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BUREAU OF RECLAMATION

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HYDRAULIC STUDIES OF A WATER JET-PUMP FOR THE KESWICK DAM FISH-TRAP CENTRAL VALLEY PROJECT CALIFORNIA

by

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Denver, Colorado

September 26, 1944

FOREWORD

The need for water in excess of the natural precipitation and stream flow in the arid and semi-arid regions of the western United States was created in the early part of the twentieth century by development of lands and cities along the streams. Many storage reservoirs were built to impound the spring run-off and freshet-flow, and the water later released to augment the natural supply of water during the dry season.

As the development of land progressed westward, many structures were planned on streams inhabited by the migratory fish. These presented an impassible barrier to the natural migration of the fish.

Since Keswick Dam is being constructed on a stream of this type, it was necessary to provide a means whereby the natural habits of the fish, whose migration would be disrupted by the barrier, could be carried out by the aid of artificial methods and the existence of the species perpetuated.

The artificial aids to fish migration usually consist of fish-ladders which are a series of ascending weirs sufficiently low to permit the fish to progress upstream by leaping from one weir to the other until the top of the dam is reached. This method will suffice if the dam is low but if the structure is high the fish-ladder becomes a major structure and the cost is extremely high. When a condition such as this exists a combination fish-trap and ladder is used. It consists of a series of low weirs which conduct the fish to a trap where they are removed and transported above the dam or to a hatchery where they are milked for spawn.

All of these types of structures require a constant water supply during the season in which migration occurs. If the total water supply is taken from the reservoir above the dam a considerable amount of water which would otherwise be valuable for power would be wasted. For this reason, it was proposed to utilize part of the outflow from the turbines to supply the fish-trap. Since the upstream end of the fish-trap is above the normal tailwater level in the river, it was necessary to pump all of the supply from the tailrace or utilize part of the flow from the reservoir to operate a jet-pump having the suction intake located in the tailrace. The latter proposal was selected for the Keswick installation because it was as sound economically as the turbine and centrifugal pump arrangement. It had the added advantage that the wear due to moving parts was negligible.

This paper was submitted as a thesis to the Faculty of the Graduate School of the University of Colorado in partial fulfillment of the requirements for the Degree, Master of Science. A copy of the thesis is available in the library of the Bureau of Reclamation.

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BUREAU OF RECLAMATION

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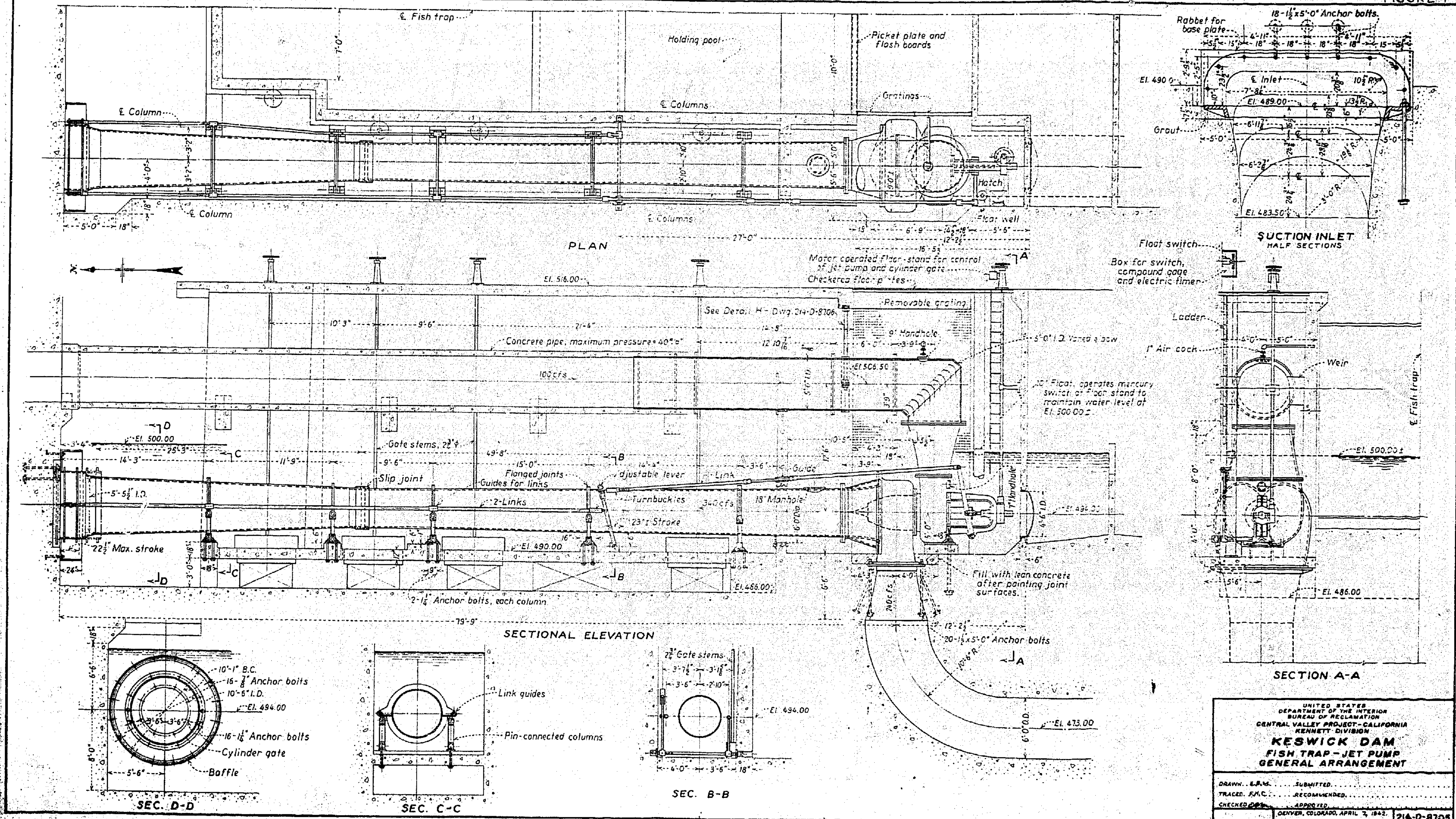
I - INTRODUCTION

General. The jet-pump is a hydraulic apparatus in which a high velocity jet carrying a comparatively small volume impacts into the central axis of an annular jet moving at a low velocity. The two masses mix, and the combined flow can then be raised to a head intermediate between the two. In this action the principal loss is due to shock, or sudden enlargement of the high-velocity jet.

A jet-pump will be used in connection with the fishway at Keswick Dam where it will supply the necessary water for operating the fishway by utilizing the reservoir head to pump tailwater from below the turbines. The combined flow (reservoir and tailwater) will discharge into a distribution chamber at the upstream end of the fishway where it will travel through the structure and return to the river.

The prototype structure, figure 1, had a mixing tube 40 feet long, which tapered from 50.6 inch diameter at the entrance to 42.7 inches at the downstream end, where it connected to a cylinder section

FIGURE 1



PLAN
SECTIONAL ELEVATION
SUCTION INLET
HALF SECTIONS
SECTION A-A
SEC. D-D
SEC. C-C
SEC. B-B
℄ Fish trap
Holding pool
Picket plate and flash boards
Gratings
℄ Columns
℄ Column
Hatch
Float well
18-1½"x5'-0" Anchor bolts
Rabbet for base plate
℄ Inlet
El. 490.00
El. 489.00
El. 483.50
Grout
Float switch
Box for switch, compound gage and electric timer
Ladder
1" Air cock
Weir
℄ Fish trap
El. 500.00±
El. 486.00
Motor operated floor stand for control of jet pump and cylinder gate
Checkered floor plates
Removable grating
El. 516.00
See Detail H - Dwg. 214-D-8706
Concrete pipe, maximum pressure = 40# □"
100 cfs
9" Handhole
5'-0" I.D. Vaned elbow
El. 506.50
10" Float, operates mercury switch at floor stand to maintain water level at El. 500.00±
El. 500.00
Gate stems, 2¾" ϕ
Slip joint
Flanged joints
Guides for links
Adjustable lever
Link
Guide
2-Links
Turnbuckles
340 cfs
18" Manhole
23"± Stroke
22½" Max. stroke
5'-5½" I.D.
El. 490.00
El. 486.00
240 cfs
Fill with lean concrete after painting joint surfaces.
2-1¼" Anchor bolts, each column
20-1½"x5'-0" Anchor bolts
El. 473.00
79'-9"
27'-0"
10'-1" B.C.
16-⅞" Anchor bolts
10'-6" I.D.
El. 494.00
16-1¼" Anchor bolts
Cylinder gate
Baffle
Link guides
Pin-connected columns
2¾" Gate stems
UNITED STATES
DEPARTMENT OF THE INTERIOR
BUREAU OF RECLAMATION
CENTRAL VALLEY PROJECT-CALIFORNIA
KENNETT DIVISION
KESWICK DAM
FISH TRAP-JET PUMP
GENERAL ARRANGEMENT
DRAWN
SUBMITTED
TRACED
RECOMMENDED
CHECKED
APPROVED
DENVER, COLORADO, APRIL 7, 1942.
214-D-8705

85.4 inches long. A diffuser, 25 feet in length and 64.56 inches in diameter at the downstream end, was attached to the cylindrical section and terminated at the cylinder gate. The gate was to be used for priming the pump and regulating pressure and flow throughout the apparatus. However, the primary control for regulating the total discharge was in the nozzle, where a needle made it possible to regulate the total flow up to the maximum of 340 cubic feet per second. The circular baffle surrounding the gate was intended to reduce boiling and splash resulting from the flow that impinged on the opposing concrete wall.

List of symbols.

P_1 = pressure across the plane of the nozzle.

P_2 = pressure at downstream end of the mixing tube.

Q_T = total discharge = $(Q_N + Q_s)$.

Q_N = discharge from the nozzle.

Q_s = discharge from the suction side of pump.

V_T = velocity at the downstream end of mixing tube.

V_N = velocity of jet emitting from the nozzle.

V_s = velocity of suction flow in the plane of the nozzle.

A_s = area provided for the suction flow in the plane of nozzle.

A_t = area of mixing tube at the downstream end.

A_N = area of the nozzle.

h_d = pressure at the discharge end of the diffuser, referred to center line of nozzle.

h_s = suction head referred to center line of the needle.

H_d = total head at discharge end of diffuser.

H_s = total suction head referred to center line of pump.

H_N = total head on center line of the nozzle, not including the pressure across the plane of the nozzle.

Theoretical considerations concerning the design. The action of the jet-pump, viewed from a purely theoretical standpoint, is as follows: The two jets issue at different velocities from two entrance nozzles into the mixing chamber, figure 1. The pressure P_1 across the mixing chamber is assumed to be uniform over the entire area. The jets then combine with inelastic impact, and at the other end of the mixing chamber they are supposed to be completely combined, having a uniform velocity and pressure P_2 over the entire area. If the mixing chamber is cylindrical, the equations of energy and momentum combined with continuity give the equation

$$\frac{Q_T V_T \gamma}{g} - \frac{Q_N V_N \gamma}{g} - \frac{Q_s V_s \gamma}{g} = a_2(P_1 - P_2)$$

which will solve the problem completely. If the mixing is not complete at the end of the mixing chamber and a large part takes place in the diffuser, the apparatus is beyond the reach of an exact theoretical analysis.

When the mixing chamber is conical, the force acting on the mass is not that due to the difference in pressure at the two ends; but there is a component of the pressure on the side walls which acts along the axis, and the momentum equation becomes

$$\frac{Q_T V_T \gamma}{g} - \frac{Q_N V_N \gamma}{g} - \frac{Q_s V_s \gamma}{g} = (a_1 + a)\, P_1 - a_2 P_2 + \int_{r_1}^{r_2} 2\pi r\, P\, dr$$

There is no way of knowing P as a function of r, or as a function of the position along the axis; hence the integral cannot be evaluated, and only an approximate solution can be obtained with this method.

The length of the mixing process varies with the diameters, viscosities, and velocities of the two jets. A partial theoretical analysis concerning mixing length has been done by Kuethe[1] and Tollmein[2]. While these are excellent papers they are concerned chiefly with two dimensional mixing and because of the complexity of the problem they ignore three dimensional mixing. Moreover, under certain conditions cavitational phenomena occurs during the mixing which further complicates the problem by adding another indeterminate factor.

O'Brien and Gosline[3] conducted a series of tests at the University of California with a small jet-pump having a cylindrical mixing chamber, to determine the agreement between the momentum equation and the actual results. In their published results, "The Water Jet-Pump," a formula has been evolved which agrees with the tests. However, results were based on a cylindrical mixing tube, and no data was available for pumps with conical tubes.

II - PURPOSE OF MODEL STUDIES

Need for model tests. In view of the uncertainties of the theory concerning the design, in combination with the unprecedented size of the prototype, it was considered advisable to build a 1:10 scale model with which it would be possible to determine the adequacy of the original design and to correct any deficiencies that were apparent.

Summary. The original design, as submitted for tests, figure 1, was intended to cause a suction flow equal to three times the

[1] References in numerical order in bibliography.

nozzle discharge (a 3:1 pumping ratio) when the differential pumping head was approximately 16 feet, prototype. This also corresponded to the head used in the model, because these tests were made with prototype heads in order to obtain prototype pressures on the model.

From the preliminary tests it was apparent that the area of the suction scroll in the region surrounding the nozzle was too large, causing vortices to form which extended into the mixing tube. This condition was remedied by placing fillers in the scroll case as shown in figure 2A. Subsequent tests of the design revealed that instead of delivering water at the 3:1 ratio in combination with the design heads, it pumped an amount so small that it could not be measured with the laboratory meters. Because of this it appeared that the proportions of the mixing tube were in error. Considerable improvement in performance was obtained by shortening the mixing tube. But, as the anticipated results were not obtained, it was concluded that the 3:1 pumping ratio was unobtainable with the design heads.

