RECLAMATION

Managing Water in the West

Modeling Spatial Water Allocation and Hydrologic Externalities in the Boise Valley

A component of the Boise Valley Water Use Planning Program

U.S. Department of the Interior
Bureau of Reclamation
Pacific Northwest Region
Boise, Idaho

February, 2009

U.S. Department of the Interior
Bureau of Reclamation
Pacific Northwest Region
Boise, Idaho

University of Idaho

February, 2009

Modeling Spatial Water Allocation and Hydrologic Externalities in the Boise Valley

Prepared for the Snake River Area Office by:

R.D. Schmidt
Bureau of Reclamation, Snake River Area Office, Boise ID

Leroy Stodick and R.G. Taylor
Department of Agricultural Economics and Rural Sociology
University of Idaho, Moscow ID

February, 2009

Mission Statements

The mission of the Department of the Interior is to protect and provide access to our Nation's natural and cultural heritage and honor our trust responsibilities to Indian Tribes and our commitments to island communities.

The mission of the Bureau of Reclamation is to manage, develop, and protect water and related resources in an environmentally and economically sound manner in the interest of the American public.

Table of Contents

TABLE OF CONTENTS 3

FIGURES 5

TABLES 7

EXECUTIVE SUMMARY 9

INTRODUCTION 13

SUPPLY MANAGEMENT AND DEMAND MANAGEMENT 14

Market-Based Management 14

Supply Functions and Demand Functions 15

Third-Party Impacts and Hydrologic Externalities 16

BOISE PROJECT HYDROLOGIC EXTERNALITIES 17

SPATIAL WATER ALLOCATION MODELING 19

Conjunctive Hydrologic Modeling using GFLOW 20

Partial Equilibrium Economic Modeling using GAMS 20

THE BOISE PROJECT SUB-AREA MODEL 21

The Sub-Area Hydrologic Model 22
- GFLOW Model Runs 25

Sub-Area Water Supply Functions 31
- Forms of Empirical Response Functions 31
- Curve-Fitting to Empirical Response Functions 32
- Converting Response Functions to Supply Functions 36
- Canal Transportation Cost 40

Sub-Area Water Demand Functions 42
- Estimating Agricultural Water Demand 43
- Elasticity of Agricultural Water Demand 43

The Sub-Area Partial Equilibrium Model 50
- Partial Equilibrium Model Scenarios 51

SPATIAL WATER ALLOCATION MODEL RESULTS 51

Model Results that include Agricultural Pumpers 52
Scenario 1 53
Scenario 2 54
Scenario 3 54

Model Results that include M&I Pumpers 55
Scenario 1 56
Scenario 2 56
Scenario 3 57

Transportation Costs and Constraint Costs 57

Valuation of the Canal Seepage Externality 59
Valuing Canal Seepage in Terms of Supply Cost 60
Valuing Canal Seepage in Terms of Demand Price 60

SUMMARY OF MODEL RESULTS 62

REFERENCES 63

Hydrology 63

Economics 63

APPENDIX A - ANALYTIC ELEMENT MODELING THEORY AND APPLICATION TO CANALS, WELLS AND DRAINS 67

APPENDIX B - PARTIAL EQUILIBRIUM MODELING THEORY AND APPLICATION TO WATER SUPPLY AND DEMAND 79

APPENDIX C - AVERAGE VERSUS MARGINAL DEMAND PRICE SPECIFICATION FOR AGRICULTURAL AND M&I WATER USERS 87

Figures

Figure 1: Examples of inelastic and elastic demand functions. 16
Figure 2: Average annual hydrologic impacts of irrigation activity in the Lower Boise Valley. 18
Figure 3: Boise Project sub-area that includes a portion of Lake Lowell, the New York Canal, Elijah, Aaron and Wilson drains, and surrounding agricultural and M&I wells. 22
Figure 4: USGS seepage measurement points along the 39 mile New York Canal and 8 mile segment with highest seepage rate. 23
Figure 5: GFLOW model hydrologic boundary features. 24
Figure 6: Aquifer head conditions full canal with 54 acre-feet of pumping from each of 149 ag. wells. 25
Figure 7: Aquifer head conditions, lined canal with 54 acre-feet of pumping from each of 149 ag. wells. 26
Figure 8: Aquifer head conditions, full canal with 165 acre-feet of pumping from each of six M&I wells. 26
Figure 9: Aquifer head conditions, lined canal with 165 acre-feet of pumping from each of six M&I wells. 27
Figure 10: GFLOW results showing average pumping lift as a function of the agricultural pumping rate and canal diversion rate. 28
Figure 11: GFLOW results showing average pumping lift as a function of the M&I pumping rate and canal diversion rate. 28
Figure 12: GFLOW results showing average canal seepage as a function of the agricultural pumping rate and canal diversion rate. 29
Figure 13: GFLOW results showing average canal seepage as a function of the M&I pumping rate and canal diversion rate. 29
Figure 14: GFLOW results showing average drain return as a function of the agricultural pumping rate and canal diversion rate. 30
Figure 15: GFLOW results showing average drain return as a function of the M&I pumping rate and canal diversion rate. 30
Figure 16: Fitted and modeled results showing average pumping lift as a function of total M&I pumping and canal diversion. 33
Figure 17: Fitted and modeled results showing average pumping lift as a function of total agricultural pumping and canal diversion. 33
Figure 18: Fitted and modeled results showing average canal seepage as a function of total M&I pumping and canal diversion. 34
Figure 19: Fitted and modeled results showing average canal seepage as a function of total agricultural pumping and canal diversion. 34
Figure 20: Fitted and modeled results showing average drain return as a function of total M&I pumping and canal diversion. 35
Figure 21: Fitted and modeled results showing average drain return as a function of total agricultural pumping and canal diversion. 35
Figure 22: Shifting supply functions for M&I pumper due to changes in canal diversion. 37

Figure 23**:** Shifting supply functions for the agricultural pumper due to changes in canal diversion. 37
Figure 24: Shift in supply functions for the drain water irrigator due to canal diversions and M&I pumping. 39
Figure 25: Shift in supply functions for the drain water irrigator due to canal diversion and agricultural pumping. 39
Figure 26: Increasing canal transportation costs due to increased M&I pumping. 41
Figure 27: Increasing canal transportation costs due to increased agricultural pumping. 41
Figure 28: Scaled demand function for canal diverters in the Boise Project sub-area. 45
Figure 29: Scaled demand function for agricultural pumpers in the Boise Project sub-area. 46
Figure 30: Scaled demand function for drain water irrigators in the Boise Project sub-area. 47
Figure 31: Scaled demand function for M&I pumper's in the Boise Project sub-area. 49
Figure 32: Meade and Cheung components of total canal seepage from model results that include agricultural pumpers. 52
Figure 33: Canal diverters' equilibrium supply and demand (with agricultural pumpers). 58
Figure 34: Agricultural pumpers' equilibrium supply and demand. 58
Figure 35: Drain water irrigators' equilibrium supply and demand (with agricultural pumpers). 59

Tables

Table 1: Irrigation Water Demand Elasticities. 44
Table 2: M&I water demand studies that used marginal prices.......... 48
Table 3: Spatial water allocation model results for canal diverters, drain users and agricultural pumpers. 53
Table 4: Spatial water allocation model results for canal diverters, drain users and M&I pumpers. 55

Executive Summary

Spatial water allocation model development involves linking spatial elements of a conjunctive hydrologic model with those of a partial equilibrium economic model. The hydrologic model describes the spatial distribution of surface water and groundwater interactions. The partial equilibrium economic model simulates distributions of surface water and groundwater among suppliers and demanders which maximize the economic utility of water use (as measured by the sum of consumer and producer surpluses). The coupled model can be used to evaluate both hydrologic and economic outcomes of various water management alternatives such as the construction of new water storage, new water conservation measures, and/or market-based water management.

Spatial water allocation modeling can be conducted on a basin-wide scale, however this report describes the development and application of a prototype model for a comparatively small portion of the Boise Project, in the Lower Boise Valley. The model includes only those canal diverters, groundwater pumpers and drain water irrigators within a 12 mile square sub-area of the Project, centered on an eight-mile section of the New York Canal, just east of Lake Lowell. The model area includes portions of the New York Canal, the Nampa Meridian Irrigation District (NMID), and the Elijah, Aaron and Wilson drains. It also includes 149 agricultural wells and six M&I wells that are part of the City of Nampa's municipal supply system.

The hydrologic model of the Boise Project sub-area describes the spatial distribution of surface water and groundwater interactions resulting from canal seepage, groundwater pumping, and drain return. The partial equilibrium economic model describes the same interactions as trades between spatially distributed water suppliers and demanders.

The process of coupling hydrologic and economic models is best explained in terms of partial equilibrium economic model inputs and outputs. Three exogenous data inputs are required by the partial equilibrium economic model: (1) water supply-price functions (2) water demand-price functions, and (3) transportation or conveyance costs.

The conjunctive hydrologic model provides water supply functions and transportation costs based on hydrologic response data. Hydrologic response functions describe canal seepage, drain return, and groundwater level responses to varying amounts of groundwater pumping and canal diversion. Response functions are converted to supply functions and transportation costs by incorporating unit cost-of-supply data.

Exogenous water demand functions are developed separately, by fitting prices to current agricultural and municipal and industrial (M&I) water demands. Demand function elasticities, which express the unit change in quantity demanded for a unit change in price, are derived from recent agricultural and M&I water use surveys.

Endogenous outputs from the partial equilibrium economic model include quantities of water supplied to and demanded by canal diverters, groundwater pumpers, and drain water irrigators along with equilibrium supply and demand prices. Equilibrium water prices are made up of several price components including supply cost, transportation cost, and opportunity cost.

The calculation of equilibrium prices is determined in part by site-specific hydrologic conditions, including the proximity of wells to drains and canals, and the connectivity of canals to the underlying aquifer. Equilibrium prices are also influenced by the existence of hydrologic externalities (un-priced economic impacts to third-parties) resulting from the influence of canal seepage on groundwater levels and drain returns.

Three water management scenarios were developed for the prototype spatial water allocation model. The scenarios are aimed at describing the hydrologic and economic impacts of water conservation measures and/or market-based water management approaches which would either eliminate or internalize the externalities that result from Boise Project canal seepage.

Scenario 1 is a base-case simulation of current conditions, including existing externalities resulting from canal seepage. The results of the base case model scenario indicate that canal seepage is an important factor in limiting the marginal supply price of water for groundwater pumpers and drain water irrigators in the Boise Project.

Scenario 2 simulates internalizing these externalities through pricing. Requiring groundwater pumpers to reimburse canal diverters for the impacts of groundwater pumping on canal seepage increases the marginal price of groundwater and reduces groundwater demand. On the other hand, canal diverters who are compensated for some of the transportation costs associated with canal seepage experience a reduced marginal price and increased water demand. Drain water irrigators benefit from the pumpers reimbursement because the impacts of groundwater pumping on drain return are reduced, resulting in increased drain water supply.

Scenario 3 simulates elimination of these externalities through canal lining. Lining the canal eliminates canal seepage entirely and therefore all of the canal diverters’ transportation costs associated with seepage. The result is a significant increase in the canal diverters’ demand. Lining the canal also increases the marginal supply price of groundwater pumpers and therefore greatly reduces demand for groundwater. Canal lining has an even greater impact on the supply price of drain water irrigators, because canal seepage is the primary source of drain water supply.

Spatial water allocation model results are also used to calculate valuations of water by canal diverters, groundwater pumpers and drain water irrigators. Water valuations are calculated in terms of average supply cost and in terms of marginal demand price. For canal diverters, the supply-cost valuation of an acre-foot of water at the head of the canal (which includes seepage) is fixed by irrigation district O&M and project repayment costs at $13.27 per acre-foot. For agricultural pumpers on the other hand, the supply-cost value of an acre-foot of water depends on pumping rate and on how canal seepage affects pumping lift. Model results indicate that based on average pumping costs, an acre-foot of canal seepage is valued by agricultural pumpers at $10.95 per acre-foot of groundwater pumped. For drain water irrigators the supply-cost value of canal seepage is $9.46 per acre-foot of drain diversion (as long as there is water in the drain). In terms of marginal demand price, canal water has a value of $89.85 per acre-foot to canal diverters (if delivered to the end of the canal). Groundwater has a value of $35.92 per acre foot to agricultural pumpers, and drain return water has a value of $74.71 per acre foot to drain water irrigators. The differences in valuation are due mainly to differences in demand functions, and to the fact that groundwater is a secondary source of supply for most agricultural pumpers in the modeled area.

Model results can also be used to calculate the value of hydrologic externalities. The marginal value of canal seepage to agricultural pumpers in the model area is $1.82 per acre-foot of groundwater pumped. The marginal value of canal seepage to drain water irrigators in the model area is $5.53 per acre-foot of drain water diverted.

Finally, it is important to recognize that the prototype model results presented in this report are site-specific, since they are based on the (site-specific) hydrologic conditions of the Boise Project sub-area. A follow-on model development project is aimed at coupling the partial equilibrium economic model with a basin-wide (Lower Boise Valley) hydrologic model.

Introduction

Surface water and groundwater hydrologic models have long been used by Reclamation water managers as tools for decision making. However integrated hydrologic and economic models, in particular models which can be used to assess the economic value or utility of water use are relatively new to Reclamation.

This report describes the development and application of a prototype *spatial water allocation model* for a portion of the Boise Project, in the Lower Boise Valley. Spatial water allocation model development involves linking spatial elements of a conjunctive hydrologic model with those of a partial equilibrium economic model. The hydrologic model describes the spatial distribution of surface water and groundwater interactions resulting from canal seepage, groundwater pumping, and drain return. The partial equilibrium economic model describes the same interactions as trades between spatially distributed water suppliers and demanders.

Spatial water allocation modeling has the potential to provide Reclamation and other water management agencies with a predictive tool for evaluating both the hydrologic and the economic outcomes of various strategies for meeting growing water needs in the Boise Valley, including, but not limited to, new reservoir storage, new water conservation measures, and market-based water management.

The integration of a conjunctive hydrologic model with a partial equilibrium economic model takes place via the development of water supply-price functions. A partial equilibrium economic model maximizes the utility of an economic commodity such as water by determining the price-quantity equilibrium positions of water suppliers and demanders, based on exogenously determined supply functions, demand functions, and transportation costs. However, since the spatial distribution of surface water and groundwater supply and demand, and the costs associated with moving water from suppliers to demanders are hydrologically determined, a separate conjunctive hydrologic model is needed to generate the exogenous water supply functions.

Economic externalities are defined as un-priced economic impacts to third-parties not directly involved in an economic activity or transaction. Externalities associated with irrigation activities are referred to in this report as hydrologic externalities. Externalities can be positive or negative in the sense that the un-priced impacts can be either beneficial or detrimental to the third-parties. Positive hydrologic externalities resulting from irrigation activities can include expanded groundwater resources and increased flows in drains. Negative hydrologic externalities can include increased canal seepage and reduced drain return due to increased groundwater pumping. Canal lining or other water conservation measures can create new hydrologic externalities by increasing pumping lifts and reducing drain returns. Hydrologic externalities can also be internalized through water pricing.

This report on spatial water allocation modeling is focused mainly on describing a methodology for coupling conjunctive hydrologic and partial equilibrium economic models, and on representing hydrologic externalities in these models. However, in addition, the results of three model scenarios are presented. The model results, which are preliminary and site-specific, describe the relative value of existing hydrologic externalities to groundwater pumpers and drain water irrigators in the Boise Project. The model results also describe possible hydrologic and economic outcomes associated with either eliminating or internalizing these hydrologic externalities.

Supply Management and Demand Management

Traditional approaches to water management are classified as either supply management or demand management. Supply management approaches focus on increasing water supply to meet new demands, such as by building new reservoirs. Demand management approaches concentrate on limiting demand for water through conservation, or by regulation based on prior appropriation.

Historically, supply management has been the perspective followed by Reclamation in the construction of new reservoir storage. Reclamation first assesses a requirement for water (say 3 AF per acre for 10,000 acres of an irrigation project) and then seeks to recover the cost of the Project in the repayment plan. The economic value of water to the end user is not considered in the cost recovery. The only costs considered in the repayment are those associated with storing water and delivering it to the end user.

Market-Based Management

Balancing supply and demand by water trading or water allocation is termed market-based water management. Market-based water management provides a mechanism for inter-regional and inter-sector water trading between suppliers and demanders. A market enables the holders of water rights to transfer water to other users willing to pay for it. Potential buyers may include M&I water users, agricultural users, or environmental programs.

Most water trades and transfers occur between users within a single economic sector however. This precludes changes in the form, place, and timing of water use, which would adversely affect third-party water users. Water transfers between economic sectors are more likely in situations where the transfer can occur without major changes in form, place, and timing of use.

In economics, market failure is a term which describes the condition wherein the allocation of goods and services by a market is not efficient. Market failures can result from limited competition for water, high transaction costs associated with water trades, inadequate information about markets among water users, or water allocation strategies that fail to consider impacts to third-party water users (Bator, 1958).

Supply Functions and Demand Functions

Water supply functions describe the relationship between the quantity of water supplied and the marginal supply price of water. Water demand functions describe the relationship between the quantity of water demanded and the marginal demand price for water. Price elasticity expresses the unit change in quantity supplied or demanded for a unit change in price.

Supply management and demand management approaches incorporate different assumptions about the price elasticity of water demand functions. Traditional evaluation of a water supply project from the supply perspective assumes that demand is fixed, i.e. that demand is completely inelastic and therefore not responsive to price. Water demand is therefore a requirement that must be met by increasing the supply of water. In effect, there is a fixed amount of water demanded and that water has infinite value. In this case, the price-quantity relationship in the water users demand function appears as a vertical line (i.e. it is a function with infinite slope).

Economic evaluation from the demand perspective assumes the exact opposite; supply is fixed (i.e. quantity is limited). It follows that demand management strategies require water use to be regulated or curtailed, or that water be conserved. Water demand is assumed to be price responsive. Water pricing can be used to reduce the quantity of water demanded so as not to exceed the limited quantity available. From the demand management perspective, the price-quantity relationship in the water users demand function is not a vertical line, instead it is a downward sloping function which reflects the water user's willingness to pay for increasing quantities of water.

Figure 1 illustrates the price-quantity relationships underlying water user demand functions from supply management and demand management perspectives. From left to right, a) is a completely inelastic demand function representative of the traditional supply management approach; b) is a linearly elastic demand function with price-quantity response that varies with the quantity of water demanded; and c) is a constant elasticity demand function with price responsiveness that is constant regardless of the quantity demanded. The last two functions are suggestive of a market-based demand management approach.

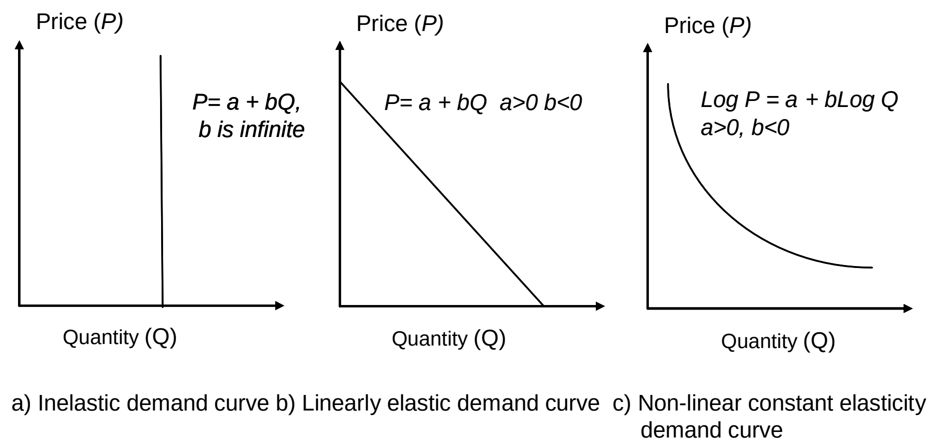


Figure 1: Examples of inelastic and elastic demand functions.

Third-Party Impacts and Hydrologic Externalities

In economics, jointness of production occurs when the economic activity of one party impacts the production possibilities of another. When these impacts are also un-priced, they are referred to as economic externalities, or simply externalities. The term externality is often used in reference to adverse environmental consequences of economic activity that are borne by society as a whole, such as deforestation or water pollution. However not all externalities are negative. Externalities can be positive in the sense that the un-priced influence of production by one party is beneficial to third-parties.

Economic externalities that are associated with groundwater and surface water interactions are referred to in this report as hydrologic externalities. Hydrologic externalities resulting from irrigation activities (both positive and negative) often occur as a result of unanticipated hydrologic interactions between surface water and groundwater systems. Some examples of hydrologic interactions that can lead to externalities include:

- raised groundwater levels due to farm infiltration and canal seepage
- increased drain returns that result from raised groundwater levels
- reduced flow in drains and increased canal seepage due to new groundwater pumping
- reduced drain returns and groundwater levels due to canal lining

Hydrologic externalities can have one-way or two-way cause and effect consequences. A hydrologic externality with one-way cause and effect results from canal seepage when the water table is hydrologically unconnected to the canal. Under these conditions, canal diverters create a positive externality for groundwater pumpers through canal seepage. However since there is no direct hydraulic connection between the canal and the water table surface, water table drawdown induced by groundwater pumping does not induce additional canal seepage, and thereby create a negative externality for canal diverters.

A hydrologic externality with two-way cause and effect results from canal seepage when the water table surface is in direct contact with the canal. Under these conditions, canal seepage creates a positive externality for groundwater pumpers, and groundwater pumpers, by creating a steeper hydraulic gradient beneath the canal, induce additional seepage from the canal, thereby creating a negative externality for canal diverters.

Externalities that have a one-way cause and affect are known to economists as Meade externalities, while those having a two-way cause and affect are known as Cheung externalities (Meade, 1952; Cheung, 1970).

Other examples of one-way and two-way hydrologic externalities arise in connection with drain water irrigators. Drain users who rely on groundwater discharge to a drain require a direct hydraulic connection between the drain and the underlying water table surface. Groundwater pumping can create a negative one-way (Meade) externality for drain users by drawing down the water table surface and reducing drain discharge. Deepening or widening a drain to prevent high water table conditions can create a two-way (Cheung) externality that is negative for canal diverters because it induces additional canal seepage, but positive for drain users because it increases flow of groundwater into the drain.

Canal diverters are not required to "waste" water to sustain the externality that provides water for drain return users and/or groundwater pumpers. Unlike other kinds of property, water rights do not convey ownership, but rather a highly regulated right to use and sell water. In the preceding examples, groundwater pumping, by lowering the water table, increases canal seepage and reduces drain returns, thereby creating negative externalities for both canal diverters and drain users. If this causes the canal operator to line the canal, the negative externality is removed but both positive externalities are also eliminated, perhaps rendering the ground water and drain water rights unusable.

Finally, water management strategies that do not account for all of the costs and benefits associated with water use often invoke the "public goods" aspect of water supply. Pure public goods are non-excludable and non-rival, which means that no one can be effectively excluded from consuming them, and consumption by one individual does not reduce the quantity available to others. However irrigation water is very often excludable and rival. In the previous example, canal lining increases the supply of water to canal diverters, but prevents consumptive use by groundwater pumpers and drain water irrigators.

Boise Project Hydrologic Externalities

Irrigation activity in Lower Boise Valley has greatly enhanced and enlarged the underlying aquifer system which functions as both water storage and water distribution system for agricultural and M&I water users. Infiltration from Reclamation's Boise Project canals, laterals and reservoirs in the early part of the 20th century boosted aquifer recharge and drain returns throughout much of the Lower Boise Valley, raising groundwater levels in places south of the Boise River by as much as 140 feet (Nace, 1957), and in many other areas by between 20 and 50 feet (USGS, 2008). Much of the increased drain return flow that resulted was re-diverted for additional irrigation use. In the 1950s and 1960s, the abundant aquifer storage encouraged the development of deep well irrigation.

The impact of the Boise Project on the groundwater hydrology of the Lower Boise Valley is conveyed in a recent water budget report (USBR and IDWR, 2008). Figure 2 shows average annual rates of on-farm infiltration, canal seepage, and drain return, in relation to groundwater pumping and base flow (sub-surface gains) to rivers in the Lower Boise Valley. In an average water year, over a million acre-feet of irrigation water seeps into the underlying aquifer as a result of on-farm infiltration and canal seepage. Of this, more than 600 thousand acre-feet is subsequently discharged to drains. About 165 thousand acre-feet of this drain discharge is then re-diverted by drain water irrigators. In addition, more than 100 thousand acre-feet of groundwater is pumped from shallow and deep aquifers for irrigation or M&I uses. In all, over 600 thousand acre-feet of groundwater that originates as canal seepage or on-farm infiltration is discharged to rivers annually (mainly to the Boise River), much of it during winter months. From the standpoint of Boise Valley water users, the hydrologic impacts of the Boise Project are mostly positive. They are also mostly un-priced.

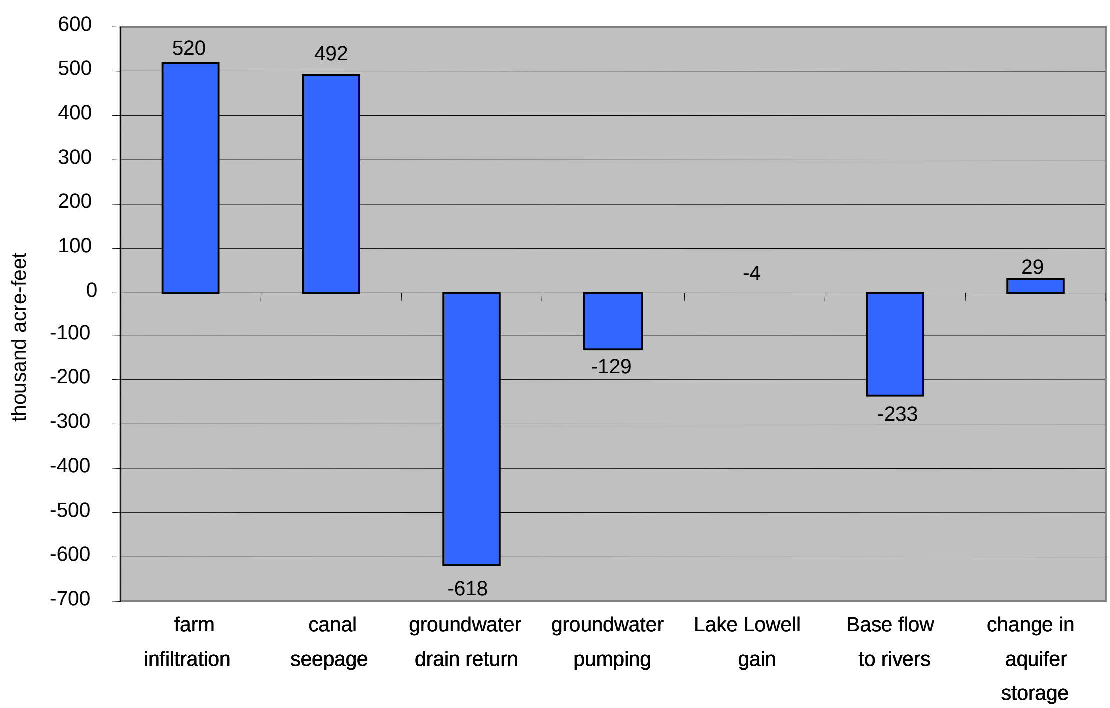


Figure 2: Average annual hydrologic impacts of irrigation activity in the Lower Boise Valley.

Changes in hydrologic externalities are frequently associated with changes in the form, place, and timing of water demand. Changes in land use in the Boise Valley will likely lead to changes in hydrologic externalities that have been fixtures of the Boise Project for many decades. In the 1980s and 1990s traditional agricultural landscapes in the Boise Valley began to be converted to residential uses. Since then, the rate of land use change has increased rapidly. Between 2000 and 2025 irrigated agricultural land in the Boise Valley is expected to be reduced by about 23,000 acres as a result of urbanization. An additional 17,500 acres of irrigated farm land is expected to be gone by 2050 (IDWR, 2000). Increased M&I groundwater withdrawals have accompanied many of these land use changes. In 2000, groundwater pumping for M&I use in the Boise Valley totaled about 110 thousand acre-feet. By 2025, M&I demand is expected to grow to about 204 thousand acre-feet, and by 2050 to about 338 thousand acre-feet (IDWR, 2000).

Spatial Water Allocation Modeling

A *spatial water allocation model* can be used to describe allocations of surface water and groundwater based on its hydrologic distribution, and on the economics of supply and demand. Spatial water allocation modeling provides a means of evaluating the hydrologic and economic outcomes of various water management alternatives, including but not limited to, new water storage facilities, new water conservation measures, and/or market-based water management.

Spatial water allocation modeling is also a means for understanding water valuations. Understanding water valuations is particularly important for gauging the economic impacts of water management alternatives which would alter hydrologic externalities resulting from Boise Project operations.

The process of spatial water allocation modeling is one of coupling a conjunctive hydrologic model with a partial equilibrium economic model. The conjunctive hydrologic model describes the distribution of water between surface water and groundwater users, hydrologically. The partial equilibrium economic model describes the same water exchanges in economic terms, as trades between water suppliers and demanders. Model coupling occurs via the development of site-specific water supply functions, which are based on hydrologic model response functions.

The coupled model simulates a distribution of surface water and groundwater among suppliers and demanders which maximizes the economic utility of water use, as measured by the sum of consumer and producer surpluses. Model outputs include equilibrium quantities of water supplied and demanded in water exchanges, along with equilibrium supply and demand prices.

Conjunctive Hydrologic Modeling using GFLOW

Conjunctive hydrologic models which simulate surface water and groundwater interactions may use either direct analytical solutions or grid-based numerical methods to represent hydrologic features such as canals, rivers, drains and wells.

An analytical modeling method that has been widely applied to regional modeling problems involving groundwater interactions with rivers, reservoirs, canals, drains and wells is known as analytic elements (Strack, 1989; Haitjema, 1995). GFLOW (USEPA, 2007) is a widely used analytic element modeling software package which solves the governing differential equation for steady-state, two-dimensional groundwater flow. With the GFLOW package, a boundary condition (either a fixed elevation head or a fixed flow rate) is associated with each canal, drain and well boundary in the model. Boundary conditions with a fixed elevation head are referred to as head-specified, and those with a fixed flow are referred to as flow-specified. Typically, head-specified boundary conditions are used to represent canals and drains, whereas pumping wells are represented by flow-specified boundary conditions.

Regardless of which type of boundary condition is used, the GFLOW package provides the solution for the unspecified condition, referred to as the hydrologic response. Thus for head-specified canal or drain boundaries, the hydrologic response is the canal seepage rate or drain return rate. For flow-specified well boundaries, the hydrologic response is the aquifer head condition in the well bore, from which pumping lift can be calculated.

Appendix A of this report describes some well known analytic element modeling concepts, including various types of interactions between pumping wells and canals. Of particular interest are canals that are perched above the water table surface as in the condition of a one-way externality, versus canals that are in direct contact with the water table surface as in the condition of a two-way externality.

