Probabilistic Approaches to Limit-State Analyses

Best Practices in Dam and Levee Safety Risk Analysis

Part A – Risk Analysis Basics

Chapter A-7

Last modified July 2018; presented July 2019

US Army Corps
of Engineers®

U.S. Department of the Interior
Bureau of Reclamation

Department of Energy
Federal Energy Regulatory Commission

TVA

Objective

- To develop an understanding of available methods for conducting a probabilistic analysis of a traditional limit-state problem.

Key Concepts

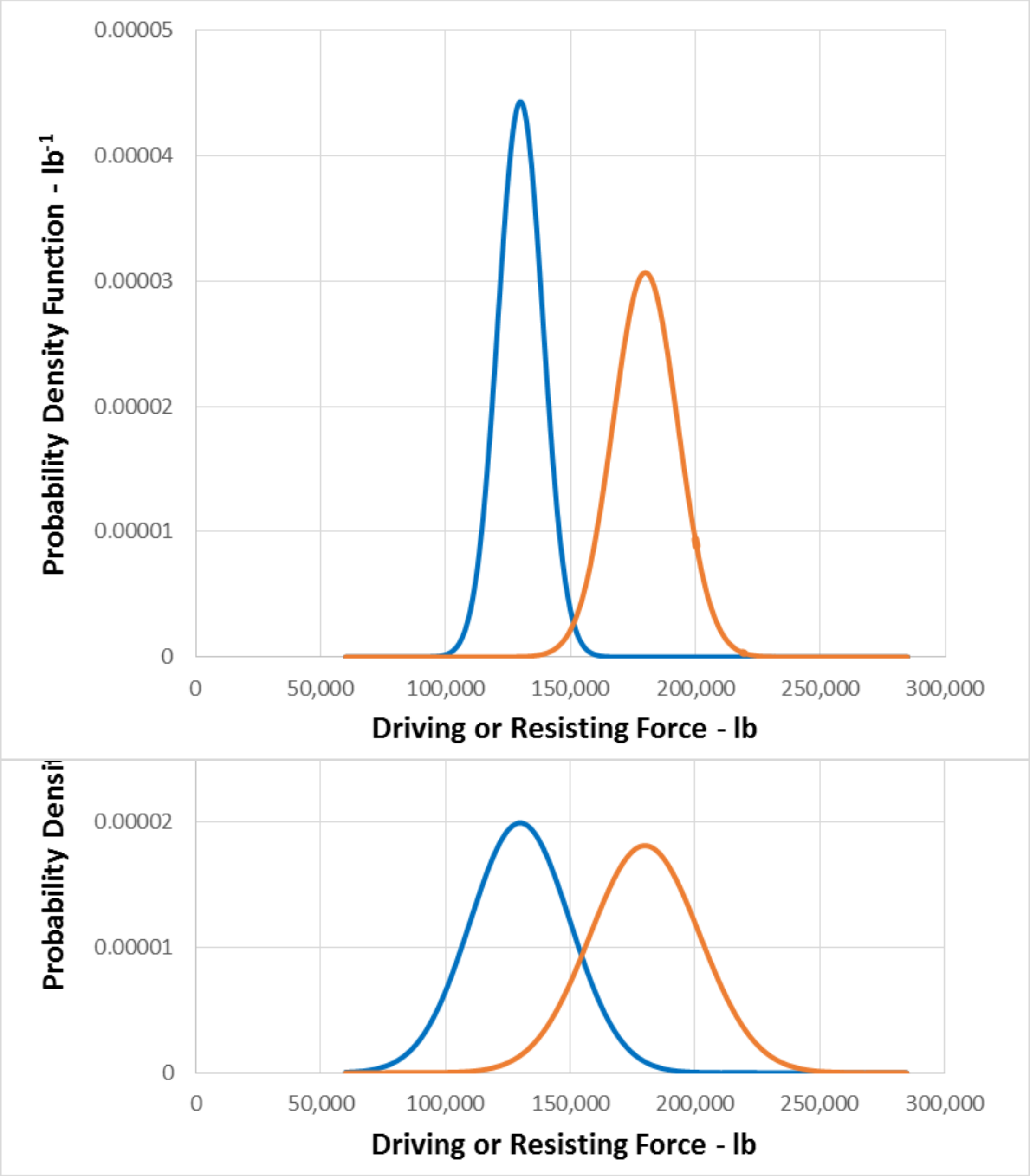
- The variables in any formula that can be programmed into a spreadsheet can be treated as random variables (distributions).
- This allows the solution or output parameter (e.g. a calculated safety factor) to be treated as a distribution as well.
- Likelihood of the output parameter being greater than the “limit state” can be used to assess likelihood of poor performance.

Topics

- This presentation focuses on the Monte Carlo simulation approach
- However, reliability methods such as First Order Second Moment methods can also be used
- The examples in this section are focused on:
    - Post liquefaction embankment stability
    - RCC gravity dam sliding stability
    - Foundation rock wedge stability
    - The general use of simulated safety factors in a risk analysis context
- Other applications are also possible

Safety Factor as a stability index

- Safety factors lower than 1.0 are theoretically associated with the loss of limit equilibrium stability
- In limit equilibrium based design, the “required” safety factor would typically be greater than 1.0 in order to account for uncertainty in the stability analysis inputs
- When deterministic factors of safety are used as information in a risk analysis, their meaning is typically weighed in the context of evaluating “more likely” and “less likely” factors
- In this context, a safety factor close to 1.0 could be used to argue both for and against stability, depending on the application



Safety Factor as a Random Variable

- When the safety factor is treated as a random variable, the uncertainty of the analysis inputs is explicitly accounted for
    - e.g., the average safety factors in both of the plots are the same, but the spread of the driving and resisting force distributions is different
- This changes the way that the information can be used in a risk analysis context

Overview of approach

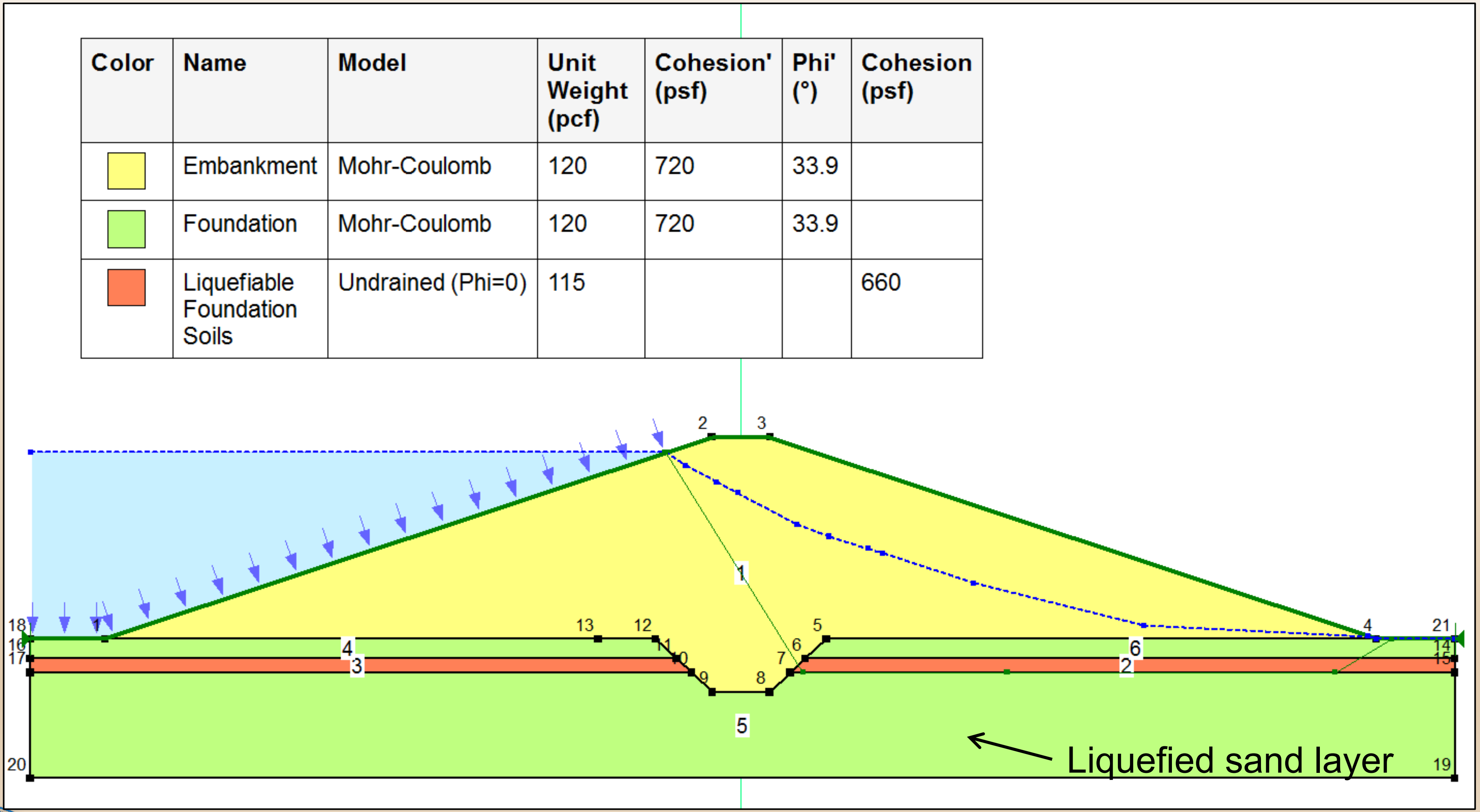
- Program deterministic analysis in Microsoft Excel or other program
- If using Excel, activate @Risk or other commercially available Macro add-in
- Instead of defining the input parameters as point values, define them as distributions
- Perform Monte Carlo analysis to generate a distribution of output safety factors by repeatedly sampling input distributions
- Use the output distribution of safety factors to evaluate the probability of unsatisfactory performance
- One option: p(F.S.<1.0) = (Number of F.S. hits<1.0) / (Total no. trials)
- Or, use the information more qualitatively

