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MECHANICS OF THE HYDRAULIC JUMP

by

L. G. PULS, SENIOR ENGINEER

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Denver, Colorado
Oct. 1, 1941.

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DEPARTMENT OF THE INTERIOR

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MEMORANDUM TO CHIEF DESIGNING ENGINEER

SUBJECT: MECHANICS OF THE HYDRAULIC JUMP

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BY L. G. PULS, SENIOR ENGINEER

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Under the Direction of

K. B. KEENER, SENIOR ENGINEER, DAMS

- - -

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MECHANICS OF THE HYDRAULIC JUMP

Synopsis

The purpose of this paper is to present a method of routing stream flow through the hydraulic jump in open channels and thereby assist in (a) the economical design of spillways by mathematical analysis, (b) organizing available experimental data to support a practical theory, and (c) attempting to direct future investigations toward a common avenue of effort. A theory of resistance to stream flow in open channels by action of the roller body and a formula which evaluates this resistance have been developed. The procedure in the solution of jump problems by the proposed mathematical process is described.

Introduction

It has long been general knowledge that neglecting friction, and assuming the channel floor to be horizontal, the sum of the pressure and momentum of the stream at the start of the jump is equal to the sum of these forces at the end of the jump. Recently the benefits of creating the jump on inclined aprons, introducing the additional force of floor reaction, have been demonstrated. Hydraulic model studies have been used with much success in the design of spillway structures, but model investigations are expensive and require considerable period of time for construction, operation, modification, and analysis of results. There is apparent need for an analytical tool in the hands of the designer who investigates, for preliminary plans and estimates, the economy of alternate profiles and proportions of energy dissipating structures. A rapid method of calculation of routing a stream through the jump body by successive processes would be of inestimable value in determining the maximum benefits with the minimum amount of excavation and concrete construction. It is believed that indications of cost trends of alternate designs may advantageously be obtained on the drafting board, provided that a satisfactory mathematical solution is made available. Although consideration has been given to the mechanics in action within the jump body continuously since the discovery of the pressure-momentum theory, these efforts have contributed very little additional knowledge toward the objective described above. No doubt the complexity of behavior of the jump body, including its turbulence and surges, have discouraged the earnest efforts of many investigators, but, perhaps the perspective on the main issues of the problem has been obstructed by minor inconsistencies of flow behavior. It is suspected that past difficulties have been experienced largely by efforts to submerge the phenomenon into complex theory, whereas guidance by the simple fundamental laws of mechanics and the application of empirical formulae based upon analyses of experimental data would have probably yielded substantial progress.

In this study it has been assumed that the hydraulic jump is a system of two distinct streams; one, the principal stream occupying the lower portion of the jump body, and the other a rotating mixture of water and air supported both in motion and position by forces imparted to it by the principal stream. The principal stream is assumed to remain intact as a fluid train during its travel through the jump body, expanding in cross section by action of the retarding forces which include: (a) differential hydrostatic pressure, (b) friction of the channel boundaries, (c) shear applied to the top boundary of the stream, and (d) differential boundary reaction introduced by the weight of the jump body. The system of flow may be considered analogous to an inclined, rubber conveyor belt in motion and loaded with a quantity of fine-grain sand. The angle of inclination of the conveyor is so chosen as to prevent the transportation of the sand beyond the end of the belt but to raise the grains adjacent to the belt to an elevation limited by the action and magnitude of mechanical friction, from which elevation the sand grains flow downward by gravity over the top of the mass. The body of sand is in equilibrium when the friction is equal to the component of weight of the sand resolved in the direction of the belt motion.

The study is introduced by the review of theory to the extent of demonstrating the conversion of the law of pressure-momentum conservation to the law of energy conservation applied to fluid flow with and without jump action. This is followed by an analysis of the hydraulic properties of the jump body, the derivation of formulae for resistance to flow and the reactive momentum of the roller in terms of the hydraulic properties, and an explanation of their use in calculating properties of the jump.

REVIEW OF THEORY

Conservation of Pressure Plus Momentum and Energy

The derivation of the equation of energy conservation may be demonstrated as follows:

Case I - Accelerated Flow on Inclined Plane

In this case a prism of water, which is a portion of a continuous stream, has acquired motion on an inclined plane by the net influence of gravitational and boundary frictional actions. The regimen of the stream has been selected so as to preclude the formation of a jump. Referring to figure 1, the forces (in units of the weight of a unit volume of water) acting on the stream prism of length L are as follows:

(a) In downstream direction,

$$\text{Momentum} = M_1 = \frac{Q}{g} v_1$$

$$\text{End pressure} = P_1 = \frac{d_1^2}{2} \cos \alpha$$

$$\text{Component of prism weight} = W_c = \left(\frac{d_1 + d_2}{2}\right) L \sin \alpha$$

(b) In upstream direction,

$$\text{Momentum} = M_2 = \frac{Q}{g} v_2$$

$$\text{End pressure} = P_2 = \frac{d_2^2 \cos \alpha}{2}$$

$$\text{Friction} = F_f = Ah_f$$

Equating forces for condition of equilibrium,

$$M_1 + P_1 + W_c = M_2 + P_2 + F_f \qquad (1)$$

or,

$$\frac{Q}{g} v_1 + \frac{d_1^2 \cos \alpha}{2} + \left(\frac{d_1 + d_2}{2}\right) L \sin \alpha = \frac{Q}{g} v_2 + \frac{d_2^2 \cos \alpha}{2} + Ah_f \qquad (2)$$

Combining terms

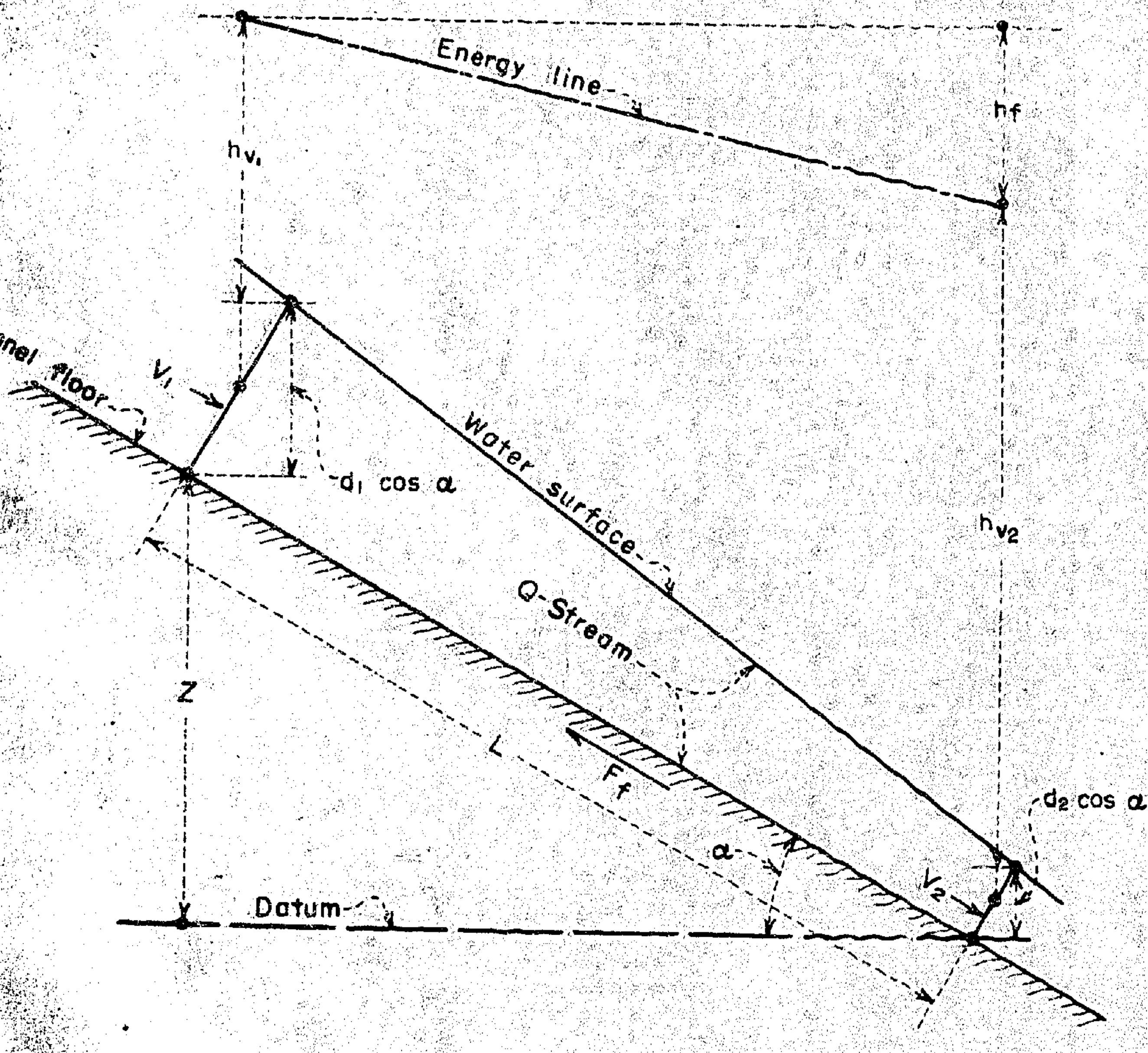

FIGURE 1

ACCELERATED FLOW ON INCLINED PLANE

$$\left(\frac{d_1^{\,2} - d_2^{\,2}}{2}\right) \cos \alpha + \left(\frac{d_1 + d_2}{2}\right) L \sin \alpha$$

$$= \frac{Q}{g} (v_2 - v_1) + Ah_f$$

Substituting A for $\frac{d_1 + d_2}{2}$ and z for $L \sin \alpha$,

$$(d_1 - d_2)\, A \cos \alpha + Az = \frac{Q}{g} (v_2 - v_1) + Ah_f$$

Dividing through by A and substituting V for $\frac{Q}{A}$

$$(d_1 - d_2) \cos \alpha + z = \frac{V(v_2 - v_1)}{g} + h_f$$

$$= \frac{(v_2 - v_1)(v_2 - v_1)}{2g} + h_f = \frac{(v_2^{\,2} - v_1^{\,2})}{2g} + h_f$$

Transposing terms, the equation for conservation of energy will be developed as follows:

$$z + d_1 \cos \alpha + h_{v1} = d_2 \cos \alpha + h_{v2} + h_f \quad \ldots\ldots\ldots\ldots\ldots (3)$$

<u>Case II - Decelerated Flow on Inclined Plane</u>

The considerations of this case are similar to those of case I except the regimen of the stream has been selected to permit the formation of the jump and thereby subject the Q-stream prism to the action of two additional forces, namely, shear at the top boundary of the principal stream and the component of the weight of the roller. Referring to figure 2, the forces acting on the principal stream prism of length L are as follows:

(a) In downstream direction,

Momentum $= M_1 = \frac{Q}{g} v_1$

End pressure $= P_1 = \left(\frac{d_1^{\,2} - a_1^{\,2}}{2}\right) \cos \alpha$

Component of prism weight $= W_c = AL \sin \alpha$

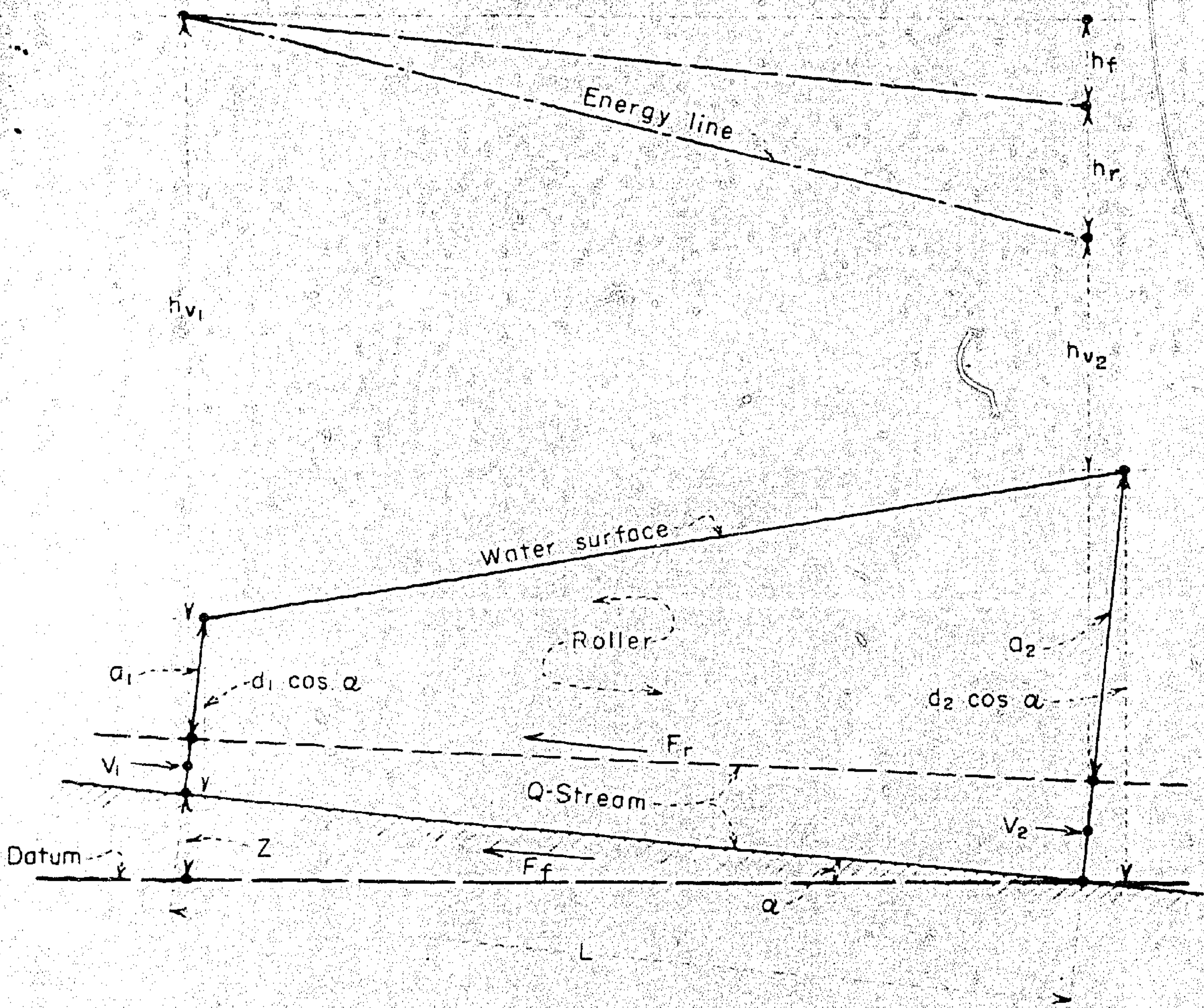

FIGURE 2

DECELERATED FLOW ON INCLINED PLANE

Reaction of roller body = P_r

$$= \left(\frac{a_1 + a_2}{2}\right) \left[(d_2 - a_2) - (d_1 - a_1) \right] \cos \alpha$$

(b) In upstream direction,

Momentum = $M_2 = \frac{Q}{g} v_2$

End pressure = $P_2 = \left(\frac{d_2^2 - a_2^2}{2}\right) \cos \alpha$

Friction = $F_f = Ah_f$

Roller traction = $F_r = Ah_r$

Equating forces for condition of equilibrium,

$$M_1 + P_1 + W_c + P_r = M_2 + P_2 + F_f + F_r \quad \text{.......................(4)}$$

or,

$$\frac{Q}{g} v_1 + \left(\frac{d_1^2 - a_1^2}{2}\right) \cos \alpha + AL \sin \alpha$$

$$+ \left(\frac{a_1 + a_2}{2}\right) \left[(d_2 - a_2) - (d_1 - a_1) \right] \cos \alpha$$

$$= \frac{Q}{g} v_2 + \left(\frac{d_2^2 - a_2^2}{2}\right) \cos \alpha + Ah_f + Ah_r \quad \text{..........(5)}$$

Combining terms,

$$\frac{Q}{g} (v_1 - v_2) + AL \sin \alpha = \left(\frac{d_2^2 - d_1^2}{2}\right) \cos \alpha$$

$$- (d_2 - d_1) \left(\frac{a_1 + a_2}{2}\right) \cos \alpha + Ah_f + Ah_r \quad \text{............ (6)}$$

or,

$$\frac{Q}{g}(v_1 - v_2) + AL \sin \alpha = (d_2 - d_1) \cos \alpha \left[\left(\frac{d_2 + d_1}{2}\right) - \left(\frac{a_1 + a_2}{2}\right) \right] + Ah_f + Ah_r$$