Partial Equilibrium Economic Modeling using GAMS

Partial equilibrium economic models examine the conditions of market equilibrium that exist when dealing with a single economic commodity, assuming all other economic variables remain constant in value. Applications of partial equilibrium modeling are generally associated with problems of utility maximization, as for example in maximizing the sum of consumer and producer surpluses (Takayama and Judge, 1971).

Partial equilibrium models maximize economic utility by determining the equilibrium position between supply and demand for a commodity, through trading. Equilibrium quantities supplied and demanded and the equilibrium prices differ among trading entities because of differences in willingness to sell or buy the commodity. These differences are reflected in the trading entities (exogenously determined) supply functions and demand functions.

The modeling software package that is used to obtain the equilibrium position in a partial equilibrium modeling problem is GAMS (General Algebraic Modeling System) (GAMS, 2007). The GAMS package solves the utility maximization problem using mixed complementary programming and the method of Lagrange multipliers. With the Lagrange method, certain equality constraints in the maximization problem are replaced by inequality constraints containing multipliers. Collectively, these are referred to as complementary slackness conditions, or the Kuhn-Tucker conditions (Kuhn, Tucker, 1951). The Lagrange multiplers describe the rate at which the output of an economic entity increases or decreases as the availability of a single constrained resource increases or decreases.

Appendix B of this report contains a more detailed discussion of partial equilibrium modeling theory, including complementary slackness Kuhn-Tucker conditions for utility maximization, and there representation in a partial equilibrium model using the GAMS software.

The Boise Project Sub-Area Model

The Boise Project sub-area model is a prototype, intended mainly to demonstrate the spatial water allocation modeling concept, along with a procedure for mathematically coupling a conjunctive hydrologic model with a partial equilibrium economic model.

The sub-area model deals with water exchanges that take place between canal diverters, groundwater pumpers, and drain water irrigators within an area of the Boise Project that is about 12 miles square and is centered on an eight-mile section of the New York Canal just up gradient from Lake Lowell (Figure 3). The model area includes portions of the New York canal, the Nampa Meridian Irrigation District (NMID), and the Elijah, Aaron and Wilson drains. It also includes 149 surrounding agricultural wells, and six M&I wells that are part of the City of Nampa’s municipal supply system.

This area of the Boise Project was chosen for a prototype model because it features the kinds of hydrologic interactions and water exchanges between surface water and groundwater users (including both Meade and Cheung externalities) that are common throughout the Lower Boise Valley. In addition, detailed canal seepage data is available for this area.

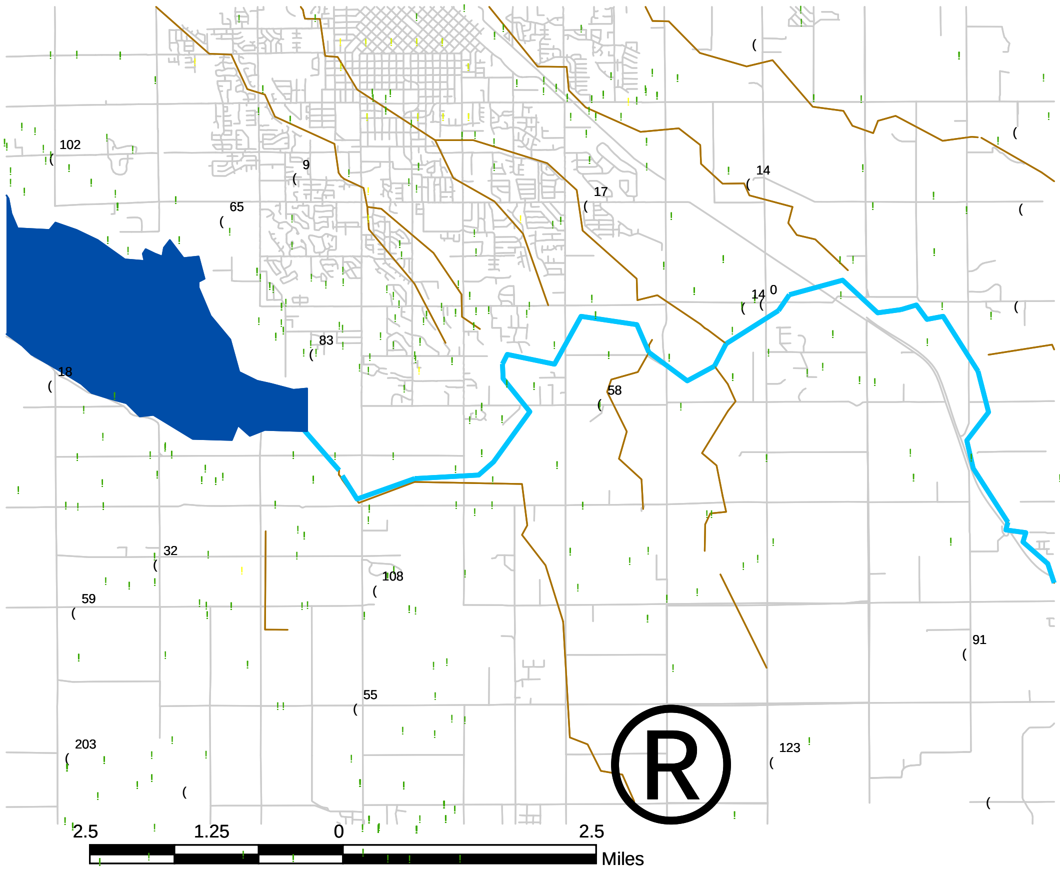


Figure 3: Boise Project sub-area that includes a portion of Lake Lowell, the New York Canal, Elijah, Aaron and Wilson drains, and surrounding agricultural and M&I wells.

The Sub-Area Hydrologic Model

The hydrology of water distribution between surface water and groundwater users in the sub-area is modeled using the GFLOW analytic element modeling package. The sub-area hydrologic model simulates typical irrigation season conditions of canal diversion and groundwater pumping, and the model is calibrated using groundwater level, drain return, and canal seepage conditions that are mid-season averages. Model calibrating parameters include aquifer transmissivity and the bed conductance of the New York Canal.

Figure 4 shows locations along the New York canal where in 1997, 1998, and 2004 the USGS estimated canal seepage losses (USGS, 2004). Over its 39 mile length (which includes a section of Indian Creek) the New York canal seeps between 26,000 and 51,000 acre-feet of water annually into the underlying aquifer. The rate of seepage depends on diversion. Higher seepage rates occur when the canal is full. Seepage rates are lower when diversion is reduced. Almost half of total New York canal seepage losses occur within the model area shown in Figure 3.

The capacity of the New York Canal at the Boise River diversion dam exceeds 800,000 acre-feet per year, however after the Mora Canal and NMID feeder canal diversions, maximum capacity is reduced to about 250,000 acre-feet. Actual irrigation season diversions average about 183,000 acre-feet. Seepage losses from the last eight miles of the canal before it discharges into Lake Lowell represent between 9 and 12 percent of the canal diversion at this point.

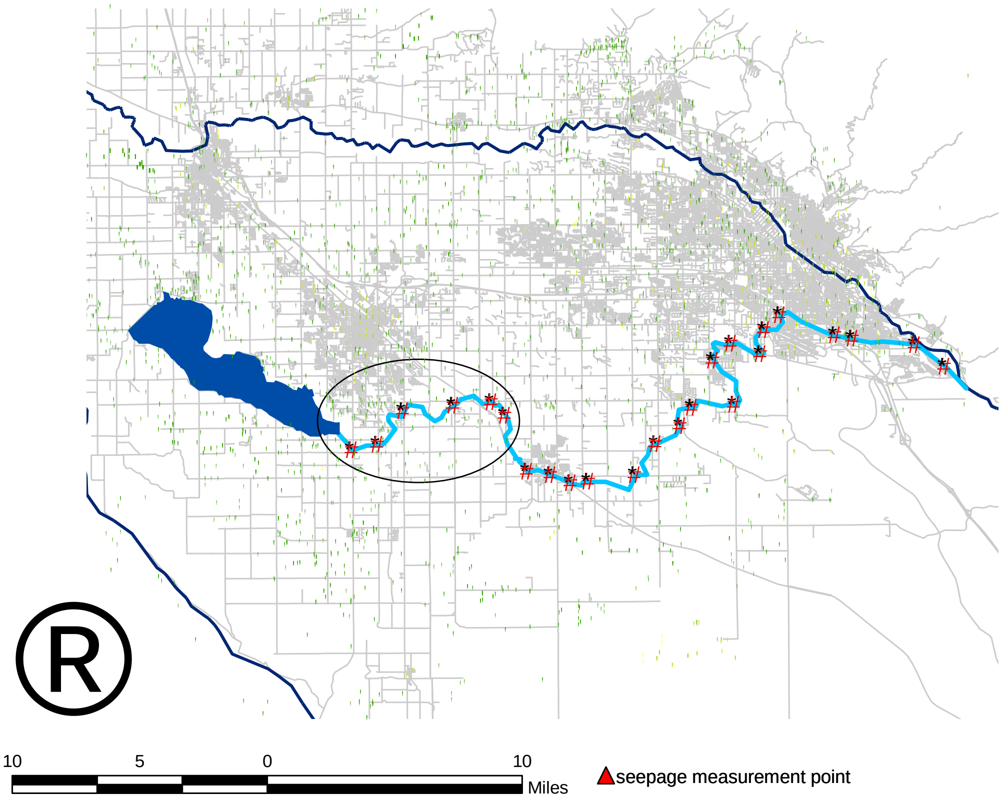


Figure 4: USGS seepage measurement points along the 39 mile New York Canal and 8 mile segment with highest seepage rate.

The distribution of analytic elements in the sub-area hydrologic model is displayed in Figure 5. The model contains 709 head-specified line source-sink elements representing a portion of the New York Canal and portions of the Elijah, Aaron and Wilson Drains. The model also contains 155 flow-specified point-sink elements representing the surrounding agricultural and M&I wells. (Far-field boundary conditions representing Lake Lowell, the Boise River and the Snake River are also included in the model but not shown in this figure.) Elevation heads of canal and drain features are obtained from digital elevation maps. Agricultural and M&I well pumping rates are based on Boise Valley water budget data (USBR and IDWR, 2008).

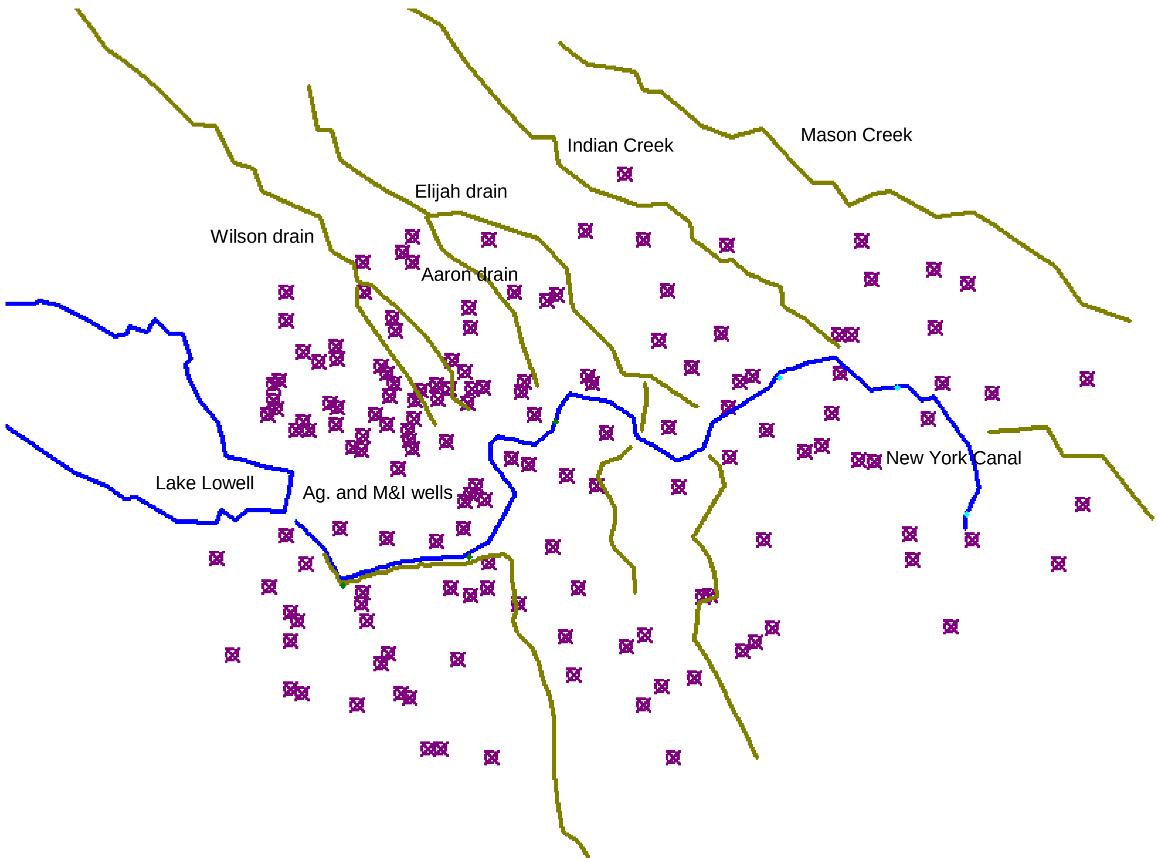


Figure 5: GFLOW model hydrologic boundary features.

The hydrologic response data produced by the GFLOW sub-area model describe the quantity of water that seeps from the New York Canal and then either returns to the Elijah, Aaron and Wilson Drains or is pumped from agricultural and M&I wells. The response data is used to develop supply functions for canal diverters, drain water irrigators and groundwater pumpers.

In order to generate the necessary supply-cost relationships, multiple GFLOW model runs are necessary. Model runs are made with three different canal diversion rates and 36 different groundwater pumping rates (16 with M&I wells and 16 with agricultural wells), for a total of 96 runs.

The canal diversion rates range from a full canal diversion (250,000 acre-feet per year) to no diversion. A full diversion is assumed to be representative of 180 days of canal operation; half diversion is 90 days of operation etc. No diversion is representative of zero days of canal operation or of a lined canal.

The total (irrigation season) agricultural well pumping rate for the 149 agricultural wells located in the sub-area ranges from zero to 9,200 acre-feet per year. The total (year around) M&I well pumping rate for the six Nampa municipal wells ranges from zero to 2,000 acre-feet per year.

GFLOW Model Runs

A sampling of GFLOW model results is shown in Figures 6 through 9. The figures are contour maps showing aquifer head conditions in the sub-area resulting from four of the 96 GFLOW model runs that were made. Figures 6 and 7 include only agricultural pumpers, figures 8 and 9 include only M&I pumpers. In Figures 6 and 8, the canal has a full diversion. In Figures 7 and 9 the canal has an impermeable liner installed to prevent seepage. The difference in head conditions in these figures (the contour interval is five feet) demonstrates that canal seepage plays a major role in sustaining groundwater levels in the presence of both agricultural and M&I pumping.

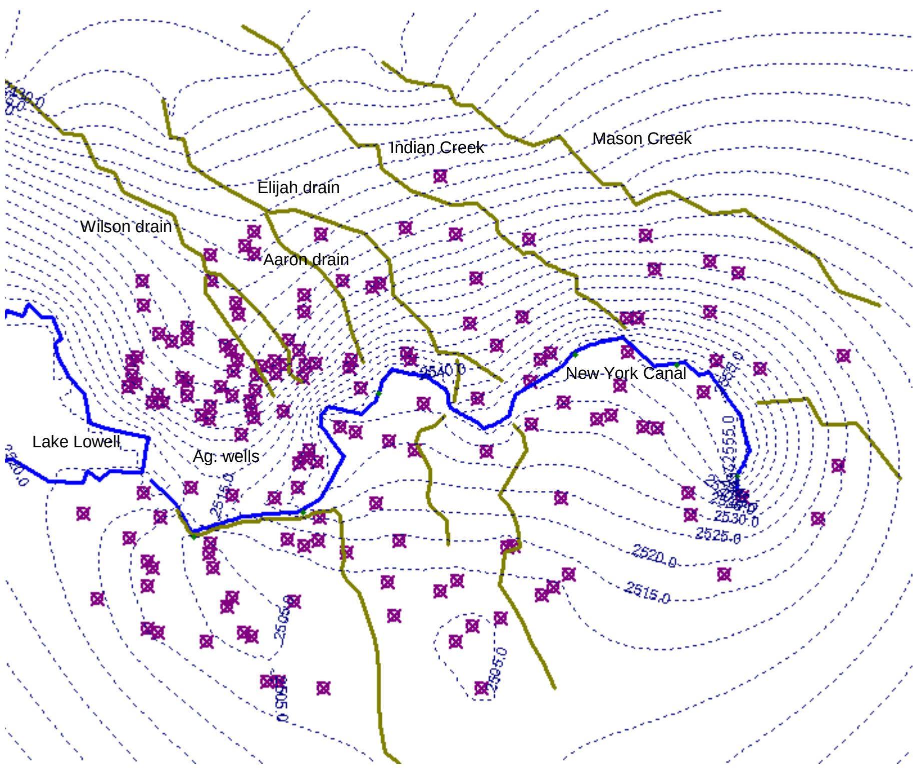


Figure 6: Aquifer head conditions full canal with 54 acre-feet of pumping from each of 149 ag. wells.

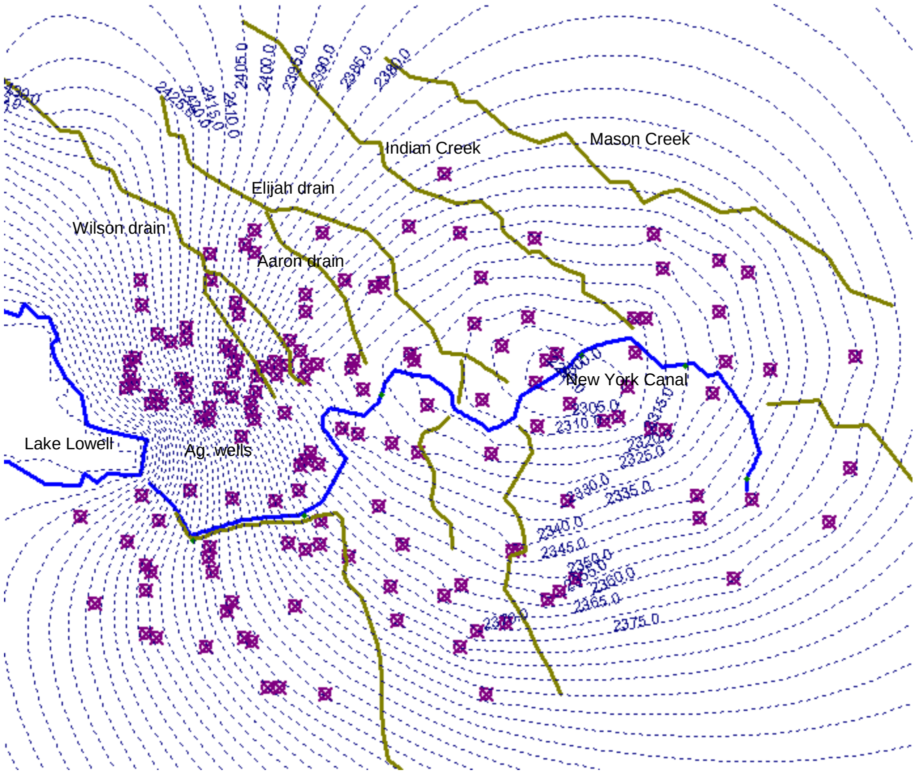


Figure 7: Aquifer head conditions, lined canal with 54 acre-feet of pumping from each of 149 ag. wells.

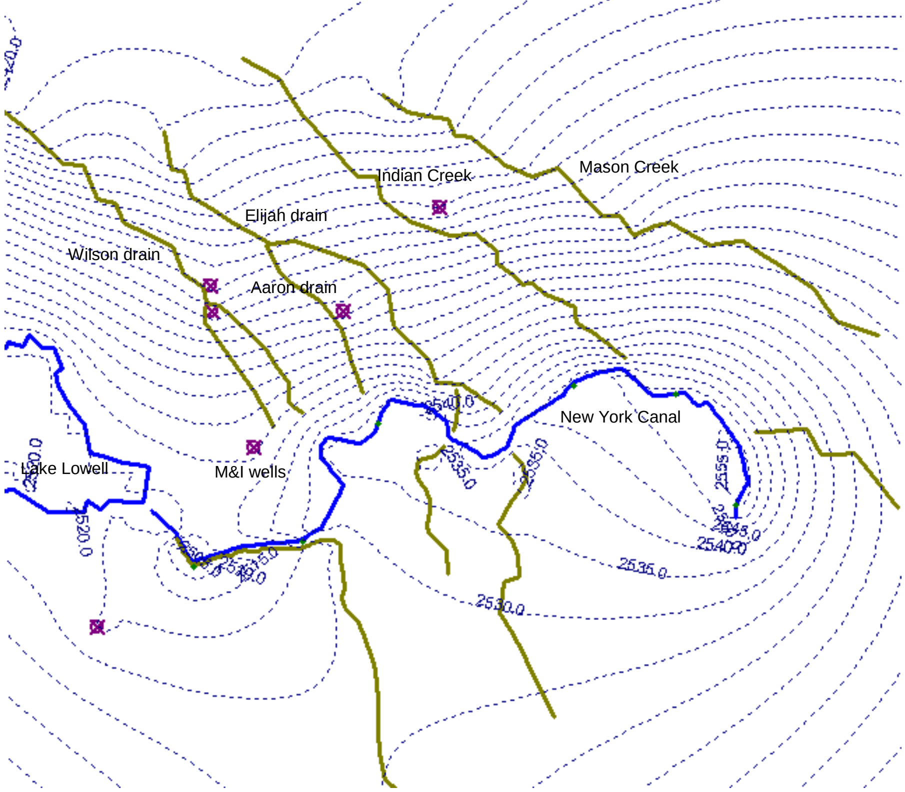


Figure 8: Aquifer head conditions, full canal with 165 acre-feet of pumping from each of six M&I wells.

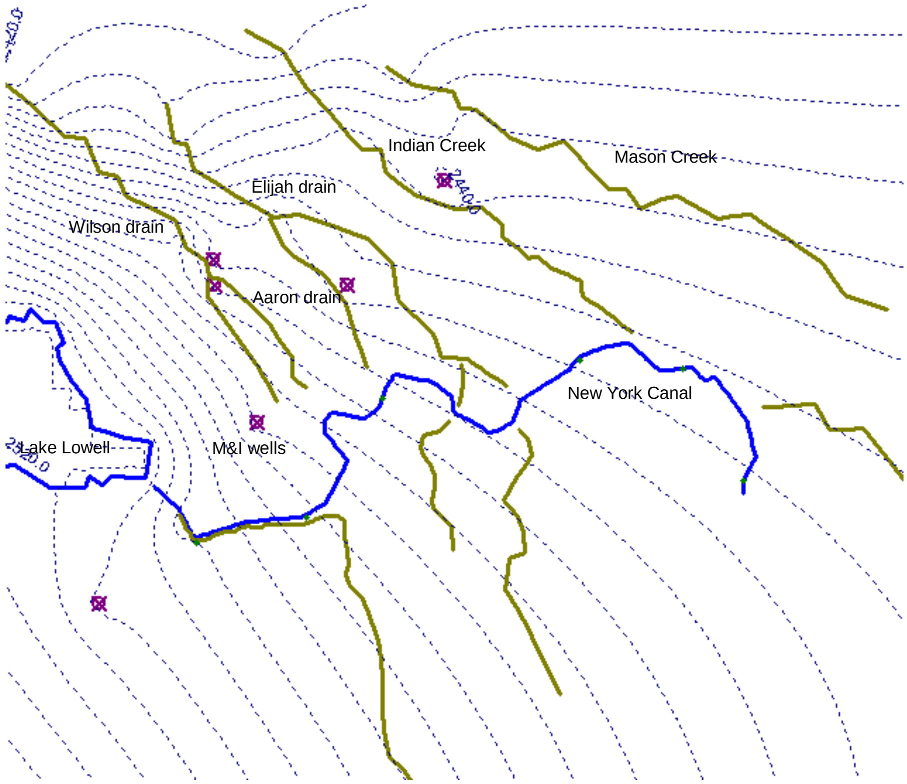


Figure 9: Aquifer head conditions, lined canal with 165 acre-feet of pumping from each of six M&I wells.

The details of how canal seepage affects pumping lifts and drain returns; and how groundwater pumping affects canal seepage and drain returns are best illustrated in chart form. Figures 10 and 11 are GFLOW results showing, respectively, average pumping lift in agricultural wells and M&I wells as a function of well pumping rate and canal diversion. Figures 12 and 13 are GFLOW results showing canal seepage rate as a function of canal diversion and, respectively, agricultural pumping and M&I pumping. Finally, Figures 14 and 15 are results showing drain return as a function of canal diversion and, respectively, agricultural pumping and M&I pumping.

The hydrologic response data in these charts show what would logically be expected in the relationship between these hydrologic variables, i.e. that pumping lift increases as pumping rate increases and canal diversion decreases; that drain return decreases as pumping rate increases and diversion decreases; and that canal seepage increases as pumping rate increases and diversion increases.

In the absence of groundwater pumping, canal seepage and drain return are influenced only by canal diversion, so canal seepage results in figures 12 and 13 are identical, as are drain return results in figures 14 and 15. However differences arise as pumping rates increase. While gradients in pumping lift and canal seepage are steeper in the model runs that include 149 agricultural wells, gradients in drain return are steeper in the model runs that include the six M&I wells. This is because most of the agricultural wells are located near the New York canal, while most of the M&I wells are located further north, closer to the three drains.

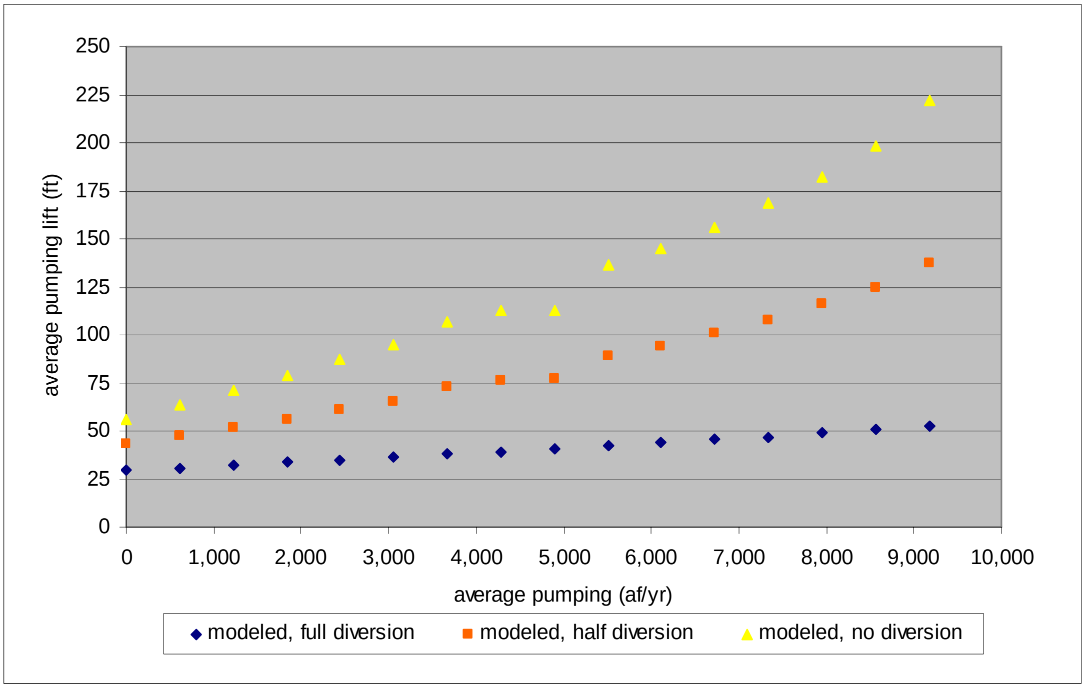


Figure 10: GFLOW results showing average pumping lift as a function of the agricultural pumping rate and canal diversion rate.

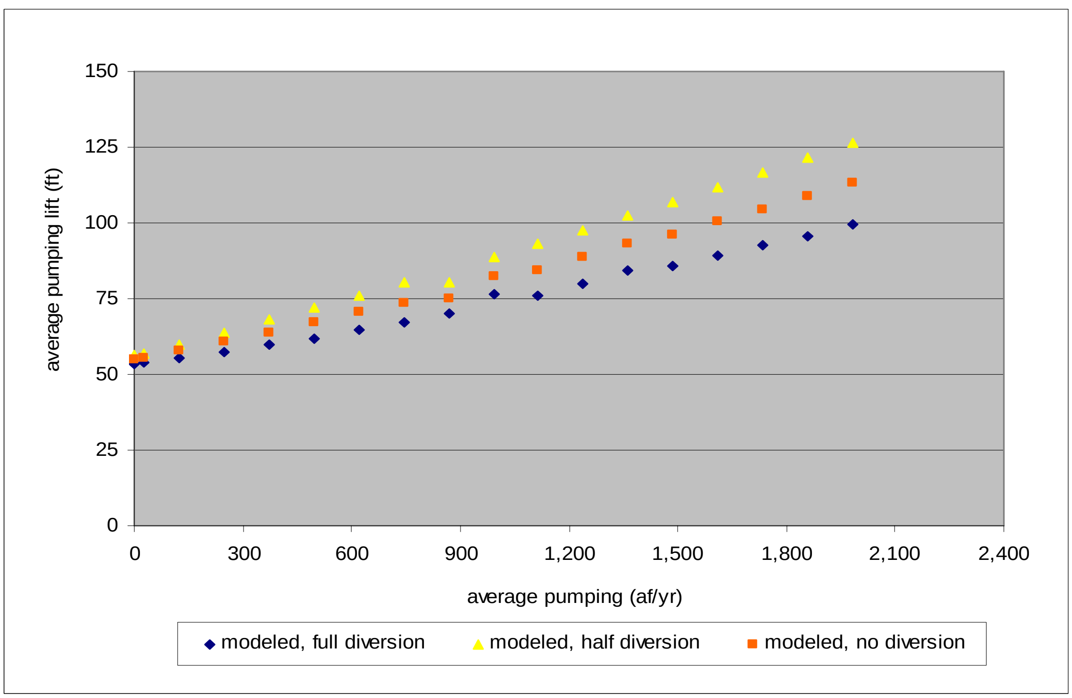


Figure 11: GFLOW results showing average pumping lift as a function of the M&I pumping rate and canal diversion rate.