In view of this, a new mixing tube and a more efficient diffuser were designed, figure 2C, to effect a 2:1 pumping ratio at the same differential pumping head of 16 feet. This design also failed to perform as anticipated. It was therefore assumed that the size of the pump was insufficient. By considering the model as being to a scale of 1:11.5 instead of the previous scale of 1:10, it was possible to obtain the desired total discharge into the fish-trap for any differential pumping head up to 16 feet and a minimum driving head of 114 feet, absolute, at the center line of the nozzle.

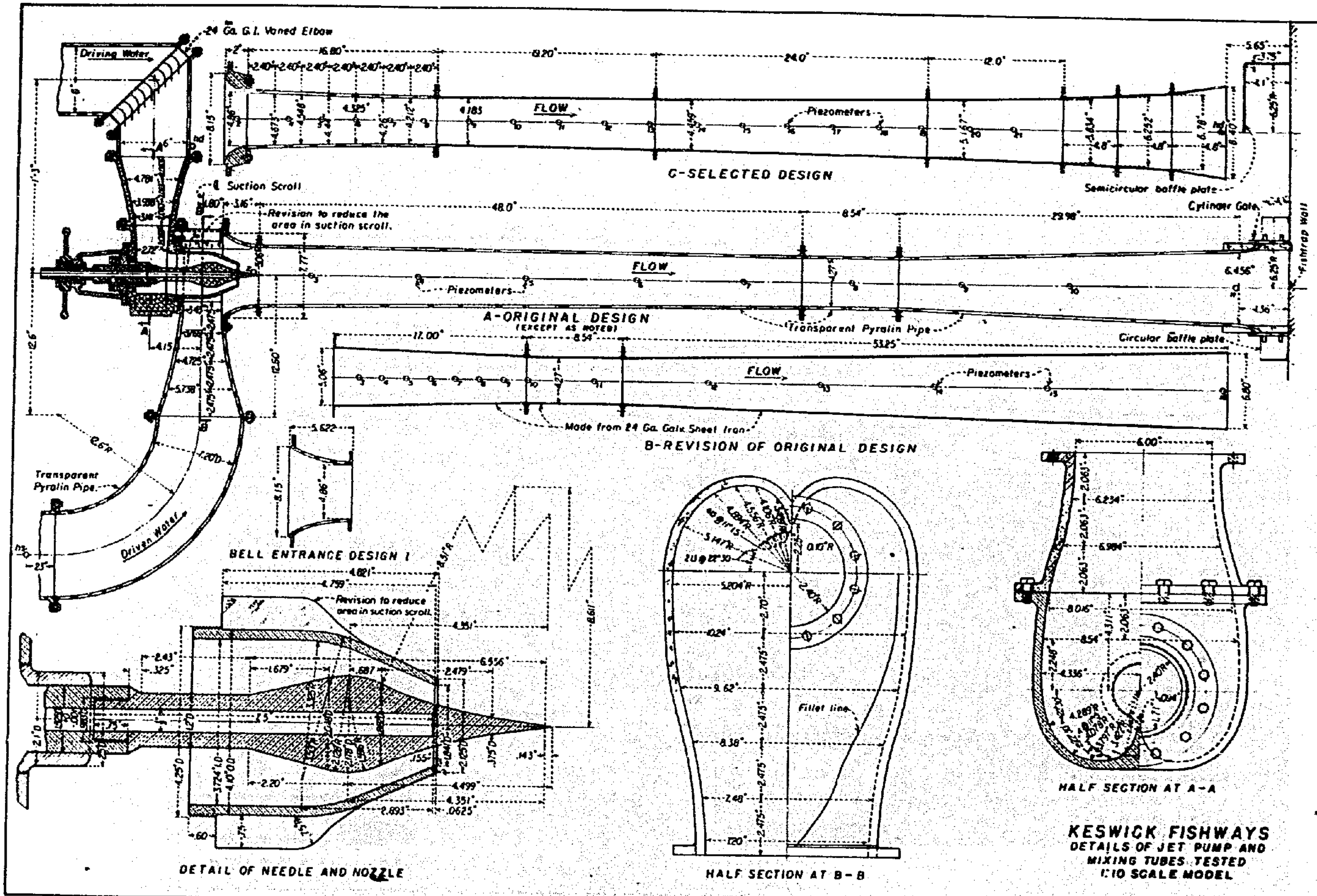


FIGURE 2

KESWICK FISHWAYS
DETAILS OF JET PUMP AND MIXING TUBES TESTED
1:10 SCALE MODEL

During the testing it was shown that the jet-pump could be primed satisfactorily without the aid of the cylinder gate. This was effected by allowing a very small discharge from the nozzle to flow into the fish-trap until the diffuser was completely submerged. Then, by increasing the flow from the nozzle, the pressure in the suction scroll reduced sufficiently to allow the water from the tailrace to flow into the pump, thereby priming the pump.

Another refinement was made in the design by changing the baffle from a shallow circular hood to a deep semicircle with a lip at the open end that deflected the flow downward from the upper half of the diffuser, figures 2A and C. This minimized the boiling and the splash that occurred when the jet from the diffuser impinged on the opposing wall of the distribution chamber.

Analyzing the data with the aid of the momentum principle, revealed that the momentum equation gave results of sufficient accuracy for design purposes. From a further study, a preliminary design curve was developed which provided a quick method of determining the applicability of the jet-pump to a particular installation.

Cavitation in a jet-pump is always a potential source of trouble even in the most carefully designed installations. This is due to the shearing action between the two jets casting off rapidly whirling vortices which, under certain conditions, form vapor pockets at their centers. The collapse of these pockets at the boundaries can cause pitting. This condition can be alleviated by admitting air to the shear plane between the jets. The quantity of air required is small and does not materially effect the efficiency of the pump.

III - PRINCIPLES OF SIMILITUDE

The π Theorem[4]. To interpret the results of model experiments it is necessary to have an equation which describes the behavior of the fluid under actual operating conditions and one which contains as variables the dimensions of the hydraulic structure and quantities such as viscosity, velocity, pressure density, surface tension, elastic modulus, etc., which suffice to determine all of the essential circumstances of flow. An equation of this type is called a physical equation.

Fluid flow is a general function of n variables and may be represented by the physical equation

$$f\ (a_1, a_2, a_3 \ldots\ldots a_n) = 0 \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

The π theorem states that if these n variables can be expressed in terms of m dimensional units, the general equation may then be expressed as a function of n-m dimensionless π terms, and that each π term will have m+1 variables of which only one need be changed from term to term. A dimensionless term being defined as one of which the numerical value does not change when the magnitude of the fundamental units are changed, when the relations between the fundamental units are kept unaltered. In accordance with this, equation (1) is generally written

$$f\ (\pi_1, \pi_2, \pi_3 \text{---} \pi_n) = 0 \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

The principal of dimensional homogeneity states that any correct and complete physical equation must have all of its terms of the same dimensions. For equation (2) to be dimensionally homogeneous, each term when expressed in dimensions of the F-L-T on M-L-T system must contain identical powers of each of the respective dimensions, according to the system being used.

All of the variables that have an effect on fluid motion are: (1) the physical properties, of density ρ, specific weight γ, viscosity u, surface tension σ, elastic modulus k, (2) the dynamic and kinematic quantities, of mean velocity V, a pressure increment Δp, and (3) a group of linear dimensions a, b, c, d representing length, width, etc., which define the boundaries of flow. The general equation of flow in these terms is

$$f\,(a,\ b,\ c,\ d,\ V,\ \Delta P,\ \rho,\ \gamma,\ u,\ \sigma,\ K) = 0 \ldots\ldots\ldots (3)$$

which contains eleven terms and each term is expressible in the three dimensional units of mass length and time. Since n is equal to the number of variables which are equal to eleven, and m is equal to 3, (the number of dimensional units mass, length and time), then according to the π theorem there will be n-m or 8 dimensionless π terms in the function

$$f\,(\pi_1,\ \pi_2,\ \pi_3 \text{ --- } \pi_8) = 0 \ldots\ldots\ldots\ldots\ldots (4)$$

As the theorem states that each π term will have m + 1 variables, then each of the π terms will have four variables. These may be selected arbitrarily, but it is expedient to select three variables, which between the three which contain all of the units in the M-L-T system or the F-L-T system according to the system used. This always results in a set of three simultaneous equations in the three exponents of the dimensions, which makes it possible to determine the value of the exponents easily. To determine the variables and to assist in the solving of the π terms, the dimensional equation of each term of equation (3) are written in the M-L-T system.

a = a length . . . . $a = L$

b = a length . . . . $b = L$

c = a length . . . . $c = L$

d = a length . . . . $d = L$

V = a velocity = $\frac{\text{length}}{\text{time}}$ . . . . $V = LT^{-1}$

P = pressure = $\frac{\text{force}}{\text{area}}$ . . . . $\Delta P = ML^{-1}T^{-2}$

ρ = density = $\frac{\text{mass}}{\text{volume}}$ . . . . $\rho = ML^{-3}$

γ = specific weight = $\frac{\text{force}}{\text{volume}}$ . . . . $\gamma = ML^{-2}T^{-2}$

u = dynamic viscosity = $\frac{\text{force x time}}{\text{area}}$ . . . $u = ML^{-1}T^{-1}$

σ = surface tension = $\frac{\text{force}}{\text{length}}$ . . . . $\sigma = MT^{-2}$

K = elastic modulus = $\frac{\text{force}}{\text{area}}$ . . . . $K = ML^{-1}T^{-2}$

From the dimensional equation select three equations which between them, contain all of the dimensional units. This procedure is not necessary, as any three terms could be selected but the method expedites the work. Three such equations are $V = LT^{-1}$, $A = L$, $\rho = ML^{-3}$. Each π term then contains these three variables with the remaining variables of equation (3) appearing singly in each group. Then

$\pi_1 = A^{x} V^{y} \rho^{z} b$

$\pi_2 = A^{x2} V^{y2} \rho^{z2} c$

$\pi_3 = A^{x3} V^{y3} \rho^{z3} d$

$\pi_4 = A^{x4} V^{y4} \rho^{z4} \Delta P$

$\pi_5 = A^{x5} V^{y5} \rho^{z5} \gamma$

$\pi_6 = A^{x6} V^{y6} \rho^{z6} u$

$\pi_7 = A^{x7} V^{y7} \rho^{z7} \sigma$

$_8 = A^{x8} V^{y8} \rho^{z8} K$

The exponents x, y and z are to be determined such that π_1, π_2, π_3, etc., shall have no dimensions, and therefore will be independent of the sizes of the fundamental units of M-L-T. This is accomplished by substituting the dimensional relationships in the π terms. Thus for

$$\pi_1 = A^x V^y \rho^z b$$
$$= L^x L^y T^{-y} M^z L^{-3z} L$$
$$= L^{x+y-3z+1} T^{-y} M^z$$

For the π_1 term to be dimensionless the sum of the exponents of each fundamental unit of M, L, or T must equal zero. Thus for π_1,

$$\left.\begin{aligned} x + y - 3z + 1 &= 0 \\ -y &= 0 \\ z &= 0 \end{aligned}\right)$$

which when solved, gave x = -1.

Then $\pi_1, = a^{-1} V^o \rho^o b = \frac{b}{a}$

Likewise, for $\pi_2 = a^{x2} V^{y2} \rho^{z2} c$, the dimensional equation is

$L^{x+y-3z+1} . T^{-y} . M^z$ from which was obtained

$$\left.\begin{aligned} x + y - 3z + 1 &= 0 \\ -y &= 0 \\ z &= 0 \end{aligned}\right) \text{ when } x = -1.$$

From this $\pi_2, = A^{-1} V^o \rho^o C = \frac{C}{a}$.

Similarly $\pi_3 = \frac{d}{a}$, and continuing

$$\pi_4 = a^{x4} V^{y4} \rho^{z4} \Delta P$$
$$= L^x L^y T^{-y} . M^z L^{-3z} ML^{-1} T^{-2}$$
$$= L^{x+y-3z-1} M^{z+1} T^{-y-2}$$

Then $$\left.\begin{aligned} x + y - 3z - 1 &= 0 \\ z+1 &= 0 \\ -y-z &= 0 \end{aligned}\right) \text{ when } x = 0$$

and $\pi_4 = V^{-2} \rho^{-1} \Delta P = \frac{\Delta P}{\rho V^2}$

In a similar manner, it can be shown that

$$\pi_5 = \frac{\Upsilon/\rho}{V^2/_a};\ \pi_6 = \frac{u/\rho}{Va};\ \pi_7 = \frac{\sigma/\rho}{V^2a};\quad 8 = \frac{K/\rho}{V^2}$$

Then from the equation (4) a general equation for the movement of fluids can be expressed as

$$f\left(\frac{b}{a}, \frac{c}{a}, \frac{d}{a}, \frac{\Delta p}{\rho V^2}, \frac{\Upsilon/\rho}{V^2/_a}, \frac{u/\rho}{Va}, \frac{\sigma/\rho}{V^2a}, \frac{K/\rho}{V^2}\right) = 0 \ldots\ldots\ldots\ldots (5)$$

This equation may also be written in the form

$$f\left(\frac{a}{b}, \frac{a}{c}, \frac{a}{d}, \frac{\rho V^2}{\Delta P}, \frac{V^2/a}{g}, \frac{Va}{u/\rho}, \frac{V^2a}{\sigma/\rho}, \frac{V^2}{K/\rho}\right) = 0 \ldots\ldots\ldots\ldots (6)$$

where $\Upsilon/\rho = g$. The form of the terms in this equation could have been obtained directly from the π terms if the variable from term to term, that is b, d, Υ, σ, c, ΔP, u and K, had been given the exponent (-1). The last four terms of equation (6) have been given names. The first term $\frac{V^2/a}{g}$ is the Froude number F and since a is a linear dimension, it may be taken as the depth y and the term takes the form $\frac{V^2}{gy}$ which is as it is generally written. The second is Reynold's number in which a can be taken as a diameter or hydraulic radius. The quantity $\frac{u}{\rho}$ is equal to Υ the kinemater viscosity. When rewritten in this form the π term takes the form of $\frac{VD}{\Upsilon}$ which is the usual way of expressing Reynolds number R. The third term is the Weber number in which σ/ρ is the kinematec capillarity. The fourth term is the Cauchy number $C = \frac{V^2}{E}$ where $E = K/\rho$, the kinematic elasticity.

