UNITED STATES
DEPARTMENT OF THE INTERIOR
BUREAU OF RECLAMATION

CRITICAL TRACTIVE FORCES ON CHANNEL SIDE SLOPES

Hydraulic Laboratory Report No. Hyd-366
Supersedes Hyd-295

ENGINEERING LABORATORIES BRANCH

DESIGN AND CONSTRUCTION DIVISION
DENVER, COLORADO

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Supersedes Hyd-295
Hydraulic Laboratory Section
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Reviewed by: E. W. Lane

Subject: Critical tractive forces on channel side slopes in coarse, noncohesive material

Synopsis

The purpose of this study is to determine the magnitude of the tractive forces exerted by flowing water which will cause impending motion of the material comprising the sloping side of a channel. Only the simplest case of coarse, noncohesive material acted on by clear water is treated. The force necessary to cause motion on the sloping side of a channel is less than that required for movement on a horizontal canal bed. The ratio of the force necessary to cause impending motion on the slope to that required on the level is shown to be a function only of the slope of the side and the angle of repose of the material. A simple expression involving these variables has been derived to give the value of the ratio, and a diagram is presented to aid in using it.

Introduction

In the United States the design of earth canals to insure freedom from scour has usually been approached from the standpoint of the mean velocity which could safely be permitted. In Europe the criteria of limiting shear or tractive force has sometimes been used. Studies using the tractive force criteria have recently been made in this country 1/. It has been shown, however, that the slope of the sides of the channel is a factor in the resistance to scour of the sides by the flowing water 2/. A satisfactory method of design of earth channels must therefore consider this factor. The results of the study described herein have shown that for coarse, noncohesive material, the effect of the slope of the sides is a very important factor in canal design. As the cohesion in the material increases, this factor becomes less important, and for highly cohesive soils, it is probably

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1/ Progress Report on Results of Studies on Design of Stable Channels, United States Bureau of Reclamation Hydraulic Laboratory Report No. Hyd-352.

2/ Stable Channels in Erodible Material, E. W. Lane. Trans. ASCE Vol. 102, 1937, p. 134.

of little or no importance. Up to the present, an analysis has been developed only for the case of coarse, noncohesive material, acted on by clear water.

Forces Acting on a Particle on a Sloping Canal Side

As is pointed out in the previously mentioned reference 2/, a particle resting on the sloping side of a canal in which water is flowing is acted on by two forces (1) the force due to the action of the flowing water on the particle, which acts in the direction of the flow, and (2) the force due to the weight of the particle, which tends to cause the particle to roll down the side slope. Movement of the particle will begin when the resultant of these two forces is large enough to start it.

The force on the particle downward in the direction of the sloping side, for particles below the water surface, as shown in Figure 1, is equal to $W_s \sin \phi$ where W_s is the submerged weight of the particle and ϕ is the angle, with the horizontal, of the sloping side. The force on the particle due to the velocity of the water is aT_s, in which "a" is the effective area of the particle, and T_s is the tractive force on the side of the canal. T_s acts in the direction of flow and is expressed in units of weight per unit area. It would be difficult to determine the value of "a" for any particle, but for the purpose of this study it is not necessary, as it is canceled out in the solution.

The resultant of these two forces, which are at right angles to each other, is

$$\sqrt{W_s^2 \sin^2 \phi + a^2 T_s^2},$$

and when this force is large enough, the particle will move.

Magnitude of the Critical Tractive Force on Sloping Side

A determination of the conditions for impending motion can be made by analyzing the forces which act on particles on a surface sloping at the angle of repose for the material involved. Consider the condition of a particle on the side of a canal filled with water, in which there is no flow, when the side slope is equal to the angle of repose θ. If W_s is the submerged weight of the particle, the force acting on the particle downward in the direction of the surface slope is $W_s \sin \theta$ and that acting normal to the surface is $W_s \cos \theta$, and the ratio of these two components is $\tan \theta$. Since for a given material the angle of repose is nearly independent of the particle size, motion of the particles is independent of whether W_s is large or small but depends only on whether the ratio of the component of the force down the slope to that normal to the slope is greater than $\tan \theta$.

Since motion is impending, the resistance to motion of the particle is equal to the force tending to cause motion, or to $W_s \sin \theta$, which is also equal to $W_s \cos \theta \tan \theta$. It is reasonable to suppose that resistance to motion of a particle on any other slope or on the level is also equal to the normal force times $\tan \theta$, and this hypothesis is therefore adopted 3/.

