THE FLOW OF WATER IN OPEN CHANNELS

WITH HIGH GRADIENTS

by

Charles Walter Thomas

B.S.M.E., Colorado State College of Agriculture

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Charles Walter Thomas

has been approved for the

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by

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Date ____________________

FOREWORD

The data contained in this thesis were assembled by the author while in the employ of the Bureau of Reclamation, Department of the Interior, Denver, Colorado. To this organization, I am grateful for permission to use the material and for furnishing the illustrations in this paper.

Mr. J. E. Warnock, Director of the Hydraulic Laboratory, under whose direct supervision the studies were made added much constructive criticism for the entire program.

Particular credit is extended Dr. Edward F. Wilsey, who translated pertinent articles and assisted in the conduct of the tests, and Dr. Victor L. Streeter for his assistance in the mathematical application of the data.

All statements contained herein are the views of the author and should not be construed to represent the opinion of the Bureau of Reclamation.

The material for this thesis was principally completed in June 1940, but service in the Armed Forces of the United States negated consummation by the author until December 1946. Hence, all research reported since the former date has not been included.

Thomas, Charles Walter (M.S., Civil Engineering)

The Flow of Water in Open Channels with High Gradients

Thesis directed by Warren Raeder and Roderick Downing

A comprehensive study of the methods of measuring high-velocity flow, supplemented by laboratory tests, made possible the development of a technique for utilizing the salt-velocity method to determine the magnitude of existing velocities in the field on an existing steep wasteway. Data included in the field measurements, relative to discharge, cross-sectional area and velocity permitted calculation of air content in the flow. The mechanics by which the air is entrained in the flow were also studied.

These data, combined with results of other tests conducted on similar structures, gave information which made possible the proposing of an additional term in the Manning formula to compensate for the air resistance. The values of the coefficients for this term are evaluated from the field test data. A definition of the hydraulic radius is presented as the area of the cross-section, assuming no air in the flowing water, divided by the total wetted perimeter including the air. An application of the Manning formula modified by the air resistance term and the new hydraulic radius, is made to the design of a typical steep chute.

This abstract of about 180 words is approved as to form and content. I recommend its publication.

Signed ______________________________
Instructor in charge of dissertation

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I INTRODUCTION

Open-channel Flow

The phenomenon of open-channel flow has long been an interesting subject, particularly to the hydraulic engineer, and when this flow is in a channel with a high gradient it is fascinating even to a layman. The freedom of the stream to change section makes possible in open-channel flow many peculiar phenomena that cannot occur in closed conduits flowing full.

Fluid flow with a free surface is complicated as compared with closed-conduit flow. The two cases are comparable only in the simple case of uniform flow in channels where the cross-section is constant in area and form. Even if uniform flow is assumed, the wide range of forms of cross-section and channel surface conditions makes it practically impossible to evolve a single formula that describes the motion.

In open-channel flow the free surface is under constant pressure since it is exposed to the atmosphere. The pressure is thus constant over the entire surface and the only force causing flow is the weight of the fluid. The resisting forces are usually considered to be only viscosity forces in laminar flow and viscosity combined with inertia forces in turbulent flow. Thus it may be seen that uniform flow in a channel is always accompanied by a change in elevation of water surface.

Let us consider uniform flow in a channel with a smooth, free surface. For this condition the mean velocities at successive cross-sections are equal. Otherwise the flow is nonuniform. To produce this flow the channel must have constant bed slope and uniform cross-section. Then and only then will the slope of the water surface be equal to the bed slope. In most practical cases the flowing water undergoes many small accelerations, retardations and deviations. These minor disturbances result from imperfect alinement of the channel. In practice uniform flow is considered to include instances where these minor deviations exist.

The problems arising in connection with the flow of water in channels more commonly relate to moderate gradients, low to moderate velocities, and steady conditions of flow. As a rule we are not concerned with the introductory period during which the velocity is increasing from zero, or from some initial velocity, to its final or steady flow value. Under these latter conditions the forces causing acceleration are balanced by the resistance to flow and steady conditions prevail, that is, over any given cross-section of the stream, the distribution of velocity remains sensibly constant in time. Furthermore the moving medium may be assumed to remain continuously in the liquid phase without mixing sensibly with air, and again, the resistance opposed to the flow may be considered as developed between the moving liquid and the solid boundaries which constrain its motion, fluid friction so called.

The final or steady motion velocity which is acquired under these conditions is, then, the item of principal interest and forms the subject of the well-known classical treatment of the subject.

Formulas for Uniform Flow in Open Channels

Being mindful of the nature of the resistance to the flow and the results of this resistance it may be seen that formulas dealing with open-channel flow must be composed of elements expressing the mean velocity, the slope of the water surface, the proportions of the channel, the hydraulic radius and the relative smoothness of the fixed boundaries. So far as is known there is no entirely theoretical derivation for the relationship between these variables. Mathematical expressions of the relationship must be assumed and exponents and coefficients determined by experiment. The usual basic expression is:

$$V = KR^yS^z \qquad (1)$$

in which K, y and z are empirical elements, V is mean velocity, S is the slope of the water surface and R the hydraulic radius.

The earliest known formula for open channels was developed by Chezy[1*] from experiments and was published in 1775. The formula is used extensively, primarily because nearly 100 years elapsed before results of further experiments were published.

Ganguillet and Kutter[2] published in 1869 a formula for determining the value of C in the Chezy formula. This formula is commonly referred to as the Kutter formula or the Kutter-Chezy formulas, when combined with Chezy's formula.

In 1890 Manning[3] published a formula for deriving the value of C in the Chezy formula and 7 years later, in 1897, Bazin[4] proposed another means of evaluating C in the Chezy formula.

The above-mentioned formulas are the most widely used in open-channel design. Much data have been collected and published fixing the values of the various coefficients of roughness for different channel materials. Other formulas substantiated by less experimentation, have been evolved. These formulas are in general of a more complicated form and are not readily adaptable to quick solution. Many formulas and discussions of their particular merit are to be had in numerous text and reference books on hydraulics, hence, only brief mention is made here. King[5] gives a number of formulas and devotes several pages to a discussion of their

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*All references cited are listed in numerical order of occurrence in the bibliography of the appendix.

value and use. Tables of values for solution of the most used formulas are also given. A similar discussion is given by Houk[6].

Presence of Air in Flow

In the case of water flowing under steep gradients the simple conditions of flow no longer prevail. The velocities acquired may be high and the resistance to flow can no longer be restricted to the solid boundaries of the stream. The air boundary now begins to play a part, and gradual insufflation with air brings a change in the density of the flowing medium and in the mean hydraulic radius of the section. The surface of the flow becomes broken and irregular due to wave formation caused by action between the air and water and due also to the marked velocity gradient between the bottom of the flow and the top. The latter is especially marked in case the flow is relatively shallow in depth. As a result, the flow may become transformed from a liquid phase into an extremely turbulent foamy phase with a rough and irregular upper surface involving a greatly increased resistance by the air boundary of the stream.

Under these conditions of flow, we are not only concerned with the ultimate velocity which may be acquired in a long channel but we are also interested in the velocity which may be acquired in a given length of channel or in a given interval of time. We are therefore concerned with relations between velocity, distance, and time, during the period of acceleration from an initial velocity toward a final velocity.

A number of scientists, including Bellasis[7], Dodge and Thompson[8], King[9], and Hill[10] have proposed equations for determining the relationship between the slope of the water surface and the hydraulic properties of the solid boundary. No terms have been included in these formulas to reflect the influence of the surface in contact with the air or the presence of air in the flow.

Need for Additional Experiments

Structures now being designed are of such proportions that it is necessary for the hydraulic engineer to possess information other than which has been available if the most economical sections are to be had.

In the design of steep chutes and spillway channels the proper freeboard allowance is one of the many problems. Calculations of the channel dimensions must be altered to include entrainment of air at high velocities and the change of frictional resistance caused by this air entrainment.

A rational approach to the solution of the problem would be a mathematical analysis supplemented by experimental data to supply the empirical elements. In the solution of this and other engineering problems much observation is necessary in conjunction with the application of the exact sciences.

With this in mind an investigation of the characteristics of flow in open channels with high gradients was proposed to supplement existing design data.

Scope of Thesis

This thesis covers; (1) the results of a study of the literature of this and foreign countries to ascertain the extent of reported material available on the subject; (2) development work to evolve a method of accurately measuring high-velocity flows; (3) a program of field measurements on an existing structure; (4) an evaluation of all obtainable data; and (5) the application of these data to design calculations.

II LIBRARY RESEARCH

Study of the Literature

A study of the literature of this and foreign countries revealed that the science of flow in open channels with high gradients is an almost unexplored field. Although it is not a new subject the difficulties encountered and the number of variables present in such flow conditions have greatly retarded the progress toward a comprehensive analysis of the problem. Also the formulas developed by the older experimenters have grown into very general usage and have been accepted as standard. This condition renders the propagation of any new formulas or methods of attack difficult. During the time that the problem has been considered, conversation with many design, construction and operating engineers disclosed that at various times some field measurements have been attempted but due to lack of suitable equipment, difficulties encountered in conducting the tests, and apparent discrepancies, the results have not been published. A much better conception of the subject could probably be obtained if this condition did not exist.

The study disclosed very little on the subject and the explanations by the authors differ greatly. The mode of attack on the problem has been dealt with primarily from the empirical standpoint although mathematical solutions have been attempted.

No attempt will be made to give a lengthy discussion of the views of the different authors, other than to say that they may be divided generally into two schools of thought. The first group are of the opinion that the hydraulic properties of channels on high gradients may be calculated by the use of the same formulas as used for determining the hydraulic properties of channels on flat grades providing of course that the correct values of the component parts of the formulas are used. This procedure assumes the presence of air to be negligible or its effect to be included in the friction factor used.

The other group prefers the hypothesis that water flowing in an open channel begins to absorb or entrain air at a velocity which is stated to be between 6.6 and 14.8 feet per second depending upon the type of structure under consideration. At higher velocities the percentage of air by volume in the flow may be as much as 80 percent. Due to this entrainment of air in the flow, the velocity is somewhat less than that calculated by standard formulas. The area of the cross-section of the flow is found to be much greater. Also the presence of air in the flow with a consequent increase in internal friction and decrease in density may result in a maximum or terminal velocity.

The exact magnitude of this velocity has not been determined although Ehrenberger[11], states:

"The maximum velocity heretofore observed at which a water-air mixture flows in a steep chute is 77.1 feet per second, and that portion of the water-air mixture occupied by the water amounts to from 20 to 55 percent of the total volume."

The mechanics by which this air enters the flow has not been satisfactorily explained due to lack of sufficient observations. Opinion differs as to the rate of absorption of air.

Regardless of the mechanics by which the air enters the flow, its presence in the discharge is certain to cause a difference in the hydraulic properties of the channel. Many authors recognize the presence of air in the flow but feel that since "n" is a relative value of friction it can contain the losses due to this air. However, some of these experimenters find a relatively high value of "n" for concrete on a steep slope. Since the formulas were intended for use with a value of "n" determined experimentally for each type of material, it does not readily follow that concrete channel lining placed under similar conditions on a flat slope should have a roughness factor less than that laid on a steep slope.

Previous Work by Bureau of Reclamation Laboratory

The problem had been recognized previously by members of the staff of the Hydraulic Laboratory but had not been developed to any great extent because of the urgency of more specific problems. Data were collected from time to time as opportunity presented itself.

The data collected was for clarification of the following questions:

(1) Is air present in high-velocity flow and, if so, to what extent?

(2) What is the mechanics by which air might enter the water?

(3) What factors enter into the retarding of the velocity of the flow in the channel?

The results of these studies are summarized briefly. In regard to (1), air was present in the flow, but at the location tested the amount as measured by two different methods was found to be a very small percentage of the total flow[12]. In (2), the mechanics by which the air enters the flow was not satisfactorily explained. A study of L. Prandtl's[13] analysis was made. His suggestion is that if a difference in velocity occurs between two layers of fluid flowing past one another, the boundary surface does not remain smooth, but first assumes a wave form, then curls back on itself and finally assumes the shape of a vortex. In the case of open-channel flow, where the water surface is in contact with the atmosphere, these vortices form at the contact plane and carry air into the water. Turbulence within the flow then distributes the air.

This turbulence was explained by E. W. Lane[14], then head of the laboratory, to be a factor in the air distribution. Also turbulence propagated from the sides and bottom of the channel was instrumental in trapping the air in the water. He states:

"As the water starts down the steep section of a chute, the portion of it which is not close to the bottom or sides, is rapidly accelerated and soon attains a high velocity. There is a narrow zone on the bottom and sides, however, in which the velocity adjacent to the walls and floor is zero and that at the outside of the zone reaches that of the center section. In this narrow boundary layer there is, therefore, a rapid increase in velocity with increasing distance from the sidewalls, or in other words, there is a high-velocity gradient. In the side strips the flow is very turbulent and air is entrained, giving the water a white appearance. ... There is a similar zone in contact with the bottom but it is not as apparent since it is not in contact with the atmosphere, and air is therefore not entrained. This boundary layer, both on the bottom and sides, is narrow at the top of the chute but widens as the water flows down. The water in the central swift-flowing portion has a relatively smooth surface, and the acceleration resulting from moving down the chute is retarded but little by the effect of friction on the sides. This central portion becomes narrower and thinner as the boundary layer increases in thickness, and if the chute is long enough a point is reached where the velocity throughout the entire cross-section is considered retarded by side friction. When this point is reached the surface becomes rough, since the turbulent zone has extended through to the water surface."

Further study in relation to this phenomenon was advised at that time.

Considering (3), in regard to factors opposing the flow, first consideration was given to the dimensions and characteristics of the channel. Some tests were made in an existing structure and the results were analyzed on the basis of determining a value of "n" in the Manning and Kutter formulas. The values were found to agree very closely with values previously determined for similar canal lining on flat slopes[15].

Friction between the surface of the flow and the atmosphere was known to exist and was recognized as a retarding factor to the velocity of the flow. An outline of tests was prepared but the program was not completed because of lack of time for the laboratory personnel to conduct it.

Since the previously described work was accomplished, small amounts of data have been collected by the author. These consist primarily of personel observations and photographs of flow.

III MEASURING HIGH-VELOCITY FLOW

Methods of Measuring Velocity

Some of the early experiments failed completely and others were incomplete or questionable because of the equipment or methods used in obtaining the data. Accurate measurements of velocity, elevation of water surface, and discharge are necessary to properly analyze flow conditions in channels and the means employed to secure these measurements must be reliable.

The currentmeter has been generally adopted in this country for use in measuring low velocities. The pitot tube has found general use in measuring the higher velocities. Each of these two instruments requires calibration under conditions similar to those for which it is intended to be used. The degree of accuracy of the results depends upon the care exercised in the calibration.

With these thoughts in mind the planning of a series of field measurements necessitated a careful choice of methods and equipment. Water flowing at high velocity, even in relatively small quantities, presents a rather unruly mass to handle. The kinetic energy of this mass is great, consequently any equipment that is used must be designed to withstand severe action.

Equipment designed for field use must be compact and possess a degree of portability. It must be made such that a minimum of component parts are employed which are easily accessible and may be repaired with a small amount of equipment and under field conditions. The principles upon which the equipment functions must be basically sound and the technique employed in handling it must be such that a maximum return is gained from a minimum of effort and time.

The use of commercial currentmeters in high-velocity flow is not practical. They will not withstand the abuse to which they are exposed. Special types of meters have been developed to meet certain requirements and still others might be evolved. Were it possible to obtain a sturdy instrument, it is still necessary to obtain a rating. As has been said before, the accuracy of the instrument depends upon the degree of care exercised during the calibration and upon the similarity of conditions under which the instrument is rated and those existing where it will be used. The latter of these two requirements is hard to meet.

Since air is present in high-velocity flow it was necessary to design for the presence of the air. The extent and distribution of air in the flow became part of the problem. It was obvious that existing conditions could not be foretold nor duplicated. The rating obtained for a properly designed currentmeter or pitot tube, although carefully made, may not be applicable to field conditions if the commonly used practice of moving the instrument through still water at a known rate of speed is used. This method is not basically wrong but the fallacy lies in the fact that the still water would normally be devoid of air. The water would have a density somewhat above that of a water-air mixture depending upon the percentage of air present. Since this percentage is not known it is impossible to correct for the difference in density and hence the rating will be in error. Furthermore, in the case of a pitot tube, the presence of air in the flow is a source of annoyance, and unless extreme care is exercised, air may trap in the leads or in the instrument and cause erroneous readings.

Another factor that must be considered is what effect, if any, the turbulent water found in high-velocity flow will have upon the accuracy of the instrument. It is not logical to expect exactly the same results as those obtained in quiet water where streamline flow past the instrument prevails.

The introduction of color into the flow and a measurement of the time required for the passage of the color between two or more points, separated by known distances, has been used. This method is not entirely satisfactory because of the error introduced in the time measurement and the difficulty of the human eye to detect the exact boundary of the colored mass. In very turbulent water where the surface is mixed with air, the flow appears very white and the presence of spray above the surface renders it almost impossible to see any foreign body in the flow. Users of this method report inconsistencies in the results.

Timing surface or subsurface floats over a known reach of channel, chemical titration, impact devices, electrical dissipation from a hot wire in the flow, traveling screen, and in closed conduits, a salt-velocity method of measuring velocity have been used. All, except the latter, were considered impractical for the type of flow to be measured.

The Salt-velocity Method

The salt-velocity method has been used extensively in determining the flow through closed conduits[16]. A test technique and method of analysis have been fairly well perfected. In closed conduit studies, the method consists of the introduction of a brine solution into the conduit by a quick-acting pop valve. The time is recorded at the time this valve opens. The brine solution is carried downstream in the form of a cloud within the flow. At a selected station downstream electrodes are placed near the center of the conduit and connected to an electromotive force. An ammeter is included in the circuit. As the brine cloud passes the station, the flow of current between the electrodes is increased due to the presence of brine. The time at which the brine cloud passes the electrode station is recorded. By carefully determining the volume of the conduit between the point of introduction of the brine and the electrode station, and the elapsed time of travel from the valve to the electrode, the discharge per unit of time may be determined.

If the velocities in the conduit are relatively low or the distance between pop valve and electrode is sufficient, the measurement of time may be made with an accurate stop watch. However, if high velocities are encountered or short reaches are used, the accuracy of the flow measurement depends largely upon the timing element employed.

A careful study of this method and the results obtained through its use led to the belief that the general principles involved could be applied to velocity measurement in open-channel flow. The technique employed in the application must necessarily be different. Since the presence of air in open-channel flow is evident, it was not advisable to use the method as a means of measuring discharge. Moreover, the volume occupied by the flow in an open channel may vary between wide limits depending upon the velocity and discharge. In a closed conduit one variable is eliminated since the volume remains constant. The portion of the method applying to velocity measurement, with certain changes in technique, seemed equally applicable to open or closed conduits.

Initial Tests in the Laboratory

A technique applicable to open-channel flow and equipment to execute the tests was necessary before any field measurements could be attempted. This development was started in the laboratory.

In addition to measuring the velocity, it is necessary to secure water surface measurements in the channel for computing air content and energy gradient. Some satisfactory means of obtaining these measurements was also sought.

A hydraulic model of a chute was being tested in the laboratory. The chute had a cross-section approximately 6 inches deep by 9-1/2 inches wide. Velocities in this model were from 10 to 15 feet per second and the section was of sufficient length to permit preliminary investigation of the method.

Copper electrodes ranging in width from 1/4 inch to 3/4 inch were spaced from 6 inches to 2 feet apart, measured parallel to the centerline of the chute. Two electrodes were used at each point, one on each side of the flume. The original strips extended from the floor to the top of the sides but some of these were later shortened to extend only part way up the sides in order that they would not project above the water surface. Others were extended part way across the floor and up the sides. Electrodes were also made by using No. 7 flathead, brass, wood screws with only the surface of the head exposed to the flow.