From the results of the π theorem, a general equation for the velocity of fluids may be expressed in the form

$$V = f\left(\frac{a}{b}, \frac{a}{c}, \frac{a}{d}, F, R, W, C, \right)\sqrt{\frac{\Delta P}{\rho}}$$

The form of the function f except in the most simple cases must be found by experiment.

The explanation of equation (6) regarding hydraulic models is simply this, for the fluid motion to be similar in any two structures, each of the dimensionless terms in equation (6) must have the same value in each structure. That is, the value of $\frac{a}{b}$, $\frac{c}{d}$, F, R, etc., in the first structure, must equal the values of $\frac{a}{b}$, $\frac{c}{d}$, F, R, etc., in the second structure. This relationship is the fundamental law governing the design of hydraulic models. The terms $\frac{a}{b}$, $\frac{a}{c}$, and $\frac{a}{d}$ are ratios of the various structural dimensions and for these ratios to be the same between model and prototype, the two must be geometrically similar. The ratio of a prototype dimension to a similar dimension on the model is called the geometric scale ratio and is designated by the letter n. The fifth term or the Froude number, automatically becomes the same in geometrically similar structures, if the variation in the force of gravity and atmospheric pressure are neglected, and the same fluid is used in both structures. The same is true of the fourth term. The Weber and Cauchy numbers may be neglected because the effect of the capillary forces are small, provided the physical dimensions of the model are not too small, and it is obvious that the compressibility of water does not enter unless water hammer is being considered. The remaining term which is Reynolds number, deals with viscous forces and for exact similarity of flow to exist between model and prototype, the value of Reynolds number must be the same in both model and prototype.

By considering only geometrically similar structures, only two terms of equation (6) need be satisfied to produce exact similarity of flow and since the values of both must be the same in model and prototype the terms may be written

$$\frac{V_m \, l_m \, \rho_m}{\mu_m} = \frac{V_p \, l_p \, \rho_p}{\mu_p} \quad \ldots\ldots (7)$$

and

$$\frac{V_m^2 \, \rho_m}{\gamma_m \, l_m} = \frac{V_p^2 \, \rho_p}{\gamma_p \, l_p} \quad \ldots\ldots (8)$$

The subscripts m and p refer to model and prototype. Since $\gamma/\rho = g$, equation (8) can be written $\frac{V_m^2}{g_m a_m} = \frac{V_p^2}{g_p a_p}$, which has been called the law of Froude, it is a kinematical relationship and states of the conditions for similarity of flow of any liquid regardless of density. Usually the ratio of $\frac{g_m}{g_p}$ is taken as one, hence the density of the liquid used in the model is immaterial.

For the Froudian law of motion the transfer equation in terms of the linear scale ratio n, are derived as follows: For any length L, then $L_p = nL_m$ or $n = \frac{L_p}{L_m}$. The area in the prototype $= L_p W_p$, and the area in the model $= L_m \, W_m$, then $\frac{L_p \times W_p}{L_m \times W_m} = \frac{A_p}{A_m}$. Since $\frac{L_p}{L_m} = n$, and $\frac{W_p}{W_m} = n$, then $A_p = n^2 A_m$. In a similar manner, volume $_p = n^3$ volume in the model. Equations for time, discharge, velocity, etc., can be derived by setting up the ratios for model and prototype but for purpose of this paper only, the results with water in the model are listed:

$\text{Weight}_p = n^3$ (weight in model)

$\text{Time}_p = \sqrt{n}$ (time in model)

$\text{Velocity}_p = \sqrt{n}$ (velocity in model)

$\text{Acceleration}_p =$ (acceleration model)

$\text{Force}_p = n^3$ (force model)

$\text{Discharge}_p = n^{5/2}$ (discharge model)

$\text{Slope}_p =$ (slope model)

$\text{Power}_p = n^{7/2}$ (power model)

$\text{Momentum}_p = n^{7/2}$ (momentum model)

$\text{Work}_p \quad = n^4$ (work model)

$\text{Head}_p \quad = n$ (head model)

The dynamical quantities as force, pressure, power, etc., bring in either the density or specific weight. With water in both model and prototype the ratios are identical with the gravity ratio which is one.

A model is a real flow system in which friction plays an important part and it is necessary to investigate the effect of viscosity or to apply the Reynolds law of similarity expressed by equation (7). If equations (7) and (8) are to be satisfied simultaneously with gravity the same in both, then

$$\frac{V_m \, l_m \;\; _m}{u_m} = \frac{V_p \, l_p \;\; _p}{u_p} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (7)$$

$$\frac{V_m^2}{g l_m} = \frac{V_p^2}{g l_p} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (8)$$

Then from equation (8)

$V_m = V_p \sqrt{\dfrac{l_m}{l_p}} = V_p \sqrt{n}$. Substituting this in equation (7), it becomes

$\dfrac{\frac{V_p}{\sqrt{n}} \, l_m \;\; _m}{u_m} = \dfrac{V_p \, l_p \;\; _p}{u_p}$, from which it is possible to obtain two solutions. The obvious one in which $l_m = l_p$, $u_m = u_p$, $_m = \,_p$, merely states that the model is of the same size as the prototype.

The other solution occurs when $n^{3/2} = \frac{\rho_m u_p}{\rho_p u_m}$. This condition is in general impossible to fulfill because of the difficulty in obtaining fluids with these properties. For low velocities the resistance of the fluid does not depend on the density and the resistance to motion is almost entirely due to the viscosity and velocity of the liquid. In model operation, if the model flow or the prototype flow are in this zone, that is Reynolds numbers below 750 for open channels and 3000 for closed conduits, it is not possible to obtain similarity of flow in the model and the transfer equations do not apply.

For a flow system of high velocities, the density of the fluid plays an important part as some of the force acting on the boundaries is due to the momentum carried away from the surface of the boundaries by the fluid in the form of eddies. Since momentum equals mass x velocity, and massequals $\frac{w}{g}$, the momentum carried away obviously depends on the density of the fluid. Thus in turbulent flow, momentum transfer is a vorticity function and does not depend upon the viscosity of the fluid. Actually, in fully developed turbulent flow, the friction coefficient is constant and independent of the viscosity or Reynolds law. Under these circumstances the friction losses fulfill the Froudian model law and the head loss due to friction in either the model or the prototype, are proportional to some constant k times the velocity squared in the respective structure.

The Reynolds number at which fully developed turbulence occurs varies with the coefficient of roughness of the boundaries. For rough boundaries, fully developed turbulence occurs at much lower values of Reynolds number than it does when the boundaries are relatively smooth. In models of open channels, especially those of river beds, it is generally impossible to obtain fully developed turbulence throughout the model, and discrepancies between model and prototype can be expected. A good general agreement between model and prototype can be expected in open channels if the Reynolds number based on the hydraulic radius is above 4,000.

In models of closed conduits, such as the jet-pump, it is usually possible to operate the model either at or very near fully developed turbulence and obtain results, provided the relative roughness in the model is proportional, which are nearly identical with the prototype. This can be accomplished in the model by increasing the head to a point where the increased velocity gives a value of Reynolds number which falls in the range of fully developed turbulence.

The jet-pump was operated with prototype heads partly because it was desirable to increase the value of Reynolds number and thus obtain nearly fully developed turbulent flow in the critical regions; however, the main reason was to obtain actual prototype pressures on the model. The transfer equations between model and prototype under these conditions are related as follows:

$\frac{Q_p}{Q_m} = \frac{A_p V_p}{A_m V_m}$, but $\frac{A_p}{A_m}$ is equal to n^2. Then $\frac{Q_p}{Q_m}$ $\frac{n^2 V_p}{V_m}$.

Since V_p is equal to $\sqrt{2gH_p}$ and $Vm = \sqrt{2gH_m}$, and since $Hp = Hm$, the condition that the model and prototype heads be equal, then

$$\frac{Q_p}{Q_m} = n^2\sqrt{\frac{2gH_p}{2gH_p}} \quad \text{or } Q_p = n^2Q_m.$$

Since Hp is equal to Hm, it follows that $V_m = V_p$. The time relationship is obtained from

$$\frac{t_p = \sqrt{\frac{2H_p}{g_p}}}{t_m = \sqrt{\frac{2H_p}{g_m}}}$$

Since g_m and g_p can be assumed equal, it follows that $t_p = t_m$. Other relationships can be obtained in a similar manner, but as the results of the jet-pump tests can be transferred to prototype data with the aid of equations already derived, it will be unnecessary for purpose of this paper to derive the remainder of the relationships.

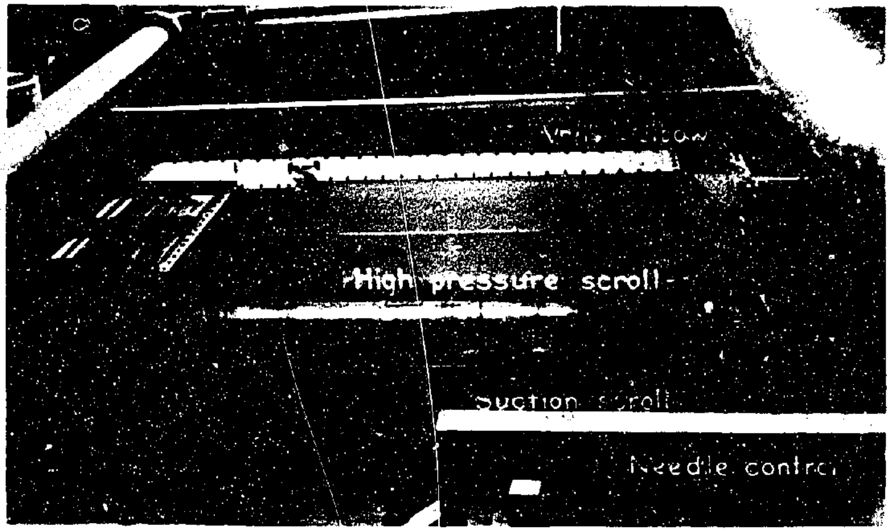
The total prototype discharge was fixed at 340 second-feet. From $Q_p = n^2Q_m$, the discharge for 1:10 model was 340 = 100 Q_m or 3.40 second-feet. With H_p equal to H_m all pressures throughout the model were equal to the prototype values.

IV - INVESTIGATION BY HYDRAULIC MODEL

Model construction and test method. The general arrangement of the model as shown in figures 3A and B, was the same for all tests. The only changes were in the types of bellmouth entrances, mixing tubes, and diffusers that were used in the course of the tests, and the removal of the cover on the distribution chamber when the model was operated under proportional heads.

In the operation of the model the high-pressure or driving water entered the bronze pressure scroll through a 6-inch, transparent, vaned elbow, whence it was directed to the transparent mixing chamber through a bronze nozzle. The suction, or driven water, came from a supply tank and flowed through a 7.2-inch

A. Model for testing with proportional heads



B. Model for testing with prototype heads

MODEL ARRANGEMENT SHOWING SELECTED MIXING TUBE DESIGN diameter pipe which connected to a transparent elbow leading to the suction scroll. The flow then passed through a plastic elbow to the suction scroll and then into the mixing chamber where it combined with the driving water. The combined flow entered the fish-trap through a sheet-metal diffuser and was returned by conduit to the laboratory system. The fish-trap, with the exception of 2 plastic windows, was constructed of 3/16-inch sheet steel because of the pressure obtained by operating the model with prototype heads.

When the model was operated with prototype heads, the fish-trap gates were closed and the distribution chamber was covered, as shown in figure 3B. This diverted the fish-trap flow to a controlled 8-inch drain in the bottom of the tank, making it possible to regulate the pressure in the distribution chamber to correspond to the equivalent water surface expected in the prototype structure. Sufficient head was obtained for the driving water by connecting in series an 8-inch and a 12-inch centrifugal pump. The driven water was supplied either by one or two 8-inch pumps; the use of one pump, or both of them, depended upon the quantity of water and the head required at the suction scroll.

With the exception of the tests necessary to study the turbulence in the fish-trap, all of the tests on the model were made with prototype heads and model discharges varying as the square root of the scale ratio. This method made it possible to obtain prototype pressures in the model, and, in addition, any adverse conditions, such as cavitation or undesirable flow, were easily detected.

Due to the difference in elevation between Denver and Keswick it was considered advisable to use absolute pressures instead of gage pressures in the operation of the model. Thus, with the center line of the prototype pump at elevation 494.00 and a variation in the water surface of the reservoir from elevation 570 to 585, the driving head on the pump varied from 75 to 91 feet, gage, or 109 to 124 feet, absolute (atmospheric pressure assumed to be 33-feet of water at Keswick). Likewise, the suction head varied from plus 6 feet to minus 9 feet, or a corresponding 39 to 24 feet, absolute. The delivery head was plus 7.0 feet, gage, or 40 feet, absolute. These absolute pressures, referred to gage at Denver, gave driving heads of 95.7-feet to 80.7-feet, suction heads of plus 10.7 feet to minus 4.3 feet, and a delivery head of 11.70 feet. While the differential between any two of the three heads does not vary, the gage values differ by the difference between the atmospheric pressures of the two locations.

Piezometers were installed at various points, as shown on figure 2, to obtain the pressures throughout the apparatus and to indicate when a desired operating condition was obtained. The suction water was supplied to the pump via a closed tank, the floor of which was several feet below the center line of the pump. This made it possible to obtain the various suction heads either by varying the water surface in the tank or by putting it under pressure. The head and the flow from the nozzle were controlled by the needle and a throttling valve. The back pressure on the diffuser was regulated by a gate valve in the 3-inch drain. This drain was located in the center of the equalizing conduit at the bottom of the fish-trap.