Applying the hypothesis to the case of a particle on the side of a canal in which water is flowing, the resultant of the two forces which act on the particle should cause impending motion when the ratio of this resultant force to the normal force acting on the particle is equal to $\tan \theta$. Then

$$\frac{\sqrt{W_s^2 \sin^2 \phi + a^2 T_s^2}}{W_s \cos \phi} = \tan \theta$$

and

$$T_s = \sqrt{\frac{W_s^2 \cos^2 \phi \tan^2 \theta - W_s^2 \sin^2 \phi}{a^2}}$$

Also, motion of a particle on a level surface is in a state of impending motion due to the tractive force when

$$a\, T_L / W_s = \tan \theta, \text{ and } T_L = \frac{W_s \tan \theta}{a}$$

In this case $[a\, T_L]$ is the tractive force which will cause impending motion of a single particle on the level surface, "a" is again the effective area of a single particle, and T_L is the average tractive force per unit area.

Ratio of Force on Sloping Side to that on Level Surface Necessary to Cause Impending Motion

For purposes of design it is desirable to know the ratio of the tractive force per unit area, T_s, which will cause impending motion on the canal side to T_L which will cause impending motion on a level surface. If this ratio is K, then

$$K = \frac{T_s}{T_L} = \frac{\sqrt{\dfrac{W_s^2 \cos^2 \phi \tan^2 \theta - W_s^2 \sin^2 \phi}{a^2}}}{\dfrac{W_s \tan \theta}{a}} = \cos \phi \sqrt{1 - \frac{\tan^2 \phi}{\tan^2 \theta}}$$

It will be seen that this ratio is a function only of the inclination of the sloping side, ϕ, and the angle of repose of the material, θ. Figure 2 is a diagram giving the value of K for various values of ϕ and θ.

3/ This hypothesis has also been used by A. A. Kalinske in Movement of Sediment as Bed Load in Rivers. Transactions, American Geophysical Union, Vol. 28, August 1947, p. 616.

A solution for the value of K was independently derived by C. H. Fan 4/, which is

$$K = \sqrt{1 - \frac{\sin^2 \phi}{\sin^2 \theta}}$$

Although the two equations are not identical, the difference between the values of K obtained by them is negligible.

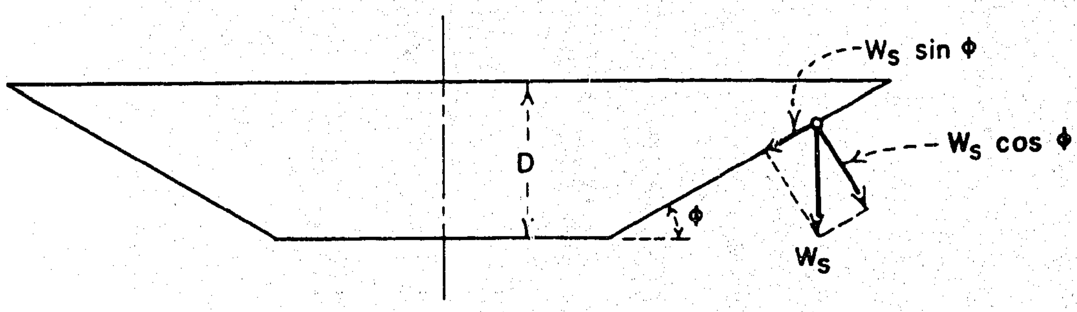
In designing a canal in coarse, noncohesive material, so that the material on the side slopes will not be moved, it is necessary to establish that the shear at any point on the sides is less than K times the shear that will cause impending motion for that material on a level surface. As the shear on the sides and bottom of a channel is not uniformly distributed, it is necessary also to know this distribution. For trapezoidal cross sections, this distribution is given in a report previously mentioned 1/. The magnitude of shears which are safe on a level surface for various sizes of coarse, noncohesive material, and data on angles of repose, are also given in reference 1/. Observations on angles of repose failed to detect any material difference between values under water and those in the dry condition.

Acknowledgment

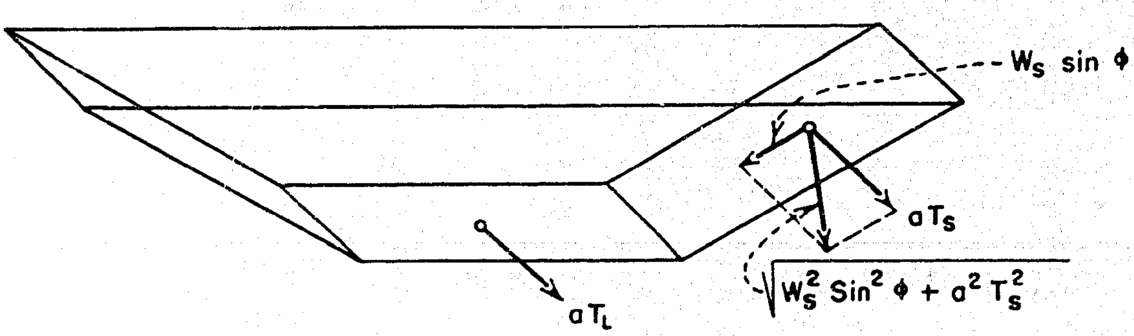
The writers of this report wish to acknowledge the assistance in preparing it of Messrs. R. E. Glover, D. J. Hebert, and P. W. Terrell.

