1.70	25.38
	25	13.3	U	10.00	0.18	4.2	0.0021	11.24	1.70	25.38
	26	20.0	U	10.00	0.18	4.2	0.0021	11.24	1.70	25.38
5	27	6.5	А	10.00	0.36	4.2	0.0021	9.24	1.02	29.99
	28	13.3	А	10.00	0.36	4.2	0.0021	9.24	1.02	29.99
	29	20.0	А	10.00	0.36	4.2	0.0021	9.24	1.02	29.99
6	30	6.5	W	10.00	0.18	4.2	0.0021	7.97	2.38	26.65
	31	13.3	W	10.00	0.18	4.2	0.0021	7.97	2.38	26.65
	32	20.0	W	10.00	0.18	4.2	0.0021	7.97	2.38	26.65
7	33	10.0	U	8.80	0.17	9.8	0.0033	12.58	2.05	22.97
	34	20.0	U	8.80	0.17	9.8	0.0033	12.58	2.05	22.97
	35	30.0	U	8.80	0.17	9.8	0.0033	12.58	2.05	22.97
8	36	10.0	А	8.80	0.34	9.8	0.0033	9.41	1.69	29.54
	37	20.0	А	8.80	0.34	9.8	0.0033	9.41	1.69	29.54
	38	30.0	А	8.80	0.34	9.8	0.0033	9.41	1.69	29.54
9	39	10.0	W	8.80	0.17	9.8	0.0033	9.35	2.76	23.16
	40	20.0	W	8.80	0.17	9.8	0.0033	9.35	2.76	23.16
	41	30.0	W	8.80	0.17	9.8	0.0033	9.35	2.76	23.16
10	42	13.3	А	6.87	0.28	15.38	0.0047	10.69	2.40	26.52
	43	26.6	А	6.87	0.28	15.38	0.0047	10.69	2.40	26.52
	44	40	А	6.87	0.28	15.38	0.0047	10.69	2.40	26.52
11	45	13.3	W	6.87	0.14	15.38	0.0047	11.00	3.13	19.98
	46	26.6	W	6.87	0.14	15.38	0.0047	11.00	3.13	19.98
	50	40	W	6.87	0.14	15.38	0.0047	11.00	3.13	19.98

were constructed with an anchor wall, foundation footer rock, and header or sill rock as shown in Figure 4. The anchor walls comprised concrete blocks extending from the flume floor to the foundation rock footer. The rock footer was grouted to the anchor wall immediately atop the blocks. The header/sill rock was grouted atop of the rock footer, but was offset approximately one-third of the rock diameter upstream (Reclamation, 2007). The weirs were grouted for structural stability and prevented interstitial flow between the stones resulting in all flow overtopping the weir sill.

TESTING PROCEDURE

A prescribed weir configuration was identified from the test matrix as indicated in Table 1. The weir was constructed, the bed material appropriately placed and leveled to the designated slope, and the entire test section contoured. Flow was then introduced at a very low discharge to fill the test section and establish the appropriate tail water condition. The discharge was then increased to one of three benchmark flows; 1/3 bankfull, 2/3 bankfull, and bankfull as presented in Table 1. The test clock began when the flow reached the benchmark flow. Each test extended for a 12-hour duration. At the 6- and 12-hour marks of each test, the water-surface elevations were measured and recorded. The flow was stopped and the bed drained after each 12-hour test. Figure 5 illustrates the steady state flow conditions during the testing of the A-weir at 1/3 bankfull discharge.

## TEST RESULTS AND ANALYSIS

A series of 31 tests was performed evaluating 11 unique rock-weir configurations. Table 2 presents a



FIGURE 4. Schematic of Offset Header and Footer Rocks (Scurlock, 2009).



FIGURE 5. A-Weir Testing at 1/3 Bankfull Flow.

summary of the data recorded from the experimental program to include discharge (Q), effective weir length  $(b_i)$ , flow depth measured upstream of the weir crest  $(y_{us})$ , crest rock size  $(d_{50})$ , and the mean weir height  $(z_i)$ .

A comparison of the stage-discharge expressions from Equations (4, 6, 8, and 9) indicates that there is a consensus in the general form of the stagedischarge rating expressions. The primary difference in these relations is how the weir specifics (i.e., geometry, shape, roughness, etc.) influence the value of the coefficient,  $C_d$ , in each rating equation. It is apparent that a general stage-discharge expression may be warranted, and efforts should be concentrated on the coefficients unique to each weir shape and the design details of the weir. Therefore, similar to Equations (4, 6, 8, and 9), a general, dimensionless stage-discharge rating relationship was formulated for this analysis and is expressed as:

$$Q = \frac{2}{3}b_{\rm i}C_{\rm d}(2g)^{0.5}(y_{\rm us} - z_{\rm i})^{3/2}, \qquad (10)$$

where  $b_i$  is the effective length along the weir crest,  $z_i$  is the mean weir height, and the  $y_{us}$  is the up streamflow depth. The effective weir length is defined as the total length along the crest of the weir arm. The mean weir height is the width-averaged height of the weir crest divided by the effective length.

