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THE REGULATION OF SMALL STREAMS BY THE
CONSTRUCTION OF DROPS

By

C. KEUTNER

Translation
by

E. F. Wilsey

Denver, Colorado

October 15, 1937

UNITED STATES
BUREAU OF RECLAMATION
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BY DROPS

A translation of
DIE REGELUNG KLKINERER WASSERLÄUFE
DURCH ERRICHTUNG VON GKFALLSTUFEN

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1. Generalities on the Regulation of Small Streams with Particular Reference to the Fore-Alps

The following account of the problems involved in the regulation of small streams was motivated by an inspection trip.¹

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This inspection trip was made in 1934 under a stipend of the William G. Kerckhoff Foundation, Bad Nauheim.

Extensive systems of drainage canals introduced as a result of agricultural expansion into marsh land and areas covered with sour grasses, which often are of little value for pasturage, cause a lowering of the water table. These canals must be large and deep enough to carry ordinary floods without overflowing the banks. Only during unusually high floods is the flooding of fields permissible. Crooked, unruly streams which meander through the countryside should be shortened by excavating cutoffs, and their bottom slopes should be reduced. However, this may involve building entirely new streams with fixed banks (canals).

The primary requirement for regulating streams passing through towns and cities is the prevention of floods. Wide, shallow streams with easily eroded banks should be replaced by canals which can carry the total flood discharges and whose water surfaces lie at low enough elevations to effectively drain the districts through which they pass. Furthermore, the shape of the cross section of the canal must be so chosen that the velocity of flow, during low water periods, is high enough to prevent all but a small amount of silting.

If the existing soil properties will not permit the new bottom slopes resulting from shortening streams, deep erosion of the bottom will ensue. This produces underscouring and caving in of the banks and re-alignment of the streams. In case the character of the soil will not permit the requisite slopes, the bottom and sides of the canal must be lined with riprap, rubble, concrete or bitumen. However, in most cases a special lining is not economical. Therefore, in general, the maximum permissible bottom slope is chosen and the surplus fall is taken care of by drops. The nature of the terrain governs the location of both the canal and the drops. The latter may either be built as over-fall structures, as is most frequently the case, or as chutes - also known as "slides" (Rutschen). Drops often involve a large part of the total cost of the work of regulation, which must be kept as low as possible, since a considerable part of the cost must be borne by associations of property holders who from time to time have little financial resources. Since the amount of excavation is governed by the properties of the soil, the most likely source of saving is in the cost of special structures. Drops must have the best design from a hydraulic standpoint and, at the same time, should require the least outlay in money, this being dependent on the most economical choice of building materials.

During the past year, a number of distinct types of structures have been built by the various boards of works, the outstanding examples of which will be considered in what follows.

Essentially different requirements and viewpoints govern regulation of streams passing through cities where freedom from floods in the thickly settled mining and industrial districts, is the essential requirement.² Here, in order to combat floods, the

2
Bulow, F. V.; Die Leistungsfähigkeit von Fluss-, Bach- und Werkkanal- und Rohrquerschnitten unter besonderer Berücksichtigung der von

der Emschergenossenschaft in Essen zu künstlichen Wasserläufen
ausgebauten Emscher und ihrer Nebenbäche (The Capacity of River,
Brook, Canal, and Pipe Cross Sections with Special Reference to the
Emscher River and its Tributaries Rebuilt as Artificial Canals by
the Emscher Board of Works at Essen. Gesundheits - Ingenieur, 1927.

most economical canal must have a large cross section and be excavated
to a considerable depth. For example, the Emscher River was lowered
about 4 meters (13.1 feet) as a consequence of regulation. These
streams are required to transport quickly and safely the drainage of
cities and industrial plants, which contains a considerable amount
of silt and other refuse, and sometimes, in addition, ground water
pumped to the surface from mines. These demands are met by relatively
small channel sections and relatively steep slopes; thus such streams
must be completely lined with concrete or some other material. The
design of drops employed here also merits consideration.³ The type of
construction used in

³ Ramshorn, A.; Die Energievernichtung bei Abstürzen und Schussstrecken
in offenen Abwasserkanälen (Energy Dissipation at Drops and Chutes in
Open Channels). Die Bautechnik, Vol. 10, 1932, page 139.

industrial districts is not suitable for agriculture regions because
of the cost involved.

The design of canals and drops may be considered separately,
although the design of one is influenced to a large extent by the
other.

II. The Canal between Drops

1. Form of Cross Section

The discharge, Q , is always given for the design of a regulation
project. The cross-sectional area of a canal governs the amount of
excavation; the canal depth governs the amount of drainage; the width

of the canal governs the land area required; the wetted perimeter governs the amount of bottom and slope protection; and the soil properties govern the permissible bottom slope. The most economical design is to be found from these factors.

A trapezoidal section is by far the most popular section for a canal (figures 2, 1), and its area is easily computed from tables found in most hydraulic construction handbooks.⁴

⁴ Weyrauch-Strobel; Hydraulisches Rechnen (Hydraulic Formulas and Tables). Konrad Wittwer, Stuttgart. 1930.

Bela von Kenessey concludes from his observations on artificial canals that a trapezoidal section does not retain its original shape because of silting, bank damage, etc.⁵ Similar to

⁵ Kenessey, Bela; Mederszelvények es Mederszelvénytablazatok (Bed Profiles and Bed Profile Tables). Vizügyi Közlemények, Vol. 15. 1933, page 298.

natural streams, artificial canals almost with no exception change their form so as to resemble conic sections. He proposes that such changes be anticipated, and instead of a trapezoidal section, a conic section, corresponding to the expected change, be used in design. If this is not done, the coefficient in the velocity formula must be adjusted.

The circle and parabola are the two conic sections most easy to compute.

For a trapezoid superposed on a circular segment (figures 2, 2) we have:

Area

$$A = \left(\frac{t}{n} + s\right)t - \frac{n s^2}{4(\sqrt{1+n^2} - 1)^2} \quad \left[2\sqrt{1+n^2} - 1 - n \arccos \frac{1}{\sqrt{1+n^2}} \right]$$

Wetted perimeter

$$P = \frac{2t \sqrt{1+n^2}}{n} - S \left[1 - \frac{n \operatorname{arc} a}{\sqrt{1+n^2} - 1} \right]$$

Radius of the circular bottom

$$r = \frac{sn}{2(\sqrt{1+n^2}-1)} = \frac{S(1+\sqrt{1+n^2})}{2n}$$

Height of the junction of the circular bottom with the tangential side slope.

$$t_1 = \frac{sn}{2\sqrt{1+n^2}}$$

For a section consisting of a trapezoid superposed on a parabolic bottom (figures 2, 3) we have:

Equation of the parabola with the origin at the vertex

$$y^2 = \frac{2s}{n} X$$

Parameter

$$P = \frac{s}{n}$$

Area of cross section

$$A = \left(\frac{t}{n} + s \right) t - \frac{sn^2}{12}$$

Wetted perimeter

$$P = \frac{2t \sqrt{1+n^2}}{n} + 2.303 \frac{s}{n} \log(n + \sqrt{1+n^2})$$

Height of the junction of the parabolic bottom with the tangential side slope

$$t_1 = \frac{sn}{2}$$

In most cases the most economical section is also the most efficient from a hydraulic standpoint. Furthermore, by adopting such a section, the excavation costs, the land area required, and the amount of side slope protection, depending on the slope of the sides, are all reduced.

The discharge for a given bottom slope is a maximum if:

$$\frac{dA}{dP} = 0 \quad \text{and} \quad \frac{d^2A}{dP^2} > 0 \quad (\text{figures 2, 4})$$

The mathematical derivation for the best dimensions will be dispensed with here. Kenessey analyzed most favorable sections, hydraulically speaking, consisting of trapezoids superposed on circular and parabolic segments, respectively. He demonstrated that the best section is one having a circular bottom. A parabolic bottom becomes less favorable the greater the slope of the sides of the trapezoid. In general, the difference is not large; with a slope of 1:3, it amounts to about 11 percent in sectional area and 3 percent in hydraulic radius; with a slope of 1:1, the difference is only 3 percent in area and 1 percent in hydraulic radius.

If a parabolic bottom is chosen, it should be noted that the angle of inclination of the sides should not be more than 60 degrees. With a greater slope, a parabolic bottom cannot be used, for then its vertex lies within the trapezoid.

Kenessey tabulated the dimensions of various cross sections in relation to the slope of the sides. These tables facilitate computations particularly for curved unsymmetrical sections.

It should be mentioned in this connection that computations for true parabolic cross sections at curves and in the straight parts of meandering streams are also given in Kenessey's paper.

2. The Discharge Capacity of Canals

Numerous so-called "velocity formulas" are available for computing the discharge of open channels.

Using the Chezy-Brahms equation as a basis, other investigators have devoted their attention either to determining the size of the roughness coefficient for various types of beds (sand, gravel, etc.) or to deriving new equations in which the roughness of the channel is expressed as an exponent.

Of the various formulas for the value of C , the Ganguillet-Kutter formula is probably the most widely used especially after Schewior's tables⁶ and a graphical chart appeared.

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Schewior, G.; *Hilfstabellen zur Bearbeitung von Meliorations - entwürfen, Kanalisationen und anderer wasser - und tiefbautechnischen Aufgaben* (Tables for Reclamation Canalization and Other Hydraulic and Foundation Problems), 3rd Edition, Paul Parey, Berlin, 1930.

Lindquist investigated Manning's formula in the light of numerous additional data, devised a nomogram for it and recommended it as the most usable formula.⁷

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Lindquist, E. G. W.; *On Velocity Formulas for Open Channels and Pipes*, Ingeniörsvetenskapsakademiens, No. 130, 1934, or Nemenyi, Paul; *Wasserbauliche Stromungslehre* (Hydraulic Textbook), J. A. Barth, Leipzig, 1933.

Forchheimer adopted Manning's procedure and obtained a discharge equation based on measurements in large canals.⁸

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Forchheimer, Ph., *Grundriss der Hydraulik* (Fundamentals of Hydraulics). 2nd Edition, Teubner, 1926.

In Austria and Switzerland, Strickler's formula of the Manning type is preferred.

R. Winkel started from the Chezy formula, arrived at a new equation for the roughness coefficient, and obtained a discharge formula without arbitrary coefficients.⁹

⁹ Winkel, R.; Die Grundlagen der Flussregelung (The Fundamentals of River Regulation). Wm. Ernst and Son, Berlin, 1934.

Among the equations without roughness coefficients is that of Matakiewicz¹⁰ which was derived from data on rivers, canals

¹⁰ Matakiewicz, M.; Die Geschwindigkeitsformel und ihre Anwendungen. (The Velocity Formula and Its Applications), Annales de l'Academie des Sciences Techniques a Varsovie, Vol. 2, 1935, page 115.

and mountain streams.

These formulas afford a relatively speedy calculation of the discharge and are satisfactory for the ordinary conditions met in practice.

