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A STUDY FOR THE DESIGN OF CRESTS  
FOR OVERFALL DAMS

by

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B. S. C. E., Purdue University, 1925

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A Thesis submitted to the Faculty of the Graduate  
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fulfillment of the requirements for the Degree

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MEMORANDUM TO CHIEF ENGINEER  
(J. E. Warnock)

Subject: Thesis for submission to the graduate school of the University of Colorado in partial fulfillment of the requirements for the degree Master of Science.

1. In partial fulfillment of the requirements of study for the degree Master of Science in Civil Engineering in the graduate school of the University of Colorado, I have prepared a thesis entitled, "A Study for the Design of Crests for Overfall Dams". The material has been derived from experimental studies made in the Bureau of Reclamation laboratories and will subsequently be included in the final reports on the Boulder Dam Project in Part VI, Hydraulic Investigations, Bulletin 3, Studies of Crests for Overfall Dams.

2. Bulletin 3 will contain a complete treatment of the subject of overfall crests as studied in connection with the design of the crests for the Boulder Dam spillway. This thesis treats only of crest with vertical upstream faces.

3. In compliance with office memorandum No. 186, dated March 2, 1939, the ~~second~~-final draft of the thesis is submitted for your approval and release.

J. E. Warnock

J. E. Warnock

OK  
J. E. Warnock  
7/26/39

This Thesis for the M. S. degree, by

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not proof-read, has been approved for the

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Date

Aug 12 1939

### FOREWORD

The hydraulic experimental program described in this thesis was conducted in the hydraulic laboratory of the Bureau of Reclamation of the United States Department of the Interior in Denver, Colorado, under the direct supervision of the author.

Particular credit is extended J. N. Bradley, Associate Engineer, who has been associated with the author from the instigation of this program, and whose untiring efforts and interest have made possible the comprehensive treatment given this subject. Appreciation is also extended to Associate Engineers C. W. Thomas and J. W. Ball; Assistant Engineers J. H. Douma, H. G. Dewey, Jr. and A. N. Smith; and Junior Engineers R. A. Goodpasture and D. M. Lancaster for their assistance in the performance of the experiments and analysis of the data.

These studies were made under the general supervision of Arthur Ruettgers, Senior Engineer, and J. L. Savage, Chief Designing Engineer. All engineering work of the Bureau of Reclamation is under the direction of R. F. Walter, Chief Engineer, and all the activities of the Bureau are directed by John C. Page, Commissioner.

The author particularly appreciates the courtesy and cooperation of the Bureau in granting him permission to use the material contained herein as a graduate thesis. Responsibility for all statements and conclusions presented are assumed by the author as entirely personal.

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## I - INTRODUCTION

### Development of Profile for Spillway Crest

With the trend in recent years toward greater depths of flow over flood spillway crests, the importance of providing the correct profile has been materially increased. It has been the accepted practice for many years to design the crest of an overfall dam with the assumption that the space beneath the jet from a sharp-crested weir is filled with the building material used in the dam. Despite all the studies previously made on the characteristics of flow over a weir, the under surface of the nappe, being difficult to observe, has received a relatively small amount of attention.

A deficiency of section at any point under the nappe will result in the formation of a negative pressure between the downstream face of the dam and the nappe of water. With this condition, three undesirable conditions will obtain:

(1) The resultant pressure in the spillway section will be increased due to the reduction of back pressure;

(2) The instability of the negative pressure with its intermittent pressure change will cause a vibration accompanied by a localized disintegration of the boundary, known as cavitation;

(3) The intermittency of the negative pressure caused by the unstable condition prevailing beneath the flow sheet will cause a state of vibration in the dam. While the amplitude of this vibration is exceedingly small, the accumulation

of forces within the dam can produce secondary forces, particularly if the natural frequency of the structure bears a particular relation to that of the vibration of the nappe. This event may give rise to a movement resembling an earthquake in the proximity of the structure.

The under surface of the nappe as it leaves the weir crest rises slightly, then becomes horizontal, and finally falls, following a path approximating a parabola. To the uninitiated, this curve of travel is insignificant and scarcely noticeable, but it constitutes an extremely important phase of the fundamentals of the contraction of a jet of flow over a sharp-edged control. While this shape has been the subject of considerable study in the past, in most cases the results have been based on theory entirely or on rather meager experimental data.

The first studies of nappe shapes were those of Bazin<sup>1</sup> made in 1886-88, in which he reduced his observations to unit head and constructed a "base curve" representing the results of his experiments. In the present studies on this subject, Bazin's results have been used for comparative purposes.

The term "nappe" as applied to the sheet of water passing a weir apparently originated with Bazin. In the translation of his works from the French to the English, the translator's comment on the use of the word "nappe" is "For want of a convenient English equivalent,

<sup>1</sup> References are listed in numerical order of occurrence in the bibliography in the appendix.

we shall designate this sheet by its very appropriate French name, the nappe, a name applied primarily to a table-cloth, the form of which, as it passes from a horizontal to a vertical plane in passing over the edge of the table, is well imitated by the sheet of water passing over the weir".

Bazin, in discussing the subject, states, "The upper surface of the nappe has already been studied by certain experimenters, but the under surface, while less easy to observe, is, perhaps, of greater interest from a theoretical standpoint; for its form shows accurately the contraction at the crest. In forming this contraction, the under side of the nappe leaves the crest at a certain angle, rising at first, then becoming horizontal, and finally falling. This upward curve of the under side of the nappe, scarcely noticed until now, constitutes, nevertheless, one of the fundamental data of the phenomenon, and M. Boussinesq<sup>2</sup> has made it the basis of a new theory of the flow over weirs."

So far as is known, the first attempt made in American literature to develop the shape of an overfall dam to fit the overflowing sheet was that of Muller<sup>3</sup> in 1908. He attempted to extend a curve from the upper section of the lower nappe through Bazin's data. His expression for the curve in the terminology of this treatise was

$$x^2 - 2.3h_g y = 0 \quad (\text{figure 1A})$$

for the thread of mean velocity with the origin of coordinates at approximately 0.35h<sub>g</sub> above and 0.09h<sub>g</sub> downstream from the theoretical weir crest. He measured downward one-third the thickness of the nappe normal to the thread of mean velocity to locate the curve of the

lower surface. Parker<sup>4</sup> reproduced Muller's curve and demonstrated that it does not fit well with Bazin's curve at the upper section. Parker's comment was "The errors in details are plain. Bazin's curves refer to sharp-edged notches, under heads not exceeding 1.7 feet; and Muller applies them to thick notches, under heads of 5 or 10 feet. The principle is a good one, and the process leads to a nice curve."

Morrison and Brodie<sup>5</sup> offer a parabolic equation for the lower surface of the nappe of

$$x^2 = 1.80h_0y,$$

where  $h_0$  is the head measured from the highest point of the lower nappe surface (figure 1B). The origin of the coordinates is at the highest point of the lower nappe surface. As a factor of safety for dam design, they recommend that the equation be increased to

$$x^2 = 2.55h_0y.$$

The equation  $x^2 = 1.80h_0y$  was also used by the Miami Conservancy District<sup>6</sup> in the design of their spillways. It was used without any increase for factor of safety. It can be shown that actually the lower nappe surface can be only approximately represented by a parabola.

Woodward, in his Miami Conservancy District report, says, "The profiles of the ~~these~~ weirs were designed to conform approximately to the profile of the lower nappe of the overflow from a sharp-created weir as determined by Bazin's experiments. The profiles as designed agree approximately in their upper portions with the formula

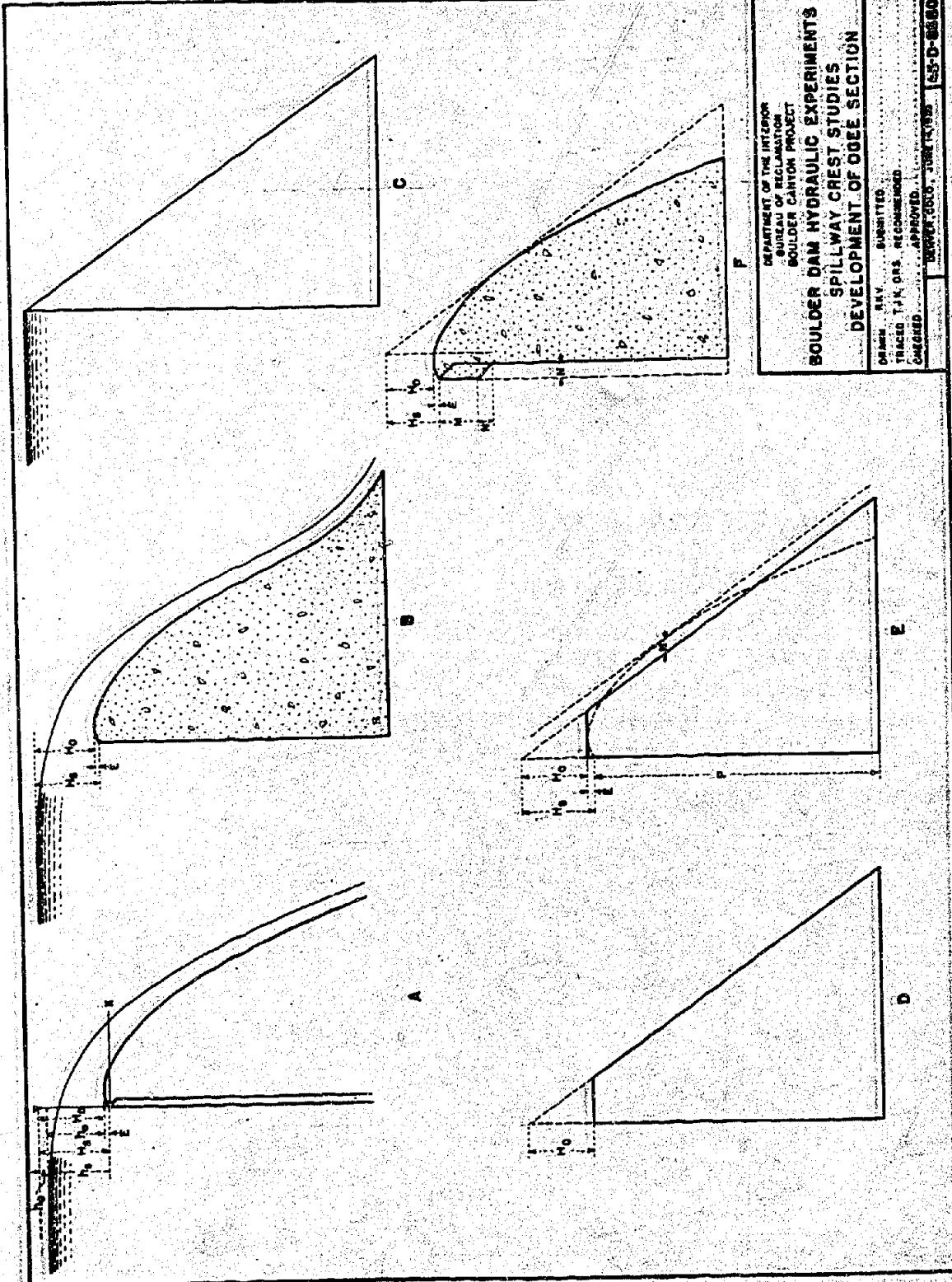


FIGURE 1 - DEVELOPMENT OF OVERFALL CREST SECTION

$x^2 = 4.8 hy$ , where  $x$  and  $y$  are horizontal and vertical coordinates measured from the crest of the weir and  $h$  is the maximum effective head on the weir including that due to velocity of approach. The discharge over the spillway weirs per foot of length was computed by the formula :  $q = 5.8h^{3/2}$ .

Creager<sup>7</sup> proposes an equation

$$x^2 = 2.732y$$

for the line of average velocity in the nappe with the origin of coordinates 0.063 units upstream from the face of the weir and 0.261 units above the highest point of the lower nappe surface. Scimemi<sup>8</sup> shows that the equation given by Creager locates a line which will fall below even the lower surface if it is continued far enough. This discrepancy, however, is less than the increase of cross-section recommended by Creager as a factor of safety.

Scimemi established an equation for a portion of the lower surface beyond  $x = 0.050$  which is

$$y = \frac{(x - 0.10)}{1.55} + 0.06x - 0.186$$

in which the origin of the coordinates is at the sharp crest of the weir. Unfortunately, this equation does not cover the most important portion of the crest, namely, the portion between the springing point and the high point of the trajectory.

An empirical equation for the lower surface of the nappe was derived by R. R. Randolph, Assistant Engineer, Panama Canal Zone, in which

$$y = 0.523 H^{-0.882} x^{1.822}$$

with the origin of the coordinates at the highest point of the lower surface. A similar equation with the same origin of coordinates derived by Assistant Engineer H. L. Davis, Bureau of Reclamation, is

$$y = 0.485 H^{-0.875} x^{1.875}$$

There evidently must be a rational equation for the shape of the nappe of an ideal fluid. Lamb<sup>9</sup> derived a set of parametric equations for the shape of the surface of a jet issuing from a sharp-edged orifice. With some modifications and the inclusion of the effect of gravity, equations might be derived which would fit the observed data. The development of such equations offers a fine opportunity for research to some engineer or mathematician.

#### Need for Additional Experiments

In the earlier days of dam design, the practice in the design of the so-called "ogee" crest of a spillway was to use a vertical upstream face. The dam cross section was designed to fit exactly or to be slightly larger than the space beneath the lower nappe of a sharp-crested weir with a flow equal to the maximum discharge. The shape was, in most instances, computed in accordance with Bazin's classic experiments, which covered a considerable range of nappe shapes.

Recently large dams having other than plain vertical upstream faces have been built for which the data of Bazin is not applicable. In one form, the crest overhangs the upstream face of the dam. (figure 1F). This type has been evolved as a result of its saving of material. The theoretical form of stable non-overflow dam is the triangle with the water surface at the apex as shown in figure 1C. For

an over-flow dam, the shape would be approximately the trapezoid formed by removing the apex from the upper part of the triangle to a distance equal to the design head,  $h_o$ , on the crest as shown in figure 1D. With an overflow having a depth of  $h_o$ , however, the downstream side of the trapezoid does not have sufficient width and the nappe would jump clear of the dam, as shown in figure 1E, falling on a curve extending a maximum distance  $N$  outside the trapezoid. To insure freedom from undesirable vacuum effects this space between the nappe and the dam should be filled with concrete. This concrete is not located in the most efficient position to resist overturning of the dam. By moving the point corresponding to the weir crest upstream a distance  $N$ , the nappe can be brought tangent to the downstream face of the trapezoid. This results in a horizontal offset in the upper portion of the spillway section equal to the distance  $N$ . The upstream edge is commonly connected to the upstream face of the dam either by a single inclined surface or by a short section of vertical face below which is an inclined face. The first form, outlined by the heavy broken line on figure 1F, was used on the Conowingo Dam on the Susquehanna River and the Rock Canyon Barrier on the Arkansas River. The second form, the solid lines on figure 1F, was used on the Wilson Dam in the Tennessee River, the Safe Harbor Dam on the Susquehanna River and the Bull Run Dam on the Bull Run River in Oregon. In this form, the concrete required under the nappe is placed where it is most effective to resist overturning and therefore results in a more economical dam. The projecting upstream portion of the dam usually alters the shape of the nappe, and causes it to no longer follow exactly the form which would result from a weir with a vertical up-

stream face.