Time measurements were made by means of an oscillograph. The oscillograph used consisted essentially of three moving coil galvanometers, an optical system, and a motor-driven camera. The current supply for the fields of the galvanometers was derived from a 6-volt storage battery and the moving coils were excited from dry cell batteries in the test circuit. The vibration of the moving coils was damped by oil. The optical system employed two small flashlight bulbs as light sources, a cylindrical lens to reduce the light supply to a horizontal band, three small cylindrical lenses (one in front of each galvanometer) to focus the horizontal band on tiny mirrors fastened to the moving coils of the galvanometers and to change the reflected bands to vertical bands, and a cylindrical lens to focus the vertical bands as points on the film in the camera. The camera was belt-driven by a 6-volt shunt motor. The record was made on standard No. 122 camera film in 37-inch lengths, wound on daylight loading spools. One length was used for each record. The film was drawn through the camera at right angles to the vibrating light beam.

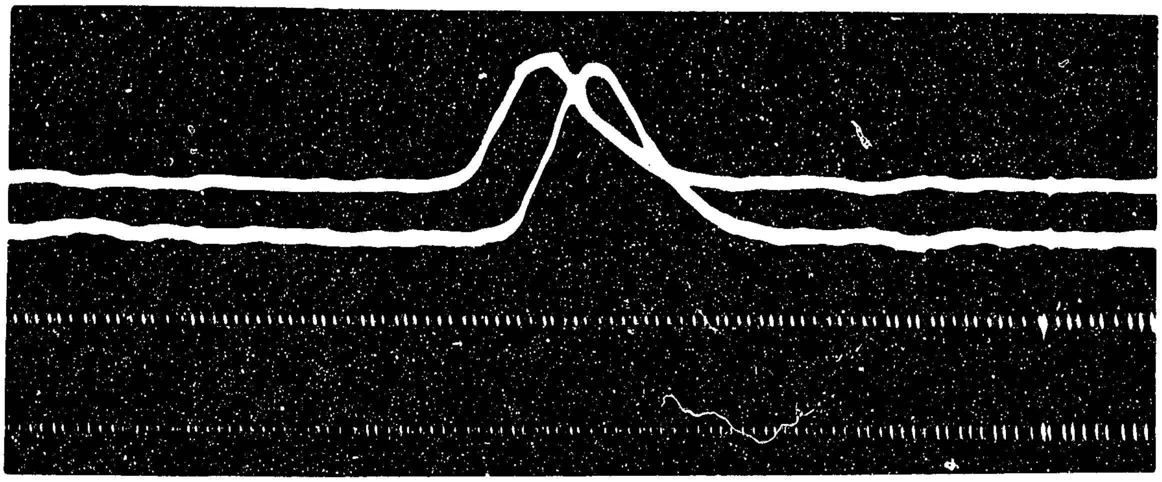
Two pairs of electrodes were separately connected by wiring systems to two of the galvanometers in the oscillograph. The third galvanometer was connected to the 110-volt, 60 cycle, alternating-current supply to provide a time base on the film record. The power supply to operate the galvanometers included one or two dry cell batteries, depending upon the base current needed, and the circuits were closed by the water between the electrodes. After all circuits were properly balanced the camera was started and salt solution was poured into the flow from glass containers. A saturated solution was used and the amount introduced varied during the tests from one-fourth pint to one quart. It was usually introduced at the upper end of the flume but in some instances the introduction was made close to the first set of electrodes. From three to five measures of brine could be emptied into the flow during the 10 seconds required for the film to pass through the camera.

Since the oscillograph was arranged to provide visual observation, very few film records were made. The type of electrodes, the number of dry cells necessary in the circuit, the amount of salt, and the point of induction were studied visually on the ground glass of the oscillograph.

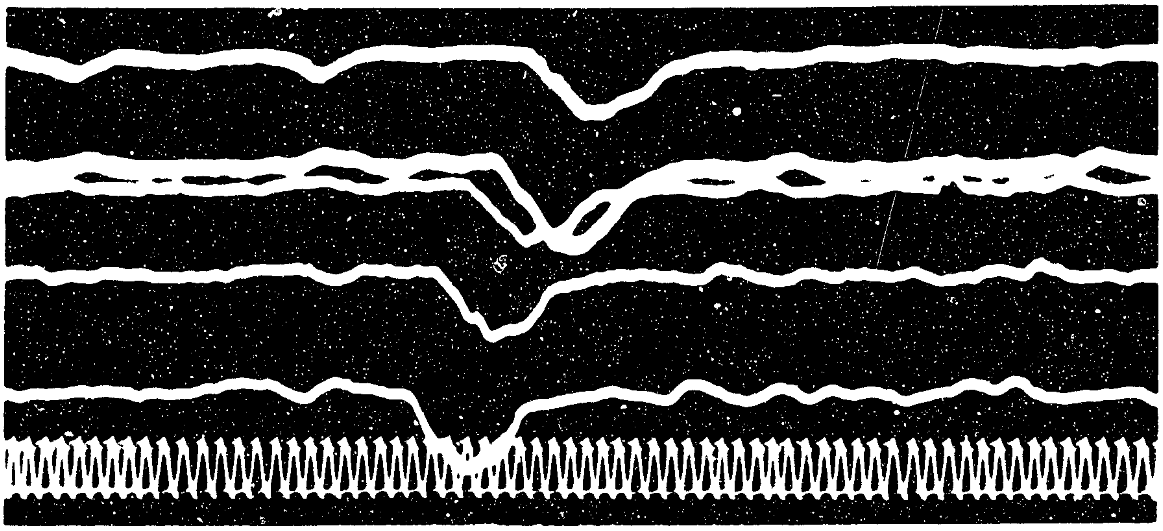
When the test circuit was connected to the oscillograph the galvanometers registered a base deflection due to the passage of some current through the flowing water. The amount of this current was dependent upon the area of the electrode exposed to the water and the amount of electromotive force supplied by the dry cell batteries. As the brine solution passed between the electrodes, the conductivity of the water was increased and the flow of current increased.

The windings in the galvanometers were calculated to give a deflection of the light beam on the film of 1 inch per 5 milliamperes change in current. The base current amounted to from 1 to 3 milliamperes and during the passage of the brine was increased to approximately 8 milliamperes. The instantly responding moving coil galvanometer and the optical system produced a current-time record on the film, in which distance along the film was proportional to time; and the deflection of the beam of light was proportional to the instantaneous value of current passing through the galvanometer vibrator.

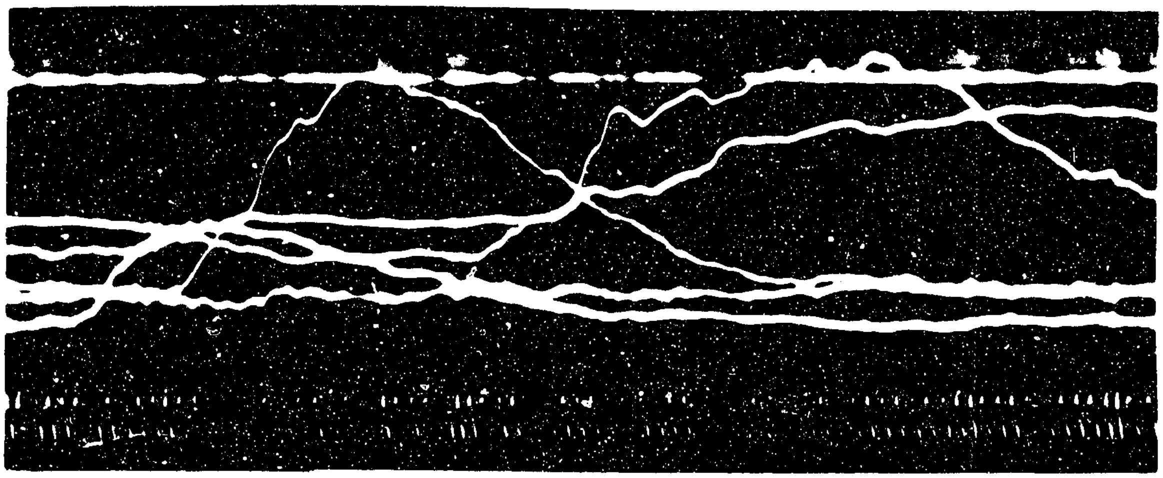
The record obtained by such an arrangement is so long that it is not feasible to reproduce it here. However, a portion of an actual record is shown on Figure 1A. No attempt was made to measure accurately the value of the current since it had little bearing on the results.



A. OSCILLOGRAM FROM SUN RIVER CHUTE MODEL



B. OSCILLOGRAM FROM 45-DEGREE LABORATORY CHUTE



C. [illegible]

FIGURE 1

The distance between the curves produced by the passage of the salt, as measured by the time scale imposed on the film by the 60-cycle alternating current, corresponded to the time necessary for the flow to travel the distance between the pairs of electrodes. This distance was accurately measured in the flume. The distance between the curves on the film was taken in three different ways. The distance between the centers of gravity of the curves, the distance between the points where the curves left the base lines, and the distance between similar points on the curves were measured. The distances all agreed very closely and no trend was evident of one measurement indicating a higher velocity than another. The measured velocities checked the calculated average velocities very closely.

The electrodes with greater exposure to the flow produced a greater base deflection in the galvanometers but this could be diminished by decreasing the voltage in the circuit. The electrodes extending above the water surface were sensitive to waves on the surface of the flow. This caused extraneous deflections in the galvanometers which were at first considered to be objectionable but were later found to be readily distinguishable from the deflections caused by the brine.

It was concluded that strip electrodes fastened to sides of the flume or circular electrodes on the sides or bottom of the flume would give satisfactory results. The area of electrode was immaterial since it was necessary to balance the current at the instrument before each test, using one, two or three 1-1/2-volt dry batteries.

The amount of salt necessary to produce a legible deflection was determined by trial. The quantity of salt solution influenced the deflection produced in two ways. The maximum deflection was increased by an increase in salt solution and the curves extended over a greater elapsed time. Too great a deflection caused the curve to swing beyond the edge of the film and an excessive time interval made the calculations difficult. By determining the correct amount of brine and by pouring it into the flow quickly very readable curves were obtained.

Final Tests in the Laboratory

A technique necessary to secure and interpret the records obtained by this method of measuring velocity was developed for the low-velocity flume. To expand this technique, a wooden flume was constructed on a 45-degree slope. The inside dimensions of this flume were 9.6 inches wide by 10.5 inches deep. The length was 17 feet 2-1/2 inches, measured along the slope. The water was delivered to the flume by a 12-inch centrifugal pump and was admitted to the flume under pressure from a rounded-edge slide gate. The maximum discharge of 8-1/4 second-feet produced velocities of approximately 50 feet per second.

The electrodes installed in this flume were 1/2-inch brass disks, placed flush with the inside surface of the chute. Nine sets, two disks for each set, were placed in the floor of the chute, 18 inches between sets, measured along the centerline of the floor. Nine sets of electrodes were placed in the sides of the chute at the same spacing and in line with the bottom electrodes.

The brine was introduced into the flow from a pressure tank with an outlet immediately downstream from the slide gate and controlled by a quick-acting valve. The introduction was made at the surface of the flow.

The technique followed during these tests was essentially the same as that in the low-velocity chute. Three additional elements installed in the oscillograph made it possible to measure the velocity over longer reaches and to trace the action of the salt cloud through a greater distance. The base current through the circuits was found to be slightly less than that in the low-velocity chute, probably due to the relatively small electrode area, hence a larger amount of brine was necessary to produce a suitable record.

The brine could be introduced more rapidly with the quick-acting valve so the initial cloud in the flow was evidently quite similar to the cloud in the low-velocity chute. However, there was more elongation as it passed down the chute. This was expected since the turbulence in the flow was greater and higher velocities prevailed.

Near the close of the test program finely ground salt was used instead of a brine solution. The cloud, as registered by a milliammeter, was slightly longer than the brine cloud but was very little greater in concentration. Evidently the highly turbulent flow dissolved the salt rapidly and distributed it throughout the cross-section in a short distance. Although it increased the work necessary in making an analysis of the oscillogram curves, this method was considered more desirable because of the simple manner in which the powder could be introduced into the flow. Since the method was being developed for field use, simplicity of equipment was essential.

The series of tests although not extensive led to some definite conclusions. Oscillograms produced were easy to interpret, Figure 1B. The brine cloud was carried the length of the flume in a sufficiently compact mass to give good records.

The results obtained from the electrodes placed in the side of the chute checked those from the bottom electrodes so closely that there was no evidence of disagreement. This indicated that the brine was well distributed throughout the cross-section and the equipment employed was sensitive to the presence of the brine regardless of the location of the electrodes.

Measurements of velocity by the use of either side or bottom electrodes checked the calculated velocity within 1 percent. The velocity was calculated by the formula $V = Q/A$, where V is velocity in feet per second; Q, discharge in cubic feet per second, and A, area of the cross-section in square feet. The discharge was measured over the calibrated laboratory weir and the area of the cross-section was carefully measured at the gate.

The oscillograms obtained from this chute showed a persistent pulsation in the base current. This pulsation was not periodic and the resulting deflections were not of equal magnitude. Several explanations were offered for this occurrence. It was possible that splash from the flow wet the insulation of the wire leads and caused a part of the leads to act intermittently as electrodes. Visual observations of flow conditions, however, showed pulsations in the flow itself. There appeared to be regions of high-velocity water separated by regions of lower velocity. These flow pulsations were evidently reflected in the photographic record. Velocity determined from the film by using these minor deflections of current gave consistent results when compared with the deflections caused by the passage of salt.

For the purpose of making field measurements, it was concluded that electrodes, if installed in a structure at the time of its erection should consist of small plates embedded flush with the interior surface of the channel. The wire leads could then be carried through the walls and to some suitable location for the recording equipment.

Electrodes for installation in an existing structure should consist of metal strips securely anchored to the sidewalls in such a manner that they would offer a minimum resistance to the flow. They should be installed perpendicular to the floor of the chute and be of sufficient length to extend above maximum water surface. The wire lead should be attached to the upper end. The distance that the electrodes should be separated, either longitudinally or transversely, would necessarily depend upon the section to be tested.

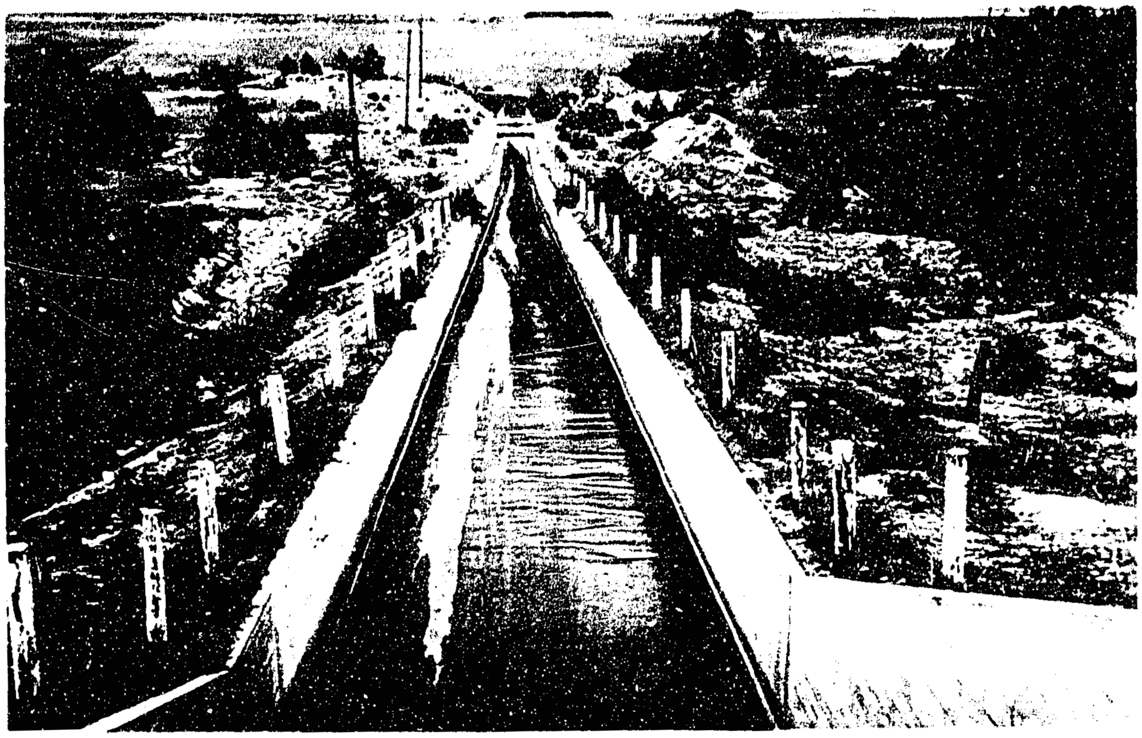
From the results obtained in the small chute, it was concluded that finely divided dry or moist salt would dissolve in the turbulent flow very rapidly and would be satisfactory for use in the field. Some development work was done to secure a means of accurately determining the water surface in extremely turbulent flow. The basic principles applied appeared to be satisfactory but insufficient time negated the completion of equipment suitable for field use.

IV TESTS ON KITTITAS WASTEWAY

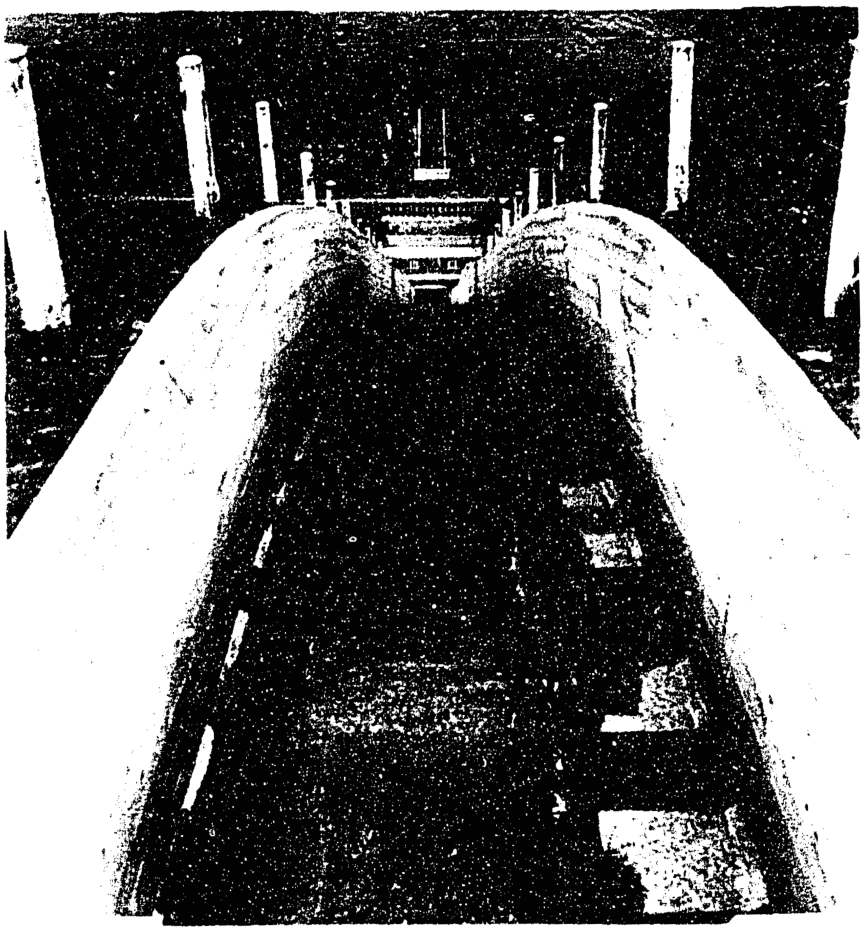
Description of Wasteway and Test Equipment

The wasteway at Station 1146+30 on the Kittitas Main Canal, Yakima Project, Washington was selected as suitable for conducting the test program.

In this structure there is a drop of 340.5 feet in a horizontal distance of 1213 feet. All but a small fraction of a foot of this fall occurs in a horizontal distance of 1135 feet, Figures 2 and 3. With two bottom slopes available for testing in the same structure, considerable data could be secured from a single installation.

