UNITED STATES

DEPARTMENT OF THE INTERIOR

BUREAU OF RECLAMATION

MEMORANDUM TO CHIEF DESIGNING ENGINEER

SUBJECT: SOME FACTORS AFFECTING THE

STABILITY OF EARTH DAMS AND

A METHOD OF ANALYSIS

------

by

D. P. BARNES, ASSISTANT ENGINEER

Under direction of

E. W. LANE, RESEARCH ENGINEER

TECHNICAL MEMORANDUM NO. 463

Denver, Colorado

June 29, 1935

(PRICE $0.90)

SOME FACTORS AFFECTING THE STABILITY OF EARTH DAMS
and
A METHOD OF ANALYSIS

The purpose of the following discussion is to present a concept of the stresses in earth materials and of the causes and analysis of slides which it is hoped will point the way toward the rational solution of a number of the problems connected with the design of earth dams. The material is not new, but with one or two notable exceptions has received little attention in English engineering literature. As far as possible mathematical language and reference to some of the more intangible principles of mechanics have been avoided in an effort to maintain easy readability. The treatment is frankly elementary and incomplete, the intention being merely to furnish to those unfamiliar with the subject an insight into its possibilities.

EMBANKMENT FAILURES AS SHEAR FAILURES

In earth dams the stresses of most concern are shearing stresses. All materials fail either in tension, compression, shear, or combinations of these, but in embankments shear is the critical stress because (1) tensile stresses are either very small and localized or are entirely absent, and (2) compression failures (consolidation) are not in general injurious to the structure.

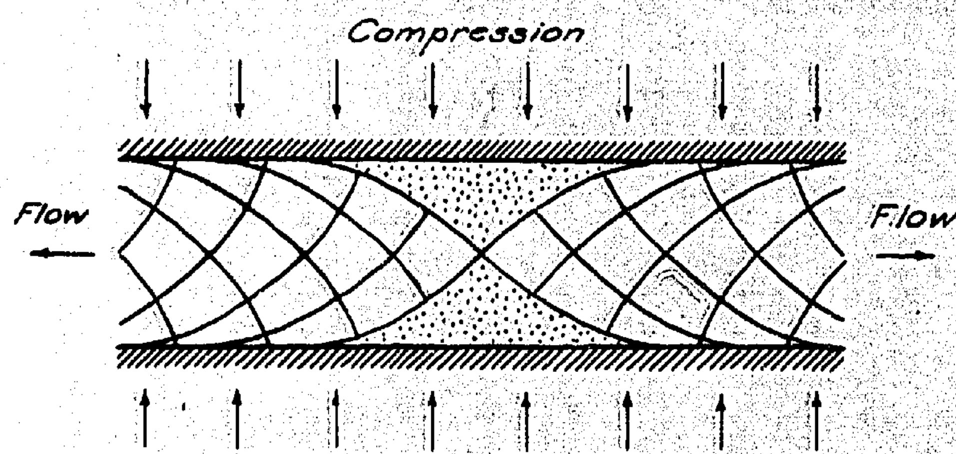
The use of the term shear to describe failure by sliding along some straight or curved surface is perhaps generally admitted, but that plastic and viscous flow usually involve forms of shearing failure may be less well known. That this is the case can be illustrated by considering the movement of any thick liquid in a trough. If it moves faster in the center than at the walls of the trough, then the center particles are sliding past the exterior particles in a way which is closely similar to shearing movement between two surfaces in a more rigid material. The very definition of viscosity may be interpreted as the shearing force required to produce unit shearing strain in unit time.

It might be contended that the settlement of a dam resulting from the squeezing out of foundation material is a compression failure, and that for this reason compression failures can not be disregarded. Such squeezing, however, is primarily a manifestation of yielding in shear. The surfaces along which sliding occurs when a plastic material is compressed between two plates are clearly shown in fig. 1, taken from Nadai's book, "Plasticity."[1] Jürgenson[2] has already adapted the equations for the stresses along such slip lines to the determination of the stresses in a layer of soft material underneath an idealized earth dam.