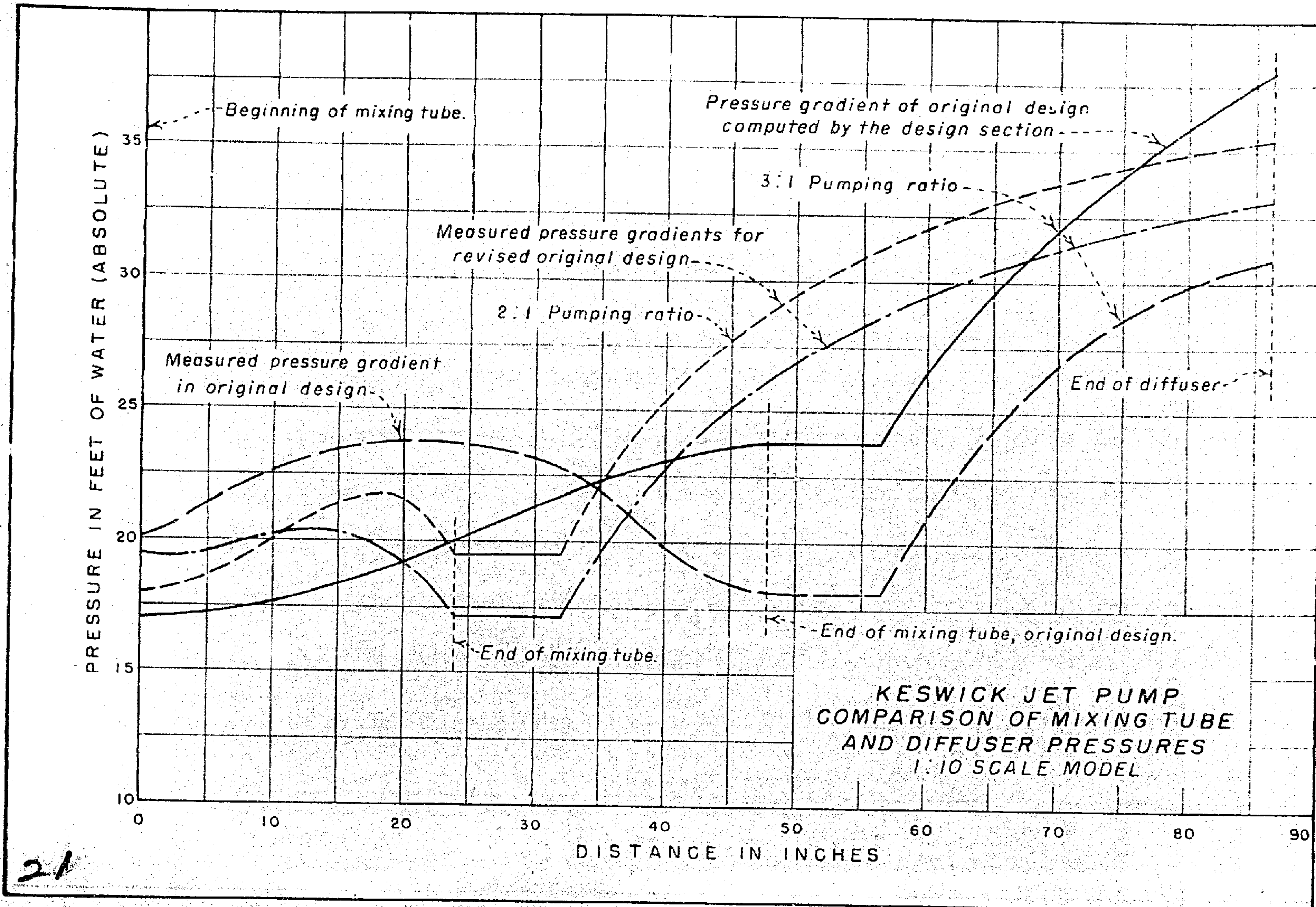
When the model was tested with proportional heads, the fish-trap gates were opened to allow part of the flow to discharge over a weir at the downstream end of the structure.

Initial tests. The tests of the original design, as shown on figure 2A, were made on a 1:10 scale model constructed as shown on figure 3. The first test was made to study the general character of the flow through the pump, from which it was found that two vortices formed in the suction scroll at the horizontal center line of the pump and extended downstream into the mixing tube. The vortices were eliminated by inserting fillers in the suction scroll case as shown on figure 2A; consequently the contour of the water passages upstream of the bellmouth entrance to the mixing tube were, from visual observations, considered satisfactory.

The original prototype design was to deliver 340 cubic feet per second with the needle 9.7 inches open, a head of 97.4 feet on the nozzle, with a back pressure of 6.0 feet at the end of the diffuser, and with a suction lift of 8 feet below the center line of the pump. As the model was to be tested with prototype heads in order to obtain prototype pressures, it was supposed to deliver 3.40 second-feet with the combination of heads mentioned above. To verify this the next test was made by setting the needle at the correct opening (0.97-inch) and carefully adjusting the controls on the model until the stated prototype heads were obtained. Then the pressures obtained from piezometer taps, located as shown in figure 2A, were recorded as were the individual discharges Q_S and Q_N.

The discharge Q_N was very near the expected value, 0.895 second-feet as compared with the computed value of 0.85 second-feet, but the actual Q_s was only 0.045 second-feet instead of the anticipated 2.55 second-feet. From this, it appeared that the design was in error. As an additional check, another test was made with the discharge and suction heads adjusted to values such that the pump would actually deliver the designed Q_N and Q_s. The piezometric pressures obtained were then plotted as shown on figure 4, on the same scale as were the pressures computed by the design section. Again it was apparent that there must have been an erroneous assumption in the design, for in addition to the pressures not corresponding to the computed values, the characteristics of the two curves were dissimilar.

In view of these results it was decided to make another test with a shortened mixing tube and a more efficient diffuser, figure 2B. The tests on this design were conducted with a constant head of 97.4 feet, gage, on the nozzle and with a variety of suction and discharge heads. Different combinations of suction and discharge heads in combination with a constant driving head produced the desired variety of pumping ratios. For this arrangement and design heads Q_N was 0.893 second-feet and Q_s was 0.70 second-feet, which was an improvement over the original design but was still far from the anticipated performance. Further testing showed that when the back pressure h_d was set at 3.19 feet, gage, with an h_s of -3.15 feet, gage, the pumping ratio Q_s/Q_N became $\frac{2.55}{0.85}$, or 3. This was a differential pumping head of 6.34 feet as compared to 5.22 feet for a similar flow in the original design. In a like manner the pumping ratio


KESWICK JET PUMP
COMPARISON OF MIXING TUBE
AND DIFFUSER PRESSURES
1:10 SCALE MODEL
PRESSURE IN FEET OF WATER (ABSOLUTE)
DISTANCE IN INCHES
Beginning of mixing tube.
Pressure gradient of original design
computed by the design section
3:1 Pumping ratio
Measured pressure gradients for
revised original design
2:1 Pumping ratio
Measured pressure gradient
in original design
End of diffuser
End of mixing tube, original design.
End of mixing tube.
10
15
20
25
30
35
0
10
20
30
40
50
60
70
80
90

became 2 with a total discharge of 3.4 second-feet, when the needle was opened to permit 1.13 second-feet to flow from the nozzle and the differential pumping head was adjusted so that the values of h_d and h_s were 5.52 and 6.17 feet, respectively.

The piezometric pressures obtained with the 3.1 and 2:1 flow ratios in the mixing tube were plotted as shown in figure 4. From an inspection of these curves it was noted that they were similar in form to the measured pressure gradient of the original design in that there was an increase in pressure from the beginning of the mixing tube to some intermediate point, after which there was a rapid decrease in pressure until it reached a minimum at the cylindrical section of the pipe.

The slope of the curve in the decreasing pressure zone roughly approximates Bernoulli's law for flow in a conical section. Therefore, it was assumed that the mixing of the inner and the outer jets was completed before the flow reached the end of the mixing tube and that better performance could be obtained if the mixing tube terminated at the point of maximum pressure.

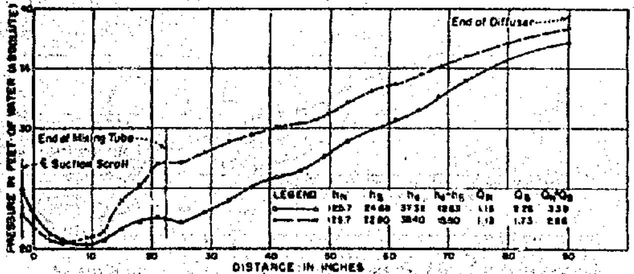
The data pertaining to pumping heads and mixing ratios, secured during the initial tests, indicated that the 3:1 pumping ratio was unattainable with the differential pumping head designated in the design because the revised design of figure 2B, which was efficient according to the present standards of jet-pumps, did not approach the optimum of the design.

In accordance with these findings, and with the aid of data secured from the tests, a new mixing tube and diffuser were evolved, figure 3B, which was to have a 2:1 mixing ratio at the maximum differential pumping head and a constant increase in pressure from the beginning of the mixing chamber to the end of the diffuser.

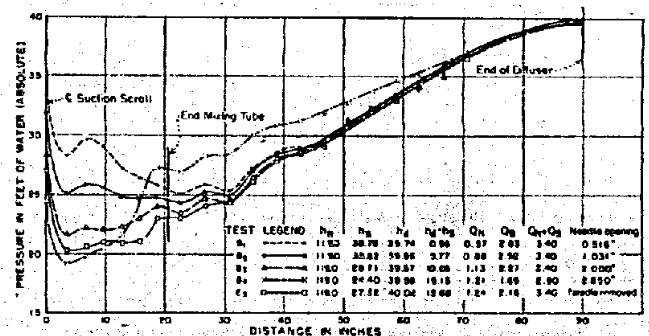
Tests of 2:1 mixing ratio design. The mixing tube and the diffuser were constructed as shown in figure 2C. The transition to the mixing chamber was effected by the bellmouth design 1, figure 2, the general arrangement being similar to that shown in figure 3B. The results of tests on this arrangement, conducted in a manner similar to those previously made, are shown on figure 5A. The tabular data accompanying the pressure curves show that with the maximum differential pumping head, $h_d - h_s$ equal to 16-feet Q_N was 1.13 second-feet, and Q_S was 1.73 second-feet, making the pumping ratio 1:1.53 instead of the expected 2:1. The 2:1 ratio was obtained when $h_d - h_s$ was reduced to 12.63 feet. In connection with this test, it is interesting to note that the constant increase in pressure expected in the mixing tube was not obtained in the model.

Other tests were performed with different throat areas in the bellmouth entrance in an attempt to increase the over-all efficiency. From these tests, the bellmouth-entrance design shown in figure 2C was selected, more because of practical considerations than for any increase in efficiency that was obtained. Another attempt was made to increase the efficiency by inserting a turbulator (a disk of 1/2-inch expanded metal lath) in the suction line at the entrance to the suction scroll. The results of the tests are shown on figures 5B and 6 which show that there was no increased efficiency.

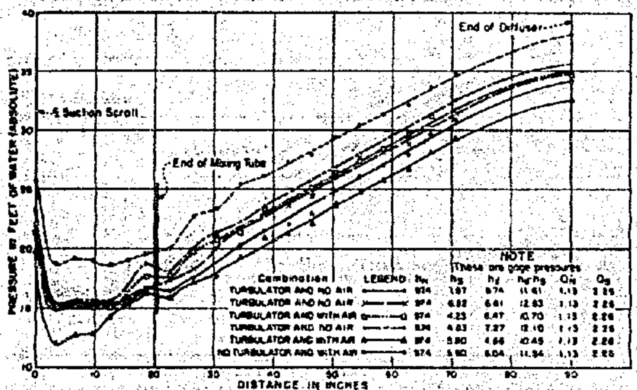
FIGURE 5



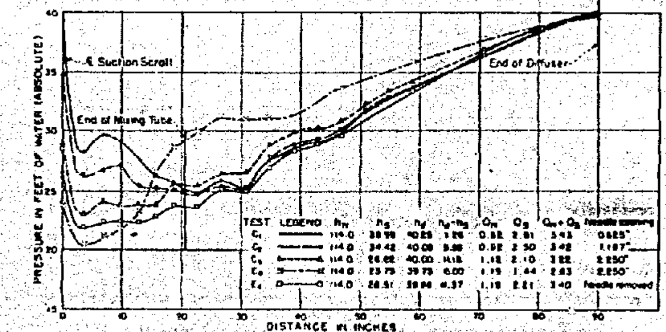
A.-MIXING TUBE AND DIFFUSER PRESSURE FOR BELL MOUTH DESIGN I AND SELECTED MIXING TUBE AND DIFFUSER 1:10 SCALE MODEL



D.-MIXING TUBE AND DIFFUSER PRESSURES AN INTERMEDIATE DRIVING HEAD 1:10 SCALE MODEL

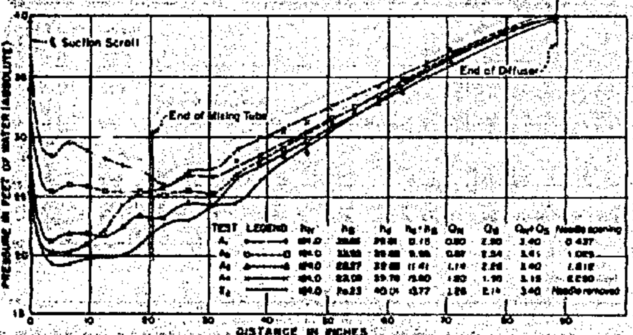


B.-MIXING TUBE AND DIFFUSER PRESSURES FOR SELECTED DESIGN IN COMBINATIONS WITH TURBULATOR AND AERATION 1:10 SCALE MODEL

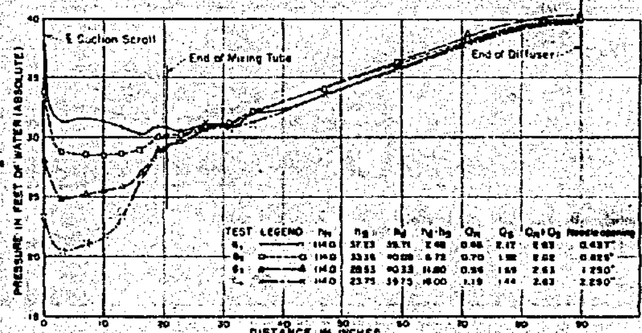


E.-MIXING TUBE AND DIFFUSER PRESSURES 1:10 SCALE MODEL AND A MINIMUM DRIVING HEAD

NOTE
All pressures are absolute pressures unless otherwise noted



C.-MIXING TUBE AND DIFFUSER PRESSURES 1:10 SCALE MODEL AND MAXIMUM DRIVING HEAD



F.-MIXING TUBE AND DIFFUSER PRESSURES FOR THE MODEL TO SCALE 1:15

MIXING TUBE AND DIFFUSER PROTOTYPE PRESSURES FOR VARIOUS MODEL ARRANGEMENTS AND MODEL SCALES

When the pump was operating with the maximum design head, there was considerable crackling and other evidence of cavitational phenomenon in the mixing tube which was not disclosed by the piezometric pressures. It was thought that pressures in the cavity at the downstream edge of the nozzle might be sufficiently low to cause the trouble. A piezometer was installed in this region, but it failed to register critical subatmospheric pressure.

