HYD 296

HYD-296

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Report No. HYD 296

Denver, Colorado
May 9, 1950

Memorandum

To: E. W. Lane

From: A. C. Carter

Subject: Critical tractive forces which start movement of sediment in a channel


Summary

1. In order to design a channel in noncohesive material which will be stable, it is necessary to know the nature and magnitude of forces causing scour and silting.

2. This investigation was made of all available literature on tractive force studies of models and natural channels in an attempt to find a relation between particle size of granular sediment and the critical tractive force which will start movement of the material.

3. The analysis of experimental data reviewed did not result in a usable solution to the problem, because traction flume data apparently cannot be correlated to prototype conditions.


Introduction

4. Reference is made to "Principles of Stable Channels in Erodible Material" by E. W. Lane, February 10, 1950. In this report it was suggested that a study be made of available literature in an attempt to determine a general relation between securing forces and particle size.

5. In the design of a stable channel in erodible material the flow characteristics must be such that scour of the banks and bed and deposit of sediment does not occur. The velocities at every point on the wetted perimeter should be sufficient to convey bed load and suspended material entering the system to the lower end without moving the material composing the banks. Particles resting on the level bottom of a channel are acted upon by forces due to the fluid in motion. If scour is to be prevented, these forces must be less in magnitude than the critical forces which will cause the particles to start to move.

6. Unfortunately, very little usable data on the forces due to velocity causing particle movement are available because of the difficulty in measuring velocities near the bed. However, many studies have shown that the movement of particles by flowing water is related

to tractive force. Tractive force is the component of weight of a volume of water acting in the direction of flow or the force due to velocity causing this volume of water to flow and is balanced by the resistance exerted by the wetted perimeter. This principle was discovered by M. P. du Boys[1] in 1879.

7. All available work on critical traction was assembled and classified into three parts for the purposes of comparison:

I. Experimental results of flume traction studies including tractive force and quantity of bed load movement for various sized particles.

II. Observed critical tractive force--Particle size relations based on flume and natural stream studies.

III. Equations expressing the critical tractive force--Particle size relation based on experimental results of traction flume studies.

An analysis of the experimental data in Parts I and II was then made in order to evaluate the proposed equations in Part III. This investigation should indicate which, if any, of these equations might be used in stable channel design to express the general relation between critical tractive force and particle size.

## Procedure

### Part I--Tractive Force Experiments

8. An analysis was made of a compilation of published and unpublished data on bed load transportation by J. W. Johnson.[7] This report summarized the basic hydraulic and sedimentary data from flume experiments in 10 laboratories in a consistent system of units. In addition, observations were corrected for side wall effect using the Einstein[4] method where this had not been done by the experimenter. Another source of data reviewed was the National Bureau of Standards'[13] report on scour of a sandy riverbed by clear and muddy water which was requested by the Bureau of Reclamation November 1, 1932.

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1/7/13/ References refer to bibliography.

9. The procedure followed using the Johnson data was to plot values of friction velocity ($V_* = \sqrt{\frac{T_c}{\rho}}$), which is a function of tractive force, against quantity of bed load movement for each size of sand or average size of sand mixture tested. The expression for tractive force may be expressed:

$$T = wRS$$

T = tractive force--lb/ft$^2$

w = unit weight of water--62.4 lb/ft$^3$

R = hydraulic radius--ft

S = energy gradient

Representative curves were then drawn through the plotted points and extended to zero discharge to obtain critical values of friction velocity. In addition, envelope curves were drawn above and below all of the plotted points to indicate the range of possible values of critical friction velocity. The point and range of values of friction velocity for zero sediment discharge were then converted to critical tractive force and plotted against mean particle diameter logarithmically. For clarity, points in the same experiment were connected. See Figure 1 for plotted values and Table 1 for tabulated values.

10. The procedure followed in analyzing the National Bureau of Standards' data was essentially the same, except that quantities of sediment transported were plotted directly against tractive force. Envelope curves were then drawn and extended to zero sediment discharge to obtain the range of critical tractive force for each grain diameter. These values were listed in Table 2 and also plotted against particle diameter logarithmically in Figure 1. The minimum value of each range of tractive force in the same experiment was connected. Indri's equations were plotted in Figure 1 for the purpose of comparison with other data.

## Part II--Tractive Force Observations

11. A number of investigators found critical tractive forces for various sizes of sediment by visual methods. In general, two criterions of critical conditions were used: (1) initial movement of grains in the bed and (2) general movement of the bed. Observed critical tractive

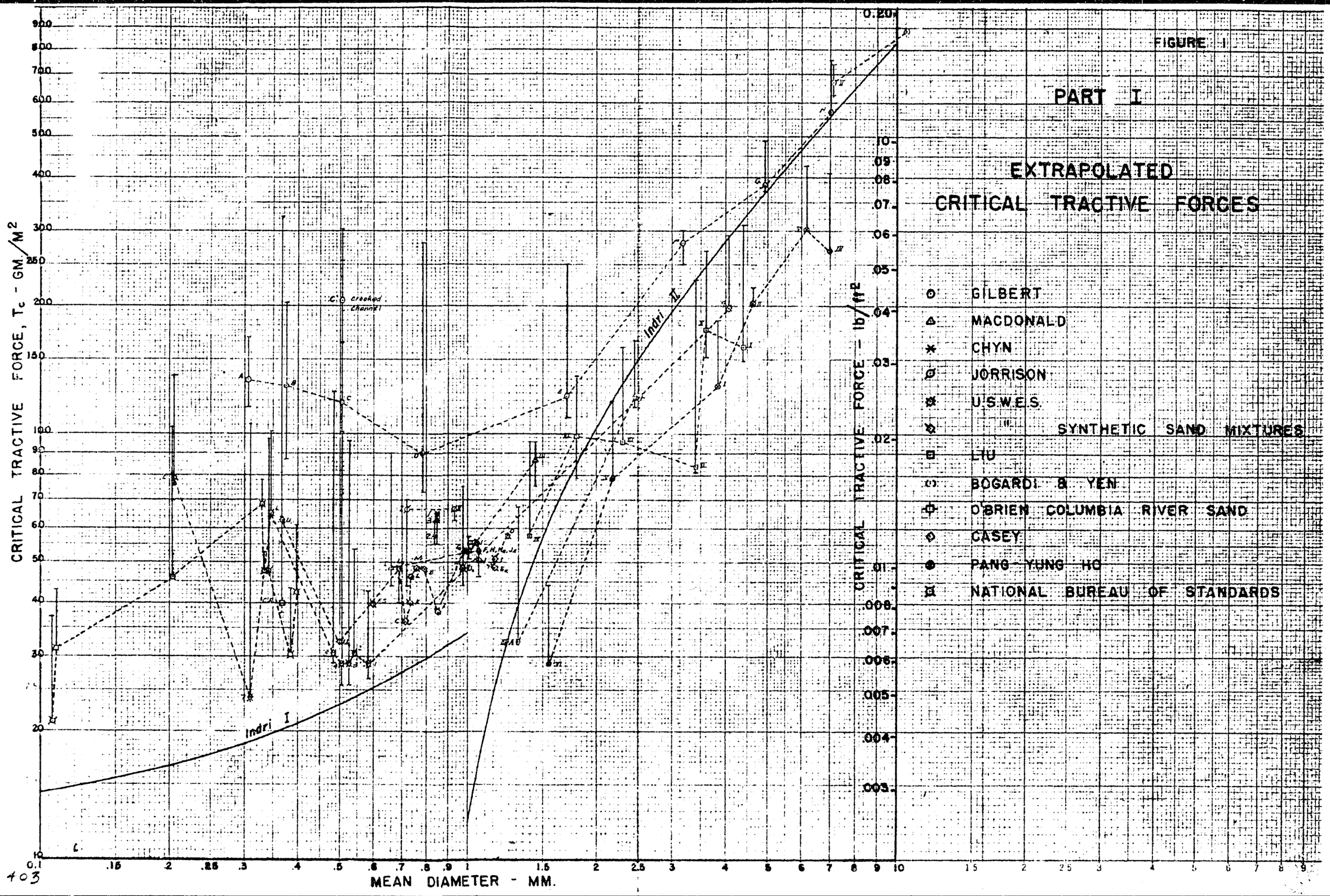
FIGURE 1
PART I
EXTRAPOLATED
CRITICAL TRACTIVE FORCES
GILBERT
MACDONALD
CHYN
JORRISON
U.S.W.E.S.
" SYNTHETIC SAND MIXTURES
LIU
BOGARDI & YEN
O'BRIEN COLUMBIA RIVER SAND
CASEY
PANG-YUNG HO
NATIONAL BUREAU OF STANDARDS
CRITICAL TRACTIVE FORCE, $T_c$ - GM./M$^2$
CRITICAL TRACTIVE FORCE - lb/ft$^2$
MEAN DIAMETER - MM.
Indri I
Indri II
Crooked Channel

