Subjective Probability

Best Practices in Dam and Levee Safety Risk Analysis

Part A – Risk Analysis Basics

Chapter A-6 Subjective Probability

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US Army Corps of Engineers®

Outline

- Objectives
- Key Concepts
- What is Subjective Probability?
- Preparation
- Team Elicitation
- Documenting Results
- Coherence Checks
- Exercise

Objectives

- Understand what subjective probability is
- Understand how to prepare for estimating probabilities subjectively
- Understand how to evaluate evidence in estimating probabilities
- Understand ways to estimate subjective probabilities that minimize most biases
- Understand pitfalls in documenting results
- Understand how to perform coherence checks on subjective probabilities
- (Note: biases will not be discussed in detail here. Facilitators are encouraged to read the manual for a better understanding of biases and how to minimize them.)

Key Concepts

- Estimating subjective probabilities is difficult for most people initially
- Several steps can aid in estimating reasonable probabilities subjectively:
  - Selecting experienced facilitator to guide the process
  - Establishing proper group for estimating
  - Preparation
  - Problem definition
  - Unbiased identification of pros and cons
  - Establishing aids for making numerical estimates
  - Making estimates and discussing reasoning within the group
  - Performing coherence checks
  - Building the case
  - (Note: Many of the ideas in this presentation come from the book, "Degrees of Belief, Subjective Probability and Engineering Judgment," ASCE Press, by Steven G. Vick

What is Subjective Probability?

Why?

- For many dam safety applications there is limited statistical data to work with
- We are evaluating the conditional probabilities of events that have not been experienced (or whose precursor events have not yet actually occurred)
- We are evaluating probabilities for which there are no analytical models for computing them

How?

- How can I estimate a probability when I don’t know?
- Not knowing is the essence of uncertainty
- Subjective probability does not require us to know, only to honestly consider what we don’t know and what we know
- A subjective probability estimate is an expression of our state of knowledge at the moment

Inductive vs. Deductive

- Estimating probabilities requires inductive reasoning
    - Weighing the evidence and arguments, evaluating the uncertainty, and estimating a number based on this information
- Most engineers and scientists are used to deductive reasoning
    - Follow the deterministic procedure and the answer is known
- A shift in thinking may be difficult for some, and procedures have been developed to help (further described in this presentation)

Frequency vs. Causal

- Frequency information deals with how often something occurs
  - One third of this inventory of 300 water retention embankments experienced internal erosion incidents
  - But be careful, how homogeneous is the inventory, what mode was in play, how does this inventory compare to the dam or levee being evaluated?
- Causal information deals with indicators that might suggest future performance
  - A similar case history, analysis results, construction details, past performance, etc.
- Assessors must be able to synthesize both types of information in making probability estimates

Preparation

Facilitation

- Selecting an experienced facilitator to guide the process is essential
- The facilitator must:
  - Have many years of experience in dam and/or levee design, analysis, construction, operational issues and safety reviews
  - Help ensure a proper team has been selected for the estimating process
  - Be able to guide a diverse team through the process of estimating probabilities
  - Be able to critically evaluate the results for coherence and reasonableness