3. The Limiting Tractive Force for Various Types of Beds

The cause of the deepening (erosion) of the bed in the upper pool at a drop is the so-called tractive force or drag. This force shoves the particles forming the sides and bottom of a stream forward. If this transportation of bed material is to be prevented, the resistance to scour of the slopes and bottom must be larger than the tractive force. Tractive force can be expressed as a function of the depth of flow and the slope of the water surface ((for uniform flow the slope of the water surface is equal to the bottom slope) thus:

$$F_t = 62.3 DS$$

where

$$F_t = \text{the tractive force in pounds per square foot.}$$

$$D = \text{the depth of flow in feet}$$

$$S = \text{slope of the water surface}$$

For narrow canals:

$$F_t = 62.3 a DS$$

According to Schoklitsch,¹¹ a is approximately unity for channels

¹¹

See footnote 4, page 79.

of a width, $b > 30d$. For narrow channels, he proposes

$$F_t = 62.3 R^3$$

where

R = the hydraulic radius in feet.

This formula for the tractive force was based on the following argument:

Krey derived a relation between the tractive force and the average diameter d_m of the bed material. He obtained for the limiting depth, D_0 , and for approximately uni-granular sand, the empirical relation¹²:

¹²

The various tractive force formulas are given in: (a) Kramer, H; Modellgeschichte und Schleppkraft (Model Bed Load and Tractive Force), 1932; (b) Casey, H. J.; Über Gescheibebewegung (On the Motion of Bed Load). Both papers appear in a publication of the Preuss. Versuchsanstalt für Wasserbau und Schiffbau, Berlin, 1935.

$$D_0 S = \frac{d_m}{2440 \text{ to } 6100}$$

where

d_m = the average diameter in millimeters.

Therefore:

$$F_0 \approx 62.3 D_0 S = 0.025 \text{ to } 0.1 d_m$$

For a canal of uniform roughness, two limiting values of the tractive force are of particular significance. The greater of the two obtains when a level bed at rest is set in motion, the smaller, when the moving bed material begins to deposit uniformly. According to Kreuter the difference between the two in natural streams amounts to about 30 percent. In laboratory tests on fine uni-granular material, according to the observations of Schoklitsch, the difference decreases almost to zero.

Transportation of bed material and the amount of erosion in canals are indeed extremely difficult problems in hydraulics to solve quantitatively. As long as no rigid formula for the movement of bed material based on measurements on natural streams is possible, past experience must suffice in practice.

If the tractive force, F_t , of the flow is greater than the limiting tractive force, F_0 , for a given type of soil or artificial lining of the channel, then the particles of the bed will be set in motion. It has been observed that various beds present different resistances to the tractive force. If the tractive force of the flow decreases to less than a certain value, deposition occurs in the channel. The size of the "erosive tractive force" as well as the "sedimentative tractive force" must be considered in design.

Kreuter and Lueger¹³ determined the "erosive tractive

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See footnote 4, page 80.

force" from observations on various types of bed material.

Measurements by the Nurenburg Agricultural Engineering Board¹⁴ gave a number of values for the limiting tractive force of

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(a) Schoklitsch, A.; Der Wasserbau (Hydraulic Construction), J. Springer, Vienna, 1930. (b) Strele, G.; Grundriss der Wildbachverbauung (Fundamentals of Checking Wild Streams), J. Springer, Vienna, 1934.

beds composed of quartz sand and gravel, loam, and stratified limestone.

The Ingolstadt Agricultural Engineering Commission bases its designs in accordance with the following values taken from past experience:

1. For firmly packed sand and fine gravel

$$F_o = 0.16 \text{ to } 0.18 \text{ pounds per square foot for normal flow}$$

$$F_o = 0.2 \text{ to } 0.24 \text{ pounds per square foot for brief floods}$$

2. For loamy gravel

$$F_o = 0.3 \text{ pounds per square foot for normal flow}$$

$$F_o = 0.4 \text{ pounds per square foot for brief floods.}$$

Ramshorn¹⁵ gives:

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See footnote 5, figure 6.

$F_o = 0.5$ pounds per square foot for a canal lined with concrete. If this limiting tractive force is exceeded, the cinder base of the slabs is removed and the entire lining is displaced.

In connection with the work in the Günsberg district of the Agricultural Board, the author observed that on the Zusam and Laugna Rivers with approximately equal bed conditions and slopes, an unimportant yet distinct deepening of the beds of over 1.97 feet occurred.

The bed of both streams consists of sand to hard loam, partly clay. At several places on the Zusam River, the sandy loam is mixed with small stones, with both beds, a significant erosion occurred during floods with a tractive force of

$$F_t = 0.254 \text{ pounds per square foot (S = 0.0007).}$$

pure sandy loam at other places was eroded during floods by a tractive force of

$$F_t = 0.206 \text{ pounds per square foot (S = 0.006).}$$

Inclusions such as small stones considerably reduce the bed's resistance to scour.

Since this erosion, for the most part, was produced by the protracted normal discharge, the limiting tractive force must lie below 0.2 pounds per square foot for this type of bed.

The "sedimentative tractive force", that is, that force at which sand and silt are deposited, is given as follows:

1. According to observations of the Ingolstadt Agricultural Engineering Commission on earth canals without linings

$$F_o^1 = 0.08 \text{ to } 0.10 \text{ pounds per square foot.}$$

2. According to Ramshorn for canals lined with concrete slabs,

$$F_o^1 = 0.05 \text{ pounds per square foot.}$$

The roughness of these two canals is different and therefore the limiting tractive forces are different. The greater the roughness of a channel, the larger the "sedimentative limiting tractive force" must be chosen in order to prevent silting.

The relation between the tractive force, F_t , and the hydraulic radius, R , for various slopes, S , according to Schoklitsch's formula for narrow channels is shown in figure 3. For example, if a value of $F_o = 0.4$ pounds per square foot is permissible for a given stream bed, for $R = 6.56$ feet a slope of $S = 0.001$ may be chosen.

For $R = 3.28$ feet, then $S = 0.002$; thus with shallow streams the slope can be almost doubled.

The tractive force may be expressed also in terms of the average velocity. A limiting velocity for various types of beds can be obtained in a similar way as the limiting tractive force was found. Figures 4 and 5 are based on Schoklitsch's tractive force equation and Forchheimer's permissible velocity formula. Using a value for the roughness coefficient equal to 48.9 in Forchheimer's equation, we have:

$$F_t = 62.3 RS \text{ (according to Schoklitsch)}$$

$$V = 48.9R^{0.7} S^{0.3} \text{ (according to Forchheimer)}$$

$$V = \frac{48.9}{15.5} \frac{F_t^{0.7}}{S^{0.2}}$$

where V = the limiting velocity in feet per second.

The functions, $f(D)$, $f(A)$, and $f(R^{0.7})$ are plotted in figure 4 for a trapezoidal canal whose sides slope 1:1.5 and for bottom widths from 4 feet to 20 feet. Similar graphs can be drawn for other side slopes and bottom widths. Figure 5 shows the relation between the hydraulic radius, R , and the bottom slope, S , for various values of the mean velocity and also for various values of the tractive force, F_t . The F_t curves intersect the V curves. The point of intersection of any two such curves gives both the tractive force and the corresponding mean velocity for a given slope and hydraulic radius.

Example given:

Depth of flow, $D = 6$ feet

Bottom width, $B = 10$ feet

Slope of sides = 1:1.5

Choose the limiting tractive force, $F_0 = 0.4$ pounds per square foot for the given type of bed and choose the bottom slope, $S = 0.001$. To find the discharge and the tractive force, F_t .

From figure 4, $R = 3.66$ feet and $A = 115$ square feet; from figure 5, $V = 3.8$ feet per second and hence $Q = 440$ second-feet. From the same figure, $F_t = 0.22$ pounds per square foot which is less than the limiting tractive force, $F_0 = 0.4$ pounds per square foot. Similar graphs can be prepared for different side slopes, bottom widths, and depths of flow encountered in design work.

The following are some observed values of the limiting velocity:
According to Engels¹⁶ the limiting velocities are:

¹⁶ Engels, H. ; Handbuch des Wasserbaues (Handbook of Hydraulic Construction), W. Engelmann, Berlin-Leipzig, 1914, page 94. See also: Footnote 4, page 43. The results of Reinhard and Telford are given in this reference.

1. For light sandy soil, V = 2.3 feet per second.
2. For average sandy soil, V = 2.5 feet per second.
3. For loam, V = 3.0 feet per second.
4. For gravel and compacted soils, . . V = 3.9 feet per second.

The mean velocity is greater than the bottom velocity; hence in considering the amount of erosion, the velocity nearest the bottom is the true criterion. Schaffernak¹⁷ derived the following equation from an analysis of numerous current meter measurements of the

¹⁷ Schaffernak; Neue Grundlagen für die Berechnung der Geschiebeführung in Flüssen (New Basis for Computing the Bed-Load of Rivers), Vienna, 1922.

bottom velocity:

$$V_b = 0.65 DS^{0.5}$$

Kozeny proposed the following ratio of the bottom velocity to the mean velocity:

$$\frac{V_b}{V_m} = 0.38$$

Whether or not the water was clear or laden with silt was not taken into consideration in the above values of the limiting velocity. Actually the silt load of the flow is not unimportant in connection with the limiting velocity. Budenny states that erosion of a stream bed begins if the tractive force is larger than resistive force of the bed. As the

erosion increases, the water increases its silt load, and its average velocity decreases. A certain degree of saturation is reached as a result of which the erosion ceases. Pure water has a greater erosive power than silt-laden water, hence the silt load increases until the saturation point is reached. The studies of Fortier and Scobey¹⁸ in 1926, gave various limiting velocities in relation to the silt

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Fortier, S. and Scobey, F. C.; Permissible Canal Velocities, Trans. A.S.C.E., Vol. 39, 1926, p. 940. See also: Hjulstrom, Filip; Studies of the Morphological Activity of a River as Illustrated by the River Fyris, Reprinted from the Bulletin of the Geological Institute of Upsala, 1936, page 502. This interesting dissertation, written from a nontechnical viewpoint, gives a rather comprehensive presentation of the pertinent results in foreign literature on bed-load and silt movement and accompanying questions.

properties of the water (table I).

TABLE I

Maximum Permissible Canal Velocities

Original Material Excavated for Canal	Velocity, in feet per second, after Aging, of Canals Carrying		
	Clear Water no Detritus:	Water Trans- porting Colloidal Silts	Water Transporting Noncolloidal Silts, Sands, Gravels or Rock Fragments.
1 Fine sand (noncolloidal)	: 1.50	: 2.50	: 1.50
2 Sandy loam (noncolloidal)	: 1.75	: 2.50	: 2.00
3 Silty loam (noncolloidal)	: 2.00	: 3.00	: 2.00
4 Alluvial silts(noncolloidal)	: 2.00	: 3.50	: 2.00
5 Ordinary firm loam	: 2.50	: 3.50	: 2.25
6 Volcanic ash	: 2.50	: 3.50	: 2.00
7 Fine gravel	: 2.50	: 5.00	: 3.75
8 Stiff clay (very colloidal)	: 3.75	: 5.00	: 3.00
9 Graded, loam to cobbles (noncolloidal)	: 3.75	: 5.00	: 5.00
10 Alluvial silts (colloidal)	: 3.75	: 5.00	: 3.00
11 Graded, silt to cobbles (colloidal)	: 4.00	: 5.50	: 5.00
12 Coarse gravel(noncolloidal)	: 4.00	: 6.00	: 6.50
13 Cobbles and shingles	: 5.00	: 5.50	: 6.50
14 Shales and hard-pans	: 6.00	: 6.00	: 5.00
	: :	: :	: :
	: :	: :	: :

In general, water laden with colloidal silt permits the greatest average velocity. In several cases the limiting velocity of water laden with silt is greater by about 40 percent than for clear water. However, the table shows that water laden with silt will erode loam, volcanic ash, alluvial silt, and shale beds as much as or more than clear water.