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Table 1

CRITICAL TRACTIVE FORCES EXTRAPOLATED FROM JOHNSON DATA

| Experiment | Type of material | Mean diameter mm | Friction velocity $V_*$ cm/sec | Range of $V_*$ cm/sec | Critical tractive force, $\rho(V_*)^2$ g/m$^2$ | Range of $T_c$ g/m$^2$ |
|---|---|---|---|---|---|---|
| Gilbert sand A | River sand | .305 | 3.66 | 3.4 -4.1 | 137 | 118 -171.5 |
| Gilbert sand B | River sand | .375 | 3.60 | 2.95-4.5 | 132 | 88.8-207 |
| Gilbert sand C | River sand | .506 | 3.43 | 3.25-5.5 | 120 | 108 -308 |
| Gilbert sand D | River sand | .786 | 3.0 | 2.70-5.3 | 91.9 | 74.5-287 |
| Gilbert sand E | River sand | 1.710 | 3.5 | 3.3 -5.0 | 125 | 111 -255 |
| Gilbert sand F | River sand | 3.170 | 5.3 | 5.0 -5.5 | 286 | 255 -309 |
| Gilbert sand G | River sand | 4.938 | 6.2 | 6.1 -7.0 | 392 | 379 -500 |
| Gilbert sand H | River sand | 7.010 | 7.55 | 7.5 -8.7 | 581 | 573 -772 |
| Gilbert sand C' | Beach sand | .506 | 4.55 | 4.05-5.05 | 212 | 167 -260 |
| MacDougall sand I | Beach sand | .66 | 2.2 | 2.1 -3.0 | 49.4 | 45.0- 92 |
| MacDougall sand II | Beach sand | 1.01 | 2.3 | 2.25-2.4 | 54.0 | 51.6- 58.8 |
| MacDougall sand III | Beach sand | 1.44 | 2.95 | 2.75-3.1 | 88.8 | 77.1- 98.0 |
| Chyn sand 3 | Beach sand | .847 | 2.5 | 2.35-2.55 | 63.8 | 56.3- 66.3 |
| Chyn sand 2 | Beach sand | .840 | 2.4 | 2.35-2.5 | 58.8 | 56.3- 63.8 |
| Chyn sand 1 | Beach sand | .849 | 2.55 | 2.5 -2.57 | 66.3 | 63.8- 67.3 |
| Jorrison sand I | Beach sand | .72 | 2.58 | 2.1 -2.65 | 67.9 | 45.0- 71.7 |
| Jorrison sand II | Beach sand | .93 | 2.58 | 2.5 -2.6 | 67.9 | 63.8- 68.9 |
| U.S.W.E.S. sand 1 | River sand | .586 | 1.7 | 1.63-2.07 | 28.5 | 27.1- 43.7 |
| U.S.W.E.S. sand 2 | River sand | .541 | 1.75 | 1.75-2.32 | 31.3 | 31.3- 55.0 |
| U.S.W.E.S. sand 3 | River sand | .525 | 1.7 | 1.6 -3.1 | 29.5 | 26.2- 98.0 |
| U.S.W.E.S. sand 4 | River sand | .506 | 1.7 | 1.6 -3.2 | 29.5 | 26.2-104 |
| U.S.W.E.S. sand 5 | River sand | .483 | 1.75 | 1.7 -3.55 | 31.3 | 29.5-128 |
| U.S.W.E.S. sand 6 | River sand | .347 | 2.5 | 2.5 -3.2 | 63.7 | 63.7-104 |
| U.S.W.E.S. sand 7 | River sand | .310 | 1.55 | 1.53-3.25 | 24.5 | 23.8-108 |
| U.S.W.E.S. sand 8 | River sand | .205 | 2.8 | 2.75-3.7 | 80.0 | 77.2-139 |
| U.S.W.E.S. sand 9 | River sand | 4.077 | 4.45 | 4.45-5.4 | 202 | 202 -297 |
| U.S.W.E.S. sand 10 | River sand | .976 | 2.2 | 2.1 -2.75 | 49.3 | 44.2- 77 |
| U.S.W.E.S. UO.589 (sand) | Synthetic | .691 | 2.2 | 2.0 -2.2 | 49.3 | 40.8- 49.3 |
| U.S.W.E.S. UO.833 (sand) | Synthetic | .976 | 2.2 | 2.2 -2.75 | 49.3 | 49.3- 77.2 |
| U.S.W.E.S. UO.293 (sand) | Synthetic | .367 | 2.5 | 2.35-2.85 | 63.7 | 56.3- 82.9 |
| U.S.W.E.S. sand A | Synthetic | .853 | 1.95 | No range | 38.8 | |
| U.S.W.E.S. sand B | Synthetic | .794 | 2.2 | No range | 49.3 | |
| U.S.W.E.S. sand C | Synthetic | .715 | 1.9 | No range | 36.8 | |
| U.S.W.E.S. sand D | Synthetic | 1.224 | 2.4 | No range | 58.8 | |
| U.S.W.E.S. sand E | Synthetic | 1.162 | 2.25 | No range | 51.7 | |
| U.S.W.E.S. sand F | Synthetic | 1.008 | 2.3 | No range | 53.9 | |
| U.S.W.E.S. sand G | Synthetic | .986 | 2.3 | No range | 53.9 | |
| U.S.W.E.S. sand H | Synthetic | 1.067 | 2.3 | No range | 53.9 | |

Table 1--Continued

CRITICAL TRACTIVE FORCES EXTRAPOLATED FROM JOHNSON DATA

| Experiment | Type of material | Mean diameter mm | Friction velocity $V_*$ cm/sec | Range of $V_*$ cm/sec | Critical tractive force, $T_c = \rho(V_*)^2$ g/m$^2$ | Range of $T_c$ g/m$^2$ |
|---|---|---|---|---|---|---|
| U.S.W.E.S. sand I | Synthetic | 1.052 | 2.35 | No range | 56.3 | |
| U.S.W.E.S. sand UO.417 | Synthetic | .498 | 1.8 | No range | 33.0 | |
| U.S.W.E.S. sand J | Synthetic | 1.066 | 2.25 | 2.15-2.25 | 51.7 | 47.2- 51.7 |
| U.S.W.E.S. sand K | Synthetic | .732 | 2.0 | No range | 40.8 | |
| U.S.W.E.S. sand L | Synthetic | .738 | 2.15 | No range | 47.2 | |
| U.S.W.E.S. sand M | Synthetic | .759 | 2.2 | No range | 49.3 | |
| U.S.W.E.S. sand $J_R$ | Synthetic | 1.066 | 2.3 | No range | 53.9 | |
| U.S.W.E.S. sand $E_R$ | Synthetic | 1.162 | 2.2 | No range | 49.3 | |
| U.S.W.E.S. filter sand | Synthetic | .597 | 2.0 | No range | 40.8 | |
| U.S.W.E.S. sand $H_R$ | Synthetic | 1.067 | 2.3 | 2.15-2.35 | 53.9 | 47.1- 56.3 |
| Liu material I | River sand | 4.4 | 4.0 | 3.85-5.57 | 163 | 155 -316 |
| Liu material II | River sand | 3.4 | 2.9 | 2.85-4.8 | 85.8 | 83.0-235 |
| Liu material III | River sand | 2.3 | 3.1 | 3.1 -4.0 | 98.0 | 98.0-163 |
| Liu material IV | River sand | 1.4 | 2.4 | 2.4 -3.1 | 58.8 | 58.8- 98.0 |
| Liu material V | River sand | 3.6 | 4.2 | 3.9 -5.2 | 180 | 155 -276 |
| Liu material VI | River sand | 1.8 | 3.15 | 2.8 -3.7 | 101 | 80 -139 |
| Bogardi and | | | | | | |
| Yen material I | Gravel | 10.56 | 9.4 | No range | 900 | |
| Yen material II | Gravel | 7.12 | 8.2 | 7.9 -8.6 | 685 | 637 -753 |
| Yen material III | Gravel | -- | -- | | -- | |
| O'Brien Columbia River sand | River sand | .366 | 2.0 | 1.95-5.7 | 40.8 | 38.8- 33.1 |
| Casey sand h | Sand | 2.455 | 3.5 | 3.4 -4.2 | 125 | 118 -180 |
| Casey sand III A | Sand | 1.32 | 1.8 | 1.8 -2.6 | 33.0 | 33.0- 68.0 |
| Pang-Yung mixture I | Gravel | 3.83 | 3.6 | 3.6 -4.3 | 132 | 132 -188 |
| Pang-Yung mixture II | Gravel | 4.64 | 4.5 | 4.5 -4.7 | 207 | 207 -225 |
| Pang-Yung mixture III | Gravel | 6.99 | 5.2 | 5.2 -6.4 | 276 | 276 -417 |
| Pang-Yung mixture IV | Gravel | 2.18 | 2.8 | 2.8 -3.45 | 80 | 80 -121 |
| Pang-Yung mixture V | Gravel | 6.17 | 5.5 | 5.5 -6.5 | 308 | 308 -430 |
| Pang-Yung mixture VI | Sand | 1.55 | 1.7 | 1.7 -2.1 | 29.5 | 29.5- 44.5 |

Table 2

CRITICAL TRACTIVE FORCES EXTRAPOLATED FROM NATIONAL BUREAU OF STANDARDS DATA[13]

| Type of material | Diam mm | $T_c$ lb/ft$^2$ | $T_c$ g/m$^2$ |
|---|---|---|---|
| Asphalt sand | .330 | .0142-.016 | 68.3 - 78.1 |
| Asphalt sand | .334 | .0098-.0108 | 47.8 - 52.7 |
| Asphalt sand passing 70 mesh | .204 | .0094-.0212 | 45.9 -103.5 |
| Coarse asphalt sand—clear water | .384 | .0062-.0089 | 30.25- 43.4 |
| Asphalt sand with kaolin undispersed | .398 | .0087-.0125 | 42.5 - 61.0 |
| Coarse asphalt sand with kaolin dispersed | .342 | .0098-.020 | 47.8 - 97.6 |
| Colorado sand—clear water | .107 | .0043-.0076 | 21.0 - 37.1 |
| Colorado sand with dispersed kaolin and Virginia clay | .109 | .0064-.0088 | 31.2 - 43.0 |

force data from 11 studies were taken from various sources and converted to consistent units. No correction for side wall effect was made because of a lack of sufficient information. Whenever reported, the criterion used by the observer was indicated in this review. The tabulated data with bibliography references appear in Tables 3 to 8. The data were plotted to logarithmic scale in Figure 2. Indri's equations were also plotted in Figure 2 to serve as a basis for comparison with other data.

