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FLOW OVER ROUNDED CREST WEIRS

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PREFACE TO FLOW OVER ROUNDED CREST WEIRS

By Whitney McNair Borland

This thesis was submitted to the faculty of the University of Colorado in partial fulfillment of the requirements for the degree, Master of Science in 1938.

The author's interest in the subject of rounded crest weirs was stimulated by Professor E. W. Lane, now a consulting engineer with the Bureau of Reclamation.

The author also wishes to express his thanks to Mr. I. A. Winter, engineer with the Bureau of Reclamation, for the use of unpublished data from tests performed by him while employed by the Alabama Power Company. This data served to confirm the author's conclusions.

W. M. B.

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J. N. B.

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FLOW OVER ROUNDED CREST WEIRS

Chapter I

Introduction

Robert E. Horton, in U.S.G.S. Water Supply Paper No. 200 (1)*, collected all the data available at that time (1906) on the subject of weir coefficients. It was thought that dams, particularly dams forming mill ponds in the northeastern states, could be used as gaging stations in determining the discharge of various streams. If the coefficient of discharge of such dams were known, it would be necessary only to determine the head or depth of water flowing over the crest, and then the discharge might be computed. No currentmeter rating of the gaging station needed to be made, and in the case of large floods it would not be necessary to extend the rating curve. Also the gaging station would not be subject to scour and deposition.

While this much used report presented available data at that time in an excellent style, additional data and a different method of analysis change the results obtained for coefficients of discharge for rounded crest weirs. The term "rounded crest" is used rather than the misused term "ogee" which refers to a reverse curve. The ogee curve is not necessarily in this type of crest, but it is usually present due to the downstream face of the dam joining the protecting apron in a circular curve, thus forming a reverse curve. The rounded crest is one in which there is some degree of curvature on either side of the crest. Here the word "crest" refers to the crown or highest point on the weir. The term "rounded crest" excludes the broad crests which have been recently brought up-to-date by Professor Woodward (18).

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*Small numerals in parenthesis refers to reference number in Bibliography at end of thesis.

It is of paramount importance to be able to predict the coefficient of discharge in the design of spillways for dams. When the height of the dam is increased a small increment, it results in a relatively large amount of storage. The amount of material needed to increase the height of the dam is usually large, since it must be added to the base of the dam. These considerations make it desirable to be able to pass the maximum flood over the crest of the dam with a minimum head. To do this either the length of crest or the coefficient may be increased. Cases in which the crest has been lengthened to nearly a maximum are Keokuk Dam and Grand Coulee Dam, which have over one-fourth mile of crest length. Practical considerations, such as the location of powerplants, width of damsites, suitable conditions for the dissipation of the energy of the overflowing stream, and cost of gates, piers, and protecting aprons, limit the length of the crest.

Any increase in coefficient of discharge is of economical importance. For example, Grand Coulee Dam was first designed with 12 gates, 135 feet long. After considerable experimenting on the shape of crest it was possible to increase the coefficient from 3.56 to 3.88, an increase of 9.0 percent, thus effecting a saving of the cost of one gate and pier, and making available additional space for powerplant. This increase in coefficient might have been used to increase the storage by 2.30 feet of a dam without gates, or to decrease the height of gates by the same amount.

When dams were first built, the factor of major importance was to get a dam of such a cross-section that it would stand up. Consequently, dams were first irregular in shape with usually sharp or flat crests being somewhat triangular in profile. As the importance of other factors became known the crests were modified. The coefficients obtained from these first dams were erratic, and any analysis of them is difficult. It is later shown that coefficients from such irregular shapes are more difficult to analyse, and they deviate by a greater amount from base curves that might be constructed. Any dam built in the future will be more likely to follow modern practices, and thus the cross-section of the dams will all be similar which will facilitate the analysis of the coefficient and decrease the deviation from the base curve. Many of the early dams developed excessive pressure, while vacuums existed in other places. The excessive pressures were not troublesome, but the vacuums were likely to cause disintegration of the concrete, increase pressure on the upstream face of the dam by reduction of back pressure, and most dangerous vibrations might be caused by the forming and breaking of such vacuums. Consideration of the pressure problem leads to filling in with masonry of the area underneath the nappe shape produced by the same quantity of water flowing over a sharp crest weir. It was thought that a little extra masonry should be added to the downstream face in order to make sure that no vacuum would form.

Chapter II

Factors Affecting the Coefficient of Discharge

As would be expected, the shape of the crest has the most influence on the discharge coefficient. The portion of the weir upstream from the crest is the most critical and should receive the most attention. In the past much effort has been put forth to develop parabolic equations with origin at the crest (highest point of nappe), while very little effort has been made to perfect a curve upstream from the crest in the most critical region. In many designs the critical region upstream from the crest has just been made to approximate the nappe shape by using either a slope or a radius, while the portion of the weir downstream from the crest has been very carefully designed. This is a mistake as the emphasis on the design of the crest should just be reversed.

It has been demonstrated by Bazin (9) and others (2), that any variation in the shape upstream from the crest will result in a change in the coefficient and the pressure distribution over the weir. The logical way to design a crest shape is to decide on the shape of the upstream portion, build a model of it, and measure the shape of the nappe from this model, then fill in the space beneath the nappe surface with masonry. This method has been used with very gratifying results both as to the value of the coefficient and the pressure distribution. Designers will do well to be very careful in fitting their rounded crest weirs to the nappe shape from a sharp crest weir in the critical region where the nappe springs free from the crest. Experiments have lead some to believe that the nappe does not spring free from the upstream edge of the weir, but due to surface tension springs free a short distance downstream. These experiments were on very small models where surface tension forces might become significant compared to the small head. As the head increases this effect is lessened; while this phenomenen might affect the shape of a rounded crest model, it would not affect the shape of its prototype.

Various arrangements of slope in the critical region of rounded crests have been tried. In Water Supply Paper No. 200, a slope in the upstream side of a rounded crest was found to change the coefficient. This slope extended from the crest upstream a short distance, terminating in a vertical face. As the slope flattens the crest shape approaches a broad crest weir with a correspondingly low coefficient, and as the slope steepened the crest shape would come nearer fitting a nappe shape with the resulting increase in the coefficient. Creager (5), has proposed a crest shape with a 1:1 slope,and tests conducted in Russia have shown that the coefficient of discharge may be increased by 2.5 percent over that of his vertical face weir. However, experiments by Dillman (7), with the same type of weir, have failed to show any increase in coefficient. The above slopes have terminated in a vertical face; the slope that extends from the region of the top of the crest to the floor always seems to reduce the coefficient. Experiments conducted by Mr. I. A. Winter (26) for Alabama Power Company, with a perfect shaped rounded crest, showed that with the weir face inclined 55° with the floor, the coefficient was decreased by 3.3 percent. Some dams have overhanging crests in order to accommodate large piers, drum gates installed in the crest itself, and also to increase the stability. Tests by the U. S. Bureau of Reclamation have shown that this overhang does not appreciably affect the coefficient and have been confirmed by experiments (2) at Massachusetts Institute of Technology.

The portion of the rounded crest weir downstream from the crest has received the most attention due to consideration of negative pressure and high velocities. This portion of the weir does not influence the coefficient as much as the upstream portion. In fact, any number of cases can be cited where the masonry was extended into the nappe and yet the coefficient has not suffered. When there is a radial or Stoney gate on the crest under which the water passes, it is common practice to design the downstream portion of the crest so that the jet of the water,under full head from a nearly closed gate, will not tend to spring free from the masonry,and there will be no regions of negative pressure. This necessitates a much fuller crest shape than dictated by the nappe shape from a sharp crest weir.

If the nappe shape from the sharp crest weir is measured vertically,it will be found that the critical depth (where velocity head equals one-third of the total head or depth of water equals two-thirds of the total head) occurs downstream from the vertical face a distance of .63 of the head measured above the highest point of the nappe. At the highest point of the nappe the depth is equal to .75 of the head. From a consideration of the phenomean of critical depth, any change in shape below the point where critical depth takes place cannot produce a pressure wave or back water up so that it will effect the depth over the crest, and thus decrease the coefficient. The reason for this condition is that a pressure wave travels with velocity equal to the square root of the depth times the acceleration due to gravity. Since this is also the velocity at critical depth, any pressure wave generated below the critical depth will either stand still or move downstream.

It is usually stated that a broad crest weir begins to submerge when the depth downstream is two-thirds of the total head on the weir. For a rounded crest weir that perfectly fits the lower surface of the nappe from sharp crest weir, the submergence will take place when the depth of water below the weir is just equal to the depth of the water at the position of critical head; that is $.63H_o$ downstream from the vertical face of the weir.

Since the critical depth occurs at a lower elevation than that of the crest, the depth of the tailwater is found to be .600 of the head on the weir instead of .666 when it is equal to the critical depth.

The above discussion is based on the assumption that the pressure at any point is expressed by the depth of water flowing over that point. This condition of pressure on the face of the rounded crest weir will not exist when the streamline of filaments of water are diverging or converging. Nor will it exist when the filaments are curved (36). In the case of rounded crest weirs the filaments are curved and also are converging due to acceleration. Both of these affects tend to reduce the pressure on the face of the rounded crest weir.

In submerging a rounded crest weir, this pressure reduction on its face is lessened by the rising tailwater which in turn decreases the velocity and increases the head on the weir. By actual experiments the coefficient of discharge usually begins to decrease as soon as the tailwater is greater than the elevation of the crest. The reduction of the discharge coefficient is only 4 percent when the depth of the tailwater is .600 of the total head on the weir, beyond this point the coefficient decreases rapidly with rising tailwater. This illustrates the fact that conditions downstream from the critical depth do not appreciably affect the coefficient, and also that the position of the critical depth was reckoned nearly correctly above.

The above discussion is based on the supposition that the critical depth will remain stationary. Just how much effect the pressure reduction, due to curving and converging of the stream-lines, has on the position of the critical depth is not known. It is believed that it will have very little since the critical depth was located on the nappe shape from a sharp crest weir where the pressure at the lower surface is zero.

If the shape of the rounded crest weir is such that the position of the critical depth is moved downstream, then conditions upstream from it but yet downstream from the crest, are able to produce waves which will effect the flow over the crest. Since the critical depth usually occurs a short distance downstream from the crest, the masonry of the weir, downstream from a point a distance equal approximately to the head on the weir, may project into the nappe shape without reducing the coefficient of discharge. If, however, the masonry projects into the nappe shape above this point enough to cause the position of the critical depth to move downstream, the coefficient will likely be decreased.

The height of the weir crest, that is, the depth of the approach channel, has an effect on the coefficient of discharge. This factor has been discussed at length in connection with the discharge over sharp crested weirs, and various methods have been used to correct for the factor in the formula for discharge. When the height of the weir or dam is great, the head on the crest expresses the total energy, but when the weir is low in addition to the head, the kinetic energy of the approaching water must be considered. The author believes that in computing the discharge for both sharp and rounded crest weirs that the head plus velocity head should be used. The velocities in the approach channel are never equal, and the kinetic head is always greater than the velocity head computed on the basis of mean velocity of approach (being obtained by dividing the discharge by the cross-section area of the approach channel). Only in the case where the approach velocity is uniform throughout the cross-section of the approach channel is the kinetic energy equal to the velocity head based on the mean velocity. If a dimensionless coefficient α be introduced to correct the mean velocities so that they will give the correct kinetic head, we have:

$$h_a = \alpha \frac{V^2_m}{2g} \quad \text{------------------} \quad 1.$$

The value of α will depend upon the relative size of the sections with low and high velocity which go to make up the mean. If the velocity varies from zero at the bottom of the approach channel uniformly to a maximum at the surface, the value of α is 2. Schroder (35) has shown by experiment that the coefficient of discharge is appreciably affected by the ratio of the mean velocity existing below the crest to that above the crest. By various derivations found in common texts, the theoretical way to correct for velocity of approach is:

$$H_o = \left[(h_o + h_a)^{3/2} - h_a^{3/2}\right]^{2/3} \quad \text{------------} \quad 2.$$

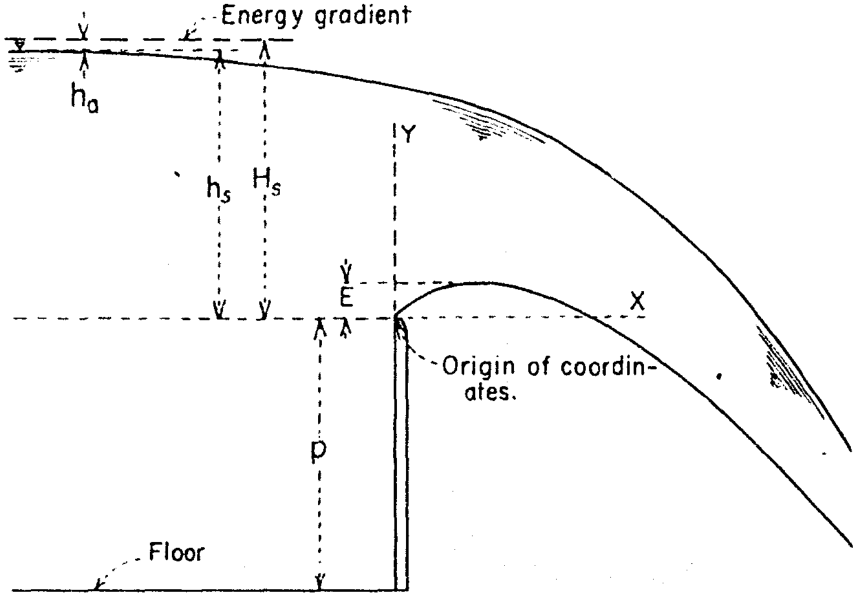
See Figure 1. for explanation of symbols. Where h_o is the measured head above the crest and H_o is the head used in usual weir formula:

$$Q = C\ L\ H_o^{3/2} \quad \text{------------------} \quad 3.$$

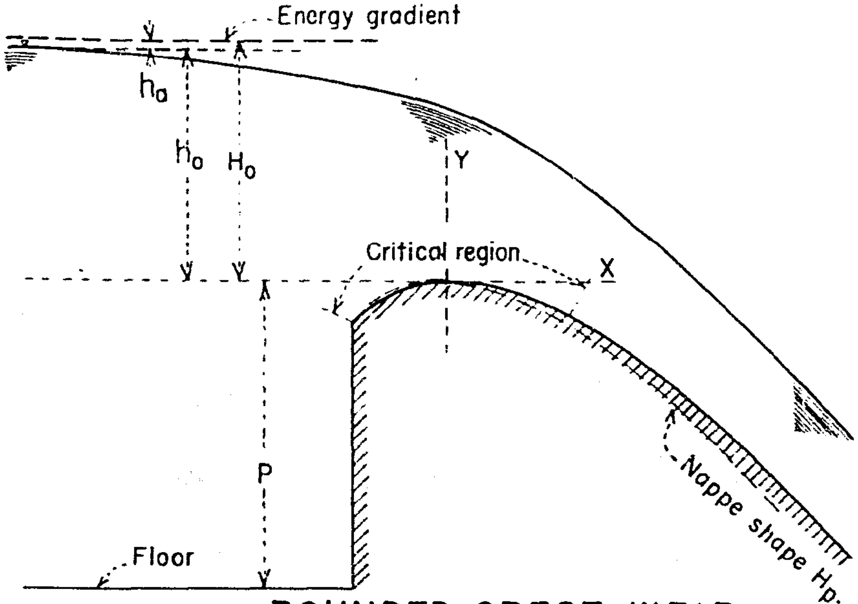
An inspection shows that the discharge is greater than if the measured head h_o is used in the formula and is less than if the measured head plus the velocity head $(h_o + h_a)$ was used.

For rounded crest weirs, since it is impossible to evaluate the factor α given variously as 1.05 to 1.34 for usual conditions, it is believed rational to use $\alpha = 1$ for general purposes. To compensate somewhat for this reduction in α the last term of $\left[(h_o + h_a)^{3/2} - h_a^{3/2}\right]^{2/3}$ is dropped so that for the total head we have:

$$H_o = h_o + \frac{V^2_m}{2g} \quad \text{------------------} \quad 4.$$



SHARP CREST WEIR



ROUNDED CREST WEIR

NOMENCLATURE AND SYMBOLS

h_s = Measure head on sharp crest weir.
H_s = Total head on sharp crest weir. $(h_s + h_a)$
p = Height of sharp crest weir.
E = Highest point of nappe shape measured vertically above sharp crest weir.
C_s = Coefficient of discharge for sharp crest weir in $Q = C_S L H_S^{3/2}$

FOR BOTH TYPES OF WEIRS

Q = Quantity of discharge.
L = Length of crest.
A = Cross section area of approach channel.
V = Velocity of approach. $= \frac{Q}{A}$
h_a = Velocity head of approach.
g = Acceleration due to gravity used as 32.16.
α = Empirical coefficient applied to $\frac{V_m^2}{2g}$ to give kinetic head. where V_m is the mean velocity in the approach channel.
μ = Dimensionless coefficient in $Q = \frac{2}{3}\sqrt{2g}\ \mu\ L H_S^{3/2}$ or $Q = \frac{2}{3}\sqrt{2g}\ \mu L H_O^{3/2}$.

FOR WEIRS WITH END CONTRACTIONS

N = Number of end contractions.

L' = Measured length of weir.
L = Effective length of weir.
h_o = Measured head on rounded crest weirs.
H_o = Total head on rounded crest weirs.
P = Highest point of rounded crest weir above floor.
C = Coefficient of discharge for rounded crest weirs in $Q = CLH_O^{3/2}$ or in $Q = CL\ (h_o + h_a)^{3/2}$ (does not include velocity of approach.)
K = Coefficient of discharge for rounded crest weirs in $Q = KLh_O^{3/2}$ (includes velocity of approach.)
C_T = Theoretical coefficient for rounded crest weir. C_T in $Q = C_T L H_p^{3/2}$.
H_p = Head in terms of h_o which produces best fitting nappe shape for cross section of rounded crest weir.
Ψ = Ratio of coefficient for low rounded crest weir to coefficient for rounded crest weir with $P = \infty$.
$1+\phi = \frac{K}{C}$ = Ratio of coefficient with velocity of approach to coefficient without velocity of approach.
K_s = Coefficient of discharge for sharp crest weir in $Q = K_s L h_s^{3/2}$ K_s includes velocity of approach.

FIGURE 1.

Thus if the height of the weir is .4 of the head, or $h_a = .10$ then $\alpha = 1.13$ for the theoretical corrected measured head to equal $\left(h_o + \frac{V^2_m}{2g}\right)$. This is not out of reason for a low weir where the difference in velocity must be great due to the relative low velocity along the bottom of the approach channel. In addition to the velocity of approach affecting the energy of the stream of water passing over the crest, the shape of the nappe itself is changed. With an increase in the velocity head there is a flattening of the nappe shape. Thus a crest shape designed for no velocity of approach will not fit the nappe shape produced by a low sharp crest weir. This change in shape influences both the pressure distribution over the rounded crest and the coefficient.

The shape and size of the approach channel modify the coefficient of discharge. If the channel is long and shallow in addition to the above modification to the coefficient, there is the friction loss due to the long channel and entrance that will be included in the usual coefficient. A great number of earth dams have been built of late years. It is impossible to to spill the floodwater over the top of them because of harmful errosion at the toe of the dam; water is usually carried around one end into an adjacent creek or some distance downstream, so as not to endanger the earth toe. This "chute" type of spillway, due to its peculiar location, often has a long approach channel terminated by a low rounded crest weir, and controlled by a set of gates. This type of rounded crest differs markedly from the one which is situated at the top of an overflow concrete dam section hundreds of feet high. In one case the height of weir is only about one-half to one-fifth of the maximum head, and in the latter it is often ten times the maximum head. If the shape of the crests are similar and the same formula for discharge is used, it is desirable that the same coefficient be used by applying some modifications. The friction loss in the approach channel is small since the length of it is not great; however, the entrance loss in the "inlet" to the gate structure may reduce the maximum discharge. The increase in cost, due to excavation for spillway gate structures in connection with the earth dam, necessitates the "inlet" being made as small as possible, many times reducing the coefficient of discharge by as much as 7 percent. Because of the variety of inlets used it is impossible to correlate the data and get a value for reduction in coefficients. Large losses in "inlet" are confined to structures where the height of the weir is equal to, or less than, the head over the weir.

