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METHOD FOR DETERMINING THE BACKWATER HEIGHT DUE TO BRIDGE PIERS WITH PURE STREAMING FLOW

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A METHOD OF DETERMINING THE BACKWATER AT BRIDGES
FOR STREAMING FLOW

By Theodor Rehbock

A translation of

Verfahren zur Bestimmung des
Brückenstaues bei rein Strömendem
Wasserdurchfluss

From

A pamphlet published to celebrate the dedication
of the new building of the Division of Engineering Structures
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A METHOD FOR DETERMINING THE BACKWATER DUE TO BRIDGE PIERS IN STREAMING FLOW

By Th. Rehbock

All attempts which have thus far been made to determine the backwater due to bridge piers by general theoretical methods have been unsuccessful. This was inevitable because the resistance so produced has been effected largely through the action of friction within the water, and because an exact mathematical interpretation of such complicated and obscure phenomena of fluid flow is still not feasible and will probably remain so for some time to come. Furthermore, the formulas of Ruhlmann and d'Aubuisson used by the River Construction Administration for determining the backwater height give completely erroneous results which may be several times greater than the true values.

In order to throw some light upon this problem the author has resorted to the use of model tests. By observing the flow under the influence of different numbers and forms of piers in a model channel it was thought that many of the extremely varied phenomena could be clarified. With the aid, therefore, of the Jubilee Foundation of German Industry, a series of tests which included more than one hundred different pier forms was undertaken in the old river laboratories of the Karlsruhe Technical University. These tests required several years since they demanded great accuracy both in the construction of equipment and in the experimental measurements. The director of the Karlsruhe laboratory, Dr. Böss, conducted the experiments with the utmost care and perspicacity. He thus laid the ground work for the elucidation of at least a few of the unknowns in the backwater problem. For that case most frequently encountered, in which water flows through an unobstructed channel in the streaming state - that is with a velocity less than the wave velocity - and in which the velocity of the flow reaches the wave velocity at no point within the openings between the piers, and for rectangular channels of any width, B, any depth of flow, d_o, and any discharge, Q, the investigation was complete and conclusive. It has been possible to derive from these experiments a general procedure for determining the backwater height produced by almost any shape of piers whose horizontal sections do not change with the elevation. Although because of the limitations of time and plan, only rectangular channels and piers whose sections were constant at all elevations could be studied, the results can be applied to other channels and pier shapes by appropriate transformation of irregular to approximately equivalent rectangular stream sections.

Because of the limitations of space the reader is referred for further details to previous articles by the author as follows: (1) "Betrachtungen über Abfluss, Stau und Walzenbildung bei fliessenden Gewässern und ihre Verwertung für die Ausbildung des Überfalles bei der Untertunnelung der Sihl durch die linksufrige Seebahn in der Stadt Zurich": (Observations on the Discharge, Backwater, and Roller Formation in Flowing Streams; Their Significance with Regard to Design of the Spillway in Conjunction with the Tunnel Under the Sihl for the left Bank Seebahn in Zurich), "Festschrift" of the Technical University at Karlsruhe, 1917 (published by Julius Springer, Berlin); (2) "Zur Frage des Brückenstaues", (On the Backwater Problem at Bridges.) Zentralblatt der Bauverwaldung, no. 37, 1919. (3) "Brückenstau und

Walzenbildung" (Backwater Caused by Bridges and the Formation of Rollers), Bauingenieur, no. 13, 1921.

Of fundamental significance for the calculation of the backwater height is information concerning the type of flow to which the water is subjected in a channel without piers and whether this type of flow changes as a result of the introduction of piers. The character and causes of backwater when the type of flow is changed by the introduction of piers are quite different from those in an unobstructed channel. This is well illustrated in figures 11 and 18 of the Festschrift article (1).

For those large flows in which the determination of the height of the backwater is of most consequence, pure streaming flow is generally prevalent; that is, at no part in the cross section does the water attain the wave velocity, $v = \sqrt{g\ d_o}$.

In most cases the velocity in a channel without piers is so far below the wave velocity that a considerably greater contraction of the channel than that caused by the introduction of piers would be required to increase the velocity between the piers to values approaching the wave velocity. The conditions under which this change in the state of flow takes place have already been determined for numerous pier forms and for other slender pointed structures usually associated with this work, and have been described in (2).

Piers for modern bridges and movable dams have a total width, Σb , which is generally so small in comparison with the total width, B , of the channel that a change from streaming to shooting flow does not take place. A procedure derived for streaming flow will therefore be applicable to by far the greater number of practical cases.

This procedure is applicable to the determination of the backwater height, Z_1, in feet for pure streaming flow in a rectangular channel and for any number of piers of any cross section, given

1. The discharge, Q, in cubic feet per second,
2. The width of the channel, B, in feet,
3. The depth of the unobstructed flow, d_o, in feet,
4. The number of piers, n,
5. The thickness of the piers, b, in feet,
6. The horizontal cross section of the piers, characterized by the form index, δ_o ;
7. The roughness of the pier surfaces.

In general, the roughness of the wetted surface on a pier can be neglected, for a special series of experiments has demonstrated that it exerts only a very insignificant influence on the backwater height which cannot even be measured in the majority of cases. The method of calculation is therefore applicable to piers of usual average roughness. For very rough piers a small increase should be added to the calculated backwater height, but in no case more than one percent. For extremely smooth structures such as concrete finished with neat cement an equal reduction should be made.

The roughness of the channel has no direct effect on the calculated backwater height. This effect is already taken into account along with the channel gradient and the shape of the channel downstream from the piers in the determination of the depth, d_o, for the unobstructed channel.

If the depth, d_o, cannot be determined by direct measurement as may perhaps be the case for particularly high flood discharges, then it must be calculated by the usual method for finding the surface curve for nonuniform flow. For this the shape of the channel below the piers, and the slope and roughness of the bed must be known.

The backwater height for a bridge may be calculated from the following four factors which will be explained below:

a. The velocity head in the unobstructed channel, k_o,

b. The contraction coefficient, α,

c. The flow ratio for unobstructed flow, ω,

d. The form index of the piers, δ_o.

The velocity head, k_o, has a linear dimension. α, ω, and δ_o, on the other hand, are ratios. Both α and ω can be computed from the given quantities (1) to (5); δ_o, however, is a function of the horizontal cross section of a pier and must be determined by model experiments.

The quantities in (a) and (b) above may be determined as follows:

a. The velocity head, k_o, is found in the usual way from the average velocity, $v = \frac{Q}{B\,d_o}$, in ~~which~~ the unobstructed channel, and then from

$$k_o = \frac{v^2}{2g}$$

b. The contraction coefficient, α, is the ratio of the area of the channel cross section filled by the piers to the total area of the channel, A, without piers. For a rectangular channel of depth, d_o,

$$\alpha = \frac{a}{A} = \frac{\Sigma(b \cdot d_o)}{B\,d_o} = \frac{\Sigma b}{B}$$

Next to the velocity head the contraction coefficient is the most important factor in determining the backwater height. The backwater height in pure streaming flow is nearly proportional to the velocity head and to the contraction coefficient.

c. The flow ratio, ω, characterizes the properties of the channel. It is obtained by dividing the average velocity head, k_o, in the unobstructed channel by the depth, d_o.