In determining the depth, slope, etc., of a regulated stream, it must previously be ascertained whether the stream flows clear or is laden with detritus.

4. The Protection and the Lining of Artificial Canals

The side slopes and bottom are not protected by a lining in the majority of cases. The foot of the side slopes is often protected by a layer of brush with wooden cross pieces laid on top; the slopes are covered with sod. If properly laid, the sod cover is an effective protection having a limiting tractive force up to $F_0 = 0.6$ pounds per square foot. According to a statement by Theuerkauf¹⁹

¹⁹Theuerkauf; Hochwertiger Rasen als Befestigungsmittel bei Wasserbauten. (High Quality Sod as a Means of Protection at Hydraulic Structures) Die Bautechnik, vol. 14, 1936, No. 42.

the cost of laying high quality sod amounts to 22 cents per square yard (in Germany) including a base of top soil about 4 inches thick.

For larger canals, the total cross section is often divided into normal discharge and flood channels. A trapezoidal channel with wide or narrow berms according to the ratio of the normal discharge to the flood discharge, is used. A compound section, however, in the course of time undergoes a change in form due to depositions of silt. This can be observed not only in canals in industrial districts (for example, the Emsher canal), but also in the Fore Alps (Isen and Rott canals). Bulow²⁰ states that the lower trapezoidal section of the

²⁰See footnote 2, figure 14.

Emsher canal has acquired a triangular form due to the progressive silting of the channel (figure 7). More specifically, layers of silt

are deposited which change the slope of the banks from 1:2 or 1:1.5 to almost vertical banks. These latter are rapidly covered with plant growth but threaten to cave in constantly. In addition, the discharge capacity decreases perceptibly as a result of the reduction of the cross-sectional area. In the Isen and Rott canals, silt deposits even extended above the rim of the normal discharge section (figure 7). In both of these cases, the decrease of the sectional area led to a lowering of the bed. Hence, for streams transporting a large amount of silt, compound sections are not to be recommended.

The foot of the side slopes is often protected by woven mattresses or mats of reeds. This type of protection is advantageous only providing that the erosion of the slopes can be repaired in a short time and providing that all wooden parts are completely submerged since wood rapidly succumbs to rot with a fluctuating water surface.

Side slopes during winter are subject to powerful attacks of floating ice. A wicker mat is a better protection of the banks than a sod cover. In this case $F_0 = 0.8$ pounds per square foot. Results found by the Wieden Company at Munich indicate that the cost of laying a wicker mat made entirely from local materials varies from 43 to 50 cents per square yard according to the location at which it is laid.

If a steep slope is demanded by the topography of the countryside and the actual tractive force is larger than the limiting tractive force of the prevailing type of soil, then the bottom of the canal must be protected by some effective means. One of the oldest remedies of this kind is the driving of piles into the bottom. The Salt Administration in the Traun district in Upper Austria has used this scheme. A row of piles is driven across the canal. This is repeated at wide intervals. Each row of piles serves as a sill in preventing rapid scour of the bottom. This type of protection is being used with good results by the forestry service in Karnten. Logs laid across the canal at maximum permissible intervals have a similar effect. As a rule, these logs are laid directly on the floor of the canal. The Ingolstadt Agricultural Engineering Commission places them on a facine mat. The distance between the logs in this case is from 3.9 to 4.6 feet. The logs serve to hold down the mat and, in addition, act as sills. Bottoms protected thus can withstand large velocities. Such a protection has a long life since the logs are covered with water. When submerged in water containing

lime, an action similar to petrification takes place. However, the use of mats in silt-laden streams is ruled out, for the transported material disintegrates them rapidly. On another stream, bottom protection was achieved by placing logs on evergreen branches. For bottom slopes from 0.004 to 0.006, the logs can be placed at wide intervals; for greater slopes they should be placed nearer together. At still another place, the bottom was protected with good results by evergreen branches held down by a layer of coarse gravel. If suitable timber is available, the bottom can be protected by split logs. The logs may be laid close together with the rounded side up. This corduroy construction can be carried out in two ways. If unskilled labor is available, the rails are nailed down to a frame close to the location where they are to be used. If skilled labor is available, the bottom is covered by the rails and the side slopes protected by reed mats at the same time.

The rapid wearing away of wood by detritus and its deterioration due to wetting and drying pointed to the possibility of using stone for protecting the bottom and part of the sides.

Paving the bottom and the base of the slopes with rubble is a means of resisting large tractive forces, and the velocity is also reduced due to the increase in roughness of the channel. Dry masonry linings are sometimes used. That part of the banks above the masonry is protected by a sod cover. The banks are protected by masonry lining up to a height at which the tractive force is less than the limiting tractive force of the sod.

In one case, a trapezoidal canal with a rounded bottom was preferred, the bottom being protected by a dry masonry lining. The cost of this protection has been estimated at 0.67 cents per square yard. Such canals permit large bottom slopes, in one case proving adequate with a slope of 6.5 percent. Masonry with joints filled by cement mortar is particularly effective if well placed. A flood canal near Deggendorf (figure 15) has a bottom slope which varies between 0.013 to 0.020 and the average velocity was computed to be from 16.4 to 19.7 feet per second. Originally this section was lined with rubble masonry to a height of from 3.6 to 4.9 feet above the bottom. A sod slope of 1:1.5 was added to the top of the 1:1.25 rubble slope. A flood in

1932 due to heavy rains over the small watershed created a heavy runoff in the streams of the region which was found to be as much as 91.4 second-feet per square mile of drainage area. Because of this many streams overflowed their banks. Water discharging from a tributary into this canal over-topped the sod banks. The sod was destroyed and the paving underscoured. Since the stones were held firmly together by cement mortar, the paving was moved and broken up in large pieces. The damage progressed until a total length of lined canal of about 0.62 mile was destroyed and parts of the side slopes were completely ruined. When the canal was reconstructed, rubble masonry was used to repair the damage to the lining and to replace the sod cover. For a more complete protection, the paving was divided into sections by concrete ribs placed across the bottom and up the banks of the canal. In the future damage to the paving should be confined to individual sections. The above flood attacked the paved slopes with such a force that stone blocks up to 8.8 cubic feet in volume were displaced from the lining. In order to prevent damage to the banks from spreading, logs were placed in the gaps and anchored. This is the quickest and most effective means of temporary repair.

Unfavorable experience with the silting of compound canal sections led to the development of simple sections for streams in industrial regions such as the one shown in figure 19, which, due to its peculiar shape, is called a "steep profile canal". The dished bottom itself is composed of two parts. The sides are protected by one or more courses of side slabs, one over the other, the number of courses depending on the local conditions. The concrete lining was laid on a cinder base 15 cm. (5.9 inches) thick. According to the results of the Emsher Board the cost of lining the canal in 1936 amounted to:

Dished bottom including side slabs per linear foot of the canal.	\$19.40
Dished bottom without side slabs per linear foot of the canal.	11.85

This price holds for the Emsher district.

The cost of laying the slab lining including storage and delivery of a cinder base 15 cm. (5.9 inches) thick is:

Without side slabs per linear foot \$ 8.54
With side slabs per linear foot 13.05

The limiting tractive force of this lining is relatively low or $F_0 = 0.5$ pounds per square foot. Scouring out of the cinder base will result, in the majority of cases, in the complete destruction of the lining.

Heavy concrete slabs on a gravel base for protecting the upper part of the slopes and rubble work for protecting the foot and lower part of the slopes, have been used with good results. Concrete slabs laid dry and the joints sealed with bitumen, have a concrete cover which resists the tendency of the side slopes to settle and to be undercut.

Canals lined with concrete are very resistant to scour by clear water, but they will not, in the long run, withstand the corrosive effect of detritus.²¹ Furthermore, they are vulnerable to frost and

²¹Schoklitsch, A.; *Stauraumverlandung und Kolkabwehr* (Siltling of Reservoirs and Prevention of Scour), J. Springer, Vienna, 1935, page 8.

marsh water. Occasionally these two factors completely destroy concrete in a short time. For small structures, necessary care is not always taken to insure that the aggregate of the concrete is free of detrimental inclusions such as sand containing iron, loamy gravel, etc. Sometimes a water-tight canal is desired. A concrete lining will meet this requirement after being in service a year when the pores have been clogged. The addition of an effective agent to the cement such as "Thurament" will also reduce the seepage.

Since experience with concrete linings is often unsatisfactory, bitumen linings have been tried. Since 1929, systematic tests on the applicability of asphalt and tar for both dry land and hydraulic structures have been performed at the Research Institute for Hydraulic Structures and Water Power at Munich. Tests on the internal structure of the materials showed that linings could be made either tight or permeable and could resist weathering for years.

The growth of plants such as thistles, willows, etc., is accelerated as a result of the absorption of heat by the dark surfaces. Plants push through the lining and loosen it. It is, therefore, necessary to treat the soil base by sprinkling with a plant eradicator before applying the bitumen cover.

In 1933 the Deggendorf Board supplied a mastic cement lining to the bottom and lower part of the sides of the circulation canal at the Saubach pumping plant. The canal is 656 feet long. The lining, 1.6 inches thick, with and without drainage holes at the foot of the side slopes. So far, this lining has been satisfactory. However, it has the tendency to flow during hot weather when the soil is dry.

Important projects, in which asphalt has been used to some extent, have been completed during the past ten years.²² However, a longer

²²Vogt; Versuche mit Asphaltdichtung beim Bau des Adolf Hitler - Kanals (Experiments on the Tightness of Asphalt for the Construction of the Adolf Hitler Canal). Die Bautechnik, Vol. 14, 1936, No. 28. Schiller-Gorges; Versuche mit Asphaltbauweisen beim Erweiterungsbau des Dortmund - Ems - Kanals (Experiments on the Structural Uses of Asphalt for the Widening of the Dortmund - Ems - Canal), Die Bautechnik, Vol. 14, 1936, Nos. 31 and 32.

experience record on large structures awaits the future.

Lining the bottom and side slopes of a stream may be economical depending on the nature of the territory through which the stream passes. It reduces the cost of excavation because a steeper bottom slope is permissible, and it prevents silting and scouring. However from the angler's viewpoint, lining a stream is not advisable because the natural hiding places of the fish are eliminated. Artificial hiding places, demanded by fishing experts, fail in their purpose when installed economically. They inevitably increase the cost of construction.