It was to determine the nappe shapes and the coefficients of discharge for such overhanging crests that an extensive series of experiments were instigated by the Bureau of Reclamation.

#### Extent of Experimental Program

In the design of various preliminary forms of spillways considered for Boulder Dam, many questions arose in connection with overflow crest shapes, concerning which data was insufficient to permit exact solutions. During the course of the design work, experimental data was collected which proved of value in the preparation of the later designs of the side-channel spillway, and this should be helpful in preparing future designs, not only of side-channel spillways, but of other forms employing overfall sections.

The aim in the original program was to correlate all available information on the subject and treat it from the practical standpoint of the designer. Admittedly, the program has fallen far short of the original intention, but such circumstances were unavoidable due to constant interruption by more urgent work. It is planned at this time to complete studies of the more common shapes as opportunity permits and finally publish the entire results as a portion of the final reports on Boulder Dam.

The principal phases of spillway design considered in the experiments by the Bureau of Reclamation were as follows:

1. Determine the shape of crest required to best fit the lower nappe of the overfalling stream using sharp-crested weirs

representing dam sections with vertical upstream faces and also with overhanging and offset forms.

2. Study the deviations of the nappe shape due to a velocity of approach. Bazin covered a portion of this field but his experiments were not extensive enough to completely analyze the effect.

3. Determine the coefficients of discharge for dams with vertical and offset upstream faces.

4. Analyze the vacuum effects which may occur on the downstream face of overflow dams.

5. Ascertain the reduction of pressure which occurs on the upstream face of a dam due to the increase of velocity over the crest. This effect, while slight, decreases the overturning moment of the dam and is conducive to a small reduction in suction.

6. Determine the discharge coefficients for models of different shaped crests with and without control gates. These studies were to include the effect of adjacent terrain, piers, and position of drum gates.

7. In the case of supplementary tests on models or actual designs, the pressures on the crest and on the drum gates were to be observed to provide information for the use of the design department in computing stresses on the structural members.

#### History of Experimental Program

The laboratory tests on this program were initiated in the Fort Collins hydraulic laboratory in 1932 as a means of providing urgently

needed material for the completion of the spillway designs for Boulder Dam. Because of the routine nature of the studies, they were subject to considerable interruption by other more urgent work. These demands prevented the complete analysis of the measurements at the time they were made. In the following years, as time and personnel were available, the material was analyzed completely to prepare it for publication. A thorough study led to the belief that parts were inconsistent and not worth publishing. Accordingly, in 1936 authority was sought and received to repeat certain of the test programs to provide a complete record. This new program was conducted in the Bureau of Reclamation hydraulic structures laboratory in the basement of the Customhouse, in Denver, Colorado. Cognizance was taken of the shortcomings of the original tests to avoid a repetition of the failures.

The principal difficulties in the original studies were lack of experience in planning a study of this type; lack of definite program; and the deficiency of time and personnel with which to analyze the material coincident with the actual measurements.

In full appreciation of these troubles, the repetition of those tests was preceded by a careful plan of procedure. As each measurement was completed, an analysis was made before proceeding to the next step. In that way, any inconsistent or irrelevant material was eliminated immediately. If the results of a particular test were not consistent, the test was repeated.

#### Scope of Thesis

In this treatise, the discussion has been limited to the studies

made on the weir with the vertical upstream face. The shape of the under nappe; the profile of the upper water surface; the deviations of the nappe due to the velocity of approach; the coefficients of discharge; and the reduction of pressure on the upstream face of the dam have all been considered.

The treatment of spillway crests with other upstream shapes would be a repetition of procedure and analysis, hence, is considered beyond the scope of this thesis.

The tests on models of actual designs to determine the authenticity of this method of approach have been treated in other papers.

A recent article by the writer<sup>10</sup> deals with the solution of a specific problem in the design of the spillway crest for Grand Coulee Dam by the measurement of the under nappe of flow over a sharp-crested weir.

## II - LABORATORY FACILITIES AND INSTRUMENTATION

Colorado State Agricultural Experiment Station Hydraulic Laboratory

The original experiments made by the Bureau of Reclamation were performed in the hydraulic laboratory of the Colorado State Agricultural Experiment Station, Fort Collins, Colorado. The weir apparatus was installed in the weir box within the laboratory building. The experiments were conducted on weirs two feet in length, this being the greatest length over which a considerable head could be maintained with the water supply available. The approach channel to the weir and the channel downstream from the weir were also two feet wide, so that the contractions at the weir were suppressed. The interior of the channel was faced with 1-inch finished lumber. It was approximately 16 feet long, the sides extending from the bottom of the concrete tank upward to a height of 6 feet 11 inches, in order that the water might enter the 2-foot channel with the least possible disturbance. The entrance was flared with a short rounded section on each side ending tangent to the channel sidewalls. The channel was fitted with a movable floor adjustable to different positions in order to vary the velocity of approach to the weir. This floor was held in place at each end by two cables, which extended from the movable floor through pulleys fastened to the bottom of the concrete tank and thence through holes in the approach channel walls to anchors near the top of the tank. The buoyancy of the floor combined with the net upward pressure due to difference in velocity above and below held the floor at the ends of the cables. At the upstream end of this movable floor, a movable inclined ramp was inserted to extend from the floor to the

bottom of the concrete tank. This ramp combined with flaring sidewalls made a converging entrance to the weir channel for all positions of the floor.

The test weir was supported by a weir plate near the downstream end of the 2-foot channel. The test weir (figure 2) was held in place atop the weir plate by a splice plate and countersunk bolts to provide a smooth face in the approach channel. The test weir was accurately made of stainless steel to avoid corrosion. Throughout the test program care was exercised to maintain a smooth upstream face and a sharp edge on the weir.

In the original studies, the top of the weir was made with a width of 1/16-inch and a slope of 45 degrees on the lower edge. A study by Rouse and Reid<sup>11</sup> at Massachusetts Institute of Technology showed that "—, surface tension causes the nappe to spring loose a short distance below the crest rather than at the sharp upstream edge, due principally to the small width of the crest, which is never made absolutely sharp." To overcome this difficulty in the tests in the Denver laboratory, the weir crest was ground to a knife edge with a vertical upstream face to eliminate the viscous effects.

The sidewalls of the 2-foot channel were continued downstream to prevent the spreading of the nappe. A hole in one wall downstream from the weir and below the crest permitted access to the lower side of the nappe for operation of the lower nappe coordinometer.

#### Water Supply and Controls

The water supply for the Fort Collins experiments was obtained

from a storage reservoir located above the laboratory. The outflow from the reservoir into the laboratory was controlled by manually-operated circular slide gates. Major adjustments of flow were made with these gates. Minor adjustments over the test weir were made by manipulation of an adjustable waste weir and valve. The waste weir was hinged at the base and was adjusted in height by rotation of a hand-wheel near the head gage. The valve could also be adjusted from the gaging station through a bevel gear and rod assembly. Large adjustments were made with the movable waste weir and fine adjustments by the waste valve.

Two float gages and one hook gage were installed in wells connected through 3/4-inch hose to the channel 10 feet upstream from the weir. The float gages were the type developed at Cornell University.<sup>12</sup> Heads were read directly to thousandths of a foot on a vernier. Two of the gages were connected to openings in the movable channel floor and the third was connected to an opening in the channel wall 9 inches below the weir crest. When the floor was placed above this opening, the third gage was not used.

#### Bureau of Reclamation Hydraulic Laboratory in Denver, Colorado

In repeating the nappe measurements in the Denver laboratory practically the same arrangement of apparatus was employed as in the original experiments. The two-foot channel was duplicated except that the approach channel was the same width as the test channel thereby eliminating the necessity for the transition walls and floor. Suitable baffles at the upstream end of the approach channel dissipated the energy from the supply pipe and distributed the flow uniformly in the

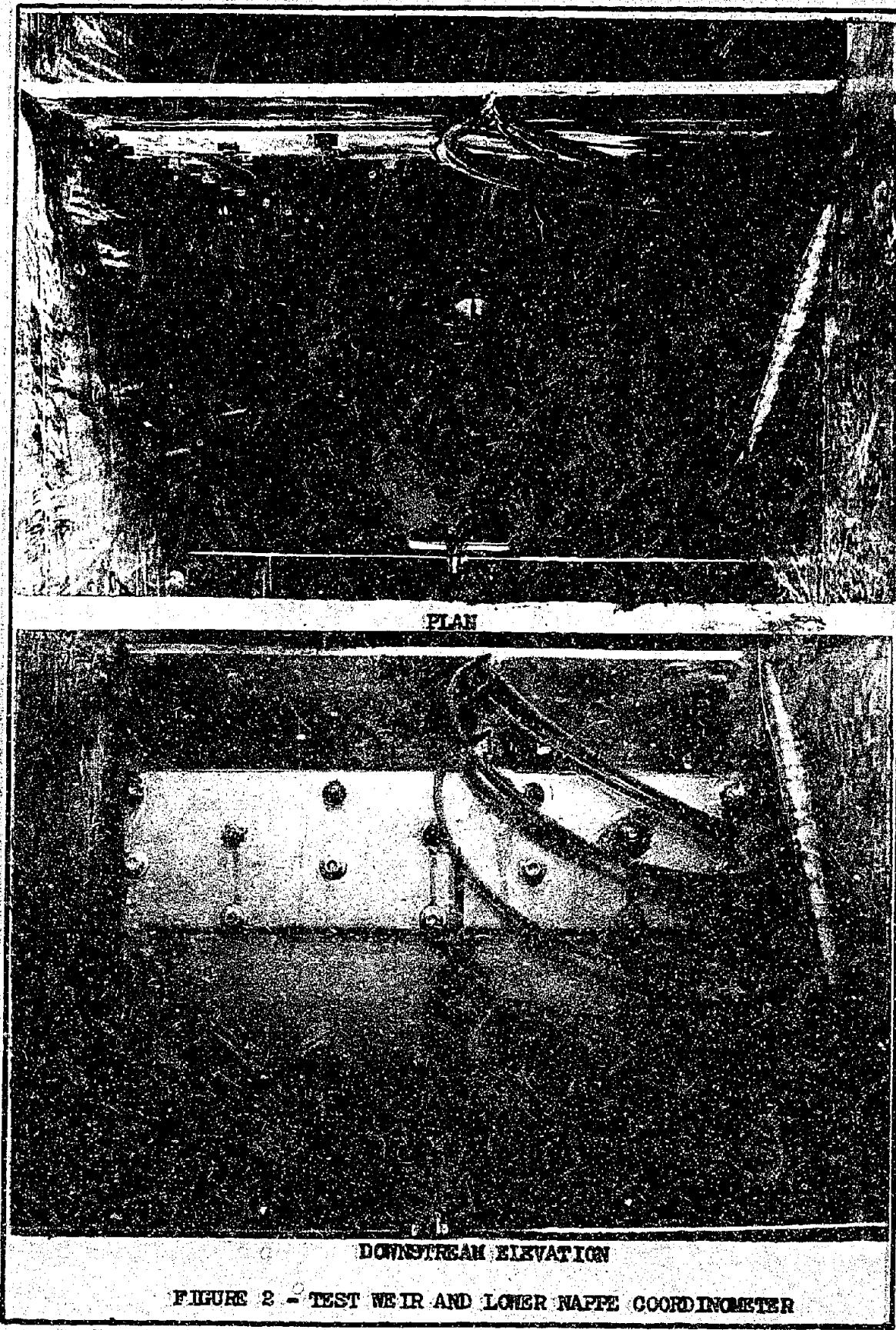


FIGURE 2 - TEST WEIR AND LOWER NAPPE COORDINOMETER

channel. The water was supplied through a 12-inch pipe directly connected to a 12-inch centrifugal pump and a particular flow was adjusted by observation of the differential head on a 12-inch Venturi meter in the supply line. Accurate measurements of the flow were made by observing the head on a 90-degree V-notch weir.

The coordinometer used in the original tests to measure the profile of the upper and lower nappes was installed in the Denver laboratory to measure the entire upper nappe and the lower portion of the lower nappe. In the re-runs, an additional coordinometer designed especially to measure the critical portion of the lower nappe was installed on the downstream face of the weir.