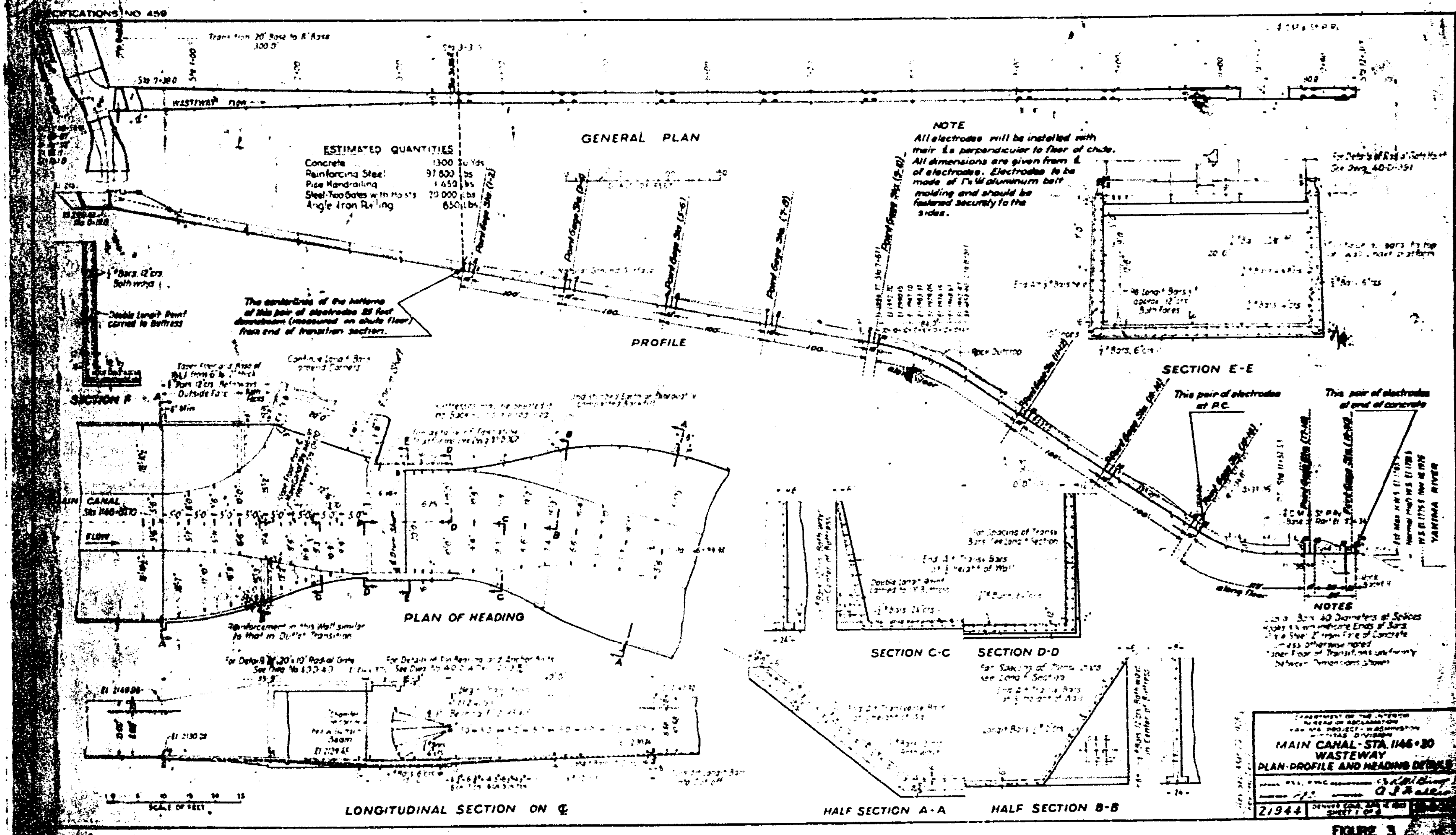


A. UPPER SECTION OF WASTEWAY - LOOKING DOWNSTREAM



B. LOWER SECTION OF WASTEWAY - LOOKING DOWNSTREAM

FIGURE 2



SPECIFICATIONS NO. 459
GENERAL PLAN
PROFILE
ESTIMATED QUANTITIES
Concrete 1300 cu. yds.
Reinforcing Steel 97 800 lbs.
Pipe Handrailing 1 650 lbs.
Steel Tee Gates with Hoists 20 000 lbs.
Angle Iron Railing 850 lbs.
NOTE
All electrodes will be installed with their ℄s perpendicular to floor of chute. All dimensions are given from ℄ of electrodes. Electrodes to be made of 1½"x¼" aluminum belt molding and should be fastened securely to the sides.
The centerlines of the bottoms of this pair of electrodes 25 feet downstream (measured on chute floor) from end of transition section.
SECTION E-E
This pair of electrodes at P.C.
This pair of electrodes at end of concrete
YAKIMA RIVER
NOTES
SECTION F
MAIN CANAL
FLOW
PLAN OF HEADING
Reinforcement in this Wall similar to that in Outlet Transition
LONGITUDINAL SECTION ON ℄
SCALE OF FEET
SECTION C-C
SECTION D-D
HALF SECTION A-A
HALF SECTION B-B
MAIN CANAL-STA. 1146+30
WASTEWAY
PLAN-PROFILE AND HEADING DETAILS
21944

FIGURE 3

The approximate location of electrodes is shown in Figure 3. It was considered advisable to install the pairs of electrodes at the two different intervals because of the uncertainty of the stability of the salt cloud. Should the dispersion occur quite rapidly the 10-foot reaches could be used to determine the velocity. The 90- 100- or 110-foot reaches were preferable, if the dispersion should be sufficiently small, because the percentage of error in reading the time would be less. The actual distances between pairs of electrodes were measured after installation and these distances were used for all calculations.

The wiring system differed slightly from that used in the laboratory. All electrodes on the left side of the wasteway were connected in series by a single insulated wire leading to the switchboard at the oscillograph. A separate insulated lead connected each of the electrodes on the opposite side of the structure to the board. The dry cell batteries were introduced between the switchboard and the oscillograph.

Discharge Measurements

The discharge through the wasteway was determined by gaging the flow in the main canal with a currentmeter. During the greater part of the tests, the total flow of the canal was diverted into the wasteway. For this condition only one gaging station, located above the wasteway turnout, was necessary. The remainder of the time some water was passed down the canal for irrigation purposes. A gaging station located in the main canal downstream from the turnout was used to obtain the quantity passing. The difference in discharge as measured at the two gaging stations was used as the quantity passing through the wasteway. The currentmeter measurements for each test were made simultaneously with the velocity and point gage measurements. An experienced hydrographer from the project office made all the currentmeter measurements. Standard currentmeter practice was employed and care was taken to attain a high degree of accuracy.

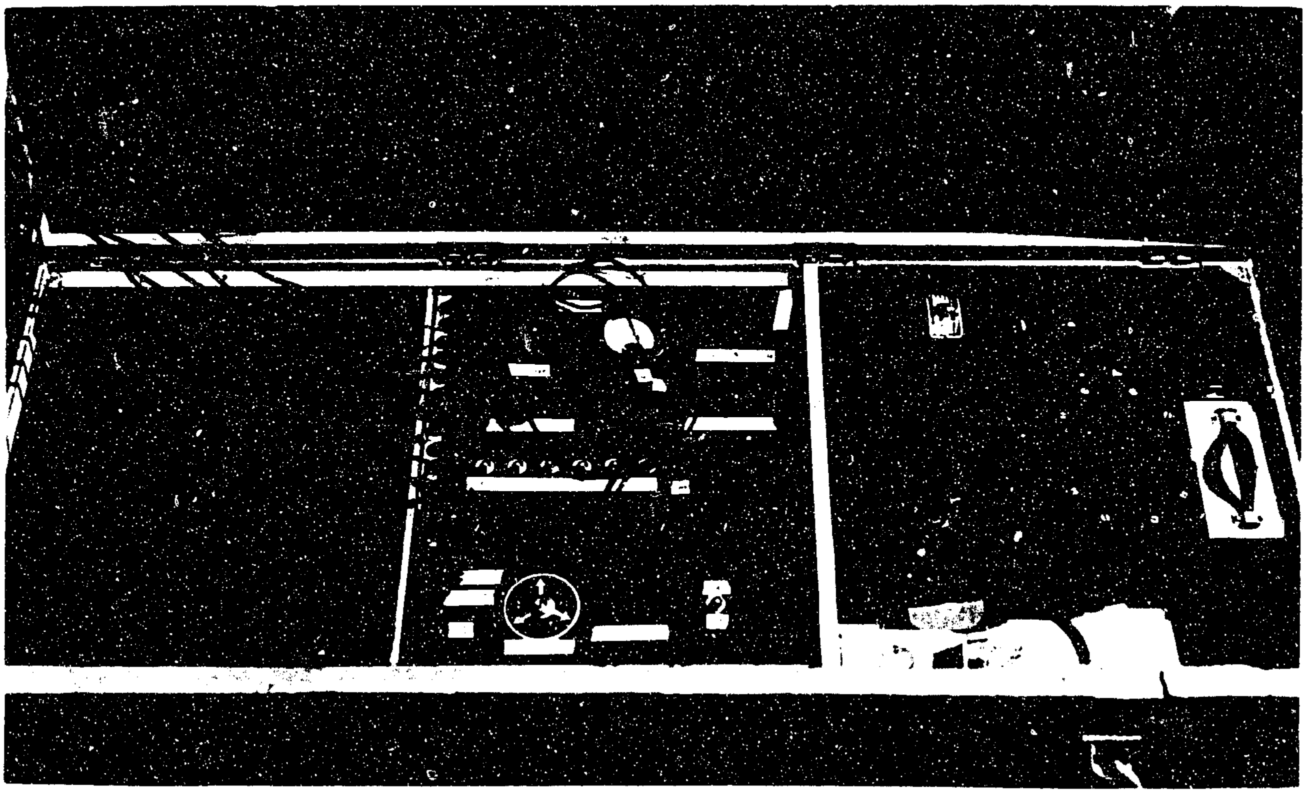
Depth Measurements

All measurements of depth of flow were made with a point gage, Figure 4A. Ten cross-sections were measured, one at the mid-point of each of the 10-foot electrode stations. Nine observations approximately 1 foot apart, were made at each cross-section. This same procedure was followed throughout the program except at the higher discharges when excessive spray made it necessary to abandon the first station below the convex vertical curve. At the maximum discharge, it was also necessary to abandon the second station below the curve.

After the tests were completed, the width of the channel at each station was carefully measured. Point gage readings of the bottom of the channel were taken with the same instrument and at the same locations at which the water surface observations were made. The depth and area of cross-section occupied by the flow was calculated from these measurements.

The results of the depth measurements are given in Table 1. The table consists of ten divisions, each division showing the properties of one of the sections observed. In the first column of each division is given the discharge as measured in the main canal during the time the depth observations were being made. The next nine columns give the actual observed depths of flow in feet at distances from the left sidewall. In the column headed "mean depth," the average of the nine readings across the section is given. The next column, flow area,

A. POINT GAGE USED FOR OBTAINING DEPTH OF FLOW



B. OSCILLOGRAPH

FIGURE 4

TABLE I

DEPTH MEASUREMENTS — KITTITAS WASTEWAY

Station 1-2

Discharge second-feet	0	1	2	3	4	5	6	7	8	Mean depth D, feet	Flow area A, sq. ft.	Wetted perimeter feet	Hydraulic radius feet
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

(Distances from left sidewall in feet; depth in feet.)

Station 3-4

Discharge second-feet	0	1	2	3	4	5	6	7	8	Mean depth D, feet	Flow area A, sq. ft.	Wetted perimeter feet	Hydraulic radius feet
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Station 5-6

Discharge second-feet	0	1	2	3	4	5	6	7	8	Mean depth D, feet	Flow area A, sq. ft.	Wetted perimeter feet	Hydraulic radius feet
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Station 7-8

Discharge second-feet	0	1	2	3	4	5	6	7	8	Mean depth D, feet	Flow area A, sq. ft.	Wetted perimeter feet	Hydraulic radius feet
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Station 9-10

Discharge second-feet	0	1	2	3	4	5	6	7	8	Mean depth D, feet	Flow area A, sq. ft.	Wetted perimeter feet	Hydraulic radius feet
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Station 11-12

Discharge second-feet	0	1	2	3	4	5	6	7	8	Mean depth D, feet	Flow area A, sq. ft.	Wetted perimeter feet	Hydraulic radius feet
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Station 13-14

Discharge second-feet	0	1	2	3	4	5	6	7	8	Mean depth D, feet	Flow area A, sq. ft.	Wetted perimeter feet	Hydraulic radius feet
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	-	-	-	-	-	[illegible]	[illegible]	[illegible]	[illegible]

Station 15-16

Discharge second-feet	0	1	2	3	4	5	6	7	8	Mean depth D, feet	Flow area A, sq. ft.	Wetted perimeter feet	Hydraulic radius feet
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
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Station 17-18

Discharge second-feet	0	1	2	3	4	5	6	7	8	Mean depth D, feet	Flow area A, sq. ft.	Wetted perimeter feet	Hydraulic radius feet
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
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[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Station 19-20

Discharge second-feet	0	1	2	3	4	5	6	7	8	Mean depth D, feet	Flow area A, sq. ft.	Wetted perimeter feet	Hydraulic radius feet
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
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[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
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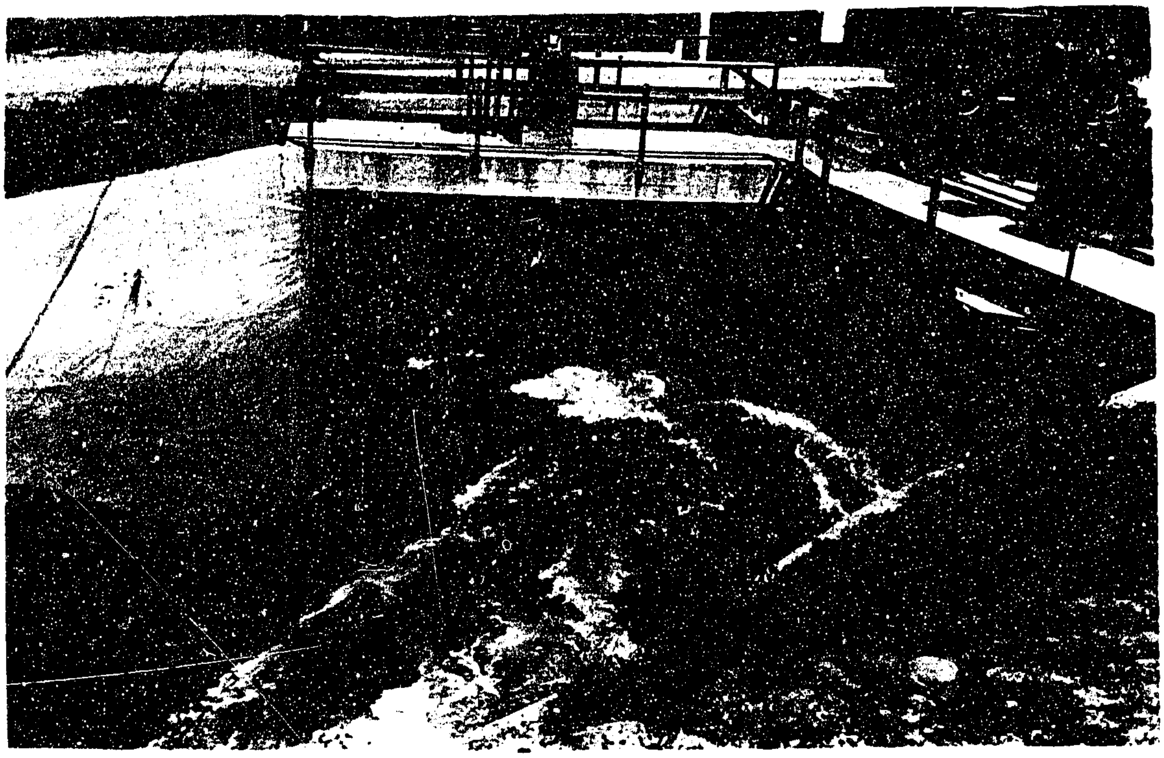
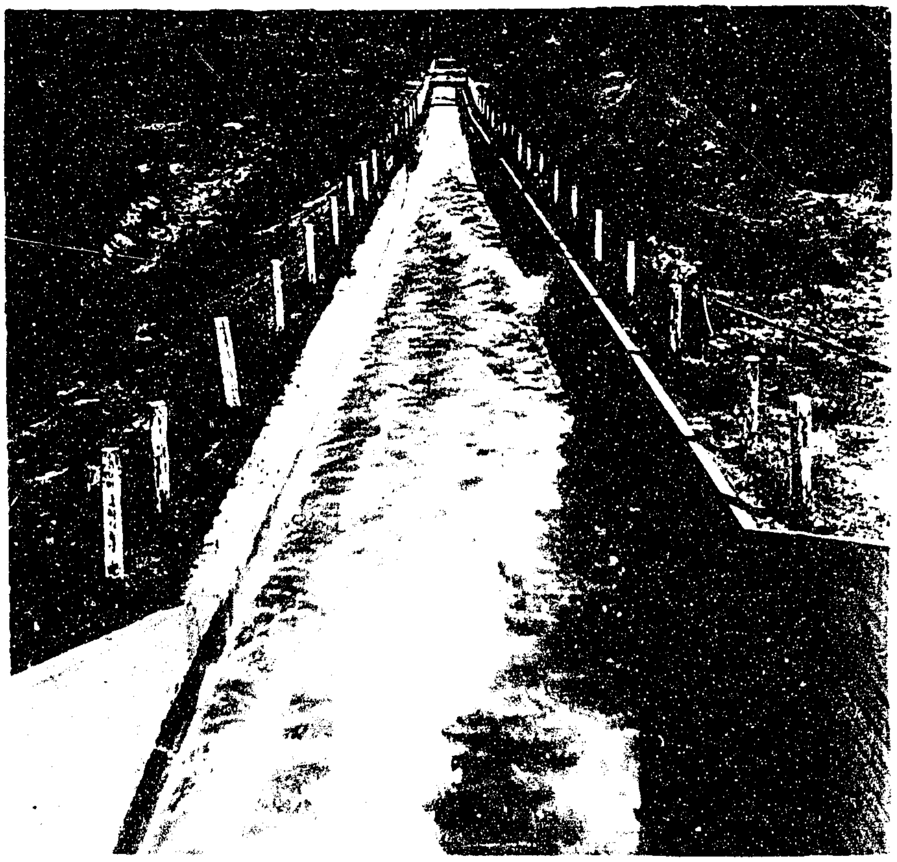
gives the area of the cross-section of flow. This is the product of the depth given in the preceding column and the average width of the channel as measured. This area is corrected for the cross-sectional area of fillets between the sides and bottom of the channel. The last two columns show the wetted perimeter and hydraulic radius as calculated from the preceding values.

All observed depths have been included in the table to show the consistency of the results. The choppy water surface and the large amount of spray rendered it difficult to determine where the point of the gage should be to give a reading that would be indicative of the actual depth of flow. The surface conditions also made it difficult to observe the point of the gage. The depth of flow was considered to be at the base of the loosely flying spray and drops of water. The top of the main portion of the flow included numerous small waves or rollers. The water depths given in the table are approximate mean values between the crest of the waves and the troughs between them. The vibration of the point gage was relied upon more than visual observation to insure that the point was at relatively the same position in the flow for successive readings.