When air was admitted through the piezometer opening the violence of the phenomenon was reduced, indicating that while no boundary pressures were sufficiently low to cause cavitation, the phenomena was occurring out in the stream. The quantity of air entering the apparatus through the 1/16-inch diameter piezometer was small and had no noticeable effect on the general pumping efficiency, figures 5B and 6. Since the reduction in cavitational noise was so marked when the air was admitted at this point, it was considered advisable to provide similar aeration on the prototype.

Since the completion of these tests, a number of 8-inch jet-pumps with the air vent have been installed at Shasta Dam for supplying cooling water to the generators. This vent supplied air to the downstream face of the nozzle and was regulated by an automatic valve which admitted air only when the pressure at the nozzle was reduced to approximately one-half an atmosphere. Experience with this installation has proven the merits of the vent.

The reservoir at Keswick Dam will supply water for storage and power; consequently, there is an expected variation of approximately 15 feet in the water surface. Likewise the tailwater level will

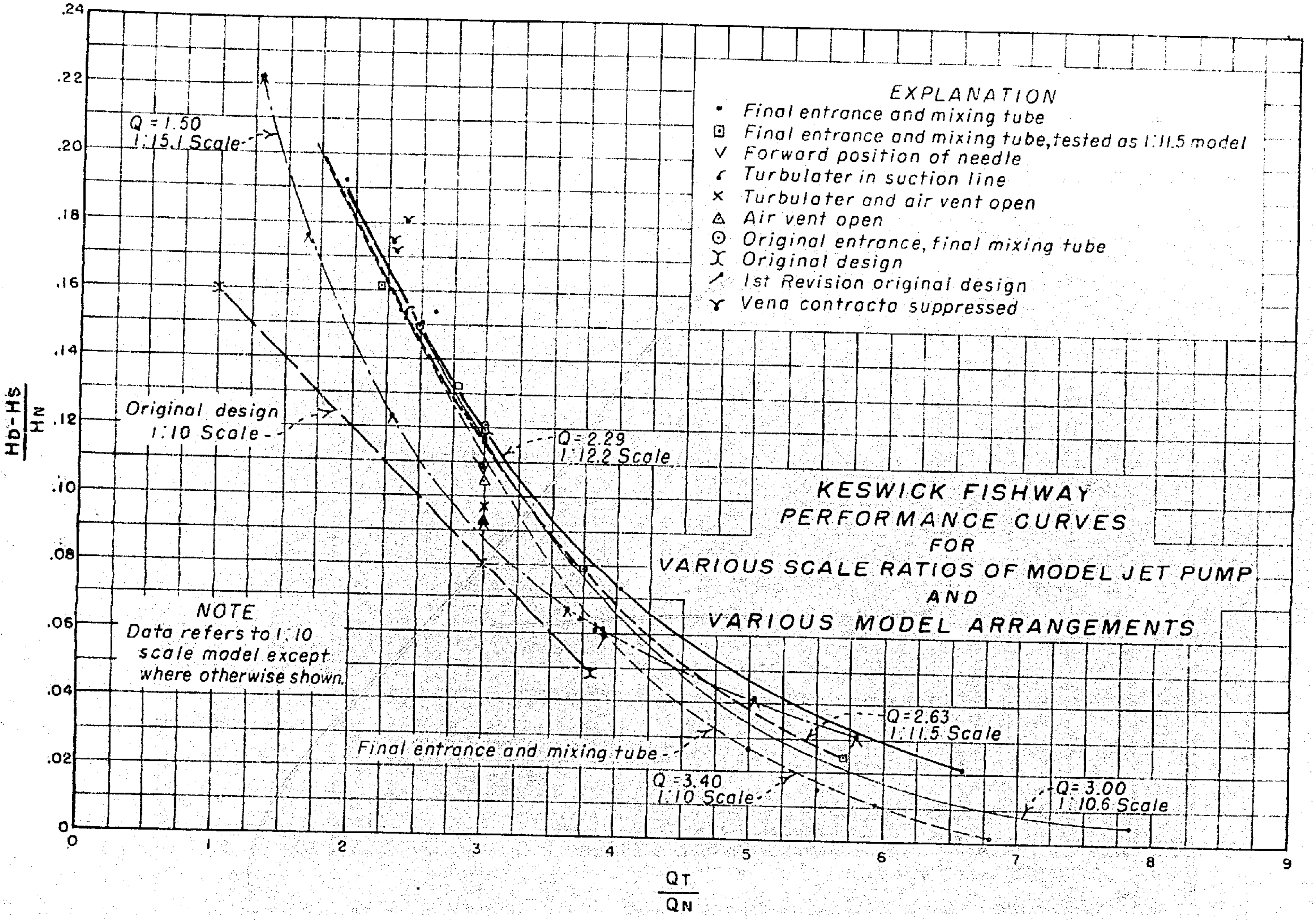

EXPLANATION
Final entrance and mixing tube
Final entrance and mixing tube, tested as 1:11.5 model
Forward position of needle
Turbulater in suction line
Turbulater and air vent open
Air vent open
Original entrance, final mixing tube
Original design
1st Revision original design
Vena contracta suppressed
KESWICK FISHWAY
PERFORMANCE CURVES
FOR
VARIOUS SCALE RATIOS OF MODEL JET PUMP
AND
VARIOUS MODEL ARRANGEMENTS
Q = 1.50
1:15.1 Scale
Original design
1:10 Scale
Q = 2.29
1:12.2 Scale
NOTE
Data refers to 1:10 scale model except where otherwise shown.
Q = 2.63
1:11.5 Scale
Final entrance and mixing tube
Q = 3.40
1:10 Scale
Q = 3.00
1:10.6 Scale
$\frac{H_D - H_S}{H_N}$
$\frac{Q_T}{Q_N}$
.24
.22
.20
.18
.16
.14
.12
.10
.08
.06
.04
.02
0
1
2
3
4
5
6
7
8
9

FIGURE 6

vary with the amount of water passing the turbines and the quantity being released downstream for irrigation purposes. These two factors directly affected the performance of the jet-pump because a fluctuation in the reservoir level will change the driving head at the nozzle and a variation in tailwater elevation will increase or decrease the total lift of the driven water. The feasibility of the design depended upon delivery by the jet-pump of the required quantity of water to the fish-trap for any combination of heads resulting from the operation of the dam and the turbines.

To study the performance and to obtain the operating characteristics over the range of heads expected, four series of tests were made on the model whereby the discharge head h_d was held nearly constant at 40 feet, absolute, and in any one series the driving head was held constant and the needle position and suction head varied until a total discharge of 3.40 second-feet was obtained. Obviously, within the range of the pump, and with a particular driving head, there were an infinite number of combinations of suction heads and needle positions wherewith a total discharge of 3.40 second-feet could be obtained. To facilitate testing, four suction heads varying between 39 feet and 24 feet, absolute, were selected for each series of tests. The needle position was varied, in each test, until the correct total discharge, or the maximum obtainable, was obtained with the particular driving head. The results of these tests, along with the pressures in the mixing tube, are shown in figures 5C, D, and E. From these tests it was evident that the required discharge of 3.40 second-feet, in combination with the higher lifts, was beyond the capacity of the pump.

The correct discharge and a design for the prototype were obtained from the model by considering its geometric dimensions to a scale ratio other than the 1:10 used heretofore. The new scale ratio of 1:11.5 was determined from the laws of similitude and the data shown in figure 5E, test C4.

The pump characteristics and the performance curves obtained from the 1:10 scale tests were not applicable to the 1:11.5 ratio; so this information was obtained from a new series of tests. The results are shown in figures 5F and 6, which show that there was an improvement in the performance, probably due to the fact that the new discharge caused a new needle setting for each suction head, thus changing the ratio of the diameters of the driving and the driven jets to a value more compatible with the phenomenon of mixing.

Space limitations at the prototype installation made it impractical to use a scale ratio larger than 1:11.5; however, other tests were made to determine the performance with scale ratios of 10.6, 12.3, and 15.1. The results, as shown on figure 5, indicated an improvement in efficiency over the 1:10 ratio for the 10.6 and 12.2 model scales, but a marked decrease in performance for the 15.1 ratio. The cause of the decrease is not clearly understood; however, it might have been the result of passing to a region where the ratios of jet diameters were unfavorable for good efficiency with the range of pumping heads used in the test.

<u>Tests with proportional heads</u>. The model as originally designed was equipped with a cylinder gate at the end of the diffuser,

figure 2A, to facilitate priming of the pump. Tests with this gate and with the model operating under proportional heads showed that the gate materially aided priming, and, for a time, it was believed to be necessary. However, subsequent testing showed that priming could be effected without use of the gate by proper manipulation of both the water surface in the fish-trap and the needle, which controlled flow from the nozzle. The priming procedure for the model was as follows: The 8-inch discharge line from the model was closed, thus causing all the inflow to the model to pass over the upstream weir in the fish-trap. Then the needle was opened 1/16-inch on the model, allowing a very small jet to issue from the nozzle and to fill the fish-trap gradually with a minimum of splash. The rate at which the fish-trap can be filled to the proper level is governed entirely by the amount of splash that can be tolerated. When the water depth in the distribution chamber became sufficient to seal the end of the diffuser, a jump formed in the mixing tube, thereby causing the pump to prime. The flow from the nozzle was then increased, and, when a sufficient vacuum formed, due to the increase, the driven water began to flow through the pump.

The priming may be accomplished in the prototype structure by closing the fish ladder gates and allowing a small flow from the nozzle to fill the fish-trap to elevation 497.5. When the water reaches this elevation, the flow from the nozzle will form a jump in the mixing tube but will not create a sufficient vacuum to pump water. By increasing the flow from the nozzle a sufficient vacuum will be created in the suction scroll to allow the tailwater to rise in the pump and flow into the fish-trap.

The final tests, to determine if the discharge emitting from the diffuser produced any adverse flow conditions in the distribution chamber or the fish-trap, showed that flow in the fish-trap was satisfactory, whereas the flow in the distribution chamber was very turbulent, and that objectionable splash occurred when the jet from the diffuser impinged on the opposing wall of the distribution chamber. To remedy this condition, several sizes of semicircular baffles were tested, the most satisfactory one being that shown in figure 2C. It eliminated the splash and minimized the turbulence.

The final design as it will be constructed in the field is shown on figure 7, where only the dimensions of the water passages are shown. There is some variation from the design tested in the model which probably will reflect in the operation of the prototype.

Analysis of the results. The jet-pump characteristics can be analyzed from a power-input viewpoint if the experimental data concerning the apparatus is available. The method is as follows: The input of power in foot-pounds through the inner annular nozzle is $Q_n \gamma H_n$ and through the outer nozzle $Q_s \gamma H_s$. The power at the exit is $(Q_N + Q_s)H_d$. The power balance then becomes

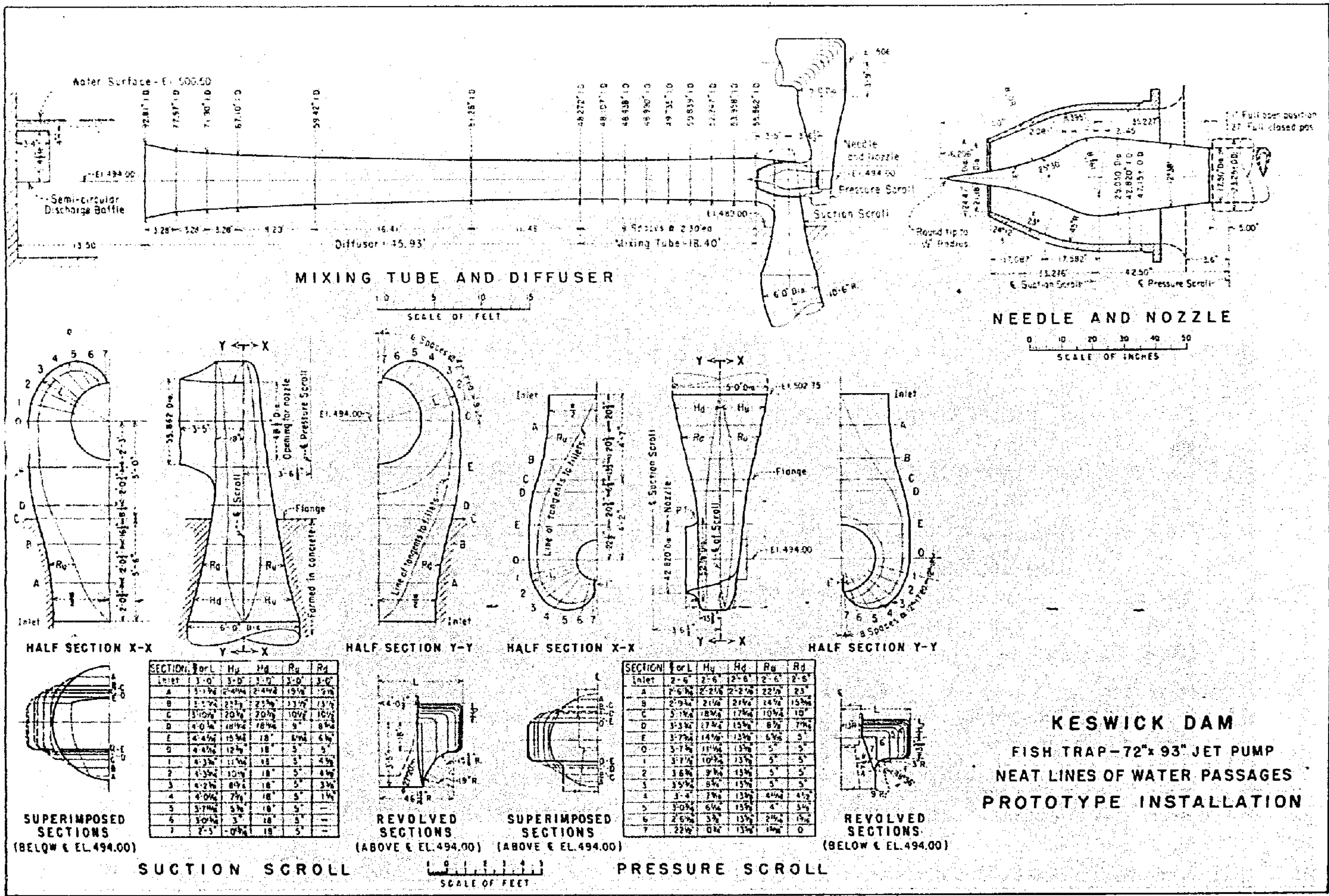
$$Q_N \gamma H_N + Q_s \gamma H_s = (Q_N + Q_s) \gamma H_d + \text{power loss.}$$