---

1 Engineering Societies' Monographs, McGraw-Hill, 1931.

2 "Shearing Resistance of Soils," Journal of the Bost. Soc. of Civil Engineers, July, 1934.



Slip lines in a plastic mass.
from "Plasticity" by Nadai

Fig. 1

In soils, then, sliding (or shearing) failures will take place along those surfaces on which the shearing stresses exceed the shearing strengths of the material. If it were possible to calculate the intensity and direction of the shearing stresses at all points in the cross-section of an earth dam and if the strengths of the material could also be calculated or predicted at all points, it would be relatively easy to pick out any region in which stresses exceeded strengths and in which failure might therefore be expected.

No rational analytical method (as opposed to empirical methods) comparable in accuracy to those applied to concrete and steel structures has yet been devised for determining the stresses in earth dams and their foundations. If it were certain that the material acted sufficiently like an elastic solid, a completely analytical solution would be possible. Probably the most accurate determination of foundation stresses thus far made is that given by Jürgenson[3] and based upon partial elastic assumptions. However, for determining stresses within the body of the dam, to the writer's knowledge, no method has yet been used which is not dependent upon some form of the assumption that pressures are in direct proportion to the depth.

3 "Application of Theories of Elasticity and Plasticity to Foundation Problems," Journal, Boston Soc. of Civil Engineers, July, 1934.

Before the accuracy of any assumption can be definitely established some kind of pressure tests under embankments will need to be made. In the meantime, analyses based on the idea of depth times density, proper allowances being made for the effects of water, have yielded results which appear to agree quite reasonably with cases of actual failure.[4]

4 Instances of such agreement are given by:
Terzaghi (Tech. Memo. No. 397).
Tiedemann - "Die Bedeutung des Bodens im Bauwesen," Handbuch der Bodenlehre, Vol. X, p. 178.
Loos - "Anwendung der neueren Baugrundforschung," Bautechnik, April 5, 1935.
Fellenius - "Erdstatische Berechnungen," Wilh. Ernst und Sohn, Berlin, 1927.

The specific method referred to is that developed by the Swedish and described by Terzaghi in Public Roads.[5] It is presented briefly on p. 4.

The factor which greatly increases the difficulty in comparing stresses with strengths is that the shearing strength of a soil varies with the pressure and consequently with the depth. Thus it is not sufficient merely to find the surface along which the shearing stresses

5 "The Mechanics of Shear Failures in Clay Slopes" Public Roads, Dec., 1929.

are greatest or the surface along which strengths are least, but rather, that surface must be found along which the difference between strength and stress is a minimum. A number of attempts have been made to find this surface analytically beginning with various approximate assumptions,[5,6] but none has been entirely successful. The only practicable

6 M. M. Frontard, "Méthodes de recherches permettant de reconnaître si un matériau donné est apte à être employé pour la construction d'un barrage de terre," or Congres des Grands Barrages, Vol. III, Stockholm, 1933.

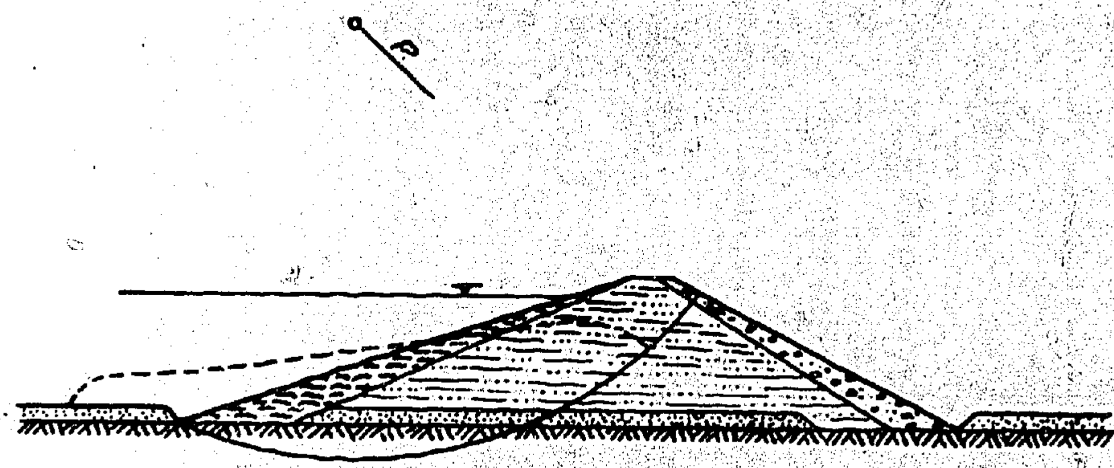
approach yet developed is the Swedish semi-graphical analysis.