Substituting A for $\left[\left(\frac{d_2 + d_1}{2}\right) - \left(\frac{a_1 + a_2}{2}\right) \right]$ and z for $L \sin \alpha$,

$$\frac{Q}{g}(v_1 - v_2) + Az = (d_2 - d_1) A \cos \alpha + Ah_f + Ah_r.$$

Dividing through by A and substituting V for $\frac{Q}{A}$

$$\frac{V(v_1 - v_2)}{g} + z = (d_2 - d_1) \cos \alpha + h_f + h_r.$$

But

$$V = \frac{v_1 + v_2}{2}$$

Then

$$\left(\frac{v_1^{\,2} - v_2^{\,2}}{2g}\right) + z = (d_2 - d_1) \cos \alpha + h_f + h_r.$$

Transposing terms, the equation for conservation of energy of the principal stream will be developed as follows:

$$z + d_1 \cos \alpha + h_{v1} = d_2 \cos \alpha + h_{v2} + h_r + h_f \ldots\ldots\ldots\ldots (7)$$

This energy equation may also be derived by considering the forces on the jump body as follows:

(a) In downstream direction,

Momentum of principal stream $= M_1 = \frac{Q}{g} v_1$

Momentum of roller stream $= m_1$

End pressure $= P_1 = \frac{d_1^{\,2}}{2} \cos \alpha$

Component of prism weight $= W_c = \left(\frac{d_1 + d_2}{2}\right) L \sin \alpha$

(b) In upstream direction,

Momentum of principal stream $= M_2 = \frac{Q}{g} v_2$

Momentum of roller stream $= m_2$

End pressure $= P_2 = \frac{d_2^{\,2}}{2} \cos \alpha$

$$\text{Friction} = F_f = Ah_f$$

Equating forces for condition of equilibrium,

$$M_1 + m_1 + P_1 + W_c = M_2 + m_2 + P_2 + F_f \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

or

$$\frac{Q}{g} v_1 + m_1 + \frac{d_1^2}{2} \cos \alpha + \left(\frac{d_1 + d_2}{2}\right) L \sin \alpha$$

$$= \frac{Q}{g} v_2 + m_2 + \frac{d_2^2}{2} \cos \alpha + Ah_f \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

but

$$(m_2 - m_1) = \text{the net force on the roller prism}$$

$$= -\left\{\left(\frac{a_2^2 - a_1^2}{2}\right) \cos \alpha + \left(\frac{a_1 + a_2}{2}\right) \left[(d_2 - a_2) - (d_1 - a_1) \right] \cos \alpha - \left(\frac{a_1 + a_2}{2}\right) L \sin \alpha - Ah_r \right\}$$

$$= -\left[(d_2 - d_1)\left(\frac{a_1 + a_2}{2}\right) \cos \alpha - \left(\frac{a_1 + a_2}{2}\right) L \sin \alpha - Ah_r \right]$$

$$= -(d_2 - d_1)\left(\frac{a_1 + a_2}{2}\right)\cos\alpha$$

$$+\left[\left(\frac{d_1 + d_2}{2}\right) - A\right] L \sin\alpha + Ah_r$$

Substituting this expression of $(m_2 - m_1)$ in (9), transposing and combining terms gives again equation (6),

$$\frac{Q}{g}(v_1 - v_2) + AL\sin\alpha = \left(\frac{d_2^{\,2} - d_1^{\,2}}{2}\right)\cos\alpha$$

$$-(d_2 - d_1)\left(\frac{a_1 + a_2}{2}\right)\cos\alpha + A\,(h_f + h_r)$$

The derivation following equation (6) has already been described.

Case III - Decelerated Flow on Horizontal Plane

This case is similar to case II except, since $z = 0$, $\sin\alpha = 0$ and $\cos\alpha = 1$, the equation of forces acting on the stream prism is as follows:

$$\left(\frac{d_2^{\,2} - d_1^{\,2}}{2}\right) - (d_2 - d_1)\left(\frac{a_1 + a_2}{2}\right) + A(h_r + h_f)$$

$$= \frac{Q}{g}(v_1 - v_2) \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

Thence proceeding as in case II, the energy equation will be developed as follows:

$$d_1 + h_{v1} = d_2 + h_{v2} + h_r + h_f \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

It is also informative to develop the equation for conservation of energy of the roller stream, which may be demonstrated as follows:

From equation (10) the forces acting on the roller stream portion of the prism are

$$-(d_2 - d_1)\left(\frac{a_1 + a_2}{2}\right) + A\,h_r = \Delta_m = m_2 - m_1 \quad \ldots\ldots\ldots\ldots (12)$$

in which Δ_m = the difference in momentum of the roller stream at the terminals of the reach.

Dividing by the average depth of the roller, a,

$$-d_2 + d_1 + \frac{A\,h_r}{a} = \frac{m_2 - m_1}{a}$$

Transposing and assigning $J = \frac{a}{A}$, will develop the roller stream energy equation in head units:

$$d_1 + \frac{m_1}{JA} + \frac{h_r}{J} = d_2 + \frac{m_2}{JA} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

This equation could also be used to solve jump problems in lieu of the energy equation of the principal stream because it simply states that the energy given up by the principal stream has been absorbed by the roller stream.

It will be noted from the above derivations that the equation of pressure-momentum and the equation of energy express similar theories based upon the fundamental law of motion; that is, force equals the product of mass and acceleration. The unit of measure of the terms of one equation is force and the expression is converted to terms in head units by the algebraic processes of substitution, transposition, and using common multipliers. The following equations are evident from figure 3, which shows a graphic assembly of the component parts of energy and force conserved within the jump body.

Conservation of energy equation:

$$E_o = d_o + h_{vo} = h_p + h_v + \Sigma\,h_r + \Sigma\,h_f$$

$$= d_n + h_{vn} + H_r + H_f = E_n + H_r + H_f \quad \ldots\ldots\ldots\ldots\ldots (14)$$

Conservation of pressure-momentum:

$$P_o + M_o = M + P + (\Sigma\,F_r - \Sigma\,P_r) + \Sigma\,F_f = P_n + M_n + F_f \quad \ldots (15)$$

The energy equation requires no comment and can be solved by supplying the value of h_r which will be described later. Referring to the pressure-momentum equation it should be explained that the terms $(\Sigma\,F_r - \Sigma\,P_r)$ represent internal forces of the jump body, the algebraic sum of which is equal to zero at the conjugate

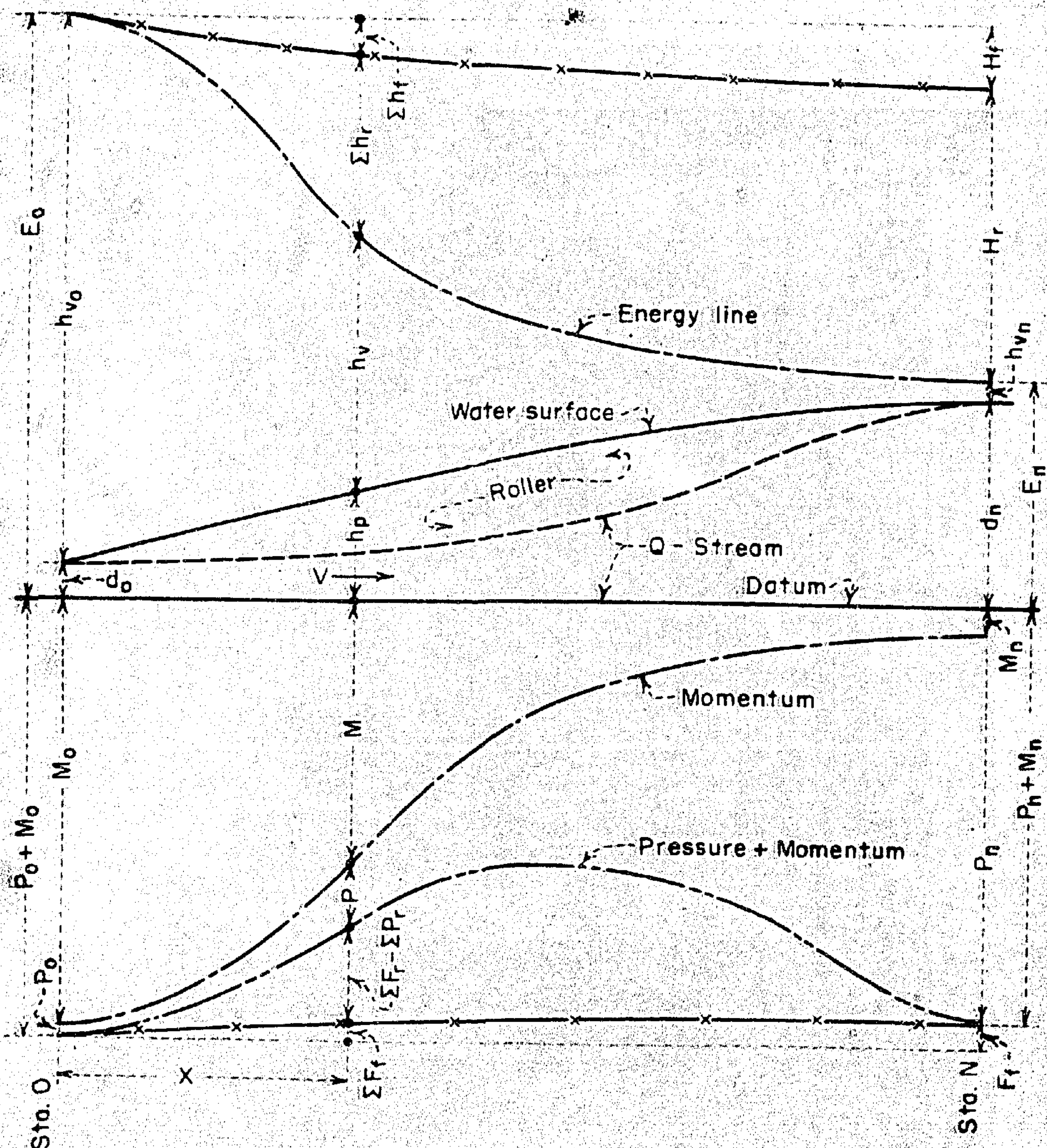

FIGURE 3

DIAGRAM OF CONSERVATION
OF ENERGY AND PRESSURE MOMENTUM

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points of the jump and of such magnitude between these points to compensate for the necessary change of the total pressure-momentum. Accordingly the equivalent value of these terms for the various reaches of case III would be as follows:

Reach No. 1 $$(\Sigma F_r - \Sigma P_r)_1 = (Ah_r)_1 - \left(\frac{Q}{v_1} - \frac{Q}{v_o}\right)\left(\frac{a_1}{2}\right) \quad \ldots(16)$$

Reach No. 2 $$(\Sigma F_r - \Sigma P_r)_2 = (\Sigma F_r - \Sigma P_r)_1 + (Ah_r)_2 - \left(\frac{Q}{v_2} - \frac{Q}{v_1}\right)\left(\frac{a_1 + a_2}{2}\right) \quad \ldots\ldots(17)$$

and so on to

Reach No. N $$(\Sigma F_r - \Sigma P_r)_n = (\Sigma F_r - \Sigma P_r)_{n-1} + (Ah_r)_n - \left(\frac{Q}{v_n} - \frac{Q}{v_{(n-1)}}\right)\left(\frac{a_{(n-1)}}{2}\right) = 0 \quad \ldots(18)$$

The forces acting on the jump body which rests on a channel floor of irregular profile (figure 4) are as follows:

(a) External forces in downstream direction,

$$\text{Momentum} = \frac{Q\, v_o}{g} \cos \beta$$

$$\text{Stream end pressure} = P_o = \frac{\gamma\, d_o^2 \cos \beta}{2}$$

$$\text{Floor reaction} = P_d$$

(b) External forces in upstream direction,

$$\text{Momentum} = \frac{Q\, v_n}{g}$$

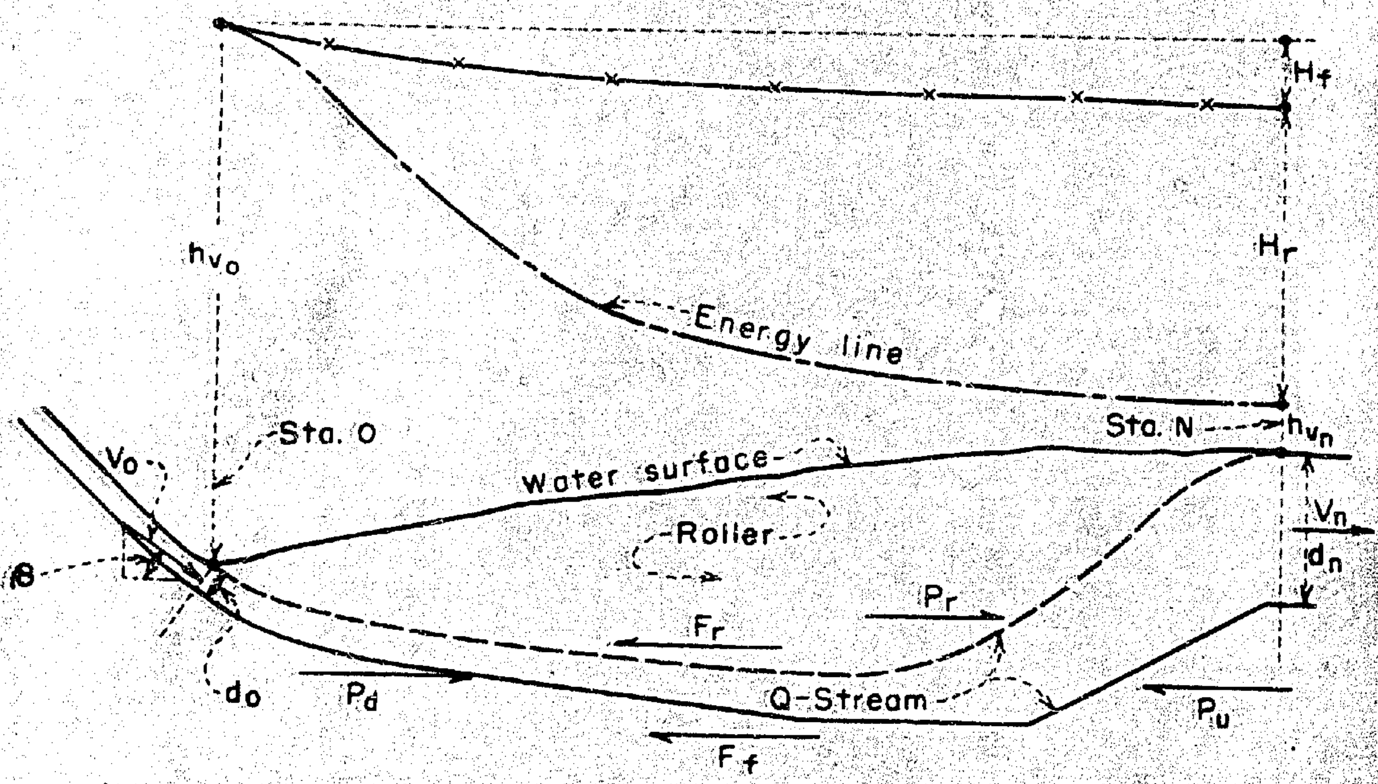

H_f
h_{v_o}
H_r
Energy line
Sta. O
Sta. N
h_{v_n}
V_o
Water surface
Roller
V_n
β
d_n
P_r
F_r
d_o
P_d
Q-Stream
P_u
F_f

FIGURE 4

Stream end pressure $= P_n = \dfrac{d_n^2}{2}$

Floor reaction $= P_u$

Floor friction $= F_f$

Equating forces for condition of equilibrium,

$$\frac{Q\,v_o}{g}\cos\beta + \frac{d_o^2\cos\beta}{2} + P_d = \frac{Q\,v_n}{g} + \frac{d_n^2}{2} + P_u + F_f \text{ --(19)}$$

The internal forces are:

The roller shear (acting upstream) on the principal stream $= F_r = H_r A$.

The net component of roller weight (acting downstream) on the principal stream $= P_r$.