If the power loss is assumed to be

$$K_1 Q_n \gamma \frac{V_N^2}{2g} + K_2 Q_2 \gamma \frac{V_2^2}{2g}$$

and it is further assumed that

$$K_1 = K_2 = A,$$



SUCTION SCROLL

SECTION	℄ or L	Hu	Hd	Ru	Rd
Inlet	3'-0"	3'-0"	3'-0"	3'-0"	3'-0"
A	3'-1 [illegible]	2'-4 [illegible]	2'-4 [illegible]	19 [illegible]	19 [illegible]
B	3'-5 [illegible]	23 [illegible]	23 [illegible]	13 [illegible]	13 [illegible]
C	3'-10 [illegible]	20 [illegible]	20 [illegible]	10 1/2"	10 1/2"
D	4'-0 [illegible]	18 [illegible]	18 [illegible]	9"	8 [illegible]
E	4'-4 [illegible]	15 [illegible]	18"	6 [illegible]	6 [illegible]
0	4'-4 [illegible]	12 [illegible]	18"	5"	5"
1	4'-3 [illegible]	11 [illegible]	18"	5"	4 [illegible]
2	4'-3 [illegible]	10 [illegible]	18"	5"	4 [illegible]
3	4'-2 [illegible]	8 [illegible]	18"	5"	3 [illegible]
4	4'-0 [illegible]	7 [illegible]	18"	5"	1 [illegible]
5	3'-7 [illegible]	5 [illegible]	18"	5"	—
6	3'-0 [illegible]	3"	18"	5"	—
7	2'-3"	-0 [illegible]	18"	5"	—

PRESSURE SCROLL

SECTION	℄ or L	Hu	Hd	Ru	Rd
Inlet	2'-6"	2'-6"	2'-6"	2'-6"	2'-5"
A	2'-6 [illegible]	2'-2 [illegible]	2'-2 [illegible]	22 1/2"	23"
B	2'-9 3/4"	21 1/4"	21 1/4"	14 [illegible]	15 [illegible]
C	3'-1 [illegible]	18 [illegible]	17 [illegible]	10 [illegible]	10"
D	3'-3 3/4"	17 [illegible]	15 [illegible]	8 1/2"	7 [illegible]
E	3'-7 [illegible]	14 [illegible]	13 [illegible]	6 [illegible]	5"
0	3'-7 [illegible]	11 [illegible]	13 [illegible]	5"	5"
1	3'-7 [illegible]	10 [illegible]	13 [illegible]	5"	5"
2	3'-6 [illegible]	9 [illegible]	13 [illegible]	5"	5"
3	3'-5 [illegible]	8 [illegible]	13 [illegible]	5"	5"
4	3'-4"	7 [illegible]	13 [illegible]	4 [illegible]	4 1/2"
5	3'-0 [illegible]	6 [illegible]	13 [illegible]	4"	3 [illegible]
6	2'-6 [illegible]	3 [illegible]	13 [illegible]	2 [illegible]	1 [illegible]
7	22 1/2"	0 [illegible]	13 [illegible]	1 [illegible]	0

KESWICK DAM

FISH TRAP—72" x 93" JET PUMP

NEAT LINES OF WATER PASSAGES

PROTOTYPE INSTALLATION

FIGURE 7

then the power balance becomes

$$Q_N\, H_N + Q_s\, H_s = (Q_N + Q_s)\, H_d + A\left(Q_n\, \frac{V_N^2}{2g} + Q_s\, \frac{V_2^2}{2g}\right).$$

If A is a known constant, the solution is as follows: From the preceding equation

$$Q_N\, V_N\, H_N + a_s\, V_s\, H_s - Aa_H\, \frac{V^3}{2g} - Aa_s\, \frac{V_s^3}{2g} = a_H\, V_N\, H_d + a_s\, V_s\, H_d.$$

As $\quad V_n = \sqrt{\dfrac{2g\,(H_n - x)}{1 + K_N}}$

and $\quad V_s = \sqrt{\dfrac{2g\,(H_s - x)}{1 + K_s}}$

the equation becomes

$$\frac{(H_N - H_d)}{\sqrt{1 + K_N}} \sqrt{H_N - x} - \left[\frac{a_s}{a_N}\; \frac{H_d - H_s}{1 + K_s}\right] h_s - x$$

$$- \left[\frac{A}{(1 + K_N)^{3/2}}\right] (H_N - x)^{3/2}$$

$$- \left[\frac{a_s}{a_N}\; \frac{A}{(1 + K_s)^{3/2}}\right] (H_s - x)^{3/2} = 0 \;.....(1)$$

where x is the pressure across the plane of the nozzle.

From the four tests conducted with the needle removed from the nozzle, an average value of A equal to 0.705 was determined. The values varied between 0.674 and 0.735, being 0.674 for $H_N = 81.44$; 0.702 for $H_N = 86.39$; 0.707 for $H_N = 91.34$; and 0.735 for $H_N = 96.34$. By substituting the values $K_N = 0.42$, $K_s = 0.031$, $A = 0.705$, $H_N = 91.34$, $H_d = 12.04$, and $H_s = +\,0.08$ into equation (1), and solving it graphically, gave $x = -7.73$ which, when substituted into the equations $\quad Q_N = a_N \sqrt{\dfrac{2g(H_N - x)}{1 + K_N}} \quad$ and $\quad Q_s = a_s \sqrt{\dfrac{2g(H_s - x)}{1 + K_s}}$

gave $Q_N = 1.238$ against a 1.24, measured, and $Q_s = 2.162$ against a 2.16, measured. The computation was made across the plane of the beginning of the mixing tube, and it was assumed that the area of the driving jet at the point was equal to the area at the end of the nozzle.

The chief difficulty with this method of approach lies in determining the empirical coefficient A. Its value will vary with the type of nozzle, length of mixing tube, type of diffuser, the ratio of the diameter of the mixing tube to the nozzle diameter, and the head on the nozzle and suction lines. It can be determined accurrately only by testing on actual installation. The close agreement obtained with this method was due to using coefficients determined from the tests.

A more satisfactory solution to the jet-pump problem can be obtained by equating the energy relations existing in the arrangement. The basic equations are

$$H_N = \frac{P_a}{\gamma} + (1 + K_N)\frac{V^2_N}{2g} \quad \dots\dots\dots\dots \quad (2)$$

$$H_s = \frac{P_a}{\gamma} + (1 + K_s)\frac{V^2_s}{2g} \quad \dots\dots\dots\dots \quad (3)$$

$$\frac{P_t}{\gamma} + \frac{V^2_2}{2g} = H_d + K_d\frac{V^2_t}{2g} \quad \dots\dots\dots\dots \quad (4)$$

$$\left.\begin{array}{l} Q_s = A_s V_s \\ Q_N = A_N V_N \\ Q_t = A_t V_t \end{array}\right\} \quad \dots\dots\dots\dots \quad (5)$$

The energy loss per unit of time in a mixing tube with parallel sides as given by Lorenz[5] is

$$L = Q_N \gamma \frac{(V_N - V_t)^2}{2g} + Q_s \gamma \frac{(V_s - V_t)^2}{2g}$$

The loss of energy due to frictional resistance of the boundaries in the mixing tube is approximately

$$K_t \, \gamma \, Q_t \quad \frac{V^2{}_t}{2g}$$

where K_t is a resistance factor computed in the same manner as for flow in a converging tube. The loss of energy in the diffuser is expressed as

$$\gamma K_d \, Q_T \quad \frac{V^2{}_t}{2g} \, .$$

Equating the power supplied by the nozzle to the sum of the work done per second and the friction and shock losses gives

$$Q_N(H_N - H_d) = Q_s(H_d - H_s) + K_N \, Q_N \frac{V^2{}_N}{2g} + K_s \frac{Q_s V^2{}_s}{2g}$$

$$+ (K_d + K_t) Q_T \frac{V^2{}_t}{2g} + Q_N \frac{(V_N - V_t)^2}{2g}$$

$$+ Q_s, \frac{(V_s - V_t)^2}{2g} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (6)$$

Subtracting equations (3) from equation (2) and inserting the relations of (5) gives

$$H_N - H_s = \frac{V^2{}_N}{2g} \left[1 + K_N - (1 + K_s) \right] \frac{V^2{}_s}{V^2{}_N} \ldots \ldots \quad (7)$$

Rearranging equation (6) gives

$$\frac{H_N - H_d}{H_d \quad H_s} = \frac{Q_s}{Q_N} + \frac{\frac{B}{2g}}{H_d - H_s} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (8)$$

where

$$B = K_N \, V_N{}^2 + K_s \frac{Q_s}{Q_N} \quad V_s{}^2 + \quad (K_d + K_t) \quad \frac{Q_T}{Q_N} \, V^2{}_t$$

$$+ (V_N - V_t)^2 + \frac{Q_s}{Q_N} \quad (V_s - V_t)^2 \ldots \ldots \ldots \quad (9)$$

subtracting equation (7) from equation (6) and collecting terms, gives

$$H_d - H_s = \frac{Q_N V^2_N + Q_N K_N V^2_N - (1 + K_s) V^2_s Q_N - Q_N B}{2g (Q_N + Q_s)}$$

Substituting this in equation (8) gives

$$\frac{H_N - H_d}{H_d - H_s} = \frac{Q_s}{Q_N} + \frac{(Q_N + Q_s) B}{Q_N \left[V^2_N (1 + K_N) - (1 + K_s) V^2_s \right] - Q_N B} \quad \ldots(10)$$

In these equations the basic assumptions are that the mixing tube has parallel sides, the area coefficient of the nozzle is negligible, the nozzle velocity is the average velocity at the vena contracta and the mixing is complete, or nearly so, at the throat. The nozzle coefficient K_N is a velocity coefficient; K_d and K_t are friction coefficients based on the throat velocity. K_t is obtained by computing the friction loss in the mixing tube from the ordinary friction equation

$$H_L = f \frac{L}{D} \frac{V^2}{2g}$$

and equating this to $K_t = \frac{V^2_t}{2g}$.

If the sides of the mixing tube are parallel, K_t is equal to $f \frac{L}{D}$.
K is obtained by equating all of the diffuser losses to $K_d \frac{V^2_t}{2g}$ and solving for K_d. K_s is a friction coefficient representing all of the losses in the suction line. It is based on V_s at the upstream end of the mixing tube. If the nozzle has an area coefficient, the V_N and A_N refer to the conditions existing at the vena contracta, and A_s is the area of the ring surrounding the vena contracta.

As equation (1) is not entirely correct for a jet-pump having a converging mixing chamber (the energy loss equation (4a) applies to a mixing tube with parallel sides), a comparison of the test results to the results computed from equation (10) was made to determine if a suitable agreement could be obtained. From test E_2, where the needle was removed from the nozzle, the test data was as follows:

$Q_s = 2.14$

$Q_N = 1.26$

$H_N = 97.31$

$H_s = -1.15$

$H_d = 12.95$

The velocities obtained from equation (5) were

$V_N = 72.41$

$V_t = 35.63$

$V_s = 20.70$

V_N is the average velocity at the vena contracta of the nozzle, and V_s is the velocity of the suction flow in the area surrounding the vena contracta at the beginning of the mixing chamber. The actual area at the end of the nozzle was 0.01849 square feet. With a coefficient of area equal to 0.943, the area of the vena contracta as 0.0174. Subtracting this area from the area of mixing chamber at the upstream end gave A_s equal to 0.1034 square feet. A_t was equal to 0.0967 square feet. The computed friction coefficients were

$K_t = 0.062$

$K_d = 0.126$

$K_s = 0.067$

$K_N = 0.278$

where $K_n = \frac{1}{C^2} - 1$. Substituting these values into equation (10) gave

$$\frac{H_N - H_d}{H_d - H_s} = 6.13$$

against the test value of 5.99, or approximately 2.3 percent variation. The results of other tests did not vary more than 6 percent from the values obtained from equation (10). The percentage of variation from the actual probably varies with the amount of divergence or convergence of the mixing tube. However, no attempt was made to determine this in the test program.

