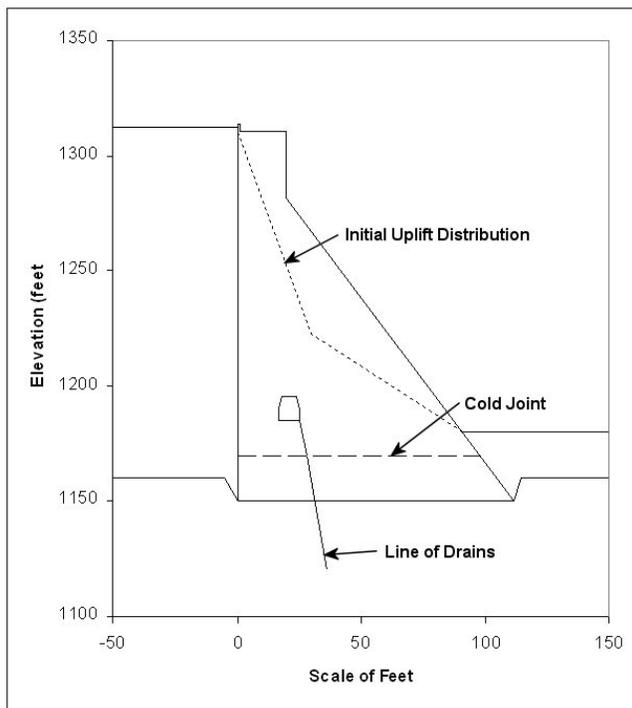


# RECLAMATION

*Managing Water in the West*

## Water Operation and Maintenance Bulletin

No. 222



### In This Issue . . .

Probabilistic Stability Analysis — You Can Do It



U.S. Department of the Interior  
Bureau of Reclamation

December 2007

This *Water Operation and Maintenance Bulletin* is published quarterly for the benefit of water supply system operators. Its principal purpose is to serve as a medium to exchange information for use by Bureau of Reclamation personnel and water user groups in operating and maintaining project facilities.

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For further information about the  
*Water Operation and Maintenance Bulletin*, contact:

**Jerry Fischer, Managing Editor**  
**Bureau of Reclamation**  
**Technical Service Center (86-68360)**  
**PO Box 25007, Denver, CO 80225-0007**  
**Telephone: (303) 445-2748**  
**FAX: (303) 445-6381**  
**Email: [jfischer@do.usbr.gov](mailto:jfischer@do.usbr.gov)**

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*Cover photograph – Geometry of RCC gravity dam.*

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Water Operation and Maintenance Bulletin  
No. 222 – December 2007

**CONTENTS**

	<i>Page</i>
Probabilistic Stability Analysis — You Can Do It.....	1

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# Probabilistic Stability Analysis — You Can Do It<sup>1</sup>

*By Greg A. Scott, P.E., Senior Technical Specialist, Bureau of Reclamation, Technical Service Center, Denver, Colorado*

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## Abstract

The dam safety profession is moving toward risk-based decisionmaking. The likelihood of failure (or unsatisfactory performance) is a key input to such evaluations. This paper focuses on simple methods for estimating the probability of sliding instability using standard spreadsheet programs and commercially available add-in macro software. The deterministic equations for factor of safety are programmed into a spreadsheet, but the input parameters are defined as distributions instead of single point values. The so called “Monte-Carlo” simulation process is used to automatically generate many values of factor of safety by sampling each of the input distributions. For the purposes of this paper, the probability of failure is determined as the area under the output factor of safety distribution for values less than 1.0 (although another value defining unsatisfactory performance could also be used). Examples are presented for sliding of a concrete gravity dam on a poorly bonded lift joint during flood loading and for post-liquefaction slope instability of an embankment dam.

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## Introduction

The dam safety profession is gradually, but consistently, progressing toward probabilistic and risk-based methods for decisionmaking. The Bureau of Reclamation (BOR) has instituted risk-based guidelines for dam safety decisionmaking (BOR, 2003). The Corps of Engineers (COE) has published guidance on probabilistic methods (COE, 2006) and has performed portfolio risk analyses for a number of their dams. The Federal Energy Regulatory Commission (FERC) has instituted the Potential Failure Mode Analysis (PFMA) process for all high-hazard dams under their jurisdiction, which is, in essence, a qualitative risk analysis (FERC, 2005). Several States have also used this process. Thus, the direction the profession is headed is clear, and it is important that regulators and practitioners understand the methods that will be employed.

It is the author’s belief that the first quantitative risk analysis for a dam was conducted at least as early as 1912 (Merriman, 1912). Yet, the profession has been slow to adopt probabilistic methods. Most engineering curricula provide limited exposure to probabilistic methods, and most publications on probabilistic methods are written for those with a more advanced understanding of probability

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<sup>1</sup> This paper was presented at the 2007 Association of State Dam Safety Officials Annual Conference (ASDSO), Austin, Texas, September 9–13, 2007.

theory. Hence, most practicing engineers are not comfortable delving into these methods and prefer the more commonly taught and understood deterministic approaches. Yet, most engineers are familiar with “sensitivity” or “parametric” analyses, and it is an easy step from these to probabilistic analyses.

With the development of new computer analysis tools, times are changing. Now, if you can program a deterministic analysis into a spreadsheet, you can use it to perform probabilistic analyses. The author began performing such analyses for sliding of concrete structures in the late 1990s and published some of those results in 2001 (Scott et al., 2001). At about the same time, similar analyses for embankment stability analyses were published (El-Ramly et al., 2002), supporting the fact that the methods can be applied to a variety of practical problems.