Equating the forces of the roller body acting on the principal stream,

$$P_r = F_r \text{ ....(20)}$$

The equations of energy and pressure plus momentum are, of course, expressions of basic theories and must be satisfied in the simultaneous algebraic calculations required for the solution of jump problems. It is necessary, however, to obtain for substitution in the basic equations independent functions of (1) the energy, h_r, transferred by shear from the principal stream to the roller stream, and (2) the momentum gained by the roller stream as the result of this transfer of energy, in terms of the hydraulic properties of the principal stream. As described in the following pages, efforts have been made to furnish the independent functions, and although based upon limited experimental data, the proposals appear to be reconcilable to universal application. The proposed function of the energy, h_r, has been derived by the process of selecting empirical influences and determining their effects when applied to examples of widely varying flow characteristics. An attempt has been made to evaluate the momentum of the roller stream by analytical derivation based upon an assumed velocity distribution and the Prandtl-Von Karman formula for turbulent shear stress.

ENERGY GRADIENT OF STREAM THROUGH THE HYDRAULIC JUMP

The energy gradient of the stream through the hydraulic jump may be evaluated as follows:

h_r = the head loss due to roller traction within the reach of L stream length.

$\frac{h_r}{L}$ = the head loss per foot of stream travel.

When the stream prism AL moves forward one foot, there is annihilated $ALw \times \frac{h_r}{L}$, or Ah_rw foot-pounds of energy. Since the length of travel is one foot, this value is also the total shear force F_r in pounds, acting along the line of contact between the roller and the principal stream.

It appears that the unit shear $\frac{Ah_rw}{L}$ varies directly with the summation of two principal influences. One influence is proportional to the ratio of the roller stream area to the Q-stream area and the five-halves power of the mean velocity of the Q-stream, $(k_rJV^{5/2})$. The other influence is proportional to the weight of a unit column of the roller body resting on the Q-stream (c_raw). These influences were revealed by trial calculations applied to several jumps of known profiles, the stream properties of which vary over a wide range in magnitude and flow conditions. The experimental and calculated data, accompanied by drawings, which substantiate the proposed empirical law of resistance between the roller and Q-streams, follow in appendix B.

Now, equating the force active in annihilating the energy, within the reach, to the roller resistance gives

$$Ah_rw = (k_rJV^{5/2} + c_raw)\,L \quad \text{.................................} (21)$$

in which

k_r = a universal constant

c_r = a coefficient of proportionality

J = ratio of roller stream to Q-stream areas = $\frac{a}{A}$

The head loss due to roller traction

$$h_r = \left[\frac{k_r V^{5/2}}{Aw} + c_r \right] JL \text{ .... } (22)$$

or

$$h_r = JKL \text{ .... } (23)$$

in which

$$K = \frac{k_r V^{5/2}}{Aw} + c_r \text{ .... } (24)$$

Based upon the limited experimental data available, values have been assigned to the constants, as follows:

$$\frac{k_r}{w} = 0.00001$$

$$c_r = 0.07$$

Substituting in (24), $K = \dfrac{0.00001\ V^{5/2}}{A} + 0.07$ .... (25)

It is believed that the coefficient $c_r = 0.07$ is appropriate for all examples of jumps, the properties of which are so proportioned to obtain maximum dissipating efficiency. In instances where the tail-water depth is either excessive or deficient to obtain the condition of maximum efficiency, it will be necessary to select the value of c_r by trial (suggested extreme limits are 0.02 and 0.08) in order to make the calculated energy line coincide with the energy line of the freely flowing stream at the downstream terminal of the jump action. In any case, it is intended that the value of c_r so chosen shall be used as the constant applicable to the property calculations of all the stream reaches within the jump body. It appears that the constant $\frac{k_r}{w} = 0.00001$ is satisfactory for all examples thus far examined, regardless of the condition of flow, and can be used without qualification.

The determination of the ratio J, when buckets of appreciable curvature are being considered, is as follows. Referring to figure 5, let

G = volume of roller body (wedge-shaped) bearing upon the stream prism of length, L

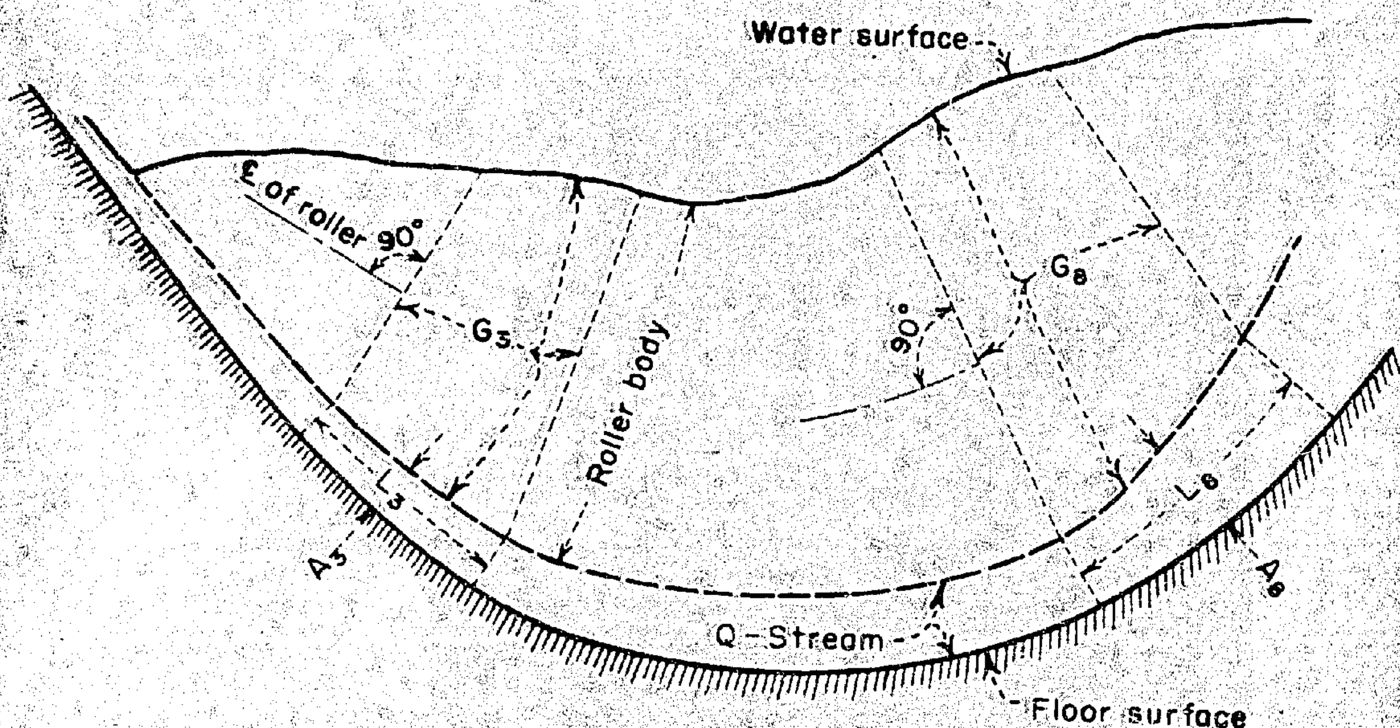

Water surface
℄ of roller 90°
G3
Roller body
G8
90°
L3
A3
L8
A8
Q-Stream
Floor surface

FIGURE 5

$a = \frac{G}{L}$ = volume of roller per foot of stream

A = average thickness of stream

then $J = \frac{a}{A}$.

It is convenient in making calculations to obtain first the value of $\left(\frac{G + AL}{L}\right)$, which is the volume of the jump body per foot of stream travel, then divide by A.

$$J = \frac{G + AL}{AL} - 1 = \frac{G}{AL} = \frac{a}{A} \qquad (26)$$

MOMENTUM OF THE ROLLER

At any transverse section of the roller, the rate of flow in the upstream direction is equal to the rate of flow in the downstream direction. The motion upstream is actuated by gravity and must be retarded to zero and accelerated downstream by the net force acting on the roller body.

Referring to figure 6, let a be the depth of the roller, normal to the direction of the Q-stream, and let point 0 at the water surface be selected as the origin of the axes. It is assumed that the distribution of roller velocity is described by the parabola that satisfies the two conditions as follows: (1) when $y = 0$, $u = -\frac{U}{2}$ and (2) when $y = -a$, $u = U$. Under these conditions the areas, separated by the Y-axis, representing the roller flows, are equal. The equation of the parabola is found to be

$$y^2 = \frac{2a^2}{3U}\left(u + \frac{U}{2}\right) \qquad (27)$$

or

$$u = \frac{U}{2}\left[\frac{3y^2}{a^2} - 1\right]$$

The momentum of the roller is determined by integration as follows:

$$m = \int_0^a \frac{w'u^2}{g}\,dy = \frac{w'U^2}{4g}\int_0^a \left[\frac{9}{a^4}y^4 - \frac{6}{a^2}y^2 + 1\right] dy$$

$$= \frac{w'U^2 a}{5g} \qquad (28)$$

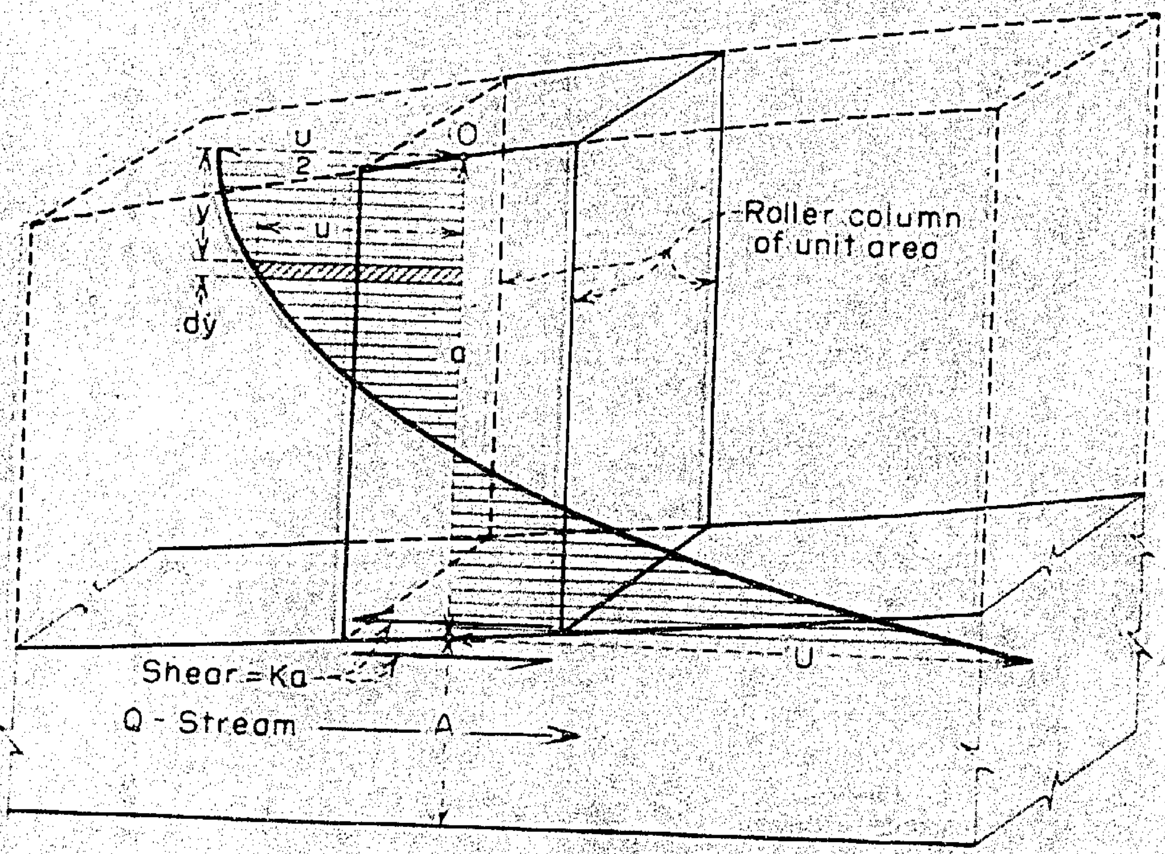


FIGURE 6

It remains now to evaluate U in terms of the shear at the point $y = -a$, $u = U$ which is on the line assumed to separate the roller and Q-streams. This is accomplished by the use of the Prandtl-Karman formula. Prandt has given the following expression for the turbulent shear stress:

$$\tau = \rho \ell^2 \left(\frac{du}{dy}\right)^2 \qquad (29)$$

where

τ = the shearing stress at point y, u

ρ = the density of the fluid

u = the velocity at the point

ℓ = the mixing length of the momentum exchange

As a consequence of his principle of similarity of turbulence, Karman derived the following expression for the mixing length ℓ at any point y in terms of the velocity gradient at y:

$$\ell = k \frac{u'}{u''} \qquad (30)$$

where the primes indicate differentiation with respect to y, and k is a universal constant characterizing the turbulence.

The several factors in (29) have the following equivalents:

$$\tau = Kaw \qquad \text{from (21)}$$

$$\rho = \frac{w'}{g}$$

$$\left(\frac{du}{dy}\right)^2 = \left\{\frac{d}{dy}\left[\frac{U}{2}\left(\frac{3y^2}{a^2} - 1\right)\right]\right\}^2 = \left(\frac{3U}{a}\right)^2 = \frac{9U^2}{a^2}$$

$$\ell^2 = \left(k\frac{u'}{u''}\right)^2 = (ka)^2$$

Substituting these values in (29) gives:

$$Kaw = \frac{w'}{g}(ka)^2\left(\frac{9U^2}{a^2}\right) = \frac{9k^2w'U^2}{g}$$

or

$$U^2 = \frac{Kawg}{9k^2w'}$$

Substituting this expression of U^2 in (28)

$$m = \frac{Ka^2 w}{45\ k^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (31)$$

Equation (31) states that the momentum of the roller at any section is proportional to the unit shear Ka, and the depth of the roller a. This relation appears appropriate when J is unity, that is, when the depth of the roller is equal to the thickness of the Q-stream. An inspection of examples calculated in accordance with equation (31) shows the necessity of modifying the relation to the extent of introducing the influence of the additional factor J. The modified equation would be

$$m = 0.1\,KJa^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (32)$$

It will be noted that since $KJ = \frac{h_r}{L}$ the momentum of the roller at any section is proportional to the slope of the energy line and the square of the depth of the roller.

SUMMARY OF FORMULAE

The formulae necessary for the solution of hydraulic jump problems are as follows: (Refer to figure 7)

I. Conservation of Pressure-Momentum

(The units are in terms of the weight of one cubic foot of water).

$$\text{Constant} = \frac{d_o^2 \cos \alpha}{2} + \frac{q}{g} v_o$$ = hydrostatic pressure plus the momentum of the q-stream plus the momentum of the roller stream at any section plus total friction minus the component of the jump body weight from the start of the jump to the section under consideration,

$$= \frac{d_1^2 \cos \alpha}{2} + \frac{qv_1}{g} + m_1 + F_1 - d_{m1} L_1 \sin \alpha$$

$$= \frac{d_2^2 \cos \alpha}{2} + \frac{qv_2}{g} + m_2 + (F_1 + F_2)$$

$$- (d_{m1} L_1 + d_{m2} L_2) \sin \alpha$$

$$= \frac{d_n^{\,2} \cos \alpha}{2} + \frac{q v_n}{g} + (F_1 + F_2 + \text{--} F_n)$$

$$-(d_{m1} L_1 + d_{m2} L_2 + \text{--} dm_n L_n) \sin \alpha$$

II. Conservation of Energy

$$L_1 \sin \alpha + d_o \cos \alpha + h_{vo} = d_1 \cos \alpha + h_{v1} + (h_{r1} + h_{f1})$$

$$L_2 \sin \alpha + d_1 \cos \alpha + h_{v1} = d_2 \cos \alpha + h_{v2} + (h_{r2} + h_{f2})$$

$$L_n \sin \alpha + d_{n-1} \cos \alpha + h_{v_{n-1}} = d_n \cos \alpha + h_{vn} + (h_{rn} + h_{fn})$$

$$K = \frac{.00001\, V^{5/2}}{A} + c_r$$

$$J = \frac{d_m}{A} - 1 = \frac{a}{A}$$

$$L = \frac{h_r}{KJ} = \frac{C^2 R\, h_f}{V^2} = \frac{h_r + h_f}{KJ + \frac{V^2}{C^2 R}}$$

$$F = \frac{L A V^2}{C^2 R}$$

$m = 0.1\ KJa^2$, in which K, a, and J have values at particular sections, indicated by subscript of m.