The general coefficient,  $C_d$ , can be unique to reflect the geometry of each rock weir structure and the stream. Applicable to this study, a unique coefficient can be developed for each of the rock weir structures. Therefore, coefficients for the U-, A-, and W-rock weirs will be referenced as  $C_U$ ,  $C_A$ , and  $C_W$ , respectively. A dimensional analysis was performed, using geometric components common to U-, A-, and W-rock weirs, yielding a general expression enabling the computation of the appropriate weir coefficient as:

$$C_{\rm U,A,W,d} = a [d_{50}/z_{\rm i}]^{b} [b_{\rm i}/B]^{c}, \qquad (11)$$

where *B* is the stream width,  $d_{50}$  is the median crest stone size, and *a*, *b*, and *c* are regression constants. The rating coefficient is a function of the ratio of crest rock size to weir height and of the ratio of the effective weir length and stream width.

A multivariant, power regression analysis was conducted using the data from Table 2 to determine the stage-discharge rating coefficients for each of the U-, A-, and W-rock weirs expressed in the form depicted in Equation (11). The resulting coefficient relations for each rock-weir type is expressed as:

U-rock weir: 
$$C_{\rm U} = 0.652 [d_{50}/z_{\rm i}]^{-0.708} [b_{\rm i}/B]^{0.587}$$
 (12)

A-rock weir: 
$$C_{\rm A} = 22.109 [d_{50}/z_{\rm i}]^{-1.789} [b_{\rm i}/B]^{-7.952}$$
 (13)

W-rock weir: 
$$C_{\rm W} = 0.002 [d_{50}/z_{\rm i}]^{1.868} [b_{\rm i}/B]^{4.482}$$
. (14)

Utilizing the test data from Table 2, the appropriate discharge coefficient was computed with either Equations (12, 13, or 14), and then inserted into Equation (10) (replacing  $C_d$ ) yielding the predicted discharge. The predicted vs. observed discharges were then plotted for the U-, A-, and W-rock weirs tested as illustrated in Figure 6. The coefficients of determination ( $R^2$ ) for Equations (12-14) are 0.970, 0.987, and 0.989, respectively. The mean error is 8.1, 10.6, and 13.7% for the U-, A-, and W-rock weir rating relationships, respectively.

In an attempt to develop a composite, stagedischarge rating expression applicable to the U-, A-, and W-rock weirs, a multivariant, power regression analysis was performed, in a manner similar to that