Example – Screening level evaluation of post liquefaction stability for an embankment dam

- 76-foot-high homogeneous earth fill embankment
- Constructed in late 1940’s
- SC material compacted in thin lifts with sheepsfoot roller
- Cutoff trench through 20 feet of alluvium down to rock
- 3 borings have been drilled through d/s shell into foundation alluvium
- Continuous clean sand layer 4’ to 6’ thick @ 8’ below dam foundation contact - $(N_1)_{60cs}$ ranges from 13 to 15
- Wet area at toe of dike indicates sand layer is below phreatic surface
- Dam is located in a seismically active area

→ Given that the sand layer liquefies, what is the probability of post-seismic slope instability?

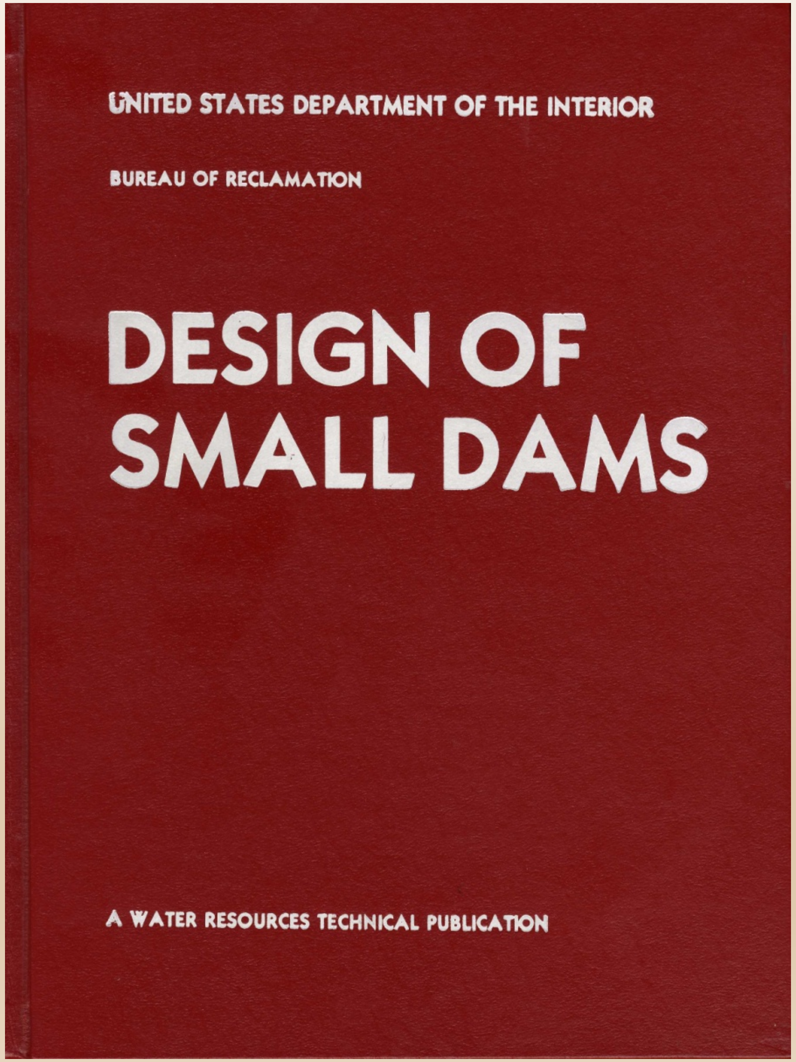
Embankment Geometry



Color	Name	Model	Unit Weight (pcf)	Cohesion' (psf)	Phi' (°)	Cohesion (psf)
🟨	Embankment	Mohr-Coulomb	120	720	33.9	
🟩	Foundation	Mohr-Coulomb	120	720	33.9	
🟧	Liquefiable Foundation Soils	Undrained (Phi=0)	115			660

Liquefied sand layer

No site specific information on embankment properties



UNITED STATES DEPARTMENT OF THE INTERIOR

BUREAU OF RECLAMATION

DESIGN OF SMALL DAMS

A WATER RESOURCES TECHNICAL PUBLICATION

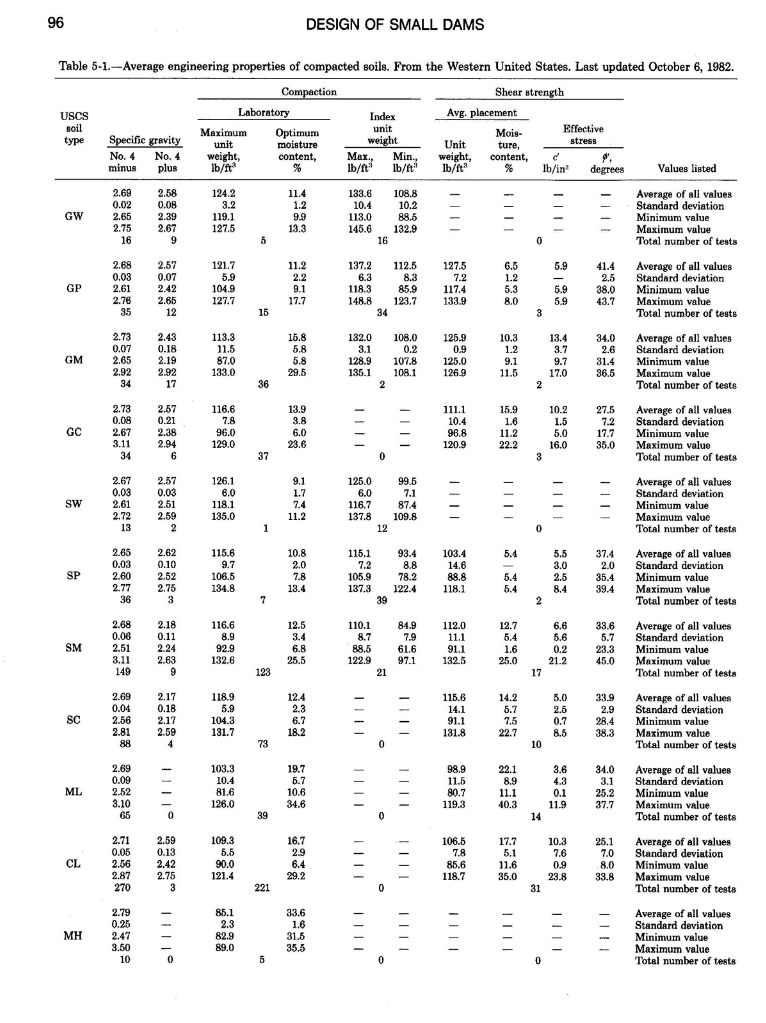


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Table 5-1.—Average engineering properties of compacted soils. From the Western United States. Last updated October 6, 1982.