Explained more fully on page 4, the method is essentially a "cut and try" procedure. A surface is assumed according to predetermined criteria, and the difference between stress and strength is calculated by simple graphic statics. This process is repeated for a number of surfaces until the most dangerous one is found (fig. 2). If the stress exceeds the strength on this surface, the embankment is concluded to be unstable. If the contrary is true, after allowing for a reasonable margin of safety, then the slope might be made steeper.

Before enlarging upon the actual application of this analysis to the stability of slopes, a brief discussion of the behavior of soils in shear is desirable.

BEHAVIOR OF SOILS IN SHEAR

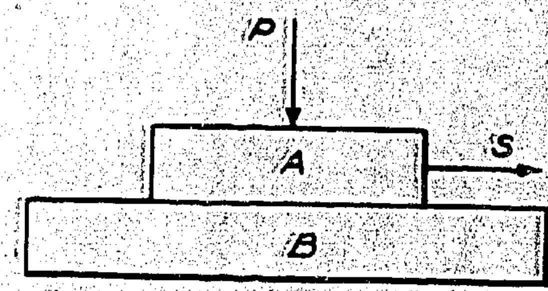
The resistance to sliding along some internal surface of a soil may be likened to the resistance developed between two wooden blocks when one is pulled across the face of the other (fig. 3). Neglecting the weight of the block, A, and if no pressure, P, is applied downward to press the blocks together, no force should be required to keep A moving along over B, once inertia had been overcome. If, however, block A is pressed against B with an appreciable force, P, the shearing force, S, required to keep it moving will become considerable. We would, no doubt, find by experiment that S would increase in the direct proportion to P according to some such line as OA in fig. 4. In this case, of course, the shearing resistance is entirely due to friction. Since OA is a straight line, S = fp where the constant, f, is known as the coefficient of friction. However, $\frac{S}{P}$ is also the



Trial sliding surface.