INVESTIGATION OF JUMP ON PLANE FLOOR

The suggested procedure in the calculation of the hydraulic properties of the jump is as follows: (Refer to figure 7)

Given d_o, q, v_o, α and elevation N.

1. Assume $c_r = 0.07$
2. Solve for pressure-momentum constant

$$= \frac{d_o^{\,2}}{2} \cos \alpha + \frac{q v_o}{g}$$

3. Select velocity v_1 at end of first reach and solve for $(h_v, \frac{q}{v}, A, V, K_2, K_m, R, \text{and } C)_1$

4. Assume trial d_1 at end of first reach and solve for $\left[d_m, j, J, L, (h_r + h_f) \text{ and } F\right]_1$

5. Substitute values in pressure-momentum equation of the first reach and obtain algebraic sum of the terms. Compare sum with constant (1).

6. Assume additional trial values of d_1 and repeat calculations. The correct d_1 will be obtained when the pressure-momentum equation is satisfied.

7. Repeat the procedure for successive reaches.

8. The end of the jump at station n will be obtained when $j_n = 0$. It is believed that the proportions of the jump so calculated will give the maximum dissipating efficiency.

9. Compare the elevation of the calculated water surface at station n with the given elevation N and, if higher, then assume another trial value of c_r slightly greater than 0.07, or conversely, if lower, and repeat the calculations for the full length of the jump.

10. The correct length of the jump will be obtained when the calculated water surface is tangent to the given tail-water surface at station n where $j_n = 0$.

INVESTIGATION OF JUMP ON FLOOR OF IRREGULAR PROFILE

The properties of a jump on a floor of irregular profile must also satisfy formulae numbers (23) and (32). Although the solution cannot be organized as simply as described above, the investigations may be accomplished advantageously by the following procedure: (Refer to figure 4)

1. Make a layout of the floor profile to be investigated.

2. Draw the surfaces of the approaching and departing streams in proximity of the conjugate points.

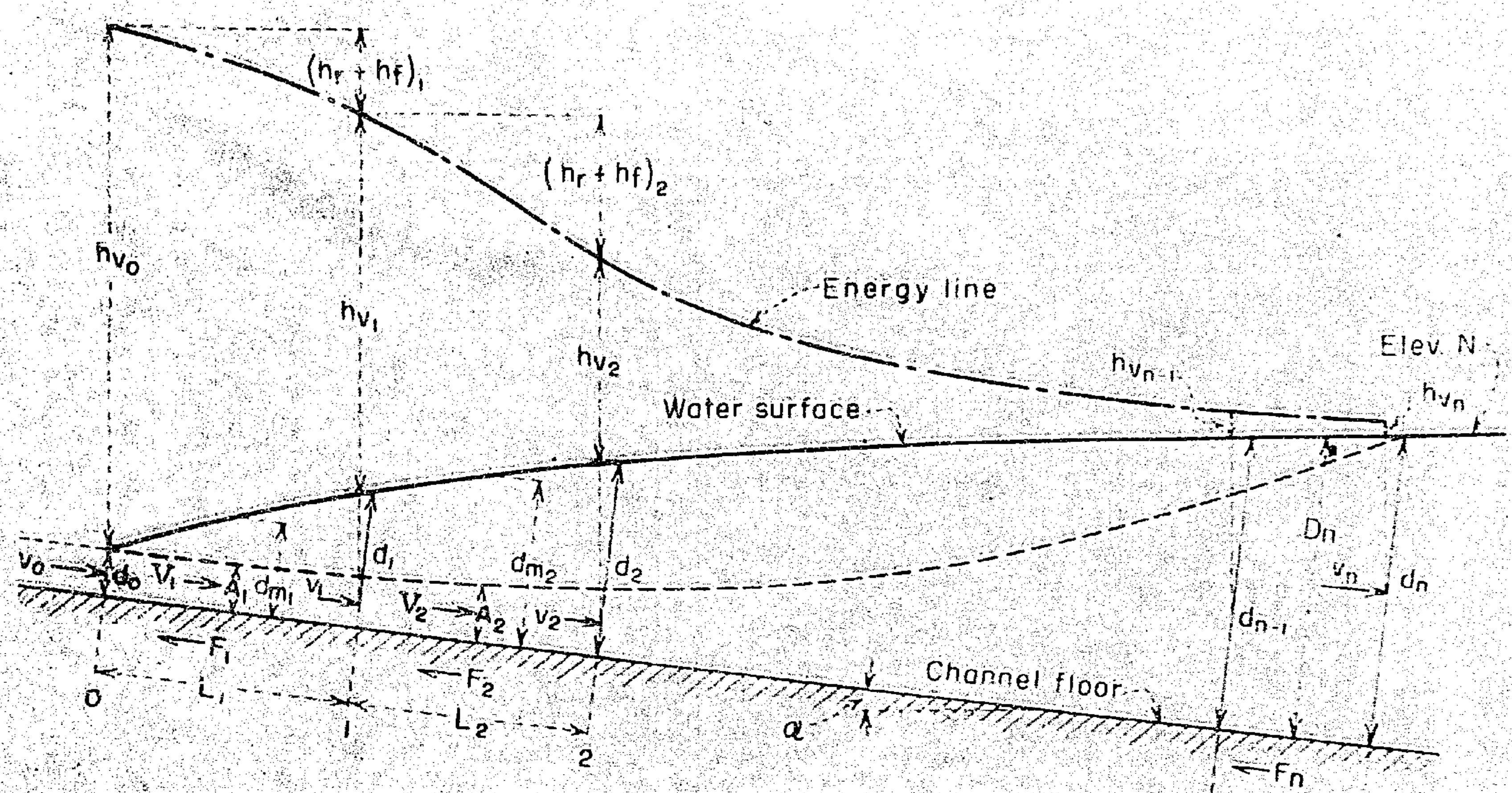


FIGURE 7

X-D-2877

3. Draw an assumed water surface, guided by judgment and trial calculations, with a profile of such curvature as to satisfy the equilibrium of forces acting on the jump body- equation (19).

4. Divide the trial jump body into reaches, the proportions of which will be adaptable to straight line variation of the hydraulic properties without appreciable error.

5. Calculate the mean velocity and resistance due to roller action of each reach by formula (23) and lay out the energy line.

6. The calculated energy line must necessarily coincide with the energy lines of the approaching and departing streams. If the trial calculations fail to obtain the coincidence, either the length or depth of the jump body must be modified to accomplish the required dissipation of energy. The modifications can easily be made after some experience in working with the resistance formula.

7. It is possible to satisfy the resistance formula by a deficiency in depth and an excess in length, or conversely, and therefore it is necessary to employ the roller momentum formula (32) to obtain the correct proportions of both depth and length. The roller momentum at the several sections of the trial jump body should be calculated and applied to effect the stability of each reach in conjunction with the other forces acting on the partial body.

8. The investigation of jumps with channel floors of appreciable curvature will require the additional examination of the effect of centrifugal force. A portion of the kinetic energy of the principal stream will be transformed to potential energy. The reduction of velocity near the center of the curved floor length will decrease the value of the coefficient K in the resistance formula and consequently reduce the slope of the energy line. As the elements of the principal stream progress toward the end of curvature, potential is transformed to kinetic energy and may result in an increase of dissipation, depending upon the value of the ratio J at this point.

9. It is suggested that the trial proportions of the jump be tested to satisfy separately the three formulae: (1) roller resistance, (2) roller momentum, and (3) transformation of energy by the curvature. Adjustments and repeated calculations will be necessary until the hydraulic properties have been evaluated in accordance with these functions.

APPENDICES

A. Notation.

B. Examples of hydraulic jumps used in investigations to determine the empirical formula of roller resistance.

C. Examples of hydraulic jumps calculated in accordance with the proposed theory to demonstrate the workability of the formulas.

D. Graph of five-halves powers of velocities.

APPENDIX - A

Notation

The following letter symbols, used in this paper, conform essentially to the "Letter Symbols and Glossary for Hydraulics," Manual of Engineering Practice No. 11, of the American Society of Civil Engineers:

A = average cross-sectional area of principal or Q-stream.

a = average cross-sectional area of roller stream.

(subscripts denote areas at particular sections)

α = angle of inclination of channel floor.

b = width of channel (considered as equal to unity).

C = Chezy friction factor.

c_r = coefficient of proportionality in roller resistance, formula (21).

d = depth of water in channel.

d_1, d_2, and so forth = depth at particular sections.

d_m = mean depth.

E = total energy.

F = total force.

F_f = friction.

F_r = roller traction.

F_1, F_2, and so forth = friction in particular reaches.

G = volume of roller resting upon the principal stream prism of length L.

g = acceleration of gravity = 32.2 ft. per sec. per sec.

H = total head.

h = minor head.

h_v = velocity head.

h_f = friction head.

h_p = pressure head.

h_r = roller resistance head.

J = mean depth of roller stream per unit mean depth of the principal stream. (The standards manual states that J equals height of hydraulic jump expressed as a ratio of final to initial conjugate depth. This definition has no significance except when comparing two jumps of different magnitude resting on a channel floor of identical inclination, and its use is limited in jump investigations. The definition as used in this paper is preferred because it permits an alphabetical sequence in the formula (23), $h_r = JKL$).

j_1, j_2, and so forth = depth of roller stream per unit depth of the principal stream at particular sections.

K = coefficient in formula (23) for head loss due to roller resistance. (Subscript m denotes mean within the reach and $_{1,\,2}$ denote particular sections).

k = the universal constant in Karman's formula for mixing length.

k_r = a universal constant in the roller resistance, formula (21).

L = length of principal stream prism.

l = the mixing length of the momentum change.

M = momentum of principal stream.

m = momentum of roller stream. (Subscripts denote momenta at particular sections).

n = coefficient of hydraulic friction in Kutter's and Manning's formulas; also a subscript to denote number of reaches.

P = end pressure on either roller of Q-stream prisms. (Subscripts denote pressures at particular sections.)

P_r = reaction of roller body.

P_u = floor reaction acting in upstream direction.

P_d = floor reaction acting in downstream direction.

Q = mean discharge in cubic feet per second.

q = discharge in cubic feet per second per foot of width.

q_1, q_2 = discharge at particular sections.

R = mean hydraulic radius.

ρ = density of the fluid.

τ = shearing stress in the roller stream.

u = velocity of any point in the roller stream.

U = velocity at lower boundary of the roller stream.

V = mean velocity of principal stream.

V_1, V_2, and so forth = velocity of principal stream at particular sections.

W_c = weight component in direction of flow.

w = unit weight of principal stream fluid.

w' = unit weight of roller stream fluid.

z = elevation head of channel floor above datum.

APPENDIX - B

The following examples of the hydraulic jump were used in the investigation to determine the principal influences which produce the shear between the roller and the Q-stream.

Example and table numbers	Name of hydraulic structure	Model or prototype	Flow per foot width of spillway c.f.s.	Energy head destroyed feet	Power annihilated per foot width of spillway hp.
1	Experiment by Bakhmeteff	model	1.564	0.25	0.0462
2	Shasta Dam	prototype	667.0	363.97	27,600.0
3	Grand Coulee Dam	model	2.4	6.29	1.715
4	Grand Coulee Dam	prototype	606.0	251.46	17,320.0
5	Friant Dam	prototype	90.36	136.36	1,400.0
6	Friant Dam	prototype	271.0	191.47	5,900.0
7	Madden Dam	prototype	112.5	119.0	1,520.0
8	Madden Dam	prototype	588.63	118.0	7,900.0

In each example, the profile of the water surface and the channel floor were obtained from results of model operation. The method used in the investigation was the process of selecting and discarding assumed influences until the dissipating action in the several examples could be reconciled and reasonable results in the calculation of the hydraulic properties obtained. It will be noted in each case that the slope of the energy line, disregarding friction, is proportional to the coefficient K and the roller ratio J. Velocity measurements were available in examples 3 and 7 and this data agreed satisfactorily with the calculated velocities.

TABLE NO. 1

Example No. 1 - Model experiment run No. C5, described in paper entitled "The Hydraulic Jump in Sloped Channels," by B. A. Bakhmeteff and A. E. Matzke - Published in Trans. of the Am. Soc. M. E., Section 1, February 1938 - Refer to figure 12 of that paper.

Q = 0.782 c.f.s.
q = 1.564 c.f.s. per foot width of channel.
d_o = 0.227 foot.
d_n = 1.05 feet.

v_o = 6.888 ft./sec.
v_n = 1.49 ft./sec.

Width of channel = 0.5 foot.
Kutter's n = 0.01
C_r = 0.07

Energy head destroyed = 0.26 foot.
Power destroyed = 0.046 hp./ft.

Sta.	v	V	A	$a+A$	$V^{5/2}$	J	K	L	V^2	R	C^2	Elev. total energy	Elev. floor surface	h_p	h_v	h_r	h_f	Calculated elevation total energy plus lost head	$\frac{q}{v}$
0	6.888											1.75	0.79		0.736				0.227
		6.364	0.246	0.334	102.0	0.36	0.0742	0.66	40.5	0.1245	10,000					0.0176	0.0215		
1	5.84											1.7109	0.74	0.44	0.53			1.7491	0.268
		5.19	0.3015	0.54	61.0	0.791	0.0720	1.04	26.9	0.1366	10,404					0.0592	0.0197		
2	4.54											1.632	0.67	0.64	0.32			1.7089	0.345
		3.95	0.396	0.71	30.5	0.792	0.0706	1.00	15.6	0.154	11,450					0.0564	0.00998		
3	3.36											1.5657	0.604	0.78	0.175			1.6253	0.464
		3.01	0.52	0.833	15.7	0.602	0.0703	1.00	9.06	0.169	12,100					0.0402	0.00443		
4	2.66											1.5211	0.535	0.885	0.11			1.5746	0.588
		2.312	0.676	0.93	8.0	0.375	0.0701	1.00								0.0263	*		
5	1.965											1.4948	0.465	0.975	0.06			1.5263	0.796
		1.728	0.905	1.013	3.9	0.12	0.0700	1.00								0.0084	*		
6	1.49											1.4864	0.395	1.05	0.0345			1.4879	1.05

*negligible

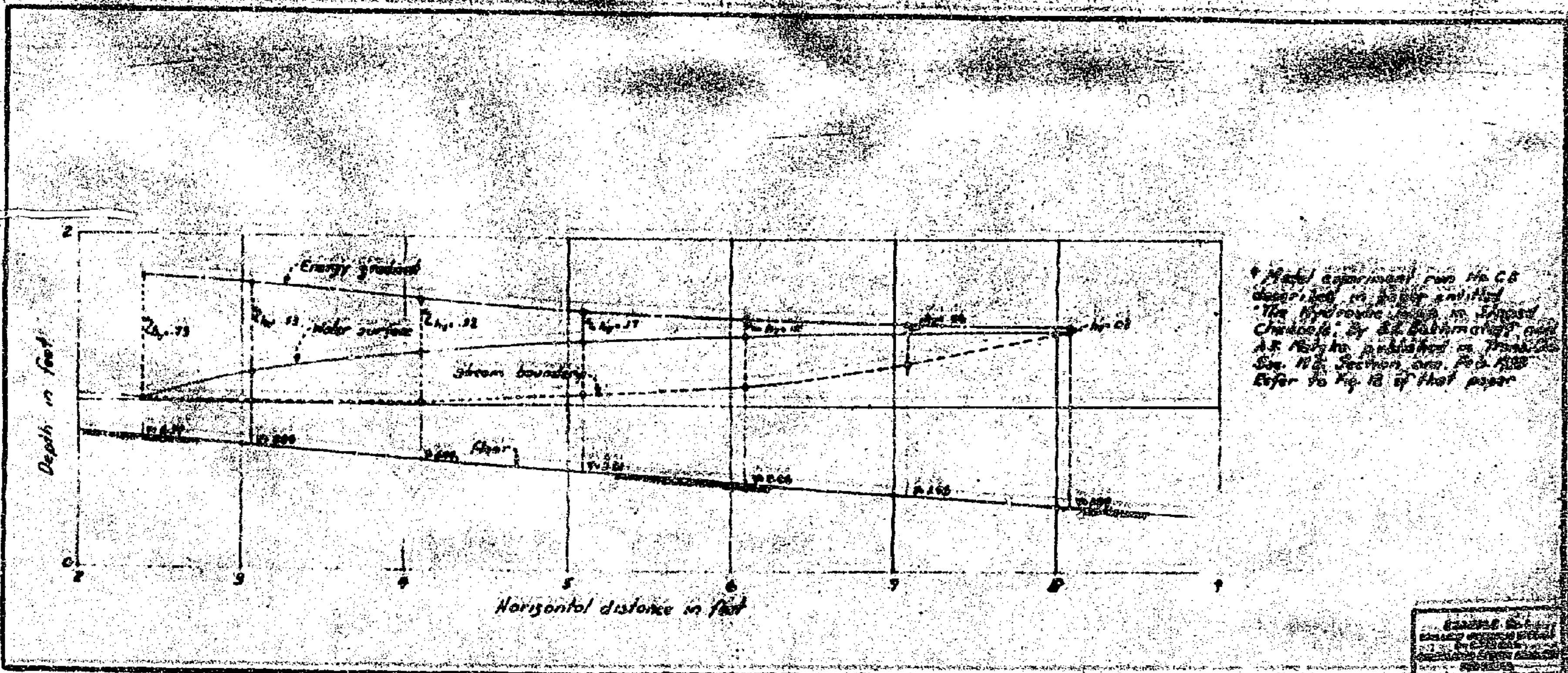

Energy gradient
Water surface
Stream boundary
Floor
Depth in feet
Horizontal distance in feet
Refer to Fig. 18 of that paper

TABLE NO. 2

Example No. 2 - Shasta Dam spillway - Prototype.