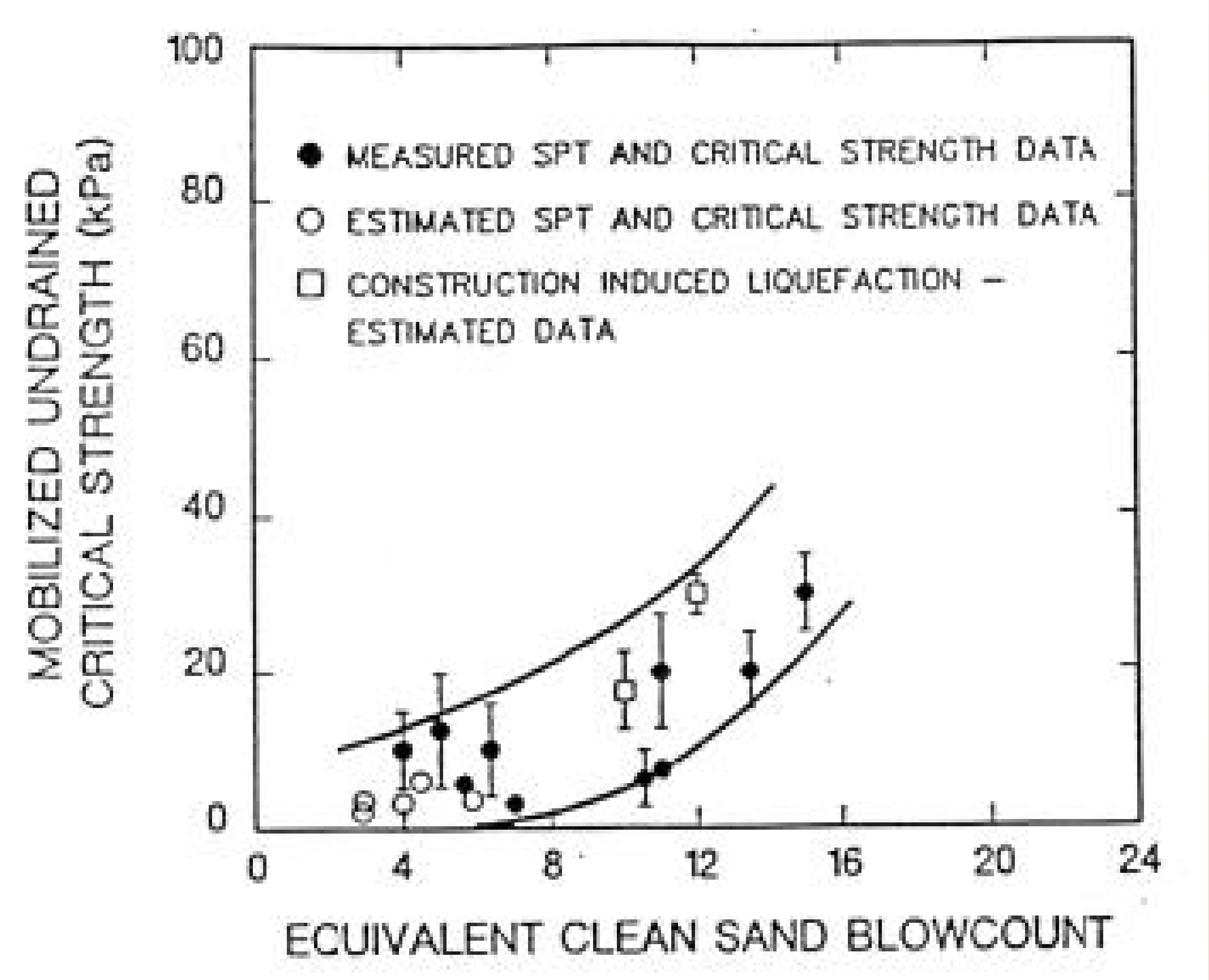
USCS soil type	Specific gravity		Compaction				Shear strength				
			Laboratory		Index unit weight		Avg. placement		Effective stress		
	No. 4 minus	No. 4 plus	Maximum unit weight, lb/ft^3	Optimum moisture content, %	Max., lb/ft^3	Min., lb/ft^3	Unit weight, lb/ft^3	Moisture content, %	c' lb/in^2	ϕ', degrees	Values listed
GW	2.69	2.58	124.2	11.4	133.6	108.8	—	—	—	—	Average of all values
	0.02	0.08	3.2	1.2	10.4	10.2	—	—	—	—	Standard deviation
	2.65	2.39	119.1	9.9	113.0	88.5	—	—	—	—	Minimum value
	2.75	2.67	127.5	13.3	145.6	132.9	—	—	—	—	Maximum value
	16	9	5		16		0				Total number of tests
GP	2.68	2.57	121.7	11.2	137.2	112.5	127.5	6.5	5.9	41.4	Average of all values
	0.03	0.07	5.9	2.2	6.3	8.3	7.2	1.2	—	2.5	Standard deviation
	2.61	2.42	104.9	9.1	118.3	85.9	117.4	5.3	5.9	38.0	Minimum value
	2.76	2.65	127.7	17.7	148.8	123.7	133.9	8.0	5.9	43.7	Maximum value
	35	12	15		34		3				Total number of tests
GM	2.73	2.43	113.3	15.8	132.0	108.0	125.9	10.3	13.4	34.0	Average of all values
	0.07	0.18	11.5	5.8	3.1	0.2	0.9	1.2	3.7	2.6	Standard deviation
	2.65	2.19	87.0	5.8	128.9	107.8	125.0	9.1	9.7	31.4	Minimum value
	2.92	2.92	133.0	29.5	135.1	108.1	126.9	11.5	17.0	36.5	Maximum value
	34	17	36		2		2				Total number of tests
GC	2.73	2.57	116.6	13.9	—	—	111.1	15.9	10.2	27.5	Average of all values
	0.08	0.21	7.8	3.8	—	—	10.4	1.6	1.5	7.2	Standard deviation
	2.67	2.38	96.0	6.0	—	—	96.8	11.2	5.0	17.7	Minimum value
	3.11	2.94	129.0	23.6	—	—	120.9	22.2	16.0	35.0	Maximum value
	34	6	37		0		3				Total number of tests
SW	2.67	2.57	126.1	9.1	125.0	99.5	—	—	—	—	Average of all values
	0.03	0.03	6.0	1.7	6.0	7.1	—	—	—	—	Standard deviation
	2.61	2.51	118.1	7.4	116.7	87.4	—	—	—	—	Minimum value
	2.72	2.59	135.0	11.2	137.8	109.8	—	—	—	—	Maximum value
	13	2	1		12		0				Total number of tests
SP	2.65	2.62	115.6	10.8	115.1	93.4	103.4	5.4	5.5	37.4	Average of all values
	0.03	0.10	9.7	2.0	7.2	8.8	14.6	—	3.0	2.0	Standard deviation
	2.60	2.52	106.5	7.8	105.9	78.2	88.8	5.4	2.5	35.4	Minimum value
	2.77	2.75	134.8	13.4	137.3	122.4	118.1	5.4	8.4	39.4	Maximum value
	36	3	7		39		2				Total number of tests
SM	2.68	2.18	116.6	12.5	110.1	84.9	112.0	12.7	6.6	33.6	Average of all values
	0.06	0.11	8.9	3.4	8.7	7.9	11.1	5.4	5.6	5.7	Standard deviation
	2.51	2.24	92.9	6.8	88.5	61.6	91.1	1.6	0.2	23.3	Minimum value
	3.11	2.63	132.6	25.5	122.9	97.1	132.5	25.0	21.2	45.0	Maximum value
	149	9	123		21		17				Total number of tests
SC	2.69	2.17	118.9	12.4	—	—	115.6	14.2	5.0	33.9	Average of all values
	0.04	0.18	5.9	2.3	—	—	14.1	5.7	2.5	2.9	Standard deviation
	2.56	2.17	104.3	6.7	—	—	91.1	7.5	0.7	28.4	Minimum value
	2.81	2.59	131.7	18.2	—	—	131.8	22.7	8.5	38.3	Maximum value
	88	4	73		0		10				Total number of tests
ML	2.69	—	103.3	19.7	—	—	98.9	22.1	3.6	34.0	Average of all values
	0.09	—	10.4	5.7	—	—	11.5	8.9	4.3	3.1	Standard deviation
	2.52	—	81.6	10.6	—	—	80.7	11.1	0.1	25.2	Minimum value
	3.10	—	126.0	34.6	—	—	119.3	40.3	11.9	37.7	Maximum value
	65	0	39		0		14				Total number of tests
CL	2.71	2.59	109.3	16.7	—	—	106.5	17.7	10.3	25.1	Average of all values
	0.05	0.13	5.5	2.9	—	—	7.8	5.1	7.6	7.0	Standard deviation
	2.56	2.42	90.0	6.4	—	—	85.6	11.6	0.9	8.0	Minimum value
	2.87	2.75	121.4	29.2	—	—	118.7	35.0	23.8	33.8	Maximum value
	270	3	221		0		31				Total number of tests
MH	2.79	—	85.1	33.6	—	—	—	—	—	—	Average of all values
	0.25	—	2.3	1.6	—	—	—	—	—	—	Standard deviation
	2.47	—	82.9	31.5	—	—	—	—	—	—	Minimum value
	3.50	—	89.0	35.5	—	—	—	—	—	—	Maximum value
	10	0	5		0		0				Total number of tests

SC material properties for stability analysis

Property	Minimum	Maximum	Mean	Standard Deviation
Unit Weight (lb/ft^3)	91.1	131.8	115.6	14.1
c’ (lb/ft^2)	101	1224	720	360
φ’ (degrees)	28.4	38.3	33.9	2.9