This paper describes the basic concepts of performing probabilistic analyses using a standard spreadsheet program (Microsoft® Excel) and commercially available macro add-ins for probabilistic analysis (Palisade Corporation’s @Risk). Other companies sell similar software (e.g., Lotus 1-2-3 and Crystal Ball by Decisioneering, Inc.).

## **Analysis Methodology**

The standard deterministic equations for calculating the factor of safety are programmed into a spreadsheet, but instead of defining the input parameters as single constant values, they are defined as distributions of values. Thus, instead of calculating a single value for the output factor of safety, a distribution of factor of safety is generated from numerous iterations using the so-called Monte-Carlo approach, whereby each of the input distributions are sampled in a manner consistent with their shape. This output distribution is used to determine the probability of failure. In most cases, this can readily be determined from the percentage of simulation points that are less than the factor of safety that represents failure. For the purposes of this paper, the probability of failure is defined as the probability of a factor of safety (FS) less than 1.0. However, other values can be used. For example, if the dam is particularly susceptible to deformation damage, a larger value of safety factor may appropriately define the state at which “unsatisfactory performance” occurs (El-Ramly et al., 2002).

## **Post-Liquefaction Embankment Stability**

### **Pertinent Background Information**

A dike was constructed in the early 1940s just prior to World War II to close off a wide drainage on one side of a large reservoir. The dike is about 76 feet high from the original channel, impounding a maximum water depth of 70 feet under normal conditions. It has 2:1 (H:V) upstream and downstream slopes and a 20-foot-wide crest. The section is homogeneous with no filters or transition

zones, but a cutoff trench was excavated through about 20 feet of alluvium to bedrock. Not much is known about construction of the dike except that it was compacted with a sheep's-foot roller using material from a borrow area in the reservoir. Embankment soils are primarily clayey sand (SC). Since the reservoir is in a seismically active area, three exploratory borings were drilled through the downstream face to explore foundation conditions. What appears to be a continuous clean sand layer, approximately 4 to 6 feet thick, was encountered in all three borings, approximately 8 feet below the dam-foundation contact. The minimum corrected  $(N_1)_{60}$  blow count values encountered in this layer varied from 13 to 15 depending on the boring. The toe of the dike is wet, indicating a high phreatic surface and saturated foundation materials in that area. Piezometers installed in the embankment indicate differences in the phreatic surface of about 9 feet from one hole to another at the same distance downstream of the centerline.

### Geometry and Calculation Method

Due to seismic concerns, post-liquefaction stability analyses were undertaken, using the cross section shown in figure 1. The sliding surface was assumed to follow the liquefied sand layer and intersect the upstream face below the reservoir surface at normal full pool such that no embankment remnant would be left to retain the reservoir. Although there may be slip surfaces with a lower factor of safety, judgment is needed to select a surface that will not result in a crest remnant capable of retaining the reservoir. It may be appropriate to examine other slip surfaces or use a factor of safety greater than 1.0 to represent unsatisfactory performance to cover the possibility of a more critical slip surface. The simplified Bishop method of analysis (Scott, 1974) was programmed into a spreadsheet, as shown in figure 2. The "allow circular reference" feature in Excel is used to iterate to the solution. Eleven slices were used to define the potential sliding mass.

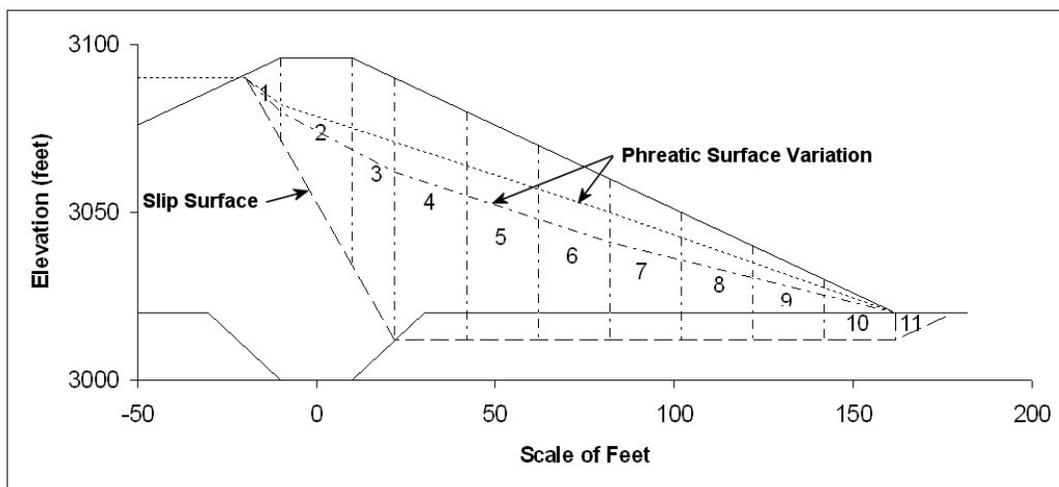


Figure 1.—Embankment and analysis geometry.