#### The Coordinometers

The profile of the nappe was measured by specially constructed instruments known as coordinometers. The instrument by which the surfaces were measured in the Fort Collins tests is shown in detail in figures 3, 5 and 6. It consisted essentially of a 4-inch horizontal beam mounted over the weir channel parallel to the direction of flow, upon which was mounted an adjustable vertical bar having at its lower end a point to contact the water surface. The coordinates of the position of the point were read on the horizontal and vertical scales. The instrument was supported on two I-beams spanning the test channel and normal to the direction of the flow. The beams were made sufficiently heavy to support a live load of several hundred pounds without a measurable deflection. The carriage supporting the vertical beam was connected through a horizontal slow-motion screw to a clamp on the I-beam by which fine adjustments of the horizontal position could

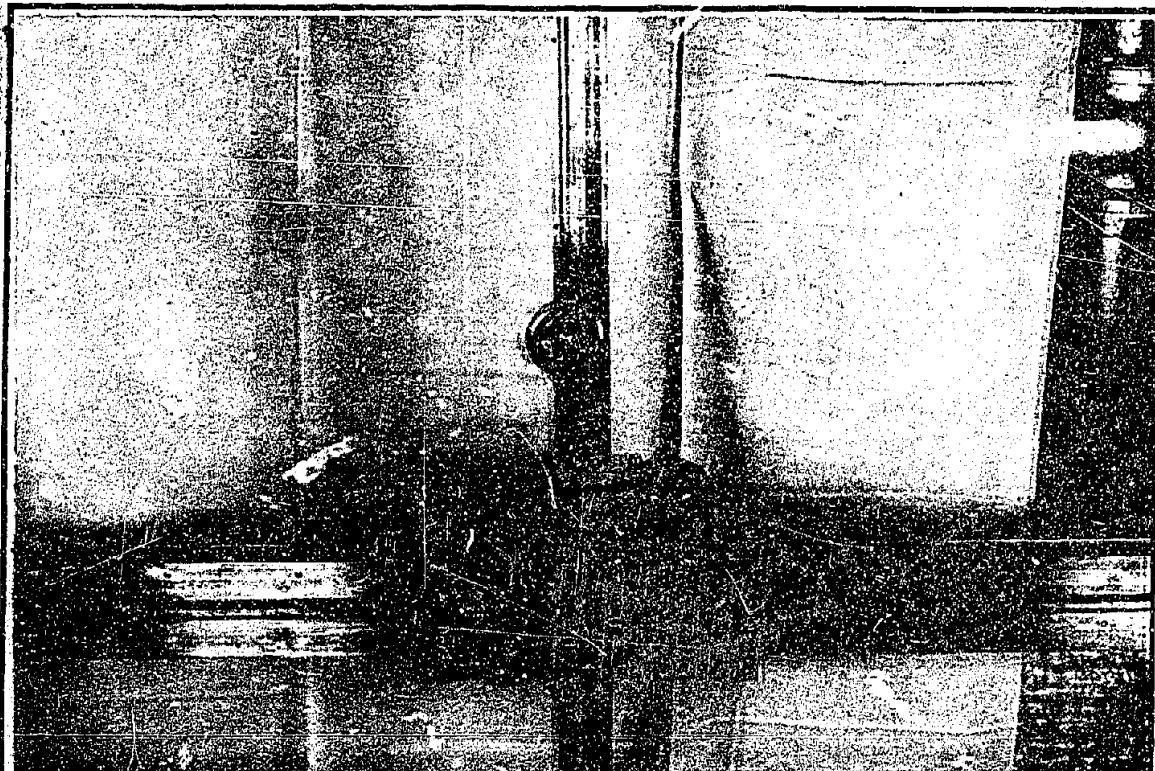


FIGURE 3 - UPPER NAPPE COORDINOMETER

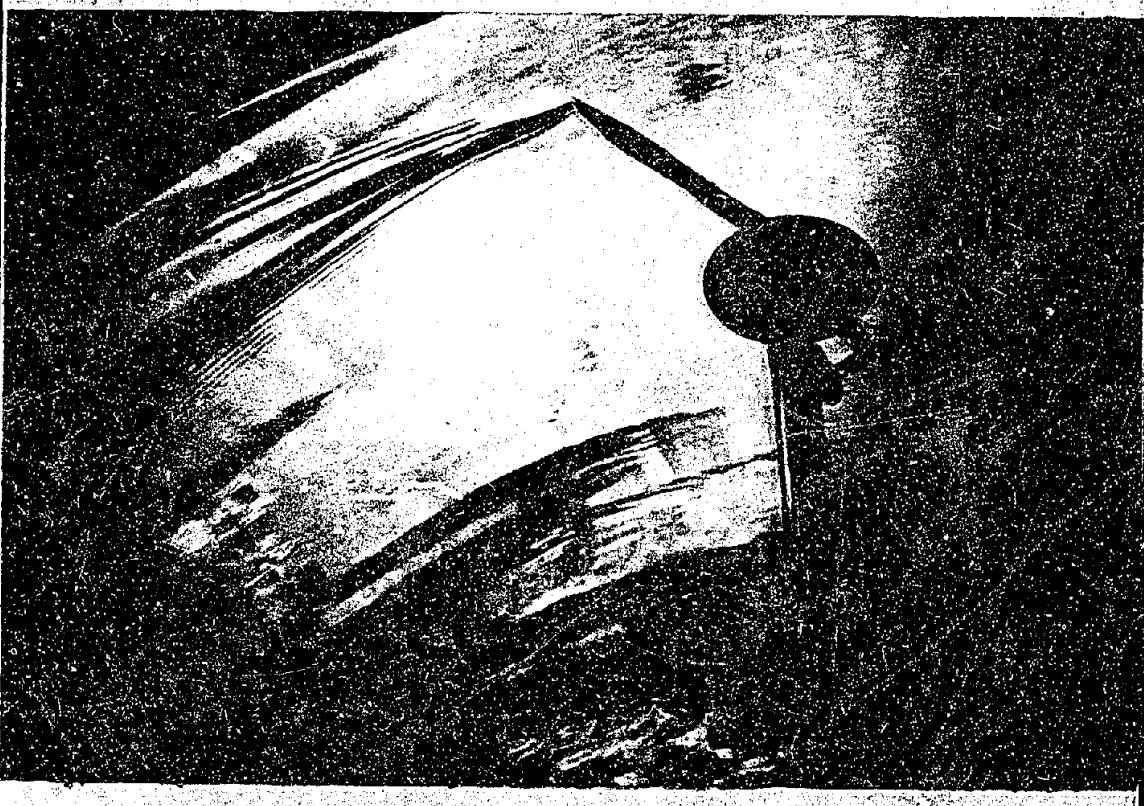


FIGURE 4 - LOWER NAPPE COORDINOMETER IN USE

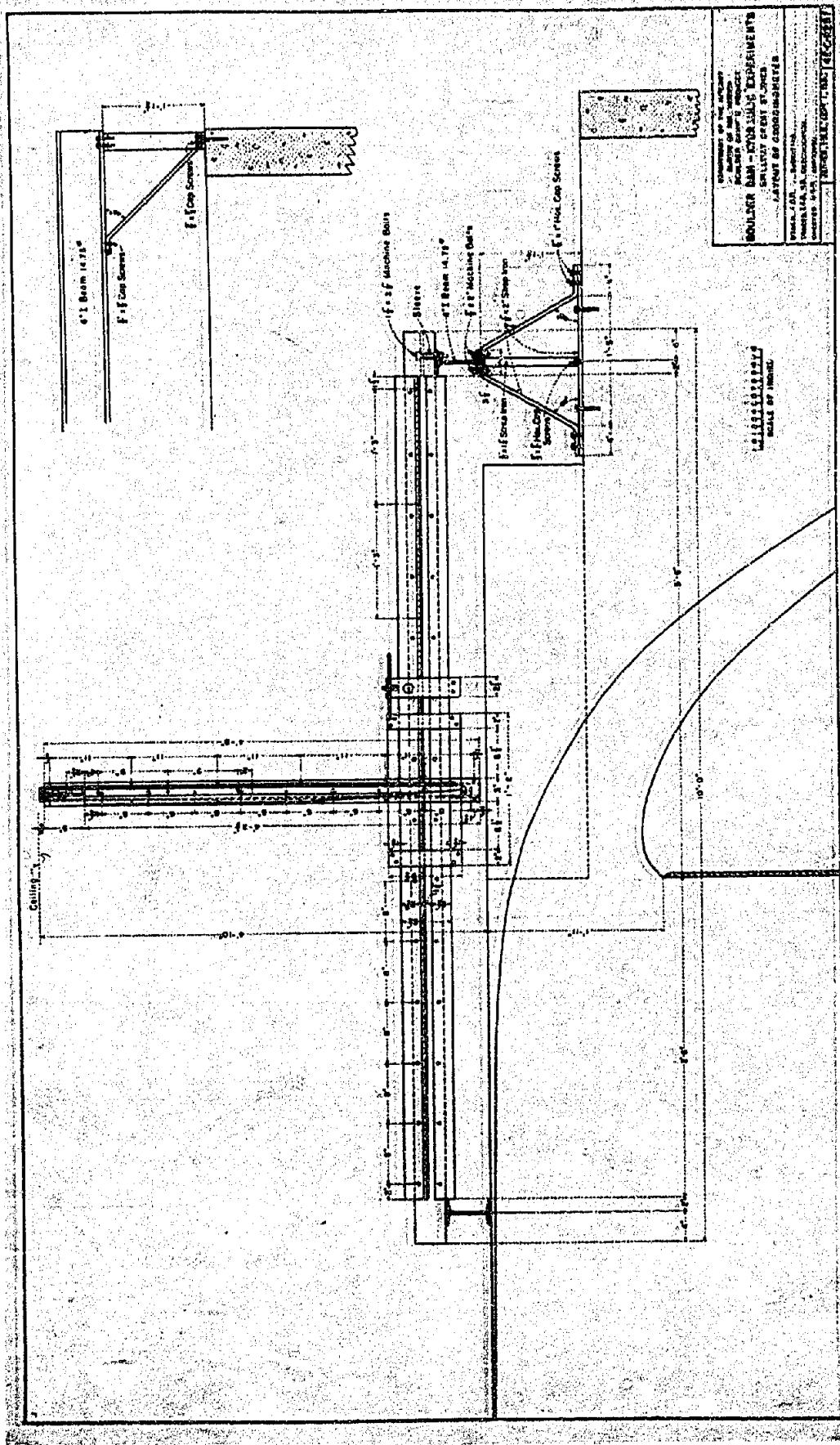


FIGURE 5 - ASSEMBLY OF UPPER NAPF COORDINATE METER



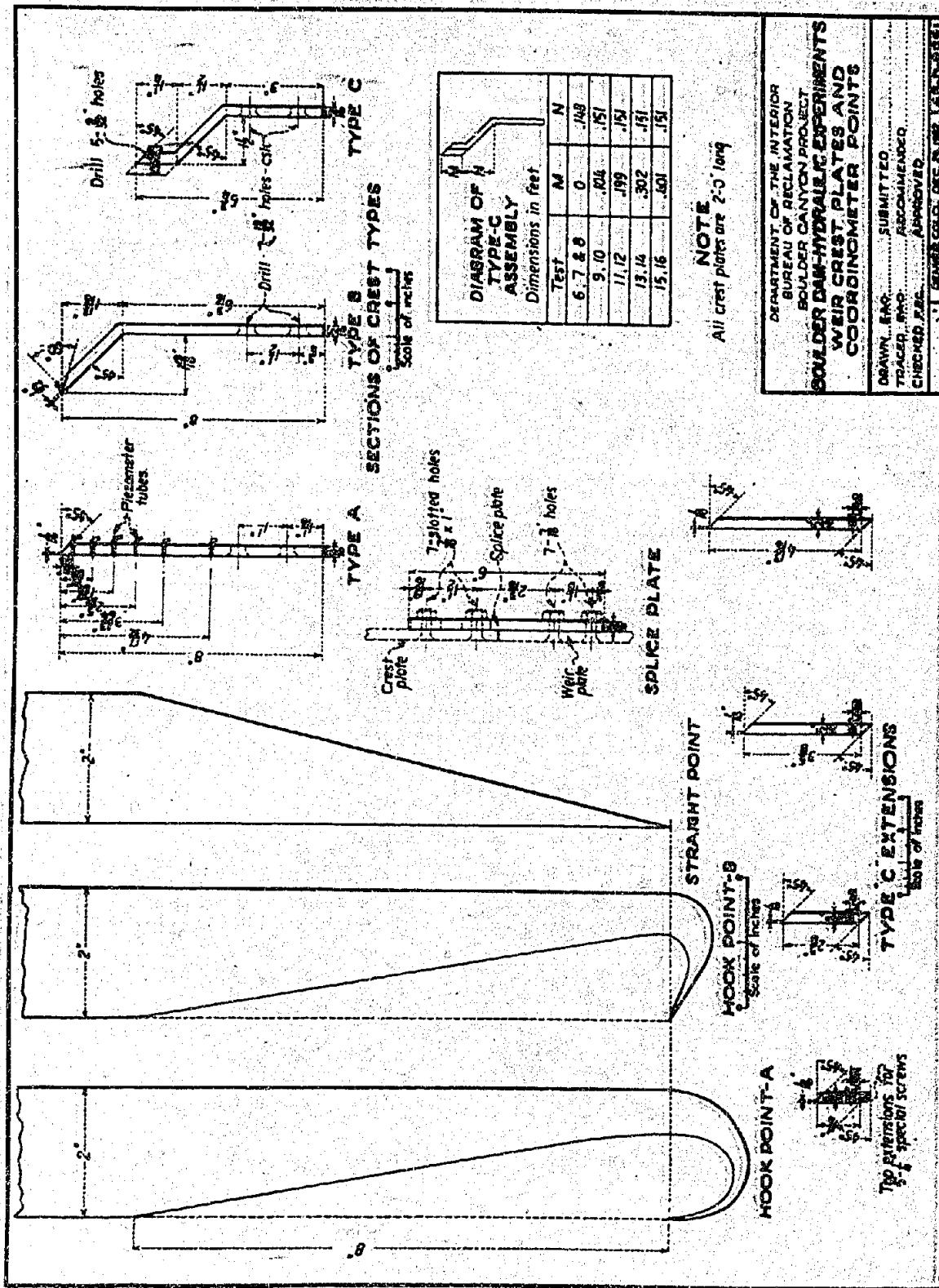


FIGURE 7 - WEIR CREST PLATES AND COORDINOMETER POINTS

be made. A vernier was provided to read to thousandths of a foot. The vertical bar moved in a groove in the vertical beam and was fitted with a clamp, slow-motion screw and vernier.

The lower part of the vertical bar was constructed to accommodate hooks of different styles. The point was used to survey the upper nappe and the hooks to point-gauge the lower surface. Two types of hooks were made as shown in figure 7, one with the point inclined upstream for measuring the section of the lower nappe upstream from the vena contracta and one with the point directed vertically upward for measuring the remainder of the lower surface. In use this coordinometer was set at a given horizontal position and the tip of the hook or point, as the case might be, was lowered until it touched the water surface. The vertical reading was made, the carriage moved to a new position and the process repeated until the entire nappe was traversed. In making the measurements of the lower surface, the instrument was inserted in the jet until the tip of the hook touched the lower surface. For the lower heads, this contact was clearly visible from above, but for the higher heads it was necessary to view the hook through the observation port in the sidewall.

The coordinometer shown in figures 3, 5 and 6 was used in the original tests to traverse both the upper and lower nappes. It was not entirely satisfactory, particularly in the section of the lower nappe between the sharp crest of the test weir and the vena contracta, which is the important portion of the crest. If that portion of the spillway does not fit the jet within a reasonable degree, quite high

negative pressures will be developed. This was proven in a series of studies on a model of the final cross section of the crest for the Boulder Dam side-channel spillways. A profile developed from the original nappe studies, when incorporated into a crest, fitted satisfactorily beyond the vena contracta but produced negative pressures between the spring line and the peak. Numerous trials failed to produce a completely satisfactory pressure distribution. There were two possible solutions: (1) as previously mentioned, Rouse and Reid found that the water would cling to the flat top of the test weir and produce erroneous results, and (2) the original coordinometer was not adapted to the exceedingly careful measurements needed in that region. The first difficulty was remedied by grinding the top of the test weir to a knife edge; the second, by constructing a new coordinometer to be fastened to the downstream face of the weir as shown in figure 2.