Fig. 2

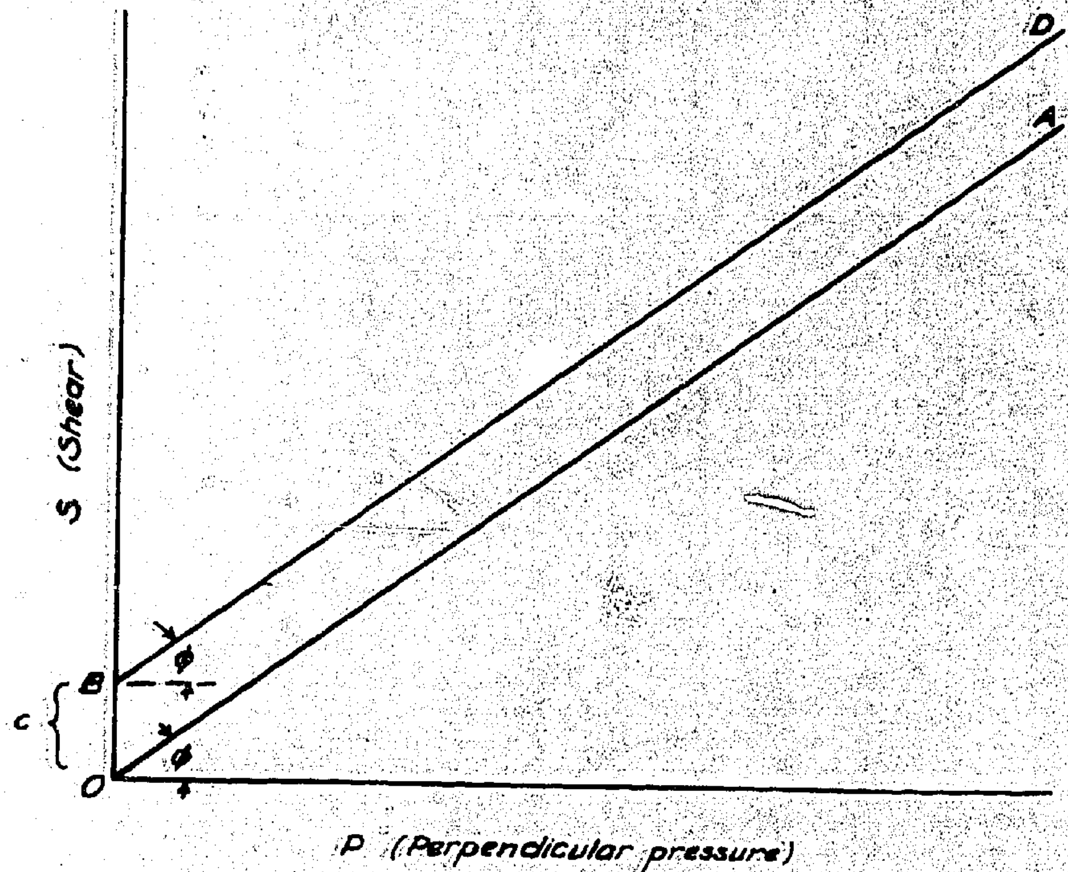
XD 1542



Friction between two blocks.

Fig. 3

XD 1543



Friction Diagram

Fig. 4

XD 1544

tangent of the angle, ∅, (fig. 4) so that $f = \tan \phi$ and ∅ is known as the angle of friction.

Suppose now that in addition to the pressure, P, a layer of sticky paste is applied between the blocks. It is reasonable to suppose that the force, S, required to pull A along B would now have to be increased by an amount corresponding to the stickiness of the paste. The line BD in fig. 4 illustrates this condition; c is the amount by which the paste increases the force necessary to move the upper block, and is entirely independent of the normal or perpendicular pressure between the blocks. The shearing force, S, is now equal to $fP + c$.

Basically, soils subjected to shearing stresses behave in much the same way. For example, if four or five samples of identical material are made up in exactly the same manner and each one loaded with a different normal pressure; then if each sample is tested to determine the shearing force required to start and maintain sliding on some plane within the sample, it has been found that when these shear values are plotted against normal load, the points will fall very much as shown by the points along curve "A" in fig. 5. The portion of the shear strength, c, which is independent of the pressure, P, is generally called the cohesion although it does not correspond very closely to what is meant by cohesion in the field of physics.[2,7] The coefficient of friction, f, is most frequently written "tan ∅."