φ’ and c’ entered into SLOPE/w as truncated normal distributions

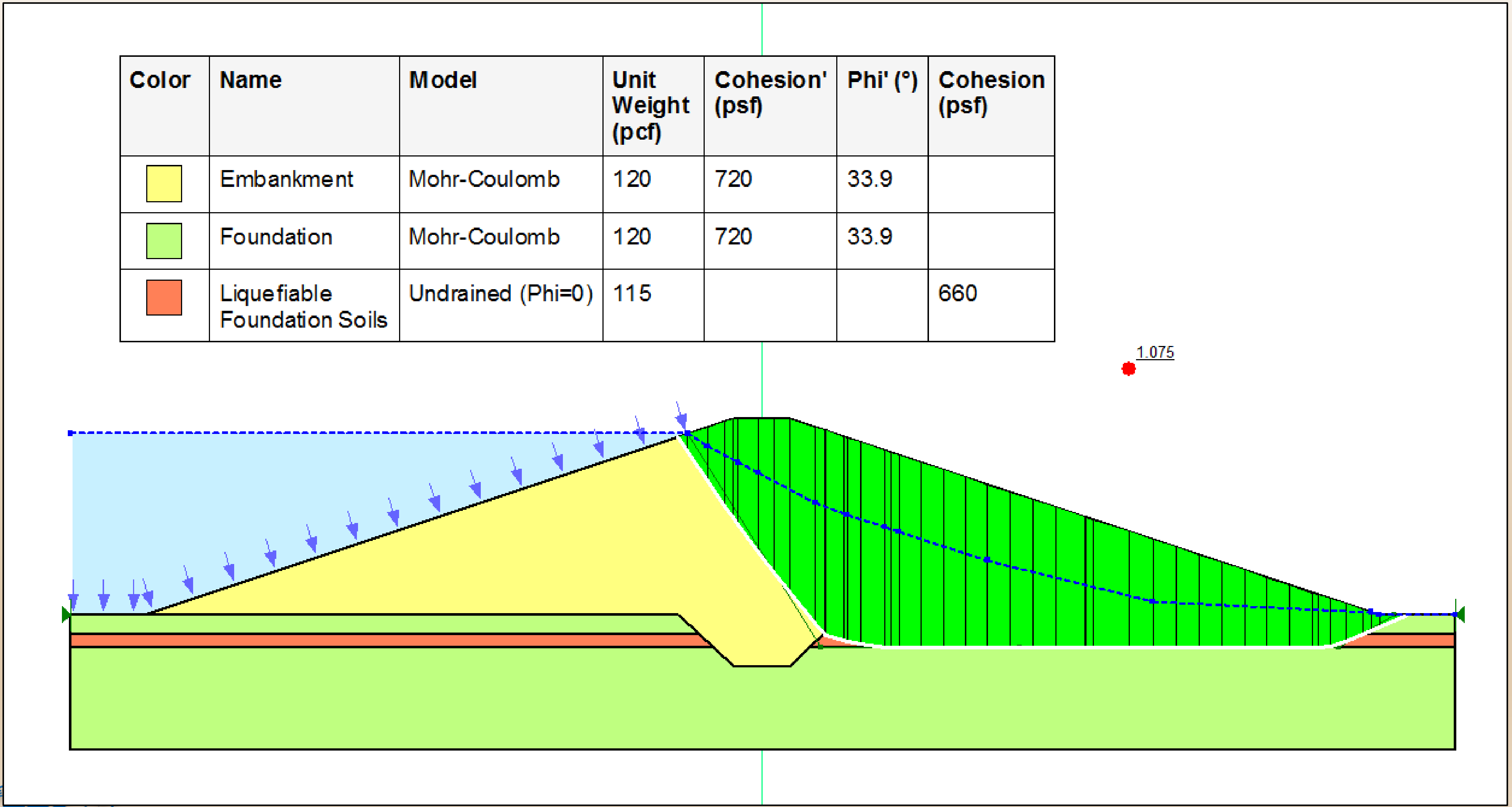
Undrained Residual Shear Strength



From Seed et al (2003)

S_u entered into SLOPE/w as a triangular distribution ranging from 19 kPA (400 psf) to 44 kPA (920 psf), with a mode of 660 psf

Limit Equilibrium Results



Color	Name	Model	Unit Weight (pcf)	Cohesion' (psf)	Phi' (°)	Cohesion (psf)
🟨	Embankment	Mohr-Coulomb	120	720	33.9	
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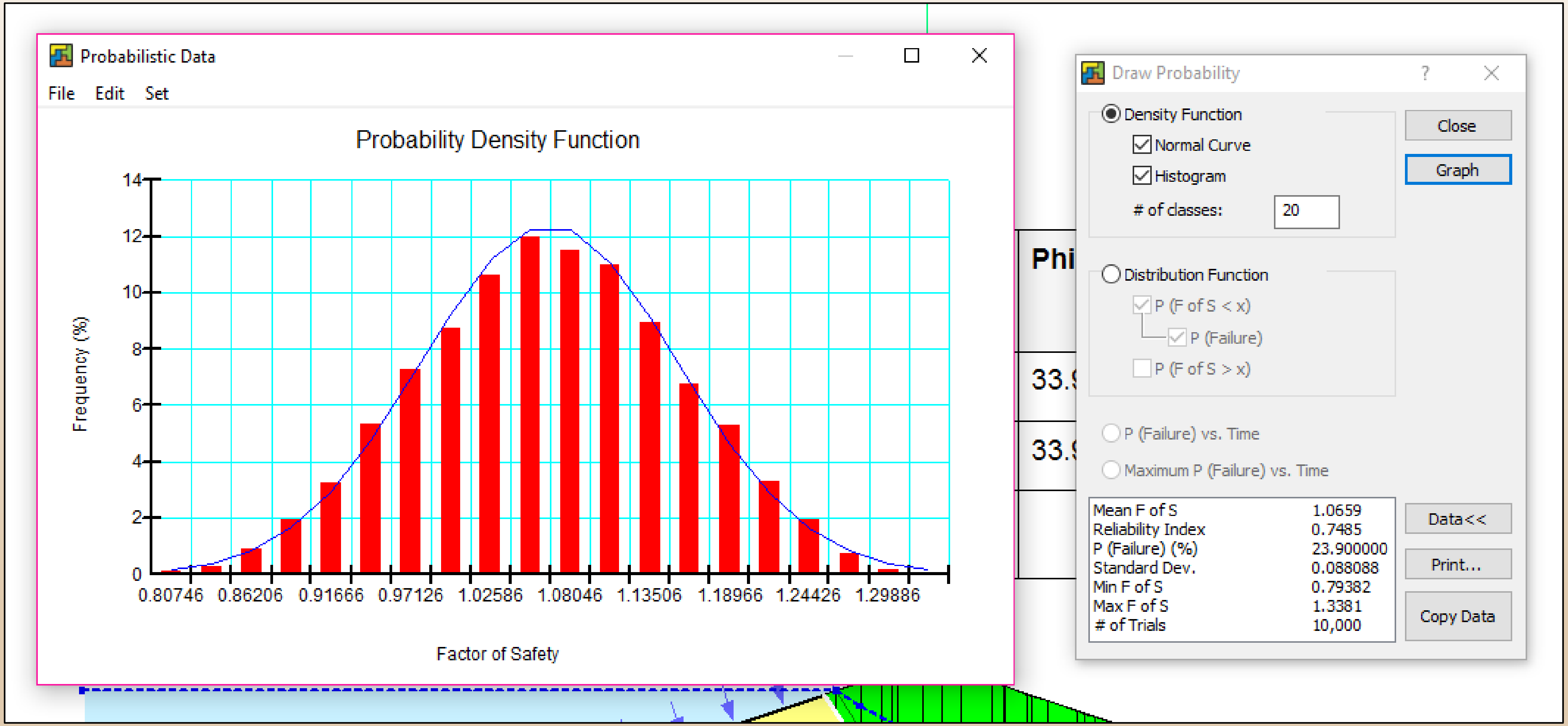
Monte Carlo Results - 10,000 simulation trials



● Density Function
☑ Normal Curve
☑ Histogram
# of classes: 20

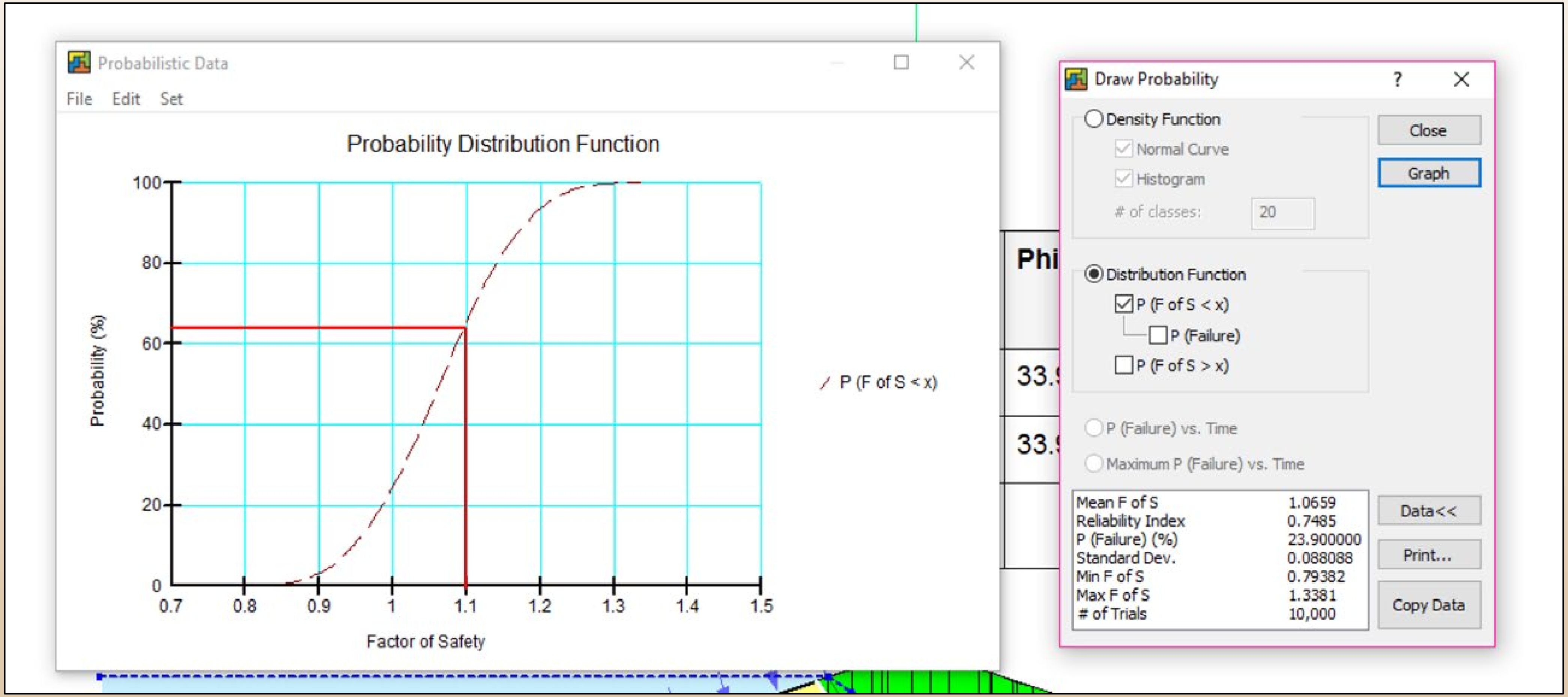
○ Distribution Function
☑ P (F of S < x)
☑ P (Failure)
☐ P (F of S > x)