Two factors may be observed in the depth measurements as given in the table. For the high discharges, the wave due to entrance conditions, Figure 5, was reflected in the point gage readings at Station 1-2 and 3-4. This wave was not observed beyond that point. The water surface at the sides of the chute was higher than that in the central portion, Figure 6. For the higher discharges, this high portion extended slightly over a foot from each wall and was as much as 0.4 foot higher than the central portion of the flow. For this reason, observation of

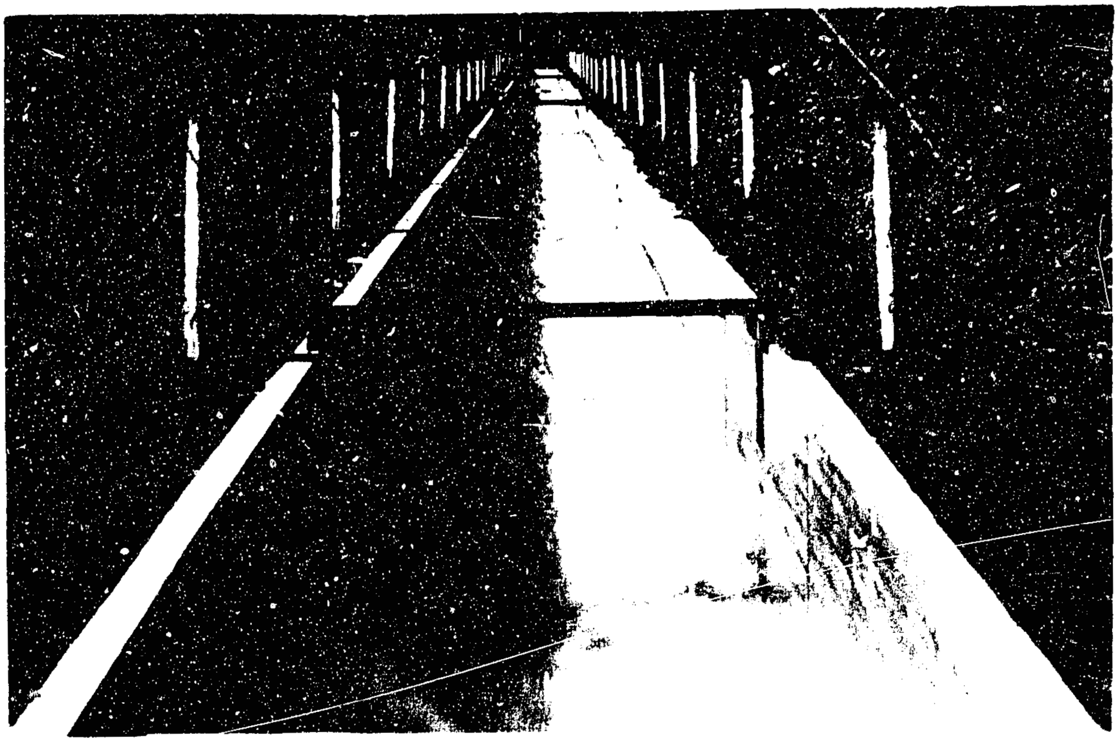


A. WASTEWAY ENTRANCE - DISCHARGE 922 SECOND-FEET

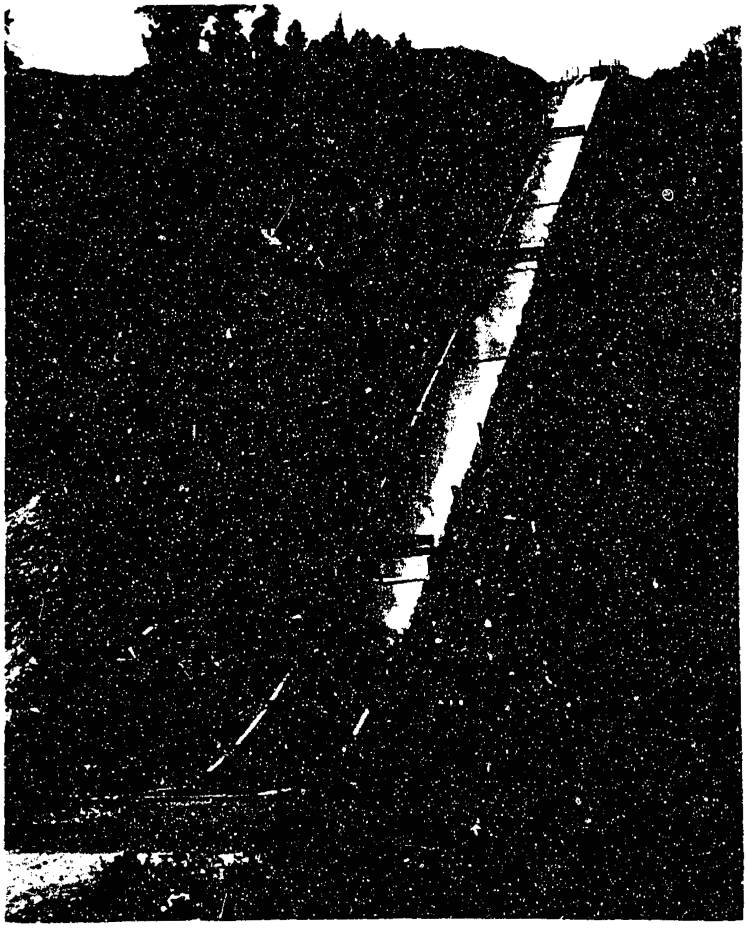


B. WAVE IN UPPER SECTION OF WASTEWAY CAUSED BY ENTRANCE CONDITION. DISCHARGE 922 SECOND-FEET

FIGURE 5



A. FLOW CONDITIONS ON 10°-12' SLOPE. DISCHARGE 777 SECOND-FEET.



B. FLOW CONDITIONS ON 33°-10' SLOPE. DISCHARGE 922 SECOND-FEET

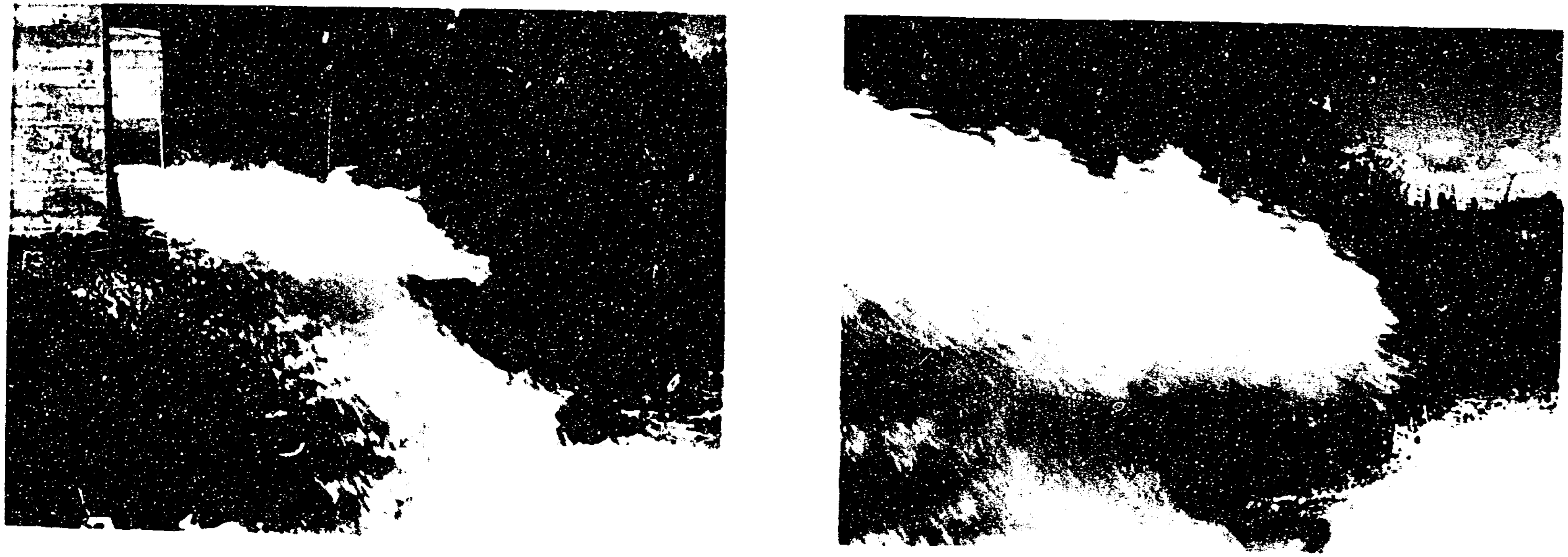
FIGURE 6

depth by staff gages fastened to the walls of such a structure would be in error.

For the two lower discharges, well-developed traveling waves were persistent in the flow, particularly on the steep slope. They were not obvious in the higher flows. These waves formed in the upper transition section and increased in magnitude and velocity as they progressed down the chute, Figure 4A. They are the type of wave observed in numerous structures at low discharges and have been referred to by some authors as "slugs" or "balls" of water.

Some interesting observations were made in regard to these waves. Although 25 waves were observed to pass Electrode No. 18 in 50.8 seconds with a discharge of 193 second-feet, they were not periodic. At times two would pass in one second and then a lapse of two or more seconds would occur before another was observed. The average rate of occurrence of these waves at Electrode No. 20 for the same discharge was one every 1.6 seconds. Observation and study of motion pictures showed that the velocity of these waves was considerably greater than the velocity of the intermediate water. This may also be seen in Figure 7. The front of the wave was nearly vertical, while the back had a long slope that extended almost to the front of the succeeding wave. The wave front was almost a straight line perpendicular to the direction of flow.

The cause of the waves was not definitely observed but the conclusions are that the friction in the fluid being less than that between the fluid and the solid chute caused the upper part of the flow to slide over the lower part and thus attain a higher velocity. This velocity water overtakes the water flowing at a lower velocity and tends to pile up forming waves. The mechanics of these waves is similar to that of a hydraulic jump.



A. DISCHARGE 89 SECOND-FEET

B. DISCHARGE 491 SECOND-FEET

C. DISCHARGE 777 SECOND-FEET

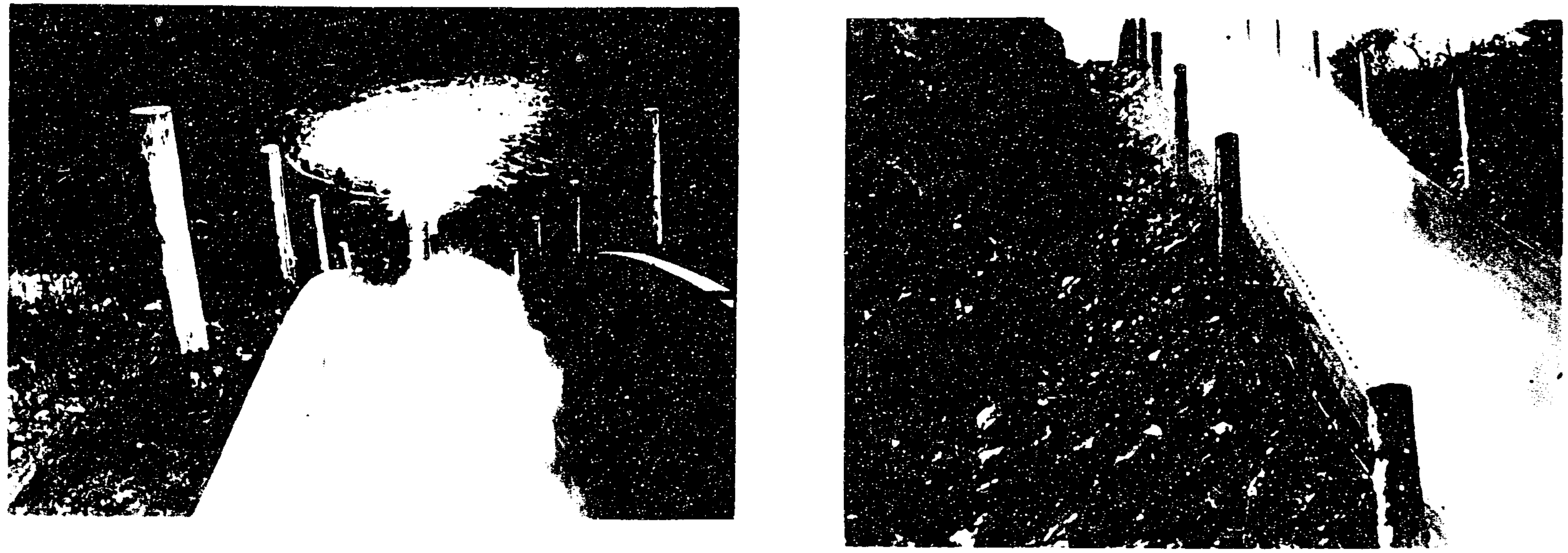
JET FROM WASTEWAY ENTERING YAKIMA RIVER

FIGURE 7

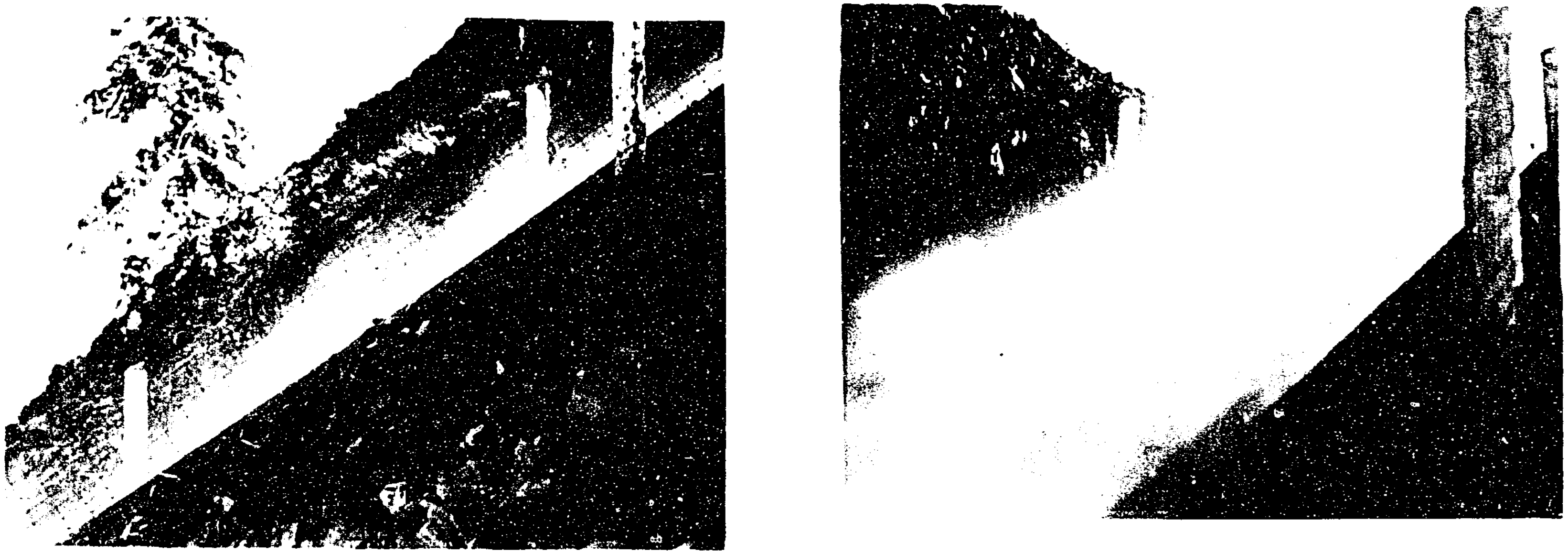
The waves made it difficult to obtain an average depth of flow for the discharges at which they occurred. Observations were made at the crest of the wave and at an intermediate point and the mean of these readings considered as the depth of flow. Observations of depth for the 89 second-foot discharge have been omitted from the table except for the two upper stations. The magnitude and rate of occurrence of the waves at stations farther down the channel rendered the observations doubtful. It will also be noted that the data is not complete for Stations 11-12, 13-14, 15-16. As has been mentioned before, adverse conditions due to spray negated completion of the data, Figure 8.

Velocity Measurements

The six-channel oscillograph used in the laboratory was used for making the field tests, Figure 4B. The only change in the instrument was the addition of an electrically driven, 50-cycle tuning fork to provide a time base. The vibrations of the tuning fork were recorded on the film by connecting one of the galvanometers in the tuning fork circuit. The remaining five elements were available for connection to the electrodes. Data were obtained from ten discharges ranging from 89 to 1005 second-feet. The circuits and elements were adjusted until the light beams would register the trace of the current of the film. Flour salt, of a fineness to pass a 140-mesh sieve, was moistened until the particles would cohere when molded by hand into balls. The size of the ball necessary for satisfactory registration on the film was determined by trial and error. When measuring velocities over a reach of 200 feet with the lower discharges, balls 2 inches in diameter were sufficient. For longer reaches or larger discharges, the size of the balls was increased. At the maximum discharge, with velocities measured over a 400-foot reach, the diameter of the ball was approximately 7 inches.



A. DISCHARGE 922 SECOND-FEET



DISCHARGE 1005 SECOND-FEET

FLOW CONDITIONS AT CHANGE IN GRADE

FIGURE 8

After the balls were formed, the recording apparatus was set in motion. The balls were then tossed into the flow a short distance upstream from the test reach. The number of balls used varied from one to five. On short sections five salt balls were used for a single film record. It was found that all the balls did not record because of the relatively short film, so the number was reduced to two for short reaches and a single ball for the longer reaches.

Since only five galvanometer elements were available, five records were necessary to cover the entire length of the flume at each discharge. The first five sets of electrodes were connected for the first record; the next five sets for the second, and so on down the wasteway. For all tests, except the first three discharges, one record was made with Electrodes 1, 3, 5, 7, and 9 connected and another with Electrodes 12, 14, 16, 18, and 20 connected. It was found that by introducing one large ball of salt, the cloud carried over the longer distance in ample time and in a sufficiently compact cloud to give a legible record. Visual observations with a milliammeter showed that the salt was carrying the full length of the wasteway in a sufficient concentration to produce records, however, the cloud was greatly elongated.

The velocities, as calculated from the oscillograms and measurements made in the wasteway are given in Table 2. In determining velocities from the oscillograms, the time distance required for the salt cloud to pass from one set of electrodes to another was obtained by averaging the time distances between the beginning points on the curves, and the time distances between other characteristic points on the curves. This probably resulted in a velocity approaching that of maximum, perhaps slightly less.

TABLE 2

OBSERVED VELOCITY—KITTITAS WASTEWAY

SLOPE 10°-12'

Distance from Electrode No. 1	0	10'	100'	110'	200'	210'	300'	310'	400'	410'
Number of electrode	1	2	3	4	5	6	7	8	9	10

Discharge second-feet — Numbers in parentheses are the observed velocities in feet per second. Numbers to right and left of each observed velocity indicate the electrodes between which the velocity was measured.

Discharge second-feet									
89	1	(35.4)	3	(33.9)	5				
			3	(34.0)	5				
193	1	(37.8)	3	(37.0)	5	(39.1)	9		
					5	(37.8)	7		
	1	(46.2)	3	(50.3)	5	(51.3)	8	(61.0)	9
362	1	(53.4)	3	(54.6)	5	(47.5)	8	(61.7)	9
	1	(46.8)	3	(53.0)	5				
401	1	(46.3)	3	(51.4)	5				
	1	(52.2)	3	(48.6)	5	(54.4)	8		
491			3	(56.0)	5	(56.4)	8		
			3	(51.4)	5	(57.4)	9		
	1	(55.2)	3	(54.7)	5	(57.5)	7	(62.7)	9
587			3	(58.3)	5	(61.0)	7	(65.9)	9
					5	(57.3)	7	(64.0)	9
	1	(57.9)	3	(57.7)	5	(51.2)	7	(69.5)	9
719	1	(57.6)	3	(50.2)	5	(62.0)	7	(67.4)	9
					5	(70.1)	7	(53.6)	9
	1	(64.6)	3	(62.2)	5	(63.1)	7	(63.0)	9
777			3	(64.1)	5	-	7	(64.4)	9
	1	(72.6)	3	(74.6)	5	(66.1)	7	(68.0)	9
922	1	(61.0)	3	(66.9)	5	(60.3)	7	(68.7)	9
			3	(60.7)	5	-	7	(64.7)	9
	1	(72.2)	3	(63.4)	5	(72.3)	7	(67.0)	9
1005	1	(74.2)	3	(71.3)	5	(68.8)	7	(67.5)	9
					5	(56.7)	7	(67.1)	9

SLOPE 33°-10'

Distance from Electrode No. 11	0	10'	100'	110'	200'	210'
Number of electrode	11	12	13	14	15	16

Discharge second-feet — Numbers in parentheses are the observed velocities in feet per second. Numbers to right and left of each observed velocity indicate the electrodes between which the velocity was measured.

