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HYDRAULICS OF STEPPED SPILLWAYS FOR RCC DAMS AND DAM REHABILITATIONS

BY

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ABSTRACT

Stepped spillways are natural extensions of roller-compacted concrete (RCC) placement techniques. Stepped spillways were first used with concrete dams, and designers are eager to use the same technology to pass flows over the top of embankment dams by providing erosion protection for the downstream slope of the dam. This paper will discuss the use of stepped spillways for both applications. Case histories of stepped spillway applications worldwide and the hydraulic advantages are given. Results from U.S. Bureau of Reclamation's (Reclamation) current research program to define the hydraulics of stepped spillways are presented. The forces and velocities produced by flow over the stepped spillway are quantified and an example of the benefit of reduced stilling basin lengths is presented.

INTRODUCTION

RCC has easily become the most popular method for building new concrete dams and/or rehabilitating many types of existing dams. Modern day RCC dam construction began in the U.S.A. in 1982 when the U.S. Army Corps of Engineers built Willow Creek Dam in Oregon, for flood control. Although constructed of RCC a typical smooth surface chute type spillway passes flow down the center of the dam. Earlier, Tarbela Dam, in Pakistan, had experienced major erosion damage in two spillway plunge pools. This was also repaired using huge amounts of rollcrete, a lean form of RCC, quickly and cost effectively.

Reclamation's first RCC experience was the construction of Upper Stillwater Dam about 80 miles east of

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Salt Lake City, Utah. This dam was built for irrigation storage. A stepped spillway covering a portion of the downstream dam face is capable of passing the probable maximum flood (PMF).

These early successes with RCC placement, producing quick results and low costs, have made RCC extremely popular in the dam building industry. Application has now extended from new dam construction to dam rehabilitations, including existing concrete, crib, embankment, and rockfill dams. Reclamation now has increased interest in applying concrete stepped overlays to protect high embankment or rockfill dams during overtopping events. This interest is a direct result of increased PMF requirements producing deficiencies in present dams and the high costs of traditional Dam Safety remedies. A cooperative research program funded by Reclamation's Dam Safety (SOD) and Water Technology and Environmental Research (WATER) programs, and the Electric Power Research Institute (EPRI) is now being conducted to determine the hydraulic properties of stepped spillway applications.

PURPOSE AND ADVANTAGES OF STEPPED SPILLWAYS

The use of stepped spillways is not a new technology. The ancient Romans first designed low head structures where water flowed down steps. Also early masonry dams (circa 1900) in the U.S. featured stepped spillways. Reemergence of stepped spillways is attributed to the RCC horizontal lift placement techniques of which a stepped surface is a natural outcome. Usually the secondary reason is the potential for dissipation of the flow energy as it travels down the steps to the toe of the dam. Energy dissipation also provides a cost benefit due to the reduced stilling basin length or entire elimination of the required basin. The step shape has been obtained in many ways. Steps have been shaped from unformed or formed RCC, and standard formed or slip-formed conventional concrete with or without reinforcement.

The problem with using stepped spillways has been, and continues to be, the lack of general design criteria that quantifies the energy dissipation characteristics of the steps for a given unit discharge, flow depth, and hydraulic dam height. Steps have proven effective for small unit discharges, where the step height clearly influences the flow. The need to pass larger flows has pushed designs beyond the limitations of the present data base.

The main objective of Reclamation's stepped spillway research program is to define energy dissipation properties

of steps for concrete and for embankment dams while ensuring a stable, protective overlay for the embankment.

STEPPED SPILLWAYS FOR RCC DAMS

Table 1 lists many concrete dams where a stepped spillway on the downstream face of the dam is used as either the service or emergency spillway. Only those dams that have formed steps specifically for providing reduced flow velocities are listed. Many of the dams have incorporated a stepped spillway without the benefit of hydraulic model investigations. Those site-specific stepped spillways with model study data on energy dissipation will be discussed further.

Table 1. - RCC or Rehabilitated Conventional Concrete Dams with Stepped Spillways on the Downstream Face.

Dam and location (Reference, date)	Design unit discharge (ft ³ /s/ft)	Hydraulic Height (ft)	Head (ft)	Downstream slope (H:V)	Downstream concrete facing and placement technique
Upper Stillwater,* UT (Houston, 1987)	123.33	202	9.8	0.32:1 top 0.6:1 toe	Conventional slip-formed
Monksville,* NJ (Sorenson, 1985)	100	120	8.6	0.78:1	Conventional formed
Stagecoach,* CO (Stevens)	39	140	4.72	0.8:1	Conventional formed
De Mist Kraal,* South Africa (Jordaan, 1986)	110	59	9.8	0.6:1	Conventional formed
Zaaihoek, South Africa	55	120	6.2	0.62:1	Conventional slip-formed
Lower Chase Creek, AZ	35.95	59	4.50	0.70:1	Conventional formed
Milltown Hill,* OR (Frizell, 1990)	154.2	180	11.24	0.75:1	Conventional slip-formed
Middle Fork, CO	overtops for events > 500 yrs.	124	?	0.8:1	Conventional formed
Knellpoot, South Africa	90	141.4	7.8	0.60:1	Conventional formed
Santa Cruz, NM	43	120	5.5	0.65:1	Conventional formed
Bucca Weir,* Australia	598	39	37.0	0.5:1	Conventional formed
Jequitai,* Brazil	98.9	118.8	6.87	0.80:1	Proposed formed RCC
Junction Falls Dam, WI	123	29.5	7.5	0.875:1	Conventional formed

Dam and location (Reference, date)	Design unit discharge (ft ³ /s/ft)	Hydraulic Height (ft)	Head (ft)	Downstream slope (H:V)	Downstream concrete facing and placement technique
Les Olivettes, France	78	103.35	9.84	0.75:1	Conventional formed
Cedar Falls, WA	30	25	4.68	0.80:1	Conventional formed

* Stepped spillway designs were determined by hydraulic model studies.