The inlet loss corresponds to a type of contraction. For rounded crests in general the contractions are usually completely suppressed or at least partially so; there is seldom a square cornered entrance. Contractions are best taken care of by considering if piers are present and whether the gates operating are adjacent or alternate. Consequently an analysis will identify three states of rounded crest weirs. (In this thesis these three conditions of discharge are designated as "A", "D", and "C"). At the ends of the spillway crest the approach wall is always well rounded so that the usual contraction formula of $L = (L'-0.1N\ h_o)$ is not applicable, since it applies to square cornered entrances. L is the effective length of weir, L' is the measured length and N is the number of contractions.

Some opinions have been advanced that the magnitude or size of a rounded crest weir affected the coefficient of discharge. The data analysed by the author ranged from experiments on very small scale models on which the radii of the curves of the crest cross-section were only a few inches in length, to measurement on large dams like Wilson where the corresponding radii were as much as a hundred times as large. No trend was observed in which the coefficient tended to increase with size of crest cross-section.

It has been found that roughness, as long as it does not affect shape, will not change the coefficient of discharge by more than 2 percent. The coefficient increases with head, but only as the ratio of head to crest size increases. Thus, it is possible to have as large a coefficient on a small scale model as on a large dam like Wilson.

Chapter III

Shape of Crest

Attention has been called to importance of the rounded crest conforming to the underneath shape of the nappe from a sharp crested weir. To the author's knowledge no other experiments on the shape of nappe from sharp crest weirs have been published since Bazin's time (9) with the exception of some meager data (13) gathered by Scimemi in Padua, in Italy. In the critical region of the sharp crest Scimemi's data agrees with Bazin's, but further down the nappe his data forms a curve lying above Bazin's nappe shape. Experiments by the U. S. Bureau of Reclamation have been made to determine the nappe shape. They were much more extensive and elaborate than Bazin's while they differ from his some distance from the crest, yet in the critical region from where the nappe springs free to beyond the highest portion of the nappe the agreement is remarkable.

Mr. I. A. Winter, working for the Alabama Power Company, performed a group of experiments (26). Since these have not been published they are regarded as confidential; however, the close agreement in the critical region tends to substantiate Bazin's nappe shape. Experiments by Mr.Rouse, for a thesis at M. I. T., on sharp crested weirs, have further fixed the exact shape of the nappe as being that measured by Bazin. The nappe shape, both the upper surface and the lower surface, conforms to the principles of dynamical and geometrical similitude. The shape is a direct function of the head on the weir. For example, the shape of the ten-foot head may be accurately drawn by determining the shape of a one-foot head and multiplying all its linear dimensions by ten. The dimensions found for a one-foot head with negligible velocity of approach are given in Figure 2. It should be noted that the coordinates are given in terms of head h_o on the rounded crest instead of head h_s on the sharp crest weir. The transformation is effected as follows:

$$h_o = h_s - E \qquad \text{See Figure 1.}$$

E= vertical distance that the highest point of the nappe or crown of the rounded crest weir is above the sharp crest of a thin edged weir. For negligible velocity of approach:

$$E = .112\ H_s \text{ and } h_o = H_o,\ h_s = H_s$$

$$\text{then} \quad H_o = H_s\ (1.000-.112) = .888\ H_s \text{ or}$$

$$H_s = 1.125\ H_o \quad \text{———————} \quad 5.$$

In terms of dimensionless numbers taking the origin at the upstream edge of a sharp crest weir:

$$\frac{X}{H_o} = \frac{X}{H_s}\ 1.125 \quad \text{———————} \quad 6.$$

$$\frac{Y}{H_o} = \frac{Y}{H_s}\ 1.125 \quad \text{———————} \quad 7.$$

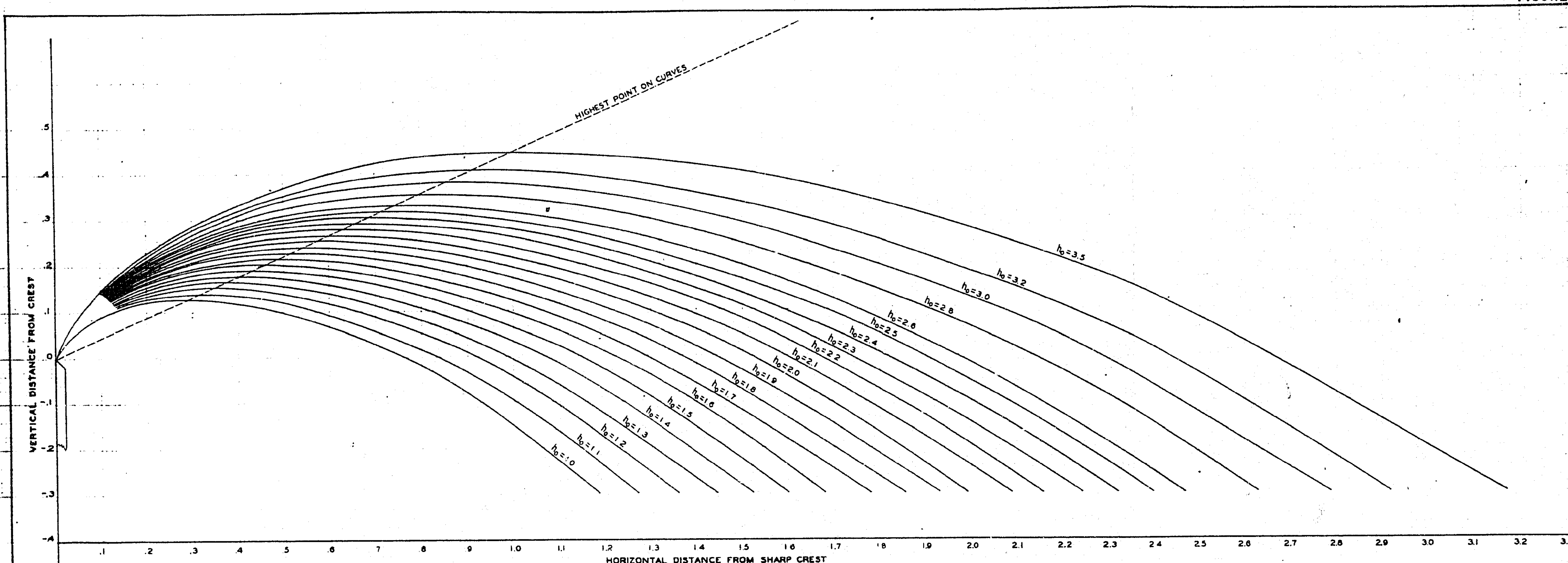
All of the coordinates for a sharp crest weir are multiplied by 1.125 to get the coordinate in terms of the head H_o on a round crest weir.

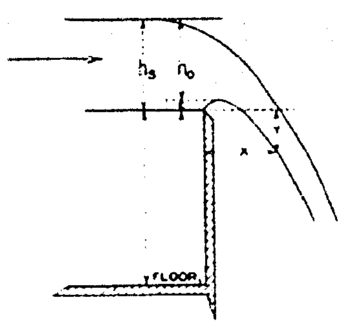
Several attempts have been made to derive an equation for the under side of the nappe shape; however, all equations omit the critical portion of the nappe shape between the points where it springs free from the sharp crest and the highest point on the nappe. When the origin is taken at the highest point on the lower surface, the equation

$$Y = 0.485\ H_o^{7/8}\ X^{15/8} \quad \text{———————} \quad 8.$$

approximately similates the low surface and is convenient to use. H_o is the head on a rounded crest weir with negligible velocity of

FIGURE 2.





EXPLANATION
UNITED STATES BUREAU OF RECLAMATION DATA BASED UPON HEAD (h_o) MEASURED ABOVE HIGHEST POINT OF NAPPE.

Coordinates for h_o =1.00

X	Y
+.00	+.000
+.10	+.090
+.20	+.121
+.30	+.127
+.40	+.118
+.50	+.097
+.60	+.068
+.70	+.028
+.80	-.020
+.90	-.080
+1.00	-.151
+1.10	-.230
+1.20	-.310

DEPARTMENT OF THE INTERIOR
BUREAU OF RECLAMATION
BOULDER CANYON PROJECT
BOULDER DAM HYDRAULIC EXPERIMENTS
SPILLWAY CREST STUDIES
LOWER NAPPE SHAPES FOR VARIOUS HEADS

DRAWN W.M.B. & D.C.W.	SUBMITTED
TRACED H.M.	RECOMMENDED
CHECKED GWMH	APPROVED
DENVER, COLO., JUNE 2, 1934	45 D-5307

approach. When the velocity head of approach becomes more than one-half of 1 percent of the total head, the nappe shape will fall outside this curve. When water flows over a rounded weir it will exert some pressure upon it. This pressure may be positive or negative (vacuum), and the sign of the pressure may be different at various positions on the crest.

On Page 7 it was found that the pressure on the face of a rounded crest weir is determined by the depth of the water over any point and by the convergence and curvature of the streamlines above that point. The magnitude of the depth produces a positive pressure, while the concave curvature of the streamline produces negative pressures, and converging streamlines due to acceleration also produce negative pressures. The sharper the degree of curvature and the greater the angle of convergence of the streamline, the greater the negative pressure will be. In general, there are five different conditions of pressure distribution which affect the coefficient of discharge. See Figure 3.

<u>Condition I</u>. The nappe may adhere to the surface of the weir and exert a pressure on it, in which case the coefficient will never be greater than 3.98 and may be less than 3.08 if the friction loss in the approach channel and other losses in the "inlet" are large compared to the head. This is the condition under which the designed rounded crest weir should usually operate, only reaching Condition II at maximum discharge. Here the pressure caused by the depth of water is larger than the reduction in pressure caused by the curvature and convergence of the streamlines.

<u>Condition II</u>. The nappe may adhere to the crest surface and exert no pressure upon it except on the vertical face. This condition will only occur if the rounded crest is perfectly shaped for that particular quantity of water passing over it. The reduction of pressure due to curvature and convergence of

streamlines just balance the static pressure due to depth of water. Regardless of what head they operate under, most rounded crest weirs will not experience this condition. With negligible velocity of approach the coefficient is 3.98 and is founded as follows:

Discharge per foot of crest of a sharp crest weir is

$$Q = C_s H_s^{3/2} \quad \text{———————} \quad 9.$$

C_s is taken as equal to 10/3. Experiments by the U. S. Bureau of Reclamation found this value to be approximately correct for values of $\frac{H_s}{H_s + P}$ equal from .04 to .70. The head above highest point of nappe is $h_o = h_s - E$ or $h_o = h_s \left(1 - \frac{E}{h_s}\right)$ but for negligible velocity of approach $\frac{E}{h_s} = .112$ and $h_o = H_o$ and $h_s = H_s$.

then $$H_o = .888 H_s = H_p$$

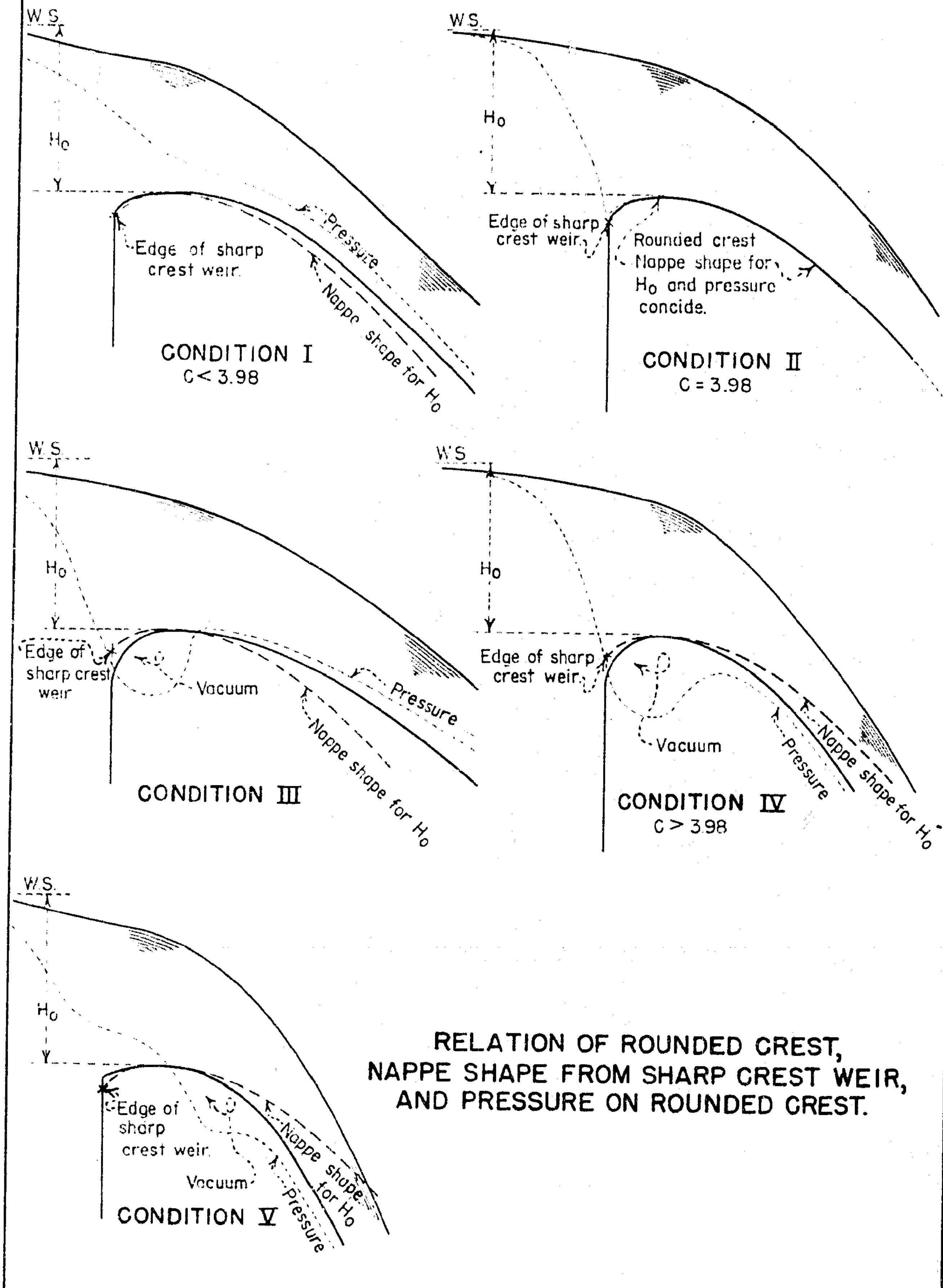
$$Q = \frac{10}{3} \left(\frac{H_p}{.888}\right)^{3/2} = \frac{10 \times 1.1932}{3} H_p^{3/2}$$

$$Q = 3.977 H_p^{3/2} \quad \text{———————} \quad 10.$$

That is $$C_t = 3.98$$

<u>Condition III</u>. The nappe may adhere to the crest and exert a negative pressure upstream from the crest with positive pressure downstream. Under these conditions the coefficient of discharge is usually less than 3.98; however, if the negative pressures are excessive and the positive pressures downstream are small the coefficient may be larger than 3.98. Under this condition the sharp degree of curvature of the streamlines upstream from the crest cause the reduction in pressure to be greater than the static depth. Downstream from the crest the static depth is greater than the reduction in pressure.

FIGURE 3



RELATION OF ROUNDED CREST, NAPPE SHAPE FROM SHARP CREST WEIR, AND PRESSURE ON ROUNDED CREST.

Condition IV. The nappe may adhere to the crest and exert a negative pressure upstream and downstream. It often happens that in the critical region of the crest the pressures are negative and then downstream from the crest quite a distance the pressure becomes positive, such cases are also listed under Condition IV, since it is the distribution in the critical region that has the greatest effect on the coefficient. Here the reduction in pressure due to curvature and convergence of streamlines completely control, being much larger than the static pressure due to depth of water. In order for negative pressures to exist downstream from the crest, the nappe must not be areated; since, if it is, it will spring free from the crest. With negative pressure throughout the critical region the coefficient is always greater than 3.98. If the nappe springs free from the downstream surface of the crest, the coefficient will decrease, approaching that for a sharp crest weir, depending upon the amount of rounding upstream from the crest as shown by Fteley and Stearns (27). In this article it states that if there is any appreciable rounding the coefficient is greater than that for a sharp crested weir.

This Condition IV offers a means of increasing the coefficient of discharge until under favorable conditions it may reach the value of 4.60; however, for all values above 3.98 there will always be negative pressures. If the head at which the nappe shape springs free from the crest be referred to as the critical head, then the maximum negative pressure before the nappe springs free is said to be four times the critical head (7). With a barometric pressure of 30 feet of water the critical head should not be larger than 7.5 feet, so that dangerous conditions (cavitation) will not result. These figures are based upon data obtained from a rounded crest shaped to approximately fit the underneath surface of the nappe from a sharp crested weir. For irregular shaped weirs the negative pressure would be greater.

Condition V. The nappe may adhere and exert positive pressure upstream from the crest but exert negative pressure downstream from the crest. With small positive pressure upstream and large negative pressure downstream the coefficient may be slightly greater than 3.98. While with large positive pressures upstream, small negative ones downstream, the coefficient will be less than 3.98.

With any given rounded crest shape, the pressures will vary with the head on the crest. If the head is increased to a large quantity, somewhere on the crest the pressure will become negative. If the negative pressure area occurs upstream first, a further increase in head will also cause a negative pressure downstream. When the negative pressure becomes large enough downstream from the crest, the coefficient will become greater than 3.98.

When the change in curvature on a rounded crest weir occurs too abruptly a roller will form. This is illustrated by the roller forming just downstream from the sharp upstream edge of a broad crested weir. The roller always relieves negative pressure which might exist at that point, and when occuring downstream from the crest is accompanied by a decrease in coefficient. This formation of rollers on irregular shape crests have the effect of filling out the crest to a more regular cross-section which will better fit the shape of the nappe from a sharp crested weir.

Chapter IV

Method of Fitting Nappe Shape

A consideration of above facts concerning pressure distribution suggests that if nappe shape be fitted to the cross-section of a rounded crest weir, it would be possible to predict the occurrence of positive or negative pressure area and thus determine something of the value of the coefficient.

A family of nappe shaped curves, Figure 2, was drawn up based on the head measured above the rounded crest h_o. By drawing the cross-section of the rounded crest weir or dam to a suitable scale on transparent paper, it can be compared with the various nappe shapes. For example, if the family of curves, Figure 2, were drawn up to a scale of 1 inch equals 10 feet and the cross-section of the rounded crest were drawn to a scale of 1 inch equals 50 feet, then if the heads producing the family of nappe curves were multiplied by 5, (the shapes remaining the same) they will give the heads that produce nappe shape on the transparent drawing of the crest cross-section, that is the 1 inch equals 50 feet, scale drawing. By selecting the proper scale for the cross-section of the dam or weir, a single family of nappe shape curves can be made to compare with any crest shape regardless of its actual size.

It is found that for any weir cross-section there was a nappe shape which fitted it best. The head that produces this best fitting nappe shape is termed H_p. When the actual head on the rounded crest is equal to the head producing the best fitting nappe shape, that is $H_o = H_p$, the nappe will exert the least pressure on the crest, if irregular in shape the negative pressure will approximately equal the positive. Under these conditions the coefficient of discharge will be close to 3.98. It is logical that the coefficient for any head on a rounded crest weir would be a function of the ratio of that head to the head that produced the best fitting nappe shape $\frac{H_o}{H_p}$. It is possible to take any rounded crest weir, find the best fitting nappe shape and, if it has a determined coefficient relation, to plot it up in terms of the ratio of head, H_o, to head producing best fitting nappe shape, H_p. Consequently, the coefficient head relation of different weirs are reduced to a comparable basis for analysis. Figure 4 shows a typical work sheet for one of the rounded crests tested for Boulder Dam Spillway. It was impossible to use a perfect nappe shaped crest due to the 16-foot drum gates that were installed in the crest requiring a broader crest shape than would be produced by the same quantity of water flowing over a sharp crested weir. In fitting the nappe shape to the rounded crest, the highest point of the nappe shape is made to correspond to the highest point of the rounded crest. It should be kept in mind that the critical portion of the crest, from where nappe springs free, to just beyond the highest point, should receive more attention than the downstream portion. For the Boulder Spillway the best fitting nappe shape is that produced by $h_o \approx 30$. Two other shapes have been drawn, $h_o = 28$ and $h_o = 32$. The nappe shape produced by $h_o = 28$ fits fairly well upstream, but is thought to fall too far below the crest downstream. The nappe shape produced by $h_o = 32$ is considerably above the crest upstream in the critical region and is above downstream, indicating that it is too large. A little practice will enable one to become quite skilled in fitting nappe shapes to the rounded crests, even though they are very irregular.