Team Approach

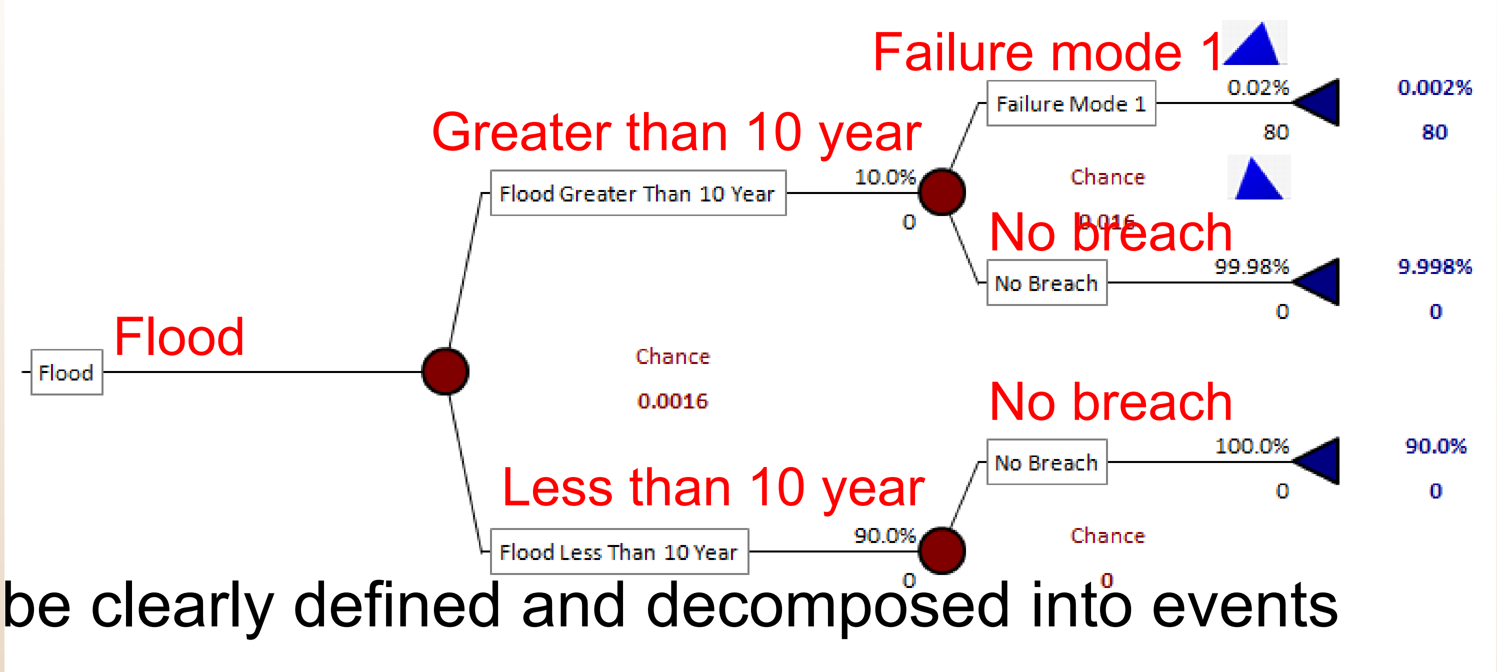
- A diverse group is preferred for making subjective probability estimates
  - e.g. geotech, instrumentation, geology, materials specialist for an internal erosion issue – each will look at the information from their own perspective
- A group will typically come up with a better estimate than any single individual

Preparation

- Retrieve and review as much original information as possible (there is often more than you think)
  - Instrumentation, construction, design, analysis, performance
- Be careful of others' summaries and interpretations of the original information
- Develop a basic understanding of probability concepts (covered in another presentation)
- Become familiar with relevant case histories
- Understand any applicable frequency information

Team Elicitation

Decomposition



- Potential failure modes must be clearly defined and decomposed into events that constitute an event tree
- Decomposition is key to ensuring estimates are within the well-calibrated range (discussed later)
- Critically evaluate “what could terminate a failure sequence” and make sure it is represented in the event tree
- Common event trees typically lead to more consistency
- A clear and unambiguous definition of the event probability to be estimated must be written down to ensure all estimators are on the same page

Describe the Event

- A branch of an event tree is labeled “Pore Pressures Rise”
- Will all members of the estimating team interpret this the same?
- How will the team members interpret that description?
  - Where do pore pressures rise?
  - By how much do they rise?
  - What is the range of possible outcomes from this rise?

Collect the Evidence

- Evidence making an event "More Likely" and "Less Likely" must be thoroughly identified and collected.
- This is typically done in a two column table format.
- Discuss the evidence as needed including the "Strength" and "Weight" of each piece. Be careful of "Hearsay" evidence – corroborate information and cite the written documentation.
- Document the evidence thoroughly. Ask, "why did we say that?" and write down the answer in the table. Repeat until the evidence will be clear to someone outside the group reading in 5 or 10 years time.

Strength vs. Weight

- Strength – How convincing is it?
  - I heard they saw turbid water pouring into the tailrace the last time the reservoir was filled.
  - Is this a strong argument that a problem exists?
- Weight – How good is it in terms of quality, quantity, and predictive validity?
  - Is this incident documented?
  - Who said it? Are they qualified to judge?
  - Where was the water entering the tailrace?
  - What is the possible source (reservoir seepage, runoff, drain flow)?