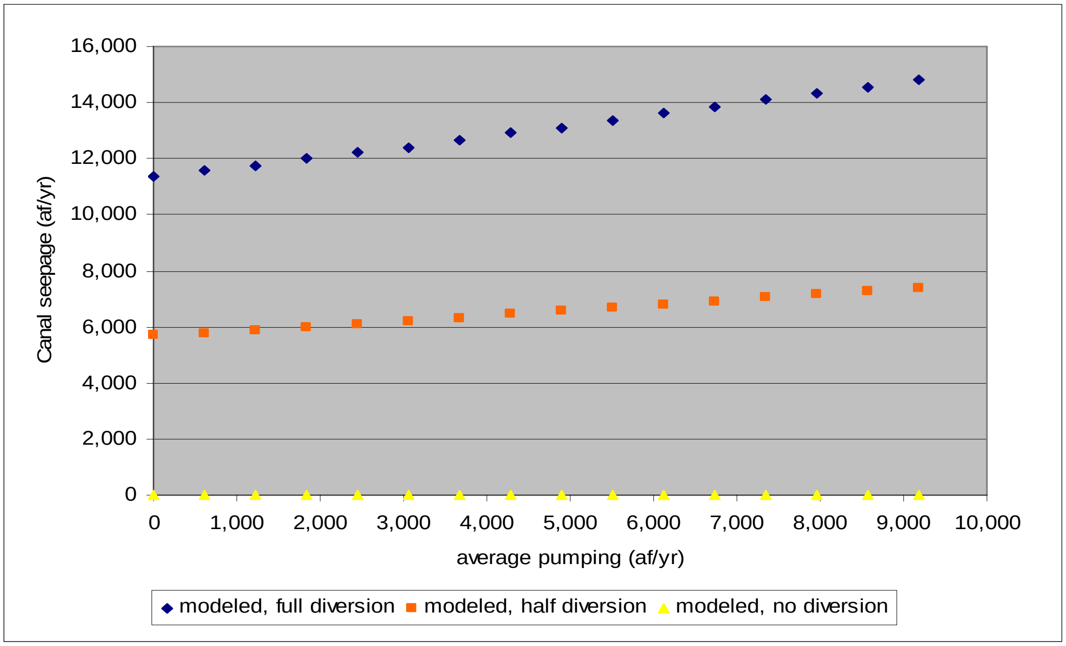


Figure 12: GFLOW results showing average canal seepage as a function of the agricultural pumping rate and canal diversion rate.

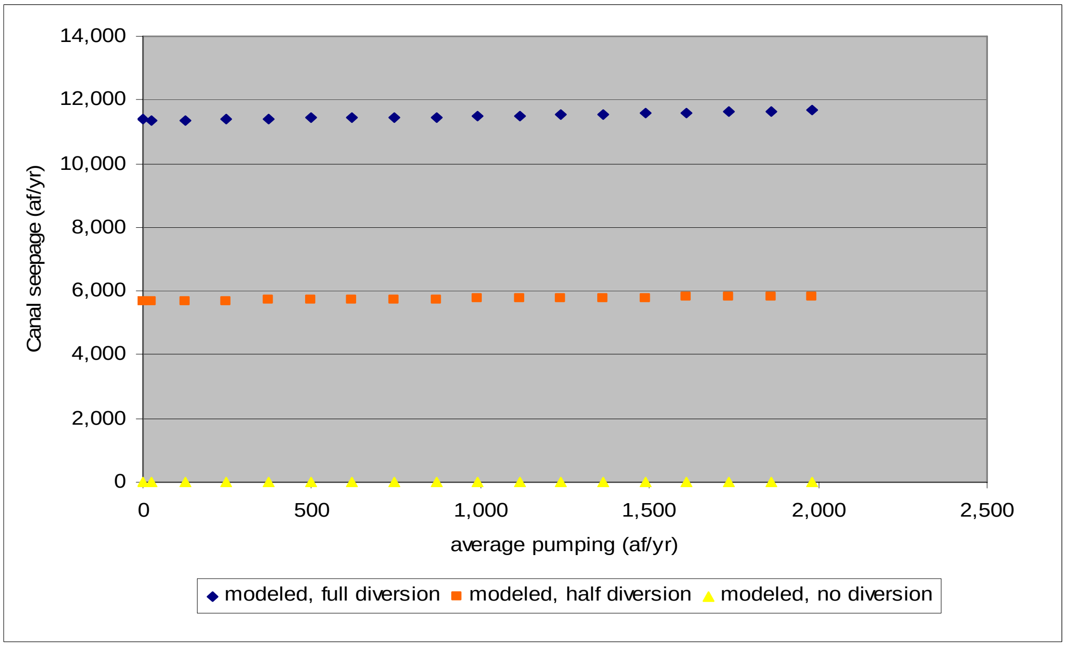


Figure 13: GFLOW results showing average canal seepage as a function of the M&I pumping rate and canal diversion rate.

Figures 14 and 15 also demonstrate that drain return is at a maximum when there is a full canal diversion and no pumping, but if there is no canal diversion there is no drain return, regardless of how much (or how little) pumping is occurring.

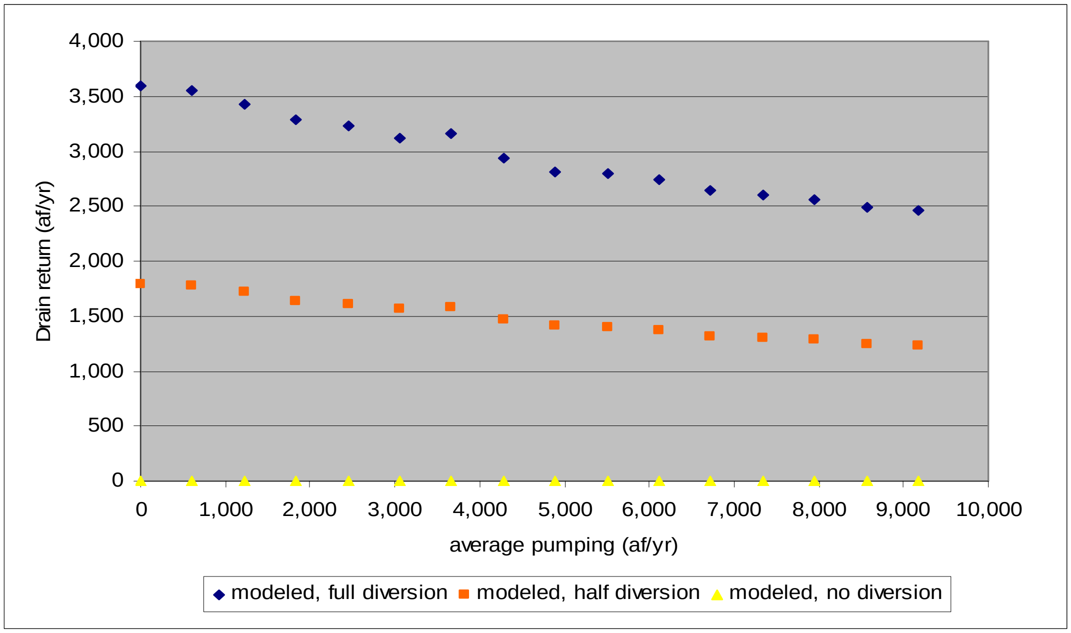


Figure 14: GFLOW results showing average drain return as a function of the agricultural pumping rate and canal diversion rate.

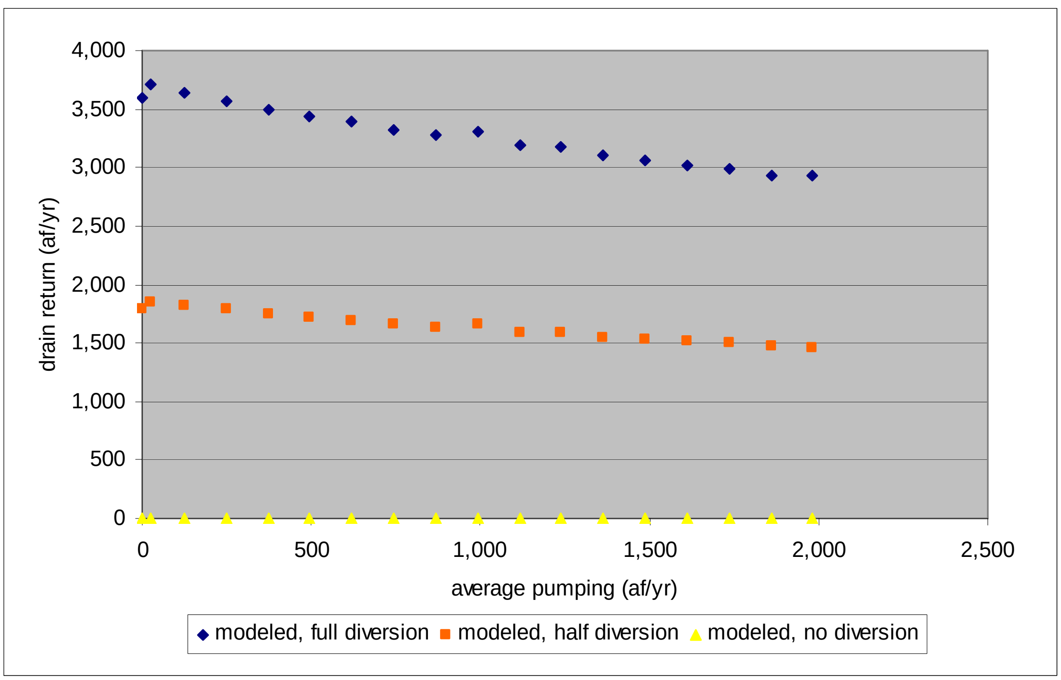


Figure 15: GFLOW results showing average drain return as a function of the M&I pumping rate and canal diversion rate.

Sub-Area Water Supply Functions

Developing supply functions for water suppliers in the Boise Project sub-area involves two steps. The first requires fitting a functional form to the GFLOW response data shown in Figures 10 through 15, in order to create empirical response functions that describe the hydrologic interactions between canal diversions, canal seepage, groundwater pumping and drain returns. The second step involves applying unit-cost data to the response functions in order to create price-quantity relationships for water suppliers.

The GAMS partial equilibrium model requires that supply functions be continuously differentiable. While the analytic element modeling procedure used in GFLOW is based on analytic functions, the functions themselves are quite complex. For this reason, a curve-fitting procedure is used to develop relatively simple empirical response functions. These response functions are in turn used to develop supply-price functions for input to GAMS.

Central to developing supply-price functions is the distinction made between the actions of one-way (Meade) and two-way (Cheung) hydrologic externalities. As described previously, a one-way externality occurs when the canal is perched above the water table surface, and a two-way externality occurs when there is direct contact with the water table surface.

The GFLOW response data indicates under what (diversion and pumping) conditions canals and drains are in direct contact with the water table, and under what conditions they are perched above it. Canal seepage and drain return response functions then reproduce the essential conditions that when canals are perched above the water table, canal seepage is fixed; and when drains are perched, drain return is zero.

Forms of Empirical Response Functions

The forms of empirical response functions used to estimate pumping lift, canal seepage and drain return, based on canal diversion and pumping rate are as follows.

$$\textbf{pumping lift} = (A_0 - A_1 * \textbf{diversion}) * \exp((A_2 - A_3 * \textbf{diversion}) * \textbf{pumping}) \quad \textbf{(1)}$$

$$\textbf{canal seepage} = B_1 * \textbf{diversion} + B_2 * \textbf{diversion} * (1 - \exp(-B_3 * \textbf{pumping})) \quad \textbf{(2)}$$

$$\textbf{drain return} = C_0 + C_1 * \textbf{diversion} + (1 - \exp(-C_2 * \textbf{pumping})) \quad \textbf{(3)}$$

Pumping lift, canal seepage, and drain return are assumed to be linearly related to canal diversion and exponentially related to well pumping rates. In (1), when both diversion and pumping rate are zero, pumping lift = $\mathbf{A_0}$, where $\mathbf{A_0}$ can be thought of as the static depth to water in the aquifer. When pumping is zero but diversion is not, pumping lift = ($\mathbf{A_0}$-$\mathbf{A_1}$*diversion), which is the pumping lift reduced by the influence that canal diversion has on static head. Finally, when diversion is also zero, pumping lift depends only on the static head condition and the pumping rate, i.e. pumping lift = $\mathbf{A_0}$*exp($\mathbf{A_2}$*pumping).

In (2), when pumping is zero, canal seepage depends only on diversion, i.e.
canal seepage = $\mathbf{B_1}$*diversion, and when diversion is zero, canal seepage is also zero. In (3), if pumping is zero then drain return depends only on diversion, i.e.
drain return = $\mathbf{C_0} + \mathbf{C_1}$*diversion. If diversion is also zero then drain return = $\mathbf{C_0}$ a constant (which is also zero if the drain is perched), indicating that (the groundwater component) of drain return depends only on the static aquifer head condition.

Curve-Fitting to Empirical Response Functions

The curve fitting procedure applied to equations 1, 2 and 3, yields response function coefficients that are applicable to a specific hydrologic setting. Changes to the hydrologic setting, for instance the addition of new pumping wells or a new canal would require another application of the curve fitting procedure.

For each of the three response functions, two sets of coefficients are produced. One set pertains to a GFLOW model in which only irrigation wells are present (see figure 7). The other set pertains to a GFLOW model in which only M&I wells are present (see figure 8). Developing separate response functions and separate supply functions for these two groups of pumpers enables GFLOW and GAMS to independently determine the influence that canal seepage and drain users have on these two groups of pumpers.
For M&I well pumping lift, the response function that best fits the GFLOW data is,

$$\textbf{M\&I pumping lift} = (56.0-1.1\times10^{-5} * \text{diversion}) * \exp((.00042-4.0\times10^{-10} * \text{diversion}) * \text{M\&I pumping}) \quad \textbf{(4)}$$

For agricultural well pumping lift, the response function that best fits the GFLOW data is,

$$\textbf{Ag. pumping lift} = (56.0-1.1\times10^{-4} * \text{diversion}) * \exp((.00015-3.5\times10^{-10} * \text{diversion}) * \text{Ag. pumping}). \quad \textbf{(5)}$$

The response function that best fits GFLOW results for canal seepage with M&I well pumping is,

canal seepage = .04545*diversion+.006*diversion*(1-exp(-.0002*M&I pumping)), **(6)**

and the response function that best fits GFLOW results for canal seepage with agricultural pumping is,

canal seepage = .04545*diversion+.06*diversion*(1-exp(-.00003*Ag. pumping)). **(7)**

The response function that best fits GFLOW results for drain returns with M&I pumping is,

drain return = 0.0+.0145*diversion+(1-exp(-.0002*M&I pumping)), **(8)**

and the response function that best fits GFLOW results for drain returns with agricultural pumping is,

drain return = 0.0+.0145*diversion+(1-exp(-.00007*Ag. pumping)). **(9)**

Figures 16 and 17 show the fitted models for, respectively, M&I well pumping lift and agricultural well pumping lift. Figures 18 and 19 show the fitted models for canal seepage with M&I pumping and agricultural pumping, respectively. Figures 20 and 21 show the fitted models for drain return with M&I pumping and drain return with agricultural pumping, respectively.

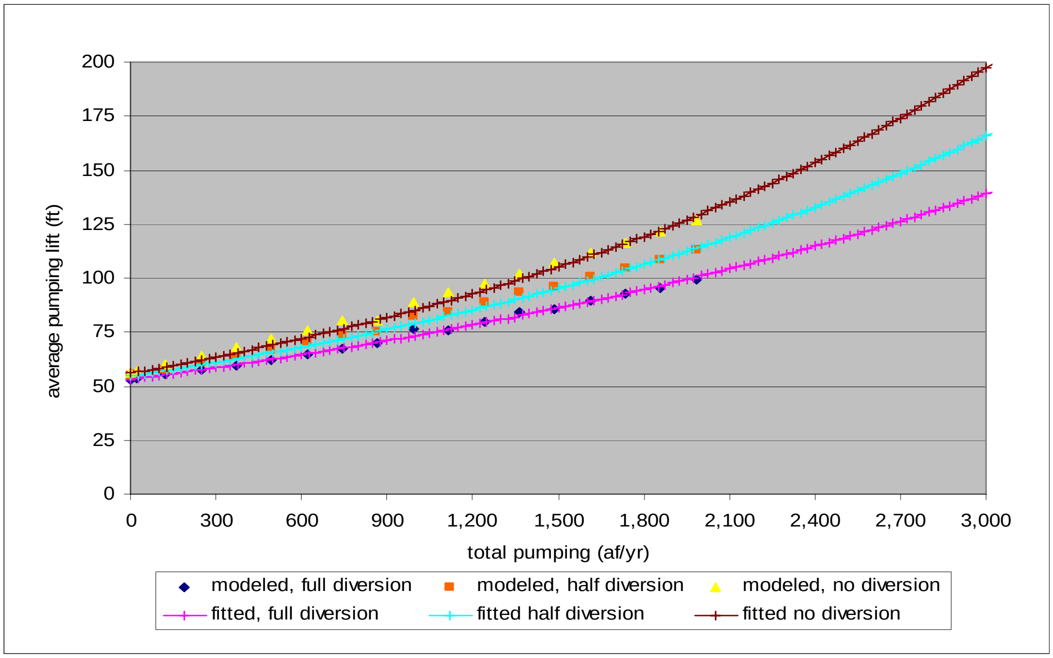


Figure 16: Fitted and modeled results showing average pumping lift as a function of total M&I pumping and canal diversion.

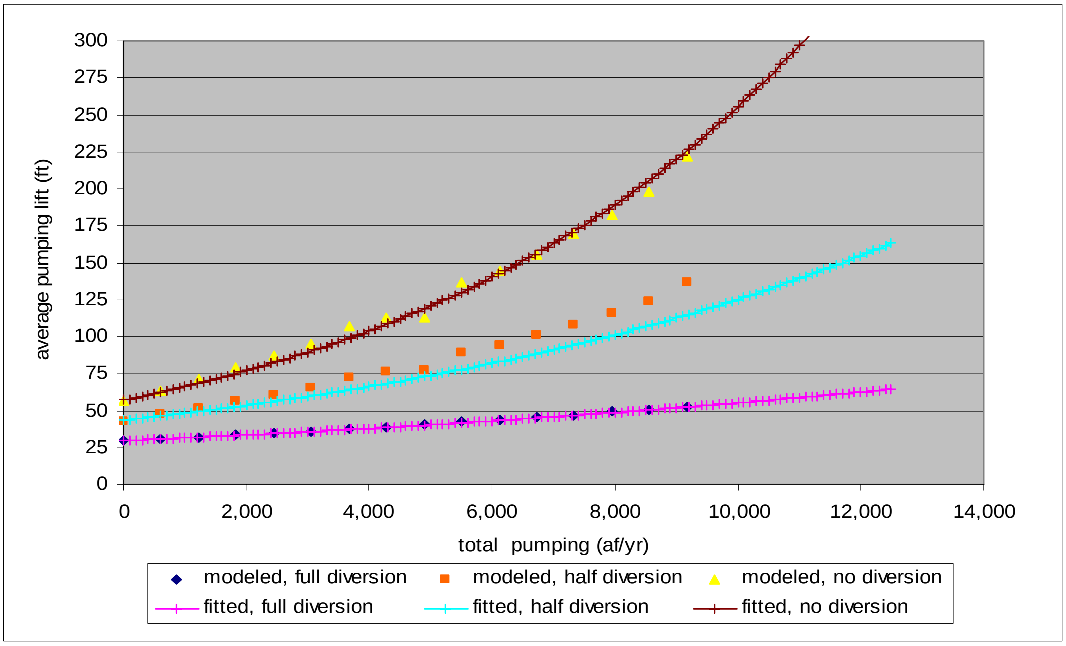


Figure 17: Fitted and modeled results showing average pumping lift as a function of total agricultural pumping and canal diversion.

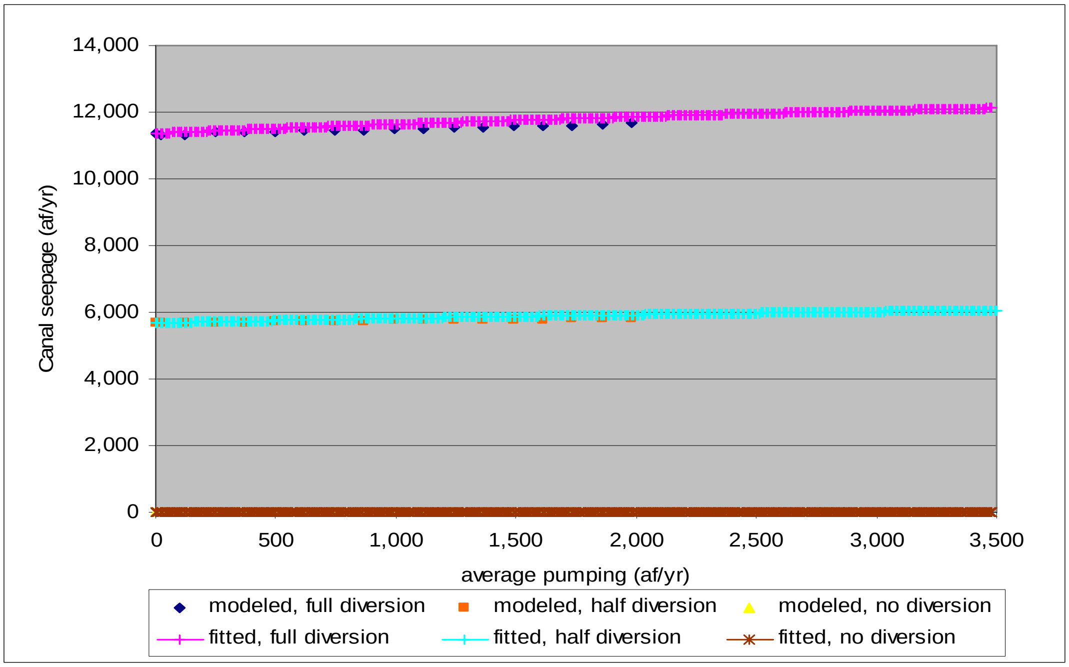


Figure 18: Fitted and modeled results showing average canal seepage as a function of total M&I pumping and canal diversion.

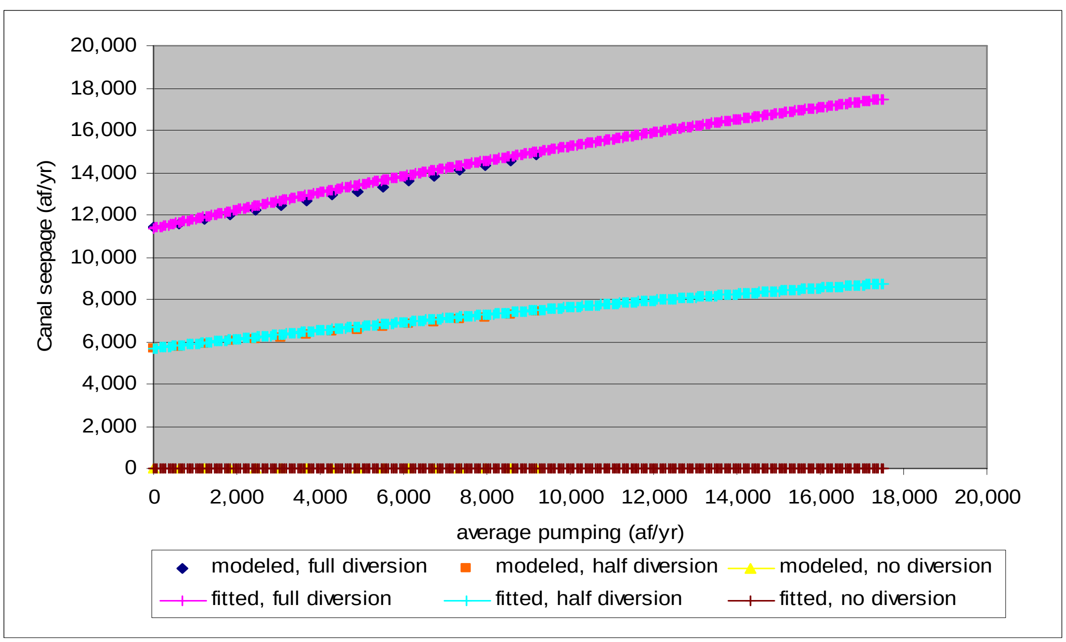


Figure 19: Fitted and modeled results showing average canal seepage as a function of total agricultural pumping and canal diversion.

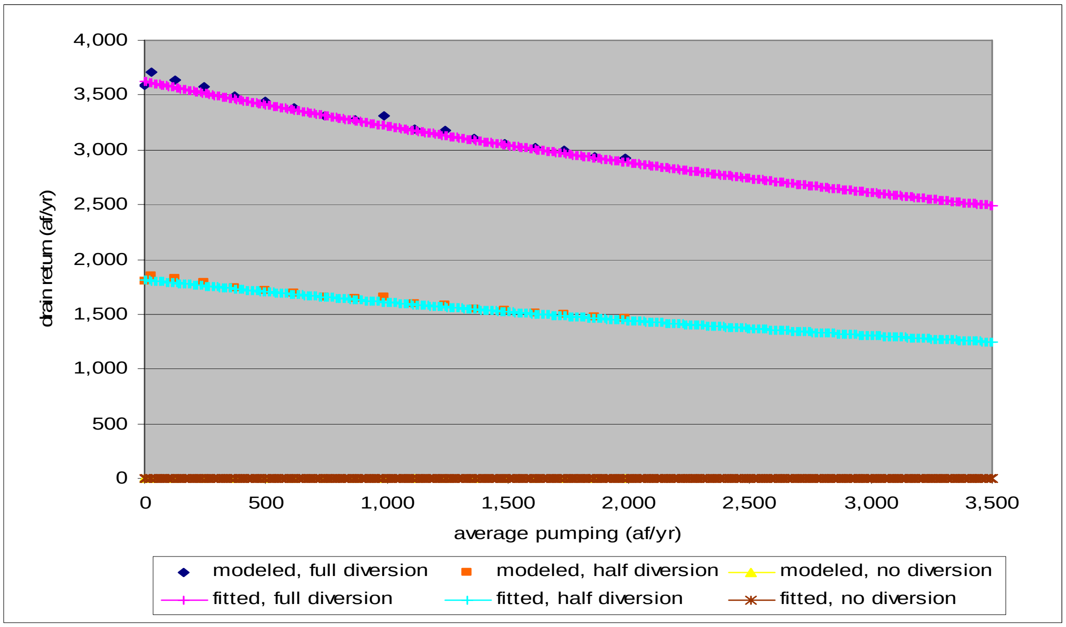


Figure 20: Fitted and modeled results showing average drain return as a function of total M&I pumping and canal diversion.

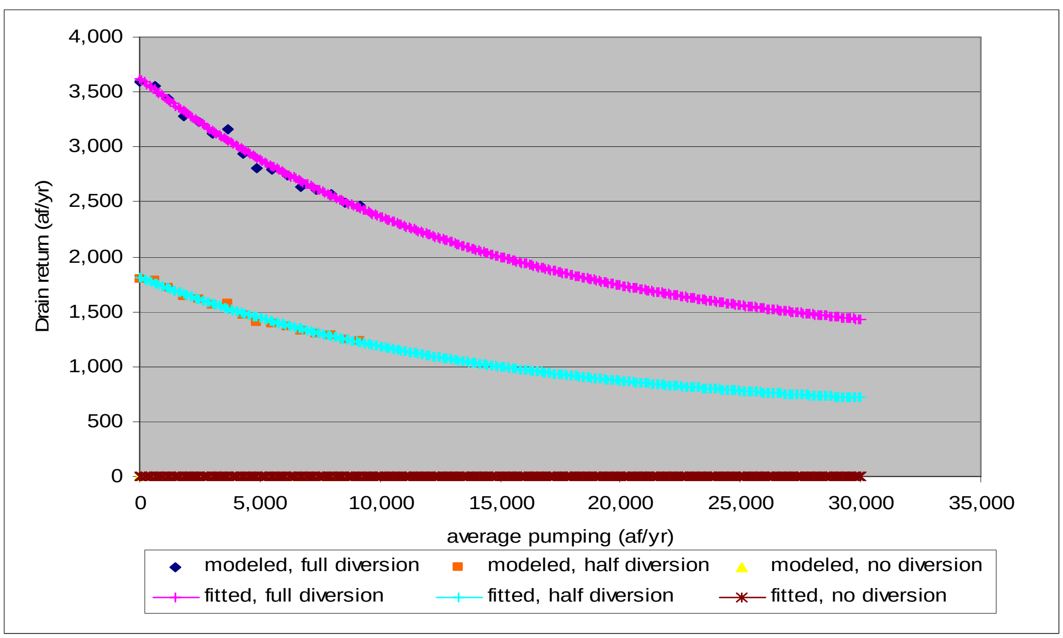


Figure 21: Fitted and modeled results showing average drain return as a function of total agricultural pumping and canal diversion.

Converting Response Functions to Supply Functions

Supply functions for the irrigation district, groundwater pumper, and drain water irrigator are developed by imposing a unit-cost (per acre-foot) on water supplied to these entities. The additional water delivery cost that results from canal seepage is formulated as a transportation cost.

Groundwater Pumpers' Supply Function

For groundwater pumpers, the unit-cost of supplying water is determined by groundwater pumping costs which are dependent in turn on pumping lift. The formula for calculating groundwater pumping cost assumes that a submersible electric pump is used which is 75 percent efficient, electric power costs are $0.06 per kwh, and 50 psi pressure is required at the well collar. Costs are assumed to be the same for both agricultural and M&I pumpers. The unit-cost of pumping as a function of pumping lift is given by the formula,

cost of gw pumping (per acre foot) = $9.46+ $0.08*pumping lift (in feet). **(10)**

Pumping lift depends on canal diversion (canal seepage) and on pumping rate. Therefore supply functions for M&I pumpers and agricultural pumpers are obtained by substituting into **(10)** either the M&I pumping lift response function (equation 4)**;** or the agricultural pumping lift response function, (equation 5). Figures 22 and 23 show the resulting supply functions developed for M&I pumpers and agricultural pumpers. The supply functions in these figures are depicted as a series of upward sloping curves. They are upward sloping because as the quantity of water supplied increases, pumping lift (and the unit-cost of supply) also increases.

The different curves in each figure depict possible shifts in the groundwater pumpers' supply function in response to changes in canal diversion or changes in groundwater pumping, either of which can induce a change in canal seepage which in turn influences pumping lift.

The magnitude of a shift in the groundwater pumpers' supply function is therefore dependent on <u>both</u> pumping rate and canal diversion. As pumping rate increases, the shift in supply cost due to a change in canal diversion also increases. For instance, if canal diversion is reduced from a full diversion (250,000 acre-feet per year) to zero (or equivalently if the canal is lined with an impermeable liner) and M&I pumping at the time is 600 acre feet per year, the marginal cost of groundwater pumping increases by only $0.60 per acre-foot. On the other hand, if average M&I pumping is 4,200 acre feet per year when the reduction in canal diversion (and seepage) takes place, the marginal cost of pumping increases by $9.81 per acre-foot. The difference in unit-costs can be traced to a two-way (Cheung) hydrologic externality resulting from the fact that (up to a point) groundwater pumping induces more canal seepage. As pumping increases the proportional contribution from canal seepage also increases and the pumper becomes increasingly reliant on canal seepage as a source of supply. When that supply is reduced or eliminated, the impact on groundwater pumper's supply cost is proportionally greater.