The effect of the flow ratio, $\omega = \frac{k_o}{d_o}$ upon the backwater height is not entirely distinguishable. The experiments have indicated that for the same piers equal velocity heads and equal contraction coefficients, with increasing values of the flow ratio, the backwater height, between the limits of $\alpha = 0.06$ and $\alpha = 0.36$, increases approximately in proportion to the quantity $(1 + 2\omega)$.

d. The form index, δ_o, is the coefficient by means of which the influence of the geometrical shape of the pier is expressed. The backwater height is by no means the same for constant values of the contraction coefficient if the shape of the piers differs. On the contrary, it varies between wide limits, largely because of changes in the pier nose or in the pier tail, and to a lesser extent as a result of changing the length.

The form of the pier nose influences the backwater height primarily by constricting the effective width of the channel. However, the effective width does not always correspond to the distance in the clear between the piers or piers and abutments. With well-rounded or pointed ~~xx~~ noses, however, the stream lines cling to the sides of the piers and the entire space between piers is filled. If, however, the transition between the body of the pier and the nose is abrupt; that is, if it is made up of a sharp edge or a curved surface of very short radius, then, as a result of the momentum of the water, the stream breaks away from the pier and a space develops between the stream lines and the pier. To be sure this space is usually filled with water which, however, does not assist in the general flow but circulates in a closed region in which rollers are formed and in which the direction of flow is at times actually upstream.

The side rollers which are formed as a result of the separation of the stream lines from the sides of a pier frequently change into channel eddies which do not remain in a fixed position but move slowly downstream. Even in this case, however, the discharge in the space between the pier faces and the main stream is so small that this space can be completely subtracted in the determination of the net cross section necessary for the discharge just as the area occupied by the piers themselves must be deducted.

In determining the net distance between two adjacent piers the space filled with rollers and vortices can be conceived as part of the piers. However, this space is more drastic on the energy content of the stream than a solid structure of equal size because the rollers dissipate considerable quantities of energy which is converted into heat. This energy must be taken from the stream itself.

Thus rollers serve to contract the cross section of flow - just as a solid pier would do - so that the drop of the water surface in the contracted opening, the velocity, and the friction loss are all increased. These last not only cause a drop in the water surface in the contracted stretch but also a raising of the water level in front of the piers. The drop in the water surface in the stretch occupied by the rollers is still further increased because of the energy dissipated by them, which is considerably more than that due to the friction at a fixed wall. Likewise, the piling up of the water is increased.

The rollers and vortices arising because of the sharp transition extending from the pier nose to its tail can be shown in model tests by taking profiles of the water surface ~~and/~~or by photographing them. (See reference 3, figures 1-9 and 11-12.) The space occupied by the rollers and vortices is a maximum in the case of a rectangular pier, extending laterally about half a pier width so that the total space withdrawn from that available for the discharge by a rectangular pier and the side rollers on both its sides amounts to approximately double the width of the pier. The piling up of the water immediately in front of the pier is considerable in this case. The more completely these side rollers can be avoided by rounding or sharpening the nose of the pier, the less the effect the nose of the pier in elevating the water in front of it. Even with a semicircular nose the side rollers occupy only about half the space as for a rectangular pier and the piling up of the water at the pier nose has been observed to be reduced to about 37 percent of that of a rectangular pier. When the nose is formed of circular arcs of a radius equal to twice the width of the pier, the side rollers completely disappear. In this case the elevating of the water surface amounts to but 28 percent of that for a rectangular pier.

Also, with true streaming flow the tail of the pier plays a role in shaping the surface profile. The downward slope of the energy gradient as a result of the energy dissipation along the tail begins upstream just above the pier nose and if streaming flow prevails, the slope of the water surface is greater, the more energy dissipated by inner friction in the water along the tail.

Furthermore, this energy dissipation along the pier tail is, for the most part, produced by rollers which originate as under rollers below the pier.

These under rollers do not form symmetrically to the axis of the pier but change from time to time from one side of the pier to the other, thus producing a fluctuating water surface. They progress upstream and with their disappearance cause an oscillating flow which continues to just below the pier. These rollers soon after their formation develop into vortices which last only a short time.

The under rollers do not completely disappear even with slender piers with pointed noses. In the case of the limiting rectangular pier they amount to almost the full width of the pier itself.

The principal backwater effect of the tail produced by the under rollers is considerable and can even surpass that of the nose.

With the same shape of the nose of a pier the backwater height varies with the length of the tail. With a given nose the smallest backwater height is observed when the total length of pier amounts to from three to five times its width. Shortening as well as lengthening the pier beyond this amount increases the backwater. Shortening the pier but retaining the same form of nose, produces side rollers thus causing a sharp increase in the backwater which may be traced to the large amount of energy dissipated in the combined side and under rollers.

The increase in backwater when the length of the pier tail is considerably increased is explained by the fact that the length of the stretch in which the surface drops as a result of the channel contraction, is increased.

In view of the complicated causes of backwater resulting from pier construction, which have already been briefly discussed for purely streaming flow, any experiment for a quantitative determination of the effect of the shape of a pier on the backwater height is hopeless. For ~~backwater~~ with purely streaming flow backwater is produced by hydraulic friction principally within the rollers, the analysis of which is still in its infancy. Today we are urgently in need of formulating the principles governing the propogation of rollers, how large they are, and how much energy they dissipate. Therefore, before we can obtain a clearer picture we, ourselves, must understand the fine points in the phenomenon of rollers and establish quantitatively their energy dissipation. Since a theoretical solution of the problem appears somewhat hopeless, and since observations at actual bridges introduced, on the whole, extraordinary difficulties, and cannot be effectively manipulated considering the impossibility of removing the structure at will and installing it again in a stream, there remains for the investigation of the bridge pier problem - in particular, for determining the influence of the shape of the pier on the backwater - only one practicable method - that of model tests. In order to obtain practical results in these tests in spite of the numerous variables to be taken into consideration, the whole backwater problem was investigated for a single arbitrary pier form, called the "standard pier". A backwater formula was derived for this "standard pier" which gives the backwater height z_o in terms of the velocity head, k_o of the unobstructed stream; the channel contraction ratio, α ; and the ratio, ω , of the velocity head below the contraction to the depth of flow below the contraction.

Then a form index determined experimentally for other pier shapes was studied by which the formula derived for the standard pier could be made applicable to any other pier form by multiplication.

These results were further complicated, as the tests show, by the fact that the form index was not solely dependent on the shape of the pier so as to remain a constant for a given pier form, but, on the contrary, varied with the channel contraction. However, the experiments also showed that, within the range of accuracy attained in the observations, the coefficients were independent of the velocity head, k_o, and the flow ratio, ω . A convex lens shape was chosen for the standard pier whose length amounted to 20/3 times the width. Since this form of pier was bounded by two circular cylindrical surfaces, it could be finished to any desired degree of accuracy on a lathe.