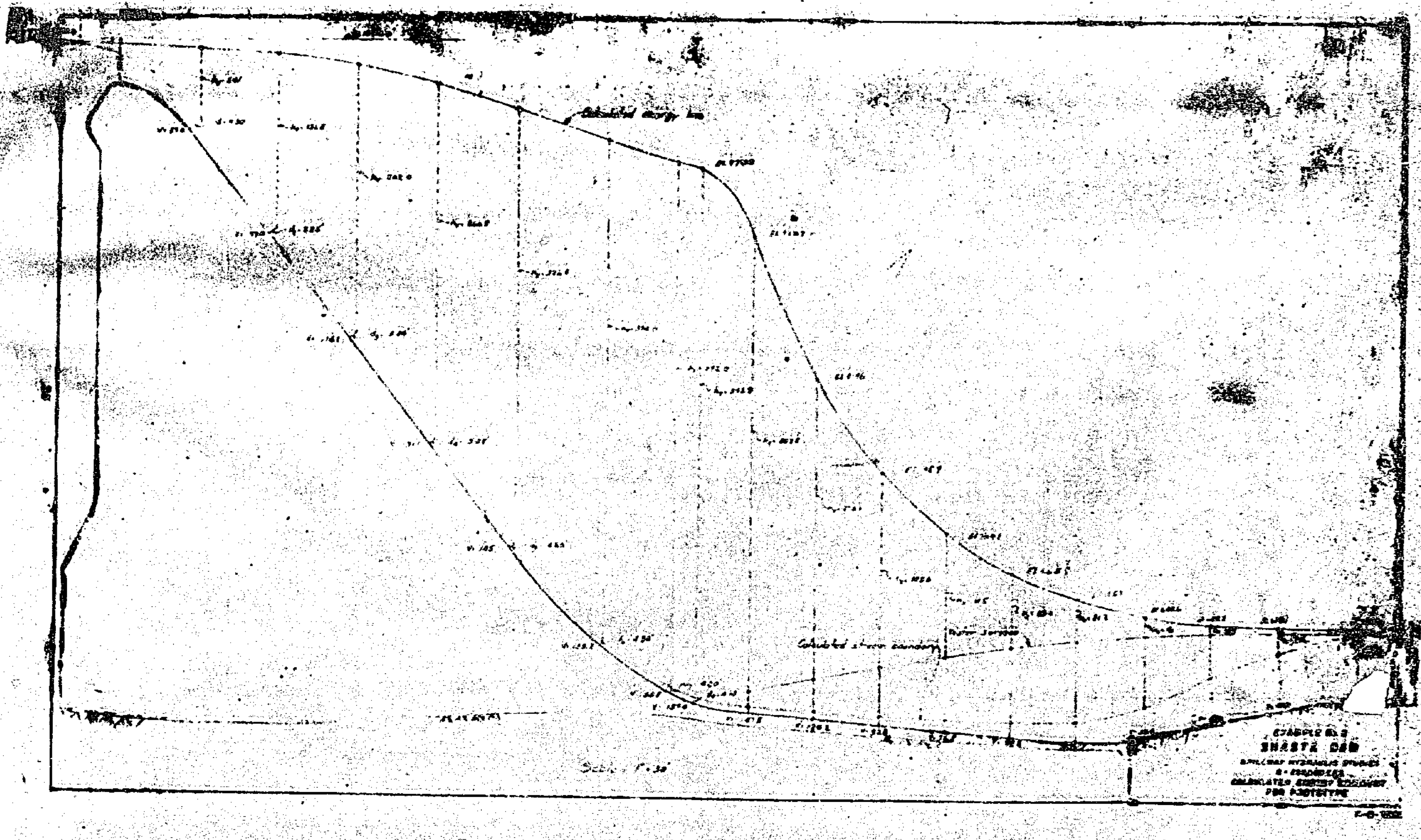
Q = 250,000 c.f.s.
q = 667 c.f.s. per foot width of channel.
d_o = 4.18 feet. v_o = 159.4 ft./sec.
d_n = 51.7 feet. v_n = 12.9 ft./sec.

Width of channel = 375 feet.
Kutter's n = 0.012 (concrete
= 0.03 (rock)

Energy head destroyed = 339.4 feet.
Power destroyed = 25,700 hp./ft.
C_r = 0.07

Sta.	v	V	A	a+A	$V^{5/2}$	J	K	L	V^2	R	C^2	Elev. total energy	Elev. floor surface	h_p	h_v	h_r	h_f	Calculated elevation total energy plus lost head	$\frac{q}{v}$
0	159.4											974.0			394.5				4.185
		153.4	4.35	10.09	298,000	1.32	0.755	42.0	23,520	4.25	23,900					41.6	9.7		
1	147.5											922.7	569.5	16.0	337.5			974.3	4.52
		133.8	4.98	24.0	212,200	3.82	0.496	50.0	17,900	4.85	24,300					95.5	7.6		
2	120.2											819.6	565.5	30.0	224.1			922.7	5.55
		108.4	6.15	36.5	123,000	4.94	0.270	50.0	11,750	5.96	25,300					66.8	3.9		
3	96.6											748.9	561.5	43.0	145.0			820.2	6.90
		86.7	7.70	49.0	70,300	5.36	0.1613	50.0	7,500	7.4	26,200					42.9	1.9		
4	76.8											704.1	557.0	55.0	91.5			748.3	8.68
		68.2	9.76	61.0	39,000	5.25	0.110	50.0	4,680	9.27	27,200					28.9	0.9		
5	59.5											674.3	552.5	67.0	55.0			704.3	11.21
		52.1	12.80	71.0	19,400	4.54	0.0852	50.0	2,715	11.98	28,600					18.2	0.4		
6	44.8											655.7	549.5	75.0	31.2			674.3	14.89
		37.7	17.68	75.0	8,700	3.24	0.0749	53.0	1,420	16.16	29,250					12.0	0.1		
7	30.6											643.6	554.0	75.0	14.5			655.6	21.80
		24.9	27.00	69.7	3,000	1.58	0.0711	52.0	610	23.0	6,710					5.7	0.2		
8	19.2											637.7	564.0	67.0	5.8			642.7	34.74
		17.6	37.20	62.0	1,270	0.63	0.0703	62.0	310	31.5	7,040					2.3	*		
9	16.0											635.4	574.5	57.0	4.0			637.8	41.70
		14.4	46.3	55.0	770	0.19	0.0702	60.0	218	37.15	7,330					0.8	*		
10	12.9											634.6	580.0	53.0	2.9			636.7	51.7

* negligible



Calculated energy line
Calculated stream boundary
SHASTA DAM
SPILLWAY HYDRAULIC STUDIES

TABLE NO. 3

Example No. 3 - Grand Coulee Dam spillway - Model, scale = 1:40.

q = 2.4 c.f.s. per foot width of channel. (Prototype q = 606 c.f.s./ft.)

d_o = 0.1171 foot v_o = 20.5 ft./sec. Width of channel = 3.0 feet Energy head destroyed =

d_n = - v_n = - Manning's n = 0.01 Power destroyed =

C_r = 0.07

Sta.	v	V	A	a+A	$V^{5/2}$	J	K	L	V^2	R	C^2	Elev. total energy	Elev floor surface	h_p	h_v	h_r	h_f	Calculated elevation total energy plus lost head	$\frac{q}{v}$
0	20.5											10.59			6.53				0.1171
		20.05	0.1196	0.414	1,800	2.46	0.2205	0.58	402	0.108	10,400					0.3145	0.2075		
1	19.6											10.07	3.53	0.56	5.95			10.562	0.1225
		18.83	0.1275	0.932	1,500	6.31	0.1876	0.63	354.5	0.1175	11,000					0.746	0.1726		
2	18.06											9.15	3.04	1.02	5.07			10.0486	0.1328
		16.66	0.144	1.523	1,120	9.575	0.1478	0.975	278	0.1314	11,200					1.38	0.184		
3	15.26											7.59	2.27	1.7	3.62			9.154	0.1572
		14.27	0.1682	1.642	762	8.76	0.1153	0.67	203.6	0.1512	11,750					0.677	0.0768		
4	13.28											6.84	1.75	2.34	2.74			7.5838	0.1808
		12.12	0.198	1.474	515	6.45	0.096	0.67	147	0.175	12,300					0.415	0.0458		
5	10.95											6.38	1.22	3.28	1.86			6.8208	0.2191
		9.815	0.245	1.57	300	5.42	0.08225	1.0	96.25	0.211	13,000					0.445	0.035		
6	8.66											5.90	0.75	4.0	1.17			6.4	0.277
		8.74	0.275	1.562	224	4.68	0.07815	1.4	76.4	0.232	13,500					0.512	0.0342		
7	8.8											5.36	1.50	2.7	1.2			5.9462	0.2727
		7.09	0.3386	1.985	133	4.87	0.07393	1.34								0.482			
8	5.38											4.88	2.32	2.08	0.45			5.332	0.4463

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TABLE NO. 4

Example No. 4 - Grand Coulee Dam spillway - Prototype.

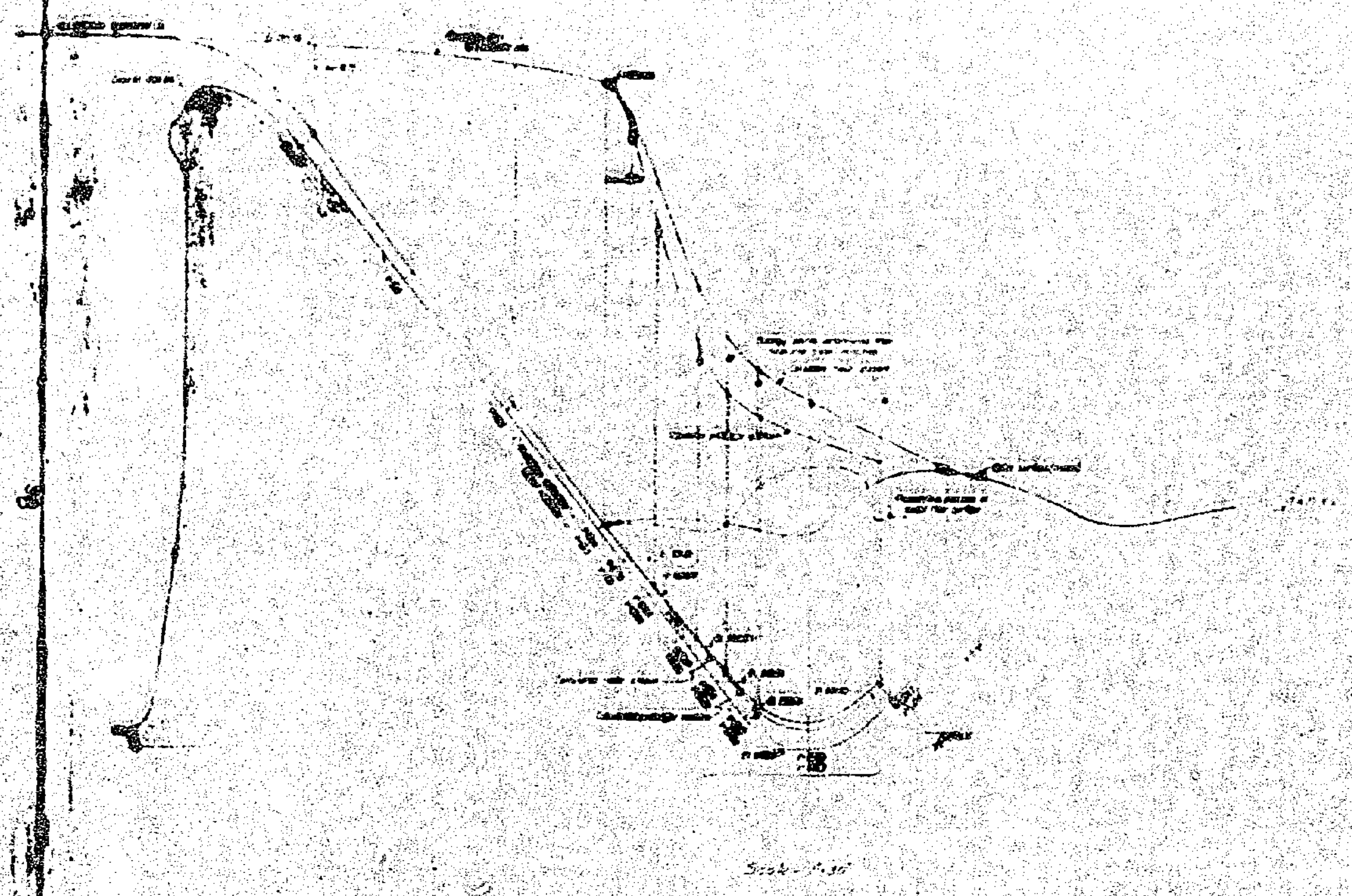
Q = 1,000,000 c.f.s.
q = 606 c.f.s. per foot width of channel.
d_o = 4.675 feet. v_o = 129.6 ft./sec.
d_n = - v_n =

Width of channel = 1,650 feet.
Manning's n = 0.012.
C_r = 0.07

Energy head destroyed =
Power destroyed =

Sta.	v	V	A	a+A	$V^{5/2}$	J	K	L	V^2	R	C^2	Elev. total energy	Elev. floor surface	h_p	h_v	h_r	h_f	Calculated elevation total energy plus lost head	$\frac{q}{v}$
0	129.6											1263.46			260.9				4.675
		126	4.81	16.35	180,000	2.4	0.444	23.5	15,850	4.78	25,700					25.05	3.03		
1	122.4											1235.38	981.25	22.0	232.5			1263.83	4.95
		114.7	5.28	37.53	144,000	6.1	0.3427	25.0	13,150	5.25	26,600					52.3	2.36		
2	107.0											1180.72	961.87	41.0	178.0			1235.53	5.66
		95.6	6.34	60.15	90,000	8.49	0.212	39.5	9,140	6.29	28,100					71.0	2.04		
3	84.2											1107.68	930.85	67.0	110.0			1180.89	7.2
		77.1	7.86	66.4	52,500	7.45	0.1368	26.5	5,940	7.79	30,000					27.0	0.674		
4	70.0											1080.01	910.00	94.0	76.0			1107.674	8.66
		61.55	9.85	58.1	30,000	4.9	0.10045	27.2	3,790	9.74	32,600					13.38	0.325		
5	53.1											1066.30	888.76	134.0	43.8			1080.265	11.41
		45.8	13.23	62.8	14,300	3.74	0.08081	40.0	2,098	13.02	36,000					12.08	0.179		
6	38.5											1054.04	870.0	161.0	23.0			1066.259	15.74
		41.6	14.56	62.5	11,400	3.29	0.07783	56.0	1,730	14.32	37,000					14.34	0.183		
7	44.7											1039.52	900.0	108.0	31.0			1053.528	13.55

X-D-2881



EXAMPLES NOS 3 AND 4
GRAND COULEE DAM
SPILLWAY HYDRAULIC STUDIES
CALCULATED ENERGY GRADIENTS
FOR MODEL AND PROTOTYPE

TABLE NO. 5

Example No. 5 - Friant Dam spillway - Prototype.