○ P (Failure) vs. Time
○ Maximum P (Failure) vs. Time

Mean F of S	1.0659
Reliability Index	0.7485
P (Failure) (%)	23.900000
Standard Dev.	0.088088
Min F of S	0.79382
Max F of S	1.3381
# of Trials	10,000

Probability of FS < 1.0 ~ 2390/10,000 = 0.24

Monte Carlo Results - 10,000 iterations



○ Density Function
☑ Normal Curve
☑ Histogram
# of classes: 20

● Distribution Function
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# of Trials	10,000

Probability of FS < 1.1 ~ 0.64

How could this information potentially be used in a risk analysis?

- Consider the following potential failure mode:
  1. An earthquake occurs
  2. A liquefiable layer exists in the foundation
  3. Continuous liquefaction is triggered
  4. Slope instability occurs
  5. Crest loss exceeds the available freeboard, resulting in an uncontrolled release of the reservoir
- The Monte Carlo results could be used directly as the probability estimate for Event 4 (not recommended)
- The Monte Carlo results could be used as a starting point for the Event 4 probability estimate, with adjustments then applied based on other factors
- The results could be used qualitatively, and taken into consideration along with other more/less likely factors.

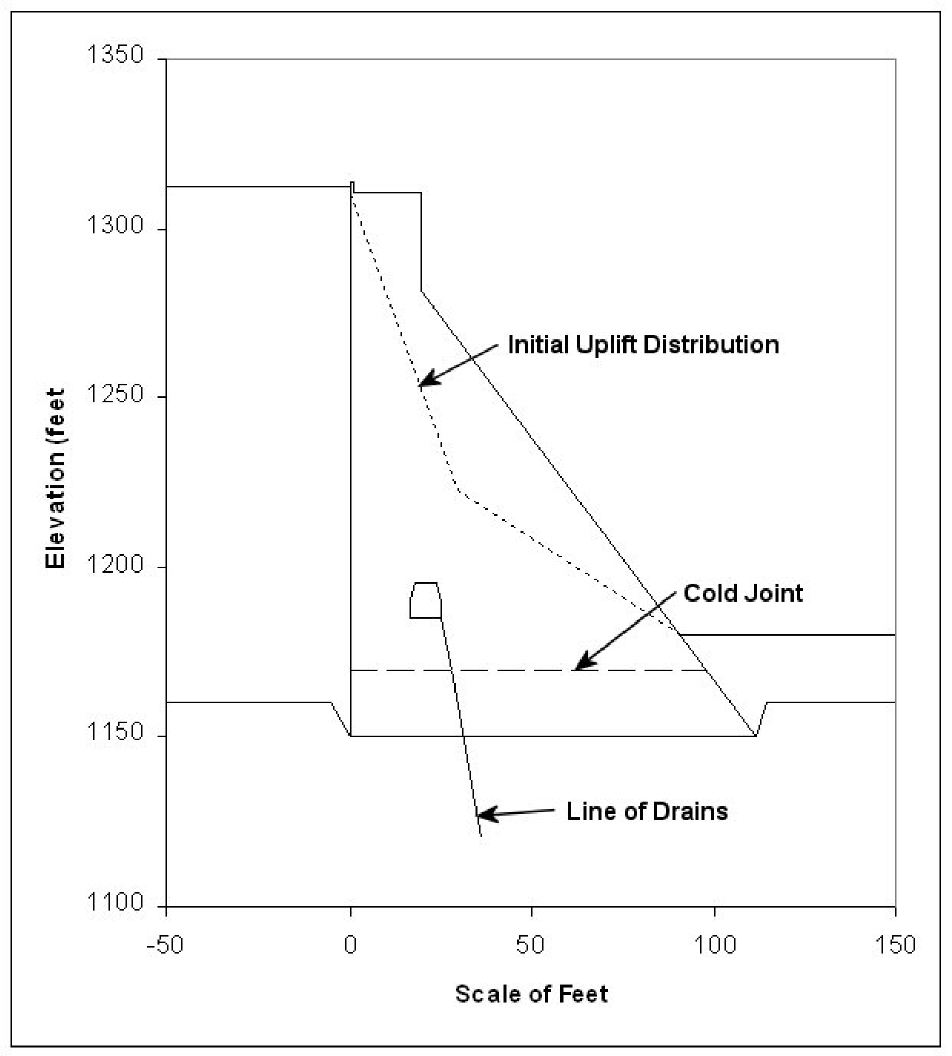
Caveats

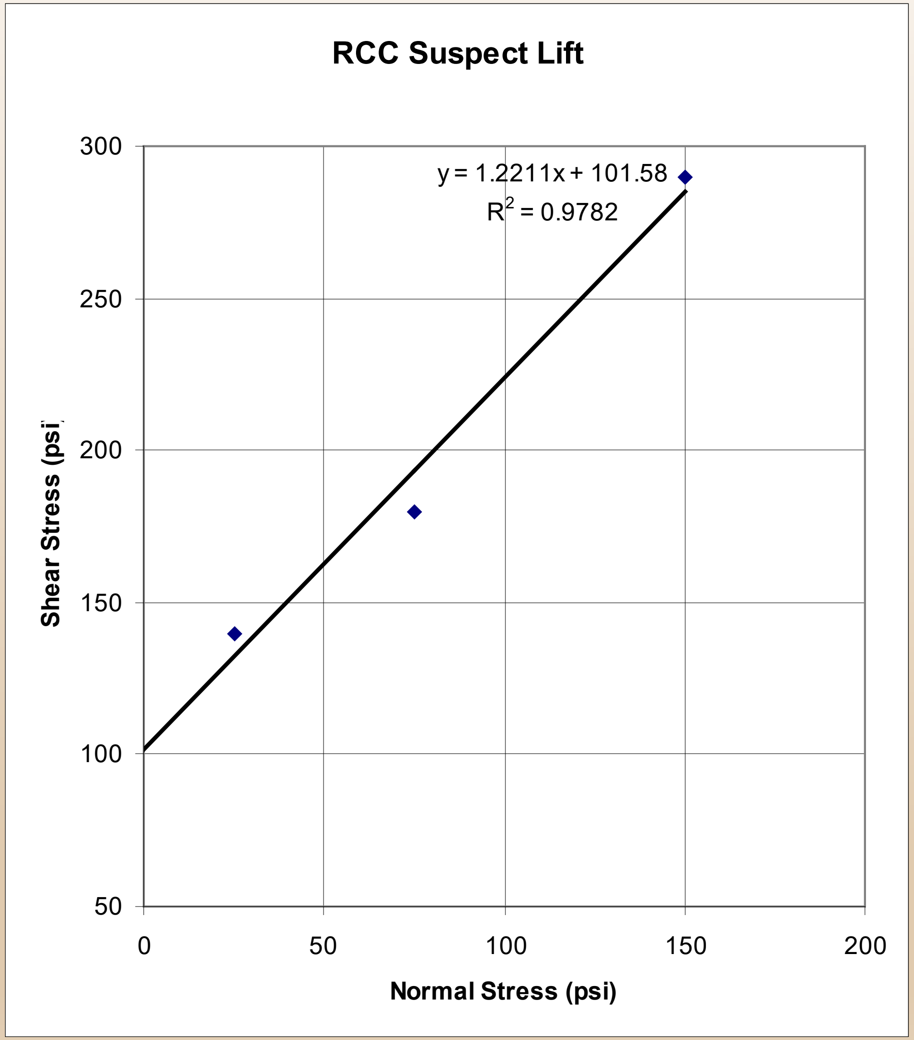
- Finite element analysis (with deformation information) to support of a higher level risk analysis
- Even if a very low probability of slope instability were indicated, this would not necessarily rule out all other seismic potential failure modes
- Not all limit equilibrium stability analysis programs include a Monte Carlo simulation capability. Some do, but provide limited flexibility in defining the input parameter distributions (e.g. the inputs may be limited to the mean and standard deviation of the input parameter)
- Be aware of what distributions are being used for the input parameters, and consider whether they are appropriate.