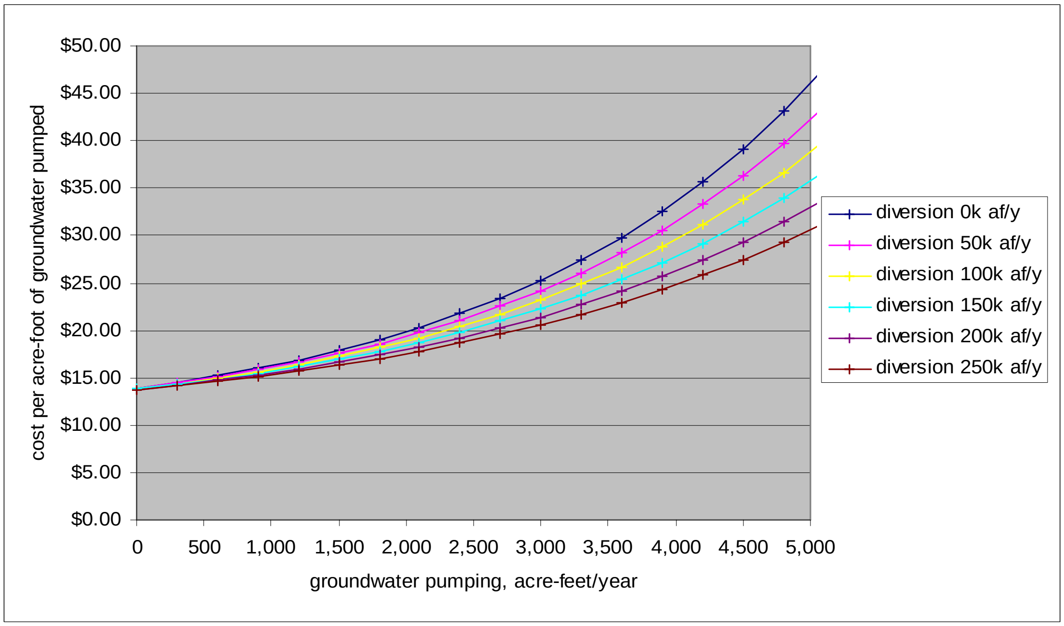


Figure 22: Shifting supply functions for M&I pumper due to changes in canal diversion.

Since there are many more agricultural wells in close proximity to the canal then there are M&I pumpers, (149 versus 6) the agricultural pumper's reliance on canal diversions as a source of supply is proportionally greater. As a result, reductions in canal diversion (or canal lining) can result in much larger shifts in the supply cost of the agricultural pumper than they do to the supply cost of the M&I pumper. For example if reduced diversion or canal lining occurs when the agricultural pumper's total pumping is 25,000 acre-feet per year, the marginal cost of pumping increases by $182.63 per acre-foot.

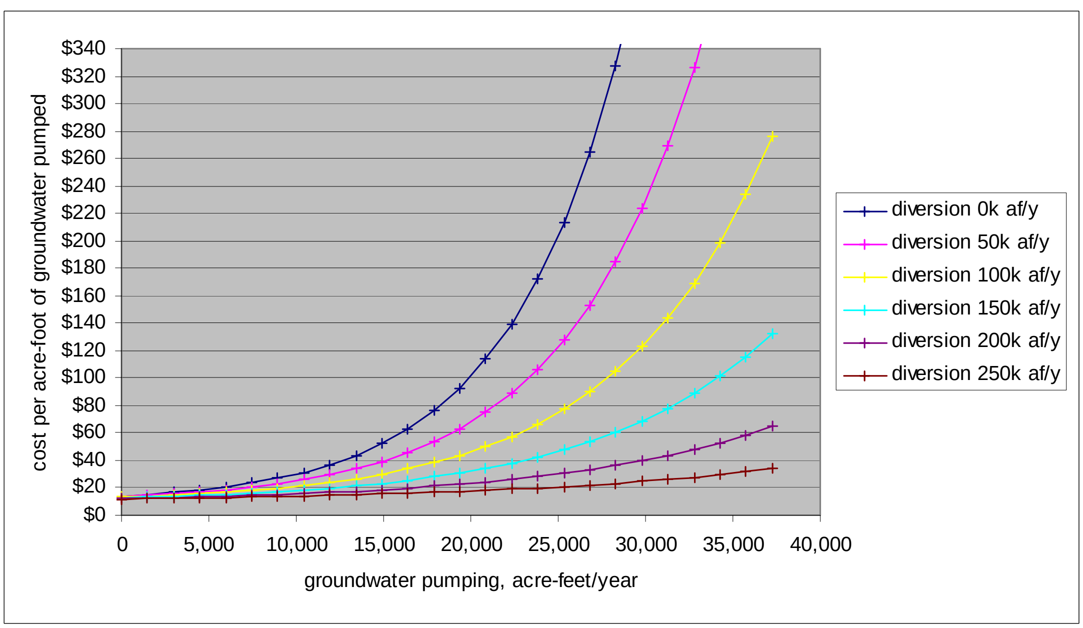


Figure 23: Shifting supply functions for the agricultural pumper due to changes in canal diversion.

Canal diversion is an important factor in limiting the marginal cost of pumping groundwater. However once pumping has increased to the point where the entire length of the canal is perched above the water table, canal seepage has reached a maximum and increased pumping cannot induce any more seepage. If the canal diversion is also at the maximum (250,000 acre-feet), no further shifts in the groundwater pumper's supply function can occur. This situation is illustrated in figures 22 and 23, when the 250k supply function is nearing vertical.

Drain Water Irrigators' Supply Function

For drain water irrigators, the formula for calculating pumping costs is the same as for groundwater pumpers except that obtaining water from the drain is assumed to require no additional pumping lift. With pumping lift set to zero in **(10)**, the cost of pumping water out of the drain is fixed regardless of how much water is pumped, as long as there is water in the drain. The drain users supply function is therefore,

$$\textbf{cost of drain diversion (per acre-foot)} = \$9.46 \qquad \textbf{(11)}$$

Drain water irrigators have no control over return flow to the drain. Drain return flow is determined by canal diversion and by groundwater pumping in the vicinity of the drain. The drain response functions, **(8**) and **(9),** (for M&I pumping and agricultural pumping respectively) are used to determine how much return flow enters the drain, and therefore the constraint that exists on the drain user's supply of water. The constraints are points where a shift in supply cost occurs if the drain user's demand exceeds his supply. If no other supply is available, the shift is from $9.46 per acre foot to an infinite cost.

Figures 24 and 25 show the drain water irrigators' supply functions, and some possible supply constraints (points where a shift in supply cost could occur) as a result of different combinations of canal diversion and pumping by agricultural and M&I users. A shift in the drain irrigators' supply function can occur at any drain diversion rate (including zero). For any combination of canal diversion and groundwater pumping, the drain irrigators' response functions, **(8)** or **(9)**, determine at what level of drain water demand the shift in supply cost will occur. For instance, with a full canal diversion (250,000 acre-feet per year) but in the absence of all groundwater pumping, figures 24 and 25 both show that the shift in supply cost occurs when drain user demand equals 3,750 acre-feet per year. As groundwater pumping increases, drain return decreases and the shift in drain irrigators'supply cost occurs at ever smaller drain diversion rates.

In figure 24, with a full canal diversion, if the average per well M&I pumping rate is 1,000 acre-feet per year, the shift in supply cost occurs when drain irrigator demand exceeds 2,002 acre-feet per year. If M&I pumping increases to 2,000 acre-feet per year, the shift occurs when drain irrigator demand exceeds 2,880 acre-feet per year. With a half diversion (125,000 acre-feet per year) and no pumping, the shift in the drain irrigators' supply cost occurs at 1,810 acre-feet per year. With 1,000 acre-feet of M&I pumping the shift occurs at 1,600 acre-feet per year. If canal diversion is zero (the canal is lined), then the shift occurs when drain demand is zero.

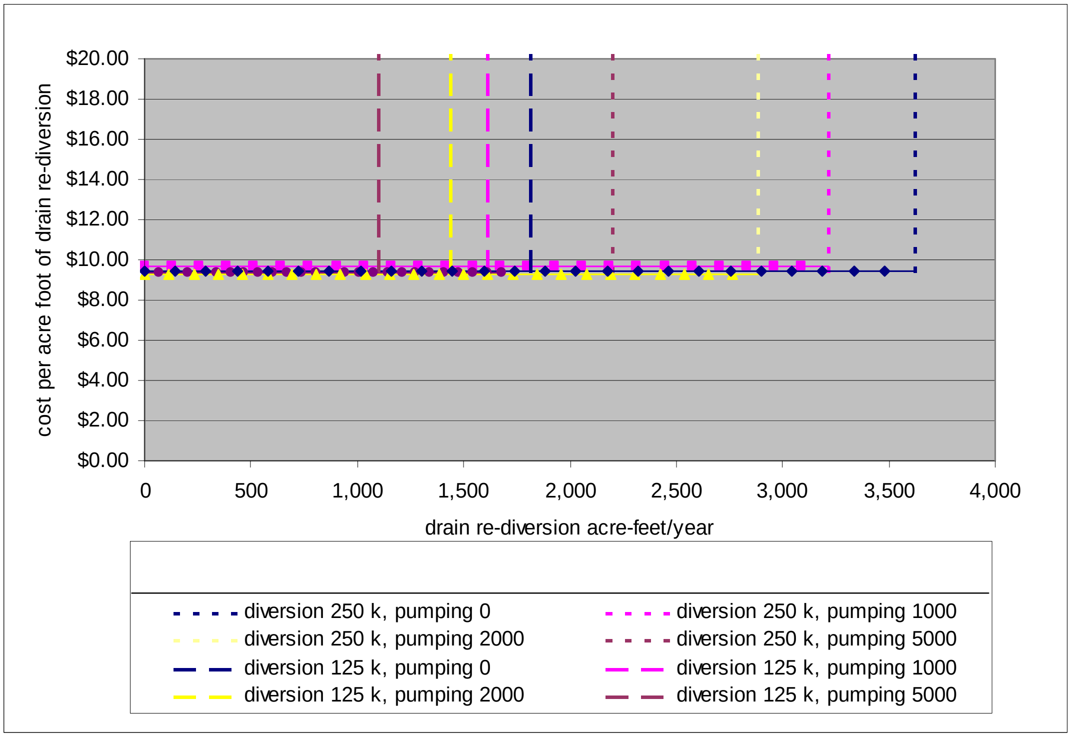


Figure 24: Shift in supply functions for the drain water irrigator due to canal diversions and M&I pumping.

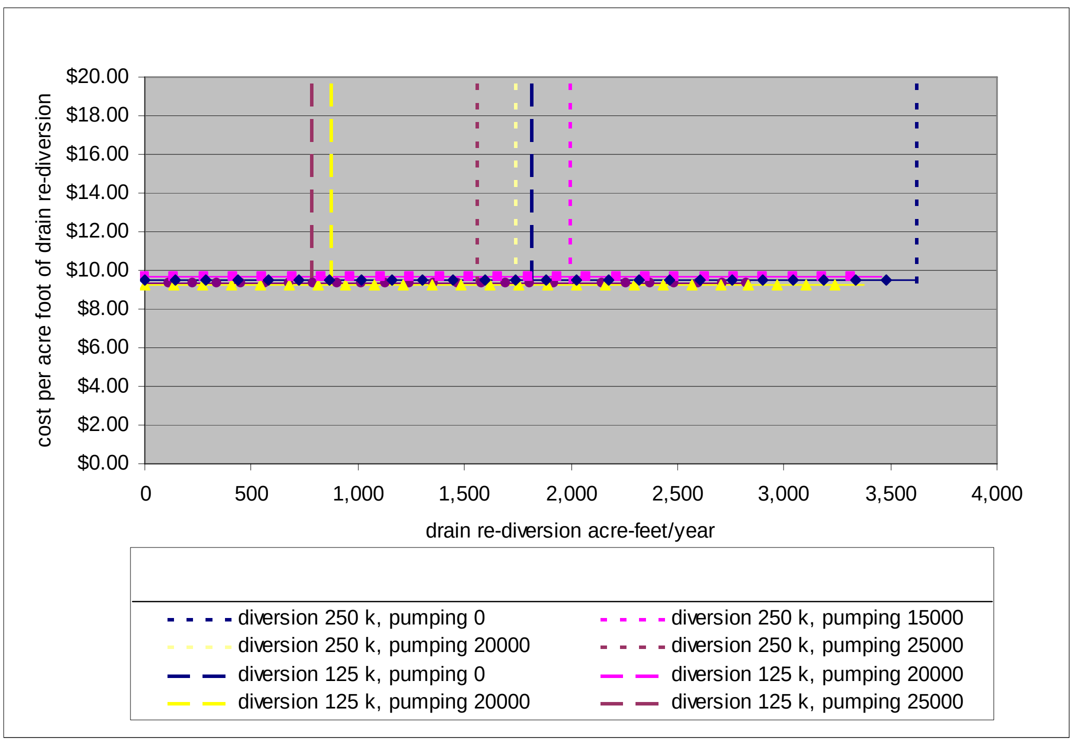


Figure 25: Shift in supply functions for the drain water irrigator due to canal diversion and agricultural pumping.

There are many more agricultural wells then M&I wells in the model sub-area. However the spatial properties of wells are important. Because agricultural wells are closer to the canal and at higher elevations (i.e. where the Elijah, Aaron and Wilson drains are perched) pumping from these wells has much less influence on drain return. Shifts in the drain irrigator supply cost as a result of agricultural well pumping occurs, but only when total pumping rates are much higher than those of the M&I wells which happen to be at lower elevations and closer to drains (see figures 8 and 9).

Irrigation District's Supply Function

For the canal diverters, the supply cost of water is determined by water district assessments. The assessments consist of O&M charges and Project repayment costs. Project repayment costs are determined by a repayment schedule for construction of the project, O&M charges are calculated based (partly) on Reclamation's operational costs. Most irrigation districts in the Boise Valley charge their members based on their irrigated acreage not on the quantity of water diverted by the district. For NMID, the total assessment in 2003 was $39.81 per acre of irrigated land. Assuming delivery of 3.0 acre-feet of water per acre of irrigated land, then the supply cost per acre-foot of water delivered to a member of the district was $13.27 per acre-foot, assuming no canal seepage. The canal diverter's supply function is therefore,

cost of canal diversion (per acre-foot) = $13.27 **(12)**

Canal Transportation Cost

The irrigation district's supply function describes the cost to the irrigation district of supplying water at the head of the canal. A canal transportation cost is associated with delivery of water from the head of the canal to canal diverters at the end of the canal. To the extent that the canal system is inefficient in its delivery of water, the cost of transporting an acre-foot of water through the canal may be greater than $13.27.

Inefficiency in the delivery of canal water is the result of canal seepage. If the canal transportation cost is assumed to be proportional to irrigation district supply cost and to canal seepage, then the canal transportation cost is given by,

transportation cost (per acre-foot) = $13.27 *(canal seepage/(diversion–canal seepage)) **(13)**

Canal seepage response functions **(6**) and **(7)** (for M&I pumping and agricultural pumping respectively) are used to determine the cost of transporting water in the canal. In the absence of canal seepage, the unit transportation cost for canal water is zero.

Figures 26 and 27 show, respectively, the increases in canal transportation cost that occur with increases in M&I and agricultural pumping. In the absence of all pumping, canal transportation cost due to seepage is $0.63 per acre-foot. As pumping increases, canal seepage also increases and cost of the transporting water to the canal diverter at end of the canal increases.

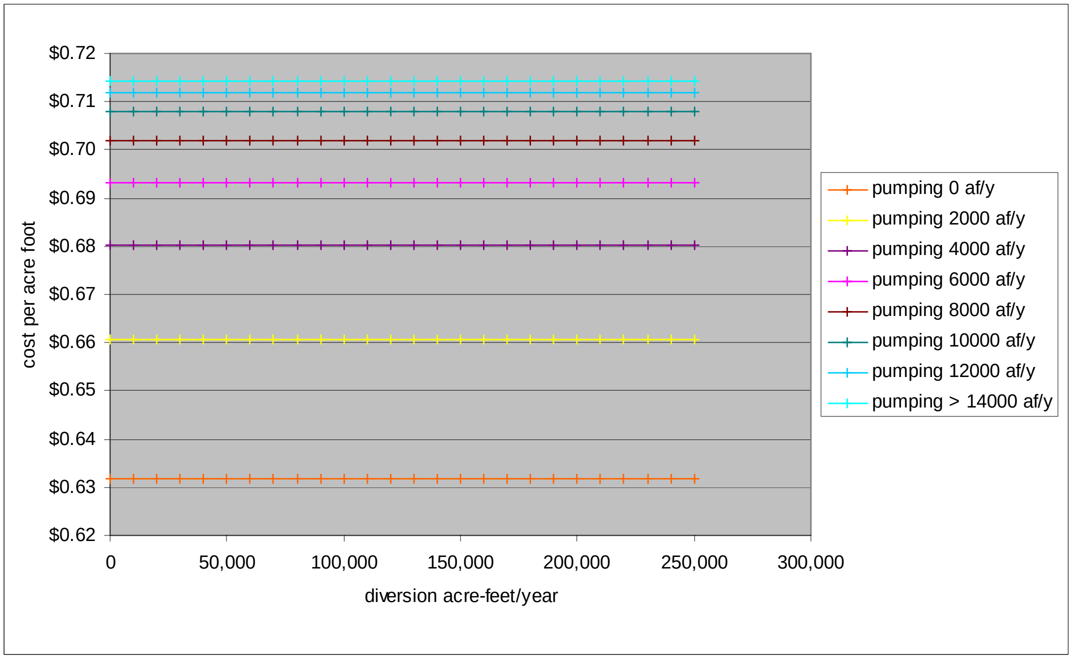


Figure 26: Increasing canal transportation costs due to increased M&I pumping.

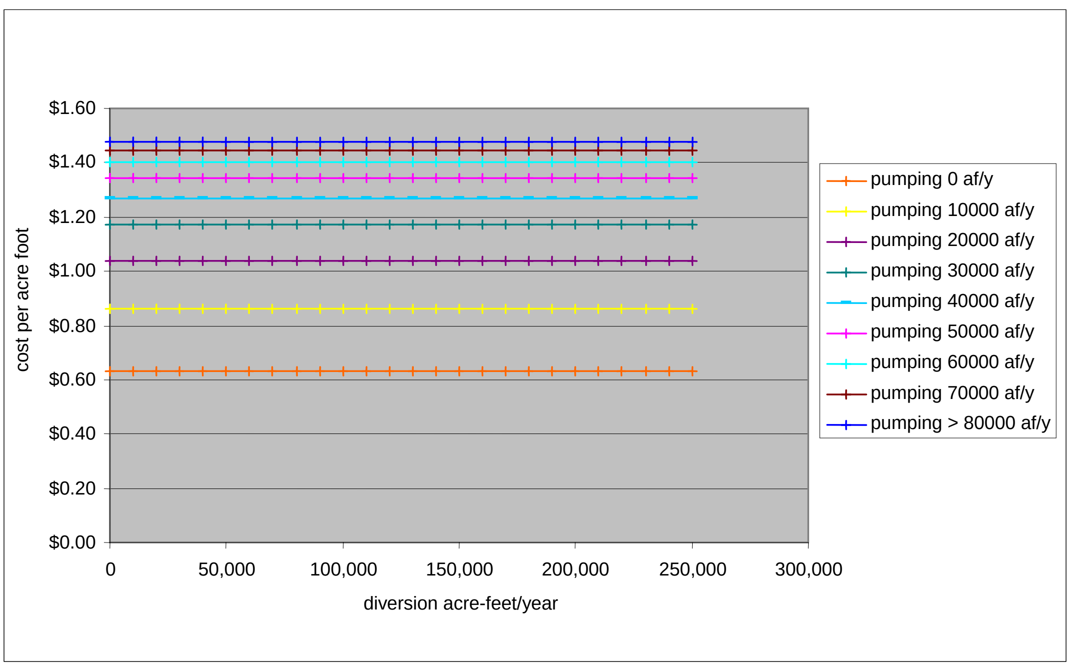


Figure 27: Increasing canal transportation costs due to increased agricultural pumping.

If pumping from the six M&I wells increases to 2,000 acre-feet per year, the cost associated with transporting water in the canal increases by $0.03, to $0.66 per acre-foot. An increase in M&I pumping to 4,000 acre-feet per year produces an additional increase in transportation cost of $0.02 per acre-foot, and a further increase to 6,000 acre feet adds $0.01 per acre-foot to the transportation cost. The increase in transportation cost decreases with increasing pumping rates since the hydrologic connection between the canal and the water table surface diminishes as pumping increases. The maximum impact of M&I pumping on the transportation costs occurs when average pumping from the six M&I wells reaches about 14,000 acre-feet per year. At which point, the transportation cost is about $0.72 per acre-foot delivered to the end of the canal.

A similar pattern of increasing transportation costs is associated with increases in agricultural pumping, except that in this case because there are many more agricultural wells than M&I wells in close proximity to the canal, the transportation costs continue to increase as pumping increases until the entire canal is perched above the water table. Again, in the absence of all pumping, canal transportation cost due to seepage is $0.63 per acre-foot. If pumping from 149 agricultural wells increases to 10,000 acre-feet per year, the transportation cost of canal water increases $0.23, from $0.63 to $0.86 per acre-foot. An increase in agricultural pumping to 20,000 acre-feet per year produces an additional increase in transportation cost of $0.18 per acre-foot, and an increase to 30,000 acre feet adds $0.13 per acre-foot to the transportation cost. The maximum increase in transportation cost occurs when agricultural well pumping totals about 80,000 acre-feet per year, at which point the transportation cost is $1.47 per acre foot. Additional agricultural pumping does not affect transportation costs further, since the canal is now perched entirely above the water table.

The canal transportation cost may be borne in whole or in part by the irrigation district (at the head of the canal) or by the canal diverters (at the end of the canal). How transportation costs are split between the supply entity and the demand entity depends on the irrigation district's supply function and on the canal diverters' demand function, and is determined as part of the partial equilibrium model solution.

Sub-Area Water Demand Functions

Water demand functions are price-quantity relationships describing the quantity of water that users will demand at a given price. As opposed to the fixed requirements demand described in Figure 1a, (exogenous) demand functions are assumed to be price responsive. Depending on the water user, demand functions have varying degrees of price elasticity, expressing the unit change in quantity demanded for a unit change in price. A common functional form used to represent price-quantity relationships is,

$$\boldsymbol{log(quantity) = log(v) - e \cdot log(price)} \tag{14}$$

obtained by taking logarithms of both sides of a multiplicative demand equation. The slope parameter, ***e***, in **(14)** then directly measures the price elasticity of demand. A log-log demand function with constant elasticity is highly inelastic in the upper left, and highly elastic in the lower right, with a point of unit elasticity in between (see Figure 1c).

The elasticity of demand, ***e,*** for Boise Project sub-area water users is derived from secondary sources described below. The ***log(v)*** term in the demand function shifts the price-quantity curve up or down and is used to scale the demand function with respect to demand quantities and demand prices actually observed in Boise Valley water transactions.

The multiplicative form of the log-log demand equation is

$$\boldsymbol{quantity} = \boldsymbol{r} + \boldsymbol{v} \cdot \boldsymbol{price}^{-\boldsymbol{e}} \ \boldsymbol{where} \ \ \boldsymbol{v} \geq 0,\ \boldsymbol{e} \succ 0, \boldsymbol{and}\ \boldsymbol{r} \prec 0 \qquad \textbf{(15)}$$

The additional constant, ***r***, is introduced in order to provide a finite price in the inverse demand function, when quantity demanded is zero.

Estimating Agricultural Water Demand

The most direct approach to estimating the demand for irrigation water by canal diverters is to observe market transactions where water rights are purchased by irrigators in a competitive market. However such transactions seldom occur. Although some irrigators sell water rights to M&I water users, and sometimes do so in relatively competitive markets, instances of open market water purchases by irrigators are rare.

Lacking a direct measure of irrigators demand for water, economists have focused on water's role as an agricultural input (see for example Moore and Hedges, 1963). Agricultural demand for water is a derived demand, derived from the value of the crops that can be grown with irrigation water. In its role as agricultural input, water affects two end products that are commonly sold in competitive markets; farm land and farm produce. However in a controlled market, prices for crops and farm land are influenced by much more than resource demand and scarcity.

In summary, calculating site-specific agricultural water demand functions is an extremely complex and costly undertaking. In lieu of conducting a survey in the Boise Valley, an agricultural demand function for canal diverters is formulated based on a review of recent literature.

Elasticity of Agricultural Water Demand

Analysis of irrigation water demand and its price-responsiveness have been presented in the literature since the early 1960s. Some authors find that irrigators are very unresponsive to changes in the price of water. Other studies indicate a more elastic demand.

A number of variables influencing the shape of demand functions as well as elasticity have been identified in the literature. During the 1970s and early 1980s, irrigation water demand functions were developed from statistical crop-water production functions based on experiments with field crops at state experiment stations. Demand function elasticity has also been estimated using econometric methods. Table 1 summarizes the range of estimates in the elasticity of irrigation water demand from recent studies.

Table 1: Irrigation Water Demand Elasticities.

Author	Number of Estimates	Range of Estimates
	Mathematical Programming Studies	
Moore, C.V. and Hedges (1963)	1	-.07
Heady, Madsen, Nicol and Hargrove (1973)	1	-0.15
Shumway (1973)	1	-1.97
Kelso, Martin and Mack (1 973)	8	-.0002 to –1.01
Moore, C.V., Snyder and Sun (1974)	1	-0.42
Hedges (1977)	1	-.04
Gisser, Landford, Gorman, Creel and Evans (1979)	2	-0.10 to –0.12
Howitt, Watson and Adams (1980)	1	-0.97
Bemardo, Whittlesey, Saxton, Bassett (1987)	1	-0.12
Hooker and Alexander (1998)	1	-0.22
Scheierling, Young and Cardon (2003)	3	-0.02 to-0.16
	Econometric Studies	
Frank and Beattie (1979)	16	-1.01-1.69
Nieswiadomy (1985)	1	-0.80
Ogg and Gollehon (1989)	1	-0.26
Moore, R.M., Gollehon and Carey (1994)	4	-0.03 to -0.10
Field Experiment Studies		
Hexem and Heady (1978)	4	-0.06 to -0.10
Ayer and Hoyt (1981)	3	-0.06 to -0.16
Kelley and Ayer (1982)	3	-0.04 to -0.56

The average elasticity from the complied studies is about -0.50. In the absence of more site-specific data, this average value is used for elasticity of water demand by irrigators in the Boise Project sub-area .

Canal Diverters' Demand Function

Scaling the agricultural water demand function requires a known price-quantity point on the canal diverter's unrestricted demand function. The demand function cannot be fitted to a price-quantity point at the current water price because water is delivered to the canal diverter at a (restricted) O&M and Project repayment price that is unrelated to the actual demand price of irrigation water.

The unrestricted price of irrigation water can be determined from crop budgets by a method known as residual imputation (RI) (Young, 1996). The residual imputation method assumes that a homogenous total product function exists, where factors of production (land, labor, capital, water) are each paid their factor share of the total marginal product price. The values (prices) of all factors of production are known except for one (water), so that all the information needed to determine the value (price) of this last factor of production is available.

For the most prevalent crop in the NMID, alfalfa, the RI method imputes a water value (price) of about $65.00 per acre-foot. And, on average, the NMID diverts about 42,600 acre-feet of water per year through the New York Canal to irrigated lands located in the model sub-area (about 7,450 acres). The price-quantity scale point for the canal diverter's demand function is therefore $65.00 per acre-foot for a quantity of 42,600 acre feet of water. In addition, it is estimated that if the demand price for water were to rise to $130/acre foot, farmers would cease irrigating crops, and the quantity demanded by NMID canal diverters would drop to zero. The scaled water demand function for canal diverters in the sub-area partial equilibrium model is therefore

$$\boldsymbol{quantity = -102845 + 1172619 \cdot price^{-0.50}} \quad . \qquad \textbf{(16)}$$

The scaled water demand function for sub-area canal diverters is displayed in Figure 28.

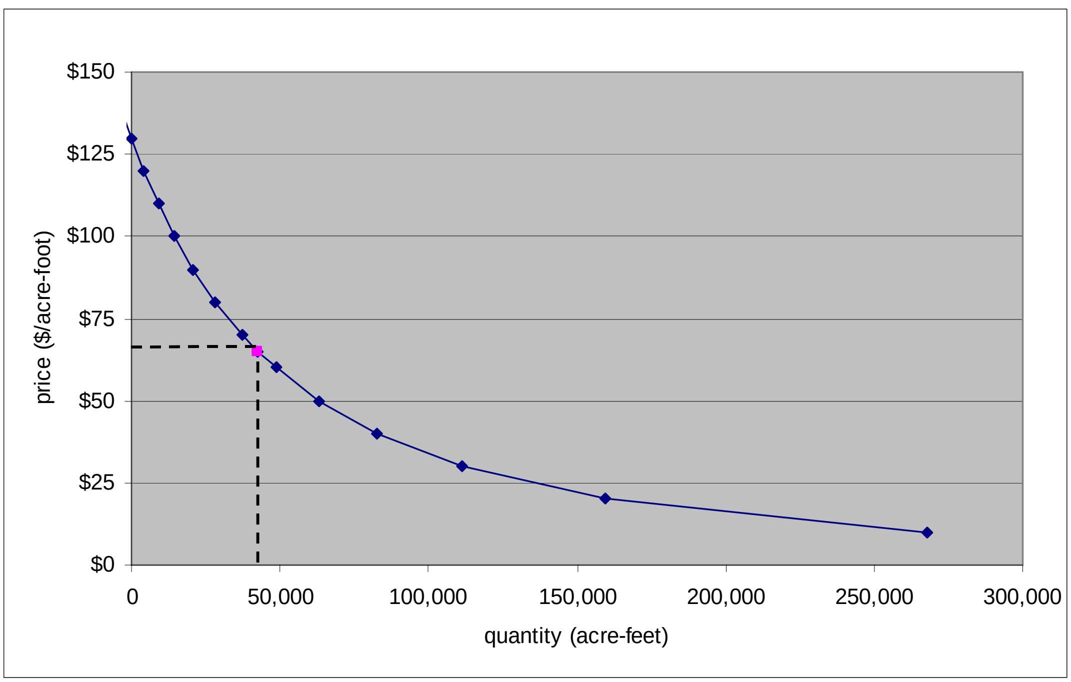


Figure 28: Scaled demand function for canal diverters in the Boise Project sub-area.

