APPENDIX B

HYPOCENTER-VELOCITY-STATION CORRECTION INVERSION

The mathematical description of the joint hypocenter-velocity-station correction inversion given below is generalized for three-dimensional velocity models. For inversions performed with one-dimensional velocity models, the formulation is the same but the velocities are only allowed to vary in the vertical direction.

The Forward Problem

The P-wave and S-wave velocity structures are represented by a 3-D, rectangular grid of nodes, with linear interpolation of the velocities between nodes. The velocity *V* at any point *(X,Y,Z)* is computed from the velocities at the eight nodes surrounding it:

$$V(X, Y, Z) = \sum_{i=1}^{8} v_i \left(1 - \frac{|X - x_i|}{dx}\right)\left(1 - \frac{|Y - y_i|}{dy}\right)\left(1 - \frac{|Z - z_i|}{dz}\right) \text{ ,(B-1)}$$

where (x_i, y_i, z_i) are the coordinates of the *i*th node, and v_i is it's velocity. The parameters *dx*, *dy*, and *dz* are the distances in the *x*, *y*, and *z* directions between the velocity nodes surrounding the point *(X,Y, Z)*.

The ray bending method of Um and Thurber (1987), as modified by Block (1991), is used to compute ray paths and travel times. This method starts with a straight ray path defined by two endpoints and one midpoint (i.e., the ray path is broken into two equal segments). The midpoint of the ray path is iteratively perturbed until the travel time is minimized. Each of the two segments is then divided in half. Each of the points along the ray path (excluding the endpoints) are then iteratively perturbed until the travel time is minimized. The process of dividing ray path segments and iteratively perturbing the ray path points is repeated until convergence of the computed travel time is achieved.

The Inverse Problem

The nonlinear hypocenter-velocity-station correction inversion is performed by iteratively solving the constrained, linearized problem. The separation-of-parameters technique is used to separate the velocity-station correction inversion from the hypocenter relocation. The inverse problem is formulated mathematically as a least squares inversion, but residual weighting is used to approximate an L1-norm optimization.

Let t_{obs}= an observed arrival time; t_{calc} = the corresponding calculated arrival time based on the current model, and r = the residual = t_{obs} - t_{calc}. The goal is to change the model parameters so that the change in calculated arrival time, $\Delta t_{calc,}$ is equal to the residual r. Expanding Δt_{calc} in terms of changes in the model parameters and keeping only the first-order terms gives

$$\Delta t_0 + \frac{\partial t}{\partial x}\Delta x + \frac{\partial t}{\partial y}\Delta y + \frac{\partial t}{\partial z}\Delta z + \sum_{j=1}^{nnodes} \frac{\partial t}{\partial v_j}\Delta v_j + \Delta sc = r , \qquad \text{(B-2)}$$

where $t_{0,}$ x, y, and z are the hypocenter parameters, v_j is the jth P-wave or S-wave velocity node, and sc is the P-wave or S-wave station correction. The partial derivatives are computed analytically. The partial derivatives with respect to the hypocenter parameters are given by:

$$\frac{\partial t}{\partial x} = \frac{-1}{v(x,y,z)}\frac{dx}{ds},\ \frac{\partial t}{\partial y} = \frac{-1}{v(x,y,z)}\frac{dy}{ds},\ \text{and}\ \frac{\partial t}{\partial z} = \frac{-1}{v(x,y,z)}\frac{dz}{ds}, \text{(B-3)}$$

where *dx/ds*, *dy/ds*, and *dz/ds* are the direction cosines of the ray path at the hypocenter, and $v(x, y, z)$ is the velocity at the hypocenter. The partial derivatives with respect to the velocity nodes are computed by summing the contributions from all ray path segments. Given a ray path of n segments, each of length Δs, the partial derivative of the travel time with respect to the velocity at the jth node is given by:

$$\frac{\partial t}{\partial v_j} = \sum_{l=1}^{n} \frac{-\Delta s}{v(x_l, y_l, z_l)^2}\frac{\partial v(x_l, y_l, z_l)}{\partial v_j} \quad , \text{(B-4)}$$

where $v(x_l, y_l, z_l)$ is the velocity at the center of the lth ray path segment. For each ray path segment, $\partial v(x_l, y_l, z_l)/\partial v_j$ is only nonzero for the eight velocity nodes surrounding it and is found by differentiating Equation B-1:

$$\frac{\partial v(x_l, y_l, z_l)}{\partial v_j} = \left(1 - \frac{|x_l - x_j|}{dx}\right)\left(1 - \frac{|y_l - y_j|}{dy}\right)\left(1 - \frac{|z_l - z_j|}{dz}\right) .\text{(B-5)}$$

Both sides of Equation B-2 are divided by $\sqrt{|r|}$ to obtain an approximate L1-norm optimization (Scales et. al., 1988). The equation is also weighted according to the quality of the arrival time pick. In addition, a residual cut-off is employed to discard extreme outliers.