Figure 5 shows the work sheet for Wilson Dam. Here the crest shape is nearly perfect and no trouble is experienced in finding the nappe shape that will fit the rounded crest.

Figure 6 shows Keokuk Dam, which because of gates on the crest is rather broad across the top, being termed a flat topped, rounded crest. For flat topped rounded crests, such as Keokuk, Cherokee Bluffs and Gatun Dams, the best fitting nappe shape will always fall inside the rounded crest, except for irregular

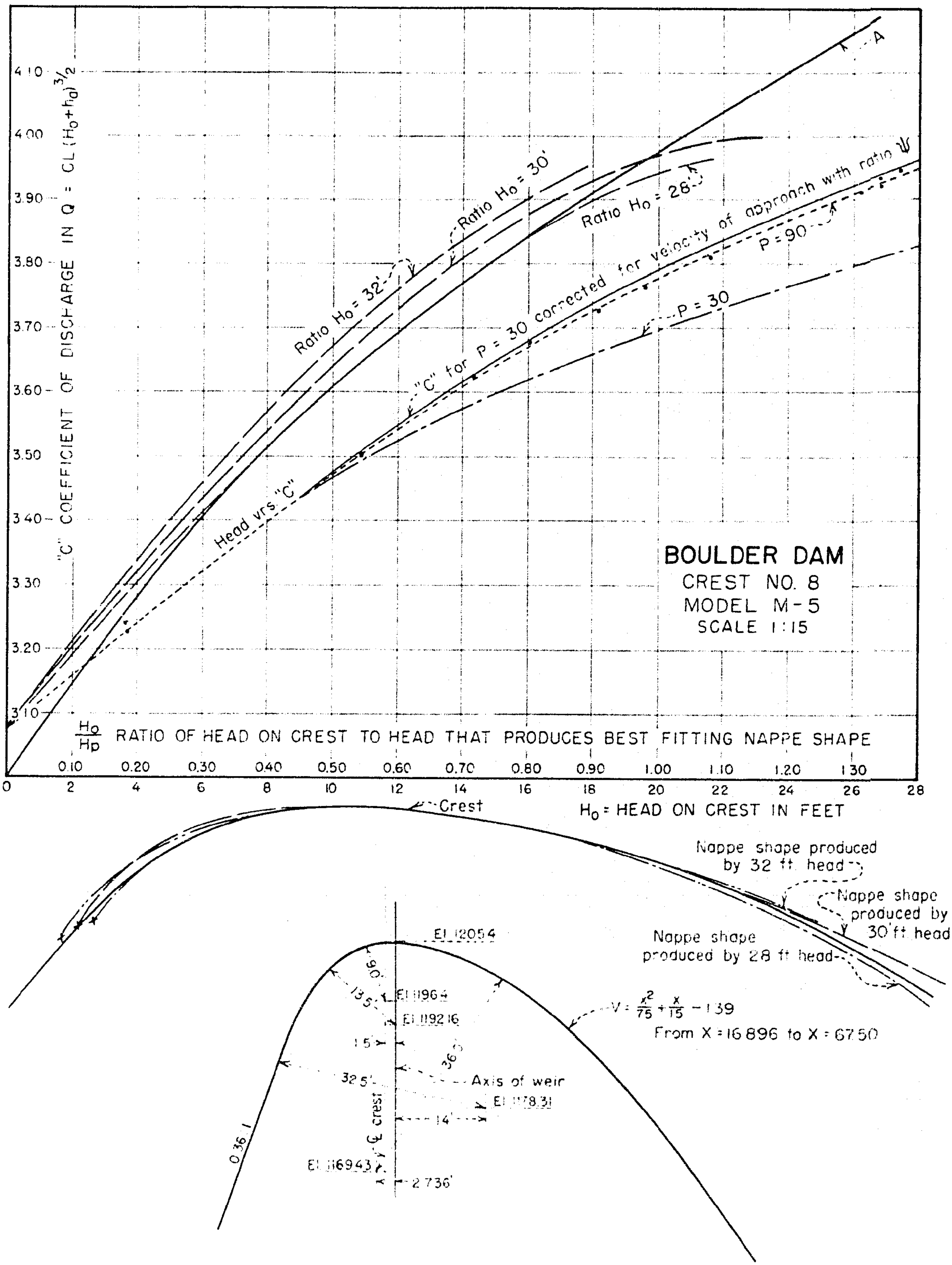
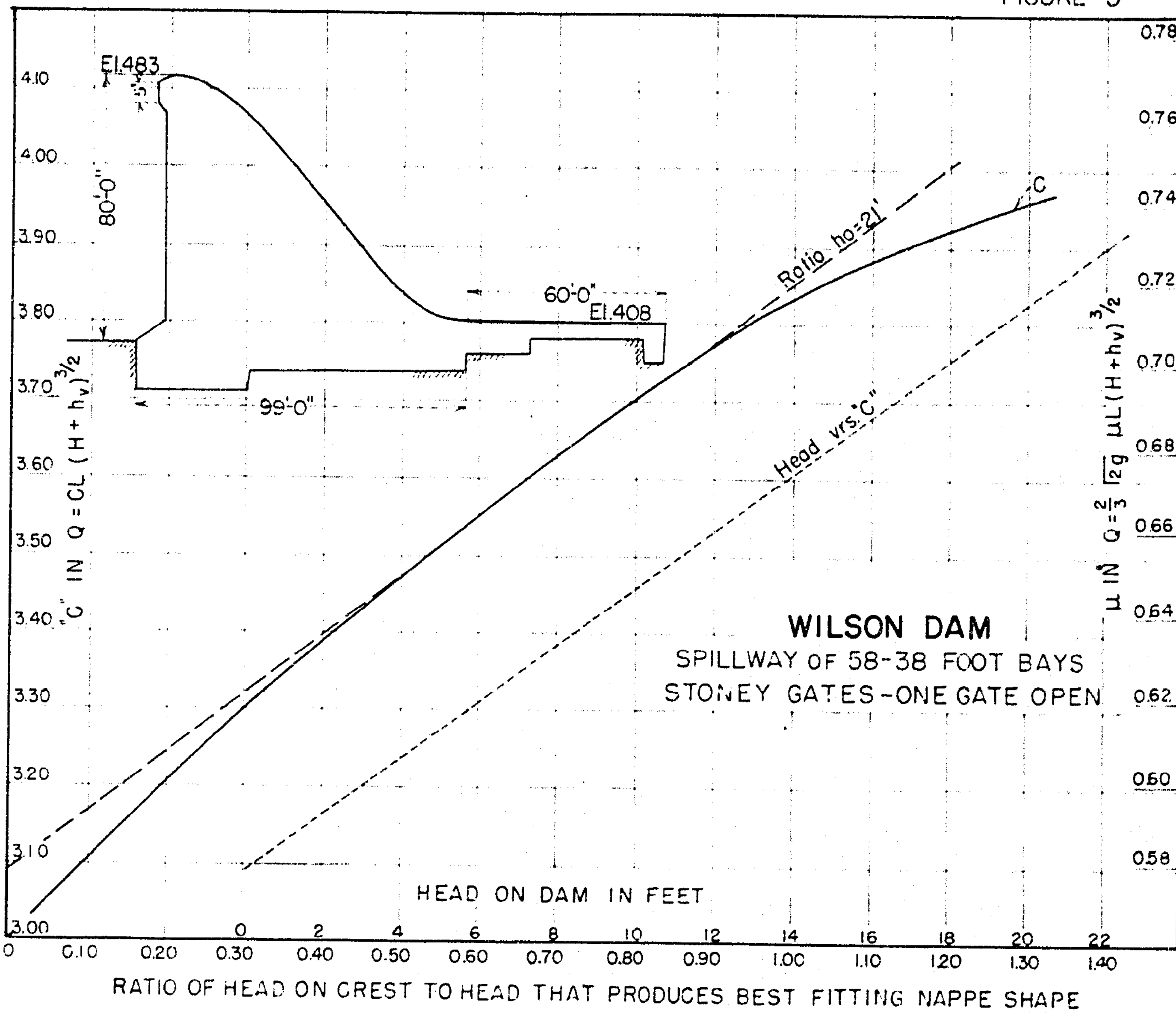
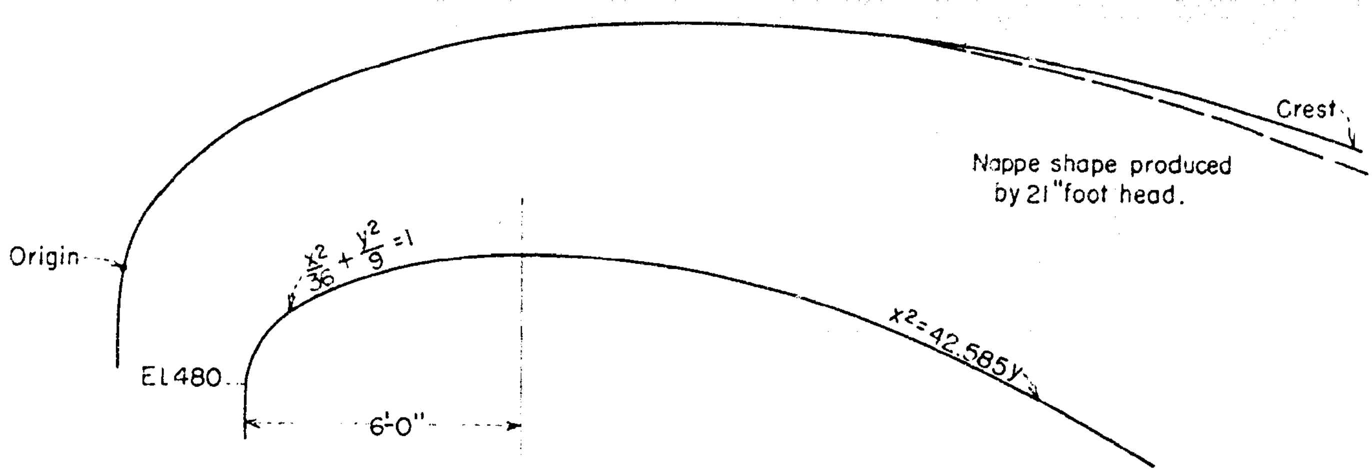

BOULDER DAM
CREST NO. 8
MODEL M-5
SCALE 1:15
"C" COEFFICIENT OF DISCHARGE IN Q = CL $(H_o+h_a)^{3/2}$
$\frac{H_o}{H_p}$ RATIO OF HEAD ON CREST TO HEAD THAT PRODUCES BEST FITTING NAPPE SHAPE
H_o = HEAD ON CREST IN FEET
Ratio H_o = 32'
Ratio H_o = 30'
Ratio H_o = 28'
"C" for P = 30 corrected for velocity of approach with ratio
P = 90
P = 30
Head vrs "C"
A
Crest
Nappe shape produced by 32 ft. head
Nappe shape produced by 30 ft head
Nappe shape produced by 28 ft head
El 1205.4
El 1196.4
El 1192.16
El 1178.31
El 1169.43
Axis of weir
℄ crest
$y = \frac{x^2}{75} + \frac{x}{15} - 1.39$
From X = 16.896 to X = 67.50
0.36:1
2.736'
14'
32.5'
1.5'
13.5'
9.0'
36.5'

FIGURE 5

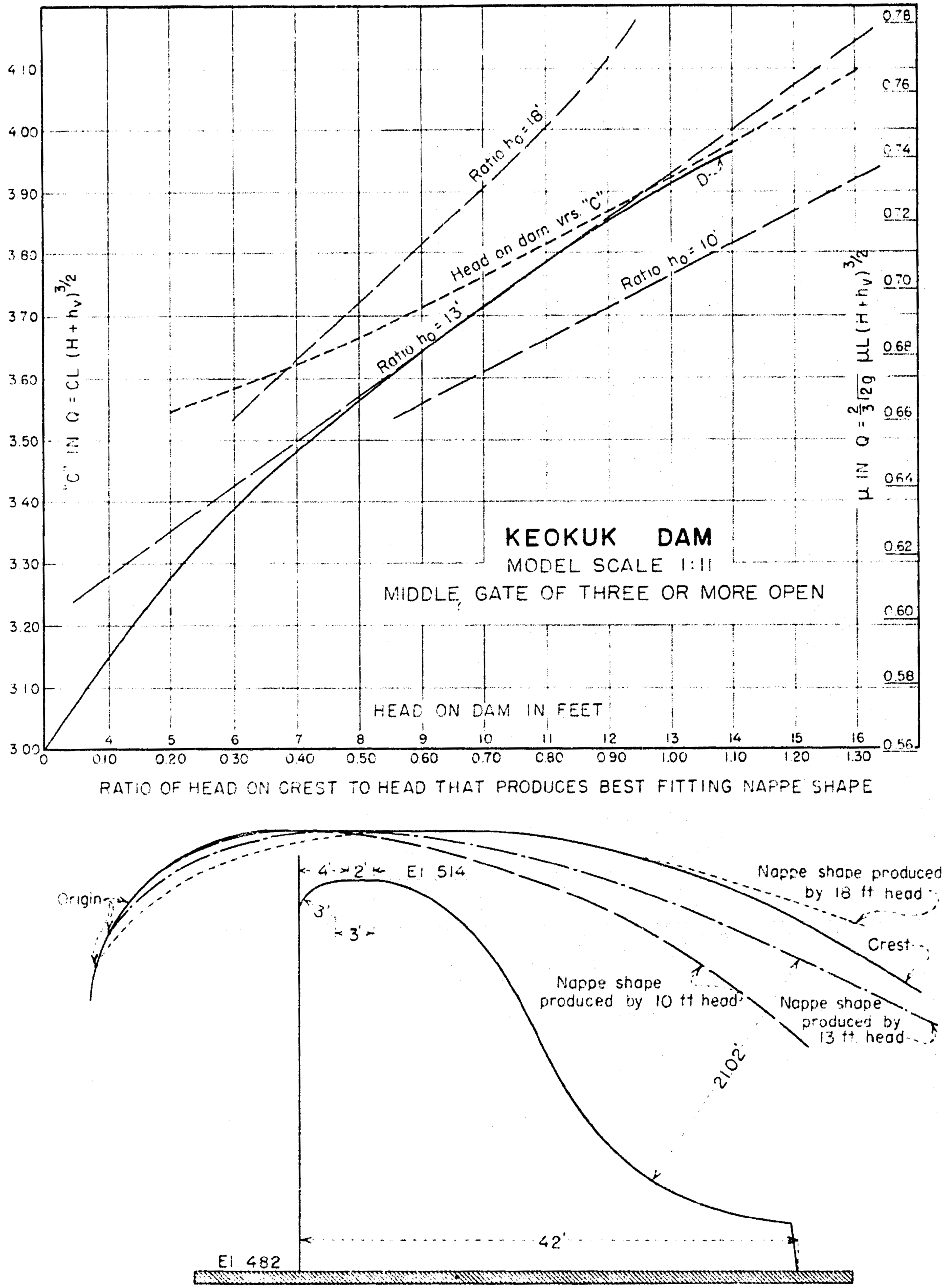


WILSON DAM
SPILLWAY OF 58-38 FOOT BAYS
STONEY GATES-ONE GATE OPEN
El.483
5'-6"
80'-0"
60'-0"
El.408
99'-0"
Ratio ho=21'
Head vrs."C"
C
"C" IN Q=CL(H+h$_v$)$^{3/2}$
μ IN Q=$\frac{2}{3}\sqrt{2g}$ μL(H+hv)$^{3/2}$
HEAD ON DAM IN FEET
RATIO OF HEAD ON CREST TO HEAD THAT PRODUCES BEST FITTING NAPPE SHAPE
3.00
3.10
3.20
3.30
3.40
3.50
3.60
3.70
3.80
3.90
4.00
4.10
0.58
0.60
0.62
0.64
0.66
0.68
0.70
0.72
0.74
0.76
0.78
0
2
4
6
8
10
12
14
16
18
20
22
0
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.80
0.90
1.00
1.10
1.20
1.30
1.40



Crest
Nappe shape produced by 21" foot head.
Origin
$\frac{x^2}{36}+\frac{y^2}{9}=1$
$x^2=42.585y$
El.480
6'-0"

FIGURE 6



KEOKUK DAM
MODEL SCALE 1:11
MIDDLE GATE OF THREE OR MORE OPEN
"C" IN Q = CL (H + hv)^3/2
μ IN Q = 2/3 √2g μL (H + hv)^3/2
Ratio ho = 18'
Ratio ho = 13'
Ratio ho = 10'
Head on dam vrs. "C"
D
HEAD ON DAM IN FEET
RATIO OF HEAD ON CREST TO HEAD THAT PRODUCES BEST FITTING NAPPE SHAPE
Origin
4' 2'
El 514
3'
3'
Nappe shape produced by 18 ft head
Crest
Nappe shape produced by 10 ft head
Nappe shape produced by 13 ft head
21.02'
42'
El 482

upstream faces, as example Gatun Dam. The best fitting nappe shape will start on the upstream portion and the high point of the nappe shape will lie between 0.3 and 0.6 of the total distance of the flat top of the crest, measured from the upstream edge of the flat portion of the crest. Three nappe shapes have been tried for the Keokuk Dam. Keeping in mind that the critical region was more important than the downstream portion, a nappe shape produced by $h_o = 13$ feet was chosen.

Besides the flat topped rounded crest weir, there is another group termed cylindrical crests. In this group a single radius is used to produce the shape of the rounded weir crest, and the upstream and downstream faces of the dam become tangent to it. The nappe shape never exactly fits this type of crest, but it has been found by analysis of several such crests and their coefficients that the head producing the best fitting nappe shape is approximately 1.6 times the radius used to form the crest; that is, $H_p = 1.6 \times$ radius. Rehbock has a formula for determining the discharge for this type of crest, which is later discussed.