The material for the standard pier was polished steel while the other piers and a second set of standard piers were made of clean, polished, seasoned pitch pine, so that they attained almost the same degree of smoothness as the steel pier. Assuming a pier width of three meters (9.84 feet) and a length of 20 meters (65.6 feet), and using a model scale of 1:100, the model piers had a width of three centimeters (1.18 inches) and a length of 20 centimeters (7.87 inches). The circular cylindrical surface of the standard pier had a radius of 34.08 centimeters (13.42 inches). In all, ten different standard piers were made so that by changing piers it was possible to obtain many channel contractions and also those of most importance in practice. The model flume was constructed with the greatest accuracy and was 40 centimeters (15.75 inches) wide (corresponding to 40 meters (131.2 feet)) in the prototype , and the discharge was maintained constant.

The following formula for the backwater height was derived from 131 different tests on standard piers for purely streaming flow:

$$Z_0 = (0.4\alpha + \alpha^2 + 9\alpha^4)(1 + 2\omega)k_0 \qquad (1)$$

For other pier forms and for purely streaming flow, the backwater height is determined as follows:

$$Z_1 = \delta_1 Z_0 = \delta_1 (0.4\alpha + \alpha^2 + 9\alpha^4)(1 + 2\omega)k_0 \qquad (2)$$

The formula for the shape index, δ_1, was found from a graphical analysis of the tests on all forms of piers with channel contraction varying from $\alpha = 0.05$ to $\alpha = 0.36$ and is:

$$\delta_1 = \delta_0 - \alpha(\delta_0 - 1) \qquad (3)$$

The final formula which is valid within the given range of channel contraction is:

$$Z_1 = \left[\delta_0 - \alpha(\delta_0 - 1)\right](0.4\alpha + \alpha^2 + 9\alpha^4)(1 + 2\omega)k_0 \qquad (4)$$

In this formula, δ_0 is a constant for any pier and is that form coefficient for the pier under consideration when the channel contraction α is 0. This ideal value is called the limiting form index. When δ_1 is plotted against the channel contraction, α, the curves obtained are approximately straight lines with the range of the ordinary channel contractions, or between $\alpha = 0.05$ and $\alpha = 0.36$. Within this range these curves form a family of straight lines all of which, when extended, pass through the point $\alpha = 1$, $\delta_1 = 1$.

Hence, δ_0 can be calculated for any value of δ_1 by means of formula 3, since $\delta_0 = \dfrac{\delta_1 - \alpha}{1 - \alpha}$.

In order to determine δ_o, for any given pier form, a previous determination of δ, was required for various values of the channel contraction, α. The form coefficient was determined for at least two different channel contractions and usually for $\alpha = 0.15$ (2 piers) and for $\alpha = 0.30$ (4 piers) so that the limiting form coefficient could be calculated from the average of two observations.

The given experimental approach had the advantage that an accurate investigation for each single pier shape by varying all three variables, namely k_o, ω, and α, was not required since it was carried out with a very significant loss of time for three forms of piers differing essentially from one another, namely, forms K, F, and A (see table III).

Many checks have shown that the above empirical method of attack gives satisfactory results throughout the given range of channel contraction.

The results of the determination of the ideal form index, δ_o, for the numerous different pier shapes shown in tables I to III are indicated by the number accompanying each pier form. Furthermore, these values of δ_o are in terms of the index for the standard pier taken as unity. For a clear comparison they are also represented by the length of the three-line band in the center of each pier.

All of the model piers were tested with the same pier width, 3 centimeters (1.18 inches). Only one form of pier was tested with other widths, namely: 0.375, 0.75, 1.5, 6, and 12 centimeters (0.147, 0.295, 0.590, 2.36, and 4.72 inches, respectively) in order to test the validity of the law of similarity and therefore the applicability of the model test results to the prototype. This investigation led to entirely satisfactory conclusions throughout the range of tests.

In most cases the length of pier was chosen equal to 20 centimeters (7.87 inches) in order to bring into sharp relief the influence of the form of the pier nose on the size of the limiting form index and thus on the size of the backwater. In numerous cases the length of pier was also varied, keeping the width constant in order to establish the influence of the length of the nose and also the tail on the height of backwater.

An absolutely general formula for δ_0 cannot be set up since the pier sections ~~that~~ were too numerous and there were too many variables depending on the pier shape. However, for a single group of piers it was possible to derive a sufficiently accurate and simple formula for determining the limiting form index δ_0.

Formulas for beveled, rounded, diamond-shaped, lens-shaped, and elliptical piers as well as for piers with a triangular, half-lens shaped and elliptical noses are shown in table I. The δ_0 indexes are expressed in every case as a function of the nose length number ε (the ratio of the length of the nose, l_k, to the thickness of the pier, b).

In a similar fashion, formulas for ten different groups of piers are given in table ~~2~~ II in which, keeping the shape of the nose and tail the same, the length of the pier was varied in order to find the effect of the pier length on the backwater height. For these groups the limiting form index δ_0 is expressed as a function of the pier length number λ - the ratio of the total length of pier, l, to the pier thickness, b. On account of the indeterminate nature of the δ_o-curve for one of the groups of piers an analysis of two subgroups was undertaken in order to establish a simple formula of sufficient accuracy.

For those forms of piers for which no formula was derived it is easy to ascertain the δ_o index with sufficient accuracy by interpolation and a critical analysis of the pier shape in question, for the numerous experiments cover a wide range of forms.

Table I also includes the limiting form index δ_o for a single set of piers which are unsymmetrical with respect to the transverse axis. It so happens that the fish-form pier, Y, possesses a smaller δ_o value than any other form and therefore indicates a smaller backwater effect, providing the broad head is directed upstream. This is explained by the relatively small dissipation of energy in the under rollers.

Values of δ_o for a large number of unsymmetrical piers of equal length and width are given in table III. The δ_o values determined for these unsymmetrical pier forms have made it possible to carry out an extended investigation of the backwater produced by the front and back halves of a single pier.

These observations made with the greatest precision have shown that with piers having a length ratio of 20:3 there is no mutual influence of the two pier halves worth mentioning as far as backwater is concerned, so that it is possible to determine the backwater action of an unsymmetrical pier from the known performance of the two halves forming a pier by simple addition. The comparison of the backwater action produced by single halves of piers is particularly instructive since it affords worthwhile conclusions as to the causes of backwater. The available space in this paper precludes a detailed study of the relation between backwater height and pier shape.

After δ_0 has been determined, it is possible in all cases to calculate the backwater height Z_1 from this value of δ_0 and the values for k_0, α, and ω. This calculation is sufficiently accurate if attention is paid to insure that the discharge at the piers is truly streaming. If a change from streaming to shooting flow occurs, entirely different causes of backwater formation enter and, in this case, it is necessary to use the formulas for backwater calculation from other reports (reference 2, formulas 6 to 10, and reference 3, formulas 8, 11, 12, and 15).

A graphical procedure is developed in table IV which permits a convenient determination of the backwater height and also indicates whether the case at hand lies within the range of purely streaming flow.

In order to find the backwater height, Z_1, for streaming flow by formula (4), it is necessary to multiply graphically the two variable factors A and B representing the two expressions contained within the large parentheses in equation (5), thus:

$$Z_1 = A \cdot B = \left\{\left[\delta_0 - \alpha(\delta_0 - 1)\right]\left[0.4\alpha + \alpha^2 + 9\alpha^4\right]\right\}\left\{(1 + 2\omega)k_0\right\} \qquad (5)$$

in which: $\omega = \frac{k_0}{d_0}$, $k_0 = \frac{v^2}{2g} = \frac{Q^2}{2gB^2d_0^2} = \frac{q^2}{2gd_0^2}$

and q = the discharge in cubic feet per foot width of uncontracted channel.