Q = 30,000 c.f.s.
q = 90.36 c.f.s. per foot width of channel.
d_o = 0.93 foot. v_o = 97.05
d_n = 40.17 feet. v_n = 2.25

Width of channel = 332 feet.
Kutter's n = 0.012
C_r = 0.02

Energy head destroyed = 136.32 feet.
Power destroyed = 464,600 hp.

Sta.	v	V	A	a+A	$V^{5/2}$	J	K	L	V^2	R	C^2	Elev. total energy	Elev. floor surface	h_p	h_v	h_r	h_f	Calculated elevation total energy plus lost head	$\frac{q}{v}$
0	97.05											459.93			146.30				0.93
		85.07	1.061	6.13	66,780	4.675	0.649	18.0	7236	1.055	16,680					54.60	7.59		
1	73.10											396.94	302.80	11.2	83.00			458.99	1.235
		61.45	1.469	14.28	29,620	8.365	0.2217	21.3	3778	1.456	18,350					39.48	3.01		
2	49.80											354.45	296.80	19.2	38.50			395.99	1.813
		43.69	2.068	20.17	12,620	8.58	0.0811	20.2	1910	2.041	21,000					14.04	0.90		
3	37.58											339.51	293.78	23.8	21.92			354.44	2.402
		33.11	2.727	25.90	6,306	8.50	0.0451	20.2	1096	2.68	21,580					7.40	0.38		
4	28.65											331.73	290.93	28.0	12.75			339.46	3.15
		24.94	3.62	30.44	3,108	7.21	0.0286	20.2	622	3.658	23,260					4.16	0.14		
5	21.25											327.48	288.07	32.4	7.00			331.77	4.253
		17.84	5.06	35.17	1,544	5.69	0.0227	20.2	328.2	4.843	24,590					2.30	0.03		
6	14.46											324.77	285.21	36.3	3.25			327.42	6.25
		11.62	7.77	38.92	460	3.107	0.0203	20.2	135.1	7.56	26,550					1.29	0.01		
7	8.79											323.47	282.5	39.8	1.20			324.60	10.27
		7.75	11.65	39.56	167.2	2.34	0.0201	10.0	60.06	10.90	27,880					0.47	0.0		
8	6.72											323.00	282.5	39.80	0.70			323.47	13.45
		5.36	16.85	43.08	66.6	1.595	0.02	10.0	28.72	15.3	29,100					0.28	0.0		
9	4.01											322.72	282.5	40.0	0.25			323.03	22.52
		3.13	28.87	39.94	17.5	0.27	0.02	20.0	9.8	24.7	31,200					0.11	0.0		
10	2.25											322.61	282.5	40.0	0.08			322.69	40.17

TABLE NO. 6

Example No. 6 - Friant Dam spillway - Prototype.

Q = 90,000 c.f.s.
q = 271 c.f.s. per foot width of channel.
d_o = 2.29 feet v_o = 118.5 ft./sec.
d_n = 44.4 feet v_n = 6.1 ft./sec.

Width of channel = 332 feet.
Kutter's n = 0.012
C_F = 0.06

Energy head destroyed = 191.47 feet.
Power destroyed = 1,966,000 hp.

Sta.	v	V	A	a+A	$V^{5/2}$	J	K	L	V^2	R	C^2	Elev. total energy	Elev. floor surface	h_p	h_v	h_r	h_f	Calculated elevation total energy plus lost head	$\frac{q}{v}$
0	118.5											524.83			218.0				2.29
		117.9	2.30	3.60	150,950	0.564	0.718	4.3	13,900	2.27	20,710					1.74	1.27		
1	117.3											521.82	303.0	5.00	213.82			524.83	2.31
		111.45	2.43	9.0	131,100	2.70	0.599	20.8	12,420	2.4	20,980					33.64	5.14		
2	105.6											483.04	296.80	13.0	173.25			521.83	2.57
		98.40	2.754	15.65	96,070	4.68	0.409	20.2	9,684	2.71	21,600					38.63	3.34		
3	91.20											441.07	293.6	18.3	129.17			483.04	2.97
		84.57	3.205	21.4	65,780	5.68	0.266	20.2	7,150	3.145	22,420					30.38	2.05		
4	77.95											408.64	290.8	23.5	94.30			441.03	3.43
		72.42	3.74	25.9	44,670	5.924	0.179	20.2	5,245	3.66	23,230					21.47	1.25		
5	66.90											385.92	288.06	28.3	69.50			408.58	4.05
		62.03	4.37	30.85	30,320	6.063	0.129	20.2	3,850	4.255	23,990					15.86	0.78		
6	57.17											369.30	285.21	33.4	50.70			385.93	4.74
		52.41	5.17	36.30	19,900	6.02	0.098	20.2	2,748	5.015	24,730					11.97	0.45		
7	47.66											356.88	282.5	39.2	35.27			369.39	5.69
		43.22	6.26	40.65	12,280	5.485	0.080	20.0	1,868	6.04	25,820					8.74	0.24		
8	38.78											347.90	282.5	42.1	23.35			356.93	6.89
		34.79	7.79	41.61	7,138	4.275	0.0692	20.0	1,210	7.44	26,630					5.91	0.12		
9	30.81											341.87	282.5	44.6	14.75			347.88	8.79
		26.70	10.15	44.72	3,688	3.308	0.0637	20.0	712.5	9.56	27,470					4.21	0.05		
10	22.60											337.61	282.5	47.2	7.93			341.89	11.98
		18.80	14.41	46.84	1,532	2.116	0.061	20.0	353.5	13.25	28,420					2.68	0.02		
11	15.01											335.01	282.5	49.0	3.50			337.60	18.05
		11.71	21.31	51.57	469	1.051	0.060	20.0	137.1	18.90	29,700					1.26	0.0		
12	8.42											333.75	282.5	50.2	1.10			335.06	32.20
		7.25	37.4	50.02	141	0.32	0.060	20.2	52.6	31.0	31,000					0.89	0.0		
13	6.1											333.36	286.3	46.5	0.58			333.77	44.4

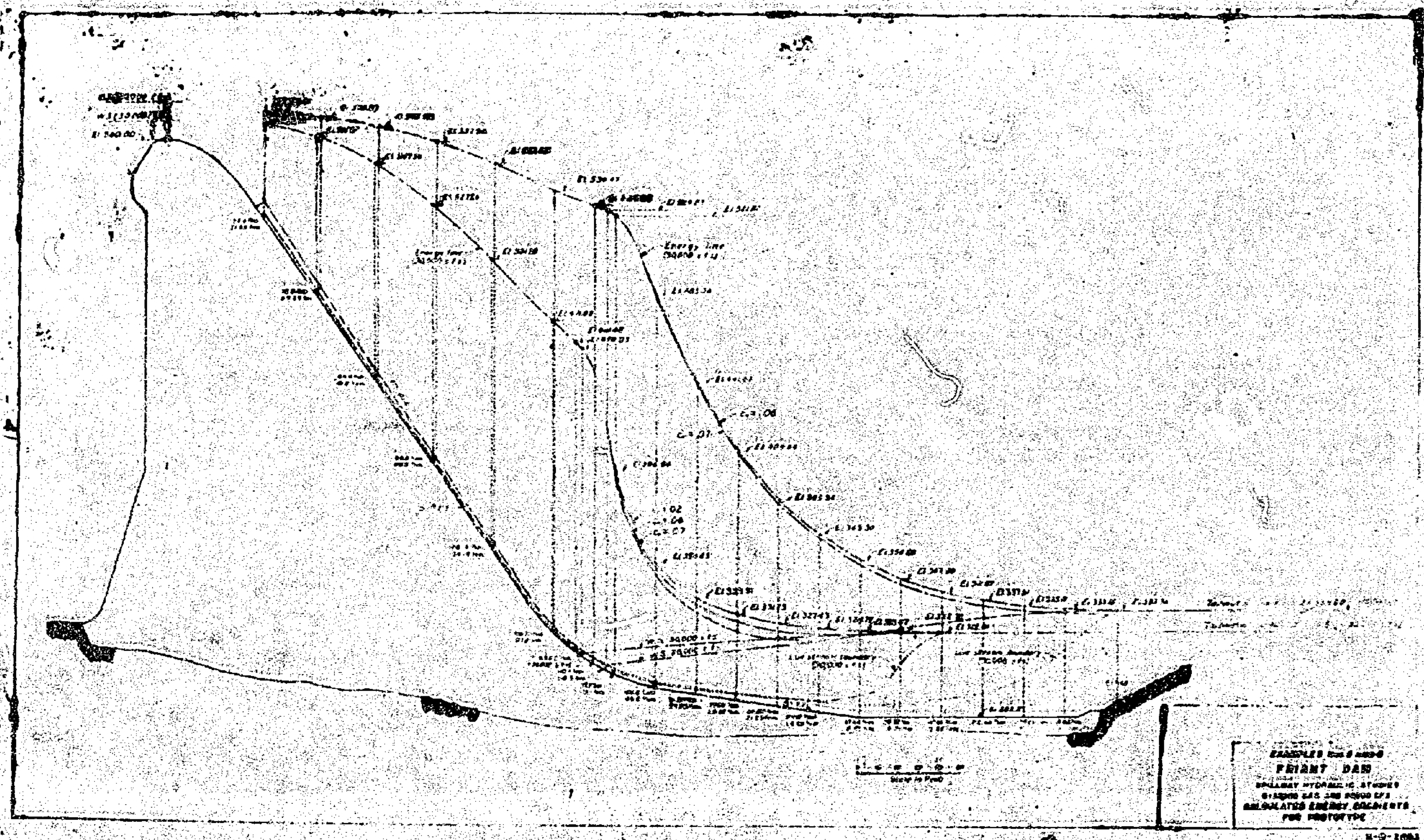

Energy line
FRIANT DAM
SPILLWAY HYDRAULIC STUDIES
CALCULATED ENERGY GRADIENTS
FOR PROTOTYPE

TABLE NO. 7

Example No. 7 - Madden Dam spillway - Prototype.

Q = 49,500 c.f.s.
q = 112.5 c.f.s. per foot width of channel.
d_o = 1.25 feet. v_o = 90.10
d_n = 47.5 feet. v_n = 2.37

Width of channel = 440 feet.
Kutter's n = 0.012
C_r = 0.04

Energy head destroyed = 118.4 feet.
Power destroyed = 671,800 hp.

Sta.	v	V	A	$u+A$	$V^{5/2}$	J	K	L	V^2	R	C^2	Elev. total energy	Elev. floor surface	h_p	h_v	h_r	h_f	Calculated elevation total energy plus lost head	$\frac{q}{v}$
0	90.10											230.0			126.0				1.25
		84.85	1.326	4.59	66,350	2.45	0.540	17.5	7,200	1.318	17,838					23.16	5.36		
1	79.60											201.48	95.30	7.80	98.40			230.02	1.415
		74.15	1.517	9.12	47,370	4.983	0.352	12.8	5,500	1.547	18,542					22.45	2.62		
2	68.70											176.51	92.20	11.0	73.30			201.47	1.638
		60.72	1.854	14.17	28,720	6.515	0.195	20.5	3,686	1.840	19,650					26.01	2.08		
3	52.75											148.42	87.80	17.5	43.16			176.55	2.134
		46.32	2.43	20.67	14,600	7.35	0.1001	20.5	2,145	2.40	21,000					15.14	0.87		
4	39.89											132.41	83.00	24.8	24.70			148.51	2.821
		34.53	3.26	27.78	7,010	7.315	0.082	20.5	1,193	3.21	22,500					9.22	0.34		
5	29.17											122.85	78.40	31.3	13.20			132.46	3.86
		24.82	4.53	32.23	3,070	5.885	0.0468	20.5	616	4.47	24,220					5.64	0.11		
6	20.47											117.10	73.80	36.8	6.50			122.85	5.50
		16.57	6.79	39.25	1,118	4.33	0.0417	20.5	274.7	6.36	25,880					3.69	0.04		
7	12.68											113.37	69.2	41.7	2.60			117.13	8.87
		10.55	10.66	37.47	361	2.37	0.0403	10.25	111.3	10.16	27,600					0.98	0.0		
8	8.42											112.39	67.0	44.3	1.10			113.38	13.35
		6.41	17.55	40.25	104	1.07	0.040	10.25	41.10	16.26	29,150					0.44	0.0		
9	4.40											111.95	64.0	47.7	0.30			112.45	25.60
		3.38	33.30	52.10	21	0.426	0.040	20.0	11.42	28.8	30,730					0.34	0.0		
10	2.37											111.61	64.0	47.5	0.09			111.93	47.5

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TABLE NO. 8

Example No. 8 - Madden Dam spillway - Prototype.

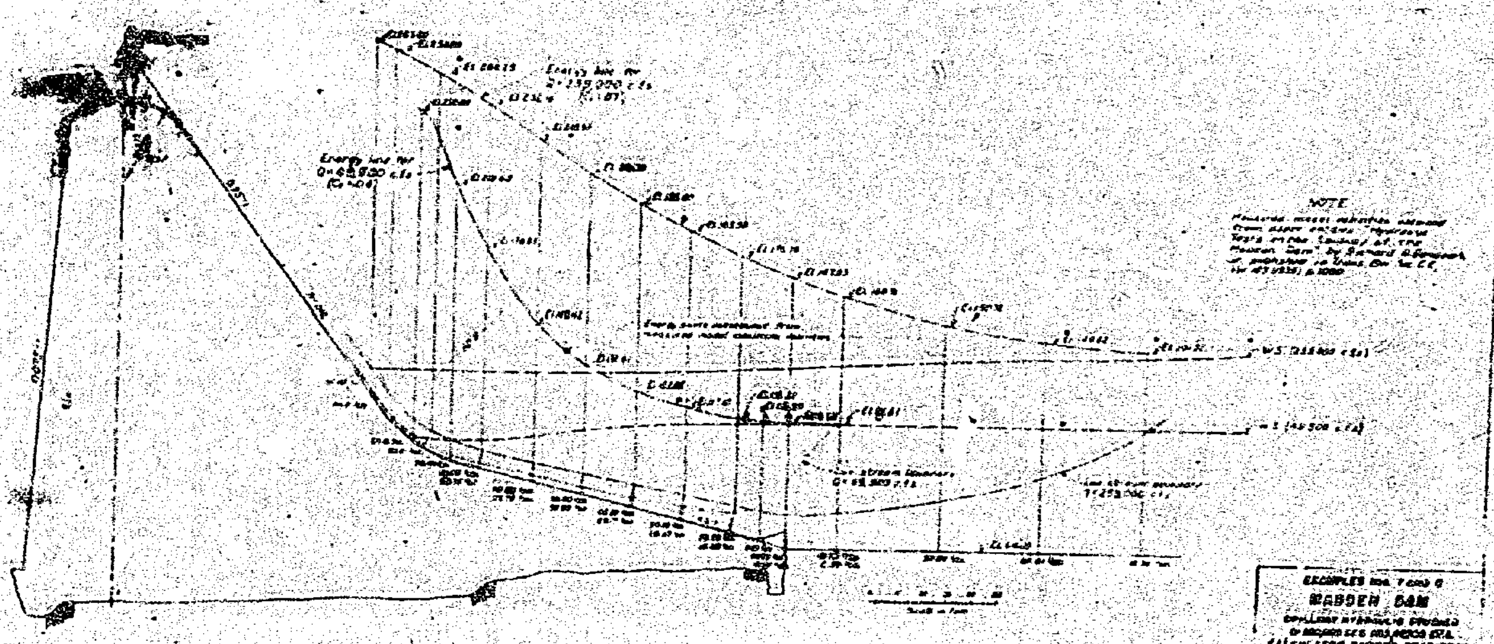
Q = 259,000 c.f.s.
q = 588.63 c.f.s. per foot width of channel.
d_o = 6.51 feet. v_o = 90.4
d_n = 46.2 feet. v_n = 12.74

Width of channel = 440 feet.
Kutter's n = 0.012
C_r = 0.07

Energy head destroyed = 115.7 feet.
Power destroyed = 3,408,000 hp.