Discharge second-feet							
193			12	(55.3)	14	(66.1)	16
			12	(52.4)	14	(67.0)	15
			12	(57.1)	14		
	11	(78.8)	14	(69.9)	15		
362	11	(67.9)	14	(70.3)	15		
			12	(71.0)	14	(65.4)	16
	11	(74.5)	13	(81.0)	16		
401	11	(74.4)	13	(78.8)	16		
			13	(72.5)	16		
			12	(73.2)	13	(83.6)	15
491	11	(79.1)	13	(79.7)	15		
			12	(77.0)	14	(70.3)	16
	11	(83.9)	13	(79.4)	15		
587	11	(79.0)	13	(82.8)	15		
	11	(74.9)	13	(83.8)	16		
			13	(86.1)	15		
	11	(95.8)	13	(93.4)	16		
719	11	(88.2)	13	(80.7)	16		
			13	(82.2)	15		
777	11	(81.7)	13	(116.8)	16		
			13	(106.2)	15		
			12	(85.3)	13	(93.4)	15
922			12	(96.2)	13	(92.5)	15
			12	(87.5)	14	(85.1)	16
			12	(113.1)	15		
			12	(93.1)	15		
1005							
					13	(101.3)	15
					13	(102.7)	15
					14	(92.0)	16

SLOPE 0°-[illegible]

Distance from Electrode No. 17	0	10'	40'	50'
Number of electrode	17	18	19	20

Discharge second-feet — Numbers in parentheses are the observed velocities in feet per second. Numbers to right and left of each observed velocity indicate the electrodes between which the velocity was measured.

Discharge second-feet			
	18	(42.2)	20
362	18	(44.0)	20
	18	(42.2)	20
	17	(73.8)	20
491	17	(55.6)	20
	18	(56.3)	20
	17	(41.8)	19
587	17	(56.2)	20
	17	(74.9)	20
	17	(83.9)	20
719	17	(74.4)	20
	17	(71.3)	19
	17	(72.4)	19
777	17	(62.7)	19
	17	(72.7)	20
922	18	(69.5)	20
	17	(77.2)	20
1005	18	(68.6)	20

It was very difficult to read the time interval from the oscillograms for the 10-foot electrode stations and the error involved was high; therefore, 90- 100- and 110-foot sections were used almost entirely for determining the velocities. Velocities could not be obtained from some of the records resulting in blank spaces in the table.

Several of the electrodes were damaged and a few were broken off and carried away by the combined action of the high velocity water and rocks and pebbles in the flow. The bent electrodes altered the distance between stations but since comparatively long reaches were used for the actual velocity determinations the error introduced from this source was small. The behavior of the salt cloud as it traveled down the wasteway is not known. It may have traveled from side to side as it progressed down the chute. This inference was gathered from observation of the electrodes after the completion of the tests. It was noted that the electrodes suffered more damage and the concrete showed more erosion on the left wall of the wasteway at one station while at the next station downstream the damage and erosion were on the opposite wall. Such action, although not a symmetrical zigzag pattern, was noted throughout the length of the wasteway. It is possible that some portion of the salt cloud traveled continuously in the high velocity center flow and it was this portion which caused the initial deflection of each galvanometer. If this is true the measured velocities approach the maximum velocities.

The velocities observed between Electrode Stations 1 and 3, with a discharge of 1005 second-feet, are slightly above the mean velocity calculated from the continuity formula, $V = \sqrt{2gh}$. The value of h used was the difference in elevation between the water surface in the canal and the water surface at a point midway between the two stations. No entrance loss or loss due to friction was assumed. These are the only two observed velocities that exceed the velocity calculated by the above formula. Since both observations made under the same conditions exceed the theoretical velocity, it may be that the measured velocity is the maximum velocity in the flow.

Other sources of error, although very small, lie in the fact that the spots from all the elements in the oscillograph cannot be adjusted to lie exactly on a line perpendicular to the centerline of the film and the rotation constants of the elements are not identical. Each element deflects slowly at first but with rapidly increasing velocity, partly because the current is increasing and partly because of its own inertia. Hence it is difficult to determine on the film the exact time that a deflection starts. These sources of error are small.

The nature of the records necessitated the foregoing analysis. As has been stated, the velocities thus determined approached that of maximum. In the design of steep chutes and spillways, the average velocity in any section is required before freeboard can be determined.

If the time distance is taken from the centers of gravity of the areas under the two curves, the resulting velocity should be an average for the section[17]. As the velocities recorded in Table 2 were computed, as described above, it appears that they are greater than the average velocities. Assuming this to be correct, sixteen of the clearest oscillograms were selected and the time distances to the centers of gravity of the curves computed. The velocities from beginning points on the curves were also computed. The ratio of the velocities from the beginning points divided by the velocities from the centers of gravity for the sixteen sets of curves averaged 1.40. The ratio of velocities as recorded in Table 2, divided by the velocities from the centers of gravity averaged 1.24.

As the oscillograms were in many cases indistinct, and in some cases the curves were not completely recorded, it was impossible to reanalyze all of them to obtain the average velocity. Instead the velocities in Table 2 were reduced by dividing each velocity by 1.24, the average ratio of velocity as obtained by the first method to the velocity as determined by using the time distance between centers of gravity. The velocities as computed in this manner are given in Table 3.

Entrained Air

The percent of entrained air as computed from velocities as shown in Table 3 is given in Table 4. Station (1-3) refers to the reach between Electrode 1 and Electrode 3; Station (3-5) refers to the reach from Electrode 3 to Electrode 5, etc., Figure 2. The mean depths, hydraulic radii and velocities refer to the mean of all such measurements taken at the ends of and within each reach.

In that portion of the table applicable to the 10° 12' slope, if the two lower discharges are disregarded because of the traveling wave type of flow and particular observation made of the velocities prevailing for discharges 362 to 719 second-feet, inclusive, there appears to be an increase in velocity for each discharge as the flow progresses downstream. This is not apparent for the three higher discharges. This condition may be only a result of errors or velocity fluctuations but seems to be too consistent for that. It may be that for intermediate discharges, constant flow conditions had not been reached in the transition section but for the higher discharges such a condition

TABLE 3

ADJUSTED VELOCITY-KITTITAS WASTEWAY

SLOPE 10°-12'

Distance from Electrode No. 1	0	10'	100'	110'	200'	210'	300'	310'	400'	410'
Number of electrode	1	2	3	4	5	6	7	8	9	10
Discharge second-feet	Numbers in parentheses are the observed velocities in feet per second. Numbers to right and left of each observed velocity indicate the electrodes between which the velocity was measured.									
89	1	(28.6)	3	(27.4)	5					
			3	(27.4)	5					
193	1	(30.3)	3	(29.8)	5	(31.6)	9			
					5	(30.5)	7			
362	1	(37.2)	3	(40.5)	5	(42.4)	8	(49.2)	9	
	1	(42.1)	3	(44.2)	5	(38.3)	8	(49.8)	9	
401	1	(37.7)	3	(42.8)	5					
	1	(37.4)	3	(42.4)	5					
491	1	(42.1)	3	(39.2)	5	(43.8)	8			
			3	(45.1)	5	(45.5)	8			
			3	(41.4)	5	(44.2)	9			
587	1	(44.5)	3	(44.1)	5	(48.4)	7	(50.6)	9	
			3	(47.0)	5	(45.2)	7	(53.1)	9	
					5	(46.2)	7	(51.6)	9	
719	1	(46.6)	3	(46.5)	5	(42.3)	7	(50.0)	9	
	1	(44.5)	3	(45.3)	5	(50.0)	7	(54.3)	9	
					5	(50.5)	7	(52.3)	9	
777	1	(52.1)	3	(50.1)	5	(50.9)	7	(50.8)	9	
			3	(51.7)	5	-	7	(51.9)	9	
922	1	(58.6)	3	(60.1)	5	(53.3)	7	(54.8)	9	
	1	(49.2)	3	(52.3)	5	(48.6)	7	(55.4)	9	
			3	(49.0)	5	-	7	(52.2)	9	
1005	1	(58.2)	3	(52.1)	5	(58.3)	7	(54.0)	9	
	1	(59.8)	3	(57.5)	5	[illegible]	7	(51.2)	9	
					5	[illegible]	7	[illegible]	9	

SLOPE 33°-10'

Distance from Electrode No. 11		0	10'	100'	110'	200'	210'
Number of Electrode		11	12	13	14	15	16
Discharge second-feet	Numbers in parentheses are the observed velocities in feet per second. Numbers to right and left of each observed velocity indicate the electrodes between which the velocity was measured						
193			12	(44.5)	14	(53.3)	16
			12	(41.5)	14	(54.0)	15
			12	(56.1)	14		
362	11	(63.5)	14	(56.4)	15		
	11	(54.7)	14	(56.7)	15		
			12	(57.3)	14	(52.7)	16
401	11	(60.1)	13	(65.3)	16		
	11	(60.0)	13	(63.5)	16		
			13	(58.5)	16		
491			12	(59.0)	13	(67.5)	15
	11	(63.8)	13	(64.3)	15		
			12	(62.1)	14	(61.5)	16
587	11	(67.5)	13	(64.0)	15		
	11	(63.7)	13	(66.7)	15		
	11	(60.4)	13	(67.5)	16		
			13	(69.5)	15		
719	11	(77.2)	13	(75.2)	16		
	11	(71.1)	13	(65.1)	16		
			13	(66.3)	15		
777	11	(65.9)	13	(94.0)	16		
			13	(85.6)	15		
922			12	(68.7)	13	(75.2)	15
			12	(77.6)	13	(74.5)	15
			12	(70.5)	14	(68.6)	16
1005			12	(91.1)	15		
			12	(75.1)	15		
					13	(81.6)	15
					13	(82.6)	15
					14	(74.1)	16

SLOPE 0°-04'

Distance from Electrode No. 17	0	10'	40'	50'	
Number of electrode	17	18	19	20	
Discharge second-feet	Numbers in parentheses are the observed velocities in feet per second. Numbers to right and left of each observed velocity indicate the electrodes between which the velocity was measured.				
362		18		(34.0)	20
		18		(35.5)	20
		18		(34.0)	20
491		17		(59.5)	20
		17		(48.1)	20
		18		(45.4)	20
587		17		(33.7)	19
		17		(45.3)	20
		17		(60.4)	20
719		17		(67.6)	20
		17		(60.0)	20
		17		(57.5)	19
777		17		(57.5)	19
		17		(50.6)	19
922		17		(58.6)	20
		18		(56.0)	20
1005		17		(62.3)	20
		18		(55.4)	20

TABLE 4

COMPUTED ENTRAINED AIR-KITTITAS WASTEWAY

SLOPE 10°-12'

Discharge Q second-feet	Station	Mean depth D feet	Mean area A feet²	Mean hydraulic radius R feet	Mean observed velocity V ft./sec.	Q/AV	Air content percent
89	(1-3)	0.49	3.88	0.44	28.6	0.79	21
	(3-5)	0.50	3.96	0.45	27.4	0.82	18
193	(1-3)	0.80	6.35	0.68	30.5	0.99	1
	(3-5)	0.81	6.44	0.68	29.8	1.00	0
	(5-7)	0.83	6.55	0.70	30.5	0.97	3
	(5-9)	0.82	6.48	0.69	31.6	0.94	6
362	(1-3)	1.25	9.90	0.96	40.2	0.91	9
	(3-5)	1.25	9.95	0.96	42.4	0.86	14
	(5-8)	1.24	9.85	0.96	39.8	0.92	8
	(8-9)	1.24	9.84	0.96	49.5	0.74	26
401	(1-3)	1.26	10.06	0.98	37.6	1.00	0
	(3-5)	1.26	9.98	0.97	42.1	0.95	5
491	(1-3)	1.54	12.37	1.14	42.1	0.94	6
	(3-5)	1.55	12.30	1.13	42.0	0.96	4
	(5-8)	1.56	12.36	1.14	44.6	0.89	11
	(5-9)	1.56	12.43	1.15	46.2	0.86	14
587	(1-3)	1.80	14.32	1.26	44.5	0.92	8
	(3-5)	1.80	14.33	1.26	45.6	0.91	9
	(5-7)	1.79	14.28	1.26	47.3	0.87	13
	(7-9)	1.80	14.36	1.26	51.8	0.93	7
719	(1-3)	2.02	16.11	1.36	46.6	0.96	4
	(3-5)	2.03	16.12	1.37	46.0	0.97	3
	(5-7)	2.05	16.33	1.38	49.4	0.89	11
	(7-9)	2.07	16.49	1.39	53.8	0.81	19
777	(1-3)	2.24	17.87	1.45	52.1	0.83	17
	(3-5)	2.22	17.68	1.45	51.0	0.87	13
	(5-7)	2.21	17.58	1.44	50.9	0.87	13
	(7-9)	2.26	17.97	1.46	51.4	0.84	16
922	(1-3)	2.52	20.02	1.56	53.8	0.86	14
	(3-5)	2.49	19.79	1.55	53.8	0.87	13
	(5-7)	2.50	19.84	1.55	51.0	0.92	8
	(7-9)	2.51	19.96	1.56	54.1	0.86	14
1005	(1-3)	2.70	21.50	1.63	59.0	0.79	21
	(3-5)	2.65	21.11	1.62	54.4	0.88	12
	(5-7)	2.66	21.15	1.62	55.8	0.86	14
	(7-9)	2.66	21.19	1.62	53.1	0.84	16

SLOPE 33°-10'

Discharge Q second-feet	Station	Mean depth D feet	Mean area A feet²	Mean hydraulic radius R feet	Mean observed velocity V ft./sec.	Q/AV	Air content percent
193	(12-14)	1.02	8.07	0.83	44.0	0.55	45
	(14-16)	1.14	9.04	0.90	53.4	0.40	60
	(14-15)	1.14	9.04	0.90	54.0	0.40	60
362	(11-14)	1.50	11.83	1.11	59.1	0.52	48
	(12-14)	1.50	11.83	1.11	57.2	0.53	47
	(14-15)	1.63	12.88	1.17	56.5	0.50	50
	(14-16)	1.63	12.88	1.17	52.7	0.53	47
401	(11-13)	1.49	11.75	1.11	60.0	0.57	43
	(13-16)	1.53	12.08	1.12	62.4	0.52	48
491	(12-13)	1.94	15.32	1.32	59.0	0.55	45
	(11-13)	1.94	15.32	1.32	63.8	0.51	49
	(12-14)	1.94	15.32	1.32	62.1	0.55	45
	(13-15)	1.90	15.02	1.30	65.8	0.50	50
	(14-16)	1.90	15.02	1.30	61.5	0.53	47
587	(11-13)	2.05*	16.23*	1.37	64.0	0.57	43
	(13-15)	2.05	16.16	1.37	66.7	0.55	45
	(13-16)	2.05	16.16	1.37	67.5	0.53	47
719	(11-13)	2.18*	17.26*	1.43	74.1	0.56	44
	(13-15)	2.20	17.42	1.43	66.3	0.62	38
	(13-16)	2.20	17.42	1.43	70.2	0.58	42
777	(11-13)	2.35*	18.61*	1.50	65.9	0.63	37
	(13-16)	2.40	19.00	1.51	89.9	0.46	54
922	(12-13)	2.63*	20.83*	1.60	73.1	0.61	39
	(12-14)	2.63*	20.83*	1.60	70.5	0.63	37
	(13-15)	2.68	21.28	1.61	75.0	0.58	42
	(14-16)	2.68	21.28	1.61	68.6	0.63	37
1005	(12-15)	-	-	-	83.1	-	-
	(13-15)	-	-	-	82.2	-	-
	(14-16)	2.94*	23.20*	1.71	74.1	.58	42

*Point gage readings made only at the end of the section on account of spray.

SLOPE 0°-04'

Discharge Q second-feet	Station	Mean depth D feet	Mean area A feet²	Mean hydraulic radius R feet	Mean observed velocity V ft./sec.	Q/AV	Air content percent
362	(18-20)	1.52	12.03		34.5	0.87	13
491	(17-20)	1.82	14.41		53.8	0.63	37
	(18-20)	1.82	14.41		45.4	0.74	26
587	(17-19)	2.14	16.95		33.7	1.00	0
	(17-20)	2.14	16.95		48.9	0.71	29
719	(17-19)	2.36	18.69		57.5	0.67	33
	(17-20)	2.36	18.69		63.8	0.61	39
777	(17-19)	2.61	20.76		54.1	0.70	30
922	(17-20)	2.91	22.93		50.6	0.68	32
	(18-20)	2.91	22.93		56.0	0.71	29
1005	(17-20)	2.90	22.98		62.4	0.71	[illegible]
	(18-20)	2.90	22.98		55.3	[illegible]	[illegible]

did prevail. The water-surface measurements show an almost constant depth for each discharge throughout this section of the wasteway.

Another reason that the water-surface measurements do not agree with the trends shown by the velocity measurements is because the point gage readings covered a considerable period of time and represent the average water surface over this period while the salt cloud employed to measure velocity represents a localized section of the flow and may attain a velocity associated with a single velocity fluctuation.

For the high discharges on the 10° 12' slope and for all discharges on the 33° 10' slope, there is no consistent increase of velocity for a given discharge and slope as the flow progresses down the wasteway. Hence it is logical to assume that uniform flow conditions prevail. This is not true for the flat slope at the lower end of the wasteway. Here the flow was decelerating and is consequently nonuniform.

V SUMMARY OF ALL TESTS ON CHUTES

Existing Data

The tests on the Kittitas Wasteway were not of sufficient extent to determine the relationship of all variables involved in high-velocity flow in open channels. In order to obtain a comparison of the results with those of earlier experimenters, a summary of all data on air content in chutes was compiled and is given in Table 5[18,20,21,22]. This table contains information on only those chutes for which the data is complete. Scattered bits of data have not been included.