Example - RCC Gravity Dam

- 160 feet high
- A winter shut-down occurred during construction after the first 20 feet of RCC placement
- The following spring, the cold joint was cleaned, a mortar layer placed, and the rest of dam constructed
- Gallery in dam with drainage curtain through potential cold joint
- Five 6 inch cores taken through cold joint
  - 3 of 5 were bonded and tested in direct shear
- Original Probable Maximum Flood (PMF) passed without encroaching on the 3.5-foot high parapet wall
- PMF recently revised, now puts 2.3’ of water on the parapet wall

Dam Geometry and Strength Results



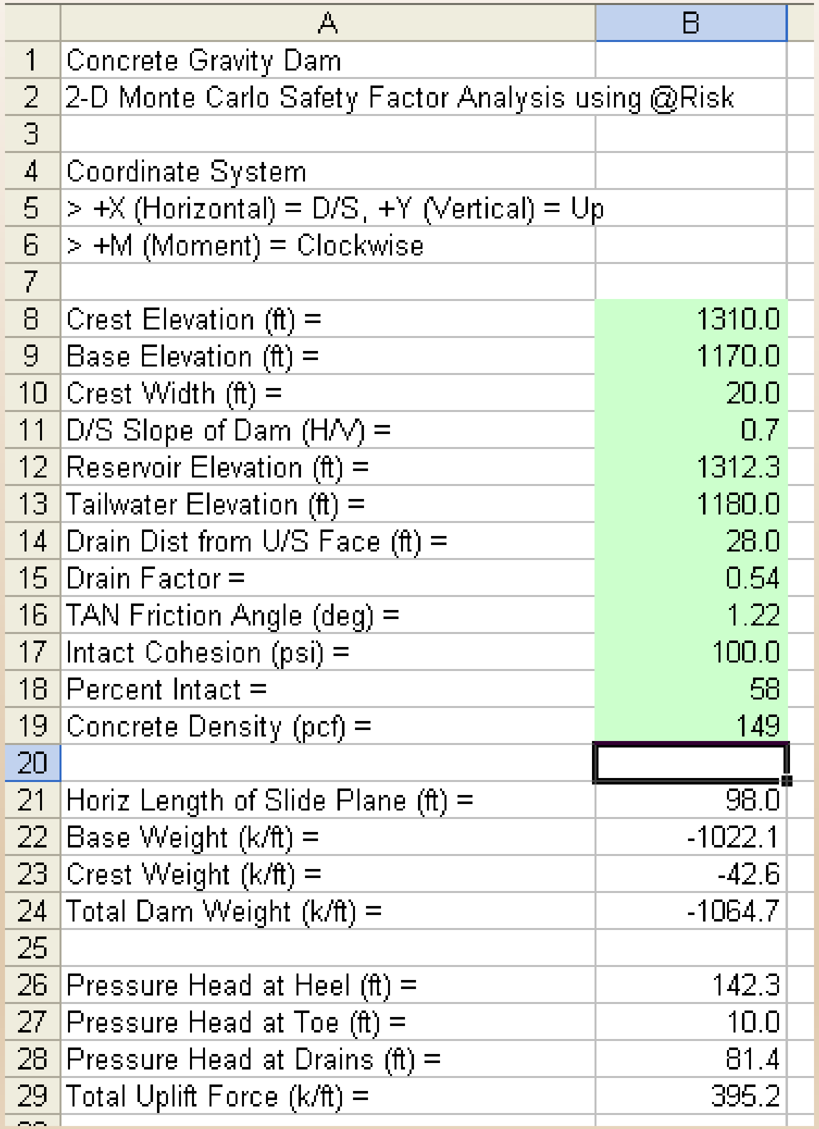


Input Distributions

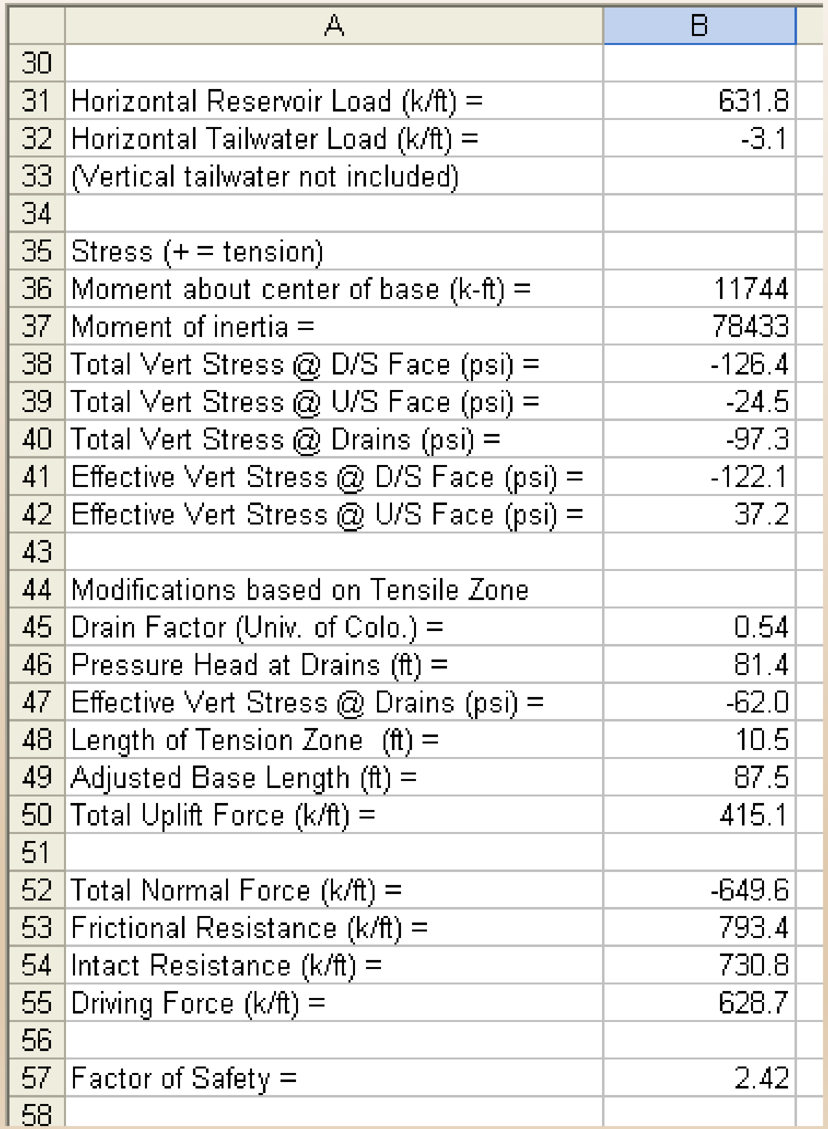
Property	Distribution	Minimum	Peak	Maximum
Initial Drain Factor	Uniform	0.33	n/a	0.75
φ’ (degrees)	Triangular	43	50	57
c’ (lb/in^2)	Triangular	50	100	150
Percent Intact	Triangular	43	60	71
RCC Unit Weight (lb/ft^3)	Uniform	146	n/a	152

note: φ’ converted to tan φ’ for calculations

Stability Calculation Spreadsheet



	A	B
1	Concrete Gravity Dam	
2	2-D Monte Carlo Safety Factor Analysis using @Risk	
3		
4	Coordinate System	
5	> +X (Horizontal) = D/S, +Y (Vertical) = Up	
6	> +M (Moment) = Clockwise	
7		
8	Crest Elevation (ft) =	1310.0
9	Base Elevation (ft) =	1170.0
10	Crest Width (ft) =	20.0
11	D/S Slope of Dam (H/V) =	0.7
12	Reservoir Elevation (ft) =	1312.3
13	Tailwater Elevation (ft) =	1180.0
14	Drain Dist from U/S Face (ft) =	28.0
15	Drain Factor =	0.54
16	TAN Friction Angle (deg) =	1.22
17	Intact Cohesion (psi) =	100.0
18	Percent Intact =	58
19	Concrete Density (pcf) =	149
20		
21	Horiz Length of Slide Plane (ft) =	98.0
22	Base Weight (k/ft) =	-1022.1
23	Crest Weight (k/ft) =	-42.6
24	Total Dam Weight (k/ft) =	-1064.7
25		
26	Pressure Head at Heel (ft) =	142.3
27	Pressure Head at Toe (ft) =	10.0
28	Pressure Head at Drains (ft) =	81.4
29	Total Uplift Force (k/ft) =	395.2



