

# RECLAMATION

*Managing Water in the West*

**Facilities Instructions, Standards, and Techniques**  
**Volume 2-2**

## **Field Balancing Large Rotating Machinery**



**U.S. Department of the Interior**  
**Bureau of Reclamation**  
**Technical Service Center**  
**Denver, Colorado**

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Volume 2-2**

# **Field Balancing Large Rotating Machinery**

*Prepared by*

**Technical Service Center**

**Mechanical Equipment Group**



**U.S. Department of the Interior  
Bureau of Reclamation  
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## **Mission Statements**

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The mission of the Bureau of Reclamation is to manage, develop, and protect water and related resources in an environmentally and economically sound manner in the interest of the American public.

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# 1 Purpose and Scope

This Facilities Instructions, Standards, and Techniques (FIST) volume is intended as a guideline to the basics of balancing large rotating machinery. An understanding of vector math is required to utilize the vector equation examples. Published vibration literature can be obtained for study in further technical detail. The objective of this balancing FIST is to aid in the reduction of hydro-turbine shaft vibration from excessive to a practical minimum. Reducing shaft vibrations generally reduces bearing loads and increases the service life of the associated machinery.

Turbine runners and pump impellers are usually two-plane balanced before they are installed and seldom require rebalancing. Normally, the rotating parts of generators and motors are balanced after installation. Maintenance work and slight shape changes with age, in some instances, can alter the balance of a generator or motor rotating parts enough to require rebalancing. This FIST is concerned with improving the imbalance masses in rotors.

The need to balance a large rotating machine should be evaluated on an individual basis. The manufacturer's tolerances (bearing and vibration), the historical vibration data, and the current available guide bearing clearance must be reviewed for each individual machine. The purpose of this document is to provide balancing guidance, and therefore, all information in this document is presented in plain text as required in Reclamation Manual FAC P-14.

## 2 Vibration Sources

Large hydraulic units are subjected to many kinds of vibrations. Vibration and imbalance can be caused by mechanical, electrical, or hydraulic problems. Excessive cavitation in the unit can cause hydraulic imbalance. However, the imbalance of rotating parts is the most common cause of excessive unit vibration. Before attempting to balance the rotating parts of a unit, determine the possible sources of vibration and imbalance.

Mechanical vibration sources are:

- Loose bolted connections present among the rotating parts (check for tightness).
- Unit alignment of the rotor or runner.
- Poor bearing condition or positioning.
- Foundation rigidity (flexure).

Check for possible electric sources of vibration such as:

- Non-uniform air gap in the generator (motor) or exciter.
- Short-circuited winding turns in the rotor poles.
- Ellipticity of the rotor.

The hydraulic passages of the unit should be checked for:

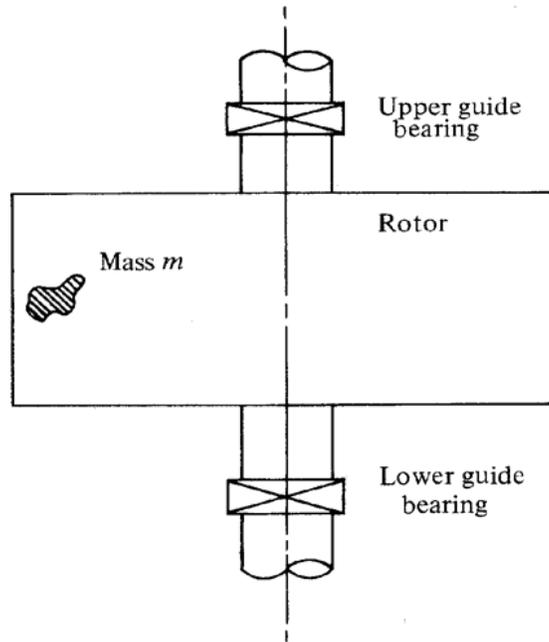
- A non-uniform pressure distribution over the surfaces of the turbine runner or pump impeller, which can cause hydraulic imbalance.
- Obstructions in the spiral case or volute.
- Large debris between the vanes.
- Incorrect vertical position of the runner or impeller (relative to the distributor).

### 3 Theory of Balancing

Imbalanced masses on rotating parts create a pulsating centrifugal force that causes the unit to vibrate. Field balancing consists of determining the amplitude (size) and location around the shaft (phase angle) of the imbalance, then placing weight on the rotor to counter the imbalance.

#### 3.1 Imbalanced Mass

As the rotor rotates (Figure 1), the imbalanced mass  $m$  tends to pull the rotor toward the bearings on the side of the imbalance, creating a “high spot” of runout on the shaft. At very low speeds, this high spot will be in phase with the imbalanced mass  $m$ . As speed increases, the high spot begins to lag the imbalanced mass  $m$ .

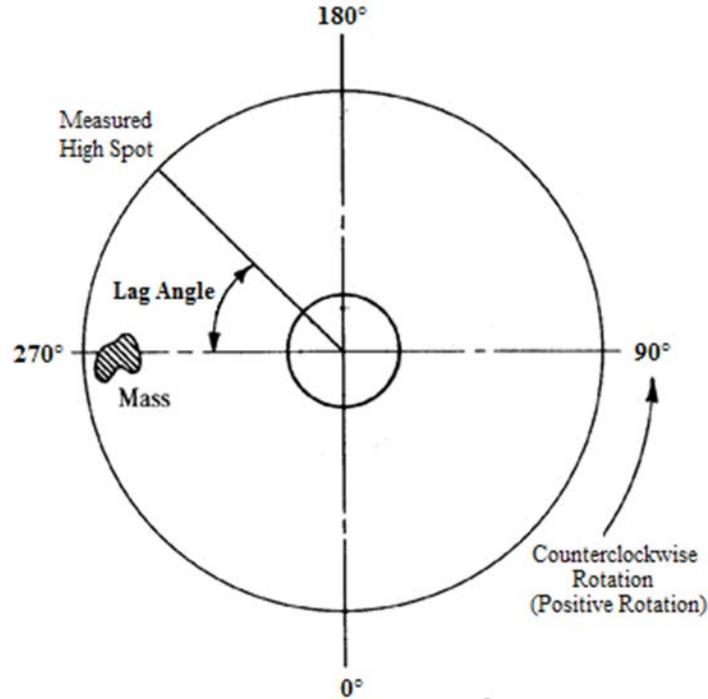


**Figure 1. Elevation view of a rotor having an imbalanced mass.**

#### 3.2 Lag Angle

For hydroelectric units, the operating speed is less than the critical speed. A lag angle between  $0^\circ$  and  $90^\circ$  should be assumed as the angle relationship between the imbalanced mass and the high spot. Figure 2 shows a plan view of the relation between the imbalanced mass and the measured high spot at typical

hydroturbine operating speeds. Note that moving counter-clockwise on the chart in Figure 2 is a positive rotation. This orientation will be used throughout this manual.



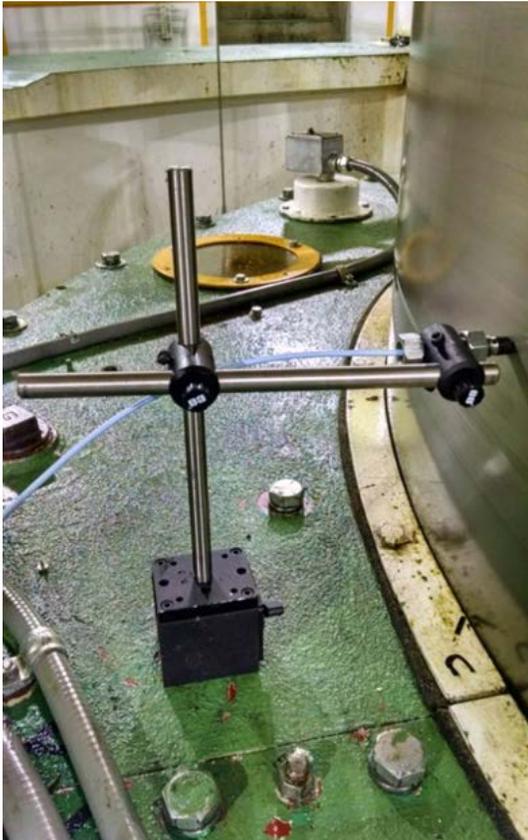
**Figure 2. Plan view of an imbalanced rotor at the upper guide bearing.**

### 3.3 Balance Weight and Location

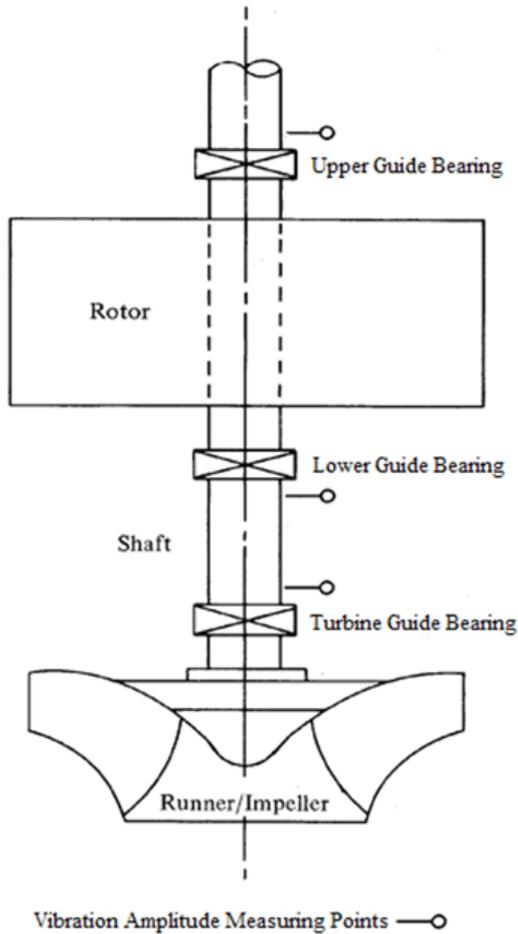
For balancing, the machine is assumed to be linear; i.e., the vibration amplitudes are in proportion to the forces causing them. For the purpose of this FIST, the term *vibration amplitude* is synonymous with *shaft deflection*, which is a measure of the imbalance present. The point where vibration amplitude (deflection) reaches a maximum is called the *high spot*. To balance a unit it is necessary to determine the vibration amplitude and the location of the high spot. From this information, a balance weight can be determined and positioned, which gives a counter-effect and balances the unit.