The volume of water entering the structure, the velocity of flow, and the area of the cross-section must be measured in order to determine the volume of air in the flow. Data other than that given in Table 5

TABLE 5

COMPILATION OF EARLIER CHUTE MEASUREMENTS

	Discharge Q second-feet	Depth d feet	Hydraulic radius feet	Measured velocity V_1 ft./sec.	AV percent water
1. Ehrenberger's Model	.353	0.057	0.050	9.61	79.0
Chute I - wood	.706	0.090	0.074	11.94	79.0
width = 0.82 foot	1.06	0.123	0.094	13.2	80.7
sin θ = 0.153	1.57	[illegible]	[illegible]	[illegible]	[illegible]
2. Ehrenberger's Model	[illegible]	0.054	0.048	[illegible]	[illegible]
Chute II - wood	.706	0.087	0.072	14.06	75.0
width = 0.82 foot	1.06	0.120	0.093	[illegible]	[illegible]
sin θ = 0.242	1.57	.150	0.113	16.64	74.8
3. Ehrenberger's Model	0.353	0.041	0.047	[illegible]	74.9
Chute III - wood	0.706	0.082	0.068	14.47	71.5
width = 0.82 foot	1.06	0.115	0.090	16.31	70.2
Sin θ = 0.305	1.57	0.151	0.111	17.55	70.7
4. Ehrenberger's Model	0.353	0.068	0.043	13.22	67.5
Chute IV - wood	0.706	0.079	0.066	16.86	64.4
width = 0.82 foot	1.09	0.112	0.088	17.73	63.1
Sin θ = [illegible]	1.57	0.128	0.109	20.28	63.5
5. Ehrenberger's Model	0.353	0.052	0.047	17.78	51.3
Chute V - wood	0.706	0.085	0.071	19.46	52.2
width = 0.82 foot	1.06	0.118	0.092	21.85	51.0
Sin θ = 0.600	1.57	0.146	0.113	23.94	51.3
6, 7, & 8. Rusts wasteway	27.7	0.10	0.097	23.5	64
Trapezoidal - wood	28.3	0.16	0.157	32.2	44
bottom width = 8.2 feet	120.6	0.49	0.446	56.1	39
side slopes = 1.5 on 1					
Sin θ = 0.600	27.9		0.210	35.5	41
(Adjusted values of d,	36.0		0.272	40.8	38
and V as given by	52.3		0.361	47.2	37
Ehrenberger)	129.2		0.512	57.0	40
	32.8	0.26	0.249	39.5	37
	56.6	0.38	0.348	48.2	37
	62.[illegible]	0.39	0.364	47.9	37
	70.3	0.45	0.407	51.2	38
	151.9	0.74	0.640	62.3	43
9. Rusts wasteway - wood	36	0.31	0.29	64.6	20
Data from Schoklitsch:	52	0.36	0.33	65.9	25
	55	0.46	0.42	53.5	27
Stauverlandung und	68	0.65	0.61	70.2	39
Kolkabwehr	132	0.84	0.72	70.2	25
	139	0.94	0.77	70.2	27
	191	--	--	70.2	64
10. Lago					
Sin θ = 0.246	49.1	0.64	0.46	33.1	64
b ... = 3.28 feet	26.1	0.43	0.34	28.2	60
11. Benkos					
Sin θ = 0.602	211.9	2.20	0.94	77.1	38
b ... = 3.28 feet	162.4	1.21	0.70	67.2	38
12. Kalinitz - Trapezoidal:					
Sin θ ..... = 0.468	166.0	1.23	0.92	68.6	—
bottom width = 6.56 feet	134.2	1.15	0.88	67.0	—
13. Fargo Drop - wood					
Sin θ = 0.124	95.0	0.57	0.48	32.3	86
b ... = 6.0 feet					
14. Mora wasteway - Concrete					
Sin θ = 0.081	27.5	0.32	0.28	22.0	78
b ... = 5.0 feet					
15. Valley Round Chute - Concrete					
Sin θ = 0.156	22.4	0.27	0.22	26.3	63
b ... = 5 feet					
16. Arena Chute - Concrete					
Section 1: Sin θ = 0.202	23.2[illegible]	[illegible]	[illegible]	20.4	79
b ... = 6 ft.					
17. Arena Chute - Concrete					
Section 2: Sin θ = 0.151	23.26	0.27	0.21	19.6	90
b ... = 6 ft.					
18. Arena Chute - Concrete					
Sin θ = 0.200	5.40	0.32	0.29	[illegible]	89
b ... = 6 feet					
19. Lizard Chute No. 1 -	0.64	0.06	0.06	4.57	73
Concrete	2.04	0.11	0.10	5.33	56
Sin θ = 0.182	2.80	0.12	0.11	10.9	71
b ... = 3 feet	5.72	0.17	0.16	14.3	72
	7.97	[illegible]	[illegible]	[illegible]	79
	10.22	0.36	0.29	17.2	75
20. Lizard Chute No. 2 -	[illegible]	0.04	0.05	6.9	42
Concrete	2.04	0.10	0.09	7.8	87
Sin θ = 0.194	2.80	0.10	0.10	11.1	85
b ... = 3 feet	5.72	0.16	0.14	19.8	60
	7.97	0.21	0.18	15.0	86
	10.22	0.32	0.26	23.8	78
21. Kittitas Wasteway - Concrete					
Sin θ = 0.547	231	0.80	0.67	52.2	69
b ... = 8 feet	682	1.3	0.98	66.0	77.5
(Measurements made downstream from chute)					
22. Hammerhill Flume - Metal					
Canadian Pacific Ry.					
Alberta, Canada	61.0	—	0.573	47.5	65
Sin θ = 0.057	26.5	—	0.433	18.6	88
b ... = 5.10 feet					
23. Leilroy Flume - Metal					
Canadian Pacific Ry.					
Alberta, Canada	59.2	—	0.993	15.2	94
Sin θ = 0.032	151.4	—	1.311	17.6	76
b ... = 10.2 feet					
24. Lateral C-11 Flume - wood					
Canadian Pacific Ry.					
Alberta, Canada					
Sin θ = 0.052	1.24	—	0.107	9.82	99
b ... = 0.9 foot					
25. Secondary Canal - wood					
Canadian Pacific Ry.					
Alberta, Canada	6.26	—	0.155	9.25	95
Sin θ = 0.025					
b ... = 4.18 feet					
26. South Canal Chute					
Milepost 2 - Concrete	382	1.15	0.98	40.24	87
Uncompahgre Project	665	1.30	1.07	30.17	94
Sin θ = 0.275					
Trapezoidal					
Bottom width = 8.84 ft.					
27. South Canal Chute					
Milepost 2 - Concrete					
Uncompahgre Project	267	1.06	0.87	28.13	103
Section 2	400	1.39	1.09	31.93	102
Sin θ = 0.070	463	1.53	1.17	32.58	104
Trapezoidal					
Bottom width = 8.47 ft.					

have been found, but in most instances, only two of the above values were actually measured and the third calculated from the formula $Q = AV$.

The discharge was usually calculated from measured cross-section and measured velocity. This, of course, gives a quantity that includes the air and hence is in excess of the true water volume. For this reason, numerous experimenters have found a value for "n" for Manning's formula that is very close to the expected value, whereas if the actual water discharge were used, some other value might have been found.

It is possible that some of the observations were made in regions where stable flow conditions had not been attained. The data under No. 26 undoubtedly belong in this class. Observations by the author made at this location show that white flow prevailed only near the sides of the structure while the center portion of the flow was darker colored. The data under No. 27 were taken on a slope preceded by a steeper slope (No. 26). A comparison of the measured velocities indicates that the flow was decelerating at this section. Other data may have been influenced by similar conditions but details are not available.

Velocity of Flow in Chutes

The experiments of Ehrenberger[18] yielded the most complete data on chutes. He attempted to establish a relationship between the hydraulic properties of the channel with models and found that

$$V = 97R^{0.52}(\sin \theta)^{0.4} \qquad (2)$$

where V is velocity in feet per second, R is the hydraulic radius, and θ is the angle that the bottom of the chute makes with the horizontal.

This relationship was established by model tests made in a wooden chute 0.82 foot wide. The depth varied from 0.048 to 0.161 foot and the hydraulic radius from 0.043 to 0.116 foot.

Experiments reported by Ehrenberger[19] on the Ruetz Wasteway in Austria show that the velocity varies with $R^{0.53}$. The bottom width of this wasteway was 8.2 feet. The depth for the flows varied from 0.10 to 0.74 foot and the hydraulic radius from 0.097 to 0.640 foot. This wasteway had a wooden lining. Data from this same wasteway reported by Schoklitsch[20] give depths from 0.31 to 0.94 foot and hydraulic radii from 0.29 to 0.79 foot. They showed no consistent relationship between V and R. The experiments made by Steward[21] in 1913 do not show a consistent relationship to exist between velocity and hydraulic radius.

The experiments on the Kittitas Wasteway covered depths from 0.5 to 2.9 feet and hydraulic radii from 0.5 to 1.7 feet. The data on velocity and hydraulic radius for these tests have been plotted on logarithmic paper and a straight line drawn through the points pertaining to each slope, Figure 9. The equations of these lines are:

$$V = 40.4R^{2/3} \qquad \theta = 10° \ 12' \qquad (3)$$

$$V = 54.1R^{2/3} \qquad \theta = 33° \ 10' \qquad (4)$$

If Ehrenberger's formula, the data from the Ruetz Wasteway, and the data from Kittitas Wasteway are considered, it appears that the exponent of R increases as R is increased. There is a meager amount of data upon which to base such a conclusion.

Since the data from Kittitas was obtained from only two slopes, a general equation, such as Ehrenberger developed, was not attempted.

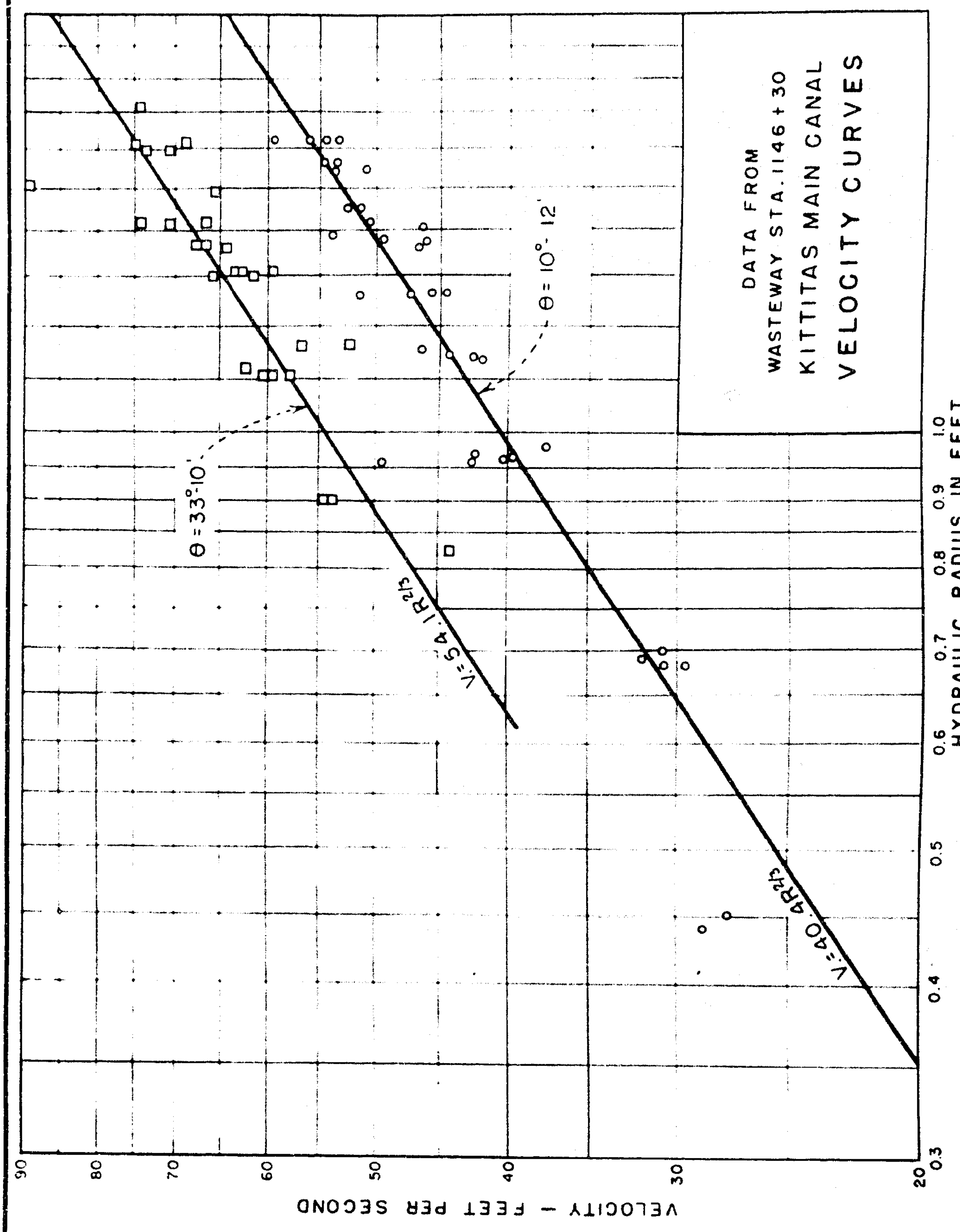

DATA FROM
WASTEWAY STA. 1146 + 30
KITTITAS MAIN CANAL
VELOCITY CURVES
θ = 10°- 12'
θ = 33°-10'
V = 54.1 R^2/3
V = 40.4 R^2/3
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
HYDRAULIC RADIUS IN FEET
90
80
70
60
50
40
30
20
VELOCITY — FEET PER SECOND

FIGURE 9.

In equations (3) and (4) the exponent of R agrees with the exponent of R in the Manning formula. This suggests that "n" may be computed for uniform flow on steep slopes by expressing the loss of head per unit length of flow as sin θ, where θ is the angle of inclination of the bottom of the channel with respect to the horizontal.

Thus the Manning formula for steep slopes becomes

$$V = \frac{1.486R^{2/3}}{n}(\sin \theta)^{1/2} \qquad (5)$$

Computing "n" by use of equations (3), (4), and (5) we find:

n = 0.0155 (θ = 10° 12')

n = 0.0203 (θ = 33° 10')

This would indicate that the value of "n" varies directly with the slope. This is contrary to the definition of "n". The value of "n" for the 10° 12' slope is slightly high for the type material in the lining, but on the steeper slope the value of "n" is entirely too high.

<u>Air Content of Flow in Chutes</u>

Ehrenberger[23] also developed formulas for determining the water portion, μ, in a unit volume of high-velocity flow. His equations are

$$\mu = 0.42R^{-0.05}(\sin \theta)^{-0.26} \quad \text{for } \sin \theta < 0.476 \qquad (6)$$

and

$$\mu = 0.30R^{-0.05}(\sin \theta)^{-0.74} \quad \text{for } \sin \theta > 0.476 \qquad (7)$$

These formulas are based on tests made on five different slopes ranging from θ = 8° 49' to θ = 37° 18' and hydraulic radii ranging from 0.043 to 0.116 foot. From the formulas it appears that with equal wall roughness, the slope is the most important factor, and the hydraulic radius, and hence the depth, are only minor factors.

A comparison of the velocities and air content observed in the Kittitas Wasteway and corresponding values calculated by Ehrenberger's formulas are given in Table 6. The observed velocities are consistently lower than the calculated velocities but the air content of the flow at Kittitas shows very poor agreement with the calculated values.

The mean air content for the two slopes observed at Kittitas and the data from Table 5 were plotted on logarithmic paper, Figure 10. Two parallel lines have been drawn through the plotted points. The equations of these lines are:

$$\alpha = 0.81 \sin \theta^{0.6} \qquad (8)$$

$$\alpha = 0.62 \sin \theta^{0.6} \qquad (9)$$

where α is the volume of entrained air in a unit volume of aerated water. The numbers appearing in the figure correspond to the numbers given the data in Table 5. Two lines were drawn because the points representing data from the model chutes did not appear to be closely associated with those from prototype structures. The upper line may be said to represent values from prototype tests, except for the flatter slopes where the points are badly scattered.

Discussion of Chute Data

The difficulties encountered and the methods utilized in securing measurements in high-velocity water cause a wide variation in results. The data on velocities show closer agreement than those on air content of the flow.

The velocity data, in general, are more consistent and a method of application of these data will be shown in a later section of this thesis.

The data on air content, with the exception of Ehrenberger's, vary widely. Attempts have been made to obtain an exponential formula relating air contents, average velocity, hydraulic radius, and the angle of the bottom of the chute with the horizontal, but no relationship

Table 6

Comparison of Observed Velocity

and Air Content with Those

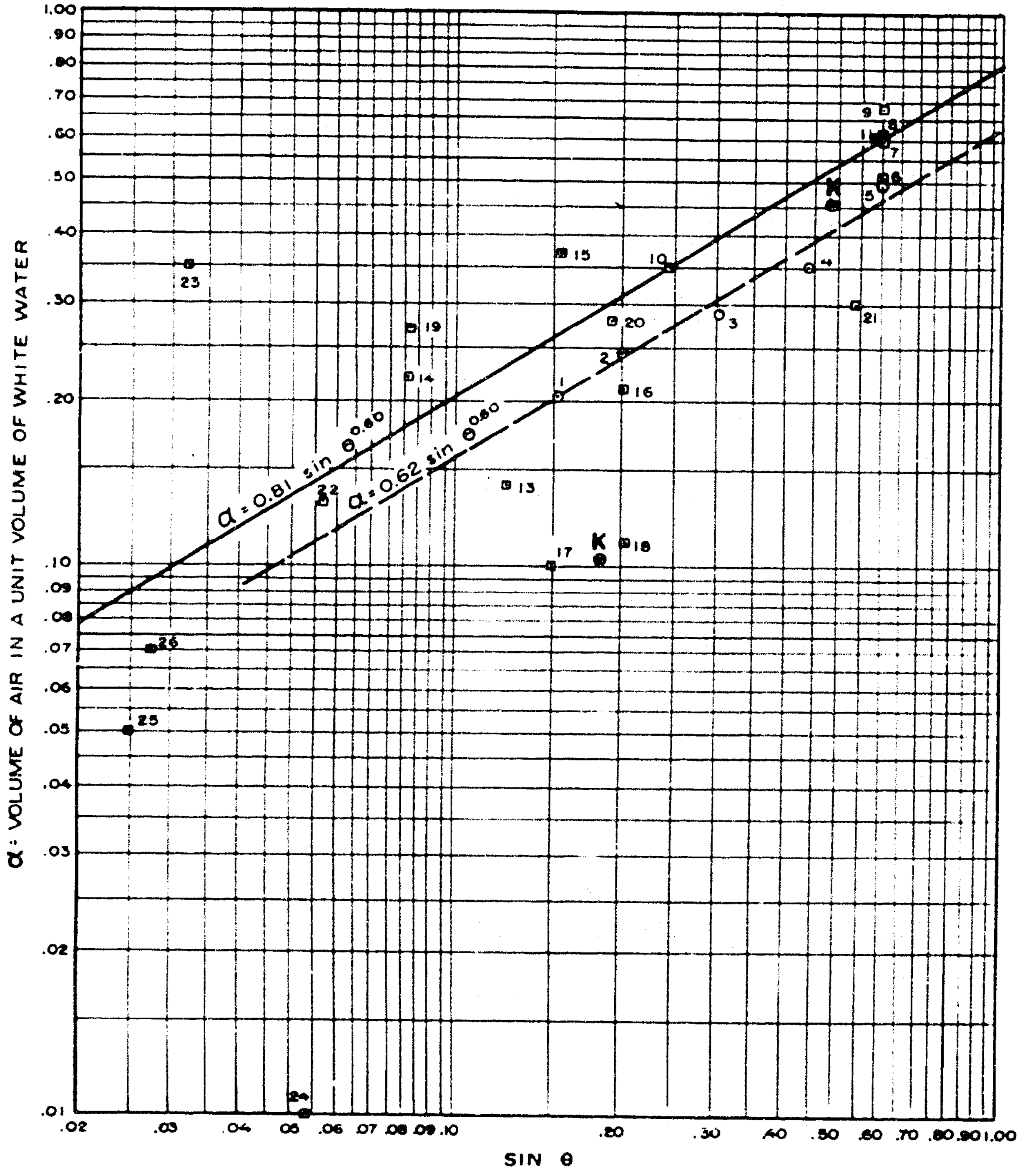
Calculated by Ehrenberger's Formulas

SLOPE 10° - 12'
$\sin \theta = 0.177$

Discharge Second-feet	Observed Mean Hydraulic Radius ft.	Observed Mean Velocity ft./sec.	Calculated Velocity ft./sec.	Observed Mean Air content Percent	Calculated Air Content Percent
89	0.45	28.0	32.0	19.5	31.4
193	0.69	30.6	40.0	2.5	32.9
362	0.96	43.0	47.5	14.3	34.0
401	0.97	39.9	47.8	2.5	34.1
491	1.14	43.7	52.0	8.8	34.5
587	1.26	47.3	54.7	9.3	34.9
719	1.38	49.0	57.4	9.3	35.2
777	1.45	51.4	[illegible]	[illegible]	[illegible]
922	1.55	53.2	61.0	12.3	35.5
1005	1.62	55.6	62.4	15.8	35.7

SLOPE 33° - 10'
$\sin \theta = 0.547$

Discharge Second-feet	Observed Mean Hydraulic Radius ft.	Observed Velocity ft./sec. (Mean)	Calculated Velocity ft./sec.	Observed Mean Air Content Percent	Calculated Air Content Percent
193	0.90	50.5	72.2	55.0	52
362	1.17	56.4	82.6	48.0	53
401	1.12	61.2	80.8	45.5	53
491	1.30	62.4	87.3	47.2	54
587	1.37	66.1	89.8	45.0	54
719	1.43	70.2	91.7	41.3	54
777	1.51	77.9	94.4	45.5	54
922	1.61	71.8	97.6	38.8	54
1005	1.71	79.8	100.7	42.0	54



θ = Angle of inclination of chute measured from horizontal.

RELATION OF AIR CONTENT IN FLOW TO SLOPE OF CHUTE

FIGURE 10

has been found which can be confirmed within satisfactory limits by the experimental data.

Air-content data from the Kittitas Wasteway and those data available from other structures do not follow the relationship established by Ehrenberger, Figure 10. An attempt to relate the air content to the Froude number did not yield satisfactory results.