Chapter V

Height of Weir

Having indicated how to determine the coefficient of discharge for various shapes of rounded crests, it is next necessary to correct the coefficient for the influence of the height of weir. Two corrections are necessary, the first for the kinetic energy contained in the velocity of approach, and the second is a correction of change in shape of nappe, due to the high velocities of approach. Considering the first correction, the discharge for a rounded crest weir is given by equation (3):

$Q = CLH_o^{3/2}$ and the value of H_o by

equation (4) $H_o = h_o + h_a$ then

$$Q = CL(h_o + h_a)^{3/2} \qquad 11.$$

In this formula the value of h_a depends upon the discharge of Q, so that it has to be solved by trial. Usually the value of h_a is first neglected and the Q is obtained from $Q = CL\ h_o^{3/2}$. Having determined the approximate Q, h_a is determined. Since:

$$h_a = \frac{V^2}{2g}$$

then $$h_a = \frac{Q^2}{2g(P + h_o)^2}$$

See Figure 1 for nomenclature and sketches. Many coefficients are determined from the formula $Q = KL\ h_o^{3/2}$. Here the effect of velocity of approach is included in the coefficient K. In order to reduce all coefficients to a comparable basis, it is necessary to modify this coefficient K so that it does not include the effect of velocity of approach.

$$KL\, h_o^{3/2} = CL(h_o + h_a)^{3/2}$$

$$\frac{K}{C} = \frac{(h_o + h_a)^{3/2}}{h_o^{3/2}}$$

Let the ratio $\frac{K}{C} = R$

Then $R = (1 + \frac{h_a}{h_o})^{3/2}$

But $h_a = \frac{Q^2}{2g(P + h_o)^2} = \frac{(K\, h_o^{3/2})^2}{2g(P + h_o)^2} = \frac{K^2\, h_o^3}{2g(P + h_o)^2}$

$$\frac{h_a}{h_o} = \frac{K^2}{2g}\left(\frac{h_o}{P + h_o}\right)^2 \qquad \text{------------------ 12.}$$

Substituting in the equation for R.

$$R = \left[1 + \frac{K^2}{2g}\left(\frac{h_o}{P + h_o}\right)^2\right]^{3/2}$$

Now let $\emptyset$ be a function of K and $\frac{h_o}{P + h_o}$ such that

$$R = 1 + \emptyset$$

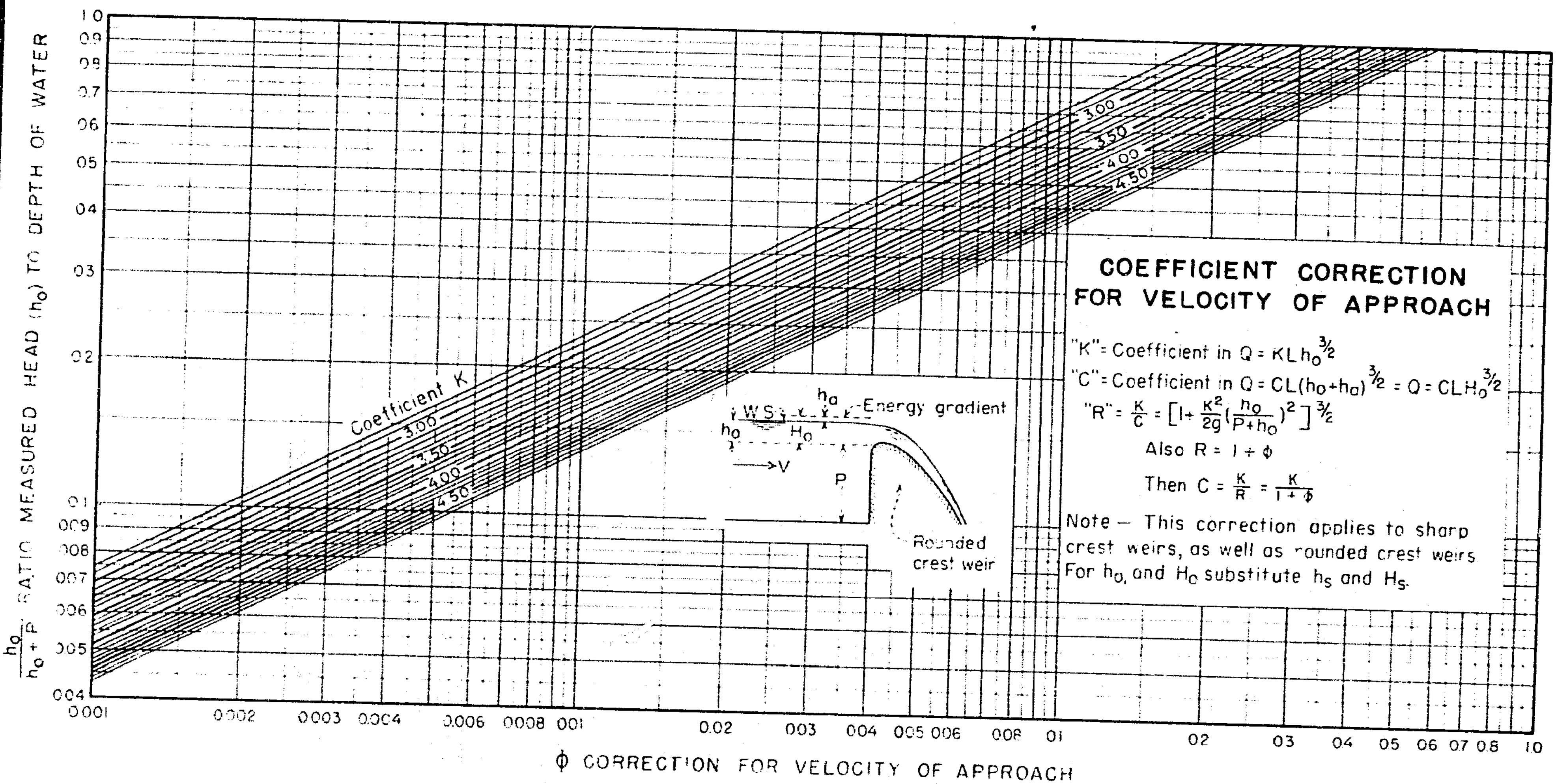
or $$\emptyset = \left[1 + \frac{K^2}{2g}\left(\frac{h_o}{P + h_o}\right)^2\right]^{3/2} - 1 \qquad \text{------------------ 13.}$$

Then $$C = \frac{K}{R} = \frac{K}{1 + \emptyset} \qquad \text{------------------ 14.}$$

In Figure 7 a graph is plotted for values of $\emptyset$ in terms of K and $\frac{h_o}{h_o + P}$. This method has been used to reduce computed coefficient K which included the velocity head of approach to the coefficient C, which does not include velocity head of approach.

The second correction for the height of weirs involves the variation in shape of the nappe from a sharp crest weir due to velocity of approach. Since the nappe becomes flatter and does

FIGURE 7


COEFFICIENT CORRECTION FOR VELOCITY OF APPROACH
"K" = Coefficient in $Q = KLh_o^{3/2}$
"C" = Coefficient in $Q = CL(h_o+h_a)^{3/2} = Q = CLH_o^{3/2}$
"R" = $\frac{K}{C} = \left[1+\frac{K^2}{2g}\left(\frac{h_o}{P+h_o}\right)^2\right]^{3/2}$
Also $R = 1+\phi$
Then $C = \frac{K}{R} = \frac{K}{1+\phi}$
Note — This correction applies to sharp crest weirs, as well as rounded crest weirs. For h_o and H_o substitute h_s and H_s.
Coefficient K
3.00
3.50
4.00
4.50
W.S.
h_a
Energy gradient
h_o
H_o
V
P
Rounded crest weir
$\frac{h_o}{h_o+P}$ RATIO MEASURED HEAD (h_o) TO DEPTH OF WATER
ϕ CORRECTION FOR VELOCITY OF APPROACH

not rise as high above the sharp crested weir for high velocities of approach, the relation between the head measured above the highest point and the head on the weir is affected.

The variation of $\frac{E}{h_s}$ is plotted against $\frac{h_a}{h_s}$ on logarithmic paper, Figure 8. The points shown are taken from Bazin's data (9). Experiments by the Bureau of Reclamation were much more extensive, giving values of $\frac{h_a}{h_s}$ up to .14, confirming the extension of the curves to that value.

It is shown on Page 16 that for negligible velocity of approach the coefficient of discharge for a perfect rounded crest weir was 3.98. This gives us the value of C when $\frac{h_o}{h_o + P} = 0$. Here $P = \infty$. The other extreme is when $P = 0$ or $\frac{h_o}{h_o + P} = 1.00$. This is the condition of a control section, and if friction is neglected, the value of C is as follows:

$$h_a = 1/2\ h_o \quad \text{or } h_a = 1/3\ H_o$$

The water is flowing at critical depth upstream from the edge, where the head h_o is measured.

Then $\frac{V^2}{2g} = 1/2\ h_o = 1/3\ H_o$. Considering a unit wide strip

$V = \frac{Q}{h_o}$ since

$$h_a = \frac{V^2}{2g} = \frac{Q^2}{2g\ h_o^2} = 1/3\ H_o\ , \text{ also } h_o = 2/3\ H_o$$

$$Q^2 = 2g \cdot \frac{4}{9}\ H_o^2 \cdot 1/3\ H_o$$

$$Q^2 = \frac{8}{27} \cdot g \cdot H_o^3$$

$$Q = \sqrt{\frac{8}{27} g}\ H_o^{3/2}$$

$$Q = 3.087\ (h_o + h_a)^{3/2} \tag{15.}$$

Thus the coefficient of discharge varies from 3.981 when $P = \infty$ to 3.087 when $P = 0$.

To calculate the theoretical coefficient C_t for various values of $\frac{h_s}{h_s + P}$ assume that the discharge over a unit length of sharp crest weir given by:

$$Q = K_s\ h_s^{3/2}$$

Then $$H_o = h_s + h_a - E = \left(1 + \frac{h_a}{h_s} - \frac{E}{h_s}\right) h_s$$ See Figure 1.

$$C_t = \frac{Q}{H_o^{3/2}} = \frac{K_s\ h_s^{3/2}}{\left[1 + \frac{h_a}{h_s} - \frac{E}{h_s}\right]^{3/2} \cdot\ h_s^{3/2}}$$

$$C_t = K_s \left[\frac{1}{1 + \frac{h_a}{h_s} - \frac{E}{h_s}}\right]^{3/2} \tag{16.}$$

Also $$Q = C_s\ (h_s + h_a)^{3/2}$$

or $$K_s = C_s\left(1 + \frac{h_a}{h_s}\right)^{3/2}$$

The coefficient of discharge for sharp crest weirs has been given by various authors. Here the value of 10/3 for C_s is used. This value has been found to fairly well approximate the coefficient for weirs of different heights. That this value is not exactly correct is shown by the fact that when $P = 0$ the discharge coefficient would be 3.333 instead of 3.087.

Then $$K_s = 10/3\left(1 + \frac{h_a}{h_s}\right)^{3/2} \tag{17.}$$

It is necessary to assume various values of $\frac{h_a}{h_s}$, and from Figure 8, the corresponding value of $\frac{E}{h_s}$ are found and the values of C_t may be computed. It is required to determine C_t in terms of $\frac{h_o}{h_o + P}$.

$$A = p + h_s. \quad \text{But } AV = Q \text{ and } Q = K_s h_s^{3/2}$$

$$p = \frac{K_s h_s^{3/2}}{V} - h_s. \quad \text{But } V = \sqrt{2gh_a}$$

$$\frac{1}{p} = \frac{\sqrt{2gh_a}}{K_s h_s} - \frac{1}{h_s}.$$

$$\frac{h_s}{p} = \frac{1}{K_s}\sqrt{\frac{2gh_a}{h_s}} - 1.$$

$$\frac{h_s}{h_s + p} = \frac{1}{K_s}\sqrt{2g\frac{h_a}{h_s}} \qquad \text{------------------ 19.}$$

From consideration of Figure 1.

$$\frac{h_o}{h_o + P} = \frac{h_s - E}{h_s - E + P}$$

$$\frac{h_o}{h_o + p} = \frac{h_s}{h_s + p}\left(1 - \frac{E}{h_s}\right) \qquad \text{------------------ 20.}$$

Also $\qquad H_o = h_s - E + h_a = h_s\left(1 - \frac{E}{h_s} + \frac{h_a}{h_s}\right)$

$$H_o + P = h_s + h_a + p = h_s\left(1 + \frac{h_a}{h_s} + \frac{p}{h_s}\right)$$

Then $\qquad \frac{H_o}{H_o + P} = \dfrac{1 - \frac{E}{h_s} + \frac{h_a}{h_s}}{1 + \frac{h_a}{h_s} + \frac{p}{h_s}} \qquad \text{------------------ 21.}$

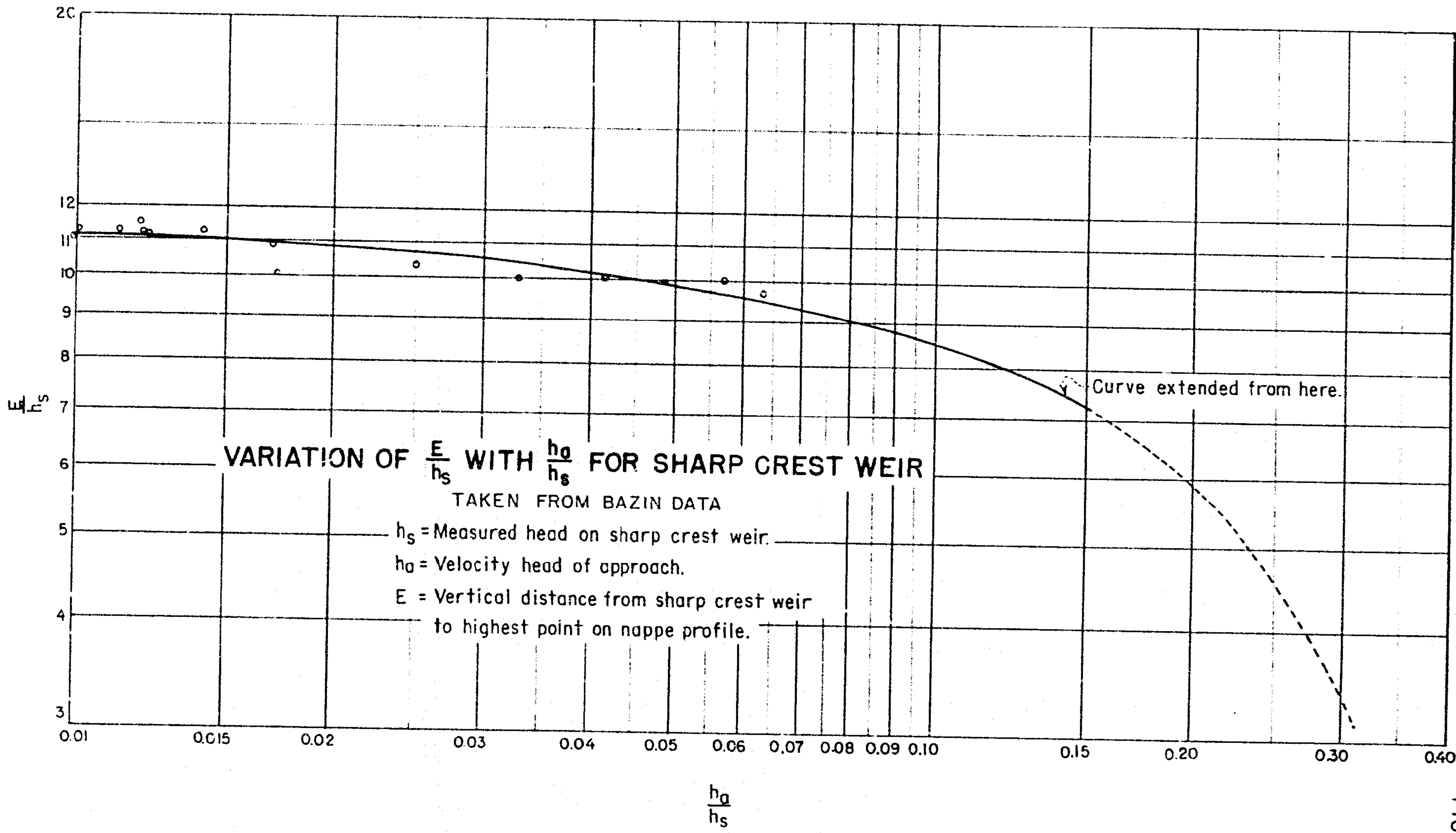

VARIATION OF $\frac{E}{h_s}$ WITH $\frac{h_a}{h_s}$ FOR SHARP CREST WEIR
TAKEN FROM BAZIN DATA
h_s = Measured head on sharp crest weir.
h_a = Velocity head of approach.
E = Vertical distance from sharp crest weir to highest point on nappe profile.
Curve extended from here.
$\frac{E}{h_s}$
20
12
11
10
9
8
7
6
5
4
3
0.01
0.015
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.10
0.15
0.20
0.30
0.40
$\frac{h_a}{h_s}$

FIGURE 8

Since $\frac{h_s}{h_s + p}$ has been computed, then $\frac{p}{h_s}$ can be calculated. The calculations are shown in Table 1, and the value of C_t plotted against $\frac{h_o}{h_o + P}$ is shown in Figure 9.

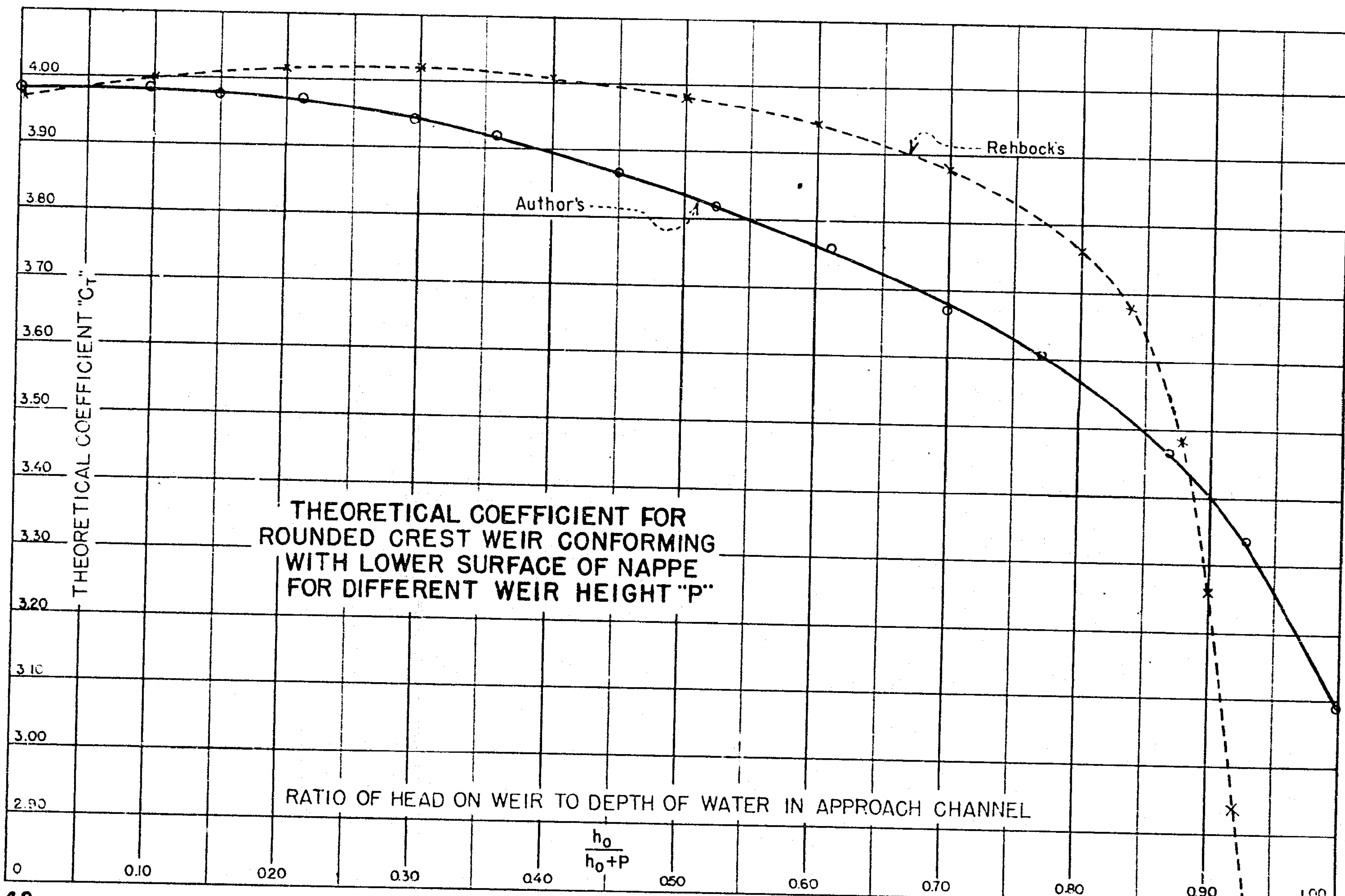
The theoretical coefficient of discharge varies due to a change of shape of nappe with different velocities of approach. This variation is computed on the basis of a rounded crest weir fitting the nappe shape for different velocities of approach. This theoretical coefficient C_t is the maximum coefficient that can be obtained without obtaining negative pressures on the face of the weir. This relation of variation of theoretical coefficient with the height of rounded crest weirs may be expressed as a ratio of coefficient C_t, with the value of 3.98, the theoretical coefficient without velocity of approach considered as a standard. This ratio $\frac{C_t}{3.98}$ or Ψ is plotted against both $\frac{H_o}{H_o + P}$ and $\frac{h_o}{h_o + P}$, in Figure 10. It is logical that all coefficients for high weirs without velocity of approach, regardless of the head, could be reduced by the ratio Ψ if the dam silted up or the P of the rounded crest weir was reduced. The ratio Ψ to use would be that obtained from Figure 10, the new P being used in calculating $\frac{H_o}{H_o + P}$. The converse of this is also true , given the coefficient for a low rounded crest, by dividing it by the ratio Ψ the coefficient will be obtained, that the crest would have had, had it been a high weir with negligible velocity of approach. The model of Boulder Spillway crest, shown in Figure 4, was tested with two different heights of floor in the approach channel. The coefficient, where P equals 30 feet, is about $2\frac{1}{2}$ percent lower than for P equals 90 feet. In order to make the two coefficients curves comparable, it is necessary to correct those with P equals 30 feet for the effect of a larger velocity head of approach produced by the lower

TABLE I

CALCULATION FOR REDUCTION RATIO ψ DUE TO CHANGE OF NAPPE SHAPE WITH VELOCITY OF APPROACH.