## 4 Instrumentation and Measurement

Hydroturbine assemblies spin at relatively slow speeds, and typically the rotating assembly cannot be removed from the installed position once an unacceptable balance is realized. Vertical hydroturbine assemblies typically have a “free shaft” design, which means that the unit rotor, shaft, and runner hang from a thrust bearing, and the guide bearings do not contact or locate the shaft in its static condition. Rather, they are in place for restraint of the shaft while the unit is spinning. This configuration necessitates the use of a data acquisition system, a key phasor (tachometer), and pairs of orthogonally oriented (90°) proximity probes in order to gather balancing data. A single proximity probe in a given vertical location can only read runout perpendicular to the head of the probe, and cannot read runout parallel to the head. Therefore pairs of orthogonally oriented proximity probes should always be used, and they should be mounted with the sensing heads close to the shaft, in the same vertical plane, and in vertical locations where deflection measurements are desired. See Figures 3 and 4. The key phasor is typically mounted in the same vertical plane as the No. 1 rotor pole. This allows for an easy reference and also repeatability in the event future testing is needed.



**Figure 3. Proximity probe on a magnetic mount, above turbine guide bearing, at Grand Coulee Unit G21.**

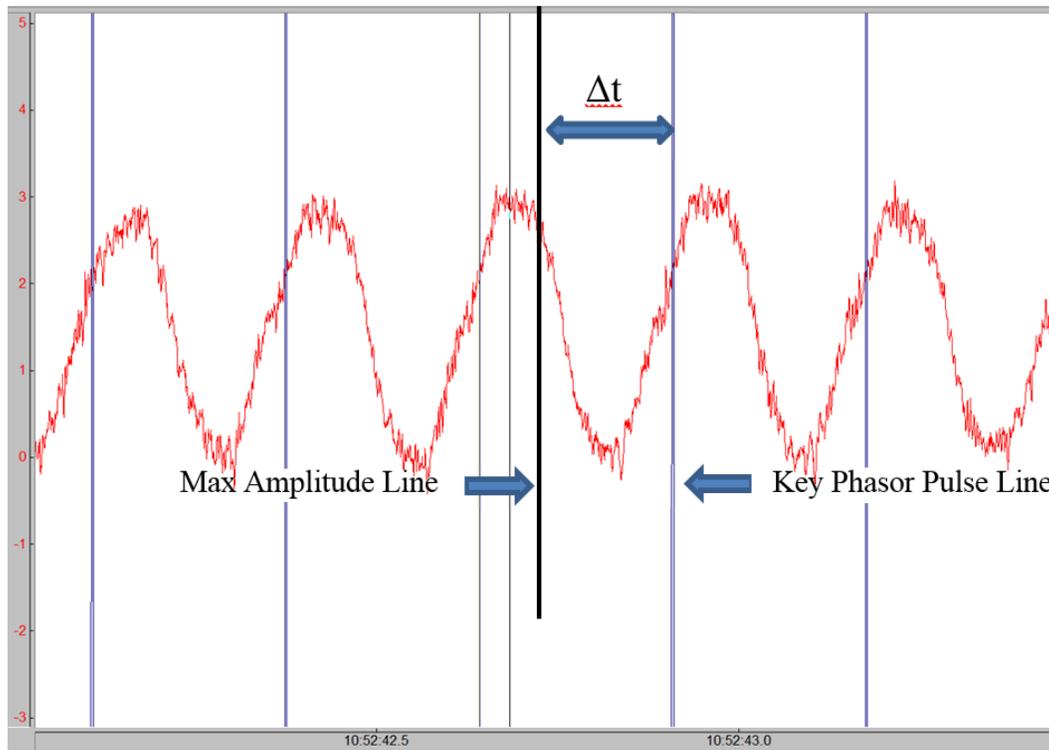


**Figure 4. Location of vibration amplitude measuring points.**

Shaft runout readings are taken simultaneously at:

- upper guide bearing
- lower guide bearing
- turbine guide bearing

Data from the upper and lower guide bearings is used to balance the unit. Data from the turbine guide bearing is monitored to assure that it is not adversely affected by the rotor balancing. The maximum amplitude, or deflection, of a unit is relatively simple to establish. The location, or phase angle, of the imbalance is generally the most difficult parameter to establish. An understanding of the relationship between proximity probe placement, key phasor placement, and the shaft's radial location is required. See Figure 5 below. The red sine wave indicates unit runout, and the blue lines indicate the key phasor pulse. The key phasor pulse is a signal occurring one time per revolution at the No. 1 rotor pole. To find the phase angle where maximum imbalance occurs, find the time lapse ( $\Delta t$ , blue arrow), which is the span between the blue "key phasor pulse" line and the black "maximum amplitude" line. Then calculate the angular location of the max amplitude in relation to the key phasor pulse (No. 1 rotor pole) using the known speed of rotation (revolutions per minute).



**Figure 5. Typical sine wave showing unit runout (red sine wave) and key phasor pulse (blue line).**

Figure 5 presents an unfiltered sine wave. It should be noted that the filters used in the DASyLab data acquisition software can alter the output of the phase angle. It is very important to obtain a correct phase angle when balancing, and therefore the data should not be filtered.

Ideally, balancing should be performed so that the unit spins with minimum runout during both online and offline conditions. However, some units do exhibit significant runout changes between their online and offline runout conditions. Most units are online most of the time, however, balancing strictly to optimize runout in the online condition may yield high levels of runout when the unit is offline. In these instances, the balance for the online runout may need to be compromised somewhat so that a reasonable offline runout can be achieved.

Specific balancing software is available, such as IOtech's eZ-BALANCE and Bently Nevada's BALANCE software packages. When combined with data acquisition hardware both can perform single and multi-channel data collection and can calculate optimal locations for one or more balance weights.

## 4.1 Iterative Testing

Three trial runs are made to obtain data necessary to balance a unit. The first run is made in the as-found condition to determine the vibration amplitude and location of the existing shaft deflection. For the second run, a trial weight is attached at the top of a rotor arm. The third run is made with the trial weight removed from the top of the rotor arm and attached to the bottom of the same rotor arm.

## 4.2 Calculating Weight and Location

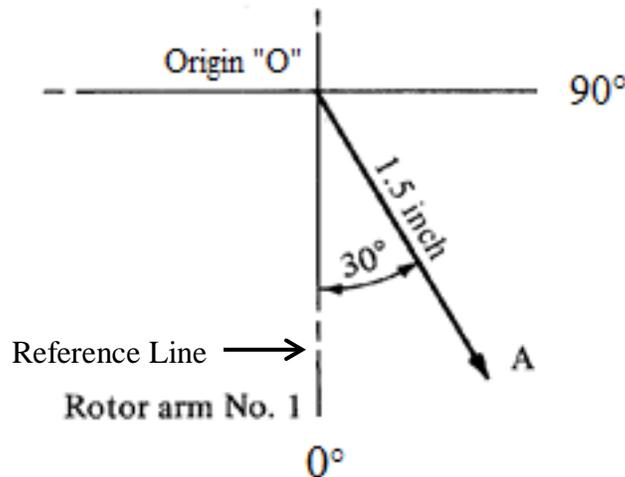
The standard for calculating the initial balance weight is equated by dividing the weight of the rotating parts by 10,000. This weight should be located  $180^\circ$  plus a lag angle of  $0^\circ$  to  $90^\circ$ , in the direction of rotation, from the high spot. The weight should be attached to the rotor arm nearest this location. A good rule-of-thumb is to use a lag angle of  $45^\circ$  for hydroelectric units. Accuracy in placement of an initial balance weight is important but not critical. It is likened to an educated guess. Vibration monitoring following the placement of the initial balance weight will demonstrate the effect the balance weight had, providing the data necessary for accurate weight placement. Throughout this FIST, angles will be measured in the direction of unit rotation with respect to rotor arm No. 1 – the reference point. Note that counter-clockwise is a positive movement of rotation when using polar coordinates.

## 5 Vector Techniques

The process of determining the required balance weights and locations can be simplified by using vectors to represent the imbalance. Definitions and examples are shown below to explain the vector procedure. A blank polar coordinate sheet is included in this FIST at the end of the document.

A vector is defined as a quantity having both magnitude,  $|v| = (x^2 + y^2)^{1/2}$ , and direction, angle  $\Theta$ . A vector of “ $|v|$ ” magnitude and “ $\Theta$ ” angle will be notated as follows in this manual:  $|v|/\Theta$ . Restated, a vector having magnitude 0.010 inch and direction  $120^\circ$  will be presented as: 0.010/120°. The directions of different vectors will be measured with respect to the same reference line.

**Example:** Measurements at the upper generator guide bearing indicate a shaft deflection of 0.006 inch at  $30^\circ$  from the No. 1 rotor arm. The measurement of 0.006 inch is the magnitude of vibration amplitude, and  $30^\circ$  counter-clockwise is the location (direction). Using a scale (such as 0.001 inch equals 0.25 inch, so 0.006 equals 1.5 inches), draw the deflection as a vector. In Figure 6, below, vector 0.006 inch at  $30^\circ$  has been drawn as vector OA. Point O is the tail of the vector and point A is the head. Note that the arrowhead always points in the direction that the vector acts, in this case, away from point O and toward A at an angle of  $30^\circ$  from the reference line. The vector is represented by the notation OA, which is not the same as AO.



**Figure 6. Vector OA direction is  $30^\circ$  having a magnitude of 0.006 inch.**

## 6 Vector Addition

A vector can be added to another vector (original) using graphical techniques. Draw the vector to be added using the head of the original vector as the starting point. A vector drawn from the tail of the original vector to the head of the added vector is the graphical sum (resultant) of the two vectors.

**Example:** In this manual, the origin “O” will always be located at the center of the Cartesian graph, but in the remainder of this document it will no longer be displayed for the purpose of simplicity and to avoid confusion with other vector letters. Vector OB having a magnitude of 0.005 inch and a direction of 150° is added to vector OA in Figure 7.

Both vectors OA and OB are shown in Figure 7. To add the two vectors, translate vector OB as described above and shown in Figure 7 as line AC. Note that line AC is parallel and equal in length to OB. Complete the triangle by drawing a vector OC from the tail of OA to the head of line AC. This vector OC is the *graphical sum* of OA plus OB. The magnitude and direction of OC can be scaled as 0.0056 inch and 81°, respectively.