Agricultural Pumpers' Demand Function

Elasticity of demand for agricultural pumpers in the sub-area model is assumed to be the same as that of canal diverters (-0.50). Demand price is also assumed to be the same as for canal diverters ($65.00 per acre foot), and if the demand price for water were to rise to $130/acre foot, the quantity demanded by agricultural pumpers would also fall to zero. Agricultural wells in the sub-area are used to irrigate about 23,000 acres, however for the most part groundwater serves as a secondary source of water for this acreage. Based on the Boise Valley water budget data, average annual pumping for 149 irrigation wells in the sub-area is about 18,000 acre-feet per year. The scaled water demand function for agricultural pumpers is therefore

$$\boldsymbol{quantity} = -\boldsymbol{43456} + \boldsymbol{495473} \cdot \boldsymbol{price}^{-0.50} \quad . \qquad \textbf{(17)}$$

The scaled water demand function for sub-area agricultural pumpers is shown in Figure 29.

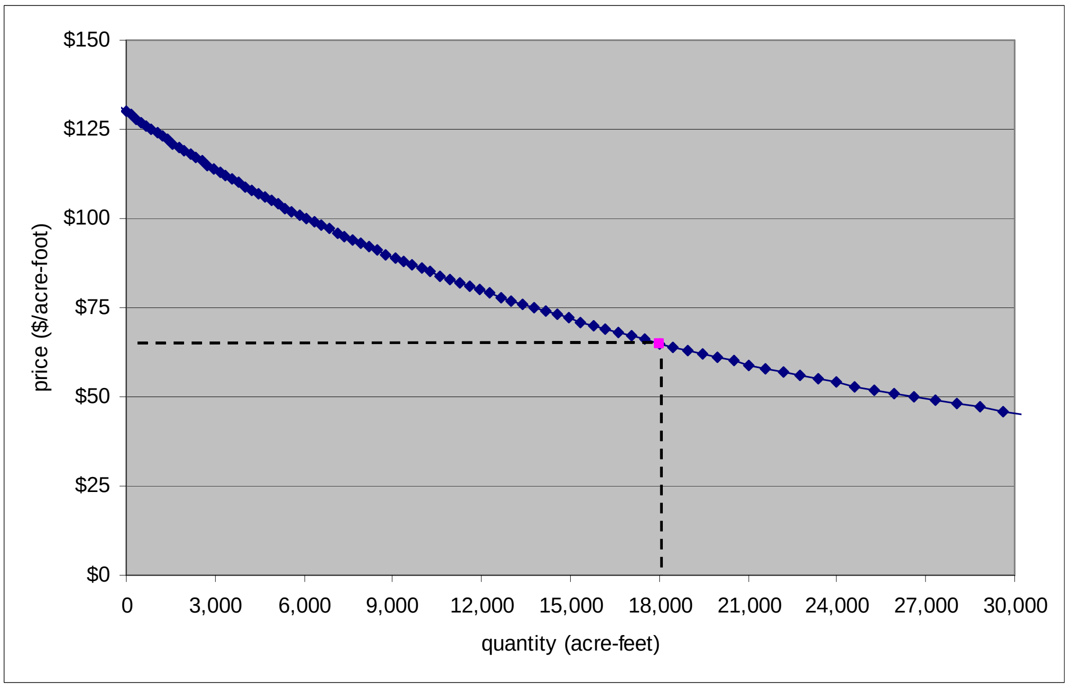


Figure 29: Scaled demand function for agricultural pumpers in the Boise Project sub-area.

Drain Water Irrigators' Demand Function

Most re-diversions from the Elijah, Wilson and Aaron drains occur in the Pioneer Irrigation District at the north end of these drains. Elasticity of demand for drain water by these irrigators is assumed to be the same as that of canal diverters (-0.50). Demand price is also assumed to be the same as for canal diverters ($65.00 per acre foot), and if the demand price for water were to rise to $130/acre foot, the quantity demanded by drain users is expected to be zero. Based on the

Boise Valley water budget, approximately 830 acres in the sub-area are irrigated with water diverted directly from these three drains before they discharge into Indian Creek. The annual re-diversion rate from these three drains is about 1,660 acre feet. The scaled water demand function for drain water irrigators is therefore

$$\boldsymbol{quantity} = \boldsymbol{-4008} + \boldsymbol{45694} \cdot \boldsymbol{price}^{\boldsymbol{-0.50}} \quad . \qquad \textbf{(18)}$$

The scaled water demand function for sub-area drain water irrigators is shown in Figure 30.

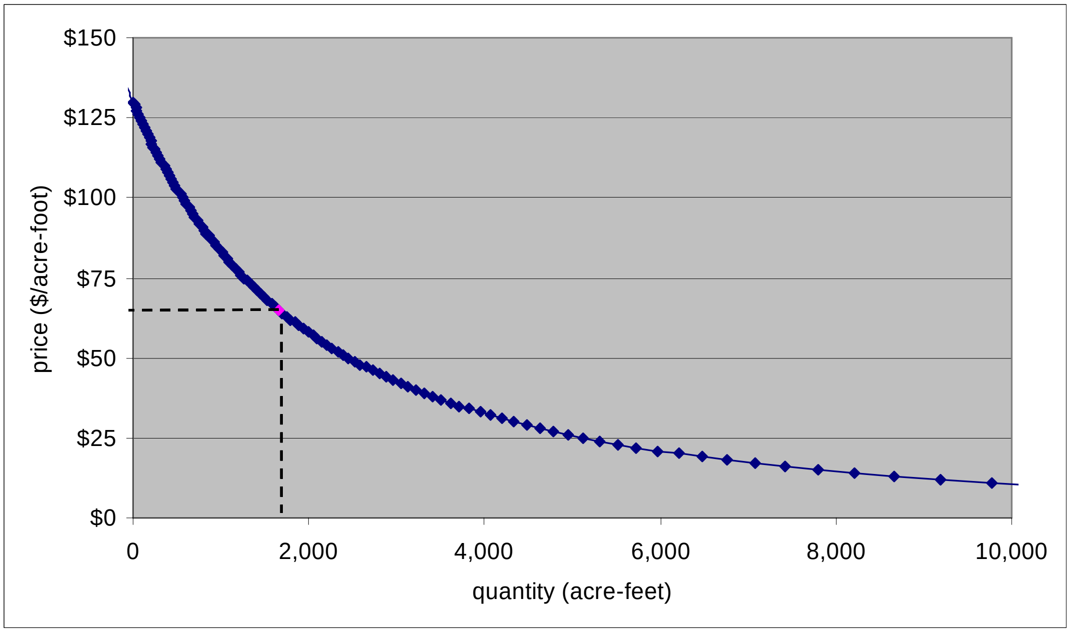


Figure 30: Scaled demand function for drain water irrigators in the Boise Project sub-area.

M&I Pumpers' Demand Function

Public utility water suppliers typically administer a rate schedule. Administered rate schedules complicate the estimation of water demand, because price varies with the amount of water consumed. The choice in developing an M&I demand function is therefore between using an average demand price for water or a marginal demand price for water. Appendix C of this report discusses some of the arguments related to choosing average demand price or marginal demand price in modeling M&I water demand.

In either case, empirical studies have shown that M&I water use varies inversely with price, as M&I consumers in the short run adjust their seasonal water use, and in the long run modify or replace water-wasting appliances or capital stocks with more efficient equipment. For the Boise Project sub-area model, the Nampa M&I water demand function is based on the marginal demand price for water.

Table 2 displays some recent estimates of M&I water demand elasticity. Elasticity estimates for M&I water demand have ranged from highly inelastic (-.003 to -.01) to elastic (-1.57 to -1.63). The estimates are from studies that used marginal prices to estimate elasticity. With the exception of Nieswiadomy (1992), these studies show remarkable convergence in their estimates of elasticity.

Table 2: M&I water demand studies that used marginal prices.

Author	Price Elasticity
Nieswiadomy and Molina (1989)	-0.09 to -0.86
Barkatullah (1996)	-0.23 to -0.28
Agthe and Billings (1997)	-0.39 to -0.57
Renwick and Archibald (1998)	-0.33 to -0.53
Martin and Wilder (1992)	-0.32 to -0.60
Nieswiadomy (1992)	-0.02 to -0.17
Taylor, McKean, and Young (2004)	-0.30

From these studies, the elasticity value from the Taylor et. al., 2004 study (-0.3) was selected to represent the elasticity of M&I demand for water in the Boise Project sub-area. Again, the M&I demand function is scaled by fitting it through a single point representing the current level of M&I pumping from the six M&I wells within the model domain, and the assessed price for water in the Nampa municipal supply area.

The M&I wells in the Boise Valley model area are part of the Nampa municipal supply system. Nampa wells provide 157 gallons of water per day per capita, (57,305 gallons per capita per year) to a total residential population of 44,550, using 47 supply wells. Total annual M&I supply is about 7,835 acre-feet.

Nampa charges a fixed monthly fee of $5.64 for hookups within the city and $11.28 for hookups outside the city. In addition the city charges $0.78 per 100 cubic feet for the first 4,000 cubic feet of water delivered inside the city and $0.46 for every 100 cubic feet over that. Outside the city, Nampa’s fixed charge is $11.28, and the incremental rates are $1.86 for the first 4,000 cubic feet and $0.92 for every 100 cubic feet above that.

For the sub-area, it is assumed that the average fixed monthly charge for Nampa M&I water is $8.46 per hookup, and that (based on four persons per hookup) there are 11,000 hookups total. It is also assumed that for half of the year, the marginal cost of water is the higher of the two incremental rates, and for the other half of the year it is the lower of the two rates. Therefore the average annual charge for water is $1.01 per 100 cubic feet, or $439.95 per acre-foot.

Nampa’s total annual fixed charges for water are estimated to be about $1,116,720, and total annual incremental charges are estimated to be about $3,447,055. The combined charges for 7,835 acre feet of water delivered to M&I users is $4,563,775, or $582.49 per acre foot, which is equivalent to paying $582.49 per acre foot for 166.7 acre feet of water delivered from each one of Nampa’s 47 wells.

The M&I demand function for Nampa wells within the sub-area is developed assuming that a homogenous total product function exists for supplying M&I water, and that factors of production, including a "pumping factor", are each paid their factor share of the total marginal price.

Given a per well M&I pumping rate of 166.7 acre feet per year, average pumping lift is about 76.5 feet (assuming a full canal diversion). The price of the "pumping factor" of production can be calculated using **(10)**. Nampa M&I water users pay $582.49 per acre-foot of water, but the "pumping factor" of production for M&I water is just $15.58 per acre-foot, all other factors of production are lumped together as a $566.91 transportation factor. The price-quantity scale point in the M&I pumpers demand function is therefore $15.58 per acre-foot for a total quantity of 1,000 acre-feet pumped from the six M&I wells located within the model domain.

In addition, if the pumping factor of demand price were to rise to $175 per acre foot, the quantity of groundwater demanded by the M&I supplier is expected to be zero. Assuming an elasticity of -0.3, the calibrated M&I demand function is,

$$\boldsymbol{quantity = -938 + 4417 \cdot price^{-0.30}} \qquad \textbf{(19)}$$

The scaled water demand function for M&I pumpers is shown in Figure 31.

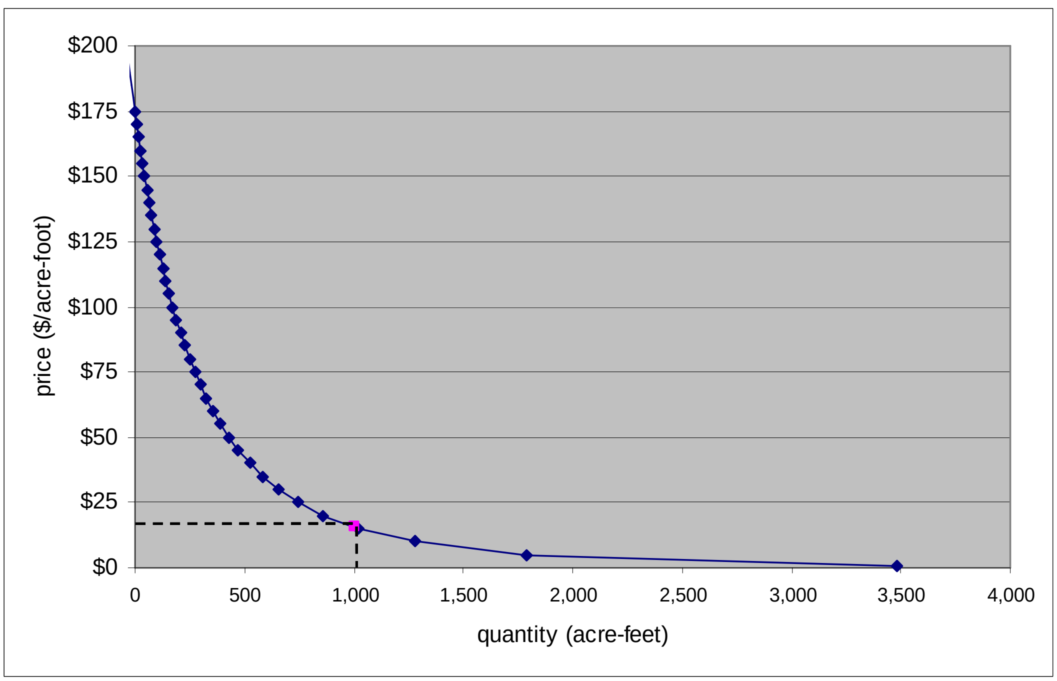


Figure 31: Scaled demand function for M&I pumper's in the Boise Project sub-area.

It is important to note that the M&I water demand function is actually the pumping factor of demand price, and applies to the Nampa municipal water supplier not to the Nampa M&I water users. Implicit in this demand function is the assumption that if the factor price of groundwater becomes too high (i.e. $175 per acre-foot) the M&I suppler would switch to some other source, perhaps surface water, to supply Nampa users.

The Sub-Area Partial Equilibrium Model

The sub-area partial equilibrium model balances water supply and demand while maximizing the utility of water use among four spatially distributed water supply or demand entities:

1. The Bureau of Reclamation and the Nampa Meridian Irrigation District (NMID) as a water supply entity;
2. Agricultural water users in the NMID who divert water from the New York Canal as a water demand entity;
3. Private agricultural pumpers and Nampa (M&I) pumpers located near the New York canal as both a supply and demand entity;
4. Elijah, Aaron and Wilson drain water irrigators located north of the New York Canal as both a supply and demand entity.

The Boise Project sub-area partial equilibrium model differs from the original partial equilibrium model developed by Takayama and Judge (see Appendix B) in some important ways.

1) Rather than have a supply-cost function and demand-price function for each of the water trading entities, some entities have only supply functions and some have only demand functions. Transportation-costs are associated with transporting water from the irrigation district supply entity to the canal diverter demand entity.

2) All water demand entities in the model have constant elasticity demand functions rather than linear demand functions (see figure 1). The demand functions include a constant term, which reflects the fact that at some very high price level, water demands by irrigation and M&I entities will be zero. On the other hand, no matter how low the demand price for water drops, the agricultural and M&I demand entities are never satiated. As the demand price falls and water supplies increase, there will always be irrigators and M&I users willing to buy these supplies of water.

3) The supply function for drain water irrigators reflects the fact that the drain user has a fixed supply price regardless of the quantity of water diverted from the drain. However the maximum quantity that can be diverted from the drain is limited by the drain return rate, which is controlled (hydrologically) by canal seepage and groundwater pumping.

4) Supply functions for canal diverters, groundwater pumpers and drain water irrigators can be manipulated in the model in order to either include or exclude the impact of hydrologic externalities.

Partial Equilibrium Model Scenarios

Three different GAMS model scenarios are developed for the Boise Project sub-area: a base-case scenario, a priced-externalities scenario, and a canal-lining scenario.

The base-case scenario represents current water supply conditions (including existing hydrologic externalities). The priced-externalities scenario alters water supply in a way that is representative of the market-based approach to water management (i.e. by internalizing hydrologic externalities). The canal-lining scenario alters water supply conditions in a way that is representative of the conservation management approach (i.e. eliminating hydrologic externalities).

Besides these three scenarios, a variety of other supply and demand management strategies could be represented using the GAMS model, including managed aquifer recharge, curtailment of groundwater pumping, or conversions from gravity to sprinkler irrigation.

Spatial Water Allocation Model Results

Sub-area spatial water allocation model results are divided into two groups. Each group of results involves four of the five water supply and demand entities described previously; an irrigation district, a canal diverter, a drain water irrigator, and either an agricultural pumper or an M&I pumper. Agricultural pumping has much greater influence on canal seepage than does M&I pumping because agricultural pumpers are supplied mainly by the shallow aquifer system, while M&I pumpers are supplied mainly by the deep aquifer system (Petrich, 2004).

For each group, the results of three model scenarios are presented. Scenario 1 is a base-case representing current conditions with respect to availability and pricing of canal diversion and canal seepage, i.e. canal diversions are at historic averages, canals are unlined, and canal seepage externalities are un-priced. Scenario 2 internalizes the canal seepage externalities through pricing and by requiring a reimbursement by the recipients of externalities. Scenario 3 eliminates the externalities altogether by lining the canal and eliminating seepage.

For each scenario, model results include the total annual quantity of canal seepage and the equilibrium quantity of water supplied to and demanded by each entity. They also include the equilibrated supply price and demand price for water at the margin. In addition, for those scenarios that include agricultural pumpers, the average supply cost of water is also presented.

Total canal seepage is broken down into Meade seepage and Cheung seepage, indicating a one-way or two-way externality. Meade seepage is the quantity of seepage that is uninfluenced by, or occurs in the absence of, groundwater pumping. Cheung seepage is the quantity of seepage that is induced by groundwater pumping. Figure 32 illustrates how the two components of canal seepage can be calculated using canal seepage response functions.

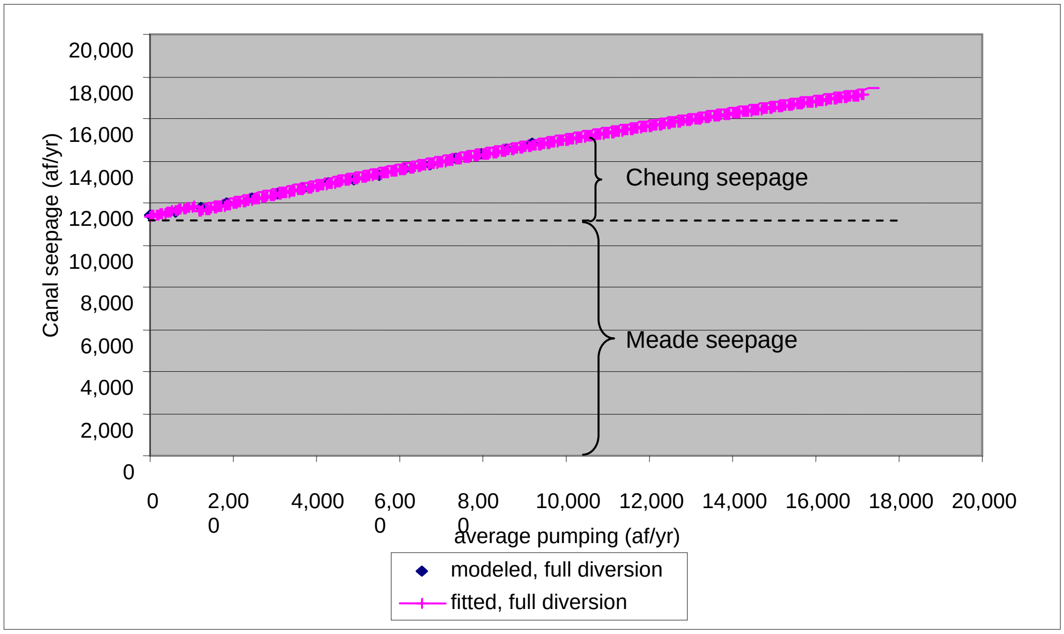


Figure 32: Meade and Cheung components of total canal seepage from model results that include agricultural pumpers.

Note that since supply costs for both canal diverters and drain water irrigators are fixed regardless of the quantity supplied, average supply cost is equal to marginal supply price for these two entities. This is not true for groundwater pumpers, whose supply price increases with increasing quantity supplied, due to increases in pumping lift.

Model Results that include Agricultural Pumpers

Table 3 summarizes the results of the three spatial water allocation model scenarios for the group of water suppliers and demanders that includes agricultural pumpers.

Partial equilibrium modeling assumptions require that the quantities of water supplied to and demanded by each trading entity be equal, and that supply costs plus transportation costs plus constraint costs be equal to demand prices. As described earlier, transportation costs are associated with water that is lost to the canal diverter because of canal seepage. Constraint costs are associated with limits on canal diversion due to canal capacity (or to limitations of water rights). Constraint costs are also associated with limits on drain return, which are hydrologically determined.

Table 3: Spatial water allocation model results for canal diverters, drain users and agricultural pumpers.

Conditions at equilibrium	Base-Case Scenario 1: with canal seepage externality	Scenario 2: canal seepage priced (externality Internalized)	Scenario 3: canal lined (externality eliminated)
Total canal seepage (af)	21738	20664	0
Meade seepage (af)	11363	11363	0
Cheung seepage (af)	10375	9301	0
Quantity supplied (af)			
irrigation district	42600	42600	42600
Ag. pumper	39220	32261	17031
drain water irrigator	1286	1386	0
Quantity demanded (af)			
canal diverter	20862	21936	42600
Ag. pumper	39220	32261	17031
drain water irrigator	1286	1386	0
Average supply cost (per af)			
irrigation district	$13.27	$13.27	$13.27
Ag. pumper	$19.32	$16.82	$30.27
drain water irrigator	$9.46	$9.46	$9.46
(Marginal) Demand price (per af)			
canal diverter	$89.85	$88.31	$65.00
Ag. pumper	$35.92	$42.82	$67.10
drain water irrigator	$74.51	$71.75	$130.00

Scenario 1

In scenario 1, the quantity of water supplied annually by the irrigation district to the canal diverters in the model area is 42,600 acre-feet (the water right limit). Canal seepage that occurs within the model area as a result of a full canal (the canal diversion capacity is 250,000 acre-feet per year) supplying 42,600 acre-feet annually to the canal diverters, is 21,738 acre-feet. (The remaining 207,400 acre-feet goes to irrigators outside the model area.) Of this seepage, 11,363 acre-feet is Meade seepage, uninfluenced by groundwater pumping, and 10,375 acre-feet is Cheung seepage, induced by agricultural pumpers in the model area who are pumping 39,220 acre-feet of groundwater annually. Canal seepage also creates 1,286 acre-feet of drain return which is being used by drain water irrigators in the model area.

The irrigation district's supply cost for any quantity of water up to 42,600 acre-feet is based on O&M charges and Project repayment costs which total $13.27 per acre-foot, at the head of the canal. The canal diverters' equilibrium demand is for 20,862 acre-feet of water, the canal diverters' equilibrated demand price, which includes canal transportation costs and constraint costs, is $89.85 per acre-foot, at the margin.

The drain water irrigators' supply cost for 1,286 acre-feet of water pumped from the drain is fixed at $9.46 per acre-foot. The equilibrated demand price for drain water is $74.51 per acre foot, at the margin.

At equilibrium, agricultural pumpers demand 39,220 acre-feet of water. The groundwater pumpers average supply cost, based on pumping lift, is $19.32 per acre-foot. The equilibrated demand price for agricultural groundwater is however $35.92 per acre-foot, at the margin.

Scenario 2

In Scenario 2, canal seepage externality is internalized. Agricultural pumpers and drain water irrigators reimburse the canal diverters for the canal seepage externality that was un-priced in Scenario 1. The reimbursements are based on the equilibrated demand price for water by groundwater pumpers and drain water irrigators.

The irrigation district supplies the same quantity of water as before to the canal diverters (42,600 acre-feet per year). As in Scenario 1, 11,363 acre-feet of canal seepage is unaffected by groundwater pumping (the Meade Externality). Canal seepage induced by agricultural pumping (the Cheung Externality) declines however, from 10,375 acre-feet (in Scenario 1) to 9,301 acre-feet per year. The decline in seepage can be traced to the decline in the agricultural pumpers' demand for water from 39,220 to 32,261 acre-feet per year as a consequence of now having to pay for some of the transportation costs associated with canal seepage. The reduction in transportation costs results in a reduction in the canal diverters' demand price for water, from $89.85 (in Scenario 1) to $88.31, and an increase in the quantity of water demanded from 20,862 acre-feet (in Scenario 1) to 21,936 acre-feet.

The reduced demand for water by groundwater pumpers results in increased drain return and a reduction in drain water irrigators' demand price from $74.51 per acre-foot (in Scenario 1) to $71.75 per acre-foot. As a consequence, the quantity of drain water demanded increases from 1,286 acre-feet (in Scenario 1) to 1,386 acre-feet.

Finally, the agricultural pumpers' reimbursement for canal transportation costs results in a $6.90 increase in the demand price for groundwater from $35.92 per acre-foot (in Scenario 1) to $42.82 per acre-foot.

Scenario 3

In Scenario 3 the canal is lined, thus eliminating the canal seepage externality entirely. As before, the irrigation district supplies the same quantity of water to canal diverters (42,600 acre-feet). In the absence of all canal seepage (and transportation costs) the canal diverters' demand increases to 42,600 acre-feet, which is the maximum allowable diversion.

The agricultural pumpers' demand declines significantly from 39,220 acre-feet (in Scenario 1) to 17,031 acre-feet as a result of the greatly increased supply cost associated with increased pumping lift. In the absence of all canal seepage, the drain water irrigators' water supply disappears entirely.

In the absence of all transportation costs, the canal diverters' equilibrium demand price for water declines from $89.85 per acre-foot (in Scenario 1) to $65.00 per acre-foot. On the other hand, the agricultural pumpers' supply cost increases, from $35.92 to $67.10 per acre-foot due to increased pumping lift. The drain water irrigators' marginal demand price exceeds $130 per acre-foot, which is the point in the drain water irrigators' demand function (see figure 30) at which the demand for drain water drops to zero.

Model Results that include M&I Pumpers

Table 4 summarizes the results of spatial water allocation model scenarios for the group of water suppliers and demanders that includes M&I pumpers. As before, there are three model scenarios; the base case with externalities, externalities internalized, and externalities eliminated. The quantity of water supplied by the irrigation district is equal to total canal seepage plus the water demanded by canal diverters, and the quantity of water supplied to drain water irrigators and M&I pumpers is equal to the quantity demanded by these entities. Note that table 4 shows equilibrated supply prices (not average supply costs as in table 3). At equilibrium, supply prices equal demand prices for all entities.

Table 4: Spatial water allocation model results for canal diverters, drain users and M&I pumpers.

Conditions at equilibrium	Scenario 1: with canal seepage externality	Scenario 2: canal seepage priced (externality Internalized)	Scenario 3: canal lined (externality eliminated)
Total canal seepage (af)	14,475	13,312	0
Meade seepage (af)	11,363	11,363	0
Cheung seepage (af)	3,112	1,949	0
Quantity supplied (af)			
irrigation district	42,600	42,600	42,600
M&I pumper	1,163	0	1,034
drain water user	3,158	3,625	0
Quantity demanded (af)			
canal diverter	28,126	31,238	42,600
M&I pumper	1,163	0	1,034
drain water user	3,158	3,625	1,034
Supply price (per af)			
irrigation district	$80.16	$76.48	$65.00
M&I pumper	$11.91	$253.06	$14.69
drain water user	$40.66	$35.84	$130.00
Demand price (per af)			
canal diverter	$80.16	$76.48	$65.00
M&I pumper	$11.91	$253.06	$14.69
drain water user	$40.66	$35.84	$130.00

Scenario 1

Again in Scenario 1, the quantity of water supplied by the irrigation district, at the head of the canal is 42,600 acre-feet. The canal diverters' water demand at the end of the canal is 28,126 acre-feet. Canal seepage totals 14,475 acre-feet, of which 11,363 acre-feet is uninfluenced by M&I pumping (Meade Externality) and 3,112 acre-feet is induced by M&I pumping. The M&I pumpers demand 1,163 acre-feet of groundwater, and the drain water irrigators pump 3,158 acre-feet from the drain.

Because of differences in aquifer systems, well locations and pumping rates, M&I pumping has a much smaller influence on canal seepage and drain return than does agricultural pumping. As a result, transportation costs associated with canal seepage are lower. The 28,126 acre-foot base-case demand for water, by canal diverters is 7,266 acre-feet greater than before, and the $80.16 per acre-foot demand price is $9.69 per acre-foot less than before.

Scenario 2

Again in scenario 2, the canal seepage externality is internalized. M&I pumpers and drain water irrigators reimburse the canal diverters for transportation costs associated with the Cheung portion of canal seepage. The irrigation district supplies the same quantity of water as before (42,600 acre-feet) to the canal diverters. The reduction in transportation costs results in an increase in the quantity of water demanded by canal diverters from 28,126 acre-feet in (Scenario 1) to 31,238 acre-feet.

As in Scenario 1, 11,363 acre-feet of canal seepage is unaffected by pumping (the Meade Externality). As a result of internalizing the canal seepage externality the Cheung portion of canal seepage declines from 3,112 acre-feet (in Scenario 1) to 1,949 acre-feet,. The decline in seepage can be traced to the virtual elimination of M&I pumpers demand for groundwater. As a consequence of now having to pay the transportation costs associated with the Cheung portion of the canal seepage externality, M&I pumping demand drops to zero. In the absence of all M&I pumping, drain water supply and demand increases from 3,158 acre-feet in (Scenario 1), to 3,625 acre-feet. The equilibrium supply price for M&I groundwater rises from $11.91 per acre-foot (in Scenario 1) to $253.06 per acre foot, which exceeds the price that M&I pumpers are willing to pay for groundwater (see figure 31).

The total elimination of M&I pumping demand in this scenario can be traced to the demand function of M&I water users. The price-quantity relationship in this function reflects the demand for groundwater by an M&I supplier, and does not include "transportation costs" associated with delivering M&I water to the end users. The scenario results represent a situation where the M&I supplier has another source of water besides groundwater that is available to meet end user demand.

The canal diverter's demand price declines slightly from $80.16 per acre-foot (in Scenario 1) to $76.48 per acre-foot because the absence of M&I pumping has reduced transportation costs. The drain water irrigators' demand price for water declines from $40.66 to $35.84 per acre-foot also, as a result of the absence of M&I pumping.

Scenario 3

As before in Scenario 3, the canal is lined thus eliminating the canal seepage externality entirely. The irrigation district supplies the same quantity of water to canal diverters (42,600 acre-feet), and in the absence of all canal seepage (and transportation costs) the canal diverters' demand increases to 42,600 acre-feet.

In the absence of canal seepage, the M&I pumpers' demand declines from 1,163 acre-feet (in Scenario 1) to 1,034 acre-feet due to increased pumping lift, at the same time the drain water irrigators water supply drops to zero. In the absence of transportation costs the canal diverters' demand price declines from $80.16 (in Scenario 1) to $65.00 per acre-foot.

Transportation Costs and Constraint Costs

Supply prices in Table 4 are the sum of supply costs, transportation costs, and constraint costs. At equilibrium therefore, transportation costs and constraint costs account for the difference that exists between supply cost and marginal supply price.