This lower coordinometer consisted of a standard laboratory point gage mounted on a carriage with the point up. The carriage was mounted on a horizontal bar with hand wheels to control the horizontal and vertical movements. Both the vertical and horizontal bars were graduated to hundredths of a foot with verniers to read the thousandths of a foot. Two points were provided for the vertical bar. One point was straight and constituted an extension of the bar in the same vertical line. The second point was bent to an angle of approximately forty-five degrees and could be placed in position sloping toward or away from the weir. In figures 2 and 4, the bent point is in position away from the weir.

A neon glow lamp was connected by a single insulated wire to a set screw on the point in use which was electrically insulated from its mounting. By connecting the neon glow lamp to the ungrounded side of a 110-volt, single-phase, electrical circuit, the lamp would glow as the coordinometer point was brought in contact with the under-side of the overflow sheet. This glow lamp was especially useful in traversing the lower nappe when the flow was fluctuating considerably as was the case when the quantity of flow was large and when the velocity of approach was great. Since water would drivel down the point as it came in contact with the water surface, and establish an electrical short-circuit around the insulated mounting, a drip cup was placed around the point just above the base. The water discharging from this cup, if sufficiently steady, would cause a short-circuit to the hand or arm of the operator, so a length of rubber hose on the outlet from the cup rendered the instrument harmless.

The traverse of the upper nappe was performed in one step with the straight point (figure 7), beginning at a point on the water surface sufficiently far upstream to be beyond the effect of the drawdown curve over the weir and extending down to the steep portion of the trajectory to the limit of the instrument.

The under nappe was traversed in four steps. Using the bent point in the lower coordinometer with the slope upstream, the lower side of the jet was carefully surveyed from the crest of the weir to a random point near the vena contracta. The bent point was replaced by the straight point, the instrument shifted horizontally toward the

weir so as to obtain one or more observations in the region already covered by the bent point. The observations were continued to the limit of the scale on the horizontal bar. The straight point was then replaced by the bent point but this time with the slope downstream and the observations continued to the limit of either the horizontal or vertical scale depending on the trajectory of a particular test. The survey was completed utilizing the upper coordinometer equipped with the hook point A shown on figure 7.

The observations were recorded on suitable form sheets. Seven observers were required to make a complete set of observations; two to operate and record the lower coordinometer readings; two to operate and record the upper coordinometer readings; one to make a current meter traverse of the test channel flow for velocity computations; one to read and record the head on the weir and the pressures on the upstream face of the weir; and one to observe the head on the V-notch weir for the flow measurement.

#### Relation between Gages and Weir Crest

It was not practicable to make the zeroes of the scales on the coordinometers, head gages and pressure piezometers coincide with the crest of the test weir. Prior to the first test, and at frequent intervals throughout the program, the elevation and horizontal position of the test weir were accurately determined with reference to each of the coordinometer points. The elevation of the weir with reference to the channel head gage and the manometer tubes connected to the piezometers opening on the upstream face of the weir was also determined at

frequent intervals.

#### Regulation and Measurement of Discharge

The water for the Denver experiments, as previously mentioned, was supplied by a 12-inch centrifugal pump. At the beginning of a test, the flow was adjusted by observation of the differential gage connected to a 12-inch Venturi meter in the supply line to the test channel. The water flowed over the test weir into the storage channels under the laboratory floor. From the storage channel, the water flowed over a V-notch weir into the pump sump. The V-notch weir had been previously calibrated volumetrically and was used for the accurate measurement of the flow.

The approximate flow for a test was established by the use of the Venturi meter and the whole circulation system in the laboratory allowed to stabilize. By small adjustments of a valve in the supply line, the head on the weir was brought to the desired value. The stability of the flow through the entire system was determined by observation of the steadiness of the water surface in the test channel head gage and the V-notch weir head gage. When the flow had been maintained stable for fifteen or twenty minutes, the traverses of the upper and lower nappes were begun. The gages on the test flume and V-notch weir were recorded at one-minute intervals throughout the test.

### III - ANALYSIS OF EXPERIMENTS

#### Compilation of Test Results

Since the principal purpose of these experiments was to determine the effect of the approach velocity on the profile of the lower nappe, the depth of approach was varied by placing the movable floor at a different elevation relative to the weir crest for each series of tests. During a series of tests, the discharge over the weir was varied to produce a range of relations between the head due to velocity and the total head on the crest.

The terminology used in compiling the results is shown on figure 8. A summary of the test results is shown in table 1, with the exception of the original coordinates of the upper and lower nappe which are tabulated in table 2. Nappe traverses were not made on the entire series, as they were not needed to complete the analysis.

Those tests in which only weir calibration measurements were made are indicated in column 20 of table 1 without asterisks; those in which both calibration and nappe measurements were made are designated by asterisks.

By varying the depth of approach below the sharp crest of the test weir,  $P$ , from one series of tests to the next and by varying the discharge quantity,  $Q$ , within a series of tests, a range of velocity of approach,  $V$ , and hence, head due to velocity of approach,  $h_{av}$  was obtained. By varying the discharge through a series of tests, a range of observed heads on the crest,  $h_s$ , was obtained.

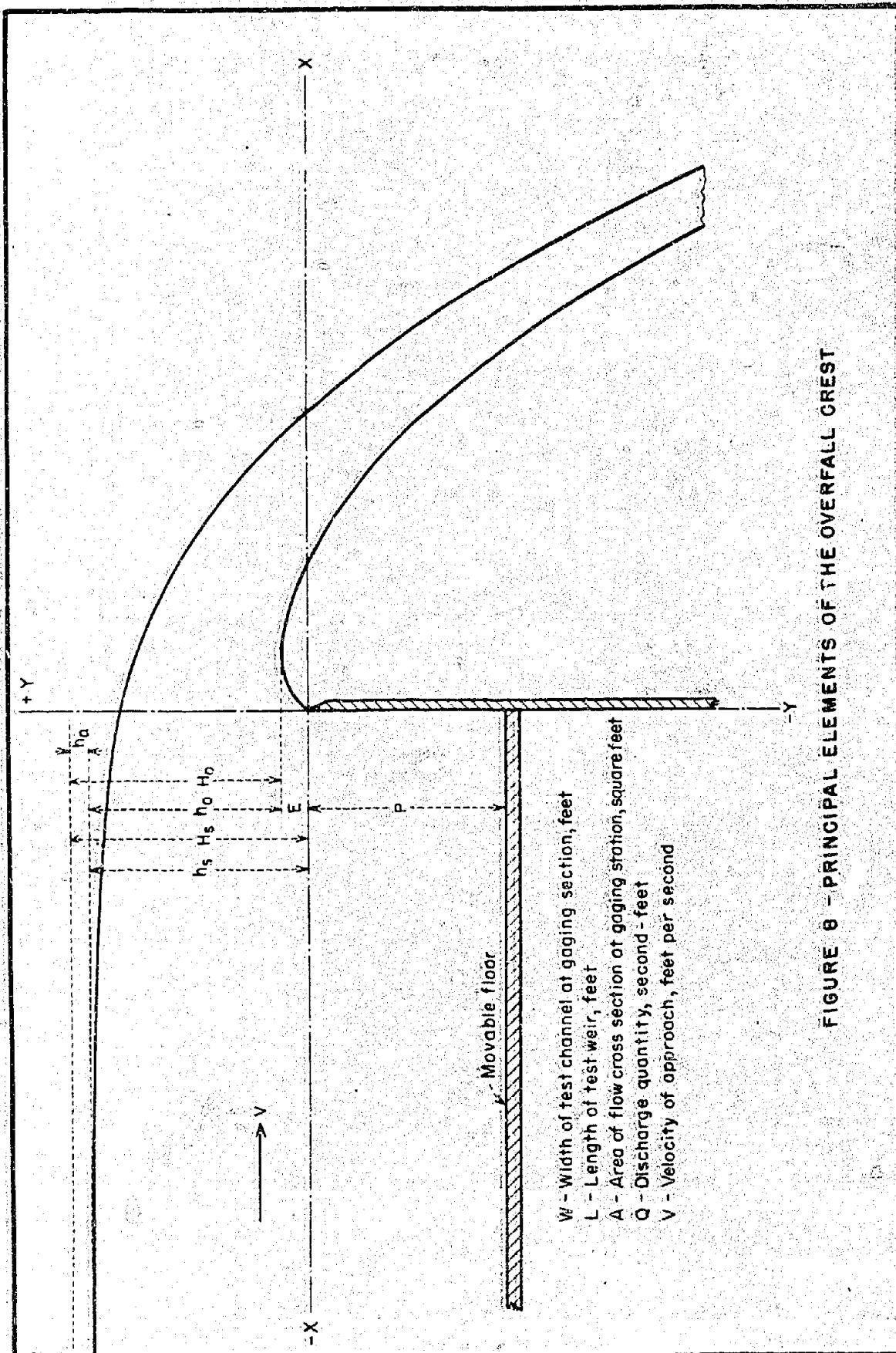


FIGURE 8 - PRINCIPAL ELEMENTS OF THE OVERFALL CREST



The depth of approach,  $P$ , in column 2 (table 1); the discharge quantity,  $Q$ , in column 3; the width of the channel at the gaging station,  $W$ , in column 4; the actual observed head on the weir,  $h_s$ , as measured at the gaging station, in column 5; and the length of the test weir,  $L$ , in column 12, have been explained or are obvious.

In column 6, the area of flow cross section or area of approach was obtained by

$$A = W(P+h_s).$$

The velocity of approach in column 7 was determined by

$$V = Q/A.$$

The velocity head,  $h_a$ , in column 8, was computed by the velocity head formula

$$\frac{h_a}{a} = \frac{v^2}{2g}.$$

The rise of the lower nappe,  $E$ , above the sharp crest of the weir as shown in column 9 was taken from the coordinometer traverse of the lower nappe. The total head above the crest of the lower nappe,  $H_o$ , in column 10, was computed by the equation

$$H_o = (h_s + h_a) - E.$$

Columns 11 and 13 are steps in the computation of the coefficient of discharge,  $C$ , for the overfall crest in column 14. The coefficient of discharge was computed by the formula

$$C = Q/IH_o^{3/2}.$$

In like manner, column 15 is a step in the computation of the ratio  $H_c/P+E$  in column 16, which is the ratio of the total head on the overfall crest to the approach depth below the crest. The total head

on the sharp-crested weir,  $H_s$ , in column 17 was obtained by adding  $h_s$  and  $h_o$ . The ratios  $h_o/H_s$  in column 18 and  $E/H_s$  in column 19 are self-explanatory.

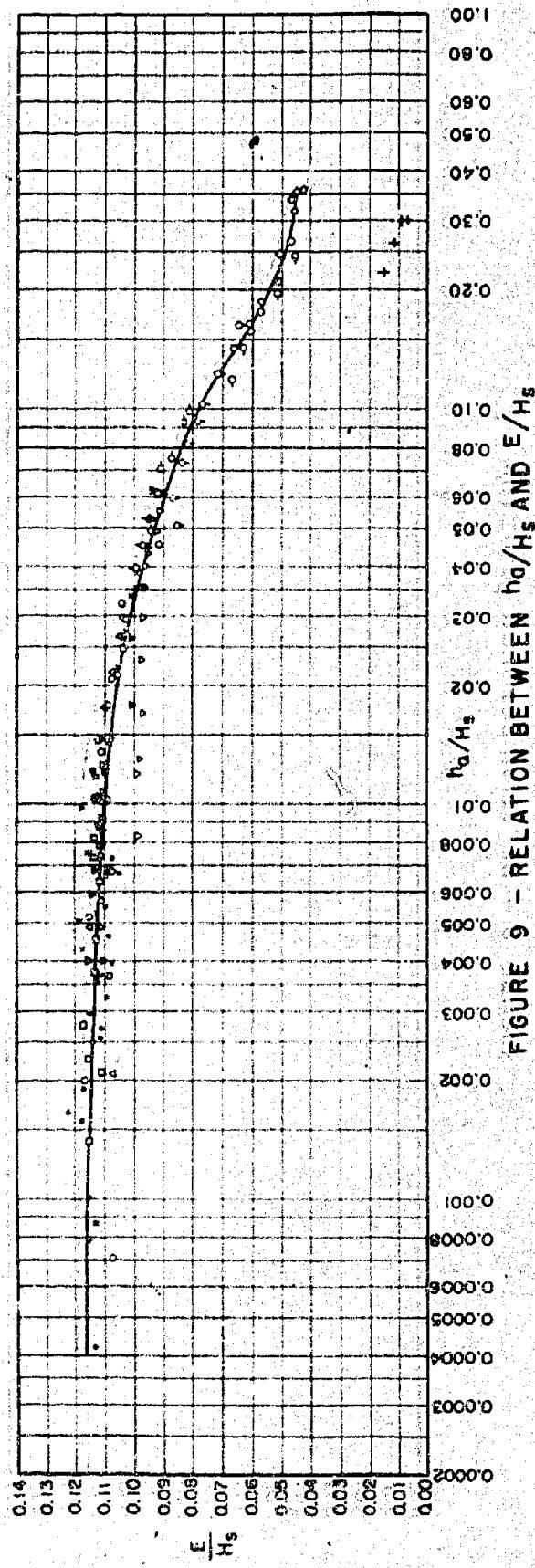
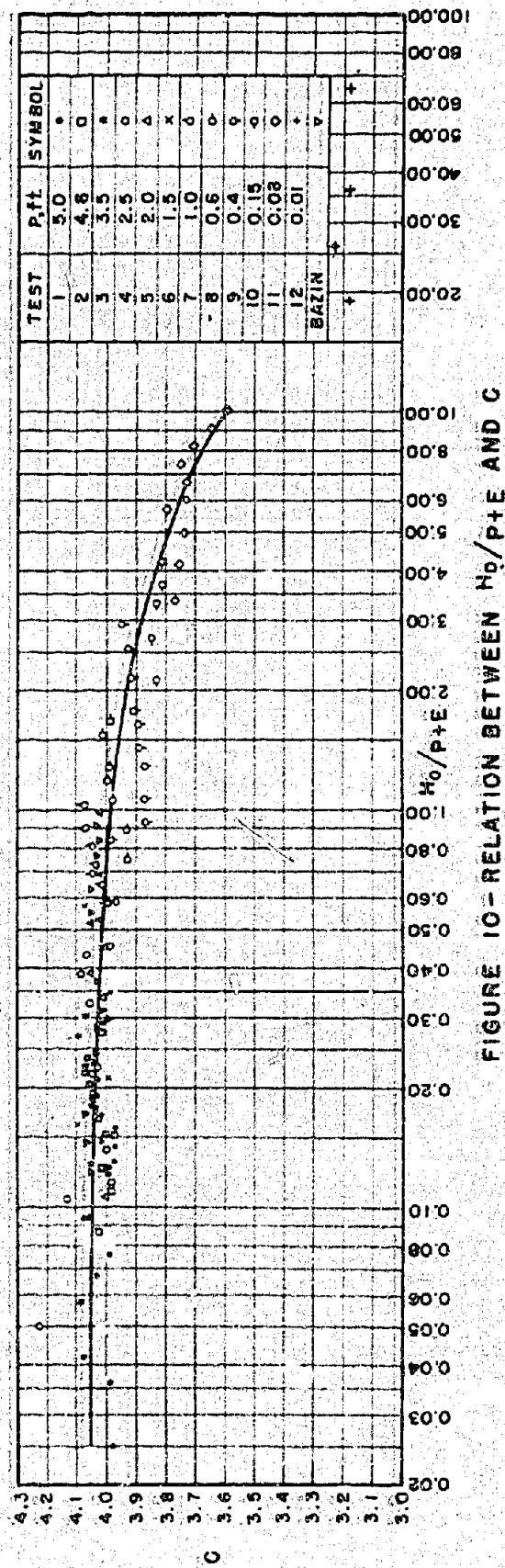
#### Relation between $h_o/H_s$ and $E/H_s$

A considerable number of dams are designed which have a height quite small in comparison with the depth of water flowing over them. The velocity of approach in these cases is appreciable and should be considered in the design. With this in mind, the computations in table 1 were so arranged as to include velocity of approach keeping the results in terms of dimensionless ratios.