Note that the term “graphical sum” has been stressed. Calculation of the sum or difference of vectors is beyond the scope of this paper. Thus, 0.005 plus 0.006 equals 0.011 in ordinary arithmetic, however graphical vector addition yields:

$$0.005/150^\circ + 0.006/30^\circ = 0.0056/81^\circ$$

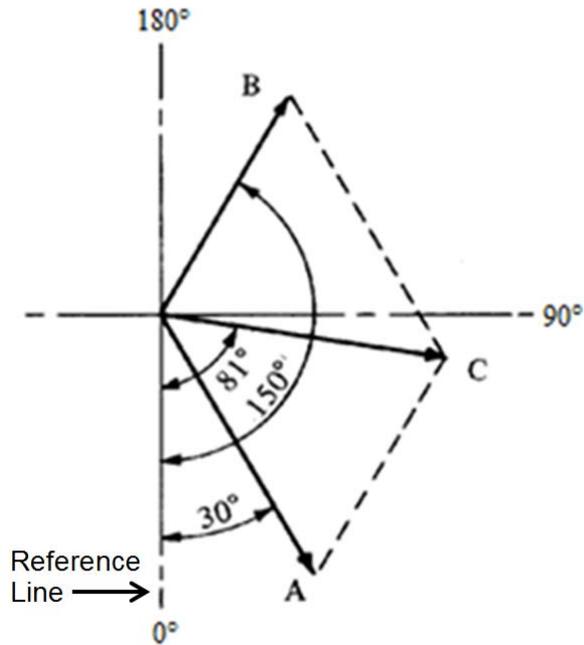


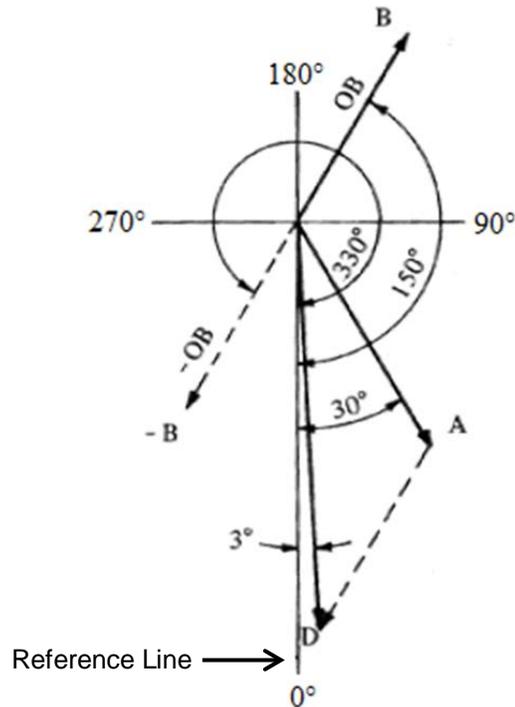
Figure 7. Vector addition.

## 7 Vector Subtraction

To subtract vector B from vector A, add the negative of vector B to vector A. The negative of a vector is equal in magnitude to the vector but opposite in direction.

In Figure 8, vector OB is to be subtracted from vector OA. The vector  $-OB$  is the negative of OB. Placing the tail of  $-OB$  at the head of OA and completing the triangle yields the resultant of  $OA - OB$ , which is OD. Note that  $OA - OB$  is equal in magnitude but opposite in direction to  $OB - OA$ . The magnitude OD is:

$$\begin{aligned} 0.006/30^\circ - 0.005/150^\circ &= \\ 0.006/30^\circ + 0.005/330^\circ &= 0.0095/3^\circ \end{aligned}$$



**Figure 8. Vector subtraction.**

## 8 Component Vectors

Often it is necessary to reduce a given vector into two component vectors of known directions. These vectors will produce the same effect as the given vector and, when added, will yield the given vector. To graphically obtain the magnitudes of the two component vectors, construct a parallelogram in which the two sets of parallel sides are parallel to the known directions and which has the given vector as the diagonal. Two sides (one from each set of parallel sides) are the component vectors.

**Example:** It is determined that a 50-pound weight placed  $15^\circ$  from rotor arm No. 1 (at  $0^\circ$ ) will balance a rotor. However, the closest locations for installing weights are at  $0^\circ$  and  $60^\circ$  (rotor arm No. 2). Therefore, two weights must be installed at  $0^\circ$  and  $60^\circ$  that will have the resultant effect of a 50-pound weight at  $15^\circ$ .

The 50-pound weight at  $15^\circ$  is scaled in Figure 9 as the vector OE. To reduce OE into a pair of vectors located at  $0^\circ$  and  $60^\circ$ , construct a parallelogram with sides at  $0^\circ$  and  $60^\circ$  and OE as the diagonal at  $15^\circ$ . Beginning at the head of vector OE (point E), draw a line parallel to the  $0^\circ$  line (rotor arm No. 1) and extend it to intersect the  $60^\circ$  line (rotor arm No. 2). This intersection is X. Also, beginning at the head of OE, draw a line parallel to the  $60^\circ$  line and extend it to intersect the  $0^\circ$  line. This intersection is Y. The pair of vectors, OX and OY, are components of the given vector OE. Therefore, the rotor should be balanced by placing a weight equal in magnitude to OX at  $60^\circ$  and a weight equal in magnitude to OY at  $0^\circ$ . By scaling, OX and OY are 15 and 42 lbs., respectively.

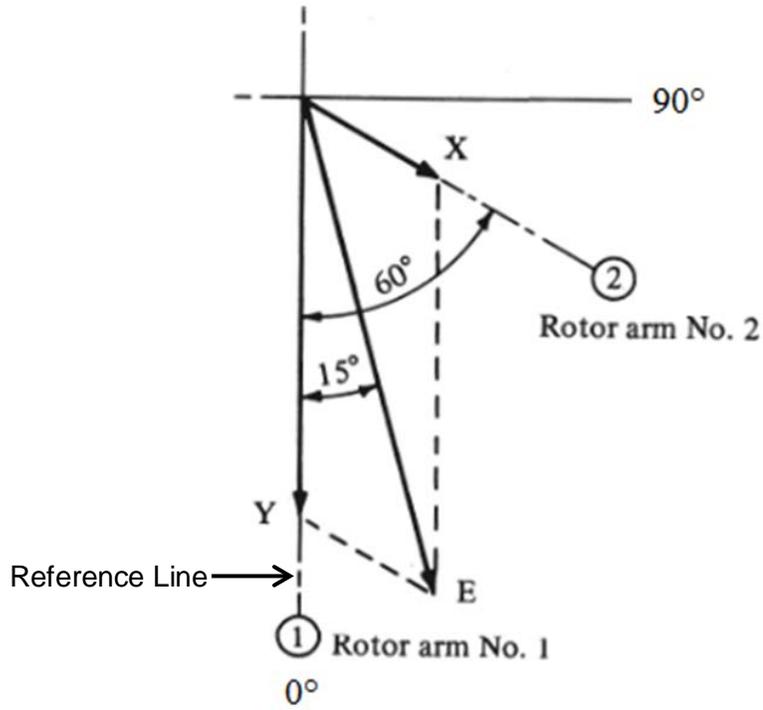


Figure 9. Component vectors.

## 9 Single-Plane Balancing

Single-plane balancing, also known as static balancing, is used to balance large motors and generators. In such cases, the balance weight has nearly the same effect when placed on either the top or bottom of the rotor arm. Therefore, weight applied to a single-plane (top or bottom of rotor arm) will satisfy imbalance correction. Single-plane balancing is used when only a negligible difference in vibration amplitude and phase angle (lag angle) exists after the weight is changed from the top of the rotor arm to the bottom. In most hydroturbine units the single-plane balancing technique is adequate to reduce the imbalance to a satisfactory level.

The required balance weight and its location can be determined either graphically or analytically. The graphical method will be used for single-plane balancing, as it only requires drafting aids and does not use trigonometry. An example of the analytical method is given later.

The graphical method requires a scale, protractor, triangles, pencil, and paper. If polar coordinate graph paper is used, the protractor can be omitted and the scale replaced by a divider.

## 10 Single-Plane Balancing—Example

Single-plane balancing technique. The unit rotates counter-clockwise, the rotating parts weigh 200,000 lbs. and the rotor has six arms equally spaced.

Maximum deflections for <i>as-found</i> condition		
Guide bearing	Shaft deflections (inch)	Location (degrees)
Upper	0.009	/150°
Lower	0.008	/150°
Turbine	0.005	/150°

Assuming a lag angle of 45°, the trial weight should be attached to the rotor arm nearest 225° (180° plus 45°) away from the high spot to counter the imbalance.

$$\begin{aligned}\text{Location for weight} &= 150^\circ + 225^\circ \\ &= 375^\circ \text{ or } 15^\circ\end{aligned}$$

Rotor arm No. 1 (reference line) at 0° is the nearest arm to 15°.

$$\begin{aligned}\text{Trial weight} &= \frac{\text{weight of rotating parts}}{10,000} \\ &= \frac{200,000}{10,000} = 20 \text{ lbs.}\end{aligned}$$

Maximum deflections with the trial weight on <i>top</i> of rotor arm No. 1		
Guide bearing	Shaft deflections (inch)	Location (degrees)
Upper	0.006	/200°
Lower	0.006	/200°
Turbine	0.004	/200°

Maximum deflections with the trial weight on <i>bottom</i> of rotor arm No. 1		
Guide bearing	Shaft deflections (inch)	Location (degrees)
Upper	0.007	/200°
Lower	0.006	/200°
Turbine	0.004	/200°

Since the shaft deflections with the weight on top of the rotor arm are similar to the deflections with the weight on the bottom, single-plane balancing may be sufficient. The largest deflection occurs at the upper guide bearing. Because the weight has slightly more effect on the deflection when it is on top of the rotor arm, the balance weight should be attached there.

The upper guide bearing condition from the as-found deflection and of the trial weight on top of rotor arm No. 1 is plotted in Figure 10. The as-found deflection is plotted as vector OA (0.009 inch/150°) with the arrow pointing in the direction of the deflection. Likewise, the deflection with top trial weight is plotted as vector OB (0.006 /200°). Construct a vector from A to B. Vector AB represents the effect of trial weight. The magnitude of AB and the angle  $\alpha$  between OA and AB are scaled as 0.0069 inch and 42°, respectively.

By studying the triangle of vectors OA, OB, and AB and applying the vector techniques introduced earlier, a better understanding of this method can be made. Note that the vectors are drawn such that:

$$OA + AB = OB$$

Where:

Vector OA is the as-found deflection

Vector AB is the effect of the trial weight

Vector OB is the deflection with the top trial weight (resultant)

To obtain a single-plane balanced condition requires that the resultant deflection OB be equal to zero so that:

$$OA + AB = 0$$

Since it is not zero, some imbalance still remains. The following procedure can be used to counter the remaining imbalance:

Beginning at the origin O, draw a vector equal and opposite to OA and label its head C (Figure 10). Vector OC represents the effect required to balance (reduce the imbalance to zero). For simplification, vector AB is shifted to the origin O and shown as OB'. An effect equal to OC is needed to balance the unit. To make OB' equal to OC, OB' must be rotated 42° to the same angular position as OC and increased by the ratio OC/OB'.