Previous Hydraulic Model Studies of Stepped Spillways for Concrete Dams

Energy dissipation characteristics of several site-specific hydraulic model tests are shown in figure 1.

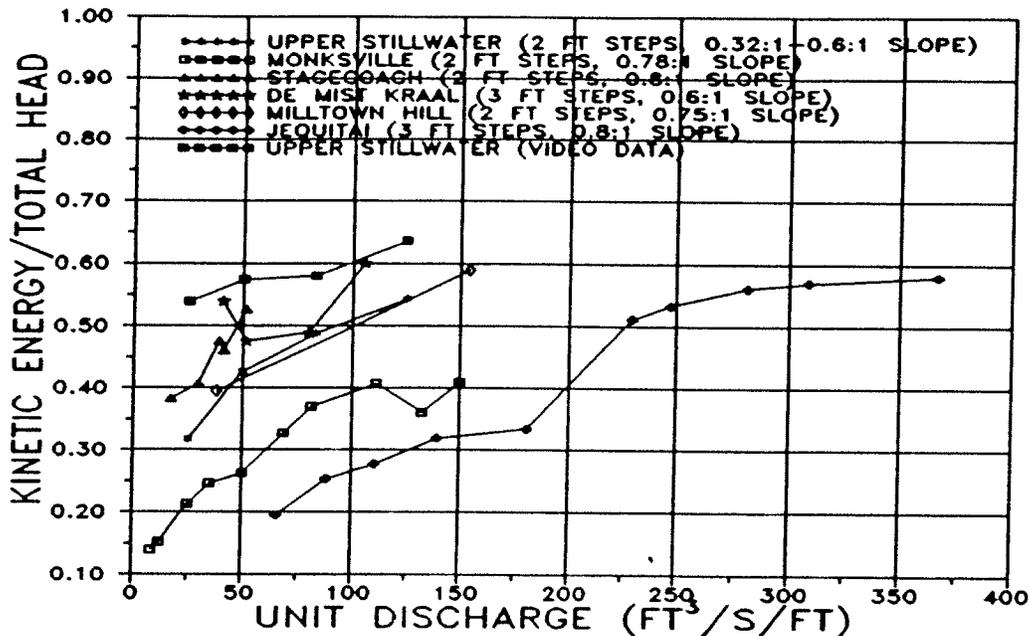


Figure 1. - Energy Dissipation Characteristics at the Toe of Stepped Spillways on Steeply Sloped RCC Dams.

These data are plotted in prototype values and show the ratio of the kinetic energy at the dam toe to the total available head versus the unit discharge. The kinetic energy was calculated using measured velocities on the steps near the dam toe. The ratio reduces to the average stepped spillway velocity over the theoretical maximum velocity, V_s/V_t , for a given dam height. Most of the information is for dams with 2-ft step heights.

There is obviously a great deal of scatter in these data (note the two curves for Upper Stillwater Dam data). Various techniques were used to measure the stepped spillway velocities. These included high speed video, pitot tubes, velocity meters, calculating continuity using measured flow and depth, and calculating the entrance velocity based on a forced hydraulic jump. Velocity data obtained using video recorded only the surface velocity, thus the higher amount of energy remaining at the toe for the Upper Stillwater video data in figure 1. The very turbulent and aerated flow makes obtaining velocity data extremely difficult and produces much of the data scatter. This figure will give the designer some general idea of the velocities at the toe of a stepped spillway taking into account the dam height and staying within the range of data shown.

One objective of Reclamation's research program is to better quantify the velocity or energy remaining in the flow at the toe of a concrete dam with a stepped spillway. This will improve safe designs of these spillways with appropriate and cost effective stilling basin lengths.

STEPPED SPILLWAYS FOR EMBANKMENT DAMS

Dam Safety inspections have concluded that a large number of both small and large embankment dams are unsafe due to predicted overtopping during extreme flood events. Construction of RCC protection for overtopping flow on existing or new embankment or rockfill dams has proven to be very cost effective. The present emphasis of the research program on stepped spillways is on the hydraulic properties produced by the steps on flatter slopes more common to embankment dams. This also has led to determining the step geometry that provides the most stable overlay with energy dissipation, characteristics of secondary importance. Table 2 is a list of several embankment dams that have been protected or are planned for rehabilitation with RCC. These range in height from about 20 ft to as much as 119 ft.

Table 2. - Stepped Spillways for Protection of the Downstream Slope of Embankment Dams.