12. Chang[3] and O'Brien[15] listed a number of observations of critical tractive forces for different sediments which were apparently defined by a zero rate of sediment transportation, although this information was not clearly indicated. Rather than include these results in Part I or the observed critical tractive forces above, they were presented separately in Tables 9 and 10 with bibliography references and plotted in Figure 3. Indri's equations were also shown in Figure 3.

## Part III--Tractive Force Equations

13. In most instances, the investigations of sediment transportation led to an equation which related critical tractive force to grain size. In fact, as many equations were derived to fit experimental results as there were investigators. In order to make a comparison with basic data and evaluate the results, 13 suggested formulas were selected and converted to consistent units. Certain assumptions were necessary regarding some of the terms involved. These were made to agree with usual practice and kept the same in each case. Each equation was reduced to the form:

$$T_c = C_1 d^n + C_2$$

$T_c$ = critical tractive force

$C_1 C_2$ = empirical constants (dimensional)

$d^n$ = mean diameter of sediment to some power

The calculations with references are presented in Appendix I, and a summary of all equations used is given in Table 11. The relations are shown graphically in Figure 4.

OBSERVED CRITICAL TRACTIVE FORCES FROM LABORATORY FLUME AND NATURAL CHANNEL STUDIES

| Table | Observer | Page |
|---|---|---|
| 3 | Nuernberg Kulturamt | |
| 4 | Straub | |
| 5 | U.S.W.E.S. | |
| 6 | Chang Summary | |
| | Chang | |
| | Krey | |
| | P.E.I. | |
| | Engels | |
| 7 | O'Brien Summary | |
| | Chitty Ho | |
| | Kramer | |
| 8 | National Bureau of Standards | |
| 8a | Indri | |

Table 3

NUERNBERG KULTURAMT OBSERVED CRITICAL TRACTIVE FORCE DATA[16] — CRITERION UNKNOWN

Limiting transporting forces $T_c$ for various kinds of detritus in its natural state in rivers, as measured by Government engineers in Nuernberg:

| Kind of detritus | Diameter | $T_c$ kg/m$^2$ | $T_c$ gm/m$^2$ |
|---|---|---|---|
| Ordinary quartz sand | 0.4 to 1 mm | 0.25 to 0.30 | 250 to 300 |
| Ordinary quartz sand | 0.2 to 0.4 mm | 0.18 to 0.20 | 180 to 200 |
| Ordinary quartz sand | 1.0 to 2 mm | 0.40 | 400 |
| Roundish quartz gravel | 0.5 to 1.5 cm | 1.25 | 1,250 |
| Clayey soil | | 1.0 to 1.2 | 1,000 to 1,200 |
| Coarse quartz stones | 4 to 5 cm | 4.8 | 4,800 |

Table 4

STRAUB'S OBSERVED CRITICAL TRACTIVE FORCES OF VARIOUS GRADES OF SEDIMENT BASED ON RESEARCH WORK OF SCHOKLITSCH, SCHAFFERNAK, GILBERT, AND STRAUB[20] — CRITERION UNKNOWN

| Mean diam in mm | General classification | Value of $T_c$ lb/sq ft | Value of $T_c$ g/m$^2$ |
|---|---|---|---|
| 1/8 | | 0.0162 | 79.[illegible] |
| | Fine sand | | |
| 1/4 | | .0172 | 84.[illegible] |
| | Medium sand | | |
| 1/2 | | .0215 | 104.[illegible] |
| | Coarse sand | | |
| 1 | | .0316 | 154.[illegible] |
| | Very coarse sand | | |
| 2 | | .0513 | 250.[illegible] |
| | Granule gravel | | |
| 4 | | .089 | 434.[illegible] |

Table 5

U.S.W.E.S. OBSERVED CRITICAL TRACTIVE FORCE DATA[22]

| Sand No. | Grain diam mm | $T_c$ Model criterion* lb/ft$^2$ | $T_c$ Model criterion* g/m$^2$ | $T_c$ Visual criterion** lb/ft$^2$ | $T_c$ Visual criterion** g/m$^2$ | Average $T_c$ g/m$^2$ |
|---|---|---|---|---|---|---|
| 1 | .5861 | .0077 | 36.3 | .0084 | 39.6 | 45.9 |
| | | .0072 | 33.93 | .0092 | 43.4 | |
| | | .0081 | 38.2 | .0107 | 50.5 | |
| 2 | .5409 | .0130 | 61.3 | .0080 | 37.7 | 42.9 |
| | | .0180 | 84.9 | .0090 | 42.4 | |
| | | .0170 | 80.2 | .0095 | 44.8 | |
| 3 | .5246 | .0198 | 93.4 | .0085 | 40.1 | 42.9 |
| | | .0215 | 101.3 | .0090 | 42.4 | |
| | | .0160 | 75.4 | .0090 | 42.4 | |
| 4 | .5056 | .0213 | 100.4 | .0080 | 37.7 | 41.0 |
| | | .0219 | 103.2 | .0083 | 39.1 | |
| | | .0222 | 104.7 | .0090 | 42.4 | |
| 5 | .4828 | .0234 | 110.3 | .0080 | 37.7 | 39.0 |
| | | .0230 | 108.4 | .0075 | 35.35 | |
| | | .0248 | 116.9 | .0085 | 40.1 | |
| 6 | .3170 | .0236 | 111.3 | .0060 | 28.3 | 29.3 |
| | | .0252 | 118.8 | .0060 | 28.3 | |
| | | .0223 | 105.2 | .0060 | 28.3 | |
| 7 | .3104 | .0228 | 107.5 | .0070 | 33.0 | 32.2 |
| | | .0227 | 107.0 | .0058 | 27.35 | |
| | | .0246 | 116.0 | .0070 | 33.0 | |
| 8 | .2053 | .0210 | 99 | .0051 | 24.03 | 22.9 |
| | | .0210 | 99 | .0048 | 22.63 | |
| | | .0244 | 115 | .0042 | 19.8 | |
| 9 | 4.0769 | .057 | 268.8 | .057 | 268.8 | 28.3 |
| | | .057 | 268.8 | .057 | 268.8 | |
| | | .0600 | 283 | .0600 | 283 | |

*Model criterion:

(1) Material in transportation is reasonably similar in composition to the material composing the original bed.

(2) The rate of movement is equal to or exceeds one pound (dry weight) per foot width of channel per hour.

**Visual criterion: General movement.

## Table 6

OBSERVED CRITICAL TRACTIVE FORCE DATA
COMPILED BY Y. L. CHANG[3]

| Y. L. Chang—General Movement Criterion | | | | | | |
|---|---|---|---|---|---|---|
| Sample No. | Diam mm | wSD lb/ft$^2$ | wSD g/m$^2$ | wSD/2 g/m$^2$ | $T_c$ lb/ft$^2$ | $T_c$ g/m$^2$ |
| 6 205 | 8.090 | .3620 | 1766 | 883 | .185 | 903 |
| 6 204 | 6.540 | .2780 | 1356 | 678 | .139 | 678 |
| 6 203 | 5.290 | .2220 | 1083 | 542 | .111 | 542 |
| 6 202 | 3.770 | .1830 | 893 | 446 | .091 | 444 |
| 6 201 | 2.540 | .0970 | 473 | 236 | .0485 | 236.7 |
| 6 200 | 1.760 | .0699 | 341 | 170 | .0349 | 170.4 |
| 6 199 | .886 | .0327 | 159.6 | 79.8 | .0166 | 81.0 |
| 6 198 | .396 | .0143 | 69.8 | 34.9 | .00757 | 36.9 |
| 6 197 | .226 | .0114 | 55.7 | 27.8 | .00635 | 31.0 |
| Pumice | .289 | .0098 | 47.8 | 23.9 | .00510 | 24.9 |
| Emery | .210 | .0142 | 69.3 | 34.6 | .00767 | 37.4 |
| Sand ∞ | .134 | .0075 | 36.6 | 18.3 | .00395 | 17.86 |

| H. Krey—Criterion Unknown | | | |
|---|---|---|---|
| Sample No. | Diam mm | $T_c$ lb/ft$^2$ | $T_c$ g/m$^2$ |
| G | .376 | .00492 | 24.0 |
| H | .526 | .00574 | 28.0 |
| J | .800 | .00656 | 32.0 |

| Prussian Experiment Institute—Criterion Unknown | | | |
|---|---|---|---|
| Sample No. | Diam mm | $T_c$ lb/ft$^2$ | $T_c$ g/m$^2$ |
| K | 1.154 | .01046 | 51 |
| L | .846 | .00861 | 42 |
| M | .836 | .01190 | 58 |
| N | .744 | .01148 | 56 |
| O | .244 | .00492 | 24 |
| P | .806 | .01148 | 56 |
| Q | .686 | .01025 | 50 |

| H. Engels—Criterion Unknown | | | |
|---|---|---|---|
| Sample No. | Diam mm | $T_c$ lb/ft$^2$ | $T_c$ g/m$^2$ |
| R | 1.484 | .0205 | 72.4 |

Table 7

OBSERVED CRITICAL TRACTIVE FORCE DATA
COMPILED BY O'BRIEN AND RINDLAUB[15]