III. Drops

1. Calculation of the Discharge

The discharge of drops is usually computed from Weisbach's formula. If the discharge is free, that is, if the tailwater lies below the

elevation of the crest of the drop, and if the cross section at the drop is equal to the cross section of the canal so that contractions and expansions are suppressed, then:

$$Q = \frac{2}{3} \mu \sqrt{2g} b \left[\left(h + \frac{v^2}{2g} \right)^{3/2} - \left(\frac{v^2}{2g} \right)^{3/2} \right]$$

in which

- Q = the discharge in second-feet.
- μ = the drop coefficient.
- b = the mean width of the drop in feet.
- h = the head on the drop in feet.
- V = the mean velocity of approach in feet, per second.

Only in exceptional cases of stream regulation, is it necessary to know the discharge at the drop, for the discharge depends on the capacity of the canal. The flow in the canal produces a certain head, h, on the crest of the drop. This static head produces a drop-down of the water surface from the canal to the crest of the drop. As the depth decreases, the velocity increases, and, in certain cases, the bottom of the canal is eroded. Therefore, it is important to know the value of h for determining the type and extent of the lining just upstream from the drop. The Weisbach formula yields only approximate results since it does not take into consideration the flow conditions at the drop.

A rational formula is only possible on the basis of the potential theory. However, such a formula is too complicated for use in practice.

Boss²³ developed a method for calculating the discharge and the

²³Boss; Berechnung der Abflussmengen und der Wasserspiegellage bei Abstürzen und Schwellen unter besonderer Berücksichtigung der dabei auftretenden Zusatzspannungen (Calculation of the Discharge and Water Surface Curve at Drops and Sills by Considering the Back-Pressure Occurring there), *Wasserkraft und Wasserwirtschaft*, Vol. 24, 1929, page 13.

drop-down curve by considering the pressure on the floor just upstream from the drop. From a condition of uniform flow upstream from the drop, the water surfaces fall and pass through the so-called critical depth, d_{cr} (figure 22). If the stream lines are still straight and parallel to each other at section 1-1, then

$$d_{cr} = \sqrt[3]{\frac{Q^2}{gb^2}}$$

The height of the energy gradient at 1-1 is:

$$H = \frac{2H}{3} + \frac{H}{3} = d_{cr} + \frac{v^2}{2g}$$

Downstream from 1-1, the pressure on the bottom becomes noticeably less than the static pressure due to a certain back pressure. As this back pressure increases the water surface falls rapidly. The magnitude of the back pressure can be expressed in terms of a number, Z . By assuming a straight line pressure distribution the equation for the discharge becomes

$$Q = b\sqrt{2g} \frac{2}{3Z} \left\{ [H - d(1-Z)]^{3/2} - (H - d)^{3/2} \right\}$$

The two limits of z are $z = 0$ (straight parallel stream lines) and $Z = 1$ (maximum discharge). If $Z = 1$, the total static pressure is transformed into velocity head; this can only happen when the overfalling nappe is completely aerated. In general, EBSS obtained the following equation for the depth at maximum discharge, the back pressure being the independent variable:

$$d = \sqrt[3]{\frac{Q^2 (2 - Z)^3}{b^2 \frac{8}{9} g (Z^2 - 3Z + 3)^2}}$$

From this analysis and the results of experiments, EBSS concluded that for the maximum discharge, the form of the drop has little influence on the discharge. His equation, however, presupposes unknown values of z and d in the calculation of Q . Hence the discharge can be computed only by trial and error.

Somewhat more difficult is the determination of the depth (static head) at a drop when it is submerged. The tail-water elevation in this case lies higher than the crest of the drop. The discharge is usually computed from the following equation:

$$Q = \frac{2}{3} \mu_1 b\sqrt{2g} \left[(h_2 + K)^{3/2} - K^{3/2} \right] + \mu_2 (h_1 - h_2) b\sqrt{2g (h_2 + K)}$$

in which

h_2 = elevation of the tailwater above the crest of the drop.

h_1 = the head on the drop measured upstream.

$k = \frac{v^2}{2g}$ = velocity head.

The use of this formula gives only approximate results since μ_1 and μ_2 cannot be determined with sufficient accuracy.

The author was able to show by means of model tests that a submerged weir can be considered in the light of a weir with a free nappe, that is as the tailwater rises, the flow conditions for the latter change gradually into the flow conditions for a submerged weir.²⁴ The

²⁴Keutner, C.; Herleitung eines neuen Berechnungsverfahrens für den Abfluss an Wehren aus der Geschwindigkeitsverteilung des Wassers über der Wehrkrone (Development of a New Procedure for Computing the Discharge of Weirs Based on the Velocity Distribution over the Weir Crest), Die Bautechnik, Vol. 7, 1929, Page 575.

distribution of velocity directly over the crest was measured and the data was analyzed according to Weisbach's formula. Figure 23 shows the results of these tests for a given discharge but with different tailwater elevations. The water surface drops from h to h' for every condition of submergence. For the unsubmerged weir in (1) of figure 23, the area 1, 2, v_n , 3, 4, shows the theoretical Q . (Weisbach's equation with $\mu = 1$.) The experimental Q -area is given by 5, 6, v_v , 3, 4. The ratio of these two areas is the coefficient, μ , in Weisbach's formula. The tests showed that the theoretical velocity according to Torricelli's law,

$$v = \sqrt{2gh}$$

is valid only at a point on the water surface directly over the weir crest. As the tailwater rises, the head, h , is not affected until a ratio of h_1 to h_2 equal to 2.5 is reached. This value will be considered as the point of transition from the free weir to the submerged weir.

As the tailwater rises still farther, the velocity in the nappe, V_v , decreases as a consequence of the back pressure of the tailwater. The theoretical velocity is attained only at the water surface for all tailwater elevations. As h_2 increases, the velocity in the nappe decreases and the V_v -curve becomes flatter. As the velocity decreases, the head, h' , at the crest of the weir increases. The sums of the kinetic- and pressure-areas as well as the total areas bounded by the V_v -curves, remain constant. If the weir coefficient, μ , is computed as the ratio of the actual Q -area to the theoretical Q -area, a series of values will be obtained, their size being uniformly dependent on the tailwater elevation. Among other things in these graphs, the Q -areas representing the theoretical discharge for a submerged weir (equation without coefficients) are also shown. In (5), this area is given by 1, 2, 6, V_v , 7, 4, and although they represent equal discharges, they are not equal in area. Therefore, the equation for the submerged weir cannot be computed using a uniform relation between the head, h , measured upstream from the weir and the submergence, h_2 . A submerged weir possesses two types of nappes, a diving nappe when $\frac{h_1}{h_2} > 1.17$ as in (3) and a wave nappe when $h_1/h_2 < 1$ as in (4) and (5). The critical depth for straight, parallel stream lines was computed from

$$h_{cr} = \sqrt[3]{\frac{Q^2}{b^2 g}}$$

and is also given in the graphs. With a free nappe, the head, h' , on the crest of the weir is significantly smaller than h'/h_{cr} . With the type of discharge shown in (3), h' is still less than h_{cr} , but with a wave nappe, $h' > h_{cr}$. The ratio h'/h_{cr} is dependent on h_2 .

General equation for these flow relations can be determined from the results of measurements on various shaped weirs having different heights of crest.²⁵ The function

$$\mu = f\left(\frac{d_E}{d_{cr}}\right)$$

²⁵Keutner, C.; Der Einfluss der Krümmung der Wasserfäden auf die Energiebilanz und das Wasserabführungsvermögen von abgerundeten und scharfkantigen Wehrkörpern (The Influence of Curvature of the Stream Lines on the Energy Balance and the Discharge Capacity of Rounded and Sharp-Crested Weirs); Wasserkraft und Wasserwirtschaft, Vol. 28, 1933, Page 25.

is plotted in figure 24 from which values of μ can be read. d_{cr} corresponds to h_{cr} in figure 23; d_1 is the depth before the drop-down curve begins. These μ -values are the new weir coefficients in Weisbach's equation. In figure 25, the following function for the submerged weir is represented:

$$\mu = f\left(\frac{d_1}{d_2}\right)$$

The boundary between the two types of nappes is also shown in figure 25. From these two graphs, both μ and $\frac{d_E}{d_{cr}}$ can be quickly determined for a

given value of the ratio, d_1/d_2 . After computing d_{cr} from the ratio, $\frac{d_E}{d_{cr}}$, θ can be calculated. For a first approximation set $d_E = d_1$.

According to Musterle, the curve of the water surface can be correctly computed by means of the impulse law.²⁶ It was observed in

²⁶Musterle, T.; Abflussberechnungen bei Wehren mit breiter Krone mit Hilfe des Impulssatzes (Calculation of the Discharge for Broad-Crest Weirs Using the Momentum Principle), Die Wasserwirtschaft, Vol. 23, 1930, Page 441.

model tests that the critical depth for straight parallel flow does not occur directly at the brink of the drop, (section II-II figure 22) but at some distance upstream. If the discharge is held constant and the tailwater is raised, there is a limit up to which the water level upstream is not changed (defined by the author as the transition from a submerged weir to an unsubmerged weir). In Musterle's analysis, the critical depth is placed at the brink of the weir (figure 26), and can be computed by means of the minimum dynamic capacity. The sum of the

actual dynamic capacity, $S_w + D_u$, upstream must equal its value S_u downstream from the drop. Further

$$S_w = S_{min} = \frac{3}{2} d_{cr}^2 b \sigma$$

$$D_u = \frac{2d_{cr} + w}{2} w b \sigma$$

$$S_u = b \left(\frac{d_u^2}{2} + 2d_u K_u \right) \sigma$$

in which $K_u = \frac{V_u^2}{2g}$

$$d_{cr} = 2K_{cr} = \frac{2}{3} R_{min} = \sqrt[3]{\frac{Q^2}{bg}}$$

d_u can be computed, or Q may be calculated from d_{cr} .

The analysis for an unsubmerged weir is similar, only in this case the dynamic capacity at a cross section of the canal upstream from the drop must be set equal the dynamic capacity at the drop.

Jacoby²⁷ developed a method of calculation based on model tests which is especially well adapted to determining the discharge over a ground sill.

²⁷ Jacoby, E.; Die Berechnung der Stauhöhe bei Wehren (The Calculation of the Head on Weirs); Wasserkraft and Wasserwirtschaft, Vol. 28, 1933, Page 79.

2. The Calculation of the Drop-Down Curve

After determining the depth at the brink of the drop, it is necessary to find the extent of the drop-down curve and from this the extent of the increased velocity. Calculations of the drop-down curve by the most popular formulas usually do not agree with measurements. In particular with narrow trapezoidal canals, the difference between computed and measured curve is very considerable. Kozeny²⁸ developed a semi-graphical

²⁸Kozeny, J.; Über die ungleichformige Bewegung des Wassers in elementarer Darstellung. Mit einem Zusatz über die nicht stationäre Bewegung (On Non-uniform Motion of Water Represented Elementarily. With an Appendix on Non-steady Motion) Die Wasserwirtschaft, Vol. 22, 1929, Nos. 29 and 30.

method for computing the drop-down in narrow channels, which shows good agreement with measurements.