Dam and location	Unit discharge (ft ³ /s/ft)	Hydraulic height (ft)	Head (ft)	Downstream slope (H:V)	Downstream concrete facing and placement technique
Lahontan, NV	68	110	6	2:1	Conventional formed
Brownwood Country Club, TX (Reeves, 1985)	24.7	19	5.5	2:1	RCC unformed

Dam and location	Unit discharge (ft ³ /s/ft)	Hydraulic height (ft)	Head (ft)	Downstream slope (H:V)	Downstream concrete facing and placement technique
Kerrville, TX	335	21	24	0.8:1	RCC unformed
McClure, NM (Frizell, 1990)	97.26	119	10.26	2.18:1	RCC unformed
Spring Creek, CO	28.25	50	4.46	2.3:1 to 3:1	RCC unformed
Goose Lake, CO (Birch, 1990)	9.1	35	2.4	1:1	RCC formed
Upper Las Vegas Wash Retention Dam	230	37	18	2.5:1	RCC
Ringtown No. 5 Dam, PA	48	60	6.0	2.75:1	RCC unformed

HYDRAULIC RESEARCH OF STEPPED OVERLAYS

Increased PMF forecasts have resulted in numerous low and high dams with inadequate spillway capacity in all business sectors. As a result, Reclamation's Dam Safety and Research Programs have taken the lead in providing funding for investigation of stepped spillway dam overtopping protection.

Laboratory Research Facility

Reclamation's laboratory research facility includes two 1.5-ft-wide Plexiglas-walled flumes--one for steep 0.5:1 to 0.8:1 slopes appropriate for concrete dam applications and one for embankment dam slopes ranging from 2:1 to 4:1. The facility allows investigation of model unit discharges up to 14 ft³/s/ft under reservoir heads up to 2.8 ft. The total drop from the laboratory reservoir to the controlled tailwater is 15.5 ft. One reservoir serves both flumes.

Emphasis is currently focused on providing stepped protection for 2:1 sloping embankment dams. The flume facility for stepped protection of embankment dam slope investigations has been in operation since January 1990, figure 2. The flume facility for stepped spillways on steep concrete dams is 50 percent complete and is scheduled for testing in June 1992.

Stepped Spillway Protection on a 2:1 Slope

Primary importance for embankment dam overtopping protection is placed upon the stability of the stepped

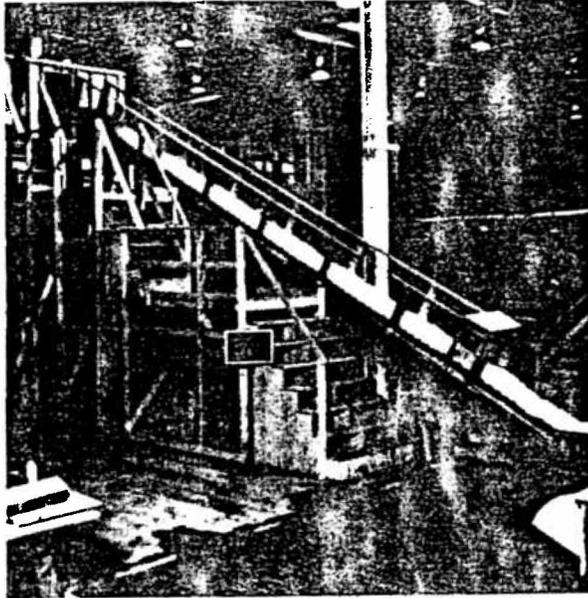


Figure 2. - Overall View of Sloping Flume Facility

concrete overlay. Research is focused on enhancing stability by providing continuous aspiration of subgrade seepage by virtue of the flow characteristics over the stepped surface. Aspiration is suction of the fluid from underneath the overlay. Suction is produced by the pressure differential created by the high velocity flow over the step offset area. For embankment dam protection, the benefit of energy dissipation and reduced velocities at the dam toe are of secondary consideration. The step geometry will be optimized to produce a zone of subatmospheric pressure to relieve buildup of seepage pressure under the overlay. A series of three step geometries is currently under investigation. For each geometry, the flow depth, pressure profiles on the steps at chosen locations, and velocity profiles (every ten steps beginning at the third step downstream from the crest) are recorded. For presentation purposes, a model unit discharge of $6.21 \text{ ft}^3/\text{s}/\text{ft}$ and overtopping head of 1.67 ft will be used throughout the paper to demonstrate and compare the test results.

Step geometry. - Flow over horizontal steps was investigated first. Model steps had a 4-in horizontal tread with a 2-in vertical rise. Following these tests the step tread was sloped downward, at 15° and 10° . The sloped tread was shown in tests by Pravdivets (Pravdivets, 1989) and Clopper (Clopper, 1989), who used individual wedge shaped blocks, to aspirate the subgrade through vent ports. The shapes tested in the flume will be continuous in width with 15° and 10° slopes below horizontal on the tread, figure 3.