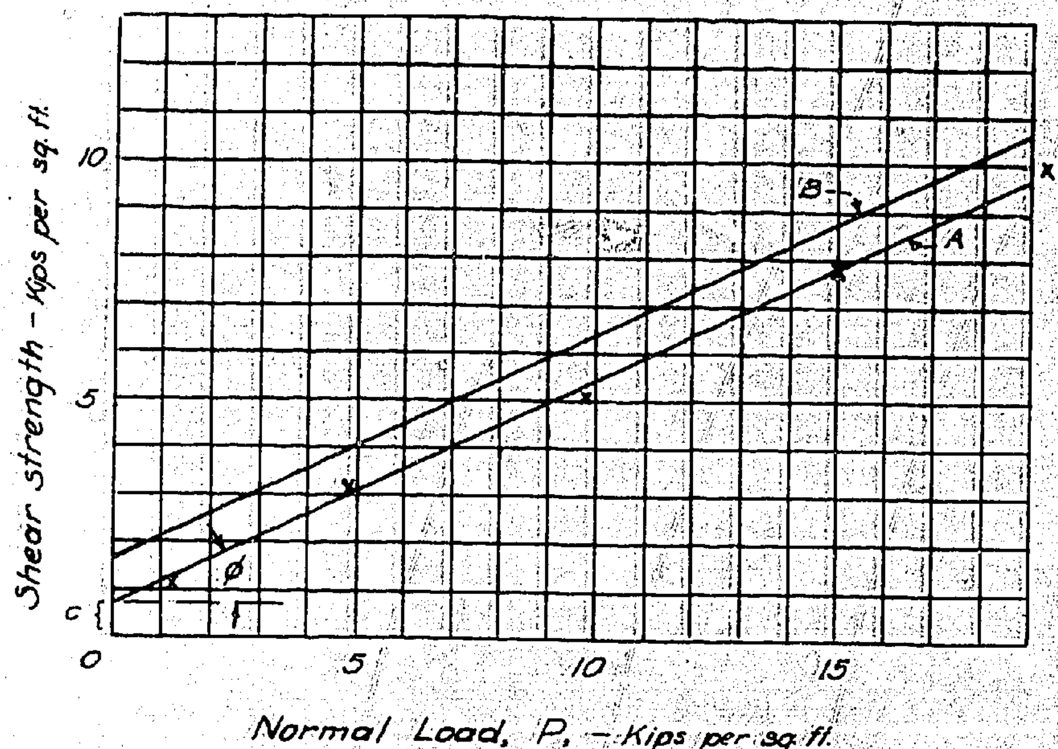
[7] Detailed discussions by Gilboy, Krey, and Krynine concerning the meaning of friction and cohesion in soils have been collected in Appendix A of Tech. Memo. No. 409.

When other samples of the same material are prepared by different methods (i.e., under different degrees of compaction or at different moisture contents) or when they are steeped in water after loading, the normal load-shear strength lines will in general show different slopes and different values of c, and may even show some curvature. Clearly, if the conditions under which a material is placed in a dam can be duplicated in the laboratory, and if the effects of settlement and saturation can be duplicated equally satisfactorily, a strength diagram can be plotted which should be a fair indication of the shearing strengths to be expected under various pressures corresponding to the depths.[8]

METHOD OF ANALYSIS

An example of the application of the results of shear tests to the analysis of an embankment will serve to illustrate the principles and some of the refinements involved. Consider an earth dam of uniform

[8] Reports covering these points in greater detail are now in preparation.



Typical

Normal Load — Shear Strength

Relations for Soils

Fig. 5

XD 1549

material and having the section and proportions shown in fig. 6. Guided by previous experience and by certain empirical rules,[8] a trial sliding surface is drawn in and a saturation line assumed. The area outlined by this circular arc is then divided into a convenient number of vertical sections, and the pressures and shearing forces on the bottoms of these sections calculated.

Consider the section ABCD. Scale off the areas of the portions above and below the saturation line, and obtain the total weight of the section (effective in producing pressure between the particles along the surface DC) by multiplying these areas, respectively, by densities of 118 lb. per cu.ft. and 62 lb. per cu.ft. (the latter is dry density diminished by uplift or $100 - \frac{100}{2.6}$). This total weight is represented by og in fig. 6. The component of this force tending to produce sliding is fg, parallel approximately to CD. The sum of all such forces along the sliding surface is the total shear, S, which must be resisted by the combined shearing strength of the material.