## Water Operation and Maintenance Bulletin

	A	B	C	D	E	F	G	H	I	J	K	L
1	Bishop's Slope Stability Analysis											
2	Drained Strengths			Undrained Strengths			Density					
4	c' =	696	psf	c =	637	psf	γ =	114	pcf			
5	tan φ' =	0.672		tan φ =	0							
6	φ' =	33.9	degrees	φ =	0							
7												
9	Slice	Δxi	c*Δxi	ui	ui*Δxi	Wi	θi	(Wi-ui*Δxi)*tanφ	C + H	cos(θi)*[1+(tanθi*tanφ/FS)]	I / J	Wi*sinθi
10		(ft)	(kips)	(ksf)	(kips)	(kips)	(deg)	(kips)				(kips)
12	1	12	8.35	0.28	3.37	17.72	60.6	9.64	18.00	0.92	19.63	15.44
13	2	20	13.92	1.45	29.02	99.97	60.6	47.68	61.60	0.92	67.21	87.10
14	3	12	8.35	2.90	34.82	94.06	60.6	39.81	48.16	0.92	52.55	81.95
15	4	20	12.73	0	0.00	165.86	0.0	0.00	12.73	1.00	12.73	0.00
16	5	20	12.73	0	0.00	143.14	0.0	0.00	12.73	1.00	12.73	0.00
17	6	20	12.73	0	0.00	120.42	0.0	0.00	12.73	1.00	12.73	0.00
18	7	20	12.73	0	0.00	97.70	0.0	0.00	12.73	1.00	12.73	0.00
19	8	20	12.73	0	0.00	74.98	0.0	0.00	12.73	1.00	12.73	0.00
20	9	20	12.73	0	0.00	52.26	0.0	0.00	12.73	1.00	12.73	0.00
21	10	20	12.73	0	0.00	29.54	0.0	0.00	12.73	1.00	12.73	0.00
22	11	16	11.13	0.19	3.00	7.27	-26.6	2.87	14.01	0.68	20.74	-3.26
23	Sum ->										249.26	181.23
24	FS = ΣK/ΣL =	1.38										

Figure 2.—Spreadsheet for calculating factor of safety using the Bishop method.

## Material Properties

Input variables that were defined as distributions included: (1) embankment soil unit weight ( $\gamma$ ), (2) effective stress cohesion of the embankment material ( $c'$ ), (3) effective stress friction angle of the embankment material ( $\phi'$ ), (4) undrained residual shear strength of the liquefied sand layer ( $S_u$ ), and (5) water forces at the base of each embankment slice for which effective stress parameters were defined. No test results were available for the embankment materials. Therefore, the mean, standard deviation, maximum, and minimum values listed in *Design of Small Dams* (BOR, 1987) for SC material (see table 1) were used to define truncated normal distributions. The @Risk function for the effective stress cohesion as an example, is RiskNormal(720,360,RiskTruncate(101,1224)). The friction angles were converted to  $\tan \phi'$  for the spreadsheet calculations. It should be noted that in many cases these types of embankment materials are treated as undrained or “friction only” strengths and are used based on the shear strength curves. However, both  $c'$  and  $\phi'$  are used in this example.

Table 1.—Summary of embankment input properties

Property	Minimum	Maximum	Mean	Standard deviation
Moist unit weight (lb/ft <sup>3</sup> )	91.1	131.8	115.6	14.1
$c'$ (lb/ft <sup>2</sup> )	101	1224	720	360
$\phi'$ (degrees)	28.4	38.3	33.9	2.9

For simplicity, moist soil unit weight was used for the entire soil mass, including the foundation alluvium. It is recognized that the saturated embankment unit weight (below the phreatic surface) will actually be slightly higher and that the alluvial materials could also be somewhat different. It is also assumed that the effective stress parameters listed in table 1 are equally applicable above and below the phreatic surface. A variation in phreatic surface of up to 9 feet under the downstream face was used to estimate water pressures and forces at the base of each slice where the sliding surface passes through the embankment, as shown in figure 1. A uniform distribution between the upper and lower values was assigned, for example, RiskUniform(1.22,1.68) in kips/ft<sup>2</sup>, indicating any value between the upper and lower value is equally likely. Finally, undrained residual shear strength of the liquefied foundation sand was estimated using the curves developed by Seed and Harder (1990). Upper and lower bound curves are provided as a function of corrected blow count. It was assumed that midway between the curves represented the best estimate value. A triangular distribution between the upper and lower bound values, with a peak at the best estimate, was used to define this input parameter, RiskTriang(360,630,920) in lb/ft<sup>2</sup>. It is recognized that more recent guidance suggests a relationship proposed by Stark and Mesri should carry a small weight in combination with the Seed and Harder relationship (Seed et al., 2003). However, it is expected the Seed and Harder relationship would be slightly more conservative, and, for simplicity, it was used alone.

## Calculations

After entering the input distributions in the spreadsheet cells, the factor of safety cell is selected as the output and the simulation settings are adjusted. In this case, 10,000 iterations were specified using the Latin Hypercube sampling method. Using the Latin Hypercube method just means that the same combination of values is not selected more than once, and the results should converge to a smooth distribution more quickly. Then, the simulation is run with the click of a button. For each iteration, the input distributions are sampled in a manner consistent with their shape or probability density function, and a factor of safety is calculated. This results in a listing of the calculated factors of safety for the simulation. It is a simple matter to sort the listing of output factors of safety in ascending or descending order using the sort command of the spreadsheet program. The probability of FS<1.0 is the number of iterations whose calculated factor of safety is less than 1.0, divided by the total number of iterations. In this case, 228 iterations produced a factor of safety less than 1.0. Therefore, the probability of FS<1.0 is 228/10,000 or 0.0228.