More Likely and Less Likely Tables Event – Initiation of Internal Erosion

Factors making the event More Likely	Factors making the event Less Likely
Uncontrolled seepage exits exist	Depth of seepage exits unchanging
Less seepage reduction above D-Shale	Historic seeps carry no sand
Lower portion of sandstone more friable	No evidence of internal erosion on first filling
Caving occurred in vertical drill holes	No piping observed in gypsum solutioned areas
Evidence of some gypsum solutioning above D-Shale	Seepage observed for 30 years since tunnel without piping incident – Conditions are stable
Horizontal drill holes made sand	No completely uncemented discharge faces
Some tunnel drain holes made sand upon drilling	Friable sandstone medium and coarse grained
Some recent changes in seepage areas with decrease in tunnel flows	Measured seepage not increasing
	Changed seepage pathway required to cause significant observable particle movement
	Most rock fractures not open
	Current tunnel drains no sand and not deteriorating

Verbal Descriptor	Defined Convention	Experimental Data Reagan et al, 1989
Virtually Impossible due to known physical conditions or processes that can be described and specified with almost complete confidence	0.01	0.02 (0.0 - 0.05)
Very Unlikely although the possibility cannot be ruled out	0.1	0.10 (0.02 - 0.15)
Equally Likely with no reason to believe that one outcome is more or less likely than the other (when given two outcomes)	0.5	0.50 (0.45 - 0.55)
Very Likely but not completely certain	0.9	0.85 (0.75 - 0.90)
Virtually Certain due to known physical processes and conditions that can be described and specified with almost complete confidence	0.99	0.90 (0.90 - 0.995)

Verbal Transformation (Vick, 2002)

Modified

Descriptor	Associated Probability
Virtually Certain	0.999
Very Likely	0.99
Likely	0.9
Neutral	0.5
Unlikely	0.1
Very Unlikely	0.01
Virtually Impossible	0.001*

*Use sparingly – People are not well calibrated below about 0.01

Eliciting the Probabilities

- It is important that each team member make an initial probability estimate on their own before hearing others estimates.
  - This helps avoid anchoring bias and forces each team member to critically think about the evidence and likelihood of the event.
- Each team member is asked to reveal their estimate. They should be encouraged to be honest as there is no “wrong” answer.

Adjustments to Probability Estimates

- Estimates might all clump together about some common number, they might spread over a wide range, clump in two or more groups, or there might be a common group with one or two outliers.
- Facilitator should call upon representatives of differing groups (e.g. high estimates, low estimates, middle estimates) to explain why it was they held a particular belief in light of the evidence common to all
- This should generate additional discussion

Adjustments to Probability Estimates

- Agreement between the estimators might indicate everyone is interpreting the information in the same way.
- Disagreement might indicate that some estimators are mistaken about the importance of particular evidence or that they hold different views in mind about geologic models or design or construction details.
- Disagreement might arise because some estimators may have a difficult time converting degree of belief to a numerical value.

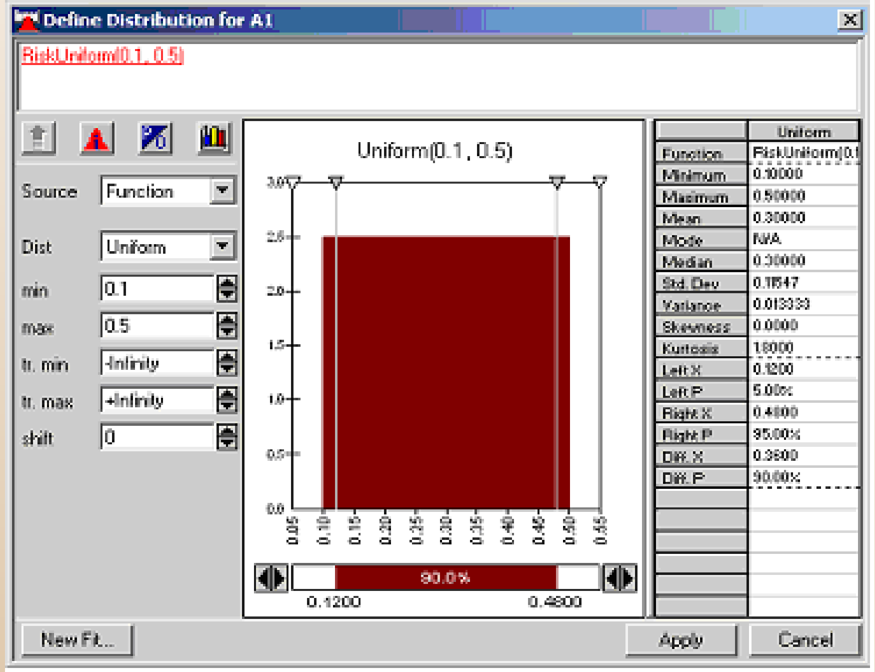
Eliciting the Probabilities

- After the discussion, determine if the group can come to a consensus. If not, carry both (or multiple) estimates through documenting the reasons for each.
- The team median is usually a good place to start for a consensus. Averages can be unreasonably skewed by one estimate and are typically not a good starting point.
- Not every single individual estimate needs to be captured by the uncertainty range
- Do not use the range of team estimated values to form a distribution for input to a Monte Carlo analysis. If a distribution is required, use the methods described in the next slides.