$\frac{h_a}{h_s}$	$\frac{h_s}{h_s + p}$	$\frac{h_o}{h_o + P}$	$\frac{H_o}{H_o + P}$	K_s	$\frac{E}{h_s}$	C_t	$\frac{C_t}{3.983} = \psi$
000	.000	.000	.000	3.333	.1120	3.983	1.0000
002	.107	.195	.107	3.343	1119	3.981	.9995
005	167	148	.148	3.358	.1118	3.978	.9987
010	.238	.212	217	3.383	1112	3.970	.9967
020	332	.296	.301	3.432	1087	3.945	.9905
030	.399	357	.365	3.484	.1056	3.930	.9842
050	.499	.449	.463	3.586	.0993	3.869	.9714
070	.576	523	.540	3.689	.0929	3.819	.9588
.100	.666	.611	634	3.847	.0846	3.760	.9440
.150	.754	.699	.730	4.110	.2725	3.625	.9227
200	.821	.773	.804	4.381	.0587	3.583	.9021
.250	.862	.823	.855	4.658	0451	3.521	.8340
.300	889	.868	.888	4.941	0332	3.465	.8699
500*	.925	.925	.948	6.123	.0000	3.333	.8368
500*	1.000	1.000	1.000	5.667	.0000	3.087	.7750

* Two values of h_a/h_s arise from fact that for $p = 0$ the coefficient of discharge C_s for sharp crest weir has two values $C_s = 3.087$ being for broad crested weirs and is correct. $C_s = 10/3$ is value assumed to apply throughout range of $h_s/(h_s + p)$, but variations when $h_s/(h_s + p)$ is equal from .80 to 1.00 is not well understood.



THEORETICAL COEFFICIENT FOR ROUNDED CREST WEIR CONFORMING WITH LOWER SURFACE OF NAPPE FOR DIFFERENT WEIR HEIGHT "P"
THEORETICAL COEFFICIENT "C_T"
RATIO OF HEAD ON WEIR TO DEPTH OF WATER IN APPROACH CHANNEL
$\frac{h_0}{h_0+P}$
Rehbock's
Author's
4.00
3.90
3.80
3.70
3.60
3.50
3.40
3.30
3.20
3.10
3.00
2.90
0
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.80
0.90
1.00

FIGURE 9

weir. This is done by assuming various H_o calculating the value of $\frac{H_o}{H_o + P}$, then determining the reduction ratio Ψ from Figure 10, and then dividing the coefficient corresponding to the assumed head by the reduction ratio. This has been done for the coefficient obtained with P equals 30 feet and the resulting curve falls nearly on top of the one with P equals 90 feet.

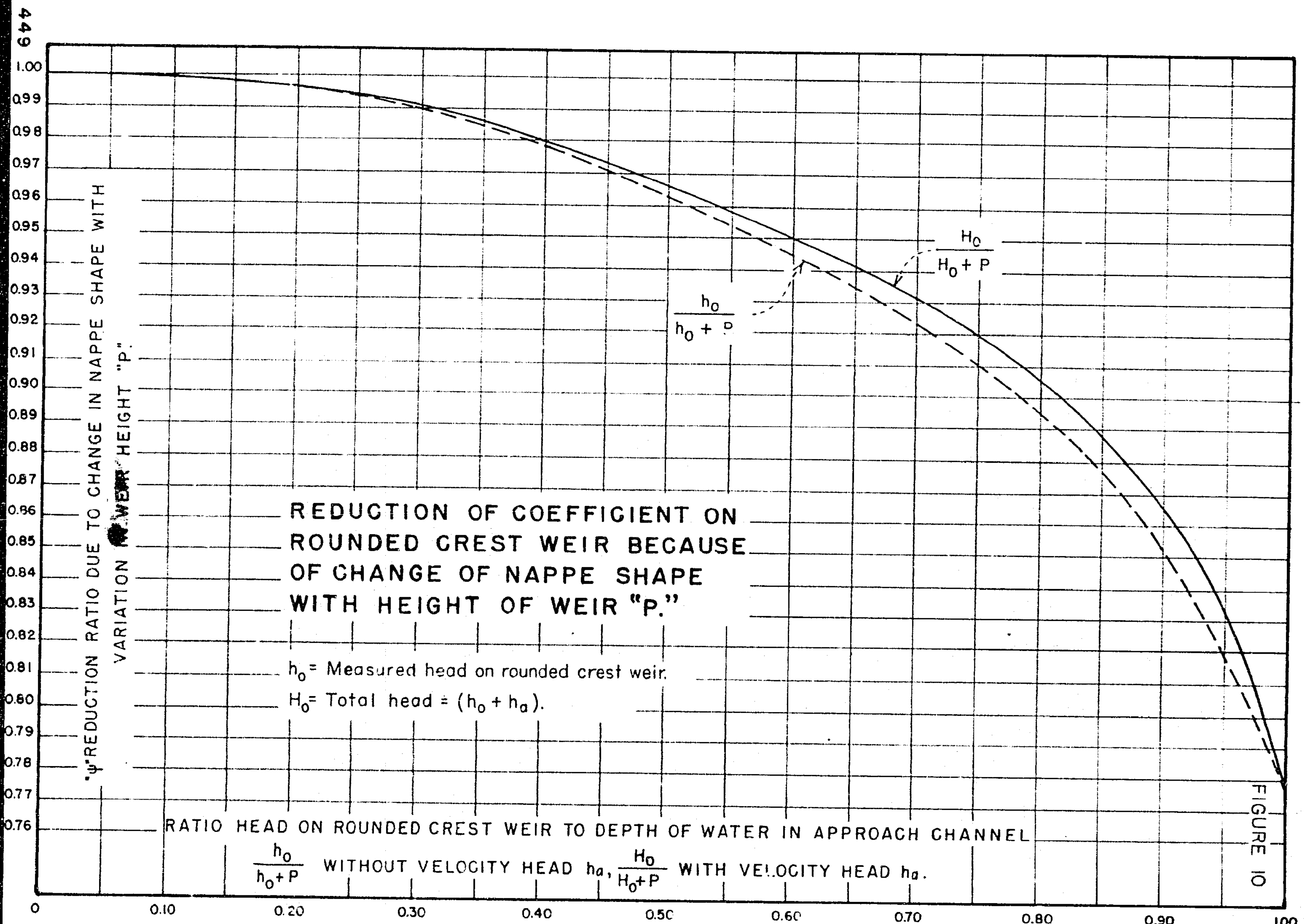

REDUCTION OF COEFFICIENT ON ROUNDED CREST WEIR BECAUSE OF CHANGE OF NAPPE SHAPE WITH HEIGHT OF WEIR "P."
h_0 = Measured head on rounded crest weir.
H_0 = Total head = $(h_0 + h_a)$.
$\frac{H_0}{H_0 + P}$
$\frac{h_0}{h_0 + P}$
"ψ" REDUCTION RATIO DUE TO CHANGE IN NAPPE SHAPE WITH VARIATION IN WEIR HEIGHT "P."
RATIO HEAD ON ROUNDED CREST WEIR TO DEPTH OF WATER IN APPROACH CHANNEL
$\frac{h_0}{h_0+P}$ WITHOUT VELOCITY HEAD h_a, $\frac{H_0}{H_0+P}$ WITH VELOCITY HEAD h_a.
1.00
0.99
0.98
0.97
0.96
0.95
0.94
0.93
0.92
0.91
0.90
0.89
0.88
0.87
0.86
0.85
0.84
0.83
0.82
0.81
0.80
0.79
0.78
0.77
0.76
0
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.80
0.90
1.00

FIGURE 10

Cnapter VI

Discussion of Results

There are 78 different rounded crest shapes analyzed, composed of 47 free rounded crests without piers or gates, and 31 rounded crests with gates. Many of the crest shapes were tested with piers, and then were tested again without piers. Such cases were counted as one crest shape. The number of coefficients versus head relations analyzed was greater for the reason stated above and also many of the crest shapes were modified by a change in height of weir or slope of the upstream or downstream face. Fifty-two coefficient head relations were from free crests, eight from rounded crests with only one gate discharging, and forty-two were from rounded crests with three or more gates discharging, making a total of one hundred and two coefficients versus head relations analyzed. Of these figures about 15 percent of the data was confidential, and consequently this data does not appear in this thesis; however, it shows close agreement with the results shown herein. Most of the rounded crests analyzed were from models; however, seven of the crest shapes tested were from prototype structures. Here, as in all hydraulic work, there is a lack of data from prototype structures.

As previously explained, the data were divided into three general classes, depending upon the nature of contractions.

Class 1. Containing free rounded crests, without piers, and was again divided into "A" curves and "B" curves, because of the large number of curves (see Tables II and III). In general the "A" curves have less actual head on the crest than do the "B" curves.

Class II. Contains all the data from rounded crests with only one gate discharging. The curves are termed "C" curves (see Table IV).

Class III. Contains all rounded crests with three or more gates discharging. This class is divided into two groups. The data from crests with radial or Stoney gates on them are termed "D" curves, and those from crests with drum gates are termed "E" curves. It was first thought that because the drum gates had to be set into the crest, that crests larger than usual would be needed. Later it was found that the face plates of the drum gates could be rolled according to compound radii, which in turn would very nearly approximate the lower nappe shape from a sharp crest weir. There is no marked difference between the "D" and "E" curves.

In the Tables II to V an explanation of some columns is appropriate. In Column 4, the head to produce the best fitting nappe shape is listed. In the case of models, the H_p is listed in terms of the prototype structure, except here the model was not of any particular prototype structure, in which case, the actual head on the model is listed.

Column 5 gives the coefficient, unless otherwise stated, when the $\frac{H_o}{H_p} = 1.00$. When the coefficient curve does not extend to $\frac{H_o}{H_p} = 1.00$, the coefficient is given as the value of $\frac{H_o}{H_p}$, stated after the coefficient.

Column 11 gives the type of curves used to make up the profile of the rounded crests in the critical region; beginning with the upstream edge first.

Column 12 gives the source of the data and refers to the reference number listed in the Bibliography.

In analyzing the available data on rounded crest weirs, it was first necessary to classify it according to the conditions of of contraction. Next a work sheet was prepared similar to those shown in Figures 4, 5, and 6, first plotting the coefficient versus head relations, then taking a cross-section of the rounded crest

TABLE II

WEIRS WITH FREE ROUNDED CRESTS – "A" CURVES

(WITHOUT PIERS OR GATES)

1 CURVE NO.	2 NAME OF MODEL OR DAM	3 SIZE OF MODEL	4 HEAD ON NAPPE H_p	5 COEFFICIENT AT $\frac{H_o}{H_p} = 1.00$	6 NATURE OF NAPPE FIT	7 UPSTREAM FACE OF DAM	8 RELATION OF COEFFICIENT RATIO TO BASE CURVE	9 P	10 $\frac{H_p}{H_p+P}$	11 TYPE CREST	12 REFERENCE NO.
1A	Karlsruhe	Model	.29	4.052	Poor	Small slope	Above	.994	.225	Cylinder	11 Fig. 16 No. 6
2A	Laboratory	1:19.30	11.5	4.08	Poor	Slope 1.1	Same to 4: then above	8.46	.585	Slope cylinder	11 Fig. 16 No. 2
3A	Laboratory	Model	.30	3.963	Poor	Vertical	Below	0.735	.290	2 Radii	11 Fig. 16 No. 3
4A	Uni. of Munich	1.20	4.5	4.05	Poor	Vertical	Above	8.76	.339	60 Cylinder	14 pp. 447
5A	Uni. of Munich	1.20	4 5	3.92	Poor	Vertical	Below	8.76	.339	60 Cylinder	14 pp. 447
6A	Karlsruhe	1.20	4.8	4.00	Poor	Vertical	Below to .80: then same			45 Cylinder	11 Fig. 17
7A	Uni of Munich	1.20	6.4	4.02	Good	Vertical	Below to .70: then above	7.938	.446	Radii & Parabola	14 pp. 447
8A	Boulder Dam	1:60	11.0	3.880	Good	Vertical	Below	120.0	.084	Shaped to Nappe	28
9A	Boulder Dam	1:60	10.5	3.895	Good	Vertical	Above to .75: then below	120.0	.080	Shaped to Nappe	28
10A	Martin Dam	1:54.9	24.0	4.100	Good	Vertical	Below to .05: then above	64.0	.272	Broad Crest & Radii & Parabola	26
11A	Martin Dam	1:54.9	24.0	4.040	Good	Vertical	Below to .70: then above	64.0	.272		26
12A	Lock 12 Dam	1:48	14.0	3.958	Poor	Vertical	Below	58.5	.193	Radius & Broad Crest	26
13A	Mitchell Dam	1:51.4	20.0	3.952	Fair	Vertical	Below to 1.10: then above	59.0	.253	Radius & Broad & Parabola	26
14A	Upper Tallassee	1:68.6	28.0	4.043	Fair	Vertical	Below to .50: then above	83.8	.251	2 Radii & Parabola	26
15A	Empirical (1.5)	Model	.3133	3.970	Good	Vertical	Same	1.167	.211	Radius & Parabola	26
16A	Model (S.C.) Crat	Model	.30	3.98	Near perfect	Vertical	Same to .60: then below	1.167	.204	Nappe shape	26
17A	Cherokee Bluffs	1.549 & 1:100	24.0	4.160	Good	Vertical	Below to .60: then above	77.0	.238	Radii & Broad & Parabola	26
18A	Boulder Dam	1:60	45.0	3.73 @ .50	Fair	1:18 36:1	Below to .30: then above	35.0	.499	972 Comp. Radii	28
19A	Holyoke Dam	1:35:6	13.0	4.075	Poor	Stepped	Same to .80: then above	30.0	.302	Slope & Radii	11 Fig. 16 No. 4
20A	Iowa Uni.	Model	1.2	3.82 @ .70	Poor	1 on 1.1 slope	Below to .67: then above	1.55	.436	Radius	18
21A	Imperial Dam No.(1)	1:30	20.0	3.90 @ .80	Fair	Slope	Above	26.0	.435	Radius & Parabola	23
22A	Imperial Dam No.(2)	1:30	15.0	3.90 @ .80	Fair	Slope	Above	27.0	.357	Radius & Parabola	23
24A	Taylor Park	1:50	6.00	3.97	Very good	Vertical	Same to .95: then below	3.0	.666	Radius & Parabola	22

TABLE III

WEIRS WITH FREE ROUNDED CRESTS - "B" CURVES

(WITHOUT PIERS OR GATES)

1 CURVE NO	2 NAME OF MODEL OR DAM	3 SIZE OF MODEL	4 HEAD ON NAPPE H_p	5 COEFFICIENT AT $\frac{H_o}{H_p} = 1.00$	6 NATURE OF NAPPE FIT	7 UPSTREAM FACE OF DAM	8 RELATION OF COEFFICIENT RATIO TO BASE CURVE	9 P	10 $\frac{H_p}{H_p+P}$	11 TYPE CREST	12 REFERENCE NO.
1B	Water Supply # 200 *19	Model	6.00	3.94	Poor	1:1 Slope	Below	5.28	.530	Slope & Radii	1
2B	Madden	1:72	48.0	3.86 @ .70	Poor	1:0.075	Above	167.5	.223	Cylinder	29
3B	Karlsruhe	Model	1.05	3.70 @ .50	Poor	Cylinder	Same to .25; then above	6.56	.566	Cylinder	11
4B	Keokuk	1:11	13.0	4.042	Poor	Vertical	Below to .80; then above	32.0	.289	Radius & Broad	16
5B	Uni of Wis	Model	.95	3.82	Good	Slope 2:1	Below	1.24	.432	Nappe Shape	30
6B	Uni. of Wis	Model	.95	3.81	Good	Slope 2:1	Below	2.13	.308	Nappe Shape	30
7B	Uni of Wis	Model	.95	3.96	Good	Vertical	Below	1.25	.432	Nappe Shape	30
8B	Uni of Wis	Model	.95	3.84	Good	Vertical	Below	2.13	.308	Nappe Shape	30
9B	LaGrange	Proto	19.5	3.62 @ .50	Poor	Vertical	Below	129 *	---	Radii & Broad	19
10B	Uni of Wis	Model	1.3	4.122	Good	Vertical	Same to .50; then above	6.11	.175	Nappe Shape	30
11B	Uni of Wis	Model	1.3	3.936	Good	2:1	Below	6.11	.175	Nappe Shape	30
12B	Water Supply # 33	Model	3.0	3.71 @ .90	Poor	Vertical	Below	14.25	.174	Slope & 2 Radii	1
13B	Austin	Proto	16.7	3.32	Good	Vertical	Below	60 *	---	2 Radii	1
14B	Diversion Yakima	Proto.	5.00	4.04	Poor	Vertical	Below to .65; then above	7.5	.400	Cylinder	31
15B	Boulder Dam	1:20 & 1:60	45.0	3.90 @ .80	Fair	0.3.6:1	Same to .40; then above	90.0	.333	4 Radii	28
16B	Boulder Dam	1:20	30.0	3.99	Good	0.3.6:1	Above	94.0	.242	4 Radii	28
17B	Lock 18 high	1:24	15.0	4.030	Good	Vertical	Below to .20; then above	28.3	.347	3 Radii & Parabola	26
18B	Lock 18 low	1:6.68	32.0	4.005	Good	Vertical	Below to .50; then above	75.4	.298	2 Radii & Parabola	26
19B	Dillman Munich	Model	.06	4.03	Good	Vertical	Below to .40; then above	0.99	.070	Elipse & Parabola	7
20B	Dillman Munich	Model	.133	3.87	Good	Overhang	Below	1.00	.117	Elipse & Parabola	7
21B	Dillman Munich	Model	.148	3.89	Good	Short Slope	Below	1.00	.129	Elipse & Parabola	7
26B	M.I.T. Series IV	Model	.232	3.99	Perfect	Vertical	Same	1.31	.148		2

* Probably silted up

TABLE IV

WEIRS WITH ROUNDED CRESTS

(DISCHARGE FROM ONLY ONE GATE)

1 CURVE NO.	2 NAME OF MODEL OR DAM	3 SIZE OF MODEL	4 HEAD TO PRODUCE NAPPE H_p	5 COEFFICIENT AT $\frac{H_o}{H_p} = 1.00$	6 NATURE OF NAPPE FIT	7 UPSTREAM FACE OF DAM	8 RELATION OF COEFFICIENT RATIO TO BASE CURVE	9 P	10 $\frac{H_p}{H_p+P}$	11 TYPE CREST	12 REFERENCE NO.
1C	Gatum	Proto	26	3.91	Fair	Vertical & Stepped	Above	60.0	.303	Stepped & Broad & Parabola	25
2C	Keokuk	Proto	13	3.75	Fair	Vertical	Below	0.32	.289	Radius & Broad & Parabola	16
3C	Cherokee Bluffs	Model	1:11 & 1:36	3.705	Good	Vertical	Below	77.0	.238	Radius & Broad & Parabola	26
4C	Wilson	Proto	21	3.853	Good	Overhang	Same	.80	.208	Ellipse & Parabola	17
5C	Wheeler	1:36	17	3.72@ .70	Fair	Vertical	Above	43.0	.266	Compound Radii	21