To help understand which input distributions have the greatest effect on the results, the @Risk program prints out a list of ranking coefficients. Those input distributions with the highest positive or negative ranking coefficients affect the results most. For the example just described, the coefficients are shown in table 2. It can be seen that the drained cohesion of the embankment and the

Table 2.—Embankment dam sensitivity rank coefficients

Rank	Name	Cell	Regression	Correlation
1	c'	\$B\$4	0.726344	0.732725132
2	c	\$E\$4	0.590719261	0.575407848
3	$\gamma$	\$H\$4	-0.292465055	-0.272376816
4	$\phi'$	\$B\$5	0.137192535	0.130003719
5	u slice 2	\$D\$13	-0.072522808	-0.070526537
6	u slice 3	\$D\$14	-0.052556307	-0.051631753
7	u slice 11	\$D\$22	-0.020666858	-0.020920598
8	u slice 1	\$D\$12	-0.018467738	-0.004675745

undrained residual shear strength of the sand layer affect the results the most. A negative ranking coefficient just means that the variable is negatively correlated with the result. For example, an increase in water force results in a decrease in factor of safety, as expected.

Although probabilistic analyses attempt to account for uncertainty, when dealing with dam safety engineering, it is unlikely that there will be sufficient data to define the input distributions with extreme confidence. Therefore, it may be appropriate to perform sensitivity studies using variations to the input distributions. For the case of the slope described above, two additional simulations were run with the following variations:

1. Examination of test values for SM soils from *Design of Small Dams* (BOR, 1987) indicates a higher mean and more variation in  $c'$  than for SC material. Since some siltier zones were observed in the embankment during sampling, more variation in this property may be warranted. However, the mean value used appears to be appropriate. The standard deviation of the drained embankment cohesion,  $c'$ , was increased by 50 percent to 540 lb/ft<sup>2</sup>. In addition, rather than truncating the maximum and minimum values for  $c'$  at the soil test values shown in table 1, these values were allowed to vary between 20 and 2,000 lb/ft<sup>2</sup> to account for the fact that the full range of possible values may not have been captured by the limited testing.
2. In lieu of No. 1 above, the undrained residual strength of the sand layer was taken as RiskTriang(310,560,790) based on an  $(N_1)_{60}$  value of 13 (lower value from the exploration) rather than 14 (mid-range value).

The results of all three simulations are summarized in table 3. Increasing the standard deviation and upper limit for  $c'$  also increased the location of the distribution centroid, resulting in a higher mean factor of safety. However, it can be seen that, although the mean factor of safety increased when more variation was allowed, the probability of  $FS < 1.0$  also increased. Decreasing the residual undrained shear strength of the sand layer decreased the mean factor of safety and increased the probability of  $FS < 1.0$  as expected. It is generally accepted that a post-liquefaction factor of safety of about 1.2 to 1.3 represents an adequately stable condition. These analyses provide a quantitative indication of what this actually means in terms of failure likelihood for the embankment studied. They also provide an indication of the likely range in failure probability, given uncertainty in the input distributions. It should be noted that even if the embankment remains stable, deformations could result in transverse cracking through which seepage erosion could take place. This must also be considered in evaluating the overall risks posed by the dam and reservoir.

Table 3.—Results of embankment post-liquefaction simulations

Case	Mean F.S.	Probability F.S.<1.0
Original input distributions	1.38	0.0228
Increase standard deviation and limits	1.44	0.0345
Lower undrained residual strength	1.32	0.0605

## RCC Gravity Dam Sliding Stability

### Pertinent Background Information

Construction of a 160-foot-high roller-compacted concrete (RCC) dam in a wide canyon on the west coast of the United States was suspended for winter shutdown after the RCC reached a height of 20 feet. The following construction season, the cold joint surface of the previous year was thoroughly cleaned and coated with mortar, and the remainder of the dam was placed. A gallery was constructed such that the gallery floor would be about 5 feet above tailwater during PMF conditions. A line of 3-inch-diameter drains, spaced at 10 feet, was angled downstream from the gallery, intersecting the cold joint about 28 feet downstream of the axis. Although a 3.5-foot-high parapet wall was constructed on the upstream side of the dam crest, the spillway was sized to pass the probable maximum flood (PMF) without encroaching on the wall. Due to concerns about the strength of the cold joint, five 6-inch-diameter cores were taken 1 year later. Two of the five cores were not bonded at the lift joint. The remaining three were tested in direct shear at varying normal stresses. Although only three data points were generated, the results were well behaved, as shown in figure 3. Accounting for about 40 percent de-bonded area of the joint, it was determined that the design