Hence: $$Z_1 = \underbrace{\left\{\left[\delta_0 - \alpha(\delta_0 - 1)\right]\left[0.4\alpha + \alpha^2 + 9\alpha^4\right]\right\}}_{A}\underbrace{\left\{\left[1 + \frac{q^2}{gd_0^3}\right]\frac{q^2}{2gd_0^2}\right\}}_{B} \qquad (6)$$

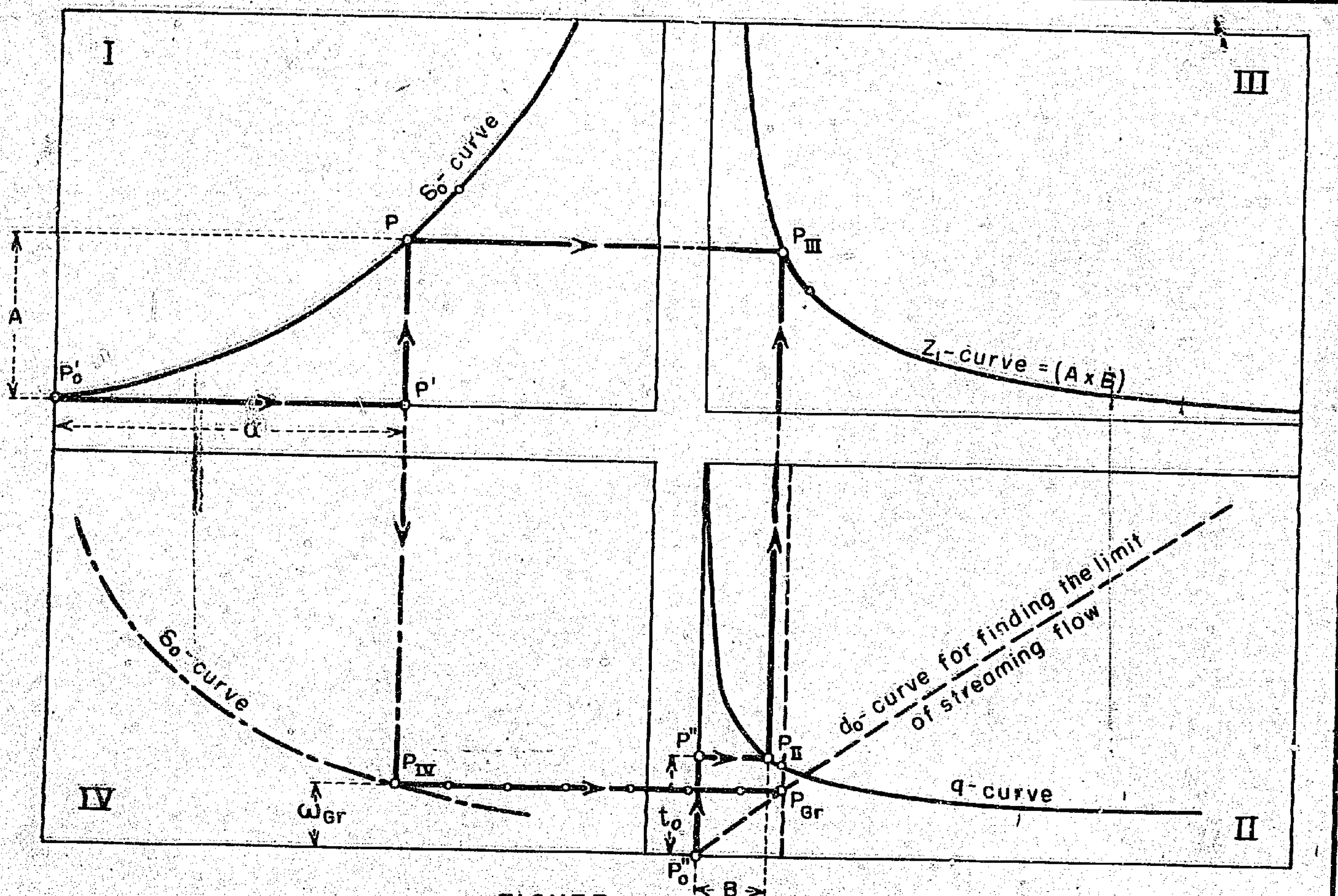

FIGURE I

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(b)

The longer of the two factors, A, is found as is illustrated in ~~group~~ graph I in the upper left-hand corner of figure 1 where A is the ordinate corresponding to the abscissa α of the intersection P_I on the δ_o-curve. In a similar way factor B is determined as shown in graph II in the lower right-hand corner of figure 1 and is the abscissa of point PII, the intersection of the ordinate d_o with the q-curve.

A horizontal line through point P_I, and a perpendicular line through P_{II} intersect in graph III at point P_{III} (upper right-hand corner). This point, P_{III}, permits a direct determination of the backwater height Z_1 for genuine streaming flow from the product curve (A · B curve) which is an equilateral hyperbola.

A is a pure number, since α and also δ_o are dimensionless quantities. They are independent of any variation in the model scale ratio.

B has a dimension of length; since the unit discharge, q, has the dimension $\frac{ft^3}{sec.\ ft} = \frac{ft^2}{sec.}$, the depth d_o has the dimension ft, and the acceleration of gravity g, the dimension $\frac{ft}{sec.^2}$. Therefore, the product of A and B has a dimension of length, namely the backwater height, Z_1.

Since the backwater height is proportional to B, the determination of small values of the backwater height, Z_1, can be made more sensitive by increasing the scale n times. The value d_o must then be multiplied by n and the quantity q by $n^{3/2}$ while the backwater height Z_1 in graph III must be divided by n.

To establish the limits for the existence of genuine streaming flow with any channel contraction, α, and flow ratio ω, is a difficult and time-consuming task by experiment. Since this could not be done for all of the tests performed an accurate determination was made for three pier shapes, all of which possessed a pier length ratio, $\lambda = \frac{20}{3}$.

The piers studied were:

1. The standard pier, K
2. The pointed-arch shaped pier, F
3. The rectangular pier, A.

The determination of the limit for genuine streaming flow was performed in such a manner that for different channel contractions, α, the limiting value of the flow ratio ω_{Gr} for the unobstructed channel was determined which if exceeded a part of the water beneath a surface roller had a larger velocity than the wave velocity and thus had shooting flow.