These experiments were performed on a sharp-crested weir for which the head,  $h_s$ , was the basic measurement. In the design of a spillway crest, it is more convenient to start the design with the actual head on the top of the crest,  $h_o$ . It is therefore necessary to transfer the experimental results from the first to the second point. This has been done by evaluating the rise of the lower nappe above the sharp crest in terms of the total head on the sharp crest, which not only takes into account the velocity of approach, but also relates  $h_o$  to  $h_s$  or  $E_o$  to  $E_s$ . With the ratio  $h_o/H_s$  in column 18, table 1, as the abscissa and the ratio  $E/H_s$  in column 19, table 1, as the ordinate, the curve shown on figure 9 was plotted.

#### Relation of C to $H_o/P+E$

The curve in figure 10 was also plotted from table 1. The dimensionless ratio,  $H_o/P+E$ , in column 16, was used as the abscissa and the coefficient of discharge,  $C$ , in column 14, as the ordinate.

FIGURE 9 - RELATION BETWEEN  $H_0/H_s$  AND  $E/H_s$ FIGURE 10 - RELATION BETWEEN  $H_0/P+E$  AND  $C$

The nappe relationships in figures 9 and 10 compare favorably with those obtained by Bazin in his experiments in 1826. The results of certain of his experiments with a weir with a vertical upstream face have been included in table 1 and plotted on both figures 9 and 10. Bazin, in his experiments on the vertical weir, covered only the range of  $h_a/H_s$  between 0.001 and 0.080, which has proven to be too small for a number of spillways designed by the Bureau of Reclamation.

#### The Limits of $h_a/H_s$

The relationships shown in figures 9 and 10 have been carried to practical limits of applicability. As  $h_a/H_s$  and  $H_o/P+E$  approach zero as a limit, the values of  $E/H_s$  and  $C$  become constants for all practical purposes. In figure 9, the average value of  $E/H_s$  is 0.116 for negligible velocity of approach, and in figure 10,  $C$  becomes approximately 4.06 for the same condition. As the depth of approach is decreased and the velocity of approach increased, the values of  $E/H_s$  and  $C$  decrease and values of  $h_a/H_s$  and  $H_o/P+E$  are approached where the relations are no longer valid. This limit is the critical depth of flow in the test channel. King<sup>13</sup> defines critical depth as "(1) the depth at which, for a given energy content of the water in the channel, maximum discharge occurs; or (2) the depth at which in a given channel a given quantity of water flows with the minimum content of energy." In his discussion of critical depth, he says "It will be observed from both of the figures (figures 91 and 92, King's Handbook of Hydraulics, 3rd edition, page 371) that at or near critical depth a relatively large change in depth corresponds to a relatively small change in energy. Flow in this region is therefore rather unstable, as is usually indicated by character-

istic water-surface undulations."

This limit was very readily detected in the test weir flume and was manifested by the surface undulations described by King. As the test results were analyzed, this limit became apparent from the inconsistency of the results beyond a certain range of  $h_a/H_s$ . A limit of  $h_a/H_s$  of 0.220 in table 4 was therefore set beyond which it was not considered safe to evaluate the results for design purposes.

In figure 9, the curve reverses beyond  $h_a/H_s = 0.20$  and shows a tendency to become asymptotic. This was the condition in test 11, but in test 12 the relation changed completely. In test 11, the depth of approach,  $P+H_s$ , was approaching the critical depth, while in test 12 it had passed critical depth and was flowing below it. In figure 10, test 12 shows a behavior consistently different from preceding tests.

This limit is also shown in figure 11 where  $Q$ , the discharge over the sharp-crested weir, has been plotted as the abscissa against  $h_s$ , the actual observed head above the sharp crest of the weir, as the ordinates. The curves for tests 1 to 10, inclusive, plot consistently, but a portion of test 11 and all of test 12 are erratic. The first three points of test 11 are quite consistent but beyond that the curve breaks and shows little consistency. In plotting the nappe shapes for individual tests, the curves followed a regular pattern until test 11 was reached, after which the trajectory flattened very markedly and behaved differently from the previous tests. Tests 11-2, 11-3, 11-4, and 11-5 are shown in table 3 as having been used to establish

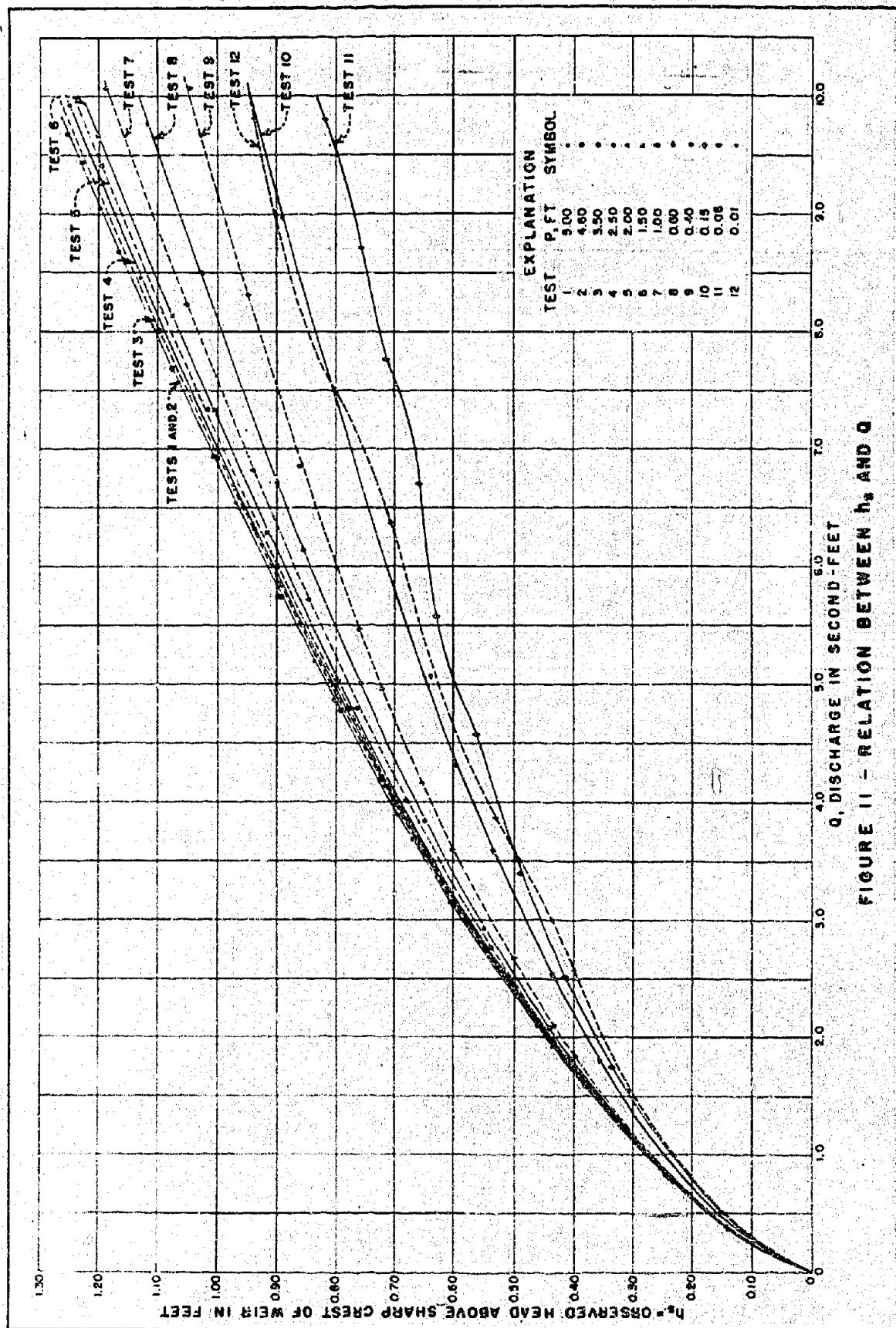


FIGURE 11 - RELATION BETWEEN  $h_s$  AND  $Q$

the final nappe shapes. Only test 11-5 with  $h_a/H_g = 0.2500$  exceeded the limit established. It was these various indications that led to the decision to establish  $h_a/H_g = 0.220$  as a conservative upper limit.

#### Reduction of Nappe Profiles to Unit Head

It has long been recognized that the nappe of water flowing freely over a sharp-crested weir has a similar shape for different heads. This was demonstrated by Bazin, who reduced all his experiments to the same bases by expressing the dimensions in terms of the head on the weir. When this is done the results are expressed in dimensionless units as in the cases of figures 9 and 10, equally applicable to any system of units.

The upper and lower nappe traverse measurements as observed on the goniometers were related to the rectangular coordinates  $x$  and  $y$  (figure 8) with their origin at the sharp crest of the weir by subtracting the reference values or zeroes. The results thus obtained and still considered to be original data are compiled in table 2 for both the upper and lower nappe. The tests so recorded are those noted in column 20 of table 1.

The coordinates of the lower nappe for each test shown in table 2 were plotted to a large scale and a smooth curve drawn through them. This step eliminated irregularities which naturally existed due to the fluctuation of the water surface. This fluctuation increased as the depth of approach was decreased. Coordinates were then read from the smooth curve for each test and divided by  $H_g$ , the total head on the experimental weir for that particular test, to obtain values of  $x/H_g$ .

















TABLE 3

## OVERFALL DAM CREST EXPERIMENTS

Corrected Test  
No.

## METHOD OF COMBINING LOWER MAPPE SHAPES

Test	P, ft.	Actual $h_2/H_s$	Plotted $h_2/H_s$
1-5 1-4	5.0	0.0010	
2-4 2-1	4.6	0.0014	
1-4 1-6	5.0	0.0019	
2-2 2-3	4.6	0.0022	
1-2 1-8	5.0	0.0027	0.0020
2-5 2-11	4.6	0.0032	
3-2 3-7	3.5	0.0103	
4-4 4-6	2.5	0.0105	
5-3 5-3	2.0	0.0105	
6-7 6-3	1.5	0.0118	0.0100
5-6 5-6	2.0	0.0214	
4-1 4-9	2.5	0.0207	
6-5 6-5	1.5	0.0220	
7-3 7-3	1.02	0.0212	0.0200
6-3 6-7	1.50	0.0311	
7-4 7-4	1.02	0.0295	
8-8 8-1	0.65	0.0296	0.0300
7-6 7-6	1.02	0.0397	
8-7 8-2	0.63	0.0395	0.0400
7-8 7-8	1.02	0.0323	
8-6 8-3	0.63	0.0491	
9-2 9-2	0.41	0.0509	0.0500
8-5 8-4	0.62	0.0606	
9-3 9-3	0.41	0.0594	0.0600
8-1 8-5	0.31	0.0704	
9-4 9-4	0.40	0.0732	0.0700
8-3 8-6	0.61	0.0754	
9-5 9-5	0.40	0.0817	0.0800
8-2 8-7	0.58	0.0928	
9-6 9-6	0.38	0.0956	0.0900
9-7 9-7	0.38	0.1014	0.1000
9-8 9-8	0.35	0.1213	
10-5 10-1	0.14	0.1193	0.1200
9-9 9-9	0.33	0.1417	
10-4 10-2	0.14	0.1422	0.1400
9-10 9-10	0.32	0.1626	
10-3 10-3	0.14	0.1641	
11-5 11-1	0.09	0.1591	0.1600
10-6 10-4	0.14	0.1774	
11-3 11-2	0.09	0.1858	0.1800
10-2 10-5	0.14	0.1970	
11-2 11-2	0.09	0.2091	0.2000
10-1 10-6	0.13	0.2441	
11-5 11-4	0.09	0.2436	0.2500

and  $y/H_s$ . With the relation  $h_a/H_s$  as a criterion the tests were combined in groups and plotted in terms of unit head on the weir crest. The range of  $h_a/H_s$  in the tests was from 0.001 to 0.30 as shown in table 1. The grouping used in combining the results is shown in table 3. For example, there were five tests which had an approximate value of  $h_a/H_s = 0.010$ . In like manner, groups of  $h_a/H_s$  approximately equal to 0.020, 0.030, 0.040, etc. were formed. From each group, a new curve was constructed for a given value of  $h_a/H_s$ . These new curves were then combined into the final grouping. Since all of this adjustment was of necessity performed on a scale so large as to prohibit inclusion of the drawing in this report in any form, three typical curves were plotted on figure 12 to show the effect of velocity of approach. After the adjustment of the entire set of tests had been completed on the large scale drawing so that the results were consistent, the values of  $x/H_s$  and  $y/H_s$  were read and tabulated in table 4, in a form for use in the design of a particular overfall crest with a vertical upstream face.

The observed coordinates of the upper nappe were referred to the origin at the sharp crest and are included in table 2. These coordinates were not reduced to unit head as they were not needed for the analysis of this particular problem. This information has been included for future analysis as the material may be needed.

#### Coordinates of Lower Nappe for Different Values of $h_a/H_s$

The values of  $x/H_s$  and  $y/H_s$  in table 4 are based on  $H_s$  equal to unity. In the solution of a specific problem, the head due to velocity of approach,  $h_a$ , and the total head above the springing line of the



overall crest,  $H_S$ , may be determined. With the ratio  $h_a/H_S$  thus obtained, the  $x/H_S$  coordinates may be read in column 1 of table 4 and the  $y/H_S$  coordinates in the column corresponding to the particular  $h_a/H_S$  being considered. For instance, if  $h_a/H_S = 0.090$ , the  $y/H_S$  coordinates are those listed in column 11 of table 4. By multiplying the values in columns 1 and 11 by  $H_S$ , the  $x$  and  $y$  coordinates may be computed. If the ratio  $h_a/H_S$  is one not shown in the table, as for instance,  $h_a/H_S = 0.083$ , interpolation for  $y/H_S$  coordinates between  $h_a/H_S = 0.080$  and  $h_a/H_S = 0.090$  will produce sufficiently accurate results.