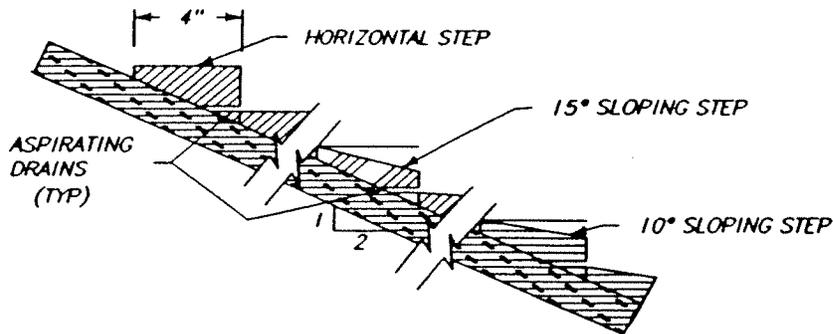


Figure 3. - Schematic of Horizontal, 15°, and 10° Sloping Steps Tested in the Flume.

Pressure Profiles. - The ability of the step shape to produce aspiration of the subgrade flows caused by passing discharges is determined by measuring the pressures on both the vertical and tread surfaces of the steps at specific measurement stations down the slope. Three stations down the flume were instrumented for the horizontal steps, two for the sloping steps. At each station, two steps are instrumented, each with 11 piezometer taps, for a total of 22 taps per station. (Note: The steps are numbered from the crest down the slope.) The mean pressure from each tap is recorded and the profiles plotted over the steps at each station.

The pressure profiles on two successive horizontal steps for 1.67 ft of overtopping head and the measured flow depth clearly show the flow characteristics, figure 4. Comparison of the flow depth with the pressure profiles shows two distinct pressure zones. One zone produces additional loading on the overlay where the jet impacts on the downstream end of the step tread, and the other zone produces reduced pressure in the offset area below the pitch line of the steps where separation of the jet off the step occurs. When considering stability of the entire stepped overlay, the impact provides additional downward force when added to the flow depth; however, in the case of the horizontal tread, the pressure in the offset area is not low enough to provide continuous aspiration of the underlying filter zone.

Sloping the step tread causes a sharp reduction in the low pressure region of each step as compared to the horizontal step. Pressure profiles (1.67 ft of head) measured on the 15° sloping steps at both stations are shown on figure 5. These profiles indicate decreased pressure, compared to the horizontal tread, at the upper

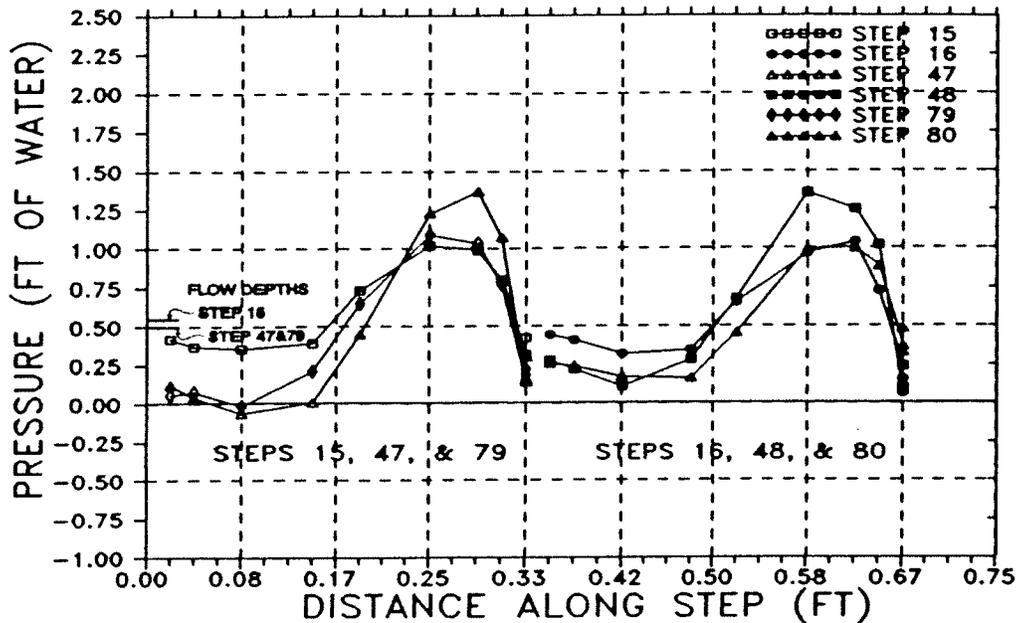


Figure 4. - Pressure Profiles on Horizontal Steps, $q = 6.21 \text{ ft}^3/\text{s}/\text{ft}$, $H = 1.67 \text{ ft}$.