The magnitude and direction of the effect are directly proportional to the amount and location of the balance weight. Therefore, the balance weight also must be rotated 42° in the direction of rotation and its weight increased by the same ratio OC/OB'.

$$\text{Required ratio} = \frac{OC}{OB'} = \frac{0.009}{0.0069} = 1.3$$

Required length for OB' = OC

$$\begin{aligned} OC &= (1.3)OB' \\ &= 1.3 (0.0069) \\ &= 0.009 \text{ inch} \end{aligned}$$

$$\begin{aligned} \text{Required balance weight} &= 1.3 (20) \\ &= 26 \text{ lbs.} \end{aligned}$$

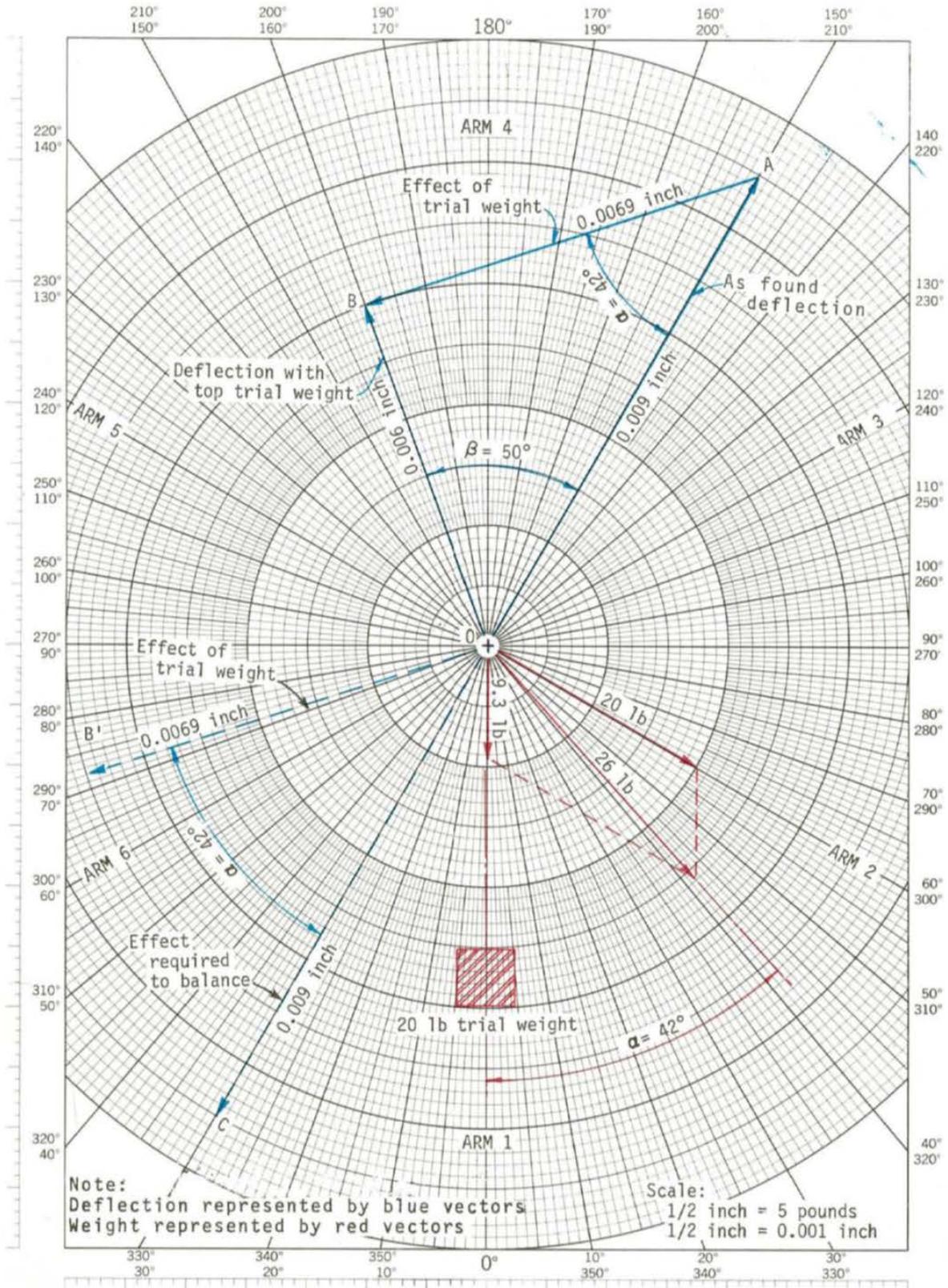


Figure 10. Static balancing using vectors.

Since the required location ( $42^\circ$ ) does not coincide with a rotor arm, the balance weight must be divided into component weights to be placed on the two adjacent arms. Draw the required balance weight as the vector, 26 lbs/ $42^\circ$  (Figure 10), and divide that vector into components corresponding to the positions of rotor arms No. 1 and 2. By scaling or reading divisions on the graph paper, the components are:

Weight on rotor arm No. 1 = 9.3 lbs.

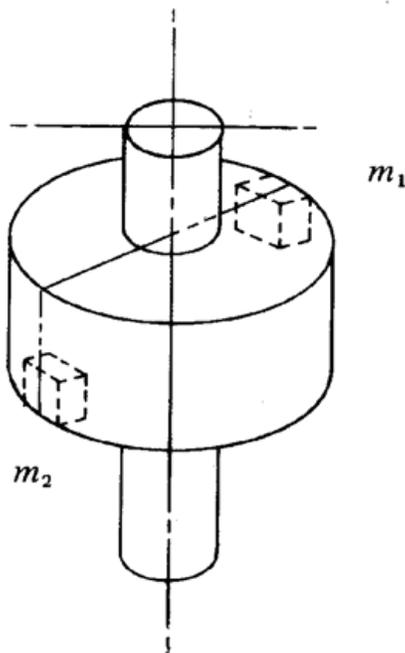
Weight on rotor arm No. 2 = 20 lbs.

Remove the 20-pound trial weight from rotor arm No. 1 and place a 9.3-pound weight on rotor arm No.1 and a 20-pound weight on rotor arm No. 2. The unit balance should be checked by running at normal speed before the weights are attached permanently to the rotor arms.

# 11 Two-Plane Balancing

Two-plane balancing, also known as dynamic balancing, involves the addition of weight at both ends of the rotor to correct the imbalance. It is generally used after the single-plane balancing technique has been tried and has failed. In such cases, a singular balance weight does not have the same effect when placed on the top of the rotor arm as it does when placed on the bottom of the rotor arm. Weight applied in a single-plane fashion may improve the balance at the top of the rotor arm but may worsen the balance at the bottom of the rotor arm, or vice versa. Two-plane balancing is used when results of a trial run to verify applicability of two-plane balancing, have shown a significant difference in vibration amplitude and phase angle (lag angle) when the weight is changed from the top of the rotor arm to the bottom. In most hydroturbine units a single-plane balancing technique is suitable and, hence, the two-plane balancing technique is unnecessary. The two-plane balancing technique generally involves more iterations of stopping and starting a unit for weight adjustment. On a hydroturbine unit this typically requires obtaining a clearance, which can take a substantial amount of time.

Figure 11 illustrates a rotor which has a two-plane imbalance. Weight added at the top of the rotor to counter  $m_1$  will create a single-plane imbalance on the  $m_2$  side of the rotor. To correct two-plane imbalance, weight must be added to both the top and the bottom of the rotor to produce a couple that will counter the effect of the couple causing the vibration. The difficult part of two-plane balancing is determining the correct combination of weights and locations.



**Figure 11. Two-plane imbalance rotor.**

The required weights and locations can be determined graphically, analytically, or by a combination of the two. Computer programs which determine the required combination are available. However, the graphical method presented here should be studied to give some insight into the forces involved in two-plane balancing. An example of an analytical two-plane balancing is included at the end of this FIST.

The same equipment used for single-plane balancing is used for the graphical method of two-plane balancing. It is noted that the graphical method is iterative and requires reasoning and experience.

# 12 Two-Plane Balancing—Example

**Two-plane balancing technique.** The unit to be balanced rotates counter-clockwise, the rotating parts weight 250,000 lbs., and the rotor has six arms equally spaced.

Maximum deflections for <i>as-found</i> condition		
Guide bearing	Shaft deflections (inch)	Location (degrees)
Upper	0.008	/170°
Lower	0.007	/0°
Turbine	0.006	/0°

Plot the maximum deflections for the upper and lower guide bearings on separate sheets and label the heads of the vectors A. Refer to Figures 12 and 13. Subscripts t and b refer to upper and lower bearings, respectively.

Add a trial weight at the top of the rotor. In the absence of other information, assume a lag angle of 45°. To counter the imbalance at the upper guide bearing, the weight should be attached to the rotor arm nearest 225° (180° plus 45°) from the high spot.

$$\begin{aligned} \text{Location for weight} &= 170^\circ + 225^\circ \\ &= 395^\circ \text{ or } 35^\circ \end{aligned}$$

Rotor arm No. 2 at 60° is the arm nearest 35°

$$\begin{aligned} \text{Trial weight} &= \frac{\text{weight of rotating parts}}{10,000} \\ &= \frac{250,000}{10,000} = 25 \text{ lbs.} \end{aligned}$$

Maximum deflections with the trial weight on <i>top</i> of rotor arm No. 2		
Guide bearing	Shaft deflections (inch)	Location (degrees)
Upper	0.003	/240°
Lower	0.008	/340°
Turbine	0.007	/340°

Plot the maximum deflections for the upper and lower guide bearings on the respective sheets and label the heads of the vectors B.