The following figures, which combine exogenous supply functions and demand functions, show the relationship between supply costs and supply prices for canal diverters, agricultural pumpers and drain water irrigators. Figure 33 combines the supply function of the irrigation district with the demand function of the canal diverters. Figure 34 combines supply function and demand function for the agricultural pumpers, and figure 35 does the same for the drain water irrigators. Red markers located at the intersection of the supply curve and demand curve in these figures indicate the quantity of water that would be supplied to and demanded by each entity in the absence of transportation costs and constraint costs (i.e. with supply-cost only). Green markers along the demand curves show the equilibrium quantities supplied and demanded given the transportation costs and constraint costs that are imposed in each scenario.

In figure 33, the difference between canal diverters' supply cost and supply price in all three scenarios is due to transportation costs stemming from canal seepage and to constraint costs stemming from limited canal capacity and/or water rights. In the absence of all canal transportation and constraint costs the canal diverter would divert about 219,000 acre-feet. However actual diversion is much less, mainly because of constraint costs. Depending on the scenario, the canal diverter's transportation costs and constraint costs range between $51.73 and $76.58 per acre-foot of diversion.

On the other hand, in figure 34, the agricultural pumpers' supply cost and supply price for water are the same, since transportation costs are fully incorporated into pumping costs (and there are no other constraint costs imposed on groundwater pumpers in the model). The agricultural pumpers' supply function fully accounts for the influence that canal seepage has on pumping lift.

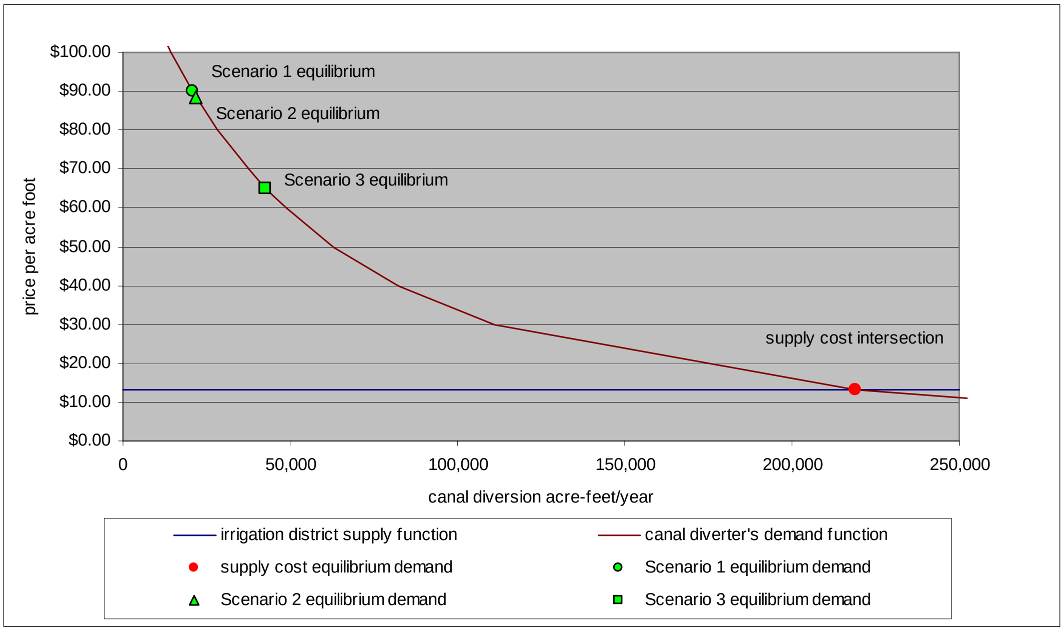


Figure 33: Canal diverters' equilibrium supply and demand (with agricultural pumpers).

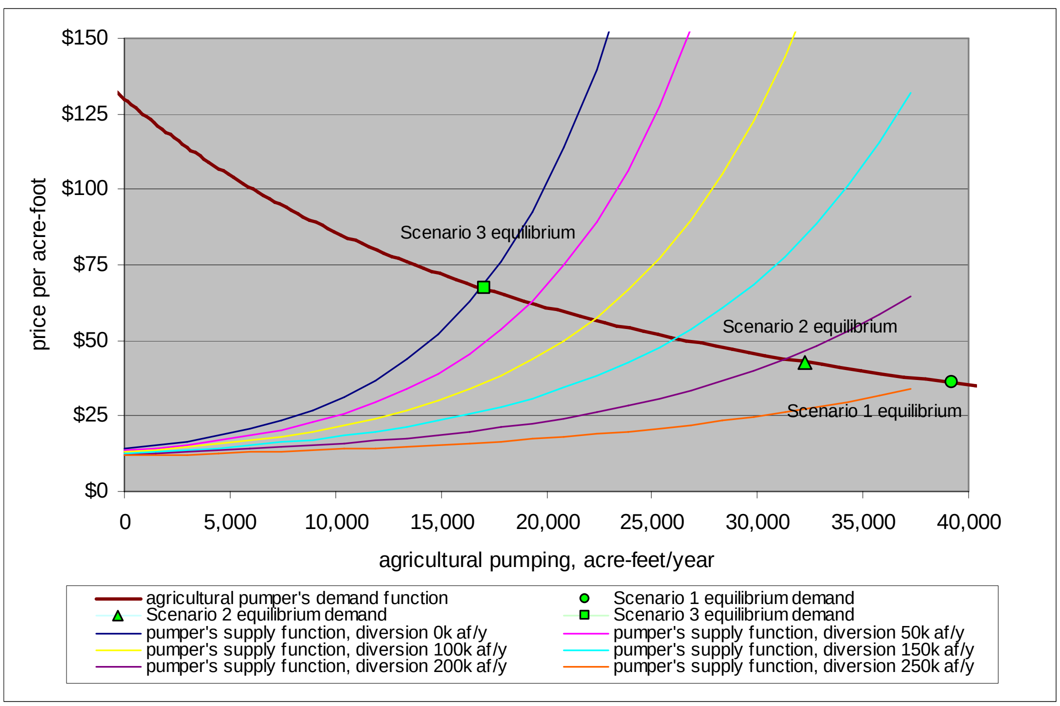


Figure 34: Agricultural pumpers' equilibrium supply and demand.

In figure 35, the difference between the drain water irrigators' supply cost and supply price is due to constraint costs stemming from the limited availability of drain return. In the absence of drain return constraints, the drain water irrigator would demand almost 11,000 acre-feet of drain return. Again, actual drain diversion is much less because of constraint costs associated with limited drain water availability. Depending on the scenario, the drain water irrigators' constraint cost ranges between $62.29 and $120.54 per acre-foot of drain return.

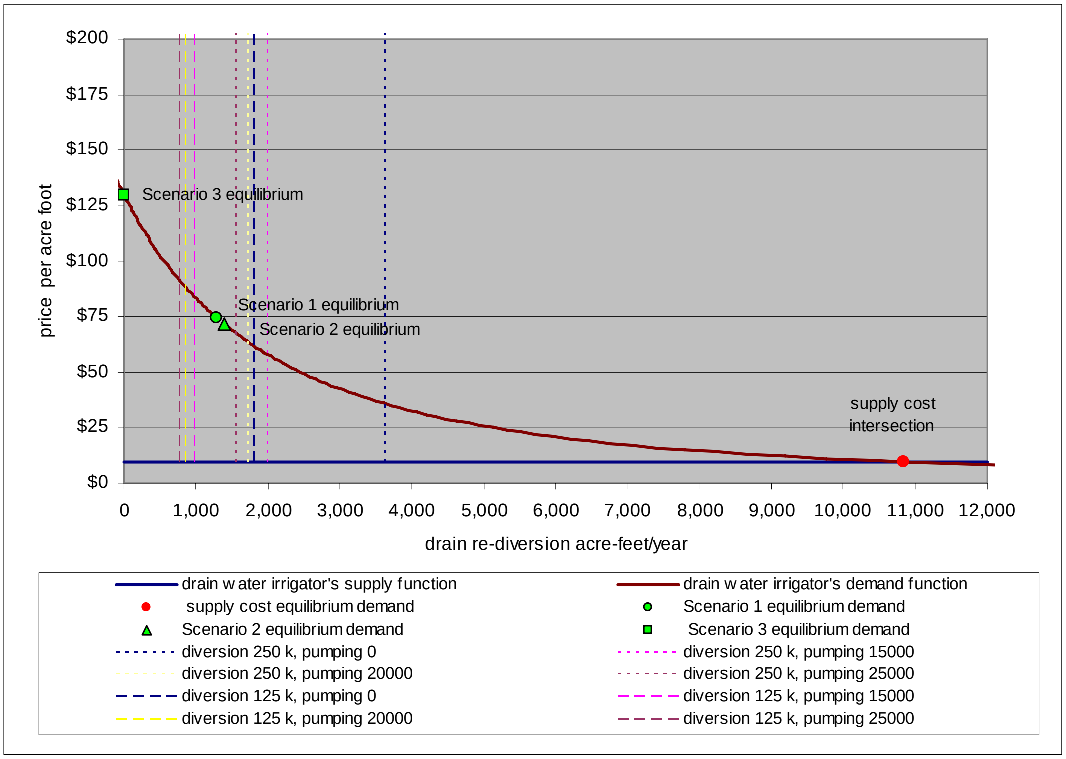


Figure 35: Drain water irrigators' equilibrium supply and demand (with agricultural pumpers).

Valuation of the Canal Seepage Externality

One of the central questions for this application of spatial water allocation modeling has to do with the valuation of the canal seepage externality by canal diverters, agricultural pumpers and drain water irrigators.

There are two ways that the value of canal seepage can be calculated; in terms of average supply cost of water, or in terms of (marginal) demand price for water. Valuing seepage in terms of average supply cost means that only the cost of supply to canal diverters, groundwater pumpers, and drain water irrigators is considered. Valuing seepage in terms of demand price means that canal seepage is valued based on equilibrated supply prices and demand prices i.e. the price of water at the margin.

Valuing Canal Seepage in Terms of Supply Cost

Determining the value of canal seepage based on supply cost of water requires that average supply costs of users be calculated in each scenario. Although the quantities of water supplied to canal diverters and drain water irrigators are constrained, their supply costs are fixed in all three scenarios. For the available supply, the canal diverters' average supply cost is $13.27 per acre-foot, and the drain water irrigator's average supply cost is $9.46 per acre-foot.

For agricultural pumpers, the average cost of groundwater is determined by integrating the pumper's (marginal) supply function (between zero and the quantity supplied) and then dividing by the quantity supplied (Table 3). In Scenario 1, this results in an average pumping cost of $19.32 per acre-foot. In Scenario 2 the average cost of pumping declines to $16.82 per acre-foot because reduced agricultural demand for groundwater results in reduced pumping lift. In Scenario 3, in the absence of all canal seepage, the average cost per acre-foot of groundwater pumped increases to $30.27 as a result of greatly increased pumping lift.

From the supply-cost perspective, the value of canal seepage to canal diverters is $13.27 per acre foot. From the same perspective, the value of canal seepage to the agricultural pumpers is, at a minimum, the difference between the average supply cost in Scenario 3 and the average supply cost in Scenario 1, i.e. $10.95 per acre-foot of groundwater pumped. This is considered the minimum value because the difference would be greater than $10.95 per acre-foot if demand for groundwater had not declined in Scenario 3 due to increased pumping lift.

Canal seepage has no impact on the supply cost of the drain water irrigators, since their supply cost is a fixed $9.46 per acre foot of water pumped from the drain. However seepage does affect the quantity of water that is available in the drain. The value of canal seepage to the drain water irrigator can therefore only be specified in terms of the drain water irrigator's demand price, which includes the drain return constraint cost.

Valuing Canal Seepage in Terms of Demand Price

The demand price value of an acre-foot of seepage to the canal diverter is the canal diverters' marginal demand price for water at the end of the canal. The canal diverters' demand price in Scenario 1 is composed of irrigation district supply cost, price of Cheung seepage, price of Meade seepage, and price of canal constraint.

In Scenario 1 these terms have the following values;

$$\$89.85 = \$13.27 + \$3.73 + \$4.08 + \$68.77 \qquad (20)$$

In words, the marginal value of one additional acre-foot of water to canal diverters, after having 20,862 acre-feet delivered by the irrigation district, is $89.85. For one additional acre-foot of water, canal diverters are willing to pay the supply cost of $13.27 to the irrigation district. They are also willing to pay a canal transportation cost that amounts to $7.81. They would also be willing to pay an additional $68.77 to avoid the constraint on water delivery that arises from limited canal capacity or water right. The constraint cost is essentially an opportunity cost.

In Scenario 2, where the canal seepage externality is internalized, payments for the seepage externality by groundwater pumpers and drain water irrigators are included in the canal diverters' demand price equation. The terms that make up demand price in this scenario have the following values;

\$88.31 = \$13.27 + \$3.29 + \$4.01 + \$70.77- \$2.68 - \$0.35 **(21)**

In Scenario 2, the marginal value of one additional acre-foot of water to canal diverters, after having 21,936 acre-feet delivered by the irrigation district, is $88.31.

The groundwater pumpers' reimbursement to the canal diverter for the additional acre-foot of water delivered to the end of the canal is $2.68 per acre-foot. The drain water irrigators' reimbursement is $0.35 per acre foot.

For one additional acre-foot of water, canal diverters are willing to pay the supply cost of $13.27 to the irrigation district. After receiving reimbursements from agricultural pumpers and drain water irrigators, they would be willing to pay $4.27 in canal transportation costs and $70.77 to avoid constraints on water delivery (i.e opportunity cost).

The reimbursements made by agricultural pumpers and drain water irrigators in Scenario 2 can be re-characterized in terms of a price per acre-foot of groundwater pumped and a price per acre-foot of drain water pumped, by multiplying them by the canal diverters' demand in Scenario 2 and then dividing them by, respectively, the agricultural pumpers' demand and the drain water irrigators' demand. Doing so results in a marginal payment for induced canal seepage of $1.82 per acre-foot of groundwater pumped, and a marginal payment of $5.53 per acre-foot of drain return diverted. These two payments reflect the marginal demand price (i.e. the value) of induced (Cheung) canal seepage to the agricultural pumper and the drain water irrigator.

The value of an acre-foot of seepage to drain water irrigators is also characterized by the drain water irrigators' marginal demand price for water. The two components which make up drain water irrigators' demand price are the drain pumping cost and the price of drain return constraint (i.e. the opportunity cost associated with drain water).
In Scenario 1 after pumping 1,286 acre-feet of water from the drain, the marginal demand price for an acre-foot of drain water is,

\$74.51= \$9.46 + \$65.05 **(22)**

In Scenario 2 after pumping 1,386 acre-feet of water from the drain, the marginal demand price for an acre foot of drain water is,

\$71.75 = \$9.46 + \$62.29 **(23)**

Internalizing the canal seepage externality actually reduces the drain water irrigators' marginal demand price by $2.76 per acre-foot. This is because internalizing the seepage externality also reduces agricultural demand for groundwater. While the drain water irrigators are paying $5.53

per acre-foot to the canal diverter, they are at the same time receiving a marginal benefit worth $2.76 per acre-foot because of reduced agricultural pumping.

Summary of Model Results

Valuations of canal seepage to canal diverters, groundwater pumpers, or drain water irrigators differ depending on how they are calculated and who they are valued by. The value of canal seepage can be calculated in terms of average supply cost of water, or in terms of the equilibrated demand price for water. Valuing seepage in terms of average supply cost means that only the cost of supply to the canal diverter, groundwater pumper or drain water irrigator is considered. Valuing seepage in terms of marginal demand price means that canal seepage is valued based on equilibrated supply prices and demand prices, and includes opportunity costs.

The supply-cost value of an acre-foot of canal seepage to canal diverters in the model area is fixed by irrigation district O&M and project repayment cost at $13.27 per acre-foot. On the other hand, the supply-cost value of an acre-foot of canal seepage to agricultural pumpers in the model area depends on how canal seepage affects their pumping costs. In this regard, an acre-foot of canal seepage is valued by agricultural pumpers at $10.95 per acre-foot of groundwater pumped. Canal seepage does not alter the supply cost of drain water irrigators in the model area which is fixed at $9.46 per acre-foot as long as there is water in the drain.

Demand-price valuations of canal seepage are derived from equilibrium model calculations of water quantities supplied and demanded by canal diverters, drain water irrigators and agricultural pumpers. Based on demand-price, the marginal value of canal seepage to canal diverters is $89.85 per acre-foot of water delivered to the end of the canal. The agricultural pumpers' payment of $1.82 per acre-foot of groundwater pumped reflects the marginal value of canal seepage to the agricultural pumper. Similarly, the drain water irrigators' payment of $5.53 per acre-foot of drain water pumped reflects the marginal value of canal seepage to drain water irrigators. All of these prices include opportunity costs.

Finally, it is important to restate the fact that the water valuations described previously are entirely dependent on site-specific hydrologic conditions as represented by the prototype model. Different hydrologic conditions, for instance different degrees of hydraulic connection between canals and the underlying aquifer, different proximities between wells, drains and canals, and different aquifer transmissivities all have the potential to significantly alter these canal seepage valuations. Canal seepage valuations could also be altered significantly by using different exogenous demand functions for canal diverters, drain water irrigators and groundwater pumpers representing different crop values or different M&I water uses.

References

Hydrology

Batu V., Aquifer Hydraulics , John Wiley and Sons, 1998.

Haitjema H.M., Analytic Element Modeling of Groundwater Flow, Academic Press, 1995

IDWR, Idaho Department of Water Resources spreadsheets and interim report prepared as part of Lower Boise Valley Water Use Planning Program (written communication from Zena Cook) 2000.

Lipscomb S. (written communication from USGS) spreadsheets and interim report of results of New York canal seepage studies, 2004.

Nace R.L. West S.W. and Mower R.W, "Feasibility of Ground Water Features of the Alternate Plan for the Mountain Home Project, Idaho", USGS Water Supply Paper 1376, 121pp, 1957.

Petrich, C.R., Simulation of Groundwater Flow in the Lower Boise River Basin. IWRRI Report 2004-02. 130 pp. 2004

Strack, ODL, Groundwater Mechanics, Prentice Hall, 1989.

Urban, S.M., Water Budget for the Treasure Valley Aquifer System for the Years 1996-2000., Idaho Department of Water Resources Treasure Valley Hydrologic Project research report. 60 pp. 2004.

USBR and IDWR, A Distributed Parameter Water Budget Data Base for the Lower Boise Valley, Bureau of Reclamation Report of Investigation, 105 pp. 2007.

USEPA, US Environmental Protection Agency, groundwater modeling software ftp://ftp.epa.gov/sandyrun/WhAEM_Development/Solver/, 2007.

USGS Water Data, National Water Information System: Groundwater levels for Idaho, http://nwis.waterdata.usgs.gov/id/nwis/gwlevels, 2008.

Economics

Bator, Francis. "The Anatomy of Market Failure." Quarterly Journal of Economics.782, no. 3, p.351-379, 1958.

Booker, J. F. and F. A. Ward. Instream Flows and Endangered Species in an International River Basin: The Upper Rio Grande. Amer. J. Agr. Econ. Vol. 81 No. 5, p.1262-1267, 1999.

Booker, J. F. and R. A. Young. “Modeling Intrastate and Interstate Markets for Colorado River Water Resources.” J. Environ. Econ. And Management. Vol. 26, p.66-87, 1994.

Broadway, R. W. and N. Bruce. Welfare Economics. Basil Blackwell, 1984.

CBO, Congressional Budget Office, Water Use Conflicts in the West: Implications of Reforming the Bureau of Reclamation’s Water Supply Policies, Vol. 97 pp. 1997.

CBO, Congressional Budget Office, How Federal Policies Affect the Allocation of Water, Publication Number 2589, 32 pp, 2006.

Cheung, Steven N. S. The Structure of a Contract and the Theory of a Non-Exclusive Resource. Journal of Law and Economics. Vol 13 p.49-70, 1970.

Connor, J. D. and G. M. Perry. “Analyzing the potenial for water quality externalities as the result of market water transfers.” Water Resources Research, Vol. 35 No. 9, p. 2833-2839, 1999.

Dahlman, C. J. “The Problem of Externality.” Journal of Law and Economics,Vol22, p.141-162, 1979.

Dinar, A. et. al. “Mechanism for Addressing Third-Party Impacts from Voluntary Water Transfers. Selected Paper at the American Agricultural Economics Association. Long Beach, CA. 2002.

Flinn, J. C. and J. W. B. Guise. “An Application of Spatial Equilibrium Analysis to Water Resource Allocation.” Water Resources Research, Vol 6, p.398-409, 1970.

GAMS General Algebraic Modeling System http://www.gams.com/, 2007.

Hamilton, J. R., G. Green, and D. Holland. Modeling the Reallocation of Snake River Water for Endangered Salmon. Amer. J. Agr. Econ. Vol. 81, p.1252-1256, 1999.

Hanak, E., “Stopping the Drain: Third-Party Resistance to Water Marketing in California”, American Agricultural Economics Association Annual Meeting, Montreal, Canada, July 27-30, 2003.

Howitt, R. E. et al. “Integrated Economic-Engineering Analysis of California’s Future Water Supply.” Center for Environmental and Water Resources Engineering, Univ. of CA, Davis. 1999.

Howitt R. and Hansen, K., “The Evolving Western Water Markets”, Choices; A Publication of the American Agricultural Economics Association, First Quarter, 2005.

Jenkins, Marion, Jay Lund, and R. E. Howitt. “Economic Losses for Urban Water Scarcity in California”. Journal of the American Water Works Association, 2003

Kuhn, H. W., Tucker, A. W. "Nonlinear programming". *Proceedings of 2nd Berkeley Symposium*: Berkeley: University of California Press, p.481-492, 1951.

Meade, J. E. "External Economies and Diseconomies in a Competitive Situation" Economic Journal 62 p.54-67, 1952.

Newlin, B. D. M. Jenkins, J. Lund, and R E. Howitt. "Southern California Water Markets: Potential and Limitations." Journal of Water Resources Planning and Management, ASCE, 2002.

Rao, B and Srivastava V. K, "The Econometrics of Disequilibrium Models", Greenwood Press 1990.

Takayama, T. and G. G. Judge. "Spatial and Temporal Price and Allocation" North Holland Publishing 1971.

Takayama, T. and G. G. Judge. "Equilibrium among spatially separated markets: A reformulation. Econometrica Vol. 32, p.510-524, 1964.

Vaux, H. and R. E. Howitt. "Managing Water Scarcity: An Evaluation of Interregional Transfers." Water Resources Research, Vol. 207, p.785-792, 1984.

Young, R. A. "Measuring Economic Benefits for Water Investments and Policies"(Technical Paper No. 338), World Bank, Washington, 1996.

Appendix A - Analytic Element Modeling Theory and Application to Canals, Wells and Drains

RD Schmidt

Groundwater models can provide important information regarding the type, location, and magnitude of externalities that arise in connection with hydrologic interactions between canals and wells. This paper presents some well known hydrologic modeling principles regarding the interaction between surface water conveyance devices (canals or drains) and groundwater wells, in the context of creating (or altering) hydrologic externalities.

The following discussion is based on the analytic element modeling work of Strack (1989) and Haitjema (1995). The analytic element modeling method has been widely applied in the context of regional hydrologic systems that involve aquifer interactions with streams and rivers. However, the method is also applicable to sub-regional systems involving aquifer interactions with canals (and drains), provided certain conditions regarding the spacing of canals and drains are met (see Appendix).

Basic Equations

Following Strack (1989), the governing differential equation for two-dimensional steady state groundwater flow in a confined or unconfined aquifer, is written in terms of a discharge potential Φ [L^3/T]as

$$\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}=0 \qquad (1)$$

where Φ is a function of hydraulic conductivity *k*[L/T], aquifer thickness *h* [L], and aquifer head ϕ,[L] given by the following pair of equations

$$\Phi = kh\phi-\frac{1}{2}kh^2 \quad \text{if } \phi \geq h \quad \text{(i.e confined conditions exist)} \qquad (2a)$$

$$\Phi = \frac{1}{2}k\phi^2 \quad \text{if } \phi < h \quad \text{(i.e. unconfined conditions exist)} \qquad (2b)$$

Note that in either equation, when $\phi = h$, $\Phi = \frac{1}{2}kh^2$, which demonstrates that the discharge potential Φ, is piecewise continuous at the boundary between confined and unconfined flow conditions.

Note also that the governing equation (1) is a linear with respect to Φ. This means that superposition of solutions to (1) is always appropriate, even when unconfined aquifer conditions exist.

A discharge potential function Φ which satisfies (1) is a harmonic function, which implies the existence of a conjugate harmonic function that is related to the discharge potential function by the Cauchy-Reimann conditions. In groundwater mechanics, the conjugate harmonic function is known as the stream function Ψ.

Since the discharge potential function and the stream function are related to each other by the Cauchy-Reimann conditions, it is possible to represent Φ and Ψ as the real and imaginary parts of a complex potential function Ω, where

$$\Omega = \Phi + i\Psi \qquad (3)$$

In groundwater mechanics, complex functions $\Omega(z)$, where z is a complex variable x + *i*y representing a two-dimensional spatial location, and where $\Phi(z)$ satisfies (1), are referred to as **analytic elements**.

The Well Analytic Element Solution

The steady state well function, often referred to as the Theim solution (Batu, 1998) is well known in groundwater mechanics. Using analytic element notation, the complex potential for a steady-state well $\Omega_w = \Phi_w + i\Psi_w$ is given by,

$$\Omega_w = \frac{Q}{2\pi}\ln(z - z_w) + C \qquad (4)$$

where the real variable Q is the well discharge rate, $z_w = x_w + iy_w$ is the location of the center of the well, $z = x + iy$ is a point in the aquifer where the well function is evaluated, and C is a constant of integration. That the well solution satisfies (1) can be demonstrated by separating Ω_w into real and imaginary parts, and taking derivatives with respect to *x* and *y*. Aquifer head at *z*, $\phi_w(z)$, is obtained from the real part of (4) by the use of (2a) or (2b), depending on whether the aquifer is confined or unconfined at *z*.

The Line Source-Sink Analytic Element Solution

The line source-sink function is also well known in groundwater mechanics. Conceptually it can be thought of as an infinite number of wells having a finite discharge rate, distributed along a line segment between two points.

Using analytic element notation, the complex potential for a line source-sink $\Omega_{ls} = \Phi_{ls} + i\Psi_{ls}$ is given by

$$\Omega_{ls} = \frac{\sigma_{ls}}{2\pi} e^{-i\alpha} \left[(z - z_1)\ln(z - z_1) - (z - z_2)\ln(z - z_2) + (z_1 - z_2) \right] + C \tag{5}$$

where the real variable σ_{ls} [L/T] is the line source-sink discharge rate per unit length, α is the angle of the line source-sink with respect to the positive x-axis, and $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ are the endpoints of the line source-sink.

The complex potential Ω_{ls} is path-independent differentiable with respect to *x* and *y*, and therefore *analytic*. That the line source-sink solution also satisfies (1) can again be demonstrated by separating Ω_{ls} into real and imaginary parts, and taking derivatives of Φ_{ls} with respect to *x* and *y*. Again, aquifer head $\phi_{ls}(z)$, is obtained from the real part of (5) by the use of (2a) or (2b), depending on whether the aquifer is confined or unconfined at *z*.

Analytic Element Boundary Conditions

The basic application of well and line source sink solutions in an analytic element groundwater model involves imposing either a flow condition (i.e. Q and σ_{ls} are known) or a potential condition (i.e. Φ_w and Φ_{ls} are known) on the real part of (4) and (5), and then solving the equations for whichever conditions remain unknown. Note that with (2a) and (2b) the potential condition equates to a head condition on ϕ_w and ϕ_{ls} .

In the present application, where the focus is on quantifying the hydrologic impact of groundwater withdrawals on canal seepage (or drain returns), the obvious choices are the potential condition for the line source sink solution, and the flow condition for the well solution.

It is important to note that the potential (and head) conditions that are specified for a canal (hereafter denoted Φ_c and ϕ_c respectively) refer to the canal itself, and not to the aquifer underlying it. This means that additional information will be needed regarding the resistance of the canal bed or the canal lining material.

Canal Boundary Condition, Confined Aquifer

If the aquifer is confined (i.e. $\phi \geq h$) then (by application of Darcy's Law)

$$\sigma_c = w \frac{\phi_a - \phi_c}{c} \tag{6}$$

where, ϕ_a is the head in the aquifer directly beneath the canal, w is the canal width, and c is the vertical conductance of the canal bed or canal lining. (Note: $c = \frac{\delta}{k_c}$ where δ is the thickness of the canal bed or lining, and k_c is its vertical hydraulic conductivity.) The canal boundary condition imposed on aquifer head directly beneath the canal is therefore

$$\phi_a = \phi_c + \frac{c \cdot \sigma_c}{w}. \tag{7}$$

Using (2a) to express (7) in terms of potentials gives

$$\Phi_a = \Phi_c + \frac{khc}{w}\sigma_c \tag{8}$$

which is the potential condition for a canal in a confined aquifer (figure 1).