Figure 27 shows a comparison of the drop-down curves computed by Kozeny's and Ruhlmann's methods. The difference is very noticeable chiefly because with the drop-down curve according to Ruhlmann, the depth, d' , was computed from Weisbach's equation and therefore is too large. According to Kozeny the drop-down extends about 1,150 feet upstream from the brink of the drop. The velocity of approach according to Kozeny, v_K , and according to Ruhlmann, v_R likewise are different. If, for the example shown in figure 27, a limiting tractive force of 0.4 pounds per square foot is permissible, the allowable velocity is 5.45 feet per second. The bottom of the canal must be lined to a distance of 574 feet upstream from the drop in order to prevent scour. According to Ruhlmann the lining must extend still farther. However, the drop-down curve computed by Kozeny's method agrees well with the measured curve. The assumed tractive force is also plotted in the figure. The determination of f_t for a single section is however, inaccurate, since the tangent to the water surface must be introduced for the slope of the water surface. Similar computations can best be evaluated in terms of the velocity rather than the tractive force.

Kozeny²⁹ also advanced a method for computing the drop-down curve in broad canals which likewise shows good agreement with measurements.

Kozeny, J.; Berechnung der Senkungskurve in regelmäßigen breiten Gerinnen (Calculation of the Drop-Down Curves in Uniform, Broad channels). Wasserdrift und Wasserwirtschaft, Vol. 23, 1928, Page 232. Here the values in the tables are plotted in graphs. See also footnote 4, page 248.

Figure 28 shows backwater curves as far as 131 feet above a drop computed by Kozeny's and Schaffernak's methods, respectively. The two curves deviate considerably from one another.

As already noted, a smaller depth than the critical depth, d_{cr} exists directly over the crest of the drop. According to Kozeny:

$$q = a d'^{3/2} = 34.2 d'^{3/2}$$

in which q = the discharge per foot of width in second-feet and d' = the depth on the brink in feet. This value of q agrees approximately with the value which Schaffernak found for a broad-crested, horizontal and gravel-studded weir, namely: $d = 37.6$ to 30.9 . In the example, $d' = 2.20$ feet. If the drop-down curve is extended to a depth d' , the position of the brink of the drop should be approximately given. This lies at about 6.56 feet downstream from the section at which the critical depth occurs. The point at which the water surface begins to drop is at a distance, $l_a = 8d_0$ (approximately) upstream from the computed position of the brink.

3. The Form of the Cross Section at the Drop

Since the drop-down curve begins upstream from the brink of the drop, an attempt has been made to contract the cross section at the drop, in order to localize the drop-down curve. With some suitable structure such as a sill or side contractions, it is possible to raise the surface curve upstream from the drop. To accomplish this, a vertical contraction such as a sill is preferred. Often the brink of the drop is placed a few inches farther downstream. However, the water surface curve is only affected at relatively small depths of flow when such a scheme is used.

Bössi²³ gives a method for computing the height of a sill which will flatten out the drop-down curve. He starts from the critical depths for straight and for curved stream lines (see figure 22). If the expression for the height of the energy gradient for straight stream lines and maximum discharge is set equal to the height of the energy gradient for the flow with curved stream lines, the height, h_s , of the sill is given by:

$$h_s = \frac{H}{2} \left[\frac{1}{3} + \frac{2}{3} Z - \left(\frac{Z}{\sqrt{3}} + \frac{1}{\sqrt{27}} \right)^{2/3} \right]$$

$$h_s = \frac{3}{2} \frac{d_{cr}}{Z} \left[\frac{1}{3} + \frac{2}{3} Z - \frac{Z}{\sqrt{3}} + \frac{1}{\sqrt{27}} \right]^{2/3}$$

For the computation, a cross section is chosen for the canal for which the localized drop-down is not large (figure 28). The depth of flow at the section at 49.2 feet is 4.66 feet, and the velocity head there is 0.87 feet; therefore, the height of the energy gradient at this section is 5.53 feet. Assuming that the nappe springs free, Z can be taken equal to unity and the height of the sill, h_s , from the Boss equation is 0.95 feet. For $Z = 0.5$, $h_s = 0.87$ feet. For a sill at least 0.95 feet high or about $0.145 d_o$, the drop-down curve will not extend back beyond the critical depth which will be at the same place as without a sill. However, the water surface at the drop will be raised an amount Δh . Thus if a depth on the crest of the sill equal to 2.2 feet is assumed sufficient for the given discharge, the new water surface will lie 3.15 feet above the brink of the drop. During floods, a sill of minimum height will be effective, for the most part, in preventing erosion of the bottom of the canal just above the drop. That height of sill which will not raise the backwater curve in the canal is shown in figure 28. The water surface is such that the depth of flow in the canal as far as a section near the drop, is constant, as is seen from the localized drop-down curve, and, therefore, the velocity is about equal to the critical velocity. If the head upstream from the sill is computed from Weisbach's formula with $\mu = 0.736$, a value, $h_o = 3.68$ feet, is obtained. It should be noted that in order to carry the given discharge, the maximum height of sill is, $h_s = 6.56 - 3.68 = 2.88$ feet or about $0.44 d_o$. The new depth at the drop, d_s , is found from the depth over the crest of the sill, h' , which is computed from the author's formula:

$$h' = 0.37h \left(\frac{h}{n} \right)^{0.2}$$

where h = head on the sill and n = the height of the sill. Thus $h' = 2.83$ feet or $d's = 5.71$ feet. According to observations during the model tests, the drop-down curve begins approximately at a distance $l_n = 8h_0$; the two values, l_a and l_n agree well in the order of their magnitude.

Although a sill installed at a minimum height, h_s , has no effect during flood stages, the water surface is raised considerably at small discharges. The velocity is decreased so much that silt deposits occur which are not scoured out by rare floods. In order to overcome this objection, a section of the sill is left open. One example of such a sill, 13.1 feet long, has a central rectangular section cut out, $S_x = h_s = 6.56$ feet (see figure 28). In relation to the depth of flow in the canal, the height of sill is $h = 0.09 d_0$, thus smaller than the minimum h_s in the computed example. The drop-down during floods is not affected by a sill of this height providing the nappe is free at the flood discharge. However, since the height of the drop is relatively small, a flood will submerge the drop. With such a flow condition, an appreciable and favorable raising of the water surface is produced by a small sill.

If a sill is installed with a maximum height, h_s , it will cause a uniform flow up to within a short distance of the crest of the drop. However, a complete silting-up of a large section of the headwater canal is surely to be expected. A slot must be cut out to take care of the smaller discharges. In order to obtain a uniform backwater as with an uncut sill it is necessary to increase the height of the cut sill to h_x (figure 28). Such a sill produces side contractions.

F. GÄLLICH³⁰ was the first to state the principles for designing

³⁰Gällich, F.; *Regulierung von Bächen mit starken Gefälle* (Regulation of Streams with Large Slopes), Brochure published by the author (deceased).

drops with side contractions for eliminating drop-down curves. He employed a parabolic shape for the drop and computed its outline mathematically, employing Weisbach's equation for weirs with a free nappe and the general equation for submerged weirs. Tables facilitate the calculation of the parabola. The drop-down curve was eliminated at all discharges. The larger the discharge coefficient, μ , is assumed to be, the smaller the area of the parabola. For the submerged weir, μ_1 , is put equal to μ_2 . Weir shapes computed with $\mu = 0.75$ and 0.83 according to Gállich are shown in figure 30. The sides of the weir are discontinuous as computed, as is shown by the broken line. The transition from a weir with free overfall to a submerged weir causes these sharp breaks in the curve. This can be explained by the fact that the Weisbach equation and the equation for a submerged weir are entirely different structurally. They do not merge smoothly from one to the other.

It is possible to apply the results already obtained by the author, in model tests on the maximum variation of the coefficient of discharge for submerged weirs, to Gállich's and Wagerer's methods of calculation, obtaining the best shape of the parabolic crest from a hydraulic viewpoint according to Gállich. Using this improved method, a curve for the crest can be computed which changes uniformly throughout even at the transition from submerged to free weir. The unsightly and expensive jagged crest in figure 30 can be assimilated to a parabola, the amount of assimilation being left to the designer. At such a drop on the Sulz River at Berching, a flood of 2,648 second-feet overflowed the banks of the canal; yet the position of the drop and the localized drop-down curve were clearly evident. Although with this discharge, the drop was submerged, and only a small difference in elevation existed between the headwater and tail water, yet for a considerable distance downstream from the drop, the flow was seen to be very disturbed and it violently attacked the bed and banks. The section at the drop compared to the cross section of the canal above the drop possessed a relatively small contraction. This design corresponds to a value of μ of about 0.70.

The larger μ is taken and the smaller the net flow area, the larger the velocity and depth of flow over the drop. The range of the trajectory of the nappe increases with an increasing velocity over the drop. At a drop at Wettenhausen on the Kammel River, the overfalling nappe caused the banks to cave in at the end of a retaining wall. The length of the stilling pool, about 79 feet, was too short relative to the range of the nappe. It was resolved, therefore, to shorten the range of the nappe by increasing the area of flow at the drop. Hence the top portion of the parabolic sill was removed for a considerable distance. By doing this, the backwater effect during high water due to the contraction was necessarily lost. During the following year, the sand and gravel bed of the upper canal was scoured to a depth of about 3.3 feet.

A narrow parabolic crest by creating a large contraction, requires a long and wide stilling pool. A drop on the Strogen River at Langenpreisen has the widest part of the stilling pool at some distance from the drop so as to take account of the trajectory of the nappe.

A peculiar type of drop is found on the Mindel River at Thannhausen. The sill is parabolic in form and is narrow in comparison with the width of the canal. The banks of the canal were warped to meet the side extremities of the sill. The short section of the canal just above the drop was now shaped like a nozzle. Not only was the backwater effect due to the contraction at the sill lost, but also the nozzle-shaped structure created a sharp drop-down curve which was just as large as though the canal was continued directly to the brink of the drop without a sill. Hence, a contraction in the area of flow is only effective in producing a backwater effect when the banks of the canal are not warped to fit the sill at the drop.

The Ingolstadt Agricultural Engineering Commission acquired a large amount of worthwhile data at a drop on the regulated Weilach River. The Crest of the drop was designed according to Gullich's method with $\mu = 0.85$ and 0.75 , respectively (figure 37).

The relative large contraction caused the nappe to have a long trajectory. The length of the stilling pool proved to be too short

especially with large μ -values, so that the banks at the end of the side walls of the stilling pool were eroded. Furthermore, the canal silted-up in the course of a year; thus the contraction was too great especially at normal water levels. The redesign of the crest according to Wagerer resulted in a somewhat better shape. The sill was parabolic in form only up to that elevation at which the naps became submerged. Above this elevation the crest consists of a straight line which meets the parabolic part tangentially. Up to this transition, the contraction is somewhat smaller than for Gullich's design and hence the danger of silting is greatly reduced. Afterwards, a rectangular opening was cut out of the lower part of the parabolic section. These changes in the shape of the drop have vindicated themselves during a period of a year. At other drops the parabolic design has been dispensed with at the outset and simply a rectangular slot used. However, the results do not come up to expectations. A proposal by Wagerer for the design of a drop is shown in figure 37. It contains a large rectangle at the base of the sill which takes care of the small discharges with only a very small backwater effect. A trapezoidal section is superposed on the rectangular slot and extends laterally farther than the parabolic crest. The amount of contraction is about the same so that the backwater effect is approximately the same for the two designs during flood discharges.