As a check on the accuracy of the original design, the assumed constants

$K_d = 0.150$

$K_t = 0.135$

$K_N = 0.155$

$C_a = 0.94$

$C_N = 0.893$

$A_N = 0.0118$

$K_s = 0.067$

$A_s = 0.1278$

were substituted in equation (6) along with the design heads. The equation was then reduced to the form

$$37.39\,Q_s^3 + 117.95\,Q_s^2 + 1705.59\,Q_s - 35.64 = 0$$

which gave Q_s = 0.021 cubic feet per second. As the suction flow in the model, under these conditions, was so small it would not register an accurate reading on the connected metering system, it cannot be stated that a good agreement was obtained. However, the results are significant when it is realized that according to the design, the model suction flow should have been 2.55 second-feet under the test conditions.

<u>Length of the mixing tube</u>. Equation (10) does not contain any terms which refer to the mixing length nor was there any attempt, in this study, to establish or verify a theory concerning this phenomena.

The data available in literature have been taken for the simpler cases of turbulent flow, namely, cases of wall bound turbulence such as flow in converging or diverging sections, or parallel flow and free discharge of a jet into a free fluid, which is free turbulence. It has been found in fully developed wall bound turbulence that the mixing length (L) is some function of a distance (y) from the boundary. The function is dependent on the convergence or divergence of the jet but is independent of the distance (x) along the axis of the stream. In a free jet (L) is independent of (y) but proportional to (x).

Many practical problems such as the jet-pump involve a combination of wall-bound and free turbulence. The characteristics of the two types of turbulence are distinctly different and the combined characteristics are difficult or impossible to predict.

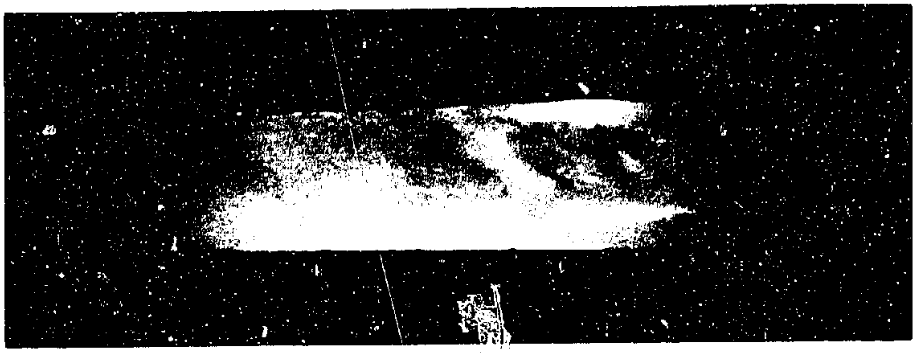
To illustrate the mixing process, a small quantity of air was admitted to the driving jet during the operation of the pump, and photographs, figure 8, were taken of the dispersion of the air. They indicate the angle of diffusion of the driving jet and that the mixing was completed before the flow reached the end of the mixing tube. Figure 9 illustrates the shearing action between the inner and the outer jets during the mixing process.

In the tests it appeared that a mixing tube, 12 driving-jet diameters in length, was sufficient. An increase of this ratio reduced the over-all efficiency.

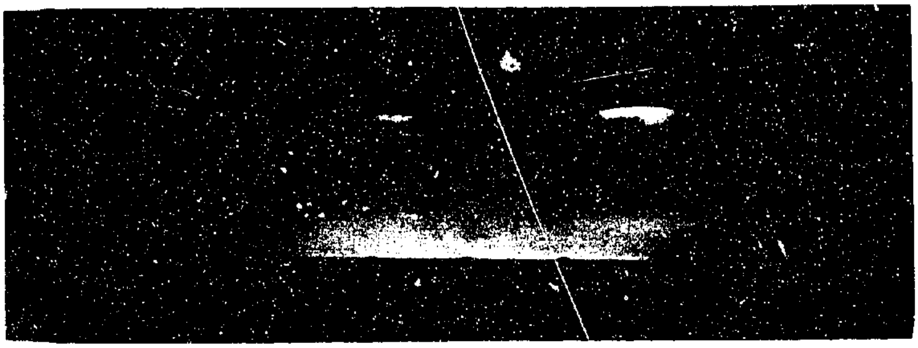
Cavitation in the mixing tube. It is generally accepted theory that cavitation in an hydraulic passage occurs only when the pressure at some point within it approaches or reaches the vapor pressure of water which would have to exist somewhere in the apparatus before damage could occur. Usually, interpretation of the pressure data obtained in the model is based upon this concept.

When the pressure at a point falls below the vapor pressure, masses of vapor are formed and they move along with the stream. As the vapor masses reach a section where the pressure is above the vapor pressure, the masses collapse, producing a sharp, metallic crackling called crepitation. By so doing they give rise to pressures of great intensity. If the action takes place in close proximity to the sides of the water passage, particles of the sides are gradually torn away. If the process is allowed to continue for a sufficient period of time, complete disintegration of sides will result. Usually, the phenomenon is due to the boundary geometry which causes a low-pressure zone that can be detected by piezometers placed at strategic points.

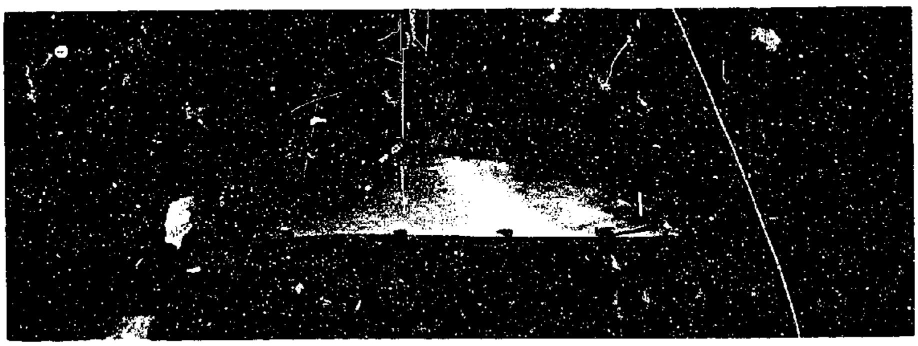
FIGURE 8



A. Nozzle discharge 1.05 sec. ft.-velocity approx. 80 feet per sec.
driven flow 1.95 sec. ft.



B. Nozzle discharge 0.91 sec. ft.-velocity approx. 80 feet per sec.
driven flow 0.60 sec. ft.



C. View showing inner and outer mixing cones

EXPANSION OF DRIVING JET

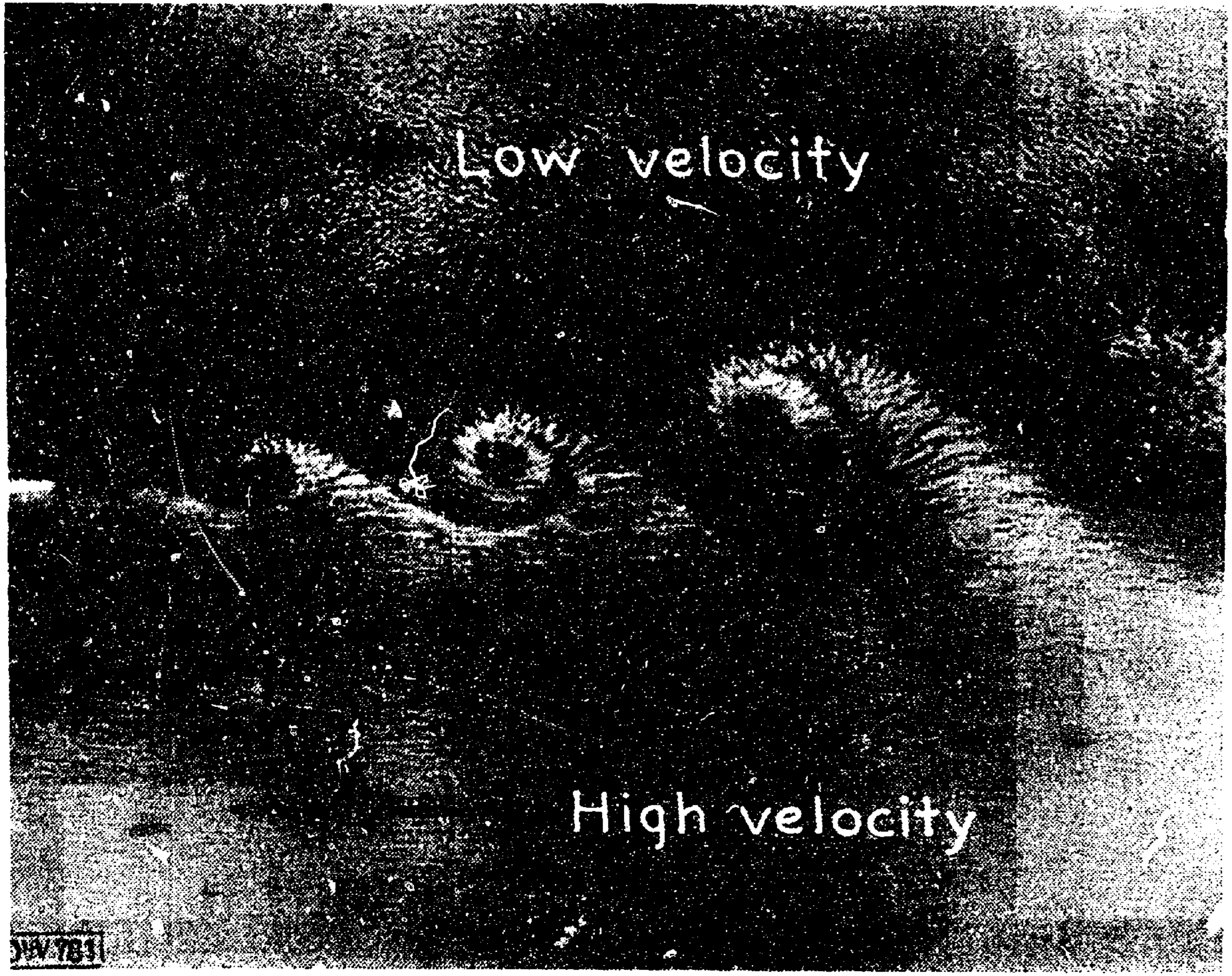


FIGURE 9

In the jet-pump mixing tube, cavitation phenomenon was indicated by sharp, metallic crackling sounds and by a movement of the mixing-tube walls similar to that which is felt when the fingers are placed on one side of the thin plate, the opposite side of which is struck a sharp blow with a sharp instrument. The cavitation which was taking place was not indicated by the piezometers, nor was it possible to locate a point where the piezometric pressures showed a pressure near the vapor pressure of water. The only explanation of the occurrence of the cavitation appears in figure 9. When two jets of different velocities combine, the mixing is accompanied by a shearing force between the two jets which causes the vortices shown in figure 9. The rapid rotation of the vortices will create a low pressure at the center of rotation. When this occurs in a region where the average pressure in the fluid is below atmospheric, it is conceivable that the whirling of the individual vortices would reduce the pressure at their centers sufficiently to cause a small vapor pocket to form. As the speed of rotation of the vortex reduced, or as it moved to a region of higher pressure, the vapor mass collapsed, causing the crackling sound. When the collapse occurred at or near the mixing-tube walls, it seems probable that the force was sufficient to cause the movement that was detected when placing the fingers on the side of the tube.

Although this type of cavitation has little, if any, effect on the average pressure in the fluid, its destructive action is just as severe as that resulting from a boundary condition. If a suitable piezoelectric pressure probe and an oscillograph had been available, it would have been possible to verify the cavitation occurring in the fluid.

Cavitation due to other causes can be expected in a jet-pump if the distance through which the water is lifted becomes sufficiently great to cause the pressure across the plane of the nozzle to reach the vapor pressure. This condition can also be obtained with relatively low lifts if the head on the nozzle is high and the back pressure on the mixing tube is relatively small. In either case the condition of vapor pressure across the plane of the nozzle limits the amount of water that can be pumped under a particular condition. The cavitating pressures due to these causes can be eliminated by proper design, or by judicious study to determine the proper elevation of the pump.

Jet-pump efficiency. Four possible methods of computing the efficiency give different results. On the basis of total energy into the pump and out from it.

$$E = \frac{(Q_N + Q_s)\, H_d}{Q_N\, H_N + Q_s\, H_s} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (11)$$

The value of E obtained from this equation depends on the datum used in computing the various heads, and it can be made nearly unity by locating the datum far below the pump, since the heads become nearly equal as the z- distance to the datum approaches infinity. This difficulty can be remedied by choosing the center line of the pump as the datum in all instances. The second method, which makes use of the total energy used and the work done, has a single value regardless of the datum plane.

The expression is $E = \dfrac{Q_S(H_d - H_s)}{Q_N\,(H_N - H_d)}$ . . . . (12)

The third method assumes the useful work done is equal to the total amount of water Q delivered between the differential pumping heads. The expression

$$E = \frac{Q_t\ (H_d - H_s)}{Q_N\ (H_N - H_d)} \quad \dots\dots\dots\dots (13)$$

is also of single value irrespective of the datum plane. This expression is probably more indicative when pumping costs are being considered than are the two previous equations. Equation (13) is cognizant of the fact that water is being delivered to some point, and, while the work done is only $Q_s\ (H_d - H_s)$, the amount of water available at H_d is not Q_s but Q_T.

A better understanding of the efficiency of the jet pump can be obtained by comparing its performances with a motor-driven centrifugal pump operating under the same conditions. If the source of energy for an electrically driven centrifugal pump is water power and 340 cubic feet per second are to be pumped through a lift of 16-feet, as is the case in the Keswick arrangement, then $\frac{340 \times 16 \times 62.40}{550} = 617.21$ hp., is the amount of energy which must be available in the discharge line of the pump. If the over-all efficiency of the pump and the motor is 70 percent and the over-all efficiency of the electrical supply is 75 percent, then the horsepower that must be available at the turbine runner is $\frac{617.21}{0.70 \times 0.75}$, or 1,175.60 hp., which is equal to the water power that must be available to operate the centrifugal pump. Under the same conditions, the Keswick jet-pump will consume water that could otherwise be used for power at the rate of 113 cubic feet per second under an 86-foot head. This flow represents 1,102.55 horsepower as compared to 1,175.60 horsepower for the centrifugal pump.