Mechanics of Air-water Mixing

The mechanics by which the air enters the high-velocity flow usually enters into the discussion of data obtained from chutes. A number of theories have been advanced concerning this phenomenon. Conversation with engineers interested in the subject and perusal of written articles, attribute the mixing to:

(1) Entrance conditions
(2) Breaking waves on the surface
(3) Roughness of the channel
(4) Curves in the channel
(5) Turbulence in high-velocity flow

Certain of these factors have been discussed in a previous section of this thesis. The tests on Kittitas Wasteway verify the hypothesis that entrance conditions and channel curvature contribute to air-water mixing. Turbulence in the flow assisted in distributing the air throughout the depth. This distribution of air did not appear to be uniform. The air content appeared much greater in the upper portions of the flow.

Observations made by the author in conjunction with a series of vibration tests at Black Canyon Dam, Idaho[24] add a limited amount of information on the mechanics of air-water mixing.

A plan and typical sections of the dam are shown on Figure 11. Flow conditions on the face of the dam were observed for different drum-gate elevations, different combinations of gate openings, and different reservoir elevations during the time the vibration tests were in progress. Air entrainment in the flow and aeration of the jet when the gates were partially raised were noted in particular and a series of pictures was made of existing conditions.

When the drum gates were completely lowered, the edges and upper surface of the jet contained considerable air. Just how far the air penetrated the jet was not determined. When the drum gates were slightly raised the conditions were greatly changed. Air entering from the underside was carried through the jet and emerged from the upper surface causing a boiling appearance. A similar condition was noted near the edge of the jet when the gates were completely lowered. The air in this case was evidently carried into the flow from turbulence caused by the piers. A comparison of surface conditions may be seen in Figure 12.

Another condition noted at Black Canyon and elsewhere is the irregular line formed where the water surface first becomes roughened and begins to entrain air. The top of the jet presents a glassy appearance for a short distance below the crest and then small waves or ripples begin to appear. The magnitude of the ripples increases as the jet continues downward. This line may be seen in Figure 12. This surface condition is undoubtedly caused by friction along the air-water boundary. The line was observed to move up the face of the dam when a wind was blowing upstream thus indicating that the condition is a function of the relative water and air velocities. The degree of

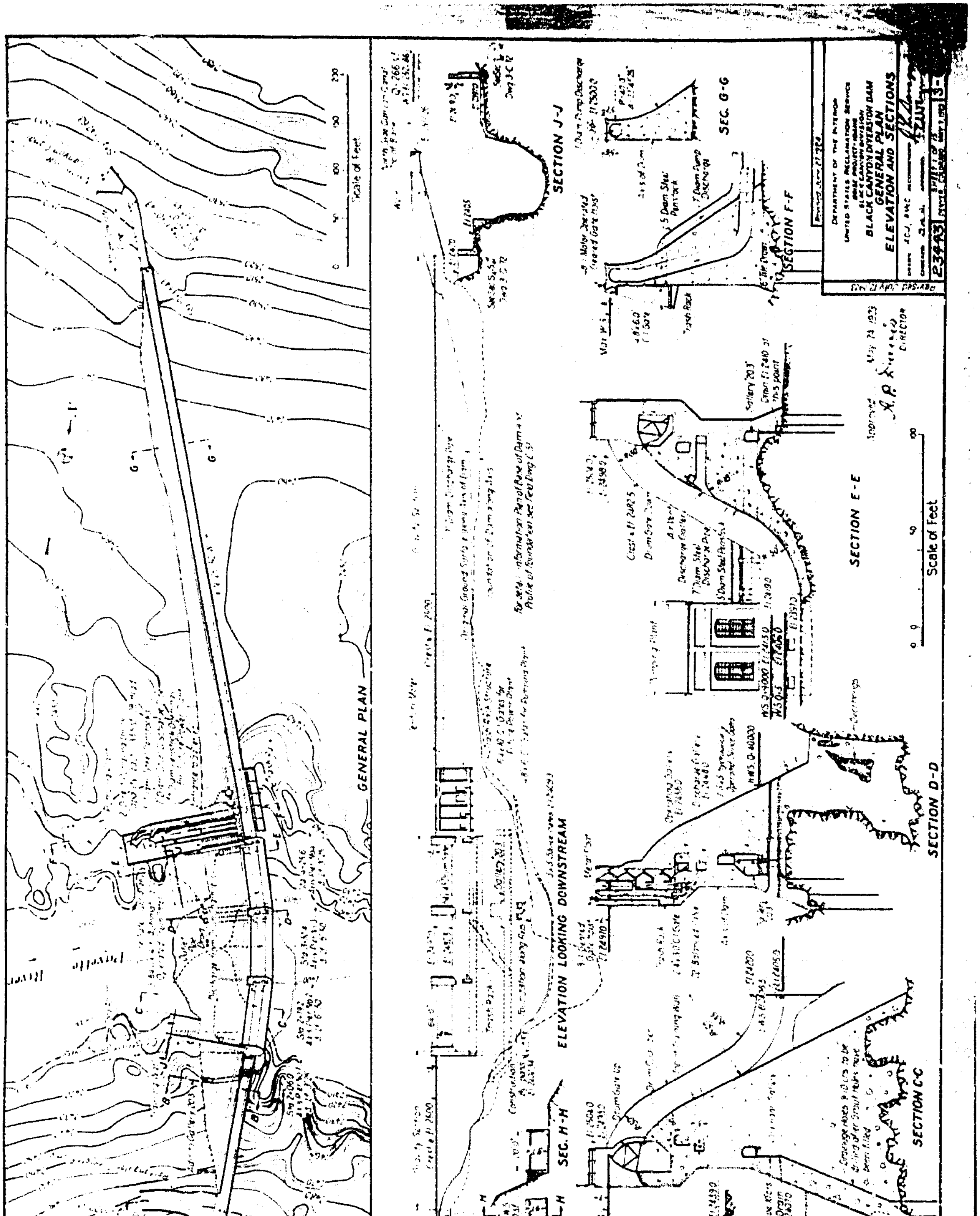
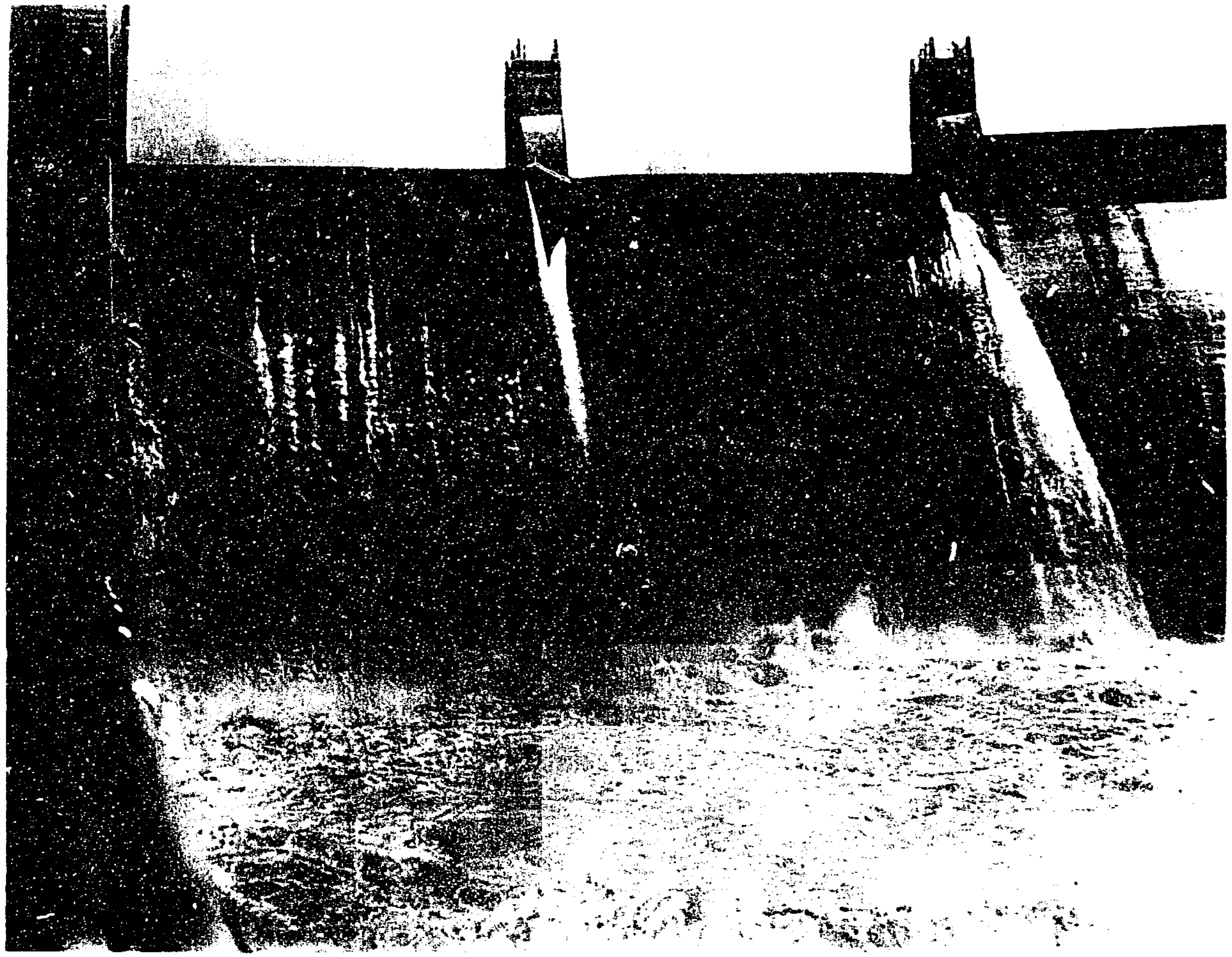

GENERAL PLAN
ELEVATION LOOKING DOWNSTREAM
SECTION J-J
SEC. G-G
SECTION F-F
SECTION E-E
SECTION D-D
SECTION C-C
SEC. H-H
SEC. A-A
SEC. B-B
Scale of Feet
DEPARTMENT OF THE INTERIOR
UNITED STATES RECLAMATION SERVICE
BLACK CANYON DIVERSION DAM
GENERAL PLAN
ELEVATION AND SECTIONS

FIGURE 11.



SPILLWAY CONTROLLED BY THREE 64 BY 14.5-FOOT DRUMGATES
CENTER GATE COMPLETELY LOWERED - HEAD 6.5 FEET
GATE NO. 3 RAISED 2 FEET
DISCHARGE TWO GATES, APPROXIMATELY 6,000 SECOND FEET
TOTAL DROP, **HEADWATER TO TAILWATER 81 FEET**

BLACK CANYON DAM

FIGURE 12

aeration was also influenced by the thickness of the jet and the position of the drum gate.

The design of the piers permitted aeration under the jets when the drum gates were in a raised position. The velocity of the air that entered under the jets was quite high at times as indicated by observing bits of paper thrown from the tops of the piers. Evidently, a large quantity of air is absorbed by the underside of the nappe. Turbulence on this lower surface was greatly increased by the projecting rivet heads on the lip of the drum gate and air entrainment began immediately. This air was carried through the jet by internal turbulence and produced the boiling effect at the upper surface. The jet leaving the drum gate is shown on Figure 13.

Motion pictures taken on 16 millimeter film at 128 frames per second show the conditions described above much more plainly than still photographs.

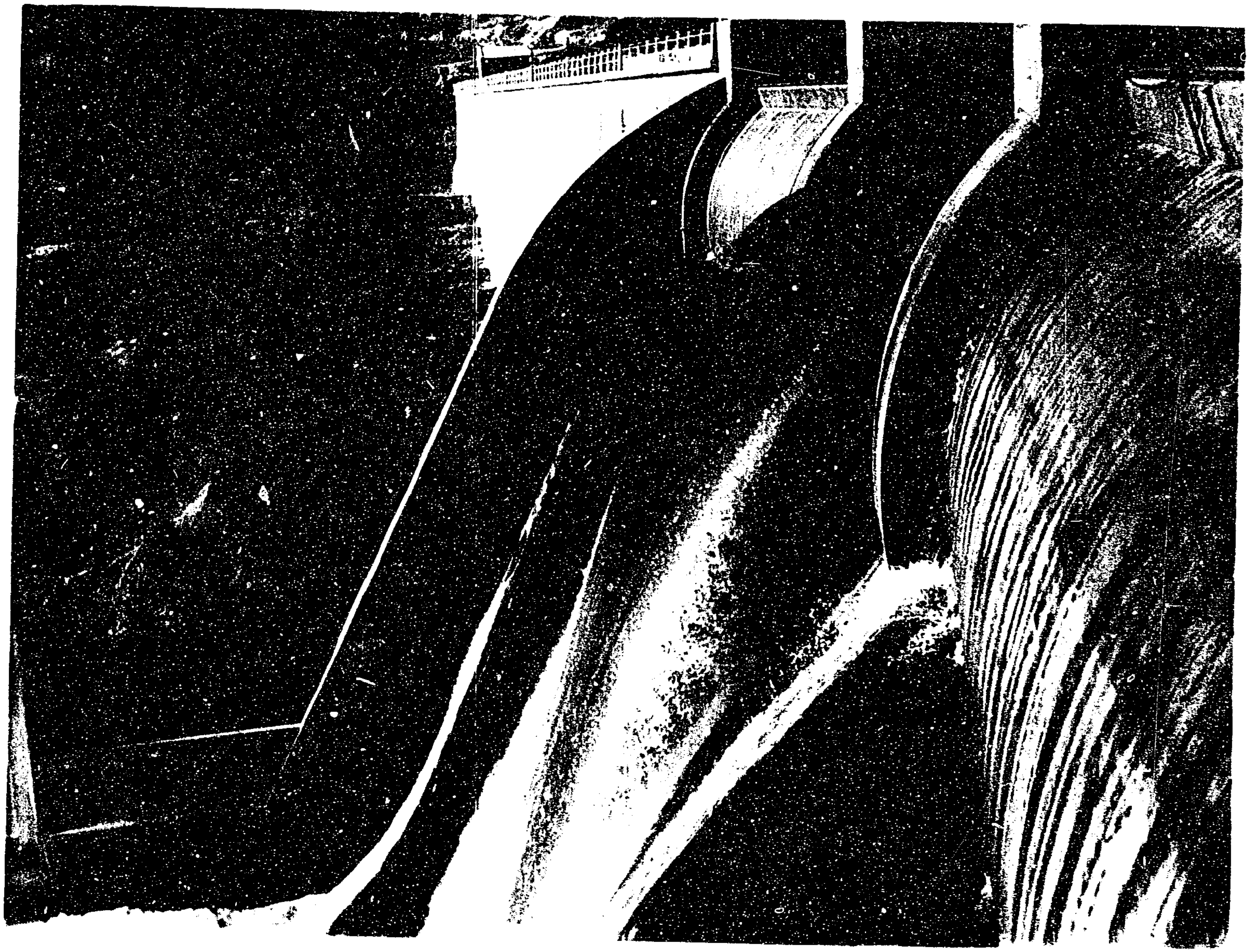
VI APPLICATION OF DATA

Applicability of Existing Open-channel Formulas to Aerated Flow

The Manning and Chezy formulas were intended for use on rivers and canals with small gradients, and comparatively small velocities. It is highly improbable that either formula is directly applicable to the solution of problems involving high-velocity flow.

A comparison of the data from Kittitas with that obtained by Ehrenberger has been made in a previous section of this thesis.

The desired objective of the tests on the Kittitas Wasteway was to justify the use of existing formulas in designing open channels or to determine empirical values for a new formula. This developed formula should give results that would be in reasonable conformity with the



BLACK CANYON DAM-BOISE PROJECT-IDAHO

GATE RAISED 4.5 FEET - HEAD ON GATE 4 FEET

FIGURE 13

measurements conducted on chutes. The data derived from the measurements at Kittitas did strengthen the belief that the Chezy and Manning formulas as written are not applicable to problems where steep chutes are involved. Insufficient data negated the possibility of extending Ehrenberger's formula over a wide range of conditions. Hence a new method of design was sought.

In the development of a design method certain assumptions are normally made. To conform as nearly as practicable to established methods of hydraulic calculations and to logically follow the observations that had been made during the tests, the following assumptions were made:

(1) Air in and above the water causes an additional resistance to flow, which is directly proportional to the square of the average velocity, and inversely proportional to some power of the hydraulic radius.

(2) The hydraulic radius may be based on a depth equal to the discharge per foot of width divided by the average velocity in the section, or the hydraulic radius, assuming no air in the water.

(3) The value of "n" in the Manning formula is consistent for the particular type of material of which the chute or spillway is constructed.

With these basic assumptions it appeared possible to formulate a method of design that would include an air resistance term.

The first assumption above is based on the following reasoning. Although little is known of the mechanics of air resistance, it has been frequently observed that the air over channels carrying high-velocity water attains velocities of considerable magnitude. The force necessary to keep this air in motion, which is an added resistance to flow, comes from the water. This force on the air would be practically independent of the depth of water but the retarding force on a pound of water would be inversely proportional to the depth. As air resistance is usually proportional to the square of the velocity, for high velocities, it is reasonable to assume this to be the case until further data can be obtained.

The second assumption may readily be justified by an examination of the Chezy formula.

$$V = C \sqrt{RS} \qquad \text{...(10)}$$

In this equation the velocity varies directly as the square root of the hydraulic radius. If air enters the flow, thus causing a greater hydraulic radius, a higher velocity would result. It has been shown[25] that the mean velocity in an open channel is independent of, or inversely proportional to, the kinematic viscosity. The kinematic viscosity of air, at 60° F, is about thirteen times that of water; hence, there is little reason to expect that velocities would be increased by addition of air. One would expect, rather a reduced velocity, and, therefore, a higher value of "n".

The following derivation of the Chezy formula shows that it is incorrect in high-velocity flow to use the hydraulic radius of the "insufflated" section, without changing the value of the coefficient C, but the net hydraulic radius may be used.

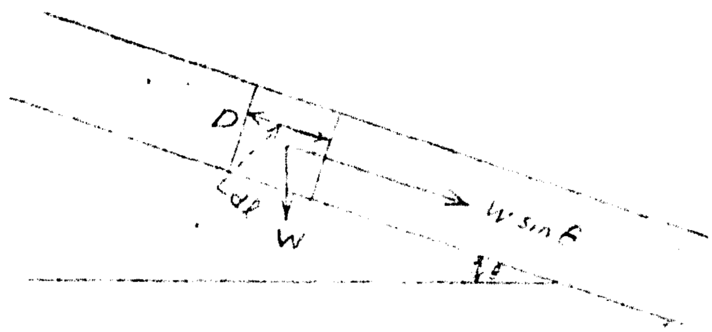


Figure 14

Let 100 μ = percent of air in the air-water mixture by volume

b = width of channel (rectangular)

w = unit weight of water

τ = shear per unit area at surface of contact between water and channel

Writing the equilibrium conditions for volume Dbdl

$$Dbdl(1-\mu)w \sin \theta = \tau(2D+b)dl$$

$$\text{or } \tau = \frac{Db}{2D+b}(1-\mu)w \sin \theta$$

$$\text{Let } \tau = kV^2$$

Assuming k to be the same in water and in air-water mixtures

$$R = \frac{Db}{2D+b}$$

$$\text{and } S = \sin \theta$$

$$\text{then } kV^2 = R(1-\mu)wS$$

$$V = (1-\mu)^{1/2}(\tfrac{w}{K})^{1/2}\sqrt{RS}$$

$$(\tfrac{w}{K})^{1/2} = \text{Chezy C}$$

$$\text{and } V = (1-\mu)^{1/2}C\sqrt{RS} \quad \text{.................................(11)}$$

In other words, the coefficient C in the Chezy formula must be multiplied by $(1-\mu)^{1/2}$ when applied to water and air mixtures. Defining R in a new way

$$R_{net} = \frac{bD(1-\mu)}{2D+b} = \frac{Q}{V(2D+b)} \quad \text{.......................... (12)}$$

in which Q = total discharge of the channel

and V = average velocity in a section

the Chezy formula remains

$$V = C\sqrt{R_{net}S} \quad \text{....................................... (13)}$$

In a similar manner it may be shown that the Manning formula should use the R_{net} as defined above when water contains air.

The third assumption is based on the definition of "n" as given by a number of authors.

<u>Analytical Form of Air-resistance Correction</u>

The coefficients in both the Chezy and Manning formulas were obtained empirically for low velocities. Under these conditions there is undoubtedly very little resistance to flow from the water-air surface. For high-velocity flow, a great deal of air is set in motion by the water with additional losses which would not be included in the Chezy or Manning formulas. In order to use either of these formulas it is necessary to make a correction for the additional losses.