Ailler³¹ proposed, for the best design, a trapezoidal notch whose sides are parallel to the sides of the trapezoidal canal (figure 30). This design has not proved satisfactory and all further details are dispensed with here.

³¹Wagerer; Zur Berechnung absenkungsfreier Abstürze (on the Design of a Drop Free from a Drop-down Curve) and Ailler; Die Wasserabführung abgetrepter regulierter Bäche (The Discharge of Streams Regulated by Drops). Both papers were published in the internal publications of Agricultural Engineering Board.

Other drops are designed differently according to their various requirements. Rectangular notches contract the nappe very markedly especially during floods so that extensive protection of the banks downstream from the drop is necessary.

In Austria, drop crests are usually lined with natural stone, if the structure is a concrete one, in order resist better the scouring action of water and silt. Small triangular notches produce only a small backwater effect during flood stages. They are satisfactory if the floods are small and of short duration and if the stream bed possesses a large limiting, tractive force.

A compound canal cross section requires a complex drop crest.

The drop on the Sulz River at Beilingries has a peculiar design. The crest is divided into a normal flow section, an average flood section and a bank-full section. The slopes of the sides are different for the different sections. The lowest section is a rectangular notch; the other two sections have side slopes of 1:1 and 1:1.5, respectively. The contraction at the drop is considerable during periods of high water. No scour was observed above the drop.

The Weilheim Board of Works when designing drops of small height departs entirely from the idea of contracting the cross section at the drop. Different designs at various locations were tested in which the cross section at the drop was expanded in comparison with the canal cross section. Such drops are submerged during flood stages. The bottom of the canal above the drop is riprapped to provide a high limiting tractive force. Expanding the cross section at the drop, insofar as the best hydraulic design of a stilling pool is concerned, contributes toward reducing the distance in which the disturbed flow is converted into quiet flow, or, in other words, the distance in which the kinetic energy is transformed into potential energy. Uniform flow began at about 65.6 feet downstream from the drop. This design should be classed among the best, hydraulically speaking.

4. The Design of the Shape of the Crest

In figure 43, discharge curves for various types of crests are shown, the nappe being free. The canal section is assumed to be trapezoidal with a bottom width of 9.84 feet and side slopes of 1:1.5.

According to curves 4 and 5, the discharge at a depth, $d = 6.56$ feet, and a slope, $S = 0.0015$, is $Q = 646.2$ second-feet. The discharge capacity of a canal for a given depth can be obtained from curve (1). The first form that will be discussed is the parabolic crest. This crest has such dimensions that it is described as a weir "without a drop-down curve." By neglecting the velocity of approach in the canal, the first approximation of a parabolic crest (1 in the sketch in figure 43) is obtained. In the majority of cases, however, it is necessary to consider the velocity of approach.

This gives parabola (2). The difference in the areas of these two parabolas becomes considerable as the depth increases. Discharge curve (3) for the parabolic crest follows the discharge curve of the canal very closely. The amount of backwater, or the amount of drop-down in the water surface for various depths of flow is given in table 2.

TABLE 2

Water Depth in Canal:	Discharge:	Backwater Height above the Parabolic Drop Crest 2:	Drop-Down Height Above a Trapezoidal Drop Crest Side slope 1:0.15:	Drop-Down Height Above a Trapezoidal Drop Crest Side Slope 1:0.15:
Feet:	Sec.-ft.:	Feet:	Feet:	Feet:
0 :	0 :	+0.000 :	+0.00 :	+0.00 :
0.5 :	6 :	+0.205 :	-0.15 :	-0.40 :
1.0 :	19 :	+0.180 :	-0.27 :	-0.62 :
1.5 :	40 :	+0.150 :	-0.38 :	-0.81 :
2.0 :	68 :	+0.114 :	-0.46 :	-0.97 :
2.5 :	100 :	+0.096 :	-0.52 :	-1.11 :
3.0 :	139 :	+0.090 :	-0.55 :	-1.26 :
3.5 :	187 :	+0.086 :	-0.55 :	-1.43 :
4.0 :	243 :	+0.080 :	-0.51 :	-1.59 :
4.5 :	306 :	+0.068 :	-0.44 :	-1.74 :
5.0 :	375 :	+0.062 :	-0.37 :	-1.88 :
5.5 :	453 :	+0.053 :	-0.28 :	-2.02 :
6.0 :	537 :	+0.036 :	-0.16 :	-2.16 :
6.56 :	647 :	+0.000 :	-0.00 :	-2.31 :

A backwater height of 0.18 feet is experienced with parabolic crest (2) when the depth of flow in the canal is 1.0 foot. With an increasing depth of flow the backwater height decreases until at a depth of 6.56 feet it becomes zero. These backwater heights are so small that this drop may be described as "free from a drop-down curve." Whether or not silt is deposited upstream from the drop as a consequence of the contraction will now be discussed. The increase of the average velocity in the canal as the depth in the canal increases is shown by velocity curve (6). The velocity curve for parabolic drop (2) is given by curve (8). This curve indicates a minimum average velocity in the canal or the velocity at the maximum backwater, that is, at the beginning of the localized drop-down curve. If the limiting tractive force for sedimentation is assumed to be $f_0 = 0.09$ pounds per square foot corresponding to an average velocity of 2.00 feet per second, then with a depth of flow, $d < 1.74$ feet or with a discharge $Q < 52.97$ second-feet, the possibility of silting exists providing the water carries silt and sand in sufficient quantities. Whether or not, danger of silting is present, can be determined only in connection with the local conditions.

A trapezoidal crest is now considered. If the cross section at the drop has the same dimensions as the canal cross section, the discharge is given by curve (5). The amount of the drop-down for various depths of flow can be determined from table 2. For a depth equal to 6.56 feet and considering the velocity of approach, a drop-down of 2.31 feet is attained.

A trapezoidal crest has been designed to discharge 646.2 second-feet without a drop-down. For a bottom width of 9.84 feet and a slope of the sides of 1:0.15, the discharge is given by curve No. 4. This deviates considerably from that of the canal itself. The mean velocity just upstream from the drop is greater than the canal velocities at all depths of flow with the exception of $d = 6.56$ feet. The drop-down for various depths is given in the table. The maximum value is $z = 0.56$ feet at a depth, $d = 3.28$ feet. In consequence of these

small values of the drop-down of the water surface, no significant increase in velocity occurred in the canal.

In all cases in which a danger of silting exists, the trapezoidal shape is to be preferred to the parabolic shape.

The discharge capacity of triangular, rectangular, trapezoidal and parabolic drop crests is easily computed by simple formulas; however, a circular crest involves a solution of an elliptic integral (figure 14). Siko³² studied the equations for various shapes of drop

³²Siko, Attila; Különféle alakú nyílásokon szabadon átbukó vízmennyiség meghatározása számítással különös tekintettel a vízmosaskőötölgéstakra (Mathematical and Graphical determination of the Discharge of weirs with free Nappes of Various Shapes with Special Reference to their Application to Check Dams for Mountain Streams), Vizügyi Közlemények, Vol. 16, 1934, No. 3. (Hungary).

crests and recommended that a parabolic crest be used in place of a circular crest pointing out that they both give approximately the same results. He developed a method for calculating the discharge for any given shape of crest which gives a rapid and reliable solution.

In general, according to figure 45,

$$dQ = \mu z \times \sqrt{2gh} \, dh$$

$$\frac{dQ}{dh} = \mu z \times \sqrt{2gh} = z$$

The area bounded by 1, 2, 3, and 4 is equal to:

$$\int_0^t f(h) dh = \int_0^t z dh = \int_0^t \frac{dQ}{dh} dh = \int_0^t dQ = Q_{th}$$

Considering the velocity of approach in the canal, the head is replaced by the total head, $h + k$, where $K = \frac{v^2}{2g}$. Area

1', 2, 3, 4 represents the theoretical discharge, Q_{th} . The actual discharge is given by:

$$Q = 2 \mu Q_{th}$$

This computation method is developed for a parabolic crest in figure 45. The numerical dimensions are taken from the preceding example. The equation of a parabola through the vertex is:

$$y = \frac{4t}{b^2} x^2$$

which may be written:

$$y = 0.268x^2$$

Table 4 shows the calculations for an average increment of head

$$\Delta x = 0.492 \text{ feet}$$

The actual discharge over the parabolic crest is

$$Q = 2 (0.63) 514.2 = 647.9 \text{ second-feet}$$

assuming a coefficient of discharge, $\mu = 0.63$. This value of corresponds to the average coefficient for a weir with side contractions.³³

³³Keutner, Wassermengenmessung an Wehren mit zwei- und dreiseitiger (Poncelet-Überfall) Einschränkung (Discharge of Weirs with Two and Three-Sided Contractions - Poncelet Weir) Die Bauingenieur, Vol. 13, 1932, P. 399.

Table 4

Parabolic Drop Crest

$$y = 0.268 x^2$$

$y=t-h$				$V = 2g(h+k)$		$Z = XV$		Δh		$\Delta C = Z_m \Delta h$	
h ft.	ft.	x^2	x ft.	h+k ft.	ft./sec.	Z	Z_m^*	ft.	sec.-ft.		
0.00	6.561	24.48	4.95	0.00	0.00	0.00					
0.230	6.331	23.62	4.86	0.617	6.30	55.4	27.7	0.230	6.4		
0.492	6.069	22.65	4.76	0.879	7.55	65.1	60.3	0.262	15.8		
0.984	5.577	20.81	4.54	1.371	9.42	77.8	71.5	0.492	35.2		
1.476	5.085	18.97	4.35	1.864	10.96	86.3	82.1	0.492	40.4		
1.968	4.593	17.14	4.14	2.356	12.34	92.8	89.6	0.492	44.1		
2.461	4.100	15.30	3.91	2.848	13.55	96.0	94.4	0.493	46.5		
2.953	3.608	13.46	3.67	3.340	14.67	97.7	96.9	0.492	47.7		
3.445	3.116	11.63	3.41	3.832	15.75	97.1	97.4	0.492	47.9		
3.937	2.624	9.79	3.13	4.324	16.70	94.7	95.9	0.492	47.2		
4.429	2.132	7.96	2.82	4.817	17.62	90.2	92.5	0.492	45.5		
4.921	1.640	6.12	2.47	5.309	18.50	83.1	86.7	0.492	42.7		
5.413	1.148	4.28	2.07	5.801	19.36	72.3	77.7	0.492	38.2		
5.906	0.655	2.44	1.56	6.293	20.18	57.6	65.0	0.493	32.0		
6.234	0.327	1.22	1.10	6.621	20.67	41.4	49.5	0.328	16.2		
6.398	0.163	0.61	0.78	6.785	20.93	30.2	35.8	0.164	5.9		
6.561	1.000	0.00	0.00	6.949	21.16	0.00	13.1	0.163	2.5		
											514.2

$$Z_m^* = 1/2 (Z_1 + Z_1 + 1)$$

SUMMARY

The height of a drop is a given quantity for the design. Using this height, a calculation is made to determine whether the nappe is free or submerged during flood discharges. The maximum velocity and maximum tractive force are also computed.