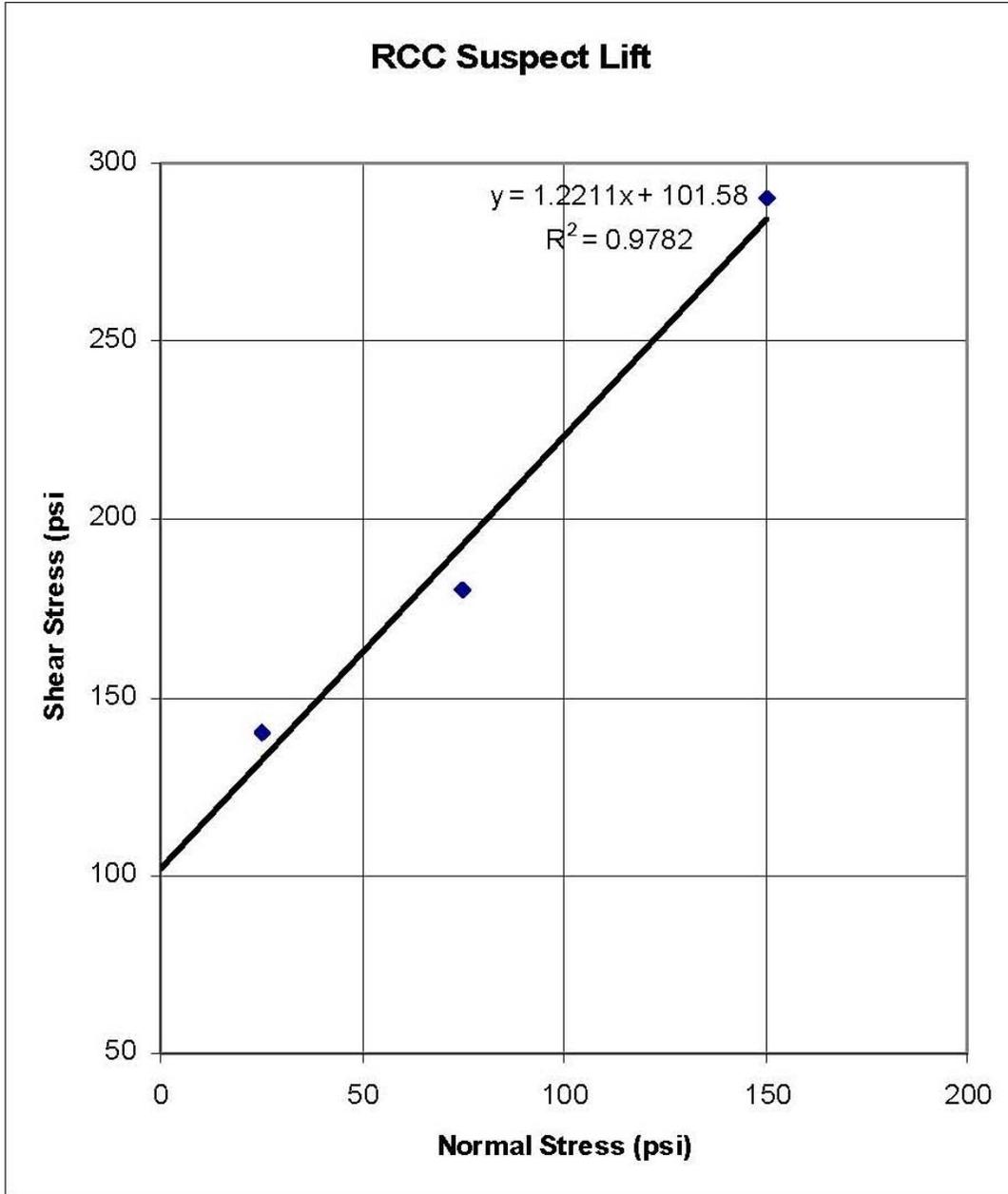


Figure 3.—Direct shear test results for suspect RCC lift joint.

intent was still met. Several years later, the PMF was revised and a flood-frequency analysis was performed. Although the new PMF did not overtop the dam, it encroached about 2.3 feet onto the parapet wall. Maximum tailwater did not change significantly. Additional stability analyses were undertaken to evaluate the likelihood of failure under the new loading condition.

## Geometry and Calculation Method

The section shown in figure 4 was used in the analysis. The vertical stress at the upstream face is calculated considering the familiar standard equation from mechanics of materials:  $P/A \pm Mc/I$  to account for the vertical load (P) and the moment (M) induced by the reservoir for the total stress condition, as indicated by Watermeyer, 2006. Initially, uplift is approximated by a bi-linear distribution, varying from full reservoir pressure at the upstream face, to a reduced pressure at the line of drains, to tailwater at the downstream face. The total head at the line of drains is defined as  $F_d * (\text{Reservoir El.} - \text{Tailwater El.}) + \text{Tailwater El.}$ , where  $F_d$  is the drain factor (1-efficiency). The pressure head is determined by subtracting the elevation of the potential sliding surface from the total head. The effective stress is calculated along the potential sliding plane by subtracting the uplift pressure from the total stress, and where the effective stress is calculated to be tensile, no resistance is included for that portion of the plane. Since the locations of potential joint de-bonding are unknown, the cold joint is also assumed to be cracked to the point of zero effective stress in this case. Full uplift was assumed in the crack until it extended past the drains. Then, approximate equations were used to adjust the drain factor to account for the crack length, based on research performed at the University of Colorado (Amadei et al., 1991). This required the “allow circular reference” feature of Excel to iterate on a crack length. The factor of safety was then ultimately calculated from the familiar equation  $FS = [c'A + (W-U)\tan\phi'] / D$ , where W is the vertical load, A is the bonded area, U is the uplift force, and D is the driving force taking into account both the downstream-directed reservoir load and the upstream-directed tailwater load. The equations for limit equilibrium analysis were programmed into a spreadsheet as shown in figure 5.