Sta.	v	V	A	$a+A$	$V^{5/2}$	J	K	L	V^2	R	C^2	Elev. total energy	Elev. floor surface	h_p	h_v	h_r	h_f	Calculated elevation total energy plus lost head	$\frac{q}{v}$
0	90.4											257.00			126.8				6.51
		89.7	6.56	11.13	76,250	0.914	0.1862	19.0	8,050	6.37	25,880					3.23	0.93		
1	89.0											252.84	111.2	18.8	123.0			257.16	6.61
		87.50	6.725	19.46	71,650	1.99	0.1765	21.7	7,660	6.52	26,005					7.61	0.98		
2	86.0											244.25	98.4	31.0	114.8			252.79	6.84
		83.72	7.03	29.7	64,100	3.33	0.1612	21.0	7,005	6.81	26,178					11.27	0.82		
3	81.45											232.16	92.2	37.0	102.96			244.25	7.22
		78.65	7.48	39.35	54,900	4.26	0.1434	20.5	6,190	7.24	26,480					12.53	0.66		
4	75.85											218.97	87.8	41.7	89.40			232.09	7.76
		73.21	8.03	44.0	45,880	4.48	0.1271	20.5	5,360	7.76	26,750					11.66	0.53		
5	70.58											206.78	83.1	46.3	77.35			218.94	8.34
		67.79	8.68	48.85	37,850	4.63	0.1136	20.5	4,595	8.36	27,010					10.78	0.42		
6	65.00											195.60	78.5	51.4	65.60			206.68	9.05
		62.15	9.47	54.20	30,430	4.73	0.1021	20.5	3,861	9.08	27,305					9.89	0.32		
7	59.30											185.39	73.8	57.0	54.60			195.61	9.90
		56.41	10.42	59.75	23,900	4.73	0.0929	20.5	3,182	9.96	27,540					9.01	0.24		
8	53.52											176.14	69.2	62.5	44.45			185.40	11.0
		50.44	11.66	65.75	18,100	4.64	0.0855	20.5	2,548	11.07	27,880					8.14	0.17		
9	47.37											167.83	64.0	69.0	34.82			176.13	12.42
		44.41	13.24	69.60	13,150	4.25	0.0799	20.0	1,973	12.49	28,250					6.81	0.11		
10	41.46											160.91	64.0	70.2	25.68			167.80	14.2
		36.13	16.28	71.10	7,850	3.37	0.0748	40.0	1,305	15.17	28,900					10.07	0.12		
11	30.82											150.72	64.0	72.0	14.75			160.94	19.10
		25.71	22.88	72.90	3,352	2.188	0.0715	40.0	661	20.77	30,090					6.20	0.04		
12	20.61											144.42	64.0	73.8	6.60			150.78	28.56
		16.67	35.30	74.30	1,135	1.105	0.0703	40.0	278	30.40	30,880					3.10	0.01		
13	12.74											141.31	64.0	74.8	2.52			144.43	46.20

X-D-2885



EXAMPLES NOS. 7 AND 8
MADDEN DAM
SPILLWAY HYDRAULIC STUDIES
[illegible]
CALCULATED ENERGY GRADIENTS
FOR PROTOTYPE

K-D-2894

APPENDIX C

The following examples of hydraulic jumps on horizontal floors will demonstrate the workability of the formulae. The energy line and water surface profile in each case have been determined with channel friction neglected. The effect of friction is shown in one case, example No. 10. It will be noted that the forces used in the pressure-momentum equation in examples 9, 11, and 12 are; pressure, momentum of the q-stream, and momentum of the roller stream at the several sections of the jump body. The additional force of accumulated friction at the sections was included in the calculations of example No. 10. If it were desired to determine the effect of placing the jump on an inclined floor, then it would be necessary to include a fourth term in the equation to obtain equilibrium. The fourth term would be the component of the jump body weight.

TABLE NO. 9

Example No. 9 - Hydraulic Jump on Horizontal Floor (Friction neglected)

$q = 2.0$ c.f.s.
$d_o = 0.1$ ft.
$v_o = 20$ ft. per sec.

$$\frac{d^2}{2} + \frac{qv}{g} + 0.1Kja^2 = \text{Constant} = 1.24722 \qquad L = \frac{h_r}{K_m J}$$

v	$\frac{q}{v}$	K	$\frac{qv}{g}$	d	$\frac{d^2}{2}$	j	a	m	Constant	V	h_v	E	h_r	A	K_m	d_m	J	L	ΣL
20.00	0.1000		1.2422	0.10	0.005		0	0	1.2472		6.2111	6.3111							0
										19.50			0.3456	0.1026	0.2358	0.230	1.242	1.180	
19.00	0.1053	0.2173	1.1801	0.36	0.0648	2.42	0.255	0.0055	1.2484		5.6056	5.9656							1.180
										18.50			0.4445	0.1081	0.2042	0.425	2.932	0.742	
18.00	0.1111	0.1915	1.1180	0.49	0.12	3.41	0.3789	0.0094	1.2474		5.0311	5.5211							1.922
										17.00			0.8659	0.1176	0.1720	0.585	3.975	1.265	
16.00	0.1250	0.1500	0.9938	0.68	0.2312	4.44	0.555	0.0205	1.2455		3.9752	4.6552							3.187
										14.00			1.4492	0.1429	0.1204	0.825	4.77	2.523	
12.00	0.1667	0.1000	0.7453	0.97	0.4705	4.82	0.8033	0.0311	1.2469		2.2360	3.2060							5.710
										10.00			1.0122	0.2000	0.0858	1.085	4.43	2.662	
8.00	0.2500	0.0772	0.4969	1.20	0.72	3.80	0.95	0.0265	1.2434		0.9938	2.1938							8.372
										6.00			0.5354	0.3333	0.0727	1.305	2.92	2.522	
4.00	0.5000	0.0708	0.2484	1.41	0.994	1.82	0.91	0.0106	1.2530		0.2484	1.6584							10.894
										3.50			0.0686	0.5714	0.0704	1.430	1.504	0.648	
3.00	0.6667	0.0702	0.1665	1.45	1.0513	1.175	0.7833	0.0051	1.2426		0.1398	1.5898							11.542
										2.50			0.0297	0.8000	0.0701	1.474	0.843	0.503	
2.00	1.0000	0.0701	0.1242	1.498	1.1220	0.498	0.498	0.0009	1.2471		0.0621	1.5601							12.045
										1.655			0.0070	1.208	0.0700	1.5123	0.252	0.278	
1.31	1.5265	0.0700	0.0314	1.5265			0	0	1.2472		0.0266	1.5531							12.323

X-D-2886

TABLE NO. 10

Example No. 10 - Hydraulic Jump on Horizontal Floor (Friction included, width of channel = 1'-0", Kutter's n = 0.01)

q = 2.0 c.f.s.
d_o = 0.1 ft.
v_o = 20 ft. per sec.

$$\frac{d^2}{2} + \frac{q}{g}v + 0.1Kja^2 + LA\frac{V^2}{C^2R} = \text{Constant} = 1.2472$$

v	h_v	$\frac{q}{g}v$	d	$\frac{d^2}{2}$	K	j	a	$0.1Kja^2$	V	A	K_m	R	C	d_m	J	$KJ + \frac{V^2}{C^2R}$	L	$LA\frac{V^2}{C^2R}$	Calculated Force Constant	ΣL	E
20.00	6.21	1.2422	0.100	0.05		0	0	0											1.2472	0	6.31
									19.50	0.1025	0.2358	0.0851	90	0.1770	0.726	0.7227	0.617	0.0348			
19.00	5.61	1.1801	0.254	0.0323	0.2173	1.414	0.1488	0.0007											1.2479	0.617	5.864
									18.50	0.1081	0.2042	0.0889	91	0.302	1.84	0.8317	0.6001	0.0302			
18.00	5.015	1.1180	0.350	0.0612	0.1915	2.150	0.2489	0.0027											1.2482	1.2171	5.365
									17.00	0.1176	0.1720	0.0952	93	0.436	2.705	0.8165	1.063	0.0439			
16.00	3.975	0.9938	0.522	0.1362	0.1500	3.175	0.397	0.0075											1.2477	2.2801	4.497
									14.00	0.1428	0.1204	0.1111	97	0.657	3.600	0.6215	2.368	0.0636			
12.00	2.237	0.7453	0.792	0.3136	0.1000	3.75	0.6253	0.0146											1.2472	4.6481	3.029
									10.00	0.2000	0.0858	0.1428	103	0.9085	3.5925	0.3742	2.710	0.0358			
8.00	0.994	0.4969	1.025	0.5253	0.0772	3.10	0.775	0.0145											1.2463	7.3581	2.019
									6.00	0.3333	0.0727	0.2000	114	1.1335	2.400	0.1884	2.606	0.0130			
4.00	0.2485	0.2484	1.242	0.7713	0.0706	1.484	0.742	0.0058											1.2481	10.1641	1.4905
									3.50	0.5718	0.0704	0.2665	122	1.267	1.218	0.0887	0.653	0.0012			
3.00	0.1397	0.1863	1.292	0.8346	0.0702	0.9375	0.6253	0.0026											1.2473	10.8171	1.4317
									2.50	0.8000	0.0701	0.3075	126	1.316	0.645	0.0465	0.3365	0.0007			
2.00	0.0621	0.1242	1.340	0.8973	0.0701	0.3400	0.3400	0.0003											1.2463	11.4536	1.4021
									1.729	1.1565	0.0700	0.349	130	1.3527	0.172	0.0125	0.2640	0.0001			
1.465	0.03335	0.0911	1.365	0.9321		0	0	0											1.2478	11.7176	1.3984

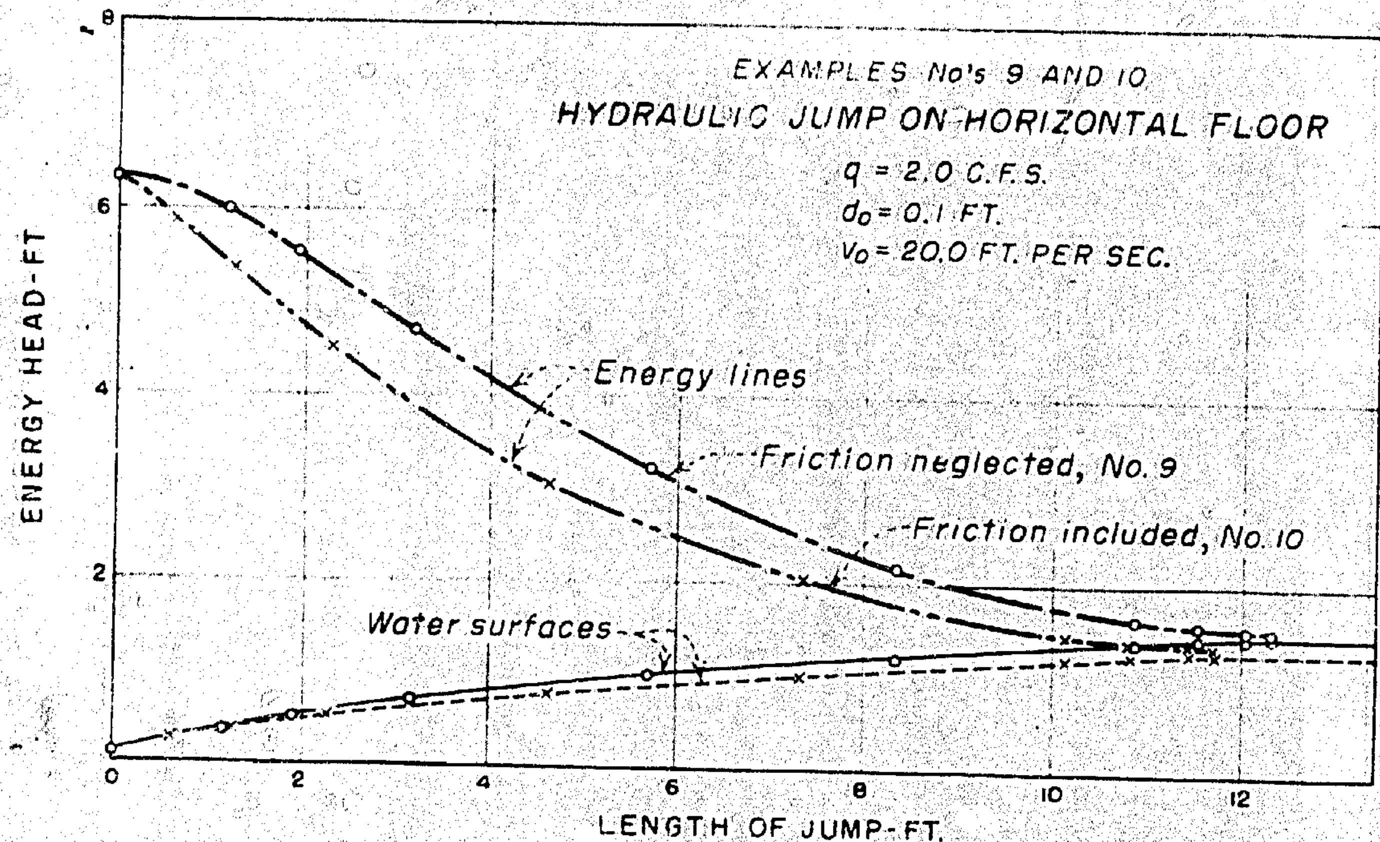
TABLE NO. 11

Example No. 11 - Hydraulic Jump on Horizontal Floor (Friction neglected)

q = 400 c.f.s.
d_0 = 3.55 ft.
v_0 = 112.7 ft. per sec.

$$\frac{d^2}{2} + \frac{qv}{g} + 0.13ja^2 = \text{Constant} = 1406.3 \qquad L = \frac{h_r}{K_m J}$$

v	$\frac{q}{v}$	K	$\frac{qv}{g}$	d	$\frac{d^2}{2}$	j	a	m	Constant	V	h_v	E	h_r	A	K_m	d_m	J	L	ΣL
112.70	3.55		1400.0	3.55	6.30	0	0	0	1406.30		197.50	201.05							0
										111.35			4.35	3.594	0.435	6.175	0.72	13.90	
110.00	3.038	0.425	1366.0	8.8	38.80	1.42	5.162	1.605	1408.40		187.90	196.70							13.90
										105.00			24.30	3.81	0.372	12.95	2.40	27.30	
100.00	4.00	0.32	1242.0	17.1	146.20	3.275	13.10	17.96	1406.16		155.30	172.40							41.20
										95.00			24.27	4.21	0.2838	19.815	3.71	23.10	
90.00	4.445	0.2455	1120.0	22.53	253.80	4.075	18.085	32.70	1406.50		125.60	148.13							64.30
										85.00			21.47	4.706	0.2124	24.895	4.29	23.50	
80.00	5.00	0.185	993.6	27.26	372.00	4.45	22.26	41.00	1406.60		99.40	126.66							87.80
										75.00			19.10	5.333	0.1619	29.36	4.50	26.30	
70.00	5.72	0.1426	889.4	31.46	495.00	4.50	25.74	42.50	1406.90		76.10	107.56							114.10
										65.00			16.41	6.153	0.1253	33.355	4.42	29.70	
60.00	6.67	0.112	745.2	35.25	622.00	4.28	28.58	39.20	1406.40		55.90	91.15							143.80
										55.00			13.56	7.273	0.101	37.00	4.08	32.60	
50.00	8.00	0.0925	621.0	38.76	751.50	3.85	30.76	33.70	1406.20		38.63	77.59							176.40
										45.00			10.71	8.888	0.0635	40.40	3.55	35.30	
40.00	10.00	0.08	496.8	42.04	883.60	3.20	32.04	26.30	1406.70		24.84	66.88							211.90
										35.00			7.84	11.428	0.0763	43.555	2.81	36.60	
30.00	13.33	0.0737	372.6	45.07	1016.00	2.38	31.74	17.70	1406.30		13.97	59.04							248.50
										25.00			4.86	16.00	0.072	46.52	1.91	35.40	
20.00	20.00	0.0709	248.4	47.97	1160.00	1.40	27.97	7.75	1406.15		6.21	54.18							283.90
										17.50			1.36	22.86	0.0706	48.65	1.13	17.04	
15.00	26.67	0.07	186.3	49.33	1217.00	0.85	22.66	3.08	1406.36		3.49	52.82							300.94
										12.50			0.63	32.00	0.07	49.955	0.56	16.10	
10.00	40.00	0.07	124.2	50.64	1282.21	0.265	10.64	0.21	1406.62		1.55	52.19							317.94
										8.91			0.07	44.89	0.07	50.906	0.135	7.49	
7.82	51.17	-	98.2	51.17	1308.10	0	0	0	1406.30		0.95	52.12							324.53