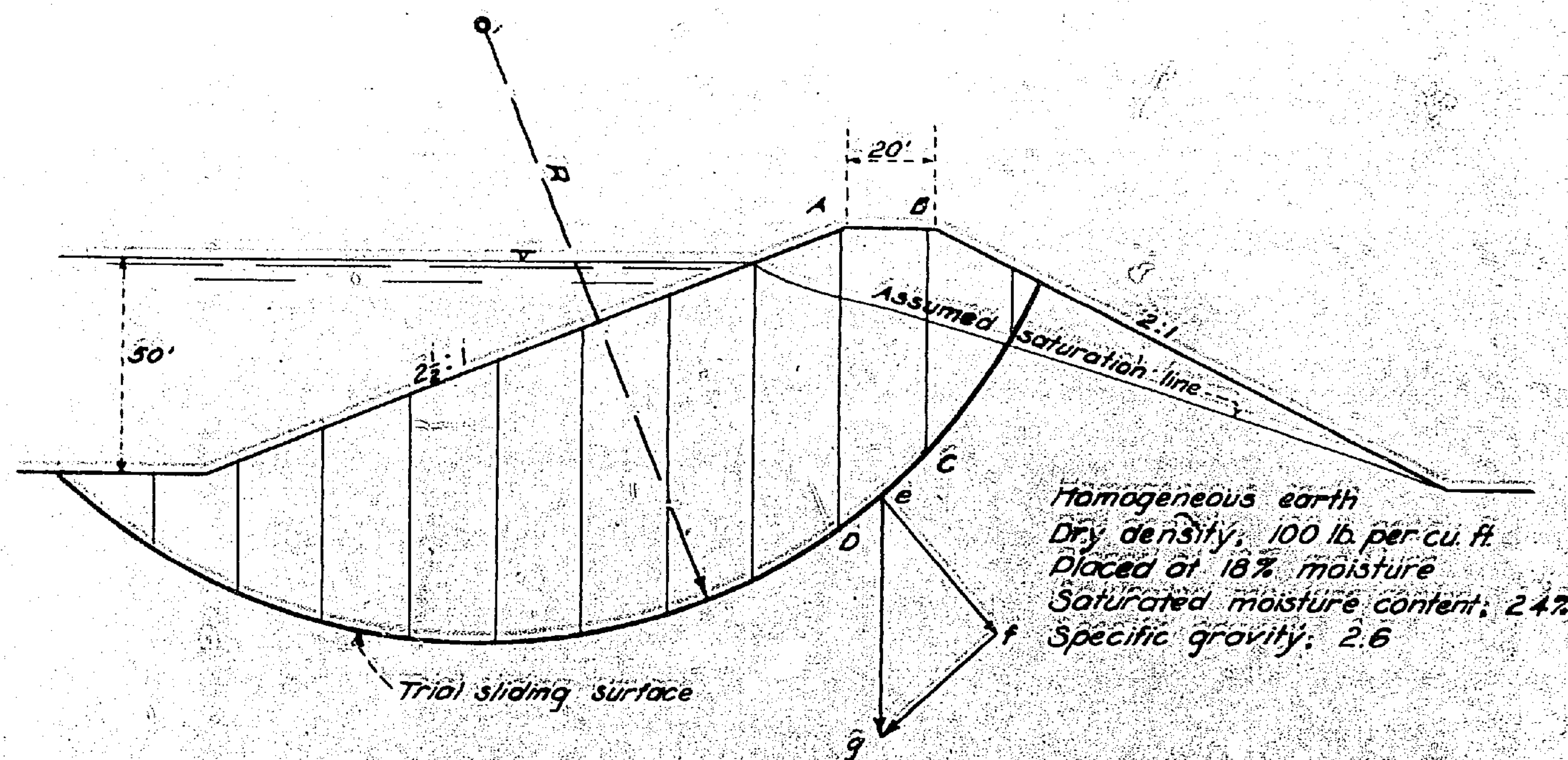
Nowsuppose that this material has been thoroughly tested in the laboratory, and that its shear strength curve is "A" of fig. 5 when saturated, and "B" of fig. 5 above the saturation line. The strength of the material along the bottom of the section ABCD is then $(c + P \tan \phi)\,\overline{CD}$ or $C \times \overline{CD} + P \tan \phi \times \overline{CD}$. In this, $P \times \overline{CD}$ is simply the total component of the weight perpendicular to the surface CD, that is, $\overline{ef}$ in fig. 6. The total strength of the material along CD is thus $c\,\overline{CD} + \overline{ef} \tan \phi$.