TABLE V

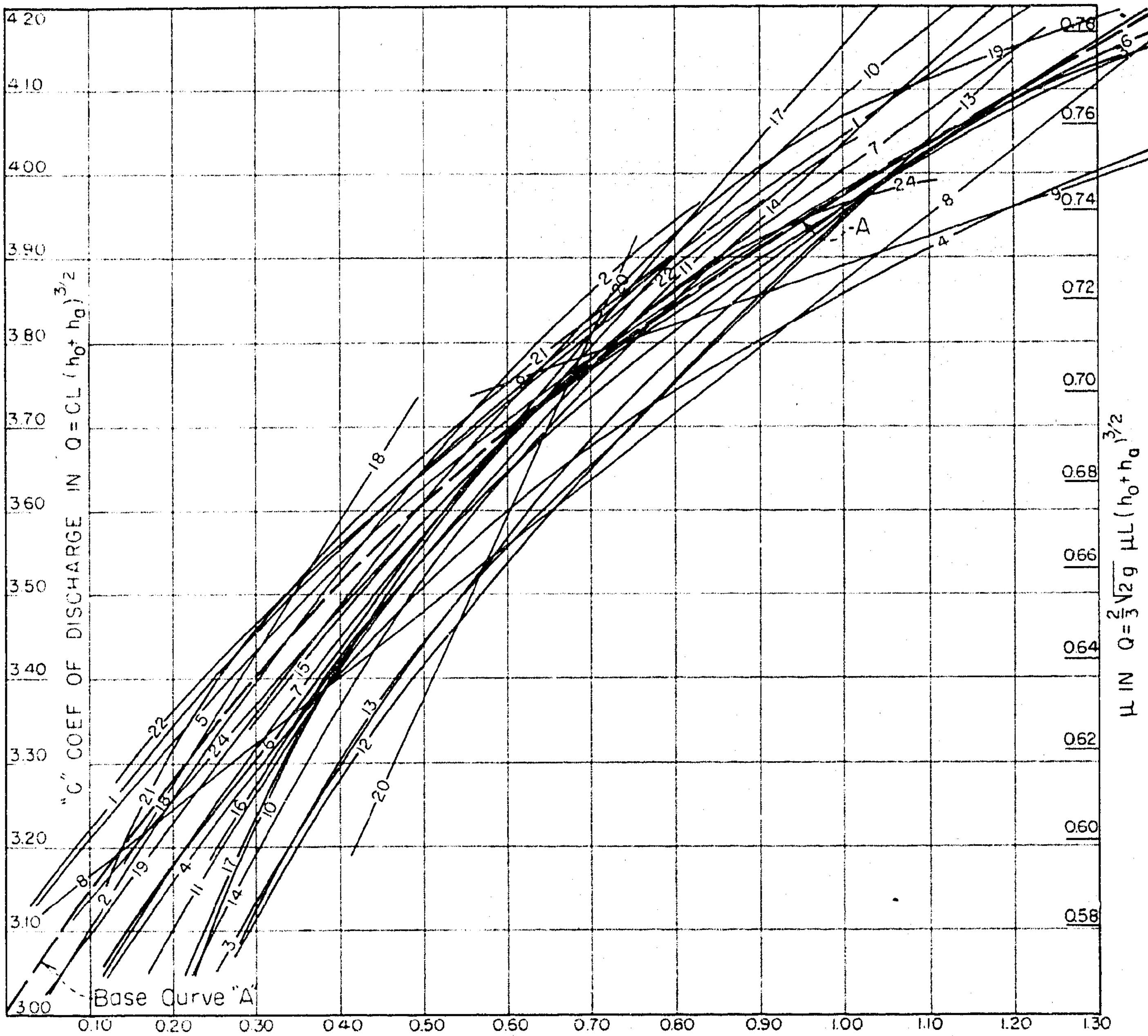
WEIRS WITH PIERS & GATES ON ROUNDED CRESTS

DISCHARGE FROM THREE OR MORE GATES

1 CURVE NO	2 NAME OF MODEL OR DAM	3 SIZE OF MODEL	4 HEAD ON NAPPE H_P	5 COEFFICIENT AT $\frac{H_0}{H_P}=1.00$	6 NATURE OF NAPPE FIT	7 UPSTREAM FACE OF DAM	8 RELATION OF COEFFICIENT RATIO TO BASE CURVE	9 P	10 $\frac{H_P}{H_P+P}$	11 TYPE CREST	12 REFERENCE NO.
1D	Cherokee Bluffs	1:30 & 1:100	24	4.00	Good	Vertical	Below to .65; then above	17.0	.238	Radius & Broad & Parabola	26
2D	Gatum	Proto.	26	4.023	Fair	Vertical & Stepped	Above	60	.303	Stepped & Broad & Parabola	16
5D	Keokuk	1:11	13	3.93	Poor	Vertical	Above to .85; then below	32	.289	Radius & Broad & Parabola	16
6D	Keokuk	Proto.	13	3.938	Poor	Vertical	Above to .90; then below	32	.289	Radius & Broad & Parabola	
13D	Wheeler	1:36	17	4.041	Fair	Vertical	Below to .32; then above	43	.266	Compound Radii	21
14D	Grand Valley	Proto	8.67	3.26 @ .50	Poor	Vertical	Above	7.5	.536	1 Radius	12
16D	Bull Lake		13	3.99	Good	Vertical	Below to .40; then above	4	.764	Radii & Parabola	32
28D	Mt Morris	1:16 & 1:64	28.8	3.82 @ .80	Good	Vertical	Below to .50; then above			Radius & Parabola	
1E	Boulder M-1	1:60 & 1:20	50	3.63 @ .50	Poor	Vertical	Above	35	.464	1 Radius	28
2E	Boulder M-3	1:20	50	3.70 @ .50	Poor	Slope .364	Above	35	.463	2 Radii	28
3E	Uni. of Munich	1:20	45	3.98	Good	Vertical	Below	7.938	.446	Radius & Parabola	14 pp. 447
4E	Madden	1:72	48	3.75 @ .80	Poor	Vertical	Above to .60; then below	167.5	.223	Cylinder	29
5E	Boulder	1:60	45	3.75 @ .60	Fair	0.36:1	Below to .35; then above	35	.445	Compound Radii	28
6E	Boulder M-5	1:20	30	3.88	Good	0.36:1	Above to .85; then below	35	.445	Compound Radii & Parabola	28
7E	Boulder C-5	1:60	30	3.88	Good	0.36:1	Above to .85; then below	35	.445	Compound Radii & Parabola	28
8E	Boulder C-6	1:100	30	3.88	Good	0.36:1	Above to .85; then below	35	.445	Compound Radii & Parabola	28
9E	Norris	1:72	36	3.92	Good	Vertical then 57:1	Same	198	.154	Slope & 2 Radii	20
10E	Grand Coulee	1:15	57.5	3.54 @ .50	Poor	Vertical	Below	95	.378	Cylinder	28
11E	Boulder	1:60	45	3.77 @ .60	Fair	0.36:1.00	Below to .40; then above Ave.	35	.445	Radius & Parabola	28

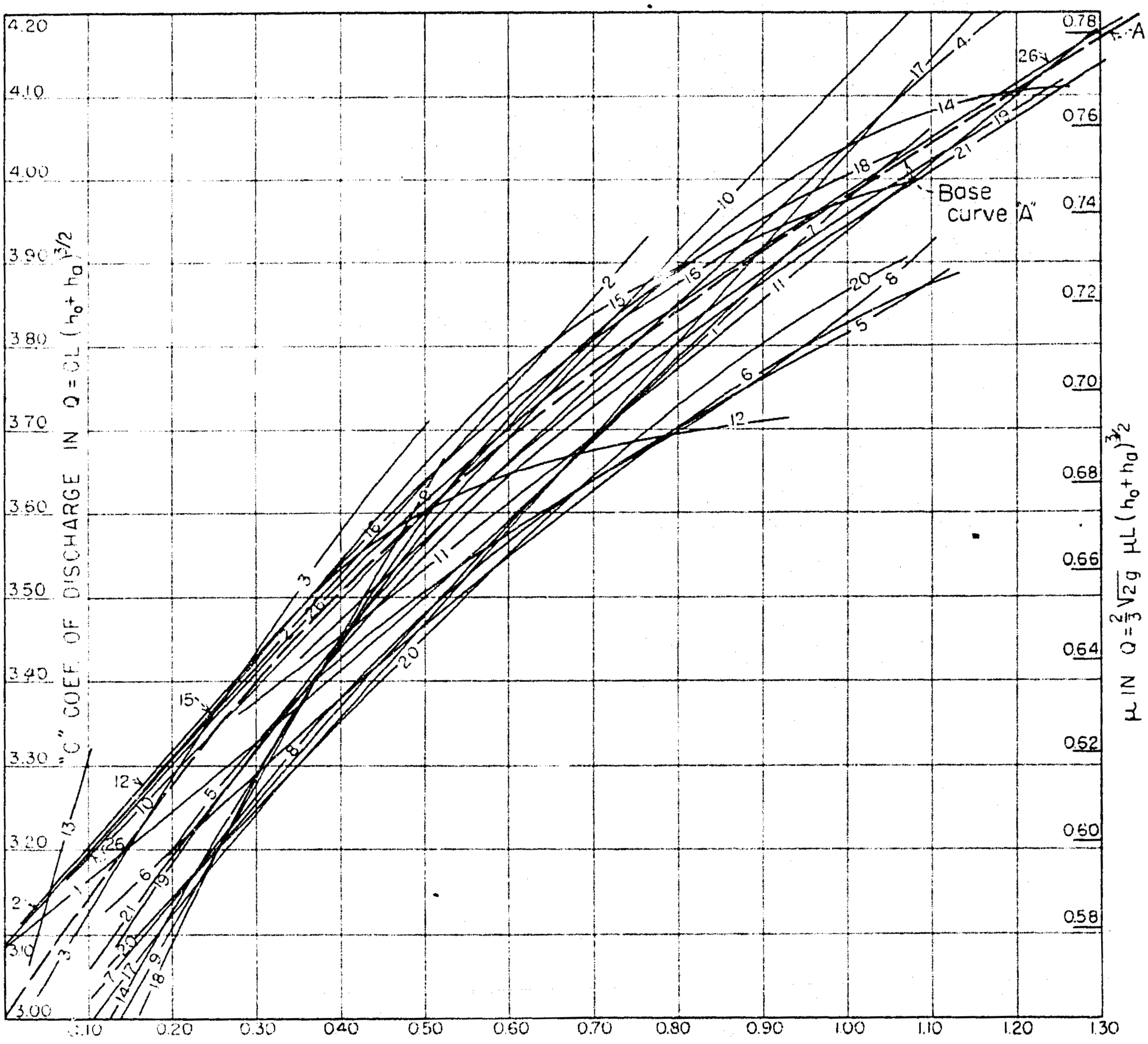
and fitting a nappe shape to it. In most coefficient curves the actual head on the crest was not extended much beyond H_p (the head producing the best fitting nappe), thus the ratio of $\frac{H_p}{P + H_p}$ will indicate the magnitude of the maximum velocity of approach. This ratio is listed in Column 10 of the Tables II, III, IV, and V, for all coefficient curves. When the coefficient K, which includes velocity of approach is given and the $\frac{H_p}{H_p + P}$ ratio is greater than .010, it was necessary to correct the coefficient as previously explained for kinetic head. It was not necessary to correct for change in nappe shape due to velocity of approach unless the value of $\frac{H_p}{H_p + P}$ became greater than .250 at which value the correction would decrease the value of the coefficient one-half of one percent. See Figure 10. After this was done the coefficient "C" was plotted again $\frac{H_o}{H_p}$ ratio of head on crest to head producing best fitting nappe shape. All the coefficients versus ratio $\frac{H_o}{H_p}$ curves from a given group were traced onto a common sheet and the base curves for that group were found by averaging the curves. That is, for various $\frac{H_o}{H_p}$ ratios the numerical values of the coefficient were added and the sum divided by the number of curves involved. The three base curves "A", "C", and "D", together with most of the coefficient curves used in making them up, are shown in Figures 11, 12, 13, and 14.

The maximum deviation of any coefficient curve from the base curve is plus or minus 4 percent when $\frac{H_o}{H_p}$ is greater than 0.50. Below this value several of the curves fall considerably below the base curves. This may be explained by the fact that small inaccuracies in measurement of discharge or head will result in greater errors in coefficient at low heads than at high heads. Also many hydraulicians are prone to carry the coefficient of



$\frac{H_o}{H_p}$ RATIO OF HEAD ON CREST TO HEAD THAT PRODUCES BEST FITTING NAPPE SHAPE

WEIRS WITH FREE ROUNDED CRESTS
(WITHOUT PIERS OR GATES)
"A" CURVES



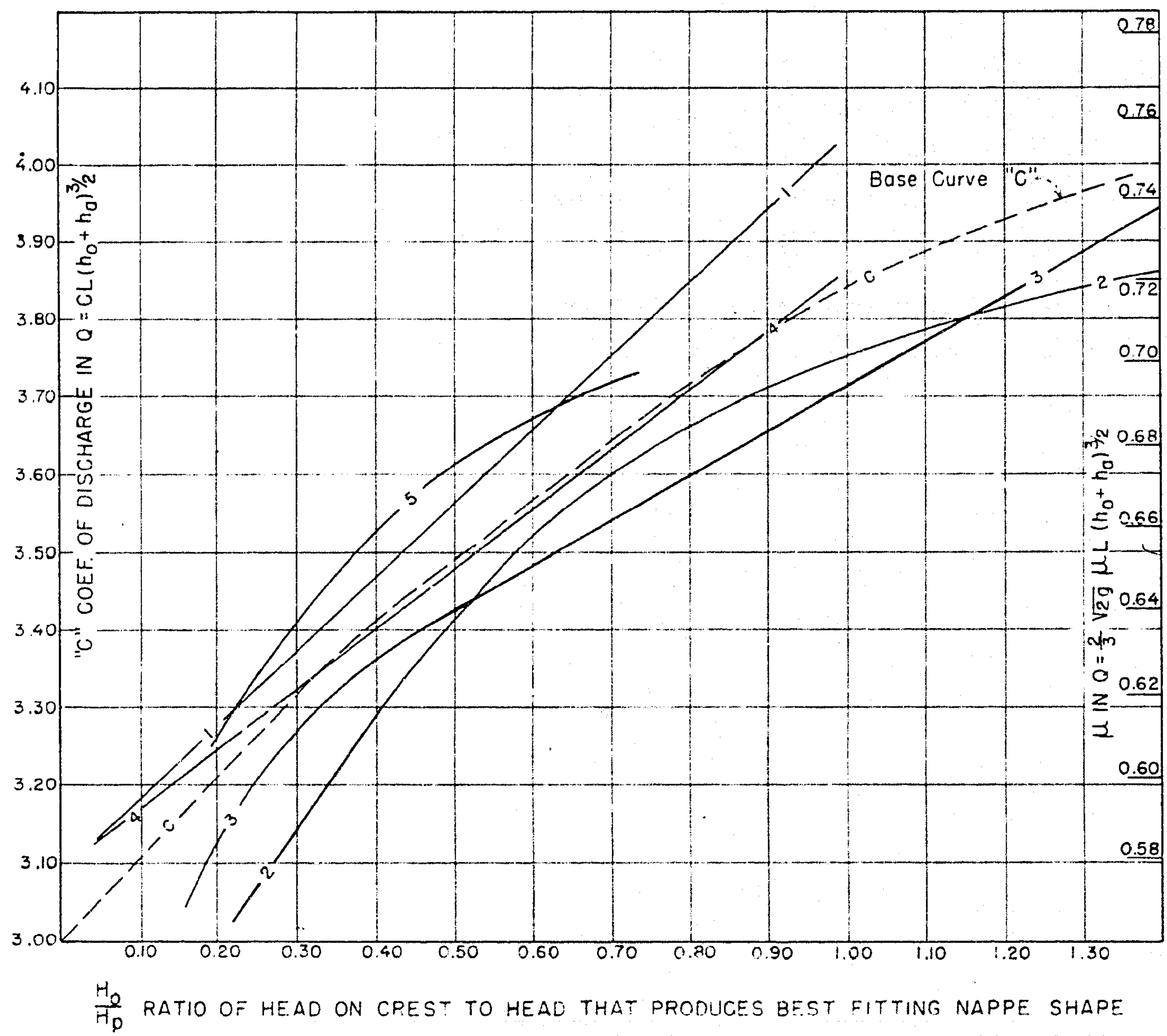
$\frac{H_o}{H_p}$ RATIO OF HEAD ON CREST TO HEAD THAT PRODUCES BEST FITTING NAPPE SHAPE

WEIRS WITH FREE ROUNDED CRESTS

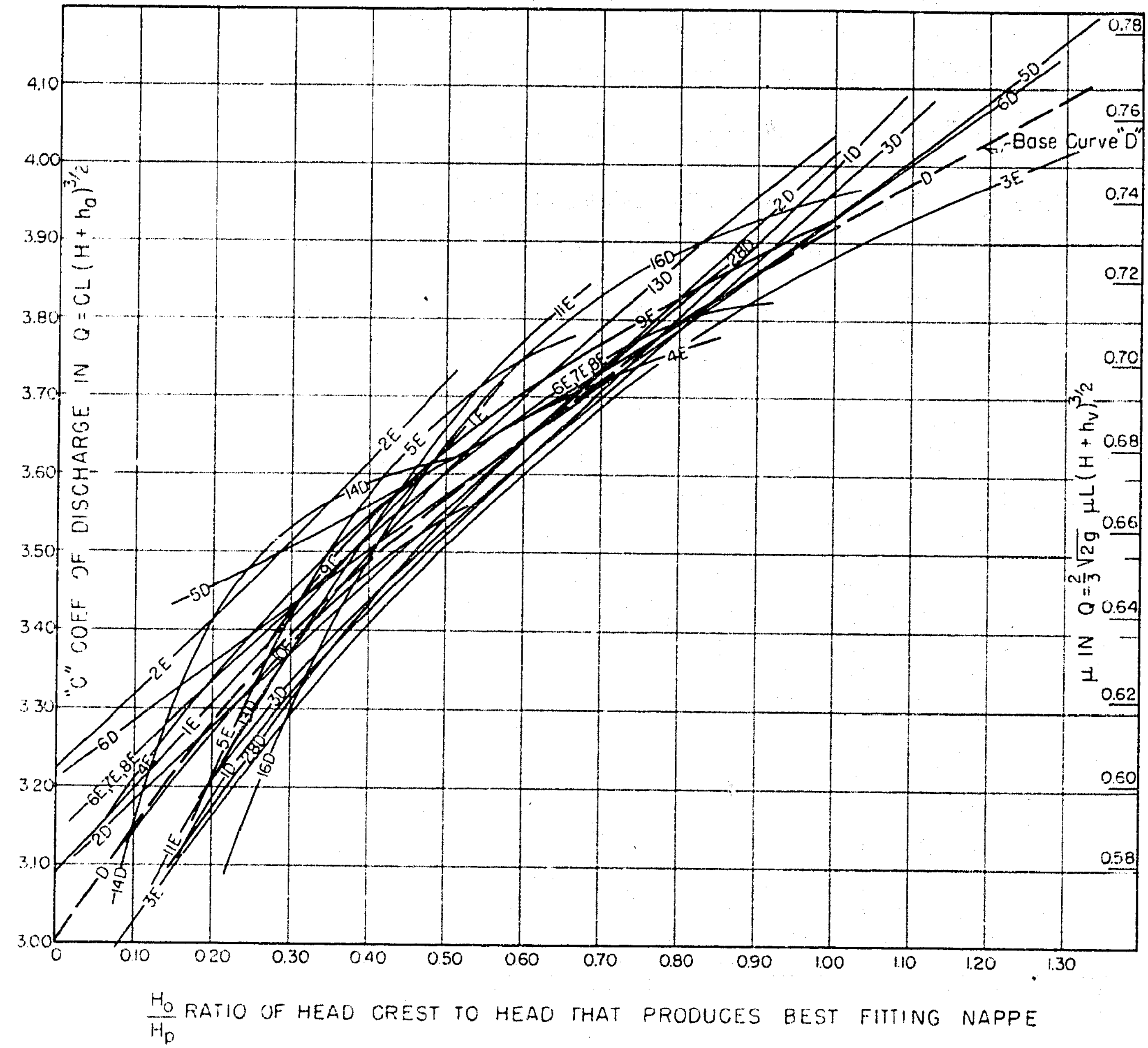
(WITHOUT PIERS OR GATES)

"B" CURVES

FIGURE 13



WEIRS WITH GATES ON ROUNDED CRESTS – "C" CURVES
DISCHARGE FROM ONLY ONE GATE



WEIRS WITH PIERS AND GATES ON ROUNDED CRESTS

DISCHARGE FROM THREE OR MORE GATES

"D" Curves with Stoney or Radial Gates on Crests

"E" Curves with Drum Gates on Crests

discharge to zero at zero head. The theoretical value of the coefficient for zero head would be 3.08; however, friction losses, depending upon the particular setup, reduces this value. For most curves studied in which no attempt was made to extend the curves through zero, an average value of 3.00 was found.