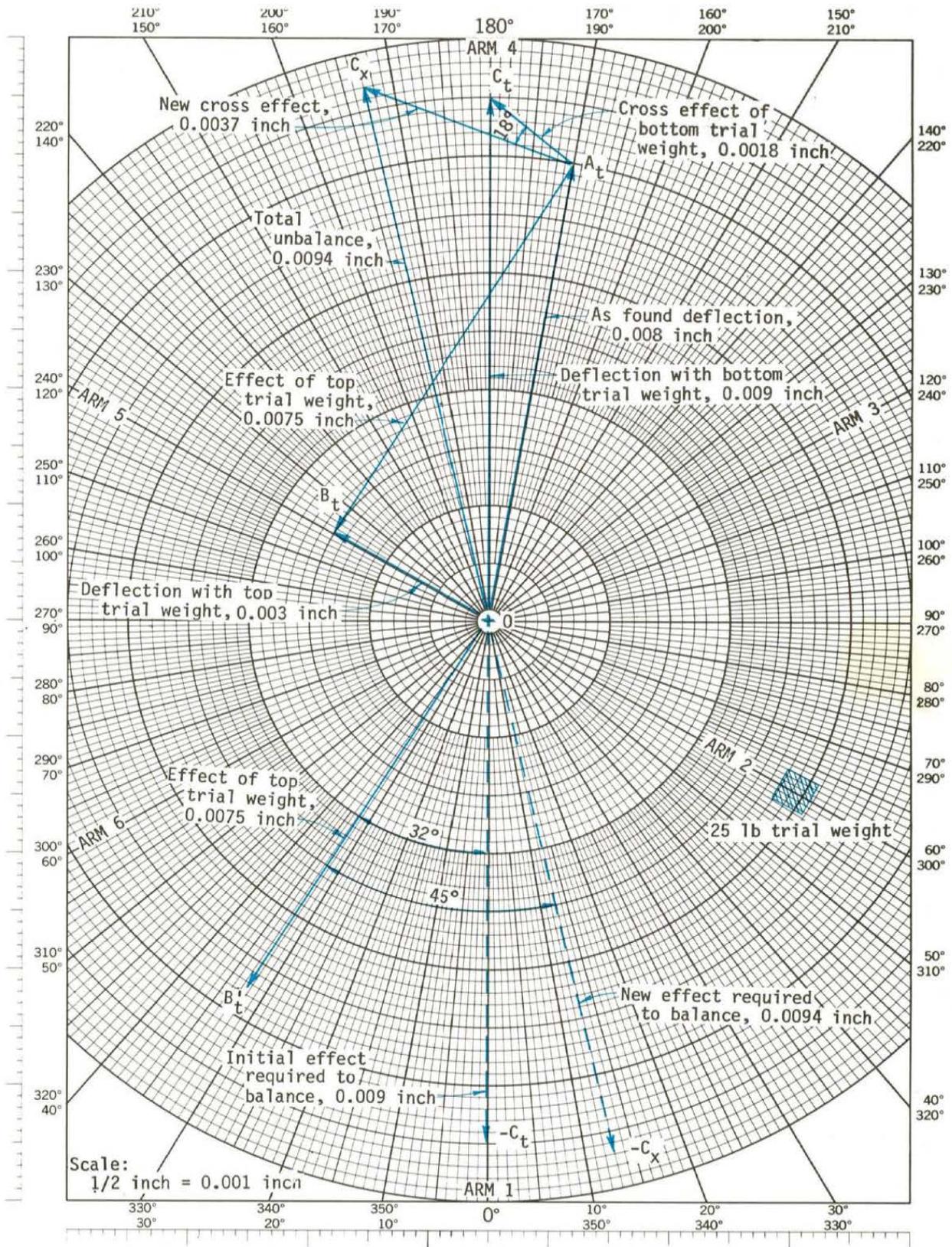


Figure 12. Dynamic balancing at upper guide bearing.

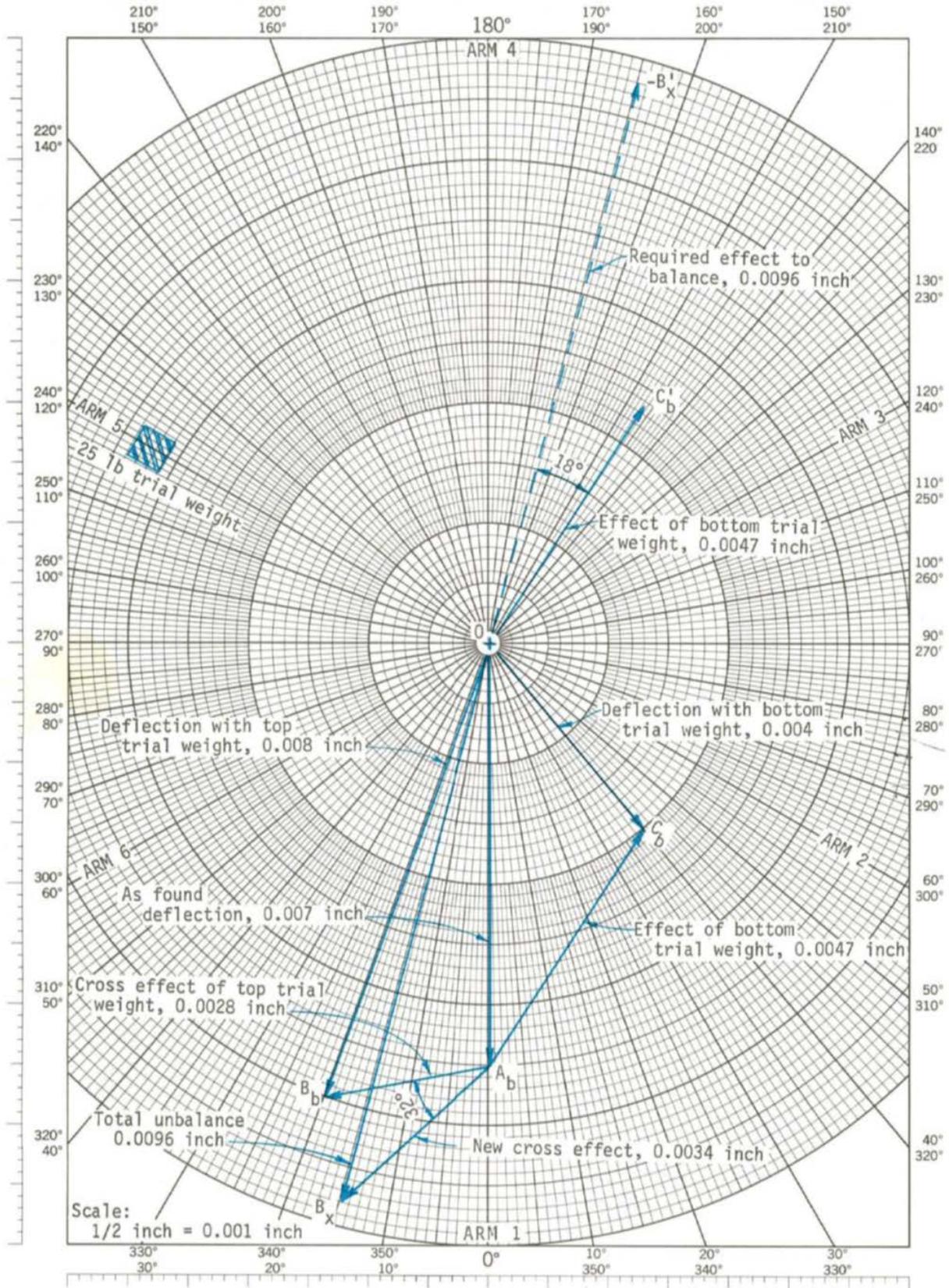


Figure 13. Dynamic balancing at lower guide bearing.

Remove the trial weight from the top of rotor arm No. 2 and put it on the bottom of the rotor arm. The trial weight now should be located to counter the as-found deflection at the lower guide bearing.

$$\text{Location for weight} = 0^\circ + 225^\circ = 225^\circ$$

Rotor arm No. 5 at 240° is nearest 225°

Maximum deflections with the trial weight on <i>bottom</i> of rotor arm No. 5		
Guide bearing	Shaft deflections (inch)	Location (degrees)
Upper	0.009	/180°
Lower	0.004	/40°
Turbine	0.005	/180°

Plot the maximum deflections on the respective sheets and label the heads of the vectors C.

Construct vectors from A to B and from A to C on both the upper and lower guide bearing sheets. The vectors OA, OB, and OC are resultant vectors, and the vectors AB and AC are effect vectors. On Figure 12, A<sub>t</sub>B<sub>t</sub> is the effect of top trial weight (on the rotor top) and A<sub>t</sub>C<sub>t</sub> is the cross effect of bottom trial weight (on the rotor bottom). On Figure 13, A<sub>b</sub>B<sub>b</sub> is the cross effect of top trial weight (on the rotor top) and A<sub>b</sub>C<sub>b</sub> is effect of bottom trial weight (on the rotor bottom).

The upper guide bearing has the highest vibration amplitude (shaft deflection). Using the single-plane balancing technique, find the top balance weight required to make the upper guide bearing deflection equal to zero. Start with OC<sub>t</sub> rather than OA<sub>t</sub>. This accounts for the cross effect of the bottom trial weight. Reference Figure 12 and translate A<sub>t</sub>B<sub>t</sub> as OB'<sub>t</sub> :

$$OC_t = 0.009 \text{ inch} / 180^\circ$$

$$A_t B_t = OB'_t = 0.0075 / 328^\circ$$

To reduce the deflection to zero, A<sub>t</sub>B<sub>t</sub> must be equal and opposite to OC<sub>t</sub>.

$$\begin{aligned} \text{Required angular rotation} &= 360^\circ - 328^\circ \\ &= 32^\circ \text{ counter-clockwise} \end{aligned}$$

$$\begin{aligned} \text{Required weight} &= \text{trial weight} \left( \frac{OC_t}{A_t B_t} \right) \\ &= 25 \left( \frac{0.009}{0.0075} \right) = 30 \text{ lbs.} \end{aligned}$$

This change will change the cross effect proportionately at the lower guide bearing. The cross effect will be increased by the same ratio OC<sub>t</sub>/A<sub>t</sub>B<sub>t</sub> and rotated 32° counter-clockwise. Referencing Figure 13, the new cross effect is shown as A<sub>b</sub>B<sub>x</sub>.

$$A_b B_x = A_b B_b \left( \frac{0.009}{0.0075} \right)$$

$$= 0.0028 \text{ inch} \left( \frac{0.009}{0.0075} \right) = 0.0034 \text{ inch}$$

$A_b B_x$  is rotated  $32^\circ$  counter-clockwise from  $A_b B_b$ .

$$A_b B_x = 0.0034 \text{ inch} / 281^\circ + 32^\circ$$

$$= 0.0034 \text{ inch} / 313^\circ \text{ as it would appear with its tail at O.}$$

The total imbalance at the lower guide bearing is the vector sum of the original imbalance and the new cross effect.

$$OB_x = OA_b + A_b B_x$$

Use the single-plane technique to find the balance weight that, when added to the bottom of the rotor, will reduce the total imbalance at the lower guide bearing to zero (Figure 13).