## Material Properties

Input variables that were defined as distributions included the following: (1) drain factor  $F_d$ , (2) tangent of the intact friction angle on the potentially weak lift joint  $\phi'$ , (3) intact cohesion on the potentially weak lift joint  $c'$ , (4) percent of the joint that is intact, and (4) the RCC density. Table 4 defines the distributions that were used. The RCC density, based on measurements from the core, was relatively constant, and a uniform distribution between the minimum and maximum values was used. The initial drain factor was taken to be a uniform distribution based on piezometer measurements and experience with other

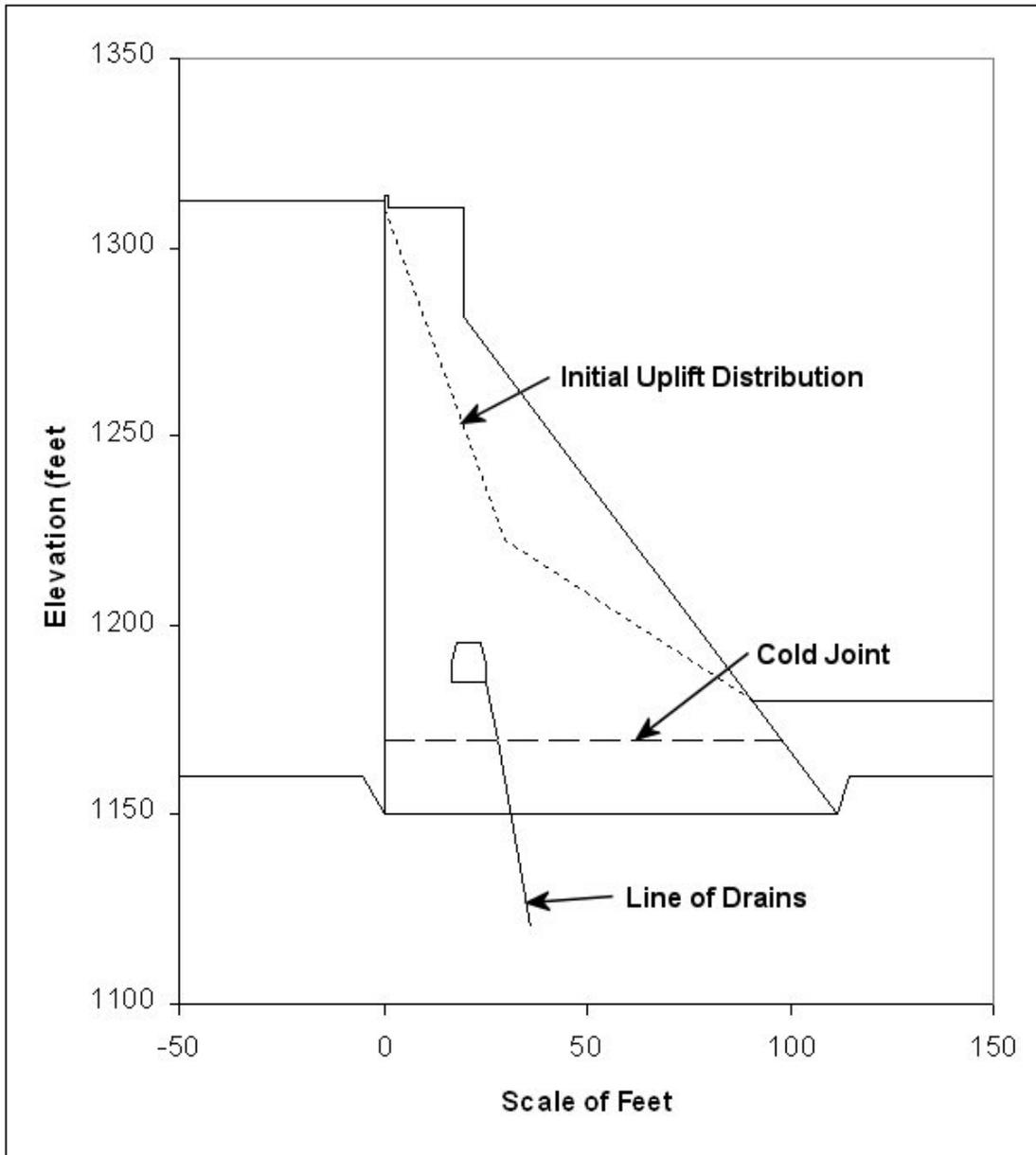


Figure 4.—Geometry of RCC gravity dam.

Water Operation and Maintenance Bulletin

	A	B		A	B
1	Concrete Gravity Dam		30		
2	2-D Monte Carlo Safety Factor Analysis using @Risk		31	Horizontal Reservoir Load (k/ft) =	631.8
3			32	Horizontal Tailwater Load (k/ft) =	-3.1
4	Coordinate System		33	(Vertical tailwater not included)	
5	> +X (Horizontal) = D/S, +Y (Vertical) = Up		34		
6	> +M (Moment) = Clockwise		35	Stress (+ = tension)	
7			36	Moment about center of base (k-ft) =	11744
8	Crest Elevation (ft) =	1310.0	37	Moment of inertia =	78433
9	Base Elevation (ft) =	1170.0	38	Total Vert Stress @ D/S Face (psi) =	-126.4
10	Crest Width (ft) =	20.0	39	Total Vert Stress @ U/S Face (psi) =	-24.5
11	D/S Slope of Dam (H/V) =	0.7	40	Total Vert Stress @ Drains (psi) =	-97.3
12	Reservoir Elevation (ft) =	1312.3	41	Effective Vert Stress @ D/S Face (psi) =	-122.1
13	Tailwater Elevation (ft) =	1180.0	42	Effective Vert Stress @ U/S Face (psi) =	37.2
14	Drain Dist from U/S Face (ft) =	28.0	43		
15	Drain Factor =	0.54	44	Modifications based on Tensile Zone	
16	TAN Friction Angle (deg) =	1.22	45	Drain Factor (Univ. of Colo.) =	0.54
17	Intact Cohesion (psi) =	100.0	46	Pressure Head at Drains (ft) =	81.4
18	Percent Intact =	58	47	Effective Vert Stress @ Drains (psi) =	-62.0
19	Concrete Density (pcf) =	149	48	Length of Tension Zone (ft) =	10.5
20			49	Adjusted Base Length (ft) =	87.5
21	Horiz Length of Slide Plane (ft) =	98.0	50	Total Uplift Force (k/ft) =	415.1
22	Base Weight (k/ft) =	-1022.1	51		
23	Crest Weight (k/ft) =	-42.6	52	Total Normal Force (k/ft) =	-649.6
24	Total Dam Weight (k/ft) =	-1064.7	53	Frictional Resistance (k/ft) =	793.4
25			54	Intact Resistance (k/ft) =	730.8
26	Pressure Head at Heel (ft) =	142.3	55	Driving Force (k/ft) =	628.7
27	Pressure Head at Toe (ft) =	10.0	56		
28	Pressure Head at Drains (ft) =	81.4	57	Factor of Safety =	2.42
29	Total Uplift Force (k/ft) =	395.2			