The fourth method of computing the efficiency and one which is used in Bureau of Reclamation designs has the form

$$E = \frac{H_d - H_s}{H_n}\left(1 + \frac{Q_s}{Q_n}\right) \quad . . . . (14)$$

This equation is also multivalued depending on the datum used for computing the efficiency. However, if the center line of the pump is used as the datum compatible results are obtained.

VI - PRELIMINARY DESIGN

A method for determining a preliminary design. The investigation thus far has not presented an approach to the problem of designing a jet-pump. This section of the report will deal chiefly with a method for determining a preliminary design and gives a means of determining if a jet-pump is applicable to a particular installation. The method is based on the momentum theorem and the results obtained are general and apply to any jet-pump if the proper losses are included.

The method is worked out on the assumption of mixing under constant pressure for simplification of the computations. This assumption will not affect the accuracy of the results or reduce them to a specific case as will be shown later. On this basis, the various momentum relationships are: nozzle momentum = $Q_n \; V_n$, suction flow momentum = $Q_s \; V_s$ and the mixing tube momentum at the throat is $(Q_n + Q_s) \; V_t$. Since V_t is equal to the average velocity at the throat, V_t becomes $\frac{(Q_n + Q_s)}{A_t}$ and the momentum at the throat equals $\frac{(Q_n + Q_s)^2}{A_t}$.

Since the momentum theorem may be expressed in words as follows: If a closed boundary is fixed in a moving fluid, the geometrical sum of the momenta crossing the boundaries per unit of time, equals the geometrical sum of the forces acting on the fluid within the boundary. The directions of the momenta should be taken in the direction of flow where the fluid leaves the boundary and opposite to the direction of flow where the fluid enters the boundary. The equation of motion in the mixing tube, neglecting losses, becomes

$$(Q_n + Q_s)^2 - Q_n V_n - Q_s V_s = (A_n + A_s)P_n - A_t P_t + \int_{r_2}^{r} 2\pi r p dr.$$

Since mixing was assumed to occur at constant pressure $P_n = P_t = P$ and the equation may be written

$$\frac{(Q_n + Q_s)^2}{A_t} - Q_n V_n - Q_s V_s = P \left[(A_n + A_s) - A_t + 2\pi r dr.\right]$$

The quantity inside the brackets on the right side of the equation becomes zero for mixing under constant pressure, and the equation reduces to $\frac{(Q_n + Q_s)^2}{A_t} - Q_n V_n - Q_s V_s = 0.$

By letting $V_n = \frac{Q_n}{A_n}$ and $V_s = \frac{Q_s}{A_s}$, substituting and simplifying, the equation is reduced to

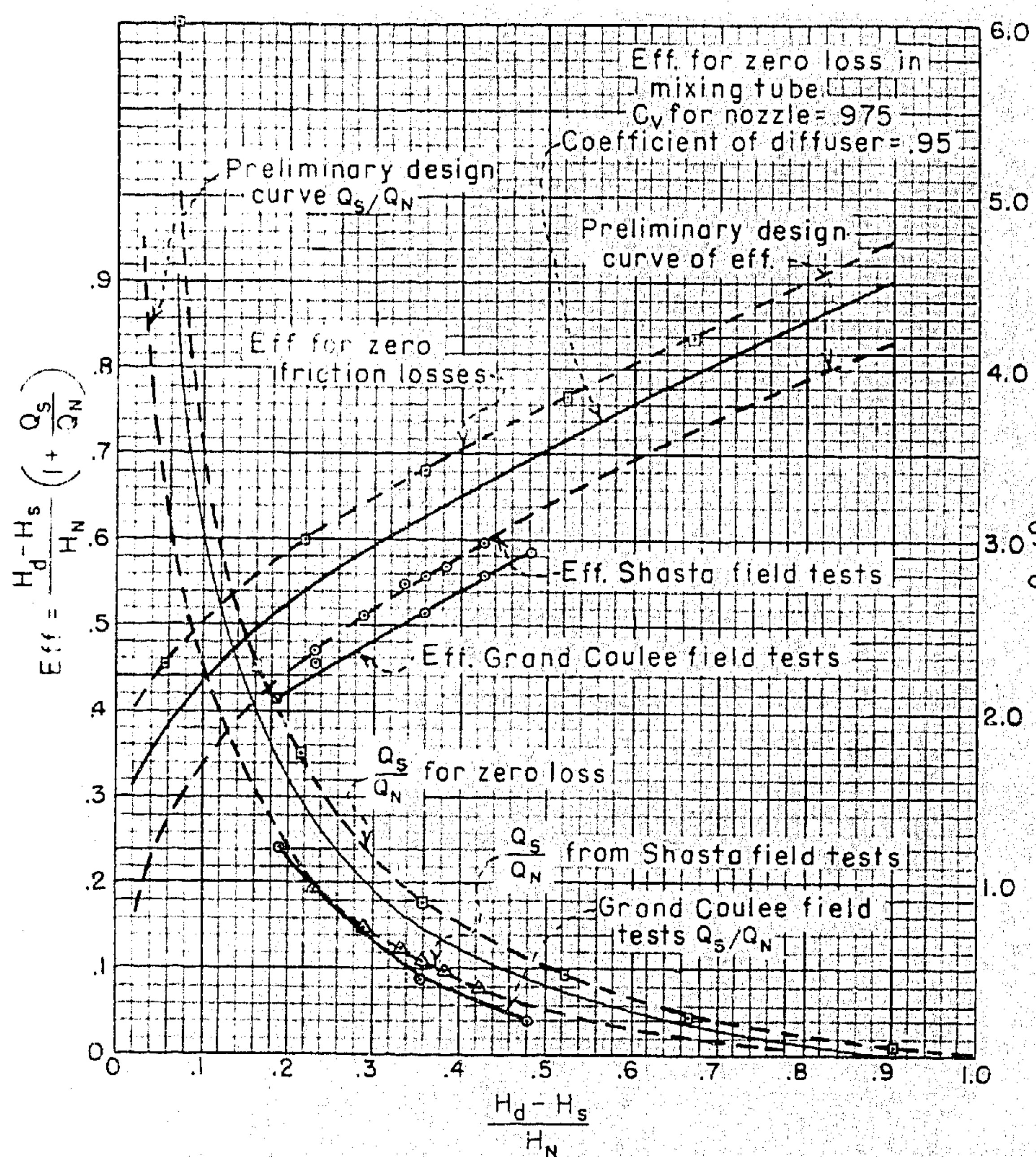
$$Q_n^2 \left(\frac{A_t}{A_n} - 1\right) + Q_s^2 \left(\frac{A_t}{A_s} - 1\right) - 2Q_s Q_n = 0 \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (15)$$

Equation 15 contains directly the area of the nozzle jet, the area of the suction jet, and the throat area. Both H_n and H_s are indirectly contained in Q_s. H_d does not enter the equation either directly or indirectly. Its value was taken as $\frac{V_t^2}{2g}$ with full recovery assumed in the diffuser. For the case of no losses H_s was taken equal to $\frac{V_s^2}{2g}$ plus or minus the datum chosen for the constant mixing pressure.

With the equation (15) as a basis and an assumed constant mixing pressure of atmospheric pressure (any other pressure could have been taken), the curve of efficiency versus $\frac{H_d - H_s}{H_n}$ for zero losses, figure 10, was computed as follows: A_n was taken as 0.10 square feet with $Q_n = 0.80$ and various combinations of A_s and A_t. These values were substituted in equation (15) and Q_s was determined. With A_n, Q_n, Q_s, Q_t, and A known, it was possible to compute H_d, H_n, and H_s. This when substituted in the efficiency equation

$$\frac{H_d - H_s}{H_n} \left(1 - \frac{Q_s}{Q_n}\right)$$

gave an efficiency value for the combination of areas chosen. By holding Q_n A_n and A_t constant, varying A and repeating the process it was possible to obtain a curve of efficiency against the ratio $\frac{A_t}{A_s}$ from which it was possible to select an area combination which gave the maximum efficiency for the particular throat area, nozzle area and discharge selected. This in turn gave a point on the efficiency curve of zero loss on figure 10. This process was repeated until the complete efficiency curve on figure 10 was obtained. The corresponding ratio of Q_s/Q_n was plotted on the same sheet. The two curves thus obtained will give the maximum efficiency obtainable as well as the maximum ratio Q_s/Q_n obtainable for a certain combination of $\frac{H_d - H_s}{H_n}$ for any jet-pump regardless of whether the mixing tube has parallel sides or whether they are converging or diverging. This is apparent from a close examination of the momentum equation



EFFICIENCY AND DISCHARGE CURVES
FOR
PRELIMINARY JET PUMP DESIGN

$$Q_nV_n + Q_sV_s - Q_tV_t + (A_n + A_s)P_n - A_tP_t + \int_{r_1}^{r_2} 2\pi rp\,dr.$$

If the mixing is at constant pressure the momentum entering the mixing tube is equal to the momentum leaving and the efficiency is at a maximum for the particular design. If a tube is designed for mixing under variable pressure the change in momentum is converted to pressure and the pumping efficiency will be the same as for mixing under constant pressure, provided variable mixing pressure design has been properly proportioned. It should be emphasized at this point that if a jet-pump is properly proportioned it will perform at maximum efficiency only at the combination of heads for which it was designed. Any variation of the H_d, H_n or H_s from the designed value will reduce the efficiency of any jet-pump design. Likewise, a mixing tube designed for mixing at constant pressure will perform in this manner only at the heads and discharges for which it was designed. Any variation of head will cause mixing to occur at variable pressures and lower efficiencies.

The efficiency curve and the corresponding curve of Q_s/Q_n for zero losses, is not applicable to a design. It is useful in that it shows an optimum condition and can be used to show the reduction in efficiency caused by various losses in a jet-pump. To illustrate the effect on the efficiency another curve is shown on figure 10 where the nozzle velocity coefficient was taken as 0.975, the coefficient of the diffuser as 0.95, and zero losses in the mixing tube. In general, this had the effect of reducing the pump efficiency approximately 5 percent. The curve was computed in the same manner as was the one for zero losses, except that H_d was taken equal to $0.95 \frac{V_t^2}{2g}$, $H_n = \frac{1}{C_v^2} \frac{V_n^2}{2g}$, and $H_s = 1.07 \frac{V_s^2}{2g}$ in the efficiency equation.

The value of 1.07 V_s made some allowance for friction and entrance losses in the suction line. The effect of these losses on the pumping ratio Q_s/Q_n was more pronounced at low values of $\frac{H_d - H_s}{H_n}$, which is an indication that the actual value of the pumping ratio in this region will be difficult to predict because of the uncertainty of the pump losses.

For comparison with these curves, the field tests of the Shasta and Grand Coulee jet-pumps have also been plotted on figure 10. Through the range for which the tests were conducted, the Shasta pumps show a reduction in efficiency of approximately 13 percent below the zero friction loss efficiency. The Grand Coulee pumps show a further reduction in efficiency; however, this probably was due to the fact that the pumps were not operated at the point of peak efficiency for the particular head available at the time of testing.

In addition to the previously mentioned curves of figure 10 two preliminary design curves have been plotted for use in determining the feasibility of a jet-pump for a particular installation. They are based on equation (10) with the addition of losses resulting from a C_v of the nozzle equal to 0.95, a diffuser efficiency of 0.95, and friction losses in the mixing tube. The curves nearly coincide with the Shasta field tests and agree closely with the model tests. They are intended to be used as follows: With a suction lift H_s of 4 feet, a delivery head H_d of 30 feet and a total discharge of 40 second-feet which will be the suction and nozzle discharges when the available nozzle head H_d is 100 feet. First compute $\frac{H_d - H_s}{H_n}$, which in this instance is 0.340. Enter the chart with this value and find the intersection with the preliminary design curve Q_s/Q_n which shows a ratio of 0.60. The corresponding efficiency is 0.54. Since $Q_s + Q_n = Q_t = 40$ or $\frac{Q_s}{Q_n} + 1 = \frac{40}{Q_n}$, the value of Q_n is found to be 25 second-feet with the ratio $\frac{Q_s}{Q_n} = 0.60$ substituted into the equation. By substraction, Q_s is found to be 15 second-feet. With this information, and knowing the characteristics of the nozzle, it is possible to determine A_n as well as the nozzle design. Anyone familiar with jet-pumps could estimate with little additional work, the approximate size and cost of the installation and thus determine the economical aspects of the design. In addition, the information concerning the water supply for the nozzle is determined with accuracy sufficient for design purposes, if the nozzle and diffuser coefficients are very near those selected for plotting the curve.

The procedure for preparing the final design will not be discussed here. It depends upon whether or not a mixing tube of parallel, converging or diverging side walls will be selected for the design.

VII - CONCLUSIONS

<u>The applicability of the momentum theorem</u>. From the results of these tests and tests by others it appears that the momentum equation is applicable to the design of jet pumps. The equation applies to either parallel, diverging or converging mixing tubes.

With the exception of a design having a mixing tube with parallel sides the use of the momentum equation in designing a mixing tube becomes quite involved and laborious. The design of mixing tubes with non-parallel sides involves a step-by-step method of selecting short increments in length of the mixing tube and analyzing each by the principle of momentum.

Another difficulty with the momentum equation is that it does not give any indication of the proper length of the mixing tube. At the present time a suitable mathematical analysis has not been developed. The length is usually fixed by the "rule of thumb" method and a mixing tube 12 driving-jet diameters in length is adequate for complete mixing.

<u>Cavitation</u>. Cavitation produced by a combination of subatmospheric pressures and rapidly rotating vortices is always a potential source of trouble in a jet-pump regardless of how carefully the pump is designed. The pressure inside the vortices cannot be determined chiefly because their angular velocity is indeterminate. Consequently, the operating point at which the destructive action of cavitation occurs cannot be determined.

The cavitation can be eliminated through a wider operating range by the admission of air to the shear plane existing between the driving and driven jets.

APPENDIX

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