For an ordinary drop with a free nappe, the depth of flow at the drop is less than the critical depth. Just upstream from the drop, the bottom and banks of the canal are eroded in consequence of the high velocity existing there.

The distance of the beginning of the drop-down curve from the brink of the drop depends on the bottom slope of the canal; the

greater the slope, the nearer is the beginning of the drop-down curve to the brink of the drop.

In order to combat erosion of the bed, it is necessary to contract the cross section at the drop relative to the cross section at the canal. A flow "free from a drop-down curve" can be established by a properly designed drop. However, in the majority of cases, this is not expedient.

If floods have a short duration - often they last for only a few hours out of a year - a tractive force larger than the limiting tractive force of the bed material is permissible. A contracted drop is not advisable if silting of the canal occurs frequently, for contracting the nappe also increases the range of its trajectory. The stilling pool must then be lengthened which increases the cost of the structure. It is proper to choose a lower limit for the contraction.

A contraction which produces a backwater effect - in contrast to a drop-down - is not permissible if a velocity lower than the limiting sedimentation velocity is thereby created. In general, a trapezoidal crest is to be preferred to a parabolic crest because with the former there is no backwater effect at low discharges.

If the height of the drop is so small that the nappe is submerged at higher discharges, then a contraction should be used. Widening the canal just upstream from the drop will insure a quiet uniform flow even with flood discharges.

The frequently used trapezoidal canal cross section is often deformed in the course of time due to the scouring of the bottom. From a hydraulic viewpoint the best form of cross section is a trapezoid superposed on a parabolic or circular bottom. Compound cross sections are not recommended since they have a tendency to silt up; also the transition from the canal to the drop increases the difficulties of construction.

An earth canal is the cheapest type. If, however, the bottom of such a canal has a steeper slope than is permissible with the given soil properties, a lining of the bottom and side slopes of

the canal is required. In general, a lining need not be impermeable; its chief function is to resist the scouring action of the flow. For silt-laden streams a lining of fascines or bitumen is not expedient, and in streams containing marsh water or carbonic acid, the use of concrete is inadvisable.

FIG-1

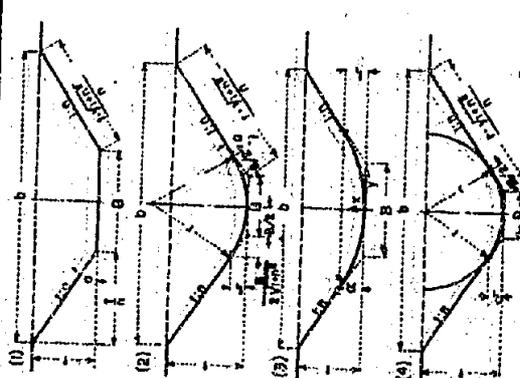


FIGURE 2 - FORMS OF CANAL SECTIONS

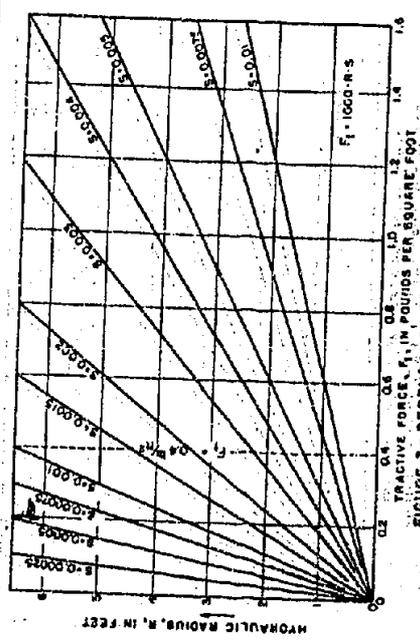


FIGURE 3 - DEPENDANCE OF THE TRACTIVE FORCE ON THE HYDRAULIC RADIUS AND THE SLOPE

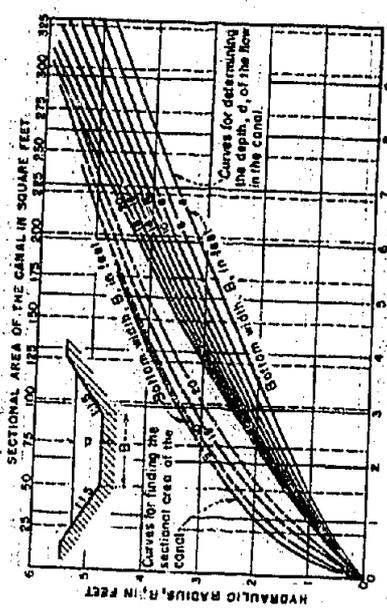


FIGURE 4 - RELATION BETWEEN THE DEPTH OF FLOW, HYDRAULIC RADIUS, BOTTOM WIDTH AND SECTIONAL AREA OF A TRAPEZOIDAL CANAL

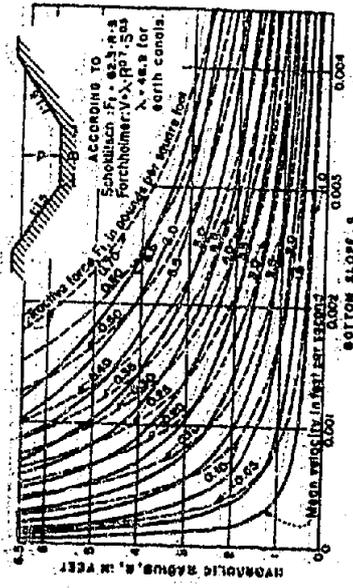


FIGURE 5 - RELATION BETWEEN THE HYDRAULIC RADIUS, MEAN VELOCITY AND THE TRACTIVE FORCE FOR A TRAPEZOIDAL CANAL

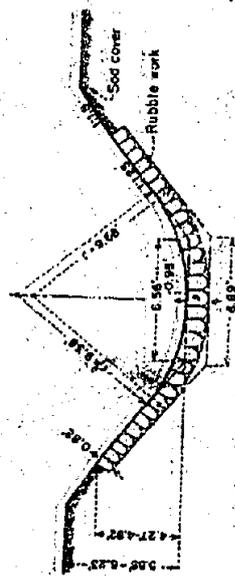


FIGURE 15 - SECTION OF THE HAMMERNDAL CREEK FLOOD CANAL, ORIGINAL DESIGN IDEGGENDORF AGRICULTURAL ENGINEERING BOARD.

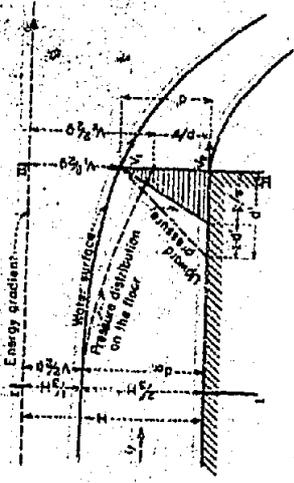


FIGURE 22 - CALCULATION OF THE DROP DOWN CURVE AT A DROP ACCORDING TO BÖSS

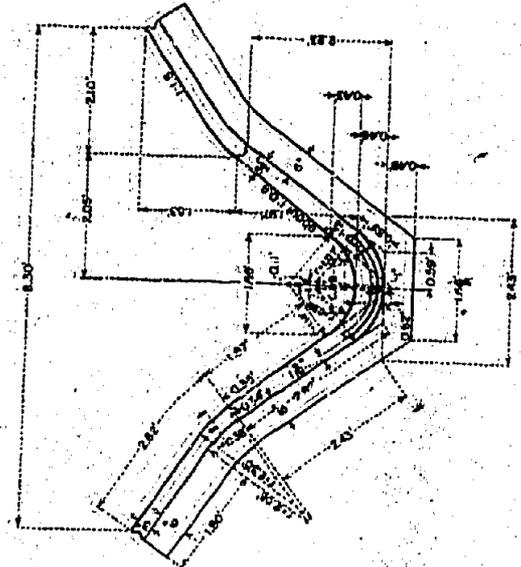


FIGURE 19 - SECTION OF A CONCRETE TOP LAYER AND SLAB LINING (STEEP PROFILE) OF A STREAM IN AN INDUSTRIAL DISTRICT (GMSCHER COMPANY, ESSEN BOARD OF WORKS)



FIGURE 7 - SILTING OF A COMPOUND CANAL SECTION

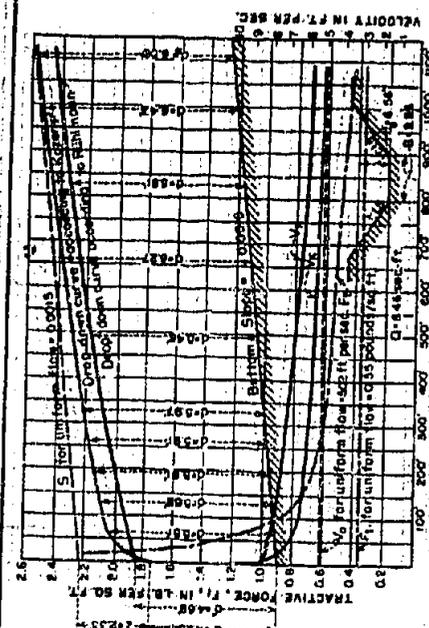


FIGURE 27- DETERMINATION OF A DROP-DOWN CURVE ACCORDING TO KOZENY AND RUHLMANN

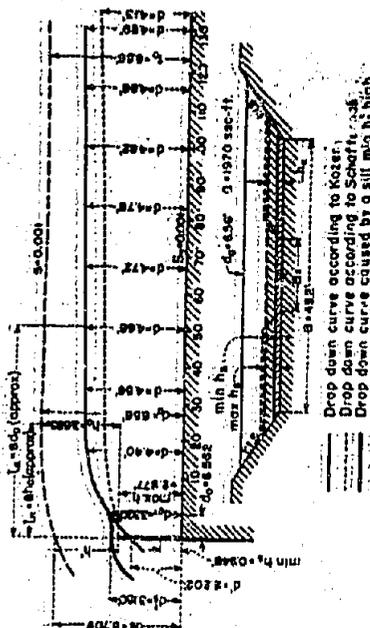


FIGURE 28- DETERMINATION OF A DROP-DOWN CURVE ACCORDING TO KOZENY AND SCHAFFERNAK FOR A WIDE CANAL AND THE PERMISSIBLE HEIGHT OF A SILL ACCORDING TO BOUSS