The sum of the shear strengths along the bottoms of all the sections is then compared with the total shear, S, tending to produce sliding. If S is greater than the total strength, obviously the condition is unstable and failure may be expected. If the strength proves greater than the stress, then additional trial curves must be drawn and similarly analyzed until it is certain that the worst case has been examined. If the material is still amply safe, a steeper slope may be tried and the whole process repeated.

The foregoing is obviously merely a skeleton of a more detailed procedure still in the process of development. In addition to the analysis shown, account must be taken of the forces introduced by percolating water. Where it is possible to sketch in even crude flow nets,[9] these forces are quickly and simply calculated (see fig. 7). The force acting on the material within any rectangle of the flow net is the average width of the rectangle multiplied by the potential interval

---

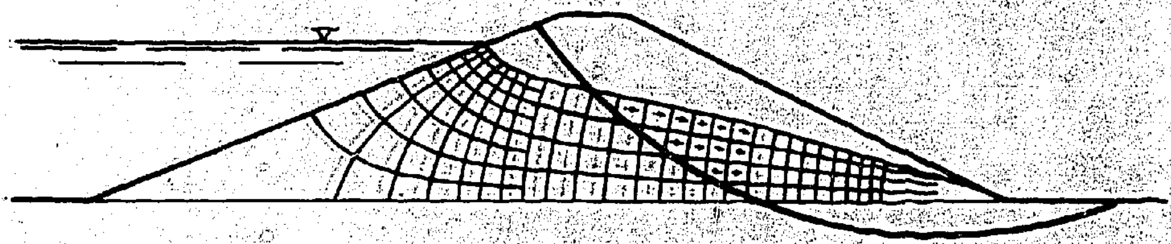
[9] Although the construction of accurate flow nets may be a difficult problem, it is believed that with a very little practice and instruction any engineer can sketch in a net of the accuracy required here in a few minutes, given the probable nature of the saturation line and certain other conditions.



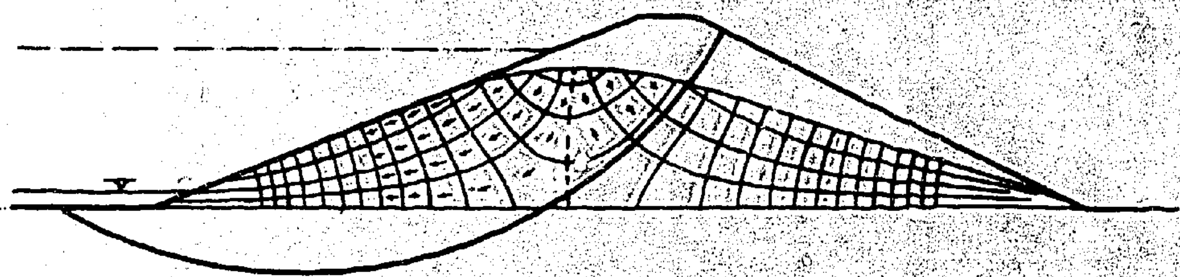
Idealized stability analysis.

Fig. 6

XD 1530



a. Full reservoir.



b. Reservoir empty.

Seepage forces.