Chitty Ho--Initial Movement Criterion

| Sand No. | Median diam feet | Median diam mm | Coefficient n* | Computed critical shear lb/ft$^2$ | Critical shear g/m$^2$ |
|---|---|---|---|---|---|
| A | .0188 | 5.73 | .0202 | .052 | 254 |
| B | .0132 | 4.02 | .0189 | .048 | 234 |
| C | .0093 | 2.83 | .0179 | .028 | 137 |
| D | .0066 | 2.02 | .0168 | .020 | 97.6 |
| E | .0047 | 1.43 | .0159 | .0133 | 64.9 |
| F | .0033 | 1.01 | .0150 | .0080 | 39.0 |
| G | .0023 | .70 | .0141 | .0034 | 16.6 |
| H | .0016 | .49 | .0134 | .0025 | 12.2 |
| I | .0012 | .37 | .0126 | .0014 | 6.83 |
| A' | .0181 | 5.52 | .0202 | .052 | 254 |
| B' | .0132 | 4.02 | .0189 | .033 | 161 |

*Strickler's $n = 0.039\ d^{1/6}$ (d = ft)

Hans Kramer--General Movement Criterion

| Sand No. | Diameter feet | Diameter mm | $T_c$ lb/ft$^2$ | $T_c$ g/m$^2$ |
|---|---|---|---|---|
| I | .00174 | .53 | .00973 | 47.5 |
| II | .00168 | .512 | .00745 | 36.4 |
| III | .00180 | .549 | .00917 | 44.75 |

Table 8

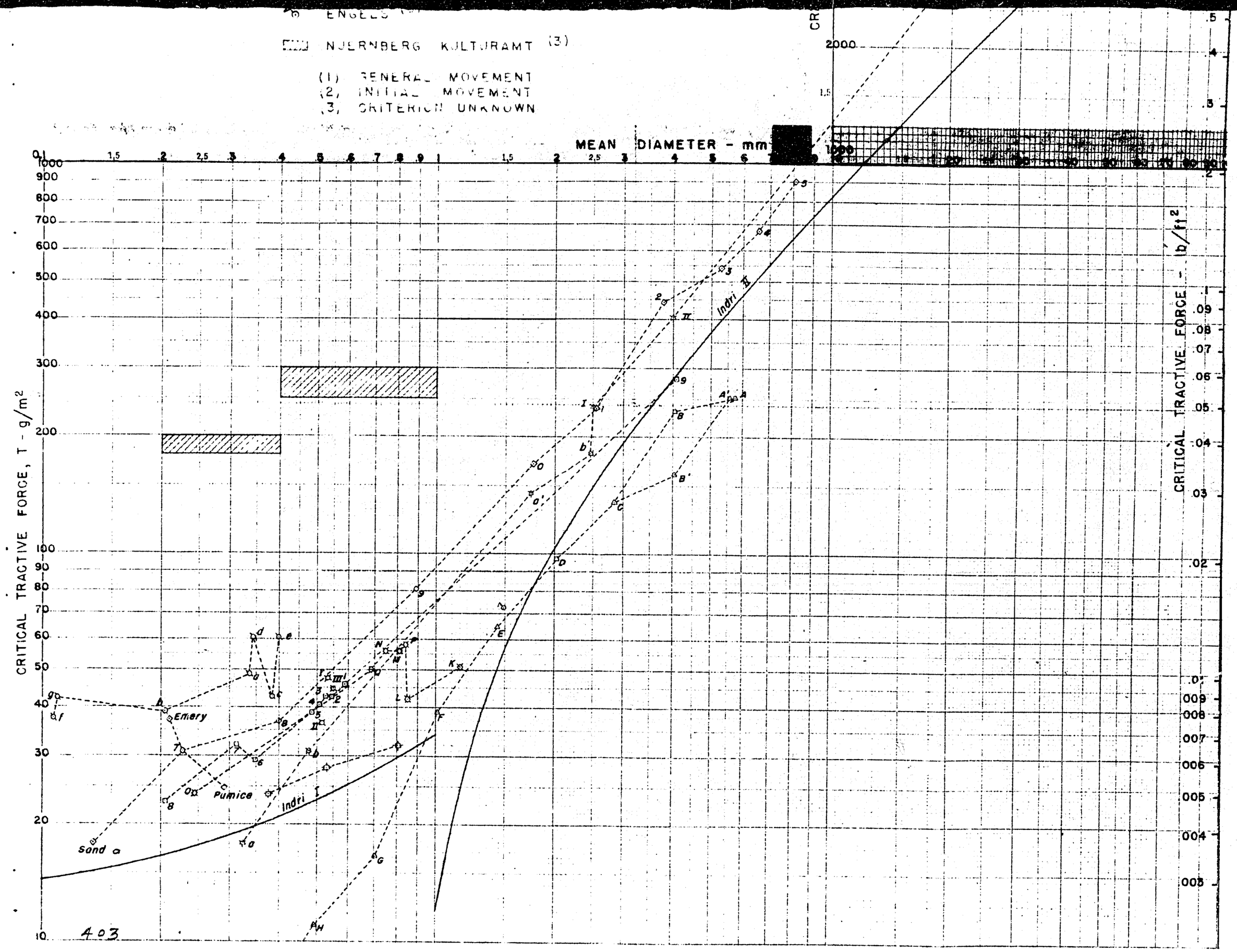
NATIONAL BUREAU OF STANDARDS[13] CRITICAL TRACTIVE FORCE DATA--GENERAL MOVEMENT CRITERION

| No. | Material | Avg diam sand $\mu$ | Diam mm | $T_c$ lb/ft$^2$ | $T_c$ g/m$^2$ |
|---|---|---|---|---|---|
| a | Asphalt sand | 336 | 0.336 | .0100 | 48.8 |
| b | Asphalt sand < 70 | 204 | .204 | .008 | 39.05 |
| c | Asphalt sand > 70 | 384 | .384 | .0088 | 42.95 |
| d | Asphalt sand + kaolin | 342 | .342 | .0125 | 61.0 |
| e | Asphalt sand + kaolin | 398 | .398 | .0125 | 61.0 |
| f | Colorado sand | 107 | .107 | .0077 | 37.58 |
| g | Colorado sand + kaolin | 109 | .109 | .0087 | 42.45 |

Table 8a

INDRI[24] OBSERVED CRITICAL TRACTIVE FORCE DATA
GENERAL MOVEMENT CRITERION

| Material | Diameter mm | $T_c$ g/m$^2$ |
|---|---|---|
| 1 | 20 | 3117 |
| 2 | 45 | 5583 |
| 3 | 80 | 10.912 |
| 4 | 125 | 15.220 |
| 5 | 58.8 | 21.688 |
| 6 | 58.8 | 21.268 |
| I | 2.5 | 240 |
| II | 4.0 | 408 |
| a' | 1.74 | 144 |
| b' | 2.47 | 182 |
| a | 0.324 | 18 |
| b | 0.474 | 31 |


ENGELS
NUERNBERG KULTURAMT (3)
(1) GENERAL MOVEMENT
(2) INITIAL MOVEMENT
(3) CRITERION UNKNOWN
MEAN DIAMETER - mm
CRITICAL TRACTIVE FORCE, T - g/m²
CRITICAL TRACTIVE FORCE - lb/ft²
Indri II
Indri I
Emery
Pumice
Sand a
403

# FRAME 2

## PART II

### OBSERVED CRITICAL TRACTIVE FORCES

- U.S.W.E.S. (1)
- CHANG (1)
- NAT'L. BUR. OF STANDARDS (1)
- KRAMER (1)
- INDRI (1)
- CHITTY HO (2)
- KREY (3)
- PRUSSIAN EXP. INSTITUTE (3)
- ENGELS (3)
- NUERNBERG KULTURAMT (3)