The limiting flow ratio so determined was found to be a function of the channel contraction, α. The following empirical equations were derived:

For pier K

$$\omega_{Gr} = \frac{1}{2.8 + 10\alpha} - 0.11 + \frac{1}{10{,}000\alpha + 13} \tag{7}$$

For pier F

$$\omega_{Gr} = \frac{1}{2.7 + 21\alpha} - 0.046 \tag{8}$$

For pier A

$$\omega_{Gr} = \frac{1}{2.5 + 5\alpha} - 0.113 + \frac{1}{2000\alpha + 10.3} \tag{9}$$

Although the three forms of piers investigated differed from one another considerably and in consequence of which possessed limiting shape indexes δ_0 which differed greatly from one another, the limiting flow ratios computed from equations 7 to 9 differed very little from one another. The limiting flow ratios for channel contraction between $\alpha = 0.06$ and $\alpha = 0.36$ between which the derived empirical equations for the determination of the backwater height should, in general, be applied, differ little from one another. Therefore, the limiting flow ω for other shapes of piers can be obtained with sufficient reliability by interpolation.

All forms of piers likely to be met in practice possess δ_r values which lie between 1.5 and 3.0, with few exceptions, and in the majority of cases fluctuate between 1.6 and 2.4, and with equal channel contraction and also with equal limiting flow ratio, it was even possible to make the computations using the simple formula (8) as a basis.

Table IV shows the limits for which genuine streaming flow applies as well as whether the backwater height Z_1 calculated from equations 4 to 6 and from their graphical representation is valid.

In order to determine whether the flow is genuinely streaming under given conditions it is necessary to proceed to graph IV (lower left-hand corner figure 1) where values of α are plotted as abscissae as in graph 1 and the corresponding limiting flow ratios are plotted as ordinates. The curves in this graph are for the limiting form indexes, δ_o = 1, 2, 3, 4, etc., obtained by interpolation and extrapolation from equations 7 to 9.

Furthermore, in graph II (lower right-hand corner) are curves for the depth of flow, d_o, with values of the limiting flow ratios $\omega_{\delta r}$ plotted as ordinates and values of B as abscissae. B should not be exceeded by the given A value if streaming flow is to exist.

The necessary limiting values of B are calculated from:

$$B_{Gr} = (1 + 2\omega_{Gr})k_o = (1 + 2\omega_{Gr})d_o\omega_{Gr} \qquad (10)$$

It must now be demanded in using the graphical procedure, that the value of B calculated from the given flow ratio ω and also the velocity head k_o of the unobstructed flow, is smaller than the limiting value B_{Gr}, or:

$$B = (1 + 2\omega)k_o < B_{Gr}$$

Whether this is actually the case or not follows from the dot and dash lines in the graphs in the following manner. A perpendicular line is drawn from point P_I in graph I down to the δ_o-line in graph II corresponding to the limiting shape index of the pier being investigated. From this point of intersection, P_{IV}, a horizontal line is drawn to the intersection point, P_{Gr}, on the ~~dashed~~ d_o-line which corresponds to the depth of the unobstructed flow.

A vertical through the point, P_{Gr}, thus found forms the limit which must fall to the right of P_{II} to insure that shooting flow does not occur. If this limit is exceeded, part of the water changes to shooting flow and the calculation of the backwater height can no longer be computed by the method in this paper, because the assumption that the flow is streaming throughout is invalidated.

Thus P_{II} lies to the left of the vertical through P_{Gr} so that the backwater height Z_1 as determined by the point P_{III} likewise lies to the left of this vertical.

Therefore, the determination of the backwater height Z_1 for genuine streaming flow with the aid of table ~~4~~ IV proceeds by the following two paths (see figure 1): $P_0' - P' - P_1 - P_3$, and $P_0'' - P'' - P_{II} - P_{III}$. The third path: $P' - P_{IV} - P_{Gr}$ is necessary in order to insure that the calculation is valid in view of the assumption that streaming flow exists and it does exist only providing P_{III} lies to the left of the perpendicular through P_{Gr}.

The determination of the backwater height for purely streaming flow is possible then, after finding the limiting form index, δ_o, given for the pier in question in tables I to III; and from the depth of water, d_o, in feet; the discharge, q, in second-feet per foot, and the channel contraction, α. By simply following the prescribed paths in table IV, no numerical work is required.

The limiting form indexes, δ_o, are given in the tables to two decimal places. However, for small backwater heights occurring with true streaming flow, it was not possible to determine δ_o accurately to two decimal places because the sensitivity of the measurements was limited.

In spite of this it was not admissible to give δ_o only to tenths because by so doing the desired accuracy could not be attained. The values of δ_o are therefore given to two decimal places as found from an analysis of the observed values.

The accuracy attained for the readings of the backwater height in the model experiments, in general, amounted to one-tenth mm. as determined by numerous duplicate tests. The data obtained by the series of check tests frequently gave the same readings as the earlier tests. Rarely did the difference amount to more than 0.02 mm. while for the standard pier, K, by averaging numerous precise single observations an accuracy of about one-twentieth mm. was probably obtained.

In order to attain such an accuracy a great amount of skill and patience was necessary.

The unavoidable small errors in the readings were smoothed out by forming equations from several observations.

The next question is whether it was justifiable to spend so much time and effort in obtaining such great accuracy in the observations in view of the fact that in hydraulic construction it is not important to know the height of backwater or dimensions of a pier accurately to a centimeter. However, in judging this question, it should be considered that experiments ought not to serve solely the needs of practical hydraulic construction; on the contrary, it has been an aim during the last twenty years in the hydraulic investigations at the Karlsruhe hydraulic laboratory to clarify the flow phenomena occurring in a channel of any shape. For these investigations accuracy in each single observation cannot be emphasized too much. Without this accuracy it would never be possible to separate clearly the different kinds of flow at piers and to establish a foundation for the theory of backwater at bridges.

The obtained data will be found useful, not only for determining the dimensions of bridge piers, but on the contrary for dealing with other hydraulic problems, and in addition, contributes in a wholly general way to the furtherance of a theoretical solution of flow phenomena. It is only necessary to recall that the important problem of the resistance of moving ships is closely related to that of bridge piers and can profit much from the published data on piers. The backwater height caused by a stationary body in flowing water stands in the same relation as the resistance of the same body moving through still water.

The author hopes that the foregoing brief description of the investigation on streaming flow at bridge piers can be later republished in a more extended form. This study is not an entirely worthless link in the long chain of research which has become necessary to clarify the complicated relations involved in the flow of water. It has as wide an application as is seemingly possible, considering the difficulty of the problem.

As far as a practical application is concerned, the important results of this investigation consist in the conclusion that the backwater at bridge piers and its detrimental action are highly overestimated in most cases, since cases in which shooting flow occurs and in consequence of which large backwater heights are in evidence, constitute exceptions.

Anxiety because of the detrimental action of the backwater at bridges, which has added to the difficulties of so many bridges and in many cases to costly changes in design and even has led to the abandonment of proposed bridges in most cases, has now become unnecessary. This proceeds convincingly from this investigation.

Special acknowledgment is due to Dr. Böss who carried out the tiresome observations on the model and the bulky calculations under the instructions of the author, and who also played an essential part in the development of the graphical procedure.