Fig. 7

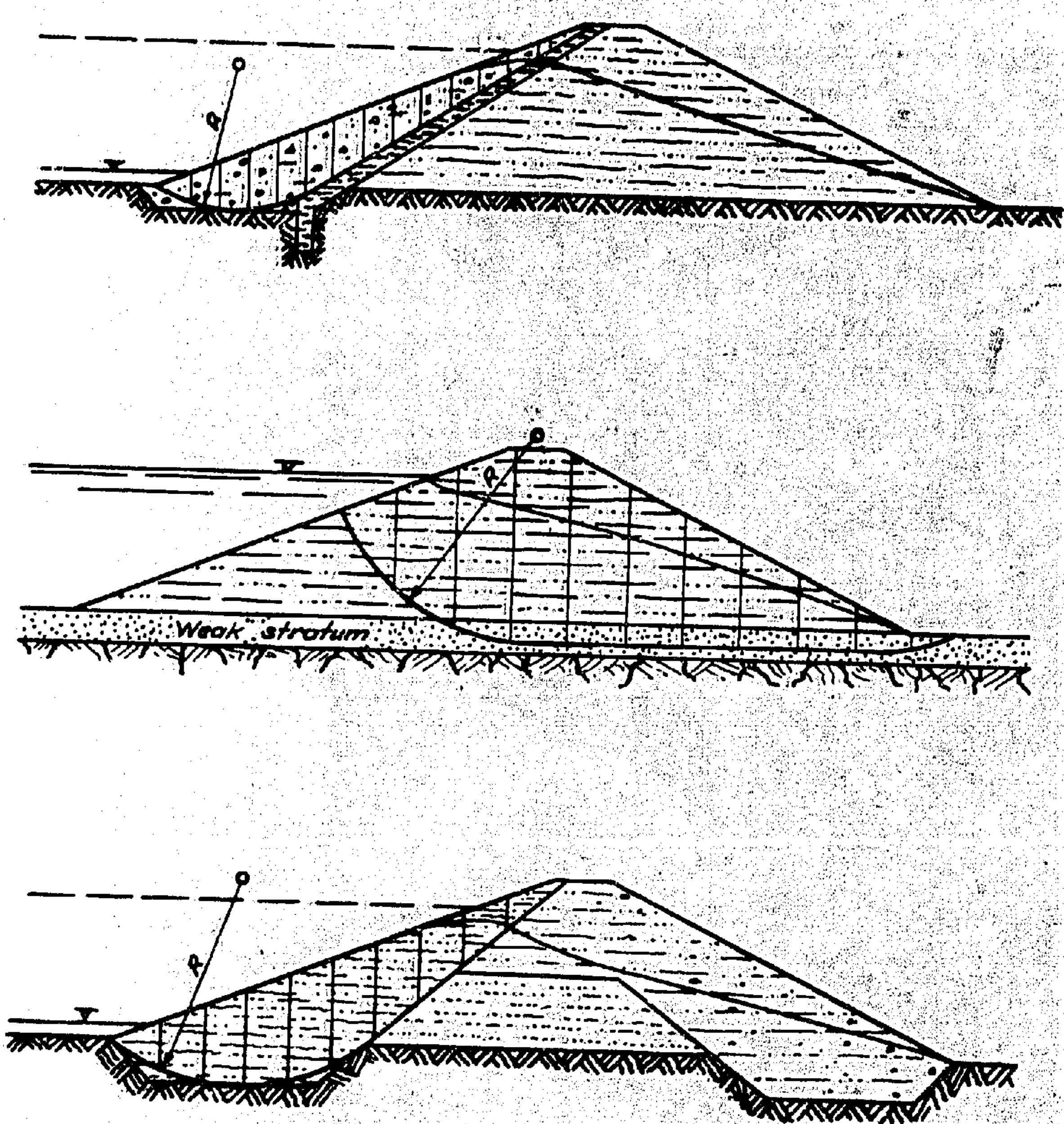
XD 1551

(the head loss indicated by the equipotential lines shown perpendicular to the directions of flow), by the void ratio, and by the density of water. These forces are graphically transferred to the trial sliding surface and their effects added to those of the gravity forces.

The method of trial sliding surfaces is adaptable to a great number of special cases as well as to the ideal. A few possibilities are illustrated in fig. 8.

Much additional testing is required to establish beyond doubt the behavior of different types of soils under widely varying conditions. Further checks between laboratory tests and field conditions are being sought. Also, every case analyzed by the trial surface method makes possible a closer initial approximation in each succeeding case. Some empirical rules for selecting the first trial have already been developed.

Reports including more detailed presentations and analyses of the results of shear testing and of the technique of stability studies are in preparation and it is hoped may be completed within the next few months.



Special cases.

Fig. 8

XD 1552