#### Effect of Approach Velocity on Lower Nappe Shapes

The lower nappe shapes for three values of  $h_a/H_S$  with  $H_S$  equal to unity have been plotted in figure 12 to illustrate the effect of velocity of approach on the lower nappe shape. The data for these curves were taken directly from table 4. The  $x$  coordinates were taken from column 1. The  $y$  coordinates for  $h_a/H_S = 0.002$  from column 2; for  $h_a/H_S = 0.080$  from column 10; and for  $h_a/H_S = 0.22$  from column 18.

The curves were selected as illustrative of cases of negligible, moderate, and high velocities of approach and the difference in characteristics is obvious. The curve of  $h_a/H_S = 0.002$ , or negligible velocity of approach, gives a greater rise,  $E$ , and a steeper slope on the downstream side than the other cases. In fact, if continued a sufficient distance this curve crosses the other curves. In the case of higher velocities of approach, the path of the under side of the jet becomes flatter and a greater horizontal distance is required to reach any particular downstream slope.

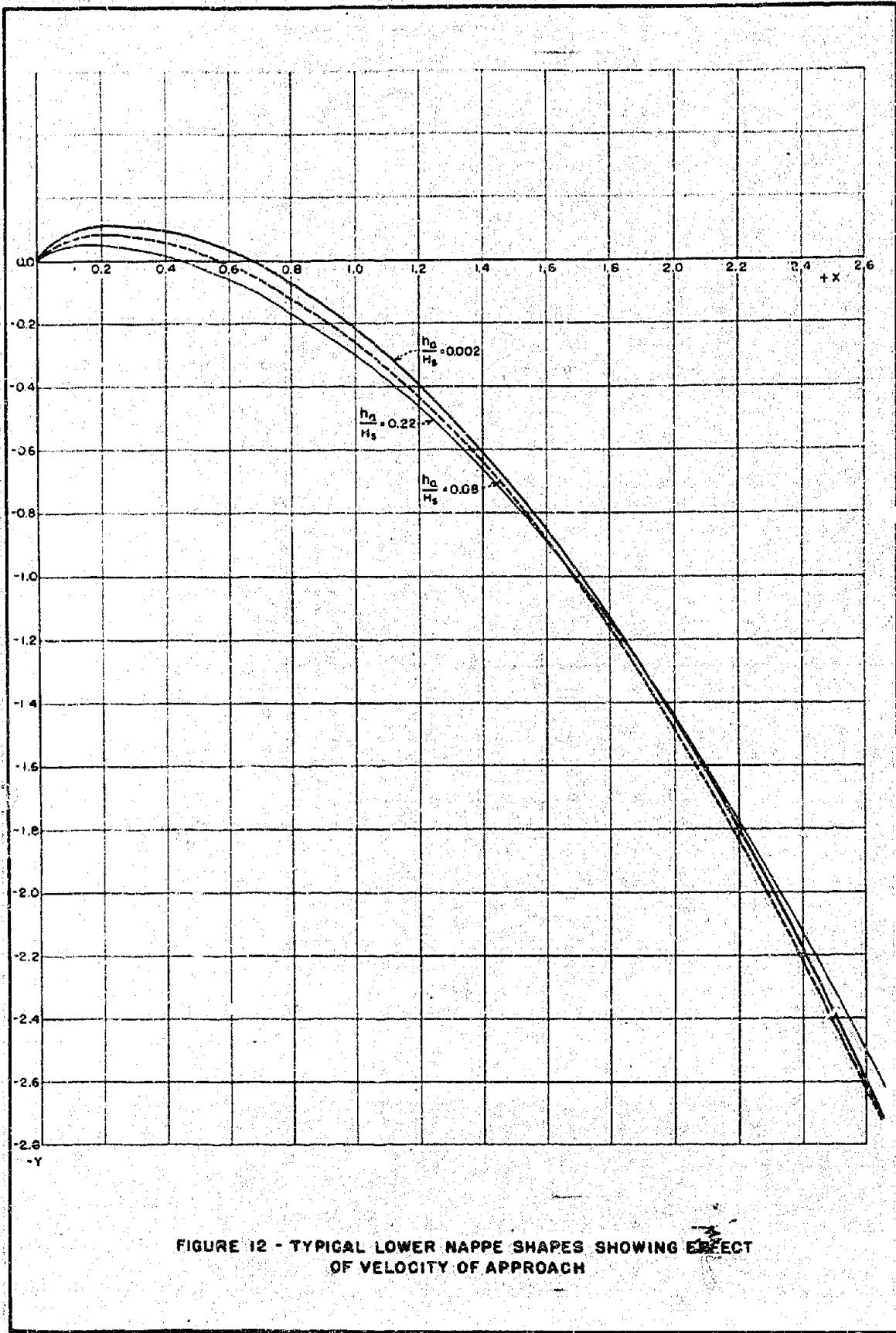


FIGURE 12 - TYPICAL LOWER NAPPE SHAPES SHOWING EFFECT  
OF VELOCITY OF APPROACH

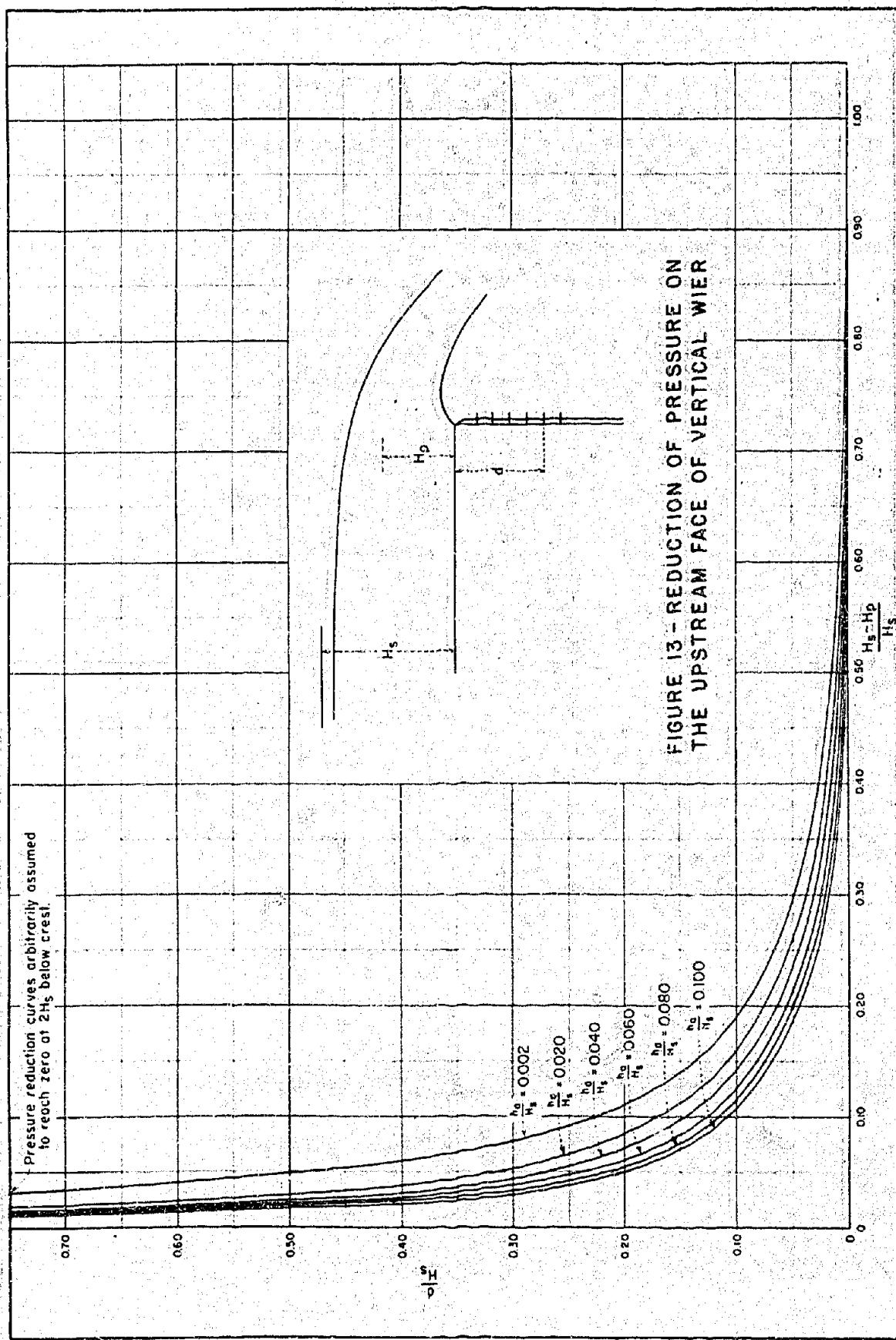
Reduction of Pressure on Upstream Face of Dam

As outlined in the introduction, one phase of this study was the determination of the reduction of pressure on the upstream face of the dam due to the conversion of static to kinetic energy as the water flows over the crest of the weir. Piezometers were installed in the upstream face of the weir and connected to manometer tubes as shown in figure 2.

The pressures at these points were recorded during each test and the results digested and shown on figure 13. The original material was voluminous and not considered of sufficient importance to include in its entirety in this treatise. The extent of the pressure reduction for different approach velocities is shown on figure 13 in terms of the total head on the weir,  $H_s$ .

The actual elevation of the water surface in the manometer tube related to the weir crest is denoted by the term,  $H_p$ , as shown in the diagram on figure 13. If there were no reduction of pressure and no velocity of approach, the elevation of the water surface in a manometer tube would be a distance  $H_s$  above the weir crest. If there were a velocity of approach but no reduction of pressure, the elevation in the manometer tube would be a distance  $H_s - h_a$  above the weir crest, or the water surface in the tube would be the same as in the reservoir or channel. To evaluate all of the variables, the ratio of the distance,  $d$ , below the sharp crest, to  $H_s$ , the total head, has been plotted as the ordinate, and the ratio of the reduction of pressure,  $H_s - H_p$ , to the total head,  $H_s$ , has been plotted as the abscissa.

With the velocity head and the total head known, the reduction



of pressure in a specific case can be determined from the curves in figure 13.

#### IV - APPLICATION OF RESULTS

##### Design of an Overfall Dam with Vertical Upstream Face

With the information now available, it is possible to design the crest of an overfall dam with a vertical upstream face and considerable velocity of approach. It will be so designed that at the maximum designed head on the spillway crest, the downstream face of the dam will fit the lower side of the overflowing sheet at maximum discharge with little or no negative pressure between the face of the spillway and the sheet of water.

Two examples will be considered, (1) a maximum head on the spillway crest, and (2) a maximum discharge per foot of spillway length. In the first example, the maximum reservoir water surface,  $h_o$ , is 20 feet above the crest of the dam and the depth of approach, P+E, is 50 feet below the overfall crest. A rectangular approach to the spillway is assumed. The length of the spillway can be varied to meet the discharge requirements.

Since the test results have been placed in terms of the total head,  $H_o$ , the design must be by successive approximations. Using  $h_o$  instead of  $H_o$ , since the velocity head is not known,

$$\frac{h_o}{P+E} = \frac{20}{50} = 0.400$$

and the coefficient of discharge, C, from figure 10 is 4.02.

Then

$$Q = C L h_o^{3/2}$$

$$= 4.02 \times 1 \times 20^{3/2}$$

$$= 359.55 \text{ second-feet per foot of spillway.}$$

The mean velocity of the approaching water will be

$$V = Q/A = \frac{359.55}{(50+20) 1} = 5.14 \text{ feet per second}$$

$$h_a = V^2/2g = (5.14)^2/2g = 0.411 \text{ feet}$$

$$H_o = h_o + h_a$$

$$= 20 + 0.411 = 20.41 \text{ feet (first approximation).}$$

For purposes of preliminary design, this value of  $H_o$  would probably be sufficiently accurate, but successive approximations will yield a more accurate discharge capacity. Using  $H_o = 20.41$ ,

$$\frac{H_o}{P+E} = \frac{20.41}{50} = 0.408.$$

From figure 10

$$C = 4.02$$

$$Q = C L H_o^{3/2}$$

$$= 4.02 \times 1 \times (20.41)^{3/2}$$

= 370.68 second-feet per foot of spillway length

$$V = \frac{Q}{A} = \frac{370.68}{(50+20) 1}$$

= 5.30 feet per second

$$h_a = V^2/2g$$

$$= \frac{(5.30)^2}{64.4} = 0.44 \text{ feet}$$

$$H_o = h_o + h_a$$

$$= 20 + 0.44 = 20.44 \text{ feet (second approximation).}$$

In this case, one additional approximation will give the same value for  $H_o$ . So the discharge capacity is

$$Q = 4.02 \times 1 \times (20.44)^{3/2}$$

= 371.48 second-feet per foot of spillway.

The next step is to determine the value of E, the rise of the crest above the springing line. Again solving by successive approximations and letting  $H_s = H_0$  for the first approximation,

$$\frac{h_a}{H_s} = \frac{0.44}{20.44} = 0.022.$$

From figure 9  $E/H_s = 0.106$ ,

then  $E = 0.106 H_s$ .

Substituting in  $H_s = H_0 + E$

$$= 20.44 + 0.106 H_s$$

$$H_s - 0.106 H_s = 20.44$$

$$H_s = \frac{20.44}{1-0.106} = 22.88 \text{ feet (second approximation).}$$

Repeating the solution

$$\frac{h_a}{H_s} = \frac{0.44}{22.88} = 0.019.$$

From figure 9  $E/H_s = 0.107$ .

then  $E = 0.107 H_s$ .

Substituting in  $H_s = H_0 + E$

$$= 20.44 + 0.106 H_s$$

$$H_s - 0.107 H_s = 20.44$$

$$H_s = \frac{20.44}{1-0.107} = 22.89 \text{ (third approximation).}$$

This last approximation did not yield a greater refinement but did illustrate the limits of the solution.

With  $h_a$  and  $H_s$  computed, the  $x/H_s$  and  $y/H_s$  coordinates of the lower nappe of the jet over the crest at maximum discharge can be read from table 4 using the ratio  $h_a/H_s = 0.019$ . The actual coordinates can be computed by multiplying the  $x/H_s$  and  $y/H_s$  coordinates by  $H_s$  as

shown in the tabulation below.