$$OB_x = 0.0096 \text{ inch} / 345^\circ$$

$$A_b C_b = OC'_b = 0.0047 \text{ inch} / 147^\circ$$

To reduce the imbalance to zero,  $A_b C_b$  must be equal and opposite to  $OB_x$ . Plot  $-OB_x$  as  $OB'_x = 0.0096 / 165^\circ$ . Shift  $A_b C_b$  to the position shown as  $OC'_b$ .

$$\text{Required angular rotation} = 165^\circ - 147^\circ$$

$$= 18^\circ \text{ counter-clockwise}$$

$$\text{Required weight} = \text{trial weight} \left( \frac{OB'_x}{OC'_b} \right)$$

$$= 25 \left( \frac{0.0096}{0.0047} \right) = 51 \text{ lbs.}$$

Changing the weight on the bottom of the rotor changes the cross effect proportionately to the upper guide bearing. The cross effect will be increased by the ratio  $OB'_x/OC'_b$  and rotated  $18^\circ$  counter-clockwise. Referencing Figure 12, the new cross effect is shown as  $A_t C_x$ .

$$A_t C_x = A_t C_t \left( \frac{0.0096}{0.0047} \right)$$

$$= (0.0018 \text{ inch}) \left( \frac{0.0096}{0.0047} \right) = 0.0037 \text{ inch}$$

$A_t C_x$  is rotated  $18^\circ$  counter-clockwise from  $A_t C_t$ .

$$A_t C_x = 0.0037 \text{ inch} / 231^\circ + 18^\circ$$

$$= 0.0037 \text{ inch} / 249^\circ \text{ as it would appear with its tail at O.}$$

The total imbalance at the upper guide bearing is the sum between the vectors  $OA_t$  and  $A_t C_x$ .

$$\text{Total imbalance} = A_t C_x + OA_t = OC_x$$

To balance, again we use the single-plane technique to find the weight needed to bring the deflection at the upper guide bearing back to zero (Figure 12).

$$\begin{aligned} OC_x &= 0.0094 \text{ inch} / 193^\circ \\ A_t B_t &= OB'_t = 0.0075 \text{ inch} / 328^\circ \end{aligned}$$

To bring the imbalance back to zero,  $A_t B_t$  must be equal and opposite to  $OC_x$ . Plot  $OC_x$  as  $OC'_x = 0.0094/13^\circ$ . Recall that  $A_t B_t = OB'_t$ .

$$\begin{aligned} \text{Required angular rotation} &= 13^\circ - 328^\circ + 360^\circ \\ &= 45^\circ \text{ counter-clockwise} \end{aligned}$$

$$\begin{aligned} \text{Required weight} &= \text{trial weight} \left( \frac{OC_x}{OB'_t} \right) \\ &= 25 \left( \frac{0.0094}{0.0075} \right) = 31 \text{ lbs.} \end{aligned}$$

Now a balance check can be made on the lower guide bearing. Again, the addition of this new weight will change the cross effect proportionately at the lower guide bearing. Referring to Figure 14:

$$\begin{aligned} A_b B_{xx} &= A_b B_b \left( \frac{0.0094}{0.0075} \right) / 281^\circ + 45^\circ \\ &= (0.0028 \text{ inch}) \left( \frac{0.0094}{0.0075} \right) / 326^\circ \\ &= 0.0035 \text{ inch} / 326^\circ \end{aligned}$$

The total imbalance at the lower guide bearing is the difference between vectors  $OB_{xx}$  and  $OB_x$ .

$$\begin{aligned} \text{Total imbalance} &= OZ = OB_{xx} - OB_x \\ &= 0.0009 \text{ inch} / 40^\circ \end{aligned}$$

Draw the desired balance weights as vectors on polar coordinate graph paper and resolve them into components corresponding to the adjacent rotor arms as shown on Figure 15.

Place the weights on the appropriate rotor arms and obtain another set of deflection readings. Some adjustment may be required due to data or analysis inaccuracies.



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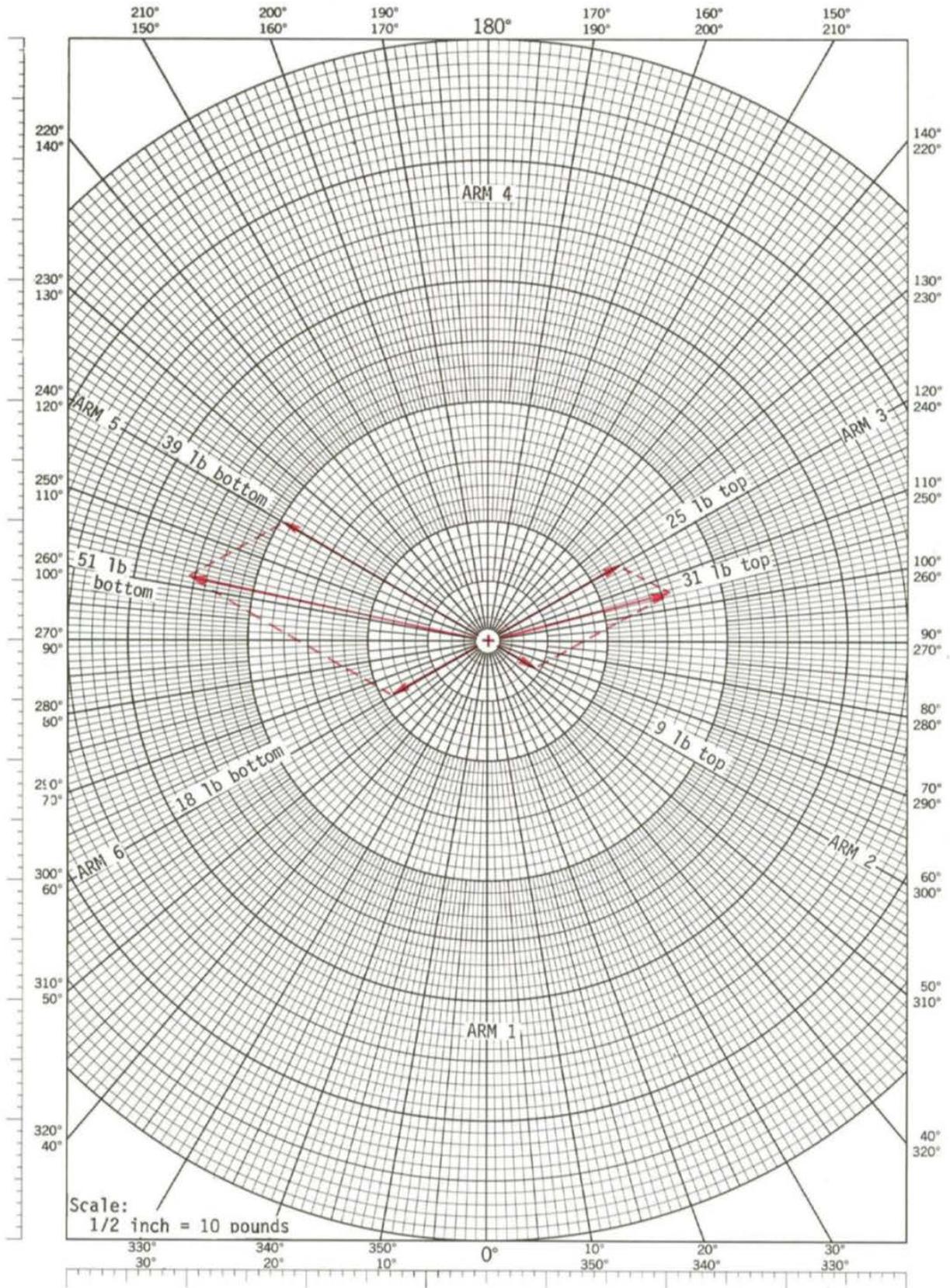


Figure 15. Dynamic balancing – balance weight distribution.

# 13 Analytical Single-Plane Balancing— Example

The example located in Section 10, Single-Plane Balancing will be used again for the analytical method of single-plane balancing.

**Data**

Weight of rotating parts      200,000 lbs.  
 Number of rotor arms         6 equally spaced  
 Unit rotation                   Counter-clockwise  
 Trial weight                      20 lbs.

Maximum deflections for <i>as-found</i> condition		
Guide bearing	Shaft deflections (inch)	Location (degrees)
Upper	0.009	/150°
Lower	0.008	/150°
Turbine	0.005	/150°

Maximum deflections with the trial weight on <i>top</i> of rotor arm No. 1		
Guide bearing	Shaft deflections (inch)	Location (degrees)
Upper	0.006	/200°
Lower	0.006	/200°
Turbine	0.004	/200°

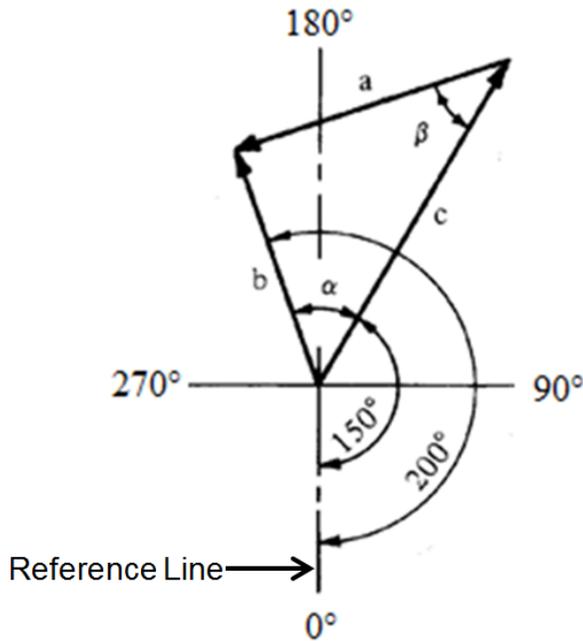
Maximum deflections with the trial weight on <i>bottom</i> of rotor arm No. 1		
Guide bearing	Shaft deflections (inch)	Location (degrees)
Upper	0.007	/200°
Lower	0.006	/200°
Turbine	0.004	/200°

The trial weight has more effect on the shaft deflections when it is on top of the rotor arm; therefore, the balance weight should be attached to the top of rotor arm.

Figure 16 shows the as-found imbalance at the upper guide bearing as well as the effect produced by positioning the trial weigh on top of rotor arm No. 1.

To balance the unit, the balance weight must have an effect equal to the as-found imbalance and in the opposite direction.

Required balance weight = weight causing imbalance.



**Figure 16. Conditions at the upper guide bearing.**

- a = Effect of trial weight
- b = Imbalance with trial weight 0.006/200°
- c = As-found imbalance 0.009/150°

Assume that the machine is linear; i.e., the vibration amplitudes are in proportion to the forces causing them:

$$\begin{aligned} \frac{C}{A} &= \frac{\text{as-found unbalance}}{\text{effect of trial weight}} \\ &= \frac{\text{weight causing unbalance}}{\text{trial weight}} \end{aligned}$$

The effect of trial weight A can be determined using the law of cosines.