(1) GENERAL MOVEMENT
(2) INITIAL MOVEMENT
(3) CRITERION UNKNOWN

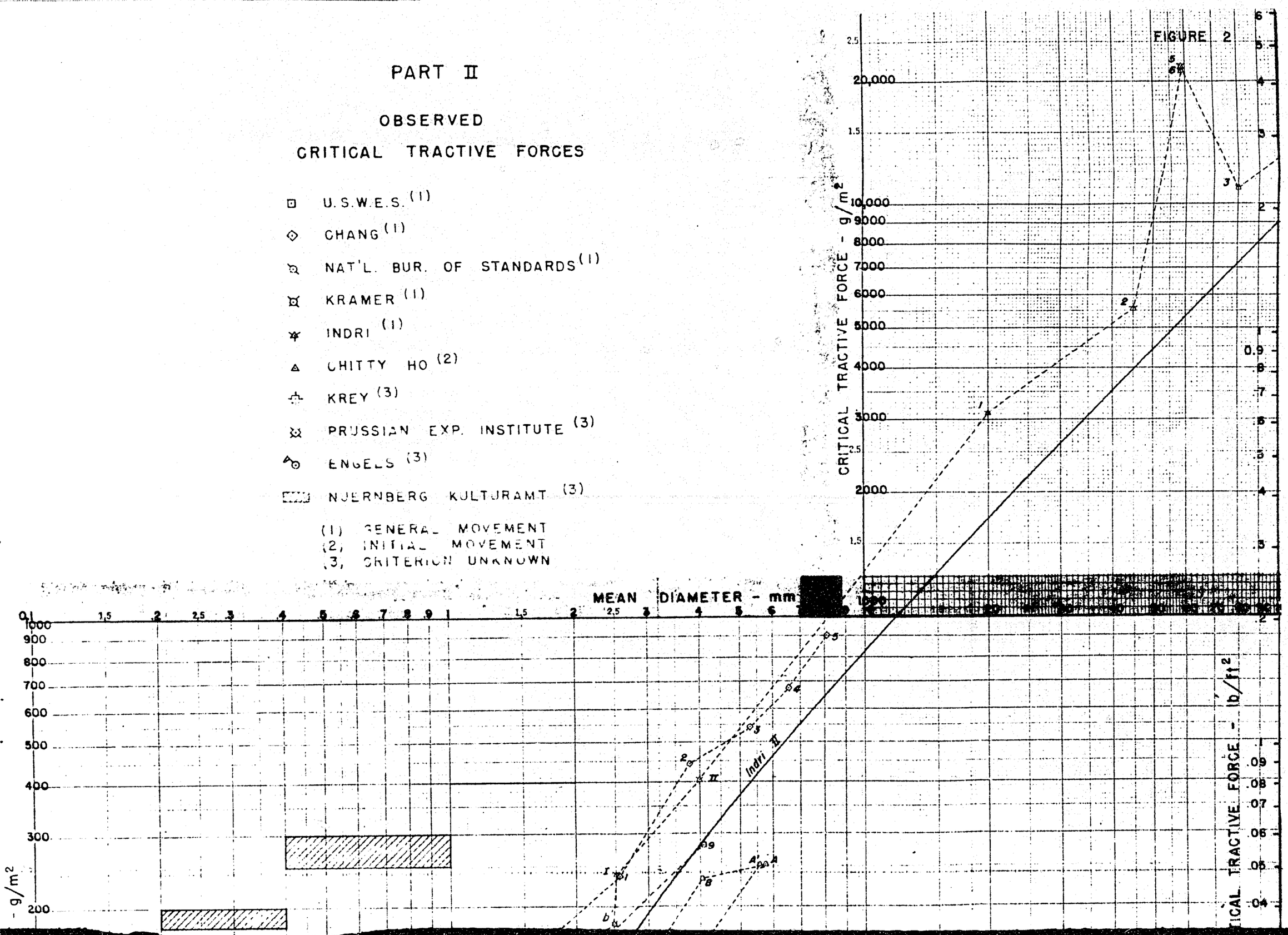


Table 9

EXTRAPOLATED CRITICAL TRACTIVE FORCE
DATA COMPILED BY Y. L. CHANG[3]

Schaffernak

| Sand No. | Diameter mm | $T_c$ lb/ft$^2$ | $T_c$ g/m$^2$ |
|---|---|---|---|
| A | 1.536 | .01538 | 75.1 |

G. K. Gilbert

| Sand No. | Diameter mm | $T_c$ lb/ft$^2$ | $T_c$ g/m$^2$ |
|---|---|---|---|
| S | .576 | .008 | 39.0 |
| T | 1.906 | .01251 | 61.0 |
| U | 3.710 | .0605 | 295 |
| V | 5.296 | .1005 | 491 |

Table 10

EXTRAPOLATED CRITICAL TRACTIVE FORCE DATA
COMPILED BY O'BRIEN AND RINDLAUB[15]

G. K. Gilbert[5]

| Sand No. | Diameter mm | Least $T_c$ used lb/ft$^2$ | Least $T_c$ used g/m$^2$ | Estimated $T_c$ lb/ft$^2$ | Estimated $T_c$ g/m$^2$ |
|---|---|---|---|---|---|
| A | .3048 | .0247 | 120.5 | .021 | 102.5 |
| B | .375 | .0195 | 95.2 | .016 | 78.1 |
| C | .509 | .0230 | 112.3 | .020 | 97.6 |
| D | .780 | .0212 | 134.0 | .0116 | 56.6 |
| E | 1.706 | .0224 | 109.3 | .020 | 97.6 |
| F | 3.17 | .0640 | 312.3 | .047 | 229.3 |
| G | 4.94 | .0750 | 366 | .062 | 302.5 |
| H | 7.01 | .0920 | 449 | .068 | 331.8 |

MacDougal

| Sand No. | Diameter mm | $T_c$ lb/ft$^2$ | $T_c$ g/m$^2$ |
|---|---|---|---|
| --- | 1.006 | .012 | 58.6 |

Schaffernak

| Sand No. | Diameter mm | $T_c$ lb/ft$^2$ | $T_c$ g/m$^2$ |
|---|---|---|---|
| I | 3.00 | .01 | 48.8 |
| II | 5.00 | .03 | 146.4 |
| III | 10.00 | .40 | 1952 |
| | 10.00 | .30 | 1464 |
| | 10.00 | .20 | 976 |
| | 10.00 | .15 | 732 |
| IV | 16.1 | .60 | 2930 |
| | 16.1 | .70 | 3416 |
| | 16.1 | .30 | 1464 |
| | 16.1 | .20 | 976 |

Table 10--Continued

EXTRAPOLATED CRITICAL TRACTIVE FORCE DATA
COMPILED BY O'BRIEN AND RINDLAUB[15]

U.S.W.E.S.[14] (Nichols)

| Sand | Size feet | Size mm | Stated $T_c$ $g/m^2$ | wDS $lb/ft^2$ | wDS $g/m^2$ |
|---|---|---|---|---|---|
| A | .0089 | .2712 | 11.91 | .00244 | 11.9 |
| B | .001105 | .337 | 14.84 | .00342 | 16.69 |
| C | .001305 | .398 | 46.80 | .00960 | 46.85 |
| D | .00143 | .436 | 193.19 | .03960 | 193.2 |

Chipps Island

| Boat pos | Diameter feet | Diameter mm | $T_c$ $lb/ft^2$ | $T_c$ $g/m^2$ |
|---|---|---|---|---|
| 3 | .00167 | .509 | .0072 | 35.12 |
| 6 | .00125 | .381 | .0055 | 26.83 |

Schoklitsch[17]

| Material | Form | Diameter feet | Diameter mm | $T_c$ $lb/ft^2$ | $T_c$ $g/m^2$ |
|---|---|---|---|---|---|
| L | Quartz sand | .0030 | .915 | .00575 | 28.07 |
| K | Quartz sand | .0041 | 1.25 | .00905 | 44.17 |
| | Slate | | | | |
| H | Thin flakes | .0041 | 1.25 | .018 | 87.8 |
| | Ore | | | | |
| F | Sharp frag | .0047 | 1.432 | .0274 | 133.7 |
| J | Porc sphere | .00553 | 1.685 | .0101 | 49.3 |
| G | Quartz sand | .0074 | 2.255 | .0231 | 112.7 |
| | Slate | | | | |
| C | Thin flakes | .010 | 3.048 | .0695 | 339 |
| D | Quartz sand | .0133 | 4.055 | .054 | 263.5 |
| A | Porc sphere | .0139 | 4.24 | .0204 | 99.6 |
| E | Porc sphere | .0139 | 4.24 | .0406 | 198 |
| B | Quartz sand | .0214 | 6.525 | .111 | 542 |

Table 10--Continued

EXTRAPOLATED CRITICAL TRACTIVE FORCE DATA
COMPILED BY O'BRIEN AND RINDLAUB[15]

Strickler[21]

| Sand | Diameter feet | Diameter mm | $T_c$ lb/ft$^2$ | $T_c$ g/m$^2$ |
|---|---|---|---|---|
| Rhine, Waldhut | .093 | 28.33 | .376 | 1835 |
| Rhine, Base 1 | .168 | 51.2 | .652 | 3182 |
| Rhine, No. 1 | .043 | 13.1 | .104 | 508 |
| Rhine, St. Margrethen | .101 | 30.8 | .213 | 1040 |
| Rhine, Holland | .027* | 8.23 | .055 | 268.5 |
| Rhine, Neastrils | .183 | 55.8 | 1.31 | 6400 |
| Aare, Aaran | .063 | 19.2 | .254 | 1240 |
| Rhone, Porte du Seine | .060* | 18.3 | .640 | 3123 |
| Miss., Vicksburg | .019* | 5.79 | .039 | 190.4 |
| Danube, Vienna | .039* | 11.88 | .228 | 1113 |
| Lutschine, Gsteig | 1.63* | 497 | .134 | 654 |
| Reuss, Seedorf | .0252* | 7.68 | .561 | 2738 |
| Seine, Poissy | .0035* | 1.067 | .040 | 195.3 |
| Saone, Racoanay | .00478* | 1.456 | .033 | 161.0 |
| Aare, Brienzwiler | .830* | 253 | .292 | 1425 |
| Aare, Brienzwiler | 6.95* | 2118 | .361 | 1762 |