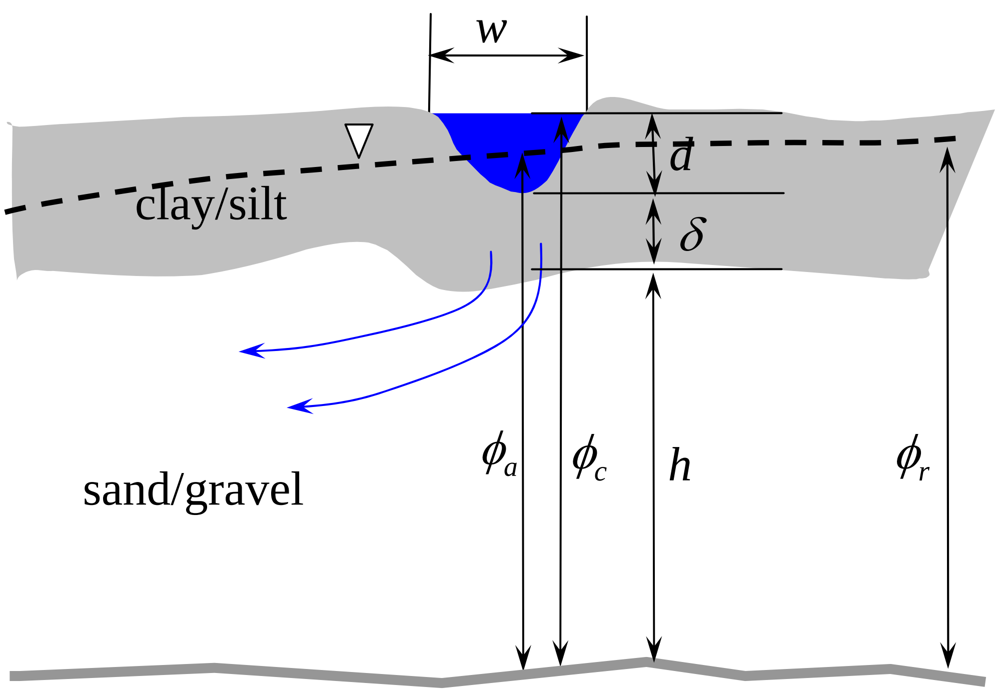


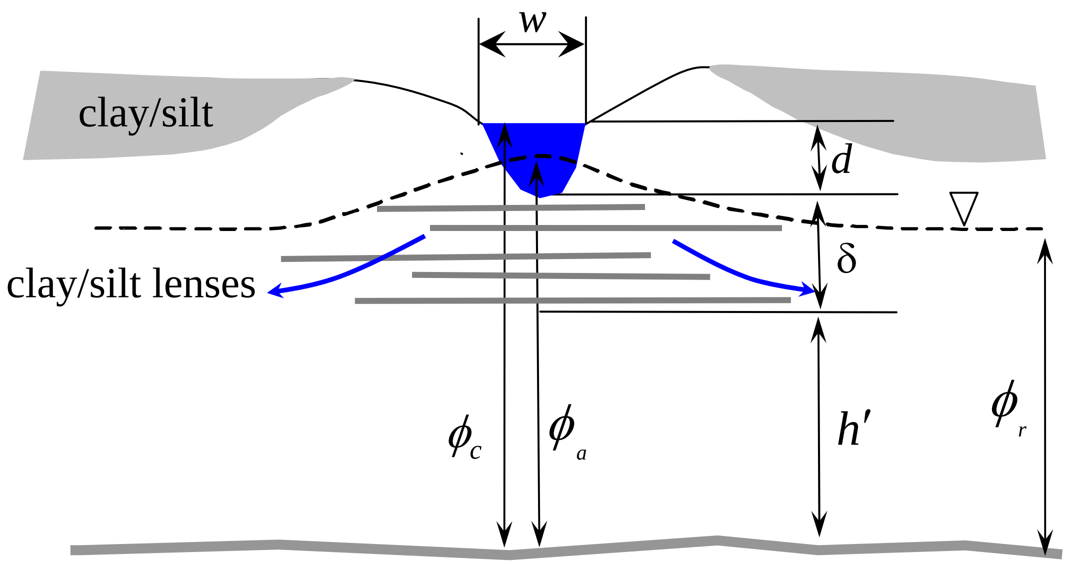
Figure 1. Conceptual diagram of the interaction between a canal and a confined aquifer

Canal Boundary Condition, Unconfined Aquifer

If the aquifer is unconfined (i.e. $\phi < h$) then (2b) is used to express the canal head condition in (7), in terms of potentials. In this case the potential condition is given by

$$\Phi_a = \Phi_c + \frac{kc\left(\frac{\phi_a + \phi_c}{2}\right)}{w}\sigma_c \tag{9}$$

Note in (9), that Φ_a is dependent on an (a priori unknown) head condition ϕ_a. Although the aquifer is unconfined, the watertable surface can still be in contact with the canal bottom (figure 2). Under these conditions it is necessary to provide an initial estimate of ϕ_a, and then solve the line sink equation (5) using iterative methods.



Canal Boundary Condition, Percolating Unconfined Aquifer

If the aquifer is unconfined ($\phi < h$) and the watertable surface is below the canal bottom (i.e. $\phi_a < h$) then the pore pressure beneath the canal is atmospheric and the seepage rate σ_c is independent of ϕ_a (figure 3). The flow specified condition for the canal in the unconfined percolating aquifer is given by

$$\sigma_c = -w\frac{\delta + d}{c} \tag{10}$$

where δ is the thickness of the resistance layer and d is the depth of water in the canal

Note that when $\phi_a = \phi_c - d - \delta$ (i.e. $\phi_a = h$), (6) and (10) give the same result.

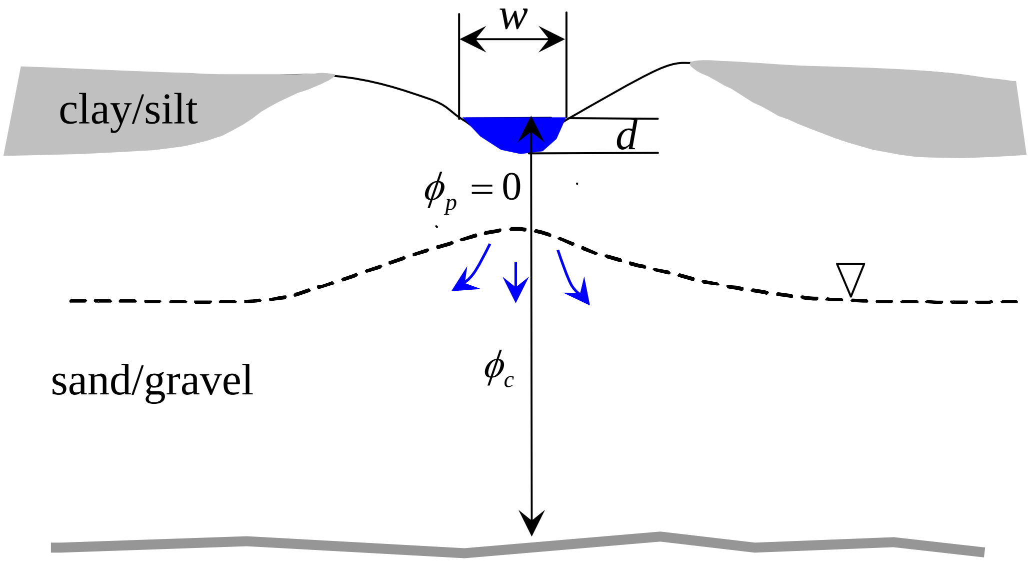


Figure 3. Conceptual diagram of the interaction between a canal and a percolating unconfined aquifer

Superposition of Analytic Element Solutions

As indicated earlier, individual analytic element solutions to (1) can be superimposed on one another in order to generate a comprehensive solution Ω for a groundwater flow problem that involves both well and canal features. The comprehensive potential solution for a single well and a single canal feature is given by

$$\Omega = \Omega_w + \Omega_c = \frac{Q}{2\pi}\ln(z - z_w) + \frac{\sigma_c}{2\pi}e^{-i\alpha}\left[(z - z_1)\ln(z - z_1) - (z - z_2)\ln(z - z_2) + (z_1 - z_2)\right] + C \qquad (11)$$

For multiple wells and multiple canal features the comprehensive potential solution is simply

$$\Omega = \sum_{i=1}^{n}\Omega_{w_i} + \sum_{i=1}^{n}\Omega_{c_i} + C \qquad (12)$$

Canal Head Conditions and Externalities

The relationship between diversion rate W_A needed to meet demand of canal user *A*, and the depth of water in the canal that supplies user A is given by

$$W_A = \frac{(a+b)d}{2} \cdot v \tag{13}$$

where *a* is the width at the bottom of a canal with a trapezoidal cross-section, and b is the width at the water surface, d is the depth of water in the canal, and *v* is the average cross-sectional flow velocity.

Rearranging terms,

$$d = \frac{2 \cdot W_A}{(a+b)v} \tag{14}$$

Relative to the datum of the underlying aquifer, the total canal head ϕ_c is given by

$$\phi_c = d + \delta + h \tag{15}$$

or with (13),

$$\phi_c = \frac{2 \cdot W_A}{(a+b)v} + \delta + h \tag{16}$$

It follows from (8) that if the aquifer is confined, then the potential condition that is needed beneath the canal in order to maintain diversion W_A in the canal is

$$\Phi_a = kh\frac{2 \cdot W_A}{(a+b)v} + kh\delta + kh\frac{c\sigma_c}{w} + \tfrac{1}{2}kh^2 \tag{17}$$

It follows from (9) that if the aquifer is unconfined, but the water table surface is in contact with the canal bottom, then the potential condition that is needed to maintain diversion W_A in the canal is

$$\Phi_a = \tfrac{1}{2}k\left(\frac{2 \cdot W_A}{(a+b)v} + \delta + h'\right)^2 + \frac{kc\sigma_c}{2w}\left(\frac{2 \cdot W_A}{(a+b)v} + \delta + h'\right) + \frac{kc\phi_a\sigma_c}{2w} \tag{18}$$

where h' is defined in the unconfined aquifer case as shown in figure 2.

If in (17) $\Phi_a \geq \frac{1}{2}kh^2$ or in (18) $\Phi_a \geq \frac{1}{2}kh'^2$, then the water table surface is in contact with the canal and the Cheung externality condition applies. If however in these equations $\Phi_a < \frac{1}{2}kh^2$ or $\Phi_a < \frac{1}{2}kh'^2$, then the water table surface is below the bottom of the canal, canal seepage is independent of the aquifer head condition, and the Meade externality condition applies.

Calculation of Canal loss Induced by Groundwater Pumping

Canal seepage induced by groundwater pumping is calculated by imposing three boundary conditions on the real part of (11).

> A far-field potential condition representing the average head in the aquifer is imposed at a distant point in the aquifer z_r.
>
> A flow condition which satisfies the demand of the groundwater pumper, $Q = W_B$, is imposed on the well.
>
> A potential (head) condition which is based on the diversion rate W_A needed to satisfy the demand of the canal user, is imposed at the center of the canal.

The far field potential condition is denoted $\Phi(z_r) = \Phi_r$ where Φ_r is based on a known (confined or unconfined aquifer) head condition at z_r and used to determine the value of *C*. Substituting the far-field expression above into the real part of (11), solving for *C*, and then substituting this expression for *C* back into the original equation yields,

$$\Phi = \mathrm{Re}\left[\frac{Q}{2\pi}\ln\left(\frac{z - z_w}{z_r - z_w}\right)\right] + \mathrm{Re}\left[\frac{\sigma_c}{2\pi}e^{-i\alpha}\begin{pmatrix}(z - z_1)\ln(z - z_1) - (z - z_2)\ln(z - z_2) - \\ (z_r - z_1)\ln(z_r - z_1) + (z_r - z_2)\ln(z_r - z_2)\end{pmatrix}\right] + \Phi_r \tag{19}$$

Imposing the well flow condition $Q = W_B$, and the canal potential condition from (17), $\Phi\left(\frac{z_1 + z_2}{2}\right) = \Phi_a$ on (19), and evaluating this expression at the center of the canal yields

$$\Phi_a = \mathrm{Re}\left[\frac{W_B}{2\pi}\ln\left(\frac{\frac{z_1 + z_2}{2} - z_w}{z_r - z_w}\right)\right] + \mathrm{Re}\left[\frac{\sigma_c}{2\pi}e^{-i\alpha}\begin{pmatrix}\left(\frac{z_2 - z_1}{2}\right)\ln\left(\frac{z_2 - z_1}{2}\right) - \left(\frac{z_1 - z_2}{2}\right)\ln\left(\frac{z_1 - z_2}{2}\right) - \\ (z_r - z_1)\ln(z_r - z_1) + (z_r - z_2)\ln(z_r - z_2)\end{pmatrix}\right] + \Phi_r \tag{20}$$

Substituting in the full expression for Φ_a from (17) and solving for σ_c yields,

$$\sigma_c = \frac{\dfrac{W_B}{2\pi}\mathrm{Re}\left[\ln\left(\dfrac{\dfrac{z_1+z_2}{2}-z_w}{z_r-z_w}\right)\right]-kh\dfrac{2\cdot W_A}{(a+b)v}-kh\delta-\tfrac{1}{2}kh^2+\Phi_r}{\left\{kh\dfrac{c}{w}-\dfrac{1}{2\pi}\mathrm{Re}\left[e^{-i\alpha}\left(\begin{array}{l}\left(\dfrac{z_2-z_1}{2}\right)\ln\left(\dfrac{z_2-z_1}{2}\right)-\left(\dfrac{z_1-z_2}{2}\right)\ln\left(\dfrac{z_1-z_2}{2}\right)-\\(z_r-z_1)\ln(z_r-z_1)+(z_r-z_2)\ln(z_r-z_2)\end{array}\right)\right]\right\}} \tag{21}$$

or with (16) and (2a)

$$\sigma_c = \frac{\dfrac{W_B}{2\pi}\mathrm{Re}\left[\ln\left(\dfrac{\dfrac{z_1+z_2}{2}-z_w}{z_r-z_w}\right)\right]-kh\phi_c+kh\phi_r}{\left\{kh\dfrac{c}{w}-\dfrac{1}{2\pi}\mathrm{Re}\left[e^{-i\alpha}\left(\begin{array}{l}\left(\dfrac{z_2-z_1}{2}\right)\ln\left(\dfrac{z_2-z_1}{2}\right)-\left(\dfrac{z_1-z_2}{2}\right)\ln\left(\dfrac{z_1-z_2}{2}\right)-\\(z_r-z_1)\ln(z_r-z_1)+(z_r-z_2)\ln(z_r-z_2)\end{array}\right)\right]\right\}} \tag{22}$$

Equation 22 could be used to evaluate the Cheung Externality condition for <u>confined aquifer settings only</u>. For unconfined settings, Φ_a would be entered from (18) instead of (17) and the far-field condition would come from (2b) instead of (2a). The non-linearity in the resulting equation would require an initial estimate of ϕ_a and solution by iterative methods.

In order to facilitate direct calculation of σ_c some simplifications of (22) are possible. First we can assume that the canal lies along the x-axis so that z_1 and z_2 are real numbers and $\alpha = 0$. We can also assume that z_r is on the x axis. Finally, it is clear from (22) that in this simple case, the distance between the well and the midpoint of the canal relative to the distance between the well and the far-field boundary is the only relevant spatial condition as far as the well is concerned. So we are free to place the well on the x axis also.

Under these conditions (22) reduces to

$$\sigma_c = \frac{\dfrac{W_B}{2\pi}\ln\left(\dfrac{\dfrac{x_1+x_2}{2}-x_w}{x_r-x_w}\right)-kh\phi_c+kh\phi_r}{\left[kh\dfrac{c}{w}-\dfrac{1}{2\pi}\left(\begin{array}{l}\left(\dfrac{x_2-x_1}{2}\right)\ln\left(\dfrac{x_2-x_1}{2}\right)-\left(\dfrac{x_1-x_2}{2}\right)\ln\left(\dfrac{x_1-x_2}{2}\right)-\\(x_r-x_1)\ln(x_r-x_1)+(x_r-x_2)\ln(x_r-x_2)\end{array}\right)\right]} \tag{23}$$

As (23) shows, under confined aquifer conditions σ_c and W_B are linearly related. However under unconfined aquifer conditions the relationship between σ_c and W_B would be non-linear.

A spreadsheet which calculates (23) shows the impact of different groundwater pumping rates on canal seepage. The spreadsheet results have been verified by comparing them with analytic element modeling results.

The GFLOW model is analytic element software developed for USEPA. It is able to simulate multiple canal and multiple well interactions with any mix of confined, unconfined, or percolating aquifer conditions.

Representing Localized 3-D Flow using 2-D Analytic Elements

Although the line source/sink solution is a two-dimensional (horizontal) flow equation, it has been widely used to represent streams and rivers which overlie an aquifer, i.e. conditions which create three dimensional groundwater flow in the vicinity of the stream or river. A 2-D simulation of localized 3-D flow is considered acceptable if 2-D flow conditions predominate throughout the rest of the aquifer. In a regional model where the spacing between streams and rivers is large relative to the thickness of the aquifer, this is generally not a problem. However neglecting the vertical gradient may be a problem in a sub-regional model if canals and drains situated above the aquifer, are very close together.

For a 2 D model application to be appropriate in a sub-regional model, the vertical head gradient that results from canal seepage or drain return should not extend more then half the distance to the nearest canal or drain.

In order to determine how far apart canals and drains should be in order for a 2-D approximation to be acceptable, a modeling experiment was conducted, using a 3-D analytical solution for partially penetrating wells (Haitjema, 1995). A row of 20 closely spaced, partially penetrating wells was used to approximate a canal or drain that is situated at the watertable surface. The 3-D model results indicated that the vertical head gradient extended outward from the row of wells to a maximum distance of about two aquifer thicknesses. Beyond this distance, no vertical groundwater gradient was apparent in the aquifer.

The model results imply that if individual canal and drain features are separated by a distance of al least four times the aquifer thickness, then representation of canals and drains using the 2-D line source-sink element is appropriate in a sub-regional model. In the Boise Valley, the upper aquifer layer varies between 200 and 300 feet thick, which implies that the 2-D line source sink element is appropriate if canal and drain features are 800 to 1200 feet apart. In actuality, in the Boise Valley, they are almost always much further apart then this.

References

GFLOW Analytic Element Modeling software http://www.haitjema.com/, 2007.

Haitjema H.M., Analytic Element Modeling of Groundwater Flow, Academic Press, 1995

Strack, ODL, Groundwater Mechanics, Prentice Hall, 1989.

Appendix B - Partial Equilibrium Modeling Theory and Application to Water Supply and Demand

Leroy Stodick

Partial Spatial Equilibrium Theory

Partial equilibrium models, following the example of Takayama and Judge, have traditionally been cast in the form of optimization problems. The modeler derives a quasi-welfare function or a net social payoff function which is maximized subject to various constraints and the equilibrium position is assumed to occur at the optimal point of the optimization problem. Few if any modelers attempt to explain why the optimum point is the equilibrium point and in some cases it may that this assumption is not justified. When Takayama and Judge published their book in the early 1970's, numerical optimization techniques were well understood but mixed complimentary programming was just beginning to be studied. With the advent of GAMS and the accompanying solvers, it is now possible to directly solve the equilibrium equations set up as complementary slackness equations or as a mixed complementary problem, instead of setting up an artificial optimization problem and assuming (in some cases wrongly) that the Kuhn-Tucker conditions for the problem coincide with the equilibrium conditions. It is now possible to define equilibrium conditions as a mixture of equations, inequalities, and complementary slackness equations and solve them using mixed complementary programming. It is no longer necessary or, in some cases, even possible to equate these equilibrium conditions to the Kuhn-Tucker conditions of an optimization problem.

The basic partial equilibrium model developed by Takayama and Judge (1971) involved spatially distributed trading entities that have both supply functions and demand functions, and included the following assumptions:

1. One homogeneous product is traded.
2. Linear supply and demand functions are defined for each entities (see Figure 1).
3. A fixed per unit transportation charge is applied to all exchange paths between trading entities.
4. No monopoly behavior exists.
5. No import or export taxes exist.

A quasi-welfare or net social payoff function was defined as the sum, over all trading entities, of consumer and producer surplus, less transportation costs. This function was maximized subject to two sets of conditions:

1. There is no excess demand.
2. Excess supply is possible

Definitions

Q_A is economic output of canal water user A

Q_B is the economic output of groundwater pumper B.

f_A is the production function for canal water user A .

f_B is the production function for groundwater pumper B

W_A is the water demand by canal water user A.

W_B is the water demand by groundwater pumper B

The Meade Externality for Canal Diverters and Groundwater Pumpers

Water demand by canal water user A enters into the production function of groundwater pumper B since water that is available to user B (via canal loss) depends partly on the water demand of user A.

$$Q_A = f_A(W_A, X_A)$$

$$Q_B = f_B(W_B, X_B, W_A)$$

This is a positive Meade externality for groundwater pumper B

The Cheung Externality for Canal Diverters and Groundwater Pumpers

Water demand by canal user A enters into the production function of groundwater pumper B since water that is available to user B (via canal losses) depends partly on the water demand of user A. In addition, when the canal is in contact with the watertable surface, water demand by user B enters in to the production function of user A, since groundwater pumping induces additional losses from the canal.

$$Q_A = f_A(W_A, X_A, W_B)$$

$$Q_B = f_B(W_B, X_B, W_A)$$

The Chung externality is positive for groundwater pumper B and negative for canal user A.

Partial Equilibrium Modeling with GAMS

In order to explicitly state the Kuhn-Tucker conditions for what Takayama and Judge call a quasi-welfare function (or net social payoff function), define the following variables:

Exogenous variables:

λ_i – intercept of the inverse of the linear demand function in region i
ω_i –absolute value of the slope of the inverse of the linear demand function in region i
γ_i – intercept of the inverse of the linear supply function in region i
η_i – slope of the inverse of the linear supply function in region i
t_{ij} – per unit transportation cost from region i to region i

Endogenous variables:

y_i – amount demanded in region i
x_i – amount supplied in region i
X_{ij} – amount exported from region i and imported into region j
ρ_i – market demand price in region i
ρ^j – market supply price in region j (subscript for ρ means market demand price, superscript for ρ means market supply price)

The Kuhn-Tucker conditions (and coincidentally the equilibrium conditions) are:

1. $\lambda_i - \omega_i y_i - \rho_i \leq 0$ and $(\lambda_i - \omega_i y_i - \rho_i)y_i = 0\ \forall\ i$
2. $-\gamma_i - \eta_i x_i + \rho^i \leq 0$ and $(-\gamma_i - \eta_i x_i + \rho^i)x_i = 0\ \forall\ i$
3. $\rho_j - \rho^i - t_{ij} \leq 0$ and $(\rho_j - \rho^i - t_{ij})X_{ij} = 0\ \forall\ i,j$
4. $\sum_{j=1}^{n} X_{ji} - y_i \geq 0$ and $(\sum_{j=1}^{n} X_{ji} - y_i)\rho_i = 0\ \forall\ i$
5. $x_i - \sum_{j=1}^{n} X_{ij} \geq 0$ and $(x_i - \sum_{j=1}^{n} X_{ij})\rho^i = 0\ \forall\ i$

where it is understood that these equations hold only at the optimum point (the equilibrium point).

The economic interpretation of each of these equations is as follows:

Equation 1: when the consumption in the *i*th region (y_i) is positive, then the regional demand price
($p_i = \lambda_i - \omega_i y_i$) is equal to the market demand price ρ_i. When $y_i = 0$, the market demand price (ρ_i) must be greater than or equal to the regional demand price (p_i). This essentially results in a kinked demand function. As long as consumption in a region remains positive, the market demand price can be found along the demand curve. When consumption is 0, the market demand price may be above the demand curve on the vertical axis. In this case, the market demand price has risen so high that consumption in the *i*th region is driven to 0.

Equation 2: when the supply in the *i*th region (x_i) is positive, then the regional supply price ($p^i = \gamma_i + \eta_i x_i$) is equal to the market supply price ρi. When $x_i = 0$, the market supply price (ρ^i) must be less than or equal to the regional supply price (p^i). This results in a kinked supply function. As long as supply in a region remains positive, the market supply price can be found along the supply curve. When supply is 0, the market supply price may be below the supply curve on the vertical axis. In this case, the market supply price has dropped so low that supply in the *i*th region is driven to 0.

Equation 3: this is the so-called price linkage equation. When X_{ij} is positive (positive transfer of goods from region i to region j), the difference between the market demand price in region j and the market supply price in region i must be the per unit-cost of transporting the goods from region i to region j. If $X_{ij} = 0$, then the difference between the market demand price in region j and the market supply price in region i must be less than or equal to the per unit-cost of transporting the goods from region i to region j.

Equation 4: this equation insures that demand is met in all regions (no excess demand). If the market demand price in the *i*th region (ρ_i) is greater than 0, then consumption (y_i) is exactly equal to the quantity imported into the region (X_{ji}). (X_{ii} is the amount produced in region i that is consumed in region i.) If the market demand price is 0, then the amount imported into the region (including the amount produced locally which is consumed locally) is greater than or equal to the amount consumed in the region.

Equation 5: this equation allows for excess supply. If the market supply price in the *i*th region (ρ^i) is greater than 0, then the amount produced in region i (x_i) is exactly equal to the amount exported to all other regions as well as the amount that is consumed locally (X_{ij}). If the market supply price is 0, then the amount supplied must be greater than or equal to exports plus local consumption.

The Boise Valley Partial Equilibrium Model

The water allocation model developed for the Boise Valley Project differs from the model described above

(Subscripts refer to demand quantities, prices, and functions.)

(Superscripts refer to supply quantities, prices, and functions.)

Model 1: The basic model with no externalities.

Exogenous variables:

1. For each region with a demand function, the functional form and parameters of a monotonically decreasing demand function must be specified. $q_i = f_i(p_i)$.
2. For each region with a supply function, the functional form and parameters of a monotonically increasing supply function must be specified. $q^i = f^i(p^i)$. (Horizontal supply functions ($p^i = \text{constant}^i$) are allowed.
3. For each allowed path between regions, the per-unit conveyance cost must be specified. t_{ij} = cost of transporting one unit of the commodity from region i to region j.

Endogenous variables:

1. x_{ij} = amount of commodity exported from region i and imported into region j.

2. q_i = amount of commodity consumed in region i.

3. q^i = amount of commodity produced in region i.

4. p_i = locally determined demand price.

5. p^i = locally determined supply price.

6. ρ_i = globally determined demand price.

7. ρ^i = globally determined supply price.

p_i is the price determined by the inverse demand function. $p_i = f_i^{-1}(q_i)$. This is the price that consumers in region i are willing to pay in order to consume quantity q_i. p_i is determined by the consumer's utility function and budget constraints. ρ_i, on the other hand, is the price that consumers must pay in order to purchase quantity q_i on the global market. This price is determined by how much other regions are willing to supply and how much demand exists in other regions. p_i does not necessarily equal ρ_i.

Similarly, p^i is the price determined by the inverse supply function $p^i = f^{i(-1)}(q^i)$. This is the price that suppliers must receive in order to supply quantity q^i. p^i is determined by the producer's profit function and production constraints. ρ^i, on the other hand, is the price that producers will receive if they sell quantity q^i on the global market. This price is determined by the demand and supply conditions in all regions. p^i does not necessarily equal ρ^i.

Equilibrium conditions for spatial price equilibrium:

A variable printed in boldface denotes the value of that variable at equilibrium.

1. $\boldsymbol{\rho}_j - \boldsymbol{\rho}^i - t_{ij} \le 0$ and $\mathbf{x}_{ij}(\boldsymbol{\rho}_j - \boldsymbol{\rho}^i - t_{ij}) = 0$, $\mathbf{x}_{ij} \ge 0$.

2. $\mathbf{p}_i - \boldsymbol{\rho}_i \le 0$ and $\mathbf{q}_i(\mathbf{p}_i - \boldsymbol{\rho}_i) = 0$, $\mathbf{q}_i \ge 0$.

3. $\boldsymbol{\rho}^i - \mathbf{p}^i \le 0$ and $\mathbf{q}^i(\boldsymbol{\rho}^i - \mathbf{p}^i) = 0$, $\mathbf{q}^i \ge 0$.

4. $\sum_j \mathbf{x}_{ji} - \mathbf{q}_i \ge 0$ and $\boldsymbol{\rho}_i(\sum_j \mathbf{x}_{ji} - \mathbf{q}_i) = 0$, $\boldsymbol{\rho}_i \ge 0$.

5. $\mathbf{q}^i - \sum_j \mathbf{x}_{ij} \ge 0$ and $\boldsymbol{\rho}^i(\mathbf{q}^i - \sum_j \mathbf{x}_{ij}) = 0$, $\boldsymbol{\rho}^i \ge 0$.

Model 2: A model with interactions which look like externalities but which have prices assigned to them by the model:

We start with the basic model and add the following endogenous variables for each externality to be incorporated into the model:

1. $EX_{i,j,k}$ – the quantity of the externality accruing at node k from the quantity shipped from node i to node j.

2. $R_{i,j,k}$ – the price to be assigned to $EX_{i,j,k}$.

We also need the following exogenous variables:

1. $c_{i,j,k}$ – the cost of transferring the externality from the route i,j to the node k.

We also need the following function for all nodes with positive externalities:

$F_{i,j,k}(X_{i,j})$ – this function describes the relationship between the quantity shipped from node i to node j and the quantity available to node k as an externality. It must have the property that. $F_{i,j,k}(0) = 0$ and must be continuous with continuous first derivatives and must be monotonically increasing.

The model now becomes:

1. $$\boldsymbol{\rho}_j - \boldsymbol{\rho}^i - t_{ij} + \sum_k \mathbf{R}_{ijk} \frac{\partial F_{ijk}(x_{ij})}{\partial x_{ij}} \le 0$$
   and $$\mathbf{x}_{ij}(\boldsymbol{\rho}_j - \boldsymbol{\rho}^i - t_{ij} + \sum_k \mathbf{R}_{ijk} \frac{\partial F_{ijk}(x_{ij})}{\partial x_{ij}}) = 0, \mathbf{x}_{ij} \ge 0.$$
2. $\mathbf{p}_i - \boldsymbol{\rho}_i \le 0$ and $\mathbf{q}_i(\mathbf{p}_i - \boldsymbol{\rho}_i) = 0, \mathbf{q}_i \ge 0$.
3. $\boldsymbol{\rho}^i - \mathbf{p}^i \le 0$ and $\mathbf{q}^i(\boldsymbol{\rho}^i - \mathbf{p}^i) = 0, \mathbf{q}^i \ge 0$.
4. $\sum_j \mathbf{x}_{ji} + \sum_k \sum_j \mathbf{EX}_{kji} - \mathbf{q}_i \ge 0$ and $\boldsymbol{\rho}_i(\sum_j \mathbf{x}_{ji} + \sum_k \sum_j \mathbf{EX}_{kji} - \mathbf{q}_i) = 0,\ \boldsymbol{\rho}_i \ge 0$.
5. $\mathbf{q}^i - \sum_j \mathbf{x}_{ij} - \sum_j \sum_k \mathbf{EX}_{ijk} \ge 0$ and $\boldsymbol{\rho}^i(\mathbf{q}^i - \sum_j \mathbf{x}_{ij} - \sum_j \sum_k \mathbf{EX}_{ijk}) = 0, \boldsymbol{\rho}^i \ge 0$.
6. $\boldsymbol{\rho}_k - \boldsymbol{\rho}^i - c_{ijk} - \mathbf{R}_{ijk} \le 0$ and $\mathbf{EX}_{ijk}(\boldsymbol{\rho}_k - \boldsymbol{\rho}^i - c_{ijk} - \mathbf{R}_{ijk}) = 0,\ \mathbf{EX}_{ijk} \ge 0$.
7. $\mathbf{F}_{ijk}(\mathbf{x}_{ij}) - \mathbf{EX}_{ijk} \ge 0$ and $\mathbf{R}_{ijk}(\mathbf{F}_{ijk}(\mathbf{x}_{ij}) - \mathbf{EX}_{ijk}) = 0, \mathbf{R}_{ijk} \ge 0$.

In order to make EX into a true externality, we must remove any price attached to the quantity. We do that by removing equation 6 from the model described above and changing equation 7 from a complementary slackness equation into an equality.

Model 3: a model with true externalities.