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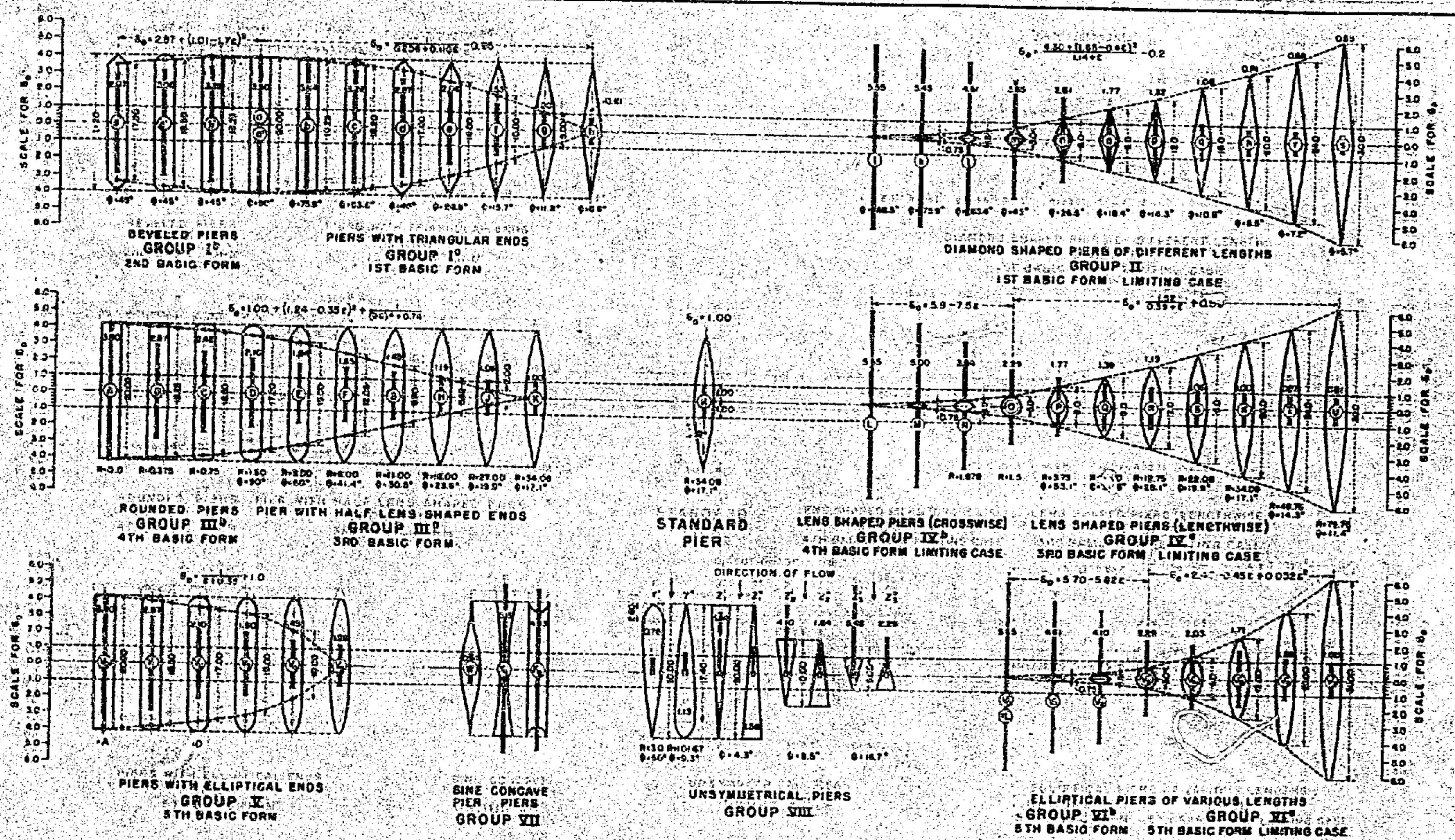


δ = Form index = ratio of the backwater height for a pier of any given form to the backwater height for the standard pier, K, for an equal α and equal ω.
δ_0 = limiting form index for $\alpha = 0$.
δ_1 = form index for pure streaming flow.

FORMULA FOR THE BACKWATER HEIGHT WITH STREAMING FLOW

$$Z_1 = \delta_1 \cdot \lambda \cdot \alpha \cdot K_e = \delta_1 [0.4 + \alpha + 9\alpha^4] [1 + 2\omega] \alpha K_e$$

WHERE $\delta_1 = \delta_0 - \alpha(\delta_0 - 1)$

$\alpha = A_1/A_2$ = contraction coefficient
$\omega = K_e/d_e$ = flow ratio
$K_e = V^2/2g$ = velocity head computed FROM THE AVERAGE VELOCITY
$\epsilon = l_n/t$ = nose length number
[l_n = length of nose; t = thickness of pier]

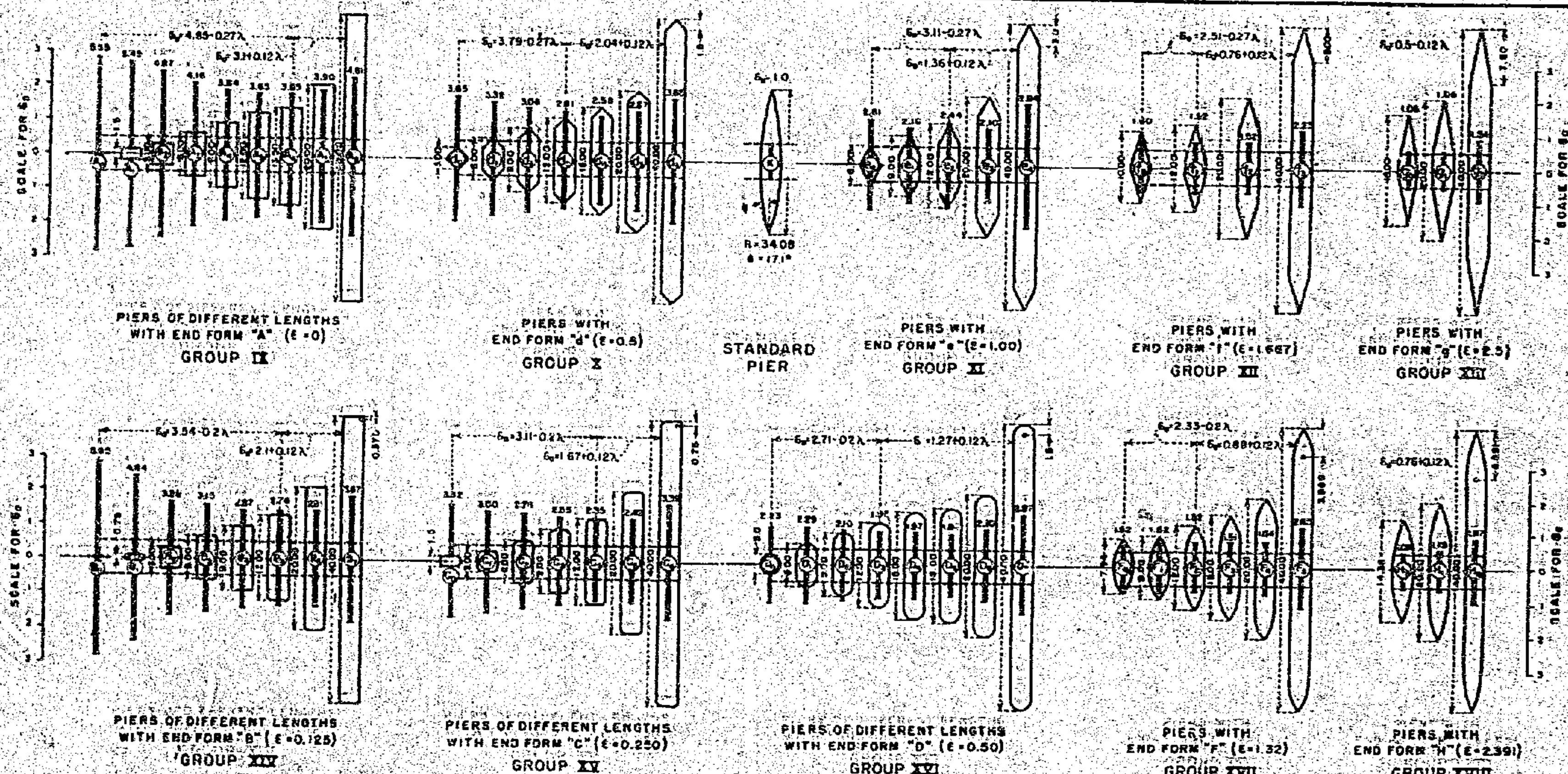
The three line symbol $\equiv$ and the number with this symbol denote the value of δ_0 calculated from the observed data.