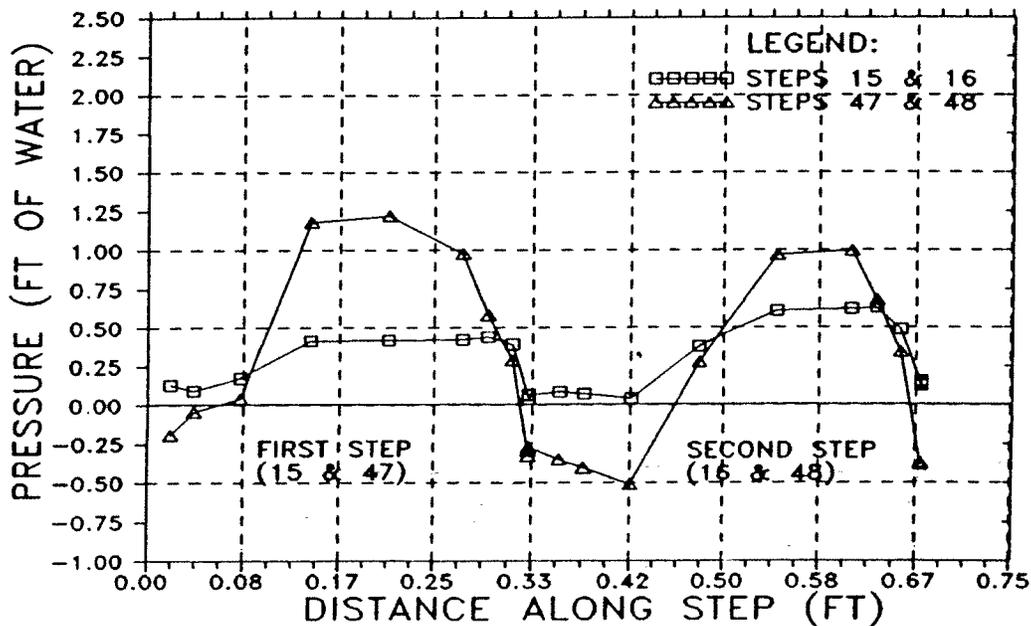


Figure 5. - Pressure Profiles for 15° Sloping Steps, $q = 6.21 \text{ ft}^3/\text{s}/\text{ft}$, $H = 1.67 \text{ ft}$.

station (steps 15 and 16) and subatmospheric pressures at the lower station (steps 47 and 48). These pressures indicate that velocities are not high enough to produce subatmospheric pressures, thus ensure aspiration, at the

upper station. The pressures are subatmospheric over a large area at the lower station and should produce excellent aspiration of the subgrade flows.

If drains are placed in areas which do not aspirate under all expected flows, two conditions may occur. First, if the seepage hydrostatic head behind the drain is greater than the pressure on the step face, the drain will function to relieve uplift pressures. Conversely, if the pressure behind the drain is lower than occurs on the step face, the drain will actively feed water into the subgrade. In general, reverse flow through a drain should be avoided. It is clear from figure 4 that placing drains in horizontal steps on a 2:1 embankment does not ensure embankment drainage. However, the pressure data measured at steps 47 and 48 from figure 5 for a 15° sloping tread show a subatmospheric pressure zone downstream of the step, thus a drain vented in this zone will aspirate for this geometry and flow condition.

Aspiration may be assured by defining the limits of active aspiration in terms of unit discharge, embankment slope, step geometry, and step position down the embankment. The designer can identify aspiration limits of different step angles and determine the placement of drains. The designer then would conduct a stability analyses, including the hydraulic forces and the required thickness of material to assure a stable overlay. The aspirating step geometry will allow the designer to place less material over the embankment and still be assured of stability. Once a stable overlay is designed, the step geometry can be optimized in terms of energy dissipation. The designer may then weigh the benefit of aspiration versus energy dissipation in determining the final step shape selection. The 10° sloping step, scheduled for the next test series, will likely provide less aspiration but have better energy dissipation properties than the 15° steps.

Velocity Profiles. - Velocity profiles are measured down the flume slope using a laser-doppler anemometer mounted on a support frame parallel to the flume slope. Measurements are taken from the tip of the step (datum) to as close to the water surface as possible. The total velocity vector and the magnitude parallel to the slope is computed from the horizontal and vertical components measured at each flow depth. A commercially available software program is then used to provide a best fit equation for each profile. The best fit equation describing the velocity profile is then used to compute the total area under the velocity profile and check for continuity. If necessary, the entire profile is adjusted by a constant, usually in the range of 0 to 3 percent, and the procedure repeated until continuity

is satisfied. Once continuity is satisfied, the velocity profiles are used to calculate the kinetic energy of the flow at each step.

Figure 6 shows the velocity profiles for a horizontal step tread at steps 3, 13, 23, and 33. Notice the near vertical profile at step 3 and the flattening and closer spacing of the profiles as the flow travels down the slope. The velocity profiles for 15° sloping steps exhibit similar traits; however, the velocities are somewhat higher. This indicates that the sloping steps do not interfere as much with the flow, particularly as the flow depth increases. Figure 7 compares the velocity profiles