TABLE 2. Summary of Test Results.	
5	

Test	Shape	$Q~({ m ft}^3/{ m s})$	$y_{us-avg}$ (ft)	Weir Rock Size (R) (ft)	$b_{i}$ (ft)	$z_{\mathrm{i}}$ (ft)	$rac{[y_{\mathrm{us}}-z_{\mathrm{i}}]}{z_{\mathrm{i}}}$ (ft)	$\frac{R}{[y_{\rm us}-z_{\rm i}]}$	C <sub>d</sub>	$Q_{ m pred}~({ m ft}^3/{ m s})$
A-weir										
27	Α	6.5	0.595	0.833	21.337	0.423	0.407	3.539	0.667	5.436
28	Α	13.3	0.741	0.833	21.337	0.423	0.751	2.186	0.667	13.641
29	А	20.0	0.834	0.833	21.337	0.423	0.971	1.757	0.667	20.055
36	А	10.0	0.654	0.733	21.488	0.447	0.463	2.338	0.873	9.444
37	A	20.0	0.764	0.733	21.488	0.447	0.710	1.730	0.873	17.926
38	A	30.0	0.907	0.733	21.488	0.447	1.031	1.293	0.873	31.353
42	Δ	13.3	0.656	0.573	22.624	0 447	0.469	1 522	0.902	10 474
12	Δ	26.6	0.817	0.573	22.024	0.447	0.400	1.022	0.002	94 577
40	A A	20.0	0.017	0.573	22.024	0.447	1 1 8 9	0.894	0.302	11 803
Moon Ø		40.0	0.315	0.070	22.024	0.447	1.102	0.024	0.302	9 196
$\frac{\text{Mean } 7}{\text{MSE}}$ $R^2$										2.595
U-weir										0.301
8	U	13.3	0.597	0.573	30.646	0.440	0.357	1.253	0.793	8.080
9	Ū	26.6	0.793	0.573	30.646	0.440	0.802	0.877	0.793	27.213
10	Ŭ	40.0	0.884	0.573	30 646	0.440	1 009	0.769	0 793	38 444
11	U	13.3	0.609	0.573	30.646	0.440	0.385	1 220	0.793	9.054
19	U	26.6	0.801	0.573	30.646	0.440	0.820	0.866	0.793	28 144
12	U	20.0	0.001	0.575	20.646	0.440	0.820	0.800	0.733	20.144
10	U	20.0	0.004	0.575	30.040 20.646	0.440	0.020	0.005	0.795	20.401
14	U	54.0 10.0	0.657	0.075	30.040	0.440	0.947	0.799	0.795	0 1 5 4
21	U	10.0	0.558	0.733	27.722	0.333	0.675	1.888	0.515	8.154
22	U	20.0	0.728	0.733	27.722	0.333	1.184	1.315	0.515	18.928
23	U	30.0	0.877	0.733	27.722	0.333	1.631	1.037	0.515	30.616
24	U	6.7	0.517	0.833	30.227	0.303	0.705	2.472	0.463	7.400
25	U	13.3	0.621	0.833	30.227	0.303	1.048	1.888	0.463	13.427
26	U	20.0	0.730	0.833	30.227	0.303	1.405	1.517	0.463	20.827
33	U	10.0	0.515	0.733	32.679	0.333	0.546	2.124	0.567	7.694
34	U	20.0	0.680	0.733	32.679	0.333	1.041	1.437	0.567	20.272
35	U	30.0	0.781	0.733	32.679	0.333	1.344	1.199	0.567	29.738
Mean % MSE	6 error									$\begin{array}{c} 10.591 \\ 4.155 \end{array}$
$R^2$										0.970
W-weir										
30	W	6.5	0.413	0.833	35.700	0.296	0.395	3.570	0.490	3.753
31	W	13.3	0.560	0.833	35.700	0.296	0.891	2.192	0.490	12.685
32	W	20.0	0.665	0.833	35.700	0.296	1.246	1.717	0.490	20.984
39	W	10.0	0.491	0.733	40.726	0.333	0.475	2.287	0.562	7.693
40	W	20.0	0.618	0.733	40.726	0.333	0.859	1.637	0.562	18.669
41	W	30.0	0.734	0.733	40.726	0.333	1.208	1.300	0.562	31.157
45	W	13.3	0.534	0.573	46.889	0.363	0.472	1.455	0.566	10.043
46	W	26.6	0.671	0.573	46.889	0.363	0.852	1.078	0.566	24.349
50	W	40.0	0.807	0.573	46.889	0.363	1.227	0.858	0.566	42.079
Mean %	6 error									13.727
$\frac{\text{MSE}}{R^2}$										4.147
Combin	ned A-, U-, a	and W-weir								01000
27	A	6.5	0.595	0.833	21.337	0.423	0.407	3.539	0.656	5.350
28	A	13.3	0.741	0.833	21.337	0.423	0.751	2.186	0.656	13.425
29	Δ	20.0	0.834	0.833	21.337	0.423	0.971	1 757	0.656	19 736
36	A	10.0	0.654	0.733	21.007	0.420 0.447	0.463	2,338	0.000 0.744	8 055
37	Δ	20.0	0.764	0.733	21.400	0.447	0.400	1 730	0.744	15 280
28	Δ Λ	20.0	0.704	0.799	21.400 91 / QQ	0.447	1 021	1 909	0.744	10.200 96 711
00 ∕10	A	อบ.U 19 9	0.907	U.100 0 579	41.400 99.694	0.447	1.001	1.273	0.744	20.741
42	A	13.3	0.000	0.070	22.024	0.447	0.409	1.022	0.074	10.140
43	A	20.0	0.817	0.573	22.024	0.447	0.829	1.067	0.874	23.805
44	A	40.0	0.975	0.573	22.624	0.447	1.182	0.824	0.874	40.576
8	U	13.3	0.597	0.573	30.646	0.440	0.357	1.253	0.799	8.145
9	U	26.6	0.793	0.573	30.646	0.440	0.802	0.877	0.799	27.431
10	U	40.0	0.884	0.573	30.646	0.440	1.009	0.769	0.799	38.752
11	U	13.3	0.609	0.573	30.646	0.440	0.385	1.220	0.799	9.127
12	U	26.6	0.801	0.573	30.646	0.440	0.820	0.866	0.799	28.369