$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc \cos \alpha \\ a &= [b^2 + c^2 - 2bc \cos \alpha]^{0.5} \\ a &= [0.006^2 + 0.009^2 - 2(0.006)(0.009) \cos(200^\circ - 150^\circ)]^{0.5} \\ a &= [0.000117 - 2(0.000054)0.64279]^{0.5} \\ a &= [47.6 \times 10^{-6}]^{0.5} = 0.0069 \text{ inch} \end{aligned}$$

The required balance weight equals the weight causing imbalance:

$$\begin{aligned} \text{Required balance weight} &= (\text{trial weight})(c/a) \\ &= 20 (0.009/0.0069) \\ &= 26 \text{ lbs.} \end{aligned}$$

The angle  $\beta$  by which the effect must be changed, equals the angle the weight must be rotated. Solve for  $\beta$  using the law of sines:

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta}$$

$$\sin \beta = (b/a) \sin \alpha$$

$$= (0.006/0.0069) \sin (200^\circ - 150^\circ)$$

$$\sin \beta = (0.006/0.0069)(0.76604) = 0.66612$$

$$\beta = 41.8^\circ$$

The required location for the weight is  $41.8^\circ$  counter-clockwise from its present location on rotor arm No. 1. The required location does not coincide with a rotor arm so the weight must be divided for placement on adjacent rotor arms.

The required weights are determined using the law of sines (Figure 17):

$$\frac{W}{\sin \gamma} = \frac{W_1}{\sin \alpha} = \frac{W_2}{\sin \beta}$$

Where:

$$W = 26 \text{ lbs}$$

$$\beta = 41.8^\circ$$

The angle between rotor arms is  $60^\circ$ :

thus:

$$\alpha + \beta = 60^\circ$$

$$\alpha = 60^\circ - \beta = 60^\circ - 41.8^\circ$$

$$= 18.2^\circ$$

The sum of triangle angles must equal  $180^\circ$ .

$$\alpha + \beta + \gamma = 180^\circ$$

$$\gamma = 180^\circ - \alpha - \beta$$

$$= 180^\circ - 18.2^\circ - 41.8^\circ = 120^\circ$$

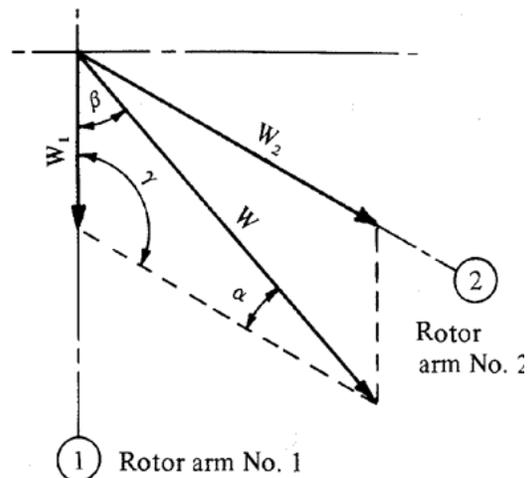
The weights for rotor arms No. 1 and 2 now can be determined.

$$W_1 = \frac{W \sin \alpha}{\sin \gamma} = \frac{26 \sin 18.2^\circ}{\sin 120}$$

$$W_2 = \frac{W \sin \beta}{\sin \gamma} = \frac{26 \sin 41.8^\circ}{\sin 120}$$

$$= 20.0 \text{ lbs}$$

Weights  $W_1$  and  $W_2$  are attached to the respective rotor arms.



**Figure 17. Weight distribution on rotor arms.**

## 14 Analytical Two-Plane Balancing— Example

Section 12, Two-Plane Balancing—Example, will be used again for the analytical two-plane balancing.

### Data

Weight of rotating parts	250,000 lbs.
Number of rotor arms	6 equally spaced
Unit rotation	Counter-clockwise
Trial weight	25 lbs.

Maximum deflections for <i>as-found</i> condition		
Guide bearing	Shaft deflections (inch)	Location (degrees)
Upper	0.008	/170°
Lower	0.007	/0°
Turbine	0.006	/0°

Maximum deflections with the trial weight on <i>top</i> of rotor arm No. 2		
Guide bearing	Shaft deflections (inch)	Location (degrees)
Upper	0.003	/240°
Lower	0.008	/340°
Turbine	0.007	/340°

Maximum deflections with the trial weight on <i>bottom</i> of rotor arm No. 5		
Guide bearing	Shaft deflections (inch)	Location (degrees)
Upper	0.009	/180°
Lower	0.004	/40°
Turbine	0.005	/180°

To divide one vector by another vector, divide the magnitudes and subtract the angles.

Example: To divide a vector of 0.006 inch at 30° by a vector of 0.005 at 150°, divide magnitudes:

$$0.006 \text{ inch} / 0.005 \text{ inch} = 1.200$$

and subtract angles

$$30^\circ - 150^\circ = -120^\circ$$

To change the negative angle to a positive angle, add 360°. The result is 1.200 at -120° or 1.200 at +240°.

To multiply one vector by another, multiply the magnitudes and add the angles.

Example: To multiply a vector of 0.004 inch at 180° by a vector of 0.010 inch at 45°, multiply magnitudes:

$$\begin{aligned}
 &0.004(0.010) = 0.00004 \\
 &\text{and add angles} \\
 &180^\circ + 45^\circ = 225^\circ \\
 &(0.004 \text{ inch} / 180^\circ)(0.010 \text{ inch} / 45^\circ) \\
 &= 0.00004 / 225^\circ
 \end{aligned}$$

The unit was assumed to be linear so the required weights  $W_{rt}$  and  $W_{rb}$  and locations will be proportional to the trial weights, that is:

$$\begin{aligned}
 W_{rt} &= \theta W_t \\
 W_{rb} &= \phi W_b
 \end{aligned}$$

Where:

$\theta$  and  $\phi$  are vector operators (having both magnitude and direction, and  $_t$  and  $_b$  refer to top and bottom subscripts, respectively)

To balance the unit, the values of  $\theta$  and  $\phi$  must be determined.

Using the notation in the graphical method where A refers to the as-found deflection, B refers to deflection with top trial weight, and C refers to deflection with bottom trial weight, the vectors are:

$$\begin{array}{lll}
 OA_t = 8/170^\circ & OC_t = 9/180^\circ & OB_t = 3/240^\circ \\
 OA_b = 7/0^\circ & OC_b = 4/40^\circ & OB_b = 8/340^\circ
 \end{array}$$

For convenience, magnitudes are in mils (thousandths of an inch) rather than in decimals.

As shown on Figures 18 and 19:

$$\begin{array}{ll}
 OB_t = OA_t + A_tB_t & OB_b = OA_b + A_bB_b \\
 OC_t = OA_t + A_tC_t & OC_b = OA_b + A_bC_b
 \end{array}$$

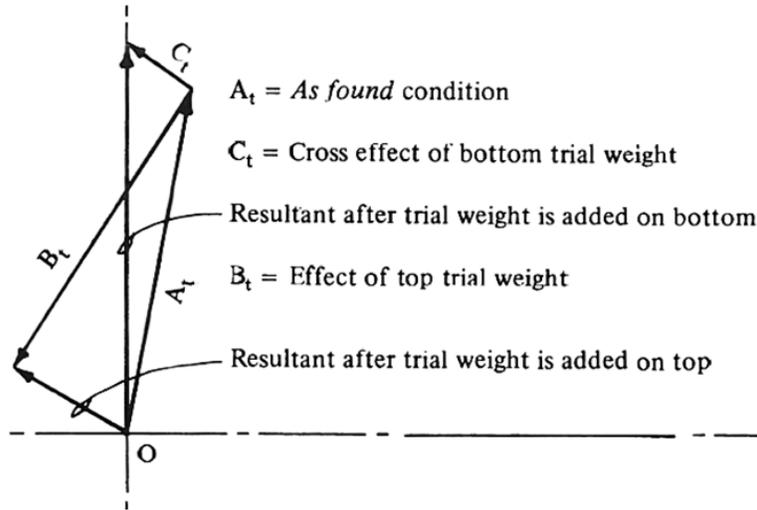


Figure 18. Deflections at upper guide bearing.

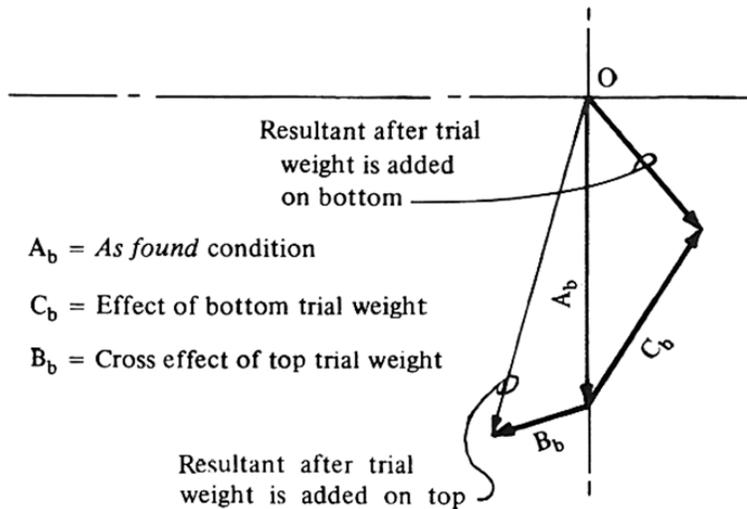


Figure 19. Deflections at lower guide bearing.