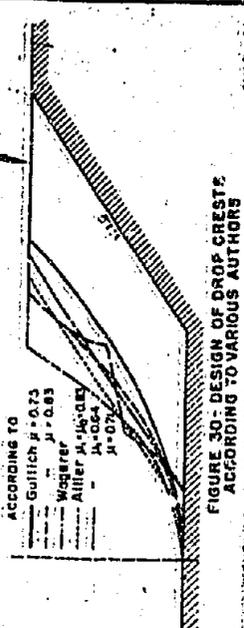


FIGURE 30- DESIGN OF DROP CRESTS ACCORDING TO VARIOUS AUTHORS

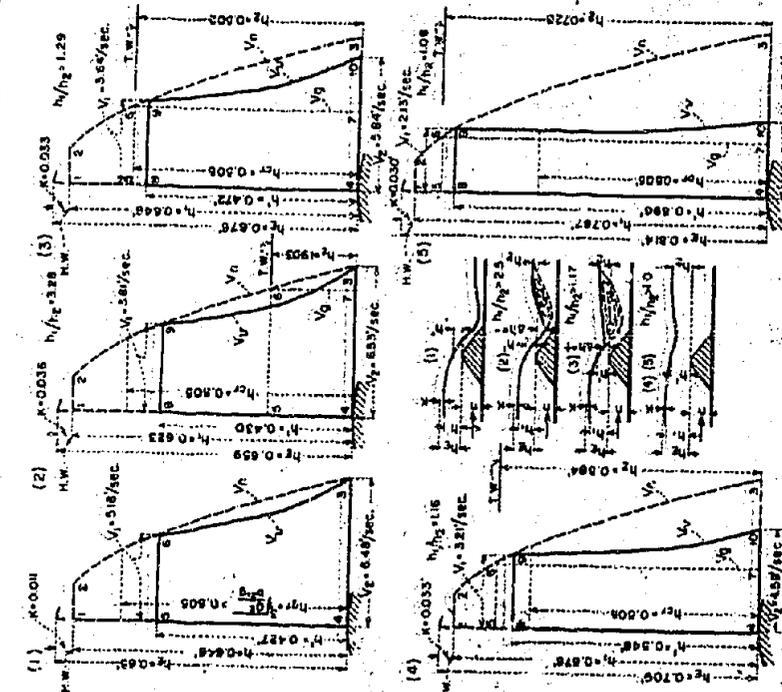


FIGURE 23- RESULTS OF MODEL TESTS ON WEIRS WITH FREE NAPPE AND SUBMERGED WEIRS ACCORDING TO KEUTNER

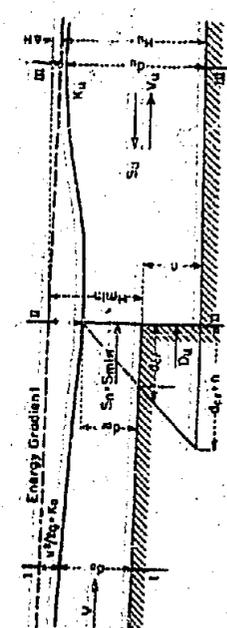


FIGURE 26- CALCULATION OF THE WATER SURFACE CURVE AT A DROP ACCORDING TO MUSTERTE

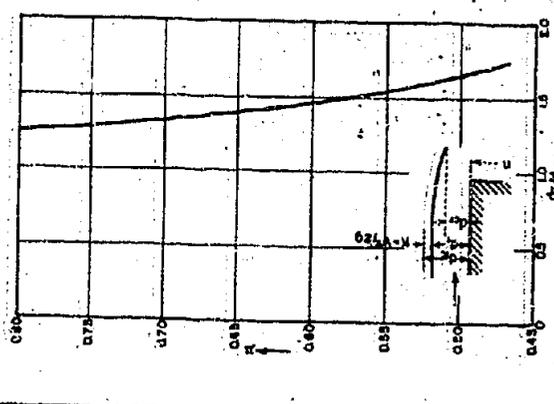


FIGURE 24- RELATION BETWEEN THE RATIO OF THE WEIGHT OF THE ENERGY GRADIENT TO THE CRITICAL DEPTH AND THE DISCHARGE COEFFICIENT OF A DROP ACCORDING TO KEUTNER

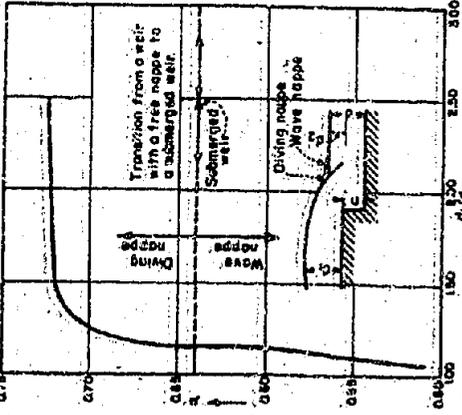


FIGURE 25- RELATION BETWEEN THE RATIO OF THE DEPTH OF FLOW ABOVE THE DROP TO THE DEPTH OF SUBMERGED WEIR AND THE DISCHARGE COEFFICIENT ACCORDING TO KEUTNER

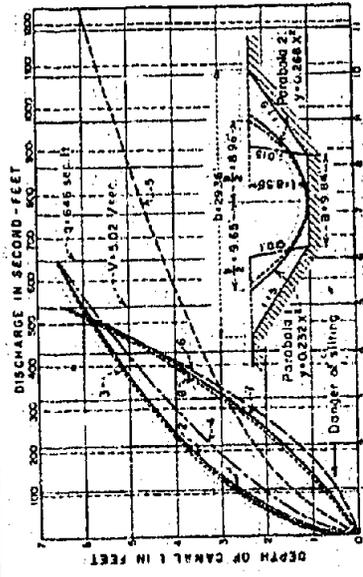
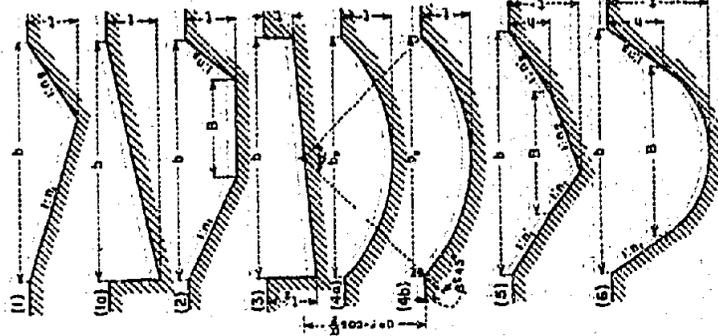


FIGURE 43 - (1) DISCHARGE CURVE OF THE CANAL; (2) DISCHARGE CURVE OF THE PARABOLIC CREST 1; (3) DISCHARGE CURVE OF THE TRAPEZOIDAL CREST; (4) DISCHARGE CURVE OF THE TRAPEZOIDAL CREST, SLOPE OF SIDES 1:0.15; (5) DISCHARGE CURVE OF THE TRAPEZOIDAL CREST, SLOPE OF SIDES 1:1.15; (6) VELOCITY IN THE CANAL; (7) VELOCITY OVER THE PARABOLIC CREST 1; (8) VELOCITY OVER THE PARABOLIC CREST 2.

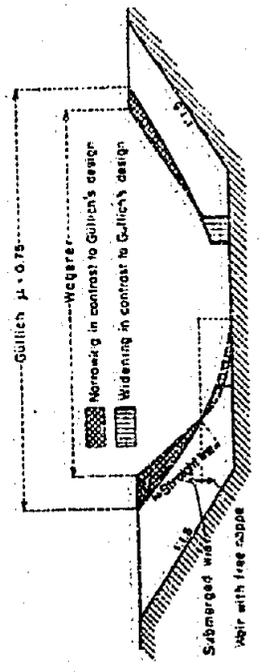


FIGURE 37 - DROP CREST ON THE WEILLACH RIVER WITH SUGGESTIONS FOR IMPROVEMENT ACCORDING TO WAGERER

TABLE 3. FORMULAS FOR CALCULATING THE DISCHARGE CAPACITY OF DROPS WITH VARIOUS CREST FORMS. (ACCORDING TO ATTILA SIKO)

NO.	FORM OF CREST	BY CONSIDERING THE VELOCITY OF APPROACH, $K = \frac{V_1}{V_2}$	BY NEGLECTING THE VELOCITY OF APPROACH
1	TRIANGLE	$Q = \frac{1}{3} \mu \sqrt{2g} [6(n_1 + n_2) \frac{1}{2} (n_1 + n_2) h^3 - (n_1 + n_2) h^3]$	$Q = \frac{1}{3} \mu \sqrt{2g} (n_1 + n_2) h^3$
10	TRIANGLE	$Q = \frac{1}{3} \mu \sqrt{2g} [4(n_1 + n_2) \frac{1}{2} (n_1 + n_2) h^3 - (n_1 + n_2) h^3]$	$Q = \frac{1}{3} \mu \sqrt{2g} h^3$
2	TRAPEZOID	$Q = \frac{1}{3} \mu \sqrt{2g} [6(n_1 + n_2) \frac{1}{2} (n_1 + n_2) h^3 - (n_1 + n_2) h^3]$	$Q = \frac{1}{3} \mu \sqrt{2g} [6(n_1 + n_2) \frac{1}{2} h^3]$
3	SLOPING CREST, VERTICAL SIDES	$Q = \frac{1}{3} \mu \sqrt{2g} [4(n_1 + n_2) \frac{1}{2} (n_1 + n_2) h^3 - (n_1 + n_2) h^3]$	$Q = \frac{1}{3} \mu \sqrt{2g} [4(n_1 + n_2) \frac{1}{2} h^3]$
40	PARABOLIC SEGMENT	$Q = \mu \sqrt{2g} b_1 \cdot h \left[\frac{1}{2} \frac{b_2^2}{b_1^2} \left(\frac{b_2}{b_1} + \frac{b_1}{b_2} \right) + \frac{1}{2} \sqrt{15} \right]$	$Q = \frac{1}{3} \mu \sqrt{2g} \pi b_1^2 h$
50	CIRCULAR SEGMENT	$Q = \mu \sqrt{2g} b_1 \cdot h \left[\frac{1}{2} \frac{b_2^2}{b_1^2} \left(\frac{b_2}{b_1} + \frac{b_1}{b_2} \right) + \frac{1}{2} \sqrt{15} \right]$	$Q = \frac{1}{3} \mu \sqrt{2g} \pi b_1^2 h$
The width and depth of parabolic and circular crests are approximately equal when $\beta = 45^\circ$			
5	COMPOUND CREST TRIANGLE PLUS TRAPEZOID	$[Q_{\text{Triangular}}]_0 + [Q_{\text{Trapezoid}}]_0$	$[Q_{\text{Triangular}}]_0 + [Q_{\text{Trapezoid}}]_0$
6	COMPOUND CREST CIRCULAR OR PARABOLIC SEGMENT PLUS TRAPEZOID	$[Q_{\text{Trapezoid}}]_0 + [Q_{\text{Circular or parabolic segment}}]_0$	$[Q_{\text{Trapezoid}}]_0 + [Q_{\text{Circular or parabolic segment}}]_0$

FIGURE 44 - VARIOUS CREST FORMS OF DROPS

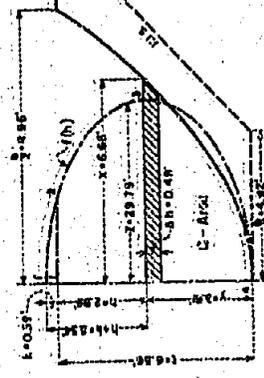


FIGURE 46 - SIKO'S METHOD FOR CALCULATING THE DISCHARGE CAPACITY OF DROPS