Test	Shape	$Q~({ m ft}^3/{ m s})$	$y_{us-avg}$ (ft)	Weir Rock Size (R) (ft)	$b_{i}$ (ft)	$z_{i}$ (ft)	$\frac{[y_{\rm us}-z_{\rm i}]}{z_{\rm i}}({\rm ft})$	$\frac{R}{[y_{\rm us}-z_{\rm i}]}$	$C_{\mathrm{d}}$	$Q_{ m pred}~({ m ft}^3/{ m s})$
13	U	26.6	0.804	0.573	30.646	0.440	0.826	0.863	0.799	28.709
14	U	34.0	0.857	0.573	30.646	0.440	0.947	0.799	0.799	35.225
21	U	10.0	0.558	0.733	27.722	0.333	0.675	1.888	0.567	8.972
22	U	20.0	0.728	0.733	27.722	0.333	1.184	1.315	0.567	20.828
23	U	30.0	0.877	0.733	27.722	0.333	1.631	1.037	0.567	33.690
24	U	6.7	0.517	0.833	30.227	0.303	0.705	2.472	0.474	7.576
25	U	13.3	0.621	0.833	30.227	0.303	1.048	1.888	0.474	13.746
26	U	20.0	0.730	0.833	30.227	0.303	1.405	1.517	0.474	21.321
33	U	10.0	0.515	0.733	32.679	0.333	0.546	2.124	0.543	7.364
34	U	20.0	0.680	0.733	32.679	0.333	1.041	1.437	0.543	19.403
35	U	30.0	0.781	0.733	32.679	0.333	1.344	1.199	0.543	28.462
30	W	6.5	0.413	0.833	35.700	0.296	0.395	3.570	0.446	3.418
31	W	13.3	0.560	0.833	35.700	0.296	0.891	2.192	0.446	11.553
32	W	20.0	0.665	0.833	35.700	0.296	1.246	1.717	0.446	19.110
39	W	10.0	0.491	0.733	40.726	0.333	0.475	2.287	0.512	7.008
40	W	20.0	0.618	0.733	40.726	0.333	0.859	1.637	0.512	17.006
41	W	30.0	0.734	0.733	40.726	0.333	1.208	1.300	0.512	28.382
45	W	13.3	0.534	0.573	46.889	0.363	0.472	1.455	0.624	11.073
46	W	26.6	0.671	0.573	46.889	0.363	0.852	1.078	0.624	26.847
50	W	40.0	0.807	0.573	46.889	0.363	1.227	0.858	0.624	46.395
Mean 9	% error									12.860
MSE										6.440
$R^2$										0.964

TABLE 2. Continued.

of the individual rock weir types, using the entire database for the three rock weir types from Table 2. The general stage-discharge coefficient,  $C_d$ , was computed using Equation (11) and then inserted into Equation (10) resulting in the composite coefficient presented in Equation (15). The coefficient of determination ( $R^2$ ) is 0.964 with mean error of 12.9%.

Composite-rock weir:

$$C_{\rm d} = 1.139 [d_{50}/z_{\rm i}]^{-0.703} [b_{\rm i}/B]^{-0.261}.$$
 (15)

The stage-discharge rating expression using  $C_d$  was compared to the weir specific rating expressions using either the  $C_{U}$ -,  $C_{A}$ -, or  $C_{W}$ -rock weir coefficients. The  $R^2$ -values for the weir specific ratings are slightly improved over the general weir rating approach, an enhancement ranging from 0.008 to 0.025. The slight reduction in  $R^2$ -value may be considered relatively inconsequential. However, a comparison of the mean square error (MSE) as portrayed in Figure 6 and Table 2 indicates that the MSE for the general weir rating (6.439) is significantly higher than the MSE for the weir specific ratings (2.595-4.154). The variability of the data scatter is reduced using the specific stage-discharge rating relationships for the U-, A-, and W-rock weirs.

Equations (10 and 11) appear to provide a reasonable approach to determine the stage-discharge rating for stable U-, A-, and W-rock weir structures. These relationships are based upon a limited laboratory testing program where the structures were scaled from a field site, and therefore the findings may be considered site specific. Further, the test discharges were limited to 1/3 bankfull, 2/3 bankfull, and bankfull, therefore Equation (10) was not evaluated for discharges below 1/3 bankfull level. However, Equations (10 and 11) are consistent in form to the relationships developed for sharp-crested and labyrinth weirs, and lend credibility that a general expression can be developed that applies to a breadth of weir structures for computing the stage-discharge.