All of the arrival times for the *i*th event yield a set of equations that can be put into matrix form:

$$\boldsymbol{H_i} Dh_i + \boldsymbol{M_i} Dm = r_i . \qquad \text{(B-6)}$$

$\boldsymbol{H_i}$ contains the weighted hypocenter partial derivatives, $\boldsymbol{M_i}$ contains the weighted velocity and station correction partial derivatives, and r_i contains the weighted residuals. The vectors Δh_i and Δm contain the hypocenter perturbations for the *i*th event and velocity and station correction perturbations, respectively. If there are more than 4 arrival times, there exists a matrix $\boldsymbol{T_i}$ such that $\boldsymbol{T_i^T H_i} = 0$ (Pavlis and Booker, 1980, Roecker, 1982). The matrix $\boldsymbol{T_i}$ is computed by QR factorization of $\boldsymbol{H_i}$ (Block, 1991). Multiplying both sides of Equation B-6 by $\boldsymbol{T_i^T}$ yields

$$\boldsymbol{M_i}' \Delta m = r_i' , \qquad \text{(B-7)}$$

where $\boldsymbol{M_i}' = \boldsymbol{T_i^T M_i}$ and $r_i' = \boldsymbol{T_i^T} r_i$. The arrival times for each microearthquake are processed in this way, and the results from each event are added as additional rows to one matrix equation:

$$\boldsymbol{M}' \Delta m = r' . \qquad \text{(B-8)}$$

To prevent extreme fluctuations of the velocity structures at poorly resolved nodes, velocity regularization is included in the velocity-station correction inversion. The regularization is implemented by minimizing either the first-order or second-order spatial velocity derivatives. (For three-dimensional velocity models, first-order derivatives are typically used. For one-dimensional models, second-order derivatives are often used so that there is no penalty for a linear increase of velocity with depth.) Similar methods have been used by others (Lees, 1989; Sambridge, 1990; Phillips and Fehler, 1991). The numerical velocity derivative for each consecutive pair of velocity nodes is set to zero. For a first-order derivative, this is represented by:

$$[(v_i + \Delta v_i) - (v_{i-1} + \Delta v_{i-1})] / d = 0 , \qquad \text{(B-9)}$$

where v_i and v_{i-1} are the velocities of 2 consecutive nodes in one coordinate direction, and d is the distance between the nodes. Equations for all consecutive nodes in the x, y, and z directions are constructed. Rearranging the terms, the equations may be expressed as a matrix equation involving the model solution vector Δm:

$$\boldsymbol{K}\Delta m = -c \, . \qquad \text{(B-10)}$$

The vector c contains the numerical velocity derivatives based on the current model, and **K** contains numerical derivative operators.

P-wave velocities from an acoustic borehole log or other type of geophysical survey may be used to constrain the velocity-station correction inversion. Each velocity data point V at coordinates (X, Y, Z) from the geophysical survey adds constraints to the 8 velocity nodes surrounding it:

$$\sum_{i=1}^{8}\left(1-\frac{|X-x_i|}{dx}\right)\left(1-\frac{|Y-y_i|}{dy}\right)\left(1-\frac{|Z-z_i|}{dz}\right)(v_i+\Delta v_i) = V \, , \qquad \text{(B-11)}$$

where (x_i, y_i, z_i) are coordinates of the ith node and v_i is its P-wave velocity value. The equations for all velocity data points are combined and given in matrix form by:

$$\boldsymbol{D}\Delta m = g \, . \qquad \text{(B-12)}$$

The matrix $\boldsymbol{D}$ contains the node interpolation coefficients, and the vector g contains the velocity residuals, i.e., the velocities from the geophysical survey minus the corresponding calculated velocities based on the current model.

The equations for the earthquake arrival time data (Equation B-8), velocity regularization (Equation B-10), and geophysical survey velocities (Equation B-12) are combined into one matrix equation:

$$\boldsymbol{A}\Delta m = \begin{bmatrix} \boldsymbol{M'} \\ \lambda \boldsymbol{K} \\ \alpha \boldsymbol{D} \end{bmatrix} \Delta m = \begin{bmatrix} r' \\ -\lambda c \\ \alpha g \end{bmatrix} = b \, . \text{(B-13)}$$

The equations for the two constraints (velocity regularization and geophysical survey velocities) are given weighting factors, λ and α, that are adjusted to control the relative importance of satisfying the earthquake arrival time data and the constraints. Equation B-13 is solved using a least-squares conjugate gradient algorithm.

The hypocenters are relocated with an iterative, approximate L1-norm (residual-weighted least squares) inversion. An undamped inversion is tried first. If the event location does not converge within 20 iterations, damping is added and the event quality factor is changed. Most events used in the hypocenter-velocity inversion are constrained well enough for the undamped relocation algorithm to converge. A progressive relocation approach is used, as in Roecker (1982). The horizontal hypocenter coordinates are recomputed first, then all four hypocenter coordinates are allowed to vary. In addition to starting the relocation with the current hypocenter elevation, the relocation may also be performed with additional specified starting elevations. The result that yields the smallest root-mean-square (rms) arrival time residual is chosen. This procedure helps prevent the hypocenter from converging to a local elevation minimum.

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