Figure 5.—Spreadsheet for gravity dam stability calculations.

Table 4.—Summary of concrete input properties

Property	Distribution	Minimum	Peak	Maximum
Initial drain factor, $F_d$	Uniform	0.33	n/a	0.75
$\phi'$ (degrees)	Triangular	43	50	57
Intact $c'$ (lb/in <sup>2</sup> )	Triangular	50	100	150
Percent intact	Triangular	43	60	71
Density (lb/ft <sup>3</sup> )	Uniform	146	n/a	152

concrete dams of similar geometry. The coring would suggest that about 60 percent of the lift surface was bonded, assuming the cores were not mechanically broken during drilling. To estimate a likely range, the percentage was adjusted assuming the drilling of two more holes yielded bonded lifts on the high side, or yielded unbonded lifts on the low side. Both the cohesion and tangent friction angle were defined as triangular distributions, with the peak of the distribution corresponding to the straight line fit shown in figure 3. High and low values were estimated based on experience with other direct shear tests on concrete joints and passing reasonable lines through the data points.

**Calculations**

The minimum safety factor calculated from 10,000 iterations was 1.43, with a mean value of 2.42. The sensitivity analysis indicated the cohesion had the largest effect on the results, as shown in table 5.

Table 5.—RCC dam sensitivity rankings

Rank	Name	Cell	Regression	Correlation
1	Intact cohesion (psi) =	\$B\$17	0.759017659	0.759702063
2	TAN friction angle =	\$B\$16	0.411501707	0.395787559
3	Percent intact =	\$B\$18	0.368619688	0.349212338
4	Drain factor =	\$B\$15	-0.311968848	-0.314501945
5	Concrete density (pcf) =	\$B\$19	0.09730957	0.085434774

Figure 3 suggests that the cohesion and friction angle are negatively correlated. That is, as the friction angle becomes greater, a line that passes through the data would intercept the vertical axis at a lower cohesion value, and vice versa. @Risk allows the user to correlate input variables such that in this case, a high value of cohesion will only be sampled with a low value of friction angle. Since there were limited data points upon which to base a correlation, a negative correlation coefficient of 0.8 was selected, meaning that the highest cohesion value does not have to be associated with the absolute lowest friction angle, but the general trend of the correlation is maintained. The minimum factor of safety calculated with this correlation is 1.79, higher than if the correlation is not maintained, indicating that ignoring the correlation would be conservative.

Since the factor of safety never drops below 1.0, it is not possible to determine the probability of failure in the same manner as for the embankment dam example. Since none of the 10,000 iterations produced a FS<1.0, it can be said that the probability of FS<1.0 is less than 1 in 10,000. However, it is possible to estimate a probability of FS<1.0 by fitting a distribution to the results without needing to run millions of iterations.

For this, the parameter “reliability index” or  $\beta$  must be introduced. The reliability index is simply the “number of standard deviation units” between the mean value and the value representing failure. Figure 6 shows the output factor of safety distribution for the first case discussed for the RCC gravity dam, with cohesion and friction angle treated as independent variables. Goodness of fit tests indicate the distribution follows a normal (bell-shaped) distribution quite well. The reliability index in this case, relative to a safety factor of 1.0, is  $(FS_{AVG} - 1.0)/\sigma_F$ , where  $FS_{AVG}$  is the mean safety factor and  $\sigma_F$  is the standard deviation of the