	A	B
30		
31	Horizontal Reservoir Load (k/ft) =	631.8
32	Horizontal Tailwater Load (k/ft) =	-3.1
33	(Vertical tailwater not included)	
34		
35	Stress (+ = tension)	
36	Moment about center of base (k-ft) =	11744
37	Moment of inertia =	78433
38	Total Vert Stress @ D/S Face (psi) =	-126.4
39	Total Vert Stress @ U/S Face (psi) =	-24.5
40	Total Vert Stress @ Drains (psi) =	-97.3
41	Effective Vert Stress @ D/S Face (psi) =	-122.1
42	Effective Vert Stress @ U/S Face (psi) =	37.2
43		
44	Modifications based on Tensile Zone	
45	Drain Factor (Univ. of Colo.) =	0.54
46	Pressure Head at Drains (ft) =	81.4
47	Effective Vert Stress @ Drains (psi) =	-62.0
48	Length of Tension Zone (ft) =	10.5
49	Adjusted Base Length (ft) =	87.5
50	Total Uplift Force (k/ft) =	415.1
51		
52	Total Normal Force (k/ft) =	-649.6
53	Frictional Resistance (k/ft) =	793.4
54	Intact Resistance (k/ft) =	730.8
55	Driving Force (k/ft) =	628.7
56		
57	Factor of Safety =	2.42
58		

Results and sensitivity to inputs

10,000 simulation trials: avg. F.S. = 2.42, min F.S. = 1.43

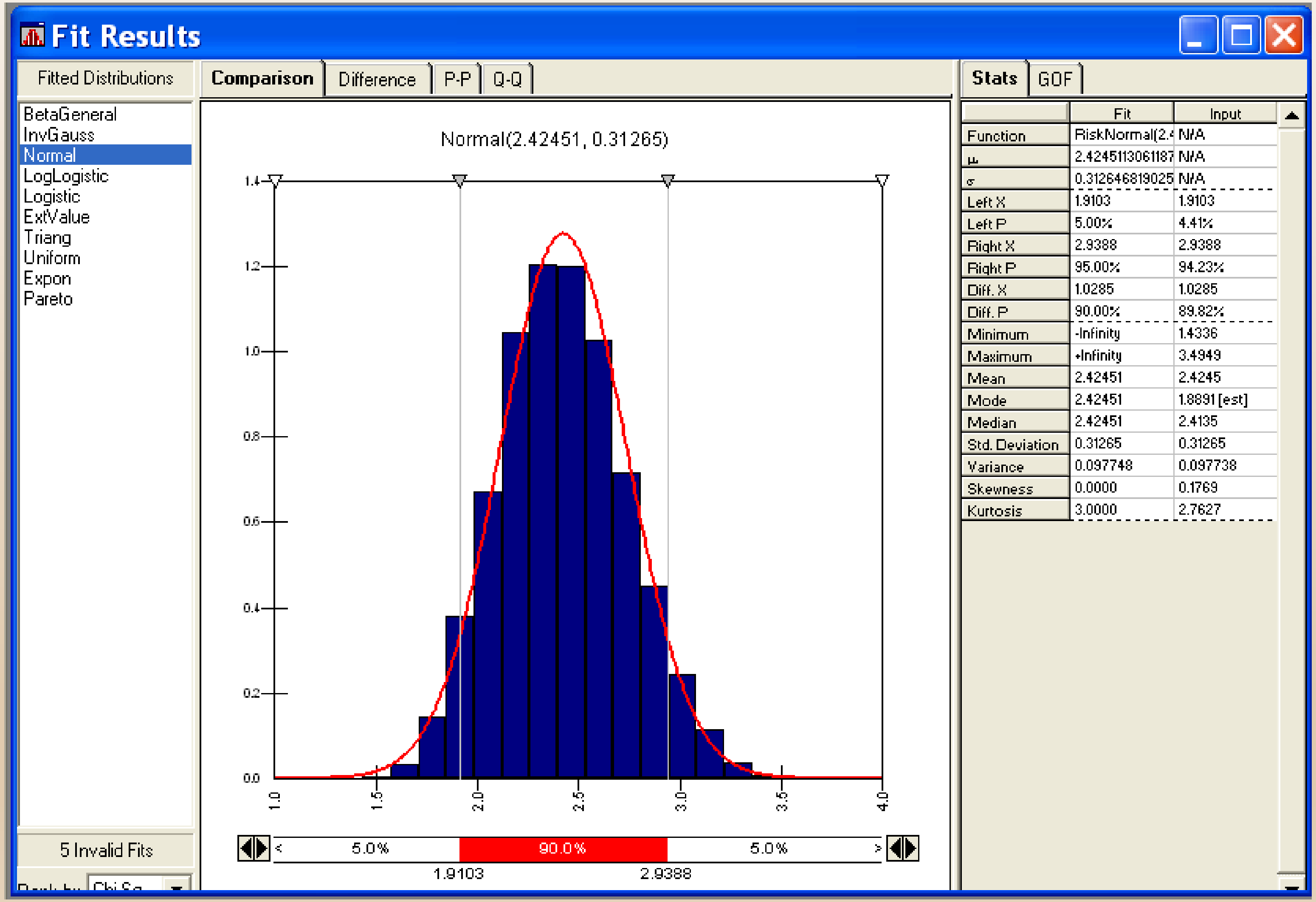
Rank	Name	Cell	Regression
1	Intact Cohesion (psi) =	B17	0.759017659
2	TAN Friction Angle (deg) =	B16	0.411501707
3	Percent Intact =	B18	0.368619688
4	Drain Factor =	B15	-0.311968848
5	Concrete Density (pcf) =	B19	0.09730957

Options when there are no FS hits < 1.0

(In order of decreasing preference)

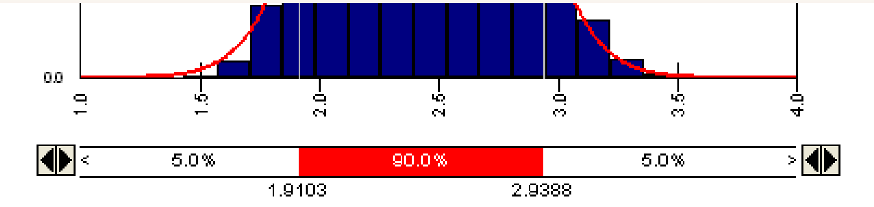
1. Use the information qualitatively (i.e. as a less likely factor for the event in question)
2. Widen the ranges of the input distributions (if reasonable)
3. Increase the number of simulation trials (will not necessarily result in any FS < 1.0 hits)
4. Use a fitted analytical probability distribution to calculate the probability of FS < 1.0

Option 4: Fitted Safety Factor Distribution



	Fit	Input
Function	RiskNormal(2.4	N/A
μ	2.4245113061187	N/A
σ	0.312646819025	N/A
Left X	1.9103	1.9103
Left P	5.00%	4.41%
Right X	2.9388	2.9388
Right P	95.00%	94.23%
Diff. X	1.0285	1.0285
Diff. P	90.00%	89.82%
Minimum	-Infinity	1.4336
Maximum	+Infinity	3.4949
Mean	2.42451	2.4245
Mode	2.42451	1.8891 [est]
Median	2.42451	2.4135
Std. Deviation	0.31265	0.31265
Variance	0.097748	0.097738
Skewness	0.0000	0.1769
Kurtosis	3.0000	2.7627

Option 4 Caveats



- Probability calculated obtained from the left tail of the analytical distribution (since there are no FS < 1.0 hits in the area where there is MC simulation data available)
- Shape of the tail depends closely on which analytical distribution was selected and on how it was fitted to the MC simulation data
- Probability of FS < 1.0 could differ by orders of magnitude depending on how the analytical distribution was fitted
- The calculated probability could change dramatically as a result of relatively minor changes to the stability analysis inputs
- Use this option sparingly and interpret results with caution. Do not use the results as a direct analog for the event probability when the results are obtained from a fitted curve.