1. $$\boldsymbol{\rho}_j - \boldsymbol{\rho}^i - t_{ij} + \sum_k c_{ijk} \frac{\partial F_{ijk}(x_{ij})}{\partial x_{ij}} \le 0$$
   and $$\mathbf{x}_{ij}(\boldsymbol{\rho}_j - \boldsymbol{\rho}^i - t_{ij} + \sum_k c_{ijk} \frac{\partial F_{ijk}(x_{ij})}{\partial x_{ij}}) = 0, \mathbf{x}_{ij} \ge 0.$$
2. $\mathbf{p}_i - \boldsymbol{\rho}_i \le 0$ and $\mathbf{q}_i(\mathbf{p}_i - \boldsymbol{\rho}_i) = 0, \mathbf{q}_i \ge 0$.
3. $\boldsymbol{\rho}^i - \mathbf{p}^i \le 0$ and $\mathbf{q}^i(\boldsymbol{\rho}^i - \mathbf{p}^i) = 0, \mathbf{q}^i \ge 0$.
4. $\sum_j \mathbf{x}_{ji} + \sum_k \sum_j \mathbf{EX}_{kji} - \mathbf{q}_i \ge 0$ and $\boldsymbol{\rho}_i(\sum_j \mathbf{x}_{ji} + \sum_k \sum_j \mathbf{EX}_{kji} - \mathbf{q}_i) = 0,\ \boldsymbol{\rho}_i \ge 0$.
5. $\mathbf{q}^i - \sum_j \mathbf{x}_{ij} - \sum_j \sum_k \mathbf{EX}_{ijk} \ge 0$ and $\boldsymbol{\rho}^i(\mathbf{q}^i - \sum_j \mathbf{x}_{ij} - \sum_j \sum_k \mathbf{EX}_{ijk}) = 0, \boldsymbol{\rho}^i \ge 0$.

6. $\mathbf{F}_{ijk}(\mathbf{x}_{ij}) - \mathbf{EX}_{ijk} = 0$

Notice that the six equilibrium conditions do not correspond to the Kuhn-Tucker conditions of an optimization problem. No net social payoff function is constructed and it is not possible to determine the total social welfare of the system. Total consumer surplus in particular cannot be calculated although it may be possible, although not necessary, to determine the net benefits of trade in terms of change in consumer and producer surplus.

References

Anania, Giovanni and McCalla, Alex F., "Does Arbitraging Matter? Spatial Trade Models and Discriminatory Trade Policies." American Journal of Agricultural Economics. 2(1991):pp. 103-117.

Brooke, A., Dendrick, D. and Meeraus, A., GAMS: A User's Guide. The Scientific Press, South San Fransisco, CA, 1998.

Ferris, M. and Munson, T., "Complementarity Problems in GAMS and the PATH Solver." GAMS – The Solver Manuals. GAMS Development Corporation, Washington, DC, 1999.

Flinn, J. C. and Guise, J. W. B., "An Application of Spatial Equilibrium Analysis to Water Resource Allocation." Water Resources Research. 6:2(1970): pp 398- 409.

Harker, Patrick T., "Alternative Models of Spatial Competition." Operations Research. 34:3(1968): pp. 410-425.

Jefferson, Robert W. and Boisvert, Richard N., A Guide to Using the General Algebraic Modeling System (GAMS) for Applications in Agricultural Economics. Cornell University, 1989.

Takayama, T. and Judge, G. G., Spatial and Temporal Price and Allocation Models. North Holland Publishing Company, Amsterdam, 1971.

Takayama, T. and Judge, G. G., "Spatial Equilibrium and Quadratic Programming." Journal of Farm Economics. Vol. 46(1964):pp. 36-48.

Vaux, H. J. Jr. and Howitt, Richard E., "Managing Water Scarcity: An Evaluation of Interregional Transfers." Water Resources Research. Vol. 20(1984):pp. 785-792.

Appendix C - Average versus Marginal Demand Price Specification for Agricultural and M&I Water Users

RG Taylor

Average versus Marginal Price

Residential water demand is often specified with the marginal price observed from a rate schedule (Howe 1998). Despite theory and empirical evidence, average price has been championed, in both early (Foster and Beattie 1981a) and recent research (Neiswiadomy and Cobb 1993; Michelsen, McGuckin, and Stumpf 1999), as the behaviorally relevant price perceived by consumers (Howe 1998). Espey et al. (1997) conducted a meta-analysis on 124 observations on the price elasticity of residential water demand. They found that the use of average price in place of marginal price resulted in higher price elasticities. If price elasticity estimates differ systematically when average price is used in place of marginal price then different methodological and policy implications ensue.

Marginal price specification in demand can be empirically equivalent to average price in two cases: (1) when firms are competitive price takers, average price equals marginal price (Edmonds 1977) and; (2) when the data result from a single equation demand function which is double log, price elasticities are invariant to marginal or average price specification (Halvorsen 1975).

As opposed to the equivalency argument in the two cases above, average price has been specified in water demand based on the assumption that consumers *perceive* price to be average price. Foster and Beattie (1981b) submit that the fixed charge is perceived as a marginal cost: " ...consumers view their choices in the fixed charge block not as a fixed cost (minimum charge) with associated zero-marginal cost for some range of water used, but as a variable cost associated with the desired level of consumption in the first block. Thus, a positive "marginal cost" is *perceived* in this block. If so perceived, marginal and average cost would be the same for the amount consumed ..." (pages 258-259)

Average price replaces marginal price so that the demand function becomes:

$$W = f\left(P_{Avg}, P_x, M\right) \qquad [1]$$

where, P_{Avg} is computed as a utility's average revenue (total revenue divided by total water sales). The perception argument justifies average price because consumers are alleged to ignore the details of the rate schedule when water represents a small portion of their expenditures (Foster and Beattie 1981b). At issue in the marginal price versus average revenue specification is the consumer's knowledge and decision mechanism. Utility bills inform customers of total expense and in many instances marginal price. Whether, on a widespread basis, consumers convert billing information to an average price to judge cost of water consumption is an empirical question. Except for the special cases described earlier, average revenue is not marginal price but average revenue *could* be a proxy or measure upon which water consumption decisions are mistakenly based.

Specification of average price in demand poses serious estimation difficulties when utilities charge a constant monthly fee sometimes in combination with either flat or block rates. Taylor's (1975) study hinted at the average price specification problem:
"Also, there is the problem that when average price is defined ex post as the ratio of total expenditures to quantity consumed, as is the usual procedure, a negative dependence between quantity and price is established that reflects nothing more than arithmetic." (p. 78).

Consider the customer bill derived from a rate schedule that includes a fixed charge coupled with a generic variable water rate;

$$TR_{it} = K_i + R(W_{it}), \qquad [2]$$

where, *TR* is total receipts, *K* is revenue derived from a monthly fee fixed by each water utility, and *R(W)* is revenue derived from the variable portion of the rate schedule; indexed over utilities (*i*), and time periods over which revenues are collected (*t*). By definition, the fixed fee charged by the i^{th} utility is fixed over all quantities consumed. Average price based on average revenue is thus;

$$P_{Avg_{it}} = \frac{TR_{it}}{W_{it}} = \frac{K_i}{W_{it}} + \frac{R(W_{it})}{W_{it}} \qquad [3]$$

Equation (3) shows that average revenue is composed of average fixed revenue and average variable revenue components when fixed fees are included in the utility rate schedule. The average revenue definition of the price is substituted into the demand function (equation 1). However, for utilities charging only a fixed fee, the average variable revenue in equation (3) equals zero, and the demand function as shown in (4) becomes an identity with W_{it} on both sides of the equation:

$$W_{it} = f\left(P_{Avg_{it}}, P_x, M\right) = f\left(\frac{K_i}{W_{it}}, P_x, M\right) \qquad [4]$$

When the only charge is a fixed fee, the price quantity relationship is average fixed revenue, which is a rectangular hyperbola with unitary price elasticity. Taylor's "arithmetic" is thus an identity. A perfect fit ($R^2 = 1$) results when this identity is "properly" estimated. For example, assume a linear demand model is estimated for equation (4), $W = b_0 + b_1\left(\frac{K}{W}\right) + b_2 P_x + b_3 M$. If ordinary least squares (OLS) was capable of returning the identity the estimated coefficients would be: $b_0 = 0$, $b_2 = 0$, $b_3 = 0$, and $b_1 = -\frac{W^2}{K}$.

Price elasticity is defined as $E_p = \left(\frac{\partial W}{\partial P}\right)\left(\frac{P}{W}\right)$ *and* $\frac{\partial W}{\partial P} = b_1 = -\frac{W^2}{K}$, when price is average fixed revenue. Thus, price elasticity is $E_p = -\left(\frac{W^2}{K}\right)\left(\frac{P}{W}\right) = -\frac{WP}{K}$, but both *WP* and *K* are total revenue and therefore price elasticity must always be minus one. The embedded identity is easily detected by OLS for a double log demand function. The double log demand equation is shown in (5) where for ease of illustration the nuisance parameters (other prices and income) are omitted:

$$W_{it} = \beta_1 \left[\frac{K_i}{W_{it}} + \frac{R(W_{it})}{W_{it}}\right]^{\beta_2} \qquad [5]$$

For the i[th] utility, charging only a fixed fee (i.e., average variable revenue = $R(W_{it}) = 0$), the observed time series of price-quantity data again is a rectangular hyperbola. The estimated price elasticity (β_2) will equal minus one and β_1 will equal a constant ***k*** representing the fixed fee. If all utilities choose the same value for their fixed fee then the fit of quantity demanded on average price will be perfect and no other explanatory variables should enter the regression. If the value of the fixed fee varies across utilities then there is a set of rectangular hyperbolas whose distance from the origin varies with the magnitude of the fixed charge. (If variations in the fixed charge across the utilities could be explained, data would again fall on a single rectangular hyperbola.) The presence of variation in the fixed fee across utilities or over time may tend to obscure the existence of the embedded identity.

The crux of the argument is that when average fixed revenue (fixed fee) is high, relative to the average variable revenue portion of the rate structure, the change in total revenue over time, will be nearly invariant to water consumption. Thus, the effect of a rate schedule that is dominated by the fixed fee is to dampen any price effects on quantity demanded. When the variance of K_i is small relative to W_{it}, the major source of variation in the average price data originates from variation in water usage, W_{it}. Thus, as average variable revenue tends to zero, estimated demand tends to an unitary elasticity identity and measures of fit will increase and price elasticity will approach minus one.

Short run substitutes for water are virtually nonexistent. Our data are cross sectional and thus portray long run consumer water consumption decisions. Expensive water-saving appliances, irrigation systems, and landscaping are long-run substitutes for water. Utility bills are notoriously vague in detailing water costs and usage. Yet, knowledge of total water costs at different points on the rate schedule and projections of costs over time is essential in making capital investment decisions. Be it from their own experience, talking with neighbors, or contacting the utility, consumers do make long run adjustments to increasing water costs.

Boise Valley Water Provider Rate Schedules and Quantities

A municipal water utility is a local monopoly, with pricing latitude to set a rate schedule and thus administer water price. Typical rate schedules of municipal water distributors include unmetered connections with fixed monthly fees, flat rate (proportional) pricing, increasing block rates, declining block rates, and combinations of fixed monthly fees with block rates. Utility customers paid varying marginal (or average) prices set by an administered rate schedule unique to each utility. The various rate schedules used by Boise Valley M&I water providers are described in Table 1. Table 2 shows the current and projected future demand for M&I water in the Boise Valley.

The econometric implications of rate schedule(s) must be considered when estimating price elasticity. Thus, identification of the demand function requires specification of an appropriate simultaneous model structure (Taylor 1975, Billings and Agthe 1980).

Price is measured at the margin in the neoclassical demand model in the presence of a rate schedule and with informed consumers (Foster and Beattie 1981a, 1981b; Howe 1998). Cost for a consumer of a given amount of domestic water consumption is:

$$TC_W = \int P_{M\arg}\, dW; \qquad [6]$$

where, $P_{M\arg|W=w} = R(W)$, the rate schedule, is a function of water quantity (W) and thus marginal price is conditional on water consumed. The resulting demand function is:

$$W = f\left(R\left(W\right), P_x, M\right); \qquad [7]$$

where, marginal price $P_{Marg,}$ selected from a given rate schedule R(W), is the appropriate price measure when the consumer maximizes utility subject to an income constraint (M) and P_x is the numeraire price. The quantity of water demanded is thus determined simultaneously with the rate schedule. When simultaneity is present, OLS under a declining block tariff will underestimate or overestimate demand elasticity depending on whether the supply schedule is steeper than the demand schedule or otherwise (Griffin and Martin, 1981).

The double–log demand function is a very popular functional form because of the ease of estimation and the demand coefficients can be directly interpreted as the constant-elasticity of demand. The constant elasticity of demand implies an equal price responsiveness to quantity of water use at high and low prices. In contrast, with a linear demand function, elasticity changes at every level of water quantity consumed and consumers are less sensitive to price the lower the price. Al-Quanibet and Johnson 1985 criticize the double-log functional form as being inconsistent with utility theory.

Table 1: Boise Valley water providers, rate schedules

Municipal Provider	Description of Charge	Summer****: monthly fixed charge	Summer**** monthly incremental rate ($ per 100 cu.ft)	Winter: monthly fixed charge	Winter monthly incremental rate ($ per 100 cu.ft)
United Water Idaho	Incremental charge levied on > 0 use	$ 6.63	$ 1.12	$ 6.63	$ 0.89
Meridian, City of	Fixed Charge only: 0 - 4,000 gal.	$ 6.48	$ 0.92	$ 6.48	$ 0.92
Garden City, City of	Incremental charge levied on > 0 use	$ 7.68	$ 0.60	$ 7.68	$ 0.60
Capitol Water Corp.	Fixed charge only	$ 24.05	$ -	$ 12.65	$ -
Eagle Water Co.	Fixed charge only: 0 - 600 cu/ft.	$ 8.17	$ 0.45	$ 8.17	$ 0.45
Kuna, City of	Incremental charge levied on > 0 use	$ 15.50	$ 0.65	$ 15.50	$ 0.65
Nampa, City of	750 - 3,999 cu. ft.	$ 5.64	$ 0.78	$ 5.64	$ 0.78
	4,000 + cu. ft.	$ 5.64	$ 0.46	$ 5.64	$ 0.46
Nampa, Outside City	750 - 3,999 cu. ft.	$ 11.28	$ 1.86	$ 11.28	$ 1.86
	4,000 + cu. ft.	$ 11.28	$ 0.92	$ 11.28	$ 0.92
Caldwell, City of	Incremental charge levied on > 0 use	$ 3.35	$ 0.55	$ 3.35	$ 0.55
Middleton, City of	Fixed charge only: 0 - 400 cu/ft.	$ 8.00	$ 0.94	$ 8.00	$ 0.94
Middleton, Outside City	Fixed charge only: 0 - 400 cu/ft.	$ 16.00	$ 1.87	$ 16.00	$ 1.87
Parma, City of	Incremental charge levied on > 0 use	$ 14.00	$ 0.97	$ 14.00	$ 0.97
Star	Fixed charge only	N/A	N/A	N/A	N/A
Melba	Fixed charge only	N/A	N/A	N/A	N/A
Eagle Water Company	Fixed charge only	N/A	N/A	N/A	N/A
Other Public Water Systems	Fixed charge only	N/A	N/A	N/A	N/A
Individual wells	Pumping costs *	TBE	TBE	TBE	TBE
Irrigation Water Use **	$30 annual per acre @ 2.4acft per acre	$ 0.01	$ -	$ -	$ -
Irrigation Water Use ***	$200 annual per developed acre	$ 0.09	$ -	$ -	$ -

Notes:

N/A: not available at this time

TBE: to be estimated

* pumping costs at various well depths in the valley are required here

** Non pressurized only. The number of gallons is calculated over the entire population.

** *Pressurized (Based on Nampa Meridian irrigation district average payments)

****Summer months are May thru September inclusive

Table 2: Boise Valley water providers, current and future demand

Municipal Provider	Total Residential Population served (02)	Total Residential Population Projected in 2025	Gallons Provided in Winter in 2000 (per capita per day)	Gallons Provided in Summer in 2000 (per capita per day)	Gallons Provided in Year 2000 (per capita per day)	Gallons Provided in Winter 2025 (per capita per day)	Gallons Provided in Summer 2025 (per capita per day)	Total Amount Provided in Year 2025 (per capita per day)
United Water Idaho	196,945	323,074	104	59	163	121	68	189
Meridian, City of	29,700	63,693	104	59	163	121	68	189
Garden City, City of	9,000	17,728	104	59	163	121	68	189
Capitol Water Corp.	7,400	9,000	104	59	163	121	68	189
Eagle Water Co. (Mun.)	4,328	6,739	104	59	163	121	68	189
Kuna, City of	4,590	9,263	104	59	163	121	68	189
Nampa, City and Outside	44,550	88,868	101	57	157	109	61	171
Caldwell, City of	23,000	50,544	101	57	157	109	61	171
Middleton, City and Outside	2,978	4,194	101	57	157	109	61	171
Parma, City of	1,817	2,091	101	57	157	109	61	171
Star	1,344	2,019	102	58	160	115	65	180
Melba	296	528	102	58	160	115	65	180
Eagle Water Company	1,000	3,411	104	59	163	121	68	189
Other Public Water Systems	30,000	55,000	102	58	160	115	65	180
Individual wells	76,052	94,052	102	58	160	115	65	180
Irrigation Water Use *	433,000	730,204	0	13	13	0	13	13
Irrigation Water Use **	433,000	730,204	0	13	13	0	13	13

* Non pressurized only. The number of gallons is calculated over the entire population with United Water data. They are likely to underestimate water use

** Pressurized. The number of gallons is calculated over the entire population with United Water data. They are likely to underestimate water use

1. Coefficient values obtained from Cook et al., December 2001
2. For irrigation company controlled water, dual use is calculated across entire population because of the way the coefficients were originally developed.
3. Water use numbers are based on assumption of real price levels remaining the same over time but income levels rising as predicted by JC. They differ among counties mainly because of differences in lot sizes (Canyon is higher) and incomes (Ada is higher).
4. Summer months are May thru September inclusive
5. Average Percentage of summer use obtained from United Water Records

Agricultural Water Demand References

Ayer, H. W., and P.G. Hoyt. "Crop-Water Production Functions: Economic Implications for Arizona." College of Agriculture, University of Arizona, Agricultural Experiment Station Technical Bulletin No. 242, September 1981.

Bateman, I. J., and A.P. Jones. "Contrasting Conventional with Multi-Level Modeling Approaches to Meta-Analysis: Expectation Consistency in U.K. Woodland Recreation Values." *Land Economics* 79 (2003)235-258.

Bernardo, D.J., N. K. Whittlesey, K.E. Saxton, and D.L. Bassett. "An Irrigated Model for Management of Limited Water Supplies. *Western Journal of Agricultural Economics* 12 (1987):164-73.

Boyle, K. J., G. Poe, and J. C. Bergstrom. "What Do We Know about Groundwater Values?: Preliminary Implications from a Meta Analysis of Contingent-Valuation Studies." *American Journal of Agricultural Economics* 76 (1994):1055-61.

Brouwer, R., I .H. Langford, I. J. Bateman, T. C. Crowards, and R.K. Turner. "A Meta-Analysis of Wetland Contingent Valuation Studies." Global Environmental Change Working Paper GEC 97-20, Centre for Social and Economic Research on the Global Environment, University College London and University of East Anglia, Norwich, 1997.

Dalhuisen, J. M., R. Florax, H.L.F. de Groot, and P. Nijkamp. "Price and Income Elasticities of Residential Water Demand: A Meta-Analysis." *Land Economics* 79 (2003):292-308.

Espey, M., J. Espey, and W.D. Shaw. "Price Elasticity of Residential Demand for Water: A Meta-Analysis." *Water Resources Research* 33 (1997):1369-74.

Frank, M.D., and B. R. Beattie. *The Economic Value of Irrigation Water in the Western United States: An Application of Ridge Regression*. Texas A&M University, Texas Water Resources Institute Technical Report No. 99, March 1979.

Gisser, M., R.R. Landford, W.D. Gorman, B.J. Creel, and M. Evans. "Water Trade-Off between Electric Energy and Agriculture in the Four Corners Area." *Water Resources Research* 15 (1979):529-38.

Hartman, L.M., and N. Whittlesey. "Marginal Values of Irrigation Water: A Linear Programming Analysis of Farm Adjustments to Changes in Water Supply." Colorado State University, Agricultural Experiment Station Technical Bulletin No. 70, 1961.

Heady, E.O., H.C. Madsen, K.J. Nicol, and S.H. Hargrove. "National and Interregional Models of Water Demand, Land Use, and Agricultural Policies." *Water Resources Research* 9

(1973):777-97.

Hedges, T. R. “Water Supplies and Costs in Relation to Farm Resource Decisions and Profits on Sacramento Valley Farms.” Giannini Foundation Research Report No. 322, University of California, Davis, 1977.

Hexem, R.W., and E.O. Heady. *Water Production Functions for Irrigated Agriculture*. Ames: Iowa State University Press, 1978.

Hooker, M.A., and W.E. Alexander. “Estimating the Demand for Irrigation Water in the Central Valley of California.” *Journal of the American Water Resources Association* 34 (1998):497-505.

Howitt, R.E., W.D. Watson, and R.M. Adams. “A Reevaluation of Price Elasticities for Irrigation Water.” *Water Resources Research* 16 (1980):623-8.

Johnston, W. E. “The Economist’s Role in Water Pricing Policy.” Paper presented at the Water Pricing Conference, University of California, Los Angeles, March 1968. University of California Water Resources Center Report No. 13, pp. 28-40, 1968.

Kelley, S., and H. Ayer. *Water Conservation Alternatives for California Agriculture: A Microeconomic Analysis*. Washington, D.C.: U.S. Department of Agriculture, Economic Research Service, National Resource Economics Division, 1982.

Kelso, M.M., W.E. Martin, and L.E. Mack. *Water Supplies and Economic Growth in an Arid Environment: An Arizona Case Study*. Tucson: University of Arizona Press, 1973.

Moore C.V., and T.R. Hedges. “A Method for Estimating the Demand for Irrigation Water.” *Agricultural Economics Research* 15 (1963):131-53.

Moore, C.V., J.H. Snyder, and P. Sun. “Effects of Colorado River Water Quality and Supply on Irrigated Agriculture.” *Water Resources Research* 10 (1974):137-44.

Moore, R.M., N.R. Gollehon, and M.B. Carey. “Multicrop Production Decisions in Western Irrigated Agriculture: The Role of Water Price.” *American Journal of Agricultural Economics* 76 (1994):859-74.

Nieswiadomy, M. “The Demand for Irrigation Water in the High Plains of Texas, 1957-80.” *American Journal Agricultural Economics* 67 (1985):619-26.

Ogg, C.W., and N.R. Gollehon. “Western Irrigation Response to Pumping Costs: A Water Demand Analysis Using Climatic Regions.” *Water Resources Research* 25 (1989):767-73.

Scheierling, S.M., R.A. Young, and G.E. Cardon. “Price-Responsiveness of Demand for Irrigation Water Withdrawals versus Consumptive Use: Estimates and Policy Implications.” Paper presented at the Western Agricultural Economics Association Meetings, Denver, CO, July 11-15, 2003. http://agecon.lib.umn.edu

Scheierling, SM., J.B.Loomis and R.A.Young. “Irrigation Demand: A Meta Analysis of Price Elasticities”, Unpublished paper, 2004

Smith, V.K., and Y. Kaoru. “Signals or Noise? Explaining the Variation in Recreation Benefit Estimates.” *American Journal of Agricultural Economics* 72 (1990):419-33.

Stanley, T.D., and S.B. Jarrell. “Meta-Regression Analysis: A Quantitative Method of Literature Surveys.” *Journal of Economic Surveys* 3 (1989):161-70.

Shumway, C.R. “Derived Demand for Irrigation Water: The California Aqueduct.” *Southern Journal of Agricultural Economics* 5 (1973):195-200.

Municipal and Industrial Water Demand References

Agthe, D. E., and R. B. Billings. 1987. "Equity, Price Elasticity, and Household Income Under Increasing Block Rates for Water." *Amer. J. of Economics and Sociology* 46(3): 273-86.

Arbues, F. M., A. Garcia-Valinas, and R. Martinez-Espineira. 2003. “Estimation of Residential Water Demand: A State-of-the-Art Review.” *The Journal of Socio-Economics* 32 (2003): 81-102.

Billings, R. B., and D. E. Agthe. 1980. "Price Elasticities for Water: A Case of Increasing Block Rates: Reply." *Land Economics* 57(2): 276-78.

Billings, R. B. 1982. "Specification of Block Rate Price Variables in Demand Models." *Land Economics* 58 (3): 386-94.

———. 1983. "Specification of Block Rate Price Variables in Demand Models: Reply." *Land Economics* 59 (3): 370-71.

Burt, O. R. 1971. “Positivistic Measures of U.S. Agricultural Supply: Some New Approaches.” *Amer. J. Ag. Econ*. 53 (4): 674-75.

Charney, A. H., and G. C. Woodard. 1984. "A Test of Consumer Demand Response to Water Prices: Comment." *Land Economics* 60 (4): 414-16.

Chicoine, D. L., S. C. Deller, and G. Ramamurthy. 1986. "Water Demand Estimation Under Block Rate Pricing: A Simultaneous Equation Approach." *Water Resources Research* 22 (6): 859-63.

Chicoine, D. L., and G. Ramamurthy. 1986. "Evidence on the Specification of Price in the Study of Domestic Water Demand." *Land Economics* 62 (1): 26-32.

Cook, Z. L., S. Urban, M. Maupin, J Church, 2001, "Domestic, Commercial, Municipal and Industrial Water Demand Assessment and Forecast in Ada and Canyon Counties, Idaho", IDWR publication.

Deller, S. C., D. L. Chicoine, and G. Ramamurthy. 1986. "Instrumental Variables Approach to Rural Water Service Demand." *Southern Economic Journal* 53(2): 333-46.

Dhrymes, P. J. 1978. *Introductory Econometrics*. New York: Springer-Verlag.

Edmonds, J. A. 1977. *The Marginal-Average Price Elasticity Problem*. Draft Report. Oak Ridge National Laboratory.

Espey, M., J. Espey, and W. D. Shaw. 1997. "Price Elasticity of Residential Demand for Water: A Meta-Analysis." *Water Resources Research* 33 (6): 1369-74.

Foster, H. W., and B. R. Beattie. 1981a. "Urban Residential Demand for Water in the United States: Reply." *Land Economics* 57 (2): 257-65.

———. 1981b. "On the Specification of Price in Studies of consumer Demand under Block Price Scheduling." *Land Economics* 57 (4): 624-29.

Goldberger, A. 1964. *Econometric Theory*. New York: Wiley.

Gottlieb, M. 1963. Urban domestic demand for water in the United States. Land Economics 39(2) 204-210

Griffin, R. C., and C. Chang. 1990. "Pretest Analyses of Water Demand in Thirty Communities." *Water Resources Research* 26 (10): 2251-55.

Griffin, A. H.., and W. Martin. 1981. "Price elasticities for water: a case of increasing block rates: A comment." *Land Economics* 26 (10): 2251-55.

———. 1991. "Seasonality in Community Water Demand." *Western Journal of Agricultural Economics* 16 (2): 207-17.

Halvorsen, R. 1975. "Residential Demand for Electricity." *Review of Economics and Statistics* 57 (1): 12-18.

Hanemann, W. M. 1998. "Price and Rate Structures." In *Urban Water Demand Management and Planning* eds. D.D. Baumann, J.J. Boland and W.M. Hanemann. McGraw-Hill Company. 137-79.

Howe, C. W. and F. P. Linaweaver, 1967 The impact of price on residential water demand. Water Resources Research. 3(1) 13-32.

Howe, C.W. 1982. "Impact of Price on Residential Water Demand: Some New Insights." *Water Resources Research* 18 (4): 713-16.

———. 1998. "Forms and Functions of Water Pricing: An Overview." In *Urban Water Demand Management and Planning* eds D.D. Baumann, J. J. Boland and W.M. Hanemann. McGraw-Hill Company. p181-89.

Jones, C. V., and J. R. Morris. 1984. "Instrumental Price Estimates and Residential Water Demand." *Water Resources Research* 20 (2): 197-202.

LaFrance, J. T. 1998. "The Silence of the Lambdas: Comment." *Amer. J. Ag. Econ*. 80 (1): 221-30.

Michelsen, A. M., T. McGuckin, and D. Stumpf. 1999. "Effectiveness of Conservation Programs in Municipal Water Demand." *J. of Amer. Water Resources Assoc.* 35 (3): 593-03.

Nieswiadomy, M. L. 1992. "Estimating Urban Residential Water Demand: Effects of Price Structure, Conservation, and Education." *Water Resources Research* 28(3): 609-15.

Nieswiadomy, M. L., and S. L. Cobb. 1993. "Impact of Pricing Structure Selectivity on Urban Water Demand." *Contemporary Policy Issues* 11 (3): 101-13.

Nieswiadomy, M. L., and D. J. Molina. 1988. "Urban Water Demand Estimates Under Increasing Block Rates." *Growth and Change* 19 (1): 1-12.

———. 1989. "Comparing Residential Water Demand Estimates under Decreasing and Increasing Block Rates Using Household Data." *Land Economics* 65 (3): 280-89.

———. 1991. "A Note on Price Perception in Water Demand Models." *Land Economics* 67 (3): 352-59.

Nordin, J. A. 1976. "A Proposed Modification of Taylor's Demand Analysis: Comment." *Bell J. of Econ.* 7 (2): 719-21.

Opaluch, J. J. 1982. "Urban Residential Demand for Water in the United States: Further Discussion." *Land Economics* 58 (2): 225-27.

——. 1984. "A Test of Consumer Demand Response to Water Prices: Reply." *Land Economics* 60 (4): 417-21.

Schefter, J. E., and E. L. David. 1985. "Estimating Residential Water Demand Under Multi-Part Tariffs Using Aggregate Data." *Land Economics* 61 (3): 272-80.

Shin, J. 1985. "Perception of Price When Price Information is Costly: Evidence from Residential Electricity Demand." *Review of Economics and Statistics* 67(4) 591-98.

Stevens, T. H., J. Miller, and C. Willis. 1992. "Effect of Price Structure on Residential Water Demand." *Water Resources Bulletin* 28 (4): 681-85.

Taylor, L. D. 1975. "The Demand for Electricity: A Survey." *Bell Journal of Economics and Management* 6 (1): 74-110.

Taylor, R. G., John R. McKean, and R. A Young. Alternate Price Specifications for Estimating Residential Water Demand with Fixed Fees. Land Economics 30(3) p463-475.

Van Helden, G. J., P. S. Leeflang, and E. Sterken. 1987. "Estimation of Demand for Electricity." *Applied Economics* 19 (1): 69-82.

Walters, L. L., and R. A. Young. 1994. *Economic Factors Affecting Residential Water Demand in Colorado*. Completion Rpt. 175, Colorado Water Resources Research Institute, Colorado State University, Ft. Collins, CO.

Williams, M. 1985. "Estimating Urban Residential Demand for Water under Alternative Price Measures." *Journal of Urban Economics* 18 (2): 213-25.

Williams, M., and B. Suh. 1986. "The Demand for Urban Water by Customer Class." *Applied Economics* 18 (12): 1275-89.