$x/H_a$	$y/H_a$	$x$	$y$
0.000	0.0000	0.000	0.000
+0.010	+0.0143	+0.229	+0.328
0.020	0.0266	0.458	0.609
0.040	0.0462	0.916	1.058
0.080	0.0723	1.331	1.655
0.120	0.0883	2.748	2.021
0.160	0.0984	3.662	2.192
0.200	0.1034	4.578	2.367
0.250	0.1054	5.722	2.413
0.300	0.1020	6.867	2.335
0.350	0.0970	8.011	2.220
0.400	0.0876	9.156	2.005
0.450	0.0751	10.30	1.719
0.500	0.0610	11.44	1.396
0.60	+0.023	13.73	+0.526
0.70	-0.027	16.02	-0.618
0.80	0.083	18.31	1.900
0.90	0.149	20.60	3.411
1.00	0.223	22.89	5.104
1.25	0.452	28.61	10.446
1.50	0.733	34.33	16.778
1.75	1.066	40.05	24.401
+2.00	-1.460	+45.78	-33.419

In cases where the depth of approach,  $P+E$ , is small, more approximations will be needed to reach the final values of  $H_a$  and  $H_o$ . In the above case, if  $P+E = 10$  feet instead of 30 feet, the results would be as follows:

Approximation	$H_o/P+E$	C	Q	V	$H_a$	$H_o$
First	2.000	3.935	381.95	11.73	2.14	22.14
Second	2.214	3.920	408.39	13.61	2.88	22.88
Third	2.288	3.920	429.00	14.30	3.18	23.18
Fourth	2.318	3.920	437.47	14.58	3.30	23.30
Fifth	2.330	3.920	441.06	14.70	3.35	23.35

In a second example, the maximum discharge per foot of spillway length and the height of dam are given and the overall crest will be

designed. With  $P+E = 50$  feet and  $Q = 500$  second-feet per foot of spillway length, the first step in the computation will be to estimate the coefficient of discharge to determine the head  $H_o$ . If an approximate value of the coefficient of discharge,  $C$ , is taken as 3.95,

$$H_o^{3/2} = Q/CL \\ = 500/3.95 \times 1 = 126.6$$

$$H_o = 25.21 \text{ feet (approximately)}$$

and  $\frac{H_o}{P+E} = \frac{25.21}{50} = 0.504.$

From figure 10,  $C = 4.01$ .

then  $H_o^{3/2} = \frac{5.00}{4.01}$   
= 124.7

and  $H_o = 24.96 \text{ feet.}$

This process can be repeated for greater accuracy but once is sufficient in this case. With the head  $H_o$  determined, the remainder of the solution is the same as in the first example.

#### Effect of Surface Tension on Nappe Shape

There is some question as to the possible effect of surface tension on the shape of the nappe in the transference of the test results to a prototype structure. Harris<sup>14</sup> considers a molecular correction for coefficients of discharge at low heads. He recommends additive molecular corrections amounting to as much as ten per cent of the coefficient at heads as low as 0.15 feet. These corrections recommended by Harris have been applied to coefficients from several tests made by the Bureau of Reclamation and by Schoder. In every case, the com-

puted coefficient was too large, hence it has been concluded that the molecular effect is not as large as Harris indicates. He mentions that the area of the nappe is affected at low heads. The tests performed by the Bureau of Reclamation on vertical weirs did not show an appreciable deviation from the law of similitude of shape for heads as low as 0.18 feet. It is possible that there may be a slight change in shape for different heads due to the failure of surface tension to follow the law of similitude, but it is considered to be negligible. Therefore, it is reasonable to assume that the basic material compiled in figures 9 and 10 and in table 4 can be safely applied to large structures.

The force of air drag upon the nappe shape was also investigated. A point was chosen 2.5 feet below the crest for a head of one foot and the air drag was considered to be the same as for a point 2.5 feet behind the leading edge of a stationary thin plate in a stream of air. Hansen's equation<sup>15</sup> for drag force was used. The horizontal component was considered to be distributed parabolically, diminishing upward and approaching zero at 2.5 feet above the point investigated. The force was integrated over the distance, and for the period which it acted was found to be sufficient to deflect the nappe toward the weir only 0.000516 foot. The assumption considers no air drag force on the upper side of the nappe. As a matter of fact, the air on the under side probably moves in a large eddy so that the drag would be diminished. The effect of air drag was considered to be of an order sufficiently small to be neglected.

## V - SUMMARY OF RESULTS AND CONCLUSIONS

### Results and Conclusions

The predominant result of this particular study is the presentation of material whereby the designing engineer can prepare a complete design of an overfall dam crest for a spillway taking into consideration the velocity of approach. In performing the experiments from which the design data has been derived, the validity of Bazin's studies in 1886-88 has been authenticated and the range of his work extended as far as the vertical weir is concerned.

The pertinent material needed in the design of a crest with a vertical upstream face is shown in figures 9 and 10 and in table 4. The method of application is illustrated by examples in chapter IV.

A practical limit was determined in the ratio of velocity head to total head,  $h_a/H_s$ . In the region of  $h_a/H_s = 0.22$ , the depth of flow approaches critical depth where the flow is undulating and unstable. For flows where  $h_a/H_s$  exceeds 0.22, the flow is likely to be unstable and no design data will apply. It is therefore recommended that no structure be designed with a depth of approach ratio in this region.

### Program of Additional Tests

This presentation of material falls short of exhausting the problem even for immediate needs. An additional program of studies will be completed whereby similar tests will be performed on weirs with downstream inclinations of 1:4, 2:4, 3:4 and 4:4. This material presented in a form similar to table 4 and figures 9 and 10, and combined with the vertical weir data will enable the designer to interpolate to complete any intermediate case such as slope on the upstream face of 2:3.

In these additional studies, it is proposed to utilize Bazin's

data where applicable and limit the experiments to those necessary to expand his range.

The entire set of studies will eventually be published in the final reports on the Boulder Canyon Project in Part VI, Hydraulic Investigations, Bulletin 3, Studies of Crests for Overfall Dams. This bulletin will contain a complete treatment of the subject of overfall crests as studied in connection with the design of the crests for the Boulder Dam spillways.

#### Mathematical Analysis of Results

A mathematical analysis of the behavior of the sheet of water over a spillway crest with different ratios of velocity head to total head is suggested as a possibility for further study on this subject. Vitols<sup>16</sup> in his paper on ~~numerical~~ dam profiles made an approach to the solution based to a large extent on Bezin's experiments. An analysis using his method, or better still a considerably simpler one, offers a good problem for a graduate student or an engineer who is mathematically adept. It was with this in mind that the original profile traverses of the upper and lower nappes were compiled in table 2.

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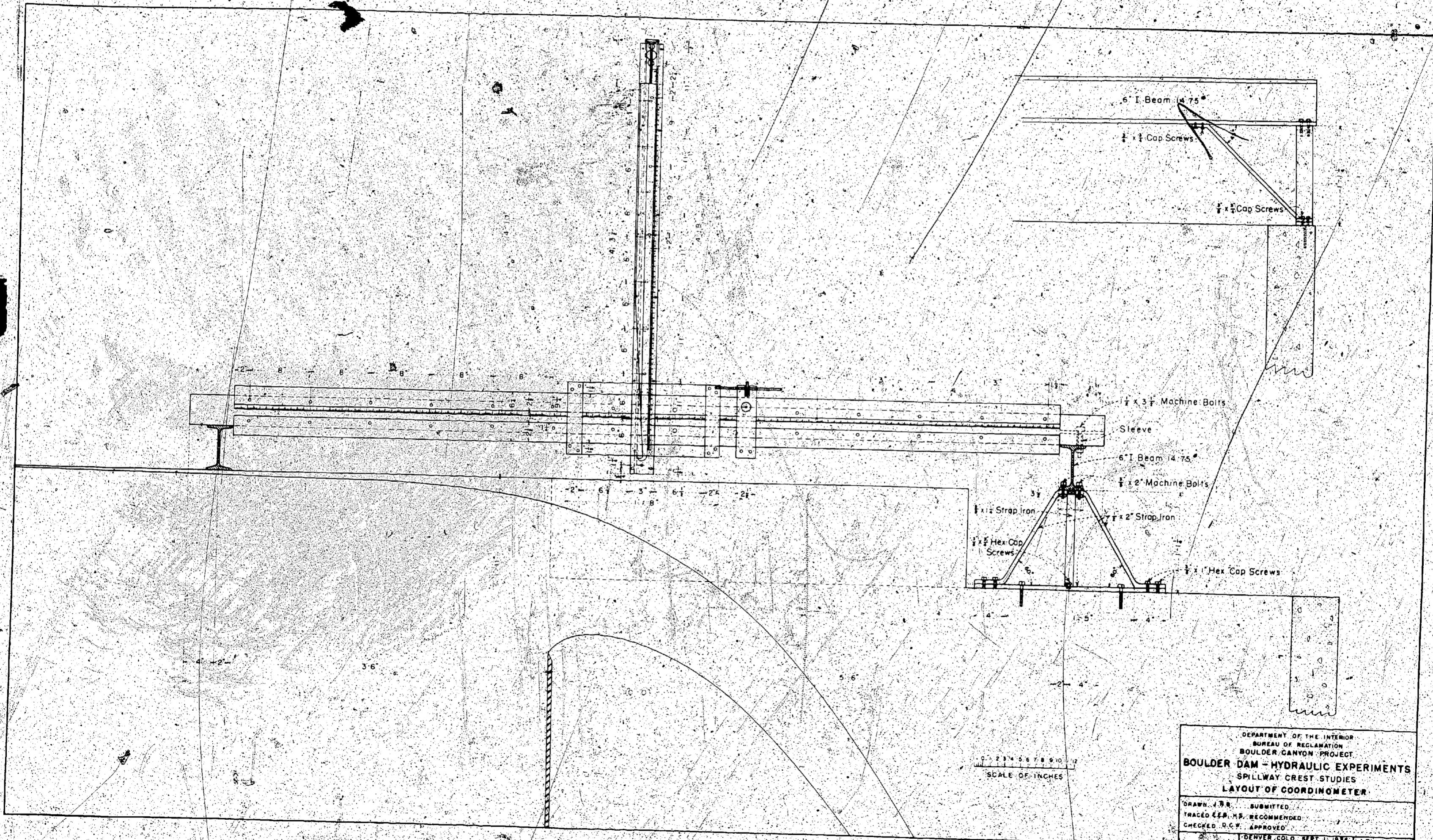
APPENDIXBibliography

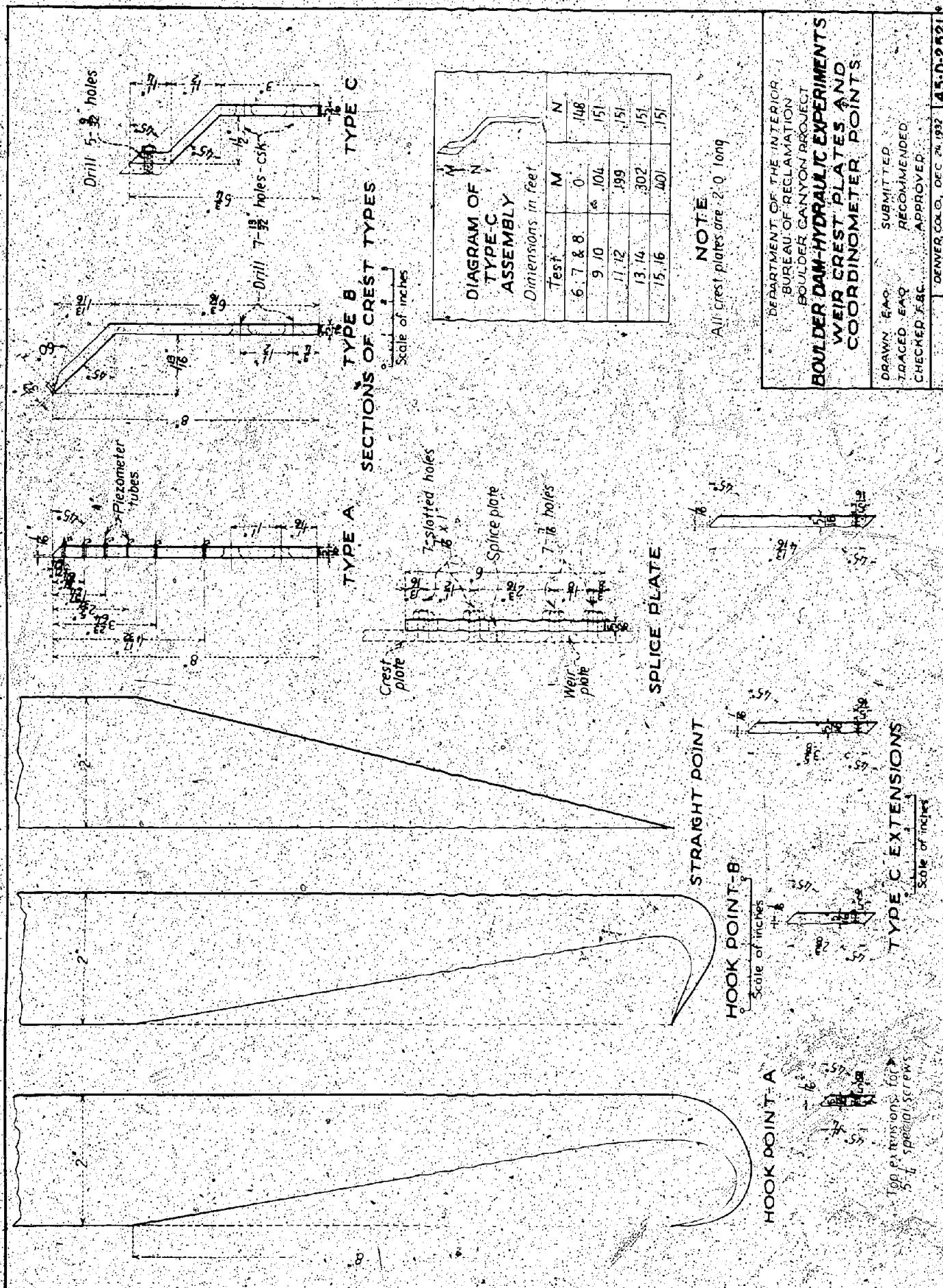
- 1 M. Buzin, "Recent Experiments on the Flow of Water over Weirs," *Annales des Ponts et Chaussées*, October 1888 (Translated by Arthur Marichal and John C. Trautwine, Jr. and published in the Proceedings of the Engineers' Club of Philadelphia, Volume VIII, No. 5, 1890, page 259, and Volume IX, No. 3, 1892, page 231).
- 2 M. Boussinesq, "Comptes rendus de l'Academie des sciences," July 4, 1887.
- 3 R. Muller, "Development of Practical Type of Concrete Spillway Dam," *Engineering Record*, Volume 58, October 24, 1908, page 461.
- 4 P. A. M. Parker, "Form of the Downstream Face of Overflow Dams," *The Control of Water*, page 399, D. Van Nostrand Company, New York, 1916.
- 5 E. Morrison and C. L. Brodie, *Masonry Dam Design*, pages 120-133, second edition, 1916.
- 6 S. M. Woodward, *Hydraulics of the Miami Flood Control Project*, Technical Reports, Part VII, page 223.
- 7 W. P. Creager, *Engineering for Masonry Dams*, pages 105-110, first edition, 1917.
- 8 Ing. Prof. Ettore Scissom, "Sulle forme delle vene trascorrenti" (On the Form of Crest Streams), *L'Energia Elettrica*, April 1930.
- 9 H. Lamb, *Hydrodynamics*, page 95, fifth edition, 1924.
- 10 J. E. Warnock, "Experiments Aid in Design at Grand Coulee," *Civil Engineering*, Volume 6, No. 11, page 737, November 1936.
- 11 Hunter Rouse and Lincoln Reid, "Model Research on Spillway Crests," *Civil Engineering*, Volume 5, page 10, January 1935.