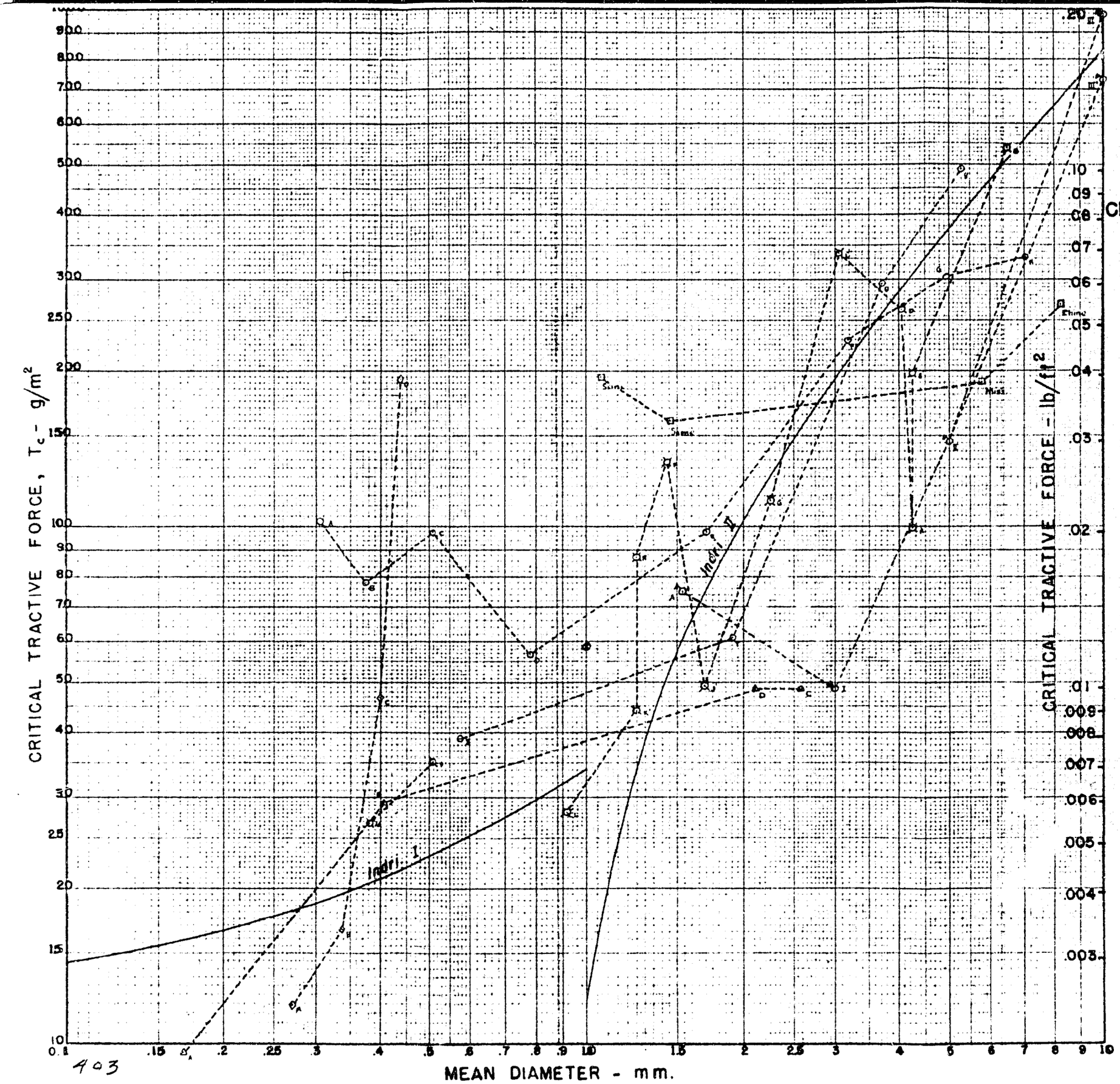
(Assumption: Critical point occurs where break appears in $V - R^{2/3} S^{1/2}$ curve)

*Computed from $d = \frac{(38.2)^6 R^4 S^3}{V^6}$ (Strickler)

Rindlaub

| Sand No. | Diameter feet | Diameter mm | $T_c$ lb/ft$^2$ | $T_c$ g/m$^2$ |
|---|---|---|---|---|
| A | .00055 | .1675 | .0025 | 9.7 |
| B | .00133 | .4055 | .006 | 29.3 |
| C | .00846 | 2.578 | .010 | 48.8 |
| D | .00693 | 2.11 | .010 | 48.8 |

FIGURE 3




PART II

EXTRAPOLATED CRITICAL TRACTIVE FORCES

(FROM O'BRIEN)

- SCHAFFERNAK, I - IV
- GILBERT, A - H
- MACDOUGALL
- STRICKLER
- RINDLAUB
- U.S.W.E.S. - NICHOLS
- CHIPPS ISLAND - SALT WATER BARRIER INVEST.
- SCHOKLITSCH

(FROM CHANG)

- SCHAFFERNAK, A
- GILBERT, S - V

## Table 11

SUMMARY OF CRITICAL TRACTIVE FORCE EQUATIONS
PROPOSED BY INVESTIGATORS

| Investigator | | Proposed equation |
|---|---|---|
| Schoklitsch | | $T_c = 1.048 \times 10^6 D_m^{3/2}$ |
| Krey | | $T_c = 99d$ |
| P.E.I. | | $T_c = 125d$ . . . . . (1) |
| | | $T_c = 126.9d$ . . . . . (2) |
| Kramer | | $T_c = 27.5d$ |
| U.S.W.E.S. | | $T_c = 37.25d^{1/2}$ |
| Indri | for | $d_g < 1.0$mm<br>$T_c = 21.94d + 12.16$ |
| | for | $d_g > 1.0$mm<br>$T_c = 90.6d - 78.48$ |
| Liu | for | $d_g = 1.4$ to $4.4$mm, $\gamma = 2.66$<br>$T_c = 45.7d$ |
| | for | $d_g = 0.2$ to $6.5$mm, $\gamma = 2.57$ to $2.70$<br>$T_c = 12.92d^{3/2} + 21.3$ |
| Kalinske | | Critical tractive force<br>$T_c = 192.3d$ |
| | | mean critical tractive force<br>$T_c = 64.1d$ |
| Chang | | for $\left(\frac{\sigma - \rho}{\rho} \frac{D}{M}\right) > 2.0$<br>$T_c = 36.22d$ |
| | | for $\left(\frac{\sigma - \rho}{\rho} \frac{D}{M}\right) < 2.0$<br>$T_c = 39.8d$ |
| O'Brien and Rindlaub | | $T_c = 56d$ |

Table 11--Continued

SUMMARY OF CRITICAL TRACTIVE FORCE EQUATIONS
PROPOSED BY INVESTIGATORS

| Investigator | | Proposed equation |
|---|---|---|
| Meyer-Peter | for | $A = 0.047$ (upper limit)<br>$T_c = 77.6d$ |
| | for | $A = 0.03$ (absolute rest)<br>$T_c = 49.5d$ |
| Shields | laminar layer | $T_c = 49.5d$ |
| | quadratic flow | $T_c = 99.0d$ |
| White | | $T_c = 74.25d$ |
| Casey | | $T_c = 63.6d^{5/4} + 15$ |

Note: Calculations shown in the Appendix. Units of $T_c$ in $g/m^2$, d. in mm except Schoklitsch D in m.

Assumptions: Uniformity modulus, M = 1.0.
Density of submerged material = 1.65.

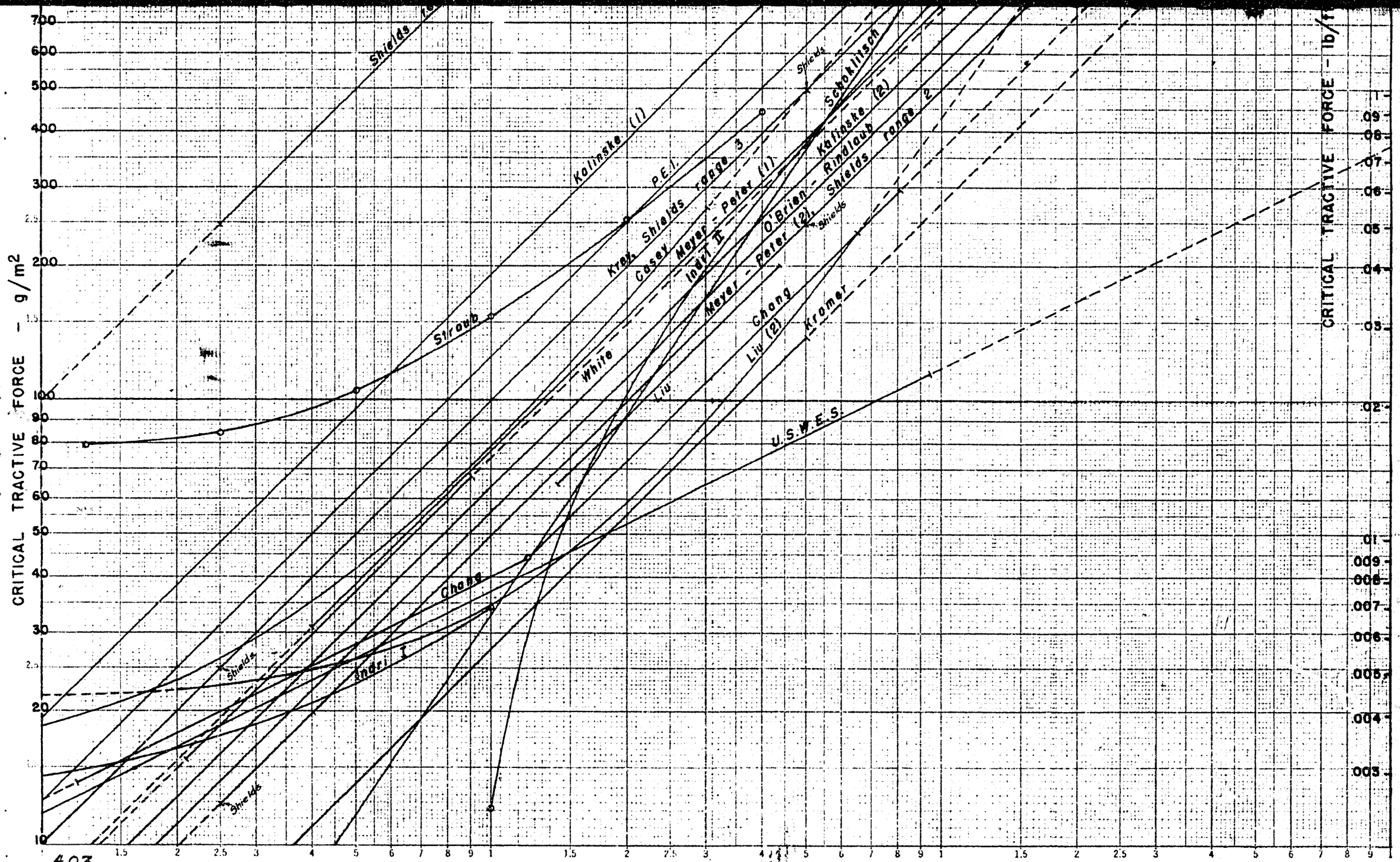
Table 11--Continued

SUMMARY OF CRITICAL TRACTIVE FORCE EQUATIONS
PROPOSED BY INVESTIGATORS

| Investigator | Proposed equation |
|---|---|
| Meyer-Peter | for $A = 0.047$ (upper limit)<br>$T_c = 77.6d$ |
| | for $A = 0.03$ (absolute rest)<br>$T_c = 49.5d$ |
| Shields | laminar layer<br>$T_c = 49.5d$ |
| | quadratic flow<br>$T_c = 99.0d$ |
| White | $T_c = 74.25d$ |
| Casey | $T_c = 63.6d^{5/4} + 15$ |

Note: Calculations shown in the Appendix. Units of $T_c$ in $g/m^2$, d. in mm except Schoklitsch D in m.