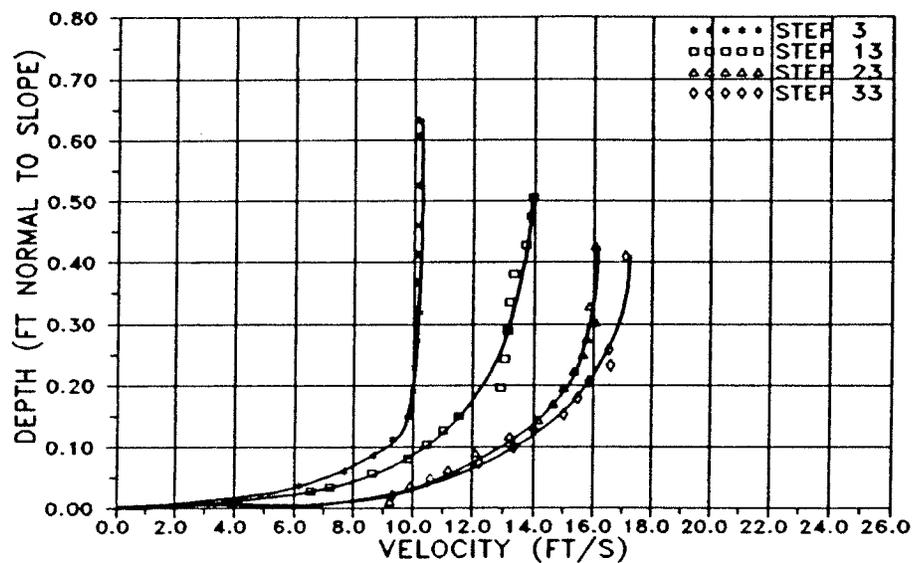


Figure 6. - Velocity Profiles for Horizontal Steps, $q = 6.21 \text{ ft}^3/\text{s}/\text{ft}$, $H = 1.67 \text{ ft}$.

of a smooth surface to those of horizontal and 15° sloping steps at the location of step 23. Of particular interest is the shape of the profiles below a flow depth of 0.2 ft. Here, close to the steps the effect of the step geometry in reducing the flow velocity is quite apparent.

Energy Dissipation. - The energy dissipation characteristics of the step geometries are compared by computing the kinetic energy per unit volume, $\frac{1}{2}\alpha\rho V^2$, to the total head available, $w(H+H_s)$, at each step location. The kinetic energy is calculated by integrating the area under the velocity profile and determining α , the coefficient of kinetic energy. The total head is calculated by adding the overtopping head, H , to the vertical drop from the crest to the step location where the velocity measurement is taken, H_s , and multiplying by the specific weight of water, w .

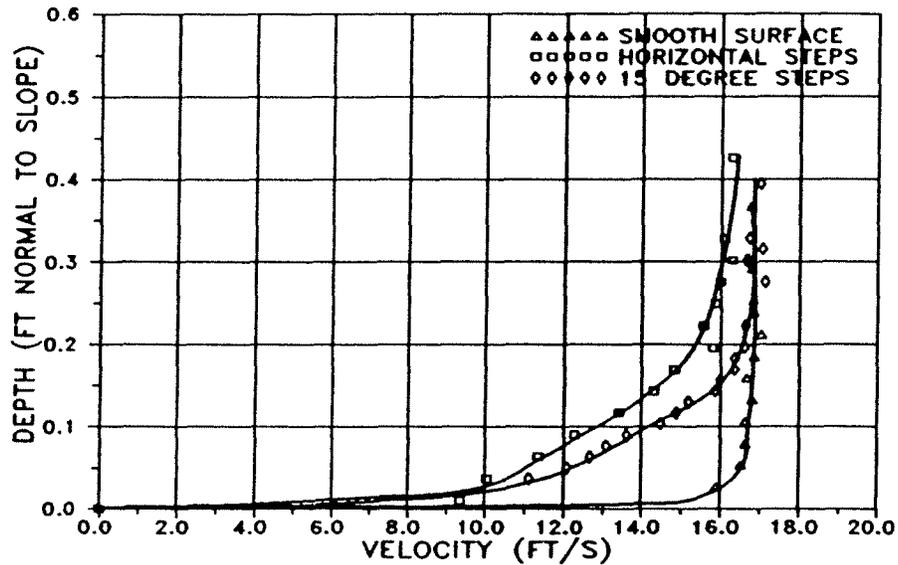


Figure 7. - Velocity Profiles at Step 23 for Smooth, Horizontal, and 15° Sloping Steps.

Figures 8 and 9 show the ratio of kinetic energy to total head along the slope for horizontal and 15° sloping steps and the unit discharges investigated. Note that the

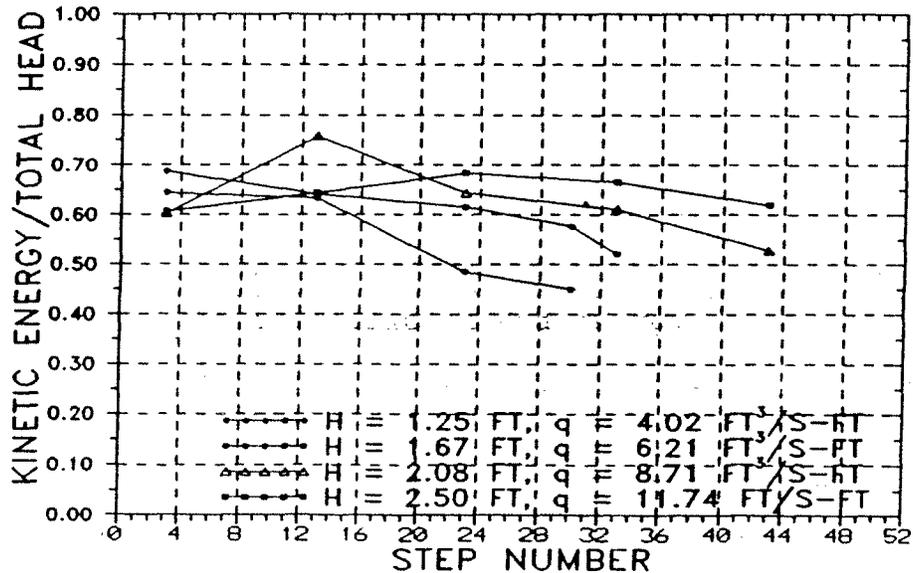


Figure 8. - Ratio of Kinetic Energy to Total Head vs. Step Number for Horizontal Steps on a 2:1 Slope.

kinetic energy remaining in the flow increases as unit discharge increases for both step geometries.