To balance the unit, the combined effect of the balance weights must be equal and opposite to the as-found imbalance at both the upper and lower guide bearings. Since vibration amplitudes are proportional to the forces causing them,  $A_t B_t$  and  $A_b B_b$  are proportional to  $W_t$  and  $A_b C_b$ , respectively, and  $A_t C_t$  is proportional to  $W_b$ . Also, the required balance weight is proportional to the as-found imbalance; i.e.,  $W_{rt}$  is proportional to  $OA_t$ , and  $W_{rb}$  is proportional to  $OA_b$ .

$$-OA_t = \theta A_t B_t + \phi A_t C_t$$

$$-OA_b = \theta A_b B_b + \phi A_b C_b$$

From these equations, the vector operators are determined by:

$$\Theta = \left( \frac{(OA_t \cdot A_b C_b) - (OA_b \cdot A_t C_t)}{(A_b B_b \cdot A_t C_t) - (A_t B_t \cdot A_b C_b)} \right)$$

$$\phi = \left( \frac{(OA_b \cdot A_t B_t) - (OA_t \cdot A_b B_b)}{(A_b B_b \cdot A_t C_t) - (A_t B_t \cdot A_b C_b)} \right)$$

This gives the following vector equations:

$$OA_t = 8/170^\circ$$

$$OA_b = 7/0^\circ$$

$$OB_t = OA_t + A_t B_t = 3/240^\circ$$

$$OB_b = OA_b + A_b B_b = 8/340^\circ$$

$$OC_t = OA_t + A_t C_t = 9/180^\circ$$

$$OC_b = OA_b + A_b B_b = 4/40^\circ$$

The effects of the trial weights are computed using vector subtraction or can be scaled:

$$A_t B_t = OB_t - OA_t = 3/240^\circ - 8/170^\circ$$

Magnitude of  $A_t B_t$

$$= [(3 \sin 240 - 8 \sin 170)^2 + (3 \cos 240 - 8 \cos 170)^2]^{0.5}$$

$$= [(-3.99)^2 + (6.38)^2]^{0.5} = 7.52$$

Angle of:

$$A_t B_t = \arctan \left( -\frac{3.99}{6.38} \right) = \arctan -0.6254$$

$$= 328^\circ$$

$$A_t B_t = 7.52/328^\circ$$

$$A_b B_b = OB_b - OA_b = 8/340^\circ - 7/0^\circ$$

$$= 2.78/280.7^\circ$$

$$A_t C_t = OC_t - OA_t = 9/180^\circ - 8/170^\circ$$

$$= 1.79/231.1^\circ$$

$$A_b C_b = OC_b - OA_b = 4/40^\circ - 7/0^\circ$$

$$= 4.70/146.8^\circ$$

Now, use vector multiplication:

$$\begin{aligned} OA_t \cdot A_b C_b &= 8/170^\circ \cdot 4.70/146.8^\circ \\ &= 8 \cdot 4.70/170^\circ + 146.8^\circ \\ &= 37.60/316.8^\circ \end{aligned}$$

$$\begin{aligned} OA_b \cdot A_t C_t &= 7/0^\circ \cdot 1.79/231.1^\circ \\ &= 12.53/231.1^\circ \end{aligned}$$

$$\begin{aligned} OA_b \cdot A_t B_t &= 7/0^\circ \cdot 7.52/328^\circ \\ &= 52.64/328^\circ \end{aligned}$$

$$\begin{aligned} OA_t \cdot A_b B_b &= 8/170^\circ \cdot 2.78/280.7^\circ \\ &= 22.24/90.7^\circ \end{aligned}$$

$$\begin{aligned} A_b B_b \cdot A_t C_t &= 2.78/280.7^\circ \cdot 1.79/231.1^\circ \\ &= 4.98/151.8^\circ \end{aligned}$$

$$\begin{aligned} A_t B_t \cdot A_b C_b &= 7.52/328^\circ \cdot 4.70/146.8^\circ \\ &= 35.34/114.8^\circ \end{aligned}$$

Vector subtraction is now used to determine the numerator and the denominator:

$$\begin{aligned} (OA_t \cdot A_b C_b) - (OA_b \cdot A_t C_t) \\ &= 37.60/316.8^\circ - 12.53/231.1^\circ \\ &= 38.73/335.6^\circ \end{aligned}$$

$$\begin{aligned} (OA_b \cdot A_t B_t) - (OA_t \cdot A_b B_b) \\ &= 52.64/328^\circ - 22.24/90.7^\circ \\ &= 67.31/311.9^\circ \end{aligned}$$

$$\begin{aligned} (A_b B_b \cdot A_t C_t) - (A_t B_t \cdot A_b C_b) \\ &= 4.98/151.8^\circ - 35.34/114.8^\circ \\ &= 31.51/289.3^\circ \end{aligned}$$

$$\begin{aligned} \Theta &= \left( \frac{38.73/335.6}{31.51/289.3} \right) = \left( \frac{38.73}{31.51} \right) / 335.6 - 289.3 \\ &= 1.23/46.3^\circ \end{aligned}$$

$$\begin{aligned} \Phi &= \left( \frac{67.31/311.9}{31.51/289.3} \right) = \left( \frac{67.31}{31.51} \right) / 311.9 - 289.3 \\ &= 2.14/22.6^\circ \end{aligned}$$

$$\begin{aligned} W_{rt} &= \Theta W_t = 1.23/46.3^\circ \cdot 25/60^\circ \\ &= 30.75/106.3^\circ \end{aligned}$$

$$\begin{aligned} W_{rb} &= \Phi W_b = 2.14/22.6^\circ \cdot 25/240^\circ \\ &= 53.5/262.6^\circ \end{aligned}$$

Because the required locations do not coincide with rotor arms, the weight must be divided for attachment on the adjacent rotor arms.

The top weight is proportioned between rotor arms No. 2 and 3 using the law of sines (see Figure 20).

$$\frac{W_{rt}}{\sin \gamma_t} = \frac{W_2}{\sin \beta_t} = \frac{W_3}{\sin \alpha_t}$$

Where:

$$W_{rt} = 30.75 \text{ lbs.}$$

$$\alpha_t = 46.3^\circ$$

$$\alpha_t + \beta_t = 60^\circ$$

$$\beta_t = 60^\circ - \alpha_t = 60^\circ - 46.3^\circ = 13.7^\circ$$

$$\alpha_t + \beta_t + \gamma_t = 180^\circ$$

$$\gamma_t = 180^\circ - (\alpha_t + \beta_t) = 180^\circ - 60^\circ = 120^\circ$$

$$\begin{aligned} W_2 &= W_{rt} \frac{\sin \beta_t}{\sin \gamma_t} = 30.75 \frac{\sin 13.7^\circ}{\sin 120^\circ} \\ &= 8.4 \text{ lbs} \end{aligned}$$

$$\begin{aligned} W_3 &= W_{rt} \frac{\sin \alpha_t}{\sin \gamma_t} = 30.75 \frac{\sin 46.3^\circ}{\sin 120^\circ} \\ &= 25.7 \text{ lbs} \end{aligned}$$

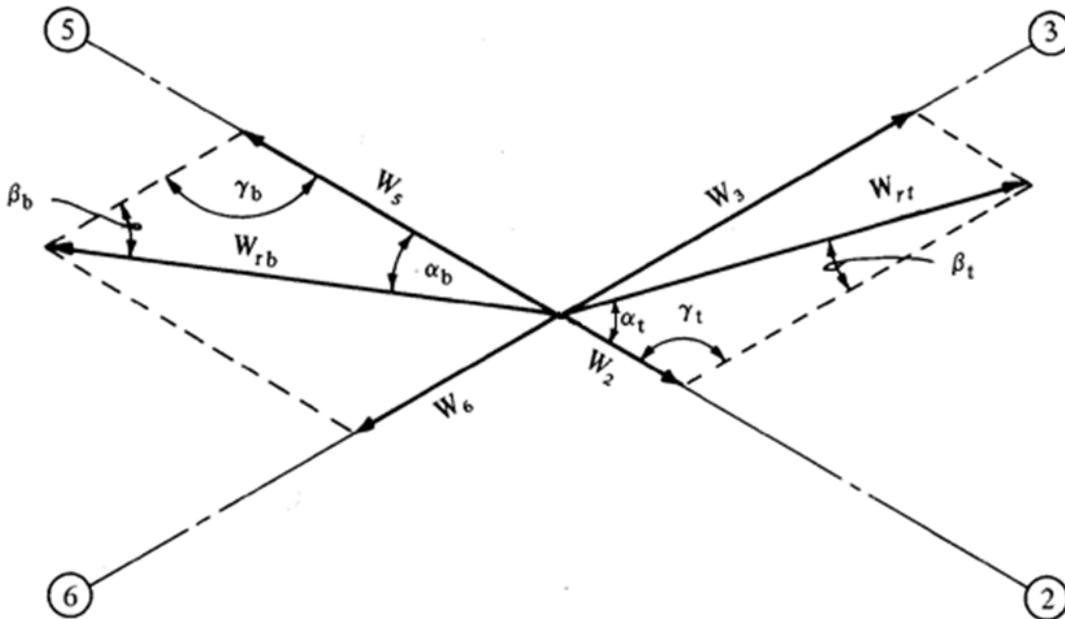


Figure 20. Divided weight for upper rotor arms No. 2 and No. 3, and for lower rotor arms No. 5 and No. 6.

Similarly, the bottom weight is proportioned between arms No. 5 and 6 (see Figure 20).

$$\frac{W_{rb}}{\sin\gamma_b} = \frac{W_5}{\sin\beta_b} = \frac{W_6}{\sin\alpha_b}$$

Where:

$$W_{rb} = 53.5 \text{ lbs.}$$

$$\alpha_b = 22.6^\circ$$

$$\alpha_b + \beta_b = 60^\circ$$

$$\beta_b = 60^\circ - \alpha_b = 60^\circ - 22.6^\circ = 37.4^\circ$$

$$\alpha_b + \beta_b + \gamma_b = 180^\circ$$

$$\gamma_b = 180^\circ - (\alpha_b + \beta_b) = 180^\circ - 60^\circ = 120^\circ$$

$$\begin{aligned} W_5 &= W_{rb} \frac{\sin\beta_b}{\sin\gamma_b} = 53.5 \frac{\sin 37.4^\circ}{\sin 120^\circ} \\ &= 37.5 \text{ lbs} \end{aligned}$$

$$\begin{aligned} W_6 &= W_{rb} \frac{\sin\alpha_b}{\sin\gamma_b} = 53.5 \frac{\sin 22.6^\circ}{\sin 120^\circ} \\ &= 23.7 \text{ lbs} \end{aligned}$$

The weights are attached to the respective rotor arms.

Literature on the subject of balancing includes many cases in the analysis of pump, turbine, and pump-turbine vibrations. As noted, one must first investigate the cause of vibrations and oscillations initiated by such conditions as vortex shedding, draft tube surges, penstock pressure fluctuations, electrical system, and mechanical anomalies.

## RECLAMATION MANUAL TRANSMITTAL SHEET

Effective Date: \_\_\_\_\_

Release No. \_\_\_\_\_

Ensure all employees needing this information are provided a copy of this release.

### Reclamation Manual Release Number and Subject

### Summary of Changes

NOTE: This Reclamation Manual release applies to all Reclamation employees. When an exclusive bargaining unit exists, changes to this release may be subject to the provisions of collective bargaining agreements.

### Filing instructions

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Filed by: \_\_\_\_\_

Date: \_\_\_\_\_