The Manning formula gives the following values of C in the Chezy formula:

$$C = \frac{1.486 r^{1/6}}{n} \quad \text{....................................... (14)}$$

The Manning formula as usually written being

$$V = \frac{1.486 r^{2/3} S^{1/2}}{n} \quad \text{.................................. (15)}$$

where n is Kutter's roughness coefficient. This equation may also be written as

$$S = \frac{n^2 V^2}{2.2082 R^{4/3}} \quad \text{....................................... (16)}$$

Following the assumptions stated above, a term to account for the air resistance may be added. The formula then becomes

$$S = \frac{n^2V^2}{2.2082R_{net}^{4/3}} + \frac{KV^2}{R_{net}^p} \quad \ldots\ldots (17)$$

where K and p are empirical values to be determined experimentally.

When air is present in the flow the hydraulic radius, R_{net}, will be slightly greater than the hydraulic radius, $R_{\mu=0}$, with no air present in the flow. Since the values of "K" and "p" are to be obtained experimentally, those values may be adjusted to compensate for the change in hydraulic radius.

R_{net} cannot be determined until the amount of air is known. However, we may write

$$R_{\mu=0} = \frac{Q/V}{b+2Q/Vb} \quad \ldots\ldots (18)$$

which is the hydraulic radius when $\mu=0$, or when there is no air in the flow.

The formula for steady flow then becomes

$$S = \frac{n^2V^2}{2.2082R_{\mu=0}^{4/3}} + \frac{KV^2}{R_{\mu=0}^p} \quad \ldots\ldots (19)$$

S = sine of angle between energy gradient and horizontal

b = width of a rectangular channel

V = average velocity in the section

Q = total discharge

K and p = constants determined from experimental data

For steady flow conditions, the available energy is represented by S. This energy is divided into two parts; the first part of the equation represents the loss due to internal friction and friction between water and solid surface. The second part, which represents the loss due to air resistance, is directly proportional to the square of the velocity and inversely proportional to some power of the depth or hydraulic radius.

The equation as stated is not directly applicable to the design of channels with steep gradients.

Dr. V. L. Streeter[26] makes the following application of the equation by combining it with Bernoulli's equation. The values of "K" and "p" were determined from the data taken on Kittitas Wasteway.

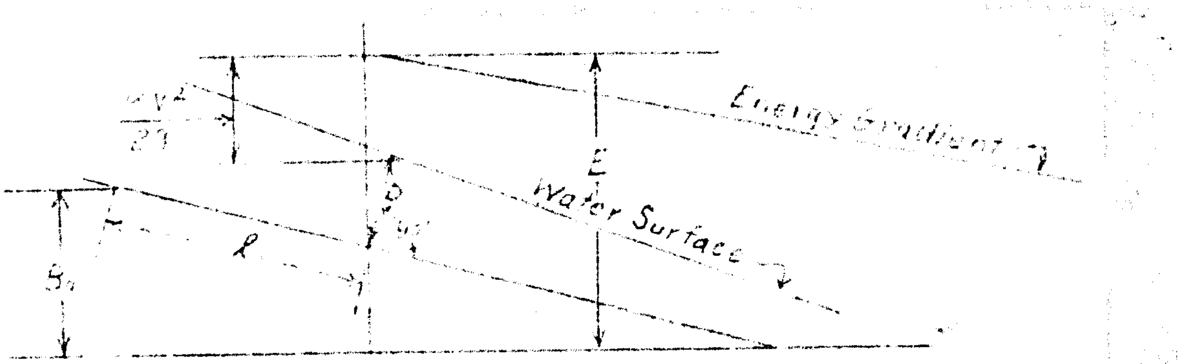


Figure 15

From Bernoulli's equation (Figure 15):

$$E = B_o - \ell \sin \theta + D \cos \theta + \frac{\alpha V^2}{2g} \quad \text{.........................(20)}$$

in which $\alpha = \int \frac{V_i^3 \, dA}{V^3 A}$

where A = cross-sectional area

V_i = velocity at elemental area, dA

V = average velocity in the section

With α defined as above, $\frac{\alpha V^2}{2g}$ is the average kinetic energy per pound of water for the section. Taking the derivative of (20) with respect to ℓ

$$\frac{dE}{d\ell} = -\sin \theta + \frac{dD}{d\ell} \cos \theta + \frac{\alpha V}{g} \frac{dV}{d\ell} \quad \text{..........................(21)}$$

with $\quad Q_o = VD$ = discharge per foot of width

$$dD = \frac{-DdV}{V} - \frac{-Q_o dV}{V^2} \quad \text{.........................(22)}$$

Combining (21) and (22):

$$\frac{dE}{d\ell} = -\sin\theta - \frac{Q_o}{V^2}\cos\theta\,\frac{dV}{d\ell} + \frac{\alpha V}{g}\frac{dV}{d\ell} \quad \text{.........................(23)}$$

Using the Manning formula with a correction for air resistance, as given in equation (19):

$$S = \frac{dE}{d\ell} = \frac{-n^2V^2}{2.2082R_{\mu=o}^{4/3}} - \frac{KV^2}{R_{\mu=o}^{p}} \quad \text{.........................(24)}$$

The negative sign is used in the right side of the equation to show a decrease in E for an increase in ℓ. Equating (23) and (24) and solving for $\frac{d\ell}{dV}$

$$\frac{d\ell}{dV} = \frac{\frac{\alpha V}{g} - \frac{Q_o}{V^2}\cos\theta}{\sin\theta - \frac{n^2V^2}{2.2082R_{\mu=o}^{4/3}} - \frac{KV^2}{R_{\mu=o}^{p}}} \quad \text{.........................(25)}$$

in which K and p are to be determined from experimental data. Although equation (25) appears complicated the right side may be expressed as a function of V for any particular canal with constant width, provided Q_o, the roughness and the slope are known. Under such conditions

$$R_{\mu=o} = \frac{bQ_o/V}{b+2Q_o/V}$$

where $\quad b$ = width

Equation (25) may be written $\frac{d\ell}{dV} = f(V)$.

Plotting $\frac{d\ell}{dV} = f(V)$ as ordinate against V as abscissa, as shown in Figure 16, the cross-hatched area is the length to that point

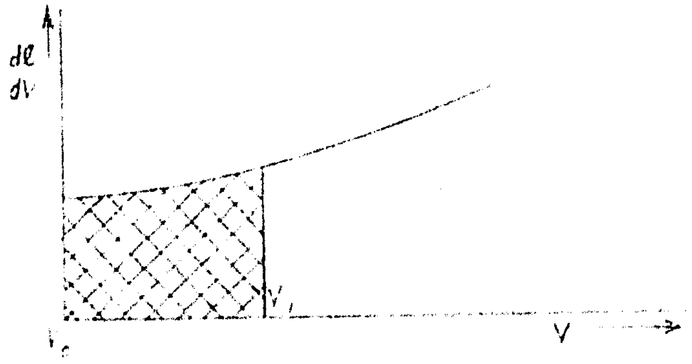


Figure 16

in the chute where the velocity is V_1. The velocity at the beginning of the slope, or any other point in the chute at which the velocity is known is V_o.

Evaluation of Coefficients

For all practical purposes it may be assumed that the velocities as determined in Kittitas Wasteway, Table 3, are terminal velocities. The values of the constants in equation (19) may be determined as follows:

$$S - \frac{n^2V^2}{2.2082R_{\mu=c}^{4/3}} = \frac{KV^2}{R_{\mu=c}^p}$$

Let $$\log\left(S - \frac{n^2V^2}{2.2082R_{\mu=c}^{4/3}}\right) = \log K + 2\log V - p\log R_{\mu=c}$$

$$\log\left(S - \frac{n^2V^2}{2.2082R_{\mu=c}^{4/3}}\right) = X$$

$$\log K = A$$

$$\log V = Z$$

and $$\log R_{\mu=c} = Y$$

then $$X = A + 2Z - pY$$

and $$X - 2Z = A - pY$$

By least squares, the two condition equations for determination of A and p are:

$$\text{I} \quad \sum(X - 2Z) = \sum A - p\sum Y$$

$$\text{II} \quad \sum Y(X - 2Z) = A\sum Y - p\sum Y^2$$

These two equations in two unknowns, result in K = 0.0000364 and p = 1.46.

Application to the Design of a Typical Steep Chute

To illustrate the use of equation (25) Dr. Streeter[26] applied it to the design of a typical steep chute. Substituting the values of "K" and "p" in equation (25)

$$\frac{dl}{dV} = \frac{\frac{\alpha V}{g} - \frac{Q_o}{V^2}\cos\theta}{\sin\theta - \frac{n^2V^2}{2.2082R_{\mu=0}^{4/3}} - \frac{0.0000364V^2}{R_{\mu=0}^{1.46}}} \quad \text{..................(26)}$$

Given: A concrete chute with rectangular cross-section, 8 feet wide, and an angle of floor with horizontal 45 degrees. The discharge equal to 1,600 second-feet.

To find: The average velocity at any section including air resistance, and the average velocity if no air resistance is assumed.

Solution: Assume n = 0.012; α = 1.16; water enters at critical depth, D_c = 10.75 feet; critical velocity, V_c = 18.6 feet per second[27].

Then $R_{\mu=c} = \frac{8D}{8+2D} = \frac{1,600}{V(8+\frac{400}{V})} = \frac{200}{V+50}$

and equation (26) becomes

$$\frac{d\ell}{dV} = \frac{0.036070V - \frac{141.42}{V^2}}{0.7071 - 0.000,000,055,755V^2(V+50)^{4/3} - 0.000,000,015,908V^2(V+50)^{1.46}}$$

The solution of this equation is shown in Table 7. The value of ℓ is determined from $\frac{d\ell}{dV}$ either by the method outlined above or by Simpson's rule. The terminal velocities with the air resistance term and without the air resistance term are found to be 101 feet per second and 117.3 feet per second, respectively. The distances down the chute at which 99 percent of the terminal velocities are reached are 946.6 feet and 1,007.2 feet for the two cases with air resistance and without air resistance, respectively. The length may be plotted against the velocity, or against $\frac{Q_o}{V} = D_{net}$. As the water contains air, the actual depth will be greater than D_{net}.

Freeboard Computations

Because of the inconsistency of the air content data a formula for freeboard computations in chutes could not be derived. At present there appears to be no reliable means whereby the amount of air or its distribution in the flow may be predicted[28]. Attempts were made to fit the available data with formulas as follows, by the method of least squares:

$$\mu = KR_{\mu=c}^{\alpha} \; V^{\beta}$$

$$\mu = K(R_{\mu=c} \cos\theta)^{\alpha}$$

$$\mu = KR^{\alpha}(\cos\theta)^{\beta}$$

in which α, β, and K are constants.

TABLE 7

[illegible]

$$\frac{dt}{dv} = \frac{0.034070 - \frac{141.48}{v^2}}{0.7071 - 0.0000000[illegible]59759\,v^2 (v-50)^{4/3} - 0.0000000[illegible]v^2 (v-50)^{1.46}}$$

Average v	v^2	$(v-50)^{4/3}$	$(v-50)^{1.46}$	0.03407v	$\frac{141.48}{v^2}$	3.5759×10^{-8} x $v^2(v-50)^{4/3}$	1.5900×10^{-8} x $v^2(v-50)^{1.46}$	Air resistance $\frac{dt}{dv}$	No air resistance $\frac{dt}{dv}$	Air resistance t	No air resistance t
[illegible].6	[illegible].96	[illegible]	479.77	0.6709	0.40870	[illegible]	[illegible]	0.3760	0.3735	0.0	0.0
[illegible]	400.0	[illegible]	[illegible]	0.7814	0.35395	[illegible]	[illegible]	0.5874	[illegible]	0.[illegible]	0.[illegible]
30	900.0	[illegible]	600.90	1.0822	0.15715	[illegible]	0.0086	1.3579	[illegible]	27.42	26.99
40	1,600.0	[illegible]	713.17	1.[illegible]	[illegible]	0.0540	[illegible]	2.0746	[illegible]		
50	2,500.0	[illegible]	851.76	1.[illegible]	[illegible]	0.0847	[illegible]	2.8471	2.[illegible]	85.51	86.16
60	3,600.0	927.06	[illegible]	2.[illegible]	[illegible]	0.[illegible]	[illegible]	3.8875	3.[illegible]		
70	4,900.0	551.89	1,085.4	2.[illegible]	[illegible]	0.3467	[illegible]	5.4168	4.57[illegible]	158.13	174.77
80	6,400.0	696.55	1,180.0	2.[illegible]6	[illegible]	[illegible]	0.[illegible]	8.[illegible]	6.[illegible]		
90	8,100.0	736.55	1,[illegible].4	3.[illegible]	0.01746	0.[illegible]	0.1758	13.8597	8.[illegible]		
100	10,000.0	796.59	1,505.46	3.407	0.01414	0.[illegible]	0.[illegible]	132.8876	13.47071	[illegible]	995.85
101											
105	11,025.0	852.41	1,577.2	3.7074	0.01283	0.5211	0.8766		19.3570		
110	12,100.0	868.[illegible]	1,[illegible].0	3.9477	0.01169	0.5[illegible]			[illegible]		560.38
112.5	12,656.25	886.8	1,[illegible].0	4.0579	0.01117	0.[illegible]			[illegible].72[illegible]		
115	13,225.0	906.59	1,781.0	4.[illegible]	0.01069	0.[illegible]			10[illegible]		816.05
116.2	13,479.2	923.04		4.1377	0.01049	0.[illegible]			[illegible]		1,087.39
[illegible]	[illegible],400.0	941.74	1,805.1	4.[illegible]	0.00982	0.7541					
117.3											
125	15,625.0	978.85	1,825.0	4.9088	0.00905						

None of the above formulas would fit the data to a satisfactory degree. Either the data are not reliable enough, or the type of formula is incorrect.

In determining the amount of freeboard, vertical curves will usually be critical points. The maximum velocity and not the average velocity, should be used in computing the trajectory to design the shape of vertical curve. The maximum velocity might well be taken as 1.5 times the average velocity of the section as an additional safeguard.

The highest air content found in the Kittitas data is 60 percent. It would seem reasonable that the air content would be directly proportional to some power of the velocity, and inversely proportional to some function of the hydraulic radius and slope of the bottom of the canal. This has not been verified, however, by the existing data.

VII CONCLUSIONS AND COMMENTS

Equipment and Methods

Many difficulties are encountered in making quantitative observations of flow in open channels with high gradients. These difficulties are reflected in the data obtained and considerable discrepancy results. The value of the data is contingent upon the equipment and methods utilized in prosecuting the program.

Measurements must be made of the quantity of water entering the chute, and the velocity and area of cross-section of flow in the chute for determination of air content in the high-velocity flow.

For accuracy of results the quantity may best be measured at a point outside the test structure where the flow is above critical depth. This procedure permits following commonly accepted methods of measuring discharge.

A satisfactory means of measuring high velocities in flow has been developed and should assist in future studies. Velocity observations taken electrically, using salt as a conductor, and an oscillograph to record, through electrodes in the flow, the time of passage of the brine cloud will yield satisfactory results and afford a permanent record. With an improved oscillograph the maximum and average velocities for a section may be obtained within reasonable limits. These two values would then give some picture of the velocity distribution.

The time distances between centers of gravity of the curves on the oscillograms may be used to compute average velocities, and the time distances between beginning points on the curves to obtain maximum velocities. Equipment can be obtained that will produce records of such quality to permit rapid interpretation.

Because of the convex nature of the water surface, observations should be made throughout the width of the structure. Observations made by means of gages on the sides of the channel would be somewhat greater than the average elevation of the surface at the section. The point gage method of measuring water surface, although satisfactory, is time consuming. Some improved method, more in keeping with the means of securing velocities, should be developed.

No satisfactory means of studying the mechanics of air-water mixing was developed. Much additional work must be done before the problem can be explained quantitatively, and in particular, a study of the surface flow exposed to the air should be made by means of an ultra-speed motion picture camera. It is possible that this problem may be studied with models in the laboratory, but it appears that the final analysis must be made in the field on large structures.

Present Data and Application

The flow of water on steep gradients is essentially different from that found in channels of ordinary slope. Velocities are greater than the critical and the kinetic energy greatly exceeds the static pressure of the water prism. Water, mixed with air, flowing down a steep channel does not follow the normal law of hydraulics as expressed by the continuity equation, $Q = AV$.

Concerted search of literature, supplemented by a program of field measurements, shows that water flowing at high velocity in an open channel does entrain air and that the existing velocities are lower than ordinarily computed or assumed.

In the experiments on the Kittitas Wasteway, the indications were that a terminal velocity existed. Furthermore, there is no question that a large amount of air was entrained in the flow. On a slope of 33° 10', an air content of as much as 60 percent by volume was found.

The results of the field tests have been compared to those obtained from previous experiments by Ehrenberger. This comparison indicated that for a constant velocity the exponent of the sine of the angle with the horizontal decreases as the hydraulic radius increases. Since data from only two slopes were available no definite conclusions could be made.

The importance of using a correct interpretation of the hydraulic radius in connection with the Manning and Chezy formulas has been discussed. The hydraulic radius must be defined as the area of cross-section, assuming no air in the water, divided by the total wetted perimeter including air. This permits the customary value of coefficient to be used. The Manning or Chezy formulas, however, have no means of expressing air resistance. The nature of air resistance is discussed and a correction term for air resistance in the Manning formula proposed. Coefficients for the new air resistance term are evaluated from the Kittitas data. Applications of the formula to the design of a typical steep chute are given.

No satisfactory relationship among the factors, velocity, hydraulic radius, and slope of channel to the air content has as yet been developed. Either the data are of questionable value or correct analysis has not been found.

The mechanics by which air enters the flow or its distribution in the flow is not definitely known. However, conclusions may be drawn in regard to contributing factors. The initial entrainment of air depends to a great extent on entrance conditions and internal turbulence of the flow. Roughness of the sides and bottom of the channel, the slope of the channel, the depth and width of flow, the air velocity and turbulence over the flow, and the alignment of the channel, all must be considered in studying the insufflation problem.

Observations and photographs made of numerous overfall dams indicate that the jets overflowing structures of this type do not contain air in the quantities observed in narrower channels. This may be because there is not sufficient length of face on the dam to establish constant flow conditions. It may also be due to some relationship between depth and width of the jet to the air content.

Visual observations supplemented by slow-motion pictures, of waterfalls and jets discharging from valves operating under high heads show that the jet disintegrates rapidly and assumes a very white turbulent appearance. Here, of course, the flow is completely surrounded by atmospheric pressure. The only force holding the jet together is surface tension and in the case of large jets accompanied by a high degree of turbulence this force is relatively small[29]. Consequently, the pressure in the flow is very close to atmospheric and there is merely an interchange of position of particles of water and air. The internal pressure may have some bearing upon the air mixture in high-velocity flow in steep channels particularly where the flow traverses a vertical curve.

On the structures observed, it may be concluded that present design practice provides ample freeboard in high-velocity channels with the exception of convex vertical curves. These curves must not be made sharper than the trajectory of flowing water falling under the action of gravity if the stream is expected to adhere to the bottom of the channel. The portion of the water prism near the surface and in the center of the channel travels at a velocity from 20 to 25 percent greater than the mean velocity in the cross-section and this maximum velocity must be used in computing trajectories.

Additional Studies

More experimental data over a wider range of channel sizes and with varying degrees of roughness must be had before the phenomenon of flow at high velocities is fully understood. In view of the present status of the problem, further studies should be directed toward establishing approximate general formulas before much effort is given to complex refinements. It is hoped that interest may be stimulated in the engineering profession to the end that additional data and analyses may be added to the small store now available. Adequate explanation and formulation must await further investigation and analysis.

It is most important that further experimental investigations be carefully planned, particularly with regard to how the data are to be used. Possibilities of laboratory research, and perhaps of theoretical investigation may not have been exhausted. Future tests should be conducted on the highest available spillways and other chutes. In particular, data should be secured from structures where an appreciable depth of flow exists. All observations to date have been made in channels with rather shallow depths.

APPENDIX

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