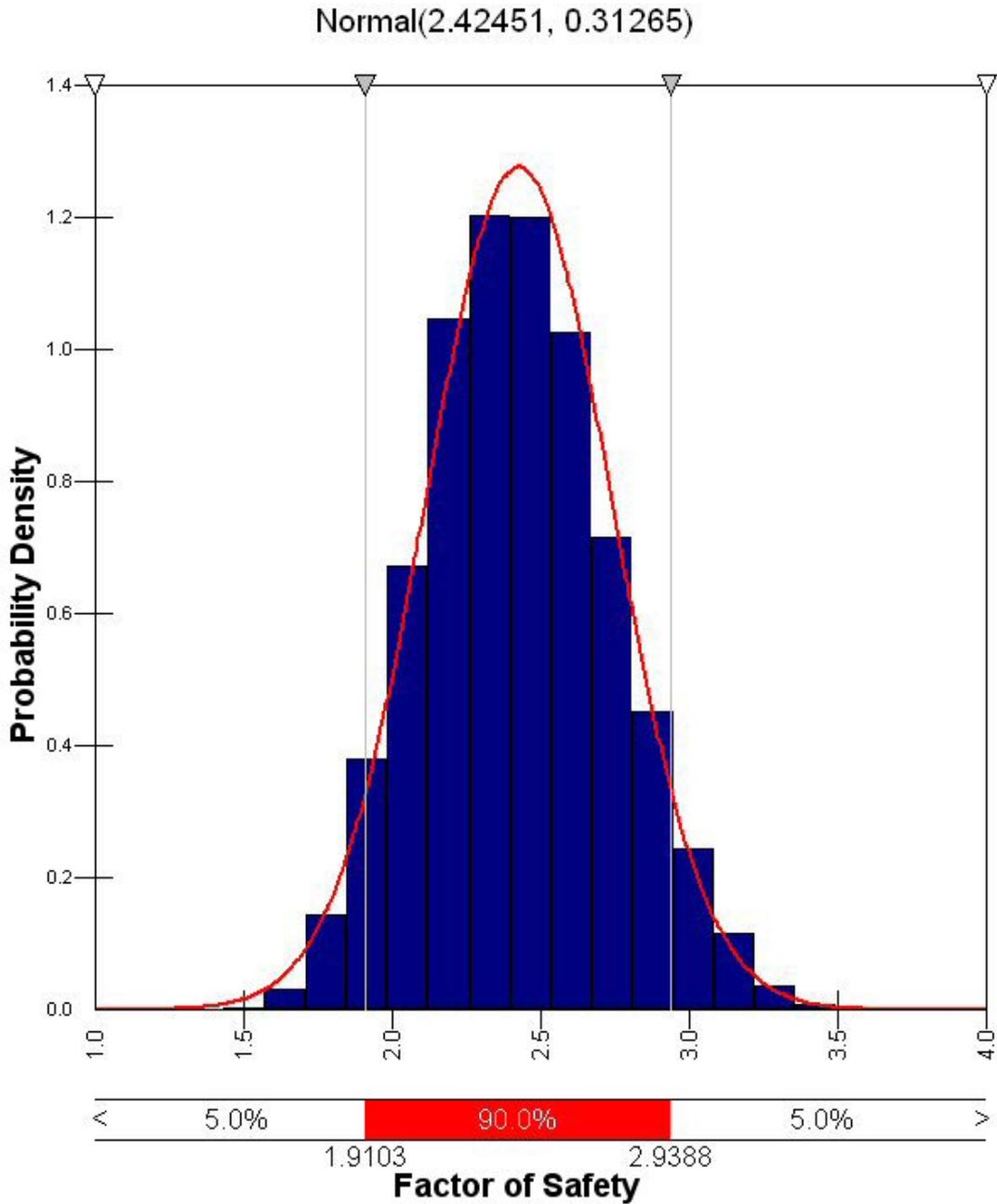


Figure 6.—Output factor of safety distribution for RCC dam.

safety factor distribution, or  $\beta = (2.425 - 1.0) / 0.3126 = 4.56$ . There is a standard function in Microsoft Excel that allows one to estimate the probability of failure directly from the reliability index, which is  $1 - \text{NORMSDIST}(\beta)$ . In this case, using this function produces a probability of  $FS < 1.0$  of  $2.61 \times 10^{-6}$ . This is a very low number, which seems reasonable given the high mean factor of safety and the

fact that the minimum value calculated in 10,000 iterations never dropped below 1.4. In many cases, the output factor of safeties may not follow a normal distribution, but rather a lognormal distribution. This same method can be used to estimate the probability of  $FS < 1.0$ . The only difference is that the reliability index is calculated with a different formula (Scott et al., 2001).

## **Additional Considerations**

“Conditional” probabilities are described in the preceding discussions. That is, given the loading conditions (earthquake or flood), they define the probability of  $FS < 1.0$ . The annual probability of failure is the probability of the loading multiplied by the probability of failure given the loading.

The probability of failure depends not only on the mean values of the input parameters, but also on their variability. Thus, two dams can have exactly the same mean factor of safety but entirely different probabilities of failure, or a dam with a lower factor of safety than another can also have a lower probability of failure. Spatial correlation can also be an important factor. For example, two holes drilled close together are more likely to sample materials with similar properties than two holes drilled far apart. Thus, variation along a potential sliding surface within a given material can be described probabilistically (El-Ramly et al., 2002), rather than treating these properties as uniform along the entire surface, as was done in the examples described here.

So, what can you do with this information? The probability of failure is useful in many regards:

- Communication with the general public is enhanced when the likelihood of a problem can be explained. Doctors have been describing the likelihood for success or complications of medical procedures in probabilistic terms for many years, and people have a concept of what they mean.
- The likelihood of failure can be used to prioritize dam safety activities, perhaps within a risk context, where risk is the annual probability of failure (which includes the probability of the loading) multiplied by the consequences of failure.
- The risk reduction benefits in terms of the money spent per unit reduction in failure probability or risk can provide a useful measure for comparing dam safety modification alternatives.

- The likelihood of failure and corresponding risk can also be used with risk guidelines developed by the dam safety industry (BOR, 2003; ANCOLD, 2003) to determine if risk reduction actions are warranted. These guidelines were generally developed by considering what level of risk society seems willing to accept in order to realize the benefits of high-hazard structures such as dams.

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The mission of the Bureau of Reclamation is to manage, develop, and protect water and related resources in an environmentally and economically sound manner in the interest of the American public.

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email: [jfischer@do.usbr.gov](mailto:jfischer@do.usbr.gov)

Vicki Hoffman, Pacific Northwest Region, ATTN: PN-3234, 1150 North Curtis  
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