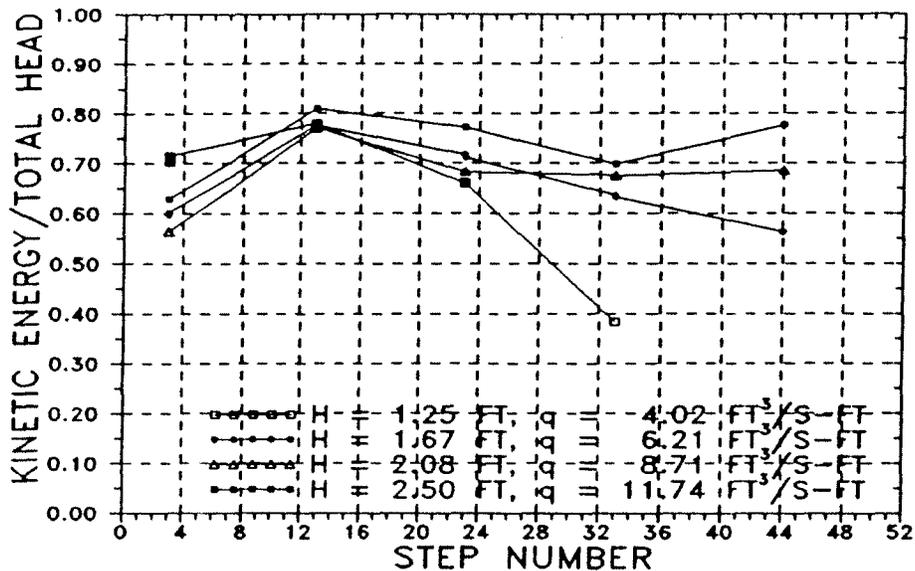


Figure 9. - Ratio of Kinetic Energy to Total Head vs. Step Number for 15 Sloping Steps.

A comparison of the smooth, horizontal, and 15° sloping steps at a unit discharge of 6.21 ft³/s/ft is shown on figure 10. This figure shows the benefits of the step geometry in reducing the total energy available after the flow has travelled down the slope for a given length. The smooth surface spillway shows the flow is still accelerating, while the horizontal steps show the greatest reduction in energy. However, since the horizontal steps do not provide continuous aspiration of the subgrade flows, a sloping step geometry must be chosen which will optimize both aspiration and energy dissipation.

For example, let's determine the length of an uncontrolled hydraulic jump below a 46-ft-high dam (23 steps 2-ft high) for each of the three flow surfaces presented in figure 10. Such a structure would represent a 1 to 12 Froude scale of the model investigation. The calculations assume that the hydraulic jump will be contained in a Type I basin which has no end sill to force the jump (Peterka, 1978). Using figure 10 and scaling up unit discharge (prototype unit discharge = 258.2 ft³/s/ft) and overtopping head (prototype head = 20.0 ft) the velocity entering the jump can be calculated as $V_1 = (KE/H) * 46$ ft. The depth entering the basin, $D_1 = q/V_1$. Figure 6 from Monograph 25 then uses the

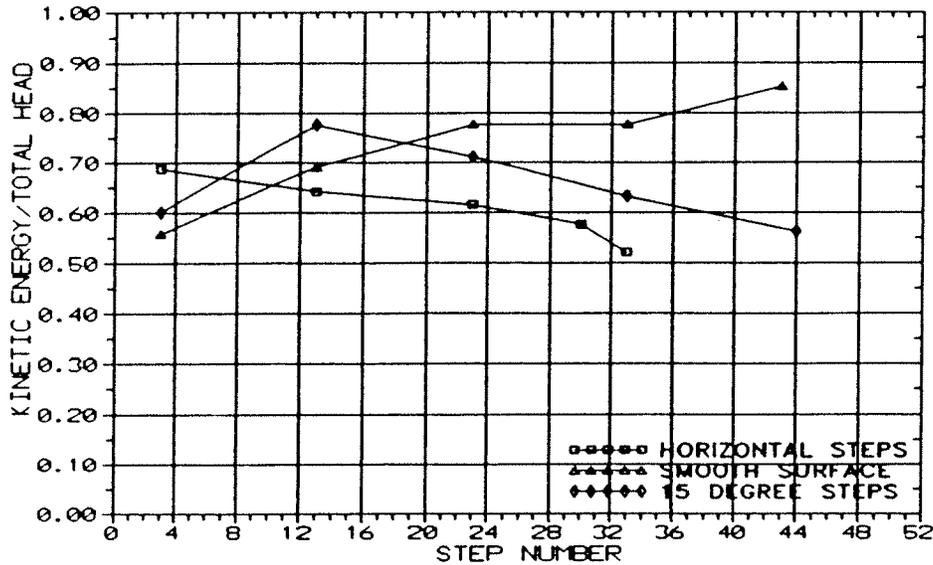


Figure 10. - Kinetic Energy to Total Head Ratio vs. Step Number for a Smooth Surface, Horizontal and 15° Sloping Steps, $q = 6.21 \text{ ft}^3/\text{s}/\text{ft}$, $H = 1.67 \text{ ft}$.