4/ A Study of Stable Channel Cross Sections, C. H. Fan, Hydraulic Engineering (published by the Chinese Society of Hydraulic Engineers), Vol. 15, 1947, p. 71.

FIGURE 1

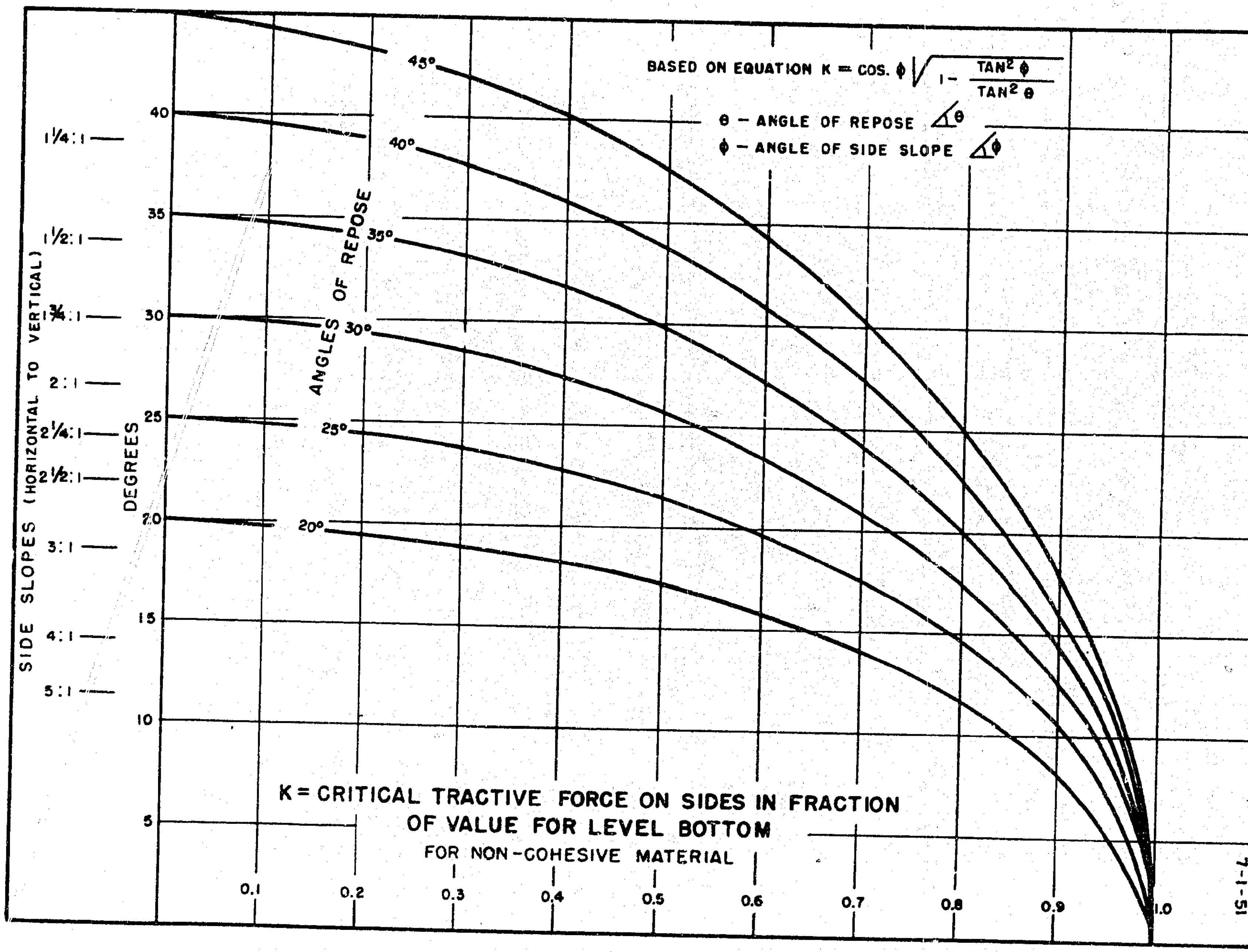


GRAVITY FORCE



TRACTIVE AND GRAVITY FORCES

FIGURE 2



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