Example - Foundation Rock Wedge Stability

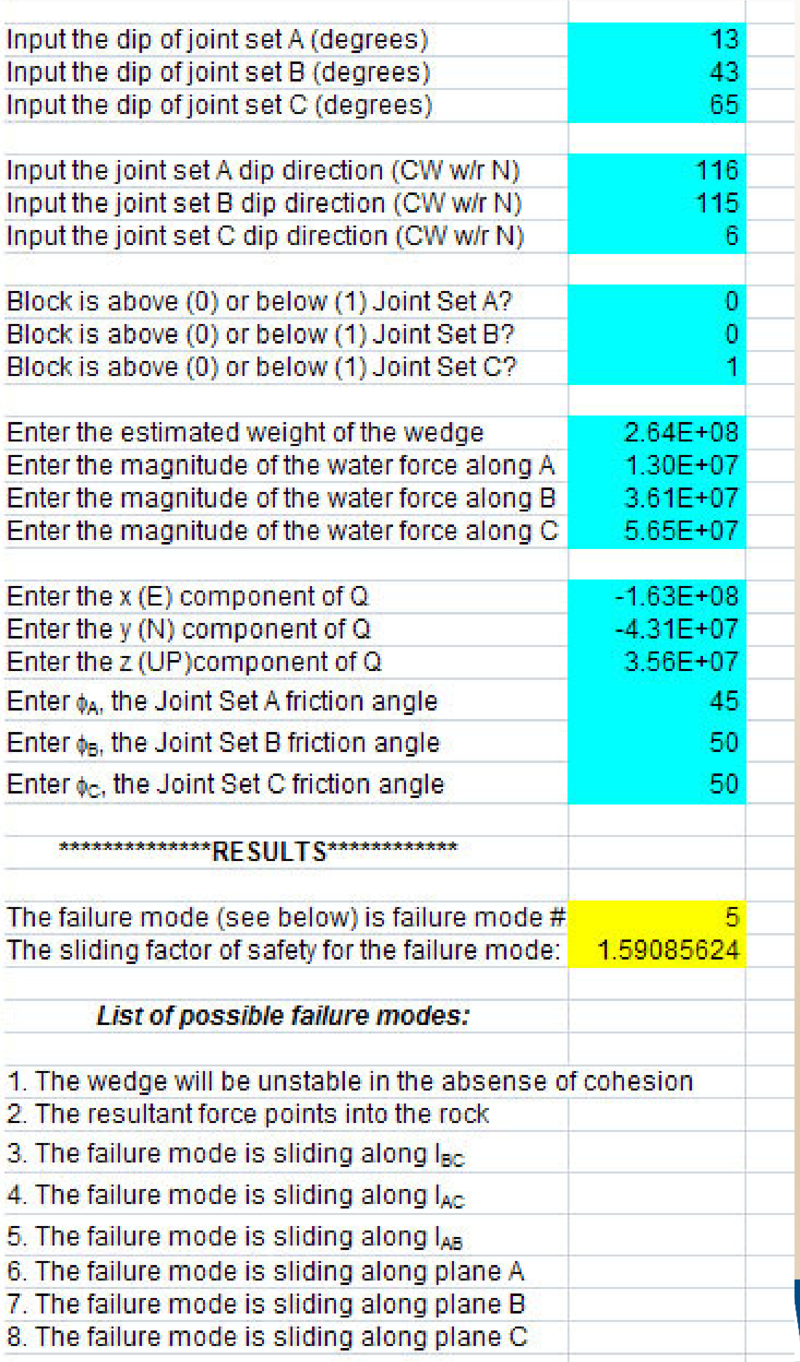
- The foundation of an arch dam constructed in the 1920s has been found to contain a large and geometrically significant rock wedge
- The risks associated with foundation wedge instability are being considered as part of a comprehensive risk analysis
- The potential failure mode involving the wedge includes the following events:
  1. Reservoir surface exceeds the critical elevation
  2. Base, side, and release planes exist in situ and are continuous
  3. Critical wedge movement initiates
  4. Movement is significant enough to cause concrete cracking
  5. Arch forces cannot be redistributed and a breach occurs
- The team is having difficulty estimating the probability of Event 3

Example - Foundation Rock Wedge Stability

- Use a probabilistic limit state approach
- Calculate resultant force on the wedge using the results of finite element analyses and a fracture-flow focused seepage analysis
- Three wedge plane uplift scenarios are developed
- A 3D wedge stability solution is programmed into Excel
- Based on the results of geologic field exploration and laboratory testing, the following distributional parameters are entered:
  - Base plane friction angle: triangular, ranging from 39° to 48°, w/ mode 45°
  - Base plane friction angle: triangular, ranging from 39° to 52°, w/ mode 50°
  - Dip and dip direction of each wedge plane: best estimate ± 3° (uniform)
  - Resultant force magnitude for each uplift scenario: best estimate ± 20 percent (uniform)

Calculation spreadsheet and Monte Carlo results

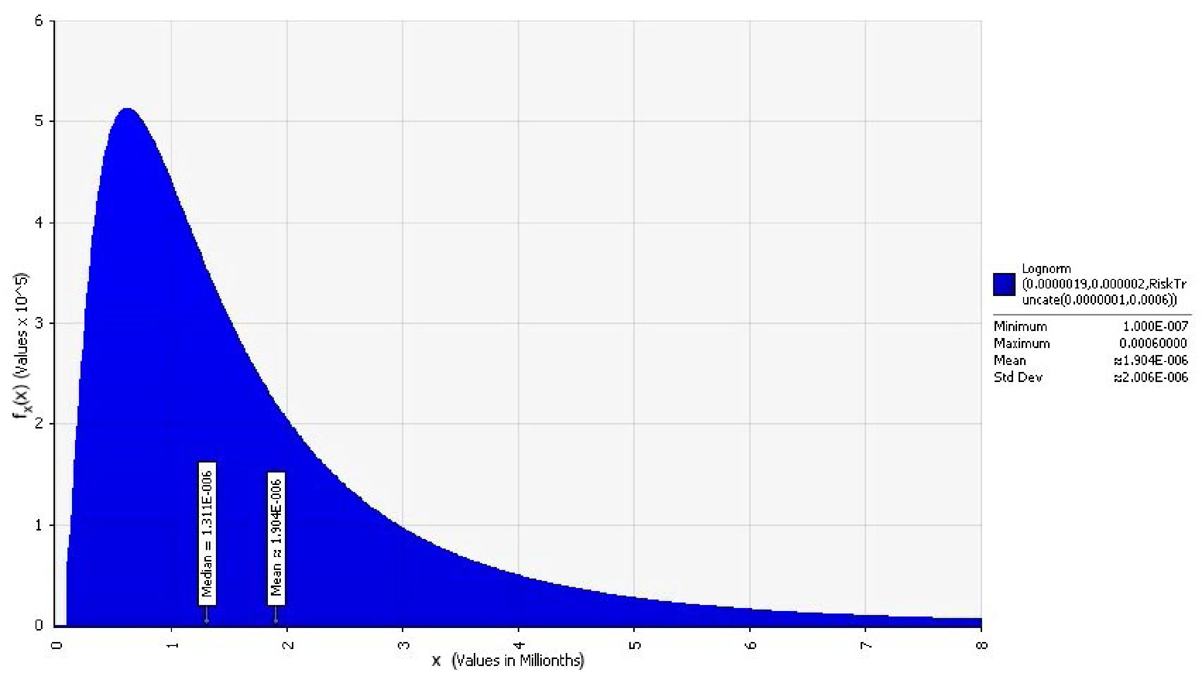
- For the “worst case” uplift scenario (static FS = 1.49), 55 of the 100,000 trials resulted in safety factors lower than unity
- For the “best estimate” scenario (static FS = 1.59), 2 of the 100,000 trials resulted in safety factors lower than unity.
- For the “best case” scenario (static FS = 2.75), none of the trials resulted in safety factors lower than unity.
- Interpreted directly, these results would suggest wedge movement initiation probabilities of 6×10^{-4}, 2×10^{-5}, and 0



Input the dip of joint set A (degrees)	13
Input the dip of joint set B (degrees)	43
Input the dip of joint set C (degrees)	65
Input the joint set A dip direction (CW w/r N)	116
Input the joint set B dip direction (CW w/r N)	115
Input the joint set C dip direction (CW w/r N)	6
Block is above (0) or below (1) Joint Set A?	0
Block is above (0) or below (1) Joint Set B?	0
Block is above (0) or below (1) Joint Set C?	1
Enter the estimated weight of the wedge	2.64E+08
Enter the magnitude of the water force along A	1.30E+07
Enter the magnitude of the water force along B	3.61E+07
Enter the magnitude of the water force along C	5.65E+07
Enter the x (E) component of Q	-1.63E+08
Enter the y (N) component of Q	-4.31E+07
Enter the z (UP)component of Q	3.56E+07
Enter ϕ_A, the Joint Set A friction angle	45
Enter ϕ_B, the Joint Set B friction angle	50
Enter ϕ_C, the Joint Set C friction angle	50
***************RESULTS*************	
The failure mode (see below) is failure mode #	5
The sliding factor of safety for the failure mode:	1.59085624
List of possible failure modes:	
1. The wedge will be unstable in the absense of cohesion	
2. The resultant force points into the rock	
3. The failure mode is sliding along l_{BC}	
4. The failure mode is sliding along l_{AC}	
5. The failure mode is sliding along l_{AB}	
6. The failure mode is sliding along plane A	
7. The failure mode is sliding along plane B	
8. The failure mode is sliding along plane C	

Caveats

- Interpreted as probabilities, the (nonzero) results would be outside the range over which most estimators are well-calibrated
- Interpret the numbers as simulated frequencies or base rates, and adjust them based on the other considerations (more/less likely factors) that apply under Event 3
- The team ultimately selected a truncated lognormal probability distribution with a lower bound of 1.0E-7, a mean of 1.9E-6, and a truncated upper bound of 6.0E-4



Takeaways

- Analysis results are frequently used as a source of information for estimating the conditional probabilities of PFM events
- When the uncertainty of the analysis results is quantified, it can provide an additional layer of information for the risk estimators
- Probabilistic limit equilibrium analysis is one method of quantifying the effects of uncertainty in the input parameters
- Use caution when interpreting the results of a probabilistic limit equilibrium analysis. Results can inform the conditional probability estimates, not define them
- Most MC software allows analytical distributions to be fitted to the data but this is not always the best approach

Comments or Questions?

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