Froude number, $F = V_1/(gD_1)^{1/2}$, to determine the ratio of the hydraulic jump length, L , to the depth entering the basin. The calculated stilling basin lengths for the three surfaces are shown in table 3.

Table 3. - Comparison of Hydraulic Jump Lengths for Smooth Surface, 15° Sloping, and Horizontal Stepped Spillways.

Spillway	KE/H, (fig. 10)	V_1 (ft/s)	D_1 (ft)	F_1	L (ft)
Smooth surface	.771	50.27	5.14	3.91	146.5
15° sloping steps	.711	46.35	5.57	3.46	133.7
Horizontal steps	.6158	40.15	6.43	2.79	115.7

Containing the hydraulic jump within a basin with no end sill (Type I) shows a decrease in stilling basin length of 21 percent is achieved for the horizontal steps versus the smooth surface spillway. From figure 10 it is also apparent that velocities entering the basin will show the

horizontal steps to be more beneficial as the unit discharge increases. A reduced stilling basin length represents a significant savings in construction time and costs.

EMBANKMENT STABILITY

The most important feature of a stepped overlay on an embankment slope is that the underlying material of the dam remains stable. Saturation of the embankment may occur due to seepage through the overlay or by flow from the reservoir. An adequate drainage system should be utilized underneath the concrete overlay to prevent buildup of uplift pressures that may cause localized failure or general sliding of the embankment or overlay.

The recommended step design will provide continuous aspiration of subgrade flows through the overlay to prevent uplift pressures. Computer modeling of the tractive shear forces imposed on the overlay and embankment during flows is being developed.

Presently, of utmost concern with stepped spillway and overtopping designs is the location of embankment drains with respect to the tailwater and location of the hydraulic jump. Drains exiting the concrete overlay in the area of the hydraulic jump are subject to the dynamic pressures associated with the violent action of the jump. Care should be taken to not locate unprotected drains in the stilling basins or in the tailwater zone of any stepped spillway or overtopping protection.

CONSTRUCTION TECHNIQUES

The stepped spillway research was initiated with the assumption that whatever shape proved to be most efficient from a hydraulic standpoint could be constructed in a continuous placement. This assumption puts no constraints on the shape of steps investigated but may produce some challenges for the construction contractors. The steps may be formed RCC, conventional concrete placed by slip forming or even reinforced concrete conventionally placed. Besides the step shape, the construction process must allow placement of drains or vents through the overlay to allow the aspiration produced by the step geometry as indicated by the drains shown on figure 3.

NEAR-PROTOTYPE TESTING

Reclamation plans to conduct near-prototype testing of embankment overtopping protection alternatives in an outdoor flume facility at Colorado State University in Fort Collins, Colorado. Construction of the facility will begin

during the summer of 1991 with completion scheduled for the winter of 1992.

The near-prototype flume will be constructed on a 2:1 slope with a drop of about 50 ft. The 5-ft flume width will allow testing of unit discharges up to about 50 ft³/s/ft.

Testing is scheduled to begin in the spring of 1992 with a test program scheduled for completion in 1994. The following tests are planned:

- Wedge shaped blocks with shape optimized from present laboratory flume tests
- Large-sized riprap
- RCC, both formed and unformed
- Smooth reinforced concrete deck
- Reinforced rockfill blanket or cable tied riprap
- Cable tied blocks

CONCLUSIONS

Early stepped spillway applications were on RCC dams with steep downstream slopes. The major benefits derived were ease of construction and energy dissipation, thus producing shorter stilling basins. These were generally high RCC dams with 2-ft step heights constructed with conventional concrete. The data available (presented on figure 1) to quantify the amount of energy dissipated by flow down these steep RCC dams can only provide general guidelines for sizing stilling basins due to the great amount of scatter. Present Reclamation research will improve our ability to predict step spillway energy dissipation.

The emphasis of present research has been on producing a stable stepped spillway overlay, that still provides energy dissipation, on 2:1 embankment dam slopes. The results reported in this paper show excellent promise toward achieving this end. The sloping step geometry will provide aspiration of seepage necessary for stability. The steps on a 2:1 slope, while not dissipating as much energy as the steep concrete dam slopes, do provide advantages over a traditional smooth surface spillway. Final results from the 2:1 laboratory flume studies should be available in the fall of 1991.

After the step geometry is finalized for a 2:1 slope the placement of drains through the formed overlay will be the next challenge. It appears that present forming and consolidation techniques could be modified to accommodate the required step geometry.

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