SECTIONS OF PIERS OF VARIOUS FORMS TESTED FOR THEIR BACKWATER EFFECT SHOWING THE VALUES OF THE LIMITING FORM INDEX, δ_0, CORRESPONDING TO THE LIMITING CONTRACTION COEFFICIENT, $\alpha = 0$.

TABLE I

HYD-24

X-D-2199



The backwater height "Z_1" for pure streaming flow, created by any number of piers of a given form, in a rectangular channel of width "B" carrying water at a depth "d_0" is computed from:-

1 – The properties of the bed and discharge characterized by:-

(a) The velocity head in the unobstructed channel, $K_0 = \frac{V^2}{2g}$ in feet.

(b) The flow ratio of the unobstructed discharge $\omega = \frac{K_0}{d_0}$

2 – The properties of the pier characterized by:-

(a) The contraction coefficient $\alpha = \frac{A_1}{A_2} = \frac{A_1}{Bd_0}$

(b) The limiting form index, δ_0, which is a number belonging to the form of the pier and denotes the ratio of the backwater caused by the pier in question to that caused by the standard pier whose $\delta_0 = 1$.

The calculation of the backwater height "Z_1" for pure streaming flow follows from the following formula:

$$Z_1 = \delta_1 (0.4\alpha + \alpha^2 + 9\alpha^4)(1 + 2\omega)K_0 \quad \text{in which,}$$

δ_1 = the form index for pure streaming flow which in turn is computed from the limiting form index, δ_0, and the contraction coefficient, α, by:

$$\delta_1 = \delta_0 - \alpha(\delta_0 - 1)$$

TO COMPUTE THE BACKWATER HEIGHT THE FOLLOWING ARE REQUIRED

d_0 = Depth of flow in the unobstructed channel in feet.
V = The average velocity in feet per second.
A_1 = The contracted channel sectional area in square feet.
A_2 = The sectional area of the uncontracted channel in feet.
α = A_1/A_2 = the contraction coefficient.
t = The width of the pier in feet.
L = The length of the pier in feet.
L_2 = The length of the head of the pier in feet.
λ = L/t = the pier length number.
ε = L_2/t = the head length number.

(t, L, L_2, λ, ε:) Values for determining the limiting form index δ_0

NUMERICAL EXAMPLE

Given: a rectangular channel 120 feet wide, the discharge Q = 9180 second-feet, the depth of flow, d_0 = 12 feet obstructed by four piers of form "F" 9 feet wide and 45 feet long.

These give:-

$V = \frac{9180}{120 \times 12} = 6.375$ ft/sec. $\quad K_0 = \frac{6.375^2}{2g} = 0.632$

$\omega = \frac{0.632}{12} = 0.053$ $\quad \alpha = \frac{36}{120} = 0.30$

δ_0 = 1.51 (see above)
δ_1 = 1.51 − 0.3 (1.51 − 1.0) = 1.36
Z_1 = 1.36 (0.4 × 0.30 + 0.30² + 9 × 0.30⁴)(1 + 2 × 0.053) 0.632 = 0.27 feet

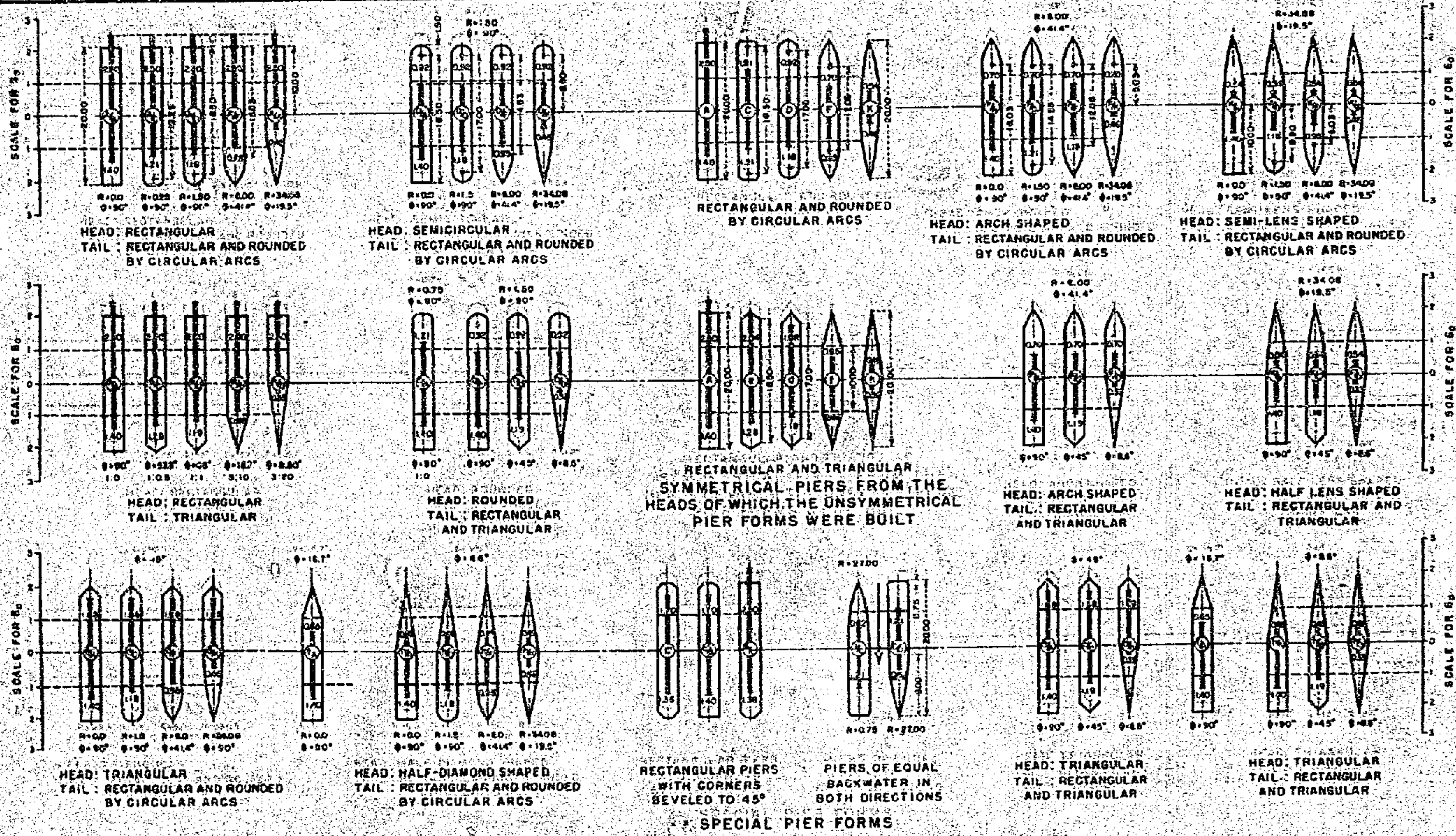
The threeline symbol III and the number above this symbol denote the value of δ_1 calculated from the observed data.