Assumptions: Uniformity modulus, $M = 1.0$.
Density of submerged material $= 1.65$.


CRITICAL TRACTIVE FORCE - g/m²
CRITICAL TRACTIVE FORCE - lb/ft
Shields
Kalinske (1)
P.E.I.
Kreitz
Shields range 3
Casey
Meyer - Peter (1)
Indri II
Schoklitsch
Kalinske (2)
O'Brien - Rindlaub
Shields range 2
Meyer - Peter (2)
Shields
White
Liu
Chang
Liu (2)
Kramer
U.S.W.E.S.
Straub
Chang
Indri I
Shields
Shields
403

FRAME 2

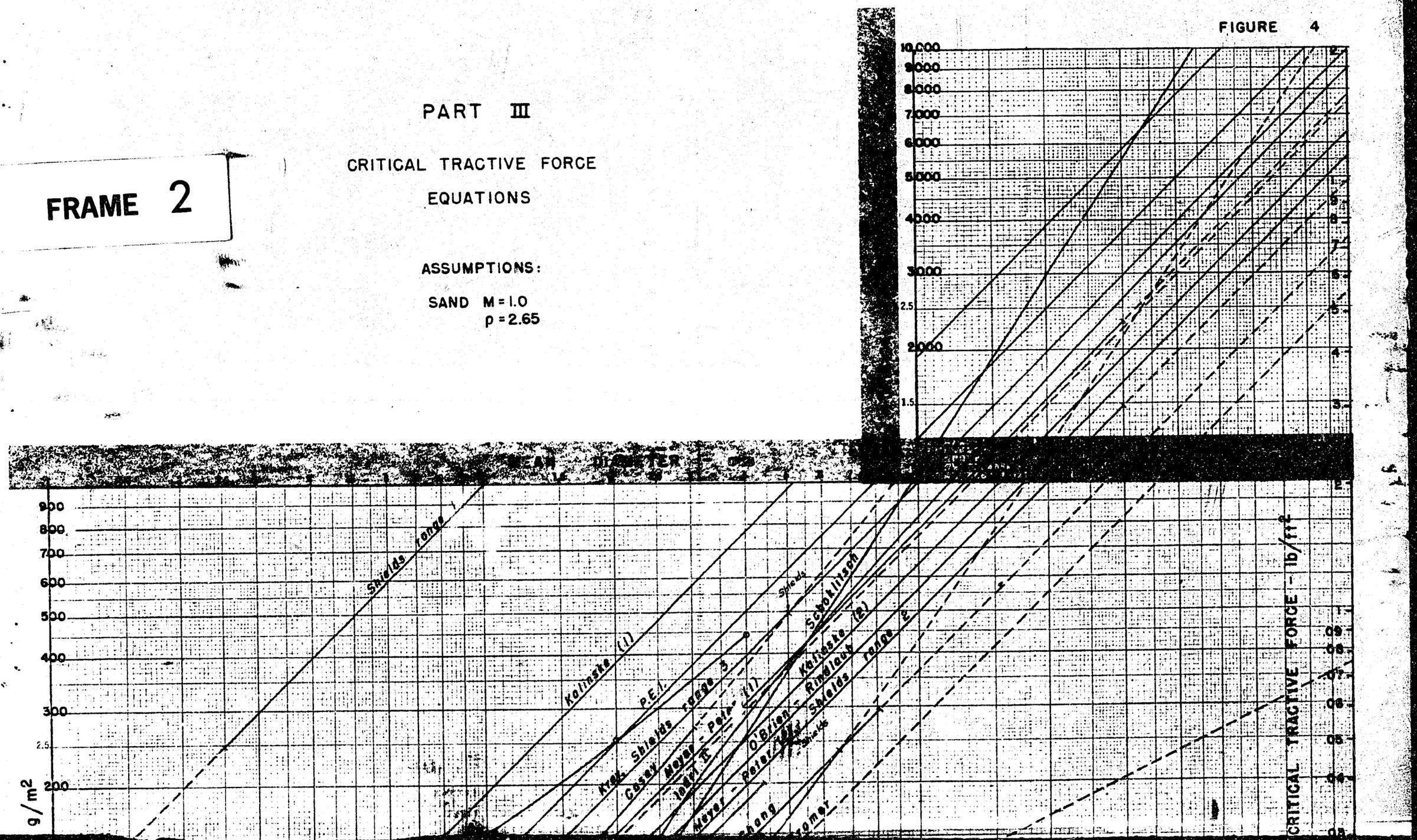
# PART III

## CRITICAL TRACTIVE FORCE EQUATIONS

ASSUMPTIONS:

SAND M = 1.0
ρ = 2.65

FIGURE 4



## Results

14. When plotted against sediment size, the critical tractive force data in Parts I and II (Figures 1 to 3) showed a certain pattern, though scattered within fairly wide limits. This variation might be accounted for by incomplete knowledge of all the variables involved in sediment transportation and their complex relations, differences in laboratory technique, and errors in observation.

15. When plotted in Figure 4, the critical tractive force formulas which were reviewed also showed a fairly wide range of possible values. The lack of agreement might be accounted for by the fact that each investigator derived a formula from different concepts of the variables involved and then introduced empirical constants to obtain agreement with experimental results. The laboratory work was of course subject to the limitations set forth in Paragraph 1.

16. Comparison of curves representing proposed expressions of the critical tractive force—particle size relation with plotted experimental results seemed to indicate that the equations of Chang and Liu defined the lower limit of critical traction in laboratory flumes somewhat better than the others. Verification of this conclusion would undoubtedly depend on additional work in the Bureau of Reclamation Hydraulic Laboratory.

17. A comparison of critical traction in laboratory flumes and the tractive forces existing in canals designed by currently accepted methods indicates a lack of correlation between model and prototype especially in the range of smaller sized sediments. The critical tractive forces in a natural channel which were observed by the Government engineers at Nuernberg, for example, were approximately 10 times greater than those observed in traction flumes by numerous investigators for the same sizes of sediment. Figure 2 shows the lack of agreement clearly.

18. The discrepancy between critical conditions of sediment transportation in models and prototypes may be accounted for by the difference in effect of a number of variables. The slopes used in laboratory experiments, for instance, were usually much greater than those found in larger channels. Also, the stability of natural channels was probably greater due to better compaction and cementation of the granular material composing the bottom and sides. This effect might be referred to as the aging of canals.

19. In order to determine the relation between critical tractive force and particle size for use in stable channel design, it will probably be necessary to conduct extensive field studies of existing canals and natural channels.

# APPENDIX I

Critical tractive force equations proposed by 13 investigators are presented in this appendix with calculations necessary for conversion to consistent units.

Scholklitsch:[19]

$$T_c = wRS = \sqrt{0.201\ w_1\ (w_1 - w)\ \lambda\ D^3}$$

$w$ = unit wt water kg/m$^3$
$R$ = hydr rad m
$S$ = energy gradient
$w_1$ = unit wt material kg/m$^3$

$\lambda$ = form factor = 1.25
$D$ = diam grain m

$$wRS = \sqrt{0.201\ (2650)\ (1650)\ (1.25)\ D_m^3} \times 1000$$

$$= 1048\ D_m^{3/2} \times 1000 = 1.048 \times 10^6\ D_m^{3/2}\ \text{g/m}^2$$

Krey:

$$T_c = wRS = (0.045 \text{ to } 0.07)\ (w_1 - w)\ D^{19}$$

$w$ = unit wt water kg/m$^3$
$w_1$ = unit wt material kg/m$^3$

$C$ = constant; assumed = 0.06
$D$ = diam grain m

$$wRS = 0.06\ (1650)\ (1000)\ \frac{D_{mm}}{1000} = 99\ D_{mm}\ \text{g/m}^2$$

Prussian Experimental Institute:

$d = 8DS$, $T = wDS$[10] . . . . . (1)

d = diam of particle
D = depth
S = slope
w = unit wt of water

$$T_c = wDS = \frac{wd}{8}$$

$$= \frac{d_{mm}}{1000} \quad (1/8) \times 10^6 = 125\ d_{mm} \quad g/m^2$$

$T_c = 0.077\ (w_1 - w)\ d$[15] . . . . . (2)