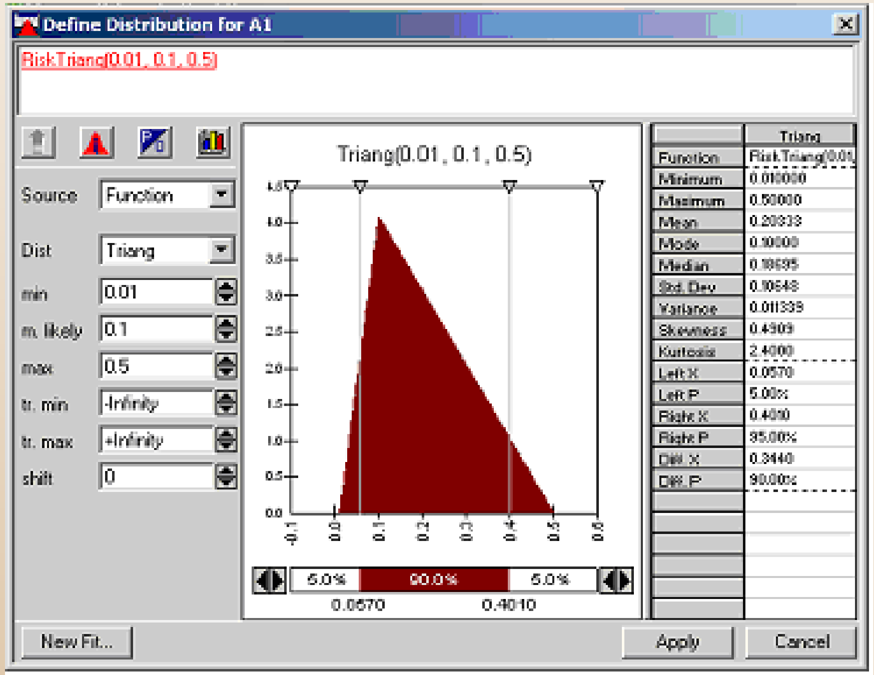
Eliciting Likelihood Distributions

- Possible questions for elicitation
  - What is the lowest reasonably plausible number you can imagine the likelihood to be?
  - What is the highest reasonably plausible number you can imagine the likelihood to be?
  - Is it more likely to be somewhere in between these values?
  - If so, what is the most likely value?