FINDING THE BACKWATER HEIGHT CAUSED BY PIERS WITH STREAMING FLOW (DISCHARGE TYPE I).
SECTIONS OF SYMMETRICAL PIERS OF VARIOUS SHAPES SHOWING THE VALUES OF THE LIMITING FORM INDEX, δ_0, CORRESPONDING TO THE LIMITING CONTRACTION COEFFICIENT, α = 0.

TABLE II

HYD-24

X-D-2266



δ = Form index = ratio of the backwater height for a pier of any given form to the backwater height for the standard pier, K, for an equal α and equal ω.
δ_0 = limiting form index for $\alpha = 0$.
δ_s = form index for pure streaming flow

FORMULA FOR THE BACKWATER HEIGHT WITH STREAMING FLOW

$$Z_1 = \delta_s \, \beta_s \, \alpha \cdot K_a = \delta_s \left[0.4\alpha + 9\alpha^3\right]\left[1 + 2\omega\right] \alpha K_a$$

WHERE $\delta_s = \delta_0 - \alpha(\delta_0 - 1)$

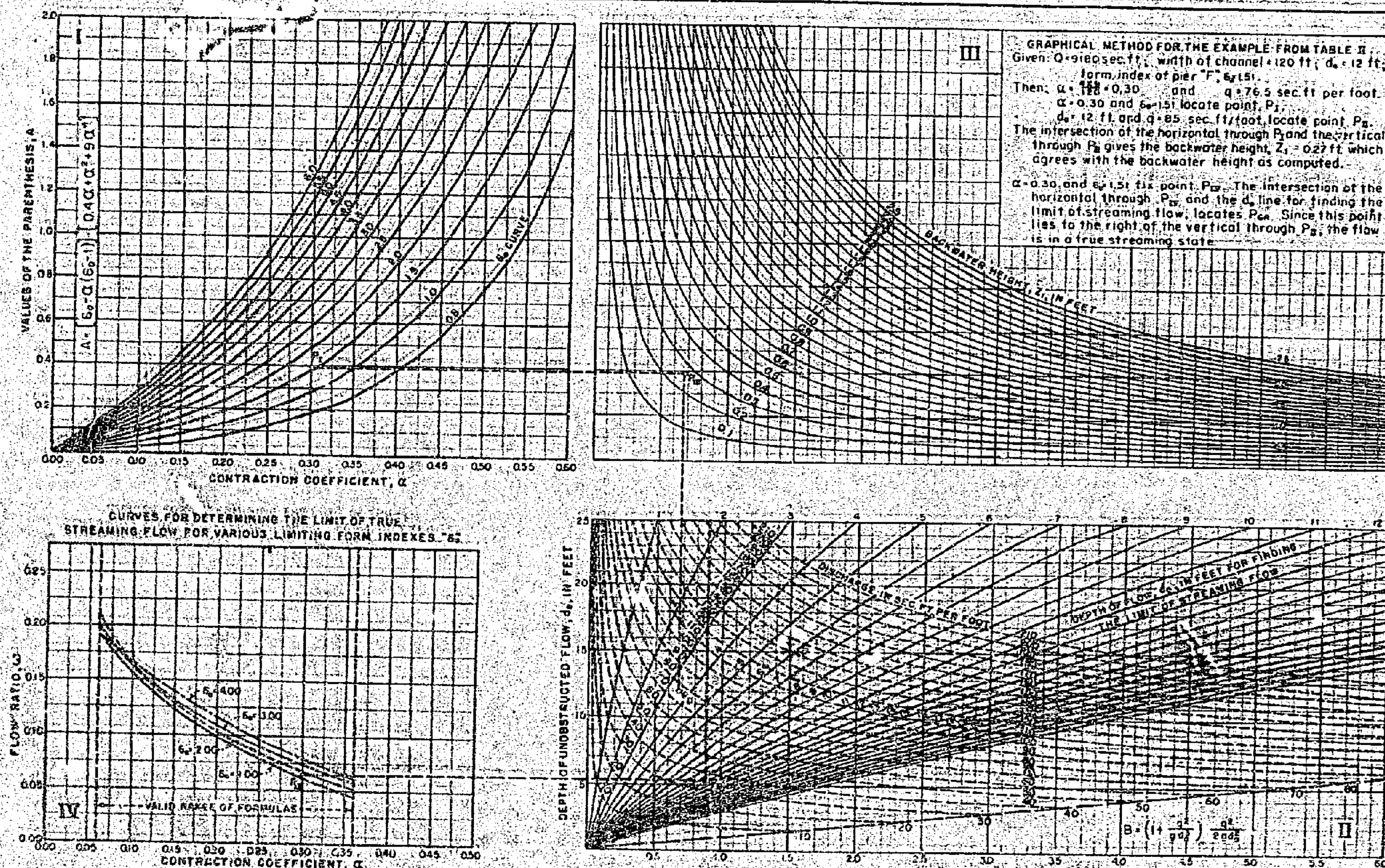
$\alpha = A/A_g$ = contraction coefficient
$\omega = h_a/d_a$ = flow ratio
$K_a = V^2/2g$ = velocity head computed FROM THE AVERAGE VELOCITY
$E = l_n/t$ = nose length number
[l_n = length of nose; t = thickness of pier]

SECTIONS OF UNSYMMETRICAL PIERS TESTED FOR THEIR BACKWATER EFFECT SHOWING THE VALUES OF THE LIMITING FORM INDEX, δ_0, CORRESPONDING TO THE LIMITING CONTRACTION COEFFICIENT, $\alpha = 0$.

TABLE III

HYD-24

X-D-2561



Formula for backwater height for true streaming flow $Z_1 = [\delta_0 - \alpha(\delta_0 - 1)][0.4\alpha + \alpha^2 + 9\alpha^4][1 - 2\omega]k_0$

CHART FOR THE GRAPHICAL DETERMINATION OF THE BACKWATER HEIGHT FOR STREAMING FLOW
GIVEN THE DISCHARGE PER FOOT OF WIDTH "q"; THE DEPTH OF FLOW "d_0"; THE CONTRACTION COEFFICIENT "α" AND THE LIMITING FORM INDEX "δ_0"

TABLE IV

HYD-24

X-D-2262