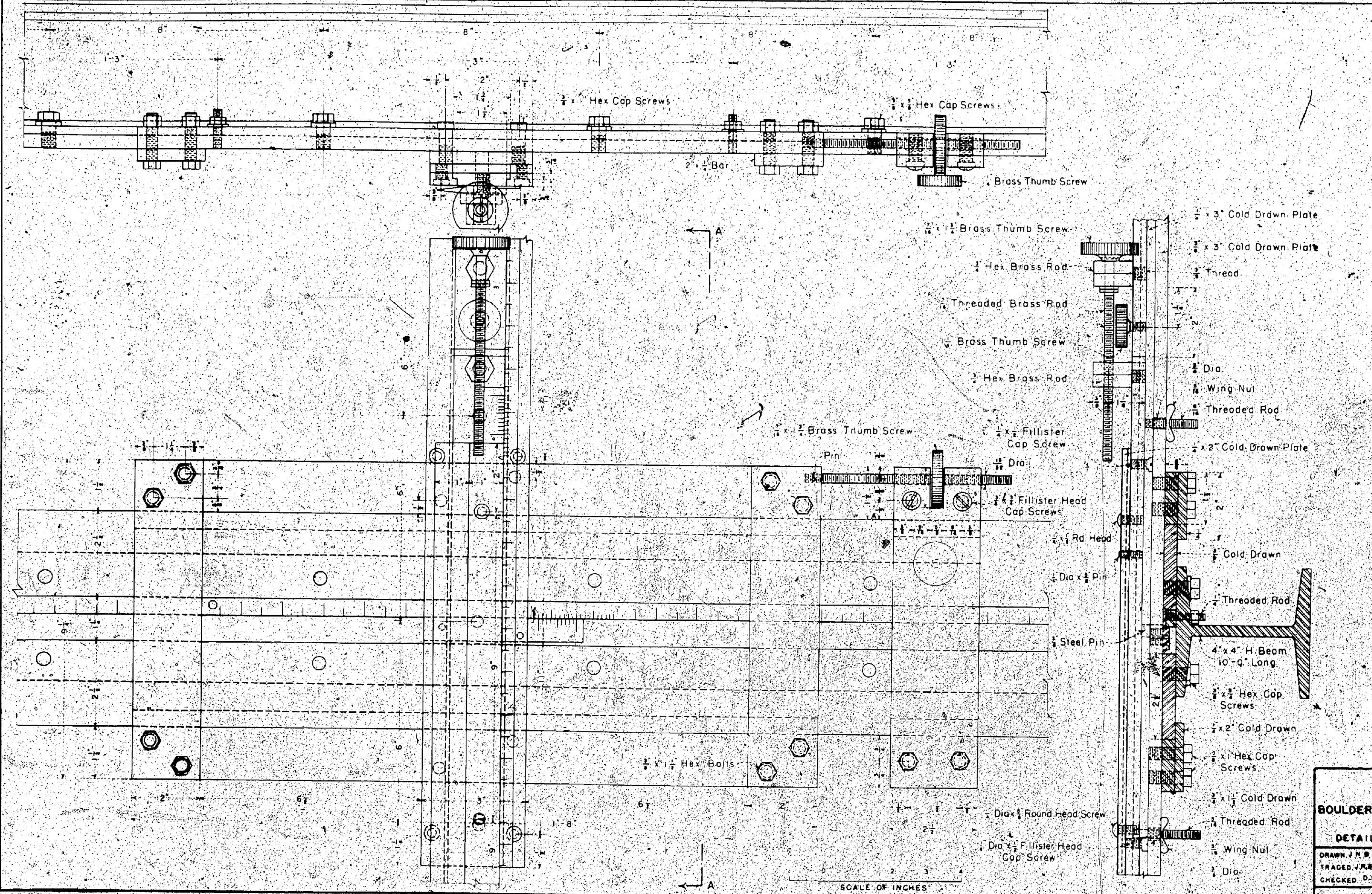
- 12 Transactions, American Society of Civil Engineers, Volume 63, 1920,  
page 1154.
- 13 H. W. King, "Weirs not Sharp Crested," pages 151-169, Handbook of  
Hydraulics, third edition, 1939, McGraw-Hill Book Company.
- 14 Harris, "An Analysis of the Weir Coefficient for Suppressed Weirs,"  
University of Washington, Experiment Station Bulletin No. 22,  
1925.
- 15 Hansen, "Velocity Distribution in the Boundary Layer of A Thin Plate,"  
National Advisory Committee on Aeronautics, Technical Memorandum  
No. 595.
- 16 A. Vitols, "Beitrag zur Frage des vacuumlosen Dammprofiles" (Vacuum-  
less Dam Profiles), Wasserkraft und Wasserwirtschaft, Volume 31,  
1936, page 207. Translated by Edward F. Wilsey, Assistant Eng-  
ineer, Bureau of Reclamation.
- Craiger and Justin, "Design of Solid Gravity Dams," Hydroelectric Hand-  
book, page 208, 1927, John Wiley and Sons, New York.
- Schoder and Dawson, "Flow over Weirs and Dams other than Sharp-crested,"  
Hydraulics, page 163, 1927, McGraw-Hill Book Company, Inc., New  
York.
- A. H. Gibson, "Flow over Notches and Weirs," Hydraulics and its Appli-  
cations, page 147-176, 1930, D. Van Nostrand Company, Inc., New  
York.
- C. S. Camp, "Determination of Shape of Nappe and Coefficient of Dis-  
charge of a Vertical Sharp-crested Weir, Circular in Plan, with  
Radially Inward Flow," University of Iowa Thesis, June, 1937.
- R. B. DuPont, "Determination of the Under Nappe over a Sharp Crested  
Weir, Circular in Plan with Radial Approach," Case School of

Applied Science 1937.

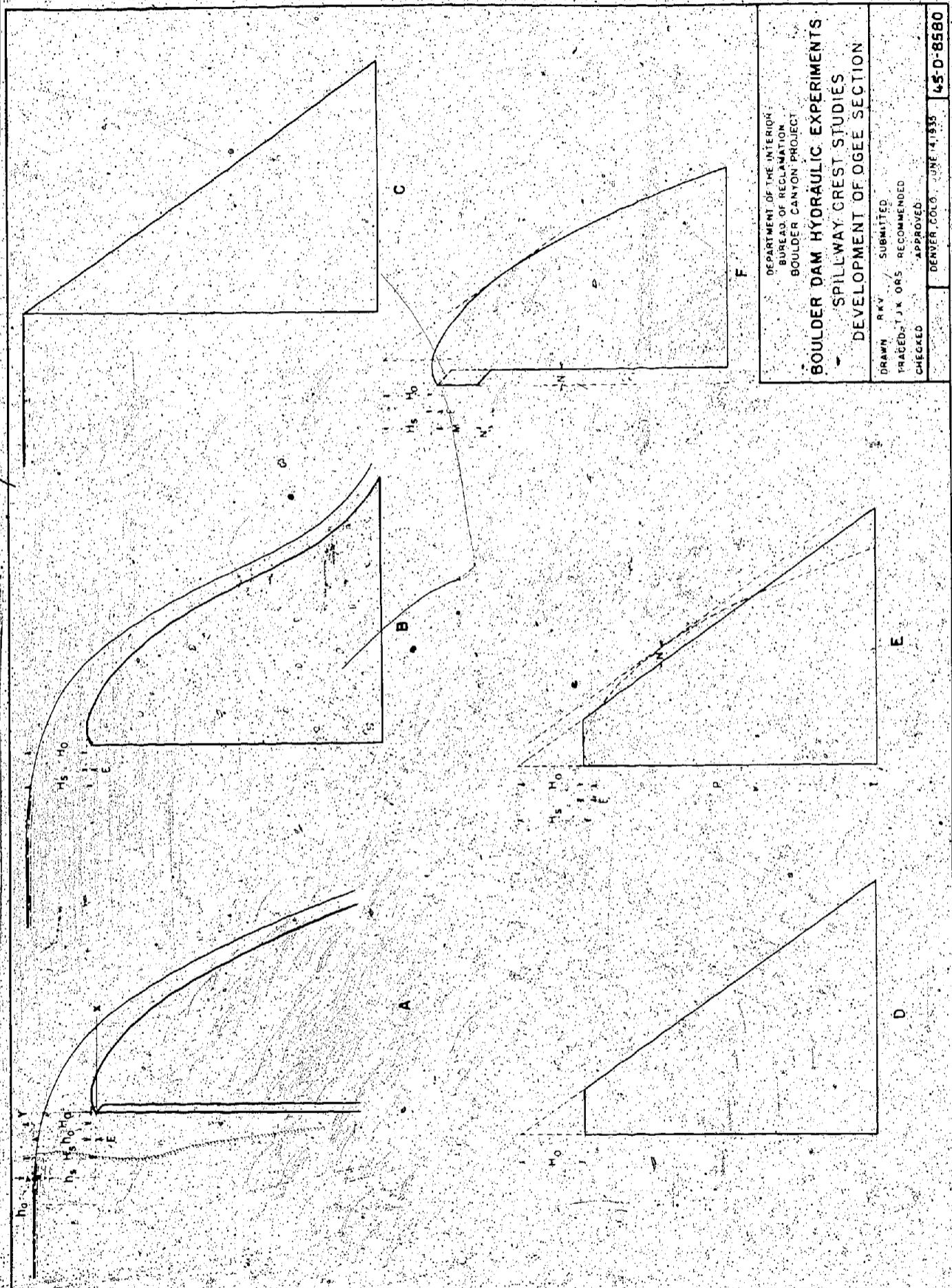
Hunter Rouse, "Spillway Design," Fluid Mechanics for Hydraulic Engineers, page 316, 1938, McGraw-Hill Book Company, Inc., New York.







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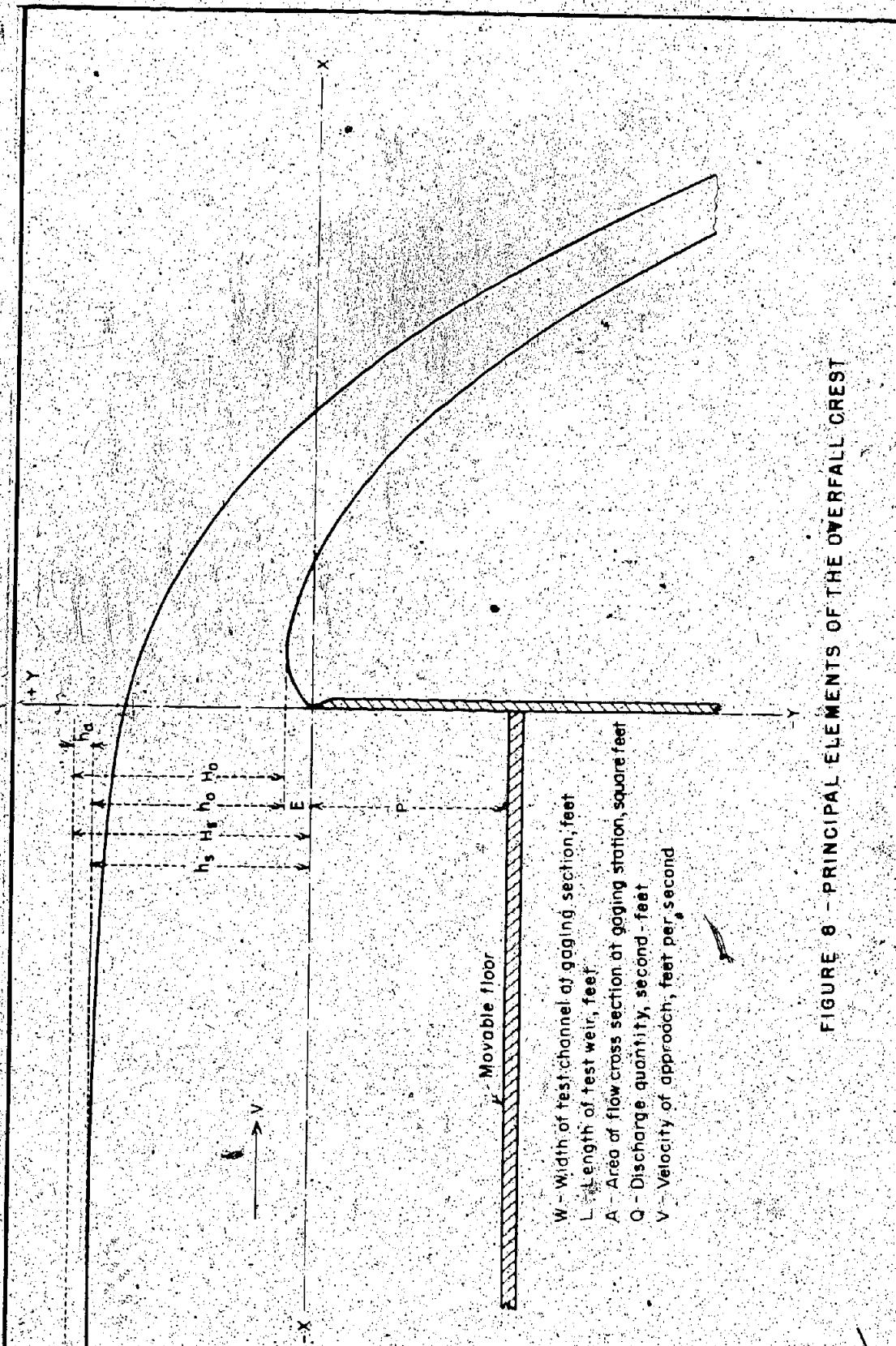


FIGURE 8 - PRINCIPAL ELEMENTS OF THE OVERFALL CREST