# CONCLUSIONS

A segment of a natural stream was modeled in the laboratory at a near-prototype, 1:5 Froude scale. A series of U-, A-, and W-rock weirs were constructed in 11 configurations in the stream bed and tested to ascertain the stage-discharge ratings for 1/3 bankfull, 2/3 bankfull, and bankfull discharges. All tests were performed in subcritical, un-submerged flow conditions.

In a manner similar to stage-discharge relations developed for broad-crested and labyrinth weirs, a dimensionally appropriate discharge rating expression was presented as a function of the stream width, flow depth upstream of the weir, weir crest height, and discharge coefficient. A unique, dimensionless





discharge coefficient was developed for each of the U-, A-, and W-rock weir configurations as a function of the crest rock size, effective weir length, and stream width. A predicted vs. actual discharge comparison for the U-, A-, and W-rock weirs indicated reasonable predictions for each weir shape. In addition, a single, composite discharge coefficient was formulated applicable to all three weir shapes. An analysis of the predicted vs. actual discharge revealed that the composite approach yielded a discharge prediction with a coefficient of determination of 0.964 and mean error of 12.9%. Although the predictive error is larger than desired (>5%), the concept that a composite stage-discharge expression applicable to

multiple shape rock weirs can be developed is reinforced.

It is recognized that the stage-discharge relationships portrayed in Equations (10-15) are founded on a limited database from controlled laboratory testing conditions for U-, A-, and W-rock weirs. Field tests are needed to validate and enhance these laboratory findings. However, these findings are encouraging in that it may be possible to develop a reliable stagedischarge rating relationship applicable to multiple rock weir shapes in the field setting.

#### LITERATURE CITED

- Brater, E.F. and H.W. King, 1976. Handbook of Hydraulics for the Solution of Hydraulic Engineering Problems. McGraw-Hill, New York, New York.
- Chin, D.A., 2006. Water Resources Engineering. Pearson Prentice Hall, Upper Saddle River, New Jersey.
- Chow, V.T., 1959. Open-Channel Hydraulics. McGraw-Hill Book Company, New York, New York. p. 362.
- Herschy, R.W., 1995. Streamflow Measurement (Second Edition). E & FN SPON, London, United Kingdom.
- Herschy, R.W., 1999. Hydrometry: Principles and Practices (Second Edition). John Wiley & Sons, Inc., New York, New York.
- Meneghetti, A.M., 2009. Stage-Discharge Relationships for U-, W-, and A-Weirs in Un-Submerged Conditions. M.S. Thesis, Department of Civil and Environmental Engineering, Colorado State University, Fort Collins, Colorado, 135 pp.
- Novak, P., A.I.B. Moffat, C. Nalluri, and R. Narayanan, 1996. Hydraulic Structures (Second Edition). E & FN SPON, London, United Kingdom, p. 278.
- Reclamation (Bureau of Reclamation), 1997. Water Measurement Manual. A Water Resources Technical Publication (Third Edition). U.S. Government Printing Office, Denver, Colorado.
- Reclamation (Bureau of Reclamation), 2007. River Spanning Rock Structures Research. http://www.usbr.gov/pmts/sediment/kb/ SpanStructs/index.htm, accessed October 2009.
- Rosgen, D.L., 2001. The Cross-Vane, W-Weir and J-Hook Vane Structures – Their Description, Design and Application for Stream Stabilization and River Restoration, Report. Pagosa Springs, Colorado.
- Rosgen, D.L., 2006. The Cross-Vane, W-Weir and J-Hook Vane Structures – Their Description, Design, and Application for Stream Stabilization and River Restoration. Pagosa Springs, Colorado.
- Schmidt, A.R., 2002. Analysis of Stage-Discharge Relations for Open-Channel Flows and Their Associated Uncertainties. Ph.D. Dissertation, Department of Civil and Environmental Engineering, University of Illinois at Urbana-Champaign, Urbana, Illinois.
- Scurlock, M., 2009. Equilibrium Scour Downstream of Three-Dimensional Grade-Control Structures. M.S. Thesis, Colorado State University, Fort Collins, Colorado, 118 pp.
- Thompson, D.M. and G.M. Stull, 2002. The Development and Historic Use of Habitat Structures in Channel Restoration in the United States: The Grand Experiment in Fisheries Management. Geographie Physique et Quaternaire 56(1): 45-60.
- Tullis, J.P., N. Amanian, and D. Waldron, 1995. Design of Labyrinth Spillways. Journal of Hydraulic Engineering, ASCE 121(3):247-255.