All units in lb, ft

$$= 0.077\ (1.65)\ (62.4)\ (4880)\ \frac{d_{mm}}{304.8}$$

$$= 126.9\ d_{mm} \quad g/m^2$$

Kramer:[9]

Quartz sand up to 5 mm in diam

$$T_c = wRS = \frac{100}{6}\ \left(\frac{D}{M}\right)\ (w_1 - w)$$

M = uniformity modulus; assumed = 1.0
w = unit wt water $kg/m^3$
$w_1$ = unit wt material $kg/m^3$

D = diam grain m
$T_c$ = tractive force $gm/m^2$

$$T_c = \frac{100}{6}\ (1650)\frac{D_{mm}}{1000} = 27.5\ D_{mm}$$

U.S.W.E.S.:[22]

$d_g = 0.04$ to $9.4$ mm

$$T_c = 29 \sqrt{\frac{d_g}{M} (\rho_1 - \rho)}$$

$T_c$ = tractive force gm/m$^2$
$d_g$ = diam grain mm
$\rho$ = density water gm/cc
$\rho_1$ = density material gm/cc
$M$ = uniformity modulus 1.0

$$T_c = 29 \sqrt{1.65\, d_{mm}} = 37.25\, d_{mm}^{1/2}$$

Indri:[6/24]

$d_g < 1.0$ mm

$$T_c = 13.3\, d_g \left(\frac{\gamma_1 - \gamma}{M}\right) + 12.16$$

$T_c$ = tractive force g/m$^2$
$d_g$ = diam grain mm
$\gamma$ = unit wt water g/cc
$\gamma_1$ = unit wt material g/cc
$M$ = uniformity modulus 1.0

$$T_c = 13.3\,(1.65)\, d_{mm} + 12.16 = 21.94\, d_{mm} + 12.16 \quad \ldots\ldots \text{I}$$

$d_g = 1.0$ mm to $150$ mm

$$T_c = 54.85\, d_g \left(\frac{\gamma_1 - \gamma}{M}\right) - 78.48$$

$$= 54.85\,(1.65)\, d_{mm} - 78.48 = 90.6\, d_{mm} - 78.48 \quad \ldots\ldots \text{II}$$

Te-Yun Liu:[11]

(1) for $d_g$ = 1.4 to 4.4 mm, $\gamma$ = 2.66

$D_c\ S_c = 0.15d$

$D_c\ S_c$ = competent slope depth ft x $10^{-3}$

d = mean diam mm
w = sp wt water 62.4 lb/ft$^3$

$T_c = wD_c\ S_c = 0.15\ wd = .15\ (62.4)\ (4.88)\ d_{mm}$

$= 45.7\ d_{mm}\ \ g/m^2$

(2) for $d_g$ = 0.2 to 6.5 mm, $\gamma$ = 2.57 to 2.70

$$D_c\ S_c = 0.02 \left\{\frac{d}{(M}\ (s - 1)\right\}^{1.5} + 0.07 \text{ ft} \times 10^{-3}$$

$T_c = wD_c\ S_c = 0.02\ (62.4)\ (4.88)\ (1.65)^{3/2}\ d^{3/2} + .07\ (62.4)\ (4.88)$

$= 12.92\ d^{3/2} + 21.3\ \ g/m^2$

O'Brien and Rindlaub:[10]

$T_c = wRS = 3.5\ d$

$T_c$ = lb/ft$^2$

w = unit wt lb/ft$^3$
R = hydr rad ft
d = med diam ft

$$T_c = 3.5\ (4880)\ \frac{d_{mm}}{304.8} = 56\ d_{mm}\ \ g/m^2$$

Kalinske:[3]

$$T_c = \alpha p \rho' g \, d \tan \phi$$

$\rho'$ = $(\rho_g - \rho)$ effective particle density
$\phi$ = angle of repose of sand grain
p = factor which indicates the proportion of the bed taking fluid shear
$\alpha$ = constant = 2/3
g = accel grav
d = grain diam

when $\tan \phi = 1.0$

$$\alpha p = 1/2 \times 2/3 \times 0.35$$

$$\rho' = \frac{(2.65 - 1.0)}{32.2/62.4} = \frac{1.65\ (62.4)}{32.2}$$

$T_c = 12\ d$ lb/ft$^2$ (critical tractive force) . . . . . (1)

$T_c = 192.3\ d_{mm}$ g/m$^2$

$T_c = 4\ d$ (mean critical tractive force) . . . . . (2)

$T_c = 64.1\ d_{mm}$ g/m$^2$

Y. L. Chang, Esq.:[3]

for $\left(\frac{\sigma - \rho}{\rho} \frac{D}{M}\right) > 2.0$

D = 0.134 to 1.212 mm

$$T_c = 0.0045 \left(\frac{\sigma - \rho}{\rho} \frac{D}{M}\right) = 0.0045\ (4880)\ (1.65)\ d_{mm} = 36.22\ d_{mm}\ g/m^2$$

$T_c$ = lb/ft$^2$
$\sigma\ \rho$ = sp gr
D = mean diam mm

for $\left(\frac{\sigma - \rho}{\rho} \frac{D}{M}\right) < 2.0$

D = 1.212 to 8.09 mm

$$T_c = 0.00635 \left(\frac{\sigma - \rho}{\rho} \frac{D}{M}\right)^{1/2} = 0.00635\ (4880)\ \sqrt{1.65}\ d^{1/2} = 39.8 d^{1/2}\ g/m^2$$

Hugh J. Casey:[2]

for diam up to 3.0 mm

$$S_o = \left[30 - 6\ (1/3 - M)\right] \left[d_g\ (\gamma_1 - \gamma)\right]^{5/4} + 15$$

$S_o$ = critical tractive force g/m$^2$

$d_g$ = mean particle diam mm

$\gamma_1\ \gamma$ = sp wt of material and water g/cm$^3$

$$T_c = \left[30 - 6\ (1/3 - 1.0)\right] \left[d_g\ (1.65)\right]^{5/4} + 15$$
$$= 63.6\ d_g^{5/4} + 15\ g/m^2$$

Meyer-Peter:[12]

$d_m$ = 0.4 to 30 mm

$T_c = (\gamma_w h J) \cong A'' \gamma_s'' d_m$

$T_c$ = limiting shearing stress tons/m$^2$

$\gamma_w$ = sg of water tons/m$^3$
$\gamma_s''$ = sg of sediment under water $(\gamma_s - \gamma_w) = \gamma_s - 1.0$
h = water depth m
J = slope
A'' = constant = 0.047 for upper limit (beginning of bed load transport) = 0.03 for abs rest

$d_m$ = effective diam = $\frac{\Sigma d \, \Delta p}{100}$ where d = avg size of particles in a fraction m

for A = 0.047 (beginning of bed load transport)

$$T_c = 0.047\ (1.65)\ (10^6)\ \frac{d_{mm}}{1000} = 77.6\ d_{mm}\ \text{g/m}^2 \quad . . . (1)$$

for A = 0.03 (Absolute rest)

$$T_c = 49.5\ d_{mm}\ \text{g/m}^2 \quad . . . . . (2)$$

Shields:[18] (d = 0.25 to 5.0 mm)

$$T_c = \alpha_1 (\gamma_1 - \gamma) d$$

$T_c = \gamma R_m J$ = shear stress $g/m^2$
$R_m$ = mean hydraulic radius cm
$J$ = slope
$\alpha_1$ = porosity ratio dependent on particle shape and thickness of layer
$\gamma_1$ = sp wt of particles $g/cm^3$
$\gamma$ = sp wt of fluid
$d$ = mean particle diam cm
$\delta$ = thickness of laminar layer cm

$R_n = \frac{V_* d}{\nu}$ = Reynolds Number

$\nu$ = kinematic viscosity $cm^2/sec$

$V_* = \sqrt{gRJ}$ = friction velocity cm/sec
$g$ = accel grav 980 cm/sec

Range 1: $\delta > d$. Bed behaves as smooth bed. Good for $R_n$ up to 2.0. $\alpha_1 = 0.6$ for uniform angular particles.

$$T_c = 0.6 (\gamma_1 - \gamma) d = 0.6 (1.65) (10^3) d_{mm}$$

$$= 990 d_{mm} \quad g/m^2$$

Range 2: $\delta \sim d$. Thickness of laminar layer same order as particle diameter. $R_n \sim 10$. $\alpha_1 = 0.03$.

$$T_c = 0.03 (\gamma_1 - \gamma) d = 49.5 d_{mm} \quad g/m^2$$

Range 3: $\delta > d$. Quadratic resistance of flow prevails. $R_n > 10^3$. $\alpha_1 = 0.06$

$$T_c = 0.06 (\gamma_1 - \gamma) d = 99.0 d_{mm} \quad g/m^2$$

C. M. White:[23]

$k = 0.21$ to $0.90$ mm

$$\tau_c = 0.18\ \rho'\ g\ k \tan \phi$$

$\tau_c$ = mean shearing stress on bed dynes/cm$^2$
$\rho'$ = effective density of sand in fluid g/cm$^3$
$g$ = accel gravity 980 cm/sec$^2$
$k$ = diam of sand grains cm
$\phi$ = angle of repose of material $\tan \phi = 1.0$

$$T_c = \frac{\tau_c}{g} = 0.18\ \rho'\ k \tan \phi$$

$$= 0.18\ (1.65)\ (10^4)\ \frac{k_{mm}}{10} = 297\ k_{mm}\ \ g/m^2$$

Using a turbulence factor of 4 for parallel flow, this equation becomes

$$T_c = 74.25\ k_{mm}\ \ g/m^2$$

# APPENDIX II

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