Answers Define Distribution



Uniform Distribution

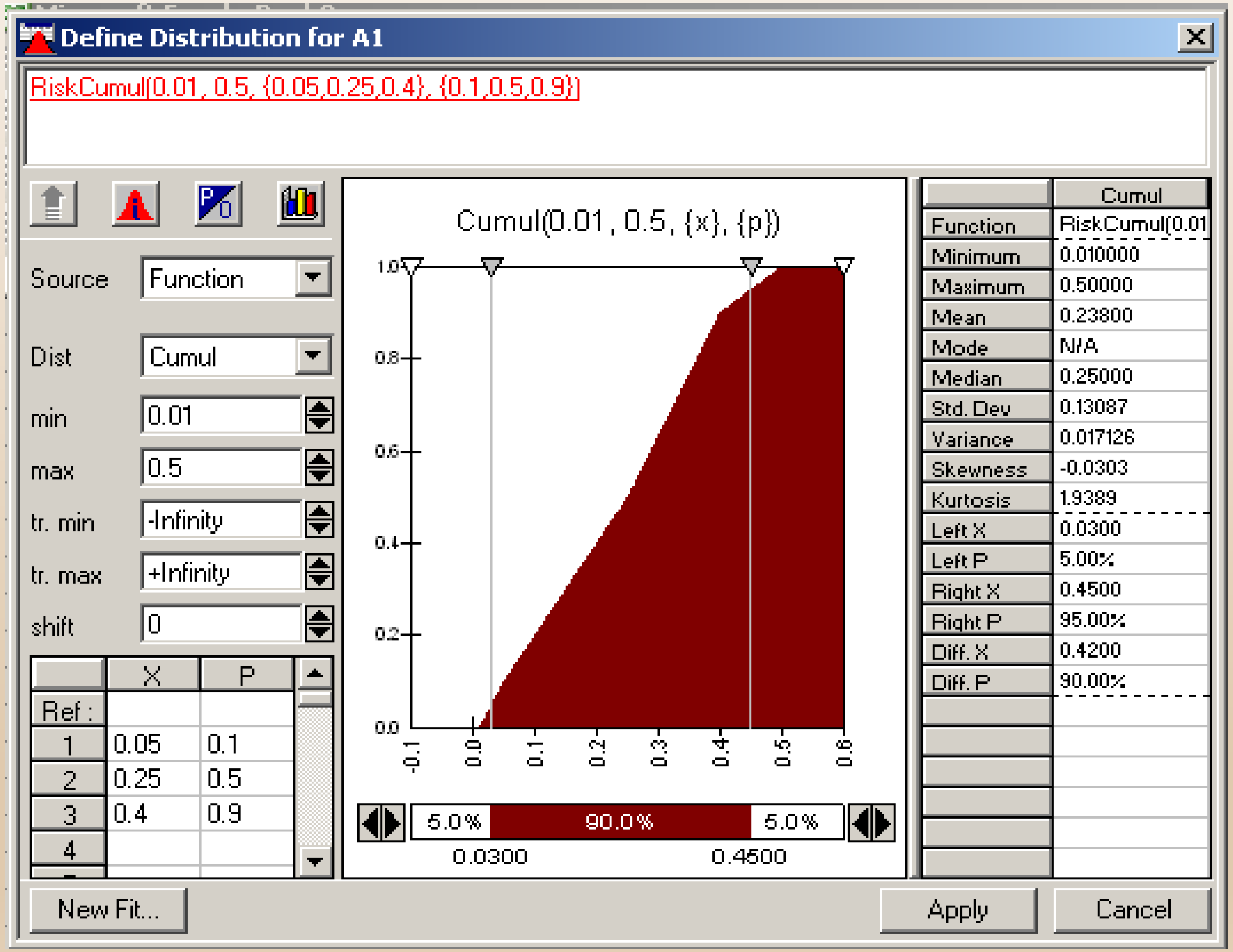


Triangular Distribution

With More Information

- The probability is not likely to be less than _?_ (10th percentile)
- The Probability is not likely to be more than _?_ (90th percentile)
- It cannot be less than _?_ (0 percentile) nor more than _?_ (100th percentile)
- It is equally likely to be more or less than _?_ (50th percentile)

Answers Define Cumulative Distribution



Documenting Results of the Elicitation

Documenting Results

- Teams have misused and misunderstood the practice of highlighting factors in the more and less likely tables.
- For example, many teams have done something like highlighting two factors on the likely side, two factors on the unlikely side, and put forward an estimate of 0.1 with no other explanation.
- So, they obviously weighted the highlighted unlikely factors more heavily, but the reader is often left to wonder why.
- In addition to highlighting, there needs to be an explanation of how the team weighted the evidence and why.

Documenting Results

- If agreement cannot be reached, the most important claims and evidence inherent to the controversy can be very helpful when it comes time to develop the dam safety case and path forward.
- Once again, if consensus cannot be achieved, both estimates are carried forward along with the reasons for each.
- The opposing viewpoints can often be turned into hypotheses which can be tested by additional investigations.

Coherence Checks

- After the team elicitations are made, it is essential to perform coherence checks:
  - Do probabilities at a branch sum to 1.0?
  - Do relative rankings makes sense (e.g. failure probabilities should generally increase for increasing loads)?
  - Are there some probabilities that do not make sense (e.g. 0.1 probability in one year = 0.96 probability in 30 years using Bernoulli relationship, would you expect the event to be almost certain in 30 years)?
  - Do overall results make sense and pass the “gut check”?
- This should be done after each failure mode is estimated and after all failure modes are estimated.

Exercise

Item	80% Confidence Band	
	Low	High
Abraham Lincoln’s age at death		
Length of the Nile River (miles)		
Number of nations in NATO		
Number of studio albums released by the Beatles		
Diameter of the moon (miles)		
Weight of an empty Boeing 747 (pounds)		
Year in which Leonardo da Vinci was born		
Gestation period of an African Elephant (days)		
Air distance from London to Sydney (miles)		
Deepest known point in oceans (feet)		