As in most all engineering work, there are steps in the analyzing where good judgment and experience acts as a guide. Here in the selection of the proper nappe shape to fit the rounded crest weir profile, the most thought and careful work must be exercised. For many of the most difficult rounded crest shapes, several different nappe shapes were tried, as was noted for Boulder Dam, Figure 4, and Keokuk, Figure 6, which were the most difficult. Three of the nappe shapes tried for the Keokuk rounded crest are shown in the figure. For each of the nappe shapes tried the corresponding coefficient versus $\frac{H_o}{H_p}$ ratio has been shown. For H_p equals 10 feet, Condition I, Figure 3, will exist. The nappe shape is almost a perfect fit upstream from the crest and a positive pressure will exist, while downstream from the crest the nappe shape falls under the masonry outline, and pressures will be produced which will decrease the coefficient. Consequently the coefficient versus head ratio is below the base curve.

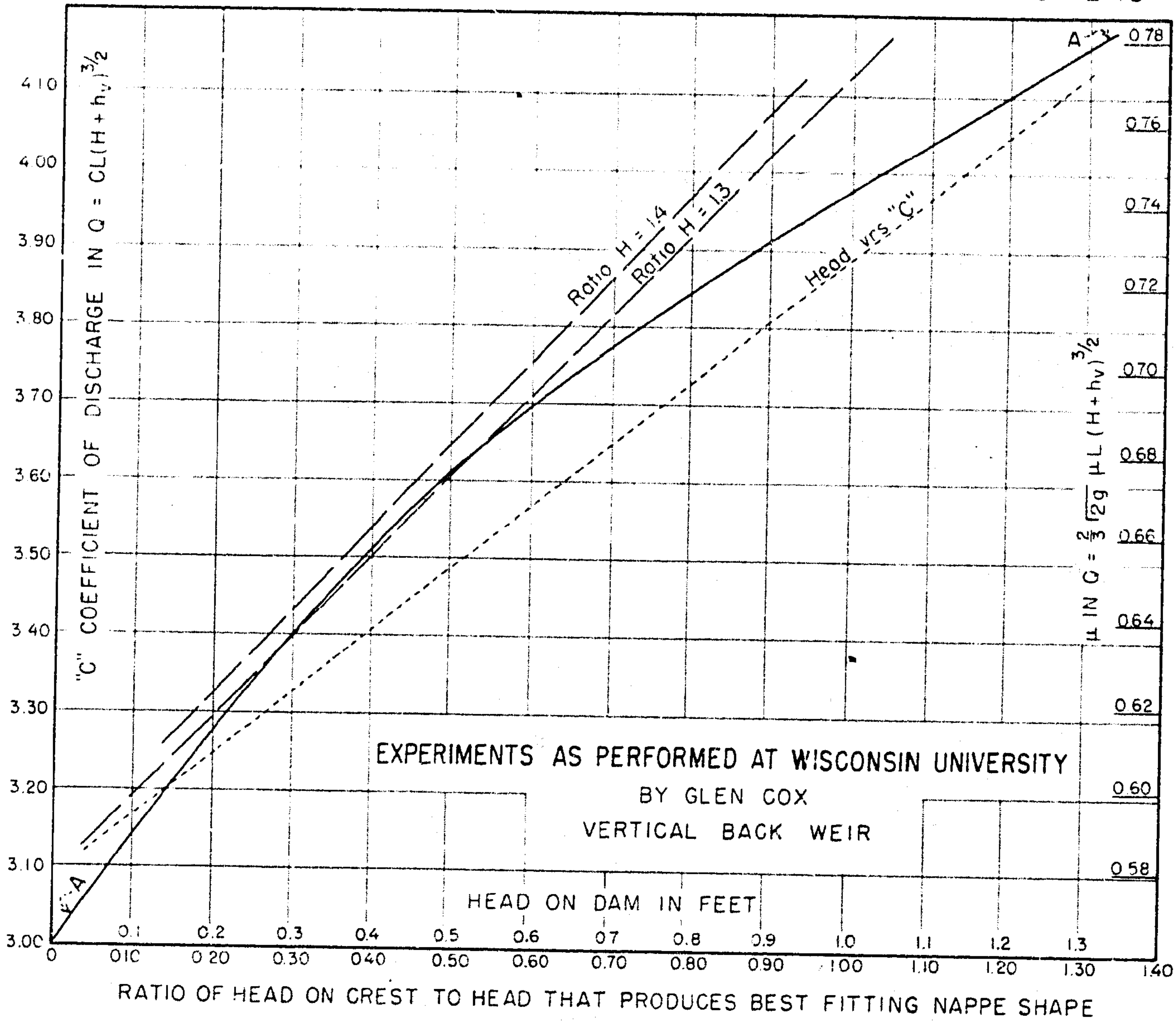
For H_p equals 18, the nappe shape is nearly a perfect fit in the critical region downstream from the crest, but if the shape were moved upstream so that the highest point of the nappe corresponded to that for H_p equals 10 or 13, the upstream portion of the nappe shape would fall outside of the rounded crest of the weir and a vacuum would produce results similar to Condition III, Figure 3. This vacuum is evidently greater than the pressure in the region downstream from the crest, and the result is a coefficient that is higher than the base curve. Keokuk Dam has a flat topped rounded crest weir and is not of recent designs where the laws governing pressure distribution and values of coefficients are well known.

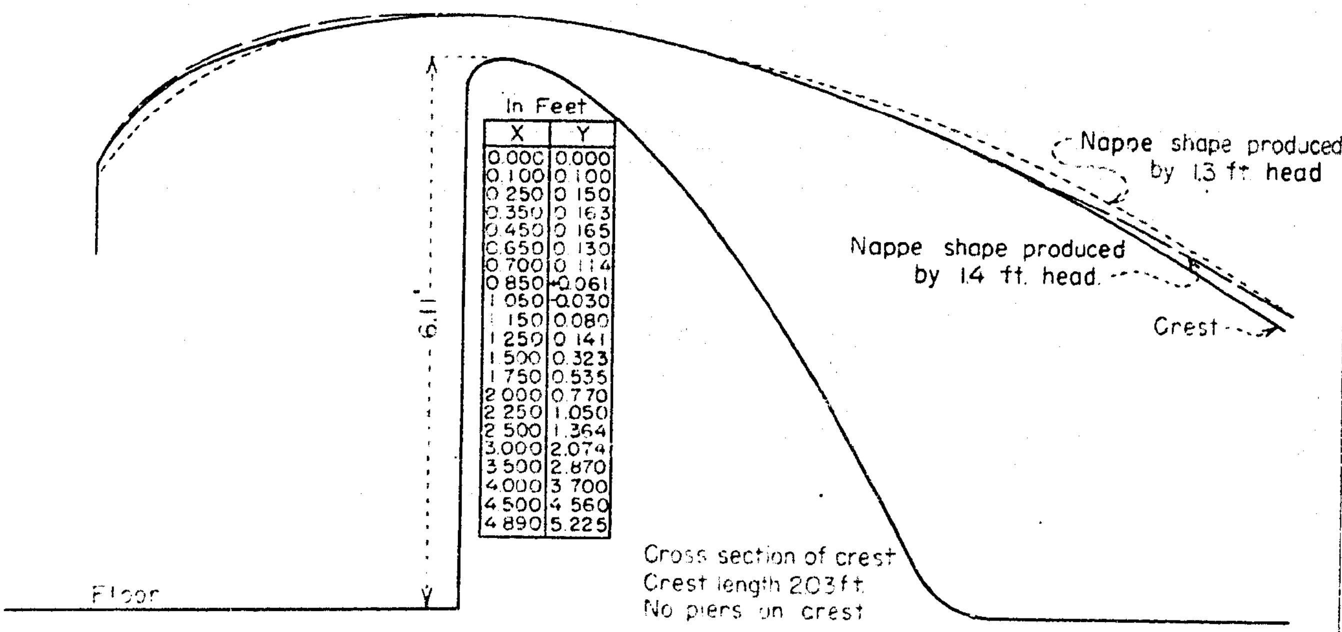
A rounded crest shape more typical of present day practices is that of Wilson Dam, Figure 5, and the rounded crest weir tested by Glen Cox at the University of Wisconsin, Figure 15. Here two different nappe shapes were tried with H_p equals 1.3 and 1.4 feet, and the corresponding coefficient versus ratio $\frac{H_o}{H_p}$ were plotted. In the case of Keokuk, the nappe shape tried varied from H_p equals 10 feet to H_p equals 18 feet, a variation of 38.5 percent from the nappe shape selected, while in the present case of the Wisconsin model, the variation is only about 7.7 percent. The nappe shapes that might be tried are much more limited and the selection is easier, thus the error that might be made in selecting a coefficient for such a rounded crest weir designed by modern practices is much less.

For the rounded crest weir tested at Wisconsin, the nappe shape produced by a head of 1.3 feet was selected, although the nappe shape produced by a head of 1.4 feet is nearly as good a fit. The nappe shape produced by a head of 1.4 feet lies slightly below the masonry line of the crest upstream, and downstream lies above it. Consequently, the distribution of pressures is similar to Condition V, Figure 3, and if the downstream negative pressure is larger will give coefficient higher than the base curve. The nappe shape produced by a head of 1.3 lies both above and below the masonry line upstream, and it is likely that the negative and positive pressure balance one another, while downstream the fit is very good until some distance below the downstream limit of the critical section where the nappe shape lies below the masonry line. The positive pressure formed in this region is too far downstream to have any appreciable effect on the coefficient.

While irregular crest shapes are difficult to analyze yet because of rollers forming in the spaces where the nappe shape forms a pocket above the masonry, the resulting coefficient versus $\frac{H_o}{H_p}$ curve compare very favorably with crests that nearly conform to the lower nappe shape.

FIGURE 15





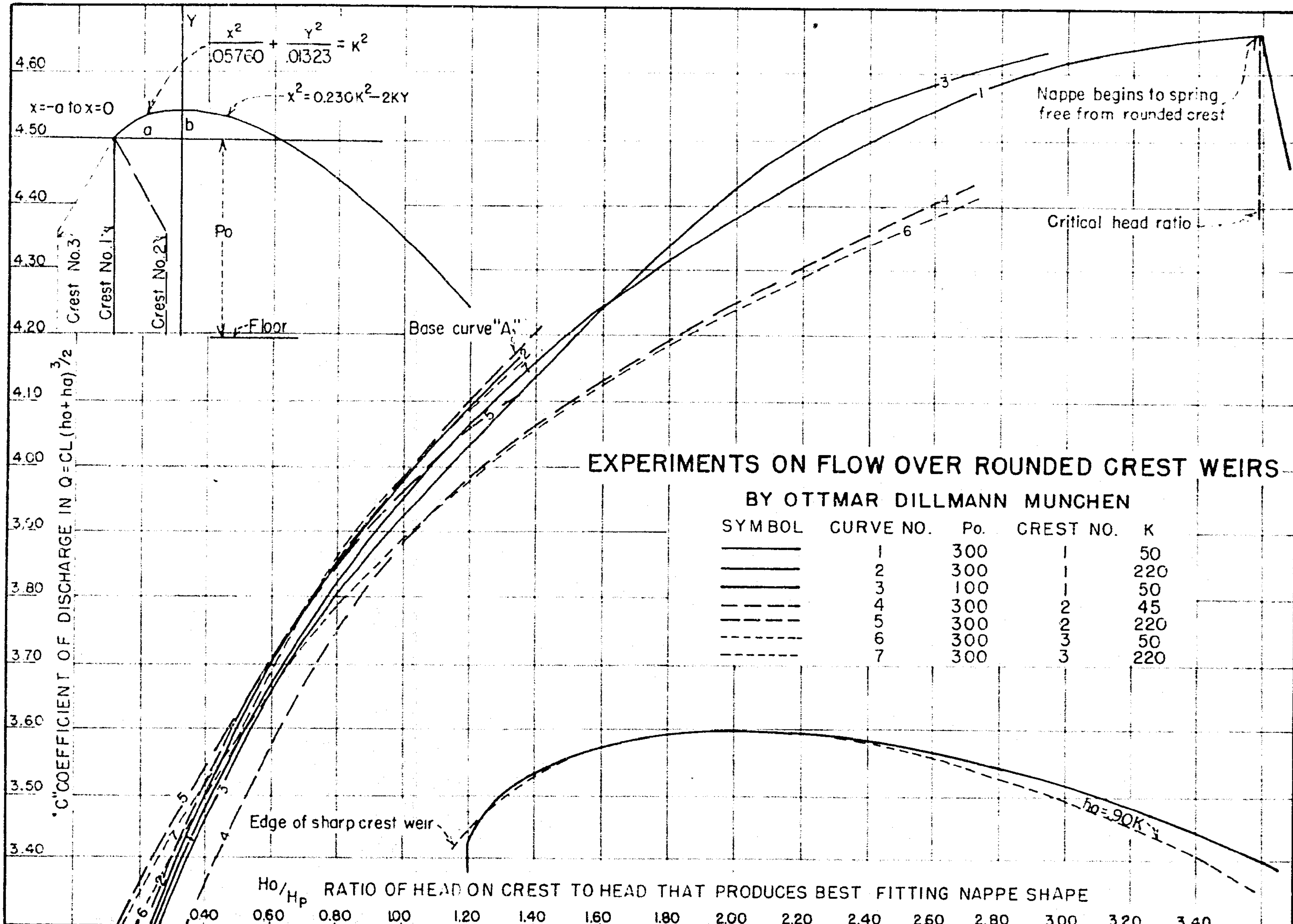
X	Y
0.000	0.000
0.100	0.100
0.250	0.150
0.350	0.163
0.450	0.165
0.650	0.130
0.700	0.114
0.850	-0.061
1.050	-0.030
1.150	0.080
1.250	0.141
1.500	0.323
1.750	0.535
2.000	0.770
2.250	1.050
2.500	1.364
3.000	2.074
3.500	2.870
4.000	3.700
4.500	4.560
4.890	5.225

In Chapter III under Condition IV, Figure 3, it was shown how the coefficient could be increased beyond the value of 3.98. Experiments by Dillmann (7), Figure 16, illustrates this phenomena. Unfortunately his crest shapes do not perfectly fit the lower nappe shape from sharp crest weirs, but the approximation is fairly good (note fitting of nappe shape to rounded crests in lower right hand corner of Figure 16). In his experiments he used rounded crest weirs that were geometrically similar, the size being controlled by a constant "K", which was the head on a sharp crest weir, producing the nappe shape that the rounded crest weir approximately fit. If his rounded crest weir had exactly fit the nappe shape, H_p would equal .888 K; however, it was found that H_p equals .900 K.

For Crest Shape No. 1, (vertical upstream face) rounded crests were made up for K equals 20, 50, 60, 120, 150, 170, and 220. For crests with K equals 20 and 50, tests were made with p_o equal to both 300 and 100, the rest were tested with p equals 300.

When the head was increased to beyond a ratio $\frac{H_o}{H_p} = 3.60$ for crests with K's of 20 and 50, the nappe springs free from the crest and there was a decrease in coefficient. Pressure measurements on the crest showed that for $\frac{H_o}{H_p}$ equal from zero to one, Condition I, Figure 3, existed, while for $\frac{H_o}{H_p}$ equal from 1.00 to 3.60 Condition III existed. The coefficients from these various crests were analysed by the method outlined above and were found to agree with Base Curve "A" as far as it extended.

These experiments were performed on very small models H_p equals 0.059 feet for K equals 20; however, they agree in the lower region $\frac{H_o}{H_p} = 0$ to $\frac{H_o}{H_p} = 1.40$ with the Base Curve "A" derived from much larger models, and even prototype structures. For Crest No. 1 the variation of size of crest did not produce any consistent variation in the coefficient. The maximum variation in coefficient



EXPERIMENTS ON FLOW OVER ROUNDED CREST WEIRS
BY OTTMAR DILLMANN MUNCHEN
SYMBOL
CURVE NO.
Po.
CREST NO.
K
1 300 1 50
2 300 1 220
3 100 1 50
4 300 2 45
5 300 2 220
6 300 3 50
7 300 3 220
$\frac{x^2}{.05760} + \frac{Y^2}{.01323} = K^2$
$x^2 = 0.230K^2 - 2KY$
x=-a to x=0
Y
a
b
Po
Floor
Crest No.3
Crest No.1
Crest No.2
Base curve "A"
Nappe begins to spring free from rounded crest
Critical head ratio
Edge of sharp crest weir
ho=90K
"C" COEFFICIENT OF DISCHARGE IN $Q=CL(ho+ha)^{3/2}$
Ho/Hp RATIO OF HEAD ON CREST TO HEAD THAT PRODUCES BEST FITTING NAPPE SHAPE
4.60
4.50
4.40
4.30
4.20
4.10
4.00
3.90
3.80
3.70
3.60
3.50
3.40
0.40
0.60
0.80
1.00
1.20
1.40
1.60
1.80
2.00
2.20
2.40
2.60
2.80
3.00
3.20
3.40

FIGURE 16

from various sized weirs was only 1.8 percent. These facts tend to prove that there is no variation in coefficients due to size of crest.

The position of Curves No. 4 and 6, Figure 16, with respect to Curves No. 5 and 7, from weirs which are over four times as large, seems to contradict the above statement. But it is believed that in the case of these crest shapes the models with K equals 50 and 45 were too small, since weirs with large "K" of 120 and 130 gave curves plotting in the immediate vicinity of Curves 5 and 7 from the weirs with K equals 220.

These curves also show that neither an upstream slope or an overhang produce any appreciable change in the coefficient of discharge for rounded crest weirs. A few engineers have attempted to express the coefficient of discharge for rounded crest weirs by either a formula or plotting of the various pertinent factors. The German engineer, Rehbock (11) has suggested the formula for cylindrical rounded crest weirs.

$$\mu' = .313 + \sqrt{0.3 + 0.01\left(5 - \frac{h_o}{R}\right)^2} + 0.09\,\frac{h_o}{P} \quad \text{---} \quad 22.$$

Here the coefficient μ' includes velocity of approach and the discharge is given by $Q = \mu' \frac{2}{3}\sqrt{2g}\, L h_o^{3/2}$. The symbol "R" expresses the radius of curvature of the cylindrical crest. The following limits are placed on the factors of formula (22):

$$P > R > 0.07 \text{ feet}$$

$$h_o < R\left(6 - \frac{20\,R}{p + 3\,R}\right)$$

For negligible velocity of approach P is very large, approaching infinity, and the last term of the equation may be dropped.

By independent analysis of coefficients for cylindrical rounded crest weirs, the author found that $H_p = 1.6R$, Page 21. Since:

$$C = \frac{2}{3}\sqrt{2g}\,\mu = 5.35\mu$$

and for negligible velocity of approach $h_o = H_o$, the formula becomes:

$$C = 5.35\left[.312\sqrt{0.3 + 0.01\left(5 - \frac{1.6H_o}{H_p}\right)^2}\right] \quad \text{—— 23.}$$

The values of C are plotted against $\frac{H_o}{H_p}$, Figure 17, and the results compared with the Base Curve "A". The agreement is very good; however, it should be remembered that Rehbock's formula is limited to cylindrical rounded crest weirs.

The last term of Rehbock's formula which corrects for velocity of approach, both due to change in shape of nappe and kinetic head, suggests that comparison might be made with ratio ϕ and Ψ obtained by the author. Since the method used to get the correction ϕ for the kinetic head contained in the water approaching the weir has been used by other hydraulicians and has proven satisfactory, it will be considered as correct.