EXAMPLES No's 9 AND 10
HYDRAULIC JUMP ON HORIZONTAL FLOOR
q = 2.0 C.F.S.
do = 0.1 FT.
Vo = 20.0 FT. PER SEC.
Energy lines
Friction neglected, No. 9
Friction included, No. 10
Water surfaces
ENERGY HEAD-FT
LENGTH OF JUMP-FT.
0
2
4
6
8
10
12

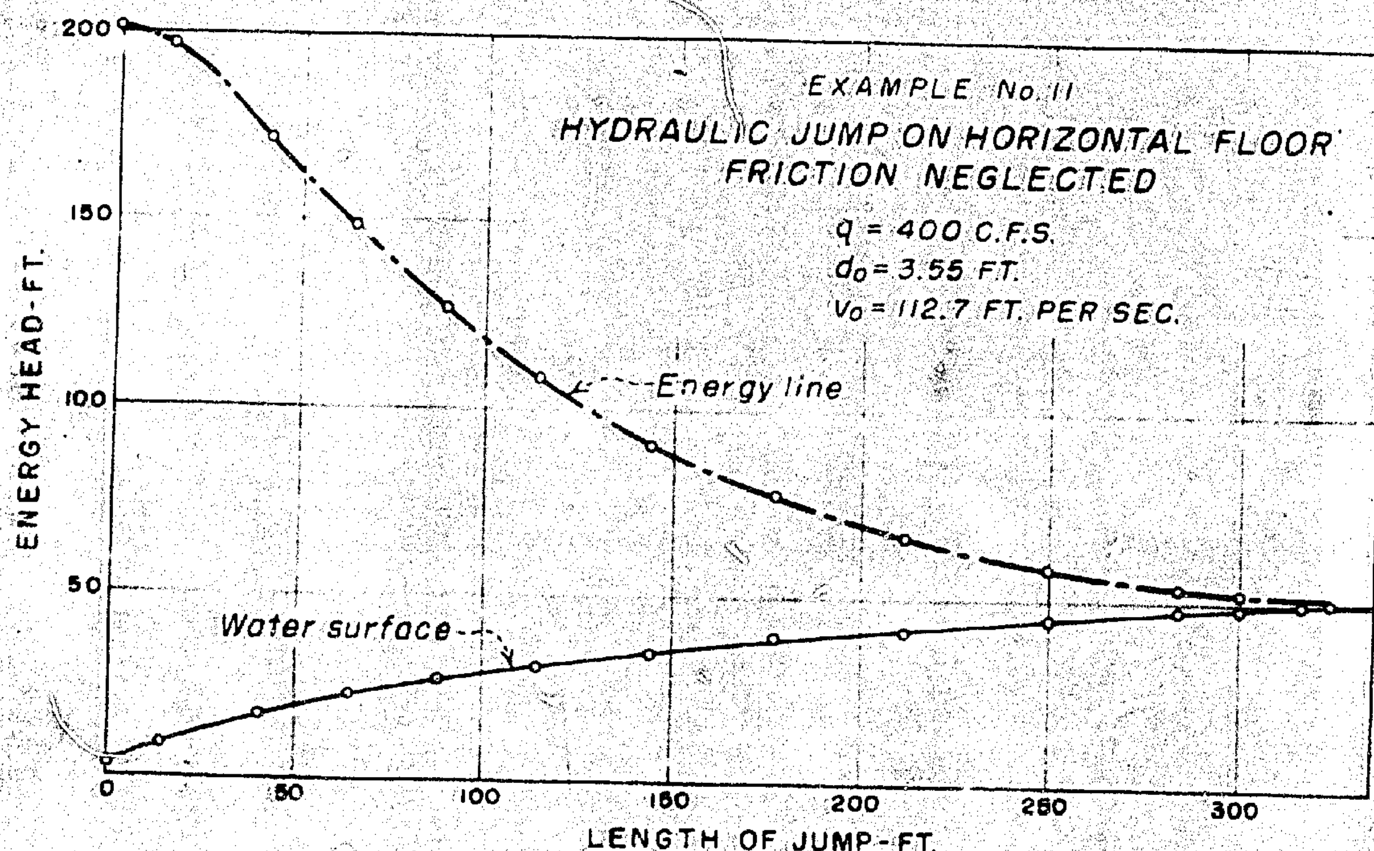

EXAMPLE No. 11
HYDRAULIC JUMP ON HORIZONTAL FLOOR
FRICTION NEGLECTED
q = 400 C.F.S.
do = 3.55 FT.
Vo = 112.7 FT. PER SEC.
Energy line
Water surface
ENERGY HEAD-FT.
LENGTH OF JUMP-FT.
0
50
100
150
200
250
300

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TABLE NO. 12

Example No. 12 - Hydraulic Jump on Horizontal Floor (Friction neglected)

q = 500 c.f.s.
d_o = 3.0 ft.
v_o = 166.67 ft. per sec.

$$\frac{d^2}{2} + \frac{qv}{g} + 0.1Kja^2 = \text{Constant} = 2592.5$$

$$L = \frac{h_F}{K_m J}$$

v	$\frac{q}{v}$	K	$\frac{qv}{g}$	d	$\frac{d^2}{2}$	j	a	m	Constant	V	h_v	E	h_F	A	K_m	d_m	J	L	ΣL
166.67	3.00	-	2480.00	3.00	4.50	0	0	0	2592.50		431.33	434.33							0
										164.33			16.18	3.043	1.208	6.925	1.275	10.52	
162.00	3.086	1.152	2516.00	10.85	58.90	2.517	7.764	17.47	2592.35		407.30	418.15							10.52
										158.50			29.585	3.155	1.072	13.308	3.22	8.57	
155.00	3.225	1.00	2407.00	15.765	124.30	3.89	12.54	61.20	2592.50		372.80	388.565							19.09
										152.50			20.875	3.279	0.946	17.08	4.21	5.24	
150.00	3.333	0.91	2330.00	18.39	169.10	4.52	15.057	93.20	2592.30		349.30	367.69							24.33
										145.00			40.295	3.448	0.805	20.743	5.02	9.98	
140.00	3.57	0.729	2174.00	23.095	266.50	5.47	19.525	152.00	2592.50		304.30	327.395							34.31
										130.00			72.105	3.846	0.571	27.393	6.12	20.64	
120.00	4.168	0.454	1863.00	31.69	502.00	6.604	27.522	227.20	2592.20		223.60	255.29							54.95
										110.00			59.94	4.545	0.349	35.92	6.91	24.86	
100.00	5.00	0.27	1552.00	40.15	805.50	7.033	35.15	234.50	2592.00		155.20	195.35							79.81
										90.00			47.80	5.556	0.208	44.15	6.95	33.08	
80.00	6.25	0.1628	1242.00	48.15	1158.00	6.707	41.90	191.70	2591.70		99.40	147.55							112.89
										70.00			36.29	7.143	0.1777	51.755	6.24	32.70	
60.00	8.33	0.1036	932.00	55.36	1531.50	5.645	47.03	129.30	2592.80		55.90	111.26							145.59
										50.00			24.82	10.00	0.0877	58.48	4.848	58.40	
40.00	12.50	0.0781	621.00	61.60	1897.28	3.928	49.10	73.90	2592.18		24.84	86.44							203.99
										30.00			12.98	16.667	0.073	64.43	2.865	62.10	
20.00	25.00	0.0707	310.50	67.25	2261.28	1.69	42.25	21.31	2593.09		6.21	73.46							266.09
										17.50			1.407	28.571	0.0704	67.905	1.376	14.53	
15.00	33.33	0.0703	232.95	68.56	2350.23	1.057	35.23	9.22	2592.40		3.493	72.053							280.62
										12.50			0.701	40.00	0.0701	69.18	0.728	13.73	
10.00	50.00	0.0701	155.30	69.80	2436.02	0.396	19.30	1.088	2592.41		1.552	71.352							294.35
										8.55			0.13	58.479	0.07	70.125	0.198	9.38	
7.10	70.45	-	110.90	70.45	2481.60	0	0	0	2592.50		0.772	71.222							303.73

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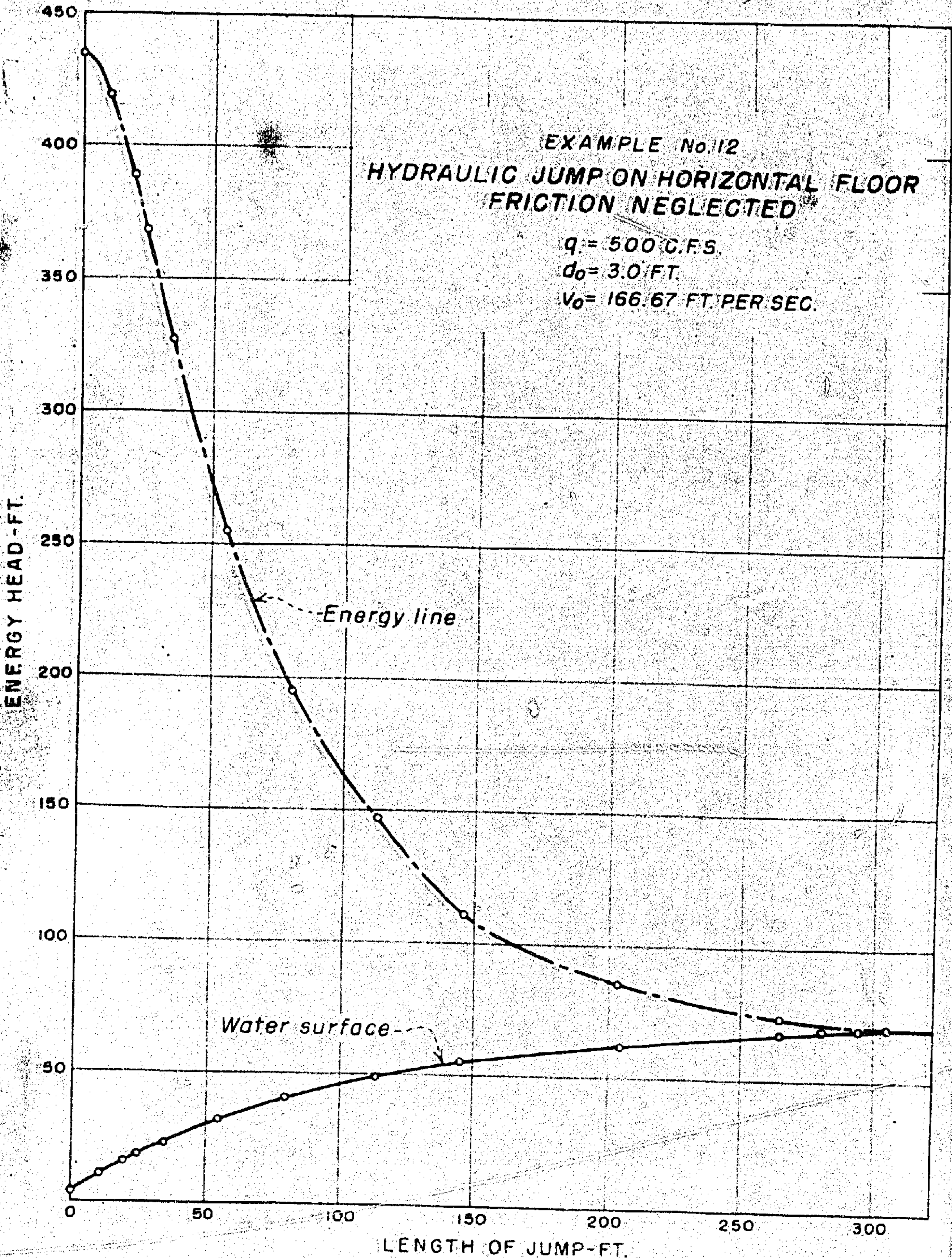
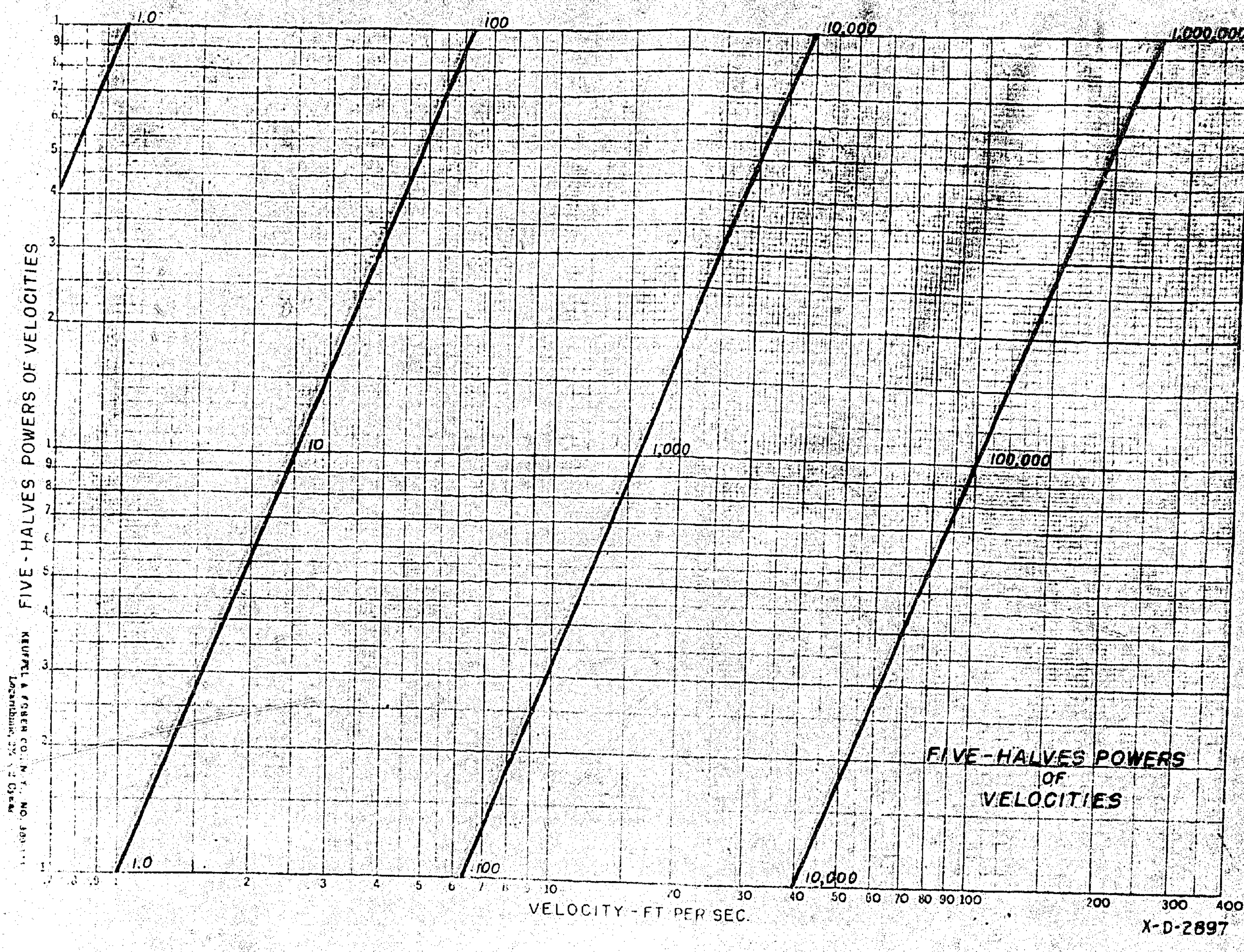

EXAMPLE No.12
HYDRAULIC JUMP ON HORIZONTAL FLOOR
FRICTION NEGLECTED
q = 500 C.F.S.
d₀ = 3.0 FT.
V₀ = 166.67 FT. PER SEC.
Energy line
Water surface
ENERGY HEAD-FT.
LENGTH OF JUMP-FT.
0
50
100
150
200
250
300
350
400
450

X-D-2896


FIVE-HALVES POWERS OF VELOCITIES
VELOCITY - FT PER SEC.
1.0
100
10,000
1,000,000
10
1,000
100,000
1.0
100
10,000
FIVE-HALVES POWERS OF VELOCITIES
X-D-2897