The last term $0.09\,\frac{h_o}{P}$ can be computed for various values of the ratio $\frac{h_o}{h_o + P}$ and added to the theoretical coefficient C_t obtained by solving the Equation 22 with $\frac{h_o}{H_p} = 1.00$, that is $\frac{h_o}{R} = \frac{1.6h_o}{H_p} = 1.60$. This means that the nappe shape that best fits the cylindrical rounded crest weir conforms to the lower surface of the nappe shape for the various heights of weirs. The Equation 22 now becomes:

$$K = 5.35\left[.312\sqrt{0.3 + 0.01(5 - 1.6)^2} + 0.09\left(\frac{\frac{h_o}{h_o + P}}{1 - \frac{h_o}{h_o + P}}\right)\right]$$

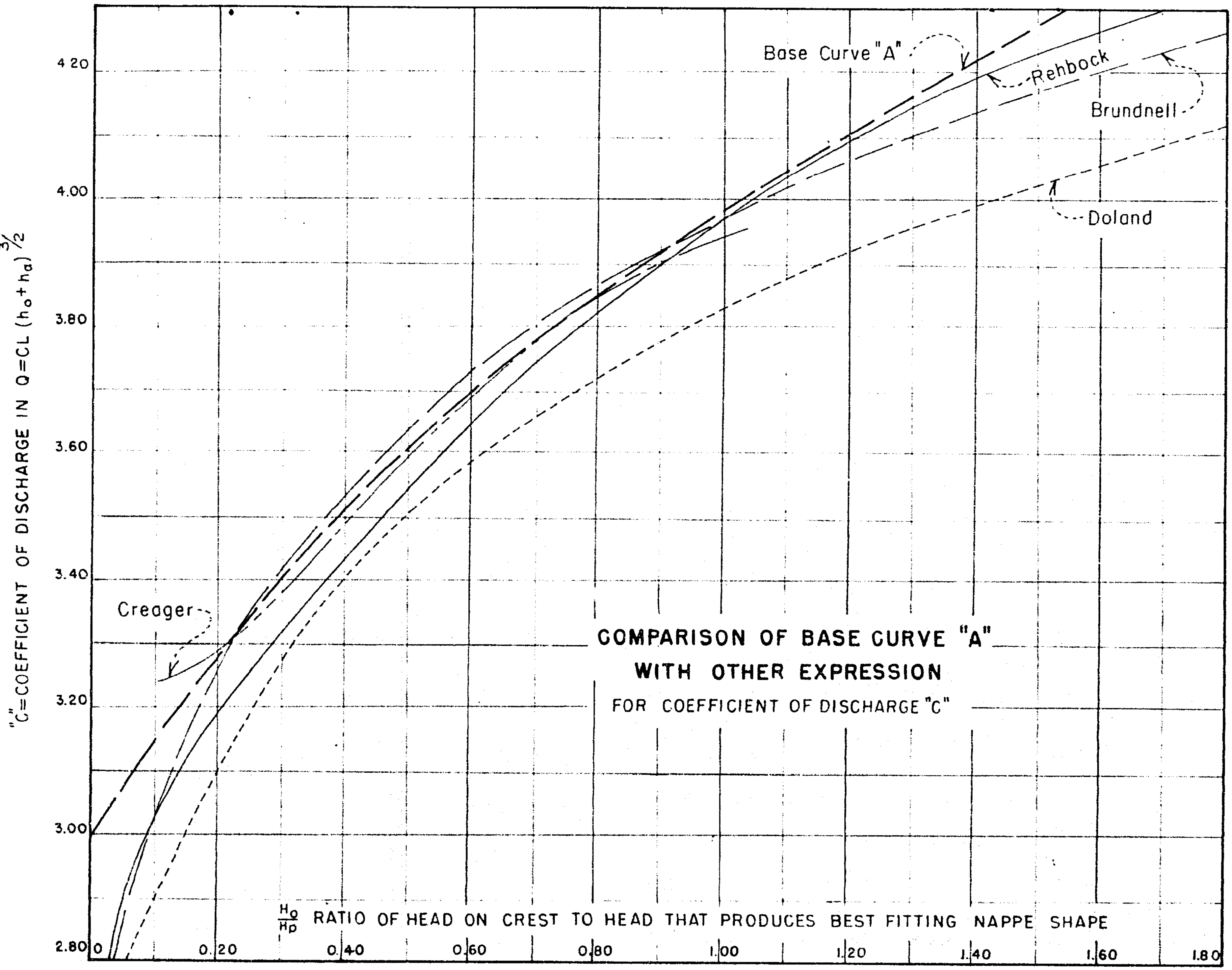


FIGURE 17

449
49

$$K = 5.35\left[\ .741 + 0.09\left(\frac{\dfrac{h_o}{h_o + P}}{1 - \dfrac{h_o}{h_o + P}}\right)\right] \text{------------------ } 24.$$

The limits placed on Formula 24 are the same as those on Formula 22, but may be expressed in different terms so as to be comprehended more readily.

Since $R = .624\ H_p$

$$P > .624\ H_p \text{ and } H_p > .112 \text{ feet}$$

also $$\frac{H_p}{H_p + P} > .616$$

$$h_o < .624\ H_p\left(6 - \frac{12.48\ H_p}{P + 1.872 H_p}\right)$$

If $P = \infty$ $h_o < 3.75\ H_p$, which is the point where the nappe shape springs free from the rounded crest. If P becomes as small as possible, that is $P = .625\ H_p$, then $h_o < .625\ H_p$.

Since the coefficient K contains both corrections it will be divided by "ϕ" correction for kinetic energy for the value of $\frac{h_o}{h_o + P}$, the result will be the theoretical C_t for that particular height of weir expressed by $\frac{h_o}{h_o + P}$. The results of the process is shown on Figure 9. This curve is compared to the one by the author obtained for rounded crest weirs conforming to lower nappe, while the Rehbock curve is for a cylindrical crest. The general trend of Rehbock's correction is the same as that derived by the author, but the coefficient obtained by Rehbock is much larger for values up to about $\frac{h_o}{h_o + P} = .85$, and drops much lower when $\frac{h_o}{h_o + P} = 1.00$. However, the limits placed on Equation 22 and 24 prohibits its use for values of $\frac{h_o}{h_o + P}$ greater than .62.

In this discussion it must be remembered that the correction for kinetic head was that used by the author, as this was the only means available to separate the two corrections contained in the last term of Equation 22 and 24.

Mr. J. J. Doland (33) began working independently on the problem of coefficients for rounded crests and obtained much of the data analyzed in this thesis from Professor E. W. Lane and the author.

Professor Doland has the same idea of reducing all coefficients versus head relations to a common denominator; however, his approach is different. The cross-section of the crest is plotted and then a circle is constructed which will best fit the rounded crest shape. The radius of this circle "R" is used to reduce the coefficient versus head relations to a common base. He plotted his results on logarithmic paper and expressed the results in the form of an equation.

$$Q = \frac{K'}{\left(\frac{R}{H_o}\right)^N} L H_o^{3/2}$$

or

$$C = \frac{K'}{\left(\frac{R}{H_o}\right)^N} \qquad \text{------------------------} \quad 25.$$

It is now necessary to determine K' and N from the logarithmic plotting. The K' is the intercept at $\frac{R}{H_o} = 1$, and N is the slope of the curve. Since the group of curves do not seem to be averaged by a straight line it is hard to determine K' and N. An attempt was made to determine the constant; however, it is realized that another person might obtain different results. Since it was found that $H_p = 1.6R$ and the average value of the slope was found to be .1331, the intercept at $\frac{R}{H_o} = 1$ was 3.60, Equation 25 becomes:

$$C = \frac{3.60}{\left(\frac{H_p}{1.6\ H_o}\right)^{.1331}}$$

$$C = 3.60 \left(\frac{1.6\ H_o}{H_p}\right)^{.1331} \qquad \text{———————— 26.}$$

This relation of C to $\frac{H_o}{H_p}$ was plotted on Figure 17. The curve falls below the Base Curve "A", the discrepancy being larger for value of $\frac{H_o}{H_p}$ greater than .60. Mr. Ross N. Brudenell (34) in a discussion of Mr. Doland's article suggested that:

$$Q = \frac{C_t\ LH_o^{(3/2 + N)}}{H_p} \qquad \text{———————— 27.}$$

or

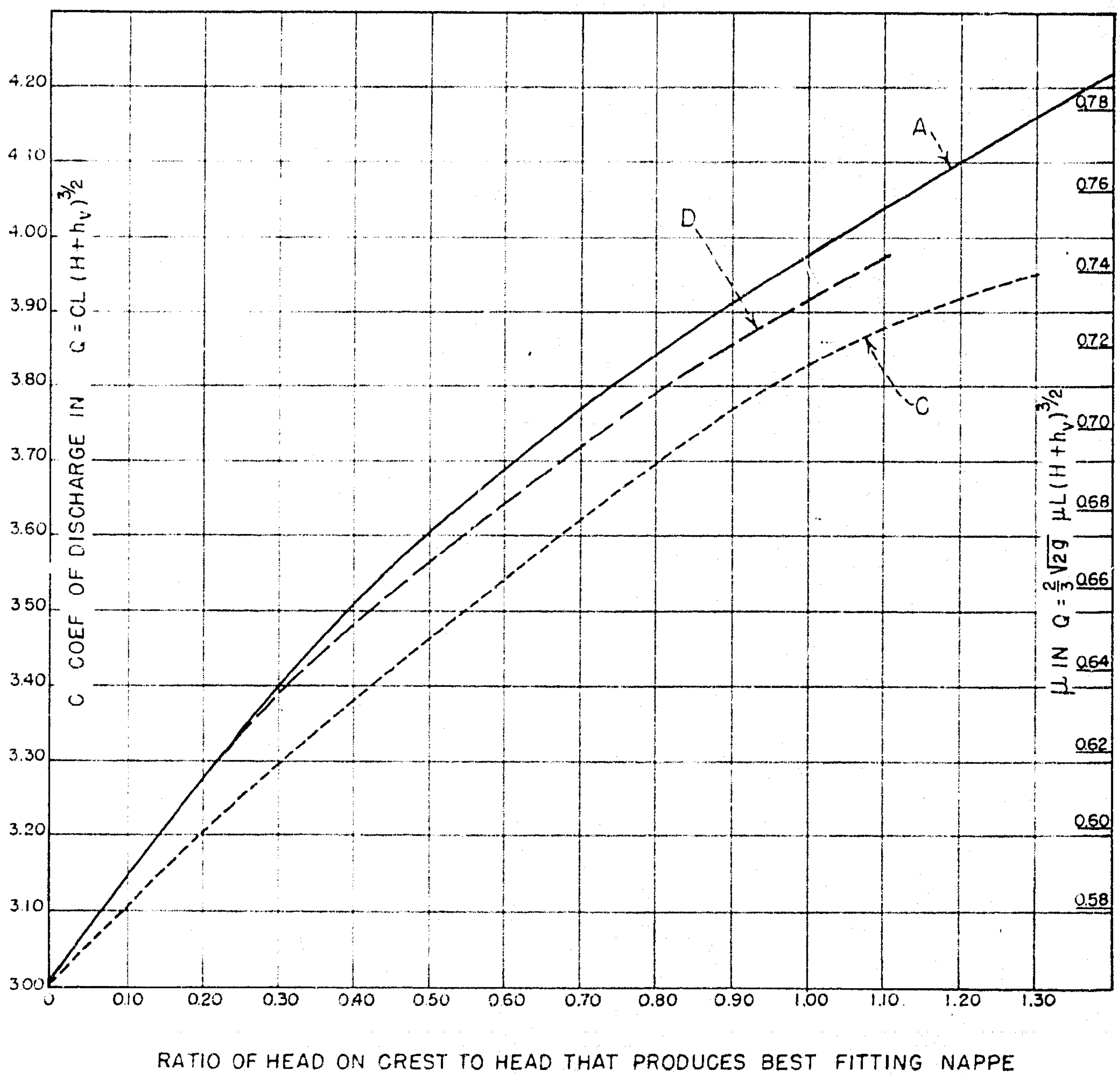
$$C = C_t \left(\frac{H_o}{H_p}\right)^{N} \qquad \text{———————— 28.}$$

He further suggested that $C_t = 3.97$ and that $N = .12$. The formula applied to rounded crest weirs which are shaped to fit the lower nappe from a sharp crest weir. Equation 28 becomes:

$$C = 3.97 \left(\frac{H_o}{H_p}\right)^{.12} \qquad \text{———————— 29.}$$

The equation was plotted in Figure 17 and agrees fairly well with the Base Curve "A", except for value of $\frac{H_o}{H_p}$ greater than 1.2. Equations 26 and 29 give a coefficient of zero when $\frac{H_o}{H_p}$ equals zero. Rehbock's Formula 22 gives a coefficient of 1.66 when the ratio $\frac{H_o}{H_p}$ equals zero. As previously pointed out the coefficient should be somewhat less than 3.08, but certainly not 1.66 or zero, whenever $\frac{H_o}{H_p}$ equals zero.

The author has expressed his results in the form of curves, Figure 18, but since some engineers prefer working with equations,



BASE CURVES
DISCHARGE COEFFICIENTS

A ——— Free Crest—no piers or gates
C - - - - Stoney gates on Crest—1 gate open
D — — — Stoney or Drum gates on Crest—3 open

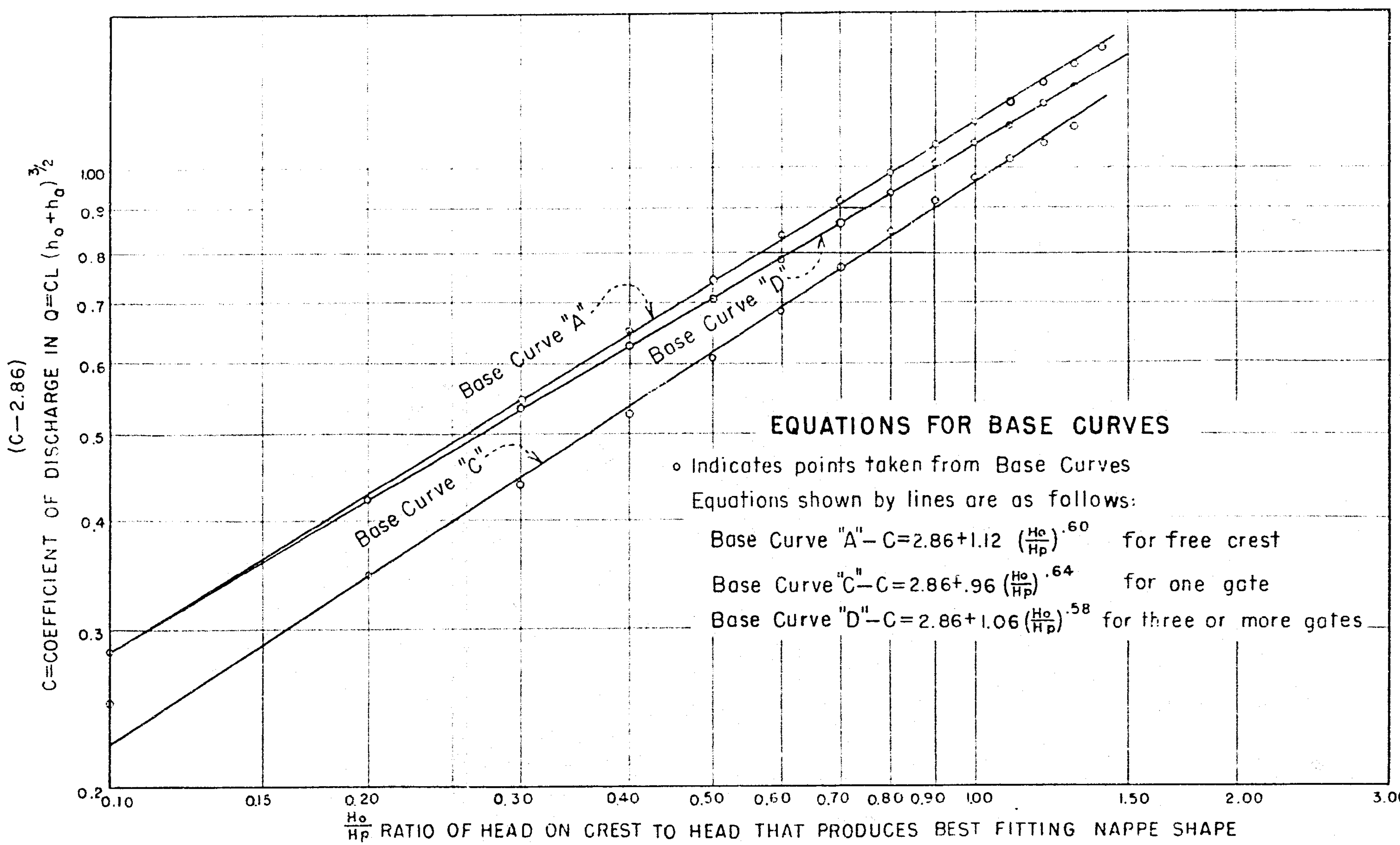
an attempt was made to express the base-curve as an equation. The general form of the equation is:

$$C = K_1 + K_2 \left(\frac{H_o}{H_p}\right)^N \quad \text{------------------------} \quad 30.$$

K_1, K_2, and N are constants to be determined. If $(C - K_1)$ is plotted against $\frac{H_o}{H_p}$ on logarithmic paper, the slope of the line will give N and the intercept at $\frac{H_o}{H_p} = 1$, will determine K_2. The value of K_1 is found by trial and error. K_1 should be of such value that $(C - K_1)$ plotted against $\frac{H_o}{H_p}$ will give a straight line. In order for C to be 3.00 when $\frac{H_o}{H_p}$ equals zero, K_1 would have to be 3.00. However, when this value was used, the value of $(C - 3.00)$ plotted against $\frac{H_o}{H_p}$ did not result in a straight line, but was slightly curved. For Base Curve "A", K_1 was found to be 2.86, and this value was found for Base Curve D, while some other values for K_1 might have been used for Base Curve C, yet the same value of 2.86 for K_1 was found to give approximately a straight line. So as to have the constant K_1 the same for all base curves, it was used for Base Curve C.

It is logical that neither the piers or numbers of gates open should materially affect the limit of the coefficient when the head on the weir approached zero. The coefficient of 2.86 for this limit is much more logical than zero or some much lower value, since it is conceivable that the theoretical coefficient of 3.08 for broad crest weirs could be reduced by friction to 2.86, especially at low head where friction would play a significant role.

With $K_1 = 2.86$ the base curves were plotted on logarithmic paper, Figure 19, and K_2 and N determined.



EQUATIONS FOR BASE CURVES
o Indicates points taken from Base Curves
Equations shown by lines are as follows:
Base Curve "A" – $C=2.86+1.12\left(\frac{Ho}{Hp}\right)^{.60}$ for free crest
Base Curve "C" – $C=2.86+.96\left(\frac{Ho}{Hp}\right)^{.64}$ for one gate
Base Curve "D" – $C=2.86+1.06\left(\frac{Ho}{Hp}\right)^{.58}$ for three or more gates
Base Curve "A"
Base Curve "C"
Base Curve "D"
(C – 2.86)
C=COEFFICIENT OF DISCHARGE IN $Q=CL\,(h_o+h_a)^{3/2}$
$\frac{Ho}{Hp}$ RATIO OF HEAD ON CREST TO HEAD THAT PRODUCES BEST FITTING NAPPE SHAPE
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.00
0.10
0.15
0.20
0.30
0.40
0.50
0.60
0.70
0.80
0.90
1.00
1.50
2.00
3.00

FIGURE 19

For Base Curve "A" $N = .60$ and $K_2 = 1.12$

For Base Curve "C" $N = .64$ and $K_2 = 0.96$

For Base Curve "D" $N = .58$ and $K_2 = 1.06$

These values give the following equations for the coefficient of discharge in the formula:

$$Q = CL\,(h_o + h_a)^{3/2} \qquad Q = CLH_o^{\;3/2}$$

For Base Curve "A" free rounded crest weirs without piers or gates:

$$C = 2.86 + 1.12 \left(\frac{H_o}{H_p}\right)^{.60} \qquad \text{———— 31.}$$

For Base Curve "C" rounded crest weirs with piers and gates (one gate only discharging):

$$C = 2.86 + .96 \left(\frac{H_o}{H_p}\right)^{.64} \qquad \text{———— 32.}$$

For Base Curve "D" rounded crest weirs with piers and gates (three or more gates discharging):

$$C = 2.86 + 1.06 \left(\frac{H_o}{H_p}\right)^{.58} \qquad \text{———— 33.}$$

The actual range of the equation is limited by their derivation to values of $\frac{H_o}{H_p} = 0.0$ to 1.40. These equations are applicable when $\frac{H_o}{H_o + P}$ is less than .30. When this limit is exceeded they should be corrected for a decrease in coefficient due to the lower height weir, by using the graph, Figure 10, for the reduction ratio Ψ which is drawn up in terms of $\frac{H_o}{H_o + P}$. Expressing this in the form of an equation when $\frac{H_o}{H_o + P}$ is greater than .30, the discharge for rounded crest weirs is given by:

$$Q = \Psi \left[2.86 + K_2 \left(\frac{H_o}{H_p}\right)^{N}\right] LH_o^{\;3/2} \qquad \text{———— 34.}$$

For free rounded crest weirs $K_2 = 1.12$ and $N = .60$. For rounded crests with one gate discharging, $K_2 = .96$ and $N = .64$. For three or more gates discharging $K_2 = 1.06$ and $N = .58$.

See Figure 10 for value of Ψ.

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